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Preface

The publication of this book marks the completion of the forty-fourth volume of the TRANSACTIONS of the American Institute of Electrical Engineers. There are contained herein papers and discussions presented at six Institute conventions and regional meetings, and printed in the JOURNAL during the year 1925. For several years it has been found impossible to include in the TRANSACTIONS all of the large amount of material presented annually before the Institute. This year a departure from custom has been made in that the papers which were reproduced in the JOURNAL but not in the TRANSACTIONS have been indexed with the TRANSACTIONS contents rather than on a separate page. The improved system of indexing which was introduced last year has been continued in this volume; the subjects are classified under general headings chosen with regard to the information contained in the papers and in many instances the subjects are also cross-referenced. The report of the Board of Directors for the fiscal year ending April 30, 1925 and lists of the officers and committeemen for the current year are also included.

Power-Factor Correction

By L. W. W. MORROW*
Member A.I.E.E.

Synopsis.—Power-factor correction has become a commercial problem of major importance, and technical knowledge and corrective equipment are available to make correction a part of the activities of the electrical industry. Field studies on several properties show power-factor correction can be obtained without difficulty.

Conditions can be improved by requiring all new business to be installed for high power-factor operation and a cooperative movement should be made to bring about this condition. The situation on existing systems can be improved in either a wholesale or retail manner depending upon conditions.

Cost analyses should be made to determine the type of correction to use and to decide upon the location of corrective equipment. But they should not be used for rate making purposes. A study of the types of systems in existence shows that in general correction is most economically and most effectively instituted at the loads. Also experience indicates the greatest effect of correction is to improve voltage regulation and service quality.

Different types of rates and billing methods have been used, but a study of the art shows the kv-a. demand charge and the kw. energy charge can be used most successfully to secure power-factor correction.

THE effects of low power factor on the operations of the electrical industry have been discussed for many years and much experience in the institution of corrective measures has been had. Yet on an industry-wide scale the evils associated with low-power-factor conditions become greater each year, even though the incentives for the institution of corrective measures increase. Low power factor has come to be recognized as the major handicap to better and more efficient electric service, and men of the industry are striving actively for a commercial and engineering method for its correction.

An analysis of the data and experiences of those properties which have instituted power-factor correction leads to the conclusion that operation under high-power-factor conditions improves both the quality and the efficiency of the service rendered by the utilities and used by their customers. Correction of power factor has its greatest influence in improving the voltage regulation and through this action enables the utility to give better service and the customers to secure better use of the service. Secondary gains resulting from power-factor correction consist of the release of system capacity with a consequent reduction in investment and a reduction in system losses with a resultant reduction in operating charges. Thus correction tunes the system and makes it cheaper and easier to give service of the quality desired in modern industry.

Actual experience shows that power-factor correction can be instituted successfully as a part of the business activities of light and power companies and as a part of the normal operating activities of industrial users of electricity. It does not require an elaborate or costly departure from customary business activities, and it secures returns to the producer and the user whose value is tangible and worth while. Yet little actual progress in power-factor correction can be seen because of many obstacles, mostly imaginary, set up in the minds of executives, engineers and customers. Some of these are: the inertia encountered when proposing any changes in a going business which is prosperous and follows time-tried methods; difficulties met in attempting to evaluate the costs of either low or high power-factor operation; fears introduced by the necessity for instituting new or changed rates and billing methods; difficulties encountered in attempts to make people understand the meaning of power factor; obstacles encountered in attempts to devise a correction scheme for universal application, and technical difficulties met in connection with the selection and location of corrective equipment. The literature of the industry is filled with objections to power-factor corrective measures and with elaborate technical and cost analysis, and very few data or opinions of a constructive character have been obtained to show that most of the obstacles to correction result from misconceptions and lack of actual experience with it in commercial practise.

The fact is that many properties have instituted correction successfully and made it a profitable part of their business. They have done this without any difficulty and did not find any great complexity or major changes introduced in their operating routine. Thus the time is ripe and the methods are available for eliminating low-power-factor conditions in the electrical industry. The rewards are ample, and the necessity for this move becomes greater with the growth of electrical systems.

Correction is a business problem and not a technical problem. The technical features of power-factor discussions begin nowhere and end at the same place, and they have little if anything to do with the business-like application of corrective measures. No one can define electricity or power factor, yet definite laws enable engineers to use resistance, inductance and capacity in circuits or in machines to secure any desired power factor at any definite place. Also, engineers understand fully the influence of lagging and leading current on system regulation and machine

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1. Managing editor, *Electrical World.*

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stability. Technical mastery of power factor prevails, and what is needed is commercial mastery.

In order to obtain commercial mastery two suggestions are offered which will apply to every electrical system:

1. Require high-power-factor operation of all new business served.

2. Correct low-power-factor conditions on existing systems either: (a) promptly and in a wholesale manner, or (b) slowly and by steps.

Make New Business Good Business

Any consideration of the first suggestion, in the light of the growth of the electrical industry and the increasing importance of service quality, shows its logic, and there remains then the use of definite methods to secure the desired results.

A light and power company is a public utility and cannot enforce any rule requiring customers to install high-power-factor equipment before it will give service, so this direct and self-evident method must be replaced by methods which are cooperative and educational in character. The job requires the cooperation of utility men, electrical manufacturers and customers and must be done for each new business prospect on each property, and in addition work must be done on a national basis. It is not a difficult task, for both means and methods are available for doing the work quickly and efficiently.

The cooperative element in the work calls for:

1. Closer contact between central-station power salesmen and prospective customers whereby guidance and expert knowledge are made available to customers.

2. Closer contact between central-station executives and electrical manufacturers so the manufacturers may be encouraged to build high-power-factor equipment and to sell this equipment on a quality rather than a price basis and with greater consideration for the engineering requirements of the purchaser.

These two cooperative suggestions are already in operation in large degree, but conscious executive attention should be given to their application in order to secure complete commercial results rapidly. The job requires that each new customer install available electrical equipment from the standpoint of his own economic production requirements. Both power factor and quality apparatus will inevitably follow every real economic analysis of an industrial electrical application, and central-station men and manufacturers' representatives must give up the slogan that any new business is good business and do more to help their customers secure quality service and cooperate more closely to sell quality equipment. It would seem easy to organize a local cooperative committee for each utility to secure the desired results. This would be composed of power salesmen, electrical manufacturers' representatives, electrical contractors, and perhaps consulting engineers, and might well be part of the activities of the local electrical leagues already instituted in so many cities. National agencies are already available in the N. E. L. A., Power Club and other associations, but their attention must be given to this problem.

There is no handicap to the manufacture and sale of electrical equipment and service on a quality rather than a price basis in an industry whose product is popular, is necessary to industry and is the cheapest and best element ever introduced into industrial processes. When the cost of power is such a small fraction of the cost of manufacture of most industrial products, it is false economy to talk first cost of service or of equipment to purchasers.

Thus very little organized effort should institute as industry-wide practise the custom of obtaining only high-power-factor business in each new load added to existing systems. The principle to apply is that service to customers should be given before they install equipment and ask for electrical energy from a utility.

The Correction of Existing Conditions

The second suggestion for power-factor correction is that existing conditions be bettered either by making corrections quickly on a wholesale basis or by improving conditions slowly and in steps. In many respects the slow and detailed method is best for those systems inexperienced in correction, but past experiences show that it is not difficult to do the whole job quickly on the entire system provided proper analyses are made and workable methods are adopted before correction is actually instituted. Whatever decision is made has little influence upon the methods used or the results obtained and predominantly influences only the time required to secure improvements in conditions. The situation on each system will determine the best procedure to use, for operating and financial requirements must be met by individual properties.

Three things only are needed to institute correction on an existing system:

1. Decisions as to the location of corrective equipment.

2. Decisions as to the corrective equipment to be used.

3. The establishment of a rate system which takes power factor directly or indirectly into account.

The first two requirements largely involve engineering and economic data and decisions to be secured within a utility organization, but the third requirement must take into consideration the utility, the customers and the regulatory commissions and involves public relations, economics and education.

Locations for Corrective Equipment

A study to determine the locations in which to install corrective equipment on existing systems

shows that the decision is somewhat influenced by the type of system and by local conditions. Usually corrective equipment should be located at all low-power-factor loads, in some cases at substations at a distance from generating stations, particularly if voltage regulation is desirable, and in very few instances in generating stations. The source of low power factor is at the load, and analyses will show that corrective equipment is best and most economically used in this location.

Some of the types of systems encountered in this country and some of the elements faced in locating corrective equipment are enumerated below:

1. A hydroelectric system containing long high-voltage transmission lines with one or more distribution systems located at the ends of the lines.
2. A combination system having both steam and hydro stations, many high-tension lines and widely distributed load centers.
3. A hydroelectric system with standby steam stations containing capacity idle a great part of the time and made up of networks of high-tension lines supplying a scattered territory.
4. A metropolitan system containing several large steam stations and made up of both overhead and underground transmission circuits.
5. A system already operating and designed for a power factor of 80 to 85 per cent.
6. A system designed to operate at a power factor of 90 to 95 per cent.
7. Systems which are growing rapidly in magnitude of load and extent of territory covered and those which have reserve capacity through interconnections.

It is seen that each system is a specific study for power-factor correction at other places than at the load sources of low power factor, and the choice of location of corrective equipment at other locations must be made after economic and operating studies. The making of cost evaluations for a particular system should be decided upon only after careful study, for they are difficult and are useful only in serving as guides for the location of corrective equipment. They serve no purpose in rate making as cost-of-service rates are now obsolete in the utility industry. However, if cost analysis are desired in some cases, the usual approach to determining fixed and operating charges is to evaluate the cost of capacity and of losses in generating stations, transmission lines, substations, transformers and distribution systems.

Generating Station Correction

The evaluation of the reduction of the reactive-kv-a. load on a generating station due to power-factor correction is difficult because so many conditions must be considered. There is much more to the problem than raising a station power factor from 70 per cent to 90 per cent in order to make available approximately 20 per cent increase in station capacity

In hydroelectric plants it is usually desirable to have reactive kv-a. available for line regulation, and in addition it is usual to find an excess of capacity installed in order to meet stream-flow conditions. Therefore, generally, no value can be placed on power-factor correction for such stations. Also, in large systems using steam and hydro stations in combination with interconnected high-tension lines, lagging current is often an operating asset, and in this case also correction is seldom or ever desirable for generating stations.

In the large steam stations feeding a power system concentrated in a comparatively small area power-factor correction may improve regulation and operation, but here again the savings in station costs are sometimes very small and must be evaluated for the conditions encountered. These plants are usually designed for 85 to 90 per cent power-factor operation at the stations, and a little study will show that cost decreases are difficult to secure. Offhand it may be said that every kv-a. of station capacity means the release of fixed charges on at least $6.00 of investment and every reduction of a per cent in losses from the normal 2 per cent of output kw-hr. can be credited in dollars and cents of operating charge. But there are some other factors that must be considered before these sums mean anything or will stand the examination of a court or commission; for example, in connection with stations in modern systems:

(a) Most systems increase their installed station capacity in units of large size and seldom permit their peak to approach a value equal to installed capacity. For example, station additions are made in units of 30,000 kw. up to 200,000 kw. and ample reserve is held at all times.

(b) Every metropolitan system often introduces large losses into its normal operations in order to insure reliability of service; for example:

(1) A station having several large units is likely to continue all of them in operation even at very light load in order to reduce the starting hazard and the time element involved in warming up a large unit and synchronizing it with the system.

(2) In order to control voltage and load division, it is common practise on systems having multiple stations to operate generating units with large lagging or leading currents.

(c) It is a difficult, if not impossible, task to determine actual station power-factor conditions during a twenty-four-hour period for the different stations in a large metropolitan network. Any method of determining power factor and of determining the value of losses is subject to criticism as to its technical accuracy, and the values obtained cannot be divided easily between power factor, load division and regulation requirements. In fact, for a given load power factor as defined and as measured by different methods will vary 10 to 30 per cent in value.

The actual capacity cost of the reactive load on a generating station not loaded to capacity is usually very little even though the cost of the increased capacity of alternators, exciters, switch gear, busses and building is determined. Seldom would this cost exceed $6.00 per reactive kv-a., and any added operating expense due to low power factor is very debatable. For some specific installations, however, where the peak load is equal to the station capacity or where the load power factor is lower than that for which the station equipment was designed, it may be well to institute power-factor correction as a temporary economical expedient for securing capacity to carry the load. Load factors, future system growth and other elements, however, must be considered.

In most other cases, however, it is cheaper and easier to buy, install and operate alternators, busses and exciters with a higher rating than it is to buy, install and operate corrective equipment and its control. For example, the increment cost of a 35,000-kv-a. alternator over a 30,000-kv-a. unit as compared to the cost of a 5000-kv-a. synchronous condenser and its control equipment is in favor of the alternator. In most stations generating capacity can usually be obtained more cheaply than by the use of corrective equipment.

Very few operating costs are affected by low power factor because the copper losses inside a station amount to an inappreciable sum. At light load the usually lowest power-factor condition increases the losses in proportion to the kw. output. This indicates the advisability of fixing a power-factor charge that varies with output. This charge should be greatest at minimum-load conditions in some cases and at peak-load conditions in other cases. A very small increment in energy charges seems the only businesslike way to cover these station losses due to low power factor as at a maximum they comprise only 2 or 3 per cent of the system losses.

The practical possibility and the dollar value of regulation by power-factor correction inside large steam stations is very dubious. The unit type of installation with the unit bus and a group of feeders for each unit has become popular, and it is difficult to conceive of this type of station operating with all busses in parallel at one voltage or to devise a switching and bus arrangement for throwing the synchronous condenser on any feeder requiring regulation. In addition, the cost of the corrective equipment used for regulation exceeds that of voltage regulators, which have the added advantages of being automatic in their operation and of having a greater voltage range.

In the general case, therefore, seldom will it be found economical to institute power-factor correction in generating stations.

Substation Correction

The installation of corrective equipment at substations reduces the kv-a. load on transmission lines and transformers between the substations and the generating stations and improves the voltage regulation. This would seem desirable in all cases, but a little study shows it is often very uneconomical and usually not the best way to secure complete correction.

In modern systems reliability of service, provision for future growth and maintenance practises dictate the use of multiple feeders to substations and multiple transformer installations. Also, modern practise tends to the use of standardized ratings for lines and transformers on the transmission system. These requirements call for the installation of a greater number and a excess of line and transformer capacity, which makes it very difficult to show the economic value of substation corrective equipment.

Moreover, it must be borne in mind that future load requirements, structural needs and legal limitations have a direct bearing on both overhead and underground line installations. The overhead line must have a certain size of pole, a conductor sufficiently large to be strong mechanically, a large number of poles per mile and very often a legal voltage limitation. These elements often must be weighed carefully when discussing the release of line capacity by the use of substation corrective equipment. In underground cable installations it has become customary to install reserve ducts and cables and to standardize greatly, so the number and the capacity of installed cables at a given time is very often determined by other things than the magnitude of the kv-a. loads. It must be considered, however, that the overload capacity of an overhead line is very great as compared with that of cables and transformers.

Thus any cable or transformer that has reached its limit in capacity while operating at a low power factor is capable of carrying an increased load and consequently earning more revenue if correction is applied at substations. The cost of the corrective equipment may be less than the installation of another line and more transformers, and regulation is improved, so that more load may be carried on a given circuit. In other words, correction will, for the same line loss, increase either the economic distance of transmission or the amount of load possible to transmit. In such cases the substation correction may be economical, but it is an expense which must be paid for in fixed charges, while a new line provides a revenue-producing and useful element for the present and future system. A system using a ring high-tension network and having multiple generating stations and several large substations on the ring may use corrective equipment in distant substations to advantage to reduce losses, release capacity and control regulation, but the economic value of this practise must be ascertained for each particular case.

Thus in very few cases will the release of system capacity justify substation correction; therefore, either decreased losses or improved regulation must be evalu-

ated to make a case. The reduction in line and transformer losses by the use of corrective equipment increases the load at which the investment cost per kv-a. transmitted is a minimum and reduces the direct operating cost of energy losses. On an average system the line energy losses will be around 10 per cent of the total output while the transformer losses should not exceed 5 per cent. These losses therefore represent a direct money loss, which, if reduced, results in an appreciable saving. Assuming that correction decreases these losses even 20 per cent, it often becomes profitable to install corrective equipment.

For example: Assume an energy cost of 1 cent per kw-hr. and a system output of 100,000,000 kw-hr. with total losses of 15 per cent in lines and transformers. If it is assumed correction will reduce the losses 20 per cent, the yearly value of correction is: $100,000,000 \times 0.15 \times 0.20 \times 0.01 = \$30,000$, or a return of 15 per cent on an investment of $200,000.

Losses in corrective equipment must be considered in a financial analysis of this character and their cost determined. If synchronous condensers should be used, this value would be changed, since the condenser has losses which must be charged against the saving produced in lines and transformers. If static condensers are used, the value of the losses in the corrective equipment is very small. In either case, particularly if the voltage at the substation is high, the losses in the transformer bank supplying the corrective equipment should be evaluated and charged against the cost of correction.

The general conclusion is that substation correction, considering the value of losses, capacity and regulation, is usually warranted for those substations located at a distance from generating stations, at least as a temporary move, until full consumer correction has been established.

Consumer Correction

The analysis for determining the proper location of corrective equipment leads inevitably to the conclusion that most of it should be placed on the premises of consumers and that consumers using high-power-factor equipment are operating most economically from their own standpoint and from that of the central station. From the angle of the central-station consumer correction relieves the system of the necessity for correction at other locations, releases the transformer and cable capacity where it is most valuable from income and load-factor standpoints, reduces losses where they are greatest and improves regulation where it is most desirable to have good regulation.

From the angle of the consumer correction of his equipment or the replacement of low-power-factor equipment with high-power-factor equipment will secure several tangible and intangible results. It will reduce the losses inside his premises, for which he pays a high rate; it will improve his voltage regulation and reduce his voltage drop, so that his machinery will operate faster and more smoothly; it will give him better-quality production, better lighting, fewer rejections of manufactured product, faster machine-starting conditions and secure for him a direct decrease in his power bill if he buys under a proper rate system.

Experience shows that operating complexity is not increased and that the industrial plant corrected for bad power-factor conditions reaps very decided tangible and intangible benefits. Needless to say, it is useless to attempt an itemized cost evaluation for correction at load locations, because the complexity of modern distribution systems, the variety of loads and services and the many assumptions necessarily made all combine to give inaccurate results. In a broad way, however, based on the actual cost of the distribution system, the cost of voltage regulation and the cost of losses, an analysis in dollars will show very decided inducements for investing power-factor correction at the source.

Kinds of Corrective Equipment

The corrective equipment to install at locations decided upon is determined largely by each local situation, but there are applications for all available types at low-power-factor loads, *i. e.:*

(a) The synchronous motor

(b) The synchronous condenser

(c) The high-power-factor commutating motor of the Fynn-Weichsel type

(d) The static condenser

Investment charges on existing installations, production requirements, fixed and operating charges on corrective equipment and the skill required to operate the equipment influence the decision to be made for each load considered.

For substation correction the same types of corrective equipment are available, but the synchronous condenser and group static condensers predominate. The decision as to which type to use at substations is influenced by cost analyses and by load and voltage regulation requirements.

Experiences on properties give evidence that the choice of equipment for correction is quite readily made for a particular location. Each system should be studied as a whole and each power customer considered if good results are desired. The utility engineers can readily determine the substation correction to use, but greater difficulty is encountered in the correction of power customers. The installation of each customer must be studied in detail if a real engineering job is to be done, and this requires time, skilled engineering and an expenditure of money. Production processes, motorization, wiring and testing are elements in the study, and a great deal of education and cooperation must be had. For these reasons a slow process of customer correction is usually advisable. Utility power salesmen, consulting engineers, factory engineers and electrical manufacturers' salesmen and engineers should cooperate in these customer studies. In actual practise no great difficulty or high costs have

been encountered in attempts to institute customer corrections, because the great majority of them can be made adequately by an over-all factory test or by competent engineering inspection. Very frequently zone studies or piecemeal areas are corrected with very little trouble.

POWER-FACTOR RATES

Electric service is a business and as such must institute power-factor correction as a part of business activity. Thus, a rate of some form must be used to secure power-factor correction, and it also follows that metering and billing accompany the institution of a rate. Experience with energy and power-factor rates over many years shows that commercial rates cannot be made upon the basis of cost of service. Experience also shows that financial inducements are the best stimuli to industrial accomplishments. Rates, billing systems and metering methods are largely matters of tradition and commercial application, but should be simple, understandable and easy to use in business.

When alternating current was first used the wonderful Thomson-Houston meter was invented and installed. This meter measured watt-hours and eliminated the necessity of measuring ampere-hours and multiplying by the normal voltage of the service as had hitherto been practised. It was an ideal instrument for measuring lighting loads, and no thought was taken on its ultimate effects on the utility business. Rates were made on a watt-hour basis and the cheap, accurate watt-hour meter became the standard measuring device for electric service. Then came the induction motor and it was early noticed that the product of current and voltage did not equal the registration of the meter. Thus power factor was discovered, and until recently the utilities decided that it was better to suffer its effects than to attempt to improve conditions or to charge for service by methods involving the use of other units than watt-hours. Power factor was difficult to define accurately, difficult to measure and still more difficult to explain to customers and executives.

However, there is a tendency today actually to measure power factor or to meter customers by methods which will take power factor into account. The first trend is indicated by the development of kv-a. meters, the use of reactive and active watt-hour meters and the use of power-factor meters. Any of these meters or metering methods are subject to objections from an economic, a technical and an operating standpoint. The cost, the complexity of the billing and the difficulty of obtaining and maintaining reliable and accurate measurements by any of these means have so far prohibited their use except for large power customers having a technical staff or sufficient consumption to warrant the cost and the trouble. Nothing very promising is on the horizon in the way of directly measuring either kv-a. or power factor on a universal commercial basis.

One metering authority has recently suggested that it would be advisable to meter customers on the old ampere-hour basis since all service is at practically constant voltage and the ampere-hour meter, besides being cheap and accurate, is a true and readily understood measure of both energy and capacity use. Multiplying ampere-hours by the voltage of the supply would permit of billing on a kv-a.-hour basis very readily. Thus a system could be developed which would use an ampere demand meter and an ampere-hour meter only for direct measurement on any three-phase or single-phase circuit. It is argued that this method is sufficiently accurate, is easy to institute, is logical and that its results compare well in accuracy with those from any existing method of measurement.

The ultimate approach to power-factor correction for customers is by means of rate making. This has generally taken the form of:

(a) The use of power-factor clauses
(b) The use of kv-a. demand clauses
(c) The use of kv-a.-hour clauses
(d) A combination of two of the foregoing

All these methods have been used and all have worked more or less successfully, but all are subject to very definite criticisms. Years of effort proved the impossibility of developing utility rate on a cost-of-service basis, and, however reluctantly, rate makers have admitted its impossibility and impracticability for modern conditions of utility service. No cost-of-service basis can be found, therefore, for the making of any type of power-factor rate, and each property must evolve a rate which can be instituted and operated to suit conditions encountered. A fundamental in power-factor rate making would be to secure a simple rate, one that could be applied to all customers, one that needed little maintenance or supervision and one that offered a financial inducement to customers to maintain and correct power-factor conditions.

Viewing the art in its technical and commercial aspects and basing opinions on practical experiences, the best form of power rate today should use a kv-a. demand charge to cover capacity costs and an energy charge to cover production costs. Power factor, as such, should not appear in the costomer rates, but should be accounted for in the kv-a. demand charge. This rate can be used to secure power-factor correction on any system very satisfactorily and does not necessarily involve the use of a kv-a. demand meter. Power customers can be tested under normal operating conditions, and the power contract can then be written for a period of time with a kw-hr. meter and a kw. demand meter to serve as checks and for energy billing. In other cases the installation of a power-factor meter normally out of circuit, in combination with ordinary watt-hour meters can be used to check customer conditions and to meet billing requirements.

However, every type of power-factor rate has been used and used successfully, so the necessity of a rate to secure power-factor correction is not a real obstacle to the institution of correction. It must be remembered that grand average results only are to be expected and that precision methods and analyses must be modified by commercial considerations.

A New Alternating-Current General-Purpose Motor

BY H. WEICHSEL[1]
Fellow, A. I. E. E.

FROM the first days of the successful commercial introduction of the a-c. system of distribution down to a comparatively recent date, all forms of general purpose a-c. motors have in one way or another been subject to serious objection from the standpoint of approaching the engineer's ideal of satisfactory design and performance.

Speaking in general terms, the types of a-c. motors available for general purpose use have been as follows:

1. The well-known squirrel-cage motor.
2. The induction motor with wound secondary.
3. The conventional form of synchronous motor.

The squirrel-cage motor, while very simple in construction, has relatively poor starting performance, and its design characteristics are frequently subordinated to the starting-load requirements rather than to best performance under normal running conditions.

The wound-secondary-induction motor is free from the defect of inadequate starting torque, but shares with the squirrel-cage motor the very serious defect of drawing from the line, under all operating conditions, a very heavy magnetizing current. This, as is well known, results in the operation of both forms of motor with a lagging power factor. When built for high or moderate speeds, the power factor at full load is reasonably satisfactory, but decreases rapidly with decreasing load. For slow-speed motors of both types, the power factor, even at full load, is very unsatisfactory, and is correspondingly lower at fractional loads.

The power factor of the circuit supplying energy to an installation of induction motors is usually considerably below unity due to the following:

The installation contains a number of slow-speed motors. Occasionally no high-speed motors are used at all. The majority of motors installed operate below full load and consequently with poor power factor. This is particularly true in cases where squirrel-cage motors are used. The size of these motors as mentioned above, is frequently larger than really required for the actual running load for the sole purpose of obtaining satisfactory starting conditions. Consequently these motors operate at low power factor, because they are only partly loaded.

The serious operating difficulties and the excess investment and operating costs resulting from poor power factor are fully understood and will not be discussed in this paper.

While the pronounced pole-synchronous motors are free of the shortcoming of poor power factor, they have unfortunately a number of other deficiencies which have prevented them from being used as general purpose motors. The main objections to this type of motor for general purpose are:

1. The starting performance is poor.
2. A separate d-c. excitation is required, which must be obtained from either a separate exciter or from an independent d-c. supply.
3. The machines are not well suited to loads which fluctuate very materially and often exceed considerably the normal rating of the machine.

The electrical industry has long appreciated the importance of the development of a new form of motor which would preserve the satisfactory features of the well-known conventional types, and, at the same time, be free from their very obvious disadvantages. In recent years the demand for such a new motor has been growing much more insistent, and the time is now at hand when the central-station section of the industry is urgently demanding such a motor.

The company with which the author is identified has, after many years of research work, developed a new type of motor which approaches to a remarkable degree, an ideal design, preserving the good characteristics of both the induction and the synchronous motor, yet free from their most serious shortcomings. This new machine, which is known under the name of the Fynn-Weichsel motor, is the subject of this paper.

The outstanding characteristics of this new motor may be stated as follows:

1. It operates over the whole normal working range with leading or unity power factor.
2. When operated in parallel with induction motors, the leading power factor of this new motor counteracts the lagging power factor of the induction motors.

1. Chief Designing Engr., Wagner Electric Corp., St. Louis, Mo.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

3. The starting characteristics are as excellent as those of an induction motor with wound secondary.

4. It is able to operate at very heavy temporary overloads.

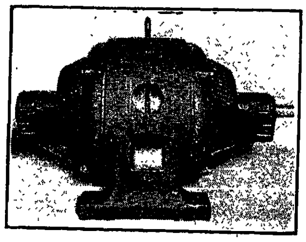

FIG. 1—100-H. P., EIGHT-POLE, 60-CYCLE, THREE-PHASE MOTOR

5. From no load to about 150 to 200 per cent load, the machine operates as a synchronous motor. When the load increases beyond these values, it operates as an induction motor, and if the load is again decreased

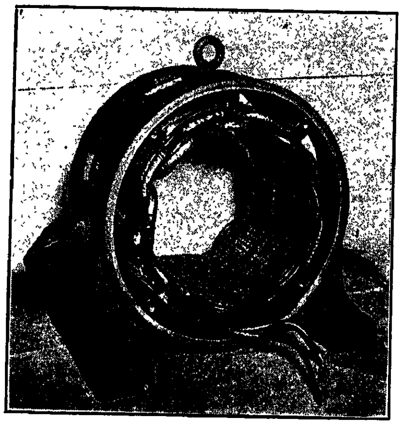

FIG. 2—STATOR OF 100-H. P., EIGHT-POLE, 60-CYCLE, THREE-PHASE MOTOR

to values in the neighborhood of 150 to 200 per cent of normal, it automatically returns to synchronous operation.

Fig. 1 presents a view of a 100 h. p., 900 rev. per min. machine of this new type.

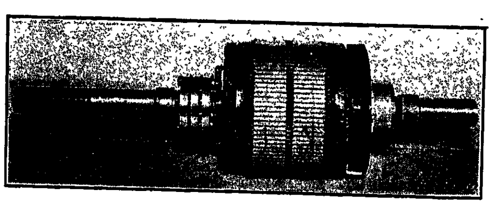

FIG. 3—ROTOR OF 100 H. P., EIGHT-POLE, 60-CYCLE, THREE-PHASE MOTOR

Fig. 2 exhibits a view of the stator.

Fig. 3 shows the rotor construction.

The rotor is provided with slip-rings and a relatively small commutator. The rotor laminations are provided with slots on the circumference and carry two windings. In the bottom of the slots, is located a small d-c winding which is connected to the commutator; and in the top of the same slots is a standard polyphrase winding which is connected to the slip-rings.

In the larger units the slip-rings and commutators are arranged on opposite ends of the armature; and in the smaller units, both of these members are located on the same side. In either case, the a-c. rotor winding is connected through the slip-rings to the a-c. supply line.

The stator is built of laminated iron with slots in the inner circumference which gives a general appearance similar to that of a standard induction motor, *i. e.*, no pronounced poles are used.

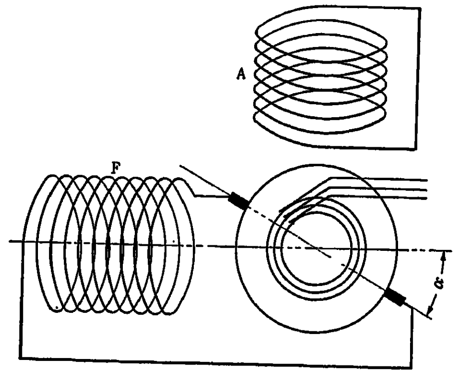

FIG. 4—RUNNING CONNECTION OF FYNN-WEICHSEL MOTORS

The stator carries two windings—the field winding, *F*, and the auxiliary winding, *A*, which are 90 electrical degrees displaced from each other. These windings are of the simplest kind possible. They are wound concentric, as is clearly depicted in Fig. 2.

The normal running connection between the individual windings is diagrammatically given in Fig. 4. It will be observed that the secondary winding *F*,

Starting Connections

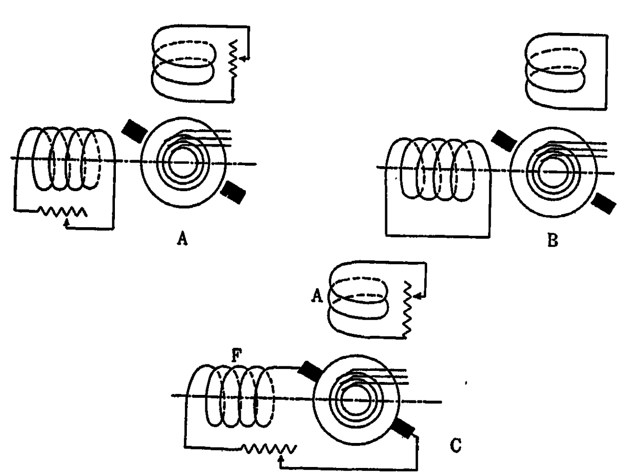

FIG. 5

located on the stator, is connected in series with the brushes riding on the commutator, and that the axis of the brushes forms an angle less than 90 deg. with the axis of the field-winding *F*. The secondary winding *A*, also located on the stator, is short-circuited. The slip-rings are connected to the line.

The machine is started by inserting ohmic resistance

in the circuits of the windings F and A. Fig. 5A illustrates a starting connection which in every respect gives the same starting performance as that inherent to a slip-ring induction motor. When this machine has reached the highest speed possible, and after all resistance in the secondary has been cut out, the windings are so reorganized as to agree with Fig. 4, which illustrates the running connection.

It is possible to start the machine in the running connection by introducing resistance in the A and F circuits and leaving the commuted winding in series with the F winding. This arrangement is extremely simple, and for this reason is used in all medium and small size machines. See Fig. 5C.

Generally speaking, this latter arrangement gives starting characteristics very similar to those of a slip-ring induction motor, but due to the commuted winding being in series with the F winding, the torque, for a given resistance in the secondary circuit, is favorably influenced.

This increase in torque grows rapidly with decreasing resistance in the rotor circuit. This is caused by the voltage, appearing across the commutator brushes, which is connected in series with the voltage induced in the winding F.

It will be proved later that the voltage appearing across the brushes has the same frequency as the voltage induced in the F winding. Therefore, by injecting this voltage in the F winding, we increase the total voltage acting on the F circuit and thereby increase the current in the circuit over the value which would exist at the same speed when the machine is operating as an induction motor. This increase in current naturally must be followed by an increase in torque, or what means the same, for a given useful torque the Fynn-Weichsel motor will operate at a higher speed than an induction motor when operating under otherwise equal conditions.

This is, roughly speaking, the explanation for the Fynn-Weichsel motor's ability to synchronize automatically.[2]

The synchronizing ability of this motor is excellent and, therefore, the machine can be used with advantage in cases where heavy mechanical torque or inertia loads have to be accelerated to synchronous speed.

The phenomena which take place during the starting and synchronizing periods will be discussed somewhat more in detail in the following:

With the armature at rest, a polyphrase voltage is impressed on the slip-rings and a rotating magnetic field is set up by the armature windings. The speed of this field, in respect to a given armature conductor, is given by the well-known formula:

$$n_0 = \frac{120 \times f_0}{p}$$

2. See paper presented before the Association of Iron and Steel Electrical Engineers, Feb. 16, 1924, "The Motor that Corrects Power-Factor." by H. Weichsel.

n_0 = speed of the field in rev. per min.
f_0 = frequency of supply circuit.
p = number of poles.

As the commuted and a-c windings are located in the same slots, the rotating field also cuts the commuted winding and consequently sets up a voltage in it. With the armature at rest, the voltage which appears across the commutator brushes has the same frequency as the line voltage. The phase of the voltage appearing across the brushes depends, however, on the relative location of the brush axis in respect to a fixed point in space.

Furthermore, the rotating field cuts the stator windings, F and A, and thereby produces a voltage in these windings which also has a frequency equal to that of the line so long as the motor is at rest.

If the axis of the commutator brushes coincides with that of the F winding, then the voltage induced in the F winding and the voltage appearing across the brushes not only have the same frequency but are also in phase. This can be seen from Fig. 6, where the com-

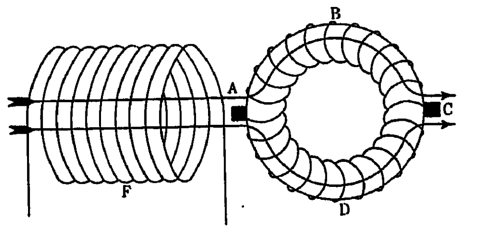

FIG. 6

muted winding is represented in form of a Gramme ring.

From this figure it will be seen that if a rotating field intersects the ring and the F winding the maximum lines are embraced by the winding A-B-C and A-D-C at the same moment when the maximum lines are embraced by the winding F, provided the brush axis coincides with F axis.

Therefore, the voltages appearing across the brushes and the induced voltage in the F winding must be in phase.

If the secondary windings F and A are closed over a resistance, a torque is set up, acting on the armature in the well-known manner. This torque, due to the fundamental induction laws, has such a direction as to rotate the armature in the direction opposite to that of the rotating field. Therefore, if the rotating field has the velocity of n_0 in respect to the armature conductors, and these latter operate with a speed n_x in direction opposite to n_0, due to the armature torque, then the rotating field has a velocity of $n_0 - n_x$ in respect to a fixed point, in space. Therefore, the speed with which the secondary windings F and A are cut by the magnetic field decreases with increasing armature speed, and the frequency of the voltage produced in the windings F and A is given by the equation:

$$f_2 = \frac{n_0 - n_x}{n_0} \times f_0$$

f_2 = frequency induced in windings A and F.
f_0 = line frequency.
n_0 = speed of rotating field in respect to armature, expressed in rev. per min.
n_x = speed of armature, expressed in rev. per min.

The voltage induced in the secondary is given by the equation:

$$E_0 \frac{n_0 - n_x}{n_0} = E_x$$

E_0 = voltage induced in the secondary when armature is at standstill.

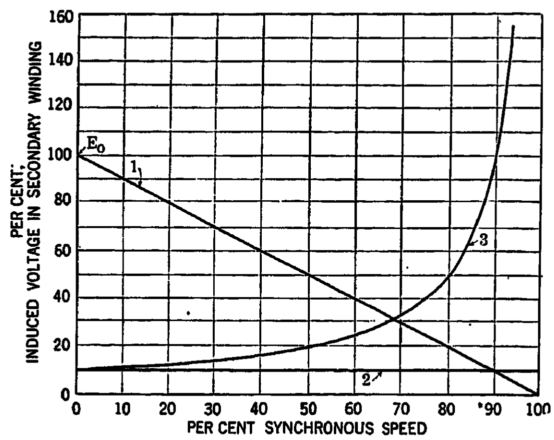

FIG. 7—OPEN CIRCUIT VOLTAGE IN SECONDARY WINDING AND ACROSS COMMUTATOR

E_0 = Induced Voltage in Field at Standstill
1 = Induced Voltage in Secondary E
2 = Volts Across Commutator in percentage of Induced Secondary Volts at Standstill
3 = Volts across Commutator in percentage of Induced Volts

E_x = voltage induced in the secondary when armature rotates with speed n_x.

This means that the voltage and frequency in the windings F and A decrease with increasing rotor speed and become zero when the armature rotates at synchronous speed. This is the same simple law which governs the voltages in the secondary of an induction motor.

However, the conditions are somewhat different if we consider the voltage appearing across the commutator brushes. The voltage appearing across the brushes has a frequency which is determined by the velocity of the rotating field in respect to the brushes, or what means the same thing, in respect to a fixed point in space. Therefore, the frequency across the brushes is, for any armature speed, equal to the frequency induced in the secondary windings, F and A.

On the other hand, the magnitude of the voltage appearing across the brushes, is determined by the velocity with which the rotating field cuts the conductors of the commuted winding. This velocity is, at any armature speed, equal to the synchronous speed of the field. Therefore, the voltage appearing across the brushes is independent of the armature speed.

These relations are graphically represented in Fig. 7. The induced voltage in the secondary falls from the value O-E_0 at standstill of the armature to the value of zero when armature operates at synchronous speed. The voltage across the commutator brushes remains fixed, as represented by the horizontal line marked No. 2. If the voltage of the F winding is connected in series with the voltage across the commutator brushes, the total voltage will differ by only a small amount from the induced voltage in F when the armature is at standstill. However, the influence of the injected voltage becomes very marked when the armature rotates near synchronous speed. The expression voltage of the commutator winding, divided by the voltage across the F winding, is given by the curve No. 3 which shows a very rapid percentage increase when the armature approaches synchronous speed. Therefore, the influence of this commutator voltage on the torque characteristics of the machine must increase very rapidly with increasing speed of the armature, or in other words, with decreasing slip of the armature.

In a standard induction motor the torque produced by a phase of the secondary winding is, at any time, proportional to the product of the current in this winding and a voltage of the same frequency as the current,

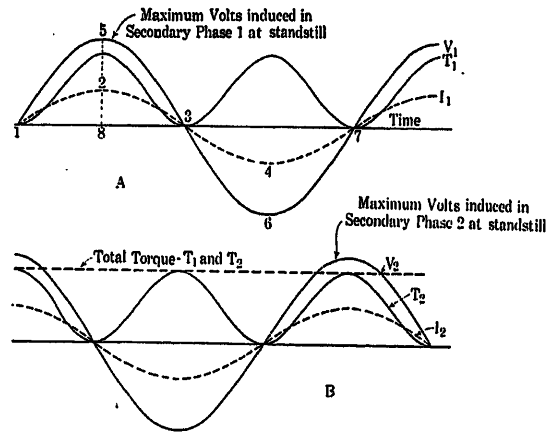

FIG. 8

A. V_1 = Volts Induced in Secondary phase 1 at Standstill
T_1 = Torque of Secondary phase 1
I_1 = Current in Secondary phase 1

B. T_2 = Torque of Secondary phase 2
I_2 = Current in Secondary phase 2

the maximum value of the voltage being equal to that of the induced voltage at standstill. The phase relation between the secondary current and this imaginary voltage must be selected equal to that of the current in respect to the actual voltage induced in the secondary at the speed under discussion.

This law is proved in Appendix No. 1.

Assuming that the secondary winding is of the two-phase type, the torque condition in a standard induction motor can be represented by the Figs. 8A and 8B. The line 1-2-3-4 gives the current flowing at any moment in the winding of the secondary phase, No. 1.

The curve 1-5-3-6-7 represents the imaginary voltage with the same frequency as the induced current I_1. The maximum value of this voltage is 5-8, and is equal to the maximum value of the induced voltage in this winding with rotor at standstill.

For the sake of simplicity, we assumed that the current I_1 and the voltage E are in phase, a condition which is always satisfied in an ideal motor without leakage reactance and is very nearly satisfied in a machine with leakage when the currents do not materially exceed the full-load currents.

The torque at any moment produced by the phase No. 1 of the secondary is, according to the above law, proportional to the product of the current I_2 and the voltage V_1 at the same moment. Making this product, the curve marked T_1, which is a sine square curve is formed.

Fig. 8B represents the same conditions as Fig. 8A, but refers to the torque produced by phase No. 2 of the secondary of the induction motor. As the secondary was assumed to be wound two-phase, it follows that the current in the second phase is 90 electrical degrees displaced from the voltage induced in the secondary phase No. 1.

Fig. 8B is identical to Fig. 8A, except that Fig. 8B is 90 electrical degrees shifted against Fig. 8A. The

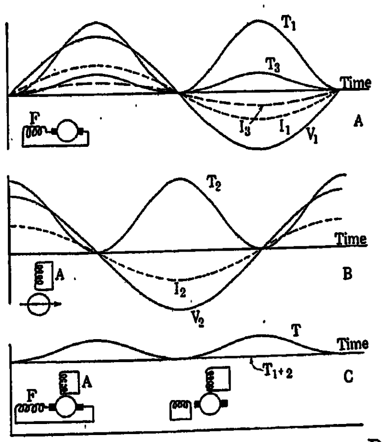

Fig. 9 —A. Torques Produced by Secondary Winding F
T_2 — Induction Motor Torque
T_3 — Torque Due to Interjected Current
I_2 — Induced Current in F Winding
I_3 — Injected Current in Secondary Winding F, by commutated Winding

B. Torque Produced by Secondary Winding A
T_1 – Induction Motor Torque
I_1 – Induced Current in Secondary Winding A

C. Torque Produced by Windings A and F
T_{1+2} – Resultant Torque $T_1 + T_2$
T – Final Torque Acting to Produce Rotation $T_1 + T_2 + T_3$

torque T_2 obtained from Fig. 8B is absolutely identical in its magnitude to the torque T_1 of phase No. 1, but when the torque due to phase No. 1 is a maximum, the torque T_2 due to phase No. 2 is a minimum. The total torque acting on the machine will be the sum of the torque T_1 and T_2. It is readily seen that the sum of these torque is a constant value at any time.

This reasoning assumes that the secondary windings have the same resistances, resistances being reduced to the same induced voltages. If this assumption is not fulfilled, then the maximum torque of phase No. 2 does not equal the maximum torque of phase No. 1. However, the respective phase relations of the torque curves T_1 and T_2 remain as given in Figs. 8A and 8B. The total motor torque, *i. e.*, the sum of torque T_1 and

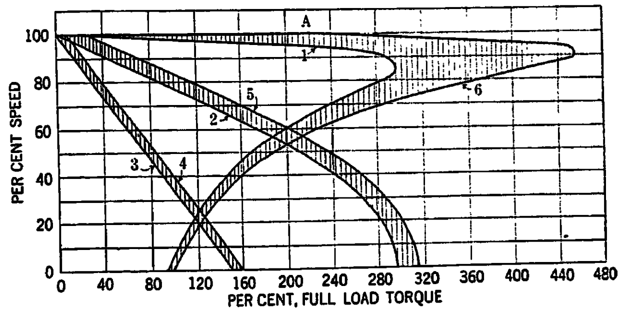

Fig. 10—Speed Torque Curves of Fynn-Weichsel Motor
1—Operating as Induction Motor—no External Resistance in Circuit
2—Sufficient Resistance to Give Maximum Starting Torque
3—Resistance Necessary to Start 150 per cent. Torque

T_2, in this case, is equal to a constant torque over which is superposed a fluctuating torque. The fluctuation is of the same frequency as that of torque T_1 and T_2. Let us assume that an additional e. m. f. is injected into phase 1 and that this injected e. m. f. has the same frequency and the same phase relation as the voltage 1-5-3-6-7 in Fig. 8A, which is redrawn in Fig. 9A. A current I_3 then flows in this circuit, caused by the injected e. m. f., in addition to current I_1 which flows due to the induced e. m. f. This injected current I_3 produces at any moment a torque equal to the product of the instantaneous value of the current I_3 and the voltage V_1. Therefore, the torque due to the injected e. m. f. is given by curve T_3 in Fig. 9A. This condition exists when the winding F and the commuted winding are connected in series, as diagrammatically indicated in Fig. 9A.

The conditions in the winding A cannot, in any manner, be altered by injecting a current in the winding F and therefore the torque produced by the winding A remains unaltered and is represented in Fig. 9B. The total torque on the armature must be the sum of the three torques discussed; Torque T_1, due to the induced current in winding F, torque T_2 due to the induced current in winding A, and torque T_3 due to the injected current in winding F.

By forming this addition from moment to moment, we obtain curve marked T in Fig. 9C, which is nothing more or less than the constant induction motor torque T_1 plus T_2, upon which is superposed a pulsating torque T_3, due to the injected current I_3 in the winding F.

Knowing the torque relations in the machine at any speed, it is possible to derive the speed-torque curves of this type of machine during the starting period, as

shown in Fig. 10. Let it be assumed that the commutator voltage e_c is 6 per cent of the induced voltage in winding F at standstill. Let the curve No. 3, represent the speed-torque curve of the machine when operating as straight induction motor with sufficient resistance in the secondary to give 150 per cent starting torque. If a voltage of 6 per cent is injected in this circuit at standstill, the torque, due to winding F, increases 6 per cent, *i. e.*, from 150 per cent to 159 per cent. For any other speed, n_x, the induced voltage

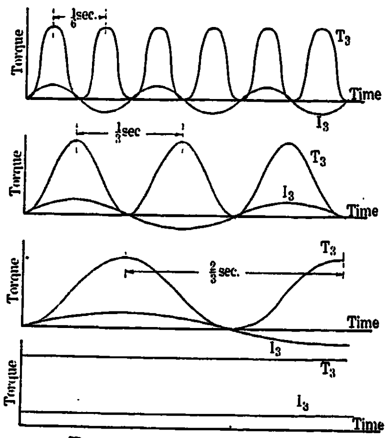

FIG. 11—TORQUE DUE TO INJECTED CURRENT AS A FUNCTION OF SLIP

a. Five per cent. Slip –three ~ per Second in Secondary
b. 2½ per cent. Slip –1½ ~ per sec. in Secondary
c. 1¼ per cent. Slip –0.75 ~ per sec. in Secondary
d. 0 Slip –0 ~ per sec. in Secondary

in the winding F is proportional to the slip speed, s_x, which equals $n_o - n_x$. If the constant voltage, e_c, is injected into the F winding, then the current flowing in the winding will be $\dfrac{e_0 \, s_x + e_c}{e_0 \times s_x}$ as great as it was originally, and the torque will be increased in the same ratio. For a slip of 20 per cent with an injected voltage of 6 per cent, the torque due to winding F will have become greater in the ratio $\dfrac{0.2 + .06}{0.2}$ which equals 1.3 times its original value.

When the resistance in the secondary circuit is reduced, the speed torque curve of the machine, as an induction motor, assumes the shape of No. 2. By performing the above calculation for a number of points, curve No. 5 is obtained.

It will be noticed that the torque of the curve No. 2 is increased more materially by the injected voltage than the torques corresponding to curve No. 3. This is simply due to the fact that the curve No. 2 is obtained with a smaller resistance in the F circuit than the resistance corresponding to the curve No. 3. Considering curve No. 1, which represents the speed torque curve of the motor when no external resistance is in the secondary, and again increasing the torque for each given slip in the ratio of injected voltage plus induced voltage over induced voltage, curve No. 6 is found. This curve is of special interest, as it shows the large increase in the maximum torque of the machine.

In other words, the remarkable condition exists that the machine with the injected voltage has a larger maximum torque than the same machine operating as a straight induction motor. This condition can readily be observed on actual tests.

Furthermore, curve No. 6 intersects the horizontal line representing the synchronous speed at the point A, corresponding to 230 per cent torque, meaning that after synchronous speed has been reached and torque T_1 and T_2 has disappeared, T_3 still exists and is equal to 230 per cent of normal full-load torque, which is the maximum torque of the motor as synchronous machine. The increase in torque, due to the injected voltage, is given in the diagram by shaded areas.

From what has been said in the beginning regarding these torques, it is known that the induction motor torque at any speed has a constant value and is independent of time. However, the torque due to the injected current, pulsates between zero and a maximum value if the injected voltage is in phase with the induced voltage. The frequency of pulsation of this torque decreases with decreasing slip. For instance, at standstill, the time between two adjacent maximums is 1/120 of a second if a 60-cycle supply is used. At 10 per cent slip, however, the time between two maximums of the pulsating torque is 1/12 of a second and at 5 per cent slip the time has increased to $^1/_6$ of a second. This has been graphically represented in Fig. 11 for 5, 2½, 1¼, and 0 per cent slip. This figure shows the torque due to the injected current when the external resistance in the secondary windings is zero, corresponding to the difference between the speed torque curves No. 6 and No. 1 in Fig. 10.

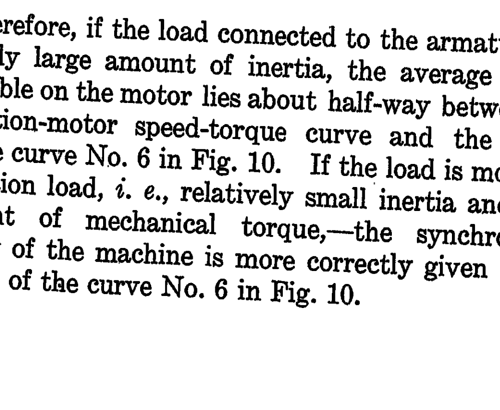

FIG. 12

Therefore, if the load connected to the armature has a fairly large amount of inertia, the average torque available on the motor lies about half-way between the induction-motor speed-torque curve and the speed-torque curve No. 6 in Fig. 10. If the load is more like a friction load, *i. e.*, relatively small inertia and large amount of mechanical torque,—the synchronizing ability of the machine is more correctly given by the torque of the curve No. 6 in Fig. 10.

The above discussion has been carried on under the assumption that the injected e. m. f. is in phase with the e. m. f. induced in winding F. As shown above, this condition exists, when the brush axis coincides with the axis of the F winding. However, if these two axes are displaced by the angle α, then the injected e. m. f. will be α degrees displaced from the induced e. m. f., and the torque curve takes the shape given in Fig. 12.

As the torque curve is formed by the product of instantaneous values of current and voltage, it must

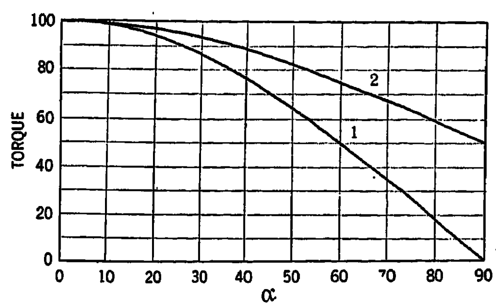

FIG. 13—1. COSINE CURVE
2. SINE² (90 DEG. − $\alpha/2$)

have the same shape as a watt curve for any circuit where the voltage and current are α degrees displaced from each other. The average watt draw of such a circuit is given by the equation:

$$\bar{E} \times \bar{I} \ \frac{\cos \alpha}{2}$$

where $\bar{E}$ and $\bar{I}$ represent peak values as distinguished from $\sqrt{\text{mean}^2}$ values. Therefore, the average torque is given by the same equation.

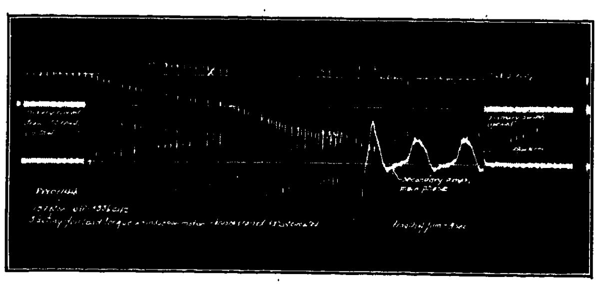

FIG. 14—10-H. P., FOUR-POLE, 60-CYCLE, THREE-PHASE MOTOR STARTING FULL-LOAD TORQUE AS SLIP-RING INDUCTION MOTOR USING REMOTE CONTROL STARTER

From this figure, it follows that the *maximum* torque is proportional to the relation:

$$\bar{E}\,\bar{I}\,\sin^2\left(90 - \frac{\alpha}{2}\right)$$

If the average torque due to the injected current is plotted as function of the brush displacement, curve No. 1 in Fig. 13 is obtained; and, if the maximum available torque is plotted as function of the brush position, curve No. 2 is obtained. In either case, the maximum value has been called 100 per cent. These curves clearly show that small brush displacements, say zero to 30 deg. do not appreciably influence the synchronizing ability for either friction or inertia load. For a larger brush displacement, however, the synchronizing ability of the machine for an inertia load decreases quite rapidly and becomes zero at 90 deg. brush displacement, while for friction load under these conditions, a synchronous torque of about 50 per cent of the maximum available torque exists.

The foregoing discussion gives in general the phenomena as they take place during starting and synchronizing periods of this new machine. Oscillograms taken on actual machines during the starting and synchronizing periods will be of interest.

Fig. 14 gives the currents in the primary and secondary circuits of the machine when connected as a straight induction motor with resistance in the secondary; *i. e.*, the commuted winding is not in circuit.

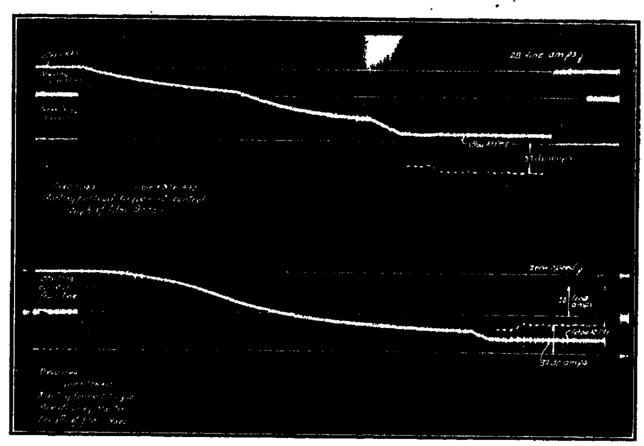

FIG. 15—10-H. P., FOUR-POLE, 60-CYCLE, THREE-PHASE MOTOR STARTING FULL-LOAD TORQUE USING CARBON-PILE STARTER

Upper Oscillogram, 15 A
Lower Oscillogram, 15 B

The machine, therefore, operated as a standard induction motor, and the current curves given in the oscillogram represented the starting conditions of a slip-ring induction motor.

Fig. 15B gives the oscillogram of the current draw in the primary and in the secondary of the same machine, as shown in Fig. 14, but with the commuted winding connected in series with the winding F.

During the starting and running periods in the tests illustrated in Figs. 14 and 15B, the same starting resistances have been used. A comparison of these two oscillograms shows that the new machine starts as a synchronous motor with a current draw not very materially different from the current draw of a motor operating as a straight induction machine.

The secondary current curve on Fig. 15B is of special interest: The frequency is high at first, then gradually decreases and finally changes into direct current when the machine has reached synchronism. In this figure, the starting resistance of the machine was purposely

selected of such a value that the machine could not run faster than 25 per cent slip when operating as an induction machine. On short-circuiting this resistance and leaving the commuted winding in series with the F winding, the machine immediately went into synchronous operation without abnormal current draw.

Fig. 15A shows the starting performance of this type of synchronous motor with a carbon-pile starter. The very low current inrushes in this diagram are of particular interest.

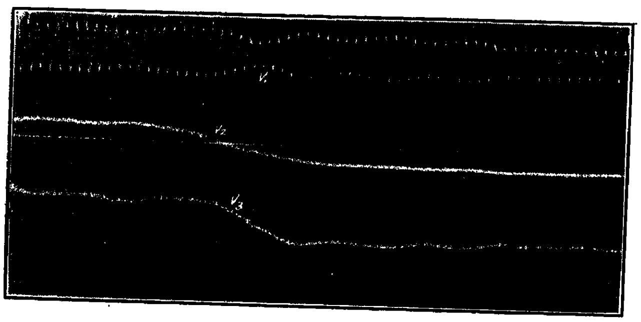

FIG. 16—V1. SHOWS CURRENT ON MOTOR CIRCUIT WHEN THE MOTOR RETURNS TO SYNCHRONOUS OPERATION WHEN LOAD IS REDUCED TO 150 PER CENT

V2. DIRECT-CURRENT FIELD-VOLTAGE WHEN THE MOTOR PULLS BACK INTO STEP

V3. DIRECT-CURRENT FIELD-AMPERAGE WHEN THE MOTOR PULLS BACK INTO STEP

Fig. 16 shows a very interesting oscillogram of the same kind of motor at the moment when the machine goes from induction-motor operation into synchronous operation with full load on the motor.

FIG. 17—CURRENT FUNCTIONS DURING STARTING PERIOD WITH NO LOAD ON THE MOTOR

V_1. Current on the Line

V_2. Current in the a-c.-Field, which becomes Zero as soon as the Motor Pulls into Step and Operates as a Synchronous Motor

V_3. Current in the d-c.-Field, Which Does not become Uniform until the Motor Pulls into Step.

Fig. 17 gives the oscillogram of the same machine, but starting idle. During this test, the starter was in circuit, but was thrown very quickly into running position in order to be within the time limit of the film.

Summing up the main conclusion of the above discussion, the following laws are obtained:

1. The brush displacement from the field axis should be made zero or as small as possible in order to obtain good synchronizing characteristics.

2. The inherent slip of the motor as an induction machine should be made as small as is consistent with the general design of the motor.

3. The injected current should be as large as possible. If it is about two times the full-load secondary current as induction motor, good commercial relations will be obtained on the machine when operating as a synchronous motor.

If these rules are followed, the machine will be found to synchronize very smoothly, and when pulled out of step, will continue to operate very satisfactorily as an induction motor.

Tests on actual machines have shown that under normal operating conditions, the synchronizing torque is from 90 per cent to 95 per cent of the pull-out torque of the machine, when operating as a synchronous motor.

The vector diagram for a synchronous motor with separate excitation and without pronounced poles, *i. e.*, with an iron circuit similar to that of an induction motor, is given in Fig. 18.

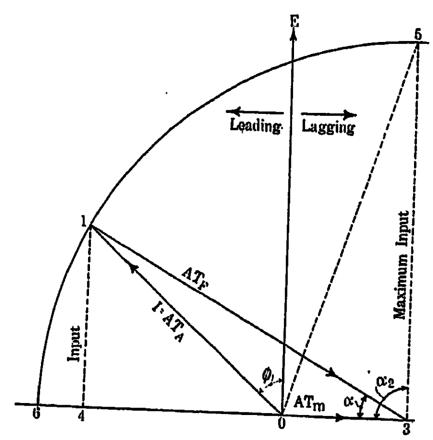

FIG. 18—VECTOR DIAGRAM OF A SEPARATELY EXCITED IDEAL SYNCHRONOUS MOTOR

For purposes of simplification, the assumption has been made that the machine is free of all ohmic resistance and leakage reactance. The machine is assumed to be built with the primary member on the rotor and the secondary member on the stator.

The resultant field in the machine is produced by the ampere-turns $A\ T_m$. These are of the same magnitude as the no-load ampere-turns of the machine when operating as an induction motor.

The armature ampere-turns in this diagram are $A\ T_a$, and the d-c. field ampere-turns are $A\ T_f$.

The resultant motor field, produced by the ampere-turns $A\ T_m$ while rotating in respect to the rotor winding, is stationary in space, as has been shown in the previous discussion on synchronizing torque. The magnitude of this field is constant if leakage reactance and ohmic resistance are zero.

If (besides the polyphase winding) the armature of this machine carries a commuted winding, then a d-c.

voltage appears across the commutator brushes due to the rotation of the commuted winding in this stationary field.

In the Fynn-Weichsel motor, the commutator brushes are connected to the field winding F (see Fig. 4) and the motor is thereby made self-exciting.

In a previous part of this paper it was shown that the angular displacement between brush and field axis materially influences the speed torque curve during the

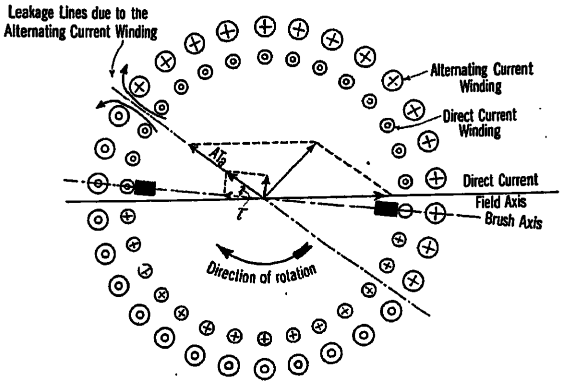

Fig. 19—Influence of Armature Leakage Flux on the Direct-Current Voltage

starting period as well as the self-synchronizing ability of the machine.

From the vector diagram Fig. 18 of the separately-excited synchronous motor, it follows that the space position of resultant motor field changes with load. Therefore, it is readily seen that the operating characteristics of a self-excited motor must be a function of the angular displacement between brush axis and field axis.

The influence of the brush displacement on the operating performance of a motor of this type will not be discussed in this paper, as the subject has been fully treated in a paper by Mr. V. A. Fynn, previously presented before the Spring Convention of the Institute, April 7-10, 1924, Birmingham, Alabama.

In the above, it was assumed that the rotor leakage reactance and ohmic resistance were zero. In reality the ohmic resistance of the a-c. winding produces a voltage drop in phase with the current and the a-c. leakage reactance produces a drop of 90 deg. displacement from the current. Under different load conditions, both of these factors tend to change the magnitude of the stationary field referred to above. These are well-known facts and can be readily considered in the calculations if desired.

The a-c. leakage reactance flux not only causes the stationary field to change its magnitude with different loads but also produces an additional voltage across the commutator brushes.

This, as well as some other phenomena, especially the interactions caused by the d-c. armature winding, will be discussed in the following.

Influence of the Leakage Flux on the D-C. Voltage

The a-c. winding of the armature produces some magnetic lines which are not interlinked with the stator winding but which are still interlinked with the d-c.-armature winding. These lines are usually called the a-c. leakage lines.

Fig. 19 shows the current distributions in the a-c.- and d-c.-armature windings, and also the general direction of the leakage flux, which is coaxial with the vector $A\ T_a$. The d-c. winding cutting this leakage flux, produces a voltage across the d-c. brushes which is given by the relation:

$$l_L = K \text{ sine } \tau$$

This value is positive when the angle, with respect to the vector $A\ T_a$ lies in a direction opposite to the direction of the armature rotation.

This additional d-c. voltage is negative when the angle τ is located in the same direction as the direction of rotation with respect to the vector $A\ T_a$.

In one case, the voltage due to leakage flux adds to the main d-c. voltage and in the other case, it subtracts from it.

Besides these leakage lines produced by the a-c. winding, there exists a group of leakage lines produced by the d-c. winding. These latter lines cut the a-c. winding and generate a reactance voltage in the a-c. winding, which is approximately in opposition phase to the reactance voltage produced in this winding by its own leakage flux.

The distribution of the leakage flux produced by the d-c. winding is given in Fig. 20.

The influence of the d-c. winding leakage flux on the operation of the machine is very small, because the armature d-c. ampere-turns are approximately 5 per cent of the armature a-c. ampere-turns.

Fig. 20

Therefore, the leakage flux produced by the armature d-c. ampere-turns is of the order of 10 per cent of the leakage lines produced by the armature a-c. ampere-turns, on account of the different field shapes. (See Fig. 20.)

In other words, the voltage produced by the d-c. leakage flux in the a-c. winding will be 10 per cent or less of the voltage induced in the a-c. winding by its own leakage flux. Therefore, the effect of the d-c.-leakage flux on the operation of the machine must be very small. It is evident that the d-c. leakage flux

cannot in any manner influence the d-c. voltage across the brushes, because the axis of the d-c. leakage flux is in line with the axis of the d-c. brushes.

Armature Reaction of the D-C. Armature Winding

The d-c.-armature winding produces ampere-turns in the direction of the brush axis, which remains fixed in space for all load conditions. Furthermore, these d-c. ampere-turns are connected in series with the stator d-c. ampere-turns. Both will increase and de-

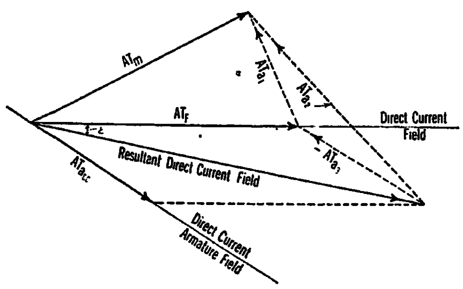

Fig. 21—Influence of d-c.-Armature Currents on the Operation of the Motors

A-c.-ampere-turns $A\ T_{a3}$ are Set up, neutralizing the armature d-c.-ampere-Turns

crease at the same rate. Therefore, the stator d-c. ampere-turns and the rotor d-c. ampere-turns add vectorially. The effect of this is that a resultant d-c. field is obtained which forms a somewhat smaller angle with the brush axis than the original angle between brush axis and field axis.

This is shown in Fig. 21. This diagram is drawn under the assumption that the magnitude of the magnetizing ampere-turns, $A\ T_m$, and their vectorial relation to the field axis F is kept constant. The total ampere-turns on the armature are represented by the vector $A\ T_{a2}$. This vector can be considered as being composed of a vector $A\ T_{a1}$ and a vector $A\ T_{a3}$, the latter being equal and opposed to the vector $A\ T_{adc}$, representing the armature reaction of the d-c. winding. This establishes the very important fact that *under all conditions, the d-c.-armature reaction ampere-turns are, at any time, counterbalanced by equivalent armature a-c.-ampere-turns*, a condition very similar to that existing in a rotary converter.

Stator Leakage Lines

As in any induction motor, there are also leakage lines around the secondary member of the machine which in this case, is the stator. As the stator is excited with direct-current, the stator leakage lines can in no manner react on the stator windings. According to their definition, these lines are not interlinked with the rotor windings, and it follows that the stator leakage lines cannot, in any manner, influence the synchronous operation of the machine. The only effect which these lines have is a slight change in the core-and-teeth induction of the stator member, but this can readily be cared for in the original dimensioning of the iron circuit when designing the machine.

Commutation

Two problems have to be considered under this heading:

1. The commutation so far as sparking is concerned.
2. The influence on the operation of the machine of the currents flowing in the short-circuited coils undergoing commutation.

The second point will first be considered. For a d-c. machine with a straight line commutation, the relations are represented by Fig. 22A. The ampere-turns, during the time of commutation zero to two, are equal and opposed to the ampere-turns during the time of commutation two to four. In other words, the coil undergoing commutation produces a very rapidly alternating magnetic flux and the average magnetic effect of this high-frequency alternating flux on the action of the machine, will be zero, as shown by sketches, 0, 1, 2, 3, representing the positions of commutation 0, 1, 2, 3 of Fig. 22A.

However, the conditions are quite different if the commutating current does not follow a symmetrical curve. Curves as given in Fig. 22B and Fig. 22C, are quite feasible. From these figures, it is seen that during the first part of commutation the positive wave of the current contains a much larger area than the negative wave representing the second part of the commutating period. The result is that the field produced by the currents flowing in the short-circuited coils undergoing commutation is of a pulsating character and in which the field in one direction predominates.

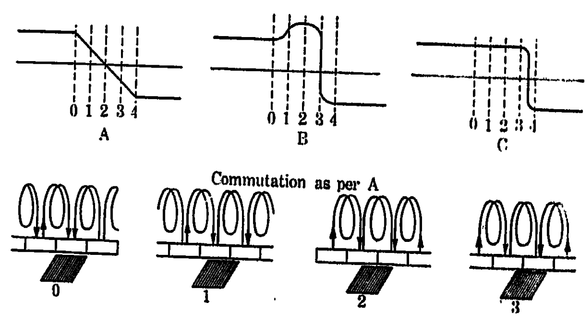

Fig. 22

Position	0	1	2	3
Time	0	1	2	3
Ampere-Turns in Short Circuit Coil	0	+ ½	0	− ½

The average ampere-turns produced by the short-circuited coil undergoing commutation must lie in a direction vertical to the axis of the brushes $A\ T_c$. (See Fig. 23). The resultant magnetizing ampere-turns in the machine $A\ T_m$, must remain constant. The vector, $A\ T_m$ can be decomposed into one vector parallel to the brush axis and one vector vertical to the brush axis.

The ampere-turns produced by the commutating current have the tendency to change the magnitude of the component $A\ T_{m1}$, and as this is not permissible,

it follows that the a-c.-armature winding must set up ampere-turns $A\ T_2$ equal and opposed to the ampere-turns $A\ T_c$ produced by the currents undergoing commutation. This has been diagrammatically shown in the vector diagram Fig. 23. From this it follows that the currents undergoing commutation have the effect of bringing the power factor nearer to unity.

Besides this magnetic effect, the currents flowing in the short-circuited coils undergoing commutation produce a certain amount of loss. This loss decreases with increasing load and is a maximum at no load. But

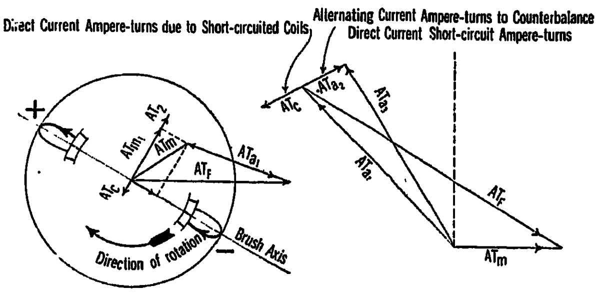

Fig. 23—Effect of Current in Coils Undergoing Commutation

even under no load, it is a very small percentage of the normal motor output, provided the commutating conditions are properly selected in the design of the machine.

It has been shown that at no load the resultant field in the machine is nearly in line with the axis of the d-c.-field excitation and at the maximum load; *i. e.*, near the breakdown point of the machine as a synchronous motor, the axis of the resultant field is nearly at right-angles to the axis of the d-c. exciting field. As the d-c.-brush axis is only slightly displaced from the d-c.-field winding axis, it follows that at no load the coils which undergo commutation stand nearly in line with the resultant field, and at maximum load are approximately 90 deg. displaced against the resultant field.

Therefore, the voltage generated in the short-circuited coil undergoing commutation is a maximum at no load and a minimum at maximum load, so that the losses due to the current in the short-circuited coils undergoing commutation decrease with increasing load.

Sparking

Under the heading "Armature Reaction of d-c.-Armature Winding," it was proved that under all conditions the d-c.-armature reaction ampere-turns are counterbalanced by equivalent a-c.-armature ampere-turns. No voltage, therefore, can be generated in the short-circuited coils undergoing commutation due to an armature reactance flux. In this respect the machine operates very much the same as a d-c. machine in which armature reaction is perfectly neutralized.

However, there remain two voltages in the short-circuited coils—No. 1. A voltage which is due to the leakage reactance flux surrounding the conductor undergoing commutation and No. 2, a voltage generated in the short-circuited coil by its movement in the main field of the machine.

This latter voltage, as has been pointed out above, is a maximum at no load and is practically zero when the machine has reached its maximum load as a synchronous machine. By properly designing the d-c. winding, it is possible to hold this voltage at such a low value that it cannot produce commutation trouble, even under no load conditions.

The leakage reactance voltage referred to under No. 1 is in every respect similar to the leakage reactance voltage in a standard d-c. machine.

Considering the three different voltages just discussed, and comparing their actions with those which occur in a neutralized d-c. machine, it will be found that this new motor from the commutating point of view operates in a manner similar to a neutralized d-c. machine in which the neutralizing winding is somewhat weaker than the d-c. armature reaction.

Due to the fact that in this new type of machine the total voltage across the brushes is in all cases considerably below 100 volts, it is evident that the commutation must be perfect.

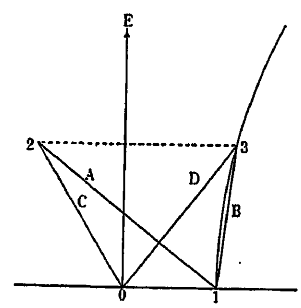

Fig. 24—Comparison of Ampere-Turn Relations in Fynn-Weichsel and Induction Motors

Losses in Fynn-Weichsel Motor

The losses can be subdivided in the following manner:

1. Primary copper losses.
2. Iron losses in the primary member.
3. Secondary copper losses.
4. Copper losses in the d-c. armature winding.
5. Friction losses.
6. Commutating losses, including brush losses.
7. Stray losses.

In order to compare these individual losses with those occurring in a standard induction motor, it is essential to compare the vector diagrams of these two types of machines:

Fig. 24 shows the vector diagram of a Fynn-Weichsel motor and a vector diagram of the same motor when running as straight induction machine.

Vectors 0-1-2 represent the operation of the machine

as synchronous motor, and the vectors 0-3-1 represent the operation of the machine as straight induction motor.

For operation as an induction motor, the vector 0-3 represents the ampere-turns on the primary member, and for operation as a synchronous motor the vector 0-2 represents the ampere-turns on the primary member.

PRIMARY COPPER LOSSES

The ratio of the length of these two vectors, D and C, gives a measure of the change in primary ampere-turns from induction motor operation to synchronous motor operation.

The primary copper loss of the machine operating as a synchronous motor, is, therefore, equal to $\left(\frac{C}{D}\right)^2 \times$ the primary copper loss of the machine when running as induction motor.

The relation of $\left(\frac{C}{D}\right)$ depends, to a large extent, on the power factor of the machine when operating as an induction motor or as a synchronous motor. Generally speaking it will be found that the primary ampere-turns are less in the Fynn-Weichsel motor than in the induction motor. Therefore, the primary copper losses in the Fynn-Weichsel motor will be smaller for a given output than the corresponding primary copper loss in a standard induction motor.

In addition to this, the mean-turn length of the primary winding in the Fynn-Weichsel motor is usually somewhat less than the mean-turn length of the primary winding of an equivalent standard induction motor, because in the latter case the winding is on the stator and in the former case the winding is on the rotor, thereby decreasing the mean-turn length.

SECONDARY COPPER LOSSES

In a similar manner, the losses in the secondary member, when running as a synchronous motor, are $\left(\frac{A}{B}\right)^2 \times$ secondary copper losses as an induction motor.

The secondary ampere-turns A of the Fynn-Weichsel motor are usually larger than the secondary ampere-turns B in a standard induction motor. This does not mean, however, that the secondary copper losses in the Fynn-Weichsel motor are materially larger than the slip loss in an induction motor, for the following reasons.

The secondary winding used in the Fynn-Weichsel motor is of the concentric type, which has a considerably shorter mean-turn length than the diamond winding usually employed in induction motors. Furthermore, the space factor for a concentric winding is somewhat better than that of an equivalent diamond winding. Both of these factors tend to decrease the secondary loss in the Fynn-Weichsel motors below the value expected by the ratio of vector length.

The above train of reasoning assumes that the d-c.-ampere-turns, 1-2, produce under otherwise equal conditions the same losses as equivalent polyphase ampere-turns of the magnitude 1-2. The correctness of this assumption is proved in the appendix, No. 2.

IRON LOSSES

For an equal number of magnetic lines, the Fynn-Weichsel motor has, as a rule, less iron losses than the equivalent standard induction motor, because in the Fynn-Weichsel motor the iron losses are located in the rotating member, while in the induction motor they are mainly located in the stationary member. But the rotating member contains less iron and therefore, for otherwise equal conditions the iron losses will be smaller than in an induction motor.

COPPER LOSSES IN THE D-C. ARMATURE WINDING

The necessary voltage for excitation generated in the d-c.-armature winding is the result of a relatively high cutting velocity between this winding and a field stationary in space. Therefore, the number of turns on this winding can be made very small. From this it follows that the resistance of this d-c.-armature winding must be very small and the loss in this winding will be found to be practically negligible.

FRICTION AND WINDAGE LOSSES

These losses are made up of:

1. Bearing losses.
2. Windage losses.
3. Slip-ring friction losses.
4. Commutator friction losses.

Items 1, 2, and 3 are in every respect identical to those in a standard induction motor.

The loss due to commutator friction, however, is inherent to this type of machine, but its value is small, as the commutator has relatively small dimensions and can be built with a small number of segments. Therefore, the friction loss due to the commutator may be considered as being counterbalanced by the decreased iron losses in this type of machine over those of a standard induction machine.

COMMUTATING LOSSES

These losses have been discussed in a previous chapter and were found to be small.

STRAY LOSSES

As a distribution of the magnetic fluxes and of the ampere-turns in this type of machine are absolutely identical to those in a standard induction motor, it follows that the stray losses are identical to those in an equivalent induction motor.

The above has shown that, generally speaking, the losses under full load on a Fynn-Weichsel motor and a standard induction motor must be very nearly the same.

From this it follows that both machines can be built in practically the same frame sizes for equal outputs and that, furthermore, the efficiencies of these new machines cannot differ materially from those of a standard induction motor. A further study of the relations shows that there is a tendency for the losses in a Fynn-Weichsel motor to be slightly heavier at fractional loads and slightly lighter at overloads than in a standard induc-

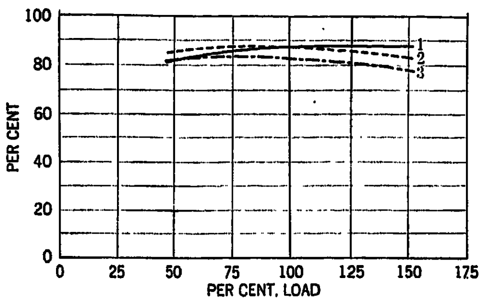

FIG. 25.—EFFICIENCIES OF 15-H. P., FOUR-POLE, 60-CYCLE, THREE-PHASE MOTORS

1. Fynn Weichsel 2. Squirrel-Cage 3. Slip Ring

tion motor. This difference in losses is the result of the difference in power factor under which these machines operate in comparison with induction motors for these extreme load conditions.

Having now discussed in general the theoretical behavior of this new type of machine, the actual behavior of the machine as found from tests will be of interest.

Fig. 25, Fig. 26, and Fig. 27 show the efficiencies of actual machines compared with those of standard in-

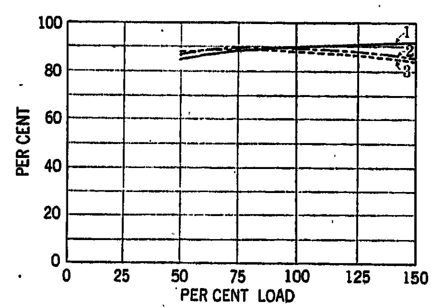

FIG. 26—EFFICIENCIES OF 30-H. P., EIGHT-POLE, 60-CYCLE, THREE-PHASE MOTORS

1. Fynn-Weichsel 2. Squirrel-Cage 3. Slip Ring

duction motors. It will be seen that these performances compare quite favorably.

Fig. 28 gives the performance of a 75-h. p. 1200 rev. per min. motor and also the performance of a 15-h. p. four-pole motor. These curves show that, similar to induction motors, the efficiencies increase with increasing h. p. capacities. They further show the general characteristic of the power-factor curve of this type of machine, which is somewhat leading at full load and becomes more leading at fractional loads. When the breakdown as a synchronous motor occurs, the power factor is near unity. After the machine has stopped operating as synchronous motor and continues to operate as induction motor, the power factor becomes lagging and reaches values similar to those of a standard induction motor. The efficiency curve of this part of the operation is also similar to that of a standard induction motor.

The change in operating characteristics of these machines under abnormal line and voltage conditions, such as high and low frequencies, follows very similar laws to those which are known to exist for standard induction motors. A detailed discussion on this sub-

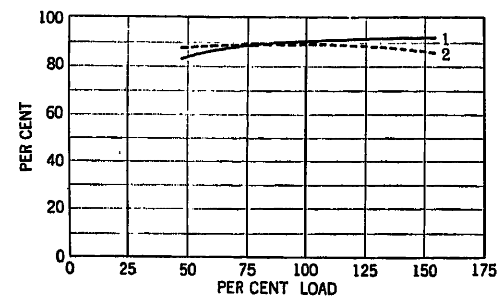

FIG. 27—EFFICIENCIES OF 75-H. P., SIX-POLE, 60-CYCLE, THREE-PHASE MOTORS

1. Fynn-Weichsel 2. Squirrel-Cage and Slip Ring

ject will be found in a paper read before the Franklin Institute, October 30, 1924, and also in the *Iron and Steel Engineer* of April, 1924.

As this new machine operates with leading power factor over its normal working range, it follows that it draws leading magnetizing current which will subtract

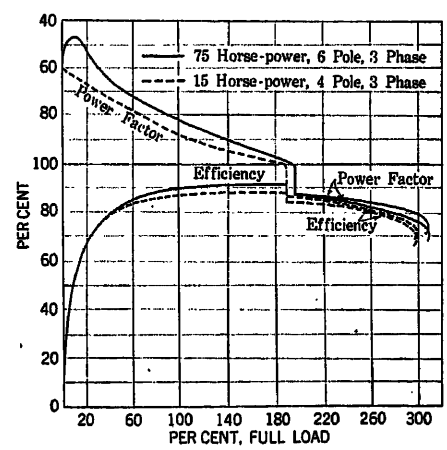

FIG. 28—COMPARISON OF POWER FACTOR AND EFFICIENCY OF FYNN-WEICHSEL MOTORS

from the lagging magnetizing current taken by induction motors, and if an installation consists of induction motors and motors of this new type, it is evident that the resultant magnetizing current of this installation can be greatly reduced. By properly selecting the relations of the Fynn-Weichsel motors and the induction motors, it is possible to obtain unity power factor

or any other desired power factor over the whole load range of the installation.

As the variation of the wattless leading current of a Fynn-Weichsel motor and of the wattless lagging current of an induction motor as a function of load follows similar laws, the power factor of the whole installation consisting of Fynn-Weichsel and induction motors will not alter materially with change in load. Tests have shown that, for instance, a 15-h. p. four-pole in-

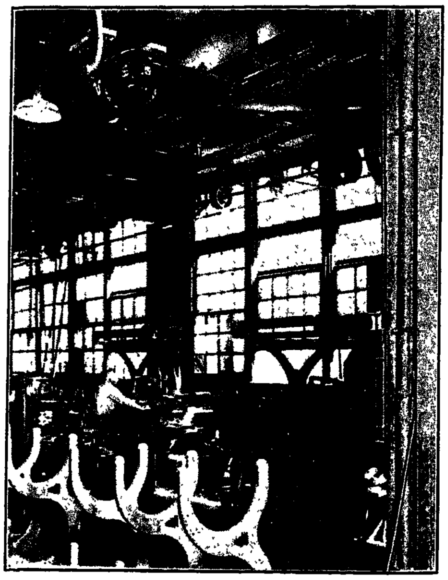

FIG. 29—15-H.P. MOTOR OPERATING WET GRINDERS

duction motor operating in parallel with a four-pole, 15-h. p. Fynn-Weichsel motor produces unity power factor over the whole load range when both motors are equally loaded as well as when both motors are loaded differently. Detail information on this subject will also be found in the above mentioned *Iron and Steel Electrical Engineer*.

APPLICATION OF THE MOTOR

From what has been said in the preceding paragraph, it follows that the most desirable manner of using these machines in an installation consists in the combination of these motors with standard induction motors and in selecting the ratios of the Fynn-Weichsel motor capacity to the remaining induction motor capacity in such a manner as to obtain the desired power factor in the installation. In laying out a new installation, it is quite simple to select this ratio in the desired manner.

In an existing installation, it is advisable to remove some of the induction motors, especially those which produce the largest magnetizing current, and to replace them by motors of the Fynn-Weichsel type.

In cases where old installations must be enlarged, it is possible to install, in the new section, Fynn-Weichsel motors, and in this manner to obtain the desired power-factor correction.

It will be found that an installation whose power factor is corrected in this manner as a rule requires less energy draw from the line than when the power factor correction is obtained by simply adding to the installation some idle-running corrective devices.

In Appendix No. 3 a calculation is given showing how much the efficiency of the Fynn-Weichsel motors can be below that of the replaced squirrel-cage motors and still give the same overall efficiency of the installation which would exist when the same installation has its power factor corrected by idle-running corrective devices.

These tables show clearly that the allowable decrease in the Fynn-Weichsel motor efficiency over the squirrel-cage motor efficiency is very large. But in a previous chapter, it has been shown that the efficiency of this new type of machine compares very favorably with the efficiencies of standard induction motors. Therefore, the overall efficiencies of an installation consisting of a combination of induction and Fynn-Weichsel motors, will be in most cases better than the overall efficiency of an induction motor installation corrected by idle-running corrective devices.

In closing, a description of a few actual installations will be given.

Fig. 29 shows a Fynn-Weichsel motor driving a set of grinders. This installation is particularly interesting, as the load on the grinders is such that very frequently the motor is greatly overloaded and thereby

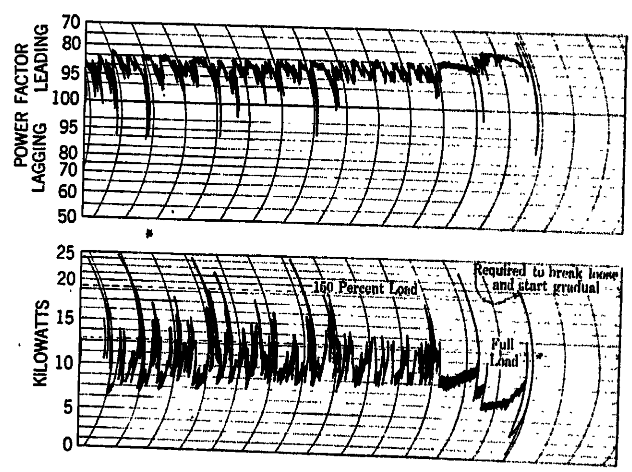

FIG. 30—POWER FACTOR AND KW. INPUT OF 15-H.P. MOTOR, OPERATING WET GRINDERS

is pulled out of step; and after the heavy overload is removed the motor immediately goes back into synchronous operation.

Fig. 30 shows the kilowatt input and power factor of this particular motor under the load conditions described.

Fig. 31 shows an installation where the motor operates a large drawing press. Here also heavy overloads occur extremely frequently.

Fig. 32 is a view of an installation in a marble cutting

plant. In this case, the constant speed of the machine has proven to be of exceptional value as the grade of polishing obtained is, to a large extent, dependent upon the uniformity of the speed with which this polishing operation is performed.

Another interesting feature of this installation is the large fly-wheel action of the grinding disks. It

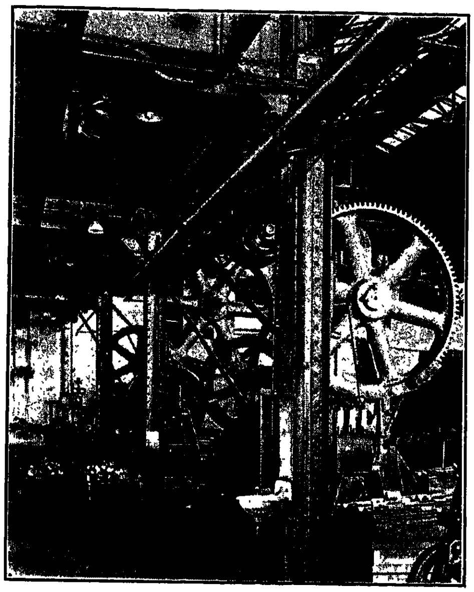

FIG. 31—7½-H.P. MOTOR OPERATING LARGE DRAWING PRESS

requires in the neighborhood of one minute to bring these large disks up to speed, and in spite of this heavy inertia, neither starting nor running difficulties of any kind have been experienced.

Fig. 33 shows a view of an installation in a cement

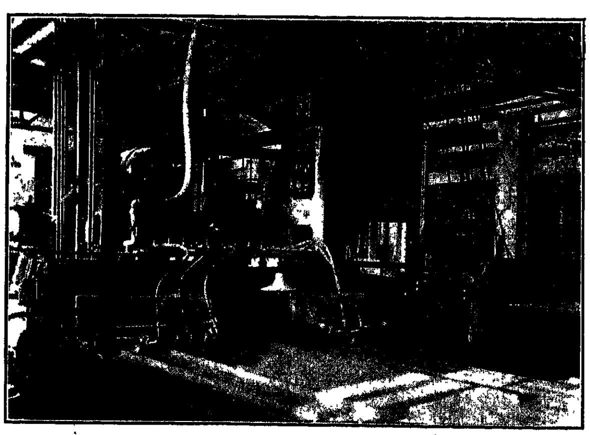

FIG. 32—75-H. P. MOTOR OPERATING MARBLE-WORKING MACHINE

mill. In this case the Fynn-Weichsel motor is operating sack-filling machines.

A great many other installations are in operation, such as motor-driven ice machines, compressors, machine shops, wire-drawing machines, and many other machines found in a variety of industries. It would lead too far to discuss in detail these installations.

Appendix No. 1

In the following, the derivation of the law is given which governs the torque of a polyphase machine at any instant. This law reads:

In a polyphase machine the torque at any moment produced by the current in a secondary phase is equal to the product of the instantaneous value of the current in this phase and the instantaneous value of an imaginary voltage whose magnitude is equal to the voltage induced in the secondary at standstill. This

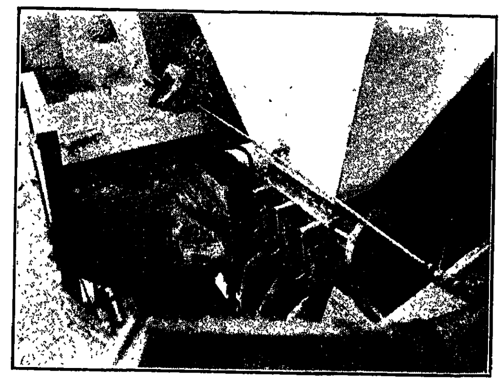

FIG. 33—25-H.P., SIX-POLE MOTOR OPERATING BAG-FILLING MACHINE IN CEMENT MILL

imaginary voltage has a frequency equal to the frequency of the secondary current. The phase relation of this voltage in respect to the secondary current is equal to the phase relation of the secondary current and the actual existing induced secondary voltage at the speed under consideration.

Use was made of this law in a previous chapter, in which the torque relations in the machine were discussed.

Assuming that the rotor is the primary member and the stator is the secondary member, and that the machine has neither ohmic resistance nor reactance in the primary, then the field produced by the primary

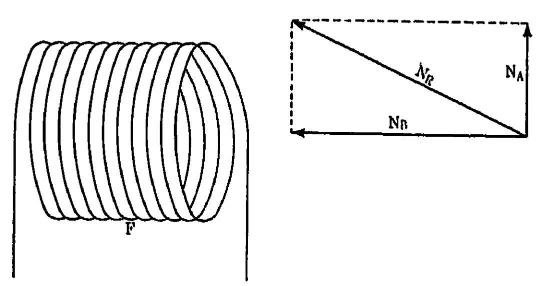

FIG. 34

has a definite magnitude and rotates in space at all rotor speeds except synchronous speed.

Fig. 34 represents the rotating field, $N\ R$, at a given moment. It can be decomposed into a component N_a, vertical to the axis of the winding F and the field N_b, parallel to the axis of the secondary winding F. The currents flowing in the winding F must produce a

torque with the field component N_a, because N_a and winding F, are at right-angles to each other, but F cannot produce any torque with the field N_b.

Both of these component fields, N_a and N_b, are alternating in time, but are 90 electrical degrees phase displaced from each other.

The torque produced at any moment by the interaction of the currents flowing in the F winding with the field component N_a is given by the following proportionality:

$$T_{F t} = i_{F t} \times N_{a t} \times s \qquad (1)$$

where $T_{F t}$ = the torque at any moment.

$i_{F t}$ = the current in the F winding at any moment.

s = the turns in the F winding.

$N_{a t}$ = magnitude of the field component in the A axis at the moment t.

Above it was shown that $N_{a t}$ follows the law:

$$N_{a t} = \overline{N}_a \text{ sine } (2 \pi f_x t) \qquad (2)$$

where $\overline{N}_a$ = the maximum value of the field component in the axis A.

f_x = the frequency of the flux N_a.

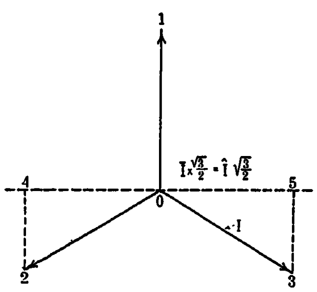

FIG. 35

This value, substituted in the equation 1 gives:

$$T_{F t} = i_{F t} \times \overline{N}_a \text{ sine } (2 \pi f_x t) \times s \qquad (3)$$

In the beginning it was shown that the maximum of the two components, N_a and N_b, are alike. Therefore, we can also write:

$$T_{F t} = i_{F t} \times \overline{N}_b \times s \times \text{ sine } (2 \pi f_x s t) \qquad (4)$$

The magnetic flux $\overline{N}_b$ can be expressed in terms of the voltage induced in the F winding when the armature is at standstill by:

$$E_0 = \overline{N}_b \times s \times 2 \pi f_0 \qquad (5)$$

where f_0 equals the frequency of the flux N_b with armature at standstill.

By solving the above equation for $\overline{N}_b$ and substituting it in equation No. 4, it follows:

$$T_{f_t} = i_{f_t} \frac{E_0}{2 \pi f_o} \text{ sine } (2 \pi f_x t) \qquad (6)$$

This equation is nothing more than a mathematical expression of the law stated in the beginning of this appendix. The correctness of this law is proven thereby.

Appendix No. 2

A statement has been made in the main article that, for otherwise equal conditions, the losses in the d-c.-exciting-winding are equal to those of a polyphase winding if the ampere-turns vectors in both cases have the same magnitude. This can be shown as follows:

Fig. 35 shows vectorially the currents flowing in the three-phase secondary of an induction motor. At the moment when one of these currents, 0-1, is zero, the currents in the two other phases have a value corresponding to their projections on the axis 0-4, and are equal, their numerical value being:

$$\hat{I} \,.\, \sqrt{3/2}$$

If the resistance of one phase is given by r, then the loss in these two windings at this moment is given by $\hat{I}^2 \times 3/2 \times 2 \times r$, but this is equal to the polyphase current loss in all three secondary windings.

If the condition for the moment given above for which the current 0-1 is zero is imitated by sending direct current through the two other phases, and selecting the direct-current so that it is of the same magnitude as the polyphase currents at the moment under discussion, then the loss must be equal to the total loss in the three-phase winding. As the magnitude of the rotating field is assumed to be of constant value, it follows that a d-c.-winding producing the same magnetization as a polyphase winding causes, under otherwise equal conditions, the same amount of loss.

A similar train of thought proves that the same relations also exist if standard two-phase windings or if any other combination of polyphase windings are used.

The copper loss due to the d-c. exciting current in a synchronous motor is, therefore, equal to the copper loss of the secondary winding of a polyphase motor provided both windings are so selected as to produce the same magnetic flux, *i. e.*, the same ampere-turns, and further assuming that both windings have the same mean-turn length and the same copper section per slot. But if the d-c. winding is so constructed as to have a smaller mean turn length and a larger copper section than the polyphase winding, then the d-c. exciting loss is less than the loss due to the polyphase exciting ampere-turns.

Appendix No. 3

An installation assumed to operate with a certain power factor and current draw, is represented in Fig. 36 by the vector $O\,I$. If this installation has its power factor corrected by having an apparatus operated in parallel which produces a leading current, $I\,C$, then the phase displacement of the resultant current is given by the vector $O\,C$ in respect to voltage vector $O\,E$.

This diagram assumes that no losses whatsoever occur in the idle running phase correcting device. In reality, a certain amount of loss exists in the phase correcting device and its loss may be represented by the line $C\,D$. The real phase displacement of the resultant current in this installation would then be given by the angle between the line $O\,D$ and the voltage

$O\,E$, but this phase displacement is so nearly equal to the phase displacement between vector $O\,C$ and $O\,E$ that no difference between these two phase displacements will be made in the following discussion. The loss in the phase-correcting device is given by the line $C\,D$, which is called a. The relation between kv-a.-input and loss in the phase correcting device is given by the expression: $a = p \times R.\,K.\,V.\,A. = p \times C$, where C equals length $C\,I$, and p is a constant.

If the same correction is obtained by removing a number of induction motors and replacing them by Fynn-Weischsel motors which have a leading phase displacement ψ_F, then the vector diagram is given by the lines O-3-C. This assumes that all induction motors operate with a power factor equal to the power factor of the installation and that the Fynn-Weichsel motors have the same efficiency as the induction motors which they replace. The kw-input of the Fynn-Weichsel motors under this assumption is given by the vector 3-1.

In order to operate this installation with the same overall efficiency, *i. e.*, the same total losses as the first mentioned installation, the power factor of which was corrected by idle-running-phase corrective devices, the input to the Fynn-Weichsel motors for the same output as before must be represented by the vector 3-2, where 1-2 is equal to $C\,D = a$.

Let 3-1 be equal to b and I-1 equal to k, and 1-C equal to m.

The efficiency of the replaced induction motors is given by $\eta_1 = \dfrac{\text{output}}{b}$. The efficiency of the Fynn-

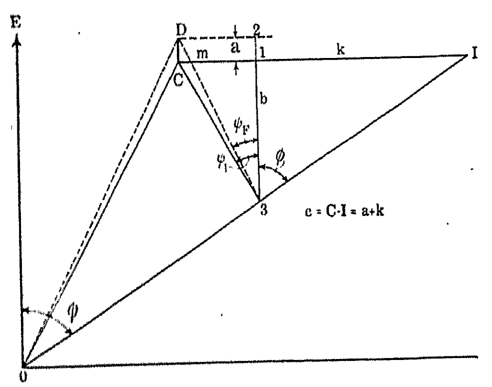

Fig. 36

Weichsel motors, in order to obtain the same overall efficiency as in the first mentioned installation, is given by $\eta_2 = \dfrac{\text{output}}{b + a}$. From this it follows that the ratio of the efficiencies is equal to:

$$\frac{\eta_2}{\eta_1} = \frac{b}{b+a} = \frac{1}{1 + \dfrac{a}{b}}$$

As $a = C \times p$, it follows that $\dfrac{a}{b} = C \times \dfrac{p}{b}$, and from the diagram it is known that $C = m + k$.

$$\text{Therefore, } \frac{a}{b} = \frac{C \times p}{b} = p\left(\frac{m}{b} + \frac{k}{b}\right)$$

$$\frac{m}{b} = \text{tang. } \psi_F \text{ and } \frac{k}{b} = \text{tang. } \phi.$$

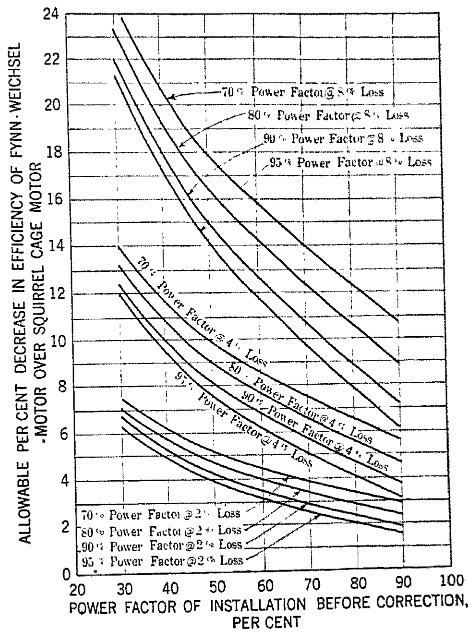

Fig. 37

This introduced in the above equation gives the relation:

$$\eta_2 = \eta_1\left(\frac{1}{1 + p\ (\text{tang. } \phi + \text{tang. } \psi_F)}\right)$$

This is a correct equation if the angle ψ_F is introduced, but a small error is made if this angle is replaced by the angle $\psi_F\epsilon$. An error made in this manner has the tendency to make the allowable efficiency calculated by this equation somewhat smaller than the real allowable efficiency. This error is, however, very small.

By aid of this equation, the curves in Figs. 37 and 38 have been calculated and the allowable drop in efficiency of the Fynn-Weichsel motor below the efficiency of the replaced squirrel-cage motor, has been plotted as ordinates and the power factor of the installation before corrective features were added, has been plotted as abscissa.

Families of curves have been calculated, assuming the Fynn-Weichsel motor operating at leading power factors of 70, 80, 90, and 95 per cent. The curve sets refer to a 2, 3, 4, 6, 8, and 12 per cent loss in the idle-running corrective device which produces the same total power factor as the Fynn-Weichsel motor installation. The use of these curves is as follows:

Assuming an installation with a power factor of 60 per cent. The power factor of the installation is to be improved to any desired value by the use of Fynn-Weichsel motors which operate with a leading power factor of 80 per cent.

The allowable decrease in efficiency of these motors below the efficiency of the replaced squirrel-cage motors is 5.9 per cent., as shown by Fig. 38, provided the overall efficiency of the installation is the same as if its power factor were corrected by the use of an idle-running phase correcting device which consumes 3-kw. for every 100-kv-a. reactive furnished.

If the idle-running phase correcting device has a 12 per cent loss and the conditions otherwise remain the same, then the efficiency of the Fynn-Weichsel motors could be 20 per cent below the efficiency of the replaced squirrel-cage machines.

These curves have been derived under the assumption that the induction motors which were replaced by Fynn-Weichsel motors operate with a power factor equal to the average power factor of the installation. In most cases, however, it will be found that, in an actual installation, some motors operate with better and others with poorer power factor than the average power factor of the installation.

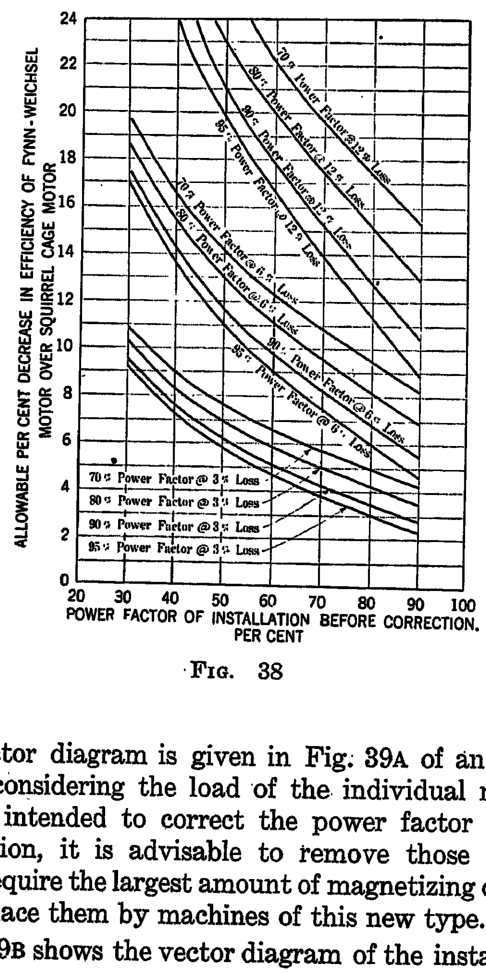

FIG. 38

A vector diagram is given in Fig. 39A of an installation, considering the load of the individual motors. If it is intended to correct the power factor in this installation, it is advisable to remove those motors which require the largest amount of magnetizing current and replace them by machines of this new type.

Fig. 39B shows the vector diagram of the installation after the induction motors requiring the largest magnetizing current have been replaced by the new type of motor. It will be seen that the capacity of the Fynn-Weichsel motors required in this case is considerably smaller than the capacity of Fynn-Weichsel motors required when all induction machines of the original installation operate at a power factor equal to the average power factor of the installation.

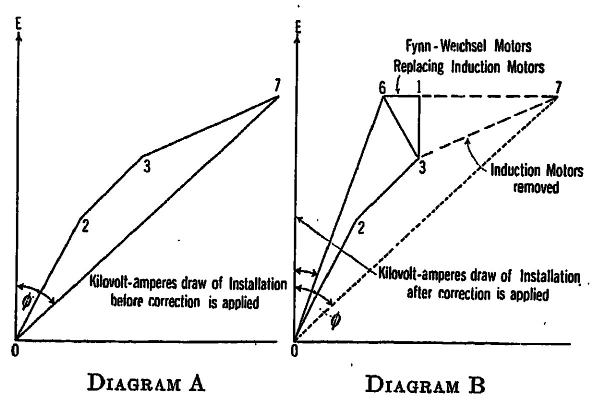

FIG. 39

From this it follows that in most cases the allowable decrease in the efficiency of the Fynn-Weichsel motors, below that of the replaced induction motors can be considerably larger than found from the curves given in Figs. 37 and 38.

For instance, if, due to the replacement of the motors which have considerably poorer power factor than the average power factor of the installation, the size of the Fynn-Weichsel motors has been reduced to one-half the value required if all motors have a power factor equal to the power factor of the installation, then the allowable drop in efficiency is twice the value which is found from the curves in Figs. 37 and 38.

In addition to this, it must not be overlooked that if the power-factor correction is obtained by replacing some of the induction motors by the new type of machine, it is possible to operate a large percentage of the wiring in the installation at unity power factor or thereabout. In this manner, the copper losses in the wiring system of the installation itself are quite materially decreased, a condition which does not necessarily exist in cases where the power-factor correction is accomplished by idle running phase-correcting devices.

A measure of the kv-a. rating of the required Fynn-Weichsel motors and of the idle running corrective device needed for accomplishing the same power-factor correction, is given by the vectors 3-6 and 6-7, respectively.

The cost of any electrical machine is a function of its kv-a. rating. Therefore, the length of the vectors 6-3 and 6-7 respectively can be readily used for determining the cost of a Fynn-Weichsel motor equipment and a corresponding idle-running corrective device.

Discussion

C. F. Scott: It happened to be my lot to be associated with Mr. Tesla in his early work in the development of his polyphase motor. I remember very well his statement that his motors were of two kinds: the synchronous motor, a splendid motor to run; and the induction motor, which he called a torque motor, that would start. The difficulty with the synchronous motor was first to get it started and second to excite it. The difficulty with the induction motor, primarily, was the lagging or magnetizing or exciting current which it required. That lagging current was a mysterious sort of thing; it was the practise to attribute anything we didn't understand in those early days to "lag."

A recent letter from Mr. Weichsel said that a score of years ago, when we were together in the Westinghouse Company in Pittsburgh, he attended one of the lectures that I gave to the students and remembered how I used yellow chalk horizontally for one kind of current and red chalk vertically for the other kind, and it gave him a clearness of conception of what was going on in circuits and an interest in it which continued. So maybe I can claim a sort of fatherly connection to the new motor, which is really a combination in one structure of those early beginnings which Tesla described as the torque motor and the synchronous motor.

The new motor has some rather striking and commendable features. The general simplicity is notable. The motor combines the starting characteristics of the induction motor and the running characteristics of the synchronous motor. It is two motors in one. It is self-contained; it has no outside exciter. There is an automatic transfer from one function ot the other, without any action of the attendant. It is simple in construction, with very simple additions or modifications to the regular induction motor.

In its construction, in its auxiliaries, in its operation, it is a simple and admirable machine, and to those of us who, back before these things were evolved, have contemplated the difficulties in the problem, and the great desideratum in getting a combination of these two motors in one simple arrangement, this solution is a most delightful one. From the engineering, inventive standpoint, it is a fine thing. And, to get a performance which is substantially that of the induction motor and of the synchronous motor, with some advantages in connection with each, is a splendid result.

W. L. Upson: This paper is a discussion of the design and operating characteristics of the now well-known Fynn-Weichsel motor and does not particularly go into the question of the demands for a power-factor-correcting motor. This latter subject has been quite fully discussed elsewhere. Fortunately this motor has now been in service long enough to establish its ability to do what is claimed for it and to demonstrate that what might seem like complications of structure are really of insignificant importance. Certainly any quantity of electrical apparatus containing as many or more complicated features is in constant use and accepted without question in practise. However, it is of interest to note that of these so-called complications the commutator, for instance, is actually much less of a problem in this motor than it is in other machines in general.

To my mind, by far the most interesting feature of the design of this motor is its small air-gap: a synchronous motor with an induction-motor air-gap. For some years I have been an advocate of smaller gaps and have felt that it was possible for the designer to obtain substantial advantages by working in this direction. In this connection I wish to quote from a discussion by Mr. H. M. Hobart contained in the TRANSACTIONS of the Institute, Vol. XXXII, p. 1595, 1913.

"Any proposition to consider the design of synchronous motors along the lines of the design of induction motors has always been handicapped by the necessity of a change of hands, as to who should design it, and bring about the evolution of the synchronous motor into a decent machine. It is at present an absurd caricature of what it might be .... I believe that the synchronous motor can be used to great advantage in much smaller sizes than has heretofore been considered desirable, in sizes which will lap over into the field that has been generally held by common consent to belong to the induction motor. If only the synchronous motor could be designed by induction-motor designers, working on the lines which have enabled them to see just what is needed for these starting and running-up conditions, the result would be for the good," and Mr. F. D. Newberry, taking part in the same discussion, admits the unsatisfactory development of the synchronous motor but imputes it to "the difference in the magnetizing current required by well a designed induction motor and a well designed synchronous motor."

It is true that a salient-pole synchronous motor would have an advantage over one with a round-rotor field providing both had the same air-gaps and both were of standard design, but in this new motor we have the restricted gap which, apparently for the first time, fulfils the desire expressed by Mr. Hobart, and in addition we have a self-exciting feature which practically overcomes the disadvantages usually encountered due to armature reaction. It would therefore seem highly desirable if we could have a comparative study of a Fynn-Weichsel motor and a standard salient-pole synchronous motor. The former exhibits such remarkable synchronizing power that it would appear that this feature might be made the basis of such a comparative study. It would be interesting to compare these motors on the basis of weight for equal capacities. On the basis of efficiency, the Fynn-Weichsel motor has every advantage, even that of commutator losses, if we take into consideration, as we should, the source of d-c. supply required by the standard synchronous motor.

There is one other feature I should like to mention, and that is that this motor requires a somewhat longer shaft between bearings than do other motors, and with the small gap, this becomes a feature in the mechanical design of considerable importance. The rotor must be truely centered, the shaft must be stiff and the bearings must not be subject to wear. These conditions might naturally be expected to add somewhat to the cost of the motor. However, there seems to be no good reason why they should not be met.

R. E. Ferris: With the operating men with whom I have talked, the cost of maintenance and continuity of service is a very important factor, of first importance, you might say. Therefore, it seems to me that all complications possible should be omitted, even at the expense of slightly reduced desirable operating characteristics.

There is one thing that has been introduced in this motor, and that is a double winding. As a d-c. designer, I have tried to avoid double windings consistently. When we got into the higher voltages, we were almost of necessity driven to double windings, especially on machines of lower capacity. However, I have even gone to the extent of designing two separate armatures, separating the commutators, and in that way the windings, in order to avoid, what seemed to me, the complication of a double winding.

L. M. Perkins: The Fynn-Weichsel motor or any high-power power-factor motor must cost more than a simple induction motor. It cannot be made to cost less, because it is exactly like the induction motor, with the addition of a commutator and extra brushes and winding.

The higher energy cost is inherent in the high-power-factor motor. If the motor has a high power factor, it means that the secondary current is thrown out of phase, very decidedly, with the field flux of the motor and, therefore, the torque for a given current and a given field is markedly decreased. The higher the power factor, the more this torque will be decreased. In addition to this, of course, there are the commutator and brush losses which again decrease the efficiency.

In his paper, Mr. Weichsel brings up a comparison of efficiencies or copper losses of the Fynn-Weichsel motor as against the plain induction motor. This appears in Fig. 24. While D is the primary current of the ordinary motor and C is the primary current of the Fynn-Weichsel motor, D is the current which flows in the stator winding and, therefore, the large winding of the normal motor, while C is the current which flows in the rotor winding or small winding of the Fynn-Weichsel motor. On the other hand, B is the current which flows in the rotor winding of the ordinary motor, while A is the current which flows in the stator winding of the Fynn-Weichsel. Therefore, A and D should be compared while C and B are also compared. C and B cannot differ very much, but A is much larger than D, and therefore entails much higher loss.

In addition to that, the point made about the concentric winding used in the stator of the Fynn-Weichsel motor can also be applied, of course, to the induction motor which can also use a concentric winding.

Further than this, the examples chosen, comparing the Fynn-Weichsel motor with the synchronous condenser, are not the most practical conditions because the Fynn-Weichsel is, in general, built in smaller sizes, and in those smaller sizes the commercial comparison will be made not between the Fynn-Weichsel and the synchronous condenser, but between the Fynn-Weichsel and the static condenser which has very low losses. For this reason, the Fynn-Weichsel motor cannot have a greater loss than the equivalent induction motor, without suffering a loss of efficiency of the system.

W. C. Kalb: In considering a motor of the type described by Mr. Weichsel there is one factor which should not be lost sight of, and that is the peculiarity of the operating characteristics of the motor when meeting overload conditions. On certain applications this represents a distinct advantage and may give the motor a preference over other types.

As a specific case, I have in mind a certain mill where it is essential that a product be ground to uniform mesh. The objection to an induction motor is that this apparatus is operated by unskilled labor, incapable of judging its operation by watching any form of indicating meter, and that the fineness of the material produced varies with the speed. As the mill becomes overloaded by too rapid feeding on the part of the operator, the drop in speed is gradual; it does not call itself to the attention of the operator by a change in tone, and variation in mesh results. The synchronous motor would be ideal from the standpoint of uniform speed, but the objection to it is that when the overload point is reached, the motor drops its load and the material circulating through the separating system drops back into the mill, stalling it completely, and making it necessary to open the mill and remove the charge.

With the peculiar characteristics of the Fynn-Weichsel motor, when this overload condition is reached the motor drops into induction operation at a sufficiently rapid rate so that the change in tone of the mill is noticeable. The operator at once recognizes that his machine is overloaded, ceases feeding until it has time to clear the load, and then proceeds without interruption and with but a momentary disturbance of the uniformity of his product.

F. G. Baum: The operating men and the designing engineers know that the induction motor as it is today and will probably continue for a long time is what might be called "the brute of the electrical system." That is, the induction motor not only throws on the kilowatt-hour load, but throws onto the system a kv-a. load which pulls down the voltage of the system. That burden that it throws onto the system, let us say, by making a power factor 0.80 in place of 1.00, may increase the current 25 per cent, which may increase the losses, say, 50 per cent in our transformers, in our transmission and in our generators. We have then to take account of the losses all the way through to the power station.

In the design of the generators, the worst thing that the generator designers have had to contend with in the last twenty-five years has been this question of the power factor. Low power factor not only adds a burden all the way through but we must carry probably 50 per cent higher excitation on the generators than we would for unity power factor. For example, we may have the generators excited for 100 amperes at open-circuit voltage, 200 amperes for unity power factor, and 300 amperes for power factor of 0.80. The high field called for by the low power factor is a menace to the system and it is, you might say, a pointed gun presented to cause trouble in case anything happens.

Anything that will tend to correct that, of course, will be beneficial. I have for years been hopeful that the synchronous-motor designers would design small units, and I believe they are doing that more and more. I think we are going to see more of that done in the future, for I believe we would have an entirely different kind of power system if we could get rid of this "brute" action of the induction motor on the power system. Such work as Fynn and Weichsel are doing is therefore of general interest to the electric power industry.

R. E. Doherty: I think there is no question whatever that in those cases of application in practise where the particular characteristics which these motors have are required and where the economics of the situation justify the investment, they have a real field. Those facts will determine, of course, the extent of the application.

With reference to the historical sketches of Professor Upson, which I believe dated back to 1913, I would call attention to the fact that very material progress has been made in the design of synchronous motors since that date. Whether in the future the synchronous motor is going to be further developed and this question of power factor solved by a simplified synchronous motor, or whether it is going to be solved by some form of commutator motor, will depend altogether upon the economic factors in the situation and the requirements of the loads.

A. M. MacCutcheon: I would like to ask Mr. Weichsel how the resistance is automatically cut out in starting this motor.

It seems to me that this type of motor surely has its place. I think Mr. Weichsel said that the cost of the motor was some 15 per cent, on the average, over that of a slip-ring motor of equal capacity. We all appreciate the increase in the price of a slip-ring motor over the very simple squirrel-cage. We appreciate that there are some disadvantages to the commutator, and to the extra windings. As a previous commentor has said, we must equate between the additional cost and the additional advantages. I suggest as a new idea that if we are going to go to the commutator, possibly we can correct the power factor either by a large size motor of this type or by a synchronous motor, if that is more economical, driving a direct-current generator and have a certain number of direct-current motors in a plant with all their consequent advantages.

Some eight years ago, I think, a good many felt that the day of the direct-current motor had passed. If I interpret the tendency in the commercial field today aright, there is a very decided tendency to use both alternating and direct current in any large plant as the ideal system. There are still many things that can be done with the direct-current motor which cannot be done with the very excellent Fynn-Weichsel motor. Therefore, if we increase the power factor by some form of large unit, either a Fynn-Weichsel motor or a synchronous motor, which could be easily maintained and inspected, it might be more economical than a large number of Fynn-Weichsel motors distributed throughout the plant, and we would have direct-current as well, with its very obvious advantages.

C. F. Scott: I don't know that we all recognize the fundamental basis of this discussion on motors. It happens that Faraday and Henry when they invented electromagnetic induction put in two things: motion, and magnetic field. Those two things—the motion and the field—for producing electromotive

force are the fundamentals of our electromagnetic machinery. Without the field, a generator or a motor is helpless.

We have been much troubled about the field. Alternators have exciters as a matter of course. The motor, too, must have its field and the question is: Can we produce that field more economically locally at the motor by having permanent magnets, by providing direct current for excitation from a battery or an exciter, or can we bring the exciting current in the form of alternating current (lagging or "wattless") from our alternator which supplies the in-phase power current? If we take magnetization for the motor from the alternator, we subtract from its magnetization and we must, as Mr. Baum says, put more d-c. excitation into the generator. We must produce somewhere the excitation for every machine in the system. One way is to produce all the excitation (as well as all the "motion") back in the power-house by putting in a bigger exciter and supplying magnetizing current through alternating mains. Another way is that of the motors described today, in which the commutator makes the motor self-exciting.

We are content to supply the motion, the power, the turbine, but everybody thinks it is all wrong that we should have to supply the magnetization. If Faraday and Henry had done differently we might do differently too, but as things stand we must supply both.

C. H. Sonntag: The Portland cement industry, with which the writer is identified, is one of those in which efficiency and high power factor in power transmission have been somewhat sacrificed to secure the greatest possible continuity of mill operation. This is particularly true in the case of the smaller motors. The tendency in recent years towards larger grinding machines has carried with it the demand for larger motors, which for application to individual machines now range in size from 75 h. p. to 500 h. p. Of these, the smaller ones are of the slow-speed squirrel-cage type, usually running at about 500 rev. per min. and so having only a moderately high power factor, while the large motors are usually of the synchronous type, which can be operated with leading current.

If these were the only motors to be run, the power factor of the system could be kept at a very satisfactory point. Unfortunately a cement mill needs a large number of small motors to drive conveyors, elevators, packers, kilns and other necessary machines. These motors will range in size from 5 to 25 h. p. or more, and to avoid excessive speed reduction, will run at a moderate speed—say about 700 rev. per. min.

The effect of these small slow-speed motors on the system power factor would be bad enough, even if they were fully loaded. But the cement-mill operator has learned from experience that such drives are frequently heavily over-loaded, due to slides of cement, accidental or necessary stoppages, and the general tendency of unskilled help to overload equipment. It is a peculiar fact that a screw conveyor handling cement, ground limestone or similar material will carry a very large load without excessive power demand as long as the material is kept moving, for the air that is mixed with the powder makes it almost as mobile as a liquid. But if the loaded conveyor is stopped for a few minutes, so that the contents have a chance to settle and pack, it will be found impossible to start it, if of any length, with a motor that is only large enough to run it as long as it is in motion. Such drives are usually over-motored at least 50 per cent, sometimes more, and the effect on the power factor can be imagined.

The cement manufacturer would welcome some way to correct for the low power factor of these small drives, and this way is now offered through the use of the Fynn-Weichsel motor that has just been described. What is needed is a machine that is reasonably simple so that the brutal treatment it will receive, and the constant presence of cement dust in the air will not put it out of business, and that does not require separate excitation. The Fynn-Weichsel motor is, in the writer's opinion, such a machine.

Some may think that the presence of a commutator makes this motor undesirable for use in dusty places. So far as the dusts found in cement-mill practise are concerned, this conclusion is not borne out by the facts. The commutator, instead of being cut and scored away by the dust, is given a high polish, and stays in excellent condition. When electric drive was first introduced into cement-mill practise, it was by the use of direct-current motors, and the excellent records made by the commutators of these old machines are still a matter of comment by those who were familiar with them.

Recently the writer had an opportunity to made a complete graphic record of the performance of one of the department circuits of a cement mill, on which were four 25-h. p., 1200 rev. per min., Fynn-Weichsel motors, together with seventeen squirrel-cage motors ranging in size from 50 to 5 h. p., most of them only half-loaded, and running at 690 rev. per min. The circuit was at 440 volts, three phase, 60 cycles. The Fynn-Weichsel motors were each driving, by direct connection through a flexible coupling, a three-tube Bates packer, which is a machine for filling bulk cement into bags. The average load on each was about 20 h. p. The other motors were driving screw conveyors, elevators, dust-collecting fans and a bag-cleaning wheel.

The sections of simultaneous charts from the graphic wattmeter and power factor meter shown herewith in Figs. 1 and 2 give a very good idea of the influence of the Fynn-Weichsel motors on the power factor of the circuit. The entire record is too long to show satisfactorily, so only the important parts are exhibited. The curves are somewhat irregular, because the entire equipment was working under commercial rather than laboratory conditions.

A number of squirrel-cage motors were started in order to get enough current through the power-factor meter to insure positive operation. The first was a 50-h. p. fan motor, partly loaded with a power factor of about 62 per cent. As smaller lightly loaded motors were started, the curves show very plainly that while the power demand increased, the power factor went progressively down until it went below 50 per cent, which was as low as the meter would register. The pen was against the stop when it drew the straight line at the point *D*. The indications on the wattmeter charge should be multiplied by 200 to get correct values.

At *E* the first Fynn-Weichsel motor was started, and the immediate increase in power factor from less than 50 per cent to about 78 per cent is very evident.

At *F* more small motors were started, bringing the load to 78 kw. and the power factor to 72 per cent.

At *G* another Fynn-Weichsel motor was started, bringing the power to 96 kw., the power factor to 83 per cent.

The load was further increased by adding two more Fynn-Weichsel motors and three small ones, with a final load of 125 kw. and a power factor of 83 per cent, which is excellent considering the underloaded condition of most of the small motors. Some of these are not needed continuously, and the effect of shutting them down is shown in Fig. 2.

At *P* a 10-h. p. squirrel-cage motor is shut down. The power falls to 118 kw. and the power factor rises to 88 per cent.

At *Q* another 10-h. p. squirrel-cage motor is stopped, the energy dropped to 110 kw., and the power factor rising to 94 per cent.

At *R* and *S* stopping two 20-h. p. and two 10-h. p. squirrel-cage motors drops the power to 63 kw. and raises the power factor to 84 per cent *leading*. At this point four Fynn-Weichsel motors and a few small ones were still running, but the load on the Fynn-Weichsel motors was falling off.

At *T* a Fynn-Weichsel motor was stopped and the power dropped to 55 kw., the power factor changing also to 88 per cent leading.

At *U* another Fynn-Weichsel motor was stopped, power falling to 39 kw. and power factor to 95 per cent leading.

Beyond this point the indications of the power-factor meter were not dependable, owing to the small current flowing in it. Taken in their entirety these charts show that the Fynn-Weichsel motor is of very real value in counteracting the poor power factor of underloaded slow-speed induction motors. Where conditions permit, this correction may be carried to the point where the resultant power factor makes the load a desirable one either for the central station or the isolated plant.

P. H. Thomas: While there can hardly be much new to be said at the present time on the subject of power-factor correction in industrial power-supply circuits, it still may be worth while to point out the fact that the indicated development of our power-supply systems is likely to throw a somewhat different emphasis on the importance of lagging current.

It goes without saying that considering the field in general, correction. The type of motor advocated by Mr. Weichsel will admirably meet many cases; sometimes the power factor may be better corrected through large synchronous motors. In this case the additional cost of the special apparatus must be balanced against any saving that can be made in the necessary correction in generators and feeders.

In those cases in which this correction must be made by installing additional apparatus, such for example, as additional generators or synchronous condensers or their equivalent, the advantage of using motors of the type proposed by Mr. Weichsel becomes very greatly enhanced, for the installation of new capacity in generating apparatus or condensers is a very different thing from operating existing machinery at a lower power factor. In other words, up to a certain point there is very little to be gained by the more expensive, higher-power-factor motor and

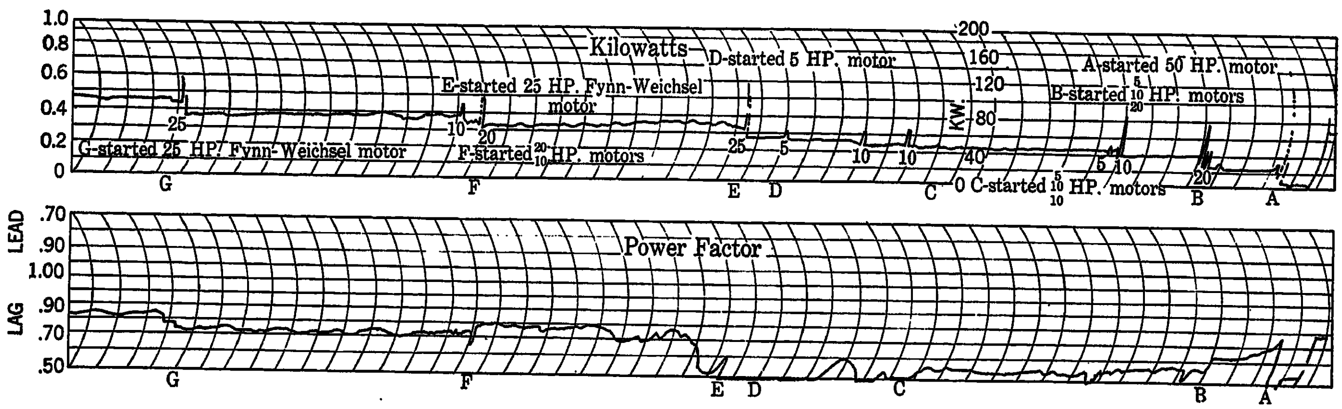

Fig. 1

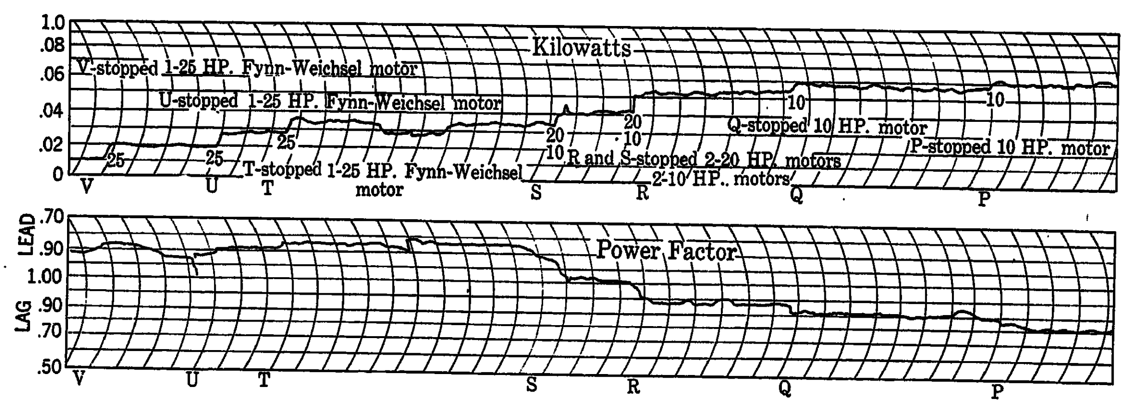

Fig. 2

Figs. 1 and 2—Load and Power Factor Charts of Mill having Fynn-Weichsel and Squirrel-Cage Motors

each actual situation must be considered by itself and they range all the way from conditions where power-factor correction is of no value to those in which power-factor correction is all-important.

Where the effect of lagging current is merely to lower the power factor of the load of a generator, leaving it still well within the proper operating range of the generator and where regulation and losses on transmission lines are not deleteriously affected, there is very little warrant for the additional expense or the lower efficiency of special forms of induction motors. This is because the use of such motors will not reduce the initial cost of the system or its operating expense. In cases, however, where the amount of lagging current is sufficient to load the generators beyond the safe current-carrying capacity of armature windings, or where the regulation is adversely affected, or in those cases where the local feeder voltage drop or line losses become excessive, power-factor correction at the load end of the feeders becomes worth while, but there is always a question as to the best method of making the other expedients may be cheaper; beyond this point the importance of the high-power-factor motor may become emphasized many times over; other remedies then become very expensive.

I would like to point out further that as interconnection and interchange of load forward and backwards between power systems grow, the situation is likely to call for high-power-factor load with considerable emphasis. To pass power forward and backward over the same line there must be a control of the power factor and if this power for both ways is anything like equal in volume it will be necessary to have it pass at leading power in one or both directions. Obviously this condition of leading power factor can be obtained only by providing means locally for carrying all of the lagging current of the local load and in addition supplying whatever additional leading current may be required. In this case the expense of bad power factor is very great because it must be corrected by carrying the lagging current on rotating

machines, at the same time carrying the load received over the interconnecting line.

As it is not possible to read the future of any particular system very far ahead, it may well be the part of wisdom to establish the policy of improving power factor of the general load from time to time as far as may be reasonable so that when the time comes when the high power factor is essential it will not be unduly expensive to secure it.

This brings forward another aspect of this matter. Since it is equitable and necessary that the purchasers of power shall ultimately pay the entire cost of furnishing the power plus a proper return on the investment and since bad power factor tends, in such cases as I have outlined, to increase very materially the cost of installation and to jeopardize the character of the service, there should be a premium in some form on a high power factor for consumers. At the present time in most cases while it is for the interest of the industry as a whole and the users in the aggregate that a good power factor should be established on a system, it is not to the interest of any individual consumer of power to have a high-power-factor load for it costs more money for him to get it and his individual bad power factor will not effect the cost of power as a whole enough to effect his rate. In other words if he establishes a high power factor he will make a benefit to the industry in general but will not improve his own rate for power. If these rates be so adjusted, as they are in some places, as to make a saving for individual consumers to establish high power factors, the burden of producing this general high-power-factor condition, which is good for the general industry, will be distributed over all the consumers of power in a more or less fair proportion. It seems to me that this is perhaps the most important aspect in the present discussion of the power factor.

G. S. Smith: The paper presented by Mr. Weichsel gives a very thorough and enlightening analysis of this new development in the line of motors. However, like most new developments, it may require some education on the part of the buying public as well as an added incentive from the power companies, before its true worth is realized.

It is needless to say that most plants using electric power are over-motored though not always without good reasons. However, there is a strong tendency for the superintendent in charge to favor a much larger motor than necessary, to avoid operating troubles, since the motor often gets less attention than the remainder of the machinery. Outside of the added investment in the first cost of the motor the power consumer is suffering no great loss unless he is penalized for the resulting poor power factor.

Since over-motoring a plant is often desirable, if not necessary, there is little doubt but that the future will see a great need for more power-factor correction, and this motor ought to supply that need since it has many desirable characteristics together with all the ease of starting found in any wound-rotor induction motor.

A series of demonstration tests were run on a 15-h. p. motor at the University of Washington, and the operation of the motor was found excellent. Its various characteristics were checked, and a number of oscillograms taken showing starting as well as various changes in operation from synchronous to induction operation and the reverse. The ease with which it synchronized even at high overloads, due to the so-called injected current, seemed the most remarkable part.

It might be desirable to afford some means of easily adjusting the power factor at which the machine operates after it is installed. Such an adjustment should be simple, though it might not be justified since every added adjustment usually means an added possibility for trouble.

There seems to be a decided fluctuation in the a-c. current drawn by the machine when it changes to induction-motor operation on overload. This, of course, is to be expected after the function of the injected current due to slip is understood, though it is not altogether desirable. The oscillogram in Fig. 3 herewith, shows this current variation and its relation to the induced currents in the two stator fields. The fluctuation indicated on the oscillogram is probably greater than would ordinarily take place since the change in load was made quickly in order to reduce the time and obtain a good record of the values both before and after the change. This doubtless resulted in some hunting of the larger machine used as load. However, the load at which it drops out of step is so high that it would seldom operate thus.

Inquiries have been made by engineers in this territory as to its operation as a generator when driving torque is applied to its shaft. It might thus be used to develop small water-power cities, floating on the line at periods of low water, and run as a self-excited alternator when water is available. Its power factor for both periods could probably be kept near to unity, or leading.

Some tests were run on a 7½-h. p. machine with the d-c. brush setting specified by the factory for motor operation. It was found that at small loads the machine would generate as a self-excited alternator but would soon drop out of step and operate as an induction generator with similar slip characteristics as given on motor operation at overload. Fig. 4 shows an oscillogram of various currents and voltages in the machine with this operation and may be of interest.

Tests were also made with several other d-c. brush settings, and with positions near 90 electrical degrees from the field axis, the machine would easily carry full load at synchronous speed, with a leading power factor, approaching unity as the load increased. At higher loads the machine dropped out of synchronism and continued as an induction generator but readily dropped back into step at about the same load it carried before when running at synchronous speed. The field current was slightly higher than for the same load on motor operation, but remained at about the same value throughout the range of load.

Fig. 5 shows an oscillogram of its performance changing from synchronous-generator to synchronous-motor operation near half load on each. The brush setting for this was about 67 electrical degrees from neutral in the direction of rotation. There is little change to be noticed except the phase difference of voltage and current. The oscillogram represents a little less than 0.5 sec. in time. The power factor was leading for both operations. Fig. 6 shows its performance as an induction generator pulling into step and continuing operation as a synchronous generator at the same brush setting as for Fig. 5. Here again the quick change required is responsible for a large part of the current variation. Oscillograms in Figs. 5 and 6 were taken on a 7½-h. p. motor, which is one of our laboratory machines.

Further tests were not made due to a lack of time, but, with the proper brush setting or some other adjustment, a generator operation might be found which is as desirable as its motor operation. The tests described simply show that it has possibilities as a generator.

It might be well to mention that at all brush positions tried, its starting and synchronizing characteristics as a motor were still very good, even though it was belted to a much larger machine.

From the educational point of view we have taken a great deal of interest in this machine since it is very illustrative of the possibilities of new developments by combination of the characteristics of well-known machines. We are interested in seeing the most made of its possibilities, as well as in the elimination of its disadvantages. It seems to be a big step forward toward supplying an increasing need which is not now satisfied.

C. R. Underhill (Communicated after adjournment): I regard this motor as a very important and timely device. Power-factor correction is an economic and operation proposition, and where motors having the general characteristics of the one described can be applied to an existing industrial-plant distribution system, particularly when placed close to motors whose power factors are to corrected, such motors should undoubt-

edly be used. However, before deciding upon any form of power-factor-correcting apparatus, a careful study of conditions should be made, a change to a higher voltage considered, and then the induction motors should be loaded to their maximum safe capacities by proper substitution, that is, by putting the right motors on the right jobs, the diversity of operation being duly considered. I have supplied induction motors for new drives without purchasing a single motor, and have put a number of motors in stock besides, while increasing the plant power factor above the penalty limit by loading the motors to their proper capacities, and that is, or should be, common practise.

In considering the use of synchronous motors, static condensers, or motors of the type described in the paper, it must be remembered that a current of abnormal strength flows between the induction motor or motors and the power-factor-correcting device or devices. For instance, connecting a synchronous motor or a static condenser across the plant terminals to correct above the penalty limit. Such managers have very poor conceptions of losses in their own plant conductors. They do not realize that higher voltages, or else larger conductors, would in many cases save them much money annually and pay good returns on the investment.

With the above reservations, I welcome the new motor, which I have studied and have witnessed in operation, as a general motor which, even if it has a commutator, is a distinct improvement over the present induction and synchronous motors. However, it should not be considered a cure-all, as in cases where plant conductors are too small or plant voltages are too low for the prevailing plant distribution system. There are instances within my own experience where there have been such excellent distribution systems and voltages that any savings in the distribution losses due to low power factor were not worth any expenditure for power-factor-corrective apparatus after the induction motors were fully loaded.

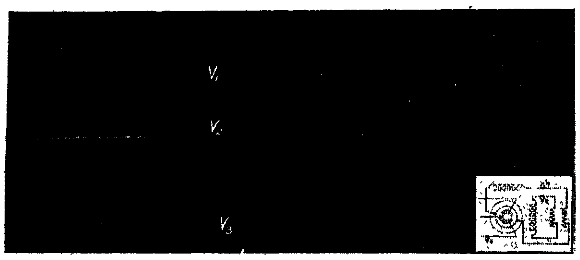

FIG. 3

FIG. 4

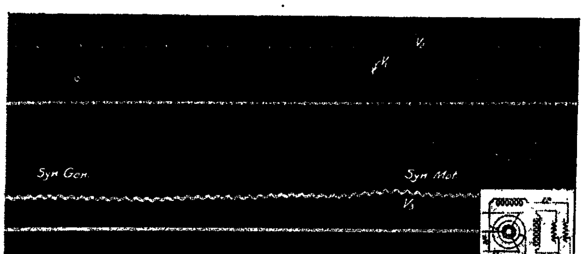

FIG. 5

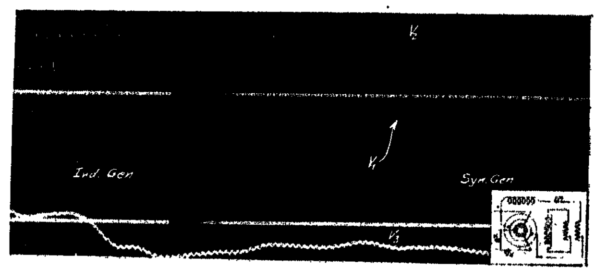

FIG. 6

FIGS. 3-4-5 AND 6—OSCILLOGRAMS SHOWING OPERATION OF FYNN-WEICHSEL MOTORS UNDER VARIOUS CONDITIONS

Figs. 4-5 and 6 are for a 7½ h. p.-motor

Fig. 3 is a 15-h. p., 220-volt motor dropping out of synchronism. V_1 is line current. It was 50 amperes before the motor dropped out of synchronism and 80 amperes afterward. V_2 is current in the auxiliary winding, and it was 41 amperes before dropping out of synchronism. V_3 is current in the field winding. The power factor was 0.81 leading before dropping out of synchronism and 0.72 lagging afterward.

Fig. 4 shows the change from motor to induction-generator operation. V_1 is line current. V_2 is current in the auxiliary winding. V_3 is field winding current.

Fig. 5 shows the change from synchronous-generator to synchronous-motor operation. The generator load is 2.7 h. p.; the motor load is 4.3 h. p. V_3 the generator field amperes are 15; the motor field amperes are 19. V_2 equals auxiliary-winding amperes. V_1 equals lines amperes.

Fig. 6 shows the change from induction-generator to synchronous-generator operation. The synchronous generator load is 5.3 h. p. The synchronous-generator field amperes, V_3, equal 11. V_2 equals line voltage.

the plant power factor does not remove the magnetizing current from the plant distribution system. Connecting static condensers across the terminals of individual induction motors minimizes the magnetizing current in the wiring system, but not in the conductors connecting a static condenser to a motor. Hence, the use of motors of the type described in the paper should be carried out with the full understanding that too great distances between induction motors and power-factor-correcting apparatus, or too small conductors, may be the cause of considerable losses in the connecting conductors.

Where two-charge rates prevail, for instance, control of the demand may be more important than correction of the power factor from a billing standpoint, and it is often difficult to impress upon plant managers the fact that further savings can be made after the demand has been minimized and the power factor raised

From my point of view, there is altogether too much stress placed on the efficiencies of motors. I prefer economy to efficiency. Would more efficient induction motors prove more economical if constructed from present available materials and and by present methods? I do not believe we could afford to buy much more efficient motors. From the industrial-plant manager's standpoint, the motor that will show the greatest saving in dollars is more important than the one that will show the greatest efficiency in per cent.

H. Weichsel: Professor Scott has presented, in a very vivid manner, the meaning of wattless and watt currents first by the "two-color" method which he devised years ago, and which has proven to be of an extraordinary help in explaining the more or less puzzling phenomena of watt and wattless currents, and second by the statement he made today that every electric

motor requires excitation, which may either be produced in the power house or at its place of consumption. It may be expected that his manner of explaining these phenomena will greatly contribute to a clearer conception of the advisability and reasons for installing power factor correcting devices.

I agree fully with Professor Upson that from the electrical engineer's point of view an electric machine should have as small an air-gap as is mechanically possible. Professor Upson is entirely correct that the distance between bearing centers in the Fynn-Weichsel motor is larger than in standard induction motors. May I add, however, that the increase in length is not very large, as the width of the commutator is usually less than that of the slip-rings of a standard induction motor. Therefore, no abnormal problems arise in the design of the shaft for sufficient stiffness to prevent abnormal deflection.

Mr. Ferris stated that, in d-c. machines, he has found it usually disadvantageous to use double-winding armatures. May I point out that, according to my judgment, the problem in standard d-c. machines is quite different from the one which presents itself in the design of Fynn-Weichsel motors. In d-c. machines, especially in those machines to which Mr. Ferris refers, a high voltage exists at the commutator and further, a high potential difference also exists between the two windings. In addition to the above, the energy carried by the commutators is quite appreciable.

On the other hand, in Fynn-Weichsel motors the voltage in the d-c. winding is extremely low and as there is no interconnection between the d-c. winding and the armature a-c. winding, no potential strain exists between these two windings. Further, the energy of the d-c. winding forms only a small percentage of the total output of the machine.

Finally, there is practically no possibility of a burnout of the d-c. winding on account of the peculiar characteristics of the exciting current of these machines. Tests, as well as theory, show that the exciting current between full load and maximum load varies only slightly.

Mr. Perkins points out that the copper losses in the stator member of a standard induction motor, when compared with the corresponding copper losses in the stator member of the Fynn-Weichsel motor, in his judgment, are materially larger for the Fynn-Weichsel motor than for the standard induction motor. This reasoning is based on the assumption that the concentric winding which is used in the Fynn-Weichsel motors can with equal advantage be used in standard induction motors. As I did not explain in detail the particular type of concentric winding which is used in Fynn-Weichsel motors, it can readily be seen why Mr. Perkins arrived at this conclusion.

The concentric winding, employed by me, cannot be recommended for standard induction motors, as it leads to uneven loading of the different phases. On the other hand, this winding is extremely advantageous in connection with Fynn-Weichsel motors, as it allows a better field distribution, shorter mean turn length, and a copper section in the axis of the main field winding which is larger than in the axis of the auxiliary winding. This results in a field winding loss materially below the values which Mr. Perkins estimated from the ratio of the vectors A and D in my Fig. 24. I may add here that this particular winding is covered by a United States patent.

Mr. Perkins further states that, in my paper, a comparison is made between an installation, the power factor of which is corrected by the Fynn-Weichsel motors and an installation with a power factor corrected by synchronous condensers. I am sorry that the paper conveyed this meaning to him. The paper makes repeated reference to "idle-running phase-correcting devices," meaning thereby either static condensers or synchronous condensers. Static condensers, as a rule, require transformers and the energy consumption of these units or any other apparatus capable of producing leading wattless current without doing useful work amounts to about three to four kilowatts for every 100 kv-a. corrected. In Appendix 3, an example shows that, with power-factor conditions as usually found in praxis the efficiency of the Fynn-Weichsel motors can be 5.9 per cent less than that of a squirrel-cage motor and still give the installation the same overall efficiency as if it would consist of squirrel-cage motors only and the correction being produced by static condensers. Elsewhere in the paper it has been shown, however, that the efficiency difference between squirrel-cage and Fynn-Weichsel motors is considerably less than 5.9 per cent. Often these efficiencies are alike, while in large units they are even sometimes better for the Fynn Weichsel motor than for the squirrel-cage motor.

Mr. Kalb mentions an experience with this new type of motor which is rather interesting. He states that the slight speed variation which occurs when these machines are sufficiently heavily overloaded to force them to operate as induction machines, has proven to be an advantage rather than a disadvantage. A condition similar to the one mentioned by Mr. Kalb has also been experienced in connection with protecting devices for these new motors. When the machines become sufficiently loaded to force them to operate as induction machines, the current draw from the line rises abruptly due to the change in power factor and to the small speed variations. This, in some instances, has resulted in a very positive operation of the protective devices.

Mr. Baum stated in a very able manner the great difficulties and dangers which arise in a system with poor power factor, which usually is caused by "the brute, the induction motor." His warning that the excitation in the generators, due to low power factor, is a point gun, and cannot be too much emphasized. According to present indications, Mr. Baum's hopes are soon to be fulfilled, as synchronous motors of small and medium sizes, especially of the type described in the paper, are becoming more popular every day.

Mr. Doherty refers to the possibility of correcting the power factor by a variety of means. There is no doubt that it is an economic question to decide which method for correcting the power factor is the most advantageous. In my way of looking at it there is no universal remedy for the ills of poor power factor. There appears to be a field of usefulness for almost any of the known power-factor corrective means. It is an economic, as well as an engineering, problem to determine from case to case the best means for achieving the desired results.

Referring to Mr. McCutcheon's discussion, I desire to state that the starting resistances for this new type of machine can be operated automatically in exactly the same manner as for standard slip-ring induction motors. He further points out the possibility of achieving the desired results of power-factor correction by using d-c. distribution and a-c. transmission, employing a converter or motor generator set as a link between the transmission and distribution systems. I believe that it will be found that such an arrangement is more expensive than a straight a-c. distribution and transmission system.

There is another point which should not be overlooked. In almost every industry there are certain places where nothing but squirrel-cage motors can be operated satisfactorily. Therefore, alternating current is required for these motors, and if direct current were to be used for the remaining machines, the wiring system would be unnecessarily complicated. For instance, this difficulty may be overcome by using straight a-c. distribution and correcting the power factor in the distribution system by using this new type of machine.

Mr. Sonntag contributed some very valuable information; his experience that the capacity of the squirrel-cage motors in cement mills must be selected in accordance with the starting torque rather than according to the running load, finds an analogy in a great many other industries. May I refer here, for instance, to the marble cutting plant which is discussed in the paper. By the use of Fynn-Weichsel motors, this difficulty is overcome and in a good many cases a smaller horse-power motor can be used. The advantageous results obtainable in this manner are very

forcibly demonstrated by the charts he presents. His statement that cement dust is not detrimental to slip-rings and commutators appears to me as rather important, since it is a conclusion based on many years of actual experience.

Mr. Thomas points out that there are cases where it is uneconomical to correct the power factor. The cases he cited may in general, perhaps, be called installations which are not working to their full capacity. The suggested method of gradually adding to the system power-factor correcting devices is, no doubt, very sound, because any system which is underloaded in the beginning will sooner or later be fully loaded, and it is then when the power-factor correction has the most beneficial effect.

Mr. Thomas' recommendation of operating transmission lines with high power factor reminds me of a statement made to me sometime ago by one of the engineers who was instrumental in bringing about our so-called "superpower system." He stated that the greatest difficulty encountered in these systems is caused by the lagging current, which is "kicked around" from one power plant to another like a football. When operator *A* finds excessive, wattless currents, he changes the excitation of his generator and shifts the current to operator *V* and vice versa.

Mr. G. S. Smith presents a very interesting oscillographic study of the Fynn-Weichsel motor when operating as a generator. These test results are extremely instructive and valuable. His test results throw a great deal of light on the somewhat puzzling conditions which arise when these machines operate as generators.

For those who are interested in this problem, a circle diagram of a Fynn-Weichsel motor is given in Fig. 7. A long time ago I derived this diagram and it has been used extensively in the actual design of machines. It is based on the assumption that the ohmic resistance in the primary member is negligible, a fact which is very nearly satisfied in actual machines. The angle found by the diameter of the circle forms and the vertical must be made equal to the angle between d-c. brush axis and d-c. field axis in the machine. The distance between any point of the circle from the horizontal line $O-1$ represents the input to the machine. When the angle, α, is zero, the brush axis coincides with the field axis. For this condition, all points of the circle lie above the base line, meaning that the machine can operate as a motor only. However, if the brush axis forms a certain angle with the field axis, one part of the circle lies below the horizontal $O-1$ and, q, therefore, represents negative input, meaning the machine operates as a generator.

If, for instance, the brush displacement is 90 deg. in direction of rotation, the machine is just as powerful as a generator as it is as a motor. If the brush axis is shifted 180 deg., then the entire circle lies below the horizontal $O-1$, meaning the machine can operate as a generator only.

This circle diagram also shows when the machine is capable of delivering magnetizing current to the line and when it draws magnetizing current from the line. As long as the points of the circle lie to the left of the line $3-4$, the machine delivers magnetizing current into the system, and when the points of the circle lie to the right of the line $3-4$, the machine takes magnetizing current out of the system. This is true whether the machine operates as a motor or as a generator.

Mr. Underhill recommends improving the power factor in an installation by properly selecting the size of the induction motors in respect to the load which they have to carry. There is no doubt that by this method a very great improvement in the power factor of the system can be obtained. I like to call attention, however, to some of its limitations. Many industries exist in which the load on the machinery is seasonal and in such cases it is not advisable to change the capacity of the motors in accordance with the seasonal business of the industry. There are also many cases where motors must carry, for a relatively short time, heavy loads and, for great periods of time, operate at no load or fractional load. Frequently the capacity of a squirrel-cage motor is governed rather by the starting requirements than by the running load.

These conditions can be particularly well cared for by installing machines which operate at unity or leading power factor for most of the time such as described in the paper. Installations of this kind also overcome the difficulty which Mr. Underhill pointed out that heavy leading currents exist in the wiring in such cases where the power-factor correction is obtained by centralized power-factor correcting devices, such as synchronous condensers or static condensers.

V. A. Fynn (Communicated after adjournment): In Mr. Weichsel's paper there appears to be an indefiniteness in his statements as to the torque conditions in general and particularly as to synchronizing-torque conditions of the machine known under the trade name Fynn-Weichsel motor.

The easiest way to avoid the numerous pitfalls scattered within this field and to gain a *true picture* and a true physical conception of what really happens in the machine is to deal separately with the torque produced by the currents *induced* in the windings *F* and *A* of Mr. Weichsel's Fig. 4 and that due to the currents *conduced* or injected into *F*.

The first is nothing more or less than the well known *polyphase*

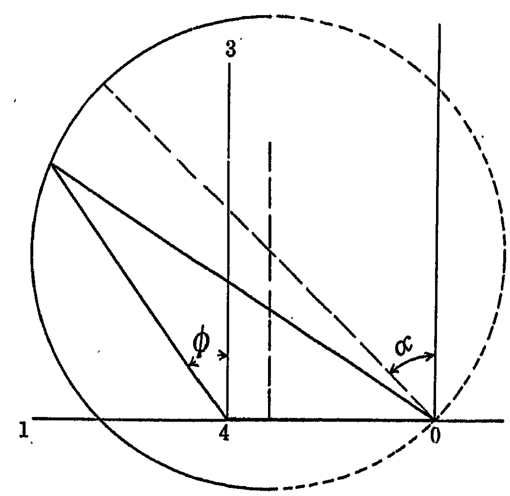

FIG. 7—CIRCLE DIAGRAM OF FYNN-WEICHSEL MOTOR

induction-motor torque. It is known that, under balanced conditions, this induction-motor torque is practically constant at subsynchronous speeds *and becomes zero at synchronism*. It is further known that its value may be varied by varying the impedance of the secondaries to which it is due. We are also advised of the fact that under unbalanced conditions, for instance with different impedances in the circuits of the several secondaries, this induction-motor torque loses its constancy and becomes undulating.

The *nature* of the second torque, that due to the currents *conduced into* the secondary winding *F* by way of the brushes cooperating with the commuted winding on the primary, and to which I refer as the *synchronizing torque*, is *entirely different*. The synchronizing torque in the motor under discussion is *never constant at subsynchronous speeds* and *does not become zero* at synchronism. At subsynchronous speeds this torque may be an alternating torque of double-slip frequency with equal positive and negative maxima or it may be a unidirectional torque; pulsating from zero to a maximum at slip frequency, all according to the magnitude of the angle α of Fig. 4. As synchronism is approached, the amplitude of the synchronizing torque *increases* and its frequency *decreases*, while the magnitude of the aforesaid induction-motor torque *decreases* and its frequency, which is zero throughout, remains *constant*. At synchronism the frequency of the synchronizing torque is zero, the magnitude of the induction-motor torque is zero and the synchronizing torque becomes the motive torque of the synchronous motor.

At the beginning the statements are indefinite and convey an erroneous idea of the function of the brush voltage in conjunction with the winding F, which incorrect idea is later fostered by the curves of Fig. 10.

A machine connected as in Figs. 5A and 5B starts and operates like an ordinary induction motor, approaches synchronism but never reaches it. Synchronism cannot be reached unless the winding F is connected to the brushes co-operating with the commuted winding which results in the production of an alternating synchronizing torque with equal or unequal positive and negative maxima, said torque being added to or superposed on the ordinary induction-motor torque.

The resistance, or more broadly, the impedance of the secondary circuits affects the induction-motor torque and the synchronizing torque in like manner and is *not* the determining factor in the situation. The difference between the two torques and that which makes it possible for the *conduced* ampere-turns in F to synchronize the motor is the fact that, while the amplitude of the voltage induced in F diminishes to zero with decreasing slip, that of the voltage conduced into F remains constant for all rotor speeds. See lines 1 and 2 of Fig. 7.

The fact that the frequency of the brush voltage is inherently the same as that of the voltage induced in the secondaries is not in itself sufficient to cause the additional torque to help the induction-motor torque. The determining factor in this case is *the phase* of said brush voltage with respect to that of the voltage induced in the secondary on which the brush voltage is impressed. The additional torque due to the brush current in F, may, according to the phase of the brush voltage, either help or oppose the induction-motor torque or alternately help and oppose same, but in no case is this torque constant and comparable to the induction-motor torque. At its best this additional "synchronizing" torque pulsates from zero to a positive maximum.

The amplitude of this superposed pulsating or alternating torque is practically independent of speed variations of the order of magnitude of the slip of an induction motor from no-load to maximum load for the reasons that the brush voltage is independent of the rotor speed and that whatever changes in the magnitude and configuration of the synchronizing torque do take place when the motor speed varies are due to changes in phase and magnitude of the brush current in F. These changes are brought about by the change in the frequency of the brush voltage, which frequency increases with increasing slip. *Such being the case, the machine described by Mr. Weischel cannot run at a constant speed as an induction motor.* The fact of the matter is that while the synchronizing torque is indispensable if the motor is to be operated synchronously, said torque *interferes* with the proper operation of the machine as an induction motor, causing the motor speed to pulsate continuously. It is therefore not correct to say simply that the Fynn-Weichsel motor will operate at a higher speed than an induction motor when operating under otherwise equal conditions. Except for the roughest kind of work this machine is unsuitable for use at other than synchronous speeds.

Judging by statements made in the paper, Figs. 8 and 9 have reference to an induction motor operating very near synchronism with a small slip, say, at full load or at less than full load; under no other conditions are secondary voltage and current nearly in phase. We all know that under these conditions the induction-motor torque in balanced circuits is constant and varies with the slip about as shown by Curve 1 of Fig. 10. All we are interested in is how the torque conditions are modified when the commuted winding located on the primary is included in the circuit of the secondary F. Mr. Weichsel's suggestion is that in case the brushes cooperating with the primary commuted winding are coaxial with F, the torque conditions are modified as shown in his Fig. 9c. This figure is qualitatively and quantitively incorrect, the quantitative error is so great as to give a quite erroneous impression of the true nature of the machine.

According to Fig. 8B and the second column on page 11, when the two secondary windings A and F are short-circuited, the resultant torque, $T = T_1 + T_2$, is constant. If so, then T_1, which is due to F, must be less when the brushes and the commuted windings are included in the circuit F, thus increasing its impedance. In Fig. 9c the sum of T_1 and T_2 must therefore be a wave and not a straight line. This is the qualitative error.

The very misleading quantitative error is found in the relative amplitudes assigned in Fig. 9c to the induction-motor torque $(T_1 + T_2)$ and to the synchronizing torque T_3. The ratio of these amplitudes scales 9.6 to 2.6.

At synchronism, or at very small slips $T_1 + T_2 = O$, or practically so and T_3 as stated in the paper, is 230 per cent of the full-load torque. The ratio of the amplitudes is then as O to 230 and Fig. 9c is clearly not drawn for nearly synchronous speeds.

At full-load asynchronous torque, the slip, according to Curve 1 of Fig. 10, is about 3 per cent. Since the amplitude of T_3 for a given brush angle α depends on the amplitude of the brush voltage which is constant and on the impedance of F which is zero at synchronism and increases with increasing slip, it is clear that for a 3 per cent slip T_3 is very little less than its synchronous value and the ratio of induction-motor torque amplitude to synchronizing-torque amplitude is practically as 1 to 2.3. Fig. 9c is evidently far from being correct for an asynchronous torque equal to the full-load torque of the motor.

But even quite near the maximum asynchronous torque, when the slip is 10 per cent according to Curve 1 of Fig. 10, the ratio in question is still as 2.9 to about 1.9. This is an extreme case quite outside the limits specified as those on which Fig. 9c is based, yet this figure no more applies here than it does at loads up to and including full load.

Making the amplitude of T_3 about 8½ times greater than shown or 2.3 times greater than that of $(T_1 + T_2)$ in Fig. 9c puts a *very* different complexion on the proposition and forcibly brings out the fact I have previously stated, *i. e.*, that such a motor *cannot* run at a constant speed when operating as an induction motor.

When the axis of the brushes cooperating with the commuted winding on the primary is displaced from the axis of F by a small angle such as α of Fig. 4, the synchronizing torque T_3 assumes the configuration indicated in Fig. 12, it becomes alternating with unequal maxima and still effectively prevents the machine from running at a constant speed when operating asynchronously. As the magnitude of the negative maxima increases, so does the asynchronous overload capacity decrease.

The Curves 4, 5 and 6 of Fig. 10 have evidently been derived on the assumption that the Curves 1, 2 and 3 represent the induction-motor speed-torque curves of the machine for given impedances of the secondary circuits, that the addition of the brush voltage e_c does not change said impedances and that e_c is constant and cophasal with the voltage induced in the winding into which it is introduced. If all this were true, which is not the case, then it would be permissible to say that the secondary current *in the phase into which e_c is introduced* and consequently *the torque due to that phase* is increased in the ratio of $e_o s_x$ to $(e_o s_x + e_c)$ *but* in his Fig. 10, Mr. Weichsel has not only represented this increased torque as if it were *constant* but as if it were applied to *both* secondary phases.

In Fig. 10, Mr. Weichsel has added *the maximum value of the single-phase, pulsating torque*, T_3, produced by the winding, F, *only* to the *constant* induction motor torque $(T_1 + T_2)$ represented by the Curves 1, 2, 3 and *offers their sum as the resultant torque of the motor!*

It is further to be noted that if the conditions were actually such as indicated by Mr. Weichsel's Fig. 10, the machine could not run synchronously unless the torque required was 230 per cent of the normal, see point A of Fig. 10. For smaller torque de-

mands the motor would run at speeds greatly exceeding the synchronous.

Errors of this kind are not so likely to occur if the synchronizing torque is dealt with quite independently of the induction-motor torque.

Pursuing this subject a little further, let us examine into the operation of the motor on the basis of the performance curves shown in Fig. 28. Further on, it is stated that the synchronizing torque T_3 for these motors as designed is from 90 to 95 per cent of the synchronous pull-out torque. For the 75-h. p. of Fig. 28 the synchronous pull-out torque is 196 per cent and the synchronizing torque is therefore at least 0.9×196 or 176 per cent. We can simplify the argument without missing the moral by assuming that T_3 remains constant down to the asynchronous breakdown point. We, of course, know that this torque actually diminishes with increasing slip. Upon the demand of a 197 per cent torque, which is slightly in excess of the maximum synchronous, the machine lapses into asynchronism and the synchronizing torque of 176 per cent reappears but is alternating with very unequal maxima. If $\alpha = 20$ deg. and the positive maximum is 176 per cent, then the negative maximum is 5.6 per cent. The positive maximum is almost sufficient to handle the load and a very small asynchronous slip corresponding to a 20 per cent asynchronous torque will supply the difference. When T_3 is at its negative maximum the slip must be sufficient to counteract this negative torque and to handle the load, which means that the slip must correspond to a 202.6 per cent asynchronous torque. The motor speed must and does vary accordingly as can be observed on any such motor.

Again, Mr. Weichsel says that the inherent slip of the so-called Fynn-Weichsel motor should be made as small as possible. This means that the winding A of Fig. 4 must have about as much copper as the winding F. Since A is *idle* at synchronism and comes into play only in asynchronous operation, at starting and under loads in excess of the maximum synchronous load, a large amount of copper in A can only be justified if the asynchronous overload capacity is actually utilized. I have shown that this asynchronous overload capacity is only available for the roughest kind of work because the speed then fluctuates continuously and Mr. Weichsel's Fig. 28 clearly indicates that the asynchronous overload capacity is not really relied upon by the makers of this machine. The synchronous overload is about 188 per cent for the 15 and 196 per cent for the 75-h. p. motors to which said figure refers. This overload is ample for all ordinary purposes and the asynchronous overload which rises to 300 per cent and 308 per cent respectively is pure waste, it cannot be and is not utilized.

Mr. Weichsel's third conclusion states that the injected current must be about twice as large as the full-load secondary current of the motor. This means that the commutator must carry at least twice the normal full-load secondary current near the synchronous break-down point and if the asynchronous overload capacity is really utilized, as suggested in the paper, then *the commutator must carry more than three times the normal full-load secondary current* when the machine operates near its asynchronous break-down point. I do not think it can be fairly said that such a commutator has relatively small dimensions, yet such is Mr. Weichsel's contention.

Much is made in the paper of a really insignificant detail and a quite erroneous impression is conveyed: Fig. 21 purports to show that the axis of the unidirectional magnetization on the secondary does not coincide with the axis of the winding F because of the d-c. ampere-turns in the primary commuted winding and it *also* purports to show that these primary d-c. ampere-turns are neutralized by the a-c. ampere-turns on the primary.

In the paper, it is also stated that the armature, *i. e.*, the primary, d-c. ampere-turns are about 5 per cent of the primary a-c. ampere-turns. It is stated that α shall be zero, or very small. From Fig. 18, and in fact without it, we know that the secondary ampere-turns in synchronous operation must be greatly in excess of the primary ampere-turns. In Fig. 21 the vector $A\ T_{adc}$ must then be much less than 5 per cent of the vector $A\ T_f$ and the angle between the two should be smaller rather than greater than that shown. How can these insignificant primary d-c. ampere-turns influence the location of the "resultant direct-current field" to any appreciable extent? In Fig. 21 the "d-c. armature field" is shown as amounting to 54 per cent of $A\ T_f$, hence the delusion.

As to the suggestion that these primary d-c. ampere-turns are neutralized by some of the primary a-c. ampere-turns,—of course they are, *but* he must either say that the "d-c. armature field" is neutralized by some of the a-c. ampere-turns and continue to figure with $A\ T_f$ only or he must figure with the "resultant direct-current field" and forget about this neutralization.

In dealing with commutation, Mr. Weichsel says that the reactance voltage is similar to the reactance voltage in a standard d-c. machine. It would be quite correct to say that it is identical, *i. e.*, identical in nature, but it is not even similar as to magnitude. The sides of the coil undergoing commutation in the standard d-c. machine lie in the open, *i. e.*, in the interpolar space; in this machine these coils are surrounded by laminations separated by an induction-motor air gap and, for otherwise equal conditions, the reactance voltage in this case is a multiple of that of a standard d-c. machine. The brush voltage and the current per conductor must be kept low.

When $\alpha = 0$ the coils undergoing commutation cut the full resultant magnetization, *i. e.*, the full field flux of this motor at no load, and cut no field flux at maximum synchronous load. For other values of α the no-load conditions improve, the maximum load conditions get worse.

Mr. Weichsel's arguments as to the advantages of a concentric over a diamond winding are beside the point for either can be used in an ordinary induction motor. As to his arguments in Appendix No. 2, it is true that by taking $^2/_3$ of the three-phase secondary and feeding into it a d-c. equal to $\hat{\imath}\sqrt{3/2}$ the flux and the loss will be the same as with an effective three-phase current $\hat{\imath}$ in each of the three rotor phases, *but* this loss will be distributed over two instead of three phases and the *heating* will be considerably greater.

Furthermore, the secondary ampere-turns in synchronous operation, with unity or leading power factor, are *considerably greater* than the secondary ampere-turns for corresponding asynchronous operation as is shown in Mr. Weichsel's Fig. 24 where the secondary ampere-turns for a certain load are $1-2$ or A for synchronous, and 1—3 or B for the non-synchronous operation. The ratio of A to B is as 30 to 19, and the d-c. in the two phases of Mr. Weichsel's Fig. 35 must therefore be $\hat{\imath} \times \sqrt{3/2} \times 30/19$ or 1.58 times greater than indicated by him.

If the amount of copper on the secondary of the synchronous induction motor is no greater than that used when the machine is designed as a straight induction motor and if two-thirds of that copper is used for the unidirectional ampere-turns in synchronous operation, then for the load conditions of Fig. 24 the secondary copper losses will be 2.49 times as great as the corresponding slip losses in non-synchronous operation and the cooling surface for this loss will have been reduced to two-thirds of that available in the straight induction motor.

The question may well be asked, why not also use the third phase of Fig. 35? One reason is that it is necessary to have a *polyphase* winding on the secondary not only for starting but also to prevent hunting in synchronous operation and to take care of loads in excess of the maximum synchronous load and give the motor a chance to work back into synchronism when a sudden overload causes it to lapse into asynchronism. Mr. Weichsel even thinks, (see his conclusion No. 2 on the eighth page,) that this polyphase winding should be such as to reduce the induction motor slip to very small values. The third phase of Fig. 35 is really the winding A of Fig. 4.

Another reason for not using all three secondary phases to

carry the unidirectional ampere-turns is the fact that when so used they form a winding distributed over *all* the pole surface and with but one axis per pole pair. To get the same flux with three phases in circuit instead of only two would require a further increase of 33 per cent in the ampere-turns and a consequent 77 per cent increase in copper losses. But utilizing the third phase

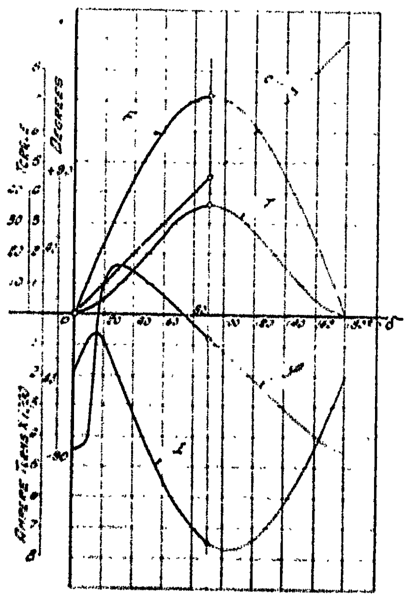

Fig. 8

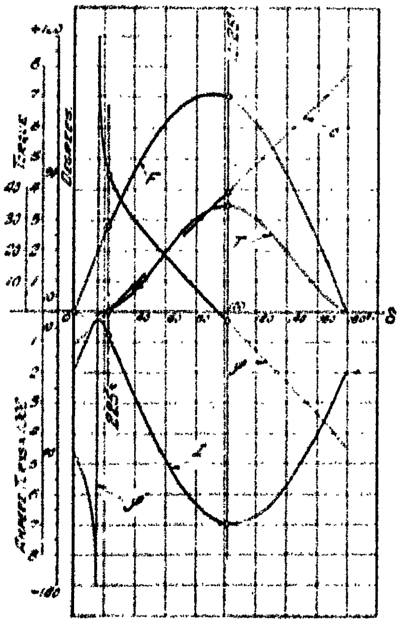

Fig. 9

Figs. 8-9 —Power-Factor-Load Characteristics of Synchronous Induction Motor

increases the amount of copper by 50 per cent so that by using all three secondary phases, *i. e.*, all of the available secondary copper, for carrying the secondary unidirectional ampere-turns the secondary copper losses become equal to $2.49 \times 1.77 \times {}^2/_3 = 2.94$. This means that for the load conditions of Fig. 24 and on the assumption of an unchanged amount of copper on the secondary, *all* of said copper being used to carry the d-c. ampere-turns, the secondary copper losses in synchronous operation are practically *three times as great* as those in non-synchronous running while the *cooling surface is the same.*

The watts loss per unity of cooling surface is some 22 per cent less when all three instead of only two of the secondary phases are used for carrying the secondary unidirectional ampere-turns but the total secondary copper loss is 18 per cent greater and no copper is available for the additional secondary winding *A* of Mr. Weichsel's Fig. 4.

The fact is that a synchronous induction motor of the form under reference *can* be built in which the secondary copper losses are not materially greater than the corresponding losses in an equivalent induction motor *but* such a machine must have *much more active material than the equivalent induction motor and must be correspondingly more costly.*

My opinion is that this so-called Fynn-Weichsel motor is much better suited for use as a synchronous condenser than as a motor.

The ordinary synchronous condenser is difficult to start and is very sensitive to line voltage or to frequency disturbances. It is very liable to fall out of step and cause oscillations throughout the system. The machine under reference is very easy to start and if it does fall out of step it will automatically go back to synchronism without fuss or trouble so soon as the disturbance is over.

The fact that the primary is on the revolving member and the primary currents are taken to it over slip rings is a serious objection to the machine as a motor, but much less so as a synchronous condenser. Slip-rings which carry current all the time

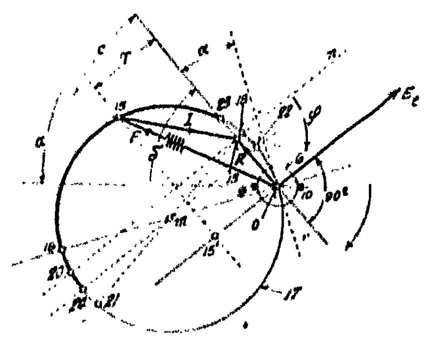

Fig. 10—Circle Diagram of Synchronous Induction Motor

are almost as touchy as a commutator. As a synchronous condenser the machine can be located without reference to any other machinery and therefore in some dry, clean and sheltered spot favorable to slip-ring operation.

The fact that the commutator carries load as well as exciting currents and the fact that any accident to the brush circuit must put the machine out of commission, since the winding *A* alone is insufficient to permit the machine to operate as an induction motor, also militate against its general use as a motor for no one cares to run the risk of an interruption in production. These same facts lose much of their significance when the machine is used as a synchronous converter. The sheltered position to which it can then aspire makes commutator operation easier and a breakdown less likely. If one does occur, it does not entail an interruption in production but merely the temporary loss of the advantages conferred by a synchronous condenser.

Another important point, relates to the power-factor-load-characteristic of such machines. The fact is that the possible inherent compounding or power-factor-load-characteristics of this machine do not permit of operation at unity power factor at all loads as clearly appears from my Figs. 7 and 8 herewith. Generally speaking, the power factor leads considerably at light loads, tends towards unity with increasing load, reaches unity near maximum synchronous torque and lags thereafter. This characteristic is suitable for a synchronous condenser but not for a general-purpose motor. It cannot be sufficiently emphasized that in so far as losses are concerned, whether in the

motor, in the transformers, in the line or in the generators, a leading current is just as objectionable as a lagging one. The one exception is in connection with the exciting current of the generators.

For the benefit of those who desire to study the compounding characteristic possibilities of these machines more closely, I append the circle diagram Fig. 10. The terminal voltage is E_t and the resultant motor magnetization is R, corresponding to $A\ T_m$ of Mr. Weichsel's Fig. 18. The brush angle is α, the location of the winding F is indicated by the coil on vector $O-15$. The locus for the unidirectional secondary magnetization F is the circle 17, the primary current is I, the phase angle is φ, while c and δ are the angular displacements between R and F and between R and the brush axis respectively. The vectors E_t and R are supposed to be stationary in space and the brushes and the winding F are moved counterclockwise through 180 deg. while retaining their proper angular relation α. This covers all possible load conditions for either polarity. The curves in Fig. 9 were calculated from the diagram of Fig. 10. The angle α is 22.5 deg. in both figures.

In my opinion, the motor described by Mr. Weichsel *is not a general-purpose motor.*

H. Weichsel (by letter): After carefully reading Mr. Fynn's later discussion of my paper, I conclude that while he and I approach the theoretical analysis of this type of motor somewhat differently, the reader will not be interested in a prolonged discussion of such differences from our respective points of view, particularly when actual commercial results secured with the Fynn-Weichsel motor bear out the analysis presented in my paper. I will confine my closing remarks in the discussion to a reference to some of the points in Mr. Fynn's discussion where his conclusions are erroneous and where the actual service performance of the motors conclusively supports my point of view. I shall make no reference to those paragraphs which I deem of minor importance.

While participating in the early theoretical development of the Fynn-Weichsel motor, Mr. Fynn has not had the advantage of contact with the commercial development and has probably not had access to actual performance test data of the character to which I refer below.

Before replying to some of the different criticisms which Mr. Fynn has made in his communication, I desire to state that those parts of my paper which deal with the working principle of this machine have, as their main object, the presentation of *fundamental* laws which govern the working of this new type of machine. It was my purpose to free these explanations, as much as possible, from any secondary considerations that would tend to obscure the main fundamental laws.

I regret, therefore, that Mr. Fynn has found it advisable to criticise several of my statements and conclusions on the ground that they lack accuracy, and also that he has entered into a discussion of various details.

In presenting the theory of the starting and synchronizing performance, my reason for the line of discussion pursued in my paper grew out of my desire to give the reader the train of thought which had led me to the discovery that a motor of this type must develop a very powerful torque when the d-c. brushes coincide with the axis of the d-c. field winding, and develop a diminishing synchronizing torque as the brushes are moved out of this position.

The first public statement giving the reasons for the remarkable synchronizing torque of this new type of motor was made by me, February 16, 1924, before the Association of Iron and Steel Electrical Engineers in Pittsburgh.

In the early part of his written communication, Mr. Fynn repeats, in different wording, the statement made in my paper in connection with Figs. 6 and 12 and also the conclusions referred to by me in regard thereto; as well as further statements made in connection with Figs. 9, 11, 12, and 13 of my paper; also, my Fig. 10 and corresponding text.

By some of these statements, Mr. Fynn conveys the impression that I set forth variations in resistance as the "determining factor" with respect to synchronizing torque. It will, however, be found that I also fully discuss the bearing of the phase of the brush voltage upon this matter.

Considerable emphasis is laid by Mr. Fynn upon the fluctuations of torque after the motor has been overloaded to a point pulling it out of synchronism and resulting in its operation as an induction motor. As stated in my paper, it is true that there are very rapid pulsations in torque under these conditions, but it must be remembered that there are not corresponding variations of speed of the motor.

The corresponding speed variations of the motor are considerably less than the torque variations, because of the inertia of the rotor and of the driven load. These relations are similar to those which hold the speed fluctuations of a reciprocating engine low. Fig. 11 herewith illustrates an oscillogram taken by the University of Washington, showing the line-current fluctuations under such conditions. These fluctuations are of somewhat of the same order of magnitude as the torque fluctuations. In practise, neither the current nor speed variations are found detrimental to the electric service, nor to the durability of the motor when such temporary overload conditions are not excessively prolonged. A very interesting demonstration of this fact is a commercial installation of the Fynn-Weichsel motor driving a high-speed grinding wheel. On every service use of the grinding wheel the Fynn-Weichsel motor is pulled out of synchronism, immediately returning to synchronism on the withdrawal of the excessive load from the face of the emery wheel. This installation has been in operation for over a year and while not a recommended type of service for the Fynn-Weichsel motor, it has proved a very interesting application, having been made solely for the purpose of testing the physical result of such service upon this type of motor, and having demonstrated two important factors—

First, that the service does not disturb adjacent motors operating from the same circuit;

Second, that there has been no deterioration of the Fynn-Weichsel motor under these conditions of service.

A further fact might be noted—that the operator of the grinding wheel is unconscious of the motor changing from the synchronous to the induction operating characteristic. Actual tests made with a tachometer on a 100-h. p., 600-rev. per min. motor, as well as on several smaller sizes, showed no measurable speed fluctuations when the machine operated as an induction motor; *i. e.*, beyond its horse power capacity as a synchronous machine.

Mr. Fynn attacks my Fig. 9c on the ground that this figure represents conditions when the motor operates very nearly at synchronism, and he states that, under such conditions, the ratio of torque $T-3$ to $T-1$ plus $T-2$ is materially larger than shown in my Fig. 9c.

The error in Mr. Fynn's statement will be found in his sentence "with a small slip at full load or at less than full load and under *no other conditions* are the secondary voltages and currents nearly in phase."

In that part of my paper which precedes my discussion of Fig. 5, it is clearly stated that the secondary voltage and currents in a machine containing leakage are nearly in phase when the currents do not materially exceed the full-load value.

Such a condition exists not only when the machine operates with small slip and no external resistance in the secondary but *also* when the machine operates with large slip and large resistance in the secondary, such as, for instance, occurs during the starting period of the machine. This is in exact agreement with the phenomena which occur in this respect in connection with standard slip-ring induction motors.

My Fig. 9c pictures correctly the conditions represented in my Fig. 10 by Curves 2 and 5 for a speed of about 80 per cent of synchronous speed and a load approximately equal to full load torque. Mr. Fynn's arguments and conclusions in this connection are, therefore, wrong, due to the faulty assumption upon which they have been based.

He states that serious mistakes exist in my Fig. 10 and asserts that I had represented the increased torque due to the injected voltage e_c not only as if it were constant but as if it were applied to both secondary phases. Further, he states that I added the maximum value of the single-phase, pulsating torque, $T-3$, produced by winding F, to the constant induction motor torque, $T-1$, plus $T-2$, and offered their sum as the resultant torque of the motor. This is a misunderstanding of the statements made in my paper. In connection with Fig. 9, I fully explained that the torque due to the injected voltage, e_c, is pulsating. In the first part of my paper there is a statement as follows: "Therefore, if the load connected to the primary has a fairly large amount of inertia the *average* torque available on the motor lies about half way between the induction motor speed torque curve and the speed torque curve which is shown in Fig. 10." This statement definitely contradicts Mr. Fynn's assertion that I had assumed the torque due to the injected voltage as constant.

Referring to his second claim, that I presented the increased torque due to injected e. m. f. as if it were applied to both secondary phases, I state: "The conditions in the winding A cannot

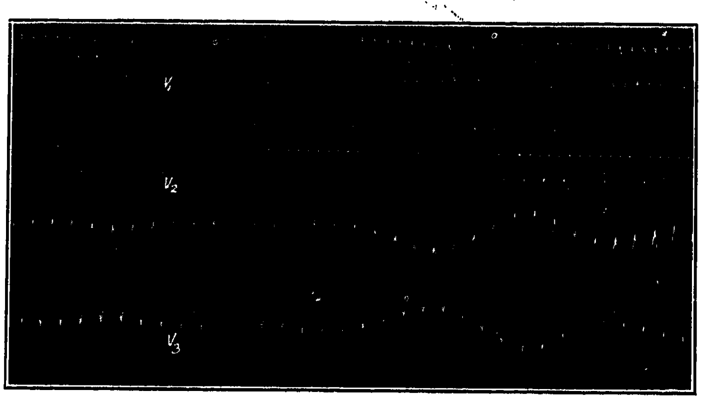

Fig. 11—Synchronous to Induction Operation of Fynn-Weichsel Motor

V_1 is the timing wave. V_2 is the line voltage. V_3 Line current.

in any manner be altered by injecting a current in winding F, and, therefore, the torque produced by winding A remains unaltered, etc."

The mistake made by Mr. Fynn lies in *his assumption* that the horizontal difference between Curves 1 and 6, or 2 and 5, or 3 and 4, represents an *average* torque. However, no such statement has been made by me, but one to the contrary in that part of the paper just cited, where I pointed out that this torque difference fluctuates. The length of the horizontal line between Curves 5 and 6 represents the time maximum of the torque due to the injected voltage. That this relation must exist not only follows from the text of my paper but also directly from Fig. 10, where the length of the line, $100-A$, represents the *maximum* torque of the motor when operating at synchronism; and the maximum torque of a motor at synchronism must, by the nature of things, be also the maximum torque immediately before synchronism is reached.

The wrong assumptions made by Mr. Fynn also led him to the misstatement in regard to the resultant torque of the motor.

The most erroneous statement is where he says that if conditions are actually as indicated in my Fig. 10 "the machine could not run synchronously unless the torque required was 230 per cent of normal. For smaller torque demands the motor would run at speeds greatly exceeding this synchronous speed." This statement is in contradiction to my Fig. 10, where the speed-torque curve of the motor, with no external resistance in the secondary, is given by the curve, $100-A-6$. The part $100-A$ of this curve corresponds to loads from zero to 230 per cent and during this part, the speed torque curve is a horizontal line passing through the 100 point, which means that the speed is independent of the load.

Statement is made that an asynchronous overload of 300 per cent is pure waste. This is completely overlooking the fact that an important factor in this type of machine is its synchronizing ability. A machine whose maximum torque as a synchronous motor is 200 per cent can synchronize a load of 200 per cent, provided the load is a pure friction load. If, however, the inertia is very excessive, the same machine can synchronize only about 100 per cent full load, as explained in my paper. However, this latter extreme condition is never accounted for in practise. Therefore, in order to provide an ample margin for safely synchronizing full-load torque, or more, under almost any kind of load which may be met in practise, it is *essential* to give these motors a maximum synchronous horse-power capacity in the neighborhood of 200 per cent. The overload capacity of the machine as an asynchronous motor is incidental and is achieved without extra expense.

Later in his written discussion, Mr. Fynn appears to create the impression that the commutator must be dimensioned for three times normal load secondary current. Anyone familiar with the design of motors knows that it is useless to dimension electrical parts of a machine in accordance with the momentary overload which the machine can carry. A glance at Fig. 3 of my paper justifies my statement there that the commutator is relatively small.

Again my Fig. 21 is criticized because it is not drawn to scale. It is a generally accepted expedient to do this in such cases in which some of the vectors would otherwise nearly coincide. The main object, as indicated by the italicized letters in the text belonging to this figure, is to show that, at any load, the d-c. armature ampere-turns are counterbalanced by equivalent a-c. ampere-turns. Mr. Fynn now thinks that this is selfevident. My diagram Fig, 21 proves that the Fynn-Weichsel motor does *not* operate as a synchronous converter, because only a *small* part of the a-c. ampere-turns is used to counterbalance the d-c. ampere-turns, while in a converter, the a-c. and d-c. ampere-turns are essentially of the same magnitude and opposed.

Further on, he criticizes my statement that this new motor operates from commutating point of view similar to a neutralized d-c. machine in which the neutralizing winding is weaker than the d-c. armature reaction. In my judgment, this statement pictures the analogy quite correctly in view of the fact that the d-c. armature field is, at any time, completely neutralized as shown in my Fig. 21, but the component of the resultant field, which is in line with the brush axis, is not wiped out. If the brushes are shifted in the direction of rotation, which is the only practical way of shifting them, the commutation conditions for the maximum load point frequently first improve, and, by a still further shift, become more difficult. By suitably selecting the magnitude of different constants of the machine, such as brush-angle, leakage-reactance, etc., it is possible to obtain the point of theoretically correct commutation for almost any load point desired.

Still further on Mr. Fynn arrives at certain conclusions regarding the heating and losses of these machines. His reasoning that, for equal losses, a winding covering only two-thirds of the circumference must be very much hotter than a winding having the same losses but with them distributed over the whole circumference, is a quite common argument, but actual experience has proved this to be wrong. If his views were correct, it would be necessary to use larger sizes of copper for certain coils in the arma-

ture of a standard synchronous converter, and this practical experience has proven to be unjustifiable.

In the "induction type of synchronous motors," such as is built in Europe, one part of the winding has four times the loss of the remaining winding, when equal copper section is used for all coils. Experience has shown that equal copper section for all coils can be used without resulting in unallowable inequality of heating.

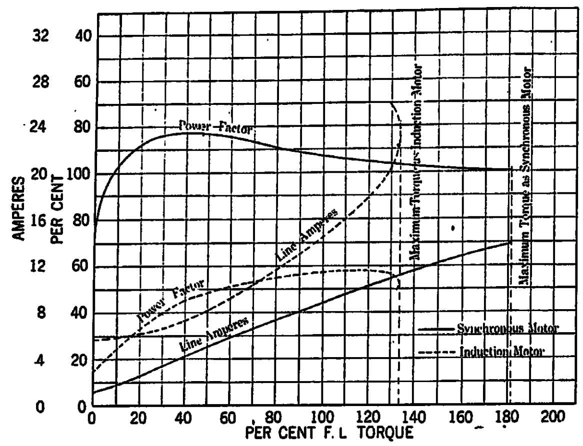

FIG. 12—7½-H. P., 60-CYCLE, FOUR-POLE, THREE-PHASE FYNN-WEICHSEL MOTOR OPERATING AS SYNCHRONOUS MOTOR AND INDUCTION MOTOR

The Metropolitan-Vickers Company, in their circular No. 1041, December, 1921, describing this type of motor, state on page 7: "For manufacturing reasons the cross-section of the conductors is kept the same in all three phases, so that the heating is slightly unequal in the different parts of the winding. But due to the relatively large heat capacity of the iron, the temperature rise is practically uniform around the rotor."

With reference to Mr. Fynn's theoretical objection to the

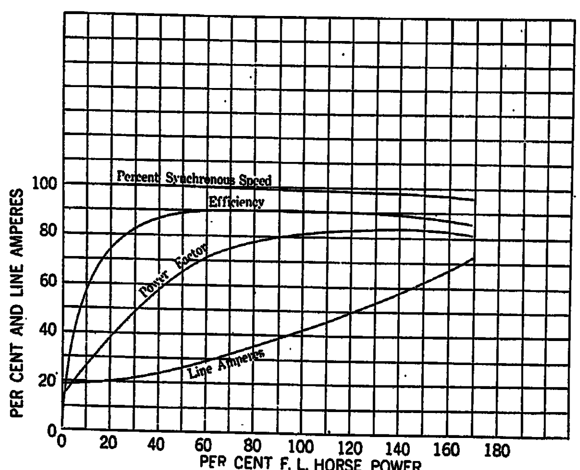

FIG. 13—PERFORMANCE CURVES FOR FYNN-WEICHSEL MOTOR

15-h. p., Four-pole, Three-phase, 60-cycles, 220-volts. Operated at normal voltage as an Induction Motor.

Fynn-Weichsel motor, that the primary currents are supplied through slip-rings, I shall merely say that extended and satisfactory experience with a large number of installations using these machines as general-purpose motors should be considered sufficient answer.

Mr. Fynn also suggests that any trouble with the commutator or brush mechanism will "put the machine out of commission." He quite overlooks, or neglects to state, that if the exciting current were suddenly broken, the Fynn-Weichsel motor would continue to operate as a polyphase induction motor with a single-phase secondary and load characteristics as shown in Fig. 12 herewith.

Normal load capacity of the motor under such circumstances can be fully restored by short-circuiting the excitation field winding, in which event, the motor will operate as a normal slip-ring, induction-type motor, as illustrated in Fig. 13. Figs. 12 and 13 are characteristic curves of all sizes of Fynn-Weichsel motors when operated under the conditions indicated.

Mr. Fynn expresses the view that the Fynn-Weichsel motor

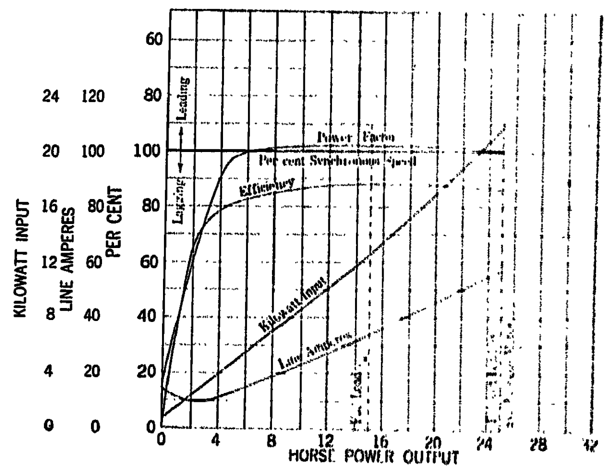

FIG. 14

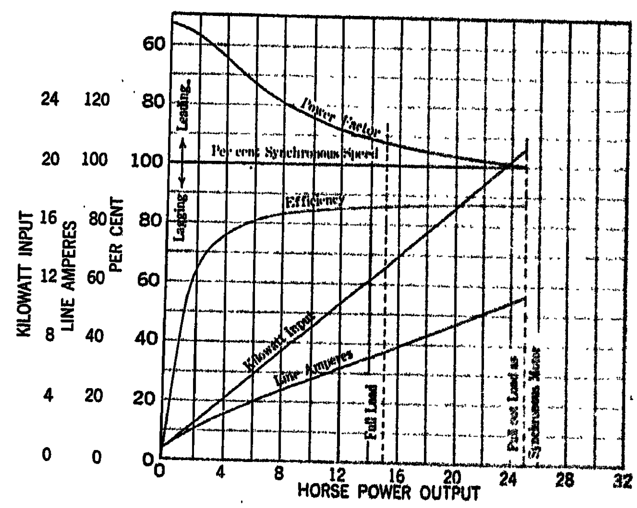

FIG. 15

FIGS. 14-15—CHARACTERISTIC CURVE OF FYNN-WEICHSEL MOTOR

This is a 15-h. p., four-pole, 60-cycle, three-phase motor. In Fig. 14 it is shown adjusted for approximately unity power factor. In Fig. 15, it is adjusted for leading power factor.

is not a general-purpose motor. It is being marketed as such and as such, is meeting with a most favorable reception. It can be installed whenever a good general-purpose d-c. motor can be installed, and the manufacturers are finding a surprising number of installations where the synchronous-speed characteristic is proving more advantageous for production purposes than the primary service of power factor correction. A large number of commercial installations in sizes of motors ranging from 5 h. p.

to 150 h. p., all made as general-purpose motor installations, suggests conclusively that the Fynn-Weichsel motor is, in fact, a general-purpose motor.

Mr. Fynn infers that the Fynn-Weichsel motor cannot operate at unity power factor. Fig. 14 illustrates a commercial test made by the Commonwealth Edison Company at Chicago, and it will be observed that the power factor is practically unity throughout the normal load range of the motor, this being a motor not designed for exact unity power factor operation but rather for operation as illustrated in Fig. 15.

Fig. 15 represents the same motor, also tested by the Commonwealth Edison Company, with no other modification than a slight change in the position of the exciting brushes, for the purpose of giving a strong leading power factor and thus compensating for the lagging power factor of an ordinary induction motor of the same size operating in the same plant, giving unity power factor on the two motors in combination.

Numerous corresponding tests have been made by a large number of the leading electric light companies of the country, and what the motor will do with respect to unity or leading power factor is not a matter of theory but of demonstrated fact, and while the above curves are test results on the 15-h. p. motor, similar characteristic performance results are given by motors of all sizes.

Mr. Fynn's employment of a diagram to establish statement that the Fynn-Weichsel motor cannot be designed to operate at approximately unity power factor over a large load range is erroneously employed, assuming design constants which would not be used were the Fynn-Weichsel motor to be designed for unity power-factor service.

Mr. Fynn's argument suggests that it would be advantageous and preferable to have these motors operate at unity power factor; in his argument he has overlooked the actual conditions which exist in present installations and probably will for many years to come. Due to the use of a relatively large percentage of induction motors in all installations, it is desirable to have the remaining machines in the installation operate with leading power factor in order to provide the necessary magnetization for the induction motors of this installation.

The Single-Phase Induction Motor

BY L. M. PERKINS[1]

Member, A. I. E. E.

Synopsis.—*The operation of the single-phase induction motor is presented according to the cross-field theory as distinguished from the theory of oppositely rotating fields. The mathematics used require only a knowledge of algebra and trigonometry and no factors, such as cross-field iron loss and cross-field magnetizing current, are neglected.*

In addition to the derivation of the vector diagram, its transformation into an accurate circle diagram, which requires no assumption except sine wave voltage and primary field distribution, is shown. The result is a circle diagram practically identical with that derived by Branson (A. I. E. E. PROCEEDINGS, *June, 1912) from comparison of the two-phase and single-phase induction motors, except that the derivation should be more easily followed and the result is a simpler diagram to construct and use.*

THE single phase induction motor consists of a primary winding connected to a source of alternating potential and placed in inductive relation to a short-circuited secondary winding which can move relative to the primary winding. A complete iron path, except for the air-gap necessary to allow relative motion between the two windings, is provided for the flux which interlinks the two windings. This interlinking flux is produced by the primary winding because of its connection to a source of alternating potential, and it is the effect of this flux on the short-circuited secondary winding in which we are interested.

The motor is shown schematically in Fig. 1. Although the secondary is considered as being wound

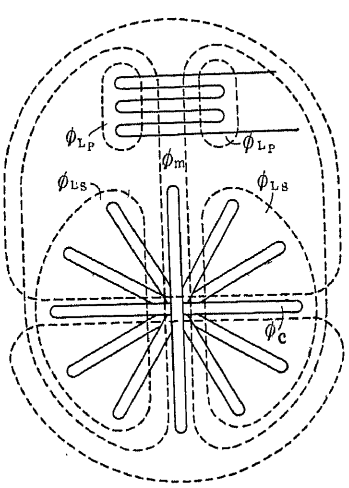

FIG. 1

with a number of individually short-circuited coils, in practise this construction is not used as it is mechanically so much easier to short circuit all the coils to each other by means of common-end rings. If the resistance of the end rings is negligible with reference to that of the bars, the two systems give identical results. Although high resistance end rings tend to distort the distribution of the secondary current from that of a true sine wave; this effect is negligible in practise so it will not be considered in this analysis. We will assume that the primary winding is so distributed as to produce a sine wave field form, and that the impressed voltage has a sine wave form.

In addition to the interlinking flux mentioned above, there are leakage fluxes around the different windings which do not link with all the windings. As shown in Fig. 1, there is a flux ϕ_{LP}, threading through the primary winding in addition to the mutual flux ϕ_m, which threads through both primary and secondary windings. Also there is a leakage flux ϕ_{LS} threading through the secondary but not the primary. Furthermore, there is another flux ϕ_C, as will be shown later, which does not thread the primary winding but only the secondary along an axis at right angles to the axis of the primary flux. The total flux threading through the primary is $\phi_{LP} + \phi_m$ which we will call ϕ_1, while the total flux threading through the secondary along the axis of the primary is $\phi_{LS} + \phi_m$ which we will call ϕ_2, and the total flux threading through the secondary at right angles to the primary axis is ϕ_C. In addition to being threaded by fluxes each coil cuts fluxes.

A voltage is generated in a coil by a change in the total flux threading the coil, whether this change is caused by a change in the total amount of flux; or whether, with a constant flux, the coil turns so as to be threaded by more or less of the total flux. The voltage induced in a coil by a change in the total flux is generally known as a transformer voltage and is proportional to the frequency of variation of the flux and to the total flux. The voltage induced in the coil as it turns so as to include more or less of the total flux is known as the cutting voltage and is proportional to the speed of cutting and to the density of the flux at the point where the coil cuts it. If the flux is varying as the coil turns the net voltage generated is the sum of the voltage generated by transformer action and cutting action.

We will assume an alternating flux ϕ_2 threading the secondary along the primary axis and will investigate the voltages generated in the various coils by their cutting this flux, and the voltages generated in the same coils due to the alternation of this flux through the coils. Also we will find what other voltages, currents, or fluxes are produced by these voltages.

1. Westinghouse Electric & Mfg. Co.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

A coil at *a*, Fig. 2, will not cut any of the flux ϕ_2, while a coil at *d* will cut the flux ϕ_2 at the point of maximum density. The voltage generated in the coil *a* will then be zero while that generated in coil *d* will be a maximum. Since the flux ϕ_2 is assumed to have sine wave distribution, the voltage generated in any coil will be proportional to the sine of the angle between that coil and the coil *a*. This voltage causes a current to flow in each coil and these currents produce a flux ϕ_c along the axis at right angles to the original flux

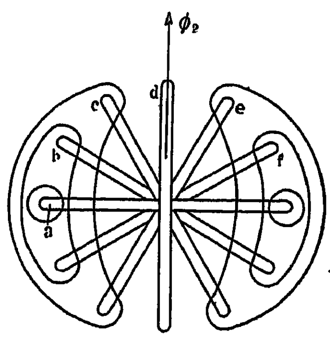

Fig. 2

ϕ_2. We will assume that this flux has a sine wave distribution, although this assumption will have to be justified later. Since the flux ϕ_2 is alternating the voltages generated in the coils by cutting this flux will also be alternating voltages, having maximum values at the time ϕ_2 is maximum and minimum values when ϕ_2 is minimum. The flux ϕ_c will also alternate because the currents producing this flux alternate with the voltages producing them. The alternating flux ϕ_c induces a transformer voltage in the coils threaded by this flux.

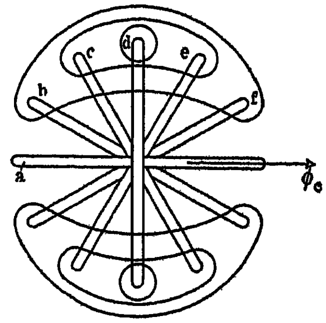

Fig. 3

Since the coil *d* includes all of this flux the voltage induced in it will be a maximum while the voltage induced in the coil *a* will be zero as it is not threaded by any of this flux. As we have assumed sine wave distribution of the flux ϕ_c the transformer voltage induced in any given coil will be proportional to the sine of the angle of the coil from the coil at *a*. Then in coil *d* there is a voltage E_3, Fig. 4 induced by the coil cutting the flux ϕ_2 and this voltage is in phase with this flux, as shown above. In addition to this voltage, another voltage E_6 is generated in this coil by transformer action of the flux ϕ_c. This voltage lags 90 deg. behind the flux ϕ_c because the transformer voltage always reaches its maximum 90 deg. later than the maximum of the flux which generates the voltage. Since the flux ϕ_c is the total flux, produced by the currents I_3, threading through the secondary along the axis at right angles to the primary axis, the only additional impedance to the current I_3 in the coil *d* is the resistance of the coil *d*. Then the $I\,r$ drop of the coil *d* will be overcome by the voltage which is the difference between the voltages E_3 and E_6, or $I_3\,r_2$ in Fig. 4. Due to iron loss, the flux ϕ_c is not exactly in phase with the current I_3 but lags slightly behind by the angle λ. For this reason, the voltage $I_3\,r_2$ is not exactly at right angles to the voltage E_6. We will assume in this analysis that the iron loss

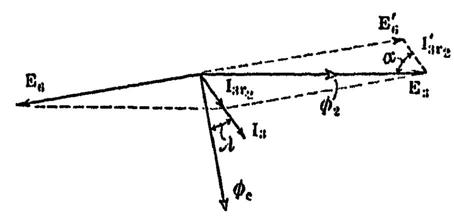

Fig. 4

is proportional to the square of the flux density in the iron, which is very close to the truth. With this assumption, the angle between the current I_3 and the flux ϕ_c is constant. Since there is no other winding threaded by the flux ϕ_c, than the secondary winding, the angle between E_3 and E_6 will remain constant as will the angle between ϕ_c and ϕ_2. Since E_3 depends on the speed, being a cutting voltage, the value of the different vectors will vary through a wide range but this will not vary their relative values nor the angles between them.

There are similar voltages to the voltages E_3 and E_6

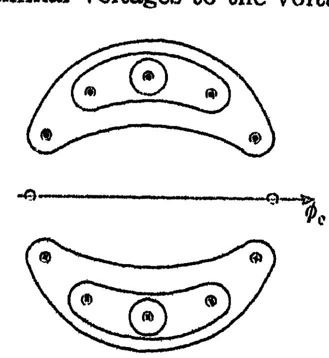

Fig. 5

generated in each of the coils *a*, *b*, *c*, *e* and *f*, although these voltages are not of the same value as the voltages generated in the coil *d* as was shown above. However, the voltage E_3' generated in any coil by that coil cutting the flux ϕ_2 will have the same phase as the voltage E_3 generated in the coil *d* by cutting the same flux. In the same way the transformer voltage E_6' generated

in any coil by the flux ϕ_c will have the same phase as the voltage E_6 in coil d but will be smaller. As shown above, these voltages are both proportional, in the different coils, to the sine of the angle between the given coil and the coil at a. It follows then that the difference between these voltages will also be proportional to the sine of the angle between the coil in question and the coil at a. But since the current I_3 in any coil is proportional to the difference between the voltages E_3 and E_6 it follows that the current I_3 in any given coil will be proportional to the sine of the angle between that coil and the coil at a. In other words the currents I_3 are in phase in the different coils and the current is distributed according to the sine law as shown in Fig. 5. It follows that the flux ϕ_c produced by these currents will have a sine wave field form. But this was the assumption made in the beginning so that all conditions for the production of the flux ϕ_c with sine wave field form have been met, and that will be its field distribution.

In addition to these voltages E_3 and E_6 there are two

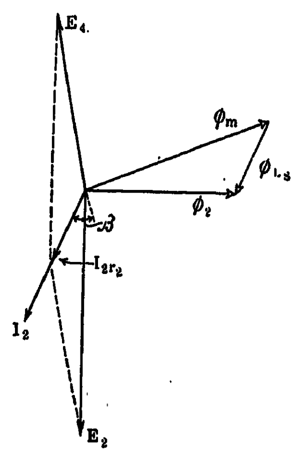

Fig. 6

other voltages generated in each coil, first a transformer voltage due to the flux ϕ_2 threading the coil and second a cutting voltage due to the coil cutting the flux ϕ_c. This transformer voltage is maximum in the coil a which is threaded by all of the flux ϕ_2 and minimum in the coil d which is threaded by none of the flux ϕ_2, and is proportional in the different coils to the cosine of the angle between the given coil and the coil at a. Also the voltages induced by the coils cutting the flux ϕ_c is maximum in the coil a and zero in the coil d, being proportional to the cosine of the angle between the given coil and the coil at a, since the flux ϕ_c has sine distribution. The vector diagram for coil a is shown in Fig. 6 where E_2 is the transformer voltage due to the flux ϕ_2 and E_4 is the voltage due to the coil a cutting the flux ϕ_c. E_2 lags 90 deg. behind ϕ_2 while E_4 is 180 deg. from ϕ_c. Since all leakage fluxes are considered in ϕ_2 the voltage difference between E_2 and E_4 will overcome the $I\,r$ drop of the coil a and force a current I_2 through the coil in phase with this voltage difference $I_2\,r_2$. Since both E_2 and E_4 are proportional to the cosine of the angle between any given coil and the coil at a, the current I_2 in that coil will be proportional to the cosine of the angle between that coil and the coil at a. It follows that the current I_2 will be distributed in the various coils according to the sine law as shown in Fig. 7.

In any other coil than a or d all four voltages E_3, E_6, E_2 and E_4 will be generated and the net current in that coil will be proportional to the net difference between all these voltages. However, this net current will be equal to the sum of the two currents I_3 and I_2 assumed

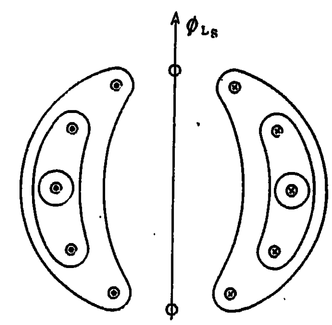

Fig. 7

to flow in the given coil in the above analysis. It makes no difference whether the net current or the two components are considered. Fig. 8 shows the vector diagram for the coil b 30 deg. from coil a. As shown, the total current I_S is the sum of I_3^1 and I_2^1, the voltages E_3^1 and E_6^1 being one half (or sine 30 deg.) of their values in coil d while the voltages E_2^1 and E_4^1 have .866 (or cos 30 deg.) of their values in coil a.

As shown in Fig. 5, the current I_3 is so distributed

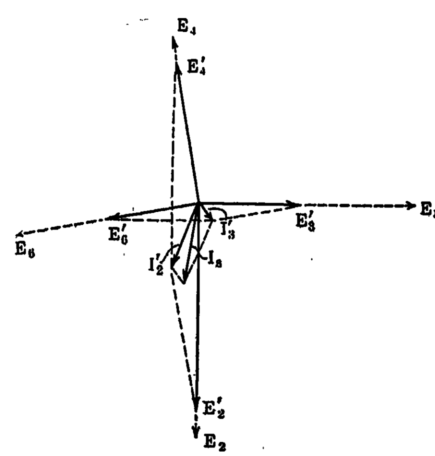

Fig. 8

in the secondary as to produce no ampere-turns along the primary axis, and will not affect the primary winding directly. However, the currents I_2 are so distributed as to act directly on the same flux path as the primary winding so that this current will be transferred to the primary by transformer action.

The flux ϕ_c is the total flux produced by the currents I_3 but the currents I_2 produce a leakage flux ϕ_{LS} in phase with I_2 along the primary axis and as we saw

above the flux ϕ_2 is the sum of ϕ_m and ϕ_{ls}. Therefore, the flux ϕ_m is the difference between the flux ϕ_2 and ϕ_{ls}, as shown in Fig. 6.

In order to force the flux ϕ_m (Fig. 9) across the air gap a resultant magnetizing current must flow in the primary winding producing ampere turns slightly leading the flux ϕ_m by the angle λ_1. (This angle is caused by the iron loss as we saw above.) But before these ampere-turns can act on the flux path the ampere-

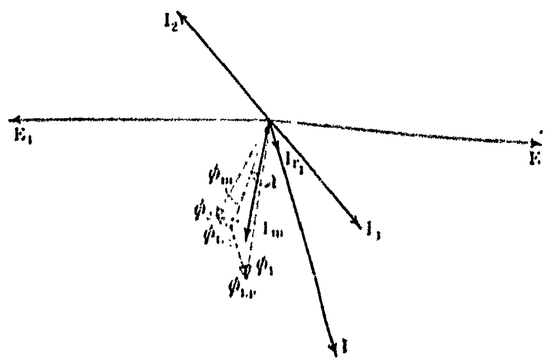

FIG. 9

turns produced by the currents I_2 acting on this same path must be neutralized by equal and opposite ampere-turns produced by the primary winding. Then the total primary current will be equal to the sum of a current I_1 (Fig. 9) equal and opposite to the secondary current (assuming one to one ratio) and a magnetizing current I_m leading the flux ϕ_m by the angle λ_1. The total current will be I in Fig. 9.

Due to this current I a leakage flux ϕ_{lp}, in phase with I is produced threading the primary winding as shown

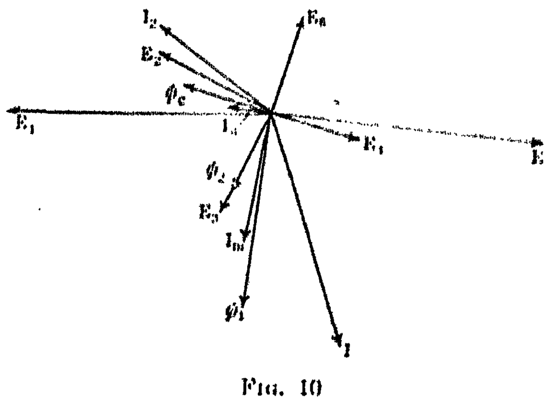

FIG. 10

in Fig. 1, and the total flux threading the primary is ϕ_1 the sum of ϕ_m and ϕ_{lp} as shown in Fig. 9. Due to this flux ϕ_1 a transformer voltage E_1 is induced in the primary winding and the voltage difference $I\,r_1$ between the line voltage and this transformer voltage, forces the current I through the resistance of the primary winding.

The complete vector diagram for the conditions assumed is shown in Fig. 10. Although Fig. 10 shows the complete diagram, it is in such form as to make it very hard to follow such changes as are produced by a change in speed. A simplified diagram which shows all the component vectors is shown in Fig. 11. As will be noted, the two component primary currents I_1 and I_m are considered instead of the total current I, and since I_2 is equal to I_1, the net result is as though the current I_2 flows through both primary and secondary windings while an additional magnetizing current, I_m flows through the primary winding only. There is an additional magnetizing current I_3 in the secondary but as was shown, it does not directly affect the primary. The voltages $I_m\,x_1$, $I_2\,x_1$ and $I_2\,x_2$ are reactance voltages

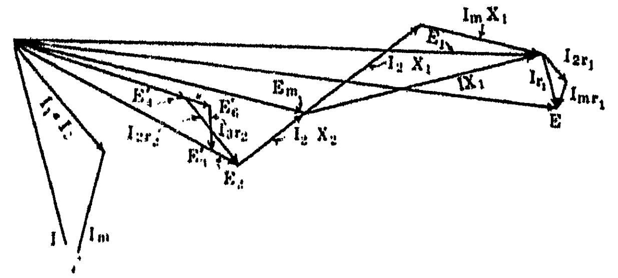

FIG. 11

generated in the windings by the leakage fluxes produced by the currents I_m in the primary and I_2 in both primary and secondary. The voltage $I\,x_1$, which is the sum of $I_m\,x_1$ and $I_2\,x_1$ is the transformer voltage produced in the primary winding by the total primary leakage flux ϕ_{lp}. $I\,x_1$ is at right angles to ϕ_{lp} (Fig. 9). In the same way $I_2\,x_1$ and $I_2\,x_2$ are at right angles to and proportional to I_2. $I_m\,r_1$ has the same phase as I_m while $I_2\,r_1$ and $I_2\,r_2$ have the same phase as I_2.

For the time being we will consider that the primary resistance is zero and that there is no primary iron loss, so that I_m is in phase with ϕ_m and λ_1 is zero. Later we will show the effect of these two losses on the diagram. With this assumption, the vectors E and $I\,r_1$ ($= I_m\,r_1 + I_2\,r_1$) drop out and the diagram which we are considering is shown in Fig. 12.

Since I_m is in phase with ϕ_m, $I_m\,x_1$ will be parallel to E_m because both $I_m\,x_1$ and E_m, being transformer

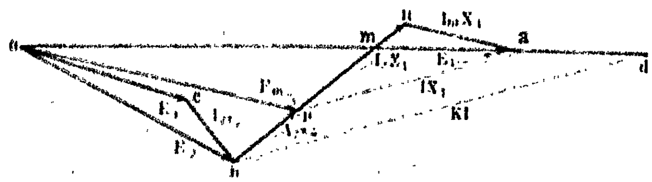

FIG. 12

voltages, are at right angles to the current producing the transformer flux. Also we know that $I_m\,x_1$ will have a constant ratio to E_m because $I_m\,x_1$ is the voltage generated by a leakage flux produced by the current I_m while E_m is the voltage generated in the same winding by the useful flux ϕ_m produced by this same current I_m. The ratio between $I_m\,x_1$ and E_m is the ratio between

the permeance of the leakage flux path and the permeance of the useful flux path. If we let $\frac{E_m}{I_m} = z_m$, $\frac{E_m}{I_m x_1}$ will equal $\frac{z_m}{x_1}$ where $\frac{z_m}{x_1}$ is the ratio between the permeance of the useful flux path and the permeance of the leakage flux path.

Since E_m and $I_m x_1$ have a constant ratio, regardless of other changes in the diagram, it follows that the lines $o\,m$ and $m\,a$ in Fig. 12 are proportional and have a constant ratio (similar sides of similar triangles $o\,p\,m$ and $m\,n\,a$) and that the point m is a fixed point on the line $o\,a$ regardless of other changes in the diagram. Furthermore, because $p\,m$ and $m\,n$ are the other sides of these same triangles, they will also be similar and, therefore, proportional

to $\frac{z_m}{x_1}$

$$\frac{m\,n}{p\,m} = \frac{x_1}{z_m} \qquad \frac{m\,n + p\,m}{p\,m} = \frac{z_m + x_1}{z_m}$$

$$m\,p = \frac{z_m}{z_m + x_1} \times p\,n$$

$$p\,b + p\,m = I_2\,x_2 + I_2\,x_1\left(\frac{z_m}{z_m + x_1}\right)$$

Therefore,

$$m\,b \text{ (which } = p\,m + b\,p) = I_2\left(x_2 + x_1\frac{z_m}{z_m + x_1}\right)$$

Since x_2, x_1 and z_m are constants, it follows that the line $m\,b$ is proportional to I_2.

The line $p\,a$ is proportional to $I\,x_1$ and therefore to I, but we do not wish to have to find the point p in the final diagram so we will find another line from b which will be proportional to I. From b draw the line $b\,d$ parallel to $p\,a$ until it intersects $o\,a$ at d. Then $\frac{b\,d}{p\,a} = \frac{m\,b}{m\,p}$. But $\frac{m\,b}{m\,p}$ is a constant, so it follows that $\frac{b\,d}{p\,a}$ is a constant and $b\,d$ is therefore, proportional to I, since pa is proportional to I.

Since the sides $m\,b$ and $m\,p$ of the similar triangles $a\,m\,p$ and $d\,m\,b$ have a constant ratio and since $a\,m$ is constant as proved above, it follows that $m\,d$ will also be constant and, therefore, that the point d is a fixed point on the line $o\,a$, regardless of any changes in the diagram due to changes in the load or speed of the motor.

To prove

$$\frac{o\,a}{o\,d} = \frac{z_m}{z_m + x_2}$$

and

$$\frac{o\,m}{o\,d} = \left(\frac{z_m}{z_m + x_1}\right)\left(\frac{z_m}{z_m + x_2}\right)$$

Let

$o\,m = z_m$, then $m\,a = x_1$ and $a\,d = x_2\left(\frac{z_m + x_1}{z_m}\right)$

$$o\,d = z_m + x_1 + x_2\left(\frac{z_m + x_1}{z_m}\right)$$

$$= \frac{(z_m + x_1)\,(z_m + x_2)}{z_m}$$

$$\therefore \frac{o\,m}{o\,d} = \frac{z_m^2}{(z_m + x_1)\,(z_m + x_2)}$$

$$o\,a = z_m + x_1$$

$$\therefore \frac{o\,a}{o\,d} = \frac{(z_m + x_1)\,z_m}{(z_m + x_1)\,(z_m + x_2)} = \frac{z_m}{z_m + x_2}$$

Redrawing Fig. 12, and leaving out unnecessary lines, we have Fig. 13, where $X_2 = x_2 + x_1\frac{z_m}{z_m + x_1}$ and $o\,d = E_1 \times \frac{z_m + x_2}{z_m}$

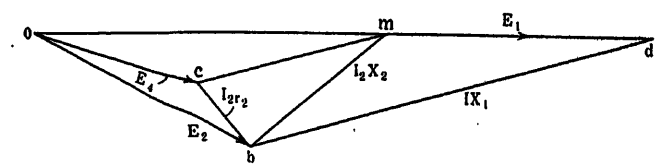

FIG. 13

It was proved in connection with Fig. 4 that the angle between the fluxes ϕ_2 and ϕc is constant regardless of other changes in the vector diagram. Since E_2 is 90 deg. from ϕ_2 and E_4 is in line with ϕc it follows that the angle between E_2 and E_4 is constant. It has also been proved that the lines $m\,b$ and $b\,c$ are both proportional to the secondary current I_2 so the triangle $m\,b\,c$ will always have the same shape and proportion of parts regardless of its size. We know further that the ratio $\frac{E_6}{E_3}$ is constant, from Fig. 4. It is also true that the ratio $\frac{E_4}{E_6} = \frac{\text{speed}}{\text{syn. speed}}$ because E_4 is the voltage generated in the coil a by putting the flux ϕc while E_6 is the voltage generated in the identical coil d by transformer action of the same flux ϕc and the cutting voltage becomes equal to the transformer voltage at synchronous speed. In the same way $\frac{E_3}{E_2} = \frac{\text{speed}}{\text{syn. speed}}$ Then it follows that $\left(\frac{\text{speed}}{\text{syn. speed}}\right)^2 = \frac{E_4\,E_3}{E_6\,E_2} = \frac{E_4}{E_2}$ times a constant. Knowing these

relations, the effect of changes of load on the diagram can be followed more easily.

If the load is decreased, the triangle $m\ b\ c$ becomes smaller while if the load is increased, the triangle becomes larger. But, regardless of its size the vertices b and c must remain on the opposite sides of the constant angle $b\ o\ c$ having its vertex at o. In order to fulfill these conditions the angle $b\ o\ c$ must swing down as the triangle $m\ b\ c$ increases in size. Fig. 14 shows how the diagram changes with reference to that shown in Fig. 13 when the load is increased over that in the case of Fig. 13. Since the line $o\ c$ is much shorter, in proportion to the line $o\ b$, in Fig. 14, than it is in Fig. 13, it follows that the speed for the condition of Fig. 14

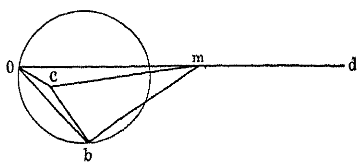

Fig. 14

will be lower than that for Fig. 13. If the line $o\ c$ is made equal to zero or in other words, the speed is zero, the diagram changes to that shown in Fig. 15. If, on the other hand, the load is decreased so that $o\ c$ becomes longer and the triangle $b\ o\ c$ swings up, at synchronous speed the diagram takes the form as shown in Fig. 16. In this case $o\ c$ represents both E_4 and E_6 while the line $o\ b$ represents both E_3 and E_2. Then triangle $b\ o\ c$ will be similar to the triangle shown in dotted lines in Fig. 4 and composed of the sides E_3, E_6^1, and $I_3\, r_2$. It is evident that, since the line $b\ c$ coincides with the line $I_3\, r_2$ and $b\ c = I_2\, r_2$, at this speed I_2 will equal I_3.

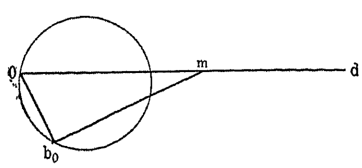

Fig. 15

In Appendix A, will be found the proof of two propositions; first that if two circles are drawn on similar parts of the hypotenuse and one leg of a right triangle, any right triangle having one vertex at the vertex of the original triangle and the other two vertices on the respective circles will be similar to the original triangle; and second, that the two vertices on the circles will lie on the two sides of a constant angle having its vertex at the intersection of the two circles. But these are the conditions to be fulfilled in the diagram above as the load changes, namely that the triangle $m\ b\ c$ must always be the same shape with its vertex at the point m and the two vertices b and c must lie on the two sides of the constant angle $b\ o\ c$, between the voltages E_2 and E_4. Therefore, the point b will follow a circle as the load changes while the point c will follow another circle as the load changes. It is then necessary to determine the location of these circles with respect to the line $o\ d$.

A circle is determined by three points on its circumference. We know three points on the circle locating the point b as follows.

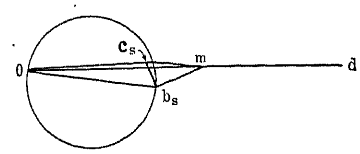

Fig. 16

First, both circles must pass through the point o, as the vertex of the constant angle must lie on their intersection.

Second, Fig. 15 gives the location, b_0, Fig. 17, of the point b for zero speed conditions.

Third, the diagram of Fig. 16 gives the location, b_s Fig. 17, of the point b for the synchronous speed condition.

Then, the first circle locating the point b can be drawn through these three points. The second circle is determined from the first by the use of the first proposition in Appendix A. A line $m\ b_2\ b_1$ is drawn from m through the center of the first circle (Fig. 17) and on this line as one leg, the triangle $m\ b_1\ c_1$ is constructed similar to the triangle $m\ b_0\ o$. Then the line $c_1\ c_2$ is laid off so that $\frac{c_1\ c_2}{b_1\ b_2} = \frac{c_1\ m}{b_1\ m}$; $c_1\ c_2$ is the diameter of the second circle.

Then any line such as $m\ b$ drawn to the first circle and the line $b\ c$ at right angle to $m\ b$ form the two legs of the right triangle showing the voltage drops due

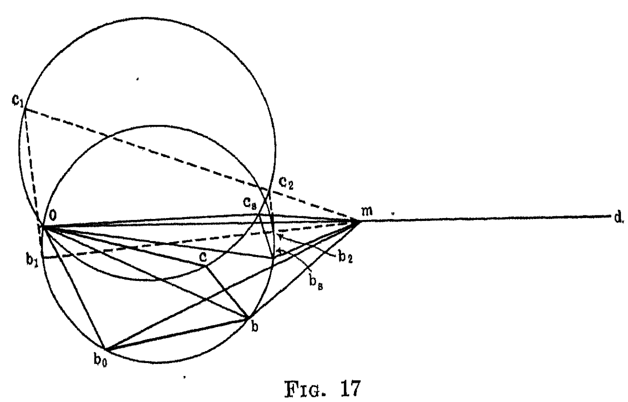

Fig. 17

to the load current and the line $o\ c$ represent E_4 while the line $o\ b$ represents E_2. Since speed is proportional to $\sqrt{\frac{E_4}{E_2}}$ it follows that the speed for the load shown by the line $m\ b$ can be found from the lengths of $o\ c$ and $o\ b$.

It will be shown later that the torque produced may also be found from the two lengths $o\ c$ and $o\ b$. It has already been shown that the primary current is represented by the line $d\ b$ in amount but that $d\ b$ is at right angles to the current I, while $m\ b$ represents the secondary current in amount, but 90 deg. out of phase. Fig. 17, therefore, shows speed, torque, primary current and secondary current and the phase of the two currents. This diagram may be very much simplified without destroying its usefulness.

Since the line $o\ c$ is the only one determined by the second circle, it is very desirable to find a line in the first

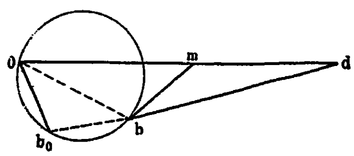

FIG. 18

circle proportional to $o\ c$. Such a line proportional to $o\ c$ is the line $b\ b_0$ Fig. 17, as is proved in Appendix *B*. The diagram may then be drawn as shown in Fig. 18, where the different lines represent the following quantities.

$o\ d$ represents E_1 to scale and in phase.

$o\ b$ represents E_2 to a different scale and in phase.

bb_0 represents E_4 to a still different scale but not in phase.

$b\ d$ represents I to a certain scale 90 deg. from correct phase.

$m\ b$ represents I_2 to a different scale 90 deg. from correct phase.

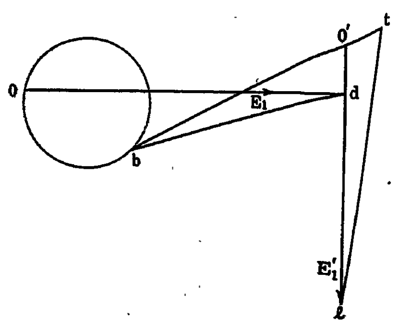

FIG. 19

So far we have neglected the primary iron loss and the primary copper loss in drawing the diagrams.

The iron loss in the stator is proportional to the flux ϕ_1 squared. Since we are assuming a constant voltage E_1 and therefore a constant flux ϕ_1 the stator iron loss will be constant. The rotor iron loss due to the primary flux is proportional to the flux ϕ_2 squared, so it will decrease as the load increases since the flux ϕ_2 decreases with an increase in load. But the motor slows down as the load increases and the rotor iron loss is very small at synchronous speed and larger at lower speeds, so the iron loss will increase for a given flux ϕ_2 at lower speeds. The increase in loss, due to the lower speed offsets the decrease in loss due to the decrease of the flux ϕ_2. For this reason, it is justifiable to assume that the total primary iron loss is constant if E_1 is constant. This loss may be considered as being supplied from the line by means of a small additional current I_{Fe} flowing through the primary n addition to the current I, considered so far. Since $b\ d$ represents I in amount but is 90 deg. from the proper phase, if we wish to add the current I_{Fe} to I we may do so by drawing a line $d\ o^1$, Fig. 19, representing I_{Fe} to the same scale as $b\ d$ represents I and at right angles to its correct phase. Then the total primary current I^1 will be $o^1\ b$ Fig. 19, to the same scale as $b\ d$ represents I.

Due to this current I^1 there is a voltage drop $I^1\ r_1$ in the primary caused by the primary resistance r_1. This voltage is in phase with the current I^1 and there-for at right angles to the line $o^1\ b$, when $o\ d$ represents the phase of the voltage E_1 and $o^1\ b$ is 90 deg. from I^1. In order to simplify the diagram, we will let $o^1\ b$ repre-

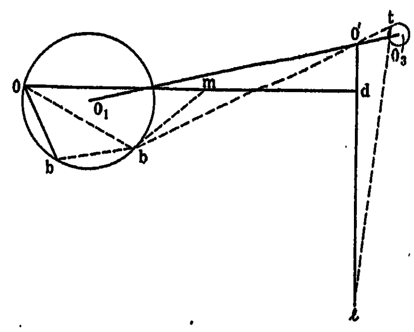

FIG. 20

sent the phase of the primary currents in which case the primary voltage must be represented by a line $o^1\ e$ at right angles to $o\ d$, and the primary resistance drop $I^1\ r_1$ is represented by a line $o^1\ t$, Fig. 19, in line with $o^1\ b$. Since the total line voltage is the sum of E_1 and $I^1\ r_1$, we will lay off $E_1 = o^1\ e$ from o^1 at right angles to $o\ d$ in which case $e\ t$ represents the line voltage to the same scale as $o^1\ e$ represents E_1.

Since $o^1\ t$ and $o^1\ b$ are both proportional to I^1 and the point b lies on a circle at all loads, the point t will also follow a circle. The two circles will have the ratio of diameters of $\frac{o^1\ t}{o^1\ b}$ and the centers of the two circles will be on the same line passing through the point o^1. The centers will also be at such distances that $\frac{o^1\ o_1}{o^1\ o_3}$ (Fig. 20) $= \frac{o^1\ t}{o^1\ b}$ (Fig. 19). Then Fig. 20 shows the complete circle diagram in which the different lines represent the following quantities.

$o^1\ b$ represents primary current in phase and to scale,

$e\ t$ represents line voltage in phase and to scale,

cos of angle $o^1\ t\ e$ equals power factor,

the lines $o\,b$ and $b\,b_0$ will give the speed and torque, the efficiency may be obtained from $\frac{\text{speed} \times \text{torque}}{E \times I^1 \times \cos o^1\, l\, e}$

If secondary current is desired it is represented by $m\,b$ to scale and phase.

It only remains to find the various constants and scales of the lines representing these quantities, so that the numerical results may be read from the diagram, and so that the diagram may be constructed when the constants of the given machine are known.

Torque

Torque is produced by the reaction of a current in a conductor on the flux cutting the conductor. Its instantaneous value is proportional to the product of the instantaneous current and the flux at that instant. If the flux and current are both alternating and in phase the maximum torque is proportional to the product of the maximum current and the maximum flux. If the current and flux are not in phase the maximum torque is proportional to the product of the maximum flux and maximum current and the cosine of the angle between them.

Since the cutting voltage is proportional to the flux times the speed, the flux will be proportional to the $\frac{\text{cutting voltage}}{\text{speed}}$ and this voltage will represent the flux in phase. It follows then that the torque is proportional to the

$$\frac{\text{cutting voltage} \times \text{current} \times \text{cos of angle between them}}{\text{speed}}$$

The secondary coil a Fig. 2, carries the current I_2 as shown in Fig. 6 and has the voltage E_4 generated in it by the flux ϕ_c. Then the torque produced will equal $\frac{E_4\,I_2\cos\beta}{\text{speed}}$. Each of the other coils will have a similar current and, therefore, will produce an additional torque. The total torque will be proportional to that in coil a. In the same way the current I_3 in coil d produces a torque with the flux ϕ_2, as do the currents I_3^1 in the other secondary coils and the total torque produced by all these currents is proportional to the torque produced by the coil d, or to $\frac{I_3\,E_3\cos\alpha}{\text{speed}}$

Since I_3 is less than 90 deg. from the cutting voltage E_3 as shown in Fig. 4, while I_2 is more than 90 deg. from the cutting voltage E_4 as shown in Fig. 6 it follows that one of these torques will be positive and the other will be negative. Then the net motor torque will be proportional to $\frac{I_2\,E_4\cos\beta - I_3\,E_3\cos\alpha}{\text{speed}}$

In order to evaluate this expression it is necessary to go back to Figs. 4 and 6 to find the relations between the various quantities. In Fig. 21, draw E_2 equal to E_2 in Fig. 6 and E_4^1 equal to but opposite to E_4 in Fig. 6. Then the line $I_2\,r_2$ will be equal to and in phase with the line $I_2\,r_2$ in Fig. 6. Along E_2 lay off a length equal to E_3 Fig. 4. Then a line at the same angle with E_3 as that between E_3 and a vector opposite to E_6 (Fig. 4) will fall along E_4^1. This is because the angle between E_2 and E_4^1 equals the angle between E_3 and E_6^1 since E_2 and E_3 are at right angles and E_4 and E_6 and also at right angles.

We see from Fig. 21, that $I_2\cos\beta = \frac{E_2\cos\theta - E_4}{r_2}$

and that $I_3\cos\alpha = \frac{E_3 - E_6\cos\theta}{r_2}$.

but

$E_3 = E_6\frac{\sin(90 + \lambda)}{\sin\alpha} = E_6\frac{\cos\lambda}{\sin\alpha}$ so that $I_3\cos\alpha =$

$$\frac{E_6\left(\frac{\cos\lambda}{\sin\alpha} - \cos\theta\right)}{r_2}$$

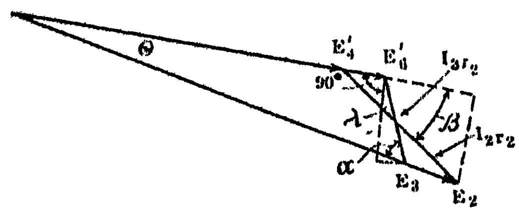

Fig. 21

and $\frac{E_3}{E_2} = \frac{\text{speed}}{\text{syn. speed}}$ while $\frac{E_6}{E_4} = \frac{\text{syn. speed}}{\text{speed}}$

so that $I_3\,E_3\cos\alpha = \frac{E_2\,E_4}{r_2}\left(\frac{\cos\lambda}{\sin\alpha} - \cos\theta\right)$

Then the net torque will be proportional to

$$\left(\frac{E_2\cos\theta - E_4}{r_2 \times \text{speed}}\right)E_4 - \frac{E_2\,E_4}{r_2 \times \text{speed}}\left(\frac{\cos\lambda}{\sin\alpha} - \cos\theta\right)$$

or $$\frac{E_2\,E_4\left(2\cos\theta - \frac{\cos\lambda}{\sin\alpha}\right) - E_4^2}{r_2 \times \text{speed}}$$

From Fig. 21, we see that $\frac{\cos\lambda}{\sin\alpha} = \frac{I_3\,r_2\cos\lambda}{E_6\sin\theta}$ but $I_3\cos\lambda$ is the current producing the crossfield flux ϕ_c by its magnetizing action ($I_3\sin\lambda$ furnishes iron loss in the same way as I_{e_r} furnishes primary iron loss.) so that E_6 is proportional to $I_3\cos\lambda$.

Since ϕ_c includes both the flux that goes into the stator (around a path having the same permeance as the path followed by the flux ϕ_m) and the flux following the path of the secondary leakage flux ϕ_{ls}, the total permeance of the path for the flux ϕ_c is proportional

to $z_m + x_2$, so that $E_6 = \dfrac{I_3 \cos \lambda}{z_m + x_2}$

$$\therefore \frac{\cos \lambda}{\sin \alpha} = \frac{r_2}{(z_m + x_2) \sin \theta}$$

and the net torque is proportional to

$$\frac{E_2 E_4 \left(2 \cos \theta - \dfrac{r_2}{(z_m + x_2) \sin \theta} \right) - E_4^2}{\text{speed}}$$

Speed

As has already been shown $\left(\dfrac{\text{speed}}{\text{syn. speed}} \right)^2$

$$= \frac{E_3}{E_2} \times \frac{E_4}{E_6} = \frac{E_4}{E_2} \times \frac{E_3}{E_6}$$

but, $\dfrac{E_3}{E_6} = \dfrac{\cos \lambda}{\sin \alpha} = \dfrac{r_2}{(z_m + x_2) \sin \theta}$

Therefore, speed = syn. speed

$$\times \sqrt{\frac{E_4}{E_2}} \times \sqrt{\frac{r_2}{(z_m + x_2) \sin \theta}}$$

Primary Current I^1

As shown above $b\,d$ represents I to a certain scale and $o^1 b$ represents I^1 to the same scale.

but $b\,d = p\,a \left(\dfrac{m\,b}{m\,p} \right)$ and $p\,a = I\,x_1$ while $\dfrac{m\,b}{m\,p}$

$$= \frac{x_1 \dfrac{z_m}{z_m + x_1} + x_2}{x_1 \dfrac{z_m}{z_m + x_1}}$$

Therefore,

$$b\,d = I\,x_1 \frac{x_1 \dfrac{z_m}{z_m + x_1} + x_2}{x_1 \dfrac{z_m}{z_m + x_1}} \text{ or } I \left(x_1 + \frac{z_m + x_1}{z_m} x_2 \right)$$

and it follows that

$$o^1 b = I^1 \left(x_1 + \frac{z_m + x_1}{z_m} x_2 \right) = I^1 X_1$$

Secondary Current I_2

As was shown above

$$m\,b = I_2 \left(x_2 + \frac{z_m}{z_m + x_1} x_1 \right) = I_2 X_2$$

so it is evident that $m\,b$ does not represent I_2 to the same scale as $b\,d$ represents I^1.

Magnetizing Current I_m

When the current I_2 is zero, the only currents in the primary will be the magnetizing + iron loss currents or I_m^1. The actual magnetizing current will be I_m represented by $d\,m$.

Then $$d\,m = I_m X_1$$

The location of the circle with reference to the line $o\,d$ is more easily found than by drawing it through the three points o, b_o and b_s, as will be shown below in Fig. 22. On the line $o\,m$ construct the triangle $o\,m\,b_o$ having $\dfrac{m\,b_o}{b_o\,o} = \dfrac{X_2}{r_2}$. Then the circle will pass through o and b_o. Unless the secondary resistance is too large, the circle will cut the line $m\,b_o$ at some other point such as b_L Fig. 22. Then for this load the triangle $m\,b_o\,o$ becomes $m\,b_L\,c_L$ and c_L will lie on the line $o\,m$. Since angle $o\,b_o\,b_L$ is a right angle and the three points o, b_o and b_L all lie on the circle, $o\,b_L$ must be the diameter of the circle. Then the angle $c_L\,o\,b_L$ is the constant angle θ between $o\,c_L$, representing E_4, and $o\,b_L$, representing E_2. Draw the line $b_L\,k$ from b_L perpendicular to $o\,m$ at k.

From Fig. 21, we know that

$$\tan \theta = \frac{r_2}{z_m + x_2 + r_2 \tan \lambda}$$

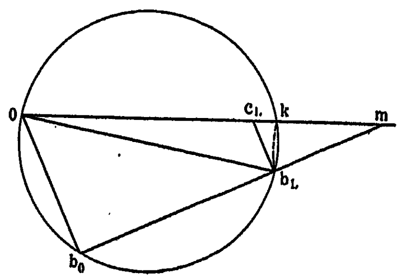

Fig. 22

so that

$$\frac{b_L\,k}{o\,k} = \frac{r_2}{z_m + x_2 + r_2 \tan \lambda}$$

also

$$\frac{b_L\,k}{m\,k} = \frac{r_2}{X_2}$$

Therefore

$$\frac{o\,k}{m\,k} = \frac{z_m + x_2 + r_2 \tan \lambda}{X_2}$$

and

$$\frac{m\,k}{o\,m} = \frac{X_2}{X_2 + z_m + x_2 + r_2 \tan \lambda}$$

but

$$o\,m = \left(\frac{z_m}{z_m + x_2} \right) \left(\frac{z_m}{z_m + x_1} \right) \times o\,d$$

so that

$$m\,k = o\,d \times \frac{X_2 \left(\dfrac{z_m}{z_m + x_2} \right) \left(\dfrac{z_m}{z_m + x_1} \right)}{X_2 + z_m + x_2 + r_2 \tan \lambda}$$

Therefore,

$$b_L\, k = o\, d \times \frac{r_2 \left(\dfrac{z_m}{z_m + x_1}\right)\left(\dfrac{z_m}{z_m + x_2}\right)}{X_2 + z_m + x_2 + r_2 \tan \lambda}$$

and

$$d\, k = o\, d \times \left[1 - \frac{(z_m + x_2 + r_2 \tan \lambda) \left(\dfrac{z_m}{z_m + x_1}\right)\left(\dfrac{z_m}{z_m + x_2}\right)}{X_2 + z_m + x_2 + r_2 \tan \lambda} \right]$$

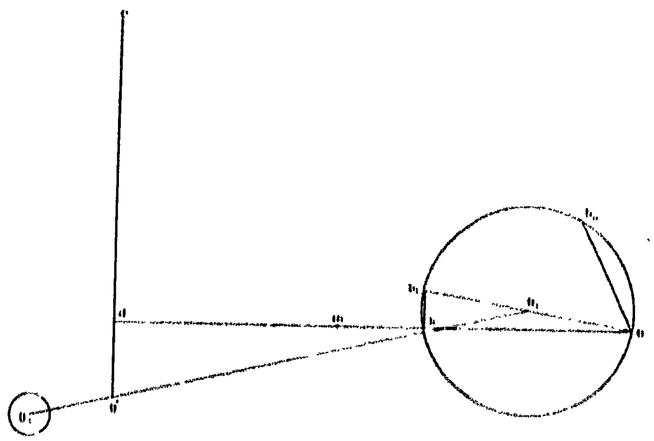

FIG. 23

THE FINAL CIRCLE DIAGRAM

Since the line $b\, b_o$ does not represent E_4 to scale, and since E_2 which is represented by $o\, c$, must be multiplied by a constant before using, regardless of whether it is to scale or not, there is no reason for drawing the part of the diagram d, o, o^1, c, b, to the same scale as the part $o^1\, e\, t$. On the other hand, it is more convenient to use a different scale for the currents than is used for voltages. The lines $o^1\, e$ and $o\, b$ will no longer represent E_1 and E_2 to the same scale.

Let S_e = volts per inch to which $o^1\, e$ represents E_1
and S_1 = amperes per inch to which $o^1\, b$ represents I.

Then

$$o^1\, e \text{ (Fig. 23)} = \frac{E_1}{S_e}$$

$$o^1\, d = \frac{I_{Ve}}{S_1} = \frac{\text{watts iron loss}}{E_1\, S_1} \quad (= \tfrac{1}{2} \text{ polyphase iron loss})$$

$$d\, o = \frac{E_1}{X_1\, S_1}\left(\frac{z_m + x_2}{z_m}\right)$$

$$d\, m = \frac{I_m}{S_1}$$

$$\tan \lambda = \frac{I_{Ve}}{I_m} = \frac{o^1\, d}{d\, m}$$

$$d\, k = o\, d$$

$$\left[1 - \frac{(z_m + x_2 + r_2 \tan \lambda)\left(\dfrac{z_m}{z_m + x_1}\right)\left(\dfrac{z_m}{z_m + x_2}\right)}{X_2 + z_m + x_2 + r_2 \tan \lambda} \right]$$

$$k\, b_L = o\, d \frac{r_2 \left(\dfrac{z_m}{z_m + x_1}\right)\left(\dfrac{z_m}{z_m + x_2}\right)}{X_2 + z_m + x_2 + r_2 \tan \lambda}$$

Then, $o\, b_L$ is the diameter of the circle locating the point b with its center at o_1.

$$o^1\, o_3 = o^1\, o\, \frac{r_1\, S_1}{S_e}$$

$$\text{diameter of circle } o_3 = o\, b_L\, \frac{r_1\, S_1}{S_e}$$

$$o\, b_o = \frac{r_2}{\sqrt{r_2^2 + X_2^2}}\left(\frac{z_m}{z_m + x_1}\right)\left(\frac{z_m}{z_m + x_2}\right) \times o\, d$$

Then on this diagram

$S_e \times o^1\, e$ = line voltage

$S_e \times e\, t$ = equivalent line voltage

$$S_1 \times b\, o^1 \left(\frac{o^1\, e}{e\, t}\right) = \text{primary amperes}$$

$$S_1 \left(\frac{z_m + x_1}{z_m}\right) \times m\, b \left(\frac{o^1\, e}{e\, t}\right) = \text{secondary amperes}$$

$$\frac{(o^1\, t)^2 + (e\, t)^2 - (o^1\, e)^2}{2\, (o^1\, t)\, (e\, t)} = \text{per cent power factor}$$

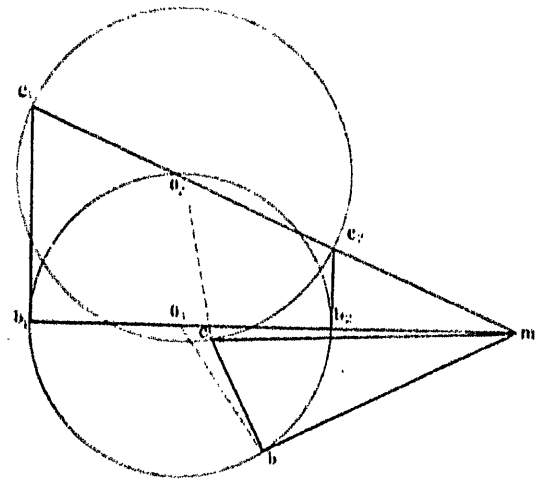

FIG. 24

$$K_1 \sqrt{\frac{b_o\, b}{o\, b}} = \text{speed (in rev. per min.)}$$

$$\left(\frac{o^1\, e}{e\, t}\right)^2 [K_2\, (o\, b)\, (b_o\, b) - K_3\, (b_o\, b)^2 .] - [\text{Friction (in h. p.) and windage}] = \text{h. p. output}$$

$$\text{Torque oz. ft.} = \frac{\text{h. p. output}}{\text{speed}} \times 84000$$

Efficiency

$$\frac{\text{h. p. output} \times 0.746}{\text{Primary volts} \times \text{primary amperes} \times \text{power factor}}$$

Where

$$K_1 = \sqrt{\frac{r_2}{(z_m + x_2) \sin \theta}} \times \sqrt{\frac{r_2^2 + X_2^2}{X_2^2}}$$

$$K_2 = 0.00134\, \frac{X_2}{r_2}\, \sqrt{r_2^2 + X_2^2} \left(\frac{z_m + x_1}{z_m}\right)^2 S_1^2$$

$$K_3 = 0.00134 \frac{r_2^2 + X_2^2}{r_2} \left(\frac{z_m + x_1}{z_m} \right)^2 S_1^2$$

Appendix A

Proposition 1. To prove that if two circles are drawn on similar parts of the hypotenuse and one leg of a right triangle, any right triangle having its vertex at the vertex of the original triangle and its other two vertices on the respective circles will be similar to the original triangle.

Construction, Fig. 24. Draw the triangle $m\, b_1\, c_1$ and the similar triangle $m\, b_2\, c_2$ and draw the two circles having their centers at o_1 and o_2 on $b_1\, b_2$ and $c_1\, c_2$ as diameters.

Then $\dfrac{m\, c_2}{m\, b_2} = \dfrac{m\, c_1}{m\, b_1} = \dfrac{c_1\, c_2}{b_1\, b_2} = \dfrac{o_2\, m}{o_1\, m}$

Draw in any line $m\, b$ and the line $m\, c$ at an angle from $m\, b$ so that angle $c\, m\, b$ = angle $c_1\, m\, b_1$, until it cuts the circle o_2 at a similar point to the position of b on the circle o. That is, if $m\, b$ does not cross the circle o_1, then $m\, c$ must not cross o_2. Connect c and b and draw the radii of the two circles $o_2\, c$ and $o_1\, b$.

Then $\dfrac{o_2\, c}{o_1\, b} = \dfrac{o_2\, m}{o_1\, m}$ (by construction)

And angle $o_2\, m\, c$ = angle $o_1\, m\, b$ (same angle $o_1\, m\, c$ added to equal angles $o_2\, m\, o_1$ and $c\, m\, b$). The triangles

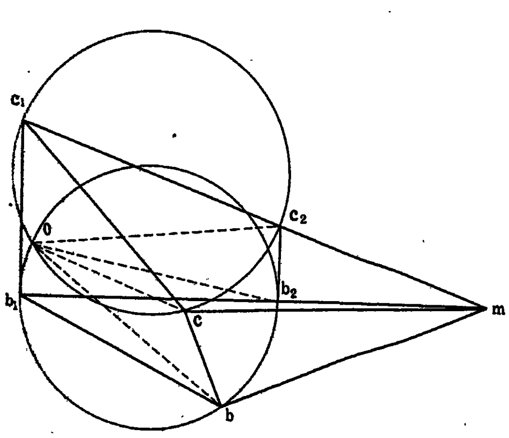

FIG. 25

$o_2\, m\, c$ and $o_1\, m\, b$ have two sides proportional and one angle equal. The two triangles are similar if the above limitation (that $m\, c$ shall not cross the circle o_2 if $m\, b$ does not cross the circle o_1) is placed on the double solution given by the triangles with two proportional sides and similar angles, (not the included angles) equal.

Then $\dfrac{m\, c}{m\, b} = \dfrac{o_2\, c}{o_1\, b} = \dfrac{m\, o_2}{m\, o_1}$ (similar sides of similar triangles)

Therefore, triangle $m\, b\, c$ is similar to triangle $m\, b_1\, c_1$ having two sides proportional and the included angles equal.

Proposition 2. To prove that the two vertices (on the circles) of the above triangle lie on the two sides of a constant angle having its vertex at the intersection of the circles.

Construction Fig. 25. Draw the circles o_1 and o_2 on the triangle $m\, b_1\, c_1$, as above, and call their point of intersection o. Draw the lines $o\, c_2$, $o\, b_2$, $o\, c$ and $o\, b$. To prove that angle $b\, o\, c$ = angle $b_2\, o\, c_2$. Draw in the lines $c_1\, c$ and $b_1\, b$.

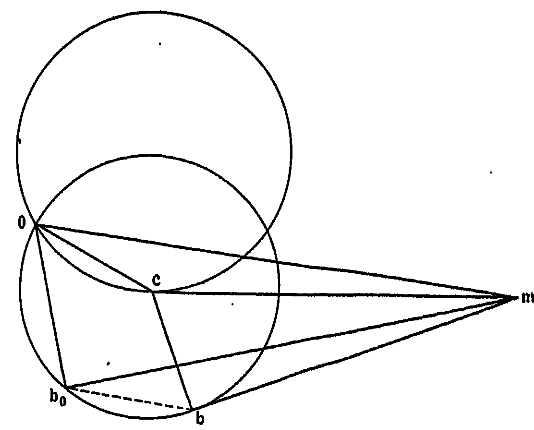

FIG. 26

Then since $\dfrac{m\, c}{m\, b} = \dfrac{m\, c_1}{m\, b_1}$ (proved above)

And angle $c_1\, m\, c$ = angle $b_1\, m\, b$ (same angle $b_1\, m\, c$ added to equal angles $c_1\, m\, b_1$ and $c\, m\, b$).

Triangles $c\, c_1\, m$ and $b\, b_1\, m$ are similar and angle $c\, c_1\, c_2$ = angle $b\, b_1\, b_2$

But angle $c\, c_1\, c_2$ = angle $c\, o\, c_2$ (each measured by ½ of same arc $c_2\, c$)

and angle $b\, b_1\, b_2$ = angle $b\, o\, b_2$ (each measured by ½ of same arc, $b_2\, b$)

Therefore, angle $b\, o\, c$ = angle $b_2\, o\, c_2$ (same angle $c\, o\, b_2$ subtracted from equal angles $b\, b_1\, b_2$ and $c\, c_1\, c_2$)

Appendix B

In Fig. 26, the two circles are drawn on similar parts of the triangle $m\, b_1\, c_1$ as in Fig. 24, and the triangle $m\, b\, c$ is any right triangle having one vertex at m and the other two vertices on the respective circles. Draw in the right triangle $m\, b_o\, o$, (similar to $m\, b\, c$ by Appendix A, Proposition 1,) and the lines $o\, c$ and $b_o\, b$.

Then, triangles $m\, c\, o$ and $m\, b\, b_o$ are similar having two sides proportional and the included angles equal.

Then $\dfrac{b\, b_o}{o\, c} = \dfrac{m\, o}{m\, b_o}$ or $\dfrac{b\, b_o}{o\, c}$ = ratio of diameters of two circles.

$\dfrac{m\, o}{m\, b_o}$ = ratio of diameters of two circles as shown in Appendix A.

Discussion

V. Karapetoff: The problem treated by the author is quite old, and a complete absence of correlation with the work of previous investigators is rather deplorable. Thus, the whole set of papers and a voluminous discussion before the Institute in 1918 are entirely ignored, as well as the subsequent contributions to the theory of this class of machines. In 1921 I showed the equivalence of the two theories of the single-phase induction machine JOURNAL, Vol. 40, page 640) and gave a new equivalent diagram, only one branch of which contains variable slip. A circular locus and a general-performance equation follow from this diagram directly. In Vol. 41 of the TRANSACTIONS there is a paper by Mr. Kostko which to me only shows that elementary mathematics is inadequate to lead to a simple theory of the machine, on the basis of two oppositely revolving fields. Mr. Perkins' paper shows that elementary mathematics is also inadequate in the cross-field theory. I hope, therefore, that this method of approach will now be definitely abandoned in favor of more advanced and shorter mathematical tools. In the JOURNAL for 1923 (Vol. 42, page 1181) I indicated the use of the scalar product of vectors in the solution of problems on locus diagrams of electrical machinery. This new use of vector analysis has already proved to be quite fruitful in a derivation of the exact circle diagram of the polyphase induction motor, without inversion or long formulas. The next step is to try this method on the single-phase machine, preferably using the unified equivalent diagram mentioned above.

Instead of a laborious step-by-step method, we ought to be able to write at once a few vectorial equations which represent all the essential relationships in the machine. For a solution of these simultaneous equations an advanced mathematical method (or some mechanical device) should be used, leaving the physical relationships "transparent" to the end. Of course, in numerical computations there will be products of complex quantities to evaluate, but there already are a few computing devices for a convenient handling of complex quantities, and probably in a few years computations with such quantities will be not nearly so tedious as they are now.

W. B. Hall: I have found considerable confusion existing in the minds of students and engineers concerning the fundamental analytical reasoning employed in the cross-field analysis of the single-phase induction motor.

We are accustomed to consider a conductor or circuit. We determine the voltage generated in this conductor, the resistance and reactance of its circuit, and we say that the current lags the voltage by an amount determined by the ratio of resistance to reactance, or we may consider the reactance as a voltage, and determine the net voltage acting in the circuit. The current flow will then be in phase with this net voltage, which is the $I\ R$ drop.

In analysis of polyphase motors, we sometimes use these methods of thought. We consider the voltage induced in conductor *d*. We say that the current will flow later, because of the reactance of the circuit, *and that d will then be in some other position*, where *e* or *f* were at first. This same attack may be applied to the single-phase motor. But it leads to unnecessary complexity.

So in single-phase-motor analysis another method of attack is used, as in this paper, but this new method has never to my knowledge, been clearly and definitely described. It has been rather assumed without distinct description, resulting in much confusion of thought on the part of many readers.

The new idea, which needs to be clearly stated, is that E_3 is not the voltage in any one conductor. If we plotted the value of the voltage in each successive conductor as it passd through the position *d*, we should obtain a sine wave, for ϕ_2 varies thus with time. A vector, E_3, may obviously be used to represent this sine wave, but it must be constantly kept in mind that E_3 does not represent the voltage in any one conductor, but represents the successive values of the voltage in each conductor as it passes through the position *d*.

Our usual ideas of reactance and resistance will not longer apply, since we have no circuit, but a succession of circuits. Hence Mr. Perkins abandons the idea of reactance and uses the method of determining the net voltage and considering that the instantaneous current flow in any of the successive circuits is directly proportional to the instantaneous value of this net voltage, which is the $I\ R$ drop at the successive instants of passing through position *d*.

Thus, in Fig. 4, E_3 represents the successive values of voltage in the rotor conductors due to cutting ϕ_2 at the instant that each passes through position *d*. E_6 represents the successive values of transformer voltage due to ϕ_c at the instant that each conductor passes through position *d*. I_3 is *not* the current in a coil *d* or in any coil, but represents the successive values of the current in each conductor as it passes through position *d*.

With this distinction clearly in mind some erroneous conclusions may be avoided, and some hazy conceptions made clearer.

P. L. Alger (by letter): Mr. Perkins has given a very complete description of a circle-diagram method of calculation for single-phase motors, based upon the cross-field theory. The clear inference is that he considers this method and this theory preferable to their alternatives, the equivalent-circuit method and the revolving-field theory. It is my opinion that the second method is greatly preferable to the first, and that the latter theory is more accurate than the former.

The equivalent-circuit method seems preferable because it avoids the time consumption and the errors of graphical construction; it permits the constants of the motor to be varied to take account of saturation, eddy currents, or heating without duplication of the work; it affords a clear visualization of the fundamental application of Kirchoff's laws for the electric circuits to the motor; and it permits the study of the motor to be carried to any degree of approximation or of refinement without change in the method. Two advantages are generally claimed for the circle diagram method, that it avoids the use of vector algebra and that it permits an easy visualization of the variation of motor characteristics with speed. The first advantage appears rather to be a handicap, as it avoids the use of a very convenient and simple means of calculation in a place where it is most fitted for use. The second advantage is an important one, but it is obtained at the expense of another sort of visualization which may be equally satisfactory. It is generally true in the history of mathematics that graphical methods are developed first, but are later supplanted by abstract calculational methods as familiarity with the work is acquired and the need for increased accuracy and decreased time consumption becomes important.

The cross-field theory is, according to all the books, precisely equivalent to the revolving-field theory, so that any result may be obtained by either method. This equivalence of the two theories was again brought out by Mr. Kimball and myself in our recent paper on "Torque Pulsations of Single-Phase Motors," A. I. E. E. JOURNAL, December 1924, page 1142, when we derived the same formulas for the double-frequency torque by the two methods independently. However, there is one phenomenon, that of eddy currents in the squirrel-cage winding, that can be better taken into account by the revolving-field theory. If a double squirrel-cage winding, or its equivalent is employed, the secondary resistance is much higher at double line frequency than at slip frequency. Thus, different values of secondary resistance must be used for the forward and backward field currents in the revolving-field theory. But in the cross-field theory, the secondary current in either axis is considered as a whole, instead of being divided into its slip frequency and double-frequency components. Thus, the resistances used in the secondary circuits of the cross-field theory must be intermediate between the high and the low value, but just what values they

should have is a problem that has not been solved. In short no method has been published of properly taking into account eddy currents in the secondary by the cross-field theory, and so in this respect the revolving-field theory is superior.

L. M. Perkins: Professor Karapetoff brought up a very desirable feature: that we use as few operations as possible in deriving our vector diagrams. But he is pointing toward Utopia, I think, on account of the difficulty of making direct analysis without use of step-by-step methods.

One point which Prof. Karapetoff brought up is that he secures by his method a single variable, which variable is of course the slip. In the single-phase induction motor this variable is slip $\times$ (2 — slip) which comes out in this paper as well as in the oppositely-rotating field theory, the slip being, of course, the slip with reference to the forward rotating field, and the (2 — slip being the slip with reference to the oppositely rotating field.

In Fig. 21 of my paper there is a diagram showing the counter e. m. f., E_4' and the e. m. f. generated in the secondary conductors by the net flux, which is E_2. If from E_4' a line is drawn perpendicular to E_4' until it intersects E_2, the intercept between this line and E_2 (that is, the end of the vector) will be the length of this variable, $s\ (2 - s)$. If this is carried through, you will find that in the final diagram the circle having the one variable s and $(2 - s)$ is the one shown.

Another point brought up was the question of considering the rotor conductor as stationary. If you want to find what the current is in the rotor as it turns, there is no better method than using the oppositely rotating fields, but if you want to know what the primary input is, what power factor you will get for a given load (since the primary is stationary and the only way the primary can transfer energy to the secondary is by a stationary field), it is best also to consider the secondary as stationary, taking care of the effects of rotation by considering the counter generated by rotation.

A good illustration of the trouble gotten into when considering the rotating-field method is apparent when one wishes to determine the effect of increasing the reactance of the secondary winding. According to the rotating-field method, there are two separate currents in the secondary winding, one of slip frequency and one of $(2 - s)$ frequency. But what happens if you increase the reactance as far as it affects the slip frequency? There is no large change, but according to the normal way of looking at it, the current of the $(2 - s)$ frequency would be cut out almost entirely. It is rather hard to see how the performance is really affected, unless one follows this method, which shows that all currents in the rotor are really of line frequency and that, therefore, an increase in reactance of the secondary will cut down or will affect all rotor currents; the slip frequency current, as well as the $(2 - s)$ frequency current due to the oppositely rotating field.

The original purpose of this paper was to bring out the theory of the single-phase induction motor in such a way that the average engineer could follow it, although of course it is still very complicated. There are no assumptions as are necessary when you consider the oppositely-rotating-field theory.

Effect of Full Voltage Starting
on the Windings of Squirrel-Cage Induction Motors

BY J. L. RYLANDER[1]
Member, A. I. E. E.

***Synopsis.**—This paper discusses the effect on the windings when starting squirrel-cage induction motors with full voltage. The data is presented in four groups: namely, (1) Special tests made with a vibrograph instrument on a 50-h. p. and a 500-h. p. induction motor: (2) Observation tests made on a number of motors when starting with full voltage: (3) The condition of some windings which failed in service: (4) Formulas which show the main factors involved in the bracing of induction motors. Induction motor coil bracing in general is discussed.*

* * * * *

It is well known that a heavy starting current is required when starting squirrel-cage induction motors with the customary auto-transformer starter. When starting these motors by connecting them directly to the line a starting current that is even heavier is drawn from the line, which, in itself, is undesirable. However, on certain applications it is desirable to have motors start from rest or at some reduced speed without the aid of an attendant when the power on the line fails and then comes again. This is accomplished by omitting the customary auto-starter and connecting the motor directly to the line, thereby impressing on the winding full-line voltage instead of reduced voltages of 50, 65, or 80 per cent.

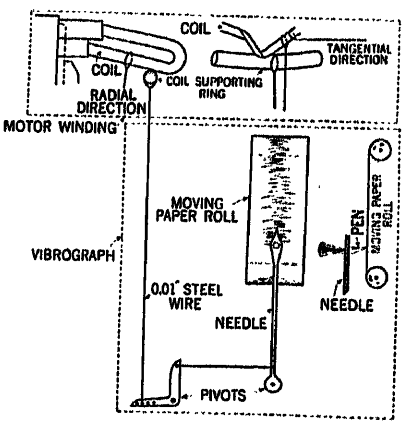

FIG. 1—DIAGRAMMATIC SKETCH OF VIBROGRAPH MACHINE AND METHOD OF CONNECTING IT TO WINDINGS

The effect of full voltage starting on the windings of squirrel-cage induction motors is discussed under four subheadings; namely—

The special tests made on two motors.

The tabulated observations on a number of motors when starting with full voltage.

The condition of windings which failed in service.

And in formulas which show the main factors involved in the bracing of induction motor windings.

Two motors were chosen for making thorough tests; one was a 50-h. p., three-phase, six-pole, 440-volt, 60-cycles, 1160-rev. per. min., squirrel-cage motor, the other a 500-h. p., three-phase, four-pole, 2220-volt, 25-cycle, 724-rev. per. min. squirrel-cage motor. The windings were of the open-slot type. There was no special coil bracing on these motors.

The deflection of the coils was measured with the aid of a special measuring device called the "vibrograph". The vibrograph is an instrument which transfers motion to a place where it can be recorded as well as magnified. It consists essentially of a moving roll of paper on which a pivoted ink-pen records the movements of the pen, which is fastened, through an arrangement of levers, to the place of motion. The purpose of the levers is to magnify the motion. The vibrograph, therefore, shows the characteristics and magnitude of the deflections. A 0.01-in. diameter steel wire was fastened around the part of coil or supporting ring on which the vibration was to be measured, and this wire

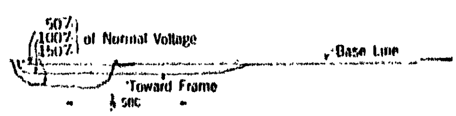

FIG. 2—DIAGRAM OF COIL MOTION, 50-H. P., 440-VOLT, SIX-POLE, 1160-REV. PER MIN. INDUCTION MOTOR (THREE SEPARATE VIBROGRAPHS GROUPED TOGETHER)

was connected to a leverage arm of the vibrograph machine. Measurements were taken at the middle and the end of the coils on the outside circumference and also on the coil supporting ring which is roped to each coil. In most of the readings taken the steel wire was in a radial position and in others the wire was in a tangential direction at right angles to the coil. Fig. 1 shows a diagrammatic sketch of the vibrograph machine and the method of connecting it to the windings. Oscillograph curves of the current were also taken at the same time.

Fig. 2 is a tracing of three separate vibrograph diagrams grouped together for convenience. This shows the effect of starting the 50-h. p., 440-volt motor with 220, 440 and 660 volts, which corresponds to 50, 100 and 150 per cent of the normal voltage. The motor was connected directly to the line with a three-pole switch. The motor was without load. With 220

1. Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

volts, the coils moved 0.002 inch, and several seconds were required to bring the motor to full speed. With 440 volts the coils moved 0.01 inch and required 1 second for the motor to reach full speed. With 660 volts the coils moved outward 0.016 inch for 0.4 second until the motor reached full speed. In all three cases the winding pulled away from the rotor and toward the frame. With 440 and 660 volts, the movements could

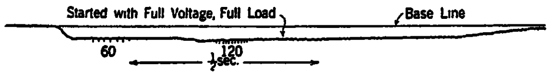

Fig. 3A—Coil Movement of 50-H. P., 60-Cycle, 440-Volt When Started with Full Voltage and Full Load

be observed with the eye and appeared to be larger than the measurements recorded by the instrument. The movement is for the radial direction.

Fig. 3A shows the coil movement of the 50-h. p., 440-volt, 60-cycle motor when started with full voltage and full load. The motor was connected directly to the line with a 3-pole knife switch. The winding moved outward toward the frame 0.01 inch, when the motor was started, and then vibrated while in this position for 1¼ seconds until the motor attained full speed. This curve indicates that the winding vibrated 60 times per second soon after the closing of the switch, but the vibrations soon changed to 120 per second. The motion is in the radial direction. Fig. 3B shows the oscillograph curve of the current which was taken simultaneously with Fig. 3A. The maximum current was 360 amperes, or 6.3 times the full-load current.

Fig. 4A shows the coil movement of the 50-h. p.,

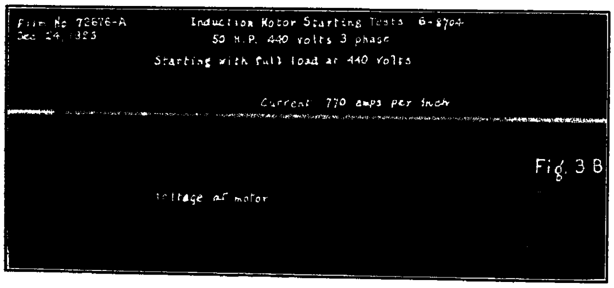

Fig. 3B

440-volt, 60-cycle motor when started with a standard auto-transformer starter. The starter was set at the 80 per cent voltage tap. The first part of the curve shows the coil movement when starting with 80 per cent voltage, and the second part of curve taken after a lapse of about two seconds shows the coil movement caused when changing from 80 per cent voltage to the full-line voltage by means of an auto-transformer starter which opens the circuit momentarily to make this change in voltage. The coil motion is for the radial direction. It will be noted that the winding moves quickly toward the frame as soon as the motor is started, and then gradually returns to the normal position and then toward the rotor; then when the change is made from the 80 per cent voltage to full voltage there is a quick movement toward the rotor followed quickly by another movement in the reverse direction of greater force than the preceding one. The winding then returned to the normal position. The cycle of movement to return to normal position required ½ second from the time the line voltage was applied. The writer has no explanation to offer for the first part of the curve for its peculiarity. This

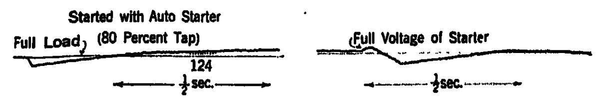

Fig. 4A—Coil Movement of 50-H. P., 440-Volt, 60-Cycle Motor When Started with Standard Auto-Transformer

peculiarity does not occur in the corresponding curve in Fig. 6B. The explanation for the movement of the winding in opposite directions as shown in the second part of Fig. 4A is that the normal direction of movement is toward the frame as in Figs. 3 and 5, and that when the circuit was momentarily opened, the residual

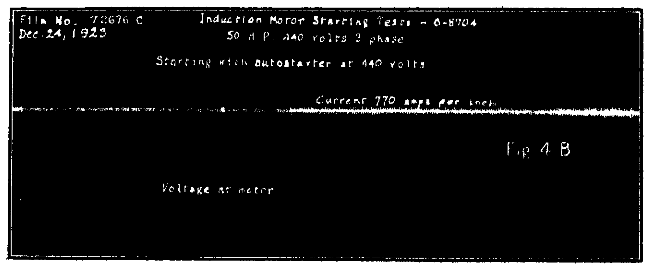

Fig. 4B

magnetization of the rotor induced a corresponding voltage in the primary, and this voltage was out of phase with the line-voltage which was then applied. Fig. 4B shows the oscillograph curve of the voltage and amperes taken simultaneously with Fig. 4A. The current at starting was 250 amperes, but the maximum current of 380 amperes flowed at the instant the line voltage was applied.

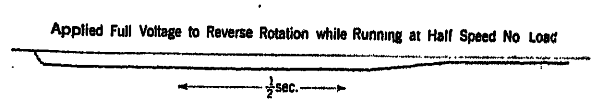

Fig. 5A—50-H. P., 440-Volt, 1160-Rev. per Min. Motor Started with No Load

In Fig. 5A the 50-h. p., 440-volt, 1160-rev. per min. motor was started with no load and the circuit was opened until the speed dropped to 600 rev. per min. Two of the primary leads were reversed while the motor was slowing down to 600 rev. per min. so that the motor would stop almost instantly and reverse its rotation. The switch was then closed. Fig. 5A shows that the windings moved 0.008 in. toward the

frame, and the total time required to stop the motor from half speed and bring it to full speed in the opposite direction was 1.3 seconds. Fig 5B shows the oscillograph curve of the current which was taken simultaneously with Fig. 5A. The current taken was 380 amperes for the first second, which is 6.7 times the normal full-load current.

Fig. 6 is a tracing of four separate vibrograph dia-

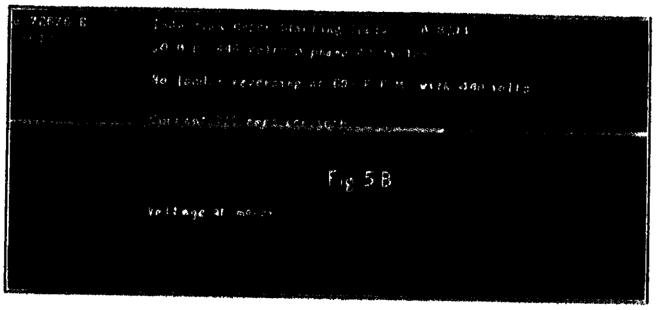

Fig. 5B

grams grouped together for convenience. These four curves are all for the 50-h. p., 440-volt motor, but in each case the motor was run on a 550-volt circuit. Curve *a* is for starting with 550 volts applied directly

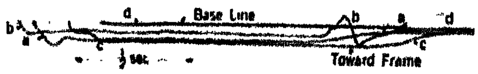

Fig. 6—Four Separate Vibrographs Grouped Together

to the winding. Curve *b* is for starting with an auto-starter on the 80 per cent voltage tap. In Curve *c* the rotation was reversed while running at half speed with no current by reversing two of the leads and then

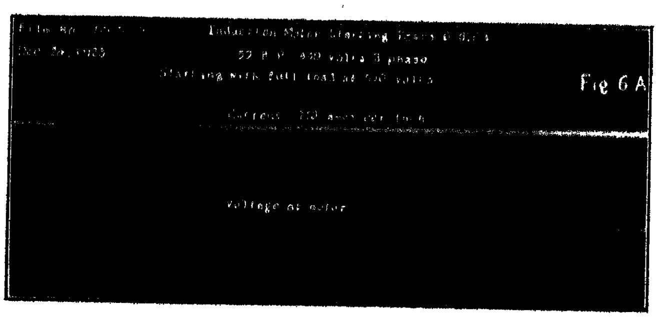

Fig. 6A

applying 550 volts. Curve *d* is the same as Curve *a* except that a coil-supporting ring was added to brace the winding. This ring was made from a ¼-inch diameter steel rod and each coil was tied to it with one turn of heavy twine. By adding the coil support the movement of the coil was reduced from 0.012 inch to 0.002 inch. The *b* curve shows the coil movement quickly reversed itself in a more marked manner than occurred in Fig. 4A. The movement was radial and toward the frame for all curves except the *b* curve, which had a momentary movement toward the rotor before changing from the 80 per cent auto-transformer tap to the line voltage. Figs. 6A, 6B and 6C show the oscillograph curves of the voltage and current for the vibrograph curves, 6 *a*, 6 *b* and 6 *c* respectively. In Fig. 6C the light in the oscillograph failed momentarily and therefore, omits the middle part of the curves.

Fig. 7 shows the coil motion for the 500-h. p., 2200-volt induction motor when it was started without load at 1100 volts. Note that the winding did not shift to either side but vibrated an equal amount about its normal position. The vibration is in the radial direction. This may be compared with Fig. 8, which is the the same except that it is for full voltage (2200) starting. With 1100 volts, the coils vibrated 1/16 inch for 2.7 seconds until motor was up to speed, and with 2200

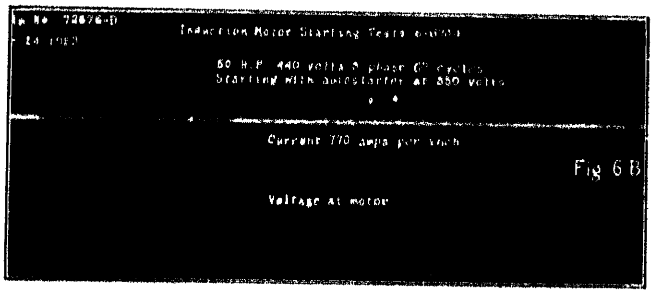

Fig. 6B

volts the coils vibrated ¼ inch (⅛ inch on each side of the normal position) for the 0.65 second required to bring motor to full speed. In Fig. 8, the coils vibrated 3/16 inch toward the rotor and 1/16 inch toward the frame and then held the position of 1/16 inch toward the frame after the motor was up to full speed. This curve gives the impression that the coils vibrated against something on the frame side 1/16 inch from normal, which prevented the winding from moving or vibrating beyond that point. This curve is for radial motion with the wire fastened at the middle of the end extension.

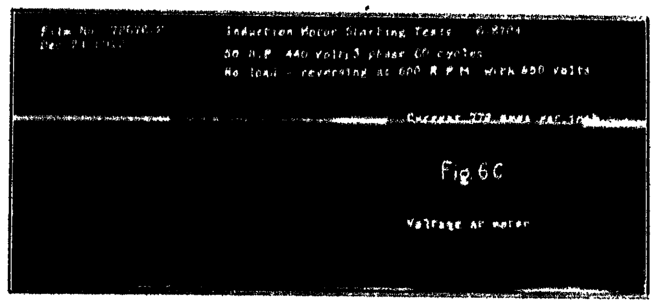

Fig. 6C

Fig. 9 is for tangential motion; otherwise it is the same as in Fig. 8. It shows that the tangential motion was almost ¼ inch (⅛ on each side of normal) and therefore about the same value as the radial motion. It also shows that the coils have a sidewise displacement of 1/16 inch to one side, which remains after the motor is up to full speed.

Fig. 10A was taken to show the effect of opening and closing the circuit similar to the action that occurs when using an auto-transformer starter. The motor was started with full voltage, then the switch was opened at the end of one second and closed again at the end of two seconds, and then opened at 2¼ seconds and closed after 3 seconds, then opened after 3⅓ seconds. The vibration was ¼ inch when the switch was closed

tained by exerting a force through a spring balance and measuring the deflections by the vibrograph.

Fig. 14 shows the radial force exerted on a coil of the 50-h. p. motor at the different voltages. It is plotted from data in Figs. 12 and 13.

General Discussion of Curves

These curves show that there was either quivering,

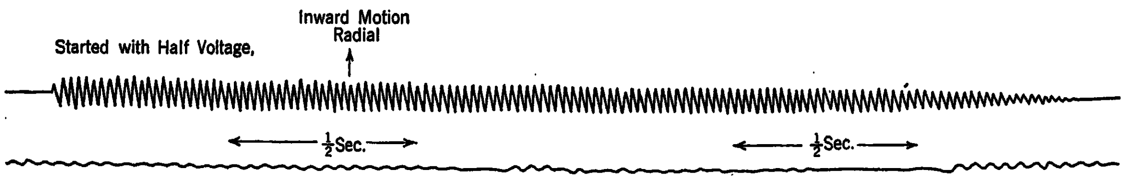

Fig. 7—Diagram of Coil Motion for 550-H. P., 2200-Volt Induction Motor When Started Without Load at 1100 Volts

the second and third time, the same as when started the first time with full voltage. Fig. 10B shows the oscillograph curve of the voltage and current which was taken simultaneously with Fig. 10A. Fig. 10B, as well as curves in Figs. 3B and 6A, shows that a voltage

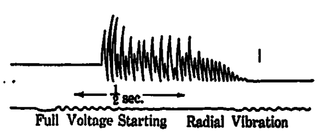

Fig. 8—Diagram of Coil Motion for 550-H. P., 2200-Volt Induction Motor When Started Without Load at 2200 Volts

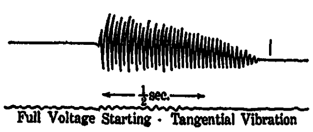

Fig. 9

is induced in the primary winding after opening the switch to the supply circuit.

Fig. 11 has been plotted from test data of the 50-h. p. motor, to show the effect of different voltages on the time in bringing the motor up to full speed. Like-

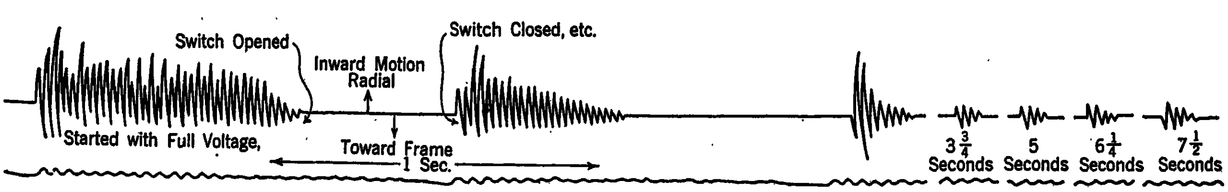

Fig. 10A—Effect of Opening and Closing Circuit Similar to Action Which Occurs When Using Auto-Transformer Starter

wise, Fig. 12 shows the actual displacement of the ends of the coils as a function of the voltage.

Fig. 13 shows that the radial displacements of a stator coil are directly proportional to the force applied to the coil within the limits measured. This was ob-

vibration or displacement of the windings when the motor was started in all of the tests on these two motors. The severity of the motion or displacement increased as a function of the square of the voltage applied to the particular winding; that is, a 440-volt or 2200-volt motor is no worse than a 220-volt motor with regard to

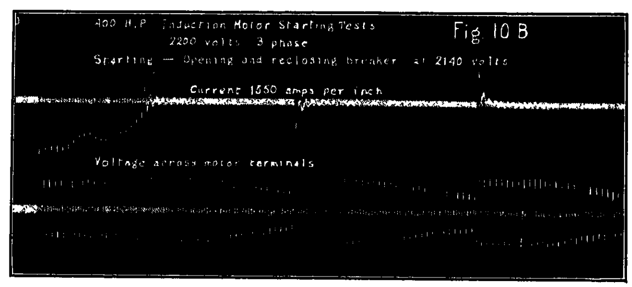

Fig. 10B

the effect of voltage alone, but 440 volts will produce four times as much coil displacement as 220 volts on a particular winding regardless of the voltage rating.

When the windings were displaced, the displacement was toward the frame in all cases except when starting the motor with an auto-transformer starter. The windings move and vibrate in both radial and tangential directions.

The amount of coil movement and its duration, as shown in the vibrograph curves, correspond with the starting current on the oscillograph curves taken simul-

taneously with them. The length of time that the windings are displaced by the starting current for any two voltages on a given winding is inversely proportional to the square of the ratio of these voltages. The that the windings vibrate, multiplied by the coil displacement in inches, will give a constant value regardless of the voltage applied. The addition of a coil supporting ring, as in Fig. 6 (*d*), shows that additional coil bracing reduces the coil movement considerably.

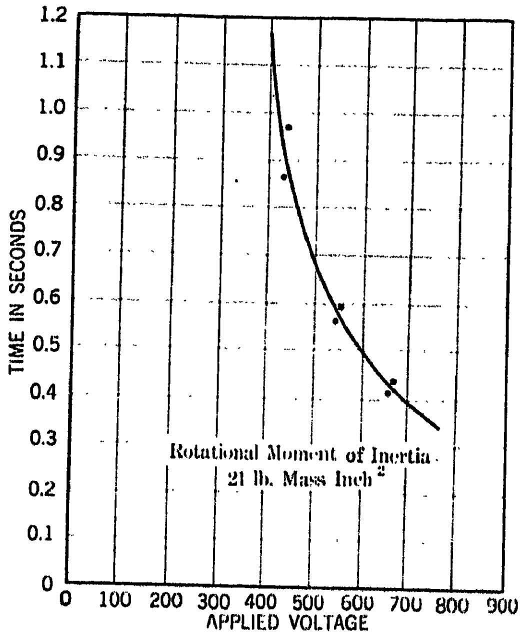

FIG. 11—EFFECT OF DIFFERENT VOLTAGES ON TIME IN BRINGING MOTOR TO FULL SPEED

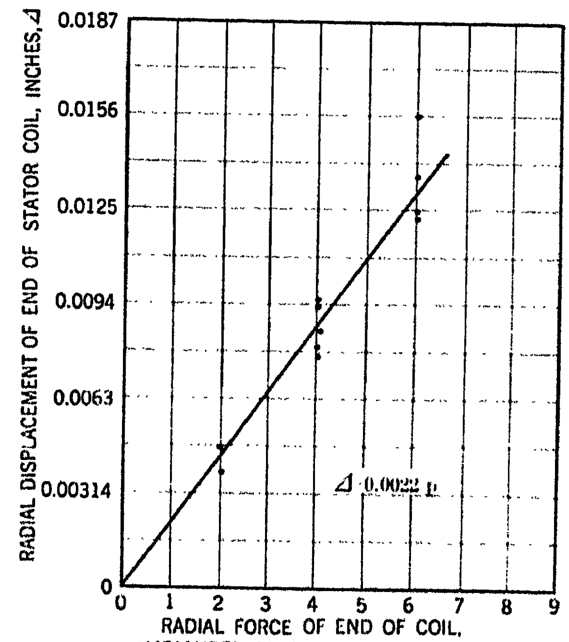

FIG. 13—RADIAL DISPLACEMENT OF STATOR COIL

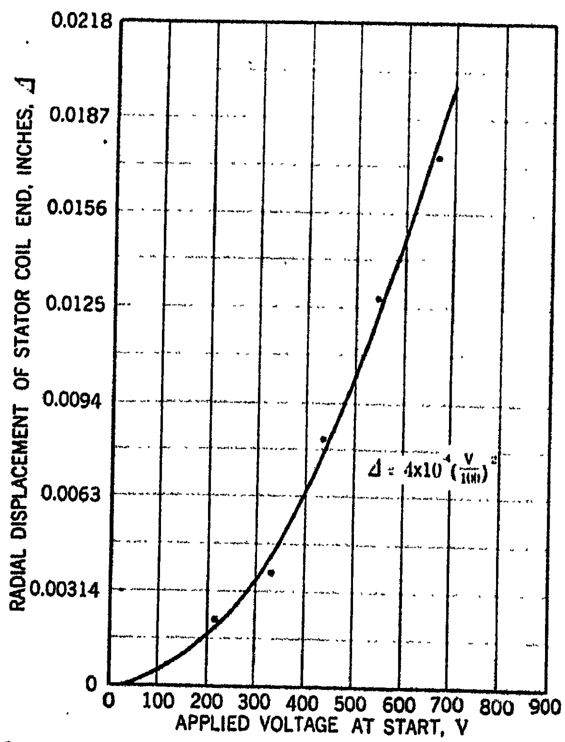

FIG. 12—ACTUAL DISPLACEMENT OF ENDS OF COILS AS A FUNCTION OF VOLTAGE

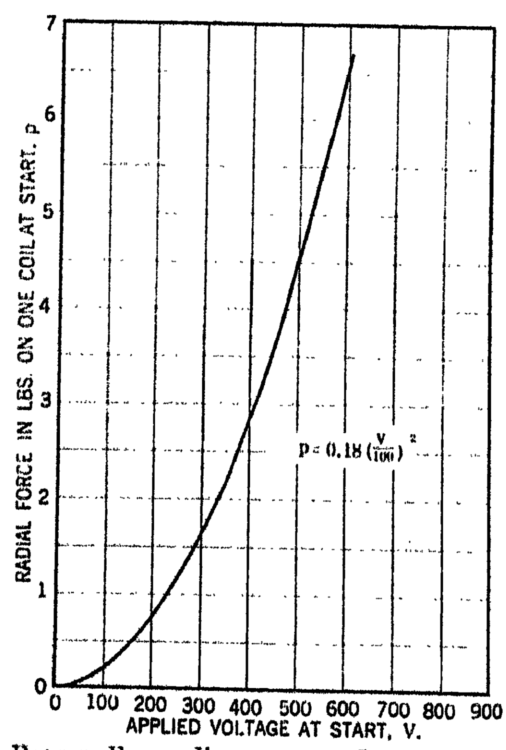

FIG. 14—RADIAL FORCE EXERTED ON COIL OF MOTOR FOR DIFFERENT VOLTAGES

radial displacement of the stator-coil ends for any two voltages applied to these windings was proportional to the square of the ratio of the voltages within the limits measured. Therefore, the length of time in seconds

Figs. 3B, 6A and 10B show that there is a voltage at the motor terminals after disconnecting the supply circuit from squirrel-cage induction motors. For the 50-h. p. motor with full load, the induced voltage is

25 per cent of normal voltage, half a second after open-the switch, and on the 500-h. p. motor with no load, the induced voltage is 50 per cent of full voltage ¾ second after opening the switch and 35 per cent after 1¾ seconds. In the primary, this voltage is induced by the residual magnetism of the rotor, and the frequency is directly proportional to the speed of the rotor which is slowing down.

Starting motors with full voltage produces a severe strain for a short period, whereas the use of an auto-transformer starter produces a less severe strain for a longer period and also a momentary strain that may be more severe than when starting with full voltage. The auto-starter may produce two severe shocks on the winding in opposite directions. This is accounted for by the voltage which is produced by the winding being out of phase with the line voltage at the instant of changing from the lower-voltage tap to the full voltage tap of the auto-starter.

What Observation of Windings Has Shown

In addition to the two motors on which oscillograph and vibrograph curves were taken, other machines as listed below were started with the full-rated voltage on the terminals and the effect observed:

H. P.	Volts	Poles	Coil Extension	Starting Current	Effect on Winding
50	440	6	4 1/2	6 × Full Load	Moved slightly
400	2200	4	11	7.5 × Full Load	Considerable movement on coils and complete winding
250	220	24	5 3/8	4.4 × Full Load	Quivered slightly
400	2200	24	5 5/8	5.7 × Full Load	Quivered slightly
600	2200	2	9	9.8 × Full Load	Appreciable movement of complete winding
10	220	4	3 3/8	5.2 × Full Load	No effect: This winding is made solid by dipping the complete winding in varnish and baking it. No other bracing used.

When the effect on the windings is slight, nothing is seen with the eyes, but any quiver or vibration of the windings is felt with the hands without difficulty. Some of these windings not only vibrated but had a definite movement which lasted as long as the heavy starting current flowed through the windings. These movements of the windings were such as to separate the windings wherever the phases change and for the winding to move away from the rotor and toward the frame. When the vibration of the winding is severe, a distinct hum is heard.

Motors Failing Due to Distortion of Windings

Some machines which had been in service for years finally failed due to distortion of the windings. These machines had used the customary auto-starter method of starting. Whether the cause of the distortion was due to the starting current or to some other disturbance, such as short circuits on the line near the motor, was not known. These windings showed the following effects:

1. The windings had separated between phases.
2. The windings had moved away from the rotor and toward the stator.
3. The top and bottom layers of coils had drawn together.

The separation between phases and the movement toward the frame were also seen on some of the windings that were observed while they were being started. The action of top and bottom layers on each other cannot very well be observed, but from the formulas which follow, it should be expected that the top and bottom layers should draw together at certain places.

Derivation of Working Formulas

There is an attraction or repulsion between each and every other conductor and also between each conductor and the magnetic field of the secondary, and also with the stray leakage flux of the primary. On account of the complications involved, and because the action of conductors on each other is the main force involved, formulas have been worked up for this action only.

The force exerted between two parallel conductors is:

$$P = \frac{4.5 \times I^2 L}{a \times 10^8}$$

where P is the force in pounds, L is the length of the conductors in inches spaced a inches apart and having I amperes flowing in both wires. This force attracts when the current is in the same direction and repels when the current flows opposite directions.

The deflection of the conductors can be derived from the formula

$$D = \frac{P L^3}{48 E M}$$

where E is the modulas of elasticity $(11 \times 10^6$ for copper) and M is the moment of inertia =

$$\frac{b h^3}{12}$$

and where h is the dimension of the beam in the direction parallel to the applied force and b is the dimension at right angles to the applied force. D is the deflection in inches when b, h and L are expressed in inches, and the deflection

$$D = \frac{1.02 I^2 L^4}{a \times b h^3 \times 10^{15}}$$

By assuming a typical value of current density in the copper conductors of 2500 amperes per square inch and using a value of the starting current as being 6 times full load, the value 15,000 $b h$ can be substituted for I, and the deflection or amount of vibration becomes

$$D = \frac{2.3}{10^7} \times \frac{b}{h} \times \frac{L^4}{a}$$

where D is deflection in inches.

h is dimension of conductor in inches in the direction

parallel to direction of applied force, and b is the dimension of conductor in inches in direction at right angles to the direction of the applied force.

L is the length of conductors in inches between supports and a is the space between conductors in inches.

Coil Bracing

The attraction and repulsion of the conductors on each other tends to move the conductors and is opposed by the mechanical fastenings which support the conductors. Many of the conductors are secured by resting against other conductors, which are secured by various means. As all armature conductors pass through the slots, the slots become an effective means of securing a large part of each conductor. Beyond the slots a number of the conductors are bound together in a unit as a coil, and this prevents the conductors from vibrating against each other. Each coil end tends to move under the effect of the force exerted on it by all other coils in the machine but is resisted by the rigidity of the coil as each coil is held by being placed in two different slots and usually at one or more other places on ends of windings by means of rope, bolts, clamps or steel bands to other more or less fixed supports.

The formula for deflection shows that the dominating factor is the length between any two supports, as the deflection varies according to the fourth[2] power of of this length. This distance should, therefore, be kept within proper limits. As the deflection is also proportional to the square of the current, the coil movement will be four times as great for a motor which has a starting current ten times the full load value, as for the same winding with a starting current of five times the full load value. As two-pole and four-pole motors have relatively large coil extensions and also have the highest ratio of starting current to full load current, their windings must be braced much more firmly than others.

In many of the small motors the coils fit closely with each other and the complete winding is reinforced with varnish or impregnating compounds; therefore, their windings are exceptionally well braced to resist the forces exerted when starting the motors.

The vibration and movement of the windings deteriorate the insulation, and if sufficiently severe, will cause an insulation failure. It may also break the coil leads.

The kind and amount of insulation and the method of bracing are the limiting factors which determine how much vibration or coil movement can be permitted. The repetition of these vibrations sometimes has a cumulative effect by loosening the rope which is usually used to secure the coils to a supporting ring.

As the amount of vibration or coil movement is directly proportional to the square of the ratio of the starting current to the full-load current, the starting of squirrel-cage induction motors is much more severe on the windings than it is with wound-rotor induction motors where the current is limited by the resistance in series with the secondary. As the amount of vibration is also directly proportional to the third or fourth power of the length of coil between supports, very thorough bracing is required where the coil extensions are comparatively large. Nevertheless, there seems to be no question but that all induction motor windings can be satisfactorily braced to stand the mechanical effect of full voltage starting.

2 Although theoretical calculations show that the coil movement should vary according to the fourth power of the length between supports, some measurements have been made that indicate that it is more nearly the third power.

Discussion

R. E. Ferris: Mr. Rylander's paper is the result of one of a series of tests being carried on covering the general subject of the effect of vibration on the insulation and windings of rotating electrical apparatus.

Insulating material may be quite highly carbonized and still retain a fairly large percentage of its original dielectric strength, provided actual mechanical breakdown does not occur. In practically all applications, however, some movement will occur. It is, therefore, necessary to keep the operating temperature of electrical apparatus well below the carbonization point of the insulation used, and also to minimize as much as possible the vibration and movement of the winding.

To accomplish the latter, it is desirable to wind all coils as tightly as practicable in the slots and to support the end portions in such a way as to minimize the effect of any possible shrinkage of insulation.

Especially for the larger size induction motors, some form of mica insulation is best, at least for the slot portions of the coils, as this material stands prolonged high temperature with very little shrinkage, thus insuring tight coils over a long period of time. Where induction motors are wound with closely fitting end-windings, *i. e.*, with no ventilation, the problem of bracing becomes simpler, but at the same time the possibilities of higher temperatures are increased, and therefore, with this type of winding, Class "B" insulation would seem to give less chance for shrinkage and consequent movement than with Class "A" insulation; in other words, the type of insulation to be used, especially on the end windings is intimately connected with the type of ventilation and method of bracing.

In the paper under discussion, the author has shown, in a quantitative way, the movements of induction-motor windings under the application of full voltage, and in this way has been able to make an intelligent study of the necessary bracings for the windings of this type of apparatus. With the type of bracing on the motors under test, the author apparently found that the use of an auto-starter did not make a great deal of difference as regards the magnitude of coil movement. The effect of ring-supported coils over those without the support is quite striking.

I might add that as I understand Mr. Rylander, he has not stated that all motors can be started under full voltage regardless of the type of bracing. In other words, he has emphasized the fact that proper and adequate bracing is necessary in order to produce motors which may be started under full voltage or over-voltage, and also that it may not be possible to eliminate the auto-starter unless the windings are properly and adequately braced.

V. Karapetoff: I think we should all welcome this new method of testing actual mechanical stresses in windings, and I am glad, therefore, to see this paper presented. I only regret the appearance of the so-called "working formula" in it.

The "working formula" on the sixth page in Mr. Rylander's paper is entirely misleading in so far as conductors in slots are concerned. The general law is that the mechanical force is equal to $I_1 I_2 d M/d x$, where I_1 and I_2 are the currents in the two circuits, and $d M/d x$ is the rate of change of their mutual inductance with the distance. (See, for example, A. Russell, Alternating Currents, Vol. I, page 40). In a very special case of two infinitely long, straight conductors carrying equal and opposite currents, the mutual inductance happens to be such that its derivative with respect to x gives the expression quoted in the paper. But with rectangular conductors in a slot, with currents flowing in nearly the same direction, and with the magnetic field determined by the iron structure, the coefficient of mutual inductance is expressed by an entirely different formula and gives a different force. For similar reasons, the formula quoted in the paper does not apply to the end connections.

R. E. Doherty: Any one who has ever tried to visualize the leakage flux in the end winding of an induction motor will have sympathy with Mr. Rylander's method of attacking this problem as contrasted with the one proposed by Professor Karapetoff. Any one of us familiar with matters of this kind, can set up the equations which express the force between coils as a function of the rate of change of mutual induction with respect to coil movement; but I would like to see the man who can apply it. Therefore, while I have the greatest respect for the wholesome and healthful effect of Professor Karapetoff's exhortation on us practical engineers toward making our treatment of practical problems a little more rigorous (we can improve it much), I nevertheless wish to express my sympathy with Mr. Rylander's practical point of view. It is necessary in many cases simply to take the larger factors involved (such as, in this case, that the force is proportional to the square of the current) and depend upon tests to determine the proportionality factors.

However, it may be that Mr. Rylander's equations can be improved to this extent: I notice that the formulas written down are for steady-state conditions. He simply balances the force exerted by the current and equates it to the force due to resilience to the coils.

I should like to call attention to the fact that this is a transient state, therefore, the accelerating force for the mass of the coils can not properly be neglected. Possibly if the effect of the mass of the coils were taken into account, it might possibly explain the discrepancy between the variation with the fourth power instead of the third power, as I believe he found in his test.

K. L. Hansen: In a paper entitled "The Starting of Polyphase Squirrel-Cage Motors" (JOURNAL A. I. E. E., Nov. 1923), B. F. Bailey discusses the effects of starting squirrel-cage motors by auto-starter, by resistance type of starter and by throwing the motor directly on the line. He considers the effects from four viewpoints:

1. Effect of starting current upon line voltage.
2. Affect upon connected apparatus.
3. Heating.
4. Power consumption.

Prof. Bailey has shown that in its effect on the voltage regulation the auto-starter offers practically no advantage over the resistance type and little advantage over direct connection to the line. This is especially true when it is necessary to use the higher voltage taps on the auto-transformer to get sufficient starting torque. With reference to possible injury to connected machinery, resulting from too rapid acceleration, there may be cases where throwing the motor directly on the line is less desirable than other methods, but such cases are rare. With reference to heating and power consumption Prof. Bailey showed that throwing the motor directly on the line is superior to both the auto-starter and the resistance type of starter.

To quote directly from Professor Bailey's conclusions:

"In view of the above facts it seems clear that it is entirely practicable to dispense with starters for polyphase squirrel-cage induction motors in a great majority of cases. No harm will come to the motor. The voltage regulation of the system in a majority of cases will be just as good as it was before and will be even better in the case of large installations"........

In his timely and interesting paper Mr. Rylander discusses the effects of various methods of starting from another very important standpoint, namely, the distortion of the windings. As pointed out by Mr. Rylander, the importance of movements and vibrations of the coils lies in the fact that they deteriorate the insulation, and, if sufficiently severe, will cause insulation failure. It, therefore, becomes of interest to observe whether or not throwing the motor directly on the line compared favorably with the auto-starter from this standpoint also.

One decided disadvantage of the auto-starter, clearly set forth in the paper, is that in going from low voltage to line voltage it is necessary to disconnect the motor entirely before reconnecting it to the line. The author states that during the moment of change-over the residual magnetism of the rotor induces a corresponding voltage in the stator, etc. This is somewhat misleading as by residual magnetism is usually understood the magnetism which remains in the magnetic circuit by virtue of hysteresis after the excitation has been completely removed. What he probably means to say is that the flux interlinked with the short-circuited winding of the squirrel cage cannot be instantly reduced to zero, because the decreasing flux induces a current in the squirrel cage tending to maintain the flux.

However, he is right in his conclusion that the voltage induced in the stator winding by the revolving flux of the secondary may be out of phase with the line voltage at the moment when the latter is applied, and that consequently there may be a heavy rush of current at this instant. That these large momentary currents are very undesirable from the standpoint of shocks to the windings is clearly brought out on the sixth page in the paper, where we read—

"Starting motors with full voltage produces a severe strain for a short period, whereas the use of an auto-transformer starter produces a less severe strain for a longer period and *also a momentary strain that may be more severe than when starting with full voltage.* The auto-starter may produce two severe shocks on the winding in opposite directions."

Thus the use of an auto-starter is liable to increase the severity of the shocks as well as the frequency with which they occur. Prof. Bailey's conclusion, that starters can be dispensed with in the great majority of cases, appears, therefore, to hold equally well, or even more so, when the effects of distortions of the windings are considered. The fact that auto-starters were used in the cases, mentioned by the author, where the insulation failed as the result of distortion of the winding, indicates that the starter is more of a liability than an asset.

The conclusions to be drawn from Mr. Rylander's experiments, taken in conjunction with the conclusions reached by Prof. Bailey, inevitably present the question, "Why are auto-starters, or compensators used in connection with starting of squirrel-cage motors, anyway?" No doubt it is desirable to reduce the starting current as well as to improve the starting torque of squirrel-cage motors, but it seems that this can be much more effectively accomplished by other means than by an auto-starter. Quite a number of years ago Boucherot proposed the double squirrel cage as a means of obtaining good starting torque, low starting current and high operating efficiency. One reason for the slow adoption of the double squirrel cage may have been that the older methods did not lend themselves readily to this type of construction. However, a recently developed process of arc welding the rotor bars seems to be readily adapted to the construction of the double squirrel cage, and there are indications that the immediate future developments of squirrel-cage motors may be along this line.

F. C. Hanker: In connection with Mr. Rylander's paper, it is of particular interest to bring out a phase that has not been

discussed. That is the elimination of the control in connection with power-house auxiliaries. This has been one of our most difficult problems because we have been confronted with sources of power far in excess of those usually found in the industrial installations. For that reason it has been very desirable to have a motor that could be thrown on the line at full voltage with satisfactory starting conditions and without control equipment other than the feeder oil circuit breaker.

The bracing of windings in the past has been largely controlled by experience in the field, and it is interesting to see the new point of attack, to observe the effects more accurately. The type of motor with the double squirrel cage gives reduced starting kv-a. and is applicable in cases where we are limited in the capacity available. In power houses we do not have that limitation and on those cases we can use the simple forms of motor with lower cost and just as satisfactory installation.

H. Weichsel: A complete understanding of the forces acting on the windings of an induction motor when the machine is switched on the line is very important in view of the fact that the electrical engineers of many industrial plants are advocating the starting of squirrel-cage motors directly across the line. The investigation carried on by Mr. Rylander reveals many very interesting points, and in some cases apparently contradictory results. In the following an attempt will be made to give a physical conception of the forces acting in a machine during switching operation.

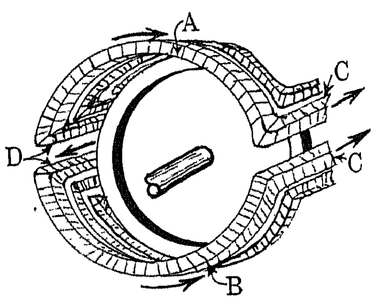

FIG. 1

For simplicity's sake, an induction motor with a so-called chain winding in the primary member will be assumed. A large part of the free ends in such a winding is located parallel to the rotor surface. Another part, the so-called straight part of the free ends, is parallel to the shaft, as diagrammatically shown herewith in Fig. 1 for a two-pole machine. Only one phase of the chain winding is shown.

At any instant, the currents in the circular sections, *A* and *B*, of the free ends flow in the same direction. Therefore, the sections *A* and *B* will attract each other. The straight parts, *C* and *D*, of the coils carry currents of opposite directions. Therefore, a repelling action will take place between *C* and *D*. The forces *C* and *D* try to bend the coils outward, while the forces due to *A* and *B* try to move the coils inward. This assumes that a constant direct current is flowing through the winding. However, if it is a low-frequency current, then the coil will swing periodically, according to the change in the forces acting on the coils. If the frequency of the currents is high, the vibration of the coils will cease and change into a steady deflection. The inertia and spring action of the coils are playing an important part in determining which of the two results will be obtained.

This phenomenon is similar to the well-known fact that the deflection of an a-c. dynamometer-type voltmeter is of a vibrating nature as long as the currents flowing through the instrument are of low frequency and with increasing frequency the swing of the needle becomes smaller and smaller and at a sufficiently high frequency the needle finally shows a steady deflection. Therefore, this example demonstrates a case where either a vibration or a definite deflection can be obtained, depending upon the relative relations of the impressed frequency and the natural periodicity of the deflected member.

It appears to me that this simple experiment gives an explanation of the fact that in Mr. Rylander's tests on the 50-h. p., 60-cycle motor a steady deflection of the coil was observed, while in the tests on the 500-h. p., 25-cycle motor a vibrating deflection was recorded.

In Fig. 1, we had assumed that the overall length of the squirrel-cage winding is considerably less than that of the stator winding and that, therefore, the current in the squirrel-cage endings can not react materially on the free ends of the stator winding.

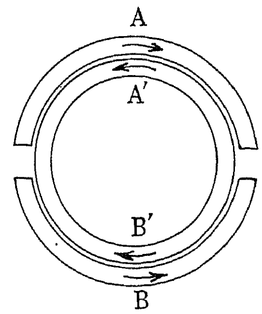

FIG. 2

Fig. 2 represents a condition where the end rings of the squirrel-cage winding are approximately in the same plane with the parts *A* and *B* of the stator free ends. The current distributions in the ring and in the stator free ends are indicated by the direction and shape of the arrows.

The currents in the ring section *A'* and in the stator free ends *A* are in opposite directions and, therefore, a repelling action exists between these members. The straight part of the stator free ends is parallel to the free ends of the rotor bars and the currents in these members are equal but opposed to each other. Therefore, a repulsion exists between these parts also. The net result is that all parts of the stator winding have a tendency to move outwardly, a condition which has been observed in most of the tests shown by Mr. Rylander. This outward movement naturally can either take place in form of a vibrating nature or in a steady deflection.

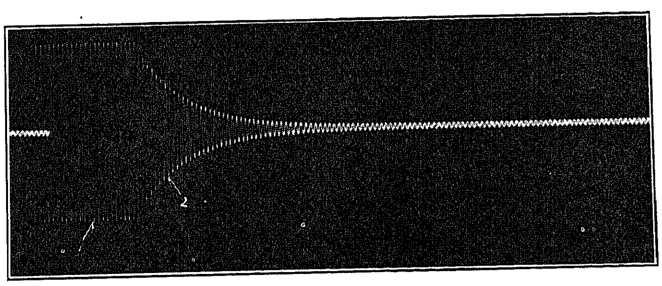

FIG. 3—DYING OUT OF TERMINAL VOLTAGE OF A SQUIRREL-CAGE MOTOR WHEN DISCONNECTED FROM LINE

The forces acting on the windings will not vary in magnitude as long as the current's flow remains unaltered. At any moment the force acting is proportional to the square of the current. During switching operations, transient-current phenomena occur which may produce momentary current flows away above the steady current expected from the constants of the machine. This heavy transient flow may be the result of one of the two following features:

1. When the motor is at rest, the first current inrush depends on the moment at which the motor is connected to the line in respect to the voltage wave on the line.

2. When the motor has been placed in motion by a supply of voltage and is then switched over to a different value of supply voltage, a very increased momentary voltage might act on the motor windings.

The case No. 1 is well understood and simply refers to the phenomena occurring when a choke coil is connected to an a-c. supply. The phenomena No. 2 are not as generally known.

If a squirrel-cage motor is running, let us say at synchronous speed, and the stator winding is suddenly disconnected from the line, then a voltage remains across the stator windings which

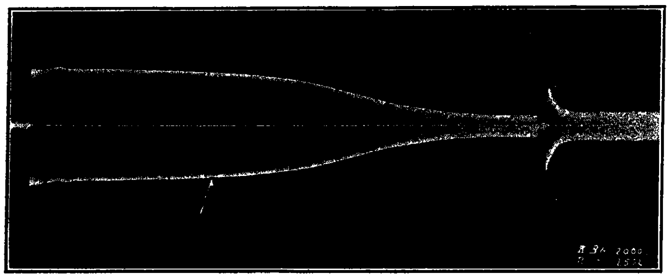

FIG. 4—SQUIRREL-CAGE MOTOR STARTING 22 PER CENT OF FULL LOAD WITH AUTO-STARTER

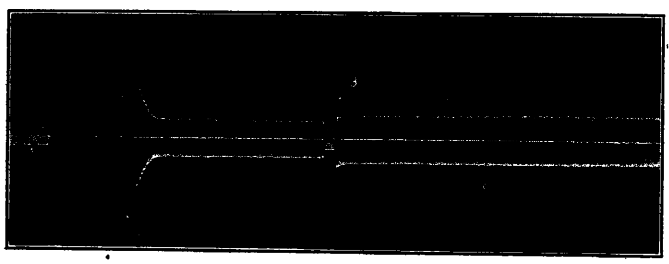

FIG. 5—SQUIRREL-CAGE MOTOR STARTING IDLE WITH AUTO-STARTER

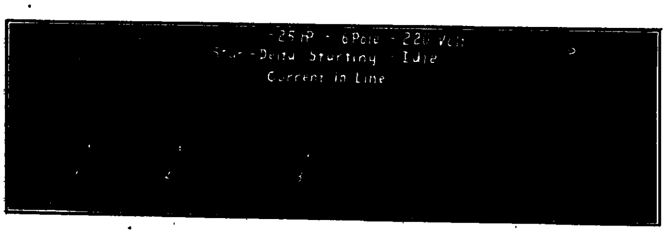

FIG. 6

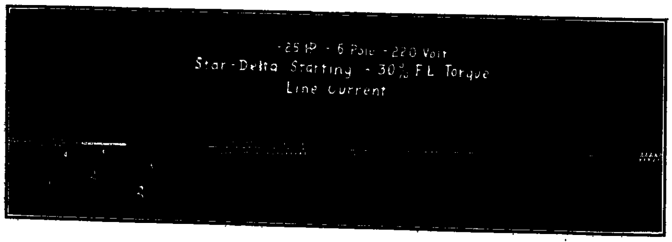

FIG. 7

gradually dies out, as given in the oscillogram Fig. 3. The reason for this remaining voltage is as follows:

According to the Lenz law, the magnetic field in a motor cannot disappear instantly. Therefore, at the moment the line is disconnected from the primary winding, currents will be induced in the rotor winding which try to maintain the original magnetic field of the machine. These are of direct current and gradually die out. The velocity with which they die out depends entirely upon the resistance of the secondary winding and the self induction of the machine. The decrease of the current and, therefore, the decrease of the generating voltage occurs according to the well-known logarithmic law, provided the speed of the armature remains constant.

Naturally the speed of the armature decreases gradually when the supply is disconnected and, therefore, the frequency of the voltage generated in the stator decreases with increasing time.

Therefore, if the line is reconnected to the motor shortly after it has been disconnected, it may occur that at the moment of reconnection the voltage generated in the machine is out of phase with the voltage impressed on the machine, which results in a very heavy current draw at the switching moment.

Oscillograms Figs. 4, 5, 6, and 7 show the heavy current inrushes which may occur during the switching period. As the forces produced by the currents are proportional to the square of the currents, it follows immediately from these oscillograms that the forces at the moment of switching might reach tremendous values and act like hammer blows on the windings.

It may be stated that these forces do not only act on the free ends of the windings but also act to produce torque on the rotor and incidently on the stator. I am familiar with cases where these torques reached such tremendous values that the stator iron shifted in the frame.

There are not only forces acting on the free ends of the stator winding in the manner discussed, but there are also forces acting on the free ends of the squirrel-cage winding itself.

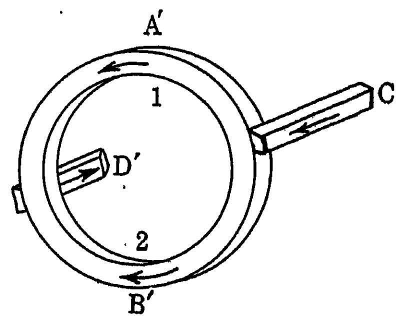

FIG. 8

Fig. 8 represents the current distribution in a squirrel-cage winding at a given moment for a two-pole machine. The ring sections, A'–B', carry a current of equal distribution flowing in the same direction. Therefore, an attraction occurs between these two sections, as indicated by the arrows 1 and 2. The extensions of the rotor bars, C and D, carry equal currents but in opposite directions. Therefore, a repelling force is created between these bars.

If we assume that the currents flowing in the rotor winding are direct currents, then the ring will take an elliptical shape under the influences of the forces just discussed. In a polyphase motor, the relative current distribution in the rotor bars and rings is the same at any moment but the picture of current distribution rotates. The velocity of rotation in respect to a given point of the squirrel cage is proportional to the frequency of the currents flowing in the secondary. Therefore, the end rings on the squirrel cage will successively take shapes as indicated in Figs. 9A to 9E inclusive. This periodic deformation of the rings and bars is quite likely to lead to a fatigue of the materials and a final breakage unless the rings are properly supported to resist effectively the forces acting on it.

In a similar manner, if the stator coils are laced to a circular coil-supporting ring, the forces acting on all the windings have a tendency to deform the ring into an elliptical shape similar to the Figs. 9A to 9E inclusive. In order to counteract this efficiently, it is advisable to brace the coil-supporting rings in twice as many places as there are poles and give these brackets for the

coil-supporting rings equal spacing. A coil-supporting ring with semi-circular cross section and a piece of heavy insulating material on the straight part of the coil-supporting ring has proven to be more effective than a circular ring. The coil would touch the circular ring in only one point and the unavoidable vibration of the coil and ring is likely to wear the insulation through in relatively short time, while with the semi-circular ring the coil makes more of a line contact than a point contact with the supporting ring and, therefore, the possibility of wearing through the insulation is materially reduced.

Mr. Rylander pointed out that the deflection of the stator coils depends to a large extent on the length of the free end coils, but as the length of the free end coils is closely related to the arc

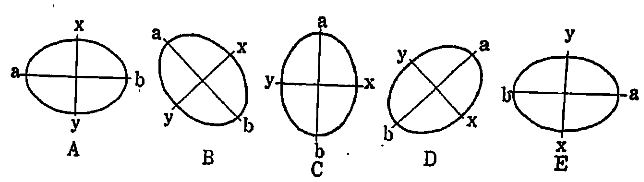

Fig. 9

spanned by the coils, it is possible to obtain simple empirical rules to determine when coil-supporting rings are needed.

My experience has shown that whenever the coil spans more than 8 to 9 in., it is absolutely essential to provide the stator winding with coil-supporting rings. The lower figure refers to 25-cycle and the larger figure to 60-cycle machines. This is explained by the fact that the 25-cycle machines usually have finer wire than the 60-cycle machines and, therefore, the coils have less rigidity.

Returning once more to the forces acting on the squirrel cage, the part of the bar which is located in the slots is subjected to a force due to the leakage lines passing over the bar, as indicated in Fig. 10. These magnetic lines have the tendency to push the bar inward, *i. e.*, in the direction towards the shaft. The force producing this tendency is proportional to the square of the current, *i. e.*, the force will reach a maximum whenever the current is of either positive or negative maximum, as given in Fig. 11. If the bar is not secured tightly in the slots, then these forces are apt to set the bar in vibration, swinging it around its

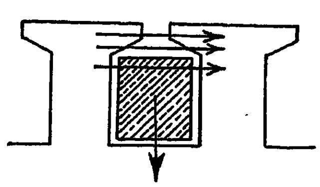

Fig. 10

two points of support on the two end rings of the machine, in the same manner as a violin string. This may result in a fatigue in the bar material at the point where the bar joins the ring with a consequent breakage. It is, therefore, essential to provide means to anchor the bars securely in the slots. Such arrangements can be made, for instance, by driving a wedge between the top of the bars and the lips of the rotor slots.

Mr. Rylander showed in his Fig. 11 that the time decreases rapidly with increasing starting voltage. In case the machine has to accelerate nothing but an inertia load, it can quite readily be proved that the starting time varies inversely to the square of the impressed voltage. If, however, a friction load is to be started in addition to an inertia load, then the starting time decreases still faster than the inverse of the square of the voltage. Therefore, it might quite readily occur that when starting a machine under full voltage the heating of the stator coils is less than when the same load is started with reduced voltage though such starting draws a heavier current than the reduced-voltage starting.

On the other hand, if a motor has sufficient starting torque to handle its load with reduced voltage, then the resistance of the squirrel-cage rotor could be reduced when full-voltage starting is employed. This would result in an increased running efficiency of the machine but also in an increased heating of the stator winding during the starting period.

Full-voltage starting, therefore, might finally lead to machines with somewhat increased running efficiencies by decreasing the rotor resistance. If, however, use is made of a decreased rotor resistance, the designer will have to safeguard against a saddle formation in the speed-torque curve. In other words, the machine will have to be designed more carefully.

J. L. Rylander: In regard to Mr. Hansen's comments about the auto-starter, it is well to mention that the type of auto-starter used was that which opens the circuit momentarily when changing from the low-voltage tap to the full-line voltage, as this is the most severe condition of the various auto-starters; and it was used for the purpose of finding out what happens when the circuit is opened momentarily before applying full voltage.

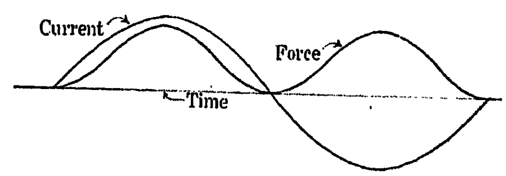

Fig. 11

In regard to Mr. Weichsel's comments about the physical conception, I think the best physical conception is to keep in mind that like currents attract and opposite currents repel. We know the direction of the current and therefore we know the reactions.

In regard to Professor Karapetoff's remarks about the formulas, it should be added that the purpose of the formulas is to show the main factors involved and use them in conjunction with tests, or the known conditions of certain motors to which comparisons can be made. There are four main factors:

1. The shape of the coil.
2. The spacing.
3. The current.
4. The length between supports.

The shape of the coil is fairly constant and the coil movement is directly proportional to it; the spacing is fairly constant and the coil movement is inversely proportional to it. But these two points are not the dominating factors. The coil movement varies according to the square of the current, which varies considerably in the different motors, varying anywhere from four to ten times the full-load current. But the dominating factor is the length, which varies as the third or fourth power. For practical purposes, the main factors are sufficient and the best way is to use them in connection with that which is known.

We can watch any motor that we are acquainted with as it is started on the 50, 65 or 80 per cent voltage tap. Then if we compare that with the formulas, we can about tell what will happen when full voltage is applied. Or if we want to compare windings that are similar except for length, it is a matter of comparing the length of the coil extensions.

Another New Self-Excited Synchronous Induction Motor

BY VAL. A. FYNN*

Fellow, A. I. E. E.

Synopsis.—One form of self-excited synchronous induction motor was described by the same author in a paper presented at the Spring Convention of the A. I. E. E., Birmingham, Ala., the present contribution describes a second form of such a motor. The starting, synchronizing, synchronous load and asynchronous overload periods are analyzed with special reference to the synchronizing torque, the automatic compounding and the weight efficiency, and it is shown that this second form is not only capable of duplicating the performance of the first form but of bettering same.

IN a paper read at Birmingham, Ala., at the 1924 Spring Convention of the A. I. E. E., the writer described the motor diagrammatically shown in Fig. 1, and dealt fully with the theory of this machine during the starting, the synchronizing, the synchronous load and the asynchronous overload periods. It will now be shown how even better results can be achieved in another way. In order to positively and readily differentiate between the two motors, let us refer to that shown in Fig. 1 as "Form No. 1" and to that here dealt with, as "Form No. 2."

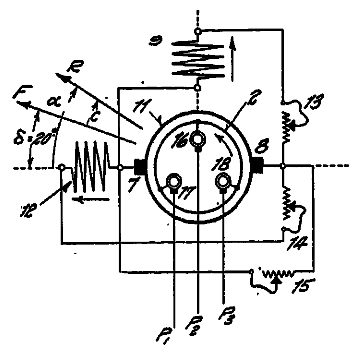

FIG. 1—SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR FYNN FORM No. 1

British Patent No. 3227 of 1913, issued to Crompton & Burge, relates in the main to synchronous motors and indicates among other things, and quite generally, a very interesting arrangement of windings and brushes which by proper modification and proportioning can, as hereinafter described, be used to cause the unidirectional excitation of such machines to increase with increasing motor load in a practically useful manner. The arrangement in question is applicable to single, as well as to polyphase, self-excited synchronous motors and is diagrammatically shown in Fig. 2; its theory, as the writer sees it, can be stated in a few words.

In the two-pole embodiment shown in Fig. 2, the revolving member is the primary, and carries a winding 2, connected to a commutator and capable of being connected to an alternating current supply by means of a suitable number of slip-rings such as 8, 9. In the figure the commutator is not shown and the brushes cooperating with same are supposed to be resting directly on the commuted winding. The same clarifying expedient is used in all the other figures. There are two sets of brushes 4, 5 and 6, 7, located along axes displaced by 90 electrical degrees. The stationary member carries two coaxial windings 11 and 12, located in the axis of the brushes 6, 7. The winding 11, is connected to the brushes 4, 5, the winding 12 to the brushes 6, 7. The British patent states that the voltage at the brushes 6, 7 is dependent on the cross flux which is approximately proportional to the primary armature reaction *A R*. Winding 11 is referred to as the "shunt" and winding 12 as the "compounding" winding. It will be shown that these terms do not correctly describe the functions of the windings 11 and 12, yet, provided this is understood, they will do as well as any others and will be used hereafter.

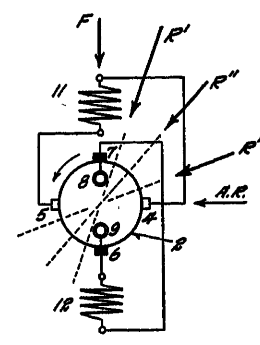

FIG. 2—CROMPTON-BURGE ARRANGEMENT OF EXCITING AND COMPOUNDING CIRCUITS FOR SELF-EXCITED SYNCHRONOUS MOTORS

During the synchronous operation of the machine shown in Fig. 2, the voltages appearing at the brushes 4, 5 and 6, 7 are both unidirectional and their magnitudes depend on the magnitude and space location of the unidirectional resultant motor magnetization *R*. One component of this resultant magnetization is *F* which is due to the arithmetical sum of the unidirectional ampere-turns in 11 and 12, its other component is the unidirectional primary or armature reaction

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

A R whose magnitude and space position vary with the load and the power factor of the motor. At no-load R may be made to nearly coincide with F. As the load increases the angular displacement c between R and F increases. In the case of Fig. 2, where the primary is supposed to revolve counterclockwise, F is stationary in space and R moves further and further away from F as the load increases, progressing in a direction opposed to the rotation of the primary and successively occupying the positions R', R'', R''' and so on. When the primary is stationary, both F and R revolve synchronously; with increasing load, F moves away from R in a direction opposed to the rotation of the secondary. At no-load the unidirectional voltage at the brushes 4, 5., is therefore nearly a maximum and it decreases with increasing load. The unidirectional voltage at the brushes 6, 7 is nearly zero at no-load and increases as the load increases. The question arises, can these conditions be utilized in order to secure a satisfactory automatic regulation of the unidirectional excitation of the motor throughout

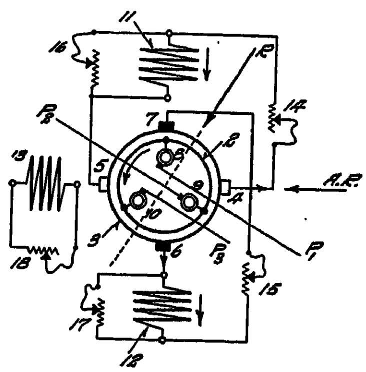

FIG. 3—SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR, FYNN FORM NO. 2

its load range? If so, then can a motor embodying this type of automatic regulation be made to show an acceptable starting, synchronizing and overload performance? What can be done in these respects will be explained in connection with Figs. 3 and 9.

The three-phase, two-pole motor shown in Fig. 3 is a self-excited, synchronous induction motor of Form 2. The primary revolves and carries a winding 2 adapted for connection to the three-phase supply P_1, P_2, P_3 through the slip-rings 8, 9, 10, and a commuted winding 3 with which the brushes 4, 5 and 6, 7 located along axes displaced by 90 electrical degrees cooperate. The stationary member, here the secondary, carries the "shunt" and "compound" windings 11, 12 located in the axis of the brushes 6, 7 and the auxiliary winding 13 located in the axis of the brushes 4, 5. The "shunt" winding is connected to the brushes 4, 5 through the adjustable resistance 14 and can be shunted by the adjustable resistance 16. The "compound" winding is connected to the brushes 6, 7 through the adjustable resistance 15 and can be shunted by the adjustable resistance 17. The circuit of the winding 13 can be closed over the adjustable resistance 18. The windings 11 and 12 are connected to magnetize in the same direction, as shown by the arrows placed alongside these windings. The arrow A R indicates the general direction of the primary armature reaction.

The auxiliary winding 13, together with the exciting windings 11 and 12, form a polyphase, here two-phase, arrangement of windings on the secondary and the machine can, therefore, be very readily and most effectively started, just like a polyphase slip-ring induction motor. In order to facilitate this starting, as well as to gain other advantages, the magnetic circuit is built exactly like that of an ordinary induction motor, without polar projections on rotor or stator and with a very small and uniform air-gap.

In the case of small machines the shunting resistances 16 and 17 can be omitted and one or both of the windings 11 and 12 closed over the commuted winding 2 for use in connection with the winding 13 at starting. The torque is regulated by suitably adjusting the resistances 14, 15 and 18. These resistances are manipulated in exactly the same manner as those in the secondaries of an induction motor. In the case of larger motors, or where small ones have to deal with particularly severe starting conditions, the windings 11 and 12 are closed over the resistances 16 and 17 and the values of the resistances 14 and 15 are kept high during the starting period in order to protect the commutator from the starting currents. When synchronism is nearly reached, the winding 18 is left short-circuited, but if they have been used the resistances 16, 17 are disconnected and the resistances 14, 15 set to their synchronizing or operating values.

The revolving flux F', produced by the polyphase currents in the primary, always revolves at synchronous speed with respect to the primary and, hence, to the commuted winding 3. As the motor starts, the primary moves in a direction opposed to that in which F' revolves. Since F' must continue to revolve synchronously with respect to the primary, its speed in space, for instance with reference to the secondary windings 11, 12, 13, must diminish as the rotor speed increases. Now the revolution of F' with respect to the secondary windings generates voltages therein, the frequency and magnitude of which diminish with increasing rotor speed, both frequency and magnitude becoming zero when the rotor reaches synchronism. These voltages are responsible for the induction motor torque-producing currents in the secondary windings and cannot synchronize the motor because they approach zero as the rotor speed approaches the synchronous. The amplitude of the voltages generated by F' in the commuted winding 3 remains constant as long as F' is constant because F' always revolves synchronously with respect to 3. The voltages generated in 3 are collected by the brushes 4, 5 and 6, 7. The amplitude of the brush voltages is, therefore, constant but their frequency depends on the speed with which F'

moves relatively to said brushes and therefore decreases as the rotor speed increases, becoming zero when the rotor reaches synchronism because F' is then at a standstill.

During the synchronizing period the windings 11 and 12 are not shunted and are connected to the brushes cooperating with the commuted winding. As the induction-motor-torque-producing currents in the secondary windings diminish, the currents due to the slip-frequency brush voltages in the windings 11 and 12 increase, because of the constant amplitude and diminishing frequency of said voltages. These brush currents react with F' and produce a synchronizing torque which brings the rotor into synchronism.

As soon as synchronism is reached the brush currents become unidirectional and provide the unidirectional excitation of the machine through the agency of the windings 11 and 12. As the load changes so does the location of the resultant motor magnetization R change with respect to the axes of the brushes, thus changing the brush voltages. During synchronism, the winding 13 is idle.

When the torque demand is in excess of the maximum synchronous torque of which the motor is capable, the rotor slips out of synchronism and continues to run asynchronously, the windings 11, 12 and 13 again doing duty as polyphase secondaries of an induction motor. Under these conditions the synchronizing torque reappears.

Such in a general way is the mode of operation of the motor, Fig. 3. After what has been said, there can be no difficulty about the conditions to be fulfilled in order to secure a good starting performance. The only question is how are the windings 3, 11 and 12 to be dimensioned in order to secure a sufficiently powerful synchronizing torque and a good compounding characteristic.

To solve the compounding problem it is necessary to divorce one's mind from the idea that 11 is a true shunt and 12 a true compounding winding, and that the voltage at the brushes 6, 7 varies proportionately with the armature reaction. As a matter of fact, increasing load causes the ampere-turns in 11 to diminish and those in 12 to increase. If a compounding action is to be secured the ampere-turns in 12 must increase much faster than those in a true compounding winding would be called upon to increase with increasing load.

Assuming that the motor of Fig. 3 is given the same general design constants as that of Fig. 1, it will be interesting to determine whether or not the performance of the motor of Form 1 can be approached or duplicated by that of the motor of Form 2.

In Fig. 45 of the author's A. I. E. E. paper entitled "A New Self-Excited Synchronous Induction Motor" are given performance curves of a Form 1 motor when designed to produce a certain maximum unidirectional excitation and to operate with its brushes 7, 8 (see Fig. 1 of the present paper), displaced by $\delta = 20$ electrical degrees from the resultant unidirectional magnetization F produced by its secondary windings 9 and 12. With a motor of Form 2 one is able to absolutely duplicate the compounding performance of Form 1. In case the brushes of Form 1 are displaced by δ degrees, this duplication is achieved by so dimensioning the circuits of the windings 11 and 12 of Fig. 3 that when the angular displacement c between F and R is zero the ampere-turns in 11 are proportional to $\sin \delta$, that when $c = 90$ degrees the ampere-turns in 12 are proportional to $\cos \delta$, and that for any value of c the arithmetical sum of the ampere-turns in 11 and 12 of Fig. 3 is equal to the vectorial sum of the unidirectional ampere-turns of Fig. 1. In Fig. 1, when $\alpha = \delta$ then R coincides with F and $c = O$. In Fig. 3 R coincides with F when α is zero. Fig. 4 shows the primary current i_1, the power factor $\cos \phi_1$ and the exciting ampere-turns M of the Form 2 motor shown in Fig. 3,

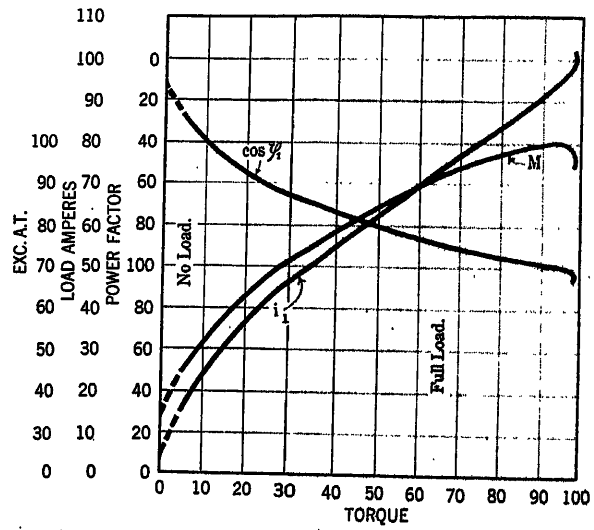

FIG. 4—SYNCHRONOUS PERFORMANCE CURVES OF FYNN FORM No. 2 MOTOR

when the "shunt" and the "compound" windings are dimensioned as just specified. It is seen that the primary current always has a very acceptable value although the power factor is leading almost throughout the synchronous operation of the machine, and that the performance of this Form 2 machine at synchronous speed is indeed identical with that of Form 1.

The commuted winding must be dimensioned with an eye to commutation and to the value of the voltages generated in 11 and 12 at the moment of starting. Both considerations lead to the selection of a low brush voltage. Values of 10 to 30 volts will in most cases answer the purpose. It should be noted that at no-load, practically all of the exciting current is supplied through the brushes 4, 5 which then stand near the neutral in so far as the resultant motor magnetization R is concerned. At full load it is the brushes 6, 7 which carry practically all of the exciting current and

at that time these brushes also stand near the neutral as referred to R.

Turning to the synchronizing possibilities of Form 2, it is seen that at subsynchronous speeds the phase of the brush voltage e_2 appearing at the brushes 6, 7, (see Fig. 5), is the same as that of the voltage generated by the primary revolving flux F' in the winding 12 to which the brushes 6, 7 are connected whereas e_1 appearing at the brushes 4, 5 is in phase quadrature with the voltage generated in 11. For these reasons, the brush current conducted into 12 will react with F' to produce a unidirectional and pulsating torque while the brush

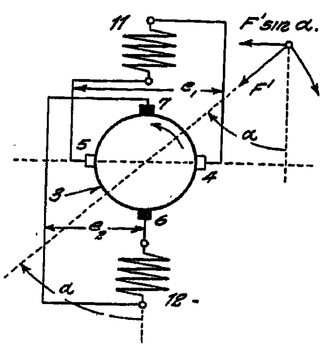

FIG. 5—SHOWING HOW THE SYNCHRONIZING TORQUE IS PRODUCED IN FORM NO. 2 MOTOR

current conducted into 11 is responsible for a double-slip frequency torque with equal positive and negative maxima. How these torques come to be unidirectional and of double frequency can be recognized with the help of Fig. 5.

If the primary member of the motor revolves counterclockwise, it is because the primary flux F' revolves clockwise. The difference between the speeds of F' and of the rotor is equal to the slip of the latter. F' moves synchronously. In Fig. 5, the flux F' has traveled α deg. from its position of coincidence with the axis of the brushes 7, 6, which position is that from which α is measured. For all values of α between zero and 180 deg., the brush voltage e_2 is of the same direction. To be able to speak of this direction as positive or negative, let it be assumed that the direction which e_2 has in normal synchronous operation is the negative one. The normal synchronous conditions are shown in Fig. 6 for both brush voltages and indicate that for values of α between zero and 180 deg. e_2 is negative. Reference to Fig. 5 shows that for values of α between 180 and 360 deg. e_2 must then be positive. This result has been plotted in Fig. 7 on the simplifying assumption adhered to throughout this paper that all voltages, currents and fluxes vary according to the sine or cosine law. The curve e_2 can also represent the current conducted into the winding 12 since at the very low brush voltage frequency existing near synchronism there is practically no phase difference between voltage and current. The current scale may, of course, differ from the voltage scale. The brush voltage e_2 and the corresponding brush current are always proportional to sine α. The other factor determining the magnitude of the synchronizing torque is that component of F' which is at right angles to the axis of the winding 12. This component is $F' \times \sin \alpha$ and the resulting positive torque T_{s-12} is, therefore, proportional to sine^2 α. It is unidirectional and pulsating, becoming zero whenever the axis of F' coincides with the axis of the brushes 6, 7 and remaining positive because the polarity of F' with respect to 12 changes concurrently with that of e_2. The maximum amplitude of this torque is proportional to the maximum conduced ampere-turns in 12. For the conditions yielding the performance curves of Fig. 4, the maximum unidirectional ampere-turns in 12 are proportional to cos 20 deg., which makes the maximum amplitude of T_{s-12} of Fig. 7 proportional to cos 20 deg.

Turning now to the brush voltage e_1 appearing at the brushes 4, 5 and impressed on the secondary 11. By reference to Figs. 5 and 6 it is clear that when $\alpha = 0$ the voltage e_1 is at a negative maximum and becomes zero when $\alpha = 90$ deg. Curve e_1 of Fig. 7 shows its variation throughout a cycle, e_1 is in phase quadrature to e_2 and is always proportional to cos α. Therefore, the brush current due to e_1 is also proportional to cos α and reacts to produce torque with that component $F' \times \sin \alpha$ of F' which is perpendicular

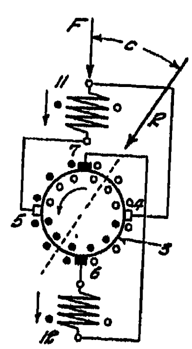

FIG. 6—SHOWING THE SYNCHRONOUS DISTRIBUTION AND DIRECTION OF THE UNIDIRECTIONAL AMPERE-TURNS IN FORM NO. 2 MOTOR

to the axis of 11. The resulting torque T_{s-11} is proportional to $F' \times \sin \alpha \times \cos \alpha$ and is of double slip frequency becoming zero whenever F' coincides with the axis of the brushes 4, 5 or with that of the winding 11. For the conditions yielding the performance curves of Fig. 4 the maximum unidirectional ampere-turns in 11 are proportional to sin 20 deg. which makes the maximum amplitude of T_{s-11} of Fig. 7 proportional to ½ sin 20 deg.

The actually available synchronizing torque is the resultant T_s of T_{s-11} and T_{s-12} of Fig. 7 and it is clear that this resultant is substantially unidirectional and therefore highly desirable. It will produce rapid and positive synchronization without tendency to hunt and will not interfere with the asynchronous overload

stage of the motor's operation to reduce its maximum asynchronous torque.

It is seen that by making the maximum ampere-turns in 12 greater than the maximum ampere-turns in 11 it is possible to secure a desirable compounding characteristic as well as very favorable synchronizing torque conditions. Here again we find no conflicting requirements to embarrass the designer.

Since the winding 11 does not materially contribute to the synchronizing torque when connected to the brushes 4, 5, its circuit need not be closed at that stage and it can also be left open at starting. However,

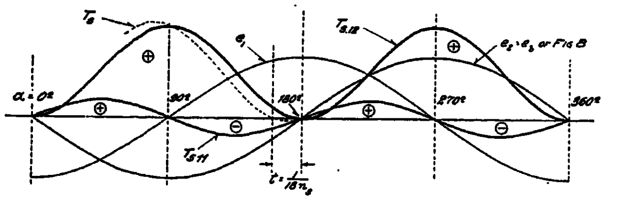

FIG. 7—SYNCHRONIZING VOLTAGES AND TORQUES IN FYNN FORM NO. 2 MOTOR

material increase in synchronizing torque can be secured by temporarily connecting 11 to the brushes 6, 7 and in parallel with 12. To this end, that terminal of 11 which is normally connected to brush 5 is connected to brush 7 and the other to brush 6. This results in a 32 per cent increase in the maximum amplitude of the resultant synchronizing torque. Another way to increase this torque is to dimension 12 for a maximum number of ampere-turns in excess of that required for compounding purposes, reduce these ampere-turns to the desired compounding value by means of the resistance 15 and reduce this resistance to zero during the synchronizing period.

It has already been stated that Form 1 motor of Fig. 1 performs, in synchronous operation, as indicated by the curves of Fig. 4 when the resultant unidirectional magnetization in the motor of Fig. 1 is displaced by 20 electrical degrees with respect to the axis of the brushes 7, 8 of said motor, and its maximum value equals the maximum arithmetical sum of the magnetizations produced by the windings 11 and 12 of Form 2 motor of Fig. 3. To satisfy these conditions, the maximum ampere-turns (disregarding saturation) in windings 9 and 12 of Fig. 1 must be proportional to sine 20 deg. and cos 20 deg. respectively, which means that these ampere-turns are the same as the maximum ampere-turns in windings 11 and 12 of Fig. 3. With this information, the curves in Fig. 8 can at once be plotted. The curve e_b in Fig. 8 must be identical as to phase and magnitude with curve e_2 in Fig. 7, since the machines are identical except for the arrangements and dimensioning of the secondary windings and the number and location of brushes on the commutator. The brush current conducted into the winding 9 of Fig. 1 must yield a double-slip frequency torque T_{s-9} because of the quadrature relation between the axes of its brushes 7, 8 and of its winding 9. The maximum amplitude of this torque is ½ sine 20 deg. and it is therefore identical with T_{s-11} of Fig. 7 except as to sign. The winding 12 of Fig. 1 yields a strictly unidirectional torque proportional to sine^2 α with a maximum amplitude proportional to cos 20 deg. This maximum amplitude corresponds with the maximum of e_b. Torque T_{s-12} of Fig. 8 is, therefore, identical with torque T_{s-12} of Fig. 7 as to direction, magnitude and configuration. Part of the resultant synchronizing torque T_s, is dotted into Fig. 8 and is clearly identical with the resultant synchronizing torque of Fig. 7. In both cases it occurs for the same value of c. The amplitude of the negative wave of T_s is less than 3 per cent of the positive one and the latter lasts eight times as long as the former. It is certainly true to say that T_s is substantially unidirectional.

The very interesting conclusion is thus reached that the synchronizing performance and the synchronous characteristics of a motor of Form 1, see Fig. 1 or any of its modifications, *can be absolutely duplicated* by the motor of Form 2 shown in Fig. 3. No demonstration is necessary to convince anyone that the starting performance of these two machines can be made identical. If the asynchronous characteristics and the synchronizing torques are identical then the asynchronous overload performance of the two machines must also be identical.

But while Form 2, Fig. 3 can do everything that Form 1 Fig. 1 can accomplish, the converse is not true.

It has been pointed out that the synchronizing torque of Form 2 could be materially increased by temporarily connecting winding 11 to the brushes 6, 7. For the compounding characteristic shown in Fig. 4, this change results in an increase of 32 per cent in the resulting

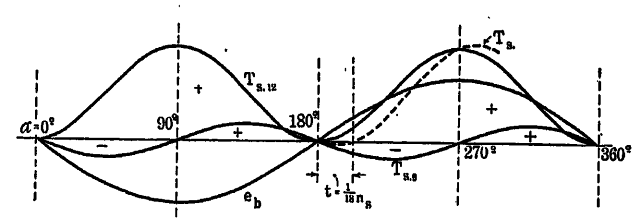

FIG. 8—SYNCHRONIZING VOLTAGES AND TORQUES IN FYNN FORM NO. 1 MOTOR

synchronizing torque. To duplicate this advantage it would be necessary to temporarily displace the brushes of Form 1 so as to bring their axis into coincidence with the resultant magnetization produced by the windings 9 and 12, a proceeding which is not as practical as a simple change of connections.

But Form 2 also permits of a wider range of variation in so far as the compounding characteristic is concerned for the reason that said characteristic can be influenced in at least three ways. The ratio of the maximum ampere-turns in 11 and 12 can be varied as explained or by changing the value of e_1 relatively to that of e_2. This is, for instance, achieved by spacing the brushes

of one set differently from those of the other. A still wider variation can be brought about by disturbing the quadrature relation of the two sets of brushes or displacing both sets from the position shown in Fig. 3 but without disturbing their quadrature relation.

In the arrangement shown in Fig. 9 the brushes 4, 5 are so placed that e_1 is less than the maximum at noload and does not reach zero at full load, while the brushes 7, 6 are so located as to make e_2 negative at no load and positive at full load. The result is that the magnetizations due to 11 and 12 oppose each other at noload and cooperate at full load. No doubt, this example will suffice to indicate the wide range of possibilities afforded by the self-excited synchronous induction motor of Form 2 as illustrated in Figs. 3 and 9. In all cases one secret of success lies in the proper proportioning of the windings 11 and 12 with respect to the limits within which the two brush voltages vary in synchronous and nearly synchronous operation.

The displacement of the brushes from the basic

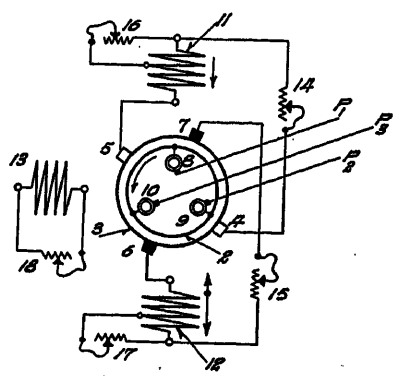

FIG. 9—SELF-EXCITED SYNCHRONOUS INDUCTION MOTOR, FYNN FORM NO. 2

position shown in Fig. 3 also has a bearing on the magnitude and configuration of the synchronizing torque. Just how this torque is affected can readily be stated in general terms. When the axis of the brushes cooperating with the commuted winding on the primary and collecting the synchronizing voltage coincides with the axis of the secondary winding to which said brushes are connected, then the resultant synchronizing torque is unidirectional and pulsating. When said axes are displaced by 90 electrical degrees, in other words, when the phase of the brush voltage conductively impressed on a secondary winding differs by 90 degrees from the phase of the voltage generated in said winding by the synchronously revolving primary flux F', then the resultant synchronizing torque is alternating, of double slip frequency and, all other conditions being equal, of half the amplitude of the pulsating torque. For an intermediate position of the axes under reference the resulting torque has one unidirectional and pulsating and one alternating, double slip frequency component. The nearer the brush and winding axes approach coincidence the larger the unidirectional and the smaller the alternating torque component. It follows that the more the quadrature relation between brush axis and winding axis can be departed from the better for the resulting synchronizing torque. This consideration at once shows one more advantage of the arrangement of brushes shown in Fig. 9.

In so far as the compounding characteristic is concerned, the windings 11 and 12 must be dimensioned with reference to the maximum brush voltages available *within the limits of synchronous operation.* These limits are determined by the travel of R under the influence of increasing load. The measure of this travel is the angle c of Fig. 6. The maximum value of c can usually be made to approach 90 electrical degrees. Reference to Fig. 9 at once shows that, for any value of c up to 90 degrees, the maximum brush voltage available within the limits of synchronous operation can very well be less than the actual maximum. But the *actual maximum* brush voltage *is always* available at subsynchronous speeds when F' travels with reference to the brush axes and it is seen that this condition is also a factor in determining the obtainable amplitude of the synchronizing torque, in this new motor it makes the brush voltage partially effective for compounding and fully effective for synchronizing purposes.

It will be noted in Fig. 9 that only part of each of the windings 11 and 12 is adapted to be shunted or short-circuited during the starting operation. Together with the winding 13, one or both of these shunted parts form a two-phase arrangement of windings. Any other combination of windings which provides a polyphase arrangement of windings on the secondary located in inductive relation to the primary can be used in order to insure the starting and the asynchronous overload operation of this motor, which particular one is chosen largely depends on the conditions under which the machine is to operate.

While the embodiments of this new motor chosen for description all have a revolving primary and a stationary secondary, it will be understood that the functions of stator and rotor may be reversed without in any way changing the underlying principles of operation and design.

In view of misunderstandings which have already occurred, and in order to forestall further questions, it should be stated that the author's inventions relating to synchronous induction motors are held by two entirely independent interests, one of which owns all but that covered by U. S. P. 1,337,648.

Discussion

C. F. Scott: The motor described by Mr. Fynn appears a sort of an extension of the motor described by Mr. Weichsel, probably with improved performance, but with some sacrifice of simplicity in construction.

Both are excellent from the standpoint of the inventor and designer, but after we get through with it all, what is it good for? It is good for improving power factor. And is it worth while to improve power factor? Is it worth while to the customer? Should he pay more for a motor in order to get a higher power factor? What he wants is power output. To make a less simple or more expensive motor attractive to him there must be inducement. That inducement probably must come through better rates for service or other advantages. Then comes the question as to whether this is the best way to improve power factor:—there are condensers of one kind or another and there are other types of synchronous motors and so on. The best way will probably again depend upon the size of motor, and conditions of the particular supply circuit, and the like. So we encounter a large and complicated economic and commercial problem of what is the right and best thing to use in different cases.

But from the standpoint of electrical engineers and of new developments in the art, it seems to me that both of these motor papers can be commended as a splendid move toward overcoming certain difficulties. Their successful application will depend on meeting certain questions in the commercial field and on the inducement given for the raising of power factor by the customer.

L. M. Perkins: If you take the simple Fynn-Weichsel motor, having a single field winding, and a single set of brushes, you may obtain any type of synchronizing torque, as it is called, from the double-frequency torque having equal positive and negative maximums, to the double-frequency torque which has a zero negative with a large positive maximum, and is designated by Mr. Fynn as the continuous positive torque.

If the brushes are shifted off from a 90-deg. position with the field winding, the torque has a so-called constant component added to the double-frequency component, until the brushes reach the point where they are in the same line as the stator winding where the constant torque has such a value that the total torque does not become negative but reaches zero. In between these two limits, there is a large range of ratios between the constant torque and the double-frequency torque. The addition of another set of brushes and another winding increases, of course, the constant torque and adds the double-frequency torque at some angle vectorally, which will increase the double-frequency torque over what it was before. The net result, however, is simply constant torque plus the double-frequency torque, which is what was secured with the simple winding. If it is desired to increase the total amount, that is easily done by changing the resistance of the field winding.

As long as the field windings, which are connected to brushes, are all in one axis it is impossible to vary the ratio between the constant torque and the double-frequency torque beyond the range of ratios that may be secured with the single set of brushes and single winding. This is true regardless of how the angle of the brushes with each other or with the field windings is varied.

V. A. Fynn: The suggestion that the theory of this machine be gone into more thoroughly will, I think, be fully met by the addition of a couple of circle diagrams. The synchronous operation of my Form 2 motor is fully discussed in paragraph four of my paper, but the circle diagrams will, no doubt, add to the value of the record. The starting and synchronizing periods are exhaustively dealt with in my paper, the former in paragraphs 5, 6 and 7, the latter in paragraphs 8 to 24 inclusive.

The presence of the two pairs of brushes shown in Figs. 3 and 9 gives the designer possibilities of varying the compounding and the synchronizing characteristics of motors of this type not available in my Form 1 motor, which is that described by Mr. Weichsel in his paper "A New A-C. General-Purpose Motor" and commercially known as the Fynn-Weichsel.

In my Form 1, there is but one set of brushes and the compounding as well as the synchronizing characteristics depend first, on the angular displacement of the axis of this one set of brushes from the axis of the conduced ampere-turns on the secondary and second, on the conduced ampere-turns per volt, in other words on the number of turns and on the resistance of the secondary winding or windings which are connected to the brushes. There are but two ways of varying the compounding or the synchronizing characteristics, one is to change the aforesaid angular relation, the other is to change the ampere-turns per volt of the secondary windings.

In my Form 2 motor, the characteristics in question can be

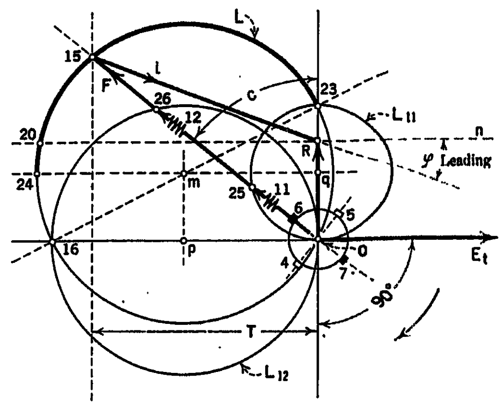

FIG. 1

varied by changing the angular relation between either brush set and the axis of the secondary ampere-turns, by changing the angular relation between both brush sets and the secondary ampere-turns and by changing the ampere-turns per volt of one or of both windings on the secondary. In addition, the maximum brush voltage can be made available for synchronizing without being available in synchronous operation.

Referring to the synchronous operation, let it be supposed that the brushes are located as shown in Fig. 3 and that the maximum ampere-turns in winding 12 are greater than the maximum

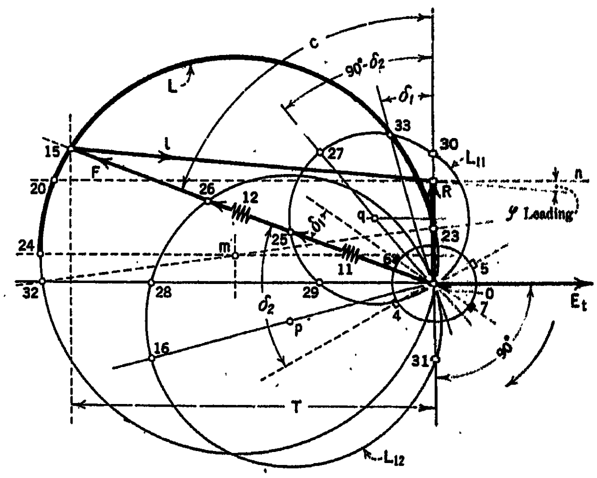

FIG. 2

ampere-turns in winding 11. To construct a circle diagram for these conditions, refer to Fig. 1 herewith and assume that R is the resultant motor magnetization in synchronous operation and that it lags 90 deg. behind the terminal voltage E_t. Further assume that R is stationary and that all the brushes, together with the windings 11 and 12 to which they are connected, are moved through 180 deg. while retaining their correct relative positions. Knowing that the magnitude of the voltage at any pair of brushes, and therefore the magnitude of the ampere-turns in the secondary winding to which they are connected, depend on the angular displacement between R and the axis of said brushes, we

can at once determine the positions for which the ampere-turns in 11 and 12 reach a maximum. When the axis of the brushes 6, 7 is at right angles to R the ampere-turns in 11 are zero and those in 12 a maximum. The vector 0–16 of Fig. 1 herewith shows location and magnitude of the maximum ampere-turns in 12. As the angular displacement between 6, 7 and R changes the end of the vector measuring the ampere-turns in 12 describes a circle, the center p of which is located half way along 0–16. The circle L_{12} is the locus for the ampere-turns in 12. Similarly, when the brushes 4, 5 are at right angles to R, which occurs when the

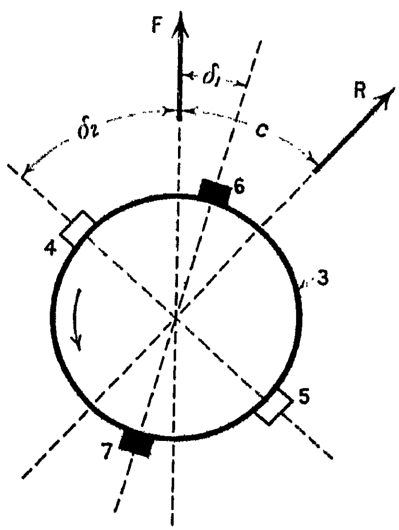

FIG. 3

brushes have moved 90 deg. from their first position, the ampere-turns in the winding 12 are zero and those in 11 become a maximum and are represented by the vector 0–23 at right angles to 0–16. The locus for the ampere-turns in 11 is the circle L_{11} about the middle point q of 0–23 as center. But we are, at each instant, interested in the arithmetical sum of the ampere-turns in 11 and 12, this sum must always be coaxial with the windings 11 and 12 and must, therefore, be measured by a vector coinciding with the axis of the brushes 6, 7. For the angular displacement of R and the brush axes specifically illustrated in Fig. 1, the vector 0–25 measures the ampere-turns in 11, the vector 0–26 those in 12 and 0–15 is the sum of the two. The vector 0–15 represents the secondary unidirectional magnetization F. The locus for F is the circle L with center at m, which center is found as the intersection of the perpendiculars on 0–16 and 0–23 at p and q respectively. The synchronous torque T is always the projection of F on the perpendicular to R. Zero torque occurs at point 23, maximum torque is reached at point 24. The primary current i is measured by R–15 and the primary phase angle by the angular displacement of i from n which is a parallel to E_t through the end of the vector R.

Fig. 2, herewith, shows the corresponding circle diagram for a case such as that illustrated by Fig. 9 of the paper. The selected brush displacements are those shown in Fig. 3 herewith and the ampere-turns per volt in the winding 11 differ from those in the winding 12. Again assuming R and E_t to be stationary and moving brushes and windings counterclockwise, when the latter have moved through δ_1 deg. from the position in which F coincides with R, the ampere-turns in 12 are zero. After a movement through $(90 + \delta_1)$ deg. the ampere-turns in 12 are a maximum and measured by the vector 0–16. The locus for the end of the vector measuring these ampere-turns is the circle L_{12} with center at p, midway on 0–16. After a movement through only $(90 - \delta_2)$ deg. the ampere-turns in 11 are a maximum and measured by the vector 0–27. The locus for the end of the vector measuring these ampere-turns is the circle L_{11}, with center at q midway on 0–27. To find the locus for F, which is the sum of the ampere-turns in 11 and 12, consider any two quadrature positions of the system of brushes. When F coincides with R the resultant secondary ampere-turns are the arithmetical sum of the vectors 0–30 and 0–31 of which the latter is negative. In this case F is represented by the vector 0–23. After the brushes have been moved through 90 deg. the ampere-turns in 11 are 0–29 and those in 12 are 0–28, their arithmetical sum F is 0–32. The locus for the end of the vector F is a circle L passing through the points 23, 0 and 32, which makes it easy to find the center m of this circle. For the rest this diagram is the same as that shown in Fig. 1.

My self-excited synchronous induction motor Form 2 is interesting from two points of view. It is quite independent of my Form 1, known under the trade name Fynn-Weichsel motor, and is not only capable of duplicating the performance of my Form 1 but of excelling same, for it has very useful characteristics not possessed by the latter.

Artificial Representation of Power Systems

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and

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Synopsis.—The size and complexity of present-day power systems have increased to the point where the prediction of the behavior of the system by analytical methods is more and more difficult. The solution of commercial networks by Kirchhoff's Laws or by cut and try methods, even with the help of star-delta transformations, leads to such involved equations that the need for simpler methods is keenly felt. Increasing attention has been given to various methods of representing power systems in miniature so that an experimental solution may be substituted for an algebraic one.

The d-c. short circuit calculating table is a satisfactory and relatively simple means of determining short circuit currents in networks, but is too inaccurate to give satisfactory solution under normal operating conditions.

An a-c. artificial representation of power networks in miniature has been developed by O. R. Schurig of the General Electric Company who used 3.75-kw. 110-volt three-phase generators as power stations. Actual transformers are used, while lines and loads are made up of conveniently arranged lumped units of inductance, capacity, and resistance. This apparatus has been in satisfactory operation for several years.

Evans and Bergvall of the Westinghouse Electric and Manufacturing Company used a test floor set-up to check experimentally the theory of long line stability. Powers of about 500 kv-a. were used.

The present paper presents a method of artificial representation on a laboratory scale, decreasing the size of the apparatus and increasing the precision of the results.

All rotating apparatus has been eliminated. Generators are represented by phase shifting transformers; transformers by their equivalent circuits, and lines by lumped constants. A description of the apparatus used by the writers is presented, together with the results which were obtained by its use in the solution of several typical problems. An analytical check on one of the examples is given, showing a precision of better than 1 per cent.

Purpose and Summary

The purpose of this paper is to describe briefly the work done by the authors under the direction of Dr. V. Bush, in the Research Laboratory of the Masschusetts Institute of Technology in designing, building, and testing apparatus for setting up miniature networks, using generating station powers of 100 watts or less. Complete voltage, current, and power solutions were made on several arbitrary networks. The representation was single-phase, using phase-shifting transformers for generating stations, and a vacuum tube voltmeter drawing absolutely negligible current for measuring potentials.

Introduction

Present power systems, comprising one to many generating stations, widely scattered loads, and the connecting lines, form complex electrical circuits. The problem of the power company is to supply its loads with dependable power at constant voltage in the most efficient way through these networks. To adequately solve this problem for an existing and constantly growing system requires thorough knowledge of its electrical characteristics. Lines and cables should be of proper size to carry present and reasonably anticipated future loads, and so placed that the minimum loss and interruption of service will occur. Future additions to lines and generating equipment should be so placed as to give the best system electrically, other conditions being equal.

What cable or line in the present system limits the load which may be added at a given point and what will be the voltage there with the new load? Where can a new feeder be placed to relieve the overload on a substation? How much may the load be shifted between stations within the system without overloading any feeder or causing excessive voltage drop? With a given load on the system, what distribution among the various generating stations gives the best voltage regulation, the minimum losses and best distribution of load over the various feeders? If a new tie line be put in, will it relieve or add to existing overload on the other lines under some conditions? The answer to these questions is found in numerous solutions of the network under normal steady state operation.

Another group of problems is presented when unusual conditions, such as short circuits, occur. Knowledge of what happens with short circuit at various points in a system is important. The proper size of current-limiting reactors must be determined to limit short-circuit currents to safe values. The magnitude of the currents again determines how much time may be allowed for relays to act without causing serious damage, and what size oil circuit breakers are required. As bus bars and other structural parts of the system have the severest mechanical stresses set up under short circuit, knowledge of short-circuit currents gives data needed in their design. Another factor is the voltage at critical points on the system, such as between generating stations, which has direct effect upon the ability of these stations to stay in synchronism. The current and voltage solution of the network under short-circuit conditions gives data from which the answer to the above problems may be obtained.

Methods of Solution

In general, the solution of a network under either normal operating or short-circuit conditions may be done analytically or by one of several experimental methods.

An analytical solution may be accomplished by using Kirchhoff's Laws, or, commonly easier on any but the simplest systems, the cut and try method. Using the former, equating the sum of all the currents entering a junction, and the sum of all voltages around any closed

1. Of Massachusetts Institute of Technology, Cambridge, Mass.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

loop, each to zero, gives simultaneous equations, the solution of which solves the network. The method becomes so tedious and complicated in a network of any complexity that the cut and try solution is quicker and simpler. By this method, a voltage is assumed at the load most remote from a generating station. Knowing the kv-a. and power-factor drawn, the current is calculated, and the drop due to it, over the line to the next load, added to the assumed voltage. This load current may then be found, the drop due to both currents in the next section of line added to obtain the next junction voltage, and so on until the generating station is reached. The discrepancy between this voltage, and that known to exist at the generating station, is used in making a new trial voltage assumption at the first load, and the problem worked through again. This procedure is continued until a sufficiently close check is obtained for the work in hand. Even the cut and try procedure is very laborious for other than comparatively simple networks and attention has been turned toward experimental solutions.

The most extensively used of these methods is the d-c. short-circuit table,[1] which serves a very useful purpose in giving approximate short-circuit currents. Its results are sufficiently accurate to be used in designing current-limiting reactors, oil circuit breakers, bus bars, and in setting relays. Two methods of representing the a-c. constants on the d-c. miniature are used; the resistance of the arms may be set equal to the reactance or impedance of the corresponding arm in the actual circuit. The former, or reactance method, is most frequently used and gives results less than 20 per cent and usually under 10 per cent in error when the generator reactance forms a large fraction of the total circuit impedance, and the power factor angle of the external circuit is roughly over 45 deg. Here the experimental currents are too large. The impedance method gives approximately the same error when used on circuits having a smaller power-factor angle, but the currents found are below the true values.

This accuracy is good enough for abnormal conditions on a network where other factors, such as generator voltage, are so uncertain. For steady state solution, where results within 1 per cent or less are desirable, the d-c. solution is powerless, and alternating currents with true impedances must be used.

In a-c. representation, the metering of the circuit determines the scale of representation, which should, for convenience and economy, be as small as is consistent with the desired accuracy. Mr. Schurig finds that with standard portable instruments, the current must be 5 or 10 amperes with 200 or 100 volts respectively to keep errors safely below 10 per cent.[2] These errors are a result of the relatively large current taken by standard dynamometer instruments. The voltmeter and wattmeter potential coils together introduce about 1.6 per cent error in the current on the 5-ampere representation. With a line drop of 10 per cent the current coils of ammeter and wattmeter cause an error of slightly over 10 per cent in this drop at 5 amperes if uncorrected. The series impedances of these instruments may, of course, be substituted for an equivalent amount of line impedance, eliminating this error entirely. Error due to potential coil admittances cannot be thus eliminated.

Mr. Schurig's apparatus, in use since 1919, gives results which are usually within 5 per cent of the true value. The error is due to unsteadiness of such small machines and to the difficulty of making the large number of simultaneous readings required, with greater accuracy.

This apparatus may be used for obtaining normal steady state solutions, usually made single-phase; or for short circuits, either three-phase, one wire to ground, or between two wires, in which case unbalance due to unsymmetrical short circuit can be studied.

This, with the work of Evans and Bergvall[3], represents practically all the work done up to the present time. It was in the attempt to get a simple, compact, accurate, easily manipulated laboratory scale means of solving networks that the present method was worked out. Since the factor limiting the reduction of scale was metering equipment, principally the voltmeters, a currentless voltmeter would largely solve the problem. In Appendix I is described the vacuum-tube instrument developed to fulfill this requirement. As constructed, it has the accuracy of the standard portable a-c. voltmeter.

With this instrument available, it was found practicable to reduce generating station capacities to a maximum of about 100 watts.

The apparatus used consisted of phase-shifting transformers representing generating stations, smooth artificial lines, resistance loads and metering equipment.

The phase shifters have the same external characteristics as generating stations, which will later be demonstrated. They alter their load by changing the phase of their terminal voltage relative to the system, the phase shift being manually adjusted in the miniature, and by the torque supplied to the generator in the actual machine.

Sections of the smooth single-phase artificial line in the research laboratory, representing No. 00 B & S solid copper at 8 ft. 9 in. spacing were used for all the lines. In some cases, two parallel circuits were used to give additional capacity. Slide wires, giving adjustable unity power-factor impedance, in series with the ammeter impedance or its equivalent, served as loads.

Voltage as high as 200 r. m. s. could be used and a maximum current of one ampere. Since the ratio of voltage to current is usually of this order of magnitude on the high-tension lines, actual line constants can be used, making current and voltage scales equal. The power scale will then be the product of the current and voltage scales.

1. For references see Bibliography

The vacuum-tube voltmeter was used to measure all voltages, both in phase and magnitude, by the three-voltmeter method. Load ammeters were made a part of the load impedance. At the generating station, the terminal voltage was held constant on the line side of the wattmeter and ammeter current coils. Correcting the wattmeter reading for the small $I^2 r$ loss in the current coils, amounting to one per cent maximum, compensates for all meter errors in the circuit.

ADVANTAGES

a. *Apparatus.* All the component parts of this network, including generating stations, are portable, *i. e.*, one man can carry a phase shifter, the heaviest, piece of apparatus. No particular set-up is required before running the apparatus.

But little space is required, the total set-up of three generating stations, some 200 mi. of line, 6 loads and metering equipment requiring only 40 or 50 sq. ft. of table space.

Since the maximum current is an ampere, very small capacity of lines and loads is required, the smooth artificial lines having ample current capacity. Lamp cord is ample for leads. While the scale is small, ½ and 1 ampere ammeters work well and 1.5-amperes, 150-volt wattmeters give satisfactory deflection. These are standard instruments.

Power requirements may not be of primary interest, still it is an advantage to have low energy consumption. The total input for three stations at full load is less than half a kilowatt.

b. *Operation.* The phase shifters can be set at phase angles unstable for synchronous apparatus and conditions will remain entirely steady. That is, any voltage at any phase within the range of the machines can be held indefinitely. The operator has far more control over the variables than in motor-generator sets.

With the absence of moving parts comes the steadiness of operation of a transformer. With steady supply voltage, the whole network remains free from current and voltage swings, which are somewhat bothersome in small rotating apparatus. Because of this steadiness and completeness of control, adjustments can be made easily, rapidly, and accurately. A network can be quickly solved. The time required for various tests, as will be shown, is surprisingly small.

Relative phases of all voltages are determined by the three-voltmeter method described later, and knowing the power factor of loads and generators, the phase of all currents can be found.

Since this system, as built, is all single-phase, unbalance cannot be studied either under normal operating conditions or in case of short circuit. It is limited to balanced polyphase or single-phase representation. A three-phase set-up can, of course, be used to analyze unbalanced conditions. For steady state solutions this method combines laboratory precision with the simplicity of test floor procedure.

ARTIFICIAL GENERATING STATION

A power network consists essentially of generators, transformers, reactors, transmission lines of one sort or another, and loads. For a representation of power generation it is not necessary to duplicate all generators, for to an observer on the line outside a generating station, completely equipped with instruments, the number of units is undeterminable at any particular time. Also, if there is a Tirrill regulator on the high-tension side of sending end transformers, the presence of the transformers cannot be detected in normal operation. One requirement of an artificial generating station is that it must be able to maintain constant voltage at its terminals or some other point.

Let us see what happens when a generating station takes on or drops load, as viewed by this outside observer. Assume two identical power stations are supplying one load jointly over separate identical lines. Let the load voltage be the vector V_L and the current

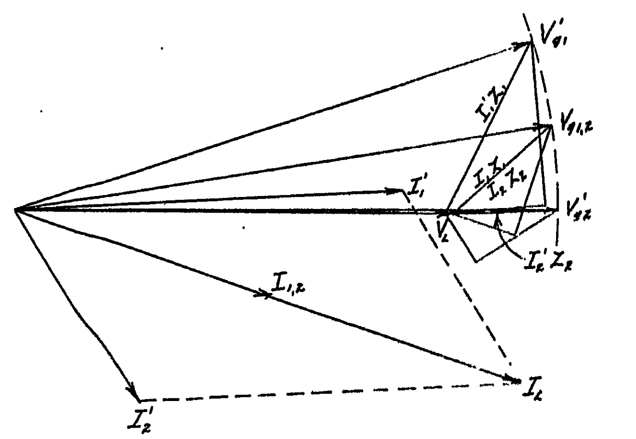

FIG. 1—VETCOR DIAGRAM SHOWING THE EFFECT OF SHIFTING LOAD BETWEEN TWO GENERATORS

taken by the load be I_L. See Fig. 1. The stations have identical governor settings so the load is divided equally. Adding the drops $I_1 Z_1$ and $I_2 Z_2$, where subscripts 1 and 2 refer to line and station 1 and 2 respectively, gives the station voltages, V_{g1} and V_{g2}, which are coincident and slightly advanced in phase from V_L, the load voltage.

Now let station 1 add 50 per cent to the load it is carrying and station 2 drop 50 per cent, the load current remaining constant in phase and magnitude and the station voltages being held constant in magnitude by the regulators. To get the new current I_1' over line 1 requires the drop $I_1' Z_1$. While the load component of I_1' is fixed by the power furnished the station by its prime mover, the reactive component is determined by holding the V_g constant. Imposing the above conditions requires that V_L be slightly decreased in the new condition. Neglecting this in its effect on load power, the vector diagram for the second condition becomes that shown by the primes in Fig. 1.

All that appears to the observer at the high-tension side of transformers at station 1 is a phase advance of V_{g1} when more steam was supplied to this station. The

normal action of a station, then, is shifting the phase of a constant voltage according to the power furnished the generators. A phase shifter therefore fulfills the entire requirements, provided sufficient power can be transformed to hold the voltage constant at the angles at which it is required. Output is a function of terminal-voltage phase relative to the system and a phase shifter output can be adjusted by varying the voltage phase and watching the wattmeter. By having taps at

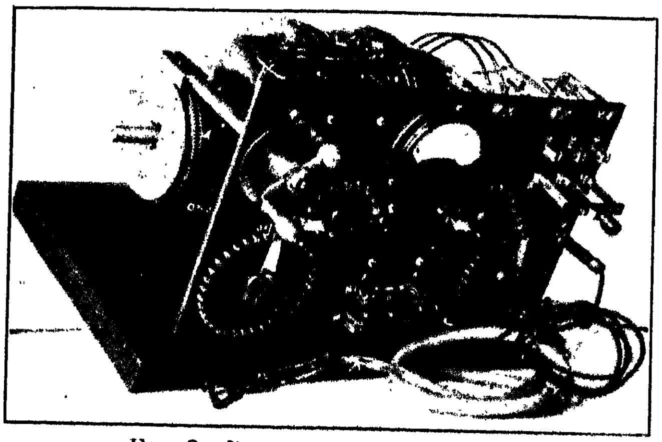

FIG. 2—PHASE SHIFTER COMPLETE

The input is through the three-phase cable in the foreground. Terminal voltage is adjusted by means of the switches on the panel. Voltage phase is adjusted by the crank on the upper left hand corner

different points on the secondary, the magnitude of the voltage can be altered independently of the phase.

To operate these artificial stations, one must, of course, know the voltage regulator settings and turbine governor speed-load curve of the machines represented and this information is necessary for the operation of any artificial station.

The above discussion of generators applies to any

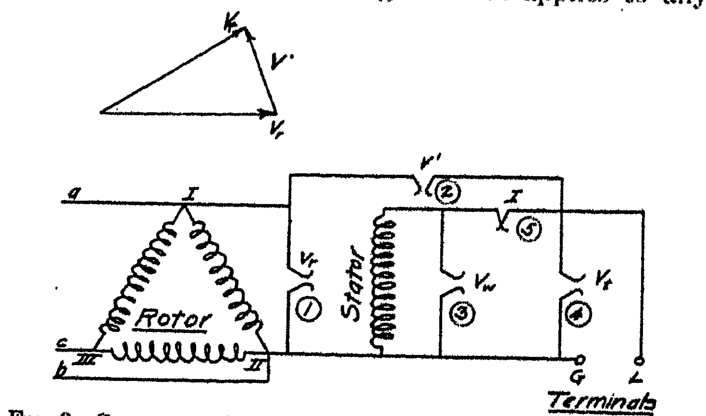

FIG. 3—SCHEMATIC DIAGRAM OF PHASE SHIFTER CONNECTIONS SHOWING METERING JACKS

The vector diagram illustrates the three-voltmeter method of determining voltage phase.

synchronous apparatus so the phase shifter may be equally well used for synchronous condensers or loads at any power factor desired.

PHASE SHIFTERS

On the basis of the above reasoning, a phase shifter, of suitable capacity to fit well with the artificial lines, was designed and two machines have been built. They are three-phase rotor, single-phase stator machines wound on half h. p. induction-motor frames. The details are very similar to an induction regulator, adapted to the voltage and current used.

Fig. 2 is a picture of the complete artificial generating station while Fig. 3 gives a schematic diagram of connections including the metering system. V_r is a voltage jack connected across one phase of the supply voltage, which is used as a phase reference. V_t gives the voltage at the machine terminals G and L. V_w supplies the wattmeter potential coils and at I the ammeter and wattmeter current coils are connected in series. Since one connection of rotor and stator is common, V' will give the third side of the voltage triangle as indicated in the vector diagram. This gives the terminal voltage phase and magnitude. By carrying the common connection throughout the entire set-up, the phase of all voltages can be determined.

STEADY STATE PROBLEMS OF THE POWER SYSTEM AND THEIR SOLUTION ON THE ARTIFICIAL NETWORK

The division of load among alternators operating in parallel is determined, in the steady state, by the governor settings of the prime movers to which they are coupled. Thus if two turbines have the characteristics shown in Fig. 4, the division of load between them would be completely determined in advance. It is not cus-

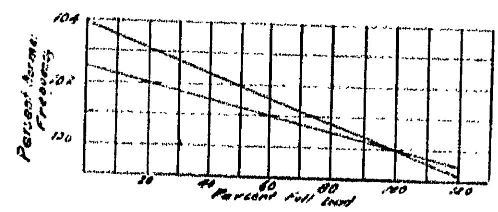

FIG. 4

tomary, however, to regard the governor setting as fixed, but to alter the spring tension so as to make the units pick up or drop load at the will of the load dispatcher. The effect of so increasing the spring tension is to translate the characteristic of the unit upward so that the load carried at any given frequency is increased. On the basis of these considerations, the apparatus described in the preceding section was used to attack some of the problems which arise in the steady state operation of power systems. Several networks were set up on the artificial system, five of which are described. The first is made simple so that an analytical solution is practicable; the others are more complex and approach more closely the problems which are continually encountered in operation practise.

Example 1. The first system set up for study in miniature consisted of two generating stations connected by 96 mi. of line. It was assumed that the generators were subject to the control of Tirrill regulators connected through potential transformers to the high-tension bus of the station, or so compounded as to compensate for the drop in the power transformers.

It was also assumed that one station was clock governed so that the frequency was held constant by that station at all loads, the other station delivering full load constantly to the system.

In solving the problem on the artificial network, the magnitude of the nominal system voltage, reduced by the scale of 1 to 707, was set at the phase shifter terminals and the phase adjusted until the stations were

TABLE I
LOADS ARE IN KW. × 10^3

Load	P_A Meas.	P_A Calc.	Error	P_B Meas.	P_B Calc.	Error
65	74.9	73.4	2.04 per cent	− 3	0	....
85	74.9	75.2	0.39 per cent	20.4	20.0	0 per cent
103	74.9	74.9	0 per cent	38.3	38.4	0.26 per cent
118.9	74.9	75.0	0.13 per cent	58.6	59.5	0.11 per cent

together. Conductance was then added at the point of load until the power absorbed corresponded to the desired value. Finally, the relative position of the phase-shifter rotors was adjusted until the stations delivered load in accordance with the original assumptions. That is, one station should deliver full load and the

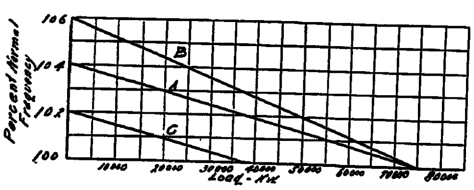

FIG. 5

other station take the balance. The results of this solution are shown in Table I and the experimental results are compared with results obtained by standard cut and try methods.

The entire time required for the solution of this problem on the artificial network was less than two

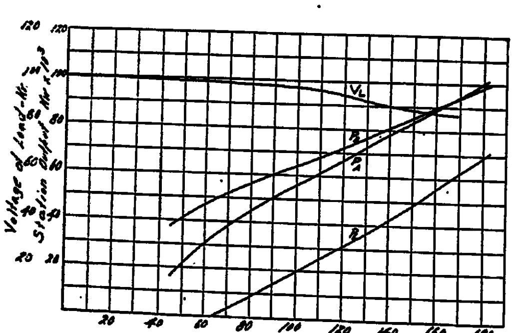

FIG. 6

hours including the time required for setting up the apparatus and applying the instrument calibrations. When the results were checked analytically, it was found that an hour was required for each point checked, although the writers knew in advance about what the answers should be.

Example 2. The second system studied on the artificial network consisted of three generating stations connected by lines, each 48 mi. long, to a load at the common junction. In this problem, as in the preceding one, it was assumed that the voltage was held constant at the high-tension side of the generating station transformer bank. The assumption in regard to governing, however, was considerably changed, the characteristics of the stations being assumed to be as shown in Fig. 5. The method of solution is identical with that used in the first example and about two hours and a half were required to obtain the results which are plotted in Fig. 6.

The characteristics which were assumed for the stations in this example must be based on the assumption

TABLE II

Station	Unit	Governor Sensitivity
A	15,000 kw.	2.0 per cent
A	30,000	3.0 per cent
A	30,000	3.0 per cent
B	37,500	2.5 per cent
B	37,500	2.5 per cent
C	22,500	2.0 per cent
C	22,500	2.0 per cent
C	22,500	2.0 per cent
C	45,000	5.0 per cent

that each station consists of one unit and one only, or else that all of the units of the stations operate all of the time. The addition of a unit in a station would have the effect of raising the frequency and would produce a discontinuous characteristic instead of the smooth curves of Fig. 5. The third example contains the added assumptions, in regard to the equipment of the stations and the order of putting the units into service, that are necessary to make the problem complete.

Example 3. The configuration of the system of this example was taken as identical with that of Example 2, but with the additional assumption that the stations

TABLE III

Station		Machines in Operation	
A		15,000	
A	15,000	30,000	
A	30,000	30,000	
A	30,000	30,000	15,000
B		37,500	
B	37,500	37,500	
C		22,500	
C	22,500	22,500	
C	22,500	45,000	
C	22,500	45,000	22,500
C	22,500	22,500	45,000

were to have the equipment shown in Table II and that the various units of the stations would be put into service in the sequence shown in Table III. In Table II the quantitative measure of governor sensitivity is the per cent regulation in frequency as the alternator goes from no load to full load. From these tables the station load-frequency curves shown in Fig. 7 were plotted.

The load voltages and outputs of the stations were measured for various values of load, but the results have not been plotted, since the discontinuity of the variables would render the curves meaningless. The results,

which were obtained in approximately two hours, are presented in Table IV.

Example 4. The purpose of this problem was to discover what time would be required to set up and solve the sort of problem which arises in determining the expansion policy of a power system. A certain power company was assumed to sell large blocks of power from

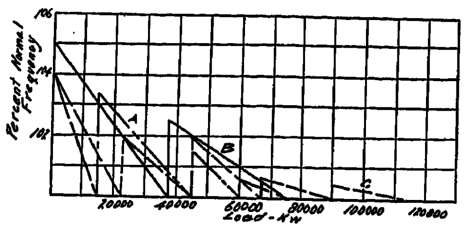

Fig. 7

its two stations A and B of Fig. 8 to the public utility corporations of two cities, L_1 and L_2. It is proposed to extend the system by building a line from J, to carry the load of L_3, a third city. The lines are all of No. 00 solid copper spaced at the corners of an 8-ft. 9-in.-equilateral triangle. L_1 and L_2 draw 93,500 kw. and 96,500 kw. respectively. The nominal system voltage is 100

TABLE IV

LOADS ARE IN KW. × 10^3. VOLTAGES IN KV.

Load	V	P_A	P_B	P_C
42.9	98.6	10.9	24.1	13.5
83.4	97.5	47.5	57.6	32.5
121.9	95.4	44.5	74.8	44.6
141.2	94.1	71.8	71.8	63.2
175.0	91.5	69.0	70.2	85.6
198.9	90.2	71.8	71.8	105.8
218.0	90.2	74.9	74.9	112.1
230.0	87.6	80.4	74.5	122.0

kv. and is maintained constant at the stations. The supplying company wishes to know what load can be supplied to L_3 without causing the voltage at any point on the system to drop more than 15 per cent from the nominal value. The division of the total load between the stations is also desired.

The solution of this problem by cut and try methods

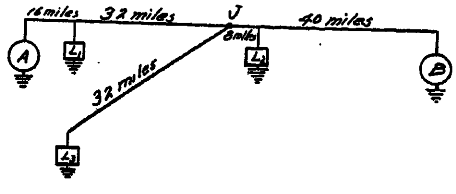

Fig. 8

is quite possible but it would require a considerable amount of time to get the correct answer. The solution of the artificial network required 45 min. of experimental work including the time required for making the set-up, and half an hour to apply the instrument calibrations and work up the data. The results show that 22,750 kw. can be delivered to L_3 and that when this load is being drawn, the other loads being as specified, stations A and B will supply 112,500 kw. and 96,500 kw. respectively.

None of the four problems which have so far been described has approached in complexity the problems which are constantly arising in commercial power networks. In such systems the problem of load dispatching, as well as the problems of voltage distribution which have been considered for the simpler cases, becomes increasingly important and difficult. The example which follows was laid out to determine whether the artificial

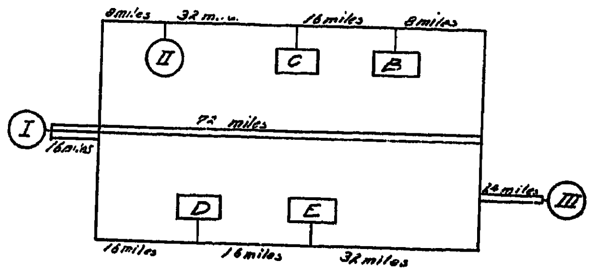

Fig. 9

network could be used as a source of information to aid the load dispatcher in the operation of the system.

Example 5. The circuit of Fig. 9 represents the high-tension network of a typical power system. Generating stations are located at I, II, and III; A, B, C, D, and E represent substations located at load centers which were assumed to have the characteristics which follow. A supplies a large steel mill, a portion of which operates on a 24-hr. day; B carries the load of several large industrial plants; C an industrial load with a small lighting load in addition; E the industrial load of a small

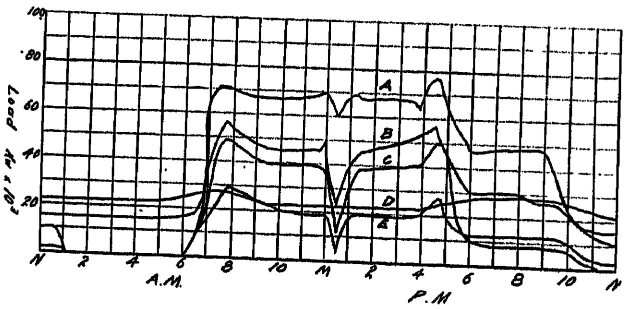

Fig. 10

town together with its lighting requirements; and D was assumed to supply a very large paper mill in a relatively small community where a small lighting load would be superimposed on the constant power demands of paper manufacture. The load cycles of these substations are plotted in Fig. 10. The problem is to determine the load distribution, during the four-thirty afternoon peak, which will give the minimum departure from the nominal system voltage at the various load centers; to determine the divisions of load at ten-thirty in the forenoon which will be satisfactory from the point of view of voltage regulation; and finally, to determine the distribution of load, as dictated by voltage maintenance, which can be used during the noon valley.

The illustration, Fig. 11, gives a general idea of the apparatus as set up for this problem, and shows the two phase shifters representing Stations I and II; between them is the vacuum-tube voltmeter.

For convenience, but one ammeter was used to measure the load currents, the equivalent impedance of the ammeter being inserted in series with the conductance, except when a reading was being taken. This made it possible to insert the instrument at will without disturbing the system. The solution of the problem, which required three hours, is given in Table V.

The A-C. Calculating Table

The work described in the foregoing section was done on smooth lines. There is no reason, however, why lines with lumped constants cannot be used in an artificial network and this network set up as an a-c. calculating table. There are two uses to which such a table might be put; it might be used by a power company in obtaining information to aid the load dispatcher, or it might be used by the consulting engineer in the solution of difficult steady state problems for his client.

In the first application the lines should be represented by lumped constant circuits. The nominal π method of representation[1] leads to an error of only 0.7 per cent with lines up to 200 mi. long, which makes this method entirely satisfactory. Transformers should be represented by their equivalent circuits; and if the assumption is made that the charging current of the line and the exciting current of the transformer produce a negligible drop in the transformer windings, it is possible to add the equivalent impedance of the transformer directly to the architrave impedance of the line or cable and the no-load admittance of the transformer to the pillar admittance. This will make the network much more compact without appreciably sacrificing the accuracy of representation.

The artificial lines and the equivalent circuits of transformers in a calculating table for the consulting engineer's use would be better designed with adjustable constants. In designing coils to represent the impedance of lines in a table of this sort, it would be well to make them up in units of five loop-miles with taps taken out of the coils to take care of the shorter sections. The resistance of the coil would be determined by the largest conductor of the lines to be represented, the inductance by the largest spacing. Thus two coils wound with 235 turns of No. 14 wire on an inside diameter of one and three-fourths inches (4.45 cm.) and one inch (2.54 cm.) thick, axially, would represent five loop-miles (8.04 loop-km.) of any line with a conductor not larger than No. 0000 and a spacing of less than 25 ft. (7.65 m.); an adjustable resistor in series with the coil takes care of the higher resistance of the smaller conductor sizes.

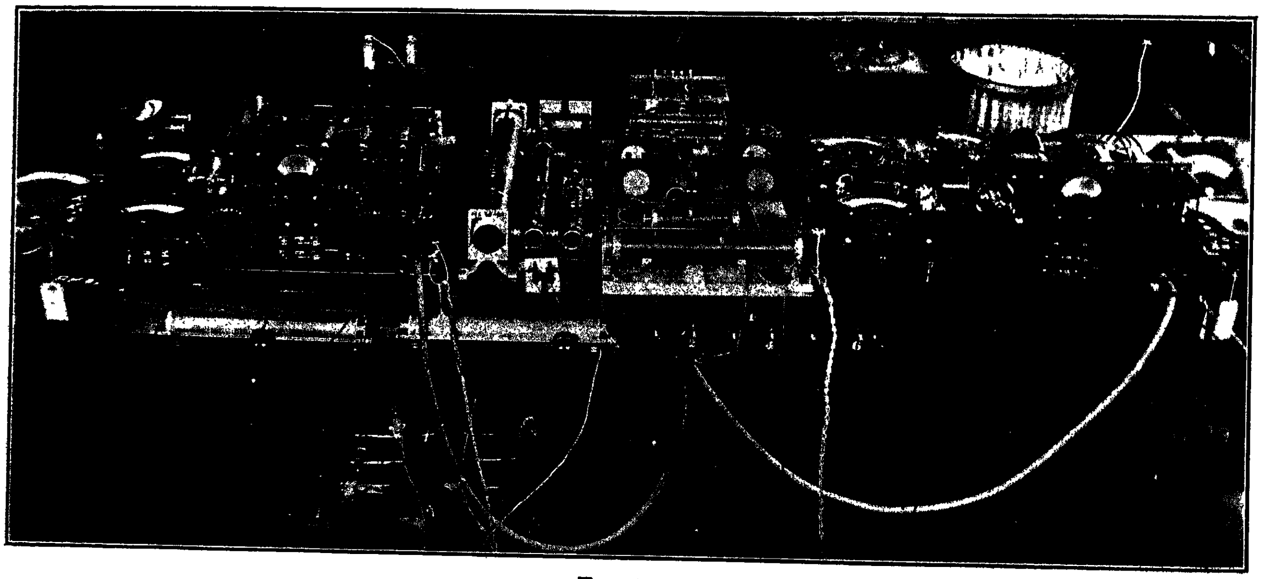

Fig. 11

Loads on the calculating table would be represented by admittances. They should be made up in boxes with point switches calibrated directly in real and quadrature mhos. Then from the voltage it would be possible to set at once the admittance to give the desired power at any power factor without the aid of any meter except the vacuum-tube voltmeter which would not draw power to introduce an error.

In conclusion, the writers wish to express their appreciation of the kindness of C. R. Oliver, of the New England Power Company, in supplying data on an actual power system.

Appendix I

The Thermionic-Tube Voltmeter

The problem of making measurements on artificial lines does not find its solution in the use of ordinary instruments. This is especially true in the case of the voltmeter as a consideration of the scale adopted will show. For example, in the work described in this paper

1. A. E. Kennelly, "Artificial Electric Lines."

the voltage scale was such that 100 volts on the miniature system corresponded to 70,700 volts on the system represented and, since there is no change in the impedances, one watt on the miniature represents 500 kw. single-phase or 1500 kw. three-phase on the actual system.

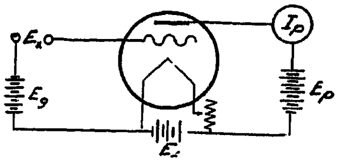

FIG. 12

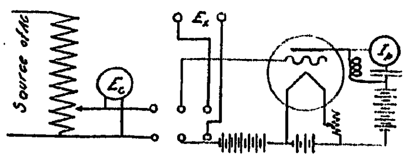

FIG. 13

The ordinary dynamometer type voltmeter takes approximately 0.660 amperes at full scale. This means, for a 150-volt meter, 9 volt-amperes at full scale; or at 141.4 volts, corresponding to 100 kv. in the above scale, 7.96 volt-amperes which is equivalent to 11,920 kv-a. three-phase. The remedy is an instrument which, like the electrostatic voltmeter, will draw no current, but stant filament temperature as the resistance of the filament of the tube increases with age.

In order to obviate these difficulties, a double throw switch was placed in the grid circuit so that the grid could be thrown over to a calibrating voltage immediately after each reading. The drawing, Fig. 13, shows how this was done. The voltages were impressed across the ends of a high resistance drop wire (2.12 megohms) and the filament and grid were placed across a portion of this wire, thus increasing the range of the instrument. The plate current of the vacuum tube consists of an alternating current superimposed on a direct current. The use of a d-c. milliammeter in this part of the circuit is impossible as it would read only the average current, the d-c. component. An a-c. milliammeter or thermo-couple instrument would be satisfactory, except that the change in alternating current is small compared to the steady d-c. component, so that the accuracy would be poor. To eliminate this trouble a 2 μf condenser was placed in series with the meter to

TABLE V

V_A	V_B	V_C	V_D	V_E	P_I	P_{II}	P_{III}	Hour
94.1	95.6	91.1	95.4	93.5	115.0	69.5	76.0	4:30 P. M.
95.9	94.9	94.3	97.0	96.0	112.0	74.8	27.0	10:30 A. M.
97.7	95.6	94.6	97.6	96.5	56.6	74.0	75.2	10:30 A. M.
97.4	95.6	95.5	97.6	93.0	111.5	22.6	74.9	10:30 A. M.
98.5	97.8	98.2	98.2	98.5	111.4	..	24.1	12:30 P. M.
97.8	97.2	96.9	98.0	98.0	59.1	74.8	..	12:30 P. M.

P_A	P_B	P_C	P_D	P_E	Hour
76.0	56.6	49.2	22.9	23.0	4:30 P. M.
67.2	45.7	38.1	18.5	22.5	10:30 A. M.
67.5	45.7	38.0	18.5	22.5	10:30 A. M.
67.1	45.7	38.1	18.5	22.5	10:30 A. M.
61.2	22.5	9.9	2.1	22.5	12:30 P. M.
61.2	22.4	9.9	2.1	22.0	12:30 P. M.

which will have the accuracy of a meter of the galvanometer type.

The vacuum tube operating as a repeater will draw no current from the input circuit so long as the grid is maintained at a negative potential with respect to the filament. This makes it possible to vary the plate current of the tube by impressing a voltage between the filament and the grid without disturbing the circuit impressing the voltage. The simplest way of doing this is shown in Fig. 12. The plate milliameter is calibrated in volts impressed.

In a circuit of this kind there are several difficulties. The alternating voltage must not reach a maximum value greater than the value of the grid bias; the grid bias must not be so great as to approach the cut-off point of the tube; and all the parameters of the circuit, as filament temperature, grid bias, and plate voltage, must be held absolutely constant. The most difficult of these problems is the maintenance of constant filament temperature. It is even difficult to hold the filament voltage sufficiently constant for work of this sort, and constant filament voltage does not mean con- block out the direct current and an inductance of 39 henries was connected across the instrument and condenser to by-pass this component.

The circuit of Fig. 13 was mounted together and made self-contained, with the exception of the batteries and some of the rheostats. The method of continuous calibration makes the precision of the vacuum-tube instrument the same as that of the voltmeter which was used in calibration. No difficulty was encountered in manipulation after the circuit was set up; as many as 120 readings being taken in an hour with ease.

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Power System Transients

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Synopsis.—The steady state load limits of a power network may be examined by familiar methods of analysis. However, the instability of a system with one or more elements working close to these limits sets lower values of power at which the system can be expected to give satisfactory operation under the sudden changes in load or connections which it must successfully sustain in operation. This paper presents methods for the analysis of power systems under transient conditions.

A qualitative discussion of the problem is given, followed by an outline of a point-by-point scheme of analysis which takes into account the inertia of machines, field transients, etc. The value of this scheme depends upon having available rapid methods of analysis applicable to conditions prevailing at a given instant during the transient. Therefore, there is also included a description of the methods which have been found advantageous, notable among which is a superposition method of solving systems by means of charts. This is a powerful means of attacking complicated systems. The method of point-to-point analysis is applied to specific types of networks as examples. There is also given an extension of approximate methods of analysis presented previously.

The paper is confined to the exposition of the methods of analysis which we have found convenient and powerful.

Introduction

With electrically long transmission lines, the ultimate carrying capacity of the line and the economic loading are of the same order of magnitude. Hence it becomes imperative to examine carefully and in detail the maximum power limit of such lines from technical considerations.

The problem is far from simple. If there were no system disturbances to be considered, no voltage fluctuations, no load variations, and so on, then a steady state analysis, using, of course, long line formulas, would readily give the load limits. It is when we attempt to consider disturbances that matters become complicated. Yet analysis is necessary, for while the margin to be allowed between operating load and ultimate steady-state load limit will always be a matter of judgment, there must be available definite facts in regard to the behavior of typical systems under the application of definite assumed disturbances on which such judgment can be based.

There are only three ways in which the knowledge necessary for a proper judgment can be gained:

1. Mathematical analysis.
2. Test of laboratory models.
3. Experience.

We shall not, for some time, accumulate sufficient experience with long lines operating close to their theoretical power limits to enable proper engineering margins of safety to be determined. So we must rely upon analysis and test.

Each of these should be made as nearly a complete treatment of the actual problem as possible, but neither can absolutely reproduce the conditions of the actual network. Test is limited because small machines cannot have the same relations between their electrical and mechanical constants as do large machines. Analysis is limited by the complexity of the problem. As in the analysis of all physical problems, something must be approximated or assumed to make a solution possible Yet each mode of attack should be pushed as far as possible in completeness. The economic importance of the problem warrants taking great pains to omit nothing from consideration that may be important. Analysis and test are complementary. The final check of theory is by test, and the final attack on the actual problems of system design must be by analysis.

In general the problem before us is this: Given a system of power stations connected by transmission lines, and operating close to the steady state power limits of some of its elements, how susceptible is such a system to disturbances of the sort which it will encounter and which it should sustain without rupture? In other words, what is the degree of stability of such a network when subjected to disturbances of the types likely to be encountered in practise? This problem is best solved at present by showing in detail how the system will react to certain definite assumed disturbances of a nature like those to be encountered in practise. For example, how will the system perform if a certain section of line is suddenly tripped out, or if a generating unit is suddenly dropped off? The answer to such questions is the definite guide needed to compare different system designs, and give the basis of judgment as to whether a given layout is satisfactory for the service for which it is intended.

The factors entering the problem are very numerous. The electrical constants of lines and machines are involved as in a steady state solution. In addition, during disturbances, the behavior of exciters, governors, and regulators comes into play, and the mechanical constants of machines as well as their electrical behavior must be considered. In other words, we are concerned not only with the distribution and time variation of voltage and current in the network, but also with the time variation of the air-gap flux of machines, and their mechanical phase relations.

Basis of Analysis

During a power-system transient we have a succession of instantaneous states which occur in sequence,

1 Both of Jackson & Moreland, Engineers, Boston, Mass.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

each of which changes into that which follows. Actually there are an infinite number of such states and the transition is entirely gradual. However, as done in many mathematical devices, we may consider only a finite number of such states occurring at finite intervals of time. If the time interval is short, we may approach the actual process in this manner as closely as is necessary for accuracy. We practically choose a time interval sufficiently short to avoid undue error, and as long as is consistent with this condition, in order to shorten the necessary computation. Hence analysis of the transient behavior involves, first, the solution of the network under the conditions prevailing at each of these chosen instants, and second, a computation of the manner in which the conditions determining these instantaneous solutions vary with the elapsed time.

When a system passes from one instantaneous state to another, the transition, so far as the transmission lines are concerned, is accomplished by means of traveling waves. These die out within a cycle or two. If the chosen time intervals are large compared with the time constant of the traveling waves, then the conditions prevailing at each instant for given terminal conditions on the line will be substantially the same as though these terminal conditions were sustained. In other words, the instantaneous solution for the transmission line will be given by formulas applying to the steady performance of the line as represented most conveniently in circle diagrams.

In machines the case is somewhat different. When the terminal voltage on a synchronous machine is suddenly altered in phase or magnitude, there is an armature transient which lasts a few cycles only. This armature transient may be neglected since it lasts for a brief interval, and its effect, if taken into consideration, would amount only to a minor correction to be applied to the field current. After this has died out, conditions in the machine are those given by curves giving the machine performance for steady conditions, provided they are entered with the actual values of field current and angle which prevail at the instant under consideration. The field current and angle change slowly, due to the large time constant of the field circuit, and the inertia of the machine; and, as we pass from one instantaneous state to another these slow changes must be allowed for and computed.

Therefore, if for any given instant we state the connections and electrical constants of any network, the flux of any connected synchronous machines, the relative phase angles between them, the characteristics of any connected load, and the voltage and current distribution in phase and magnitude, we specify to a sufficient degree of accuracy, the condition of the system at that instant. If sufficient of these data are given for a certain instant, we can solve the remainder from the circle diagrams of lines and connected machines, using these latter at the values of field current and angle actually existing at the instant considered.

Outline of Method of Analysis

Before going into the detail of computation of the complete solution, let us consider qualitatively what happens to a typical system during sudden load changes.

Consider a system consisting of a load supplied in parallel from a local steam station and a hydro station transmitting power over a long line which has synchronous condensers at the receiver end. When a load is suddenly applied at the receiver end of such a system, the increment is shared in the first instant between (1) the steam stations, (2) the synchronous condensers, and (3) the line and hydro station. Each begins to drop back in phase at a rate depending upon the load increment applied and the moment of inertia of its rotor. The field fluxes remain substantially unchanged during the first instant, and then start to alter at a rate depending upon the constants of the field windings and upon the rate at which armature currents are simultaneously changing. After an interval the regulators act and the variations of field current are, thereafter, influenced by this new factor. The governors on the various units also come into action after an interval, and the rate of phase change of each machine is thereafter correspondingly influenced by the increment shaft torque applied by the prime mover as the throttle is changed in position.

Preliminary consideration of the sequence of these events show that at first all of the rotating equipment on the system will start to change speed, and that the apparatus which supplies the larger increment of load in proportion to its kinetic energy of rotation, probably the synchronous equipment nearest to the load, will slow down most rapidly. In slowing down this equipment so changes the phase position of its field structure with respect to the terminal voltage that it delivers a smaller and smaller load increment which eventually passes through zero. It then takes power until the excess changes the direction of swing. Meanwhile the other machines must deliver the deficit in power created by the swing of the first machine, thus in turn carrying a load which makes their rate of phase change greater, so that they follow the first machine through a similar cycle. In this manner synchronous apparatus on the system oscillates about some power point. The condensers, of course, oscillate about the zero power axis and the generators tend to oscillate about their respective governor characteristic curves plotted against the time.

While this discussion relates primarily to sudden load application, there is a similar sequence of events when a generating unit is dropped at the receiver end of the line, or when a transmission circuit is suddenly tripped out. The methods of analysis presented are applicable to any of these cases, or to three-phase short circuits in any part of the system considered.

Given a power system operating under known steady conditions, and given a disturbance such as a change of load or connections suddenly impressed on this system,

it is desired to compute in detail the time variations of all factors in the system in which we are interested.

There is an immediate change, first to be computed, which follows directly after the start of the disturbance and after the lapse of a very small interval of time. This small time interval, a cycle or two, is sufficient for the subsidence of traveling waves, and for the armature transients in machines to have disappeared; but it is insufficient for appreciable change in the relative mechanical phase angles of machines, or appreciable change in the magnitude of the flux in machines.

This immediate change can hence be computed by a solution based on the characteristics of lines and machines in which the given quantities are all the mechanical phase angles and values of flux. These are sufficient to fix the system, and a solution then gives the phase and magnitude of voltage and current everywhere, and hence the new values of power and reactive volt amperes in each machine. We will assume for the present that this solution is made, and later we will return to specific examples, and show how such a solution can be obtained in a manner conveniently adapted for the transient analysis.

We now have the solution for the first instant after the beginning of the disturbance, and hence the values of power which appear immediately after the disturbance in each machine involved. The difference between these values and the initial values of power give the power increments, plus or minus, on each machine. In the first instant these increments are applied entirely to produce positive or negative acceleration of the rotors, and these accelerations may be computed, when the speeds and moments of inertia of machines are known, by well-known formulas.

If now this power increment remained constant, that is if the flux in the machine and its terminal voltage suffered no further change in either phase or magnitude, and if there were no additional power supply to the unit from governor action, we could compute its position for subsequent times from

$$\theta = \frac{\alpha t^2}{2}$$

Where α is the acceleration and θ is the mechanical change in angle. In making this computation, we may, as noted below, make as a refinement a correction for the power consumed by damper windings. Of course, quantities do not remain constant in the manner indicated above, yet, if we choose a sufficiently small time interval, the error in assuming them constant may be made as small as we please. In fact, even although a small variation in the factors involved occurs during the time interval, we may estimate the variation and allow for it. The accuracy in the estimate could be checked presently, and in case of serious discrepancy, corrected by a recomputation. It is possible though to avoid the cut and try process involved in this estimate of power increment and this is in fact necessary if the analysis is not to become unduly laborious. The superposition methods of analysis which are described below enable this to be done by a correction applied to a characteristic curve.

In the first instant after the occurrence of a disturbance, each machine will also have an increment of armature current. Due to the fact that the magnetic linkages with the field winding cannot be changed instantly the field current will also undergo a sudden change. The amount of this change can be obtained from the increment of armature reaction, since the total magnetomotive force on the magnetic circuit of the field remains constant. In this computation allowance is, of course, made for the incompleteness of coupling between armature and field circuits. It is sufficient to use the increment of armature current in quadrature to the induced voltage.

We thus obtain the value of field current in each machine at the instant after the beginning of the disturbance. If there were no further change in armature current and in the absence of regulator action, we could then compute the field current, and hence the flux, in each machine for subsequent times. The field current would return exponentially to its original value, the decrement of the exponential being given by the time constant of the alternator field circuit. In the first short interval of time the regulators will not have acted, so we can compute in this manner the flux in each machine at the end of a chosen short time interval, except for one effect. This is the result of further armature current change during the interval. Such change produces a field current increment in exactly the same manner as the initial change. This may be termed for convenience the subsidiary field transient. It could be allowed for as follows: Estimate the terminal voltage at the end of the chosen interval, from the machine characteristic determine the corresponding armature current and armature reaction, and from this the subsidiary field transient. Apply to this the field decrement corresponding to half the time interval to allow for the fact that the effect is produced continuously throughout the interval at approximately constant rate, and add the result, with proper sign, to the field current obtained from the previous computation. Upon obtaining the next solution the estimate of voltage could be checked. While this method of allowing for subsidiary field transients would work, it is laborious in that it involves a cut and try method. Again the superposition method of obtaining instantaneous solutions which we use in this transient analysis enable us to allow for this subsidiary transient without using cut and try methods. This will appear in the examples which follow.

From the above we now have at the end of a chosen time interval the values of flux in all machines and their relative mechanical phase positions. It is hence possible, by just the same methods employed for the previous solution, to solve for the voltage and current

everywhere in the system at the end of this interval. We may then compute new power increments, and new values of field current.

We then proceed with a second time interval. The procedure is exactly as before with two exceptions. First, in computing the angular space travel of machines in the second and subsequent intervals, we must not only, as before compute, that due to their angular accelerations in the interval, but must add the travel due to the velocity which existed at the beginning of the second interval with respect to a constant speed base. The angle for the second interval is given from

$$\theta = V_1 t + \frac{\alpha_2 t^2}{2}$$

where V_1 is the angular speed at the end of the first interval, obtained from

$$V_1 = \alpha_1 t,$$

except for the effect of power increment during the interval, allowed for as before. Second, it is necessary in computing the field current change in the second interval, to apply the decrement for the interval to the total field current increment at the end of the first interval due to both initial and subsidiary field transients, and to add on the curves the effect of the subsidiary transient in the second interval.

Proceeding in this manner, we can compute the variation point by point with the time, of the voltages, currents, powers, and angles in the system. This is continued until sufficient data is obtained to make evident the behavior of the system subsequent to the beginning of the disturbance. During this process we will shortly arrive at a lapse of time sufficient for the voltage regulator to begin action. When this occurs we shall need to add a new term in our computation of field currents, namely the increment produced by the regulator action on the exciter. This is obtained from a curve of main field current against time after closing of regulator contacts, drawn for the particular exciter system involved. Mr. R. E. Doherty has developed the form of these exciter transients in much detail[1], and we have checked many of his derivations. It is accurate for small increments with shunt wound exciter and brushes on neutral to add the separate effects of field decrement and exciter action. In other words, the effect of exciter field built-up occurs, as far as small increments are concerned, as though the field current were otherwise stationary in value. The consideration of compounded exciters requires that the two effects be considered in combination by a cut and try process.

It may even occur in the course of the analysis that the terminal voltage of a machine will rise to a point where the regulator contacts will again open. This is taken into account by a curve very similar to the above, again allowing a definite interval of time after arriving at the new voltage for the delay in opening of contacts due to time lag of the relays. This is one or two-tenths of a second, depending on the regulator design.

Another effect which occurs after an interval where prime movers are involved is governor action. This is readily taken into account, provided we know the governor characteristics, by including with the power increment used in calculations of angular swing a new increment, of the same or opposite sign, as the case may be, due to governor action. For this we need simply a curve for the governor giving additional power supply to the unit against the time. It is in fact best to construct this as we proceed, as the rate of response of a governor depends upon the increment in speed, or the total response depends upon the integral of the speed change. This again is a matter requiring detailed analysis of its own, and unfortunately complete information in regard to the behavior of all types of governors is not yet available in the form necessary. We have been assisted in arriving at reasonable curves for governor action by certain studies carried out by the General Electric Company. The action of steam governors is especially important. The lag of hydraulic governors is so much greater that the power transient is usually past its critical stage before they get into action.

From the above it is evident that point by point analysis of this sort depends principally upon a proper adaptation of methods of obtaining instantaneous solutions under the conditions set up by the transient analysis. Especially is it possible to proceed with facility when the charts are so modified that cut and try processes, both on power variation and on subsidiary field transients, are avoided. We will, therefore, devote much of this paper to the treatment of rapid solutions under the peculiar terminal conditions which obtain where such solutions are a part of a transient analysis. We will follow this by examples employing the methods developed.

Experimental methods of studying transients, using alternating current artificial lines, and small machines to represent the various generating and condenser stations, have been used by Evans and Bergvall to give a means of checking theory. This is a valuable procedure which will undoubtedly be extended. When it is attempted, however, to obtain from such an artificial system transient data which will apply directly to an actual system, we encounter a difficulty in that the relations between electrical and mechanical constants and the relation between armature and field constants cannot be made the same in a small machine as in a large one. One way of avoiding this difficulty is to replace the small rotating machines by stationary phase modifiers, and to then manually make the phase adjustments and generated voltage changes indicated by the point by point analysis. The artificial network then becomes simply a convenient means of arriving rapidly at the steady state solutions necessary for the complete analysis. This method of representing gener-

1. R. E. Doherty, Trans. A. I. E. E., 1922.

ating stations in an artificial network has been developed by Messrs. H. H. Spencer and H. L. Hazen at the Massachusetts Institute of Technology, and their paper (see page 72 of this volume) describes the apparatus and its use, so that it need not be further discussed here. The advantage of the method, of course, is that the solutions for points in a transient analysis may be obtained more rapidly when complicated systems are considered.

The refinements to be introduced in the transient analysis are a matter of judgment, and experience with the method soon shows the relative importance of various factors. One point of this sort deserves particular mention. During the progress of a transient the speed of a machine will change slightly, and strictly its characteristic as used in the solutions for instantaneous points should then be altered so as to apply to its actual speed. We have found, however, that in cases in which the disturbance was such as to approximately produce loss of synchronism the speed change during the portion of the transient which must be computed in order to predict results was usually well below 1 per cent, so that we have not considered this particular refinement necessary when treating transients due to sudden disturbances.

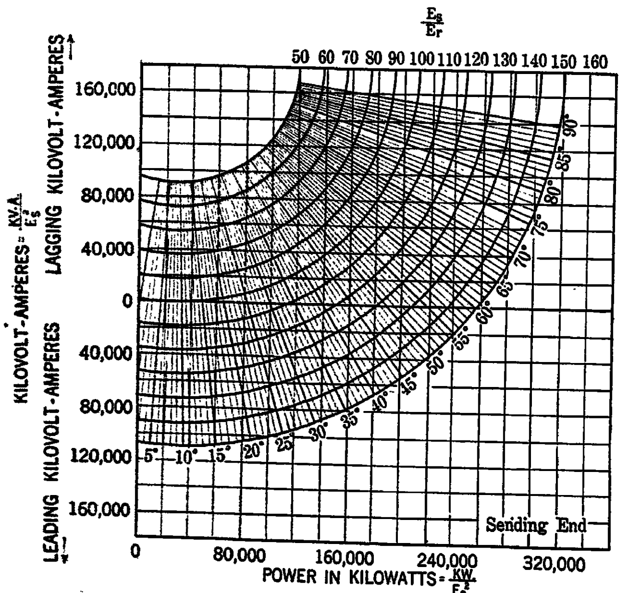

Fig. 1—Simplified Circle Diagram of Transmission Line, Sending End

Aids to Rapid Calculations

In the course of this work, a type of circle diagram has been developed which, while it does not incorporate all the features of some of the diagrams, is much simpler to construct and use, and is particularly convenient for the type of solutions we need in this transient study.[1]

1. This same form of diagram has been presented, since this paper was written, by Mr. C. A. Nicklein a discussion in Transactions A. I. E. E., 1924, Page 85. The numerical example which he gives will be found especially useful.

Hence, although it is a digression, we shall briefly describe it. This simplified diagram, shown in Figs. 1 and 2, is a modification of the Evans and Sels diagram, except that we have included the voltage, power, volt-amperes and angle relations at variable voltages at both ends, instead of at a single end of the line as has been done by previous writers. The advantages of this chart are simplicity of construction, accuracy, and the ability to depict line conditions with variable sending and receiving-end voltages on two charts only, instead

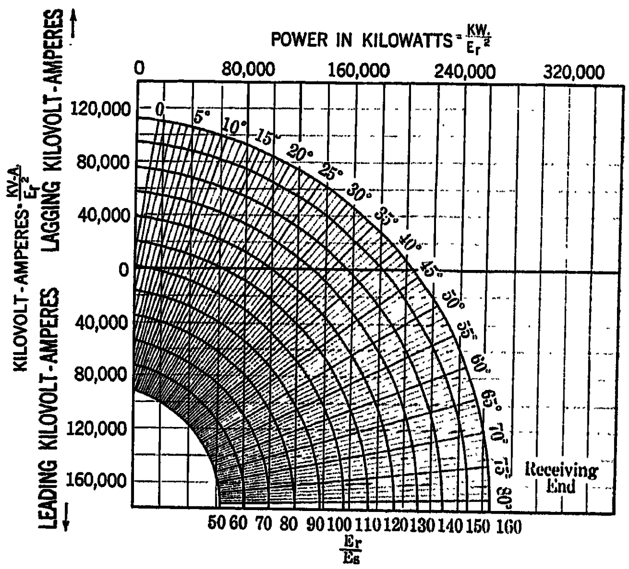

Fig. 2—Simplified Circle Diagram of Transmission Line, Receiving End

of the large number required by the ordinary diagrams to cover a range of voltage conditions. Interpolation on this diagram is rendered more accurate and easy by the uniform scale of voltages at either end of the line and by the fact that the circles are concentric. The diagram is not an approximation: it may be based upon hyperbolic formulas for a long line, and will then represent the action of the line in accordance with these formulas.

The derivation of this chart and the computations of the requisite constants for construction, are similar to those which have been presented for other circle diagrams.

The equation

$$\check{E}_s = (a + j\,b)\,\check{E}_r + (c + j\,d)\,\check{I}_r$$

is a vector equation expressing the relation between the sending voltage $\check{E}_s$, the receiving voltage $\check{E}_r$, and the receiving current $\check{I}_r$, for a given line in terms of the constants $(a + j\,b)$ and $(c + j\,d)$. This equation may be written for a simple line, a line with transformers, a pair of parallel lines, etc.[2]

Take $\check{E}_r$ at standard phase, denote the horizontal and vertical components of $\check{I}_r$ by $\bar{I}_r$ and $\dot{I}_r$, and similarly for other vectors. The scalar equation corresponding to this is then

2. Evans and Sels, *Electric Journal*, August 1921, p. 358.

$$\bar{E}_s + j\,\dot{E}_s = (a + j\,b)\,\bar{E}_r + (c + j\,d)\,(\bar{I}_r + j\,\dot{I}_r),$$

which may be written in two parts:

$$\begin{cases} \bar{E}_s = a\,\bar{E}_r + c\,\bar{I}_r - d\,\dot{I}_r \\ \dot{E}_r = b\,\bar{E}_s + d\,\bar{I}_r + c\,\dot{I}_r \end{cases}$$

Square and add these equations, and we have

$$E_s^2 = (a^2 + b^2)\,E_r^2 + (c^2 + d^2)\,\bar{I}_r^2 + (c^2 + d^2)\,\dot{I}_r^2 + 2\,(a\,c + b\,d)\,E_r\,\bar{I}_r + 2\,(b\,c - a\,d)\,E_r\,\dot{I}_r$$

Now if P_r and Q_r are the total receiving-end power and quadrature volt-amperes respectively, and if E_s and E_r are phase voltages, then

$$\bar{I}_r = \frac{P_r}{3\,E_r}$$

$$\dot{I}_r = \frac{Q_r}{3\,E_r}$$

for a balanced three-phase system. We also use the abbreviations:

$$A = a^2 + b^2$$

$$B = \frac{c^2 + d^2}{9}$$

$$C = \frac{2}{3}\,(a\,c + b\,d)$$

$$D = \frac{2}{3}\,(b\,c - a\,d)$$

Insert these above and obtain:

$$\left(\frac{P_r}{E_r^2} + \frac{C}{2\,B}\right)^2 + \left(\frac{Q_r}{E_r^2} + \frac{D}{2\,B}\right)^2 = \frac{1}{B}\left(\frac{E_s}{E_r}\right)^2$$

If $\frac{P_r}{E_r^2}$ and $\frac{Q_r}{E_r^2}$ are used as coordinates, this is the equation of a circle, and the coordinates of the center of the circle and the radii are then given by

$$\begin{cases} h_r' = \dfrac{C}{2\,B} \\[2ex] k_r' = \dfrac{D}{2\,B} \\[2ex] r' = \dfrac{1}{\sqrt{B}}\,\dfrac{E_r}{E_s} \end{cases}$$

Similar reasoning applies to charts in terms of sending-end conditions giving

$$\begin{cases} h_s' = \dfrac{J}{2\,B} \\[2ex] k_s' = \dfrac{K}{2\,B} \\[2ex] r_s' = \dfrac{1}{\sqrt{B}}\,\dfrac{E_s}{E_r} \end{cases}$$

where $J = -\,2/3\,(e\,c + f\,d)$

and $K = 2/3\,(d\,e - c\,f)$

The position of the center of the circles is now fixed independent of voltage. For any given value of $\frac{E_r}{E_s}$ we have the same radius, and hence the same circle. Therefore circles may be labelled with the ratio of the two voltages, and these circles will be concentric and evenly spaced. The angle between terminal voltages will be given by a series of evenly spaced radial lines. The diagram is thus simpler to draw and use, and more accurate when it is necessary to interpolate. Also a single chart, instead of a series of charts, is sufficient to cover the line behavior with variable voltage at both ends.

The charts shown in Figs. 1 and 2 give a graphic description of the operation of the line, but even they are not necessary, for the ideas involved in this sort of chart may be readily incorporated in a simple link motion which will avoid altogether the necessity of line charts in the usual computations.

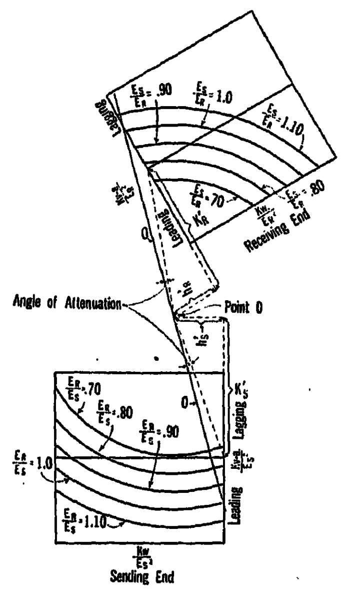

Fig. 3—Layout of Transmission Charts for Use With Link Motion

Place two pieces of rectangular coordinate paper, with axes marked in termes of the ratio of power and quadrature volt-amperes to the square of voltage, as shown in Fig. 3. It is unnecessary to add further loci to the diagram, but some of these are shown on the figure for convenience in explanation.

An examination of this figure shows that a bar pivoted at the point O will intersect the charts at equal angle values. If this bar is graduated to read values of $\left(\frac{E_s}{E_r}\right)$ and $\left(\frac{E_r}{E_s}\right)$ and equipped with a mechanism such when a rider on one end of the bar is adjusted to a certain value of $\frac{E_s}{E_r}$ or $\frac{E_r}{E_s}$ the rider on

the other end will be set at the inverse ratio, we will then have a device with which we can predict conditions at either end of the line when sufficient data are given. The inverse motion can be accomplished by the simple linkage in Fig. 4.

The analysis of the linkage is as follows: (See Fig. 5)

$$a^2 = b^2 + x^2 - 2\,bx \cos\theta_1$$
$$a^2 = b^2 + y^2 + 2\,by \cos\theta_1$$

From which

$$a^2 (x + y) = b^2 (x + y) + (x + y)\,x\,y$$
$$x\,y = a^2 - b^2 = \text{constant.}$$

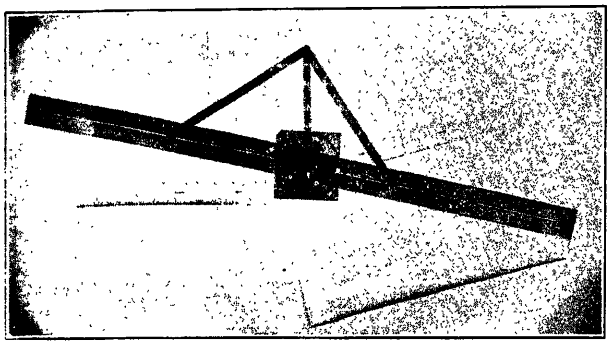

Fig. 4—Linkage and Charts for Predicting Transmission Line Performance

Let us now consider x and y for ratios of $\frac{E_s}{E_r}$ and $\frac{E_r}{E_s}$ of unity, to be themselves unity.

Then $b = 1$ and $a = \sqrt{2}$ are convenient lengths of arms. Choose convenient scales for the coordinate axes used for the sending and receiving end charts, making the length unity equal to the length unity employed on the bar, and mount these upon a board in

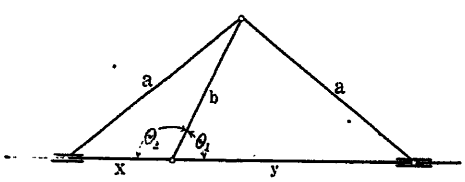

Fig. 5—Diagram of Linkage Used with Transmission Line Charts

such a manner that the angle and the distances between the axes may be varied in accordance with any line constants.

It will then only be necessary, in solving for conditions on any line, after fixing the position of the rectangular scales, to multiply the scalar value of power or volt-amperes read from the chart by the value in volt-amperes of the radius of either chart for $\frac{E_s}{E_r}$ = unity; which is a constant for any line, and by the proper value of E_s^2 or E_r^2. If the bar $x\,y$ is calibrated with a linear scale so that unity occurs at the distance unity from point O, these latter values may be obtained directly from the readings at the two ends of the bar.

The link motion with the two pieces of coordinate paper may thus be set for any given line, and will then give all the necessary relations between power, quadrature volt-amperes, voltage, and angle at both ends of the line. The angular relation between, and the relative positions of the two charts are determined by the values of h and k defined for the sending end and receiving end on a preceding page.

Machine Characteristics

Much more information in regard to the behavior of synchronous machines than is usually given is necessary for the solutions we have in hand. It is necessary to specify the complete relations between field current, generated voltage, terminal voltage, and armature

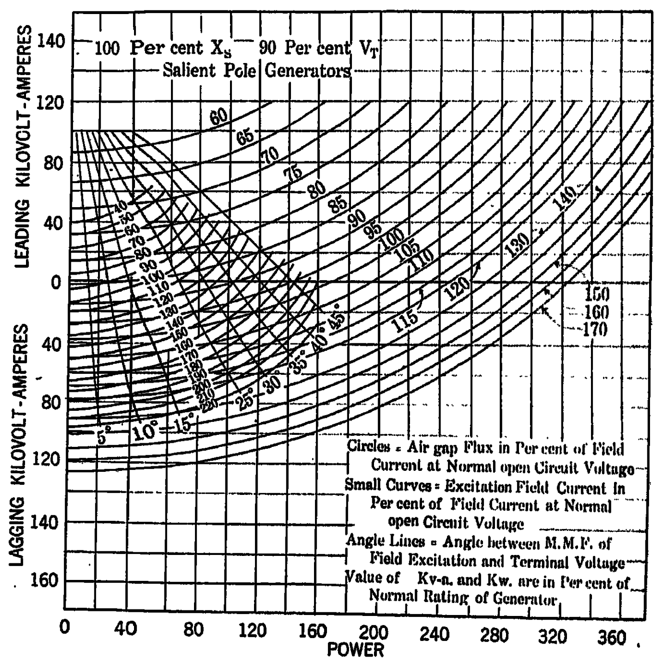

Fig. 6—Chart of Salient Pole Generator Performance

current, both in phase and magnitude. This is most readily done by a series of charts, similar to circle diagrams, constructed for various values of terminal voltage, having power and reactive volt-amperes as coordinates, and showing the loci of constant field current and constant flux in the air gap. Such a chart is shown in Fig. 6. From these may be rapidly derived the necessary curves for graphical system solutions.

Since the stability of power systems operating close to their load limits depends intimately upon the characteristics of the connected machines, it is necessary that these charts be constructed as faithfully as possible. The assumption of a constant synchronous impedance is here unallowable, since under the conditions of operation, it varies widely. Also the usual treatment of a salient pole machine by non-salient pole methods appears too loose an approximation.

We shall usually have available the excitation curve, and the full-load current, zero power-factor characteristic of the machine, together with its short-circuit curve, and the leakage reactance which may be assumed constant without serious error. We can also obtain from design data the factors which apply for armature reaction, for a salient pole machine, to the components of armature current in quadrature to and in phase with the generated voltage; that is, the cross magnetizing and demagnetizing components of armature reaction. Let us call these respectively K_c and K_d.

We can now construct a vector diagram, Fig. 7, for any assumed value of armature current as follows:

Lay off the known current I_a and the terminal voltage V_t. Compute the leakage reactance drop, and by adding this to V_t, obtain the generated or air-gap voltage, E_a'. Perpendicular to this, and of a length obtained by reference to the magnetization curve, lay off the resultant field current R. Through the end of R draw $R\,A$ opposite in phase to the current, and of a length equal to the armature reaction in terms of field current. Divide this line at B so that

$$\frac{R\,B}{R\,A} = \frac{K_c}{K_d}$$

Draw $O\,B$, and project A on this line at C. Perpendicular to $O\,C$, and of a length corresponding in voltage scale to $O\,C$ as a field current on the magnetization curve, draw E_a, the nominal or field current voltage. The length $O\,C$ gives the field current corresponding to the assumed voltage and current. The angle $I\,O\,E_a$ gives the angle of current with respect to the field structure.

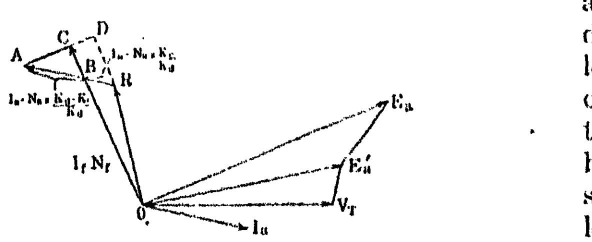

FIG. 7—VECTOR DIAGRAM OF A SALIENT POLE ALTERNATOR

The basis of this construction is as follows: Armature reaction $R\,A$ is split into components $R\,D$ and $D\,A$, in phase with and in quadrature to the center line of field structure given by the direction $O\,C$. The former is added, complete, to vector R, but the latter or cross magnetizing portion is only partially added. The portion used, $D\,C$, is that part of the total, $D\,A$, given by multiplying by the factor K_c. This follows from similar triangles. This method assumes that the reluctances of the magnetic circuits of the leakage flux and the main flux are equally affected by saturation of the iron core.[2]

2. A more complete treatment of the vector diagram of the salient pole machine will be found in *Harmonic Analysis*, by W. V. Lyon, TRANSACTIONS A. I. E. E., 1918, Page 1477.

From the above diagram, we obtain power, quadrature volt-amperes, current and power factor as usual. These are then entered on a chart, and the loci of constant field current, etc., mapped out.

REPRESENTATION OF SYSTEM CHARACTERISTICS

When a system consists of transmission lines interconnecting existing systems, some method is necessary for representing the behavior of each formerly independent system which is tied in. This representation should include the behavior of the loads in that system as well as the generating stations, and the characteristic should be reduced to the point where the system is tied into the transmission line. The generating stations on the system are considered independently as outlined in a preceding section. The load, which may include many small synchronous motors and a large percentage of induction motors, cannot, of course, be treated as so many independent rotating machines. An examination of the relative time constants of these machines and the natural period of oscillation of the rotors shows that the duration of the transients will be very short as compared with those obtaining in the large alternators. Hence, we have used the steady state characteristics of these machines in our analyses.

Actually, each system consists of distributed loads and one or several generating stations feeding into the distribution network. The action of the distributed load may be represented as a single concentrated load of proper characteristics at the load center of the system. In computing these characteristics there are to be included the constants of the lines connecting the several loads with the load center. Thus an individual load which is connected to the load center through a given line may be represented by an impedance at a particular value of voltage at the load center. This impedance includes the impedance of the load and connecting distribution lines in series. Adding the admittances of individual loads at the load center in parallel gives a resulting load admittance for the particular value of voltage assumed. Repeating this process at different voltages gives the load characteristic at the load center. This characteristic is then to be transferred to the point of connection of the system to the transmission line, which is accomplished by adding the impedance of the lines between the load center and the point of connection to the resulting load impedance at the particular value of voltage, thus constructing a new characteristic of the load as determined against voltage at the point of connection.

The above outlines the method of representing the characteristics of a system which is connected to the primary transmission system at a single point only. In case the system is connected to the primary network at two points, a modification of the method is necessary. This consists, in general, of splitting the system into two parts to be concentrated at the two points of connection. The tie, within the secondary system, be-

tween the two points, is then included as a parallel connection to the primary transmission system between the two points. In other words, instead of reducing the load to a single equivalent load, it is reduced to an equivalent load in two parts at the two points of connections. The load at the load center, which is determined as before, instead of being transferred to a single point of connection by including the impedance between the load center and that point, is divided into two parts which are transferred to the two points of connection. The general principle of this method of transfer is that the system with its loads is reduced to an artificial network with two concentrated loads, which is equivalent to the actual system in so far as its behavior at the two points of connection is concerned, in terms of the variation of both power and volt-amperes with voltage.

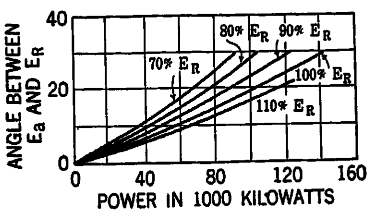

Fig. 8—Derived Power Chart of Condenser Performance Output

Derived Curves

For the solutions incident to this transient analysis, the primary tools, or reference charts, are the characteristics, preferably in the form of circle diagrams as shown above, of generators, connecting lines and transformers, condensers, and loads. From these are taken off as needed in the course of the study of a particular transient problem, special curve sheets for rapid use in obtaining solutions for a given point of time. These are, for reasons which will appear below, most conveniently put in the form of plots of kilowatts or reactive kilovolt-amperes against electrical angle with respect to some base angle as reference, for lines, generators and condensers. The vector used as a base may be the terminal voltage at some chosen point for steady state normal load conditions. On each sheet is a nest of curves for various terminal voltages. For a machine it is also necessary to prepare a separate sheet for each value of field current. Curve sheets of this sort are shown in Figs. 8 and 9 for a 20,000 kv-a. synchronous condenser. Similar charts for a line are shown in Figs. 10 and 11.

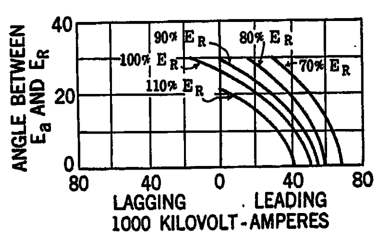

Fig. 9—Derived Quadrature Kv-a. Chart of Condenser Performance Input

These curve sheets are used by superposition methods for obtaining solutions for a network where angles and flux values are the known parameters. It has been found desirable to put kilowatt and quadrature kilovolt-ampere relations on separate sheets, and treat them separately rather than to attempt to superpose circle diagrams themselves. The reason for this is that these solutions are made as a part of a transient solution and because when done it is possible to avoid laborious cut and try processes by a single alteration in the curve sheets used in superposition. Upon superposing these curves a resultant diagram is obtained satisfying all power relations for the network, as well as a second diagram which satisfies all quadrature relations. A final superposition of these two gives a set of conditions which is common to both, and hence is a solution of the network under given conditions.

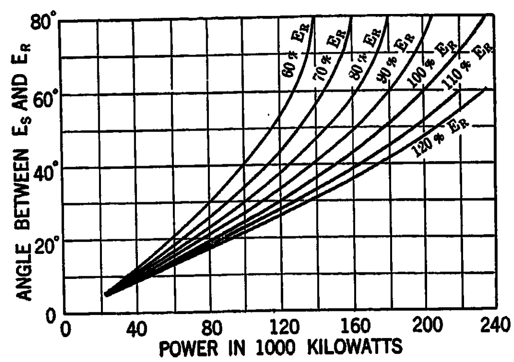

Fig. 10—Derived Power Chart of Transmission Line Performance

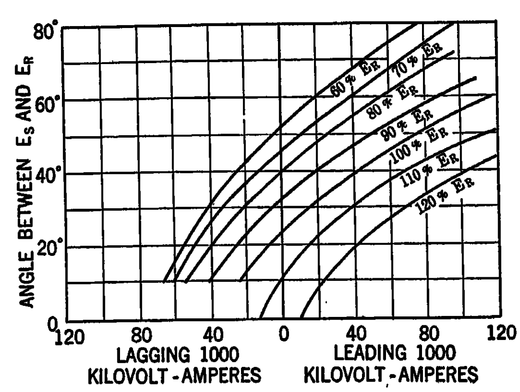

Fig. 11—Derived Quadrature Kv-a. Chart of Transmission Line Performance

This, in brief, is the scheme of the superposition method. It is difficult to explain clearly in general terms, but specific examples will be presented in some detail, and in the course of this, it will appear why these particular superposition methods, avoiding the necessity of cut and try processes, have been adopted for the solutions incident to this transient analysis.

Examples

The underlying theory of this analysis has been presented in foregoing sections and it has been noted that characteristics of machinery, and lines are most readily introduced into the calculations in the form of kilowatt vs. angle charts and reactive kilovolt-amperes vs. angle charts. This modification of the ordinary solution has been found desirable for two main reasons: first, that the intersections obtained by use of the ordinary charts were obscure, and second, that refinements to introduce mechanical movement and subsidiary transients in the fields were readily applicable to the new charts in such a manner as to eliminate the cut and try processes otherwise incident to the consideration of these factors.

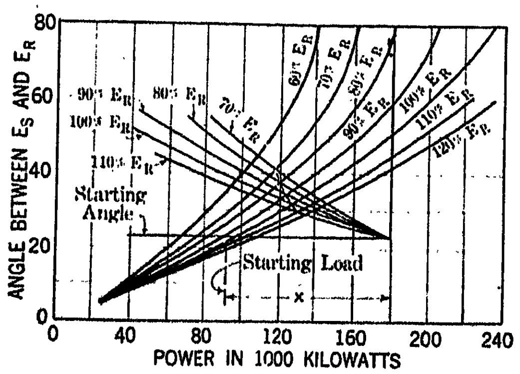

FIG. 12—SUPERPOSED CHARTS FOR DETERMINATION OF POWER CONDITION LOCUS

The method of using these new tools will be illustrated in examples. The first case will be very simple for the purpose of presenting the general method, and because of its simplicity will not involve all of the refinements necessary for solutions of more nearly representative systems. The other cases will consider more extensive systems and will, we hope, present the method and refinements in sufficient detail making the extension to more complex systems apparent.

Consider first a system consisting of a generating station of such dimensions that the high tension voltage at the sending end of a transmission line is substantially fixed in phase and magnitude, with a line transmitting power from this station to a receiver point where a condenser is operating to regulate voltage. Let us determine the operation of the system upon the sudden application of a block of load of constant kw. and kv-a. at the receiver end of the line.

The characteristics used for the solution of the first point are, Fig. 10, the receiver-end power characteristic of the line plotted vs. angle between the sending and receiving end voltages, E_s and E_r, Fig. 11, the receiver-end kv-a. characteristic plotted also vs. angle between E_s and E_r, Fig. 8, the power characteristic of the condenser for the known initial field excitation plotted against electrical angle between the voltage of the impressed field or the center line of field structure and the terminal voltage E_r, and Fig. 9, the corresponding quadrature kv-a. characteristic for the condenser.

Locate upon Fig. 10 the operating point of the line before the application of the additional load x; then place Fig. 8 upon Fig. 10 so that its 0° line coincides with this point of operation. If the zero power point of the condenser is also located at this point, the graph indicates steady state power conditions. However, since we are considering the application of an additional load of x kw., place the zero power line of the condenser on the load line equal to the total load as shown in Fig. 12. Initially, the power in the condenser is zero, so that the angle between terminal voltage and field structure is zero. Thus, the vertical distance between the displaced axes gives the angle between the voltage at the sending end of the line and the field structure of the condenser, which remains unchanged in the first instant after the application of additional load.

It is apparent from Fig. 12 that the power requirements of the system independent of the reactive kv-a. could be satisfied at any point on a locus indicated by the intersection of characteristics of the line and the condenser, for corresponding terminal voltages, shown on the dotted curve. This fixed superposition applies when the kw. of the load is independent of terminal voltage E_r. In case of a load characteristic showing a variation of kw. with E_r, it is sufficient to simply slide the two superposed curves horizontally with respect to one another, as the intersections are spotted in succession, keeping the total horizontal displacement equal to the power in the load corresponding to the particular voltage curves being considered. This same process applies to the consideration of quadrature kv-a. below.

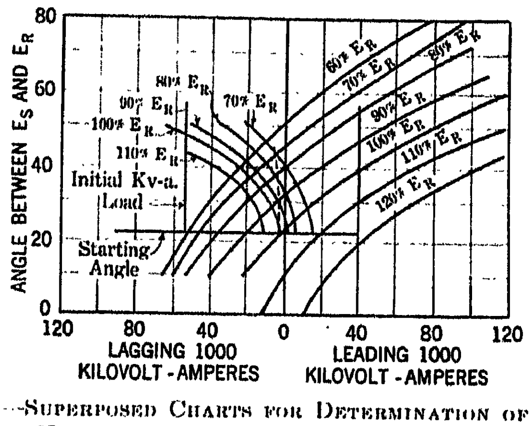

FIG. 13—SUPERPOSED CHARTS FOR DETERMINATION OF QUADRATURE KV-A.-CONDITION LOCUS FOR INITIAL CONDITIONS

We can, in very similar manner, satisfy the kv-a. requirements independent of the power. In Fig. 13, we have made a solution for kv-a. under the initial conditions.

The conditions obtaining upon the addition of a load of y kv-a. are as indicated in Fig. 14.

The dotted curve indicates the possible operating range without regard to power requirements. It now remains to simultaneously satisfy the power and kv-a.

relations. This we can do by superposition of the two resultant characteristics when plotted upon a common base. We have chosen condenser angle and E_r as the determining characteristics, but in this example line angle and E_r could be used equally conveniently.

When the two preceding loci are thus plotted, we obtain Fig. 15, the intersection indicating the operating point for the instant immediately following the application of load. Now knowing the receiver voltage and the angle of the condenser, we can readily determine line angle, line power and the other information desired.

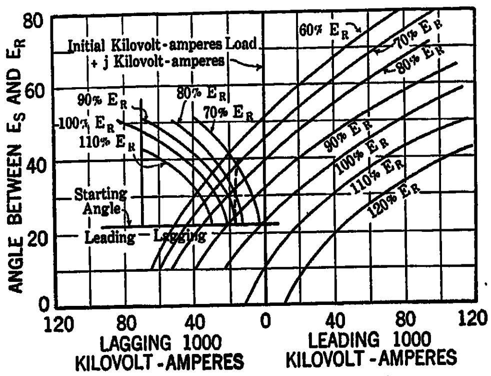

Fig. 14—Superposed Charts for Determination of Quadrature Kv-a.-Condition Locus for Conditions After Adding Load

The above indicates the approach to the problem but certain factors have as yet been omitted from the solution. The first is the transient that occurs in the condenser field as the current in the condenser armature becomes altered. The condenser characteristic used above was made for a fixed value of field current. The field transient may be taken into account by showing this characteristic on a somewhat different basis. Let

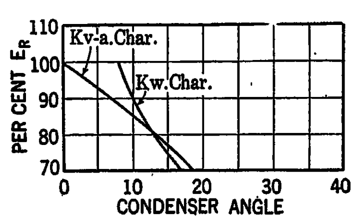

Fig. 15—Superposition of Power and Kv-a.-Loci to Obtain Common Point

us compute, in advance, the change in condenser field current due to transformer action per ampere change of each component of armature current. Now construct the condenser characteristic in such a manner that for a constant terminal voltage the curve is not drawn for constant field current, but at each point for the field current which would obtain, in accordance with the change of armature current in arriving at this point. Since this change of armature current for a given change in electrical angle is definite both in power and kilovolt-amperes for any particular terminal voltage, the resultant change in exciting current which would have obtained, is likewise definite. When in computing later points, the conditions to be determined are for an instant at an interval of time after some preceding known conditions, the increment of field current is computed as a current generated equally throughout the interval considered and is then attenuated in a corresponding manner before being applied. For convenience the time interval chosen between computed points is taken equal, in order to facilitate the computations and make curves useful for more than one point. When the condenser curves are thus constructed the solution will automatically be corrected to allow for the field transient in the machine.

It may be noted that the above method of introducing the field transient involves the assumption of field currents in the condenser, which differ at each point of the chart and do not correspond on the two characteristics. However, since the operating point obtained as a solution is that point at which the condenser angle and terminal voltages are identical on the two characteristics, the field currents involved on each characteristic at the intersection will be identical and correct for the

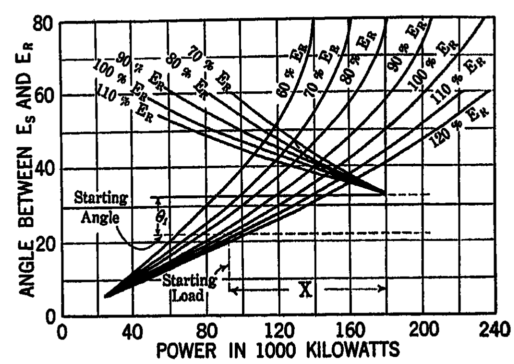

Fig. 16—Superposed Charts for Power Locus Showing Shift for Angular Travel

conditions obtained as a solution. At no other points on the curves would this obtain.

Let us now determine the conditions obtaining after a short interval of time t. The rotor of the condenser has now made an angular movement of θ deg. given by,

$$\theta_1° = \omega t + \tfrac{1}{2} \alpha_1 t^2$$

where α_1 is the acceleration due to power increment at the preceding point, and ω is the relative angular velocity at the beginning of the interval; zero in this case, but to be included for later points. Also the field current increment at the preceding point will have attenuated to a new value, even if the armature current has remained unchanged during this time t.

First construct condenser characteristics for the new attenuated value of field current, correcting the curves as before to the field current that would obtain at any point that involves an armature current other than found for the last-known condition of flux.

This curve will now be used in conjunction with the line characteristic in a manner similar to computations for the initial conditions; except that since the rotor has

been slowing down and has moved the distance θ behind synchronous speed position, the pole pieces of the condenser have now assumed an angle equal to the angle of the preceding point plus the angle of travel. This is shown in Fig. 16. It will be apparent that the condenser may be delivering or receiving power at the end of this interval different from that at the starting time, and that the movement due to this additional accelerating force during the interval considered, has not been incorporated in the calculations. This is done by moving each point of the characteristic curves by an angle θ_2. Since the difference in power between that delivered at the preceding point and the new is variable during the time interval, we must consider a resultant acceleration. If we assume the variation of power to be linear with time during the short interval, we find that

$$\theta_2 = \frac{1}{6}\,\alpha_2\,t^3$$

where α_2 is the maximum acceleration due to an additional power increment during the period, and corresponds to the difference in power delivered from the condenser at the successive time intervals. Hence, after choosing a given time interval, we can compute what additional movement the rotor would complete during the interval by considering the power at the preceding point as a base and computing the maximum acceleration which would obtain at any other power as that created by the difference between that power and the power at the preceding point. The additional movement due to this is then added to the electrical angle of the condenser for each point on the characteristic. This may be most readily done by merely rotating the axis of the power characteristics by the proper amount about the power point of the preceding interval, and then replotting the characteristic by measuring ordinates from this displaced axis. The same change in ordinate for each electrical angle of the condenser and terminal voltage is added to the kv-a. characteristic of the condenser.

Where damper winding effects are important they can be introduced into the calculations at this point. Since these effects are induction motor action which is a function of the difference between the angular velocities of the rotor and the impressed voltage, we have the data for computation of this action. This will be facilitated if we consider this action to be the resultant of a constant slip velocity equal to the velocity $\omega + \frac{1}{2}\,\alpha_1\,t$, plus a variable velocity equal to $\frac{1}{4}\,\alpha_2\,t^2$. Since this variable element can be expressed as a function of power at the end of the interval in a manner similar to space travel we can predict the damper winding effects that would obtain from operation at any point and add these effects to the characteristic of the synchronous machine in the manner indicated for power change in the interval. The field windings of the machine will give us similar effects, but in computations made for typical cases we have found these negligible.

Succeeding points are now computed in similar manner and the progress of the transient mapped out point by point. After sufficient time has elapsed for regulator action to begin, the field current increment due to this cause is also added in at each point, taken from a curve for the regulator used on the condenser. The process is continued until sufficient information is obtained to determine whether the system will or will not lose synchronism, and how much voltage fluctuation is involved.

From the above the method undoubtedly appears more laborious than it really is, although it is admittedly not simple, the problem being inherently complex. Yet in applying the superposition method, many short cuts will be seen; the necessary portion of characteristics only need be plotted, etc. For a system such as above considered, after the circle diagrams and first angle charts are prepared, two practised computers may readily determine the effect of a given disturbance in a day. Of course, more complicated cases take much more time.

Let us now apply these tools to more complex systems. Consider the system shown in Fig. 17. For

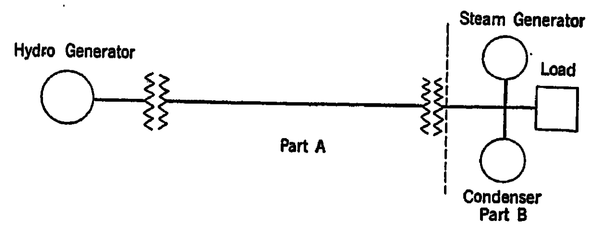

FIG. 17—SYSTEM LAYOUT FOR EXAMPLE

simplicity consider the system broken into two portions as indicated by parts A and B. We can prepare characteristics for the part indicated as A which will show independently the power and kv-a. characteristics of the line and generator at the receiver end vs. the angle between the voltage at the receiver end of this system and the voltage of the impressed field of the generators. This can be prepared quite easily in the manner outlined below for computations of the approximate analysis. These curves, in the form of constant field current characteristics, can readily be applied to any case by choosing points from curves of proper values of field current to make up the characteristics which include the subsidiary transients. These subsidiary effects of changing armature currents can be applied to the characteristics in a similar manner to that used for condensers in the previous example. It will be noted here that the curves are plotted in terms of receiver values whereas the factors which enter into the computation of the field changes are sending-end values. These sending-end values are quite easily obtained when the equation for sending-end current is noted.

$$I_s = C_o\,E_r + D_o\,I_r$$

For any characteristic at constant receiver voltage, we may compute the change in sending-end current by multiplying the change in receiver-end current by D_o. Hence the constant D_o may be incorporated with the transformation ratio of armature to field of the generator to make it a vector ratio of receiver-end current to

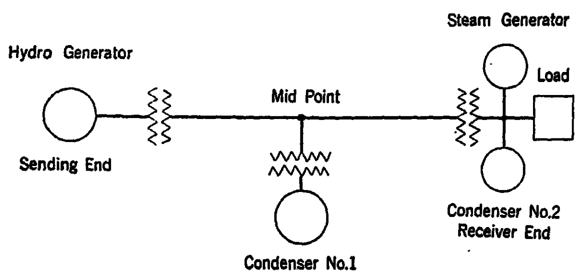

FIG. 18—SYSTEM WIRING DIAGRAM

field current. In this case we must compute and include the angular swing of the hydro generators as well as of the condensers. This is handled in the same manner for each, computing with respect to a constant speed base. With these modifications the portion of the consideration. The rate of swing of condensers and generators will not be the same, so each is computed separately, and this shift changed accordingly at each point. In this example the movement of the rotor of the turbo generator due to governor action, must also be considered. The unbalanced force to produce acceleration is the difference between electric and mechanical shaft torques. The details of this computation of the variation of mechanical shaft torque need not be discussed further at this point.

With these slight modifications we have now reduced our data for this system to a form similar to that used in the first example and no difficulties of solution should be found.

Let us now consider a system that involves transmission lines sectionalized by condensers as indicated in Fig. 18. The treatment of this case, if modified slightly, will also apply when the mid-condenser is replaced by a generator, or generator and load.

The method of attack on this problem is similar to that used for previous cases, but is more involved as the present example has one additional degree of complexity.

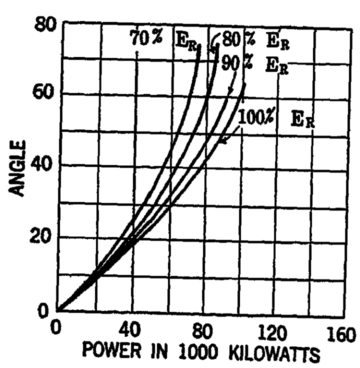

FIG. A

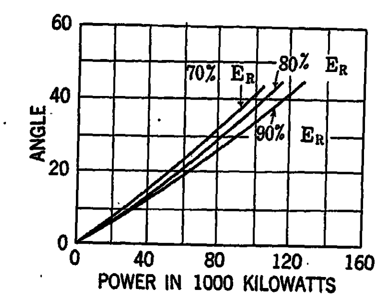

FIG. B

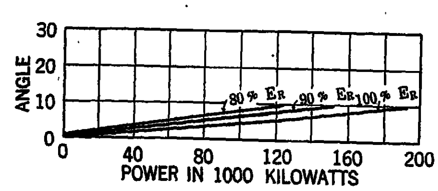

FIG. D

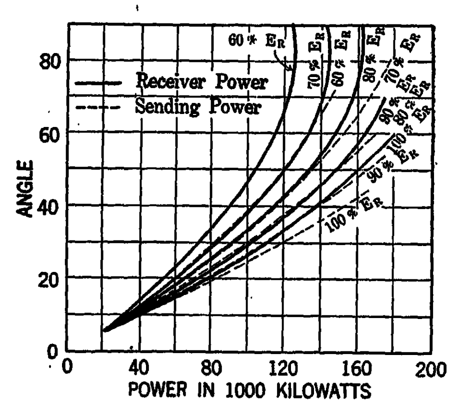

FIG. C

FIG. 19

a. Receiver Power Characteristic of Generator and First Section of Line
b. Power Characteristic of Condenser No. 1
c. Power Characteristic of Second Section of Line
d. Power Characteristic of Condenser No. 2 and Steam Generating Station

system shown in A can be readily handled in a manner similar to that of the preceding case.

The portion B of the system can be reduced to an equivalent single electric machine plus a load by adding together, as we compute each point, the characteristics of the steam generator and the condenser for equal angle displacements at the same terminal voltages. This is accomplished by superposing the individual characteristics with a shift in the direction of the axis of angles of an amount given by the angular displacement of the machines as computed for the instant under

The necessary tools for this solution are characteristics similar to those previously used. The power characteristics are shown in Fig. 19. It will be understood that these are to be modified as in previous cases, to take care of increment power and subsidiary field transients in the interval under consideration.

Fig. 19A is the receiver power characteristic of the hydro-generator and the first section plotted against angle between the voltage of the impressed field and the voltage at the mid-point E_m. This is similar to one of the characteristics used in the preceding example.

Fig. 19B is the power characteristic of the condenser installation No. 1, plotted against the angle between the voltage of the impressed field and the terminal voltage E_m.

Fig. 19C is a plot of both the sending and receiving power characteristics of the second section of the line against angle between E_m and E_r. This figure, for simplicity, shows but one set of terminal voltage conditions.

Fig. 19D is a summary plot similar to 19B for both the condenser No. 2 and the steam generator.

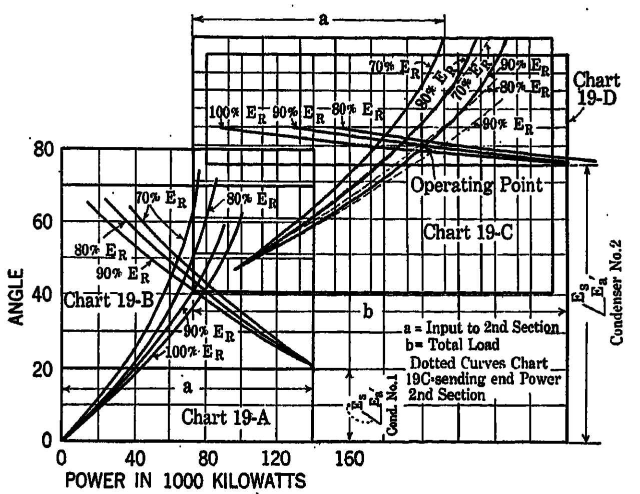

Fig. 20—Superposed Charts for Determination of Power Locus

When the various angles between the field structures of machines are known, it will be sufficient to show the manner of obtaining the solution at a given instant. From an examination of Fig. 20, it will be apparent that the power conditions will be everywhere satisfied for any set of terminal conditions given by an arrangement of the charts as shown. The kv-a. conditions could be similarly treated, but in order to obtain good intersections in this particular type of system it has been found advantageous to adopt a slightly different method than used in previous examples, although it will be apparent that a similar group of charts could have been set up for solutions for reactive kilovolt-amperes.

The power charts are moved in position, keeping the relations shown in Fig. 20 always satisfied, and from each grouping of the power charts we record the power and angle of each of the component charts together with the terminal voltages assumed. We now tabulate with this material the kilovolt-ampere wattless component which would be required in each of the two condensers, as determined from line charts, in order to make conditions fixed by these power relations an actual operating condition. This then provides us with a tabulation of required wattless component of each of the condensers against terminal voltage. We also have a tabulation of terminal voltages, fluxes and internal angles of these same machines as specified by the power components of the condenser output. Since each of these sets of data specify the condenser conditions, we now have a means of determining the operating point by finding the point at which the specifications of operating conditions from power and wattless components of load are identical. Since condensers one and two must simultaneously satisfy these requirements the solution may not be immediately apparent and will be briefly outlined.

Plot the required condenser actions as in Figs. 21A and 21B. Fig. 21A shows the required input to the mid-point condenser for various receiver voltages E_r plotted vs. E_m and 21B in a similar plot for the condenser at the receiver end. We have also shown on these plots the kv-a. characteristic of the corresponding condensers for various angles between the pole pieces and the terminal voltages. The above figures show graphically what condenser angle will be required at each station for various combinations of mid-point and receiver voltages as specified by reactive kv-a. relations. Since the power relations, first satisfied, have also specified the angles on the condensers for similar combinations of

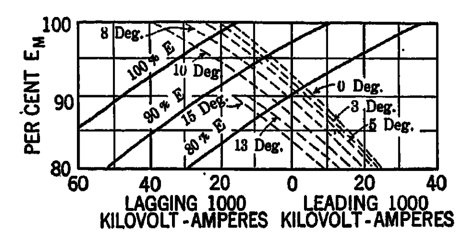

Fig. 21A—Mid-Condenser Requirements

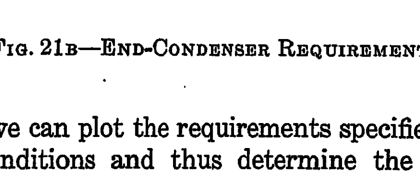

Fig. 21B—End-Condenser Requirements

voltage, we can plot the requirements specified by each set of conditions and thus determine the operating point. From superficial consideration of the above, it might appear that a plot of requisite condenser angle at each point as determined from power and reactive requirements would be sufficient to determine independently each of the condenser angles. However, since we have an additional requirement that the power and kv-a. relations for both condensers shall be satisfied simultaneously—*i. e.*, when all other determining conditions of the circuit are identical—we find it desirable to

express the angle of each condenser in terms of some function common to both.

We find that the angle of the mid-point condenser can be expressed in terms of the first section line angle, since the condenser field structure will maintain a posi-

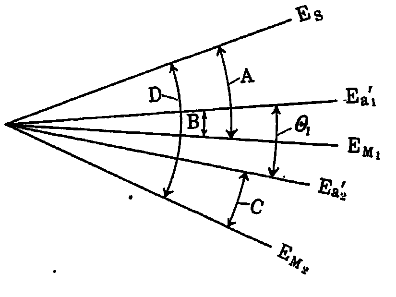

FIG. 22

a. First Section Line Angle for Preceeding Point
b. Electrical Angle of Condenser for Preceeding Point
c. Electrical Angle of Condenser Plus Space Travel Due to Acceleration
d. Total Angle of First Section of Line
e. Travel Due to Velocity of the Rotor Plus Acceleration of Power of Preceeding Point

tion with respect to the sending voltage in accordance with the computed angle θ, plus the angular movement due to acceleration during the interval which is later incorporated in the condenser characteristic. Where

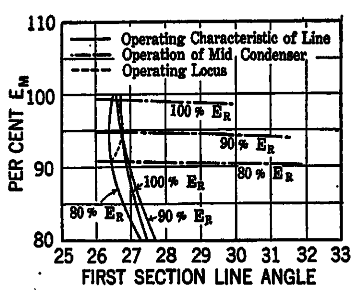

FIG. 23A—DETERMINATION OF OPERATING LOCUS OF MID-POINT CONDENSER

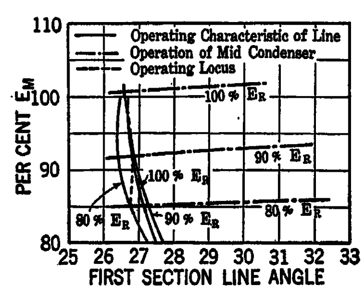

FIG. 23B—DETERMINATION OF OPERATING LOCUS OF RECEIVER APPARATUS

we have referred to the angle between the pole pieces of a machine and the terminal voltage, we mean the angle between the space position of the voltage of the impressed field at the start of the interval and the terminal voltage at the end of the interval. Hence we know that the line angle of the first section, at any time, must equal the line angle at the start of the interval, minus the electrical angle of the condenser at the start of the interval, plus the angle θ, plus the angular movement of the rotor during the interval, plus the electrical angle of the condenser. This for computations for conditions at any given time reduces to a constant plus the angle of the condenser. These relations are shown in the vector diagram, Fig. 22.

In a similar manner we can express the action of the receiver-end condenser in terms of the angle of the first section of the line by knowing the total angle of the system at the preceding point and deducting from this

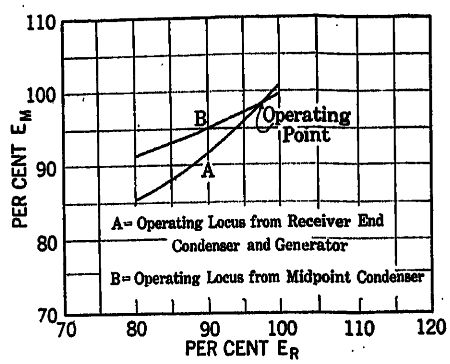

FIG. 24—SUPERPOSITION OF CONDENSER OPERATING LOCI

the angle necessary in the second section to transmit the reactive kv-a. specified for any condenser position with the values of E_m' and E_r also specified, and adding the change in receiver-end condenser angle shown on graphs. By this construction we obtain graphs which depict the requisite angle of the first section of line as specified by the mid-point condenser and by the receiver-end condenser which we can superimpose on the line angle specified by the power relations as shown in Figs. 23A and 23B. These two charts give loci of

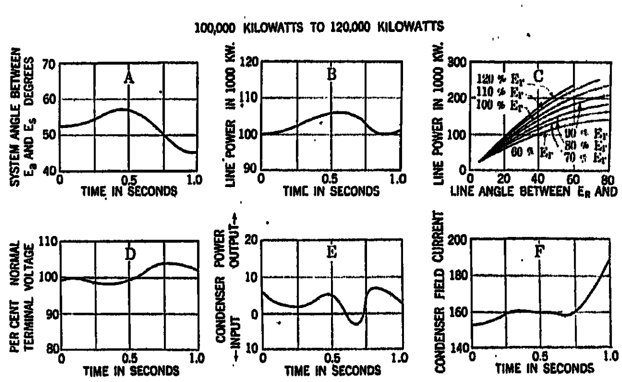

FIG. 25

operation which would be possible as specified by each of the condenser characteristics. Now, by superimposing these two loci as shown in Fig. 24, we obtain the operating point.

It may be remarked that the power characteristics shown in Figs. 23A and 23B are identical but the operating point determined from plotting both condenser characteristics on one chart is obscure. This results from attempting to determine the intersection of two lines in space, *i. e.* three dimensions—from one pro-

jection. However, by transferring the operating lines to the above Fig. 24, we obviate this difficulty.

We have found that the results of the foregoing analysis may most readily be presented in the form of plots similar to those in Fig. 25, which depict results of calculations for a system similar to that used in the second example. Fig. 25c, which is a plot of line power vs. line angle, is found especially useful. If the points are plotted upon this characteristic, it will be possible to predict whether the system will stay in synchronism or go out by the movement of the operating points. It can be demonstrated analytically that when the power over a section of line decreases as the angle increases, that element of the system has entered upon a cycle of cumulative unstability. There are many interesting features indicated in these curves, but the space available is so limited that this discussion must be omitted.

Approximate Methods

In the preceding we have presented a scheme of analysis which, while undoubtedly complicated, has at least been simplified and shortened to a considerable extent; and in this analysis we have provided for the inclusion, we believe, of all the principal factors affecting the behavior of a power system in the transient condition. If some of these factors are left out of consideration the solution becomes much easier, and, of course, a less faithful treatment of actual conditions. However, having available complete methods, which can be used on important cases as a check, it is very desirable to also have available more rapid approximate schemes for common use, testing the conclusions arrived at by their use either by test or by complete analysis.

In the discussion of the transmission papers presented at the Midwinter Convention last year, as published on pages 72-77 of the 1924 Transactions the writers presented an approximate method for determining system stability for a two section line with mid-condensers and a generating station of such size that variations in its terminal voltage could be considered negligible. This method was based upon the assumption that the field structure of the condenser followed the terminal voltage instantly and that rate of build-up of the Tirrill was so slow that the field transients, due to the change of load, were completed before any benefit of Tirrill action was experienced. Hence the characteristics of equipment considered were those of synchronous reactance. The application of the same method to leakage reactance characteristics was outlined at the same time.

This approximate method of analysis may be extended to include the action of systems involving more than one generating system feeding into a network, together with load action, also at various points. Consider, for example, a system consisting of a long transmission line with two generating stations, one at either end, and load systems connected at the mid-point through another line. Assume condensers, also, to be used here, and a transmission line from this to other power systems. The characteristics of the main generating system, together with the first section of line can in a manner similar to that previously outlined be determined from known starting conditions for various values of voltage at the junction at this point, but also upon the two lines connecting the other systems. However, since the condensers operate not only on the long line we must obtain the characteristics of the interconnected systems as generators delivering load to this junction. As previously outlined we can first determine the characteristics of the load at the point of connection to their local generating stations. This load characteristic must now be used in parallel with the generating stations, the combined effects of which, for varying system voltages, can be computed quite

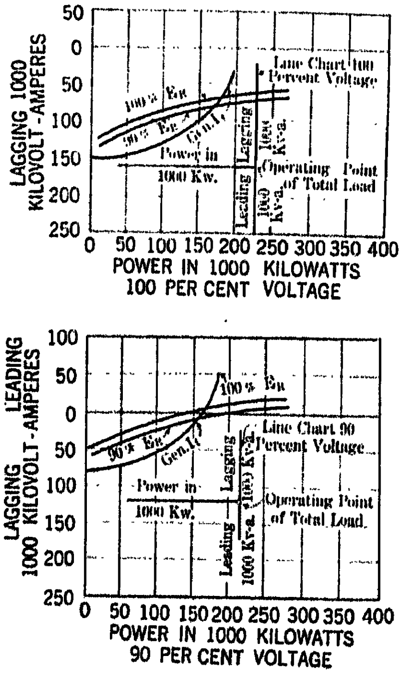

Fig. 26—Method of Obtaining Operating Point of a Line for Various Terminal Voltages When Generator, Load and Line Characteristics, are Introduced into the Calculation

readily as indicated in Figs. 26A and 26B. These figures are plotted characteristics of the generator and line power vs. reactive kilovolt-ampere for a given terminal voltage at the generating station. The generator characteristic given is one of constant field current corresponding to initial conditions and the line characteristic indicates conditions for various voltages at the junction with the main transmission system. A group of figures for various generator terminal voltages is used in the computations. The method of constructing these superimposed charts is to locate upon the generator chart a point corresponding to the total kilovolt-ampere and kilowatt of the load at the particular terminal voltage to be considered. Place upon this point the origin of the kilowatt and kilovolt-ampere axes of the line chart with proper attention to orientation

ot axes of chart. When the charts are thus arranged it will be noted that the given load has been divided between the lines and the station as indicated by the intersection of the field current line of the generator and the line characteristic corresponding to the voltage being considered at the junction with the main transmission system. The determination of the line characteristics for varying junction voltages is thus

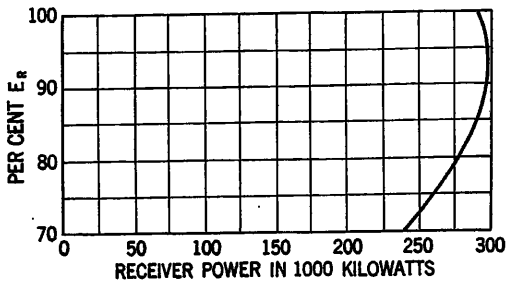

FIG. 27—OPERATING CHARACTERISTIC OF GENERATOR AND TRANSMISSION LINE ONE SECTION OUT IN INITIAL CONDITION

rendered simple and the load characteristics are properly introduced. In locating the center of the line chart for voltages other than normal, we determine the power and reactive components from the load characteristics.

Hence we can build characteristics for the several lines and stations connecting at the junction, at which point the characteristics may be combined. We can combine for common junction point voltages, but some other determining condition is also necessary, since voltage alone does not completely fix the system. We take this condition to be equal increment angle changes in all the interconnecting lines. That is, when the voltage at the junction falls back to a certain angle on the main line, we assume it to experience an equal change when viewed from any other line. This involves the assumption that the rotors of all generators in the network drop back in phase together, or that the rotors of all generating equipment remain fixed relatively during the switching disturbance.

Having obtained a combined characteristic for power at the junction, which for convenience we have plotted in the same form as a generator or line chart, it remains to superimpose the combined characteristic of the condenser and the sending-end of the next section. In previous analyses, we have superimposed the kilovolt-ampere of condenser and load on the receiver-end charts of a line to determine the stability curve, but here this would be inconvenient, since the various lines to the junction have varying sending voltages. Therefore, we determine the kilovolt-ampere at the receiving end of each of the lines for corresponding junction-point voltages and equal increment angles for various transmitted powers, from which we deduct the kilovolt-ampere taken by the condenser at any given terminal voltage. The resulting kilovolt-ampere are summarized and plotted against the transmitted kilowatt. These values are then plotted in the same manner, as are the kilowatt and kilovolt-ampere of a generator, to obtain a characteristic which depicts the equivalent generator action of the system combined at the junction.

From here on, this is used as a single generator of characteristics as found above, making combinations where necessary, and with the analysis completed as for the compound line.

From this example it will be evident that an approximate solution of this sort may be readily extended by similar methods to those here described, to give results for quite complicated networks.

These approximate methods are, of course, more easily applied than the more complete method. However, the neglecting of the effects of inertia and field transients is so loose an approximation that it may render misleading results obtained by the short method. One purpose of the point-by-point method is to determine, by the examination of specific cases, whether the approximations made in shorter analysis are justified. An example of this use was given in the discussion mentioned above and need not be repeated.

The advantage of the shorter method is that it may be applied rapidly to complicated systems. The results obtained, while they must until examined by complete analysis or test be regarded simply as indications of the probable behavior of the system, are often of much interest.

With this reservation, we present in Fig. 27 and Fig. 28, the results of such approximate analysis of a system consisting of a two-circuit long line supplied by hydro generators, and with an intermediate condenser station, as well as a condenser at the receiving end of the line. One curve sheet, Fig. 27, gives the stability for condi-

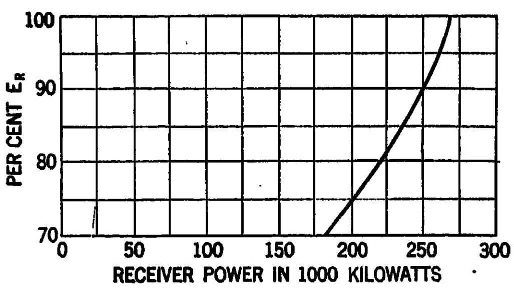

FIG. 28—OPERATING CHARACTERISTIC OF GENERATOR AND TRANSMISSION UPON SUDDENLY DROPPING ONE SECTION OF LINE WHEN DELIVERING 291,000 KILLOWATTS INITIALLY

tions obtaining with one section of one transmission circuit out of operation, field conditions of alternators being adjusted for full load operation under this condition. The other, Fig. 28, gives the curve obtaining when the section is suddenly dropped at time of full load. These curves, in terms of power against percentage of normal receiver-end voltage, give an indication of the stability of operation, since they show the way in which voltage will vary for a sudden load change.

The fact that the second curve shows the load to decrease with decrease of voltage, even at the full voltage point, indicates that this system upon suddenly dropping a section, becomes unstable. In this case the line was 240 miles long, 220-kv., 60-cycles per second, and the characteristics of apparatus were taken typical of standard apparatus, and therefore not specially designed for this service. With condensers and generating equipment more suitable for the service, a similar analysis upon dropping the section indicated stability.

The approximate analysis is valuable as an indication of performance, especially when backed by complete analysis of typical cases for comparison. Our experience, thus far, has been that results obtained by the two methods correspond satisfactorily when synchronous impedance charts are used in the approximate method. This is not, however, a general conclusion, and each system configuration is a separate problem in itself.

Conclusion and Comments

We have outlined in this paper certain methods of analysis of the behavior of power systems during transient conditions. We have confined the paper to an exposition of methods. As a result of our experience in using these methods, we have several suggestions which we believe will assist others in their application to specific problems.

The stability of systems is greatly influenced by the characteristics of the connected machines. Hence it becomes desirable to obtain comparisons between machines of different designs from the standpoint of their relative effects in the maintenance of stability in a system. We find this can be most readily accomplished by analyzing their performance when connected to a typical simple system, using approximate analyses. However, the process should be checked by occasional complete analyses in order to make sure that the result of introducing approximations is to affect each machine studied to substantially the same extent. For example, the use of an exciter system of more rapid response will undoubtedly influence the comparison between alternators, and may even render a different approximation more nearly applicable. That is, it may become advisable to use the assumption of constant air-gap flux in the approximate methods of comparison when such an exciter system is used rather than the assumption of a complete subsidence of field transient such as appears to give more dependable results when the exciter system responds slowly. A complete analysis will determine the matter. It is also advisable to analyze the effect of the machine studied in different networks in order to be sure that the simple one adopted as a basis for comparison will give results which are not misleading.

It appears, from approximate analyses we have made of complicated networks, that a disturbance may often produce instability in a single element of the networks, and that when this occurs the network as a whole will become unstable provided the element is of a magnitude comparable with the remainder of the system. Whether approximate analysis indicates correct conclusions in such a case can of course be determined by a point by point analysis. Even when the network is so complex that a complete analysis would be very laborious, the question may often be settled by the computation of two points only to give the direction in which operating characteristics begin to vary after the first instant. The approximate analysis based on transient reactance gives the solution for the first of those points applying to the instant after the application of the disturbance. The second point, if refinements are considered unnecessary may even be computed in the same manner after introducing computed angular differences.

We have found in examining the behavior during disturbances of a portion of a system, such as a trunk transmission line, that widely different results are obtained according to whether this element is treated as a separate unit or in connection with the network to which it connects. Hence, in examining the design of such an element of a system, it is advisable to make at least some complete analyses of the entire system. The influence of a connecting line depends not only upon its line constants but also upon those of any generating stations and load to which it ties the main transmission unit.

Discussion

V. Karapetoff. The method of solution of the problem of power surges, developed by the authors, may be called that of solution of differential equations by finite increments. While in the paper the differential equations themselves are not written down explicitly, they are understood to hold true. These equations express the condition that an excess of mechanical energy, applied to a synchronous machine, partly accelerates (or retards) its revolving masses, partly is delivered to the bus through a change in the torque angle, and the rest is delivered to the bus through the asynchronous action of the damper winding[1]. This is expressed analytically as

$$(J/p)\,\Omega_m\, d\,\omega/d\,t + W_s\,(\theta - \theta_m) + W_D\, d\,\theta/d\,t = \Delta\,P \qquad (1)$$

where the first term represents the rate of change in the stored kinetic energy in a rotating mass of moment of inertia, J, the second term represents synchronous power due to a change in the torque angle from its mean value θ_m to an instantaneous value θ, and the third term represents the power transmitted through the damper winding. The term Δ, P on the right-hand side, is the applied excess power which causes the surge. Such an equation may be written for every synchronous generator, motor, or condenser in the system. In addition, we have relationships of the form

$$d\,\theta/d\,t = \omega - \omega_k \qquad (2)$$

where ω_k is the angular velocity of the vector of the bus vector and ω is that of the machine, both at the instant under consideration. For each bus, the value of ω_k is different because of the properties of the transmission line. For a section of such a line we may write

$$N\,(\omega_k - \omega_k') = \Sigma\,\Delta\,P \qquad (3)$$

1. E. Arnold, *Wechselstromtechnik*, Vol. 4, p. 384.

where ω_k' corresponds to ω_k at the other end of the line and N is the slope of a power curve, such as are shown for example in Fig. 10 of the paper. In other words, N is the rate of increase in power transmitted, per degree in the change of the angle between the receiver and the generator voltages.

All the equations of the types (1), (2), (3), form together a system of simultaneous differential equations which expresses the complex phenomenon of hunting within the limits of the assumptions made. Further equations may be added to take into account centrifugal governors, voltage regulators, machine transients, etc. While the foregoing equations are linear with constant coefficients, so that their solutions may be written down directly without much trouble, the purely algebraic complexity of these solutions makes them almost useless in all but the simplest cases.

The method of finite increments consists in taking small finite values, Δt, of time and computing the values of θ and $d\,\theta/d\,t$ at the end of each interval Δt. For example, at the first instant of application of ΔP, we must put in eq. (1): $\theta = \theta_m$ and $d\,\theta/d\,t = 0$. Hence $d\,\omega/d\,t$ may be computed directly. Assuming this value of $d\,\omega/d\,t$ to remain constant over a small interval of time Δt, will give directly the values of θ and $d\,\theta/d\,t$ at the end of the interval. Substituting these values in eq. (1), a new value of $d\,\omega/d\,t$ can be computed, etc. Of course, the method is quite tedious, but is apparently the only one feasible if the results are to be derived by computation, and not experimentally.

Sometime ago the present writer proposed a similar method of solution for high-frequency transients on transmission lines and constructed a computing device to facilitate the actual obtaining of numerical results.[3]

F. C. Hanker: The problem of stability in power systems has become one of increasing importance as a result of the rapid increase of electric supply systems. In the early days, a great many of the troubles were caused by the variation in angular velocity due to use of reciprocating prime movers. It was found, however, that difficulty was experienced, even with turbine-driven units, in operating synchronous apparatus on transmission lines of high impedance values, and field tests made some twenty years ago at the suggestion of Mr. Lamme developed empirical data that were sufficiently accurate to meet conditions existing prior to a few years ago.

During the power survey made in 1920-1921 in the North East Atlantic States, studies of transmission of large blocks of power from the St. Lawrence indicated instability. Mr. Baum had been working on the same problem, and the analytical studies suggested the use of intermediate regulating stations. It was recognized that systems were operating satisfactorily under similar conditions when prime movers with drooping speed characteristics were driving the machines used for regulating purposes. There was some discussion, however, as to the stability of synchronous condensers operating in this way, and it was for the purpose of studying this and other important phases of the problem that the tests made in 1923 were conducted.

These tests were reported to the Institute in a group of papers presented at the 1924 Midwinter Convention. Since that time, we have continued our analysis of the problem, supplementing it with tests to develop data on specific phases. It is very reassuring to find that there is a very general interest in the problem, and that the importance of it is recognized. In addition to the studies that are being made from the standpoint of bulk transmission, the operating companies are making careful analysis of the conditions that exist in power systems. This will give us a great deal of fundamental data that will be extremely valuable as supporting the analytical work.

The authors state "There are only three ways in which knowledge necessary for proper judgment can be gained." It appears that they have overlooked one of the most important sources of information, and one in which the variables are considered in their proper relation. I refer to tests made on actual operating systems where the layout corresponds to the condition to be analyzed. A number of tests of this character have been made, more are now under way, and it is to be hoped that further tests will be carried on. The very active interest of the operating engineers in this problem, and their cooperation in carrying on these tests, give us the assurance that data necessary for supporting analytical studies will be available.

As the authors have pointed out, it is necessary in the theoretical studies, to make assumptions as to the characteristics of a number of the variables involved. It is this necessity in connection with analytical studies that makes it important that supporting data be made available.

The investigations that we have made have indicated that the single-phase short circuit with its subsequent switching is the most severe practical condition to be met in operation. Obviously, a system cannot be economically designed to insure continuity of parallel operation under a three-phase short circuit that would reduce the voltage on the interconnecting lines to zero. On extremely high-voltage lines, the construction is such that the occurrence of a short circuit involving all conductors is extremely rare and can be practically eliminated from consideration as the final criterion. In the present state of the art, single-phase faults from one conductor to ground are to be expected, and for such condition, we consider it reasonable that the system be designed to maintain continuity of service.

A short circuit to ground may involve a considerable increase in the true power demand on the generators, depending upon the location of the fault, and the ohmic resistance of the ground circuit. In our study of this problem, we find that under certain conditions, the usual method of governing by speed only increases the tendency to instability. It is desirable to reduce the amount of power transmitted during a disturbance, and with the relation between generator inertia and line impedances found on long lines, sufficient time is available to effect this by closure of the water-wheel gates, provided the actuating impulse is started at nearly the same time the short circuit occurs. The control device to do this would probably be a relay operating on ground current, negative-sequence current, reactive power, or increment of true power.

Tests have been made on a large power system, which show that the oscillation produced by disturbances are damped out very rapidly, hence after the control point has been passed, the gate may be opened again, the whole operation lasting perhaps 15 seconds. The deficiency in energy at the receiver end of the line, due to thes hort circuit and partial closure of water-wheel gates, can be readily supplied from the kinetic energy of the rotating apparatus there, and by temporary overload of the steam plants.

We are making a study of the normal action of steam-turbine and water-wheel governors to determine their effect on stability, when they are grouped together at the receiver end of a line. Due to the large number of stations, it is impracticable to attempt to use any auxiliary control on the governors, and only their normal operating characteristics can be considered.

C. L. Fortescue: The authors of this paper have undertaken the difficult task of presenting in detail a step-by-step method of computing the effects of power transients in transmission systems. On starting to read this paper I was quickly conscious of the inherent difficulty of presenting a subject of this nature in a paper, and I can fully sympathize with them in these difficulties.

I have gone over this paper several times with some care and believe I am safe in saying that it would require fully a week of careful study before anyone not thoroughly familiar with the authors own particular methods could apply them to the solution of transient problems. There are so-called short cuts that in

2. V. Karapetoff. Double Integraph for Electric Line Transients; *Sibley Journal of Engineering*, 1925, Vol. 39, p. 243; also the Bulletin No. 4 of the Engineering Experiment Station of the College of Engineering, Cornell University.

my estimation save little time and are confusing to use, as for example the arrangement illustrated by Figs. 3, 4 and 5. In my opinion the circle diagram without the other attachments would be much more understandable and just as easy to use.

We have made analyses similar to those described by the authors. However, our methods are, I think, more direct and simple although they take into consideration all the factors which the authors have considered. I cannot help feeling that they have made a difficult problem still more difficult. Some of the statements in the paper are very obscure, as for example the statement regarding computations for condenser angle at the bottom of the first column on the eleventh page. My understanding of this statement is that the field current is determined on the basis that during the initial transient the magnetomotive force acting on the main field remains practically constant.

The proper way of presenting a subject of this kind is of course by direct example, that is to say, carrying out the computations step-by-step with the pupil and showing the method of superimposition actually. It may be as the authors state that in a description these methods appear to be more complicated than they really are.

I believe, however, that graphical superimposition methods, however appropriate they may be for illustrating the behavior of a system under certain abnormal conditions, do not lend themselves to accurate point-by-point analysis. In point-by-point analysis, the accuracy of each succeeding computation depends upon that of the previous one, consequently small errors become cumulative resulting in quite large errors in the final results and after all, the point-by-point computations are only steps leading up to the final result which determines whether the set up under the conditions assumed is stable or not.

I would like to point out that the circle diagram which the authors use is a simple modification of the circle diagram which we have used in the past. I am quoting verbatim from our paper of last winter's convention.[3]

"Proof of the Circle Diagram

"From this point a proof of the circle diagram is readily derived. The conjugate equations of (2) may be written:

$$\hat{I}_s = (\alpha - j B)\,\hat{E}_s - (\gamma - j \delta)\,\hat{E}_r$$
$$\hat{I}_r = (\gamma - j \delta)\,\hat{E}_s - (\alpha - j B)\,\hat{E}_r$$

"Multiplying I_s by E_s and I_r by E_r we obtain the power at generator and receiver respectively.

$$P_s + j\,Q_s = E_s\,I_s = (\alpha - j B)\,E_s\,E_s - (\gamma - j \delta)\,E_r\,E_s$$
$$P_r + j\,Q_r = E_r\,I_r = (\gamma - j \delta)\,E_s\,E_r - (\alpha - j B)\,E_r\,E_r$$

but $E_s\,E_s = E_s^2$ and $E_r\,E_r = E_r^2$

If we let E_r be the datum line and $E_s = E_s\,\epsilon^{j\theta}$ then $\hat{E}_r\,\check{E}_s = E_r\,E_s\,\epsilon^{j\theta}$ and $\check{E}_r\,\hat{E}_s = E_r\,E_s\,\epsilon^{-j\theta}$. Substituting in above,

$$P_s + j\,Q_s = (\alpha - j B)\,E_s^2 - (\gamma - j \delta)\,E_s\,E_r\,\epsilon^{j\theta} \qquad (4)$$
$$P_r + j\,Q_r = (\gamma - j \delta)\,E_s\,E_r\,\epsilon^{-j\theta} - (\alpha - j B)\,E_s^2 \qquad (5)"$$

Dividing (4) by E_s^2 and (5) by E_r^2

$$\frac{P_s}{E_s^2} + \frac{j\,Q_s}{E_s^2} = (\alpha - j B) - (\gamma - j \delta)\,\frac{E_r}{E_s}\,\epsilon^{j\theta}$$

$$\frac{P_r}{E_r^2} + \frac{j\,Q_r}{E_r^2} = (\gamma - j \delta)\,\frac{E_s}{E_r}\epsilon^{-j\theta} - (\alpha - j B)$$

A circle diagram which has many advantages is obtained by dividing both equations (4) and (5) by $E_s\,E_r$ giving

$$\frac{P_s}{E_s\,E_r} + j\,\frac{Q_s}{E_s\,E_r} = (\alpha - j B)\,\frac{E_s}{E_r} - (\gamma - j \delta)\,\epsilon^{j\theta}$$

$$\frac{P_r}{E_s\,E_r} + j\,\frac{Q_s}{E_s\,E_r} = (\gamma - j \delta)\,\epsilon^{-j\theta} - (\alpha - j B)\,\frac{E_r}{E_s}$$

Divide equation (4) by E_s^2 and equation (5) by E_r^2 and we have the two systems of concentric circles which the authors used. Moreover the angular relation between sending and receiving voltage is not lost, thus equation (4) says that the locus of $P_s + j\,Q_s$ is given by $\alpha - j\,B$ and the radius vector drawn from this point $-(\gamma - j\,\delta)\dfrac{E_r}{E_s}\,\epsilon^{j\theta}$. The angle θ being the lag of $\dfrac{E_r}{E_s}$ over E_s, the data from which to measure being obtained by equating θ to zero.

I wish to say that when plain vectors are under consideration there is no excuse for using Cartesian analysis since an appropriate algebra for such vectors already exists, being merely the algebra of complex numbers. The only precaution necessary in dealing with electrical quantities represented by vectors is to remember that the vectors are not the actual quantities but merely represent them; the actual quantities are scalar and may be represented actually by the sum of two conjugate vectors. The advantage of the above method of analyzing the transmission power problem is that the angular relation between sending and receiving voltage is preserved and its relation to the power sent over this circuit is plainly seen.

While the authors have taken a great deal of pains to present a method of computing the behavior of transmission systems they have said very little in regard to methods of improving the behavior of such systems. While the Company with which I am connected has been quite busy in making extensive computations in connection with the stability of transmission systems they have been still busier in attempting to find means to improve the stability. The following is a brief résumé of some of the most promising means of accomplishing this.

(a) *High-Speed Excitation and Special Machine Characteristics.* When a sudden load is thrown on a system there is an instantaneous drop in voltage at the load point. If a regulator can be obtained sufficiently sensitive with a high-speed exciter the field of the condenser or generator at this point may be kept from decreasing and actually increased. As an actual fact during the field transient the field circuit has no effective inductance and the only resistance the electromotive force of the exciter has to overcome is the resistance of the field itself, so that there is a decided advantage in quick action outside of the effect on stability. The same exciter if it were to operate after the field transient had died out would take very much longer to build up a field to the same value.

Special machines can be designed having characteristics which lend themselves to stability. The theoretical basis of this may easily be obtained by the consideration that generators, synchronous motors, condensers, etc., are merely extensions of the transmission line; their internal characteristics will therefore have considerable bearing on stability.

(b) *Intermediate Condenser Stations.* Intermediate condenser stations having condensers of suitable characteristics are essential for economic transmission over long distances. There are other advantages which cannot be measured in dollars but which may be more important. Properly chosen characteristics for the condensers may result in considerable increase in the stability of the system.

(c) *High-Speed Circuit Breakers.* In case of short circuit in any section of the transmission system it is advantageous to cut out the faulty section as quickly as possible. Where the fault occurs near the generating station due to the sluggishness of hydraulic governors the generators may fall in speed quite considerably before the circuit breaker opens. After the circuit breaker opens on regaining equilibrium there is a tendency to overshoot due to gathered inertia to such a point that the machine gets out of step. A sufficiently quick acting circuit breaker takes care of this condition very effectively and also will take care of most conditions of trouble arising in transmission systems.

(d) *Governors.* A complete paper could be written on steam

3. *Some Theoretical Considerations of Power Transmission,* C. L. Fortescue and C. F. Wagner, A. I. E. E. Journal, February 1924, page 106.

and hydraulic governors and such a paper would I regret to say deal chiefly with their shortcomings. I feel that there is room for a great deal of improvement and such improvement will come by studying their characteristics in connection with the problem of stability.

(e) *Increasing the Number of Sectionalizing Points.* The shorter the sections of transmission line that are cut out due to a fault, the less effect the cutting out will have on stability. Therefore, increasing the number of sectionalizing stations will improve the stability of the system. There is, however, an economic limit to this method determined by the number of circuit breakers required for each additional sectionalizing station.

(f) *Neutral Impedances; Selective Impedances.* Since most of the troubles connected with transmission lines are due to grounds, power stability may be preserved by the introduction of neutral impedances or resistances. The objection to this method of preserving stability is the tendency to produce voltage transients in the transmission system endangering insulation and apparatus. Since most short circuits are single-phase or unsymmetrical therefore an impedance which will impede the negative-sequence component of current without offering impedance to the positive-phase component of current will help secure stability.

In our study of the problem of stability we have attempted to steer a course between analytical methods and practical experiment. Since the analytical solution at its best is very involved, one can hardly expect engineers of utilities or technical advisers of syndicates to accept the results without some practical backing. We are preparing papers to be presented shortly before this Institute showing our method of analyzing these problems but we prefer to hold back this information until we can get actual data in the field, substantiating our results. Tests are under way now to procure this data and so far results appear to be very promising. I may say that the results indicate that all the investigators of stability under transient conditions including ourselves have been too pessimistic. I am glad to say that we have been the least pessimistic and are therefore more nearly right.

I wish to emphasize that in a problem so involved as in the one that the authors treat of, analytical solutions alone carry very little weight and they must be backed by actual practical demonstrations on existing circuits. Demonstrations without artificial transmission lines and models are not satisfactory. The method of point-by-point analysis using an artificial transmission line described by Messrs. Spencer and Hazen offers a great deal of promise. There is also a possibility of working out kinematical models which will embrace all the essential elements of the problem. I think that work along these lines should be encouraged. I think a machine is much more adaptable in working out problems of the character than a human being.

F. G. Baum: The problem of stability is one that has been with us for more than twenty years. If you will examine the TRANSACTIONS of the Institute of twenty odd years ago you will find a great deal of space taken up by discussions of the question of hunting of synchronous motors, and the hunting between parallel stations.

The problem as presented by Professor Karapetoff was covered, I think, as early as twenty odd years ago, and for those of you who haven't followed the transmission work particularly and who wish to study it, and to begin in an elementary way, I suggest you look at a paper published in the *Electrical World*, March 29, 1902 on "Synchronous Motor Stability and Overload Capacity Curves." You will find there many of the curves given by Messrs. Booth and Bush. They are curves given in pecentage of voltage, percentage of current, and so forth, so they are applicable for any scale that you wish to take, whether it is 110,000, 150,000 or 220,000 volts or something beyond that.

It has been my pleasure and privilege to be connected with transmission work for nearly thirty years, and I have found the subject so interesting and so broad that I haven't done much of anything else. But the transmission has been rather a disappointing part of the system, I may say, until within the last few years. It seems as though transmission was always lagging behind the power-station work. The power stations and the transformers were always ahead of the transmission lines. We never could get the capacity over the transmission lines that it would appear to be necessary, from the demands of the power stations and the power-consuming loads.

About five or six years ago, realizing that that condition confronted us, I made a study of the 220-kv-a. system. And I must say that the first time I ever got a real kick out of making transmission calculation was when I calculated a 220-kv-a. system; for it appears that for the first time we have a transmission system that is commensurate with the generators, and the large-size generators, water-wheels, transformers, and loads required at the present time.

As I say, realizing that five or six years ago, I recommended that we build a 220-kv-a. system to bring power from the northern part of California down to San Francisco, as I felt that without a high-voltage system of this kind it could not be economically done.

In a large system like the Pacific Gas and Electric Company, many problems of stability are solved by Nature. For example, on our system for about twenty years we have not operated with all the governors on the system trying to take hold of the load. You can not do that, but you eliminate and block the governors, and one or two stations at the most become the clearing house for kilowatt-hours. That simplifies your governor operation very materially and resolves it, then, to an infinite bus practically and a smaller unit with a fluctuating load.

With the 220-kv-a. system, it is necessary to operate with condensers on the system. You cannot do it without them. That makes it possible to simplify the operation to a very much greater extent, for just as we have one or two stations furnishing a clearing house, so to speak, for the kilowatt-hours, we have one or two stations furnishing a clearing house for the kv-a.

It is the lagging kv-a. that pulls the power system, and if you can prevent that lagging kv-a. from pulling the voltage out of the system, and if you can keep the actual kv-a. necessary to maintain the reaction pressure tangent to the voltage circle, you are not going to have instability. It is impossible to have instability with the current and pressure in phase. It is only when they get out of phase, at cross-purposes, that you get instability.

Finally, I may say this: It appears, that now for the first time we have a transmission system which is commensurate with the needs of the industry, but apparently we also have a transmission system that is inherently suited to outdoor operation. It seems, that we can never defy the lightning or the weather conditions in other respects.

C. F. Wagner: The authors of the paper mentioned the subject of high-speed excitation but have not stressed its importance sufficiently. I wish to emphasize high-speed excitation as a means of improving the stability of long-distance power transmission systems.

Introducing the effect of generators and condensers reduces the ability of the system to withstand disturbances as compared with the line alone. Suddenly increasing the lagging current drawn from an alternator or condenser increases the demagnetizing effect. The flux, however, cannot change instantly. At the first instant current is induced in the field windings and eddies in the pole pieces of such magnitude as to overcome this demagnetizing effect. The eddy currents in machines without damper windings are relatively small and to simplify the discussion will be neglected. The voltage required to circulate the additional field current is supplied by the change in flux through the field windings and is equal to the increment of IR drop in the field circuit. With hand regulation the flux will decrease until the total IR drop in the field winding is again equal to the exciter

voltage. In effect, a hand-regulated machine introduces an artificial internal series reactance although it is in part compensated by the larger artificial internal voltage.

The voltage coils of automatic voltage regulators of the Tirrill type are connected across the terminals of the generator. On sudden application of lagging load the internal-drop reduces the terminal voltage which in turn closes the regulator contacts and increases the exciter voltage. If the exciter voltage could be built up instantly to such value corresponding to the new IR drop of the field currents the flux would remain constant. Actually, however, the exciter voltage builds up along a very definite curve. During the interval while the drop in the field winding

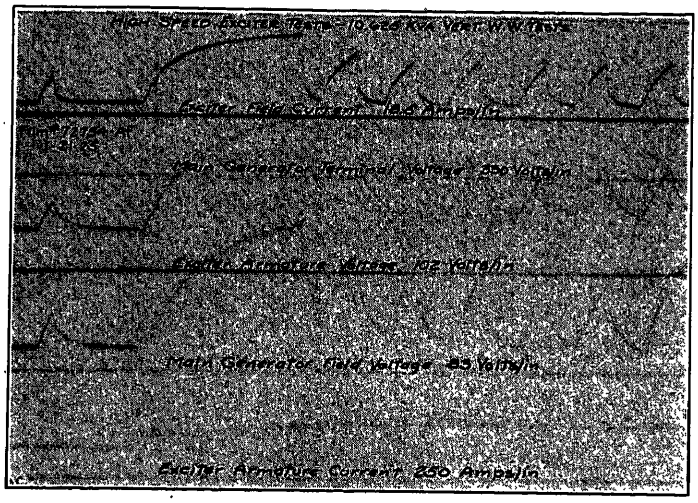

Fig. 1

is greater than the exciter voltage, the flux in the main machine decreases in a manner similar to that discussed for hand regulation. The flux will continue to decrease until the exciter voltage is equal to the drop in the field windings. An excess of exciter voltage over IR drop increases the flux.

From the foregoing it can be seen that the faster the exciter voltage increases the smaller will be the decrease in flux. This is what the high-speed exciter attempts to accomplish. Note particularly that the high-speed exciter relies not upon changing the flux in the main machine but rather upon annulling the effect of the armature demagnetizing current. Of course, the slower the field of the machine the smaller will be the flux change, but if the increase in exciter voltage is always of such value as to annul very quickly the effect of any reactive current that might reasonably occur then it would be more desirable to have a fast field and a high short-circuit ratio.

Fig. 1 is an oscillogram taken during some tests on a 10,625-kv-a. 100 rev. per min. alternator in which 4350 kv-a. leading load was thrown off. This simulates the condition of suddenly increasing the lagging load as would be the case during short circuits. Dropping leading load simultaneously decreased the terminal voltage. After an interval due to the lag in the regulator and relays the exciter voltage begins to increase as shown. This lag can be made practically zero by arranging an extra pair of contacts in parallel with those of the regulator and operating them from the fault ground current or from reactive kv-a.

The rest of the story is self-evident. The loss of magnetizing armature current induces an equivalent current in the field winding. This current decreases for an interval but the rapidly increasing exciter voltage soon annuls this action and increases the field current. In the meantime the terminal voltage is increasing with the field current until a voltage is reached which causes the regulator contacts to separate. The exciter voltage decreases and after a series of oscillations settles down to normal no-load conditions.

The effect of high-speed excitation upon the flux is better illustrated in Fig. 2. These curves were prepared from test data and indicate the change in exciter terminal voltage and flux of main machine as the armature current is increased from zero to rated-value zero power factor. Curves marked a represent the conditions for hand operation, curves marked b for standard exciters and curves marked c for high-speed exciters. The load was applied at an instant when the exciter voltage had reached a minimum value to represent approximately the worst condition. For hand operation the flux decreased enormously; with the standard exciter the decrease was still quite considerable; but with the high-speed exciter the decrease was negligible. With this type of excitation transient conditions may be calculated using the assumption of constant flux through the field windings, *i. e.*, that component of flux in phase with the rotor. It should be noted that high-speed exciters do not control the cross flux, this being a function of armature current only.

While it is true that the stability of a high-voltage transmission system can be increased by the application of high-speed exciters with fast regulators of the Tirrill type, its utility is not universal, in fact quite the contrary. Their application to generators connected to transmission lines in which the resistance is about equal to or greater than the reactance decreases the stability. For this case hand regulation or slow-speed exciters and rheostatic regulators are preferred. In this connection it should be noted that the general stability of a system cannot be increased by speeding up the excitation of synchronous equipment at the end of distribution lines, which usually have high ratio of resistance to reactance as compared with high-voltage transmission lines.

In the event of a disturbance which unbalances the voltages

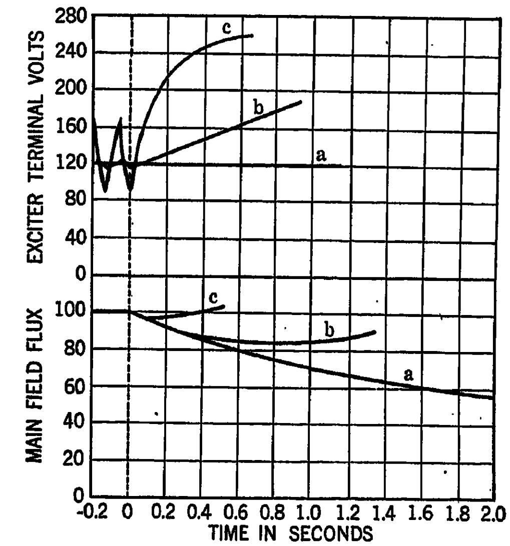

Fig. 2—Effectiveness of High-Speed Exciter in Maintaining Flux in Alternator

Armature current suddenly increased from zero to 100 per cent, zero power factor lagging.

a. Hand regulation
b. Standard exciter with Tirrill regulator
c. High-speed exciter with Tirrill regulator

such as a line-to-ground fault, it is of course necessary that the regulator contacts close and remain closed. This may not occur if the regulator is connected in the usual manner. Rather than decreasing the voltage of the phase to which the regulator is connected analysis has shown that the disturbance might increase the voltage. Proper action can be insured by connecting the regulator through a network which passes only the positive-sequence voltage.

High-speed exciters increase the duty on circuit breakers, but this is probably not an insurmountable difficulty.

R. C. Bergvall: Mr. Hanker and Mr. Fortescue have pointed out that ordinary switching operations are not the limiting conditions which have to be met in considering stability. The limiting condition is the ability of the system to remain in synchronism during short circuits and during the switching operations following the abnormal conditions set up by the short circuits. Experience has shown that line-to-ground short circuits constitute about 90 per cent of the system disturbances and particular attention must, therefore, be paid to maintaining stability under these conditions. With two systems tied together by two parallel transmission lines, it is practically impossible to maintain synchronism if a three-phase short circuit occurs near the generating station or the substation, because the voltage on the tie line is reduced to zero. Various methods have been considered in an effort to increase the possibility of maintaining synchronism, such as rapid relay operation or making the transformer a unit with the line as Mr. Thomas proposed at the 1924 Midwinter Convention, thereby interposing the transformer reactance between the bus and the short circuits. However, in most cases the necessity for flexibility in switching of the high-tension lines makes it impossible to lay out the system in this manner.

The engineers of the Westinghouse Company have been actively investigating the effect of line-to-ground short circuits during the past year. The importance of making such investigations can best be seen by considering a line-to-ground short circuit having 5000 amperes of ground current. This corresponds to a loss of 25,000 kw. per ohm of ground resistance, which is an appreciable kilowatt load to throw on a system already carrying a heavy load.

The magnitude of the ground resistance is an indeterminate factor. A bushing circuit on a transformer would have practically zero ground resistance while a tower located on rocky soil would have considerable ground resistance. The worst condition would be some value of ground resistance between the zero ground resistance and the extremely high ground resistance, provided it was desired to maintain stability under all line-to-ground short circuit conditions.

The problem of determining stability in the case of line-to-ground short circuits is complicated by the presence of unbalanced currents and voltages and by the opening of circuit breakers before the steady-state condition has been reached. When the circuit breakers open the entire load is immediately transferred to the other transmission line and pull-out is likely to result because of the sudden change in load at the same time that the system conditions have been disturbed by the short circuit.

The calculation of single-phase short-circuit currents by the commonly used methods on an extensive transmission network is difficult because the currents and voltages are unsymmetrical and each phase is inductively coupled in the transmission lines, transformers and rotating machines. Furthermore, all rotating machines, provide a distinct phase-balancing action which tends to restore symmetry in voltage and current. A solution of a problem of this type is to be published by R. D. Evans in a future issue of the *Electric World*. Essentially it is an application of the method developed by C. L. Fortescue for the general solution of unbalanced polyphase circuits. Briefly, the voltages and currents of the three-phase grounded-neutral circuit are resolved into the positive-sequence, negative-sequence, and zero-sequence components, which do not react upon each other and may be considered independently. With normal balanced loads only positive-sequence voltages and currents are present, but in the case of a short circuit to ground all three sequence components are involved.

The method employed in investigating the possibility of pull-out occurring during line-to-ground short circuits consists of substituting equivalent impedances in the circuit to represent the zero-sequence and negative-sequence components, thereby leaving only the positive-sequence components which may be handled by the usual methods.

S. B. Griscom: As already pointed out in the discussions certain disturbances on transmission systems, notably the single-phase, line-to-ground short circuit with the subsequent line switching may produce large oscillations in the phase position of the generators about the angle corresponding to their mean output. The system must be designed so as to withstand disturbances which are of common occurrence. This means that the maximum operating load must be considerably less than the power limit of the line. On the other hand, economic considerations demand the transmission of the maximum power possible per dollar of transmission investment.

Two general courses of action are available to increase the economy of transmission. The theoretical power limit of the line may be increased by proper design of the line, including the associated apparatus, or the effect of disturbances can be reduced, enabling a smaller factor of safety to be chosen.

The group of papers presented in February 1924, and the discussions on them, considered in a general way the use of synchronous condensers located at points along the line for the purpose of increasing the amount of power that could be transmitted over the circuits. This scheme was first proposed by F. G. Baum, in a paper before the A. I. E. E., in 1921.

We have since made an extended study of this system which has shown its utility. For transmitting a given amount of power over a line, a certain amount of reactive power must be supplied to it depending on the voltage conditions. The usual method is to supply this at the two extremities of the line; by the generators at one end, and by synchronous condensers at the other.

On long lines it is necessary to sectionalize at one or more points, so that in event of trouble, it will not be necessary to cut out too large a proportion of line. These sectionalizing points are ideal locations for the installation of synchronous condensers, since switch structures and attendants are already there.

By installing condensers at these points, the maximum operating load may be increased, and the voltage regulation at intermediate points greatly improved. This is especially desirable for future interconnections or loadings at intermediate points.

The theoretical limit of a long line may be made to approach the limit of the longest section between points where synchronous machinery is installed, depending upon how well the machines can maintain the line voltage. By the use of machines designed with this object in view, and by using a high-speed exciting system in conjunction with them as mentioned in one of the discussions the regulation can be made very good.

Our studies have shown that it is economically possible to increase the permissible load on a 250-mi., 220-kv. line about 25 per cent by means of suitable installation of synchronous condensers at intermediate points.

The other general method of increasing the carrying capacity of transmission circuits is to lessen the effect of short circuits in causing pull-outs between supply and receiver ends. One method which shows considerable promise is to use an auxiliary control on the governing mechanism, as already mentioned. Another method under consideration is to use circuit breakers on the main transmission circuits, which would operate at speeds considerably higher than those in use at the present time. The utility of this scheme is readily apparent when we consider that if the breaker could be opened at the instant of short circuit, the condition would be in effect only that of line switching, which is much less severe.

Up to the present time, no data is available as to the maximum speed that can be attained by circuit breakers in interrupting large currents at 220 kv. In order to get an idea of what can be gained by using different speeds of circuit breakers, some calculations were made on stability during single-phase-to-ground short circuits, when the faulty section of line was cut out

at different time intervals from the instant the short circuit occurred. The results of these calculations are presented in the form of curves, showing the variation of angular position of the generator rotors with time, for a typical transmission system.

By referring to the accompanying curves, it will be noted that if the circuit breaker opens at the instant of short circuit, there is very little relative movement in phase position of the rotors of the generators with reference to the load. If the short circuit is permitted to hang on for 0.2 seconds, a somewhat greater swing takes place. For 0.4 seconds, the swing is of such a large magnitude that on the second half cycle of mechanical oscillation the systems pull apart as indicated by the fact that the angle continues to increase with time. The same is true with any time of breaker opening up to 1.0 second. From 1.0 to 1.6 seconds stability can be maintained. From these curves, it is evident that if the period of oscillation is of the order of one second, the stability of the system can be greatly improved by the use of sectionalizing cricuit breakers which open the circuit within 0.2 seconds after a short circuit occurs.

If the natural period is greater, a correspondingly higher maximum time is permissible. These curves show clearly the element of chance, due to erratic relay or breaker operation, and explain why a short circuit may cause pull-out on one occasion and not on another when conditions are apparently identical.

Aside from the question of lessening oscillation of the generators and other synchronous machines, the use of high-speed circuit breakers would be of material assistance in minimizing voltage decrement due to demagnetization. On this account, their rupturing capacity must be increased, although the use of a special excitation system might independently impose this requirement.

It may be somewhat premature to mention the use of high-speed circuit breakers of this type, when none have thus far been developed, but the principle purpose of this discussion is to point out possible methods for improving stability.

V. Bush and R. D. Booth: A number of the discussions treat of improvements of transmission system and correlated apparatus and need no further comment in connection with a paper devoted to methods of analysis.

Prof. Karapetoff's discussion is an admirable presentation of the fundamentals upon which the methods of the paper are based, and undoubtedly adds completeness and clarity to the entire subject. It might, however, be in order to point out that the differential equations used by Prof. Karapetoff are linear and also that the coefficient N,—the rate of increase in power transmitted per degree change of the angle between the receiver and generator voltages—is constant. The methods presented in the paper permit analysis of those problems in which the equations are non-linear, and the coefficients are not constant.

Mr. Baum's discussion of the progress of the art of transmission and of operation of systems involving long transmission lines is of general interest. We presume that Mr. Baum refers to steady-state rather than transient conditions when he says that practically infinite bus conditions at the sending generators can be obtained by blocking the governors thereof.

Mr. Wagner's discussion of the possible benefits of high-speed exciter systems and Mr. Griscom's studies of the advantages of high-speed circuit breakers are of great importance to the industry. We presume that Mr. Griscom's curves of desirable operating times apply only to a particular system and for particular values of arc resistance. Also we presume that Mr. Wagner's discussion of the limitations of high-speed exciters is based upon specific data regarding arc resistances and operating times of breakers and governors.

Testing High-Tension Impregnated Paper-Insulated, Lead-Covered Cable

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Synopsis.—The increase in voltage rating of cables has necessitated that the tests to assure satisfactory cable be more adequate than as standardized at present. This has resulted in an intensive study of the tests previously standardized, development of new tests, and the design and manufacture of suitable testing equipment to meet the new testing requirements.

Measurements are made upon cables to determine the following properties of the insulation:

Insulation Resistance
Dielectric Strength
Dielectric Power Loss and Power Factor
Capacitance
Ability to Withstand Bending

Insulation Resistance: This measurement is being made in the same way on cables of all voltage rating. The results of the test on high-tension cables are of doubtful value as a criterion of the suitability of cable for use. Continued study of this measurement should be made.

Dielectric Strength: Suitable testing equipment for satisfying the requirements of the increased voltages in dielectric strength tests has been made available. This includes sine-wave generators, adequate testing transformers, appropriate cable testing terminals.

Data is given from which conclusions are drawn as to the magnitude and duration of test voltages. The adequacy of these values will become known through experience. The need for field testing is shown.

Dielectric Power Loss and Power Factor: The tendency is to extend the measurement of dielectric power factor to include each reel length to be shipped. The Schering Bridge for making such measurements is described. The need for standardizing the testing procedure for power-factor measurements is shown.

Testing Installed Cable: The study of so-called "current-time curves" for rating installed cable should be continued. Preliminary measurements made at high frequencies as a means of rating installed cable did not show the results to be immediately usable.

Testing With Direct Current: Data is given to show the d-c.- to a-c.-ratio of breakdown voltage of some samples of 12 kv.-3-conductor cable. Tests indicate that the d-c.-to a-c.-ratio will depend upon many conditions such as nature and structure of the material, thickness of the material, temperature of the material, shape and size of electrodes, and rate of application of the applied potential.

THE report of the Transmission and Distribution Committee of the A. I. E. E. for 1923-1924 contains the following significant statement:[1]

"The most important development during the past year has been the evidence that the cable specifications of the N. E. L. A. and the present Standards of the A. I. E. E. do not insure satisfactory cable for the higher operating voltages.

"This subject is now receiving attention from the manufacturers as well as the users of high voltage cable and as a result of these studies, it is hoped that it will be possible:

First, to make the necessary changes in manufacturing processes and materials so as to secure a satisfactory cable for operation at the higher voltages, and

Second, to devise a method of testing high voltage cable which will determine its operating characteristics in advance of its installation."

The present paper discusses some phases of the second of these two objectives, not that such a method for which engineers are eagerly searching has been finally devised, but because a discussion of the conditions under which the present tests are applied may be helpful in so modifying them that they may be made more adequate.

The tests at present standardized for determining the suitability of cable for use as regards the insulation are as follows:

I. A. I. E. E. Standards 1922.
- A. Tests to be made on each length to be shipped.
  1. High-voltage test
  2. Measurement of insulation resistance
  3. Measurement of capacitance
- B. Test to be made on samples (10 ft. of lead)
  1. Measurement of ultimate dielectric strength

II. N. E. L. A. Specifications for Impregnated Paper-Insulated, Lead-Covered Underground Cable, 1922.
- A. Tests to be made on each length to be shipped
  1. High-voltage test
  2. Measurement of insulation resistance
- B. Test to be made on one reel length per 15,000 ft. of cable
  1. Measurement of dielectric power loss and power factor at about 85 deg. cent
- C. Test to be made after installation when cable is installed by the manufacturer
  1. High-voltage test
- D. Tests to be made on samples
  1. Measurement of dielectric strength
  2. Bending test
     - a. Wrinkling of lead; dielectric strength; visual examination
     - b. Deformation
  3. Measurement of dielectric power loss and power factor.

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1. For references see Bibliography.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

The electrical characteristics represented in the above tests include all that are now known. The problem is to apply them so that unsatisfactory cable may be separated from satisfactory cable without harming the latter. The following discussion on this subject will, in the main, refer only to high-tension cables with paper insulation treated with mineral compound for circuits rated above 12 kv. The results of tests included in this discussion were obtained from commercial cable manufactured prior to Jan. 1, 1924, except as may otherwise be noted.

Insulation Resistance

Insulation resistance is measured after the high-voltage test on the entire length to be shipped, measuring the leakage current after a one-minute electrification with a continuous e. m. f. of from 100 to 500 volts, the conductor being maintained negative to the sheath.[2] From the results so obtained, a "Megohms Constant" is calculated so that cables of different dimensions may be directly compared. The increase in voltage rating of cables has brought about no real change in method of making the insulation resistance measurement nor have new difficulties therein been encountered. Humidity may still be a disturbing factor by increasing the leakage, though a recent suggestion of surrounding the insulation testing set and operator with a sheath wire, heated by circulating current to keep the testing set dry, has proven entirely effective in eliminating the leakage without causing discomfort through overheating of the test booth.

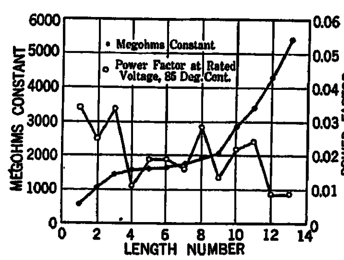

Fig. 1—Megohms Constant and Power Factor of Reel Lengths of 3-Conductor 33-kv. Treated Paper-Insulated Cable

Although measurements of insulation resistance have been made on cables for years and are still being made, the results as determined by the standardized procedure are of doubtful value as being indicative of the suitability or unsuitability of cable for use. It was recently noted that a nail hole through the lead to the paper of a 3-conductor 22-kv. cable caused no noticeable change in the megohms constant derived from measurements made daily after a period of two weeks, the reel being immersed continuously in water. Enlarging the nail hole to the size of a quarter and continuing the measurements daily for two months with the reel immersed has not materially changed the megohms constant from its original value of 3000 before the nail hole was made.

Although not sensitive to such conditions, results are obtained showing sensitiveness to other factors which are not easily determinable. Fig. 1 shows the values of megohms constant obtained from the measurement of 13-reel lengths of three-conductor 33-kv. cable, manufactured to the same specifications during a period covering three months. The ratio of the highest reading to the lowest is in the order of 5 to 1. Values of power factor for each length at 85 deg. cent. at rated voltage, are also plotted and show all lengths of the cable to be of low power factor. The ratio of the highest to the lowest power factor value is 3 to 1. Fig. 2 shows similar values for 18 reel lengths of three conductor 12-kv. cable, showing characteristics similar to those of the 33-kv. cable, Fig. 1. The relation between power factor and insulation resistance is noted to be generally inverse, though not consistently so. These results indicate a relatively wide range of insulation resistance for cable, all of which is apparently satisfactory.

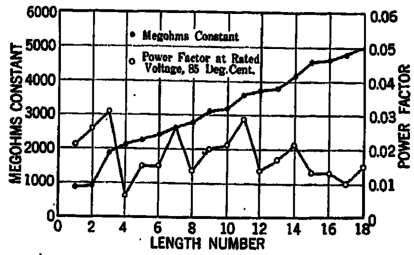

Fig. 2—Megohms Constant and Power Factor of Reel Lengths of 3-Conductor 12-kv. Treated Paper-Insulated Cable

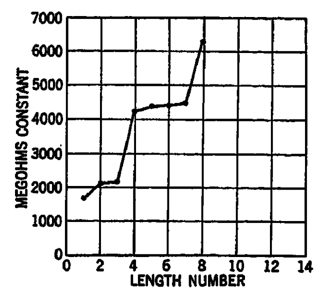

Fig. 3—Megohms Constant of Reel Lengths of 3-Conductor 22-kv. Treated Paper Insulated Cable, Treated in Same Tank at Same Time

Fig. 3 shows the megohms constant obtained from measurement of eight lengths of three conductor 22-kv. cable made from the same kind of paper, treated with the same compound in the same tank at the same time. The ratio of the highest value to the lowest is 4 to 1, yet there has never been any evidence that any one length was more satisfactory cable than another. In general, the ratio under these conditions averages more nearly 2.5 to 1. For the same kind of cable treated in different tanks, however, ratios of 5 to 1 and even 10 to 1 are not uncommon.

It appears, therefore, that insulation resistance measurements are being made on cables of the higher voltage rating just as they have been made for many

years on cables of lower voltage rating. The measurement, though comparatively simple, is of little real value as a means of distinguishing between good and poor cable. Measurements made at higher direct voltages may prove to be more useful but such have yet to be more thoroughly investigated.

Dielectric Strength

The criterion of a satisfactory cable is that it shall successfully operate at its rated voltage in the system to which it is connected and for which it is designed. This requires that every point along the entire length of the cable must be dielectrically strong and continue to be so. The only means now available for determining initially the suitability of a cable as regards dielectric strength is to determine the ultimate dielectric strength of samples of the cable by breaking them down under electrical tension, and then applying a lower electrical tension for a given time to similarly made cables in lengths intended for use, the value and time of application of the test voltage being chosen so as to separate such cable as is dielectrically weak from that which is dielectrically strong without harming the latter. The final criterion for the effectiveness of this test is experience.

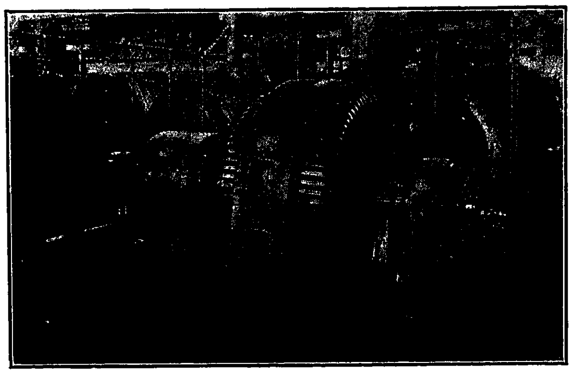

Fig. 4—Sine-Wave Synchronous Motor-Generator Set

Generator rated three-phase, six-pole, 400 kv-a. 12 rev. per min., 900 volts

Motor rated, three-phase, four-pole, 100 h. p., 1200 rev. per min., 550 volts

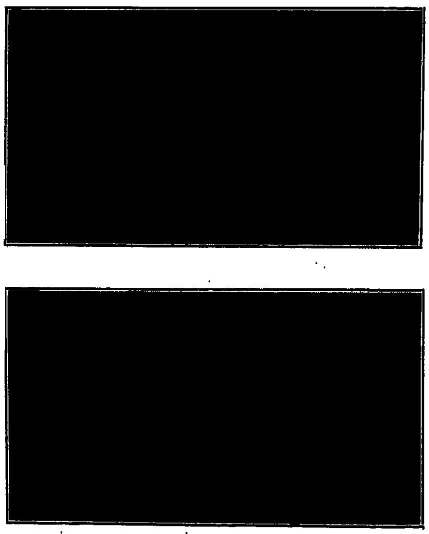

Fig. 5—Wave Shapes of Sine-Wave Generator, Rated Three-Phase, 400 Kv-a., 1200 Rev. Per Min., 900 Volts

Upper curve, no-load voltage wave

Lower Curve, voltage wave, Load of 1242 ft., three-conductor, 33-kv. cable at 85-kv.

The increase in voltage rating of cables has introduced difficulties into the testing procedure of dielectric strength tests, but these are being overcome. At the present time the best samples of single-conductor cable for 66-kv. three-phase circuits have a breakdown voltage on short-time test of from 300 kv. to 350 kv., while the best samples of three conductor cable for 33-kv. three-phase circuits have a breakdown voltage on short-time test of 200 kv. High-voltage tests in the factory on long lengths of the former cable are conducted at from 100 kv. to 150 kv.; on the latter cable at from 82.5 kv. to 95 kv., the time duration being from 5 min. to 15 min., depending upon conditions. Such values as these require testing equipment of high voltage and high kv-a. rating which must conform to standardized requirements, such that the voltage wave shall "approximate as closely as possible a sine wave." In this phase of the work substantial progress has been made.

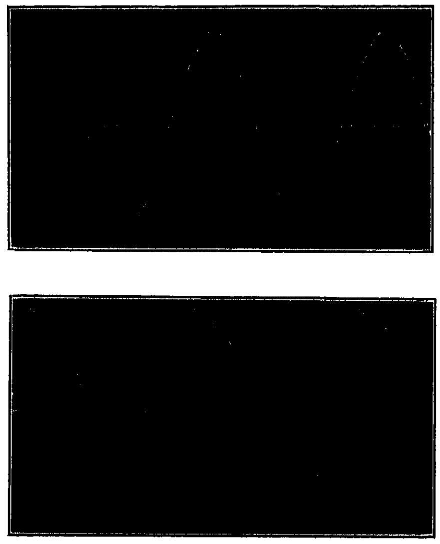

Fig. 6—Wave Shapes of Generator, Rated Three-Phase, 75 Kv-a., 500 Rev. per Min., 550 Volts, in Use for Testing Cable 15 Years Ago

Upper Curve, no-load voltage wave

Lower Curve, voltage wave, load of 702 ft., three-conductor, 12 kv. Cable at 40 kv.

Fig. 4 shows a motor-generator set rated 400 kv-a. at 1200 rev. per min. and 900 volts, of which the no-load voltage wave, and the voltage wave when connected through transformers to 1242 feet (three reel lengths) of 3-conductor 33-kv. cable at 85 kv., are shown in Fig. 5. The characteristics of these waves are:

Load	Deviation Factor	Form Factor
No load	1.2%	1.11_5
Cable load of 1242 ft. of 3-cond. 33-kv. cable at 85 kv	1.0%	1.11_4

Fig. 6 shows wave shapes as obtained from a generator which has been in use for cable testing, for 15 years, rated 75 kv-a. at 500 rev. per min. and 550 volts, of which the characteristics are:

Load	Deviation Factor	Form Factor
No load..............	2.8%	1.11_7
Cable load of 702 ft. of 3-cond. 12-kv. cable at 40 kv.......	30.9%	1.10_8

These comparative values indicate the substantial progress which has been made in the generator design. The excellent characteristics of the waves shown in Fig. 5 for the modern generator are all the more remarkable when it is considered that they are for a load with leading current and of extremely low power factor.

The wave shapes of the modern generators are well within the required limits for testing with sine wave shape and for measuring voltage with a voltmeter following any of the standardized methods, such as through an auxiliary ratio transformer or by means of a voltmeter coil placed in the testing transformer. Ratio transformers being necessarily quite large for use at the high voltages now required, the voltmeter coil placed in the testing transformer becomes a most useful means for making the voltage measurement. A paper on testing transformers has been recently presented to the Institute,[3] in which paper also is included a discussion of voltage measurement together with apparatus available for such measurement.

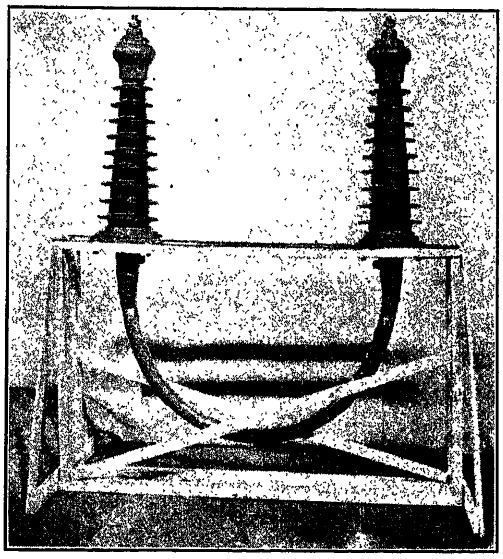

Fig. 7—Two Oil-Filled Cable Terminals, Assembled at the Ends of a Sample of Cable for a 66-Kv. Three-Phase Circuit, for Ultimate Dielectric Strength Test up to 240 Kv.

Probably the greatest single difficulty confronting the cable tester when making dielectric strength tests is that of applying the voltage to the cable, it being necessary to so prepare the cable ends that they will withstand the high voltage without breakdown or flashover, and so that the break will occur under the lead and not in the end-bell. For tests on reel lengths in the factory where the voltages applied for three minutes are not above 100 kv. between conductors for three-conductor cable or between conductor and sheath for single-conductor cable, the problem is not so difficult, there being several designs of end-bells in use. The usual method is to apply an inverted paper cone to the cable end, which is then filled with petrolatum hot enough to pour. Pressed paper or fibre tubes are frequently placed over the separate conductors of three-conductor cable.

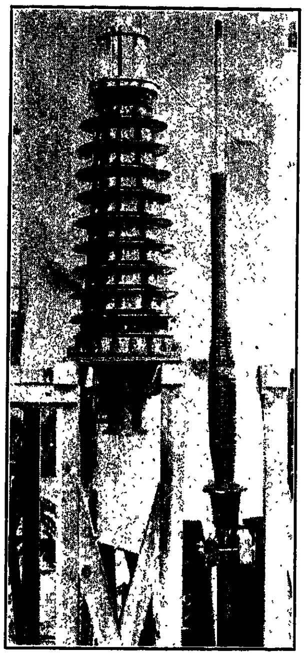

Fig. 8

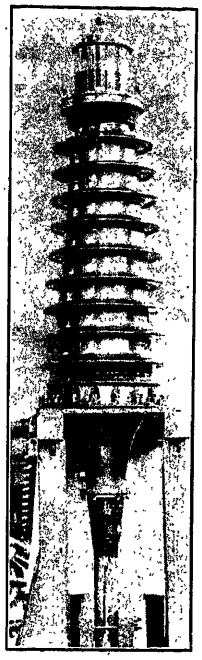

Fig. 9

Fig. 8—Oil-Filled Cable Terminal, Ready for Assembly to a Sample of Single-Conductor Cable for a 66-Kv. Three-Phase Circuit, for Ultimate Dielectric Strength Test up to 350 Kv.

Fig. 9—Oil-Filled Cable Terminal Assembled to a Sample of Single-Conductor Cable for a 66-Kv. Three-Phase Circuit for Ultimate Dielectric Strength Test up to 350 Kv.

For single-conductor cable where the test voltage is above 100 kv., a most satisfactory means of applying the electrical tension has been found to be through the application of a terminal made of a porcelain oil-filled bushing of standard design provided with suitable fittings for proper application to the cable. Fig. 7 shows two such terminals attached to a sample of single-conductor cable with 30/32 in. treated paper for a 66 kv. three-phase circuit. Terminals such as these have been regularly used in testing reel lengths of single-conductor cable in the factory where the test voltage has been 150 kv. for both five-min. and for 15-min.

application; also for ultimate dielectric strength tests up to 240 kv. which is the dry flashover voltage of this terminal. Figs. 8 and 9 show a larger terminal ready for assembling to the cable end, and as assembled to the cable end, the former showing how the cable end is prepared before the terminal is applied. This is done by removing a suitable length of the lead and then building up over the factory insulation with black varnished cloth, covering a portion of this with copper wire or flattened string solder to carry the ground potential of the sheath up over the reinforced insulation a suitable distance to properly distribute the stress.

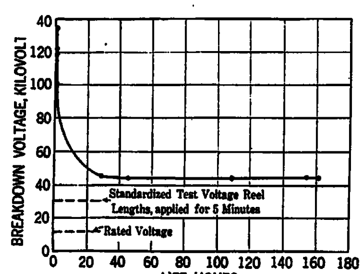

FIG. 10—BREAKDOWN VOLTAGE-TIME CURVES AT 25 DEG. CENT. FOR SINGLE-CONDUCTOR CABLE SAMPLES WITH 9/32 IN. TREATED PAPER INSULATION

A terminal such as shown in Figs. 8 and 9 has a dry flashover voltage of 350 kv. and if the cable remains sound, it is absolutely satisfactory for use in testing cable samples up to a breakdown voltage of that value. Flashover of the terminal may result at a lower value from disturbances which may occur in the cable, in which event a larger terminal may be used.

A terminal such as shown in Fig. 7 weighs 155 lbs. with fittings, and for a factory test on a reel length at 150 kv., where it is not necessary to build up the cable ends with varnished cloth, it requires the time of two men for two hours to apply the terminals and remove them. A terminal such as shown in Fig. 9 weighs 375 lbs. with fittings, and for an ultimate dielectric strength test where the cable ends must be built up with varnished cloth, it takes two men a day to apply the terminals and remove them. The use of such terminals has made possible satisfactory tests of dielectric strength without the difficulties usually attendant the use of built-up end-bells, and represents a most gratifying advance in the art. Suitable terminals are available for the entire range of voltages required in making dielectric strength tests on single-conductor cable.

No satisfactory built-up end-bell has yet been devised for testing 3-conductor cable samples for ultimate dielectric strength where the breakdown voltage is in the order of 200 kv. An average gradient of approximately 250 volts per mil seems to be about the maximum voltage where a built-up end-bell following the usual designs can be used in ultimate dielectric strength test of 3-conductor cable with satisfaction. Above this value, breakdown usually results in a crotch failure between conductors in the end-bell. Out of 12 samples recently tested where the probable failure would be in the order of 200 kv., every sample failed at substantially lower voltage in the crotch in the end-bell, which failures were preceded by excessive leakage over the conductor insulation. Efforts to prevent crotch failures have been made in practise by the use of different filling compounds, such as petrolatum, linseed oil, and transil oil, but as yet without success. The use of long treated paper tubes over the separate conductors has not bettered the results measurably.

Theoretical considerations and calculations indicate that the excessive leakage stress can be kept below safe values only by so designing the external surface of the end-bell that it follows essentially the curve of the conductors as they diverge. This is not simple to do in a built-up end-bell. From a review of all of the factors, it is felt that the best plan is to use a three-conductor oil-filled cable terminal following the practise above suggested for testing single-conductor cable. The early use of such when results will be available for report is contemplated.

Even with proper equipment available for conducting dielectric strength tests, the problem of determining a satisfactory suitability test still remains. The long established test voltage for cables of two and a half times normal rated voltage applied for five minutes[4] has apparently not been satisfactory for the high-tension cables. Such a test, of course, only establishes the condition at the time this is illustrated by the

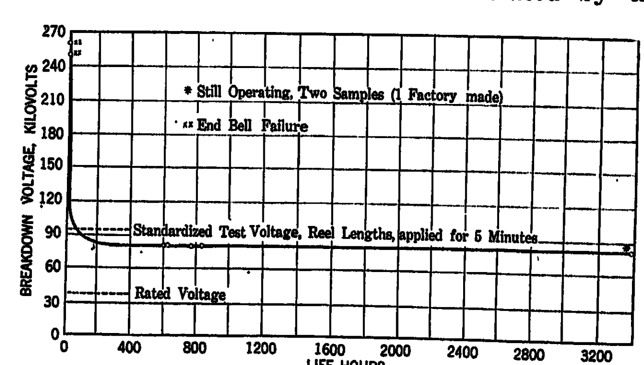

FIG. 11—BREAKDOWN VOLTAGE-TIME CURVE AT 50 DEG. CENT. FOR SINGLE-CONDUCTOR EXPERIMENTAL CABLE SAMPLES WITH 30/32 IN. TREATED PAPER INSULATION

results shown in Fig. 10. Ten samples of 2/0 A. w. g. single-conductor cable insulated with 9/32 in. treated paper insulation were tested, five for ultimate dielectric strength (short-time step-up test) and five for endurance dielectric strength at 44 kv. at 25 deg. cent. The curve connecting the resulting points takes the characteristic form of that for fibrous materials. The normal rating of the cable and standardized test voltage on long lengths of such cable are indicated. The life of the cable at 44 kv. is many hours, and at rated voltage in operation, it is several years; the initial test, however, covers only the first few minutes. Similar results are shown in Fig. 11 for single-conductor

experimental cable with 30/32 in. treated paper insulation, and Figs. 12 and 13 for three-conductor cable for 33 kv. circuit. These results show also the longer life of cables on endurance run at 50 deg. cent. as compared with life at 25 deg. cent. which results are consistently

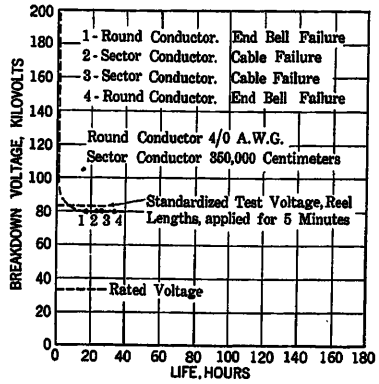

FIG. 12—BREAKDOWN VOLTAGE-TIME CURVE AT 25 DEG. CENT. FOR THREE-CONDUCTOR CABLE SAMPLES WITH 19/64 IN. TREATED PAPER INSULATION ON EACH CONDUCTOR AND 7/64 IN. OVERALL BELT

noted on treated paper cable with different treating compounds and different kinds of paper. The better performance of round conductor as compared with sector conductor with the same insulation thickness is also shown.

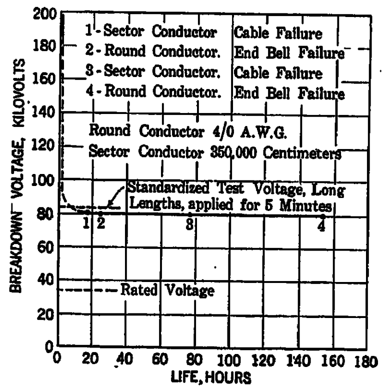

FIG. 13—BREAKDOWN VOLTAGE-TIME CURVE AT 50 DEG. CENT. FOR THREE-CONDUCTOR CABLE SAMPLES WITH 19/64 IN. TREATED PAPER INSULATION ON EACH CONDUCTOR AND 7/64 IN. OVERALL BELT

The two factors involved in the high-voltage suitability test are magnitude of test voltage and duration of application of same. It would be most impracticable to apply the test voltage on all lengths for the time durations shown in Figs. 10 to 13. Fifteen minutes has been suggested rather than the present value of five minutes. The evidence available does not indicate that such an increase would be of any real value. A long time test on a sample length from each lot of cable would, however, be of considerable value. Increasing the magnitude of the test voltage would appear to have merit. The value would have to be finally determined by experience, though an immediate increase in the order of 50 per cent. over present standardized practise for single-conductor cable, and about 20 per cent increase for three-conductor cable does not seem excessive, the lower value for the three-conductor cable being justified because of the inherent differences due to construction.

In this connection there arises again the question of the mechanism of breakdown. If the total number of cables and cable samples examined after breakdown

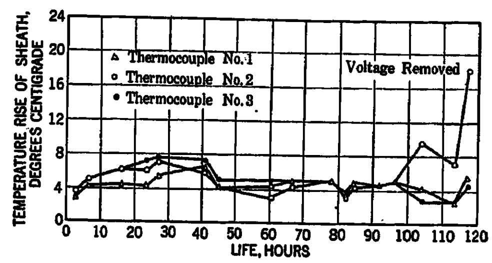

FIG. 14—TEMPERATURE RISE OF SHEATH, SINGLE-CONDUCTOR CABLE SAMPLE WITH 9/32-IN. TREATED PAPER INSULATION, 44 KV. APPLIED

on dielectric strength test to determine the reason for such, was known, the number would be appalling, yet the answer is still wanting. An examination of many cable samples examined after breakdown allows of a classification of such into two general classes; (1) those where the puncture is radial and clean, and (2) those where the puncture is accompanied by deterioration irregularly throughout the matail.

In the latter case many so called "partial failures"

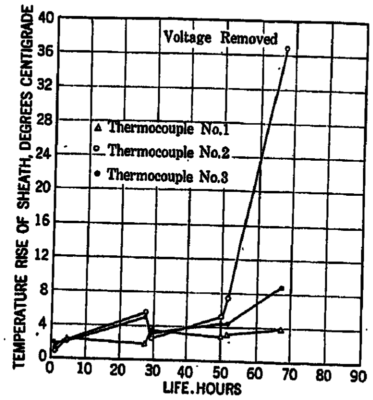

FIG. 15—TEMPERATURE RISE OF SHEATH, SINGLE-CONDUCTOR CABLE SAMPLE WITH 9/32-IN. TREATED PAPER INSULATION, 44 KV. APPLIED

or failures through only a portion of the layers of the insulation are frequently found. Invariably in single-conductor cable these extend out from the conductor part way to the sheath, while in three-conductor cable the failure seems to form in the center of the insulation between conductors and extends towards the conductors.

Efforts to determine the "germ" of a breakdown have been made by subjecting samples of single-conductor cable insulated with treated paper of 9/32 in. thickness to a voltage of 44 kv., exploring the sheath for hot spots

and removing the voltage when a hot spot is found before breakdown occurs. Figs. 14 and 15 show the temperature rise on the sheath of two such samples. When examined, the sample of Fig. 14 showed failure to have started under the hot spot. The first five layers of tape from the lead sheath in were normal. A black spot, which developed as other layers of tape were removed, was found on the sixth tape. Toward the center many small burned spots appeared and a few so-called "tree designs" were found. The sample of Fig. 15 also showed failure to have started under the hot spot. The first twenty layers of tape from the lead sheath in were normal. Extensive tree designs appeared in three regions around the hot spot, together with a dark spot on the twenty-first tape. The designs in the center region continued in for 12 layers and then disappeared. The designs in the other two regions

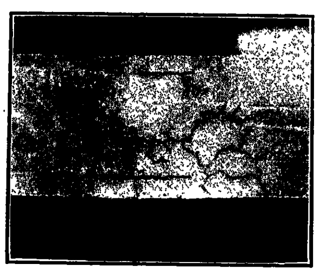

Fig. 16—Design Appearing on the Surface of the Tapes of Treated Paper-Insulated Cable Resulting from the Application of Electrical Tension

joined together 24 layers in and then faded into a burn through the remaining nine layers, the dark spot noted above continuing through to the burn. The nine inner layers had been creased and torn, and the burn extended down through the tears, but no burn was noted in the layer next to the conductor.

The tree designs noted are as shown in Fig. 16, and are as recently reported before this Institute[5] and by others[6]. These designs were traced on the surface of the tapes, the pattern coinciding for adjacent tapes but differing on both sides of the same tape. The design could be scraped off with a knife; it did not appear to follow the fibres. Such designs have been noted previously in laminated treated-paper tubes subjected to high electrical stress, their presence being found in definite fissures between the laminations.

The appearance of many more tree designs in the sample of Fig. 15, which judging by the temperature rise and the radial burned spot had approached nearer to breakdown indicate, possibly, the formation of the tree designs after the beginning of the tendency to radial breakdown. The samples represented by Figs. 14 and 15 were not tightly wrapped, there thus being less lateral dielectric strength. Similar tests, with tightly wrapped paper, have not shown tree-designs though partial radial failures were present. From the shape of the designs and the length of time required for complete failure under these conditions, it would appear that these are formed either locally in a small air space as a result of electrical discharge caused by overstress, or more extensively throughout the laminations because of low lateral dielectric strength. Their presence recalls the need for consideration of both radial and lateral dielectric strength in laminated structures. It is felt that a continued study of these phenomena is merited.

From the results shown in Figs. 10 to 15, it is seen that the initial high-voltage test is quite unable to detect faults which may later come into existence from conditions, some of which become known but many of which remain unknown. This points to the need of testing during operation. The phenomena here shown are, of course, at higher values of electrical stress than under rated conditions, yet these apply, in so far as unexplained failures occur at the rated electrical stress and the time factor is present. In this very connection there has just come to the writer's attention[7] a case in practise where a 17-kv. cable joint was poorly made, the evidence showing that an air space of 3/8 in. was left therein, which was not detected during the high-voltage test at 48 kv. The joint failed later, during operation, from lightning.

Dielectric Power Loss and Power Factor

The measurement of dielectric power loss and power factor of cables for service has in the past been mainly confined to measurements on short samples at voltages up to rated and temperatures up to 100 deg. cent. and on sample reel lengths at rated voltage and temperature of 85 deg. cent[8]. The tendency is to extend these measurements to include each reel length to be shipped, measuring the power factor thereof at room temperature at voltages varying from approximately 1/2 rated to 2 1/2 times rated. The change in power factor between these voltage limits is noted, the theory being that there should be no change in power factor if the insulation is homogenous, the measurement being made outside of the breakdown voltage range.

Such a measurement requires considerable change in application of the methods previously employed for measuring dielectric power loss and power factor. The means employed and the calculations involved must be simple enough for manipulation by the factory testing personnel under factory working conditions. The

range required for cables in use at present is approximately as follows:

Power factor, from 0.002 up, particularly in the range 0.002 to 0.04.
Voltage, 5 kv. to 100 kv.
Current, up to 1 ampere, current leading.
Accuracy, power factor to within 0.002.
Frequency, 25 to 60 cycles.

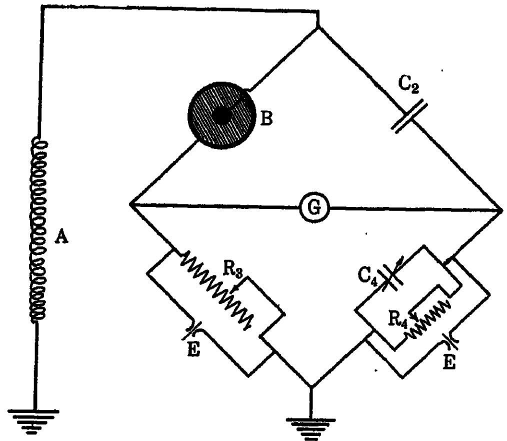

FIG. 17—DIAGRAM OF CONNECTIONS OF SCHERING BRIDGE.

A—Transformer
B—Cable
C_2 Air Condenser
R_3 Non-Reactive Variable Resistor
R_4 Non-Reactive Variable Resistor
C_4 Variable Air Condenser
G—Galvanometer Detector
E—Safety Gaps

Several methods are applicable to this measurement no one of which is entirely free from objections. One method which gives promise is that of the Schering Bridge, which has been described in various issues of the technical press[9]. The diagram of the connections is shown in Fig. 17. A description of the equipment used in such a set-up, which is proving to be satisfactory, is as follows:

The standard air condenser C_2 is a 3-plate condenser following a design previously described.[10] All of the plates are mounted on a common base and are fixed to give a capacitance of 4×10^{-10} farads. The overall dimensions are 10.5 ft. long, 4 ft. wide, and 7.5 ft. high. The non-reactive resistor R_3 is of high-resistance low-temperature coefficient alloy, wound in two sections connected for minimum reactance. The rated current is 1 ampere, and taps are provided to give approximate resistances of 5, 10, 20, 40 and 60 ohms.

The detector G is unique in that a permanent magnet galvanometer recently developed for detecting alternating currents of low value is used. This is shown in Fig. 18. An ordinary three-stage audio-frequency amplifier is used with the galvanometer, thus giving the desired sensitivity. The galvanometer moving system, entirely immersed in oil, is free from the effects of external vibration so that the required sensitivity is obtained without need for elaborate means of protecting against the effect of external vibration.

The non-reactive variable resistor R_4 is of standard design with a maximum resistance of 11,110 ohms. The variable condenser C_4 is of standard design parts, having a maximum capacitance of 0.16×10^{-6} farads.

The safety gaps consist of two contacts separated by two cigarette papers, which form of gap has in the past proven effective. Any discharge through the cable or the condenser goes to ground through the break produced in the paper. The standard air condenser in the above set-up has been short-circuited during operation at 50 kv. with no effect on the apparatus or the operator. The entire measuring apparatus, with the exception of the air condenser and detector, is mounted in a glass-covered case with manipulating handles for safety to the operator, extending through.

The operation of the bridge for a given reading is quite simple. The bridge is balanced by varying R_4 and C_4 until a balance is obtained as noted by the detector. R_3 remains constant for a given range of readings, and the power factor is calculated from the equation,

$$\text{Power factor} = 2\,\pi\, f\, C_4\, R_4$$

where F is the frequency

C_4 is the capacitance in farads of the variable condenser C_4

R_4 is the resistance in ohms of the variable resistor R_4.

If it is desired to obtain the dielectric power loss, the equation is,

$$\text{Watts} = 2\,\pi\, f\,\frac{R_4}{R_3}\,C_2\,E^2\,(\text{power factor})$$

where, in addition to the factors defined above, R_3 is the resistance in ohms of the resistor R_3, C_2 is the capacitance of the standard air condenser C_2, and E is the applied voltage in volts.

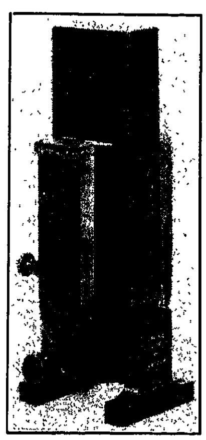

FIG. 18—PERMANENT MAGNET TYPE OF GALVANOMETER FOR DETECTING ALTERNATING CURRENTS OF LOW VALUE

All methods for making such measurements as the above usually appear well on paper but in operation, factors may be introduced which considerably affect results. The results obtained with the Schering Bridge were, therefore, compared with results obtained on the same cable, by the compensated dynamometer wattmeter method,[10] using two instruments, one of low current rating for determining the necessary compensation with the standard air condenser, the other of higher

current rating required when making the measurement on the cable. From results thus obtained, it was found that large errors might be introduced into the bridge reading because of capacitance to ground of the leads from the low voltage side of the standard air condenser and the cable. When effect of this error was reduced to a minimum, results such as shown in Fig. 19 were obtained. These show agreement to within power factor 0.002.

The use of indicating wattmeters alone for such measurements as those described above is highly

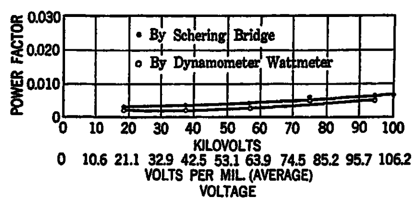

FIG. 19—VARIATION OF POWER FACTOR WITH VOLTAGE AT 25 DEG. CENT. OF A 130-FT. LENGTH OF SINGLE-CONDUCTOR CABLE WITH 30/32-IN. TREATED PAPER INSULATION FOR A 66-KV. THREE-PHASE CIRCUIT

desirable. The very unique application of a water column of varying temperature to change the resistance of a circuit without changing its reactance, recently described before the Institute[11], has great possibilities. An outfit set up with the "tank," of the wattmeter replaced by a long hose, shielded as suggested by Professor Ryan, to allow of continuous measurement has given promise of perhaps immediate application in measuring small amounts of power.

The method of applying the voltage during power-factor measurements on reel lengths of three-conductor cable, is important. A choice exists between the use of three-phase supply and single-phase supply. The former allows the test to be made under operating conditions and only one connection need be made during the test, although the measuring instrument will have to be shifted from phase to phase if only one instrument is used. The use of a dynamometer with a standard air condenser is possible, although shielding the high-voltage windings in the transformer appears to be necessary. The current element of the dynamometer must during measurement be connected between the transformer winding and the grounded neutral. Error due to leakage to ground may thus be introduced since the circuit must be changed from that during compensation with the air condenser.

The use of single-phase supply allows the use of both bridge methods and dynamometer methods. Connections to the cable may be made in several ways, of which representative connections are shown in Fig. 20. Connection A gives a measurement of belt insulation and conductor insulation next to the belt insulation. Connection B gives a measurement of conductor insulation between one conductor and the other two conductors. Two such measurements with two different conductors "high" should suffice. Connection C gives the measurement of conductor insulation between one conductor and the other two conductors and of the conductor and belt insulation between that conductor and the sheath.

Connections A and B are symmetrical as regards calculating the average voltage gradient of the insulation wall under test. In connection C the average gradient over the conductor insulation and adjacent belt insulation will differ from that over the conductor insulation between conductors if the thicknesses of conductor insulation and belt insulation are different. Because of the possible differing stress in the belt insulation and conductor insulation, connection C is not satisfactory for high values of average gradient. Measurements using connection A and connection B with each conductor "high" in succession, certainly explore the cable completely and are, therefore, advisable. Successive measurement with two conductors high in connection B may suffice, as such would include all of the conductor insulation between conductors. Experience thus far indicates that where cable is poor, it is poor symmetrically, so that all of the power-factor-voltage curves show large change. A single measurement may, therefore, prove sufficient.

The objections to the single-phase measurement are that several connections are required during test; and that the rotating field present with 3-phase supply is not present with the single-phase source.

The equivalent value of 3-phase watts and power factor may be calculated from single-phase measure-

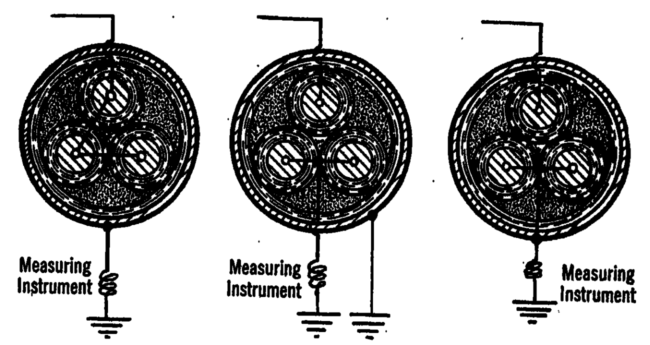

FIG. 20—CONNECTIONS OF SINGLE-PHASE SOURCE TO THREE-CONDUCTOR CABLE FOR POWER FACTOR AND DIELECTRIC POWER LOSS MEASUREMENTS

ments made successively with connections B and A, as follows:

Let W_B = watts measured, connection B, at voltage E

W_A = watts measured, connection A, at voltage $\frac{E}{\sqrt{3}}$

Then, total three-phase watts $= 1.5\ W_B + W_A$.

The three-phase power factor may be taken as the average of the power factor obtained with connection B at

voltage E, and the power factor obtained with connection A at voltage $E/\sqrt{3}$.

The above relations have been proved to check the dielectric power loss and power factor obtained with three-phase supply up to rated cable voltage. For values of voltage above rated, the results appear to be consistent, but check measurements will have to be made before they can be assumed to be reliable with certainty.

For purposes of applying the power-factor-voltage

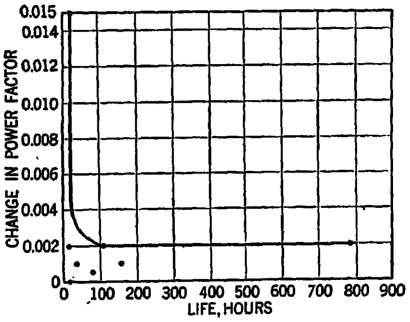

FIG. 21—RELATION BETWEEN LIFE OF SAMPLES OF TREATED PAPER CABLE WITH 9/32-IN. INSULATION, OPERATED AT 44-KV. AT 25 DEG. CENT., AND CHANGE IN INITIAL POWER FACTOR WITH VOLTAGE FROM 27 VOLTS PER MIL. (AVERAGE) TO EIGHTY-SEVEN VOLTS PER MIL. (AVERAGE) AT 25 DEG. CENT.

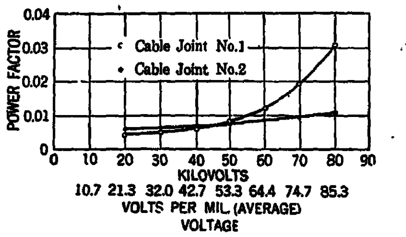

FIG. 22—POWER FACTOR-VOLTAGE RELATION AT 25 DEG. CENT. OF TWO JOINTS OF SINGLE CONDUCTOR CABLE WITH 30/32 IN. TREATED PAPER INSULATION

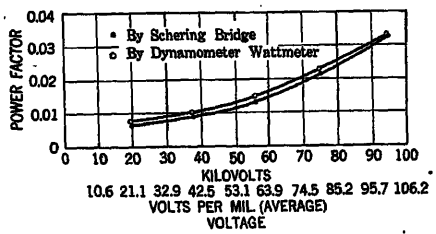

FIG. 23—POWER FACTOR-VOLTAGE RELATION AT 25 DEG. CENT. OF A 100-FT. LENGTH OF SINGLE-CONDUCTOR CABLE WITH 30/32 IN. TREATED PAPER INSULATION SHOWN BY DIELECTRIC STRENGTH TEST TO BE POOR CABLE AND UNSUITED FOR USE

test, it is, therefore, suggested that connections B and A be standardized, and that separate curves be obtained therewith, these to be on the same basis of average volts per mil.

The application of voltages during the power-factor measurement comparable in value to those of the high-voltage test will necessitate the use of larger end-bells. This will prevent the effective application of the usual guard circuits; also a relatively large dielectric power loss may be located in the larger end-bell. Some measurements which have been made in this regard indicate that as long as there is no visible leakage, the difference in result obtained with or without guard rings is negligible. When corona and current streamers are visible, differences of 50 per cent have been obtained. The unsheathed portion of the cable must, therefore, be of sufficient length to prevent this source of error.

The effectiveness of the power-factor-voltage test to separate poor cable from good will have to be determined largely from experience. Fig. 21 shows the relation between life and change in initial power factor for several short samples of cable with 9/32-in. treated-

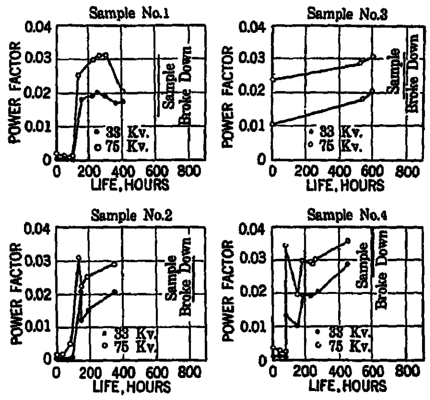

FIG. 24—CHANGE IN POWER FACTOR DURING LIFE OF SINGLE-CONDUCTOR EXPERIMENTAL CABLE SAMPLES WITH 30/32 IN. TREATED PAPER INSULATION OPERATING AT 80-KV. AT 50 DEG. CENT.—MEASUREMENTS OF POWER FACTOR MADE AT 33-KV. AND 75 KV.

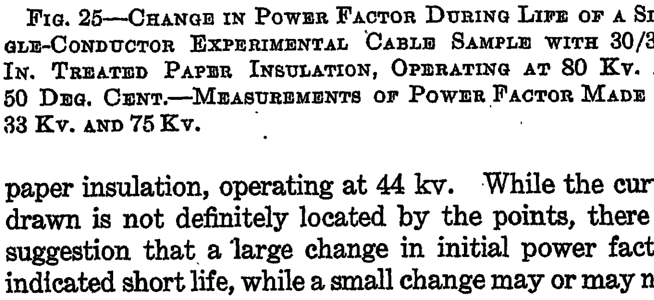

FIG. 25—CHANGE IN POWER FACTOR DURING LIFE OF A SINGLE-CONDUCTOR EXPERIMENTAL CABLE SAMPLE WITH 30/32-IN. TREATED PAPER INSULATION, OPERATING AT 80 KV. AT 50 DEG. CENT.—MEASUREMENTS OF POWER FACTOR MADE AT 33 KV. AND 75 KV.

paper insulation, operating at 44 kv. While the curve drawn is not definitely located by the points, there is suggestion that a large change in initial power factor indicated short life, while a small change may or may not indicate long life.

Fig. 22 shows the power-factor-voltage relation for two cable joints of single-conductor cable with 30/32 in. paper. Cable joint No. 1 was hand-made and lasted two min. at 175 kv.; cable joint No. 2 was machine-

made and lasted 36 hours at 225 kv. Fig. 23 shows the power-factor-voltage relation for a length of cable with 30/32 in. paper shown by dielectric strength test to be poor cable.

A 200-ft. length of cable* similar to that for which the power-factor-voltage curve is shown in Fig. 19, was connected from one line, to neutral of a three-phase 110-kv., 60-cycle commercial circuit, at a voltage from conductor to sheath of 63.5 kv. It operated continuously at no-load for 10 days when failure occurred at a point 30 feet from one end. Examination revealed absolutely no discernible reason for the failure, the cable appearing to be perfect throughout. The defective end was cut from the length, voltage again applied, and same has been running continuously for 90 days. (January 1, 1925.)

It has been noted that the power factor of a cable changes during its life. Fig. 24 shows the results of measurements of power factor made at different times during the life of four samples of single-conductor experimental cable with 30/32 in. treated-paper insulation, when operating at 80 kv. (85 volts per mil, average) at 50 deg. cent. In sample No. 3 the paper was treated with a petrolatum-rosin mixture, while the others were treated with straight petrolatum.

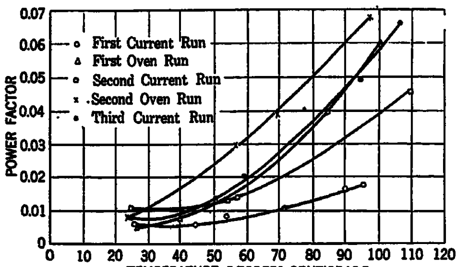

FIG. 26—POWER-FACTOR-TEMPERATURE CURVES OF SINGLE-CONDUCTOR CABLE WITH 30/32-IN. TREATED PAPER INSULATION, THE CABLE BEING HEATED BY OVEN AND CIRCULATING CURRENT

Fig. 25 shows results on a sample which has operated for over 3500 hours at 80 kv. at 50 deg. cent. In this connection it is interesting to note that a measurement of power factor made at a life of 250 hours gave a value of 0.17 at 75 kv. and 0.14 at 33 kv. No error could be found in the measurement at the time, nor have subsequent tests on other cables shown such large change as this. These values have, therefore, been omitted from the curve which has been made dotted in this region. The comparatively large change in power factor of the sample in Fig. 25 is interesting in the light of the long life of the sample.

Such results, showing change in power factor with life, are not new as they have been noted on tests with different papers under electrical tension. Similar results have also been noted in the case of armature coils as well as having been reported to this Institute[12].

Apparently the change in power factor within the limits shown has little effect upon the dielectric strength of the cable. It is evident, however, that continued electrical tension affected the cable.

When making measurements of either dielectric power loss or power factor at temperatures above room temperature, it is quite necessary that the cable or cable sample be heated uniformly throughout. If this is not done results such as shown in Fig. 26 will be obtained where the power factor of a sample of single-conductor cable with 30/32-in. treated-paper insulation was measured in order

1st, by circulating current through the conductor and measuring conductor temperature, using this value as the temperature.
2nd, by placing the sample in an oven for four hours and measuring both sheath and conductor temperature, which check.
3rd, by repeating the 1st run.
4th, by repeating the 2nd run.
5th, by repeating the 1st run.

The results show that not only is the power factor higher when the temperature throughout the cable is uniform, but they also show increasing values with continued heating. This suggests that temperature measurements on cables intended for use should not be carried to values above the operating values.

TESTING INSTALLED CABLE

The evidence available on the endurance of cable indicates the need of continued testing after the initial tests and installation of the cable. A good example of results obtained by methods departing from those standardized for testing cables has been reported to the Institute[13] by Messrs. Phelps and Tanzer in connection with tests conducted to rate cables after installation. The application of the method of rating by so-called "current-time" curves to low-loss paper-insulated cables has been carried on to a limited extent, with results which, although usually negative, still are of such a nature that the method is not considered to be absolutely inapplicable. This study should be vigorously continued.

Opportunity has been given to test single-conductor cable with 30/32 in. treated paper insulation, known to be poor cable, up to direct voltages of 450 kv. applied for 15 minutes, without evidence of impending failure as shown by current-time curves taken periodically during the test.

Suggestion has been made that high frequency might be utilized for making capacitance or power-factor measurements on installed cable, the bridge and generator equipment required being of such a nature as to make it quite easily portable. Table I shows results of capacitance and power-factor measurements made at 100 kilocycles on short samples of single-conductor cable with 9/32 in. treated paper insulation before and after endurance life run at 44 kv. The length of life does not seem to be predicted by the value of the initial power factor.

*Manufactured after Jan. 1, 1924.

TABLE I

Sample	Endurance Test at 44 Kv.		Power Factor at 100 Kilocycles	
	Temperature Deg. Cent.	Life Hours	Before Test	After Test
1	25	77	0.0105	0.0103
2	25	86	0.0103	
3	25	89	0.0106	0.0102
4	85	139	0.0114	
5	85	139	0.0106	
6	85	1055	0.0105	0.0143*
6	85	1055	0.0105	0.0093
7	85	1535	0.0116	0.0102

*Measurement made with failure in sample.
Measurements on other samples made "After Test" were made after cutting out the failure.

Values of power factor taken after test with the failure removed show practically no change from initial values.

Several measurements of power factor made at 1000 cycles on cable samples with 30/32 in. treated-paper insulation gave results from which no concord could be noted as regards life. The low voltage, about 25 volts, at which these measurements are made is probably a factor. It is interesting to note, however, that measurements made at 100 kilocycles on cable samples of different kinds of papers showed power factors differing by 50 per cent, though for samples of any given kind of paper, the variations were of the order as shown in Table I. The method appears, therefore, to be of value in detecting different materials, but does not seem to be sensitive to deterioration in a portion of a given material.

Testing with Direct Current

Experimental work is being continued in the application of direct current to the testing of cables in order

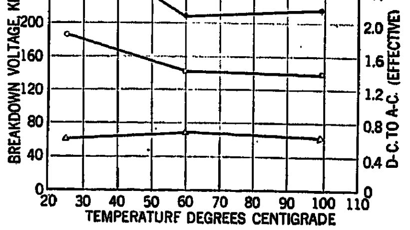

Fig. 27—Relation Between D-C. and A-C. (Effective) Breakdown Voltage Above Ground at Different Temperatures of Three-Conductor 500,000-cm. Sector Cable. with Treated Paper Insulation 9/64 In. on Each Conductor. 1/16-In. Overall Belt

that its apparent advantages may be utilized in this field. The large number of d-c. cable-testing sets now in use by the operating companies should allow of accumulation of a considerable amount of data showing the desirability of their use in the field.

Fig. 27 shows results of tests made on samples of three-conductor 500,000 cm. sector cable with treated-paper insulation 9/64 in. on each conductor, 1/16-in. overall*. The a-c.-breakdown tests were made by starting at 40 kv. applied and held for five min. and then raising five kilovolt each 15 seconds until breakdown. The d-c.-breakdown tests were made by starting at 100 kv. held for five min. and raising five kilovolt each 15 sec. until breakdown. Three tests were made on each section of the cable by applying the test voltage to one conductor with the other two and the lead sheath grounded. In this way each conductor was tested separately from copper to sheath.

These results show a ratio of d-c.- to a-c.- effective breakdown value somewhat higher than previously assigned from the results available at the time, but the evidence is accummulating that the ratio will increase as the thickness of the material increases. Recent tests on varnished cloth in sheet form up to a thickness of 10 sheets or 120 mils substantiate this conclusion.

Continued experimental work and study of the problem indicates that although for any given breakdown voltage with alternating current there is a corresponding breakdown voltage with direct current, each of these values depend upon several factors such as the nature and structure of the material, thickness of the material, temperature of the material, shape and size of electrodes, and rate of application of the applied potential. Such a condition obviously prevents standardization of a general value for the d-c.- to a-c.- breakdown-voltage ratio, but rather requires that each individual case be considered separately. This confirms what has been pointed out previously.[14]

Conclusions

1. Values of insulation resistance as heretofore obtained are of doubtful worth as a means for distintinguishing between satisfactory and unsatisfactory cable.

2. The test for dielectric strength is the most important and the best now available as a suitability test. The difficulties in testing due to the increasing voltage rating of cables are being overcome. Indications are that the present standardized values of test voltage should be increased. Increasing the time of application a few minutes is of doubtful value; a long-time on a sample length has merit. Short-time ultimate dielectric-strength tests should be made on samples taken from reel lengths. The bending tests, as at present, should be continued. Recognition should be given to the fact that many things may happen to the cable after the high-voltage test which this test cannot detect. Hence the advisability of periodic testing at voltages which will not injure sound cable and which will only affect cables where the deterioration has been abnormal.

3. Measurements of dielectric-power loss and power factor are of value, and should be continued. The evidence indicates that good cable will show low dielectric-power loss and power factor, and that the change in

*These samples were supplied by the Commonwealth Edison Co.

power factor of good cable with varying voltage at room temperature will be small. The power-factor voltage test may not always be infallible, however. There is need for standardizing the power-factor voltage test, as regards the nature of the source, the method of connecting same to the cable, the temperature of the cable, and the reporting of the results.

4. Development work towards new methods for testing and rating installed cable during life have not produced new tests which are suitable for superseding those already in use.

5. The increasing use on the part of the operating companies of d-c.-cable-testing sets, together with the experimental work being carried on, should allow of a determination of the most suitable application of direct-current to cable testing that its apparent advantages may be fully utilized.

In closing, the author wishes to acknowledge his indebtedness to others in the General Electric Company who have allowed him to use results of their work in this paper.

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Discussion

D. W. Roper: The paper by Mr. Lee (the best discussion of the subject of testing high-voltage cable that has ever been presented to the Institute) contains a great amount of interesting information all drawn from the experience of one cable manufacturer. Some of the conclusions which he draws are, therefore, rather limited in their application as they refer solely to the particular type of insulation with which he is familiar. As will appear later in this discussion, there is a wide variation in the characteristics or properties of the insulation made by the different manufacturers. For the purpose of broadening our view and thus improving our perspective, and not for the purpose of making any invidious comparisons between the products of the several manufacturers, I am going to quote some of the results that we have obtained in testing the product of a number of cable manufacturers, in the hope that I may be able to show how some of the conclusions given by Mr. Lee should be modified.

On page 114, Mr. Lee cites the case of the failure of an experimental length of cable connected to a 110-kv. three-phase overhead line, and states that the examination revealed absolutely no discernible cause for the failure.

About one-third of our failures in service result in a similar manner.

In another paragraph on the same page, the author states that the study of the current-time curves obtained with insulation-resistance measurements should be vigorously continued. On this point I thoroughly agree with him and offer him our continued assistance and bespeak for him the cooperation of other operating engineers who are in a position to assist. The development of additional methods of testing cable should result in the

discovery of causes of failure for which no explanation can now be given.

In Fig. 1 of the paper is given some data regarding the variation in insulation resistance of fourteen reels of cable, the maximum range being of the order of 10 to 1. A study of our records shows that in three shipments of similar cable ranging from thirty to sixty lengths, the maximum range in insulation resistance was 1.6 to 1 as compared with Mr. Lee's figure of 10 to 1 for fourteen reels. The author thinks that such measurements are of little value as a means of distinguishing between good and poor cable. In view of the data regarding other makes of cable which have been quoted, it appears that the results given by the author indicate that in his factory they have not secured control of all of the factors which affect the quality of their insulation. It also appears that studies made for the purpose of eliminating the present variations in insulation resistance will result in methods that will bring about an improvement in the product, and that there is an even greater chance of such a result than that the studies of current-time curves of insulation resistance will result in the discovery of improved methods of testing.

No methods of testing high-voltage cable will be satisfactory until it is possible to detect and reject at the factory all cable which contains the germ of a failure which will occur shortly after the cable is placed in service, such as the example cited by the author. This requirement, while it may appear somewhat idealistic, becomes more important as the operating voltage of the cable is increased. On a 12-kv. line we may be able to carry a maximum load of 8000 kv-a.; on a 33-kv. line, this figure will be about 15,000 or 20,000 kv-a.; on a 66-kv. single-conductor line, the load may be 40,000 or 50,000 kv-a.; and on a 110,000-kv. single-conductor line, the maximum load may be 70,000 kv-a. These figures, which are illustrative, and not intended to be strictly accurate, show that the importance of continuous service inceases rapidly with the operating voltage.

The record of a number of recent installations of high-voltage cable made in this country and abroad, as shown in Fig. 1 herewith indicates the general trend of thickness of insulation as related to the operating voltage. This record indicates that the operating voltage is increasing at a faster rate than the thickness of insulation. This means that as the operating voltage increases the dielectric stresses increase, and if the cable is to be as successful in operation as cable of the lower voltages, there must be a continuous improvement in the quality with the increase in the operating voltage. It is, therefore, quite necessary, as stated by the author at the beginning of his paper, that improvements in cable manufacture should be paralleled by improved methods or additional methods of testing, so that we may be able to distinguish good cable from cable that will prove unsatisfactory in service.

The rainbow that the engineers of some manufacturers, as well as engineers of operating companies, including the speaker, have been chasing for a number of years is the search for some single test that will discriminate between good and poor cable. But impregnated-paper insulation is such a complex structure and as made at different factories has such wide variations in its several properties, that the discovery of any single test that will prove adequate seems to be just as hopeless as the discovery of perpetual motion. Impregnated-paper insulation has a number of different qualities, as set forth by the author, such as insulation resistance, dielectric strength, dielectric loss, etc., and these qualities are so interrelated that it is difficult, or perhaps impossible, to improve the insulation in one particular, without having some effect on the other qualities, and these several qualities individually have very different values in determining whether the cable will prove successful in service.

In Fig. 2 herewith are shown the maximum and minimum dielectric strength values obtained in accordance with A. I. E. E. Standards on short lengths of cable for all of the sizes purchased by the company with which the speaker is connected, during the last two years. All of these cables have met the requirements of the N. E. L. A. Specifications and the A. I. E. E. Standards. It will be noted that the range in dielectric strength varies from 5 to 1 for low voltage cables to about 1.5 to 1 for 20-kv. single-conductor cable, and 2.4 to 1 for 33-kv. cable. It appears fairly evident from these data that the several cable manufacturers place very different weights on the importance of dielectric strength.

In Fig. 3 are shown a number of curves showing the variation of dielectric loss with temperature which are typical of the product of practically all the leading cable manufacturers in the world, each one of whom thinks that in this particular feature his cable is entirely satisfactory. With such wide variation in the forms of the curves and in the values of the losses at the maximum operating temperatures, it does not appear that all of them can be right.

Fig. 4 shows in a similar manner typical ionization curves; that is, the variation of power factor with voltage. One manufacturer contends that there should be no variation in the power factor between 50 per cent and 150 per cent of normal voltage, while another manufacturer contends with equal force that a

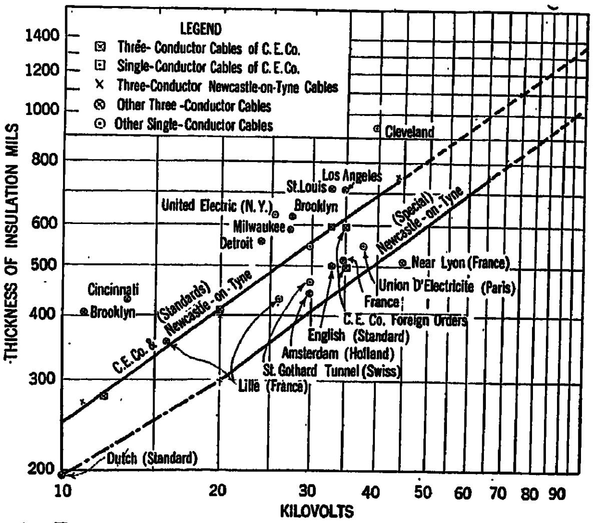

FIG. 1—RELATION BETWEEN THICKNESS OF INSULATION AND OPERATING VOLTAGE

variation of 2 per cent over about the same range of voltage is permissible. No one manufacturer makes cable which is the best in all of those qualities described by the author which collectively indicate its excellence, and, in the face of such conflicting statements presented by the various cable manufacturers that are equally skillful and reputable, what is the purchaser to do?

During recent years, improvements in cable manufacture have been so rapid that it is not possible for an operating company to buy cable of several different combinations of the qualities illustrated, and then wait for five or ten years to determine from operating experience which of the several varieties is the best before purchasing any more cable. Commercial developments and increases in load compel the operating companies to buy cable every year, and even if they should wait for five or ten years before buying more cable, they would then be told by the manufacturers that such cable was no longer made, due to their improvements in manufacture and cable of a different combination of qualities would be offered.

Table I shows the writer's suggestion for a solution of this problem. In it are listed all of the test requirements given by the author. There are also given in the third column under the heading *Weight for Best Performance* the writer's judgment as to the relative weight which should be assigned to each of those qualities or properties. As set forth in the table, all of these

cables comply with the N. E. L. A. Specifications and the A. I. E. E. Standards, but it will be noted that they pass the specifications with widely different margins in the various particulars. The cable which gives the best results in the ionization test is not the best in dielectric loss nor dielectric strength. As these

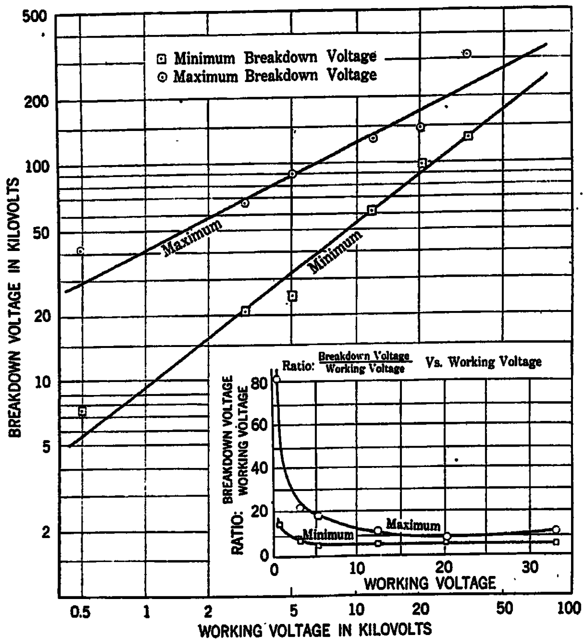

FIG. 2—BREAKDOWN VOLTAGES OF TESTS ON HOT STRAIGHT SAMPLES OF IMPREGNATED PAPER INSULATED, LEAD COVERED CABLES

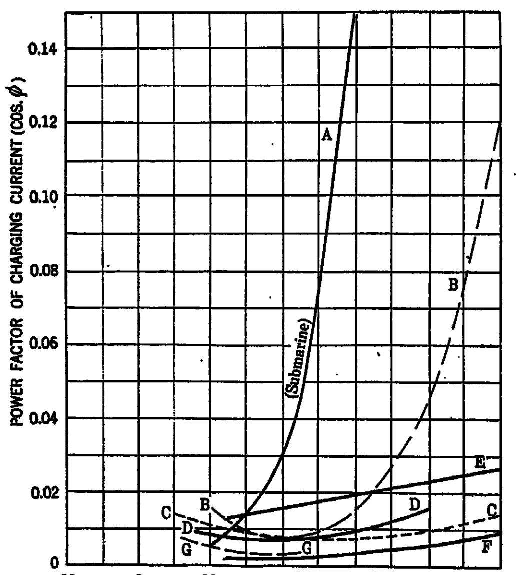

FIG. 3—VARIATION OF POWER FACTOR WITH TEMPERATURE FOR HIGH VOLTAGE CABLES

Curves *B*, *C* and *D* are guarantees. Curves *A*, *E*, *F* and *G* are of test results. All curves are for 60 cycles except *A* and *C*, which are for 50 cycles.

figures in the column called *Weight for Best Performance* are the personal opinions of the writer, it is entirely possible that they are all wrong or that a different method of calculating the relative performance of the several manufacturers from the test records should be used. A number of similar tables have been calculated, using radically different figures in the column of *Weights for Best Performance.* These calculations show that changing the figures in this column to any other figures which appear to be within reason makes but slight changes in the relative order of the manufacturers.

Such calculations indicate that the general method is correct; but some adjustment of the details may be necessary to secure greater accuracy. It is not pretended that the figures at the foot of the table correctly represent the relative merit of the cables on a percentage basis, but it appears that the table does determine, with reasonable accuracy, their relative order of merit.

It should be possible by a study of the test records and the operating records to determine with a fair degree of approximation the proper figures to be used for the weights for best performance, and after this has been done, the rating of the cables consists mainly of simple arithmetical calculations from the test records. After such a tentative assignment of weights has been agreed upon by the manufacturers and the operating companies,

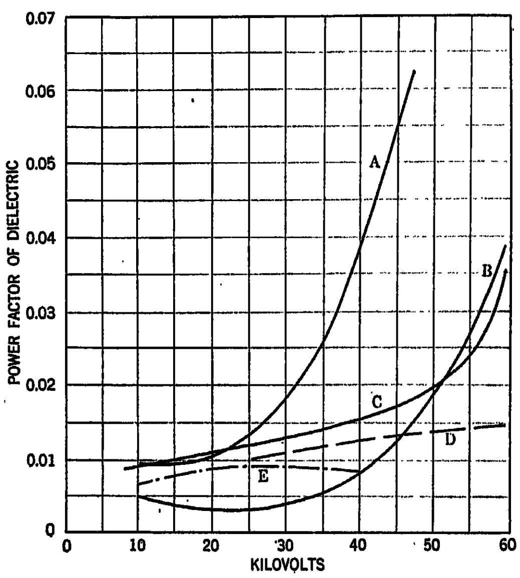

FIG. 4—IONIZATION TESTS ON THREE CONDUCTOR HIGH-TENSION CABLES

All 33-kv. cables except *D*, a 10-kv. cable

then the manufacturers have a definite statement of the desired combinations of qualities to guide them in the improvement of their product.

The operating companies, on their part, should carefully analyze all of their cable failures, with the assistance of the manufacturers, and agree upon the particular quality or property of the cable whose deficiency was responsible for each failure, and from the mass of operating records of this kind which are available for study, modify not only the weights for best performance from time to time in accordance with operating experience, but also the minimum requirements of the specifications.

There is no country in the world in which the engineers of the manufacturing companies and those of the operating companies are so well organized as they are in America, through their technical associations, like the A. I. E. E., and the commercial associations like the N. E. L. A. and the A. E. I. C. to bring about such cooperation in a proper way. If this cooperation can be secured, there appears to be no good reason why the American cable manufacturers in a few years should not be leading the world in the manufacture of high voltage cable, instead of occupying their present subordinate position.

W. F. Davidson: Mr. Lee has presented a very helpful sum-

mary of the present status of testing high-tension paper-insulated cables so far as concerns tests to be made in advance of installation, but I feel that it is unfortunate that he does not have more information to present in regard to tests intended to determine the fitness of the cable to meet the requirements of continuous operation.

The tendency to use higher test voltages and slightly longer times of application in tests on reel lengths and samples seems to be a step in the right direction, for it will tend to eliminate weak spots and it is always the weak spots that cause trouble. But there are many things which we cannot learn from short-time tests, or even from tests of several hours duration. Several operating companies using cables at the higher operating voltages have lately found signs of a slow but very noticeable deterioration. In many cases this has been accompanied by the formation of a substance variously known as "*X*", "wax," or "cheese." Efforts to produce this substance in the laboratory have generally been unsuccessful except where the time of test has been prolonged to many hours; short-time tests yield no results. This experience would seem to indicate the desirability of making tests on samples of cable using a voltage only slightly—say, 50 per cent—in excess of the rated voltage and then making a thorough examination, including chemical tests, after two hundred or three hundred hours. There is another point, which may be touched on later, and that is the need for more elaborate tests to detect the presence of injurious impurities.

The data presented to show the significance of insulation-resistance values are certainly almost enough to lead one to discard them entirely from cable specifications. However, they seem to have a value which Mr. Lee has overlooked. The user of cable is vitally interested in securing cable of uniform quality—999 ft. of superlatively good cable cannot keep a line going if 1 ft. is defective,—and it is my feeling that uniform cable will show reasonably uniform insulation resistance. When one stops to consider the very complex nature of cable insulation, it is not surprising that no simple relation has been found between insulation resistance and other characteristics of the insulation. But this does not, to my mind, afford a valid reason for rejecting them. As an example of that point in mind and illustrating what is commercially possible, I have plotted the insulation-resistance readings (see Fig. 5) on successive reel lengths of cable brought up for test in two factories. In one case the ratio of maximum to minimum values is about 1.32, while in the other case it reaches 15.6. These cables have not been in operation long enough to permit a statement to be made as to their operating performance, but it is interesting to note that the first lot of cable gave distinctly superior results on all of the other tests. As I see it, this means that the factory processes in the first case were more carefully controlled than in the second case.

Several curves are presented with the paper to show the relation between the test voltage and the time required to secure failure, but I feel that in their present form they are somewhat misleading. Take Fig. 10 as an example: We find that nearly instantaneous breakdown was secured with five variations of test voltage between 132 kv. and 100 kv. and we also note that 44 kv. produced failures in times varying from 30 hours to 160 hours. A curve has been drawn to pass through all of the points with remarkable precision. One might be led from this curve to assume that the cable would operate indefinitely at 43 kv. The experimental data seem rather too few for determining a curve and one would wish that figures were available to show the life of the cable at, say, 30 kv.

Partial failures, such as are described, have long been known to those who have studied cable insulation and we find frequent reference to them in the technical literature. I think it is going a little too far to suggest that these failures generally occur near the center of the insulation. Undoubtedly they do occur, as Mr. Lee observes, at points where the insulation is unusually weak or where the stresses are high but this point may be quite as well near the conductor as at the center of the insulation. In a number of samples taken from cable of ten different makes, burns have been observed rather more frequently at the point of high stress—that is, at the surface of the conductor. For the obvious reason that the conductor is itself practically an equi-potential surface the tree designs are not found near the surface of the conductors but other forms of partial failure are very common.

Reference is made to the direct-current or kenotron testing method outlined by Messrs. Phelps and Tanzer in a paper pre-

TABLE I

COMPARISON OF RESULTS OF INSPECTION AND TESTS ON 3-CONDUCTOR CABLE RATED AT ABOUT 13 KV. AND MADE IN 1924

All of these cables comply with the N. E. L. A. Specifications and the A. I. E. E. Standards

Item No.	Item	Weight for Best Performance	Weights for Various Manufacturers						
			A	B	C	D	E	F	G
	Mechanical								
1	Wrinkles	4	4.0	3.2	3.0	3.0	4.0	2.8	3.6
2	Creases	4	4.0	3.0	3.6	3.0	3.6	3.2	3.6
3	Tearing in cold bent sample	4	3.2	4.0	3.0	3.6	3.2	3.0	2.9
4	Fillers	4	4.0	3.6	3.4	2.0	3.4	4.0	2.6
5	Impregnation	8	8.0	6.2	8.0	6.2	6.2	4.0	6.2
6	*Other conditions	6	6.0	4.8	6.0	3.8	4.8	3.3	3.9
	Electrical								
7	Range of insulation resistance	5	3.1	2.6	5.0	2.0	3.4	4.5	2.5
8	†Increase in Power Factor—Average	5	4.0	5.0	2.3	4.9	3.9	4.8	3.4
9	Increase in Power Factor—Maximum	10	7.9	8.1	4.7	9.0	5.6	10.0	7.1
10	Dielectric loss at 80 deg. cent.—Average	3	0.9	2.2	1.0	3.0	1.0	1.5	1.2
11	Dielectric loss at 80 deg. cent.—Maximum	7	2.1	4.6	3.1	7.0	3.4	3.6	3.4
	Puncture Voltage								
12	Hot straight sample—Average volts per mil	7	7.0	6.5	6.9	6.8	7.8	6.5	5.4
13	Hot straight sample—Minimum volts	11	8.6	7.8	8.4	7.4	11.0	9.3	8.1
14	Cold bent sample—Average volts per mil	8	8.0	6.9	6.7	6.7	4.2	3.0	4.1
15	Cold bent sample—Minimum volts	14	13.5	14.0	13.1	8.9	10.0	7.2	10.1
	Total weight, mechanical	30	29.2	24.8	27.0	21.6	25.2	20.3	22.8
	Total weight, electrical	70	55.1	57.7	51.2	55.7	50.3	50.4	45.3
	Grand total weight	100	84.3	82.5	78.2	77.3	75.5	70.7	68.1

*"Other conditions" include: (1) Twisting of sectors; (2) Registered tapes; (3) Workmanship in starting and ending tapes; (4) Thickness of lead and insulation in routine and deformation tests; (5) Overall diameter; (6) Condition of lead sheath.

†Increase in power factor at room temperature between 45 per cent and 215 per cent of rated voltage.

Date: February 5, 1925.

sented before this Institute. Where attempts have been made to apply this method to low-loss cables it has been the common experience that it was difficult to get satisfactory results. Some experience along this line with the Brooklyn Edison Company seems to indicate that the difficulties are very largely the result of the rapid decay of the charging current. Whereas the old cables tested in Philadelphia showed that from two to three minutes is required for the current to reach a fairly steady value,

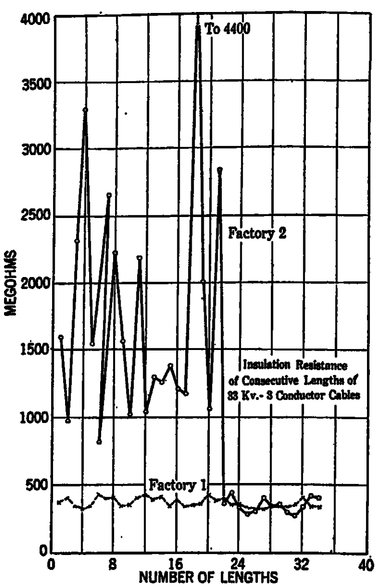

FIG. 5—INSULATION RESISTANCE OF CONSECUTIVE LENGTHS OF 33-KV., THREE-CONDUCTOR CABLES

low-loss cables show steady current values after about ten seconds. This makes it impossible to rely upon readings taken visually but special high-speed curve-drawing instruments have been used with very satisfactory results.

The illustrations shown here indicate the type of results which can be secured, although it must be admitted that the case chosen is somewhat extreme. Fig. 6 herewith shows a reproduction of the graphic millimeter record with the voltmeter record transferred to it as a solid line. It will be noted that *A* and *B* conductors behaved as good conductors should behave, while *C* conductor showed excessive leakage. Just previous to the test, this feeder had been tested at the normal value of 135,000 volts, direct-current conductor-to-conductor for five minutes and 95,000 volts, conductor-to-sheath for five minutes but, as indicated, even this high voltage had failed to remove all of the faults. Conductor *A* and *B* were grounded, and *C* conductor was raised to 98,000 volts and after five minutes failure occurred in a defective joint. When this had been removed the feeder was tested at 135,000 volts and 95,000 volts as before. Then the tests shown in Fig. 7 were obtained. A few such cases as this seem to be ample justification for the use of this method. They also point to the moral that high voltages alone do not give sufficient assurance that a piece of cable on a complete feeder can operate at the rated voltage.

In passing it may be worth while mentioning that the kenotron test method offers real advantages in the nature of sizes of testing equipment and ease of testing. A test set designed for 200,000 volts from conductor to conductor with a 250-milliampere output has been built and mounted on a 6-ton motor trailer. It would be quite impossible to move about a-c. testing equipment capable of handling the same feeders. Moreover, experience in Brooklyn with comparatively high d-c. test voltages has failed to bring forth any evidence indicating damage to the cables as a result of repeated direct-current tests.

J. B. Whitehead: There is general agreement nowadays that insulation resistance based upon the one-minute observation is of questionable value. The reason is that insulation resistance has little bearing upon the loss and upon the performance of a cable, unless the resistance happens to be extremely low. In the latter case, however, one of the puncture tests would be certain to pick it up. The loss due to insulation resistance is a negligible part of the total loss of a normal cable, being of the order of 1 per cent or 2 per cent. The one-minute test is an indication of the dielectric absorption or residual charge of the cable. Taken at one minute this particular quantity has no special interest. If the rate of absorption could be measured for a small fraction of a second we should have important information. The loss in a dielectric is almost entirely due to absorption. It would be very important if we could know the type of variation of the absorption during time intervals corresponding to fractions of the complete period of the alternating voltage. The laws of absorption are not sufficiently definite to enable us to deduce these values from the one minute test.

Will Mr. Lee please tell us whether the curves between the power factor and insulation resistance give the order in which the observations were taken? The curve for insulation resistance ascends continuously and if the points were taken in order it should be easy to trace the causes of the changes appearing. If these one-minute observations give the absorption it is easy to suggest two possible reasons for the differences shown. Absorption is most sensitive to the presence of moisture and also to temperature changes. A further factor often leading to erratic one-minute readings is the influence of the preceding state of the material. Small traces of residual charge will often cause wide variations in the one-minute reading.

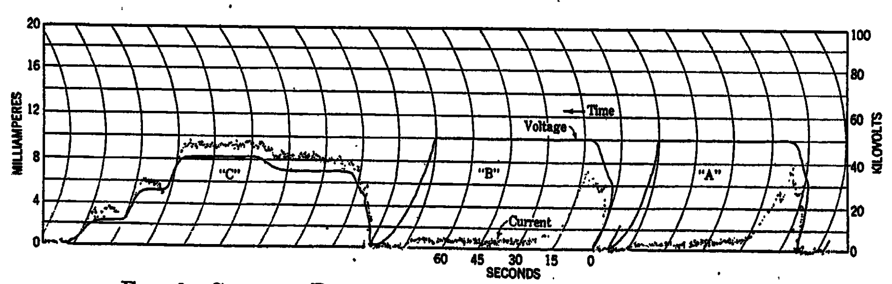

FIG. 6—GRAPHIC RECORD OF KENOTRON ON CABLE SHOWING ONE LEAKY CONDUCTOR

Breakdown of cable insulation may be divided broadly into two types; direct puncture, and slow heating. The latter may begin as due to normal absorption loss, but ultimately it is a heating due to the passing of current. It seems to me that in the case of cables breakdown must fall into the second class. Cable insulation has pronounced absorptive qualities and this accounts for practically all of the initial losses. It is of immediate importance therefore, in determining a proper test for picking out as defective cable, to consider the test from the standpoint of its influence upon the internal losses. Thus, I believe that the high-

voltage puncture test should not stretch beyond a comparatively short interval of time, as its chief function should be the picking out of flagrant imperfections. If a high puncture voltage is applied during a considerable length of time, an abnormal duty is imposed upon the insulation from the standpoint of heating, and increasing absorption due to temperature. The losses vary as the square of the voltage, and as even a higher power of the temperature. Two and one-half times normal voltage causes an *initial* rate of increase of loss of over six times normal. It seems to me, therefore, that the suggestion of the author to increase both the value and duration of the voltage test is open to serious question because of the extreme abnormal duty which may be imposed on the insulation.

I agree with the author that not too much importance is to be attached to the power-factor test. The power factor involves not only the loss but also the capacity of the cable. Both these quantities are variable and you may have a cable that is open to suspicion from the standpoint of loss, but which may not be defective as based upon the power-factor test. Naturally if the power factor is abnormally low it is a good indication of trouble.

I offer the suggestion therefore that the best method for testing cables as they come through the factory is the measurement of the loss, and to as high a degree as possible, the distribution of the loss along the length of the cable. I am quite conscious of the difficulties of such a plan but I believe that everyone will agree that cables ultimately fail due to increasing local losses. Fortunately however the rate of increase of these local losses is initially not very rapid. A measurement of the total loss on a length of cable may completely screen the presence of a dan-

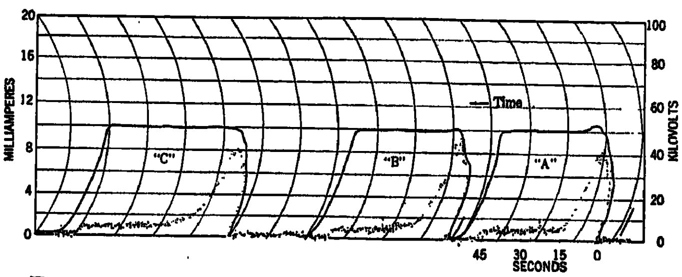

FIG. 7—RECORD OF KENOTRON TEST ON GOOD CABLE

gerous local loss. The ideal test therefore would include not only the total loss test but a measurement of the time rate of change of temperature at intervals over the entire reel length.

R. W. Atkinson: I shall limit my remarks to the method of measuring dielectric loss by the a-c. bridge. My earliest work in cable engineering was in measuring dielectric losses by a bridge method. I have retained an enthusiastic interest in that general type of measurement of dielectric loss because of the very great advantages it has over many other methods.

The bridge Mr. Lee has described undoubtedly accomplishes in a very satisfactory manner the measurements for which he has used it. We have been using a bridge method that has a greater range of adaptability than Mr. Lee has apparently expected from his method, and I think it may be of interest to give a very brief description of that method and a rough comparison of the differences between the two. This may be done briefly, because our method has recently been published in the February issue of the *Electric Journal* (1925, p. 58).

I shall describe our bridge in terms of Mr. Lee's Fig. 17. The left-hand side of Mr. Lee's bridge is shown as a condenser, the cable, in series with a resistance R_3. The right-hand side of the bridge is the condenser C_2, and a resistance R_4 shunted by a variable condenser. In our bridge, the variable condenser C_4 is absent, and we have a variable resistance in series with C_2. This of course is the Wien bridge which has been used by many and which was described in a Bureau of Standards bulletin about 20 years ago.

In order to make it available for high voltage, we have made one important change. Fig. 17 shows the cable and the high-voltage condenser connected together and the junction point connected to the source of high voltage. In our bridge we have opened this junction; the high voltage is connected to the cable, and a low voltage is connected to the right-hand side of the bridge. This requires an additional low-voltage transformer on this side, and necessitates a compensation, the value of which is determined by means of a standard or zero-loss condenser which is connected across the high-tension side of the line at all times that the cable sample is connected thereto, and which is connected in the bridge circuit in place of the unknown condenser by means fo a voltage-switching connection.

The general method of operating the bridge is to compensate by a special compensating circuit so that the loss is read correctly on the zero-loss condenser, the low-voltage switching connection is made, and the condenser or cable is then measured.

With the Wien bridge, the same voltage is connected to both halves of the bridge. If it is desired to use this method at high voltages, a great many difficulties are introduced because of the fact that the resistance in series with the condenser, C_2, is at high voltage. By opening the bridge and having high voltage on the cable only, these difficulties are avoided. With the Schering bridge described by Mr. Lee, some of these difficulties are avoided by eliminating the variable resistance at high voltage and using a variable condenser at low voltage as shown in Fig. 17.

I notice that Mr. Lee uses as much as 10,000 ohms in R_4. With the detecting circuit that we have, we are able to measure extremely small samples with a maximum resistance of 1000 ohms in the corresponding arm. If there is any capacity, as there usually is, between the right-hand corner of the bridge and ground, there will be an error in power factor, unless it is corrected, of exactly the same sort in either bridge, and of the same sort as the adjustment in power factor that is purposely made by Mr. Lee's condenser C_4. With 1000 ohms, which is the maximum resistance we use, that error is only 1/10 as much as it would be with 10,000 ohms. We have found it convenient and satisfactory to compensate for this error where it is desired to make accurate measurements. In general, the accuracy possible is to about 0.01 per cent in power factor. We often do not try to go this far, but it is quite possible even to exceed that degree of accuracy.

Another advantage of our arrangement over any other form of bridge that I have seen described is that it is suitable for the measurement of three-phase losses. The primary of one of our transformers is reversed in direction, and the bridge then becomes available in exactly the same way that a dynamometer-wattmeter is used in connection with the three-phase measurement of power factor of cables.

In conclusion, I would like to add that our resistances are independently variable, so that power-factor balance may be made without reference to the capacity balance which makes for a very wide range of adaptability of this method of measurement.

W. A. Del Mar: There are listed in Mr. Lee's paper five classes of tests, all of which are intended to indicate whether cable will be reliable in operation. These tests are all more or less accepted and standardized, in spite of the fact that no one can prove a very definite connection between most of them and the subsequent reliability of the cable.

Thus the insulation resistance test is known to be meaningless. The author has shown how an immense variation in the value of the insulation resistance occurs even in cable impregnated at the same time and in the same tank.

The factors which affect the insulation resistance of the cable are the resistivity of the oil, the resistivity of the paper, and the relative amounts of oil and paper.

Take the matter of the oil alone: The oil which is furnished

to the cable manufacturers is a very small matter to the oil refiners. To them lubricating oils, etc., are the principal things. They do not, as a general thing, pay very much attention to the very exacting requirements of the cable manufacturers. The result is that in purchasing oil with a specified minimum resistivity, the variation in resistivity is likely to be very great, sometimes of the order of 100 to 1. There is a similar situation with regard to the paper, but it is not such a big factor as the oil. Of course, the ratio of oil to paper also depends somewhat upon the tightness of the paper and the perfection of impregnation.

Hence, there is really no reason why one should get a uniform resistivity, and I rather suspect that it never is obtained except at very low values. You will have noted, in Mr. Davidson's curves, that the manufacturer who showed a uniform resistivity had a uniformly low one, whereas the one who had variable resistivity was generally high, and at one time it came down to, but not below, the other. I should rather suspect that, in the case of the manufacturer who had the low resistivity, there was either some unknown leak in the testing circuit or improperly dried or over-oxidized oil which rendered all the values constant.

With regard to the dielectric-strength test, there is considerable misunderstanding about that, as is to be seen from the arguments about the direct-voltage proof test. Experiments have been made which show that cable tested with direct voltage has a tendency to break down at about 2.4 times the voltage that causes it to fail with alternating voltage. From this fact it is commonly deduced that proof tests with direct voltage should be 2.4 times the proof tests with alternating voltage. I do not think that this is a logical deduction because we are not interested so much in the conditions at breakdown, as we are in applying a test which will not initiate injury, and we are yet lacking in information as to the relative voltages, direct and alternating, which initiate deterioration.

The dielectric-loss test has been shown to be useful in distinguishing between two classes of cable, namely, those in which accumulative heating occurs at low temperature and at high temperature; it has, however, been rather generally assumed that because a cable with 5 per cent power factor is better than one with 20 per cent, therefore a cable with one per cent power factor is better than one with 5 per cent.

I do not think that we have been justified in being so enthusiastic about very low power factors. There are experimental reasons, substantiated to a certain extent, by practical experience which indicate that very low power factors may be dangerous because of the inability of the cables to absorb transients. When a transient gets loose on a line, if there is no place for it to be absorbed, it is likely to cause damage. This effect is of little importance at 13,000 volts, but becomes of increasing importance as greater voltages are used.

Some definite information on this was published by G. W. Partridge in the *Electrical Review*, 1924, vol. 94, page 992. Some of the old Ferranti cables were on the same system as some modern cables. The old cables having high dielectric losses, especially in the joints had quite an advantage over the low-loss cables in the small number of pressure rises which occurred, especially at times of light load.

The capacitance test is in the same class as the megohm test; it does not mean anything in relation to the value of the cable. It is one of the survivals from the old days of telegraph-cable engineering.

The last of the five tests listed is the bending test, which seems to me a way of passing the sins of the cable installer and designer onto the cable manufacturer. The bending test seems to be in the nature of a confession on the part of the operating companies. It has stood and still stands in the way of progress in high-voltage cable design.

What do we need to assure the permanent success of a cable? We need five things:

First, ability to carry the normal voltage.

Second, ability to carry or dissipate probable transient voltages.

Third, chemical stability.

Fourth, high temperature of accumulative heating.

Five, adequate but not excessive flexibility.

The first and fourth problems have been fairly well solved—ability to carry the normal voltage and the making of the temperature of accumulative heating high—but the other three have not been fully solved. American cable manufacturers ought to be given an opportunity to concentrate upon solving those problems instead of on trying to get uniformity in megohms, certain values of capacity, flexibility and other things which do not mean anything at all.

E. W. Davis: There can be no doubt that the standard acceptance tests as applied to low-tension paper cables are not a safe basis for determining the quality and performance of paper cable for the higher working voltages. For low-tension cables, the mechanical and physical tests of the insulating materials are often of equal if not greater importance than the electrical tests. But as we pass from low-voltage to high-voltage cables, and especially as we approach the high operating voltages of 33,000 and 66,000 volts, the results of electrical tests assume tremendous importance. By a relatively short-time test, we must be able to determine the suitability of a paper cable for long-time operation. At the present time, we have much to find out about the electrical properties of a finished cable and the relation of these properties to the electrical properties of the various materials that go into the construction of the cable.

It has often been found that paper and impregnating compounds which show the best results for mechanical and physical properties have the lowest values for electrical properties.

While the insulation resistance of a cable as it is measured at the present time may be of doubtful value, yet the fault may not be in the method of measurement, but rather one of interpretation. We have not as yet been successful in establishing any relation between d-c. resistivity and power factor. On the other hand, when we have had paper cables that showed insulation resistance values below normal, we have invariably found the cause either in materials used or processes through which the cable was put during manufacture.

Long-time high-voltage tests are undoubtedly a necessary part of acceptance tests of to-day, to determine the suitability of a cable for high-voltage operation. On the other hand, the length of time and the proper magnitude of the voltage are unknown quantities.

Standarization of testing apparatus, especially apparatus for making dielectric-loss and power-factor measurements, is a matter that should receive attention. All the present methods give fairly satisfactory and consistant results at the higher temperatures, but results at low temperature (20 deg. cent. and 40 deg. cent.) vary considerably with the kind of apparatus used to make the tests. The Schering bridge offers, perhaps, a satisfactory solution for this problem.

The question of high-voltage d-c. testing is still open to discussion. As in high-voltage a-c. testing, the question of the proper magnitude of the voltage is of vital importance. The speaker has in mind a cable system that apparently withstood an a-c. test of 30,000 volts but failed when retested on 25,000 volts d-c. No satisfactory explanation for this phenomena has been offered.

C. L. Dawes: Mr. Lee in connection with the description of the Schering bridge mentions a three-stage audio-frequency amplifier which increases the sensitivity of the detector to the necessary value. At Harvard University, in connection with research being conducted under the N. E. L. A. Paper Cable Research Committee, we have developed an amplifier almost identical in design to the one used by Mr. Lee. In view of several recent queries concerning the design and use of such amplifiers,

Mr. Lee suggested that a description of such an amplifier might be of interest to the Institute at this time.

In some of our measurements we use a high-voltage bridge, two of whose arms have impedances of several megohms. For a detector we use a vibration galvanometer also of the iron-vane type, but unlike the one described by Mr. Lee, it has electromagnets and also it must be critically tuned. In order to function properly in our bridge the detector must have not only high power sensitivity but must also have a very high input impedance because of the high impedance of the bridge arms with which it is associated. We were unable to make the galvanometer itself fulfill these functions even when all the design factors were carried to the limit. Since the input impedance of a thermionic tube is very high, the use of such tubes in the detector circuit immediately suggested itself. Accordingly, a tube amplifier was designed and constructed, the details being worked out by Mr. G. H. Browning, a Research Fellow in Electrical Engineering.

At first, the amplifier consisted of seven stages but later as the galvanometer was improved the number was reduced to three. A diagram of the amplifier is shown in Fig. 8.

Owing to the low frequencies with which we work, it was found desirable to use resistance coupling between stages, rather than than transformer coupling. Special 102-D tubes were used for the two stages, since this type of tube has an amplification factor of 30 or 40, whereas the usual amplifier tube has an amplification factor of only 7 or 8.

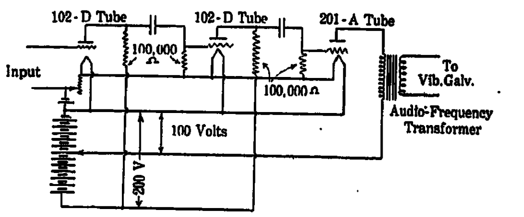

FIG. 8—AMPLIFIER FOR USE WITH GALVANOMETER

In the last stage a 201-A or similar tube is used because its plate-circuit resistance is substantially equal to the highest transformer input impedance which it was practicable to construct. This transformer between the last tube and the galvanometer is necessary in order to reduce the transition loss between the tube and the galvanometer, the galvanometer impedance being only of the order of 1500 ohms at 60 cycles.

This amplifier theoretically gives an amplification of 3000, and rough tests have shown it actually to be approximately 1000. Although at first sight this amplifier appears simple, it was necessary to overcome many difficulties before it became practicable. Considerable experimenting was necessary to determine the proper relation between grid condenser and the resistance of the grid leak. The amplifier is very sensitive to magnetic and electrostatic fields and hence must be carefully shielded from both. The shielding must be sufficient distance from the tubes to prevent capacitive feed back. The input lead and the galvanometer lead must be twisted and surrounded by grounded sheaths, and yet the capacitance between these two sets of leads must be very low or feed back will result. With a high-impedance bridge, such as we use, it is absolutely necessary to bring the two input leads to ground potential, as otherwise capacitance e. m. fs. between these leads permit stray currents to flow in the bridge, resulting in a false balance.

C. F. Hanson: At the bottom of page 112 in Mr. Lee's paper he makes the statement that three-phase power factor may be taken as the average of the power factor obtained with connection B at voltage E, and the power factor obtained with connection A at voltage $E/\sqrt{3}$. I wish to say that my experience confirms that statement.

At the bottom of page 113, Mr. Lee suggests that connections A and B in Fig. 20 be standardized. These connections require single-phase testing and consequently four different connections on the cable are necessary to obtain the power factor in various parts of the insulation. To make these four different connections consumes a considerable amount of time. I should prefer three-phase testing because only one connection has to be made. The transfer of the measuring instruments from one phase to another can be done with properly designed air switches at ground potential. The time saved and the convenience gained is desired from a production point of view.

If the ground on the sheath, connection B, Fig. 20, is removed and the sheath, instead, is connected to the middle point of the high-tension winding of the testing transformer, the current flowing in the wire connected to the lone conductor of the cable will be fairly accurately the three-phase charging current of the cable at voltage E. The three-phase dielectric power loss may accordingly be obtained with only one single-phase reading. Obviously, the lead sheath of the cable needs to be insulated from ground in order that the measuring instruments may be at ground potential.

James A. Duncan: I was very glad to note the remark made this morning by a cable manufacturer's representative to the effect that the manufacturers are now keeping an eye open to the possibility of impurities entering the cable in the oil used. I have in my hand a sample of paper given me as a fair representative piece of what had been put into a number of reels of cable made by a certain manufacturer for the Brooklyn Edison Company. This piece of paper has a resistance of less than an ohm and an average dielectric constant of infinity and a power factor which I have not measured, but which is probably somewhat excessive for so-called "insulation."

In this connection it seems well to consider some of the impurities which certainly exist in cable insulation and some of the reactions which are known to take place with such materials.

Ozone, for instance, causes a number of reactions to take place in hydrocarbon oil. Messrs. E. W. Blair, T. S. Wheeler, and W. Ledburry report in the September 1924 *Journal* of the Society of Chemical Industry an experiment in which ozone was bubbled through boiling normal saturated hexane. The formation of formaldehyde and acetaldehyde in relatively large quantities and of small amounts of water, carbon monoxide and carbon dioxide were definitely shown to result. Probably all the acids up to the hexoic and the hexyl hexoate ester are also formed.

It has long been known that certain metals catalyze the sludging process of mineral oils. Dr. Hans Staeger in the *Schweitz Elektrotechnischer Verein Bulletin* for March 1924 discusses the present state of our knowledge on this subject, including the very valuable part which his own experiments have contributed. He finds the presence of copper, brass, nickel, iron, zinc, tin, aluminum, lead, constantan and rheotan accelerate the acidulation of, and consequent sludge formation in, the oil. The magnitude of the effect varies, of course, with the metal and the time.

Four hundred per cent more sludge was formed in 1000 hours in a sample of oil in contact with copper than in a similar pure sample. The sludges usually consist of both soluble and insoluble (in oil) parts, and each of these parts may consist of both neutral and acid constituents. The oils oxidize the metals and in the case of copper, brass, zinc and lead actually partially dissolve them. Copper, zinc and lead have been detected in the sludges formed in the presence of these metals. The insoluble sludge with lead contained 20 per cent of the metal.

Both aluminum and iron have been detected in samples of the so-called "cheese" or wax formation found in three different cables. These impurities are easily traceable to the paper used in cable manufacture. The aluminum presumably goes into the paper in the form of alum, which is used as a size. I have detected it in five samples of paper, but failed to detect it in fibre just before it enters the last beating process in the paper manufacture.

The form or amount of aluminum in the "cheese" has not yet been determined. Samples of cable paper from every cable-paper mill and from every cable factory in the country have failed to produce so much as a square millimeter in which the presence of iron cannot be detected. The chemical test for iron is extremely sensitive and it is very possible that one might detect chemically iron occurring in such small quantities as to be of no importance.

To avoid making this mistake, I have also applied a physical test by means of a magnet.

It is astounding to me that the cable manufacturers are using "insulating" paper which contains pieces of iron of sufficiently large dimensions to be picked up with a magnet. I have several pieces of paper with such spots of iron in them that it is possible to ring a door bell in a battery circuit including two flat electrodes "insulated" from one another by the paper.

Aside from the conductivity of the iron, it is extremely objectionable for another reason. One could hardly imagine a better situation for the production of ionization than a region bespeckled with rough bits of metallic iron, unless it were bits of copper.

Suppose we think of a cable as consisting of a very large number of small unit lengths and think of each of these units as consisting of a condenser and a resistance in parallel. These units will then all be connected in parallel. If we admit that, for reasons outlined above or otherwise, one of these units may be very different from the average of the rest. Suppose, for example, its conductivity is very high and its capacity either high or low. It then becomes immediately obvious why one would not expect the average resistance and the average capacity, dielectric constant, or any quantity depending upon the capacity to bear any single relation to each other, for the reason that when in parallel capacities add arithmetically, while resistances add reciprocally. One very low resistance may be the predominant factor in a parallel connection, while a low capacity would offset the average for the set very slightly.

For example, in a cable supposedly divided into a thousand equal units, if one unit were altered so as to have one-thousandth the resistance and one-thousandth the capacity of its normal amount, the average capacity would be altered 0.1 per cent, while the average resistance as determined by measuring the whole cable would change approximately 50 per cent.

It seems to me, therefore, that the resistance ought to be one of our best tests because it can not be uniformly high unless every individual element of the cable is at least fairly high. In testing a length of cable in the ground power-factor measurements might not distinguish between a uniformly poor cable and one merely containing bad spots, while a kenetron test, for instance, will burn through the weakest element of a spotty cable.

Herman Halperin: Near the beginning of the paper it is stated that "the best samples of three-conductor cable for 33 kv. have a breakdown voltage on short-time tests of 200 kv." Three different makes of this kind of cable purchased by the Commonwealth Edison Company have had breakdown voltages of over 200 kv., and on one make the maximum voltage was 315 kv. In the latter case the average breakdown on about five random samples was almost 300 kv. between conductors. These data are quite different from the figure given by Mr. Lee.

The average gradient obtained in these samples was over 400 volts per mil on one sample and over 600 volts per mil on the other sample, which values are about double the value of 250 volts per mil given by Mr. Lee as being about the best obtainable.

The cable insulation is a very important part in potheads and even though we have trouble with failures in the crotch on samples, it appears to me that the voltages obtained by breakdowns in the potheads are an indication of the quality of the cable. For instance, in tests on seven makes of three-conductor, 33-kv. cable in Chicago and at the factories, we have found that old deteriorated 33-kv. cable might fail in the crotch at 125 kv., while better or new cables of the same make would fail at about 180 kv. Then the new samples of the three makes of 33-kv. cable previously mentioned withstood tests of over 200 or 300 kv., after which they had failures in the potheads.

The suggested increase of 20 per cent for full-reel high-voltage tests on three-conductor cables at the factory, seems to be insufficient. We have tested about twenty lengths of various makes of 200-ft. section of 12-kv. three-conductor cable and found that some cable which had been in service and failed due to defects in the cable, has withstood tests of 46 kv. for one, two, or three hours. These lengths had previously withstood factory tests of 30 kv. for five minutes. Increasing that factory test to 36 kv. would have easily allowed considerable unsatisfactory cable to pass the requirements. An increase of 50 per cent is more reasonable to insure satisfactory cable, but even then some unsatisfactory cable might pass this test. It is for this reason that it is necessary to have the other tests in order that all the tests together will insure cable which will be satisfactory for operation.

For instance, as pointed out in his Figs. 19 and 23, which show the variation of power factor with voltage at room temperature, if the increase in power factor is small the cable is superior to "poor cable" which has the large increase in power factor shown in Fig. 23.

On the twelfth page a large number of factors are enumerated for the d-c. to a-c. ratio, and a conclusion is given that each individual case must be considered separately. This would result in a complicated set of formulas which would probably be impracticable. It appears to me that the ratio could be changed on the basis of some major considerations, such as temperature and thickness of insulation.

Apparently the ratio now being used, that is, 2.4, is conservatively low because Mr. Lee's tests show that the proper ratio is 3.0 for cable with insulation for 12-kv. service, and he states the ratio increased for larger thicknesses.

S. J. Rosch: The author describes for the object of his paper "an attempt to devise a method of testing high-voltage cable, which will determine its operating characteristics in advance of its installation."

In looking through this paper, I find a series of statements of present practise accompanied by a series of statements showing the results of certain tests made by the company with which the author is associated. I fail however to see any real constructive criticism as to what values should be used as criteria for the determination of the suitability of cable before it leaves the factory. I am not wholly in sympathy with the author's conclusions and the discussion I shall present, is an attempt to clarify some of the points which in my opinion the author's paper has failed to do.

Of all the mysteries presented to the electrical engineer for solution, the most baffling in my estimation has been the impregnated paper cable. It seems to me that the greatest cause for this mystery has been provided by the cable purchaser. To amplify this statement, let me say that the greatest cause for the lack of a proper understanding of cable phenomena has been the vast multitude of specifications supplied by various purchasers for high tension cable.

Take for example a cable for 13-kv. service. Specifications for this type of cable range from a wall of 9/64 in. on the conductors and 5/64 in. on the belt to 14/64 in. on the conductors and 8/64 in. on the belt. One can readily conceive that of two cables made of the same materials under the same routine, the cable with the heavier wall of insulation will certainly have the greater factor of safety. If, therefore, we start comparing the operating performance of 13-kv. cables of several utilities, we find ourselves comparing cables which should be alike, but which are absolutely foreign to each other.

Until the present time, the purchaser has been specifying every criterion for the determination of cable quality, but apparently

the best of these specifications have been unable to pick out defective material. I believe it would be well to pause for a moment to see if we can trace some of the underlying causes for this fact. The testing of a cable generally consists of two phases, namely,

(a) Tests made on the full cable length.

(b) Tests made on a sample of this cable.

Right here in my opinion is the crux of the situation for there has been entirely too much testing of the samples, and not enough of the cables. Look through your N. E. L. A. reports or the proceedings of the A. I. E. E. and you will find that although cables intended for 33-kv. service have failed after only three months of service, nevertheless samples which were supposed to be representative of these cables, could not be broken down at 230,000 volts. Furthermore cables whose samples did not break down at these high values, are operating without any difficulty whatsoever. What inferences can we draw from the above? The ones that I would draw are as follows:

1. That the dielectric strength of a sample is not to be taken as indicative of the dielectric strength of the cable.

2. That there may exist a compound which has high initial values, but which deteriorates rapidly under actual service conditions.

3. That there may exist a compound which has moderate initial value, but which remains practically constant for a period of years.

If what I say be true, the solution must be obvious. The standards by which a cable is measured must be revised. In revising these standards, we must bear in mind that it is the cable and not the sample which is to be placed underground. We must therefore begin studying the cable more closely and look to the sample to give us only such additional information as cannot be revealed by the cable. I shall now take up the various tests in their order.

Full-Reel High-Voltage Test

Until such time as the thicknesses of insulation for various working voltages are standardized, I believe that the following test values should be used.

For cables up to 15 kv. the test voltage should be either 100 volts per mil or two and a half times the rated voltage, the larger of the two values being the one to be used for the test voltage. The voltage should be applied for five minutes. For cables rated at 16 kv. to 25 kv., the test voltage should be either 125 volts per mil or two and a half times the rated voltage, the larger of the two values being the one to be used. The voltage should be applied for five minutes. For cables rated at 26 kv. and over, the test voltage should be computed on the same basis as that prescribed for the 16- to 25-kv. class except that the period of application be fifteen minutes instead of five.

On cables rated above 15 kv. one reel out of every ten should receive an additional test of twice the working voltage applied for one hour. I consider all of the above tests as the best yet advocated for the determination of high-quality long-endurance cable. The values advanced are not high enough to overstress the insulation, and yet provide a very considerable factor of assurance that the cable will stand up under the rated service conditions. The last test, namely, the one applied for one hour to one out of every ten reels, is one of the most important in that it gives a fair idea of the endurance ability of the cable in general. Choosing one reel out of every ten, is almost the same as giving each reel an endurance test, since the reel selected, will practically represent the condition of each particular batch of cable as it is manufactured.

Insulation Resistance

The subject of insulation resistance has always been more or less of a joke in the paper-insulated cable industry. Every attempt at some form of standardized specification has recognized this fact in one way or another. In the Report of the Underground Systems Committee of the N. E. L. A. of 1922, embodying specifications for Impregnated Paper-Insulated Cable, under section 8, the first paragraph reads as follows, "The insulation resistance in a quantity having little significance except as a guide to the uniformity of the product; that is to say, a cable of high insulation resistance may be no better than one of comparatively low insulation resistance. Any important variation revealed by test should be investigated with a view to ascertaining whether the particular length has undesirable characteristics."

On page 105 in Mr. Lee's paper appears the following statement "Although measurements of insulation resistance have been made on cables for years, and are still being made, the results as determined by the standardized procedure are of doubtful value as being indicative of the suitability or unsuitability of cable for use." Mr. Lee further points out that in one case on thirteen lengths of 33-kv. cable manufactured during a three-months' period, the ratio of insulation resistance between the highest and lowest cable was as five to one. He goes one step further by saying that in another case, on eight reels of 22-kv. cable made from the same paper, treated in the same compound, in the same tank at the same time, the ratio of insulation resistance between the highest and lowest cables was as four to one, and yet there had never been any evidence to prove that one reel of cable was any better than the other. It is practically impossible in the light of our present knowledge to attempt to explain the cause for the variation in the last mentioned case because apparently the conditions were identical throughout.

In spite of all of the foregoing, inspectors have rejected cables for a lesser variation in insulation resistance, simply because the cable manufacturer has been unable to expound some theory that would plausibly explain away these variations. On page 306 of the *Electrical World* issue of February 7th, 1925, under an article entitled "Progress in High-Voltage Cable Manufacture," various causes are cited that led one large cable purchaser to reject cables in 1924 and among these is listed insulation resistance. I am citing this case to show that although insulation resistance has not been considered of importance in determining high-quality cable, nevertheless, purchasers have used the results of this test to reject cables.

If insulation resistance as now measured is meaningless, is there some method for making this test that will prove of greater value? I believe there is.

Fig. 9 shown here represents the present method of making insulation-resistance measurements, namely, taking each conductor against the other two and the sheath which in turn is connected to earth. The faultiness of this method lies in the fact that we do not get the resistance of the various leakage paths separately, but collectively.

Fig. 10d shows conductor No. 1 against No. 2, No. 3 and sheath and ground as in the ordinary method. Fig. 10e shows conductor No. 2 against No. 3 using both poles of the source of d-c. potential and giving an absolute resistance of the field between conductors. Fig. 10f shows conductor No. 3 against the sheath and ground which enables us to explore the field between one conductor and the sheath. Fig. 10g shows conductors 1, 2 and 3 against the sheath which in turn enables us to explore the entire belt insulation.

This method uses only one additional test per reel of cable. It is not new, having been used as far back as 1910 to the speaker's recollection, but it certainly tends to reveal considerably more concerning the leakage paths in a cable than the one shown in Fig. 9 which is in general use at the present time. Furthermore, tests *d, e, f* and *g* have a further value in that they

should bear a definite relationship to one another. For example, if the value of insulation resistance obtained on

d.................. = 100 per cent
Test *e* should be approximately = 160 per cent
Test *f* " " " = 125 per cent
Test *g* " " " = 70 per cent

With the different types of compounds in vogue today, where the compound used might be a mixture of some solid substance combined with a very fluid oil, the relationship between these tests might vary, but the same rate of variation should exist for that particular manufacturer's cables. There is one other point I would like to advance, that in the case where the insulation resistance appears to be low, the best way to determine whether it really is low, is to submit it to a three-minute electrification test. It is obvious that a conductor having a weak spot in its

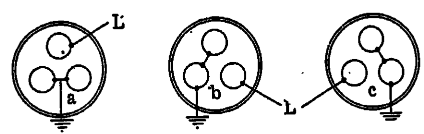

FIG. 9—PRESENT METHOD OF MEASURING INSULATION RESISTANCE

insulation cannot have the same rate of electrification for three minutes as a good conductor.

POWER FACTOR, DIELECTRIC LOSS AND IONIZATION

One of the largest cable purchasers in this country recently reviewed the results of tests made during 1924 on a very large order of 13-kv. cable. This order was divided among six different cable manufacturers, the entire order having been inspected by what is undoubtedly the best testing organization in the country. This purchaser attempted to rate the six cable manufacturers according to the tests obtained, with the following results:

The manufacturer who had the greatest number of wrinkles in his insulation, the greatest number of creases in his paper, in short the poorest mechanical conditions in his cable, this same manufacturer who had the poorest impregnated cable, the lowest dielectric breakdown on a hot sample, the lowest dielectric breakdown on a cold sample after bending, this same manufacturer nevertheless, had the best ionization characteristics, namely, the smallest variation in power factor between stresses of 20 and 100 volts per mil, and also had the very lowest dielectric loss of all the cables manufactured for this purchaser.

Let me bring up one more point. The company with which the speaker is associated, in 1900 manufactured and installed the first 25,000-volt cable ever made in this country. Tests made recently on samples of this cable, showed that it had power factor of about 30 to 40 per cent at 100 deg. cent., that it would fail miserably on the bending test at − 10 deg. cent., and nevertheless this cable is operating to this very day with a smaller number of proportionate failures than any cable of similar design manufactured within the past ten years. One must necessarily ask the question as to how important are ionization, dielectric loss and power factor in the pre-determination of the suitability of a particular cable for a certain definite service.

In Fig. 24 of Mr. Lee's paper, sample No. 3 which had the highest initial power factor of all four samples nevertheless stood up after 600 hours under the particular life test with a much smaller variation in power factor than the three others which had the lower initial values. All of the above in my estimation points clearly to the fact that power factor, dielectric loss and ionization have very little effect on the dielectric strength of a cable. In other words, a high power factor may or may not be accompanied by a high dielectric strength or vice versa. It seems to me that in the past, we have been a bit too rash in condemning certain types of cables, and overrating others on the basis of test performance on dielectric loss, power factor and ionization. In the paragraph immediately above Fig. 24 Mr. Lee states "The Effectiveness of the power factor-voltage test to separate poor cable from good will have to be determined largely from experience." I am in absolute sympathy with Mr. Lee on that score and believe that although these measurements should not be discouraged, they should not be wholly used as a means for picking good cables from bad.

There is no question that of two cables, the one having the lower dielectric loss, will have the higher current-carrying capacity. Let us confine ourselves therefore, to a reasonable value of power factors say a maximum of 5 per cent at 80 deg. cent. and 10 per cent at 100 deg. cent. and not attempt to try to make cables as perfect condensers. I can see no valid objection to any series of tests, but I do object to setting up these tests as a criterion, when their value as such has not been substantiated to the satisfaction of all concerned.

SAMPLES

A 10-ft. sample cut from the end of a reel of cable can be used to reveal the following:

(a) It may show that the cable which it represents can stand a bending test, but it cannot reveal whether or not a 10-ft. sample cut from the other end of the cable, will be as good as the first in meeting the bending test.

(b) It may show that the cable which it represents has a high dielectric breakdown strength at 85 deg. cent. but there is nothing to indicate that a 10-ft. sample cut from the middle of the cable would yield as high a value. Tests made by the speaker on a number of samples cut from cables of different makes, have shown considerable variation in dielectric strength although the particular samples were taken adjacent to each other.

(c) A 10-ft. sample can however be made to show the physical make-up of the cable, the method of application of the insulation, the tensile strength of the insulation after drying and impregnation as compared to average values obtained on the new treated paper. In this way the inspector can tell whether the extremely high insulation resistance or low power factor obtained on a certain cable has been obtained at the expense of over-drying the paper insulation or whether it is inherent in the materials of which the cable has been manufactured.

I would therefore recommend, in the absence of any better suggestions, that these physical and electrical tests on samples

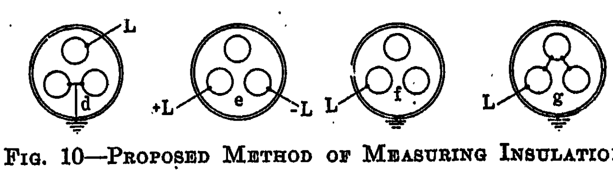

FIG. 10—PROPOSED METHOD OF MEASURING INSULATION RESISTANCE

be continued, always bearing in mind however, what the results obtained actually mean, and how far they can be applied in fixing cable quality.

RATIO OF D-C. TO A-C. IN D-C. HIGH-VOLTAGE TESTING OF CABLES

The subject of d-c. high-voltage testing has assured some importance of late and has also brought out some adverse criticism due to the arbitrary way in which a ratio of 2.4 has been fixed. We know that when testing a cable with alternating current the insulation is subjected to the maximum value of the voltage wave, we should therefore conclude that it would be able to stand an equivalent d-c. value which should be equal to 1.41 times the maximum a-c. value. Instead of this however,

we find that an arbitrary value of 2.4 times the root-mean-square a-c. voltage has been taken, which gives a value of 1.7 times the maximum a-c. value, or an increase of over 21 per cent over the theoretical d-c. value to be used. With all due respect to the splendid work done by investigators both here and abroad, we are nevertheless confronted with the fact that the true ratio does depend upon the nature and structure of the material, upon the thickness of the insulation, upon the temperature of the material, the size and shape of the conductors and the rate of application of the applied potential. In other words of two similarly designed cables, one may be impregnated with a compound where the ratio of d-c. to a-c. may be more than 2.4 but the other may be less than that value, nevertheless both cables will have equal life when passing the a-c. installed voltage. In fairness to all concerned until more knowledge has been obtained on the results of this form of testing, I would suggest that each manufacturer be permitted to specify the ratio of d-c. to a-c. with which he would allow his cables to be tested. I am a firm believer in d-c. high-voltage testing of cables and expect to see the day when this form of testing will supersede all others now in use on both the factory and installation tests of cables.

Proof Testing After Installation

I am a firm believer in proof testing of cables and would recommend that twice a year, cables be tested at one and a half times the rated voltage for fifteen minutes. This would not overstress the cable insulation, and yet would be sufficient to give assurance that the cable was going to operate satisfactorily. Above all, I would earnestly recommend that the utility engineers inform the cable manufacturer of the results obtained on his cable during service, so that he can link up the experience gained in the factory with that obtained under actual operating conditions.

I. M. Stein: I was very much interested in the new galvanometer mentioned in this paper, and am sorry Mr. Lee didn't say more about it. Very often in a-c. measurements, progress is retarded by lack of a suitable detector. When we have a new one, it may open up paths for further progress. I hope, therefore, Mr. Lee can tell us more about his new detector.

I had the pleasure of reading Mr. Atkinson's paper on Bridge Measurements of Dielectric Losses, which appeared in the February issue of the *Electric Journal.* The detector used by Mr. Atkinson is somewhat different from Mr. Lee's and doesn't require any amplification. I am wondering if Mr. Lee can tell us the advantage of his detector over the one used by Mr. Atkinson, which I believe is an iron-core dynamometer.

Bridge methods are known to have the advantage that the detector does not have to be calibrated, as in a bridge the detector is used as a null instrument. However, there are null methods which are not bridge methods, and one which is particularly suitable for measuring the power factor of cables is shown herewith in Fig. 11.

In the method shown an uncalibrated dynamometer serves as a null detector. This does not mean that there is no current flowing through one of the dynamometer coils, but rather that whatever current is flowing through one coil is in quadrature with that in the other.

The operation consists merely of rotating the inductometer until the dynamometer indicates zero; then the power factor is read directly from the scale of the inductometer, which may have a range of 0 to 5 per cent power factor.

A direct-reading power-factor scale is obtained by compensating (with a condenser and shunting resistance) for the residual inductance of the inductometer, the inductance of the potential coil and the phase angle of the potential transformer. The phase angle of the potential transformer may vary somewhat with the applied voltage, and one terminal of the compensating condenser is made adjustable to take care of this phase-angle change. The adjustable terminal of the condenser is coupled with a voltage scale, and by setting the index of this scale to the working voltage, the change in phase angle of the potential transformer is compensated automatically.

It is usual to make tests at a low voltage and at five times this voltage. A range-changing switch may be provided, which operates on both the series coil and the potential coil in such a way as to give the same sensitivity at low voltage and five times low voltage, still retaining the direct-reading power-factor scale.

Unlike Mr. Atkinson's bridge arrangement, this equipment is intended for one use only; namely, for measuring the power factor on full-reel lengths of cable. Because of the simplicity of the arrangement, together with the direct-reading power-factor scale, only a few seconds are required to make each measurement.

H. W. Eales: It is probably fair to say that the need of users still exceeds the capability of the makers taken in numbers. The data developed in the paper applies mainly to short lengths of land cables of moderate voltages, with maximum of 33-kv.

Departure from these values brings the user into untried fields. Testing equipment has been designed for short lengths and moderate voltages, the number of makers equipped to test cables of very high voltage and long lengths being very few. Those who use 66-kv. to 132-kv. cable have very little experience of others to go by.

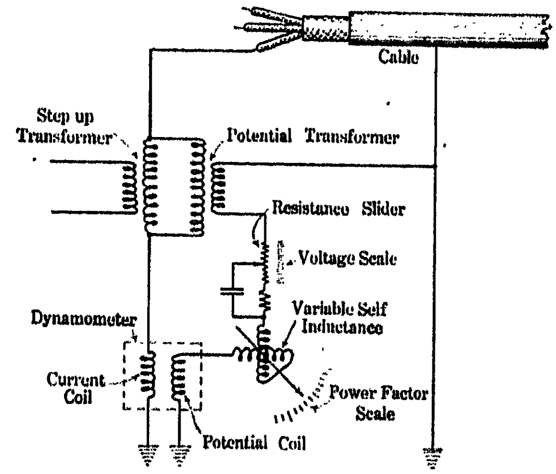

Fig. 11—Method of Measuring Power Factor of Cable

Fortunately, progress is made by trial. Unusual interest is therefore attached to the 66-kv. and 132-kv. installations at Cleveland and Schenectady respectively, and to the announced purchase of 35-kv. 350,000-cir. mil three-conductor sector submarine cables which are to be installed in the Mississippi River at St. Louis, each of which will be made up in half-mile lengths without splice, or twice the length heretofore attempted for this class of cable.

R. W. Wieseman: When alternating current is used for testing, the wave shape of the applied voltage must remain the same throughout the test if consistent and dependable results are to be obtained. The sine wave is universally recognized as the standard wave form of voltage.

In cable testing it is extremely important that the generator should have a sine wave of voltage at all conditions of load. The generator voltage wave must therefore be free from all harmonics, especially tooth ripples. Otherwise the condenser capacity of the cable will greatly amplify the harmonics and the current wave will be saw-tooth instead of smooth, also additional dielectric losses will occur in the cable insulation. Furthermore, the generator must be stable throughout a wide voltage range when delivering large leading-zero-power-factor currents.

The testing generator shown by Fig. 4 of the paper was de-

signed especially for cable testing. The excellent voltage-wave characteristics shown by Fig. 5, and by the table at the bottom of page 106, were obtained by a suitable choice of armature winding pitch and distribution. This type of generator can be short-circuited either three-phase or single-phase and the full normal-load current wave at zero voltage has a maximum deviation factor of only 1.5 per cent. A generator which has these characteristics is ideal for testing purposes.

P. W. Sothman: "Too many of our high-tension cables are coming up to specification but not up to expectation." In other words we test the proposed cable and find everything o. k. We put it into service and too large a percentage fails under operation, which shows that there is something wrong.

Quite often the consulting engineer is called into conference too late, *i. e.*, after trouble occurred which in many instances could have been avoided.

The characteristics which we know and have observed in overhead high-tension transmission lines are familiar to us. In underground cables the phenomena are often 10 to 20 times more severe in destructive effect than in overhead systems, due to the characteristics of a cable. The result of our work on overhead lines has been the design of surge-preventing or surge-reducing apparatus. In the underground systems nothing or very little has been done to make the over-potentials, surges, etc., harmless or to prevent them.

There is much room for closer cooperation between the cable manufacturer and the utility company. Each one should place the observed facts open to constructive criticism and discussion.

We learned from one discussor that the oil companies do not pay any too great attention to the quality of the raw materials delivered. The same is true of the paper manufacturers, which simply means that the cable manufacturers must treat the materials individually and cannot adopt a standard method. The treatment should be continued until tests prove the material has been made satisfactory.

The company which installs the cable also should do its part. I have seen cable pulled into duct in such a manner that the force applied was greater than the elastic limit of the cable. I even know of cases where new cables were pulled in two. Furthermore, the tools used in the laying of cables are often obsolete and not suitable. For instance, the pulleys through which the cables move often have flat grooves instead of grooves to fit the contours of the cable. The bearings in the guide pulleys are often common holes, whereas roller bearings would reduce the friction greatly and should be used. See Fig. 12. The consequence of such flat grooves is that the cable is damaged and will not perform properly.

Personally, I do not approve of our cable specifications in respect to dictating the minimum insulating thickness, as this does not spur the cable manufacturers to use the very best insulating material. My suggestion would be to say, "The insulation or distance between conductors should not be more than so many thousandths of an inch."

G. E. Luke (by letter): It is noted that the insulation resistance of the cable is measured after a one-minute application of the d-c. voltage. It is well known that such a reading is not the true ohmic resistance, since the transient current measured at this time interval may be several times greater than the final steady value due to the absorption effect. Hence, in a general way, this current will be a function of the power factor as shown in Figs. 1 and 2. It is suggested that this apparent resistance be measured at the end of a five or ten minute period since such a value would be very near to the true resistance; also a reading should be taken 10 to 30 seconds after voltage application. The ratio of the short-time reading to the long-time reading would be approximately the ratio of the absorption current to the true conduction current and would be an indication of the condition of the cable. (See paper by Phelps and Tanzer, A. I. E. E., 1923, p. 54.)

In the conclusions (1) the author says "Values of insulation resistance as heretofore obtained are of doubtful worth as a means for distinguishing between satisfactory and unsatisfactory cable." It is agreed that such measurements will not classify average-grade and high-grade cable but should in some cases be able to distinguish between defective cables and passable cable. Thus in electric machines insulation resistance is taken before the high-voltage test in order to determine if the insulation is in condition to withstand the test. In such cases a very low resistance might indicate absorbed moisture or defective insulation.

In reading the data given in Figs. 10 to 13 on breakdown voltage-time curves at various temperatures, nothing could be found as to whether the temperature specified applied to the conductor, sheath or ambient air. For example, in Fig. 10 the temperature given was 25 deg. A voltage application of several times normal value will result in an elevation of the insulation temperature, hence it would be interesting to know the maximum insulation temperature for each test. This raises the question—Was the insulation breakdown due to high temperatures, chemical action, or to a dielectric puncture at normal temperatures? Of course, combined effects may be present; however, it is suspected that the thermal action dominates.

It is suggested that the longer life of the cable on endurance run at 50 deg. cent. compared to 25 deg. cent. may be due to the lower viscosity of the impregnating compound and hence better distribution. If this is true, then it is an argument for the more fluid compounds.

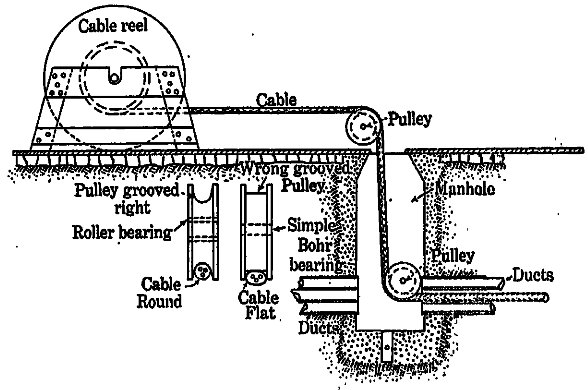

Fig. 12—Use of Pulleys for Pulling Cable

Regarding the nature of the failures as outlined in two classes, (1) those where the puncture is radial and clean, and (2) those where the puncture is accompanied by deterioration, we also experience similar failures in armature coils. Those of the first class are usually due to a sudden application of a high voltage, while those of the second type are due to a long-time application where cumulative chemical and thermal action may dominate. Failures of the first type may be due to voltage impulses of very short duration and may be classed as fundamental dielectric punctures.

The efforts to determine the "germ" of a breakdown by exploring the sheath temperatures were interesting. It would be of value to know the maximum internal insulation temperature corresponding to the sheath temperatures measured. This "hot-spot" correction would be considerably greater were it not for the equilization of the temperatures due to the conduction of heat along the heavy lead sheath.

The tree designs on some of the paper due to corona in the air spaces emphasize the fact that good impregnation is desirable. This raises the question—What per cent of the free space is filled in practise? In other words, what per cent of air space can be expected in such cables

E. S. Lee: It has been a great pleasure to have such excel-

lent data presented to us in the discussion of this paper, particularly since the general conclusions of the paper are substantiated thereby.

Dr. Whitehead asks whether the curves between the power factor and insulation resistance give the order in which the observations were taken. The answer is, no. These data are plotted with the values of insulation resistance in ascending order, the lowest value first. Power factor values are plotted corresponding with the respective values of insulation resistance. This method of plotting (in order of ascending or descending values without regard to time) has considerable merit in cases where a large number of observations of a single quantity are obtained, since a graphic picture of the variation of the observations is clearly presented.

Regarding insulation resistance, I recognize, as all do, that we would like to have every piece of similar cable that comes from the tanks to be of the same insulation resistance, and that we would like to have that value just exactly the right value. At the expense of having it thought that perhaps our processes were not absolutely controlled, I have taken data to show the wide variation that may exist, and data have been presented by others showing this same wide variation in other cables. But what I had hoped to hear in discussion was that where the variation is large the cable will not live, and that where the variation is small the cable will live. There does not seem to be evidence to substantiate such a conclusion, however, which is all that I have tried to say in my paper.

As regards dielectric strength, I think there is nothing else to say. I have merely suggested that present standardized values should be increased. In the absence of a rational basis for establishing values of proof-test voltage, we will have to arrive at such values through conference, and modify them from experience. Referring to Figs. 10 to 13, the temperatures indicated are ambient. Mr. Davidson's suggestion that the data are not extensive enough to justify lines being drawn through the points with precision is quite right. The lines might better have been drawn dotted. The purpose of the curves was to give a picture of the relation of the present standardized test voltages, rated voltages, and breakdown voltages.

Mr. Atkinson's scheme for adapting his bridge for measurement of power factor and dielectric power loss from three-phase supply is quite unique and appears to have great merit. It is quite possible that the Schering bridge may be somewhat similarly modified to allow of such measurement therewith.

Regarding Mr. Stein's question about the dynamometer, it is a remarkably fine instrument. It requires no tuning. We use it down in the factory. It stands on the table, with men working all around it, the crane running about ten or fifteen feet above it, and still it does its job. It is oil-filled. Its sensitivity is about two-fifths, I calculate, of what Mr. Atkinson gives in his paper, which is, I think, about 50 microamperes per millimeter. This would then be about 20 microamperes per millimeter. This is with the amplification, which is 100 or 125. So it isn't quite as sensitive as Mr. Atkinson's, but it does the work.

Mr. L. T. Robinson has often said: "When we used to have only one bridge in the laboratory, we could measure the resistance of anything very accurately and get its value. But when we got two bridges" he said, "we couldn't do quite as good a job in measuring the resistance."

Well, we have had the dynamometer wattmeter method and used it. Mr. Shanklin operated with it and got good results. So when we applied the Shering bridge we said, "Let's see what we shall get" and the paper shows you what we have been able to get under probably the best conditions. I show differences there in power factor of 0.002, that is, 0.2 per cent power factor, and in one case the bridge is high and in the other case the dynamometer is high, which is good. We have had results that vary as much as 0.4 per cent between the two measurements, but nothing more than that. So I feel that we are in very good agreement, and this little galvanometer does it.

As regards testing installed cable, I am gratified to have Mr. Davidson bring data which show that he has arranged that all of the d-c. tests which will be made on his system will be available in graphic form, with the voltage and the current values shown together on one instrument chart. Thus a permanent record of what has occurred will be available for study, which should enable us to learn more of the value of the d-c. test.

As regards the d-c. to a-c. breakdown voltage ratio, I wish again to call attention to the fact that while for any given breakdown voltage with a-c. there is no doubt a corresponding breakdown voltage with d-c., the relation between these two is dependent upon so many variables that each case has to be considered separately. Such a situation, therefore, requires caution in drawing conclusions relative thereto.

One more thing: We obtain the breakdown with d-c. and we obtain it with a-c., and we take the ratio. What we want, however, is a suitability test. That is, we want to put some kind of a test on the cable and not break the cable down. We want the cable to continue to live and be useful, but we want to know whether it is of such value that we ought to continue to let it live and be useful. That is the whole point.

The best we can do, is to obtain a proof-test voltage value from what data we have, and we have the breakdown ratio. If the breakdown ratio for a particular kind of insulation is 2.4 and we say that is the best we have, then we are using the best we have, though that may not be exactly what we want. We have to find out by experience. But if we will recognize how we arrive at the value of d-c. to a-c. ratio, that is, from breakdown values, and that we are applying it to a proof test, then I am sure any differences that may appear on the surface will easily be straightened out, and everybody will be in absolute agreement.

As to Mr. Roper's tabulation, such an idea has merit and it seems that if we can continue to cooperate in obtaining what is considered by all to be a fair rating of the various items presented in that tabulation, it will be a step in the right direction. I want to note in this connection, however, that we are rating a good many properties that are not directly measurable. That is, we look at a thing, and we feel of it, or something of that kind, and then give it a rating. We would like, however, to be able to measure it in some way or another, which is what we must strive to do, as has been brought out by several discussors.

In a lighter vein, if Mr. Duncan's remarks are recalled, it might be thought perhaps that the title of this paper should have been "Testing High Tension Iron-Impregnated-Paper-Insulated Lead-Covered Cable." But with all due respect to the paper samples that he may have, and with all due respect to what has been said about the variations in oil, I want to say that the folks who supply the compound and the folks who supply the paper are interested in supplying the cable manufacturers with material that is as uniform as possible. And, in turn, the cable manufacturers are taking those materials, with what variations may exist therein, and are putting them through the necessary processes to produce long lengths of uniform insulation. A satisfactory means of determining the uniformity along a 500-ft. length of cable is not available however, since all the test methods now used (which do not destroy the cable) give a summation of the whole. One of the greatest needs of the art is a test method for determining the uniformity of insulation without harming it. Such is a subject for both industrial and collegiate research which ought to be actively pursued.

Predicting Central Station Demand and Output

BY FARLEY C. RALSTON[1]

Member, A. I. E. E.

Synopsis.—For relatively long-term predictions of central station demand and output, of the order of one year or more, the use of constant (or approximately constant) yearly percentages of growth is common and well understood. This method is frequently applied by straight, or nearly straight, line projection of the plot of past data on semi-logarithmic paper.

When the term of the prediction is less than one year, or when detailed estimates are required throughout any year, this method fails on account of the seasonal variations.

In this paper is investigated the nature of the seasonal variations in the daily load curve of the company with which the author is connected, as they affect the output and the peak demand.

The variation of the kilowatt-hour output is first analyzed: and it is found that, in a year fairly free from abnormal business conditions, a plot of the "normal midweek day" outputs on semi-logarithmic paper can well be rationalized to a curve whose components are an inclined straight line and a single-frequency sine curve.

This curve is represented analytically by the following equation:

$$y = J\, e^{kr}\, [\,1 + L \cos (0.986\, r - M\,)^{\circ}\,]$$

In this equation, e is the base of the natural system of logarithms: r, the number of days (positive or negative as the case may be) counted from a given zero date: and J, k, L and M are parameters to be determined for each curve.

Factors are included for determining the output on holidays, Sundays, Mondays and Saturdays, as compared with adjacent "normal" midweek days."

For short-term peak demand predictions, the method employed is to separate into three components that portion of the daily load curve beginning at 2.30 P. M. and ending one and one-half hours after sunset.

These three components are a constant "base load," an "afternoon block" and an "evening block."

In the formula;

$$z = A\, F\,(t) + B\, f\,(t) + C$$

expressing this condition, the maximum value of the afternoon block is designated A, the maximum value of the evening block B, and the value of the base load C.

Both F (t) and f (t) are found to be exponentials in the data used and their forms are given with methods of evaluating the parameters.

However, even this stage having been reached, the formula is still in awkward shape for obtaining, analytically, the solution most often desired,—namely, the value of the peak demand and the time at which it will occur.

A graphical construction is, therefore, developed which solves simultaneously for these two quantities. This solution is equally valid if the form of F (t) and that of f (t) are defined only graphically, it is not necessary that their analytical expression should be known, nor even that they should be analytical in character.

* * * * *

For a variety of reasons, predicting future conditions is frequently necessary in most commercial enterprises. As regards the peak demand and output of central station companies, the forecasts required fall into two distinct classes.

Long Term Predictions

The first class extends usually from one year or thereabouts to a number of years in the future, admitting of latitude in the estimation of values increasing with the length of view.

The methods of making forecasts of this class are quite simple and well understood, and will not be discussed here in detail.

It should be stated that the material of this paper is almost entirely taken from the records of the company with which the author has been connected for some time; but such data from other companies as has been available support the view that the theory developed is of very general application.

In the case of the company mentioned, and in most others, it is found that, taken over a number of years, the growth of the yearly kilowatt-hour output and the maximum yearly peak approximates very closely to a constant yearly percentage increase. For this type of load growth, plotting on semi-logarithmic paper is particularly suitable, since the plot then approximates to a straight line.

To estimate the growth for any period in the future, it is merely necessary to project this straight line as far as is required, allowing for such modifying conditions as may be anticipated. Fig. 1 shows the result of such a plot for the company mentioned above.

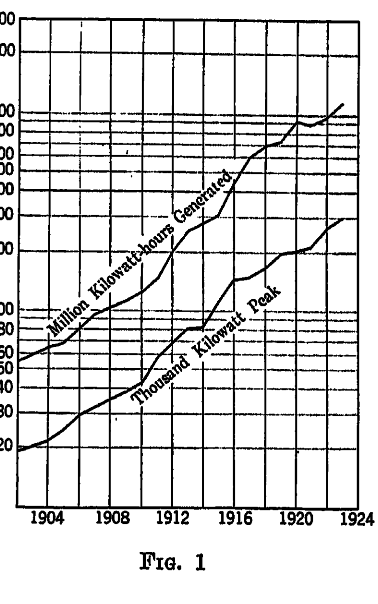

Fig. 1

If the growth does not approximate to a constant yearly percentage increase, it is only slightly more difficult to project the actual growth-curve, whatever its shape, into the future for the required period, allowing as before, for modifying conditions.

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SHORT TERM PREDICTIONS—GENERAL

The second class includes predictions of relatively short extension into the future, from one year or thereabouts down to a few hours. In this class of predictions, of course, considerably greater accuracy is desired.

To companies working under an operating budget system, it is important to predict the monthly outputs for each month of the budget year, in order that suitable allocation can be made among the individual stations and proper allowance made for fuel cost, etc. The prediction of daily peak demand for several months ahead is of value in arranging construction and major maintenance schedules. It is useful to a load dispatcher to know, on any given day, (in the middle of the afternoon, for instance) what peak demand he must expect that day, particularly so, if the day is unusually dark and stormy, making probable a high peak and preventing the direct use of the days immediately preceding as a guide.

In the following analysis the term "normal midweek day" will be used extensively. Tuesdays, Wednesdays, Thursdays and Fridays will be designated "normal

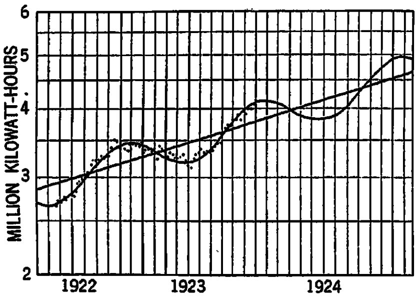

FIG. 2

midweek days;" omitting, however, any of these which happens to be a half holiday, a full holiday or the day after a full holiday. For instance, in the week containing Thanksgiving Day, there are only two "normal midweek days," Tuesday and Wednesday.

SHORT TERM PREDICTIONS—KILOWATT-HOUR OUTPUT

Probably everyone who has investigated the variation of central station output throughout the year has observed the periodic nature of this variation. On more or less rational grounds, based on the practically single-frequency harmonic variation of the principal cause of the output variation (the variation of the hours of daylight), the presence of a single-frequency harmonic component in the output variation would be suspected. But as output data are usually available by calendar months, the regular character of the periodicity is not apparent. Two factors account for this; first, the varying number of days per month, and second, the varying number (both among different months of the same year and among corresponding months of different years) of normal midweek days, Mondays, Saturdays, Sundays and holidays per month.

In eliminating the effect of these factors, the value of using only the normal midweek day outputs or, better still, the average of these for each week, is shown by Fig. 2. The points plotted on this chart indicate the average normal midweek day output for each week of 1923 and of the latter part of 1922. Semi-logarithmic paper has been used for this plot because it is natural to expect the growth during short periods of time to partake of the same exponential character as that over longer periods.

It is comparatively easy to insert by inspection an inclined base line having superimposed on it a sine curve which approximates the plotted points quite closely.

In inserting this base-line, it should be noted that its height and inclination are determined by the requirements that its alternate intersections with the group of plotted points must be exactly one year apart; also that the upper and lower lobes of the curve must be of equal heights. It is to give an additional point of intersection that the extension into the preceding year is made.

The form of the equation expressing this curve analytically is

$$y = J\,e^{kr}\,[\,1 + L\cos\,(0.986\,r - M\,)°\,] \qquad (1)$$

in which e is the base of the natural system of logarithms;

r, the number of days (positive or negative as the case may be) counted from a given zero date; and

J, k, L and M—parameters to be determined for each curve.

The coefficient 0.986 is the ratio of 360 (the number of degrees in a circumference) to 365.25 (the average number of days in a calendar year).

In the equation (1), the term $J\,e^{kr}$ represents the base line. When $r = 0$

$$J\,e^{kr} = J$$

In Fig. 2, if January 1, 1923, is taken as the zero date, J is equal to 3,190,000,—the ordinate of the base line at that date.

The value of k is given by the slope of the base line. For instance, on January 1, 1923, ($r_0 = 0$) $J\,e^{kr} =$ 3,190,000; on January 1, 1924, ($r_1 = 365$) $J\,e^{kr} =$ 3,800,000.

$$\frac{J\,e^{kr_1}}{J\,e^{kr_0}} = \frac{e^{365k}}{e^0} = e^{365k}$$

$$\frac{3{,}800{,}000}{3{,}190{,}000} = 1.192 = e^{0.176}$$

$$e^{365k} = e^{0.176}$$

$$k = \frac{0.176}{365} = .000482$$

M is equal to the number of days from the zero date to the spring intersection of the complete curve with the base line, less one fourth of 365.25; or to the number of days from the zero date to the fall intersection of the complete curve with the base line, less three-fourths of

365.25. In Fig. 2, M has been found to have the value 2.

Let r_2 be the value of r which makes the expression $(0.986\, r - M)$ equal to zero; and let y_2 be the corresponding value of y. Then

$$L = \frac{y_2}{J\, e^{k r_2}} - 1$$

In Fig. 2, $r_2 = 2.025$, and the date is January 3, 1923. On that date, $y = 3{,}460{,}000$ and $J\, e^{kr} = 3{,}193{,}000$; therefore,

$$L = \frac{3{,}460{,}000}{3{,}193{,}000} - 1 = 0.0836$$

The complete equation representing the curve of Fig. 2 is then

$$y = 3{,}190{,}000\, e^{0.000482r}\, [1 + 0.0836 \cos (0.986\, r - 2)°]$$

It would, of course, be possible to locate this curve and deduce the values of the parameters by strictly analytical methods, using least squares, etc.; but the labor of doing this would not be justified by the accuracy and consistency of the data.

If no change is expected in the rate of growth, the base-line and sine-curve are projected into the ensuing year with the slope unchanged. If, however, it is expected that other influences, not predictable from past performance, will affect the rate of growth, either upward or downward, the inclination of the base-line and consequently of the sine curve, must be modified accordingly. In either event, the ordinate of the projected curve at any point gives the value of the kilowatt hour output for an average normal midweek day at the date corresponding to the abscissa.

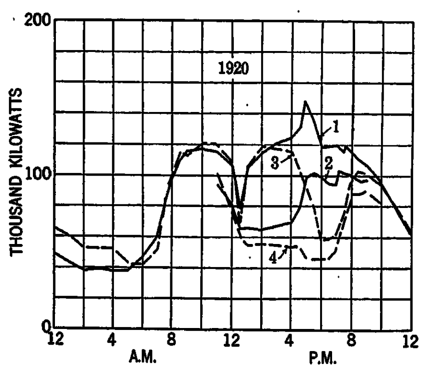

Fig. 3—Curve 1—Monday, Nov. 29
2—Saturday, Nov. 27
3—Thursday, May 27
4—Saturday, May 22

To derive the monthly outputs from the values for normal midweek day output throughout the year, it is necessary to determine the number of normal midweek days' output to which each week's output is equivalent. It has been found in the experience of the company mentioned above that the following relations hold quite closely among the days of any week:

Average of normal midweek days	1.00
Monday or day after full holiday	0 96
Saturday or half holiday	0.86
Sunday or full holiday	0.57

By combining these multipliers correctly for each week or fraction in a month, the total monthly output may be estimated.

The method here outlined may not give greatly superior results to purely empirical estimating by one thoroughly familiar with the characteristics of load variation for a number of years; but it is much more desirable in that it affords a logical analytical basis for estimating instead of placing dependence entirely upon a "trained guess."

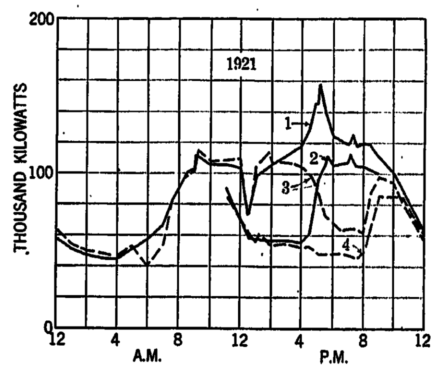

Fig. 4—Curve 1—Tuesday, Dec. 6
2—Saturday, Dec. 3
3—Friday, June 17
4—Saturday, June 18

Short Term Predictions—Peak Demand

The following analysis of daily load curves is valid only for systems whose seasonal load variation is unrestrained. For instance, it does not apply to the 25-cycle load of the company mentioned earlier, because the demand in this system is restricted by contract limitations; all demand above a definite value being carried by another system in parallel. It is applicable, however, to the 60-cycle system of this company and the accompanying illustrations are taken from load curves of this system.

The analysis is based on the separation of that portion of any daily load curve starting about 2:30 P. M. and ending about one and one half hours after sunset into three components—a constant "base load," an "afternoon block" and an "evening block." In Fig. 3 are shown typical summer and winter week day load curves for the year 1920, and portions of the load curves for adjacent Saturdays. The portions of the Saturday curves which resemble closely the week-day curves, are omitted for clearness. Figs. 4 and 5 show corresponding sets of curves for the years 1921 and 1922. In the summer curves of all these charts, the outlines of the upper parts of the afternoon and evening blocks are easily distinguishable, since the superposition of the blocks is of negligible effect, except in the lower parts

on week-days. The winter curves present greater difficulties; and it is particularly in reference to these that accurate predictions are usually of value.

Purely empirically, it has been found that the lowest value of an adjacent Saturday afternoon is suitable as a value of the constant base load for week days. Rationally this choice is not without justification, since on Saturday the afternoon block is practically negligible.

Fig. 6 shows typical winter load curves only for the year 1923, a week day and an adjacent Saturday. In this figure the separation of the week-day curves into three components has been made graphically, the base load (which will be designated C) being equal to 96,000 kw.; the maximum value of the afternoon block (which will be designated A) being 76,000 kw. and the maximum value of the evening block (which will be designated B) amounting to 105,000 kw. By inspection it is found that the highest combination of the afternoon and evening blocks which can be made occurs at 5 o'clock, when the afternoon block has the value of

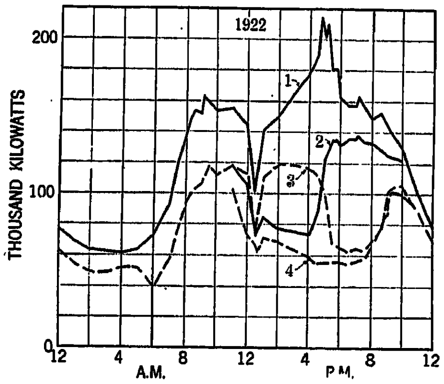

FIG. 5—CURVE 1—THURSDAY, DEC. 14
2—SATURDAY, DEC. 9
3—MONDAY, JUNE 26
4—SATURDAY, JUNE 24

56,000 kw. and the evening block the value 101,000 kw. The resultant of the three components is then 96,000 plus 56,000 plus 101,000 = 253,000 kw. The actual peak of this curve occurred at 4:57 P. M., and amounted to 250,000 kw.

For any given date, the portion of the daily load curve under consideration can be closely represented by an expression of the form

$$z = A\,F\,(t) + B\,f\,(t) + C \qquad (2)$$

in which t = hours P. M.

$F\,(t)$ is a function of t, whose analytical form is as yet undetermined, defining the shape of the afternoon load block; and similarly $f\,(t)$ with respect to the evening block. C = on a Saturday, the minimum kw. demand of the afternoon; on a week day, the minimum kw. demand of the preceding Saturday afternoon.

$$A = (\text{kw. demand at 2:30 P. M.}) - C$$

When the load curve is such that the value of the afternoon block 1.5 hours after sunset is negligible (on Saturdays through the entire year, and on week days from spring to early fall) B = (kw. demand 1.5 hours after sunset) $- C$. But the days, when the presence of an appreciable afternoon block component 1.5 hours after sunset masks the value of B, are just the days for which a knowledge of its value is most desired—namely, late fall and winter week days. For these days B must be determined in another manner, to be described later.

The demand at 2:30 P. M. has been chosen for determining A, because at that time complete recovery from

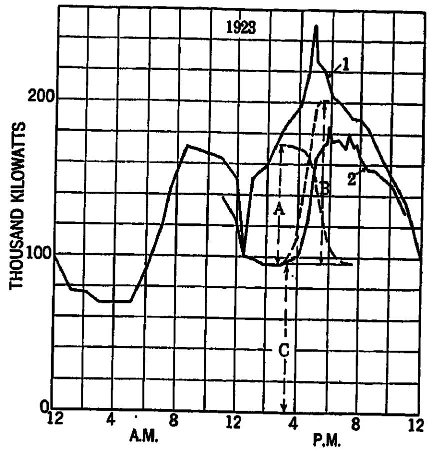

FIG. 6—CURVE 1—THURSDAY, DEC. 20
2—SATURDAY, DEC. 15

the noon dip has been made, and also the value of the evening block is negligible, even at the season of maximum overlap.

The choice of 1.5 hours after sunset as the time for determining the value of B was likewise guided by the fact that at that time the rise of the evening block to its maximum value is complete, and also, even in the

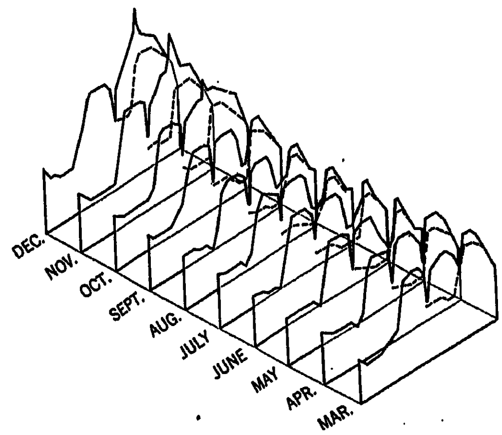

FIG. 7—TYPICAL WEEKDAY LOAD CURVES (SOLID), AND PORTIONS OF SATURDAY LOAD CURVES (DOTTED) FOR EACH MONTH FROM MARCH TO DECEMBER, 1923.

summer, no falling off from this maximum has occurred.

Fig. 7 shows the seasonal variation for the year 1923 in greater detail. It is a diagrammatic form of the Annual Load Relief Map, described in a paper by W. L. Robertson (TRANS. A. I. E. E., Vol. 36, p. 1073); using for the sake of clearness only a single pair of curves per month.

Inspection of Figs. 3, 4, 5, 6 and 7, conveys the impression that both functions, $F(t)$ for the afternoon drop-off and $f(t)$ for the evening pick-up, are exponential in character. This impression is confirmed if for each of these functions superposed curves for a large number of days, taken in various years and at various seasons, are plotted.

It is easy to derive comparable values of these functions, suitable for superposing, from the data available in the daily load curves. Division of the load curve values, $A\,F(t)$ and $B\,f(t)$, by A and B respectively, gives directly the corresponding values of $F(t)$ and $f(t)$, which may then be expressed either as decimals or in percentage for plotting.

It has been found that the composite plot of the afternoon block curves can be well represented by the curve of Fig. 8, for which equation is of the form

$$F(t) = e^{-[a(t-p)]^4}$$

in which p is the zero time at which the decrement commences; and a is the reciprocal of the number of hours from p to the time when the value of $F(t)$ is e^{-1} or 0.368, since when

$$e^{-[a(t-p)]^4} = e^{-1},\ [a(t-p)] = 1 \text{ or } a = \frac{1}{t-p}$$

The insertion of the particular values of p and a taken from Fig. 8 results in the equation

$$F(t) = e^{-[0.429(t-3.15)]^4}$$

It is found that the composite plot of the evening block curves is best fitted, not by a single curve, but by a family of similar curves, forming a band about 20 min. in width. The extreme curves of this band—those for a clear day and for a very dark day—are shown in Fig. 9.

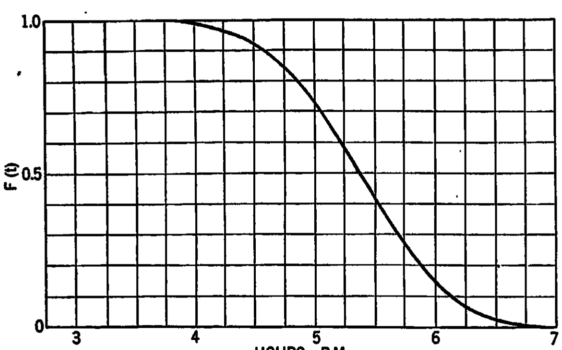

FIG. 8

For any condition of cloudiness between these extremes, a correspondingly intermediate curve would represent the evening block. The form of the equation of these curves is

$$f(t) = e^{-[b(h+q-t)]^2}$$

in which h is the time of sunset, hours P. M.; q is the number of hours from h to the zero time when the maximum value of the evening block has been reached; and b is the reciprocal of the number of hours from the time when the value of $f(t)$ is e^{-1} or 0.368 to the zero time, $h + q$.

The following particular values are met in the curve family of Fig. 9:

b is equal to 1.

q ranges from 1.25 for a clear day to 0.917 for a very dark day.

h varies nearly harmonically through the year, from about 4:20 P. M. in late November and early December to about 7:15 P. M. in late June and early July.

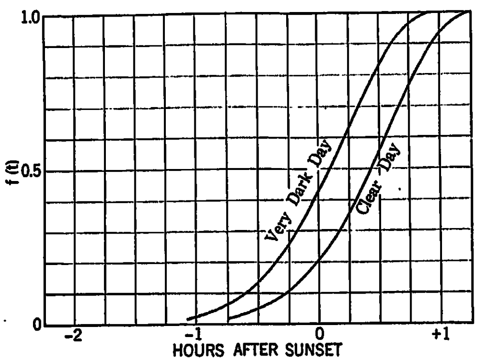

FIG. 9

The method of determining the value of B for weekdays during the period of overlap of the afternoon and evening blocks, rests on a relation observed between the Saturday and week day values of B through the remainder of the year. Reference to Fig. 7 will indicate the existence of a ratio approximately constant between the Saturday and week day values of B for the months March to September inclusive. Table I, giving numerical values of A, B and C taken from the curves of Fig. 7, confirms this indication and shows that the approximate value of the ratio is 0.75.

Assuming the ratio 0.75 to hold also for October, November and December, the week-day values enclosed in parentheses have been derived from the corresponding observed Saturday values; and these derived values have been used successfully in estimating the week-day peak.

TABLE I
VALUES OF A, B AND C IN THOUSAND KILOWATTS FOR THE LOAD CURVES OF FIG. 7

Weekday	Saturday	A Weekday	B Weekday	B Saturday	C Weekday and Saturday
March 5	March 3	75	85	65	70
April 6	April 7	80	85	63	67
May 11	May 12	80	80	60	65
June 13	June 9	85	75	55	65
July 11	July 7	85	65	50	65
Aug. 3	Aug. 4	80	65	50	70
Sept. 25	Sept. 22	85	80	65	75
Oct. 19	Oct. 20	82	(93)	70	80
Nov. 15	Nov. 10	80	(96)	72	85
Dec. 20	Dec. 15	76	(105)	79	96

Since the analytical form of each component of the portion of the daily load curve under consideration has now been determined, with suitable values of the para-

meters for various conditions, it would be possible, when it is desired to find the time and value of the peak for any particular date, to set up the complete equation, inserting for h its proper value taken from a table or curve, and for q a value depending on the state of the weather; to differentiate with respect to t; to equate the derivative to zero; and to substitute the value of t, thus obtained in the original equation. This procedure, however, would be out of the question by reason of its complexity. A graphical method, much simpler but equally accurate, has been developed to overcome this difficulty.

Equation (2) is repeated here in the functional form rather than in the analytical form, since, in the graphical method, it is not necessary that the analytical expression for each of the functions $F(t)$ and $f(t)$ be known, nor even that they should be capable of being expressed analytically.

$$z = A\,F(t) + B\,f(t) + C \tag{2}$$

Differentiating (2) with respect to t.

$$\frac{dz}{dt} = \frac{A\,dF(t)}{dt} + \frac{B\,df(t)}{dt}$$

Equating to zero and dividing by A,

$$\frac{dF(t)}{dt} + \frac{B}{A}\,\frac{df(t)}{dt} = 0$$

$$\frac{dF(t)}{dt} = -\frac{B}{A}\,\frac{df(t)}{dt} \tag{3}$$

Geometrically, the term $\frac{dF(t)}{dt}$ represents the slope of the curve of Fig. 8 at any value of t; while the term $-\frac{df(t)}{dt}$ represents the slope, not of one of the curves of Fig. 9, but of its negative—that is, a curve having ordinates of the same absolute value, but extending downward from the horizontal axis. The term $-\frac{B}{A}\,\frac{df(t)}{dt}$ correspondingly represents the slope of a curve whose ordinates are $-\frac{B}{A}$ times those of one of the curves of Fig. 9; and whose functional form is therefore $-\frac{B}{A}\,f(t)$.

Equation (3) indicates that the geometrical condition for a maximum of z is that the slopes $\frac{dF(t)}{dt}$ and $-\frac{B}{A}\,\frac{df(t)}{dt}$ shall be equal. Fig. 10 shows the graphic method of accomplishing the solution. In this figure, the solid lines indicate lines drawn on a fixed sheet, the solid curve being a repetition of the curve of Fig. 8 and the fine vertical line being drawn at the time of earliest sunset; while the dotted lines indicate lines drawn on a sheet of tracing cloth, celluloid or other non-opaque material, which is adjustable to any desired position on the fixed sheet. The curves on the movable sheet are members of the family represented by $-\frac{B}{A}\,f(t)$, taken for values of $\frac{B}{A}$ ranging, as indicated, from 0.6 to 1.4 at intervals of 0.2. On the sheet the variation in position of the curves, with respect to the time of sunset, due to weather conditions, is taken into account, not, as in Fig. 9, by having a single axis of sunset time and shifting the curves between the extremes shown, but by having a single set of curves and shifting the axis of sunset time correspondingly. In using these sheets, it is, of course, necessary to maintain the horizontal and vertical axes of the movable sheet horizontal and vertical with respect to the fixed sheet.

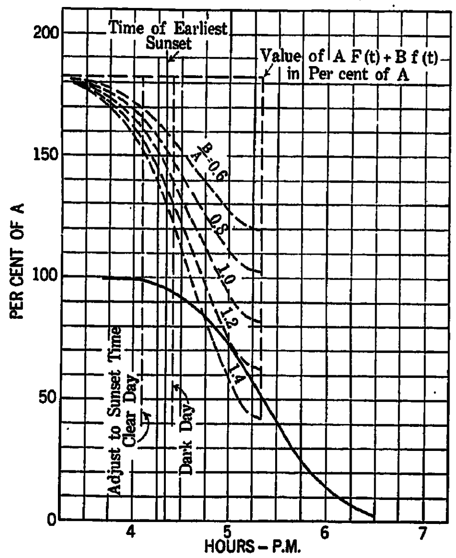

FIG. 10

In order that times on the movable sheet and on the fixed sheet shall correspond, either one of the extreme positions of the sunset axis on the movable sheet or some intermediate position (depending on the state of the weather) must be adjusted to coincide with the time of sunset as read on the scale of the fixed sheet. Then, holding this horizontal setting, the movable sheet must be adjusted vertically until the curve bearing the correct value of $\frac{B}{A}$ is tangent to the curve on the fixed sheet. At the point of tangency the slopes of the two curves are equal, and this satisfies the condition of Equation (3) for a maximum value of z.

Not only does the abscissa of the point of tangency

indicate the time at which the peak occurs; but also the ordinate of the point of tangency gives the value of the afternoon block in per cent of A at that time, and the height from the point of tangency to the horizontal axis of the movable sheet, read on the scale of the fixed sheet, indicates the value of the evening block at the same time, likewise in per cent of A. Therefore, the ordinate of the horizontal axis of the movable sheet, read on the scale of the fixed sheet, gives the value of the sum of the afternoon and evening blocks.

In the particular example shown in Fig. 10, $\frac{B}{A}$ is 1.2 (this would be the case if B were 90,000 and A 75,000 kw., for instance), the day is very dark and sunset occurs at 4:25 P. M. When the movable sheet is adjusted horizontally so that the line for a dark day coincides with the time 4:25 P. M. on the fixed sheet, and vertically so that the curve $\frac{B}{A} = 1.2$ is tangent to the curve on the fixed sheet, the abscissa of the point of tangency gives 5:03 P. M. as the time of the peak, and the ordinate of the horizontal axis of the movable sheet gives the value of $[A\, f\,(t) + B\, f\,(t)]$ as 182.5 per cent of A. The peak value of the load curve is then given by the equation

$$z = C + 1.825\, A$$

The estimation of peak demands by any method based on the values of the actual total peak for corresponding periods of past years is open to several criticisms.

In such a method, no consideration is taken of the possibility that one or more of the previous years may fail to give a reliable indication, on account of the absence of stormy weather at the time of the peak. It is also tacitly assumed that the shape of the load curve remains constant. This assumption is equivalent to the assumption that the three components, A, B and C, distinguished in the present investigation, increase in the same ratio; and this assumption is not justified.

Furthermore, it is difficult not only to make a reliable estimate of the peak at a given date in the future, but also to correlate and check such an estimate, when made, with actual peaks as the given date approaches.

On the other hand, the separation of the components of past peaks allows the growth of each component to be observed individually, regardless of whether or not it has been combined at any particular date with the maximum overlap. It is then relatively simple to project the growth of each component, at its own growth-rate, to the desired date, and to combine the estimated values properly.

The reliability of an estimate of peak prepared in this way results both from the accuracy with which the individual components may be predicted and from the probability of compensating errors in the values of the components.

A great advantage in this method is that, after an estimate has been made, it may readily be checked at any time as the date approaches by noting whether the values of the components taken from current load curves confirm the original estimate or indicate the necessity of a revision.

The Thermal Time Constants
of Dynamo-Electric Machines

BY A. E. KENNELLY*
Fellow, A. I. E. E.

Review of the Subject.—*Considering the thermal behavior of a dynamo-electric machine intended for continuous service, its acceptance tests require, at present, only a limiting temperature elevation under a continuous rated load. For the intelligent operation of a machine after it has been put in service, additional information is desirable concerning its thermal behavior under changes of load. This subsidiary thermal information concerning a machine with a continuous rating may consist of (1) its final temperature rise under some steady load other than its rated load, such as either 75 per cent or 125 per cent of the rated load, and (2) its thermal time constant.*

The thermal time constant of a machine, assumed as conforming strictly to an exponential law of temperature rise above a constant ambient temperature, after being transferred suddenly from one steady load to another, is taken as the time required to attain $1-\epsilon^{-1}$ *or 63.2 per cent of the final temperature change. This may be called the exponential thermal time constant. This is a fundamentally scientific quantity; but is very awkward to remember or to explain to a person not well versed in the mathematical theory of the subject.*

It is recommended in the paper that for all practical engineering work, a new time constant called the binary time constant be used. It would correspond to the "Period," or "Half-value period," already used in the Science of Radio-activity and in measurements of Radio active decay. A binary time constant is that time in which a machine, assumed as conforming to an exponential law of temperature change, after being suddenly transferred from one steady load to another, attains one half of the final temperature change (50 per cent). In two binary time constants, it will then attain ¾ (75 per cent) of the final temperature change, in three of them ⅞ths (87.5) and so on. This is an easy relation to remember and explain. A binary thermal time constant may be taken, for practical purposes, as 70 per cent of the classical exponential time constant. It is more strictly 69.32 per cent.

Although dynamo machines do not rigidly follow an exponential law of temperature change, for reasons discussed, yet for many purposes the deviation therefrom may be ignored. It is recommended that the binary time constant of all such machines may be adopted, where practical, for industrial use.

In rotating machines, there are two thermal time constants, the constant-loss time constant, and the variable-loss time constant. The latter, expressed as a binary constant, is the practical one presenting itself for use.

The binary thermal time constant has also useful applications in correcting the final ambient temperature during a continuous-load test, when the ambient temperature has been observed to change.

It is proposed to discuss the nature, applicability and advantages of the thermal time constants of dynamo machines, from an engineering standpoint. Although, broadly speaking, the subject is not new, it is believed that certain new branches of it are here presented for consideration.

Steady State Thermal Condition

We may assume, to begin with, that a dynamo machine is operated continuously under a steady load, and in a place where the whole environment (air, walls, floor and ceiling) is maintained at a steady ambient temperature T_a deg. cent. In practise, these conditions can only be imperfectly realised, and the temperature of the environment fluctuates with time, besides being subject to some variation in different parts at any one moment. Under the conditions assumed, however, the machine will attain eventually a steady thermal condition. We may assume that, at least as a first approximation, the thermal conductance of the steel stator framework and rotor core are sufficiently great to maintain a virtually uniform temperature in all their parts. The insulated windings, however, when steadily heated by the working currents, may develop very appreciable thermal drops, or differences of temperature, in different parts, owing to the interposition of thermal resistance in the insulating materials. Setting aside these differences for the present, we may assume that the whole machine arrives at a steady maximum temperature T_m deg. cent. or attains a rise of temperature above the environment of

$$\Theta_0 = T_m - T_a \qquad \text{deg. cent.} \qquad (1)$$

The heat dissipated from the machine to the environment, due to the elevation of temperature, escapes by three different ways,—conduction, radiation and convection. The quantitative laws of these three modes of dissipation are very different; but it is generally admitted that, within the range of working temperatures permitted in dynamo machines, the dissipation may be regarded as proportional to the temperature elevation, and also that most of the heat in modern open machines is dissipated by fluid convection; *i. e.*, by circulating air, in the case of generators and motors, and by circulating oil in the case of transformers. Consequently, if p watts are steadily generated in the machine as heat, the thermal dissipation must also amount to p watts, or

$$p = \Theta_0 \, s \qquad \text{watts} \qquad (2)$$

where s is the thermal dissipation coefficient of the machine, in watts per deg. cent. temperature elevation, including all three modes of dissipation. The dissipation coefficient s might be measured in simple cases by measuring the internally wasted power p, and the maximum temperature elevation Θ_0; but for ordinary purposes, s does not require to be known with precision. It is evident that s increases with the size of

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machine for any given type; because the thermal waste of energy p, at rated load, increases with the size, while Θ_0, the temperature elevation under rated load, remains the same, or nearly the same.

Maximum Temperature Permissible

The modern theory on which the thermal rating of dynamo machinery is based, considers that there is a certain working temperature, above which the insulating material undergoes thermolysis or active deterioration. This is the critical temperature for the type of insulation employed. Below the critical temperature, it is supposed that the working lifetime of the insulation is indefinitely long. The hottest spot on the conductor of the machine is therefore at all times to be kept from exceeding the critical temperature, which, for class A insulating materials of impregnated cellulose, is at present set at 105 deg. cent. The hottest spot in the winding of a machine may not be accessible to temperature measurement, and must ordinarily lie on the inner surface of the insulation, next to the copper conductor. By allowing a conventional drop of temperature of 15 deg. cent. between the hottest internal spot of the insulation and those external parts into or onto which exploring thermometers may be applied, the maximum permissible observable temperature under continuous rated load becomes 90 deg. cent. by thermometer. Field windings may have their average temperatures conveniently measured by resistance, and for such measurements a conventional drop of 10 deg. cent. is taken as developed between the hottest spot and the measured value.

Available Output as Dependent on Temperature Rise

It is generally accepted, as the result of experience, that for practical purposes, the ultimate temperature rise at normal sea-level barometric pressure is independent of the initial and ambient temperature within the usual range. Thus, whether we operate a machine in a cool, outdoor space at 10 deg. cent., or in a warm engine room at 40 deg. cent., the rated output of the machine, steadily maintained, will bring about the same ultimate temperature rise of say 50 deg. cent.; so that the outdoor machine would show 60 deg. cent., and the engine-room machine 90 deg. cent. This independence of the temperature rise greatly simplifies the discussion. On the other hand, the density of the air, as determined by the local reading of the barometer, has a marked influence on the dissipation constant s of rotating machinery; because although the radiation of heat and conduction of heat by solids are independent of the air density, the convection of heat by air circulation is immediately affected. By International Electrotechnical Commission (I. E. C.) agreement, no correction needs to be introduced until the elevation is 1000 meters above sea level; although a graded, rather than a sudden, correction would be logically preferable.

If the loss of energy in a machine were always in direct proportion to the output; *i. e.*, if the efficiency were constant at all loads, the ultimate temperature elevation by (1) might be expected to increase directly with the load. If the rated output ultimately produced 50 deg. cent. rise, half the rated output might be expected to produce 25 deg. cent. In practise, however, the losses increase more rapidly than the output, and different machines differ in this respect. Consequently, the ultimate temperature rise increases faster than the output. If we plot the observed maximum temperature rise against output, we obtain a rising curved line, the precise algebraic expression for which may be complicated. If, however, we confine ourselves to outputs between say 50 per cent above and 50 per cent below the normal rated output, we may expect to find that the ultimate temperature rise Θ_0 is, to a first approximation, a simple power of the output or

$$\Theta_0 = c\,P^n \qquad \text{deg. cent.} \quad (3)$$

where c is a constant, P the watts output (maintained long enough to reach substantially constant maximum rise) and Θ_0 is the rise in deg. cent. corresponding thereto. This means that if the output and temperature rise are plotted on logarithm paper, the graph will be approximately a straight line. This is indicated in Fig. 1, where the abscissas are in percentage of the rated load, and the ordinates indicate the ultimate observed temperature rise. Thus at 100 per cent of rated load, the observed temperature rise is 43 deg. cent. and at 125 per cent of rated load, it is 64 deg. cent. The straight line $A\,O\,C$ may be taken as representing the behavior of temperature rise versus output between 50 per cent and 150 per cent of rated load, for the particular machine considered. Then the exponent n of formula (3) is the ratio of $O\,B$ to $A\,B$ in the figure. This is nearly 1.8 in the case shown, so that over the range of load considered, the temperature rise varies approximately as the 1.8th power of the load. Reciprocally, the output varies as the $1/n$th power of the temperature rise, or in the ratio $A\,B : B\,O$, which is 0.56 in Fig. 1. The output in this case varies as the 0.56th power of the temperature rise.

In order to know, with reasonable precision, how the temperature rise of a machine behaves with respect to sustained outputs, we need either a diagram like Fig. 1, or the value of the *temperature—output exponent n* in (3). This value differs with the size, type and make of machine. It commonly lies between 1.4 and 2.0. Equivalent information would be the temperature rise at say 25 per cent sustained overload.

In order, therefore, that the particular machine of Fig. 1 should not develop a higher observable thermometer temperature than 90 deg. cent., the output should be 108 per cent for an ambient temperature of 40 deg. cent., 120 per cent for an ambient temperature of 30 deg. cent., and 130 per cent for an ambient temperature of 20 deg. cent. Good operating practise might call, however, for some reduction of these loads, as no margin would be allowed on them for accidental variations in

ambient temperature or other irregularities of service.

The present I. E. C. international rating of a machine, with class A insulation, calls for an ultimate temperature elevation of 50 deg. cent. under continuous load, as the test rating. The maker's rating would presumably be slightly less, so as to fall within the test rating. The machine whose thermal behavior is indicated in Fig. 1 would be 7 deg. cent. below the test rating, or its rating could be increased 8 per cent without exceeding the I. E. C. limit.

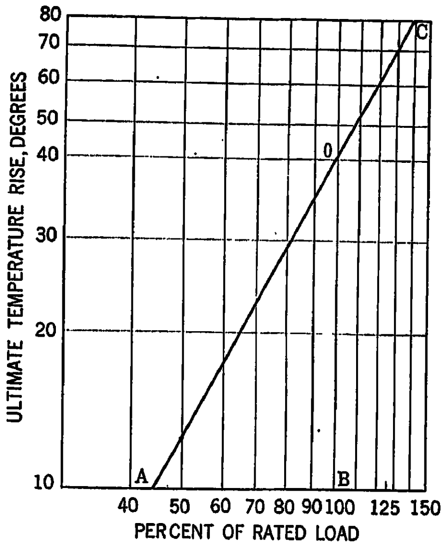

FIG. 1—APPROXIMATE STRAIGHT LINE RELATION BETWEEN ULTIMATE TEMPERATURE RISE AND STEADY LOAD FOR A DYNAMO MACHINE

THE TRANSIENT STATE OF TEMPERATURE ELEVATION

When the output of a machine is changed abruptly from one steady value to another, or when at constant output the ambient temperature changes quickly from one steady value to another, the thermal state of the machine changes slowly from the initial condition to a final condition, corresponding to the impressed change. The transition is a transient phenomenon; although it may require many hours to complete, within the limits of engineering measurements, and theoretically the full transition requires infinite time. From an engineering viewpoint, the temperature change in the machine is a transient. It has long been known[1] that under the assumed limitation of constant losses in the machine during the transition, the transient is a simple exponential transient, like that of the current strength in a simple continuous-current circuit containing both resistance and inductance. It has recently been shown by P. Girault[2] that the temperature transient is still simply exponential when the iron losses are constant, but the copper losses increase according to the resistance temperature coefficient of the windings.

It is proposed to carry this proposition here one stage further, and to show, on the assumption that the iron losses also follow a straight line law of change from the initial to the final temperature, the change in temperature of the machine will remain a simple exponential transient, with a time-constant curve.

Fig. 2 indicates a simple circuit of resistance R, and non-ferric or air-cored inductance $\mathfrak{L}$, carrying a steady initial current of I_1 amperes from a storage battery of negligible resistance, through a recording oscillograph O. A switch, S enables the e. m. f. of the battery E to be suddenly increased from 20 to 50 volts, or suddenly diminished from 50 to 20. A sensitive relay A, automatically inserts a protective resistance r in circuit with the upper part of the battery, when the switch S places that battery on short-circuit.

EXPONENTIAL TIME CONSTANT τ_ϵ

If I_1 be the initial steady current strength (amperes) and I_2 the final steady current, after throwing the switch at time $t = 0$, the transient current i at any intermediate instant, according to well-known principles, is

$$i = I_2 + (I_1 - I_2)\,\epsilon^{-\frac{t}{\tau}} \quad \text{amperes} \quad (4)$$

where ϵ is 2.71828 . . ., the Napierian base and

$$\tau = \mathfrak{L}/R = \tau_\epsilon \quad \text{seconds} \quad (5)$$

a time constant, equal to the inductance in henries divided by the resistance in ohms, which is 0.005 second

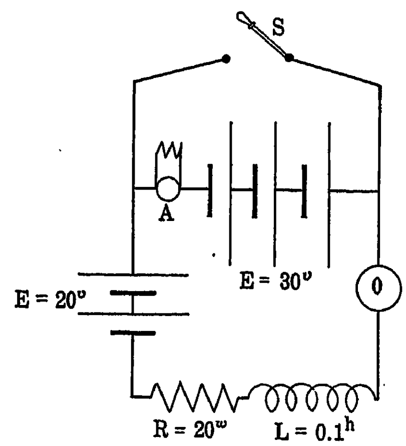

FIG. 2—SIMPLE CONTINUOUS-CURRENT CIRCUIT FOR DISPLAYING A SIMPLE EXPONENTIAL TRANSIENT

in the case of Fig. 2, and which we may call an *exponential time constant*, denoted by the symbol τ_ϵ, to distinguish it from other time constants to be considered later.

Fig. 3 indicates the transient current curve $A\ B\ C$ . . . $N\ O\ P$, on opening switch S, as we might expect to

1. "Temperature Curves and the Rating of Electrical Machinery." R. Goldschmidt, *Jour. I. E. E.* London. March, 1905. Vol. 34, pp. 660-691.

"The Heating and Cooling of Electrical Machinery," P. Grice, *Jour. I. E. E.* London. Nov. 1912, Vol. 51, 1913, pp. 840-851.

2. "Sur l'échauffement d'un organe de machine électrique soumis à des pertes dans le fer constantes, et à des pertes par effet Joule," P. Girault, *Rév. Gén de l'Electricité.* 2nd & 9th Dec. 1922, Vol. XII, pp. 873 and 874; also 28th July and 4 Aug. 1923, Vol. XIV, pp. 115 and 147.

find it recorded by the oscillograph O Fig. 2. The current starts at $I_1 = 1.0$ ampere for $t = 0$. After the lapse of τ_ϵ, one exponential time constant, it has reached C, at 1.948 amperes, having risen 0.948 out of the total change

$$\Delta = I_2 - I_1 = 1.5 \quad \text{amperes} \quad (6)$$

that it has to cover. The fraction $0.948/1.5 = 0.6321$, may be called the *attainment*, at the point C considered. The remainder Cc,

$$\delta = I_2 - i = 0.552 \quad \text{amperes} \quad (7)$$

is what is left to be overcome. The fraction

$$\frac{\delta}{\Delta} = \frac{0.552}{1.5} = 0.3679$$

may be called the *deficiency* at this point. At any instant, the sum of the attainment and the deficiency is unity. At C, one exponential time constant from A, the deficiency is always $1/\epsilon$, or 36.79 per cent, and the attainment 63.21 per cent.

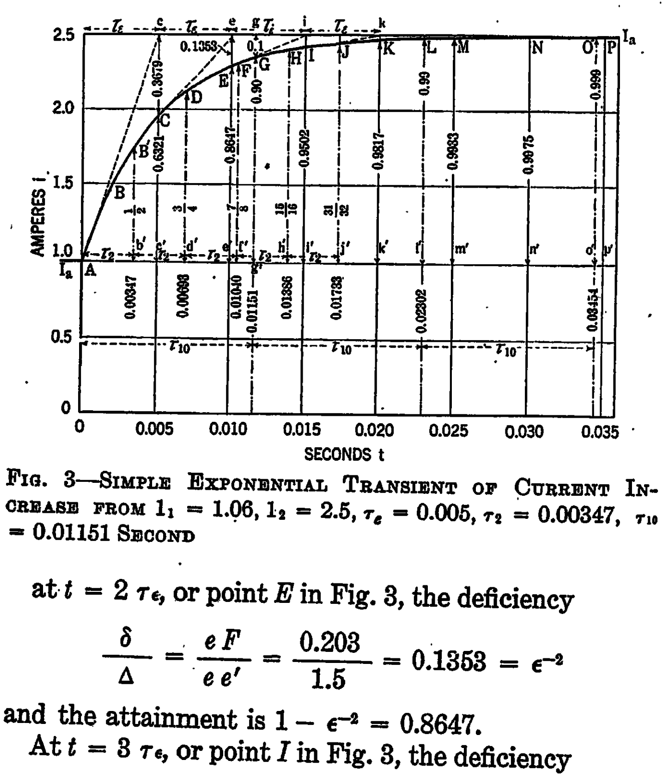

FIG. 3—SIMPLE EXPONENTIAL TRANSIENT OF CURRENT INCREASE FROM $I_1 = 1.06$, $I_2 = 2.5$, $\tau_\epsilon = 0.005$, $\tau_2 = 0.00347$, $\tau_{10} = 0.01151$ SECOND

at $t = 2\,\tau_\epsilon$, or point E in Fig. 3, the deficiency

$$\frac{\delta}{\Delta} = \frac{e\,F}{e\,e'} = \frac{0.203}{1.5} = 0.1353 = \epsilon^{-2}$$

and the attainment is $1 - \epsilon^{-2} = 0.8647$.

At $t = 3\,\tau_\epsilon$, or point I in Fig. 3, the deficiency

$$\frac{\delta}{\Delta} = \frac{i\,I}{i\,i'} = \frac{0.0747}{1.5} = 0.0498 = \epsilon^{-3}$$

and the attainment is $1 - \epsilon^{-3} = 0.9502$. This is probably the simplest statistical relation to express, or to remember, concerning an exponential time constant; *i. e., that in three of them, the deficiency is very nearly 5 per cent.*

It is to be noted that at any point of the curve, such as C, Fig. 3, the tangent $C\,e$ cuts the final current line I_2, at a time distance beyond c, of $c\,e$, equal to the exponential time constant. Consequently, if we know the final steady current I_2, and can trace the curve of the transient at any point with sufficient precision to draw the tangent, we can measure the time constant τ_ϵ graphically, the subtangent on the final line I_2 being constant, and equal to τ_ϵ.

We may rewrite equation (4) in the form

$$i = I_2 + \Delta\,\epsilon^{-\frac{t}{\tau_\epsilon}} \quad \text{amperes} \quad (8)$$

or in the case of Fig. 3

$$i = 2.5 - 1.5\,\epsilon^{-\frac{t}{0.005}} \quad \text{amperes} \quad (9)$$

Because $I_1 - I_2$ is negative, we apply the negative sign to Δ.

The exponential time constant is, from a mathematical standpoint, the fundamental property of the simple exponential transient; but it is awkward to employ practically, or to explain in simple terms. When we say that after the lapse of one exponential time constant the deficiency is 1/2.71828, we do not convey a simple conception, except to a person familiar with the mathematical subject of exponentials.

BINARY TIME CONSTANT τ_2

We may, however, restate (8) as follows

$$i - I_2 = \delta = \Delta\,\epsilon^{-\frac{t}{\tau_\epsilon}} \quad \text{amperes} \quad (10)$$

$$\frac{\delta}{\Delta} = \epsilon^{-\frac{t}{\tau_\epsilon}} = \epsilon^{0.69315 \times \frac{t}{0.69315}} \text{ deficiency ratio} \quad (11)$$

But

$$\epsilon^{0.69315} = 2 \quad (12)$$

so that

$$\frac{\delta}{\Delta} = 2^{-\frac{t}{0.69315\tau_\epsilon}} = 2^{-\frac{t}{\tau_2}} \text{ deficiency ratio} \quad (13)$$

where

$$\tau_2 = 0.69315\,\tau_\epsilon \quad \text{seconds} \quad (14)$$

That is, we may transfer the deficiency from the Napierian base ϵ to base 2, and substitute for the exponential time constant a new time constant τ_2, which is very nearly 70 per cent of τ_ϵ. This new time constant τ_2 may be called, for distinction, the[3] *binary time constant.* The binary time constant would then correspond to what has already been known for some years as the "Period," or "Half-value period," in the science of Radio-activity, and in the measurement[4] of radio-active decay.

In Fig. 3, the time $A\,B' = 0.00347$ second, is shown as equal to the binary time constant τ_2. At the time $t = \tau_2$, the current will have reached B', and the deficiency δ/Δ at this moment is 2^{-1} or ½. The attainment is likewise ½. Again, at $t = 2\,\tau_2 = 0.00693$ second $= d'$, the deficiency is $2^{-2} = ¼$, and the attainment $1 - 2^{-2} = ¾$. Similarly, at $t = 3\,\tau_2 = 0.0104 = f'$, the deficiency is $2^{-3} = ⅛$, and the attainment $1 - 2^{-3} = 7/8$, corresponding to point F. Five binary time constants are marked on Fig. 3, as far as j', where

3. "Time Constants for Engineering Purposes in Simple Exponential Transient Phenomena" by A. E. Kennelly, *Proc. Nat. Acad. Sciences.* Nov. 1924.

4. "Practical Measurements in Radio-Activity." Makower and Geiger, 1912, p. 81.

the deficiency is $2^{-5} = 1/32$, and the attainment 31/32.

The binary time constant τ_2 is just as sound mathematically as the fundamental exponential time constant τ_ϵ, and is always approximately 70 per cent of the latter. The binary time constant has the practical advantage that it is easier to remember and explain. In τ_2, the attainment and the deficiency are both ½, or 50 per cent. In the next τ_2, the attainment is to ½ of what was left over, or to ¾, and so on. This is an easy concept to remember or to explain. It is recommended that, in engineering, exponential time constants be replaced by binary time constants.

Decimal Time-Constant τ_{10}

Although from the practical and descriptive standpoints, the binary time constant τ_2 has marked advantages over the exponential time constant τ_ϵ, neither of these lends itself to ease of computation at indiscriminate values of t. We may however rewrite (11) in the form

$$\frac{\delta}{\Delta} = \epsilon^{2.3026 \times -\frac{t}{2.3026\tau_\epsilon}} \qquad \text{deficiency ratio} \qquad (15)$$

$$= 10^{-\frac{t}{\tau_{10}}} \qquad \text{deficiency ratio} \qquad (16)$$

where

$$\tau_{10} = 2.3026\,\tau_\epsilon \qquad (17)$$

By substituting the *decimal time constant* τ_{10} for the exponential time constant, we obtain in (16) an expression for the deficiency at any assigned value of t, which can be evaluated quickly with the aid of any ordinary table of logarithms.

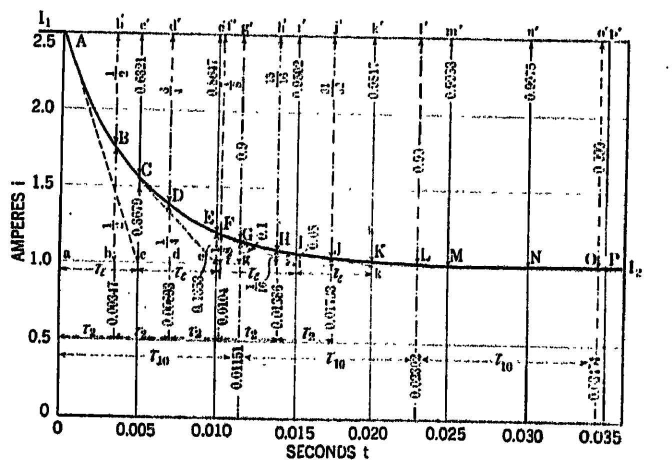

Fig. 4—Simple Exponential Transient of Current Decrease from $I_1 = 2.5$ to $I_2 = 1.0, \tau_\epsilon = 0.005, \tau_2 = 0.00347, \tau_{10} = 0.01151$ Second

In Fig. 3, the point g', where $t = 0.01151$, marks one decimal time constant. At this moment the current is at G, and the deficiency is 10^{-1}, or 0.1. The attainment is $(1 - 10^{-1})$ or 0.9. Again, at $t = 2\,\tau_{10}$, the current has reached L, where the deficiency is 10^{-2} or 0.01, and the attainment is $1 - 10^{-2}$, or 0.99. At $t = 3\,\tau_{10}$, the deficiency, at 0, is 10^{-3} or 0.001. From a practical standpoint, the decimal time constant is too long, since the attainment is then 0.9.

Fig. 4 shows the corresponding transient of decrease, when the switch S of Fig. 2 is closed. The curve in Fig. 4 is the same as in Fig. 3, but inverted. The same formulas apply and

$$i = 1.0 + 1.5\,\epsilon^{-\frac{t}{\tau_\epsilon}} \qquad \text{amperes} \qquad (18)$$

All three time constants τ_ϵ, τ_2 and τ_{10} are indicated in Fig. 4, and their applications are the same as in Fig. 3.

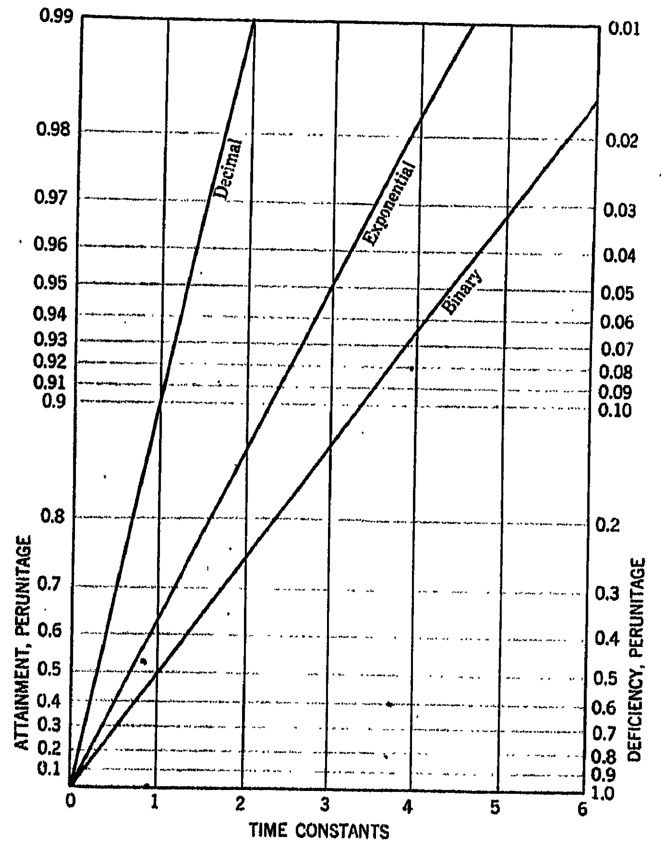

Fig. 5—Rectilinear Attainment-Time Constant Diagram on Inverted Arith-Log Paper, for Decimal, Exponential and Binary Time Constants

Straight-Line Graphs of Deficiency and Attainment on Inverted Arith-Log Paper

Fig. 5 shows that the graphs of attainment and deficiency become straight lines on inverted arith-log paper for all three time constants. Thus at one time constant, the attainment is 0.5 with the binary, 0.632 with the exponential, and 0.9 with the decimal time constant. With the aid of this chart, computations relating to simple exponential transients may be greatly simplified, for most engineering purposes. The temperature rise of any machine, under steady load and steady ambient temperature, will trace out a straight line with time as in Fig. 4, if the transient is strictly exponential and the final rise is known.

Time Constants Applied to Transient Temperatures of Dynamo Machines on Basis of Constant Losses During Transition

The following is a brief presentation of the well-known theory of transient temperatures in dynamos under steady load. It is assumed that the impressed speed is constant, in the case of a generator, impressed voltage in the case of a motor, impressed voltage and frequency in the case of a transformer. The machine is supposed to comprise a single thermal body, of perfect thermal conduction throughout. The machine starts at $t = 0$ hours, from an initial and constant ambient temperature T_a deg. cent., under continuous load, finally reaching a final temperature $T_m = T_a + \Theta_0$ deg. cent. The power losses in the machine remain constant at p_0 watts. The dissipation constant s of formula (2) is constant and also the thermal capacity k, in watthours absorbed per deg. cent. elevation of temperature. Then if θ is the instantaneous temperature of the machine at any time t, the heat generated in the machine during any short interval of time dt is $p_0\ dt$ watthours. During the same interval, the heat dissipated by the machine is $\theta\, s\, .\, dt$ watthours. If during the same, the temperature of the machine rises by $d\theta$ deg. cent. the heat absorbed in the machine will be $k\, .\, d\theta$ watthours. Equating the heat generated to the heat dissipated and absorbed,

$$p_0\, .\, dt = \theta\, s\, .\, dt + k\, .\, d\theta \quad \text{watthours} \quad (19)$$

Let

$$p_0 = \Theta_0\, s \quad \text{watts} \quad (20)$$

by (2). Then

$$(\Theta_0 - \theta)\, s\, .\, dt = k\, .\, d\theta \quad \text{watthours} \quad (21)$$

or

$$(\Theta_0 - \theta)\, .\, dt = \tau_\epsilon'\, .\, d\theta \quad \text{deg. cent hours} \quad (22)$$

where

$$\tau_\epsilon' = \frac{k}{s} \quad \text{hours} \quad (23)$$

τ_ϵ' is an exponential time constant. It may be called the *constant-loss time constant.*

Hence

$$\frac{d\theta}{\Theta_0 - \theta} = \frac{dt}{\tau_\epsilon'} \quad \text{numeric} \quad (24)$$

and

$$-\,\text{logh}\,(\Theta_0 - \theta) = \frac{t}{\tau_\epsilon'} - c \quad \text{numeric} \quad (25)$$

where logh signifies the hyperbolic or Napierian logarithm, and c is an integration constant.

Then

$$\Theta_0 - \theta = C\, \epsilon^{-\frac{t}{\tau_\epsilon'}} \quad \text{deg. cent.} \quad (26)$$

When $t = 0$, $\theta = 0$ and $C = \Theta_0$.

Thus

$$\theta = \Theta_0 - \Theta_0\, \epsilon^{-\frac{t}{\tau_\epsilon'}} \quad \text{deg. cent. rise} \quad (27)$$

$$= \Theta_0 - \Theta_0\, 2^{-\frac{t}{\tau_2'}} \quad \text{“ “ “} \quad (28)$$

$$= \Theta_0 - \Theta_0\, 10^{-\frac{t}{\tau_{10}}} \quad \text{“ “ “} \quad (29)$$

These results are in accordance with those obtained for the continuous-current transient of Figs. 2, 3 and 4, using the three time constants mentioned above.

The instantaneous temperature elevation θ will be completely specified if we know the ultimate temperature rise Θ_0, and any one of the three time constants. The easiest time constant to measure is probably τ_ϵ, from a tangent to the curve of initial heating, assuming that the ultimate rise Θ_0 is known or is measured. We do not need to know either k or s. The attainment at any time is θ / Θ_0, and the deficiency $(\Theta_0 - \theta)/\Theta_0$.

Cooling Transient

The curve of cooling of a machine will be identical with that of heating inverted (see Figs. 3 and 4), if the dissipation constant s remains the same. In the case of an air-cooled transformer, this may be expected; but in the case of a generator or motor, this can only be expected if the speed of rotation remains the same during both cooling and heating. It is evident that s will be greatly diminished if the machine is stopped while cooling.

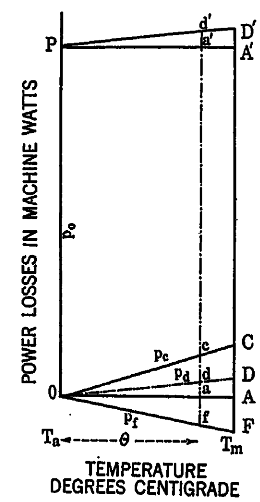

Fig. 6—Diagram of Assumed Straight Line Losses at Constant Output as Function of Temperature

Time Constants Applied to Straight-Line Change of Losses During Transition

Fig. 6 represents an ideal diagram of losses in a machine operated at a steady output, commencing at an initial and ambient temperature T_a and terminating at a final temperature T_m deg. cent. $O\,P$ represents the internal losses p_0 watts converted into heat. Of this amount, a certain part, not definitely known, is in the copper windings, another part in the steel frames of rotor and stator, while the remainder will be in frictions of various kinds. If p_0 were constant throughout the transition, the loss would reach $A\,A'$ at the final temperature. Suppose, however, that the copper losses increase by an amount $A\,C$ along the straight line $O\,C$. This would be true if the current strengths in the windings remained constant, and the copper resistances increased with temperature in the regular way. Actually, the currents may have to be re-adjusted as the temperature increases; while the electric resistance of

carbon brushes, if used, may diminish. It will be sufficient that the actual change in copper losses does not depart markedly from a straight line $O\,C$. Again, the iron losses are likely to diminish as the temperature rises, because the resistance of the iron to eddy currents will increase; while the hysteresis losses remain substantially unchanged. We may assume that the change in iron loss will amount, at the maximum temperature, to a reduction $A\,F$ watts, and that it will follow a straight line $O\,F$. Actually, the change will be more complex; but it will suffice for most practical purposes if the deviation from the assumed straight line $O\,F$ is not serious. The total change of power losses over the transition will then be say $A\,D$, plus or minus, according to the predominance of $A\,C$ or $A\,F$. The total change with temperature will then also occur along a straight line $O\,D$, so that the total losses will follow the straight line $P\,D'$.

Let p be the power liberated in the machine at temperature elevation θ deg. cent. Then

$$p = p_0 + p_c - p_f \qquad \text{watts} \quad (30)$$

$$= p_0 + c\,\theta - f\,\theta = p_0 + d\,.\,\theta \qquad \text{"} \quad (31)$$

where c is the ratio $A\,C : O\,A$ or $\tan A\,O\,C$,

and f " " " $A\,F : O\,A$ " $\tan A\,O\,F$

" d " " " $A\,D : O\,A$ " $\tan A\,O\,D$

Then at any temperature elevation θ, the power liberated in the machine being $p_0 + d\,.\,\theta$, the heat liberated in time dt is

$$p\,.\,dt = (p_0 + d\,.\,\theta)\,dt \qquad \text{watthours} \quad (32)$$

The heat dissipated in the same time will be $\theta\,s\,.\,dt$ watthours; while the heat absorbed in the machine by a simultaneous small rise of temperature $d\theta$, is $k\,.\,d\theta$ watthours. Equating the gain and loss of heat, as in (19)

$$(p_0 + d\,.\,\theta)\,dt = \theta\,s\,.\,dt + k\,.\,d\theta \qquad \text{watthours} \quad (33)$$

Let $p_0 = \Theta_0\,s$, as before.

Then $\{\Theta_0\,s - \theta\,(s-d)\}\,dt = k\,.\,d\theta$ watthours (34)

or

$$\text{or} \quad \left\{ \Theta_0\left(\frac{s}{s-d}\right) - \theta \right\} dt = \left(\frac{k}{s-d}\right) d\theta \qquad \text{deg. cent. hours} \quad (35)$$

$$\text{or} \quad (\Theta - \theta)\,dt = \tau_\epsilon\,.\,d\theta \qquad \text{deg. cent. hours} \quad (36)$$

$$\text{where} \quad \Theta = \Theta_0\left(\frac{s}{s-d}\right) \qquad \text{deg. cent.} \quad (37)$$

$$\text{and} \quad \tau_\epsilon = \frac{k}{s-d} = \tau_\epsilon'\left(\frac{s}{s-d}\right) \qquad \text{hours} \quad (38)$$

τ_ϵ may be called the straight-line variable-loss time constant, or simply the *variable-loss time constant.*

Equation (36) corresponds to (22), and its solution is

$$\theta = \Theta - \Theta\,\epsilon^{-\frac{t}{\tau_\epsilon}} \qquad \text{deg. cent. rise} \quad (39)$$

$$= \Theta - \Theta\,2^{-\frac{t}{\tau_2}} \qquad \text{deg. cent. rise} \quad (40)$$

$$= \Theta - \Theta\,10^{-\frac{t}{\tau_{10}}} \qquad \text{deg. cent. rise} \quad (41)$$

The effect of the variation of losses with temperature is to change both the ultimate rise and the time constant in the ratio $s/(s-d)$. If the losses are greater at the higher temperature d is positive; so that both Θ and τ are increased. The temperature elevation will remain a simple exponential transient, the curve of which will correspond to that of Fig. 3 for rises, and to that of Fig. 4 for falls. The exponential time constant of the curve, as obtainable from the tangent, will be τ_ϵ, and very nearly 70 per cent of this will be the binary time constant τ_2.

Cooling Curve with Normal Dissipation and Load Removed

If the load were cut off the machine and all excitation removed, while the normal dissipation was maintained by continued rotation, the power liberated in the machine would be zero. Equation (33) then becomes

$$0 = \theta\,s\,.\,dt + k\,.\,d\theta \qquad \text{watthours} \quad (42)$$

$$0 = \theta\,.\,dt + \tau_\epsilon'\,.\,d\theta \qquad \text{deg. cent. hours} \quad (43)$$

$$-\,\theta\,.\,dt = \tau_\epsilon'\,.\,d\theta \qquad \text{deg. cent. hours} \quad (44)$$

$$-\,\frac{dt}{\tau_\epsilon'} = \frac{d\theta}{\theta} \qquad \text{numeric} \quad (45)$$

the solution of which is

$$-\,\frac{t}{\tau_\epsilon'} = \text{logh}\,\theta + c \qquad \text{numeric} \quad (46)$$

$$\text{Hence} \quad \theta = \Theta\,\epsilon^{-\frac{t}{\tau_\epsilon'}} \qquad \text{deg. cent. rise} \quad (47)$$

The time constant of this falling transient is not τ_ϵ but τ_ϵ'. Consequently, if the losses vary with the temperature at constant output, the curves of transient temperature will be simply exponential, whether with load applied or with load removed; but the time constants will differ. But since it is very unusual to run a machine steadily without load or excitation, the constant-loss time constant τ_ϵ' does not present itself in practise, and only the variable-loss time constant τ_ϵ comes into operation.

Although, under the above assumptions, the transient temperature of a dynamo machine in continuous operation from one steady load to another is a time-constant transient, yet it will be found on examination of such temperature curves that they deviate somewhat from strict time-constant curves, although for many practical purposes the deviations may be ignored. A principal cause for deviation is that the windings of a machine, with their copper losses, form one thermal system, and the steel structures, with their iron losses, form another. Each system has its own k, s and time constant. If the two systems were thermally independent, they could be measured apart; or, if the two systems were united by perfect thermal conduction, they would blend into

one with a single time constant, as has been assumed above. But they are actually only semi-detached, being in mutual communication through thermal resistances; so that each tends to modify the behavior of the other, in a rather complicated way. The curves of transient temperature of the copper are affected by the influence of the associated steel structures, and reciprocally; so that neither can display a true time-constant curve.

By the courtesy of Mr. H. M. Hobart, a number of curves of transient temperature, in passing from one steady state to another, have been procured and examined for various types and sizes of machines. Two only of these are presented here, as examples.

Fig. 7 gives the observed hottest stator lamina temperature, as obtained by thermometer, on a particular 30-h. p., 3-phase, 6-pole induction motor, operated under steady load, at 500 rev. per min., from a 440-volt, 25 ~ circuit. The ambient temperature was 16 deg. cent. and remained within 1 deg. cent. of that value during the test. The small black circles, at half-hour intervals, indicate the observed temperature rise; while the curve $A\ B\ C - H$ shows a time constant computed on the basis of $\Theta = 37$ deg. cent. ultimate rise.

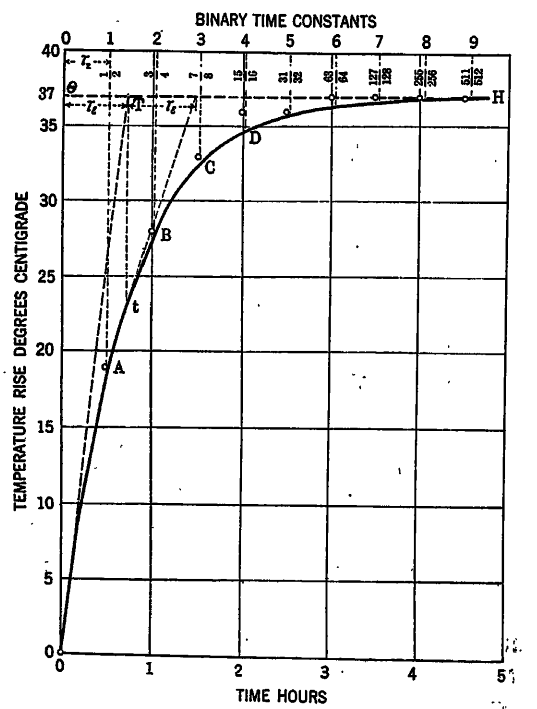

FIG. 7—TEMPERATURE RISE OF HOTTEST STATOR LAMINA IN INDUCTION MOTOR

The observations do not depart more than 1.5 deg. cent. from the computed curve at any point, but indicate a tendency to exceed the curve near the middle of the run. Two exponential time-constants of about 44 minutes each are marked off on the 37 deg. rise line $T\ H$, by tangents from the curve. The binary constant τ_2 of about 30.5 min., is obtained by taking 70 per cent of τ_ϵ. Nine of these binary time constants are marked off. At the last, the rise should be theoretically within 1/512 of the final value.

Fig. 8 gives the corresponding data for a particular 4-pole, 50-h. p., d-c. motor, operating at 1075 rev. per min., on a 230-volt circuit. The motor was operated for five and one-quarter hours steadily, at rated load, and then after 15 min. intermission, at 125 per cent of rated load, for two hours more. The black circles give

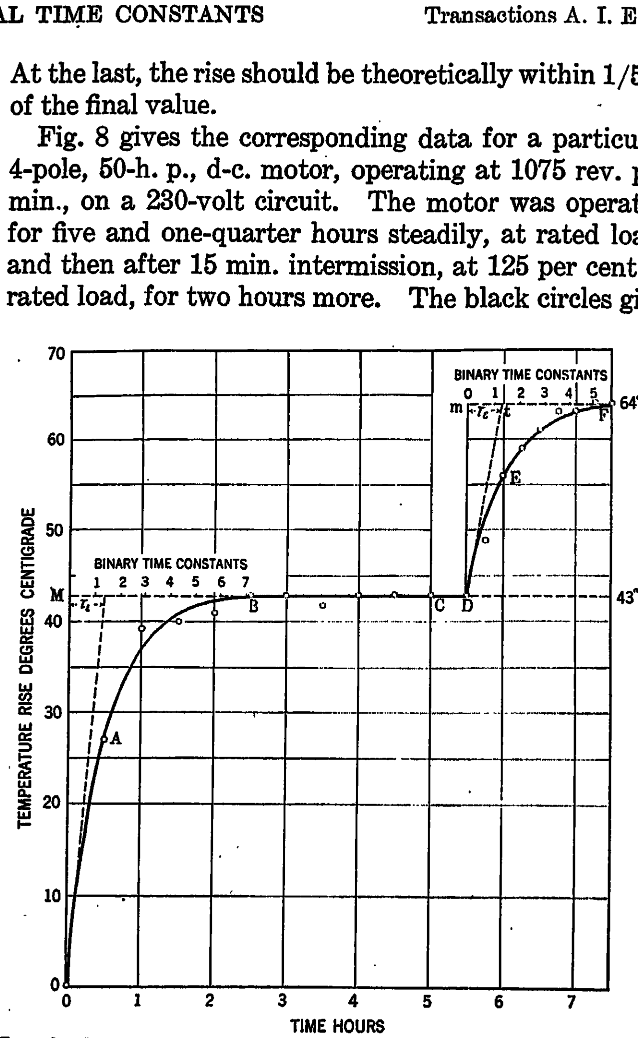

FIG. 8—TEMPERATURE RISE OF COMMUTATING FIELD WINDING IN D-C.-MOTOR

the observed temperature rise on the commutating field winding, by thermometer, above an ambient temperature of 15 deg. cent. at the beginning of the test. This ambient temperature rose steadily to 22 deg. cent. at the end of the run. The curve $O\ A\ B\ C$ is the computed time-constant curve, based upon an ultimate temperature rise of 43 deg. cent. The initial tangent $O\ T$ cuts this horizontal line at T, marking off an exponential time constant $M\ T = \tau_\epsilon$, of 30 min. The binary time constant was thus approximately 21 min. Seven binary time constants are marked off along $M\ B$, where the deficiency would be 1/128.

The overload-run temperature time-constant curve is $D\ E\ F$ based on a new ultimate rise Θ of 21 deg. cent. The initial tangent marks off an exponential time constant $m\ t = \tau_\epsilon$, of 30 min. as before, and five binary time constants are marked off along the ultimate-rise line $m\ F$. Half the ultimate rise should develop at 1, half the remainder at 2, and half the succeeding remainder at each successive binary time-constant interval.

It has recently been pointed out[5] that if after an

5. "The Temperature Rise of Electrical Machinery" by T. R. Rowlands, *The Electrical Review*, London, correspondence July 4th, 1924, page 11.

interval of t_1 hours from a starting point on an exponential heating curve like that of Fig. 3, the temperature elevation reached is θ_1, while after an interval of $t_2 = 2\,t_1$ hours, it has reached θ_2, then the ultimate rise should be

$$\Theta = \frac{\theta_1}{2 - \theta_2/\theta_1} \quad \text{deg. cent.} \quad \textbf{(47a)}$$

Correction for Change in Ambient Temperature During Run

If, as very frequently happens, the ambient temperature is observed to change during a run under steady load, some uncertainty develops as to the proper value to accept for the temperature rise Θ. The heat dis-

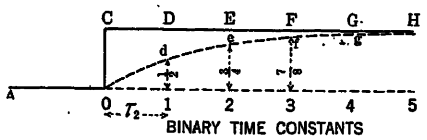

Fig. 9

sipated by the machine during the run tends to raise the ambient temperature. If the thermal time constant τ_2 were negligibly small, so that the temperature of the machine could follow impressed thermal forces without delay, the attained temperature elevation would always be equal to $T_m - T_a$; but there would be no need for carrying out a long continuous-load run, because such a machine would instantly reach its full temperature rise, on the application of the load. The longer the binary time constant, the greater the time needed for the machine to accommodate itself to a change in ambient temperature. Some correction of the observed temperature rise is therefore necessary for changes in ambient temperature. Such corrections should not require much computation, and should be

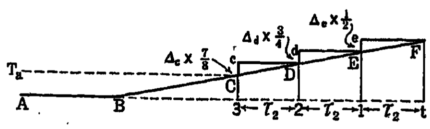

Fig. 10

easily applied; because with ordinary care, neither the change in ambient temperature nor the correction for it need be large.

In Fig. 9, the ambient temperature T_a, after pursuing the uniform stationary line $A\ B$, is supposed suddenly to rise by an amount Δ deg. cent. to C, and thereafter to continue unchanged with time, along the straight line $C\,D\,E\,F\,G\,H$. Commencing at O, the moment of change, binary time constants τ_2 are marked off along the time axis 1 2 3 4 5. After say three time constants have elapsed, the machine will theoretically have acquired at f, a thermal state corresponding to 7/8ths of the full change Δ, and no correction from the ambient temperature $T_a' = T_a + \Delta$ will ordinarily be required if Δ is small. The temperature elevation at any time after F will thus be

$$\theta = T_m - T_a' \quad \text{deg. cent. rise} \quad \textbf{(48)}$$

where T_m is the observed temperature of the machine. But immediately after the instant C Fig. 9, the ambient temperature should be reckoned as T_a and not T_a'; because the machine has not had time to respond. At D, after one time constant, the corrected value of T_a' would be $T_a + \Delta/2$. At E, it would be $T_a + 3\Delta/4$.

The conditions relating to Fig. 9 suggest the following simple approximate correction to be applied when the ambient temperature is changing. Suppose that the observed ambient temperature follows the lines $A\ B\ C\ D\ E\ F$ in Fig. 10, and that at the instant t, when the ambient is at F, and the temperature at some warm part of the machine under observation is then T_{mf}. Required the corrected ambient temperature T_{af}, so that the corrected temperature rise may be taken as

$$\theta_f = T_{mf} - T_{af} \quad \text{deg. cent. rise} \quad \textbf{(49)}$$

Find the variable-loss binary time constant τ_2 of the machine, as previously described. Measure off three of these time constants along the time axis, as at C, Fig. 10. Between C and D, the change was gradual; but assume that it occurred suddenly as $C\,c$. The corresponding sudden changes at D and E would have

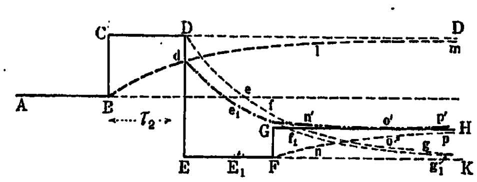

Fig. 11

been $D\,d$ and $E\,e$ deg. cent., respectively. Let the three disturbances be denoted as Δ_c, Δ_d and Δ_e, the signs of which are all the same in the case considered. Then

$$T_{af} = T_{ac} + \frac{7}{8}\,\Delta_c + \frac{3}{4}\,\Delta_d + \frac{1}{2}\,\Delta_e \ \text{deg. cent.} \quad \textbf{(50)}$$

In the case considered, with uniform ambient rise from C to F, the uncorrected ambient temperature would be $T_{ac} + 3\,\Delta_c$. The approximately corrected value would be, by **(50)**, $T_{ac} + 2\frac{1}{8}\,\Delta_c$.

In this approximate correction process, it is assumed that a change in ambient temperature which occurred 3 or more time constants back, has produced its full effect on the temperature of the machine.

Effects of Oppositely Directed Changes in Ambient Temperature

If the ambient temperature pursues in time the course $A\,B\,C\,D\,E\,F\,G\,H$, Fig. 11, changing its value suddenly at B, D and F, the effects successively produced in the inferred ambient temperature of the machine may be obtained by superposition. If the ambient temperature, after changing abruptly from B to C, were to

continue unchanged thereafter, it would follow the horizontal straight line $C\,D\,D'$, parallel to the time axis. The effective ambient temperature of the machine, with a binary time constant τ_2, would follow the time-constant curve $B\,d\,l\,m$, ultimately coinciding with $D\,D'$. Again, if the abrupt change from D to E in the minus direction, were followed by a stationary condition, the ambient temperature would pursue the straight line $E\,F\,K$. The effect of this negative change $D\,E$, acting alone, would be to produce the time-constant curve $D\,e\,f\,g$. The negative ordinates of this curve are now applied to the curve $d\,l\,m$, to produce the resultant curve $d\,e, f, g$. The third positive change $F\,G$, acting alone, would produce the time-constant curve $F\,n\,o\,p$. The positive ordinates of this curve, with respect to $F\,K$, are now applied to the previous resultant curve e, f, g, producing the final resultant $G\,n'\,o'\,p'$. The course of the effective ambient temperature is thus $B\,d\,e, G\,n'\,o'\,p'$, which happens to end close to the final actual ambient temperature H.

Approximate Correction of Ambient Temperature When Varying Oppositely

Applying the above principles to the general case of a slowly wandering ambient temperature, the changes of

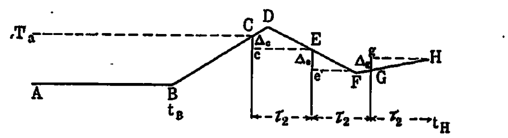

Fig. 12

which are kept relatively small, but have varying directions, we proceed as in Fig. 12; the method is the same as in Fig. 10, but the application is more general. Let $A\,B\,C\,D\,E\,F\,G\,H$ be the observed course of the ambient temperature with time. At the time t_H, corresponding to point H, it is required to find the approximate effective ambient temperature. Measure back three binary time constants along the curve, marking them off at G, E and C. It is assumed that T_{ac} the value at C, is the ambient temperature for correction. The changes in ambient $C\,c$, $E\,e$ and $G\,g$, occurring in each of the three time constants, are drawn. The hypothetical path of the ambient temperature is then $C\,c\,E\,e\,G\,g\,H$. It is assumed that the effects of these three sudden changes will be substantially the same as those of the wandering changes $C\,D\,F\,H$. Let the three hypothetical displacements be denoted by Δ_c, Δ_e and Δ_g, respectively. The first two have the $-$ sign and the last one the $+$ sign. Then

$$T_h = T_{ac} + \frac{7}{8}\,\Delta_c + \frac{3}{4}\,\Delta_e + \frac{1}{2}\,\Delta_g \text{ deg. cent.} \quad (51)$$

the proper signs of the Δ terms must be carefully followed.

In particular cases, a more elaborate correction formula may be called for. In such a case, the correction might be carried back to cover four binary time constants instead of three, and each time constant might be divided into two equal parts. There would then be eight perpendiculars of the type $C\,c$, $E\,e$ and $G\,g$, Fig. 12. Calling them $\Delta_a, \Delta_b \ldots \Delta_g, \Delta_h$, the factors to apply, with the proper sign in each case, may be taken as

$$\frac{15}{16}\,\Delta_a + \frac{9}{10}\,\Delta_b + \frac{7}{8}\,\Delta_c + \frac{4}{5}\,\Delta_d + \frac{3}{4}\,\Delta_e + \frac{13}{20}\,\Delta_f + \frac{1}{2}\,\Delta_g + \frac{3}{10}\,\Delta_h$$

Advantages and Disadvantages of a Long Time Constant in a Machine

A dynamo machine with a short time constant has the advantage that its temperature elevation follows closely upon the heels of impressed changes of load. The machine has a short thermal memory, and its previous history in service exerts a relatively small influence upon its instantaneous thermal state. If the ambient temperature changes, there is also less uncertainty as to the correction to be applied for the same, when measuring the temperature rise.

A machine with a long time constant has, however, one advantage in service. If the machine is called upon to supply a temporary overload, it may be able to do so without becoming overheated, by relying upon its heat storage capacity. If the duration of the overload is no more than one binary time constant, the overload will only be able, as we have seen, to develop half the ultimate temperature rise of that overload. Consequently, the duration that might be permitted to an overload increases with the time constant of the machine. It might readily happen that a central-station manager, desiring to purchase electric railway machines, under conditions where a certain overload period was expected, might decide in favor of a machine with a relatively long time constant, if in all other respects the machines submitted to his choice were substantially alike. It would be advantageous for him to utilize the longer time constant for carrying the overload, provided the proper maximum temperature limits were unexceeded. On the other hand, a long time-constant machine needs more careful supervision than a short-time-constant machine.

With equally effective ventilation, heavy machines tend to develop long time constants and light machines short time constants.

Complete Specification of Thermal Behavior in a Machine

Three quantities are theoretically necessary and sufficient for determining the thermal behavior of a machine under any assigned load schedule.

(1) The rated load temperature rise, or the maximum accessible temperature rise under continuous rated load.

(2) The corresponding temperature rise at some other steady load, such as 25 per cent below, or 25 per cent above, rated load. From this a chart like Fig. 1 can be prepared.

(3) The variable-loss time constant, and for practical purposes, preferably the variable-loss binary time constant.

No. 1, the rated-load temperature rise is regularly supplied by the manufacturer. No. 2 and No. 3 are not ordinarily stated for small machines. Indeed, it would be unreasonable to expect a maker to execute the tests on each machine for arriving at these two specifications. Nevertheless, the maker would probably know their numerical values approximately, from tests made on similar machines of the same type and size, so that there would probably be little difficulty in securing this information, within a degree of precision sufficient for operating purposes.

Summary of Deductions

(1) For operating purposes, it is requisite and desirable to have three specific thermal data concerning a machine, the rated load rise, another load rise, and the time constant.

(2) For practical purposes, the binary time constant is the best: but any one of the three time constants, exponential, binary or decimal, is readily convertible into the others.

(3) The binary time constant of a machine is the interval of time in which, when passing normally from one temperature to another under any steady load, the outstanding difference from the ultimate temperature is reduced by 50 per cent, or to one half.

(4) The curve of temperature of a machine under steady load, working from a steady ambient temperature, is approximately a time-constant curve.

(5) The true time-constant curve of temperature change against time makes a straight line on inverted arith-log paper when the final temperature is known.

(6) Changes in the ambient temperature observed during any steady-load run may be corrected for, in estimating the temperature rise, by a simple approximate process.

(7) A common value for the binary time constant of an ordinary generator or motor is less than one hour. In particular cases, however, and especially in air-cooled transformers, it may be several hours.

Discussion

V. Karapetoff: The use of exponential expressions for heating and cooling curves of machinery is quite old. I published a complete theory of such curves in my Laboratory Notes in 1906. See also my "Experimental Electrical Engineering," second edition, Vol. II, pp. 69 to 74. For a number of years we had at Cornell a student experiment on temperature rise in transformers using and checking exponential curves. However, the discrepancy with the observed results was so considerable that we dropped the theory and left only the measurements. As a result of study of various Institute papers on temperature rise in electrical machinery, I have come to the conclusion that the exponential theory, in the simple form used by the author, is inadequate to represent the observed facts even in simple stationary apparatus, not to speak of revolving machinery. I am therefore omitting this theory from the third edition of my laboratory book.

Some attempts have been made of late to differentiate among the various surfaces of a machine in their heat emission. See in particular G. E. Luke, A. I. E. E. Transactions, 1922, Vol. 41, p. 172; a bibliography is on p. 173. With two coefficients proposed by Mr. Luke, it is probably easier to approximate an experimental curve than with one time constant, only I would keep two separate expressions, with a finite difference of temperature between two parts of the machine, and not bunch them together.

It would hardly be wise to introduce the concept of an "over-all" thermal time-constant of a machine into our Standards, or even to encourage its use, thereby creating an impression that heating and cooling curves of machines follow a simple exponential law with any accuracy, until we have more assurance of this fact. It is desirable to continue both theoretical and experimental researches with a view to finding out discrepancies with the simple exponential law for different kinds of electrical apparatus, from a simple wire to a big turbo generator with forced draft. A somewhat more complicated law, with two or more parameters, will probably be necessary.

The value of the paper lies in (a) bringing the phenomena of temperature rise to the attention of the profession at large, in a clear and interesting manner; (b) showing the possibility of using a time constant to a base different from that of natural logs; (c) extending the previous treatment of the subject to the case when the losses in the machine are functions of the temperature; (d) calling attention to the desirability of including certain thermal constants among other characteristics of the machine.

I recently developed a theory of heating and cooling curves on the assumption of a finite variable temperature difference between the winding and the core, and hope to present it later before the Institute. It leads to a sum of two exponential terms with different parameters.

V. M. Montsinger: While I suspect that this paper, according to the subject, is intended to apply principally to dynamo electric machines, yet in several places, reference is made to air-cooled transformers and the inference is that oil-immersed self-cooled transformers are included in the subject under discussion, namely, that the manufacturer give to the operating engineer the thermal time constant as a part of the regular specification, and that this constant can be determined by the methods shown in the paper, and that the binary instead of the exponential time constant be used.

Now, let us see how the various methods of determining the time constant compare and apply to air-cooled transformers.

The author has, correctly, pointed out that when the copper and iron systems are thermally independent, it would not be necessary to resort to the graphical method of determining the time constant by drawing a tangent to the tested time-temperature line as shown in Fig. 3, or by plotting the deficiency temperature rise values against time on semi-log or arith-log paper as illustrated in Fig. 5.

Most oil-immersed transformers, with the exception of small units, where the windings are sometimes wound directly on the iron core, come within this class of apparatus in which the copper and iron parts are thermally independent. It has been my experience that when the weight of the materials, the losses and the final temperature rises are known, it is a simple matter to calculate the time constant and consequently be able to predict the temperature rise at any time during the change from one steady thermal state to another. If the final rise is known—and it usually is—for some given load, the final rise for any other load

can be estimated by any one who is experienced in this line of work as accurately as it can be determined by test.

Before giving the method or formula which I have used, I would like to point out the conditions under which the method of plotting the deficiency in temperature rise against time and the graphical method of drawing a tangent to the tested time-temperature line hold, and where they do not hold.

In order to analyze correctly the thermal conditions in a transformer it is well to remember that the temperature rise of the windings above the cooling medium is composed of two steps which are: (1) top oil temperature rise above room; and (2) winding temperature rise above top oil. Each of these steps must be dealt with separately. I have found that if the deficiency in temperature rise of the maximum or top oil is plotted against time on semi-log paper, the points fall in a straight line, regardless of the size of the transformer and amount of oil. This method, therefore, can be used to determine either the decimal, the exponential or the binary time constants for any given load. Likewise, the deficiency of the winding temperature rise over the top oil against time can be plotted as a straight line on semi-log paper. The winding rise over the ambient, however, does not lend itself to this method unless we neglect the time it requires the winding oil to become constant. But after the temperature rise of the windings over the oil becomes constant, which usually requires from 20 to 30 min. for the average transformer the points naturally fall in a straight line which line is parallel to the oil-rise curve. The line, therefore, naturally does not go through the point of zero time. For this reason, I do not believe that this method would be entirely satisfactory when applied to the winding rise over the room temperature.

The graphical method illustrated in his Fig. 3 is really not satisfactory for either the oil rise or the winding rise over the room temperature. The temperature of a large body of oil when loss is first liberated in it does not immediately start to rise but seems to have a certain time lag. This slight displacement of the curve at the beginning is enough to prevent the drawing of a tangent to the line with any reasonable degree of accuracy. Furthermore, I do not believe that this method will give entirely satisfactory results for any type of apparatus because so much depends on the first few readings being absolutely accurate, and again, even if the first test points were right it is rather difficult for one to know just where to draw the tangent. When applied to the temperature rise of the windings over the room the method falls down completely, for the reason that, as pointed out before, the winding rise increases very rapidly at first and becomes constant within 25 or 30 min. whereas the oil required approximately the same number of hours, *i. e.*, about 25 hours to become constant. Consequently, the tangent line is too nearly vertical and gives a very much smaller time constant than the correct one. And then, there is the further objection or disadvantage to both the graphical and deficiency methods as applied to the oil rise of transformers that the time constant for a given transformer has different values for different loads. The reason for this is that the temperature rise for constant conditions is never proportional to the loss but to the loss raised to some power less than unity, generally around 0.8. This means, therefore, that it would be necessary to make a test under all the possible loading conditions which is an impractical thing to do. The same objection holds for horizontal coil rises over oil, but not to vertical coil rises over oil, because in the latter case, the rise is approximately proportional to the loss.

I believe Dr. Kennelly will agree with me that where the thermal conditions are simplified as they are in a transformer and where temperature rise is not proportional to the loss, the most satisfactory method of determining the thermal time constants is by calculating them. At least, he gives this intimation in his paper at the bottom of page 143 and top of page 144.

The formula which I have used for several years with satisfactory results is similar to equation (27). The formula and the definition of the various factors in it are as follows:

$$\theta = \theta_f \left(1 - \epsilon^{-\frac{t}{B}}\right)$$

in which

θ = temperature rise at time t.

θ_f = final temperature rise with the resulting loss (*i. e.*, considering the increased copper loss due to increased temperature) for constant conditions.

B = Time constant

$$= \frac{C\,\theta_f}{L}$$

L = initial loss in watts

C = thermal capacity of mass being heated.

When calculating top oil temperature rise, and time t is expressed in hours:

$C = (2.96\,c + 3.5\,i + 105\,a\,G)/60$

where

c = lb. copper

i = lb. (core + 2/3 tank wt.)

G = gallons (U. S.) oil

a = ratio of average to max. vertical temperature gradient of tank (which ranges from about 0.75 to 0.95, depending on the design).

When calculating winding temperature rise above top oil, and expressing time t in minutes.

$$C = 2.96\left(\frac{A + a}{2\,a}\right) \times \text{lb. bare copper.}$$

where A and a are the insulated and bare cross sections, respectively, of the conductor in the winding.

No attempt will be made here to compare calculated and tested results as space does not permit. However, a paper giving a more complete discussion of this subject will be published in the near future by Mr. W. H. Cooney, who is associated with me in my work.

Dr. Kennelly strongly recommends that we discard the exponential time constant and use the binary time constant. I am in accord with the author in everything he says regarding the advantages of the binary over the exponential time constant, and would be glad to see the binary constant used.

In reference to the theoretical formula, that is, equation (47), which he gives for the cooling after load is removed, I would like to point out that it has been found that this formula cannot be used to calculate the cooling of the windings of oil-immersed transformers after shut-down. The reason seems to be that the temperature of the cooling medium for the windings, which, of course, is the oil circulating around and through the windings is undergoing a change in temperature at the same time the windings are cooling down. This subject was discussed fully in my paper on "Cooling of Oil-Immersed Transformer Windings after Shut-down," see TRANSACTIONS, A. I. E. E., 1917, p. 117. It was shown in this paper that the cooling of the windings was approximately a function of the watts per pound in the copper and of the time after shut-down. Even the insulation on the conductors, etc., does not have to be considered, consequently the problem is very much simplified. The rule for cooling after shut-down given in the present A. I. E. E. Standards was based on the data given in the above referred to paper. The formula is of the form:

$$\theta = 1.95\,W_c^{.7}\,(1 - \epsilon^{-at})$$

in which

θ = cooling in deg. cent.

W_c = Watts per lb. of bare copper

$a = 0.106\,W_c^{0.3}$

t = time in minutes.

With reference to the proposed method of correcting for a change in ambient temperature, during heat run, while I have

not tried it out, I feel that the most satisfactory means of doing this for a transformer is by the use of an idle unit having approximately the same amount of material as the unit under test. While the idle unit may not always give ideal results, it appears to be the most practical at present and it is recommended that it be used wherever possible.

In closing I would like to urge that we give serious consideration to the author's plea for the adoption of the binary time constant. There is no question but that it is preferable to the exponential constant. The fact that I advocate the method of calculating the constant for transformers does not affect in the least the question of the merits of the binary constant.

W. B. Kouwenhoven: Dr. Kennelly's paper on "Thermal Time Constants" is a valuable one, and his introduction of the "binary time constant" simplifies to some extent the use of temperature data.

I have had occasion to make quite a number of heat tests during the past several years, and I have found that the use of the theoretical temperature curves and thermal constants will often save considerable time in determining the rating of a piece of equipment. There are three principal uses to which a knowledge of these curves may be applied.

1. By means of the curve plotted from the readings secured in the first two to three hours of an eight-hour heat run, we can predict fairly closely the end temperature or maximum temperature rise that will be reached at the end of the test period.

2. By means of the same curve secured in the early part of the heat run we can determine whether we are applying the proper load for the eight-hour rating; and whether the machine will reach the specified temperature rise in the specified time.

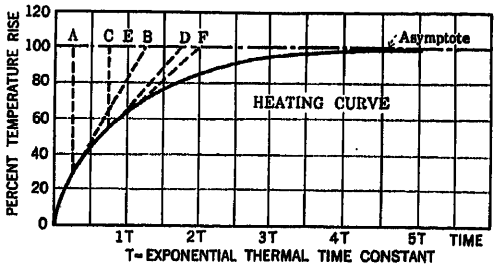

Fig. 1

3. We can determine the thermal constant of the machine, and as shown by Dr. Kennelly, use this in estimating what the machine will do under intermittent load or under some other load conditions.

Before discussing these applications it will be necessary to refer to the heating curve shown in the accompanying Fig. 1. The maximum or end temperature is the asymptote to this curve. It can easily be shown that the subtangents *A B*, *C D* and *E F* of the heating curve, referred to its asymptote as an axis, are equal.

In the actual heat run the desired load is applied, conditions being kept as nearly constant as possible, temperature readings are taken at frequent intervals and the temperature-rise curve is plotted. As soon as a sufficient length of curve has been obtained, a number of tangents to this curve are drawn and as nearly as possible, the height of the horizontal line that will give equal subtangents is determined. The point where this cuts through the Y axis gives the end temperature.

As Dr. Kennelly has shown, these subtangents are also equal to what is known as the exponential thermal time constant of the apparatus and in a length of time equal to five exponential thermal time constants the machine will reach 99.4 per cent of its maximum temperature rise. As soon as the constant is known we can not only predict the end temperature but also the duration of the heat run necessary to reach this final temperature. By this means we can tell whether the proper load is being applied for a given rating.

In using this theory in practise there are many cases where the heat curve is of such irregular shape that it is impossible to draw tangents and to obtain any idea of the constants of the apparatus under test. In other cases the early part of the curve is useless, but the middle portion may be of value and may permit us to obtain an idea of the end conditions.

My experience, however, has been that it is always worth while to attempt to apply the theoretical curve in making a heat run.

C. M. Laffoon: The simple exponential relation between temperature rise and time during the transient temperature period for any given constant-load condition is correct on the basis of the simplifying assumptions which were made. As a matter of fact, this exponential relation has been well known for a number of years. Designing engineers of electrical machinery have used similarly shaped curves as obtained from test or by calculation to determine the temperature rise of machines when operating under different load conditions.

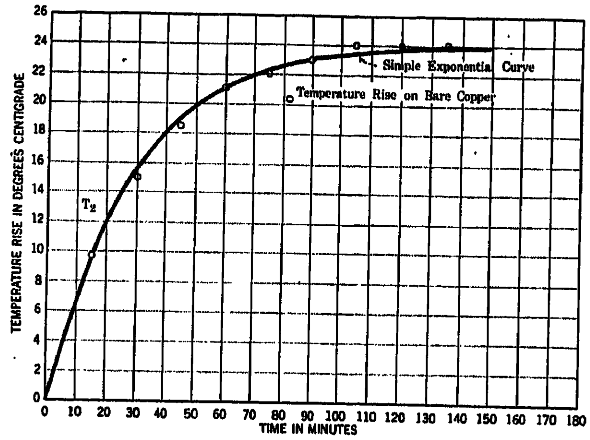

Fig. 2—Temperature Rise of 35,000 Kv-a. Turbo Generator

Generator rating—12,000 volts, three-phase, 60-cycles; operating at 75 per cent load, starting from 50 per cent load at constant temperature.

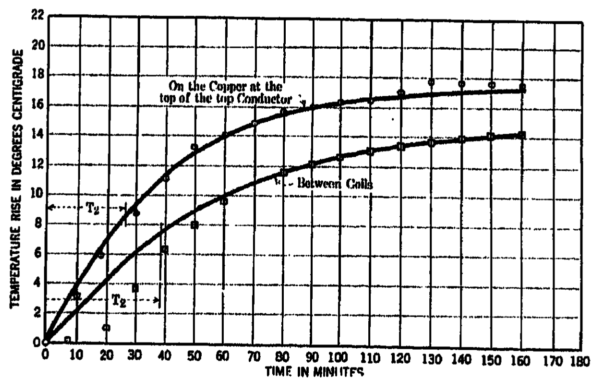

Fig. 3—Temperature Rise of 25,900 Kv-a. Turbo Generator

Generator rating—8000 volts, three-phase, 60-cycles; temperature rise under 75.3 per cent load, starting from constant temperature and 45 per cent load. Simple exponential curves used to fit experimental data.

At the present time, there is insufficient experimental data available on actual machines to determine definitely how far it is necessary to extend the analysis in order to obtain a satisfactory solution. If no simplifying assumptions are made and the analysis is based on the actual distribution of the losses, and

the losses are considered to be dissipated principally by conduction and convection, the solution becomes very complicated and involved. Fairly complete mathematical solutions have been worked out for the case of transformers and considerable progress is being made in determining a more complete solution for rotating machines.

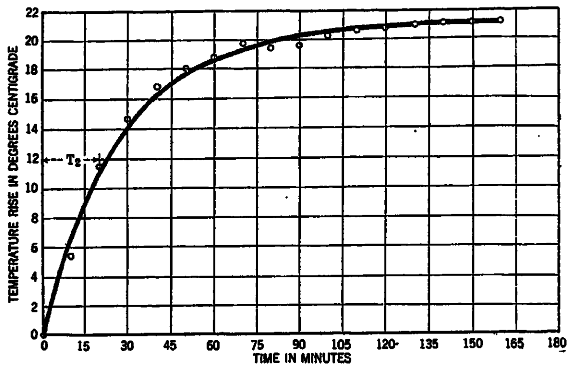

FIG. 4

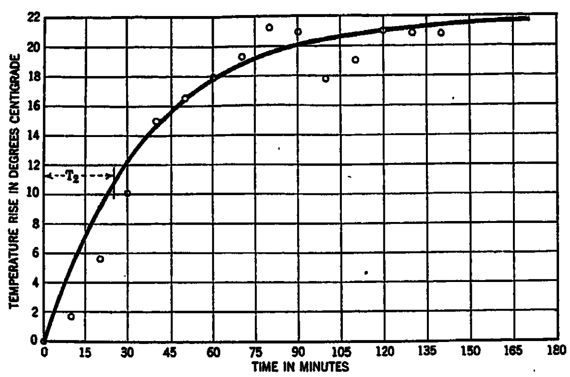

FIG. 5

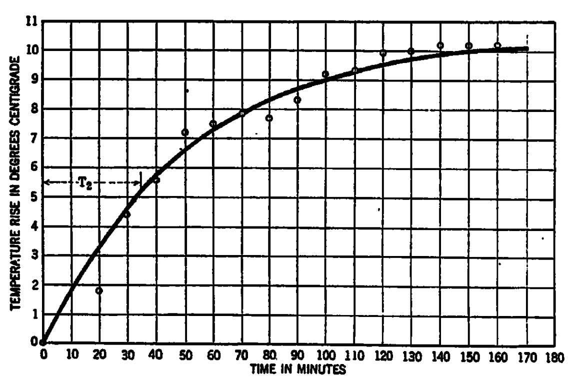

FIG. 6

FIGS. 4—5—6—TEMPERATURE RISE ON VARIOUS PARTS OF 25,000-KV-A. TURBO GENERATOR

Generator rating—66,000 volts, three-phase, 25 cycles; temperature rise under full load from constant temperature under 74.5-per cent load. Simple exponential curves are used to fit the experimental data.

Fig. 4 shows temperature at surface of top conductor as measured by thermo-couple

Fig. 5 shows temperature between coils

Fig. 6 shows temperature of field-winding, calculated from the resistance rise

The accompanying curves in Figs. 2 to 7 show the measured temperature rises of several large turbo generators at definite time intervals when operating under constant load conditions. The temperatures of the armature windings were obtained by imbedded temperature detectors which were placed in contact with the bare copper, as well as between coil sides. The temperatures of the field windings were obtained by the increase-in-resistance method. Simple exponential curves are superimposed on the test points. It is interesting to note that the simple exponential curves practically coincide with the test points for the case of the temperatures on the bare copper. In the case of the temperature detectors placed between the coil sides, the exponential curves fit the test points reasonably closely, as the temperatures approach sustained values, but during the early part of the transient state the test points drop appreciably below the exponential curves. This temperature dip or lag is due to the fact that there is an appreciable volume of insulation through which the heat, caused by the copper loss, must flow and its coefficient of specific heat is relatively large, as compared to that of copper.

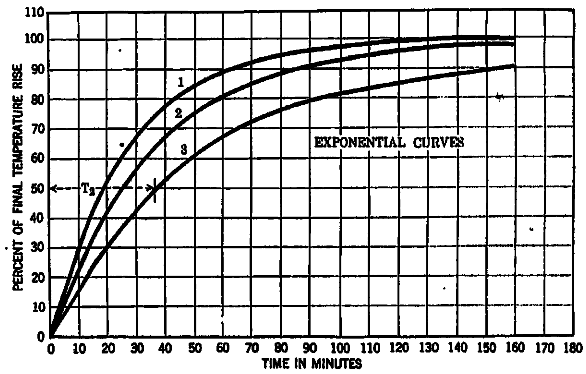

FIG. 7—TEMPERATURE RISE OF DIFFERENT PARTS OF 25,000 KV-A. TURBO GENERATOR

Generator rating—6600 volts, three-phase, 25 cycles, operating at 100-per cent load starting from constant temperature at 75 per cent load.
Curve 1 is temperature on bare copper
Curve 2 is temperature between coil sides of armature winding
Curve 3 is temperature of field winding

Another interesting and practical point to note for the last set of curves (Fig. 7) is that the thermal time constants are widely different for the different elements of a given generator. In the case of the 25,000-kv-a. turbo generator, a 50-per cent change in temperature rise corresponds to a binary time constant of approximately 20 min. for the bare copper of the armature winding, 26 min. for the temperature detectors between coil sides, and 38 min. for the field winding. (The values are based on the exponential curves.) Hence, in order to determine the rating of an electrical generator when operating under overload conditions, it is necessary to know the thermal time constants for both the armature and field windings. If exponential curves are to be used in determining the temperatures, the curves must be based on measured values obtained near the final or sustained values.

G. E. Luke: It is of special interest to electrical engineers to be able to visualize problems such as transient heating phenomena in terms of similar electrical problems. The similarity can be seen even down to the constants of the equation. Thus, the time constant in the electrical circuit is L/R and in the thermal circuit it is k/s; (L) and (k) are similar in that both represent constants of the circuit which are proportional to the stored energy. Likewise (R) and (s) both are factors proportional to the loss in the circuit.

In a previous paper[2] presented to the Institute on a similar subject, an equation of the same form as Dr. Kennelly's equation was derived, also this equation was shown in the form:

$$t = 2.3\,\tau_e \operatorname{Log}_{10} \frac{\theta_0 - \theta_a}{\theta_0 - \theta} \tag{27}$$

This is the general form where θ_a is the temperature rise at

2. Heating of Railway Motors in Service and on Test-Floor Runs, by G. E. Luke. TRANSACTIONS, A. I. E. E, 1922, p. 165.

start, the other constants being the same as in Dr. Kennelly's paper. This equation will satisfy any heating or cooling condition and can be easily solved with the aid of an ordinary slide rule. This equation can also be solved graphically with the use of special nomographic charts.

An interesting application of these heating equations is in the solution of the heating problem where a continuously applied heating and cooling cycle is found. Such a cycle is common in elevators, hoists, street cars, etc. Thus the curve in Fig. 8 herewith gives a typical cycle where one load is applied at alternate intervals of (t_1 – hrs.) and another load for the other (t_2) intervals. θ_0' would be the continuous temperature rise if the first load should be applied continuously and θ_0'' the rise on the basis of a continuous application of the second load. The

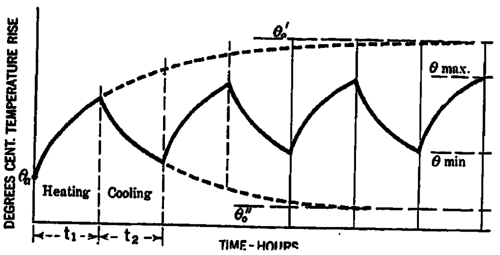

FIG. 8—TEMPERATURE CYCLE OF MOTOR ON VARIABLE LOAD

two points of interest on such a load are the maximum steady rise θ_{max} and the minimum rise θ_{min}.

It can be shown that

$$\theta_{max} = \frac{\theta_0'\,(1 - e^{a_1}) + \theta_0''\,(e^{a_1} - e^{a_1 + a_2})}{1 - e^{a_1 + a_2}}$$

$$\theta_{min} = \frac{\theta_0''\,(1 - e^{a_2}) + \theta_0'\,(e^{a_2} - e^{a_1 + a_2})}{1 - e^{a_1 + a_2}}$$

where

$$a_1 = -\frac{t_1}{T_e'} \text{ and } a_2 = -\frac{t_2}{T_e''}$$

if

$$a_1 = a_2 \text{ and } \theta_0'' = 0$$

Then

$$\theta_{max} = \frac{\theta_0'}{1 + e^{a_1}} \text{ and } \theta_{min} = \frac{\theta_0'\,e^{a_1}}{1 + e^{a_1}}$$

As mentioned in the paper, the heating of various parts of an electric machine is not uniform, and much of a machine's masses absorb very little heat. Thus in Figs. 7 and 8 of the paper if the time constant is calculated on the basis of the total weight of the motor, its value will be over twice the tested value. On the other hand, if the heating curve for the d-c. motor armature copper is calculated its time constant will be considerably less than that shown on Fig. 8. Hence the use of such heating curves and formulas for industrial motor applications may be dangerous unless they are based upon that part of the machine that has the smallest time constant, that is, heats up the quickest.

A good example of the way various parts of a motor increase in temperature is shown on in the accompanying Fig. 9. This is the tested heating curve of a typical d-c. railway motor. Thermocouples were placed in the windings, both field and armature (with slip rings). The approximate exponential time constants based upon the tangent to the curves at the origin are 0.9, 2.1 and 2.2 hours for the armature copper, field copper, and surface respectively. The same constants based upon the time corresponding to the 63.2 per cent temperature rise are 1.1, 1.8 and 2.0 hours respectively. Thus the initial rate of heating of the armature copper is over twice the rate of the field copper. This is primarily a question of ventilation and heat flow. The time constant of the copper alone imbedded in the slots is very small, being considerably less than the values given on the author's Fig. 7 and Fig. 8. This influences the "hottest spot" and when short overload of 125 to 150 per cent of full load are found the permissible time of loading should be based upon the time constant of the "hottest spot" and not upon the average heating of the motor as found by thermometers. This point was explained in greater detail in the paper previously mentioned.

It is believed that there is some error in the last sentence on page 141, since the curve referred to, in Fig. 4, is not a straight line. The equation

$$\theta = \frac{\theta_1}{2 - \theta_2/\theta_1} \qquad \textbf{(47a)}$$

holds true only if the origin is taken at the time of start ($\theta_a = 0$) and hence will not be true in general when $\theta = \theta_a$ for $t = 0$, similar to curve Fig. 3.

In the summary in the last sentence on page 147, it is assumed that air-cooled transformers means oil-insulated, self-cooled, instead of air blast transformers, since the latter usually have a small time constant.

E. B. Paxton: I believe that information based on the thermal time constant as proposed by Dr. Kennelly would be very useful if it could be made available in such simple form as to be readily used and understood, not as a rating but as information supplementary to a rating.

Electrical machines could often be applied to better advantage if there were some simple means available of knowing, if only approximately, their operating capabilities under various service conditions such as, for example, low ambient temperatures or short-time operation. An understanding of what might be expected of a machine under such conditions would often result in the installation of a smaller machine.

A criticism has been made that each different part theoretically has its own time constant and that, due to the heat of each part affecting the others, the effect is much more complicated than indicated by the simple logarithmic relation. However, test results show, as pointed out in the discussion of Mr. Laffoon, that the temperature rises of individual parts often follow quite closely a law based on a single time constant and it is only necessary, as a basis for information to serve as a guide in operation, to regard the limiting or hottest part.

O. E. Shirley: The binary heating constant gives a very

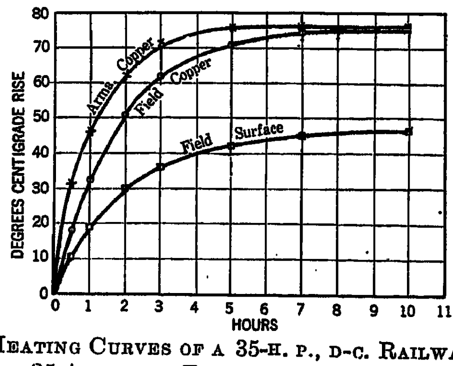

FIG. 9—HEATING CURVES OF A 35-H. P., D-C. RAILWAY MOTOR 300 VOLTS, 35 AMPERES; TEMPERATURE BY THERMO COUPLES

convenient method of determining to a practical degree of accuracy the heating of a rotating machine under a considerable number of varying conditions. I agree with the view brought out in some of the previous discussions that the results may vary from the actual tests under many conditions, but the method may be used to considerable advantage where approximate heating is required for machines operating on duty cycles.

A number of machines, for which heating curves are available, have been checked to obtain an idea of typical values of the

binary constant. The constants for several machines are as follows:

Cycles	Kv-a.	Volts	Binary Constant (Minutes) Thermometer	Temp. Detector
25	1250	6400	30	29
25	2450	13200	38	39
25	2450	6600	23	20
50	2500	11000	25	19
60	1250	2300	15	16
60	2500	4000	20	12

The agreement between results for thermometer and temperature detector is very good except for the last machine which shows a considerable difference between the constants by thermometer and by temperature detector.

In determining the constants from the test heating curves there were some appreciable variations in the time for the different intervals, but by taking the average values for the higher increments, (*i. e.*, from ½ to ¾ rise, and from ¾ to ⅞ rise) constants could be obtained that would give curves checking the tests to a very good degree of accuracy.

The binary time constant as proposed in the paper appears to increase with the size of the machine, and also with the thickness of insulation.

The machines, which have been checked, indicate that the range of values for the usual design is 15 to 45 min., but this maximum value would very likely be exceeded in very large units and with high voltages.

As I see it, the principal advantage of the proposed constant is to give an easily remembered way of approximating the effect of a varying load from the curve of final temperature and load and the time constant which can be obtained from one test heating curve. The binary constant is very much easier to apply than the exponential constant and the accuracy of the results is the same.

A. M. MacCutcheon: This subject has a tremendous interest if we intend to use our knowledge of the time constant in connection with design and test. If it is to be given to the user of electrical apparatus for assistance in applying electrical machinery, I think it is exceedingly dangerous for the reasons that Prof. Karapetoff and others have pointed out. It is difficult to determine accurately, and it is likely to vary, I believe, even for duplicate machines. We have known duplicate machines to vary 15 degrees in measurable temperature under continuous full-load operation. If there are differences like that in machines which are intended to be identical, how can a man in the field applying electrical apparatus intelligently use a time constant if he makes its application general?

I believe that successful application of electrical machinery will result to a much greater degree if more study is given on the application problem, which is a very difficult one, and I think it would be dangerous to mislead power-apparatus users into thinking that something we can give them on a time constant is going to solve all of their difficulties. I am referring particularly to smaller sizes of power apparatus, from 250 h. p. down. When a large installation goes in possibly a time constant could be well used, because on such an installation as that, the company putting it in has high-grade engineers probably to consider the time constant and to make absolutely certain that they are applying it well. But for general applications of power motors, it seems to me extremely dangerous to attempt to give to the user a time constant which we aren't sure we can accurately predict; we aren't sure that we will not have to divide it into ten parts, as Professor Karapetoff has said, and we aren't sure that it will always be the same constant for duplicate machines.

F. D. Newbury: A possible inference from Dr. Kennelly's paper is that this mathematical relationship between temperature and time affords a practicable method of defining the rating of a rotating machine for any combination of loads and times of loading. If a machine can be given, as one of its characteristics, a single time constant, then its rating can be completely and exactly defined by stating its temperature rise at one loading, applied continuously, in combination with this time constant.

Mr. Laffoon in his discussion points out two important facts that greatly complicate this practical application: the agreement between the exponential curve and the test results is close only when it is possible to measure temperatures at or near the seat of the loss; and each machine has a time constant for each major loss-producing element. In a motor, for example, a time constant, determined from externally-applied thermometer readings, would be safe to use only when the test is continued long enough for the motor to approximate its final temperature; for short times the measured temperature is considerably lower than the temperature obtained from the correct exponential curve. In all machines of any size, and notably in turbine generators, the time constant of the armature windings and field windings will be quite different. Finally, the several time constants of each size of each type of machinery will vary depending on design proportions affecting distribution of losses, heat storage capacity, and ventilation constants.

The application of this matter to the rating of electrical machinery is not as simple as it might seem from the paper under discussion, and a word of caution in connection with the practical use of the exponential relationship for the application of motors and other machinery to service conditions may not be out of place.

M. L. Keller: (by letter): Although I agree with Professor Kennelly that the binary time constant as suggested by him is the much more practical term for a simple explanation, the exponential time constant has nevertheless some importance from the practical standpoint. Instead of saying that after the lapse of one exponential time constant the deficiency is $1/2 \times 0.7182$, which for practical purposes is meaningless, we may define the exponential time constant as the time necessary to reach the maximum temperature if no heat is dissipated during the period of observation (See Fig. 10 herewith).

This definition is very simple and has also other advantages. For instance: we are able to determine the tangent of the temperature curve at the start without reference to the later period of the temperature rise.

The differential equation as set forth, is:

$$p \, . \, d\,t = G \, . \, c \, . \, d\,\theta \tag{1}$$

p = watt; G = weight (gramm), c = specific heat

Considering the temperature-rise in degrees centigrade per second of a conductor with the conductivity (ρ), specific weight (γ) and specific heat (c), at a current-density (i) amp/mm², we find

$$\theta\,^\circ/\text{sec} = i^2 \frac{\rho}{\gamma \, . \, c} \tag{2}$$

Therefore, knowing the ultimate temperature-rise to be $\theta_0 = \frac{p_0}{s}$ [Prof. Kennelly's equations (20) also (3)], the exponential time constant is:

$$\tau = \frac{\theta_0}{i^2 \dfrac{\rho}{\gamma \, . \, c}} \tag{3}$$

This means that if we know the conductor and ultimate temperature-rise at a given current density, the value of the time constant can be easily obtained.

Applying equation (3) to electrical machines, it will be found that the time constant thus determined is in most cases too small. The reason for this lies in the fact that we must take into account the thermal effect of the materials surrounding the conductor. It is apparent that the insulation around the conductor increases the time constant of the same, acting as a means of

storage of heat energy produced by the conductor. It may be added that with the increase in the heat-absorption qualities and conductivity of the insulating material, the active time constant also increases. As it takes time for the surrounding materials of the conductor to absorb heat, it is evident that the increasing influence of the insulation, etc., upon the conductor depends on the rapidity of the temperature rise, which in turn depends on the load. (For load periods the duration of which is greater than about 3/5 times the full-load time constant this need not be considered.) In other words: the time constant of electrical machines is not only a function of material and construction, but indirectly also of load periods and load variations as well.

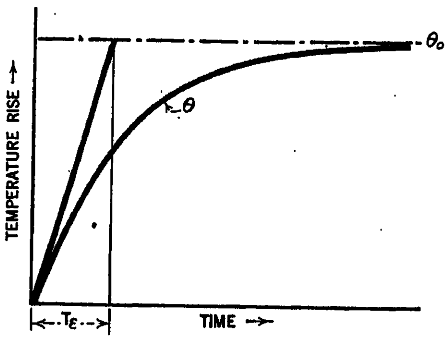

FIG. 10

This fact is of extraordinary importance where sudden load changes in quantity and duration occur and where the total time constant of the machine is large in respect to the inherent time constant of the heat producing conductors (for instance, transformers).

The above conclusions explain Prof. Kennelly's statement, why with reference to Fig. 7 in his paper "the computed curve has the tendency to exceed the temperature curve near the middle of the run." The inherent time constant of the motor analyzed by this curve—judged by its size—will probably be half (or even less) the given full-load time constant. For a general rule, it may be said that the full-load time constant of a motor depends on the conductor plus about 50 per cent on the insulation in the slot.

Prof. Kennelly mentions as a main reason causing deviation of the temperature curve from a strict time-constant curve "that the windings of a machine with their copper losses form one thermal system and the steel structures with their iron losses, form another and that each tends to modify the behavior of the other."

This conclusion is of much importance and brings forth the question: How do these different thermal systems affect the hottest—*i. e.* critical spot—of the machine and how are we able to consider them in a simple manner? As the critical spot—which is the main point of interest to us—lies at the inner surface of the insulation next to the conductor, the simplest way seems to be, to take as a basis for any temperature consideration the time constant of the conductor which is identical with that of the critical spot and to consider the other heating sources, iron losses and adjacent conductors, as distributing factors.

Recapitulating, we may say, the smallest time constant of an electrical machine is the inherent time constant of the conductor. It increases through the thermal influence of the surrounding materials (insulation, etc.). The amount of this influence depends on the rapidity of heat changes in the conductor, this means that the time constant is also a function of the load. The heating influence of the iron losses and adjacent conductors are to be considered as disturbing factors in respect to the critical spot.

F. Wenner (by letter): In the analysis of this problem Prof. Kennelly points out the similarity between it and the variation of an electric current in an inductive circuit. This procedure enables us to handle certain parts of these two problems practically as if they were the same problem, and to apply information we may have concerning one to the other. To those who are required to work with both problems this results in a real saving of mental effort.

As this subject is one of considerable importance and the method of analysis is new it may not be out of place to point a little more in detail the way in which thermal time constants may be used to advantage in the solution of a particular problem. It will be conceded without question, I think, that an exact mathematical solution is too complicated for general use. Further, nothing is gained by carrying the computations to four or five significant figures, since the data is reliable to a few significant figures only and the fundamental assumptions which must be made are only approximately correct. What is wanted is a simple procedure by which calculations can be made to two or at most three significant figures.

In the consideration of a similar though somewhat more complicated problem in electromechanics, it was observed that a considerable simplification could be made by using what, for the lack of a more appropriate designation, we have called a "weight" curve. This is the response and recovery curve with the time scale reversed and with all ordinates multiplied by such a factor as to make the area under the curve unity.

In a dynamoelectric machine heat is developed within the material of the machine simultaneously with the energy output and is dissipated slowly. If a load is applied for an instant only, the mean temperature increases suddenly and then decreases very slowly following approximately a curve similar to that shown in Fig. 4 of the paper. The variation in mean temperature, therefore, may be considered to follow the curve shown here in Fig. 11. The instantaneous rise shows the response and the part beyond the peak shows the recovery. The weight curve corresponding to this response and recovery curve is of the form shown in Fig. 12. In the latter the origin of time is taken at the end. Any convenient unit of time such as the minute or the hour may be used. However, there is a very considerable advantage in taking one of the time constants of the machine under consideration as the unit of time. When this is done both the response and recovery curve, and the weight curve are independent of the constants of any particular machine. In fact all responses and recoveries of whatever nature, which take place according to the laws assumed to apply here have the same weight curve.

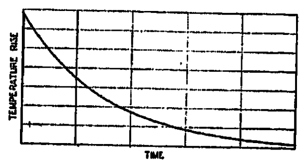

FIG. 11—TEMPERATURE RESPONSE AND RECOVERY
Load applied for an instant only

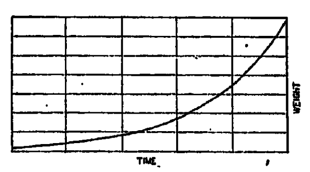

FIG. 12—WEIGHT CURVE CORRESPONDING TO RESPONSE AND RECOVERY FIG. 11

If the unit of time is taken as the binary time constant then the weight at any time t is equal to $\text{Log}_e 2\ t^3$. This gives the following data which may be used in constructing a weight curve.

Time	Weight
0	0.693
−¼	0.583
−½	0.490
−1	0.347
−2	0.173
−4	0.043
−8	0 003

Such a curve is shown drawn to scale in Fig. 13.

Now let us see how we may proceed to predict the temperature rise for a particular machine when subjected to a particular assumed load. Let this load be as shown in the curve of Fig. 14, and let the problem be the determination of the temperature rise at the end of 4¼ hours. As Prof. Kennelly has pointed out, three specific thermal data are required. These are the ultimate temperature rise for two different constant loads and the time instant. The former give two points from which a straight line curve similar to that shown in Fig. 1 of the paper may be plotted and the latter serves to convert the time scale of the assumed load to the time scale of the weight curve. We then have three curves for use in making the calculations, namely: 1. The assumed load curve with a proper time scale, 2. The ultimate rise in temperature curve of the type shown in Fig. 1 of the paper, and 3, the weight curve as shown in Fig. 13 herewith. With these three curves, a slide rule, and pencil and paper, the calculation is carried out as follows: First, read from the load curve the average load from $-\frac{1}{4}$ to 0. Second, from the ultimate temperature rise curve read the rise in temperature corresponding to this load. Third, read from the weight curve the mean weight from $-\frac{1}{4}$ to 0. The ultimate rise in temperature, times the time (in this case ¼) gives the temperature rise at the time 0 which would result from the assumed load if applied only from $-\frac{1}{4}$ to 0. This figure is the first of a series to be added. Proceeding in the same way the temperature rise which would result at the time 0 from the assumed load if applied only from $-\frac{1}{2}$ to $-\frac{1}{4}$ is obtained. This constitutes the second term of the series to be added. Third, Fourth, etc., terms of the series are obtained in the same way, except, as the weight becomes smaller, an interval of time equal to a half, a full, or even two time constants may be used. The series is extended back so as to include the time at which the load came on, or to a time previous to which the load does not appreciably affect the temperature at the time 0. The addition of the series gives the temperature rise to be expected at the time 0 (in this case at the end of 4¼ hours) for the assumed load. The calculation is simple, straightforward, and readily gives an accuracy comparable with the accuracy of the fundamental assumptions and data based upon these assumptions.

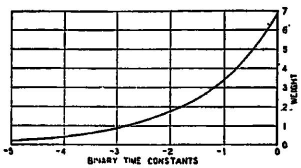

FIG. 13—WEIGHT CURVE CORRESPONDING TO RESPONSE AND RECOVERY SHOWN IN FIG. 11
Time expressed in binary time constants

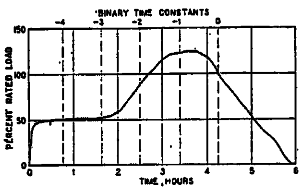

FIG. 14—ASSUMED LOAD IN TERMS OF RATED LOAD

In this discussion I have tried to show how the technique acquired in work upon a more complicated problem in electromechanics may be used in computing the temperature rise in a dynamoelectric machine, when subjected to a variable load. Fundamentally the procedure followed is the same as that outlined in the paper. It is, however, given in more detail with the hope that this may lead to a fuller appreciation of the advantages to be gained by the use of time constants, whether thermal, electrical, or mechanical, depending upon the nature of the problem under consideration.

A. E. Kennelly: The discussion has brought out many interesting and valuable points, including new mathematical material by Messrs. Luke and Wenner.

It has been pointed out that while the graphs of temperature elevations in different parts of dynamo machines may deviate considerably from exponential or time-constant curves, yet in many cases, under certain restrictions, they approximate to true time-constant curves sufficiently closely for many practical purposes. Valuable graphical data to this end have been contributed by the speakers. Moreover, it seems to be unanimously agreed that whenever a time constant can be used in practise, it should be a binary time constant.

It has been well shown that in view of deviations of heating curves from strict time-constant curves, it is unsafe to assign a time constant to a heating curve by means of a tangent drawn through the origin. Perhaps the most practical way to determine the mean binary time constant from a curve of observed steady-load heating against time, is to measure the temperature elevation θ, at successive uniform time intervals t_1. Then if θ_1 is the temperature rise in one such interval, and this has reached θ_2 by the end of the second interval, we have as the binary constant τ_2 on the assumption of exponential rise during these two intervals ($2\, t_2$)

$$\tau_2 = \frac{0.30103\, t_1}{\log \dfrac{\theta_1}{\theta_2 - \theta_1}} \quad \text{time}$$

Thus, if the chosen time interval t_1 happened to be equal to τ_2, the rise θ_1 in one such interval would be just double the further rise $\theta_2 - \theta_1$ in the second interval, and $\log \left(\dfrac{\theta_1}{\theta_2 - \theta_1}\right) = \log 2 = 0.30103$. By repeating the computation over several such successive pairs of intervals, we can easily ascertain whether τ_2 is nearly constant throughout, or whether a mean value of τ_2 can be accepted.

Squirrel-Cage Induction-Motor Core Losses

BY T. SPOONER[1]
Member, A. I. E. E.

Synopsis.—An experimental method is presented for segregating the various no-load losses of a squirrel-cage induction motor. It is shown that for insulated rotor bars the losses consist principally of stator fundamental-frequency losses, rotor surface losses and eddy-current losses in the rotor bars due to radial and tangential slot-leakage fluxes in the rotor slots caused by reluctance pulsations in the air-gap between stator and rotor teeth. The pulsation losses vary about as the square of the air-gap induction and about as the 1.2 power of the frequency. The surface losses for the particular slots described are from ¼ to 1/3 of the total pulsation losses. The pulsation losses are approximately the same whether or not the rotor bars are connected by end rings, and are approximately equal to the stator fundamental-frequency losses.

Introduction

In a recent edition of Behrend's "The Induction Motor," it is stated that certain assumptions with reference to induction-motor core losses are "justifiable only on account of a profound ignorance of the causes of core losses and the magnitude of these losses." This may be true to a degree in connection with the core losses of this type of machine, particularly concerning the high-frequency pulsation losses due to the pulsations in the tooth flux as the rotor teeth pass by the stator teeth. Some experimental work has been done to segregate these high-frequency losses for wound-rotor induction motors, but so far as we know no data have been published showing a segregation of the various types of core loss for squirrel-cage induction motors. In this paper we propose to give such data for a special experimental machine.

Types of Core Loss

Squirrel-cage induction-motor no-load core losses may be classified as follows:

1. Fundamental-Frequency Losses
   a. *Stator*
      1. Yoke hysteresis
      2. Yoke eddy current
      3. Tooth hysteresis
      4. Tooth eddy current
   b. *Rotor*
      1. Core hysteresis (Slip frequency)
      2. Core eddy current (Slip frequency)
      3. Tooth hysteresis " "
      4. Tooth eddy current " "
2. High-Frequency Pulsation Losses
   a. *Stator*
      1. Surface
      2. Tooth pulsation
      3. Copper eddy current
   b. *Rotor*
      1. Surface
      2. Tooth pulsation
      3. Rotor bar eddy current
      4. High-frequency damping current in secondary winding.

In the squirrel-cage construction some of these losses will be negligible. For instance, there will be no appreciable fundamental-frequency losses in the rotor under ordinary no-load conditions due to the very small slip. High-frequency damping currents in the rotor bars may exist not only to damp out the tooth pulsations but also to give more nearly a sine-wave distribution of flux in the air-gap. The value of these latter currents will depend upon the number of slots and type of stator winding used. The relative magnitude of these various losses will be discussed later in connection with the test results.

Test Apparatus

The induction motor used in this investigation was a special 3-phase, 4-pole, 60-cycle, 440-volt machine of about 35-h. p. capacity. The stator and rotor punchings had the following dimensions:

Stator
Punchings O. D. 19 in. I. D. 13-19/32 in.
Length 6 inches
No. Slots (open) 60
Air-gap 0.047 in.

Rotor
Punchings O. D. 13½ in. I. D. 4 in.
Length 6 inches
No. Slots (nearly closed) 78
See Fig. 1 for slot dimensions
The rotor bars were 0.484 in. by 0.3 in. by 9.25 in.

The rotor was direct-connected to a 3-h. p. d-c. motor, the set being supplied with 3 ball bearings. The induction-motor, stator and rotor punchings were made of enameled 0.0172 in. one per cent silicon sheet steel. Special pains were taken to remove the burrs before enameling the punchings. The induction-motor stator was supplied from a 440-volt 3-phase generator direct-connected to a d-c. drive motor, the set having a considerably larger capacity than the special motor under test. The d-c. motor of the special set was supplied from a storage battery.

Test Methods

Three sets of tests were made: (1) with no windings on the rotor, (2) with rotor bars in position but no end-rings, (3) squirrel-cage winding complete with brass end-rings. In order to eliminate the uncertain variations due to laminations being short-circuited by the rotor bars, as is the condition for the standard construction, slightly smaller bars than normal were used and insulated by means of 3-mil fish paper.

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

For the first two tests the results were obtained by the method described in the appendix of a previous paper.[1] The method consisted in measuring the a-c. input to the induction-motor stator and the d-c. input to the drive motor, subtracting the friction and windage, brush and $I^2 R$ losses from the inputs and plotting the curves as indicated by Fig. 2 for a given a-c. voltage and frequency and various rotor speeds. a, b, c, d, e, is the a-c. input and o, f, g, h, i, is the corresponding d-c. input. c is the midpoint of b, d, and g is the midpoint of h, f. $O - c'$ is the stator fundamental-frequency loss for the given conditions.

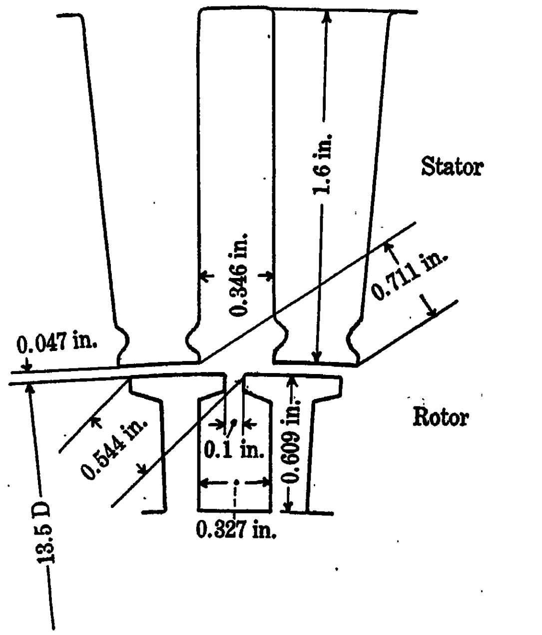

FIG. 1—DIMENSIONS OF STATOR AND ROTOR PUNCHINGS

$O - g'$ is the high-frequency pulsation loss. O, g, j, is the pulsation-loss curve for various rotor speeds and the given applied stator frequency and voltage. For a theoretical discussion of this method see Alger and Eksergian.[2]

Due to voltage limitations of the supply generator an average air-gap induction of 22 kilolines per sq. in. only was obtainable by this method. In order to extend the range direct current was applied to the stator and the input to the drive motor noted for various speeds and stator fields, as described in the previously-mentioned paper on Surface Losses. The fundamental-frequency rotor losses as obtained from the test with alternating current on the stator were extrapolated and subtracted from the total, giving the pulsation losses. Results up to 32 kilolines, air-gap induction, were obtained.

For the third set of tests with the complete squirrel-cage winding we had to resort to a slightly different method of procedure. Due to the large torque produced by the rotor windings, speeds only slightly different from synchronism could be used. Therefore tests were made with rotor speeds varying in general not more than 0.2 of a cycle from synchronism. Two points above synchronism and two below were measured

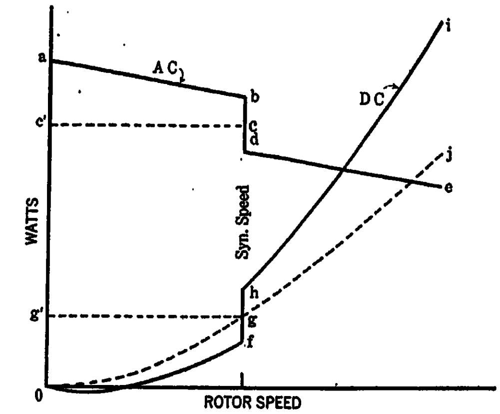

FIG. 2—INPUTS TO MOTOR-GENERATOR SET, SHOWING METHOD OF SEGREGATING LOSSES

and the results plotted as indicated by the typical results of Fig. 3. It will be noted that even for the very slight departure from synchronism of 0.1 of a cycle, the inputs and outputs are considerable. The very greatest care was necessary in measuring these

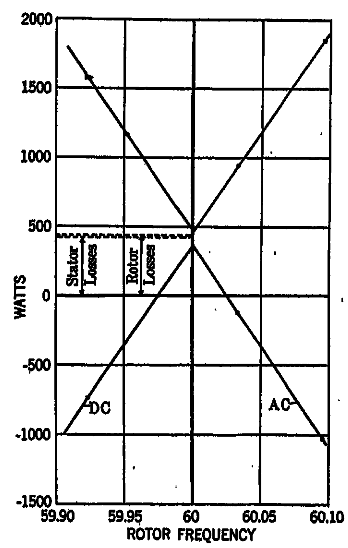

FIG. 3—A-C. AND D-C. INPUTS TO MOTOR-GENERATOR SET WITH SQUIRREL CAGE ROTOR 60 CYCLES

inputs and outputs in order to obtain reliable results, and without a very steady d-c. and a-c. supply such tests are impossible. Three synchronous speeds were used, namely, 20 cycles, 40 cycles and 60 cycles, and as wide a range of voltages as possible. It may be noted that with the scale of abscissas used for Fig. 2 the lines for Fig. 3 would be nearly vertical.

1. Surface Iron Losses with Reference to Laminated Materials, by T. Spooner and I. F. Kinnard, presented before the A. I. E. E. in Philadelphia, Feb., 1924.

2. Induction Motor Core Losses. JOUR. A. I. E. E., Oct., 1920, p. 906.

In order to measure the slip frequency, use was made of a coil which was placed around a single rotor tooth for the purpose of measuring the tooth-flux pulsations. This was connected to a d-c. galvanometer through slip rings. Whenever the d-c. galvanometer completed a cycle of oscillation, the rotor completed a cycle of slip. The time was measured by means of a stop watch.

Test Results

Fig. 4 gives the pulsation losses for a synchronous frequency of 20 cycles and the three conditions,

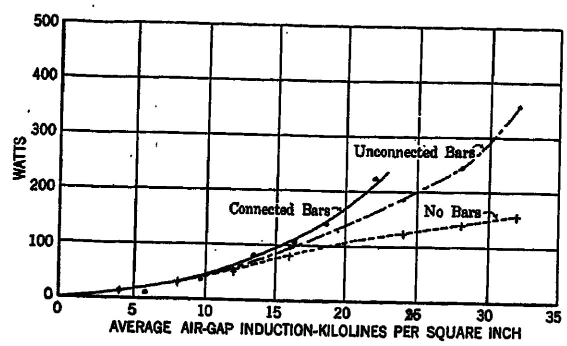

Fig. 4—Pulsation Losses—20 Cycles

namely, no rotor bars, bars unconnected and bars connected. With no bars it will be seen that the rate of increase of pulsation losses decreases for the higher air-gap inductions. This is due to saturation of the teeth and consequent reduction of the tooth pulsations as explained in a previous paper.[3] The difference between these results and the losses with the unconnected bars is due to high-frequency slot leakage fluxes caused by saturation of the teeth which produced eddy-current losses in the bars. These fluxes are

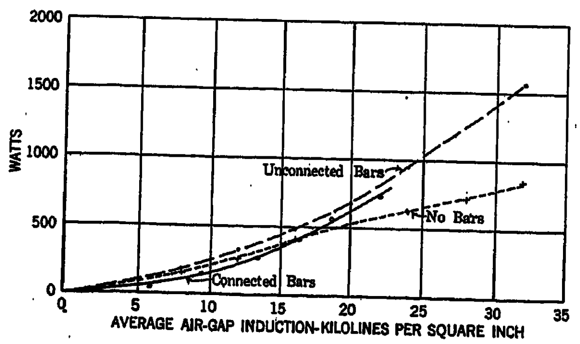

Fig. 5—Pulsation Losses—60 Cycles

both tangential and radial and while the density is only a few hundred lines per square inch, the frequency is so high that the losses become considerable.

With the bars connected, the high-frequency tooth pulsations produced currents in the bars which tended to damp out the tooth-pulsation fluxes. In fact, they do this so effectively that at 60 cycles no tooth-pulsation voltages could be detected by any a-c. indicator available when connected to the previously-mentioned coil surrounding a rotor tooth. With no rotor bars this voltage was of the order of 3 volts and with the bars connected was less than 0.1 volts. It will be noted that in spite of this damping out of the high-

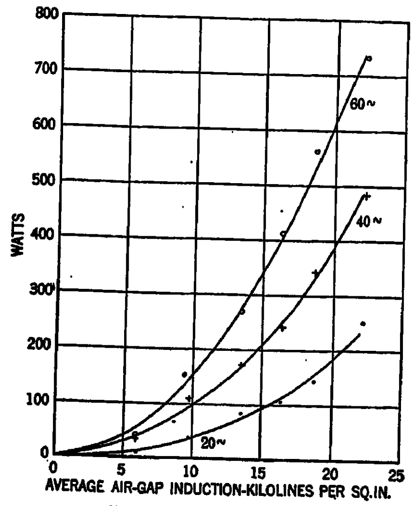

Fig. 6—Pulsation Losses—Insulated Bars Connected

frequency fluxes, the pulsation losses are not much changed. This will be discussed later.

Fig. 5 shows the corresponding 60-cycle pulsation losses. 40-cycle losses are intermediate.

Fig. 6 shows a comparison of the pulsation losses for

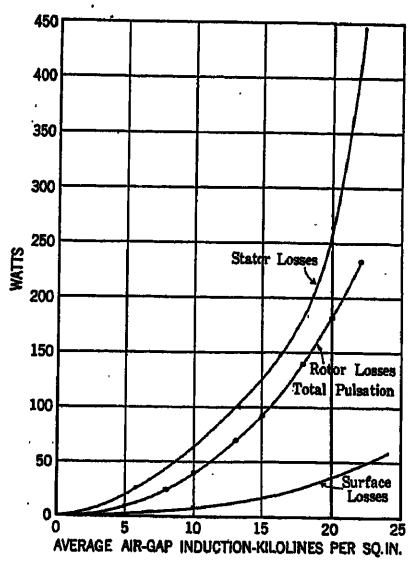

Fig. 7—Insulated Bars—Connected—20 Cycles

the three fundamental frequencies, 20, 40 and 60 cycles with the bars connected. It will be noted that in spite of the difficulties of test the points line up fairly well.

Fig. 7 shows the no-load losses for 20 cycles applied to the stator with the bars connected. It will be seen that the pulsation losses are nearly as great as the fundamental-frequency stator losses. These pulsa-

3. Tooth Pulsation in Rotating Machines, by T. Spooner. Jour. A. I. E. E., July, 1924, p. 646.

tion losses include the surface losses, which are given by the lower curve and were plotted from the results previously obtained on smooth core rotors.[1] Since the rotor slots are nearly closed, it is assumed as a first approximation that the surface losses for the rotor are the same as for a smooth-core rotor. Also due to the nearly closed rotor slots it is assumed that

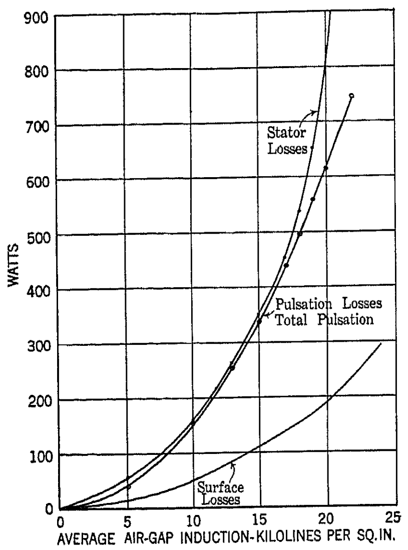

Fig. 8—Insulated Bars—Connected—60 Cycles

the stator tooth-pulsation and surface losses are negligible. The rapid increase of stator losses at high inductions is due to the fact that the stator yoke was rather narrow and began to saturate, thus forcing flux into the solid frame and hence rapidly increasing the stator losses.

Fig. 8 gives the motor core losses corresponding to an applied frequency of 60 cycles. The relative losses

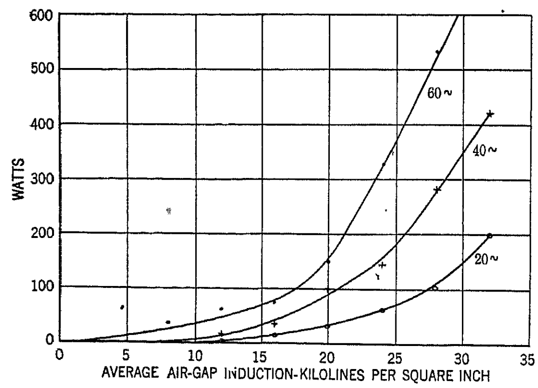

Fig. 9—Rotor Copper Eddy-Current Losses. Bars Insulated and not Connected

are much the same as for 20 cycles. The 40-cycle results lie in between the 20- and 60-cycle losses.

Fig. 9 gives the eddy-current losses in the unconnected copper bars as obtained by subtracting the test values of set No. 2 from set No. 1, namely, the difference between the losses with the unconnected bars and no bars.

Fig. 10 gives the stator losses with the connected bars and with no bars on the rotor. The increased losses in the former case should be noted.

Discussion of Results

The fundamental-frequency stator losses can be calculated in the ordinary way from the known fundamental magnetic characteristics of the material. If appreciable burrs are present or the enamel on the punchings is not in good condition, eddy losses will be

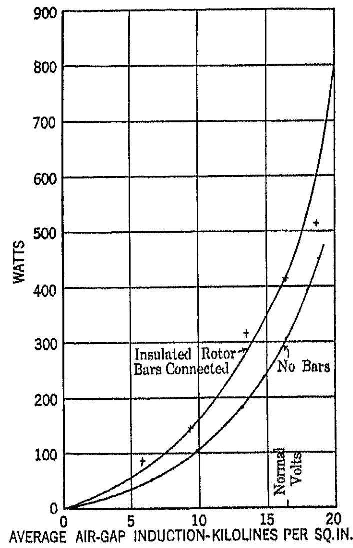

Fig. 10—Stator Losses—60 Cycles

present which are not subject to accurate calculation. Also, if it is desired to go to such a refinement, the additional losses in the yoke material due to the elliptical field may be calculated by the method developed by Alger & Eksergian.[2] This correction is fairly small in this case.

Since we are dealing only with synchronous speeds

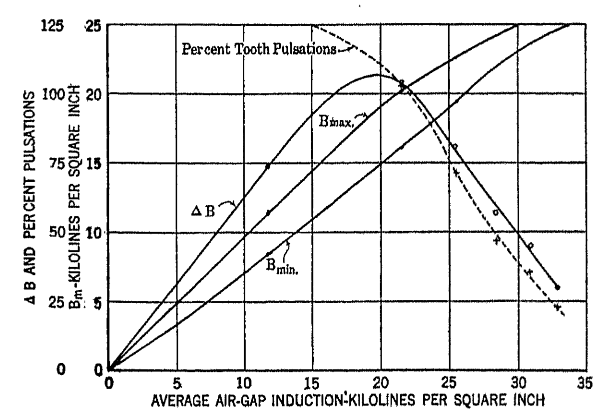

Fig. 11—Maximum Tooth Inductions (B Min.) and Tooth Pulsations

the rotor slip-frequency losses are zero. In any case they would be fairly small except under starting conditions. They may be calculated in the ordinary way for any desired slip.

As previously mentioned, for the nearly-closed slots

it is assumed that the stator surface and tooth-pulsation losses are negligible. We have left then only the rotor high-frequency losses. The method of calculating the surface losses has been described previously.

Referring now to the rotor tooth-pulsation losses, we shall give only brief attention to the condition with no rotor bars, since the losses do not correspond to working conditions though the results are of interest in connection with other types of machines having wound rotors. These tooth-pulsation losses are the result of high-frequency pulsations which penetrate the whole length of the rotor teeth and are caused by variations in the reluctance between the individual stator and rotor teeth at the air-gap. The method of calculating these pulsations has been given previously.[3] The actual magnitude of the tooth pulsations was measured ballistically with direct current applied to the stator as shown by Fig. 11. The effect of saturation in reducing the flux pulsations should be particularly noted. Inductions are expressed in net section of iron and correspond to the position of maximum air-gap flux. Remembering that for a 60-cycle fundamental frequency the tooth pulsations have a frequency of 1800 cycles, it can easily be seen how these high-frequency fluxes produce the hysteresis and eddy losses which were observed. Referring to Figs. 4 and 5, the effect of saturation on the tooth-pulsation losses for no bars should be compared with the tooth-pulsation data of Fig. 11.

For the second case with the unconnected bars we have in addition to tooth-pulsation losses, eddy-current losses in the bars due to radial and tangential slot leakage fluxes. As the teeth begin to saturate, high-frequency flux pulsations pass down the rotor slots and produce eddy currents in the rotor bars. Also, when a rotor tooth is opposite a stator tooth and therefore in position of minimum air-gap reluctance, the adjacent teeth are in a position of greater air-gap reluctance, therefore at a lower magnetic potential. Due to the saturation of the first tooth flux crosses the air-gaps to the teeth at lower magnetic potential, thus giving rise to tangential leakage fluxes of the same high frequency as the radial fluxes. When a tooth is in a position of maximum air-gap reluctance the tangential leakage fluxes flow in the opposite direction across the slots.

The magnitude of these slot-leakage fluxes was measured ballistically and found to be only from one to two hundred lines per square inch, even at fairly high tooth inductions. Nevertheless, due to the large section of the copper bars and high frequency, losses of several kilowatts would have been produced at the higher inductions with 60 cycles applied to the stator if there were no skin effect. Due, however, to skin effect, these eddy losses were much reduced, giving the values actually observed. The magnitude of these leakage fluxes can be calculated roughly from the permeability of the tooth material and the variations in tooth air-gap reluctance but the details will have to be reserved until a later date.

For the third case with the rotor bars connected we arrive at further complications. As it has been shown experimentally that the tooth pulsations are reduced to small values, it might at first be assumed that since the pulsation losses remain nearly the same, the high-frequency circulating currents in the copper bars give rise to losses which are approximately equal to the decreased iron losses in the teeth.

The magnitude of these high-frequency currents was calculated for an average air-gap induction of 22 kilolines and found to be about 100 root-mean-square amperes maximum. The corresponding losses were calculated using Field's Method[4] of calculating the effective resistance and it was found that this would account for only a comparatively small percentage of the observed losses. It was therefore concluded that these losses were the result of the eddy-current losses in the copper due to increased tangential slot-leakage fluxes, due to the increased magnetomotive force between adjacent bars. Since pulsating flux can no longer flow down the teeth to compensate for the varying tooth air-gap reluctance, a higher magnetomotive force exists between the adjacent teeth and as a consequence more leakage flux passes tangentially across the upper part of the slots, producing higher eddy losses than existed with the open-circuited rotor bars and thus compensating for the decreased iron losses in the teeth. Moreover, these tangential slot-leakage fluxes will be of considerable magnitude at lower tooth inductions since we would have the same effect produced by the short-circuited bars as would be produced by saturation of the teeth. The radial slot-leakage fluxes probably would not be greatly altered by the short-circuited bars.

The rate of increase of these losses with induction and frequency is interesting. As previously reported,[1] the surface losses increase about as the square of the air-gap induction and as the 1.5 power of the frequency. For the closed-bar rotor, the pulsation losses increase about as the square of the induction (a little less at the lower inductions and a little greater at the higher). The pulsation losses increase about as the 1.2 power of the frequency and if the surface losses are subtracted, the pulsation losses increase about as the first power of the frequency.

The increase in fundamental-frequency stator losses with closed rotor bars (Fig. 10) is probably due to the following cause. With no rotor bars the field form at the air gap is rather flat. In the presence of the squirrel-cage rotor at synchronous speed there is no fundamental frequency current in the rotor bars, but there are higher frequency currents which circulate and tend to give a sine wave distribution of flux in the air-gap, thus increasing the maximum stator-tooth

4. A. B. Field, Proc. A. I. E. E., Vol. 24, 1905, p. 659.

inductions and probably accounting partly at least for the increased fundamental-frequency stator losses.

Conclusions

The important no-load losses for a squirrel-cage induction motor having open stator slots and nearly closed rotor slots are, then, the fundamental frequency stator hysteresis and eddy-current losses in the teeth and yoke. (These may be altered somewhat due to change in field form resulting from harmonic currents in the squirrel-cage winding). There are also, of course, $I^2 R$ losses in the stator windings due to the magnetizing current.

In the rotor we have surface losses, eddy losses in rotor bars due to tangential and radial slot leakage fluxes and $I^2 R$ losses in the rotor bars due to high-frequency damping currents. The varying magnetomotive force due to the air-gap tooth-reluctance pulsations produces tangential slot-leakage fluxes which give large eddy losses in the bars but under conditions of large skin effect, namely, the eddy currents are concentrated near the surface of the bars. There are also, of course, certain losses in the iron due to high-frequency leakage fluxes which can be estimated only very roughly.

When the rotor bars are not insulated from the core new conditions arise which we hope to consider at a later date. Uninsulated bars are, of course, the standard practise.

In conclusion, we may say that for the ordinary squirrel-cage induction-motor the rotor pulsation losses are of the same order of magnitude as the stator fundamental-frequency losses and that the pulsation losses are approximately the same whether or not the rotor bars are connected.

Discussion

P. L. Alger: Mr. Spooner's paper extends by one step our knowledge of the various kinds of core losses which we have been considering for several years in Institute papers. First, we described the line-frequency losses; then, recently, Mr. Spooner described the surface losses; and now the pulsation losses are being attacked. These last losses, though, are the most complicated and the most difficult to understand of all those that occur at no load, and, consequently, it is worth while to point out some of the things that are not mentioned in Mr. Spooner's paper; which, nevertheless, are very important in this connection.

The first point I have in mind is an explanation of Mr. Spooner's statement that he finds more loss with the bars short-circuited than he can account for. I believe the reason is that when the bars are short-circuited in the rotor, the induced squirrel-cage currents shove the pulsation of flux back into the stator. That is to say, with the bars open the rotor flux pulsates; when the bars are closed the rotor flux is held constant and, consequently, the flux is forced to pulsate in the stator. Thus, losses of high frequency are produced in the stator in addition to those occurring in the rotor, and these extra iron losses account, in my mind, for the additional losses occurring beside the losses calculated to be in the copper itself.

In the second place, I will describe a convenient means of measuring the tooth-frequency losses applicable to completed machines, which Mr. Spooner has not mentioned. By measuring the running-light slip of an induction motor, the total torque developed by the flux is determined. For, the torque of the motor is proportional to the slip at light loads, and the design data give a reasonably accurate value of the torque per rev. per min. of slip. When the motor is running light, the total torque developed is absorbed in overcoming the friction and windage losses and tooth-frequency losses. The tooth-frequency losses, which are due to the passage of the rotor past the stator teeth, are precisely analogous to friction losses in their manner of origin. Like all friction losses, they must be supplied from the source of power that causes the motion, which, in this case, is the fundamental electromagnetic torque of the motor. From this, it is evident that by subtracting the friction and windage watts from the synchronous torque developed by the motor when running light, as indicated by the observed value of slip, there is obtained the value of the part of the core loss due to the tooth pulsations and surface losses. The value of tooth-frequency loss so obtained must be increased by the amount of power supplied by the rotor hysteresis torque, as obtained by calculation.

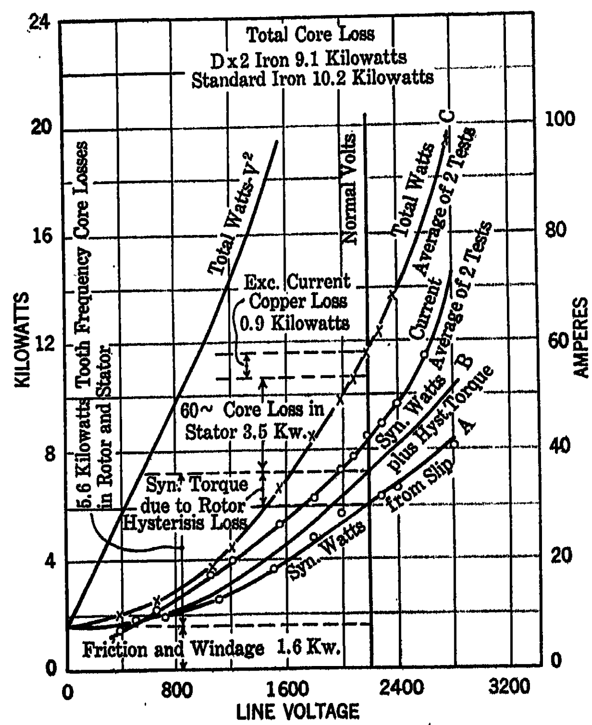

Fig. 1

The advantage of this method of measuring tooth-frequency losses is that it forms a means of segregating core loss into its two major portions as a routine part of the commercial testing. To illustrate the results obtained by the method and to make clear my third and last point, there are shown herewith three diagrams illustrating the test results on three dissimilar motors. Fig. 1 shows the total core loss as a function of line voltage on a 400-h. p., 60-cycle, slip-ring motor. There are also shown curves of the synchronous watts obtained from slip measurements, and of the synchronous watts corrected by adding the hysteresis torque. With these data available, the separation of the total loss into its components is easily made, as indicated in the figure. In this particular motor, the line-frequency core loss was only 3.5 kw. out of a total core loss of 9.1 kw., showing the motor to have an exceptionally large percentage of tooth-frequency losses.

The remaining two figures, Figs. 2 and 3, illustrate my third and last point, which is the possibility of varying the tooth-pulsation losses without affecting the major electrical character-

istics of the machine, by varying the ratio of primary to secondary numbers of teeth. When the numbers of teeth on the two sides of the air-gap are nearly equal and when the rotor slots are partly closed, each rotor tooth always spans approximately one stator tooth and one stator slot, so that its flux remains practically constant. However, when the number of rotor teeth becomes large, each rotor tooth first embraces the flux from a

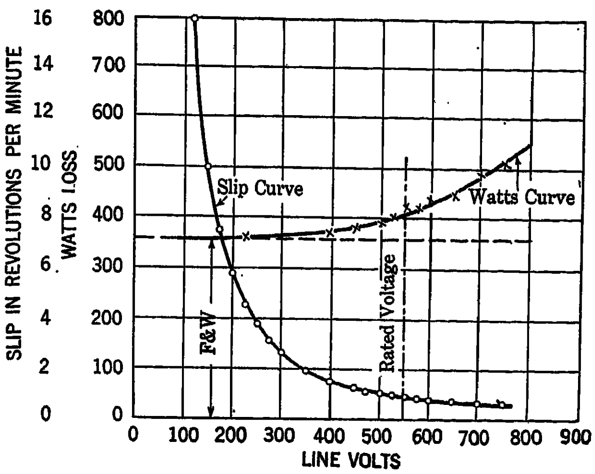

FIG. 2

stator tooth, and then that of a stator slot, and so on, so that the flux pulsates very considerably. The relations to be expected between the flux pulsations and the tooth ratios are given at some length by Chapman in the *London Electrician* for August, 1916. Here it need only be said that with a large tooth ratio the pulsation losses are high, and with a nearly one-to-one ratio the losses are small.

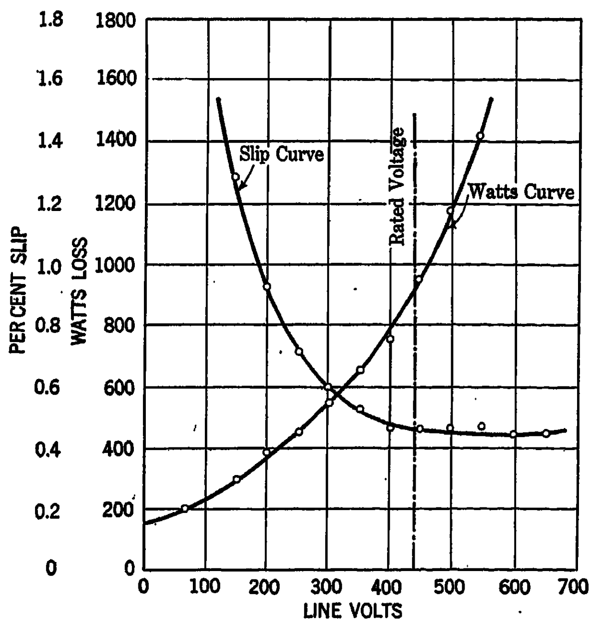

FIG. 3

Fig. 2 shows the running-light slip and the synchronous-watts curve on a 30-h. p. motor with 72 and 69 slots. It will be seen that the ratio of the tooth-frequency losses to the friction and windage is very small, indicating a very low total core loss. Fig. 3 shows a similar pair of curves on a 20-h. p. motor with a comparatively large ratio of teeth, and with magnetic wedges in the rotor. This motor has a tooth-frequency core loss of five times as much as its friction and windage, corresponding to a very high total core loss.

The obvious reason for not designing squirrel-cage motors with nearly one-to-one ratios of slots and, consequently, with low core losses, is that such a design makes the machine noisy. When a combination of slots that avoids both noise and core loss is obtained, it will be found that standstill locking or synchronous crawling will occur, so that as yet no means of solving the problem other than a good old-fashioned compromise has been discovered.

H. Weichsel: Any designer of electrical machines will

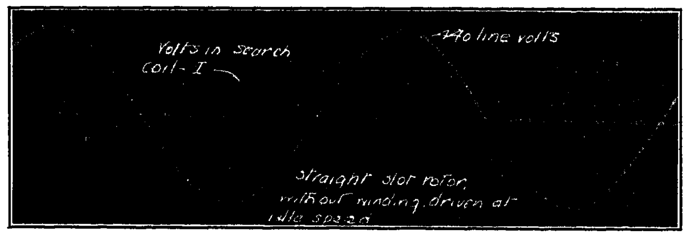

FIG. 4

welcome and appreciate the valuable contributions which Mr. Spooner has given to the engineering fraternity upon several occasions on the subject of iron losses. The phenemona which are responsible for the so-called iron losses in electrical machines are extremely complicated and are the result of a large number of factors entering into this problem. Mr. Spooner has described in various articles some of the most important factors. May I take this opportunity to point out some conditions which have not received general attention.

If a search coil is placed over a stator tooth of an induction motor and the induced voltage in this coil is recorded by an oscillogram, the following phenomena will be observed:

If the rotor is driven at approximately synchronous speed from an outside source, while the stator winding is connected to the supply line and the rotor is not provided with a winding, then certain high-frequency oscillations will be recorded, such as are shown in Fig. 4. If the rotor is then provided with a squirrel-

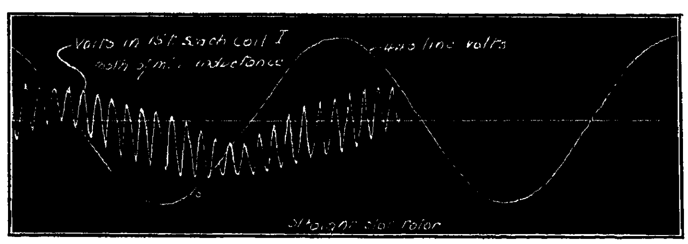

FIG. 5

cage winding and the machine is allowed to operate idle under its own power, it will be noticed that the fluctuations of the voltage in the search-coil have greatly increased, as shown by oscillogram Fig. 5. This increase of the oscillations is the result of the damping currents flowing in the squirrel-cage windings.

These damping currents, as pointed out by Mr. Spooner, attempt to reduce the fluctuation in the rotor teeth. But these currents not only react on the rotor teeth but also on the stator teeth and in this latter member the fluctuations are increased. The oscillograms Figs. 4 and 5 were taken on a machine with semi-closed slots in the stator and rotor and no skew in either member.

Figs. 6A to 6C, inclusive, represent a rotor and stator punching. The stator is assumed to have open slots and the rotor is shown

with semi-closed slots. In Fig. 6A the maximum possible flux will pass through the rotor tooth No. 1. In the position pictured in Fig. 6B, the number of lines passing through rotor tooth No. 1 have decreased and in the position represented by Fig. 6c, the number of lines passing through the rotor tooth No. 1 are a maximum.

A voltage will be set up in a loop of the squirrel-cage winding surrounding the tooth No. 1. The frequency of this voltage will be such that one cycle is completed during the time required for the rotor tooth to move one stator slot pitch. The current which is caused by this voltage to flow through the loop surrounding the rotor tooth No. 1 will be 90 electrical degrees displaced from its voltage. This current is a maximum when the rotor and stator tooth coincide and is zero when rotor tooth is in position *B*. (See Figs. 5 and 6.) The current is a maximum again when the rotor tooth has reached position *C*. All stator teeth which at a certain moment lie in the region of maximum induction of the main field, have the same magnetic polarity and are of practically the same field strength. During the short time required for the rotor to move one rotor slot pitch, the magnetomotive forces acting on adjacent stator teeth due to the stator winding can be considered as constant and of equal polarity and strength.

The whole arrangement pictured in Fig. 6 may be considered as a short-circuited high-frequency induction generator, where the stator forms the exciting member and the rotor coils represent the short-circuited induced winding.

At the first thought it might appear to be advisable to design the rotor punching of an induction motor with an open slot stator in such a manner that the fluctuations in the rotor teeth become zero. Such a condition can readily be obtained when the relation exists as shown in Fig. 7.

An arrangement as per Fig. 7 gives less losses than an arrangement corresponding to Fig. 8 on basis of equal induction and equal dimensions of the magnetic circuit. However, a machine built in accordance with Fig. 8 requires for the same maximum horse power less magnetic lines than a machine of equal dimensions built in accordance with Fig. 7.

As the losses decrease roughly with the square of the number of lines under otherwise equal conditions, it can be readily seen that in spite of the less favorable relations from the fluctuation point of view in Fig. 8 the total losses of the machine built according to Fig. 8 may be less than the total losses in a machine built according to Fig. 7. The general tendencies of the laws governing the relative changes between the necessary magnetic lines and the relative changes of the losses are approximately represented in Fig. 9. Experience has shown that generally the minimum iron losses for a given machine are obtained when the ratio of rotor tooth crown to stator slot pitch is 70 to 75 per cent, while the stator tooth crown is about 40 to 50 per cent of the stator slot pitch.

It may also appear that skewing the rotor member would have a beneficial effect on the losses of a machine. It is quite readily seen that when the fluctuations in a stator tooth are of a symmetrical nature, when the rotor is not skewed, it is possible to eliminate the fluctuation in a stator tooth by skewing the rotor one rotor slot pitch. Experience has shown that a machine with skewed rotor shows a reduced fluctuation in the voltage induced in the search coil of a stator tooth. Nevertheless the idle losses of this machine are practically unchanged. This can probably be explained by the fact that the skewing of the rotor does not influence the fluctuation of the individual stator laminations but only produces a steady flow of the total magnetic lines passing through one stator tooth. The skewing of the rotor member has further the effect of increased leakage and, therefore, for equal strength of the machine the number of lines must be increased. The only real beneficial effect obtainable by skewing the rotor is a decrease of noise and perhaps more uniform torque during starting.

The additional losses in a machine, especially the surface losses, are greatly influenced by machining operations on the magnetic circuits. Cases are known to the writer where careful reassembling of the same iron decreased the losses very materially. Other instances are known in which turning or grinding the rotor surfaces effected the losses noticeably, but it was also found that

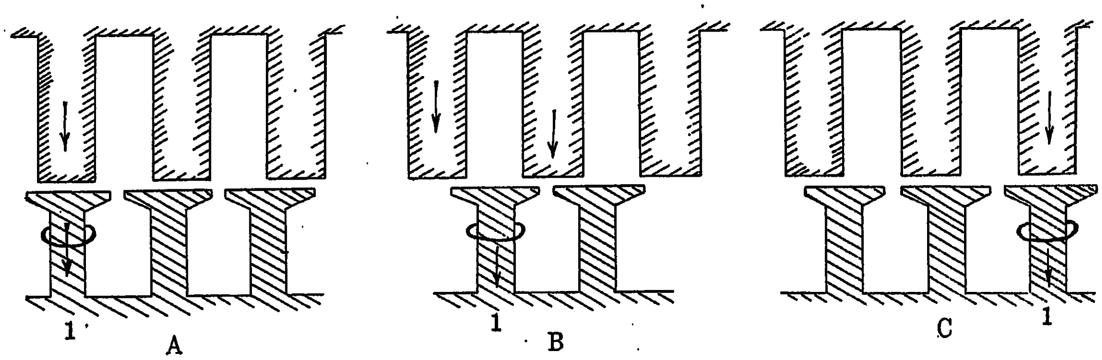

FIG. 6

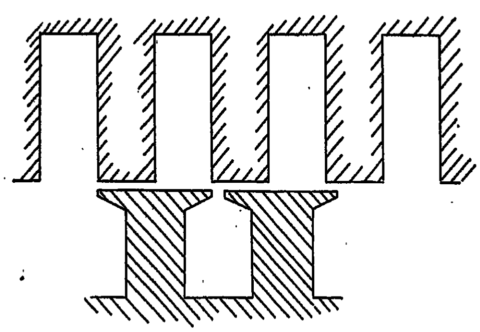

FIG. 7

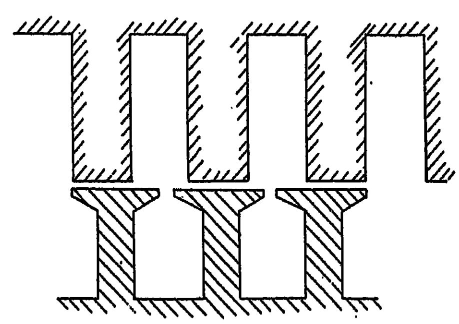

FIG. 8

when the grinding of the rotor surface is done properly the increase in losses is negligible. The magnitude of the additional losses in open-slot motors is often very large and it is not uncommon that under otherwise equal conditions open-slot motors have 1½ to 3 times the losses of semi-closed slot motors. That a large percentage of these additional losses are located in the rotor iron

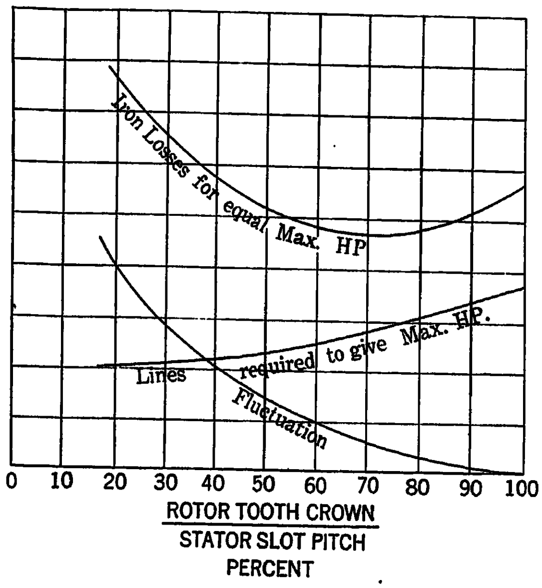

Fig. 9

can readily be demonstrated by using different grades of iron for the rotor circuit. The writer is familiar with cases where a change in the grade of rotor iron has decreased the losses in an open slot machine 25 per cent and more.

Another loss which is not negligible will be found in the end

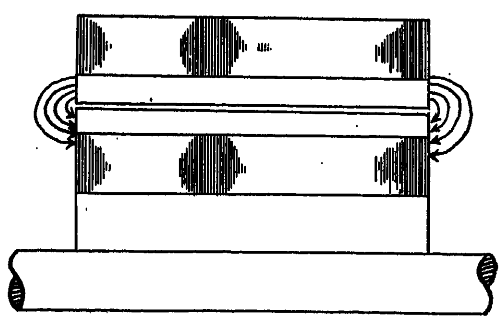

Fig. 10

punchings. There is a certain number of magnetic lines which pass from the stator, not through the air-gap into the rotor but which pass out of the end punchings of the stator into the end punchings of the rotor, as indicated in Fig. 10. These lines penetrate the end punchings at right angles to the plane of the punching and, therefore, are most favorably located to produce heavy eddy currents. Similar conditions exist in regard to the end rings holding the punchings.

That these losses are not negligible has been frequently observed by the writer by comparing the iron losses of machines built with exactly the same punchings but with different lengths of iron. Invariably the machine with longer iron showed a smaller loss per inch of iron length than the shorter machine, the comparison being made for equal induction. This clearly indicates that quite appreciable additional losses must take place in the end punchings.

Thomas Spooner: Mr. Alger and Mr. Weichsel have undoubtedly given the correct explanation for the fact that the pulsation losses are approximately the same with and without the squirrel-cage bars connected by end rings, namely, in the former case the pulsations are set up in the stator teeth as a result of the high-frequency m. m. f. due to the currents in the rotor bars and these pulsations produce stator-tooth pulsation losses which are approximately equal to the rotor-tooth pulsation losses which existed with the rotor bars unconnected. This point was overlooked by us, consequently no measurements were made of the stator-tooth pulsations.

The method which Mr. Alger describes for obtaining pulsation losses in commercial machines should be very useful due to its simplicity. It was not applicable, however, to a large proportion of the tests which we made on the experimental machines which were described in the paper because much of the time the rotor windings were either not operative or not present at all.

As pointed out by Messrs. Alger and Weischel, there seems to be no way of eliminating pulsation losses in induction motors without producing other even worse effects, such as noise, dead points, etc. A further knowledge of the factors which govern pulsation losses and these other undesirable allied effects should, however, make possible better compromise designs than often have resulted in the past when insufficient consideration has been given to tooth pulsations.

Mr. Weichsel states that the minimum iron losses occur when the ratio of the rotor-tooth crown to the stator slot pitch is 70 to 75 per cent while the stator tooth crown is about 40 to 45 per cent of the stator slot pitch. We have no specific data to show whether or not this is correct. In general, however, our experience indicates that for open stator slots, other things being equal, the tooth-pulsation losses alone are less for a given per cent difference between the number of stator and rotor teeth when the rotor teeth are less in number than the stator teeth. There are some advantages, however, not connected directly with iron losses in having the rotor slots greater in number than the stator slots.

Referring to Mr. Weichsel's comments on the effects of end punchings, these extra losses due to these punchings are partly caused by the fact that these punchings are much thicker than the ordinary material and are often made of steel which is inferior magnetically. Under these circumstances the end punchings will frequently have several times the hysteresis and eddy losses which would occur in an equal volume of ordinary thin laminated material. The increased losses for the short armatures noted by Mr. Weichsel are undoubtedly also due in part to the axial components of leakage flux and consequent large eddy currents as explained by him.

Complete Synchronous Motor Excitation Characteristics

BY JOHN F. H. DOUGLAS,[1] ERIC D. ENGESET[2] and ROBERT H. JONES[2]
Associate, A. I. E. E. Enrolled Students of A. I. E. E.

Synopsis.—*Synchronous motor excitation characteristics published hitherto have been incomplete. If theoretically determined, while forming closed loops, some doubt as to their accuracy has been entertained. When determined experimentally, thay have not been complete, in that the so-called unstable portions of the curves have not been obtained, and generally speaking, the upper portions of the characteristics have been missing.*

This paper shows complete synchronous motor characteristics, experimentally determined, differing materially from published curves determined by theory. These differences are discussed, and their cause attributed to variations in synchronous impedance.

Experimental data is included showing the factors on which synchronous impedance depends, and how it varies with current, saturation and power-factor.

THE excitation characteristics of a synchronous motor, more familiarly known as the *V* curves are well known. As determined from test they are incomplete, *i. e.* lacking portion corresponding to the larger armature current values. In Fig. 1 is reproduced a set of complete excitation characteristics, calculated by Dr. C. P. Steinmetz and shown in his "Alternating Current Phenomena", page 434. These curves are closed loops. Quoting from page 437, we read, "The upper parts of these curves, however, I have never been able to observe completely and consider it probable that they correspond to a condition of synchronous motor running which is unstable. The experimental observations usually extend about the lower portion of the curves, and in trying to extend the curves further to each side the motor is thrown out of synchronism." The writers know of no published experimental data covering the unstable portion of these curves.

Dr. Steinmetz did not regard these curves as accurate for he says, "It must be understood, however, that these power characteristics can be considered as approximations only, since a number of assumptions are made which are not, or only partly, fulfilled in practise." For instance Fig. 1 indicated that a synchronous motor will not carry any load with the field circuit open. Dr. Steinmetz says "So by decreasing gradually the excitation, and thereby the e. m. f., the curves, at light load, occasionally are extended below zero, into negative values of voltage, while the power still remains constant and positive as a synchronous motor. In other words, the motor keeps in step even if the field excitation is reversed; the lagging component of the armature reaction magnetizes the field, in opposition to the demagnetizing action of the reversed field." We do not recall any experimental data published which shows the distortion actually occuring in the left hand corner of the curves in Fig. 1.

The object of our experiments was to obtain the complete excitation characteristics of a synchronous motor, unstable as well as stable portions, and the complete closed loops if possible, and to observe the manner in which the curves are distorted for small and reversed excitations. We hoped that the instability was of a purely mechanical nature, for then we might succeed in stabilizing the motor by coupling it to another synchronous motor. We had, fortunately, in our laboratory, a 15 kv-a., 6 pole, 1200 r. p. m., synchronous set consisting of two identical machines, made by the Westinghouse Company for educational institutions. One of the machines had a stator that could be rotated around the shaft as an axis, either by a hand wheel, or freely within limits. The stator of this machine carried a two foot arm for weighing the

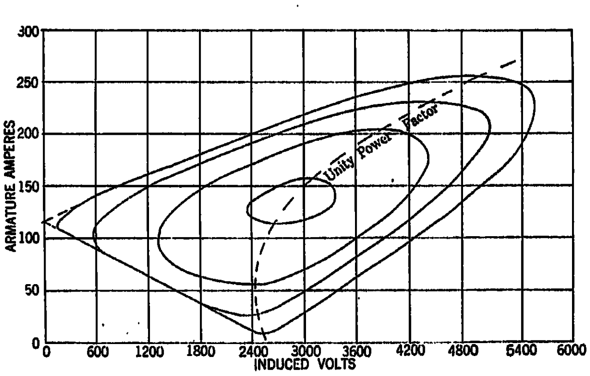

FIG. 1—SYNCHRONOUS MOTOR CHARACTERISTICS AFTER STEINMETZ

torque by the reaction method. We connected the machine according to Fig. 2. The projecting arm of the movable stator was supported on a jack-screw, which was in turn supported on small platform scales.

In Fig. 2 the motor at the right was the one tested. Although connected for 3 ϕ, *Y*, and 230 volts, it was run at reduced voltage by a bank of auto-transformers. It was run first at 115 volts, later at 55 volts, and finally at 20 volts, impressed voltage. The motor at the left was run at 230 volts, and was successful in all cases in keeping the other machine from falling out of step. When supplied with current the movable stator tended to rotate, and we were able to measure the reaction

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Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

torque successfully. The left hand motor was first started. The right hand machine was then excited to transformer voltage. S_2 was then closed. Before closing S_1 the movable stator was swung into such

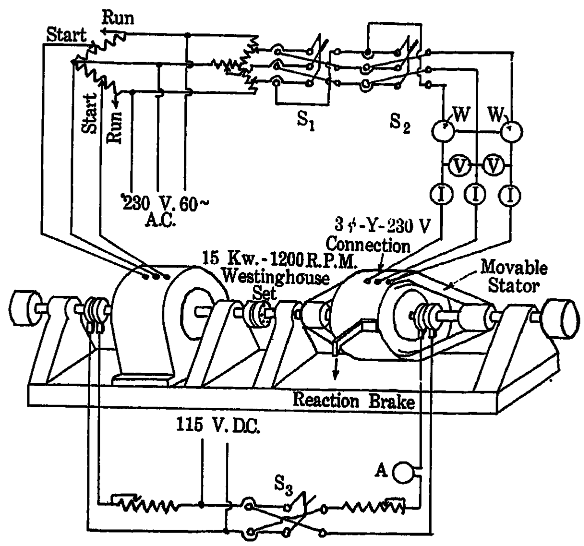

FIG. 2—CONNECTIONS FOR TEST ON COMPLETE SYNCHRONOUS MOTOR CHARACTERISTICS

a position that the phase of the induced voltage was the same as that of the line when a voltmeter across switch S_1 indicated no voltage. Then switch S_1 was closed,

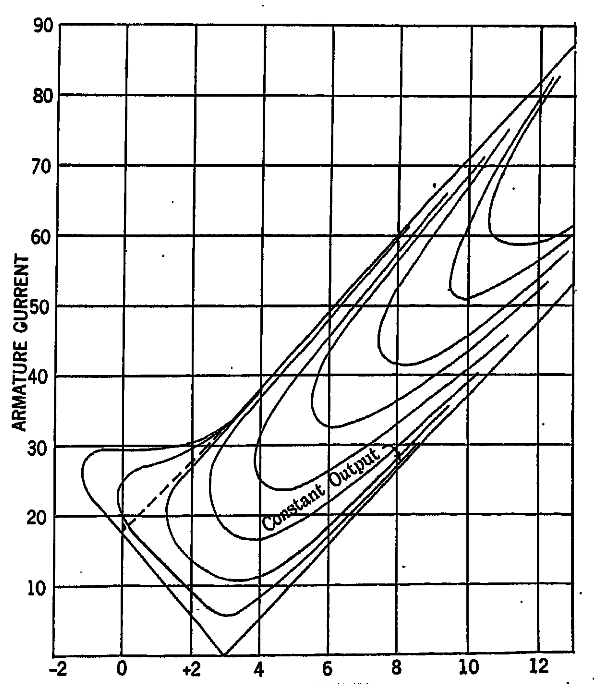

FIG. 3—COMPLETE EXCITATION CHARACTERISTICS OF 15 KV-A., 1200 R. P. M., WESTINGHOUSE SYNCHRONOUS MOTOR AT HALF NORMAL VOLTAGE

and as might be expected practically no current flow resulted, and no torque was produced.

When a synchronous motor is loaded, the rotor drops back in phase, this in turn, causes the motor to draw an increased current, and usually a larger torque is produced enabling the motor to carry its load. However, when the increased current gives a smaller torque, the motor will stop. In place of allowing the rotor to fall back in phase, we advanced the stator with the jack screw. We found that advancing the stator in the direction of rotation, the current and watts increased continuously. At first the torque also increased, but it reached a maximum, and then an advance of the stator phase resulted in a lower reading on the scales. We were able to reduce the torque again to zero in this way, in every case. This shows conclusively that we were observing points on the so-called unstable portions of the characteristics.

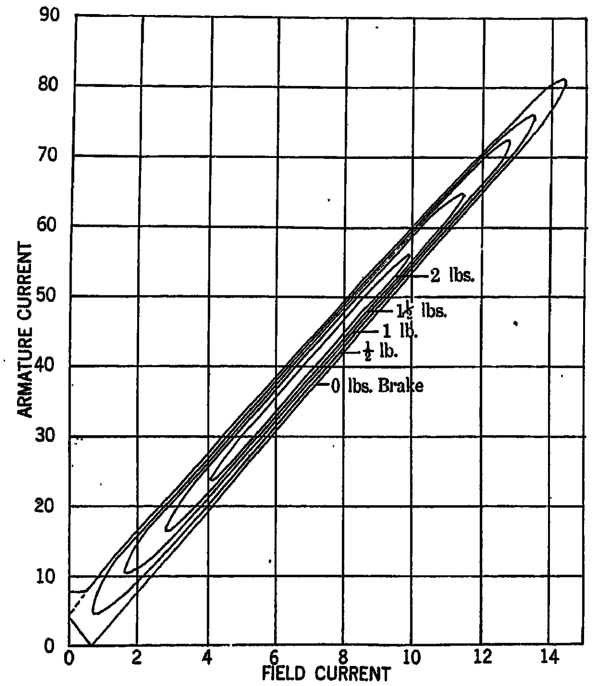

FIG. 4—COMPLETE EXCITATION CHARACTERISTICS OF 15 KV-A., 1200 R. P. M., WESTINGHOUSE SYNCHRONOUS MOTOR AT 20 VOLTS

We were able to shift the phase of the stator sixty electrical degrees by the jack-screw. By reversing switch S_1 or S_2 we were able to advance or retard the phase 120 deg., by reversing S_3 and either S_2 or S_1 we were able to advance or retard the phase 60 deg. We made a number of runs at each voltage. Most often we held the field current constant for a given run, and varied the phase reading, phase angle, torque, volts, watts, and armature current. The readings were then corrected and reduced to a standard voltage and plotted.

The characteristics obtained at 115 volts (and at 55 volts adjusted to 115 volts) are shown in Fig. 3. The nose-shaped protuberance on the curves for reversed excitation is to be noted. The upper portion or unstable portion of the curves seems to have been obtained, however, no evidence of the curves forming loops is shown.

In an endeavor to obtain closed loops, tests were made at an impressed voltage of 20 volts. The results of these tests are shown in Fig. 4. The loops are now closed. The maximum torque occurred at 7 amperes in the field, and 40 amperes in the armature, and was 2.13 lb. Fig. 4 is somewhat surprising, in that the loops are very long and narrow, and with practically straight sides. No evidence of a saturated condition is observed with large field currents. Dr. Steinmetz, in the place quoted, expressed a fear that saturation would affect the shape of the curves, which does not seem to be realized.

The peculiarities in the shape of the characteristics here shown, may be attributed possibly to the following assumptions used by Dr. Steinmetz, and acknowledged by him to be inaccurate. "While the reactance of the line is practically constant, that of the motor is not but varies more or less with the saturation, decreasing for higher values." "Furthermore, this synchronous reactance usually is not a constant quantity, even at constant induced e. m. f., but varies with the position of the armature with regard to the field; that

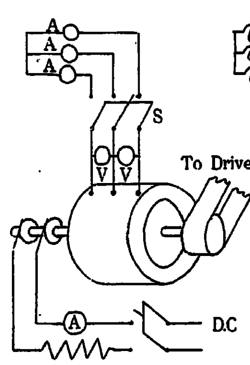

Fig. 5

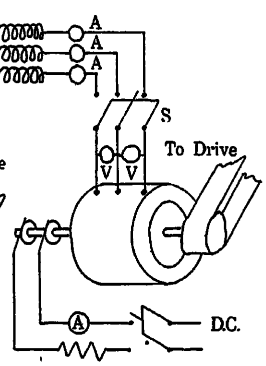

Fig. 6

Synchronous Impedance Tests

is, varies with the current and its phase angle. While in most cases the synchronous reactance can be assumed constant, with sufficient approximation, sometimes a more complete investigation is necessary, consisting in the resolution of the synchronous impedance into two components, in phase and in quadrature, respectively, with the field poles. Especially is this the case at low power factors." He here mentions the example of a synchronous motor remaining in step with light loads and reversed excitations as coming under this case, and which Fig. 3 verifies.

We draw from the above data the following conclusions:

1. The unstable portions of the synchronous motor characteristics can be obtained by directly connecting the motor to another rated at the same speed.

2. The torque can be conveniently measured by the reaction method if the stator is swung from bearings and free to turn about the shaft.

3. Complete characteristics (closed curves) can be observed if the motor is tested at a greatly reduced voltage. The curves for one voltage are similar to those obtained at another except for scale.

4. Anomalies in the shape of the characteristics have been observed at light loads, with small and with reversed field excitations.

5. Further tests to determine the amount of variation in the synchronous impedance, with saturation, current, and phase angle, were thought desirable, in view of the anamolies observed, and the attributing of variations in shape from Fig. 1 to this quantity by Dr. Steinmetz.

Variations in the Impedance of a Synchronous Motor at Synchronous Speed

A common method of measuring the synchronous impedance of a motor or generator is shown in Fig. 5. The voltage induced in the machine is measured with the switch S open, the motor being driven at synchronous speed, and separately excited. The current on short circuit is measured with the switch S closed, the field current being unchanged. The synchronous impedance is taken as the ratio of cause to effect, of open circuit voltage to short circuit current. The impedance obtained in this way is much too large. Since the armature reaction destroys the saturation of the machine, this test does not give the equivalent impedance at higher values of saturation.

Another method of finding the synchronous impedance is shown in Fig. 6. The voltage with S open is called E_o, that with the switch closed, E_1, the current being I. The impedance may be taken as the ratio of cause to effect, but the cause of current flow is the unbalanced voltage $(E_o - E_1)$. The impedance is $Z = (E_o - E_1)/I$. This method is superior to that in Fig. 5, in that we have two controls, the field resistance and the choke coils. We are thus able to obtain the synchronous impedance at different saturations, and, if we desire, at different currents as well. This is the method on which the A. I. E. E. standard is based. This method gives no clue as to how the impedance varies with power factor, since the current is always at very low power factor.

In order to determine the variation of synchronous impedance with current, saturation and power factor, three controls should be available, and these were available in the Westinghouse set, in the two field rheostats, and in the phase displacement hand wheel attached to the machine with the movable stator. The two machines were coupled together and driven by a d-c. motor at 1200 rev. per min., and connected as in Fig. 7. First both machines were excited equally, then the adjusting handwheel was turned until the voltmeter $D E$ indicated no voltage, The angular position of the movable stator was then noted on a degree scale mounted on its frame.

If either the phase position of the movable stator,

or the relative field excitations were changed, the voltmeter across the switch indicated a resultant voltage $D\,E$, which was taken as the cause of the flow of current resulting when the switch was closed. The reactions of this current equalized the fluxes in the two machines and the terminal voltage E when the switch S was closed was taken as the measure of the saturation. The synchronous impedance was taken as the ratio of cause to effect, or $Z = (D\,E/I)$. By unbalancing the voltages more and more different circulating currents could

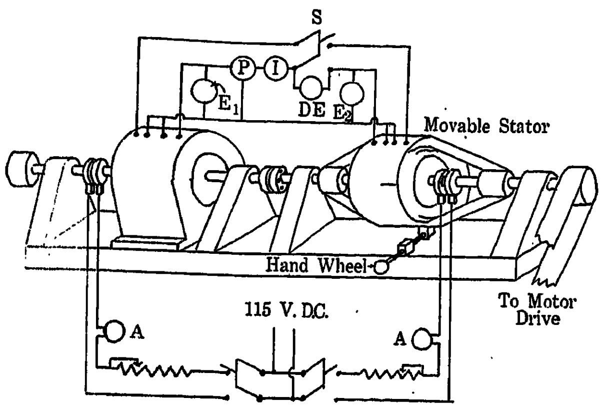

Fig. 7—Improved Synchronous Impedance Test.

be obtained. By increasing both fields together different saturations could be obtained. By controlling the phase relation of the circulating voltage through the combined use of the hand wheel and the field rheostats, currents of any power factor could be circulated. The power factor was taken as the ratio of the wattmeter reading P to the product $E\,I$. When the voltage was unbalanced by field excitation only, (the stators being in phase) the resulting current had a very low power factor, 8-10 per cent. When the

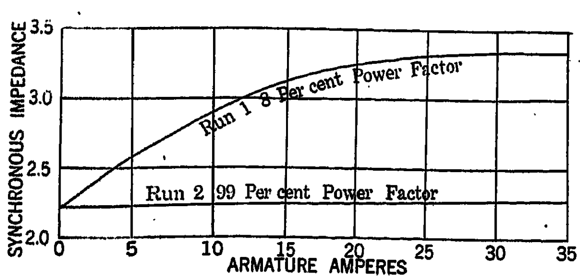

Fig. 8—Synchronous Impedance at Constant P. F. and Terminal Volts, Varying Current

voltages were unbalanced by the hand wheel only, the fields being equal, the resultant current had a very high power factor from 99-100 per cent. Six readings were taken for each point of data, the voltage E_1, E_2 and $D\,E$, with S open and E, I and P with S closed.

In the first two runs the terminal voltage on closed circuit E was held constant at rated value 180 volts, but the voltages were unbalanced more and more, circulating different currents. In one run this was done by the field rheostats, the phase displacement between the stators being zero, and resulting in an 8 per cent power factor. In the other, this was done by the hand wheel, the machines being excited equally, resulting in a 100 per cent power factor. The results of these two runs are shown in Fig. 8. An increase of

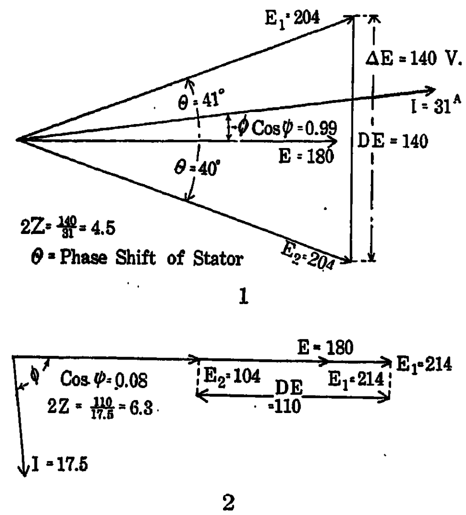

Fig. 9—Vector Diagrams Runs 1 and 2

impedance with current at low power factors is indicated. A considerably lower impedance is shown for 100 per cent power factor than for low power factors. The impedance at 100 per cent power factor seemed to be independent of the current strength. Two points are shown, one on each curve for which vector diagrams are given in Fig. 9. One curious result shown in this figure is that the terminal voltage on closed circuit E is greater than the average of E_1 and E_2 for low power

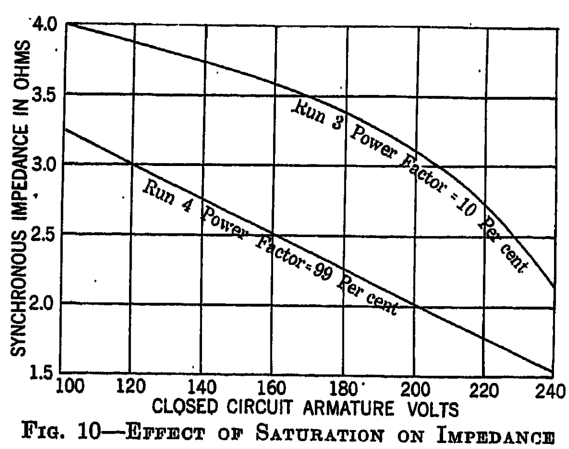

Fig. 10—Effect of Saturation on Impedance

factors, but less than the vectorial average of E_1 and E_2 for high power factors. This indicates that our test gives simply an average impedance of the two machines.

In the next two runs the power factor was again kept constant, but the saturation of the machines was varied. In the third run the two machines were kept at zero relative phase, the power factor was 10 per cent, and the circulating current was kept at rated

value on the name plate of the machine. In the fourth run the phase displacement of the machines was kept constant at 41 deg., and the power factor was 99 per cent. The results of these two runs is given in Fig. 10. A decrease in impedance with saturation is indicated, as well as a lower impedance at 100 per cent than at low power factors. The decrease in the impedance with saturation at 100 per cent power factors

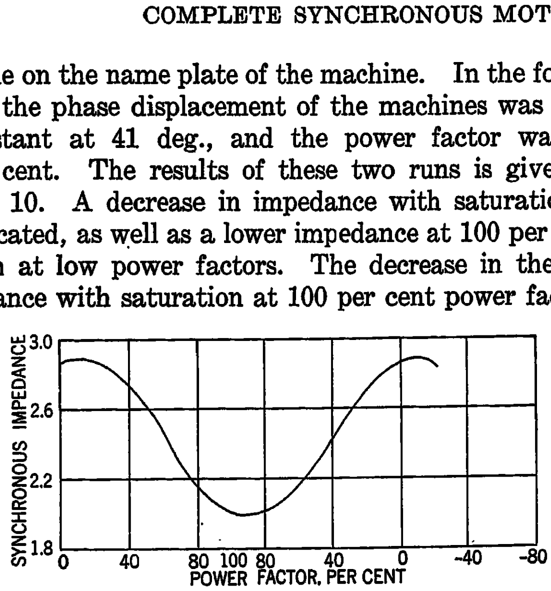

FIG. 11—EFFECT OF POWER FACTOR ON IMPEDANCE-CURRENT AND SATURATION CONSTANT

is believed to be a novel result. The ratio of impedance at normal voltage namely impedance at 100 per cent power factor to impedance at zero power factor is as 2 as to 3. This ratio is considerably higher than that obtained by purely mathematical calculation as for example in Karapetoff's "Magnetic Circuit" pages 150 and following.

In the fifth run the saturation was kept constant at a terminal voltage of 180 volts, the circulating current

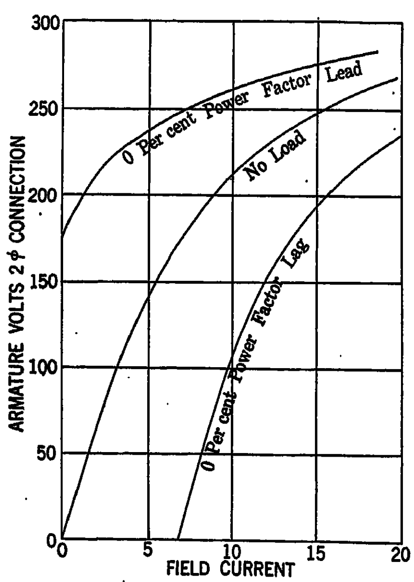

FIG. 12—SATURATION CURVES

was kept constant at (2/3) of name plate rating, but the power factor was varied from 90 deg. lag to 90 deg. leading current. The results are shown in Fig. 11. A variation of very considerable amount with power factor is indicated. The saturation curves of the machine at no load, and full load zero power factor both leading and lagging are given in Fig. 12 as a matter of interest.

The variation in synchronous reactance with saturation is satisfactorily allowed for in the magnetomotive force method, and in the A. I. E. E. method of figuring synchronous machine performance as well. The Blondel method of computing performance, explained by Professor Karapetoff in his "Magnetic Circuit" pages 150-157, provides a satisfactory method of allowing for the effects of saturation, and also power factor, with two exceptions. First, as noted above, impedances at 100 per cent power factor seem to be larger than those indicated by the "Magnetic Circuit." Secondly, according to Professor Karapetoff, the impedance at 100 per cent power factor, should be independent of of saturation. This is conclusively disproved by our experiments.

We do not have any theory to propound that will harmonize the variations here noted, but we believe that the Blondel method can be improved so as to allow for the effects of saturation with high power factor currents. Furthermore, we hope, that when such a theory is found it will completely explain the queer nose we have observed and recorded in our Fig. 3.

Discussion

Q. Graham; (communicated): To those who are familiar with synchronous motor *V* curves in the normal working region only, the shape of the curves in their remote upper portions may seem to be of little importance. Yet they do form a fascinating study and they have a certain practical importance.

A synchronous machine having negligible resistance in the armature winding and negligible saturation could be made to furnish armature current in proportion to its field current through an unlimited range. As soon as armature resistance is introduced this relation is changed and an upper limit for armature current is reached. The curves then take the shape which the authors show in their Fig. 4.

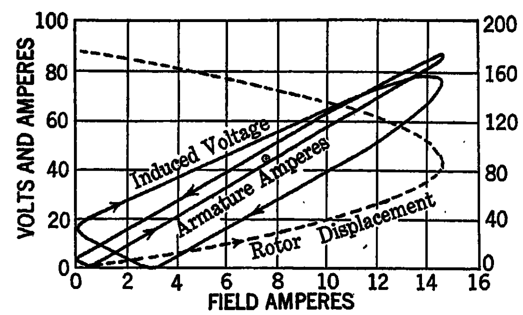

FIG. 1—THE POINT OF MAXIMUM TORQUE IS SHOWN BY THE SMALL CIRCLE

In order to point out some of the interesting relations that exist I have calculated the curve of zero torque for the authors' machine, having had access to the manufacturer's records. This calculated curve along with the corresponding curves of induced voltage and angular position of the rotor are shown in Fig. 1. The corresponding branches of the three curves are shown by the arrows. The calculated curve agrees fairly well with the test curve shown in the paper.

If we proceed from the point of zero armature current an increase in field current gives an increase in armature current until the point of maximum field current is reached. Here the displacement of the rotor is nearly 90 deg. A slight reduction of field current and an increase in displacement brings the armature current to a maximum as the displacement reaches 90 deg. At this point the power factor of the machine is unity and the resistance drop in the winding is just equal to the line voltage. A

further reduction in excitation accompanied by a greater phase displacement of the rotor reduced the armature current again. The next point of interest occurs where the induced voltage is zero. That is, the impedance drop in the winding is equal to the line voltage and the field and armature ampere turns across the air-gap are neutralized. From this point the curves proceed to the position of zero field current. To continue the increasing displacement of the rotor after zero field current is reached the excitation must be reversed. An increase of field in the reversed direction will then bring the armature current back to zero and the complete path will have been traversed. It will be noted that the rotor displacement has reached 180 deg. but since the excitation has been reversed a condition of stability exists. In

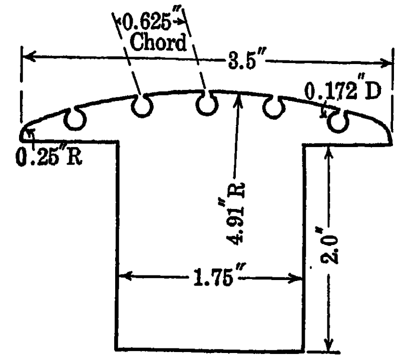

Fig. 2

passing through the point of zero excitation it would have been satisfactory to consider the rotor as going through an instantaneous shift in position of 180 deg. and the excitation as remaining positive.

It is interesting to consider these curves from the standpoint of power input and output and thereby examine the region beyond the point of maximum excitation. Under any condition the power input is divided between the $I^2 R$ loss in the winding and the output which appears as torque at the shaft. For pure condenser operation, or zero torque, the input is entirely absorbed as loss in the winding. The following conclusions may be drawn at once:

1. For any torque curve, including the zero torque curve, the power factor is unity at the point of maximum armature current.

2. The maximum torque point occurs with an armature current which is half of the maximum current on the zero torque curve. The input is then equally divided between loss and output.

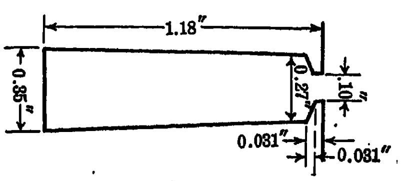

Fig. 3

3. For armature and field currents beyond the range shown by the curves the loss exceeds the electrical input and torque must be supplied to the shaft. The machine then becomes a generator loaded on its own windings.

A study of the complete excitation characteristics of synchronous motors has proved worthwhile in connection with applications requiring extremely low frequency operation. If the authors are inclined to make further experiments they might operate their machine at 25 cycles or even lower and thus have the equivalent of a higher resistance armature with its concomitant effects.

The following design data on the authors' machine may be of interest.

Stator:

15 in. outside diameter, effective (punching is not circular.)
10 in. inside diameter.
5 in. axial length.
72 slots. See Fig. 3 for dimensions.
22 conductors per slot. Each conductor consisting of two 0.57 in. diam. wires in parallel.
Winding in three parallel groups, star-connected, for 220 volts.
Throw of coils from slot No. 1 to slot No. 10 or a pitch of 9 slots.
Single air-gap 0.094 in.

Rotor:

Six poles. See Fig. 2 for dimensions.
Damper winding; 5 bars per pole, each 0.162 in. diam. copper.
End ring 0.156 in. $\times$ 0.75 in. copper.
Field winding: 185 turns per pole of 0.057 in. $\times$ 0.081 in. copper ribbon.

R. W. Wieseman: The unstable portion of the excitation curve or phase characteristic of a synchronous motor has never been obtained experimentally, so far as we know, and therefore the authors of this paper are to be commended for obtaining the complete synchronous motor excitation curve. However, these curves are apparently more valuable from the academic standpoint than from the practical standpoint because a synchronous motor can not be operated in practise at the unstable part of its excitation characteristic.

The authors state that in Fig. 4 no evidence of a saturated condition is observed in the excitation curves at large field currents. I wish to point out that practically no saturation could exist in a machine which is operated at one-eleventh voltage and consequently one-eleventh of its normal magnetic flux density. Saturation will only occur and affect the shape of the curves when the motor is operated near normal flux densities. No effects of saturation are noted in Fig. 3, which shows the excitation characteristics at one-half normal densities.

In the description of the method of finding the synchronous impedance shown by Fig. 6, it is stated that this method gives no clue as to how the impedance will vary with power factor, and that to obtain this variation an arrangement shown by Fig. 7 is necessary. The variation of impedance with power factor can be obtained very easily with the arrangement shown by Fig. 6 if resistance is connected in series or in parallel with the inductance.

This paper states that the synchronous impedance of a synchronous motor varies with the several conditions of load. It is easier and also more practical to consider the synchronous impedance made up of two components: armature leakage reactance and armature reaction. The armature resistance can be neglected. In a salient pole machine the armature reactance is maximum with unity power factor currents and minimum at zero power factor, whereas the effect of armature reaction is minimum at unity power factor and maximum at zero power factor. Therefore, the manner in which the synchronous impedance varies depends upon the inherent design of the machine as well as the load conditions.

V. Karapetoff (communicated): The authors deserve much credit for having shown that the so-called unstable portions of load characteristics of a synchronous machine may be obtained experimentally, by coupling two machines mechanically at an adjustable angle[2]. The instability is purely mechanical, and is due to the fact that in regular operation the machine (whether a generator or a motor) is allowed to change its torque angle with the load. When two synchronous machines are coupled mechanically and also connected electrically, this angle is entirely under the observer's control, and the stability is that of the set and not of the individual machines. While the practical application of the method is somewhat limited because two mechanically and adjustably coupled machines are but seldom available, the method should prove of interest in experimental studies of armature reaction and reactance, and in checking various proposed theories and performance diagrams of synchronous

machines. Just now, the stability limit in the generator range is of considerable practical importance in superpower plants in which the generator characteristics limit the amount of power which can be transmitted over a given line. Possibly, the range near and beyond the generator stability limit can also be conveniently studied by means of two adjustably coupled machines. Then, if the observed performance, on small machines checks with that predicted by theory, the designer may compute the power limit of his superpower generators with much more confidence.

The value of the paper would be much enhanced if complete design data of the machines were available. It is to be hoped that the maker of the machines will be generous enough to contribute these data to this discussion, in order that the observed curves may be checked with theoretical diagrams. Perhaps with the theory of two reactions, when properly applied, the observed hump in Fig. 3 could be interpreted or predicted. In the range under consideration, the machine is unsaturated, and Blondel's theory should apply with considerable accuracy.

I am at a loss to understand the latter part of the paper, purporting to analyze the effect of the current, saturation, and power factor upon the magnitude of synchronous impedance. The concept of synchronous reactance was introduced in the early days of a-c. engineering, and served its useful purpose in those days, but it is almost unbelievable that any one should now attempt to interpret the observed performance characteristics of a synchronous machine with salient poles in terms of a variable synchronous reactance. The authors should have separated the direct armature reaction from the leakage reactance, using the zero-power-factor load curve,[3] and then estimated the transverse armature reaction from the performance at unity power factor and the observed torque angle. This would have given them a rational basis for an interpretation of the other observed curves and for a judgement about the accuracy of the method of two armature reactions. Having instead bunched together all these heterogeneous factors into a physically irrational concept of "synchronous impedance," they have reached conclusions apt to befog rather than to clear up the issue.

Equally incomprehensible to me are their references to my "Magnetic Circuit." They speak of the observed ratio (2 to 3) of the synchronous impedance at unity power factor to that at zero power factor and state that it is considerably higher than that computed by me. My formulas and computations refer to the direct and transverse components of the armature reaction, and not to synchronous impedance, which latter I have used in my writings only in order to show its limitations and inadequacy. My values could be checked with the experimental results only by computations involving the design constants of the machines, and there is nothing in the paper to indicate that the authors have used the design data in their conclusions. The authors also ascribe to me the statement (without giving the page) that the impedance at 100 per cent power factor should be independent of saturation. Since I do not use the concept of synchronous impedance in the "Magnetic Circuit" at all, it is not clear how and where I could have made such a statement. At unity power factor at the terminals, there is not an inconsiderable internal phase angle and a corresponding direct reaction. With higher saturation, the effect of this direct reaction upon the net flux is of course less. The transverse reaction and the leakage coefficient are also affected by the saturation of the pole tips, although these factors are difficult to take into account theoretically.

Within the last six or seven years, Prof. Blondel has published in various French periodicals a number of excellent articles, further explaining and extending the application of his theory of two armature reactions of the synchronous machine. In the light of these painstaking researches, the interpretation of the results by the authors is quite out of date. They could render real service to the profession if they would further experiment with the same machines, and use their results to build up, step by step, a consistent and accurate theory of synchronous machines, and also critically check various modifications of the present theories.

J. F. H. Douglas: We wish to concur in what Mr. Wieseman has said on saturation, and on the armature leakage reactance probably being larger at high power factors. We wish to thank Professor Karatpetoff for suggestions for future tests, and for bibliographic references. We find the comment and calculations of Mr. Graham of interest and his discussion of stability illuminating. The design data furnished will be valuable. We wish to call attention to fact that Mr. Graham's and Mr. Wieseman's statements as to whether armature resistance is negligible can be harmonized. The armature resistance is negligible at normal frequency and impressed voltage, if the motor is not greatly over excited it must be taken into account otherwise.

We wish to express a doubt whether a simple load test using resistance as well as reactance, will give the synchronous reactance. For illustration, in Fig. 1, if E_o is measured at no load, one might think the synchronous reactance drop to be perpendicular to the current and might get an erroneous value for the same. Fig. 9 in our paper, indicates that the $I\,X_s$ drop is not perpendicular

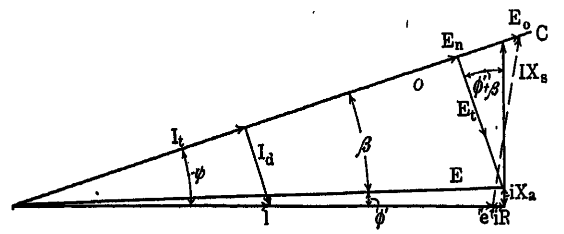

FIG. 4—VECTOR DIAGRAM COMPARING BLONDEL METHOD WITH SYNCHRONOUS IMPEDANCE

to the current in all cases. Professor Karapetoff's remark is quite to the point here that considerably more experiment is needed.

We are criticized by Professor Karapetoff for the use of synchronous impedance on various grounds. We consider the term too firmly entranced in the texts, and by implication in the A. I. E. E. rule for regulation, to be ignored. We consider it useful as a concept for the very fact that it gathers up a number of components. (1) In qualitative explanations it affords a simple way of speaking of the effects of changing the excitation of a synchronous motor, or of putting a load upon it. (2) For approximate calculations it affords a quick method of computation. (3) When allowance is made for the fact that at high power factors the measured impedance is less, (and the reason why is easily explained), it affords a simple basis for calculating period of hunting and synchronizing torque, and a simple basis for discussing the accuracy of the A. I. E. E. method for regulation at high power factors.

Defined as the ratio of cause to effect, of antecedent to consequent, as voltage across as witch to current flowing after closing it, it is in accord in reasoning with many of Professor Karapetoff's own explanations. To term it a collection of heterogeneous factors is not quite just. The distortion of flux and the drop in voltage resulting from it has only one "genesis" or origin—the armature m. m. f. We may, if we wish, divide the cause into components parallel to and perpendicular to the poles. We may divide the effect into components, those fluxes, and e. m. fs. strictly proportional to armature current and those which, owing to saturation, are not so proportional,—reactances, and reactions. This is a question of accuracy and convenience for the purposes in hand.

Synchronous impedance is criticised as physically irrational. It is empirical, while the Blondel method is highly theoretical.

2. Testing of alternators by means of two identical machines rigidly coupled at an angle was suggested by A. Blondel in 1892 and more fully described by him before the International Electrical Congress in St. Louis, 1904; TRANS. Vol. 1, p. 620.

3. V. Karapetoff, Experimental Electrical Engineering, Vol. I, Third edition, Chap. 20.

However, all theories contain irrational elements in the sense of items which are approximations of the physical reality. In this sense, the Blondel theory is also irrational. To mention one point, in the "Magnetic Circuit" it is assumed that armature leakage reactance is an absolute constant. This is the basis of the criticism that we should have worked up our data in a certain way, but if we had done that we would have come under Mr. Wieseman's criticism," in salient pole machines, the armature reactance is a maximum with unity power factor, and a minimum with zero power factor," a statement, which we think may be correct. To mention another point, the coefficient of cross reaction or relation between the voltage induced to the current, is assumed as a constant. This relation is derived in Karapetoff's Magnetic Circuit by the analogy of "fictitious poles," which show no saturation. This is an irrational element, that only brings home the point that all our theories are approximations. We have no objection to using the Blondel theory as a second approximation, until a better is found, but we think of synchronous impedance as a first approximation.

That the Blondel theory does need improvement, we feel most strongly. Our data will not agree with Professor Karapetoff's formulas. When we try to get synchronous impedance from his diagrams at 100 per cent power factor and at low saturations the result is less than that shown by our tests. The value taken off his diagrams is independent of saturation, contrary to our experiments. We did not mean to imply that he used or calculated synchronous impedance himself. However, to be specific, we take the synchronous impedance at unity power factor to be approximately equal to

$$X_{s(100)} = X_a + (E_t'/i) = X_a + (0.3\ K_b mnv)$$

the quantities appearing in part in his equation (**84**), page 156.

The Fig. 4 herewith is Fig. 40 of page 150 of the Magnetic Circuit, drawn for 100 per cent power factor with three vectors added. The solid line $O\ E_o$ is the no-load voltage along the polar axis $O\ C$, the dotted line $E\ t'$ from equation (**84**) is drawn in its proper phase position, perpendicular to the current, and terminating on the line $O\ E_o$. The dotted line joining e and E_o is what we call, and what is usually called the synchronous impedance drop. There is nothing in the Magnetic Circuit to indicate that E_t' even has a position, but by reference to the equation after Eq. (**186**), namely,

$$E_t' \operatorname{Sin}(\phi + \beta) = E \operatorname{Sin}(\beta)$$

we see that the line E_t' of our diagram does meet these conditions. In the above diagram $I\ Z_s$ is numerically very close to $I\ X_a + E_t'$.

Numerically we note that X_a may be taken from our Fig. 12 as also may the direct reaction, given by Eq. (**79**) in the Magnetic Circuit. We find from our Fig. 12 that the value of (Xa) is too small to measure. We attribute then the whole effect to reaction. We find that an armature current of 35 amperes in Fig. 12 neutralizes a field current of 7 amperes. Comparing equations (**79**) and (**84**) in the Magnetic Circuit we note the constant in one is 40 per cent of that in the other, with V the slope of the saturation curve included. We compute the slope to be 31 volts per ampere. We compute $X_{s100} = 0.4(7/35)\ 31 = 2.5$ ohms.

When we refer to our Fig. 10 we see that this value is much too small when the machine is not saturated, and that the value of the reactance is not a constant, as indicated by the above interpretation of the Theory in the Magnetic Circuit, but actually a variable affected by saturation of the machine. Fig. 4 shows clearly that the direct reaction can not be a major item being nearly horizontal but that the saturation of pole tips affects the transverse reaction. As Professor Karapetoff admits, this factor is difficult to take into account theoretically but the variation of values in Fig. 10 of our paper at unity power factor from 3.2 to 1.6 ohms indicates that the reluctance of the transverse flux path is approximately doubled, by saturation. This change is so large that it is difficult to see how one can avoid the conclusion that the theory of transverse reaction needs considerable improvement.

With the data now available from the manufacturers it would be easy to follow the suggestion made to check the shape of the hump in our curves by the Blondel theory. The following calculation was made before this data was supplied, but checks so well in a general way, that on this point Professor Karapetoff may consider himself well vindicated. Let us assume that resistance may be neglected, and that we have only to consider two reactances X_t for cross-magnetizing current, and X_d for direct reaction component of the current. This is a rather crude simplification of the Blondel theory that will hold fairly well with

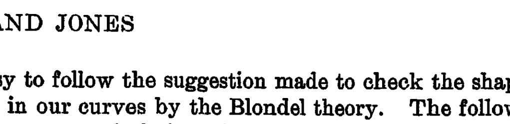

Fig. 5—Simplified Blondel Diagram

small saturation. Fig. 5 gives the vector diagram. The impressed voltage is E, the induced voltage at no load E_o, shows by its magnitude the field current, and by its phase the angle the pole has fallen behind. The current I may be resolved into components I_d and I_t in quadrature and in phase with the pole centers. By assumption I_d is caused by voltage component $E_o - E \cos\psi$ and is governed by X_d, while I_t is due to $E \sin\psi$ and is governed by X_t. Thus

$$I_t = E \sin(\psi)/X_t; \qquad I_d = (E_o - E\cos(\psi))/X_d \qquad (1)\ \&\ (2)$$

The ordinates Y of a set of V curves may be taken as the ratio the armature current to any current such as $(E/X_d) = I_o$ as a basis. The abscissas may be taken as the ratio of field currents of induced

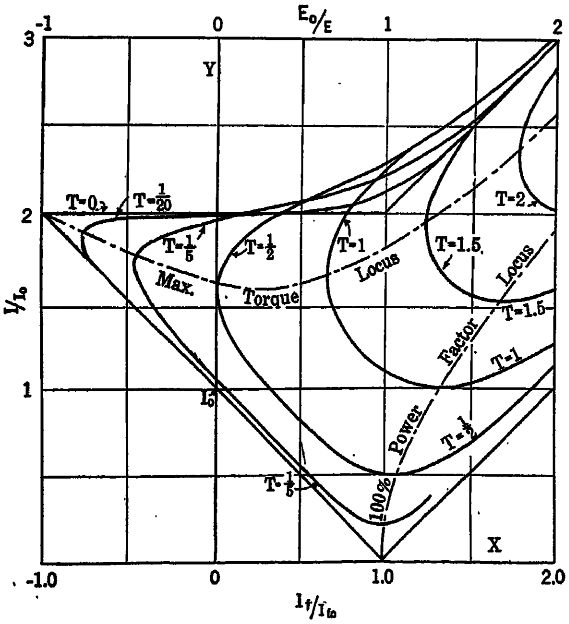

Fig. 6

no-load volts E_o/E. If we call $X_d/X_t = K$, I_p the power, and I_{qw} the wattless components of the current, we have the following:

$$I_t = I_o K \sin(\psi); \qquad I_d = I_o(X - \cos(\psi)) \qquad (3)\ \&\ (4)$$

$$T = I/I_o\, 2\sin(\psi)\ (\ (K-1)\cos(\psi) + X) \qquad (5)$$

$$I_w = I_o\,(X\cos(\phi) - K + (K-1)\cos^2\psi) \qquad (6)$$

$$I = \gamma\ (I_p^2 + I_w^2) \qquad (7)$$

We assumed $K = 2$, $T = 0.05$, 0.02, 0.5, 1.0, 1.5, and 2.0; we assumed values of $\psi = 2°$, 5°, 10°, 20°, 30°, ...170°, and solved Eq. (**5**) for X. We then solved Eq. (**6**) for I_w and Eq. (**7**) for I. We have plotted the values of I thus computed against X in Fig. 6. These curves in the main agree with our experiments, and certainly bear out the contention that the Blondel theory applies quite accurately at low flux densities.

Factors Affecting the Design of D-C. Motors for Locomotives

BY RALPH E. FERRIS[1]
Associate, A. I. E. E.

Synopsis.—The designer of motors for locomotive service is confronted with at least two limitations; space and weight. For large locomotives, the second limitation may not be of prime importance, but the first must be constantly in the mind of the designer.

The paper gives a comparison between different types of motor mounting as regards the amount of power which may be developed in the available space with direct current motors. The comparisons are largely qualitative but within reasonable limits are also quantitative.

The available space between wheels or locomotive side frames is divided into two parts. One of these parts is made up of units which are assumed to be constant within the range considered, while the other part is made up of variables. Expressions for the variables are derived, generally in terms of armature diameter, and constants and variables are then combined into a complete expression for motor output.

The voltage applied to motor commutator, voltage-to-ground, number of poles, peripheral speed and track gage, as well as type of motor mounting are considered in the comparison.

THE object of the following discussion is to develop the proportions of d-c. motors in relation to the available space between wheels or side frames of locomotives.

Two voltages only have been considered, viz., 3000 and 1000 volts, this, not because other voltages, either higher or lower, are not possible or desirable, but simply to give a basis of comparison and eliminate the almost endless number of combinations.

The results should be considered as largely qualitative, but it is believed that the effect of various factors will be shown and in a measure the results are also quantitative. Actual numerical values have been used in most cases with the full realization that exceptions could be taken to these values for special cases, or perhaps even for average conditions, but an honest attempt has been made to place all results on a basis as nearly comparable as possible. The nominal, or one-hour rating, of motors has been used as the physical dimensions conform more nearly with this rating than with the continuous rating where the type of ventilation is a determining factor.

The paper has been divided into three general sections: first 3000-volt motors; second, 1000-volt motors; and third, discussion of curves and conclusions.

3000-Volt Motors

For purposes of this paper the following classification has been chosen for the 3000-volt motors. It should be understood that the voltage given in the tables refers to the voltage across the commutator and not the voltage to ground which as before stated is in all cases either 3000 or 1000 volts.

1. Axle-hung Gear Drive

a. 750 Volts	*b.* 1000 Volts	*c.* 1500 Volts
1. four-pole	1. four-pole	1. four-pole
2. six-pole	2. six-pole	

2. Frame-mounted, Quill and Gear Drive

a. 750 Volts	*b.* 1000 Volts	*c.* 1500 Volts
1. four-pole	1. four-pole	1. four-pole
2. six-pole	2. six-pole	

3. Frame-mounted, Gear and Side-Rod Drive

a. 750 Volt	*b.* 1000 Volts	*c.* 1500
1. four-pole	1. four-pole	1. four-pole
2. six-pole	2. six-pole	2. six-pole
3. eight-pole	3. eight-pole	
4. 10 pole		

4. Gearless

a. 750 Volts	*b.* 1000 Volts	*c.* 1500 Volts
1. two-pole	1. two-pole	1. two-pole
2. four-pole	2. four-pole	2. four-pole
3. six-pole	3. six-pole	

To illustrate the method of analysis, all the cases for the axle-hung motors, 1 *a*, 1 *b*, and 1 *c*, are worked out in some detail, especially for the 750-volt four-pole motor. The first and second cases, under 4 *b* for the 1000-volt, gearless motor, will also be given in more or less detail. Of the remainder, curves will be shown for all four cases under 3 *a* and first case under 3 *b*, 3 *c*, 4 *a*, and 4 *c*.

Proportion of Space for 3000-Volt Motors, Axle-Mounted

The space between wheel flanges may be divided as follows:

L_t = Total space between wheels or between locomotive side frames

L_c = Length of commutator

L_e = Length of coil extension on ends not including cell extension.

L_g = Length of gear-face

L = Length of armature core iron

L_k = A constant which is made up of the following items:

1. Clearance between gear-case and wheel, front and rear .... 2.00 in.
2. Gear-case walls and clearance to gear, wheel side, front and rear .... 1.75 in.

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[1] Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

3.	Gear-case walls and clearance to gear, motor side front and rear	1.50 in.
4.	Gear-case to end-housing, front and rear	0.75 in.
5.	Housing walls, front and rear	2.00 in.
6.	Clearance between front-end housing and commutator V-ring	0.75 in.
7.	Creepage on V-ring	2.50 in.
8.	Commutator groove and clearance between brushes	0.75 in.
9.	Coil extension, both ends	2.00 in.
10.	Clearance between coil and rear-end housing	1.25 in.
	Total	15.25 in.

In order that the foregoing variables and constants may be understood, they are indicated in the diagrammatic long section shown in Fig. 1.

It is realized that the constants as given may be open to question in any particular case, but it is believed they are not far from average conditions, and will at least give, as before stated, a basis for comparison.

Output Equations

It can be shown that the output of a motor may be expressed as follows:

$$\text{k-w.} = C\,D^2 L\,S \tag{1}$$

where

C = A so-called output constant
D = Armature diameter
L = Length of armature iron
S = Speed in rev. per min.

Substituting values given in preceding section in equation (1), gives

$$\text{k-w.} = (L_t - L_c - L_e - L_g - L_k)\,D^2 C\,S \tag{2}$$

But

$$L_c = \text{kw.} \times C_c \tag{3}$$

Where C_c is a constant depending on brush width, voltage, number of brush arms, and current density in brush.

$$L_g = \text{kw.} \times C_g + C_2 \tag{4*}$$

$$L_e = C_e D \tag{5}$$

Where C_g, C_2 and C_e are constants. The derivations of equations 3, 4 and 5, which are a means to an end, will be shown in more detail under the special case of the four-pole, 750-volts per commutator motor.

Substituting equations 3, 4 and 5 in 2, and solving for kw. gives

*Equation (4) is only approximate, the following being more nearly correct.

$$L_g = \frac{\text{kw.} \times C_p \times P_t}{s \times N_t}$$

where
P_t = Diametral pitch of gear
S = speed of motor in rev. per min.
N_t = Number of teeth in pinion
C_p = Constant depending on P_t

A value of C_p which will give a fairly conservative gear-face width is as follows:

$$C_p = \frac{32\,P_t}{1 - 0.19\,P_t}$$

$$\text{kw.} = \frac{D^2 S\,C\,(L_t - C_e D - C_2 - L_k)}{1 + D^2 S\,C\,(C_c + C_g)} \tag{6}$$

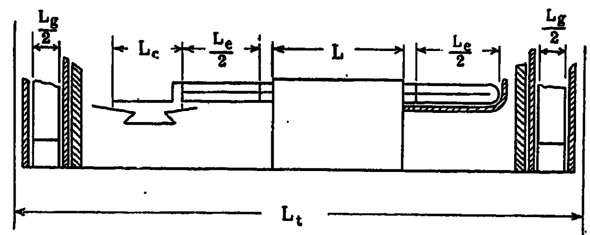
If $L_h = L_t - L_k$

Then

$$\text{kw.} = \frac{D^2 S\,C\,(L_h - C_e D - C_2)}{1 + D^2 S\,C\,(C_c + C_g)} \tag{7}$$

In order to obtain a comparison, a constant peripheral speed in feet per minute of C_s is taken, in which case,

$$S = \frac{3.82\,C_s}{D} \tag{8}$$



$L_k = L_t - L_g - L_c - L_e - L$

Fig. 1

By substitution, a final expression for kw. in terms of D is as follows:

$$\text{kw.} = \frac{3.82\,C_s\,D\,C\,(L_1 - C_e D - C_2)}{1 + 3.82\,D\,C_s\,C\,(C_c + C_g)} \tag{9}$$

General Discussion of Output Constant C

Very often in engineering literature a so-called constant is not a constant at all but is merely a convenient way of combining a number of factors which are difficult to evaluate. Such is the case with the output constant C. This constant C is affected by a number of factors, the principal being as follows:

a. Slot-space factor. The slot-space factor is the ratio of cross-section of copper in slot to cross-section of slot, and therefore with a given wall thickness of insulation to ground, this factor varies as some direct function of the cross-section of copper in the slot. For a given voltage and peripheral speed, the diameter of armature will increase with increase of total copper cross-section, or stated inversely, the slot-space factor increases with increase of armature diameter.

It can be shown that for a constant number of slots, the slot-space factor is given approximately by the following expression:[2]

$$f = \frac{K_1 D^2 - K_2 D + K_3}{K_1 D^2}$$

where f = Slot-space factor
K_1, K_2, K_3 = constants

b. Iron loss. For a given density in the teeth and core of the armature the iron loss will vary as some

2. Effect of Insulation on Design by R. E. Ferris, "*The Electric Journal*," October, 1923.

direct function of the speed. For a given peripheral speed, the iron loss per cubic inch of iron for a constant density and a constant number of poles will decrease with increase of armature diameter, or for constant iron loss per cubic inch, the density in teeth and core and therefore in the air-gap may be increased with increase of armature diameter. As the air-gap density is one of the factors which enters into the output-constant, it follows that this factor also causes the output-constant to increase with the armature diameter.

c. Pole constant. The pole constant as defined in this paper is the ratio of total pole-face bore to periphery of armature core. It is, of course, evident that the larger the pole constant, the larger will be the output constant, other things being equal.

In general, the space between main pole tips will remain approximately constant for a given number of poles within a reasonable range of armature diameters. With this assumption, therefore, if we let

a = space between pole tips
C_p = pole constant
D = armature diameter,

then

$$C_p = \frac{\pi D - a}{\pi D} = 1 - \frac{a}{\pi D}$$

This expression shows that the pole constant, and therefore the output constant, increases with armature diameter.

d. Watts copper-loss per sq. in. of armature surface. This factor in the output constant is governed largely by the method of heat dissipation and is not affected

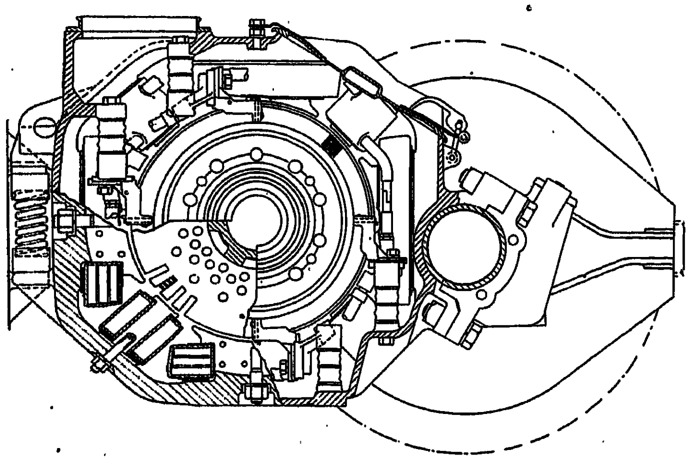


Fig. 2

materially by the armature diameter except as the armature diameter affects method of ventilation.

In general, therefore, the output constant increases with increase of armature diameter. It is, of course, practically impossible to give an expression for the output-constant in terms of armature diameter, which will more than approximate the results for any given design. The following imperical expression, however, is used as coming within the limits of accuracy of this article:

$$C = K \log (a D - b)$$

FOUR-POLE AXLE-HUNG MOTOR, 750 VOLTS PER COMMUTATOR

The axle-hung type of mounting and drive are so common that little or no description is necessary. Briefly, one side of the motor is supported by either a nose or bar suspension on the truck transom, while the other side is supported directly on the axle, through axle caps and bearings. With this mounting, a certain percentage of the motor weight is carried directly on the axle with no intervening spring.

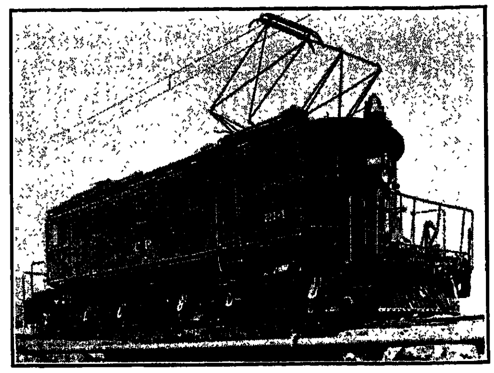


Fig. 3—Complete Locomotive with Axle-Hung Motors

A cross section of this type of motor mounting is shown in Fig. 2, and a complete locomotive with axle-hung motors in Fig. 3.

The output constant for a four-pole, 3000-volt, 750-volt-per-commutator motor may be expressed imperically by the following equation:

$$C = \log (0.385 D - 1.31) 7.65 \times 10^{-5} \qquad \mathbf{(11)}$$

Length of commutator. The length of commutator, L_c, for 750 volts per commutator and four brush-holder arms, may be derived as follows:

If

I_t = total amperes to armature
E = volts at motor terminals
f = efficiency

$$I_t = \frac{1000 \times \text{kw.}}{E F} \qquad \mathbf{(12)}$$

and, if,

$E = 750$
$f = 0.88$
$I_t = 1.52$ kw. **(13)**

The length of commutator neck cannot be expressed in terms of kilowatt unless the number of parallel circuits are taken into account; but as this introduces a further complication, such a factor will be omitted and the length of neck expressed as

$$\text{Neck} = 0.0024 \text{ kw.} \qquad \mathbf{(14)}$$

At 60 amperes per sq. in. in the brush and 5/8 in.

brush width, the expression for commutator face, including allowance for brush-stagger, is,

$$\text{Face} = \frac{I_t}{120 \times 0.625} = 0.0203 \text{ kw.} \quad (15)$$

or L_c = Neck + face = 0.0227 kw. (16)

Width of Gear-Face. As an imperical approximation of the gear-face

$$L_g = 0.0134 \text{ kw.} + 3.5 \quad (17)$$

This, for the larger sized motors, will give a gear-face

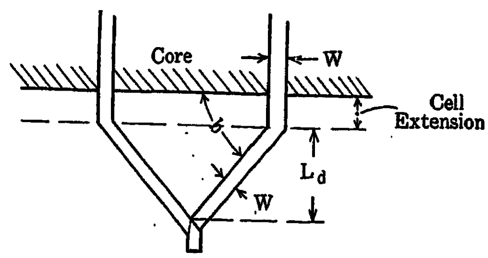


Fig. 4

which is somewhat low, and for the smaller sizes, somewhat high.

Coil Extension L_e. In general, the armature slot-depth will vary as some function of the armature diameter. For the present discussion the following relation will be used:

$$D_s = 0.1\, D \quad (18)$$

where

D_s = Slot-depth

Let

P = ½ the arc of coil-pitch at diameter $(D - 0.1\, D)$

Then for a four-pole machine

$$P = \frac{\pi\,(D - 0.1\, D)}{8}\, 0.95 \quad (19)$$

The factor 0.95 is introduced to take care of a slight amount of chording.

Referring to Fig. 4, with slot-width W equal to tooth-width at one-half depth of slot and with the coil on the diamond part of the end portion equal to slot-width, the sin of angle b, between iron and coil, will be,

$$\text{Sin } b = \frac{\dfrac{\pi\,(D - 0.1\, D)}{2}}{\pi\,(D - 0.1\, D)} = 0.5 \quad (21)$$

Tan b = 0.577

Therefore, referring to Fig. 4, (22)

$$L_d = P \times 0.577$$

But

$$P = 0.335\, D$$

and

$$L_d = 0.194\, D \quad (23)$$

If turn at end of diamond part of coil is considered as six per cent of armature diameter and both ends of the winding are included, then

$$L_e = 0.448\, D \quad (24)$$

TABLE OF CONSTANTS

The following values are, therefore, either constants or assumed constants, at the one-hour rating for a four-pole 750-volts-per-commutator axle-mounted motor:

C_c = 0.0226
C_g = 0.0134
C_2 = 3.5
C_e = 0.448
L_k = 15.25
L_t = 53.25
L_h = 38
C_s = 3500

Substituting equation No. **11** and constants given above in equation No. **9**,

$$\text{kw.} = \frac{(35.4\, D - 0.46\, D^2) \log (0.385\, D - 1.31)}{1 + 0.037\, D \log (0.385\, D - 1.31)} \quad (25)$$

Equation (**25**) gives the kw.-output of a four-pole 750-volts-per-commutator motor with double gears. The case for single-end gears will not be considered, as the only difference would be more available space between wheels for active material. In any case, as limitations are being considered, the motor sizes under consideration would, in general, have double gears.

Curve A of Fig. 5, is the graph of equation (**25**), and shows the kw.-output plotted against the armature diameter for axle-mounted motors, 750-volts per commutator, at a constant peripheral speed of 3500 ft. per min.

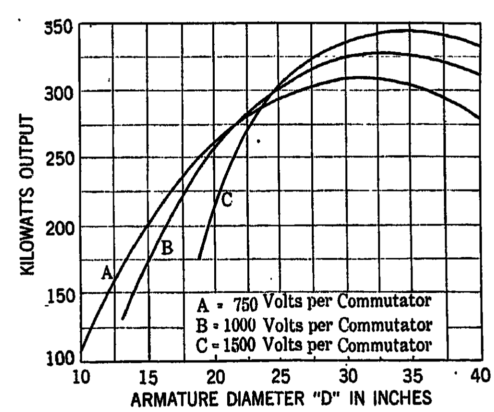


Fig. 5—Axle-Mounted 3000-Volt, Four-Pole Motors, 3500 Ft. per Min. Peripheral Speed 53.25 In. Track Gage

Effect of Track Gage. To show the effect of track-gage on maximum output, the following equation was derived from equation (**25**); assuming a constant armature diameter of 30 in. and a constant peripheral speed of 3500 ft. per min.;

$$\text{kw.} = 14.6\, L_t - 470 \quad (26)$$

Where L_t is distance between wheel flanges.

The graph of this straight line equation is shown in Fig. 6, and brings out strikingly the advantage to be gained by wide track gage.

Variable Peripheral Speed. In order to show the effect of change in peripheral speed, the armature

diameter was assumed a constant at 30 in. in which case, equation (27) was derived from the general equation to show this relation;

$$\text{kw.} = \frac{0.186\, C_s}{1 + 0.000318\, C_s} \tag{27}$$

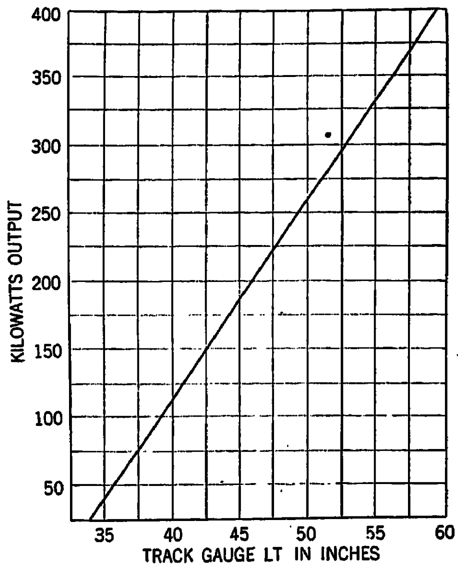


FIG. 6—AXLE-MOUNTED, 3000-VOLT, FOUR-POLE MOTOR, 3500 FT. PER MIN. PERIPHERAL SPEED, 30 IN. ARMATURE DIAMETER

The graph of equation (27) is shown in Fig. 7.

Depth of Field. It will be necessary at this point to digress somewhat from the specific consideration of

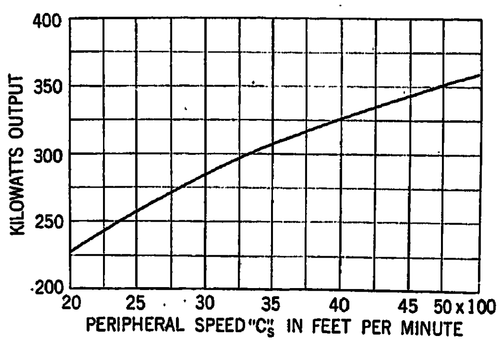


FIG. 7—AXLE-MOUNTED, 3000-VOLT, FOUR-POLE MOTOR 750 VOLTS PER COMMUTATOR, 30 IN. ARMATURE DIAMETER, 53.25 IN. TRACK GAGE

the 750-volt-per-commutator motor, to the more general discussion of the field design.

The amount of copper in the field-winding of a railway motor does not, in general, increase in direct proportion to the diameter of motor armature, mainly because the air-gap which absorbs a large share of the ampere-turns, is not increased in direct proportion to the motor diameter; for example, a 30-in. diameter armature would not, in general, have an air-gap twice the length of a 15-in. diameter armature. Furthermore, the available space for field copper increases with increase in armature diameter, assuming a constant depth of field, or, inversely, the depth of field may be less for a given amount of copper, the larger the diameter of armature. With the foregoing in mind, Figs. 8 and 9 will be understood. Fig. 8 shows roughly the ampere-turns per pole in relation to armature diameter, and Fig. 9, the depth of motor from armature surface to outside of frame at the bottom.

Wheel Diameter. From the curve shown in Fig. 9, and, assuming that the motor is raised above the center line of axle 1½ in., and with clearance-to-rail of 4½ in.,

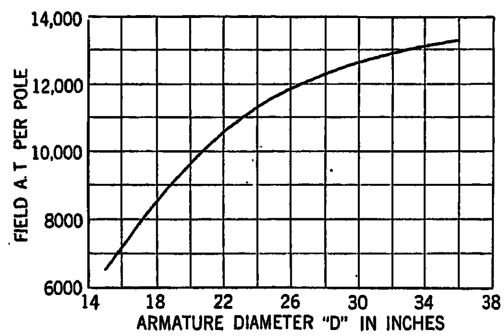


FIG. 8—FIELD AMPERE-TURNS PER POLE FOR A FOUR-POLE, AXLE-MOUNTED MOTOR

the wheel-size is plotted against armature diameter in Fig. 10.

Diameter of Gear. After the wheel diameter is determined, the pitch-line diameter of maximum size gear is approximately fixed, if a given clearance under the gear case is assumed. With 4½ in. clearance from the pitch-line of gear to rail, the pitch-line diameter of gear is plotted against armature diameter in Fig. 11.

For the size of motors under consideration, perhaps the minimum-size pinion would be 16 teeth 2 diametral

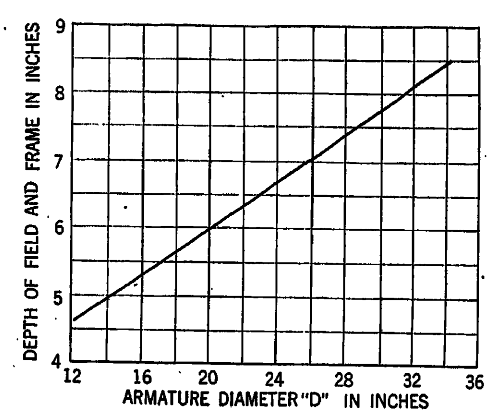


FIG. 9—APPROXIMATE DEPTH OF FIELD AND FRAME

pitch, which would give a minimum pitch-line diameter of pinion of 8 in.

With the diameter of gears and pinions given, by the foregoing assumptions the gear center distance of the motor is fixed, but this gear-center distance checks approximately with the gear-center distance derived from armature diameter, depth of field and frame, thickness of axle bearing and diameter of axle.

Minimum Miles per Hour. With a constant peripheral speed, the rev. per min. of armature may be derived for each diameter, and this, with the maximum gear reduction as given above, gives the minimum mi. per hr. at the one-hour rating for any sized armature under the conditions assumed. This is shown in Fig. 12.

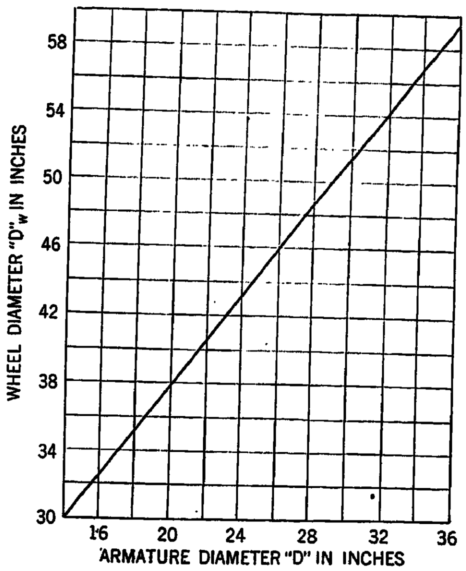


FIG. 10—AXLE-MOUNTED, FOUR-POLE MOTORS—WHEEL DIAMETER

FOUR-POLE AXLE-MOUNTED MOTOR, 1000 VOLTS PER COMMUTATOR

The constants included in L_k will be the same, for a 3000-volt motor with 1000 volts across the commutator,

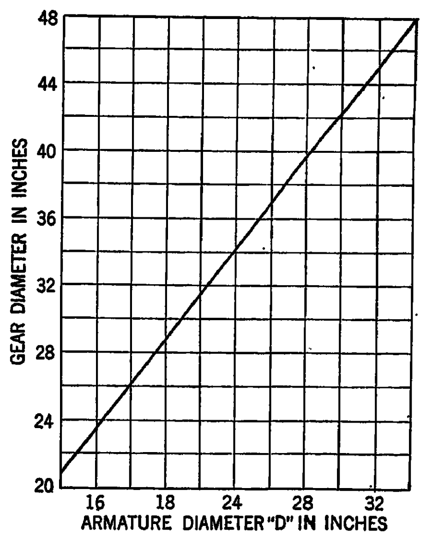


FIG. 11—AXLE-MOUNTED, FOUR-POLE MOTORS GEAR, DIAMETER

as for a similar motor with 750 volts on the commutator.

The length of coil extension L_e, for the same armature diameter and the width gear-face L_g for the same kw. output, will also be the same. The length of commutator L_c and output-constant C, will, however, be different.

The length of commutator will be less, due to the smaller amount of current, with the higher voltage.

The output constant for the smaller diameters will be less than for the 750-volt motor due, mainly, to the smaller slot-space factor, but for diameters over 26 in. there will be but little difference.

Taking all the factors into consideration, an expression similar to that given in equation (25) was derived. Curve B in Fig. 5 shows kw.-output plotted against

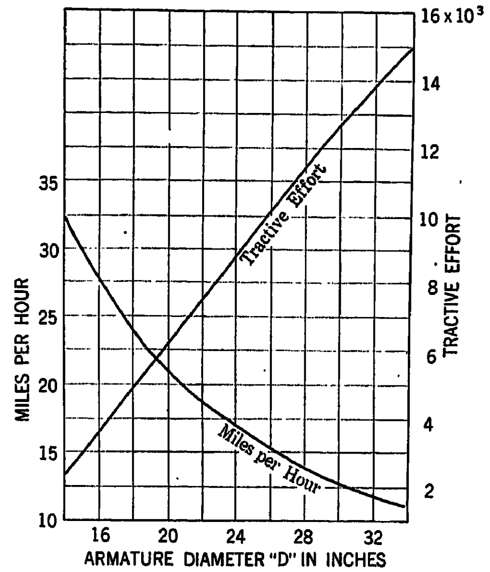


FIG. 12—AXLE-MOUNTED, FOUR-POLE MOTORS, MI. PER HR. AND TRACTIVE EFFECT

armature diameter for the four-pole, 1000-volt-per-commutator motor.

Curves for varying track-gage, or peripheral speed, are not shown, as the effect of these factors on the output is clearly displayed in the curves for the 750-volt motor.

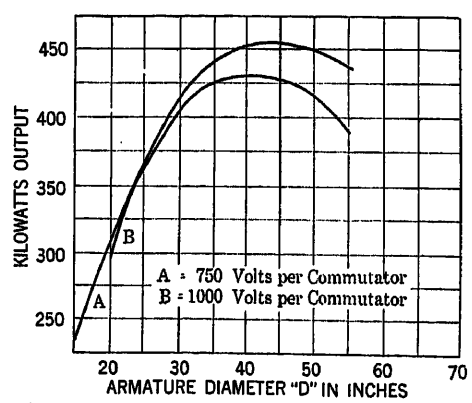


FIG. 13—AXLE-MOUNTED, 3000-VOLT, SIX-POLE MOTORS 3500 FT. PER MIN. PERIPHERAL SPEED, 56.5 IN. TRACK GAGE

FOUR-POLE AXLE-MOUNTED MOTOR, 1500 VOLTS PER COMMUTATOR

The proportion of space for the 1500-volt motor as included in L_k will be the same as for the 750-and 1000-volt-per-commutator machine.

The length of coil extension L_e, and gear face L_g will also be the same.

The output constant C, and length of commutator L_c will, however, be different than for either of the lower voltages.

Taking these factors into account, an expression was derived, as in the preceding cases. Curve C of Fig. 5, shows relation of kw.-output to armature diameter for a 1500-volt-per-commutator, four-pole motor.

Six-Pole Axle-Mounted, 750 and 1000 Volts-Per-Commutator Motor

The proportions of space for the six-pole 750-and 1000-volt axle-hung motors are the same as for the four-pole. The length of commutator L_c, and coil extension L_e, will, however, be different, due, in case of commutator length, to the larger number of brush-arms and, in the case of coil extension, to the shorter coil throw.

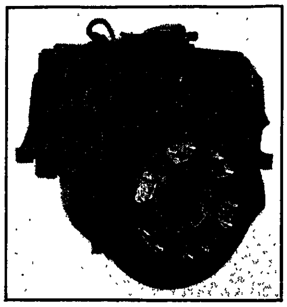


Fig. 14—Quill-Type Drive

Curve A of Fig. 13, shows the relation of kw.-output to armature diameter for a six-pole, axle-mounted, 750-volt-per-commutator motor, while curve B, in the same figure, shows a similar relation for the 1000-volt-per-commutator motor.

Fig. 15—Quill-Type of Drive and Twin Motors Mounted

Frame Mounting—Quill Drive

In the quill drive, the motor is mounted on the spring-born frame of the locomotive, and the main driving axle centered through a hollow quill. The driving gear is mounted on the quill, this quill being held in definite relation to the motor and locomotive frame. The actual drive takes place between the gear and drive-wheel by means of springs which permit the drive-wheel and axle to move in relation to the gear and motor.

In general, it is found desirable to have somewhat more clearance between the wheel-flange and end-housing of motor in quill-mounting, than in axle-mounting; otherwise the proportions of space are the same and, therefore, the curves of output will be approximately the same as those for axle-mounting.

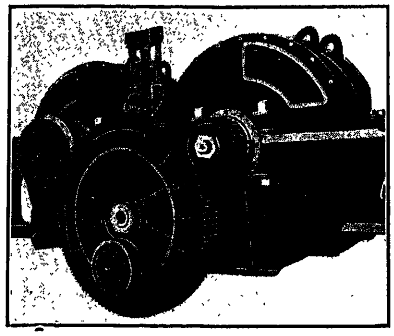


Fig. 16—Inside-Hung Frame

The quill drive does, however, permit the use of a twin motor construction and thus, unless limited by the drive, gives twice the power per axle as with an axle-hung motor.

Fig. 14 shows the quill type of drive while Fig. 15 shows a locomotive with quill drive and twin motors mounted in place.

Fig. 17—Inside-Hung Frame Motor, Mounted Complete

Frame-Mounting Side-Rod or Gear and Side-Rod Drive

In the case of frame-mounted motors, with side rod or gear and side-rod drive, the distance between the side frames of the locomotive L_t, available for the motor, is not a definite quantity, but for the case of inside-hung frames, perhaps 44 in. will be a fair average for the size of locomotives under consideration.

This type of motor mounting is shown in Fig. 16, while Fig. 17 shows the locomotive complete.

If the motor is made self-contained, so that it may be lifted out of the locomotive frame complete, there will be less room available for active material than if the armature bearings are mounted in the side-frame of the locomotive and end-housings omitted except for

comparatively thin protective covers. For the latter case, the constant L_k will contain the following:

1. Clearance between the frame and commutator V-ring	0.75 in.
2. Creepage on commutator V-ring	2.50 in.
3. Commutator groove and clearance between brushes	0.75 in.
4. Cell extension both ends	2.00 in.
5. Clearance between coil and frame at rear	1.25 in.
Total for L_k	7.25 in.

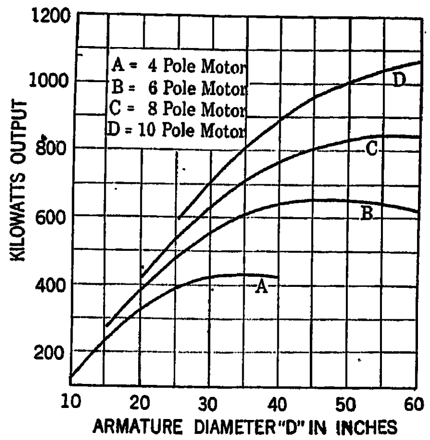


Fig. 18—Frame-Mounted, 3000-Volt Motors, 3500 Ft. per Min. Peripheral Speed, 750 Volts per Commutator

A = Four-Pole Motor
B = Six-Pole Motor
C = Eight-Pole Motor
D = Ten-Pole Motor

In this case, space need not be allowed for gears and the output equation becomes:

$$\text{kw.} = (L_t - L_c - L_e - L_k)\, D^2\, C\, S \qquad (28)$$

From which with the proper substitutions and a peripheral speed of C_s

$$\text{kw.} = \frac{3.82\, C_s\, D\, C\, (L_h - C_e\, D)}{1 + 3.82\, C_s\, C\, C_c\, D} \qquad (29)$$

Substituting the proper constants for a 750-volt, four-pole, frame-mounted motor, in equation No. **29**,

$$\text{kw.} = \frac{(37.6\, D - .46\, D^2) \log (.385\, D - 1.31)}{1 + .0231\, D \log (.385\, D - 1.31)} \qquad (30)$$

The graph of equation (**30**) is given as curve A in Fig. 18.

In a similar manner, expressions were derived for six, eight-and ten-pole, 750-volt motors, and the results shown in curves B, C and D of Fig. 18. Also, expressions for four-and six-pole motors, with 1000-volts per commutator, were derived and the results shown in curves A and B of Fig. 19.

Gearless Motors

The gearless motor may be divided into two classes:

1. Direct axle-mounting
2. Quill-mounting

Direct Axle-Mounting. In this case the armature is pressed directly on the driving axle, and therefore follows the motion of wheels and axle in both a vertical and lateral direction. This, for all practical purposes, necessitates a bi-polar construction with the pole faces eccentric to the armature, thus permitting the movement of the core vertically in relation to the poles.

With this type of motor design, it is desirable, from a mechanical standpoint, to have a comparatively large air-gap, also the pole constant C_p will be relatively small; the latter, taken by itself, would mean a low output constant, but owing to the comparatively low rotational speed and the bi-polar construction, the frequency will, in general, be quite low, and, therefore, the air-gap, teeth, and core-flux density may be high without undue iron loss or heating, thus giving a higher output constant than would otherwise be the case.

It is even more difficult to give an expression for the so-called output constants in terms of the armature diameter for the bi-polar type of construction than for the motor with a larger number of poles, but, by segregating the separate factors which go to make up the output constant, it is believed that no gross error has been introduced.

The same nomenclature will be used for division of space between wheel flanges as was used on the axle-mounted, geared motor, but in this case, the constant L_k will be made up of the following items:

1. Clearance between wheel and commutator V-ring	2.25 in.
2. Creepage on commutator V-ring	2.50 in.
3. Commutator groove and clearance between brushes	1.00 in.
4. Cell extension both ends	2.00 in.
5. Clearance between wheel and rear of armature	2.25 in.
Total L_k	10.00 in.

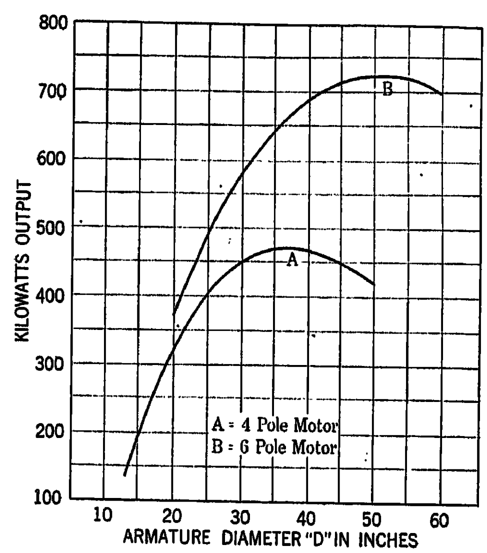


Fig. 19—Frame-Mounted, 3000-Volt Motors, 3500 Ft. per Min. Peripheral Speed, 1000 Volts per Commutator

The problem of output will be approached in a somewhat different manner for the bi-polar gearless than for the proceeding type.

It will be assumed that the clearance between armature surface and rail is constant at 7½ in., regardless of armature diameter, thus giving an armature diameter 15 in. less than the diameter of wheel.

At 25 mi. per hr. which is used as a basis of comparison, the peripheral speed of wheel is 2200 ft. per min. If

R = rev. per min. of wheel and armature at 25 mi. per hr.

D_w = Wheel diameter in inches

Then

$$R = \frac{2200 \times 12}{D_w \times \pi} = \frac{8400}{D_w} \qquad (31)$$

also

$$R = \frac{C_s \times 12}{(D_w - 15)\,\pi} = \frac{3.82\,C_s}{D_w - 15} \qquad (32)$$

where C_s = Peripheral speed of armature in ft. per min. equating (31) and (32) and solving for C_s

$$C_s = \frac{2200\,(D_w - 15)}{D_w} \qquad (33)$$

The construction of the bi-polar, gearless motor does not permit the use of commutating poles and the armature winding may, therefore, be chorded to the point where further chording would give a reduction in counter electromotive force, and increase the speed for a given flux per pole. This coupled with the wide interpolar space, gives a wide commutating zone and permits the use of a wider brush than might otherwise be be used. The chording also shortens the armature-winding end-extension.

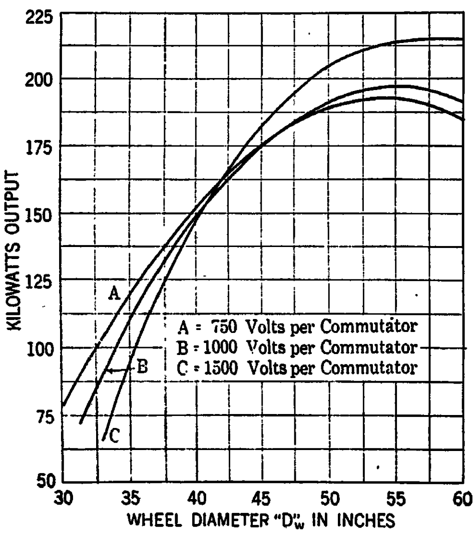


FIG. 20—GEARLESS 3000-VOLT BI-POLAR MOTORS, 25 MI. PER HR. 56.5 IN. TRACK GAGE

For the case of 1000-volts-per-commutator, brush width of 0.75 in., 60 amperes per sq. in. in brush, and 90 per cent efficiency at the one-hour rating.

the commutator face = 0.0247 kw.

neck = 0.0035 kw.

or neck + face $= L_c = 0.0282$ kw. (34)

Without giving details of derivation, the following expression for coil extension was used:

$$L_e = 0.6\,(D_w - 15) \qquad (35)$$

also the output constant within the range considered was given the following value

$$C = \log\,(1.45\,D - 13.5)\;3.43 \times 10^{-5} \qquad (36)$$

In order, therefore, to obtain an expression for kw.-output in terms of wheel-diameter for the bi-polar gearless motor equations (33) and (36) and the following values were substituted in equation (29):

$$D = D_w - 15$$
$$L_h = 43.25$$
$$C_e = 0.6$$
$$C_c = 0.0282$$

giving

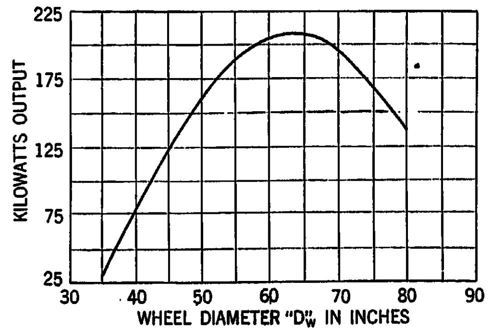


FIG. 21—GEARLESS 3000-VOLT, FOUR-POLE QUILL MOTOR 25 MI. PER HR., 1000 VOLTS PER COMMUTATOR 56.5 IN. TRACK GAGE

$$\text{kw.} = \frac{(20.3\,D_w^2 - .173\,D_w^3 - 490\,D_w + 3380)}{D_w + (0.00813\,D_w^2 - 0.244\,D_w + 1.84)}$$

$$\frac{\log\,(1.45\,D_w - 35.3)}{\log\,(1.45\,D_w - 35.3)} \qquad (37)$$

Curve B in Fig. 20 is the graph of equation (37).

If, in place of varying the wheel diameter, the peripheral speed or, in other words, the mi. per hr., is taken as the variable and the wheel-diameter held constant at 44 in. the following equation gives the approximate kw.-output in terms of peripheral speed of armature for a 1000-volt bi-polar, gearless motor:

$$\text{kw.} = \frac{0.145\,C_s}{1 + 0.000156\,C_s} \qquad (38)$$

The curve shown in Fig. 22 is the graph of equation (38) except that mi. per hr. is used in place of peripheral speed.

Expressions for kw.-output, in terms of wheel-diameter for 750 and 1500 volts per commutator, were derived and the results are shown in curve form as A and C in Fig. 20.

GEARLESS QUILL-MOUNTING

With this type of motor, the armature is mounted on a hollow quill which surrounds the driving axle with sufficient clearance between the inside quill wall and axle to allow of maximum movement of wheels and axle in a vertical or horizontal direction. The quill is supported through quill bearings and end-housings

by the motor frame, which, in turn, is supported on the spring-borne locomotive frame.

In the case of the gearless quill drive, the constant L_k will be divided as follows:

1.	Wheel flange to commutator V-ring	8.00 in.
2.	Creepage on commutator V-ring	2.50 in.
3.	Cell extension both ends	2.00 in.
4.	Rear end to wheel flange	8.00 in.
	Total L_k	20.50 in.

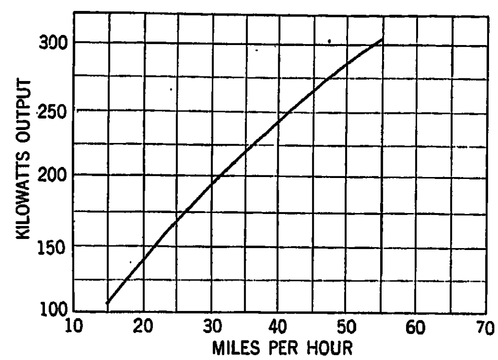


FIG. 22—GEARLESS 3000-VOLT BI-POLAR MOTOR, 44 IN. WHEEL DIAMETER, 56.5 IN. TRACK GAGE, 1000 VOLTS PER COMMUTATOR

As was done in the case of the bi-polar, gearless motor, the clearance between the armature surface and rail will be considered constant. This, of course, is not strictly true, as, on this type of motor, room must be allowed for field coil and frame between armature surface and rail, as well as for the proper clearance. However, within the range considered, the assumption will be approximately correct.

This distance between armature surface and rail will be assumed as 11 in., thus giving an armature diameter 22 in. less than the diameter of wheel.

From this point on, the same procedure was followed as for the bi-polar gearless, except for difference in number of poles, output constant, etc.

Fig. 21 shows kw.-output plotted against wheel diameter for a four-pole, 1000-volt-per-commutator gearless quill mounted motor.

1000-VOLT MOTORS

Some explanation may be in order as to why a unit of 1000 volts was chosen as an illustration of the use of lower voltage in place of—for example—600 or 750 volts.

In general, the designer of d-c.-traction motors is confronted with two difficulties,—not limits, but difficulties; that is, for 1500-volts-per-commutator motors, commutating difficulties are encountered, especially for two-circuit armature windings. On the other hand for 600-volts-per-commutator motors, the size of commutator becomes unduly large for a given number of poles. As a happy medium, therefore, a unit of 1000 volts was chosen.

It is realized, of course, that this is not a standard line voltage, but in making this choice, a higher line voltage was presupposed, either a-c. or d-c., with some method of conversion to a lower voltage for use on the traction motors.

For the purpose of this paper, the following classification has been chosen for the 1000-volt motors:

1. Axle-hung Gear-Drive
   - *a.* Four-poles
   - *b.* Six-poles
2. Frame-mounted, Quill and Gear Drive
   - *a.* Four-poles
   - *b.* Six-poles
3. Frame-mounted, Gear and Side-Rod Drive
   - *a.* Four-pole
   - *b.* Six-pole
   - *c.* Eight-pole
4. Gearless
   - *a.* Two-pole
   - *b.* Four-pole
   - *c.* Six-pole

The analysis of this class of motor was made in much the same way as for the 3000-volt motor; therefore, the giving of details would be a needless repetition.

Two factors, however, should be mentioned; viz, the better slot-space factor for the 1000-volt motor, due to the use of less insulation, this giving a higher output constant, and the smaller creepage surfaces on the commutator V-ring and shorter cell-extension outside of the core.

For the case of 1000-volt motors, therefore, the output-constant was expressed imperically as follows:

$$C = \log (0.32\, D + 0.2)\, 8.2 \times 10^{-6}$$

and L_k was made up of the same items as given for the

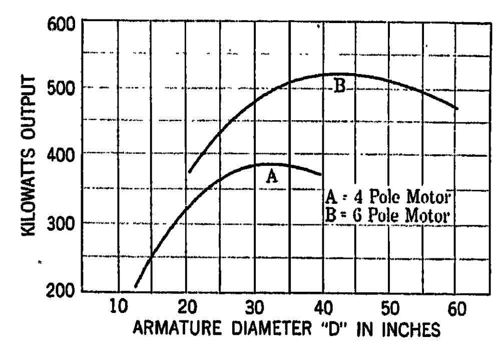


FIG. 23—AXLE-MOUNTED 1000-VOLT MOTORS, 3500 FT. PER MIN. PERIPHERAL SPEED, 56.5 IN. TRACK GAGE

3000-volt, axle-mounted motor, except that the creepage on the V-ring and the cell-extension for both ends were each given a value of one inch, giving for the total 12.75 in.

It will be noted that this value of L_k is 2.50 in. less for this voltage motor than for the 3000-volt, axle-mounted geared motor. This same reduction holds true also for the frame-mounted and gearless motors of this type.

Expressions were derived for 1 *a*, and 1 *b*, 3 *a* and 3 *b*, and 4 *a* of the classification table. Curves *a* and *b* of Fig. 23 give kw.-output in terms of armature

diameter for a 1000-volt, axle-mounted geared motor, four-pole and six-pole respectively.

Curves *a* and *b* of Fig. 24 give kw-output in terms of armature diameter for a 1000-volt frame-mounted motor, four-pole and six-pole, respectively.

Curve of Fig. 25 gives kw.-output in terms of wheel diameter for a 1000-volt, bi-polar, gearless motor.

Discussion of Curves and Conclusions

It will be noted that in the foregoing no mention has been made of commutation limits. This will actually prove the practical limiting factor in some cases, but

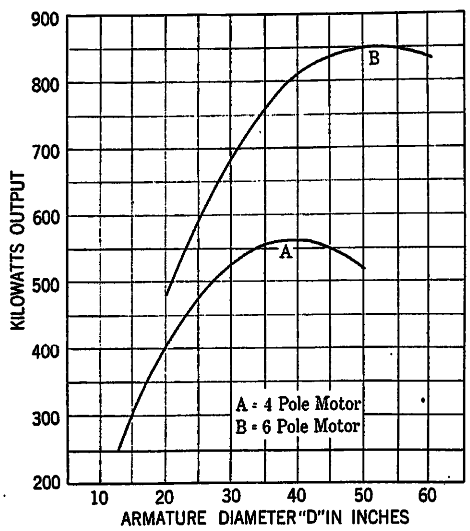


Fig. 24—Frame-Hung, 1000-Volt Motors, 3500 Ft. per Min. Peripheral Speed

in general it may be met by refinements in design. However, the flashing and commutation characteristics of a d-c. motor limit the number of poles which may be used for a given armature diameter, and it is largely for this reason that the classification given at the opening of this paper was employed.

The paper is not intended to include a discussion of types of locomotive-drive, but the application is so intimately allied with this much discussed problem that it must be touched on, at least incidentally.

Reverting now to Fig. 5 which shows output in kw. for four-pole 750-, 1000- and 1500-volt motors, plotted against armature diameter, it will be noted that for the smaller diameters, the lower voltage make the better showing. However, as the diameters increase the higher voltage approaches and finally crosses the lower voltage curve. All three voltages reach a maximum output at between 32 in. and 36-in. diameter with very little gain after 32-in. diameter.

It is evident, therefore, that there is a real limit to the amount of power which may be placed between the wheels with an axle-mounting, regardless of wheel size. Fortunately, this limit is sufficiently high to permit of a fairly heavy axle loading with speeds between 15 and 20 mi. per hr. This, coupled with the simplicity of drive and ruggedness of motor construction, place the combination well towards the front as a solution of the direct-current heavy traction problem.

The curve of Fig. 6 shows strikingly the handicap of a 42 in. track-gage, or, on the other hand, the advantage of a 60-in. gage over the standard.

With better methods of balancing, and a closer study of the stresses involved, advantage is being taken of higher peripheral speeds. The relative advantages to be gained by change in peripheral speed is shown in Fig. 7, where armature diameter is held constant and kw. plotted against peripheral speed.

The curves of Fig. 13 for six-pole, axle-mounted motors as compared with those of Fig. 5, show the advantages of a larger number of poles for a given armature diameter and also show that it should be possible to place more power between the wheels with the larger number of poles. The point of maximum output, however, is not reached in this case under 40-in. armature diameter.

The curves of Fig. 18 show the advantage of a larger number of poles in obtaining large output when faced by space limitations, and also show that the amount of power which the motor designer is able to produce is practically unlimited in the case of frame-mounted motors.

The output for the bi-polar, gearless motor is shown in Figs. 20 and 22, and is also limited. This limit may, however, be determined more by the dead-weight per axle than by maximum output limit. It will be noted from Fig. 22 that this type of motor finds, perhaps, its best application in high-speed service, for, if

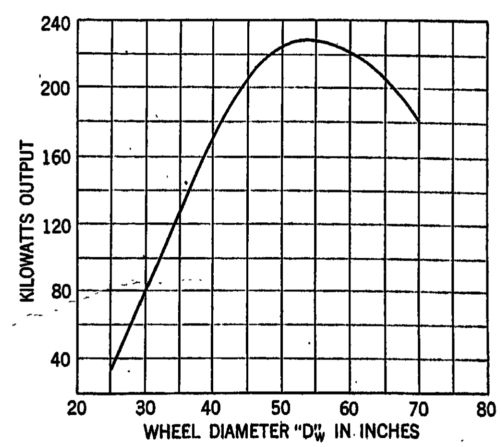


Fig. 25—Gearless 1000-Volt Bi-Polar Motor, 25 Mi. per Hr., 53.25 In. Track Gage

the curve of Fig. 22 is compared with that of Fig. 7, it will be found that the output of the bi-polar motor increases much faster with a given increase in peripheral speed than is the case with the four-pole axle-mounted motor.

The curves of Fig. 23 for the 1000-volt motor should be compared with similar curves of Figs. 5 and 13 for the 3000-volt motor. The gain in maximum output is quite pronounced, approximately 20 per cent for the four-pole and 15 per cent for the six-pole motor. The

same advantage is also shown when comparing curves of Fig. 24 with those of Fig. 19 for frame-mounted motors.

The curve of Fig. 25 for a 1000-volt, bi-polar gearless motor as compared with Curve *B* of Fig. 20 also shows the advantage of the lower voltage motor in so far as output is concerned.

It would seem, therefore, that in so far, at least, as the driving motors are concerned, a very decided advantage may be gained by the use of lower voltage,—say around 1000 volts. The method or methods of obtaining this voltage is beyond the scope of this paper, but we believe sufficient evidence has been presented to show that the direct current motor designer has something to offer in return for the lower applied voltage to the motor.

In conclusion, it may be said that d-c.-motors may be built for any type of locomotive drive. The qualifications of the more common types are summed up, as follows:

1. Axle-mounted d-c.-motors may be built which have sufficient power to permit fairly heavy axle loadings, as shown by the curves.

2. Quill drive d-c.-motors may be built, of which the power-per-axle will probably be limited by the method of transmitting power to the wheels rather than by the motors, themselves.

3. D-c.-motors for side-rod or gear and side-rod drive may be built with practically any desired power.

4. D-c.-gearless motors may be built which have sufficient power for comparatively light axle loadings, this light axle loading necessitating a larger number of axles for a given locomotive rating than would otherwise be used.

5. A lower voltage motor has a definite advantage in possibilities of greater output for a given armature diameter.

A Study of Direct-Current Corona in Various Gases

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Synopsis.—1. The critical corona intensities have been determined for helium, hydrogen, oxygen, nitrogen, air, and carbon dioxide for pressures ranging from 2 to 760 mm.

2. In each case the relation $\frac{E}{P} = A + \frac{B}{\sqrt{P}}$ *was found to hold approximately for pressures above 4 cm.*

3. The values found for air agree well with those found by Whitehead and Isshiki from their investigation of alternating-current corona.

4. No simple relation could be found giving the connection between $\frac{E}{P}$ *and* $\frac{1}{\sqrt{P}}$ *over the whole range of pressure.*

5. The influence of temperature on critical corona intensity was investigated for hydrogen, and found to consist merely of a change in δ, as was expected.

6. It is shown that the data obtained do not permit the explanation of corona as a process of ionization by collision, unless we make further assumptions. The nature of the assumptions necessary is not evident.

* * * * *

1. Introduction

The law of formation of corona on wires is an empirical relation which was first developed about 15 years ago by Whitehead. The electric intensity at which corona forms on a circular wire coaxial with a circular cylinder is given by the relation

$\frac{E}{\delta} = A + \frac{B}{\sqrt{\delta r}}$, where δ is called the density factor: $\delta = \frac{3.92\,P}{T}$ A and B are constants which depend on the kind of gas and on the polarity of the wire. P is the pressure of the gas in centimeters of mercury. T is the absolute temperature, and r is the radius of the wire, in centimeters.

The corona law was developed experimentally for alternating potentials by Whitehead, Peek and others. Whitehead,[1] was the first to determine both the exact influence of the radius of the wire and the constants. Peek,[2] considering the influence of temperature, first established the density factor δ. Farwell,[3] and later Whitehead and Brown[4] determined the constants A and B for continuous potentials by changing r. Whitehead and Isshiki[5] found that for alternating-current corona there was a break in the linear relation between E and $\frac{1}{\sqrt{\delta\, r}}$ at a definite value of $(\delta\, r)$. This value of $(\delta\, r)$ was found to be the same as the value at which the straight lines for positive and for negative corona crossed. Whitehead and Isshiki also investigated the influence of temperature for alternating-current corona, while Whitehead and Lee did the same thing for direct-current corona, Whitehead[1] had previously shown that the moisture content of the air had no effect on the critical intensity.

Townsend[7] derived the relation $\frac{E}{P} = A + \frac{B}{\sqrt{P\,r}}$ from the laws of sparking between parallel plates, introducing the assumption that ionization by collision takes place out to a certain distance from the surface of the wire, where the electric intensity is constant. Even if we admit this assumption, Townsend's analysis has no way of distinguishing between positive and negative corona, and it does not account for the departures found to exist for low values of $(\delta\, r)$.

Davis and Breese[8] investigated the continuous corona in hydrogen. They found that whereas in air, for the higher values of $(\delta\, r)$, negative corona occurs at higher values than positive corona, the reverse was true for hydrogen. The purpose of the present investigation was to determine the critical corona voltages for a number of different gases over a wide range of pressure, and to determine, if possible, what physical processes are involved in the formation of corona.

2. Description of Apparatus

The apparatus was similar to that used by Whitehead and Lee[6] and is shown in Fig. 1. In preliminary experiments it was found that there was a small fluctuation in the continuous potential at the corona tube, the amplitude of which varied with the frequency of the generator voltage. To eliminate this fluctuation, the capacity between the kenotron and the corona tube was increased to 0.1 microfarad, and an inductance of 143 henries was placed in the same circuit. Under these conditions the fluctuation was reduced to a negligible quantity in the range of frequencies used, (500 to 600 cycles). This is shown by the following table of observed corona voltages:

Frequency	Volts
60	133.0
133	135.0
600	135.6

*Of Johns Hopkins University, Baltimore, Md.

1. For references see Bibliography.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

The voltage fluctuation to be expected was calculated from the formula given by Hull[9] to be about 0.5 per cent, without the large inductance. Oscillograms taken with the inductance in series showed no noticeable fluctuation.

For convenience in manipulation the kenotron, together with the filament battery, condensers, choke coils, and the multiplier resistance for the corona-voltmeter, was separated from the remainder of the apparatus and placed on a well-insulated wooden platform, directly above the transformer and the corona tube.

The corona tube. The corona tube was the one used by Whitehead and Lee[6]. The main chamber was made from a piece of 6-in. steel piping, 19 in. long, carefully bored out in a lathe to receive the supports for the

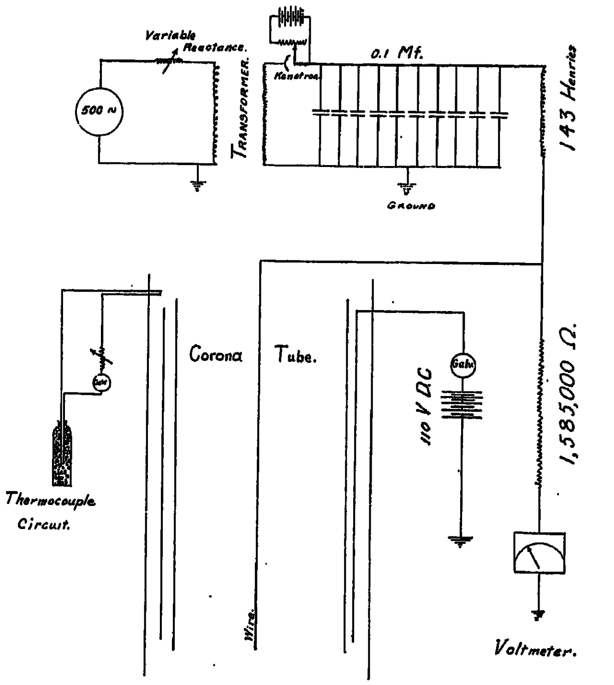

FIG. 1

concentric cylinders. The inner cylinder was of heavy brass, and had a number of small perforations. Its inside diameter was 3.75 in. = 9.52 cm. Two caps of bakelite in the form of crosses were machined to fit tightly around the ends of this cylinder, and the wire used in the corona measurements was stretched through small holes carefully centered in the caps, and was held taut by lock-nuts. In this way it was possible to remove the wire without removing the lower end of the tube.

3. Calibration of Instruments and Accuracy of Measurements

The voltmeters used to measure the corona voltage has been calibrated and found to be accurate within the error of observation. These limits were ± 3 volts for the Siemens-Halske voltmeter and ± 10 volts for the Weston (used only above 7000 volts). The limits within which the corona voltage could be determined varied from ± 5 to ± 30 volts at the two ends of the range. At pressure 1-5 mm. the initial deflection was very large, ranging from 10 cm. to complete off-scale. For pressures ranging from 20 cm. up to atmospheric pressure, and especially for positive corona, the initial deflection was only one mm. or less. Hence the accuracy of observing the critical voltage is less at the higher pressures; but the percentage accuracy is about the same, namely 0.5 per cent.

Since it was necessary to use pressures both above and below atmospheric pressure, the manometer was of the open-tube type; a wooden meter-bar mounted between the two manometer columns served as a scale. The barometric pressure was taken to the nearest hundredth of a centimeter. Since the scale of the barometer was accurate at 0 deg. cent., a correction had to be applied to the observed barometric readings to reduce them to scale readings at 0 deg. cent. A similar correction was applied to the wooden meter-bar. The final accuracy of the pressure readings is considered to be about ± 0.04 cm.

The galvanometer used to detect the beginning of corona was made by Leeds and Northrup, and had a resistance of 10 ohms. Its current constant was determined with a standard shunt and milliameter. The mean of several determinations gave one cm. scale deflection = 2.8×10^{-8} amperes. A deflection of two millimeters or more was considered an indication of corona.

4. Materials

a. Wires. Several different wires were made up from steel wire of diameter 0.0663 cm. The specimens chosen were free from kinks and surface irregularities, and were tested for variations in diameter with a micrometer caliper. They were then cut to the required length, 25 cm., and were mounted in the tube between brass rods of about two millimeters diameter. The surface was polished with fine emery cloth and soft leather. It was found that the surface conditions of the wire were affected by the corona, and so a steel wire with a chemically deposited surface of copper was tried and found unsatisfactory. The wire used in most of the experiments was a steel wire on which a thin layer of gold had been deposited electrolytically. This wire was polished with jewelers' rouge, and with soft leather. The surface so obtained was affected very little by chemical action in the corona. The data finally selected were taken partly with the gold wire, partly with the steel wire, and were always taken with a freshly polished wire.

b. Gases. In every case the only purification of the gaseous material which was attempted was the removal of water vapor. This was effected by passing the gases through a series of tubes containing calcium chloride and phosphorous pentoxide. In the earlier experiments sulphuric acid was used as a drying agent, but was found unsatisfactory.

Hydrogen and oxygen were made electrolytically in the generator shown in Fig. 2. Various electrodes, current strengths and electrolytes were used, but no corresponding variations in the corona characteristics were detected. Hence the influence of any impurities depending as to their amount on the conditions of generation was considered negligible.

Hydrogen, oxygen, nitrogen, CO_2 and helium, were obtained commercially. The hydrogen was made electrolytically, the oxygen and nitrogen by fractionation of liquid air.

The air used was laboratory air passed through the drying train, which also contained provisions for removing the coarser motes and dust particles.

The amounts of the impurities in the various gases were stated by the manufacturers to be as follows: hydrogen, nitrogen, oxygen, less than one per cent: CO_2, less than one-half per cent: helium, about five per cent nitrogen (estimated).

Pressure changes were always extended over a period of time ranging from two to five minutes, depending on the amount of gas to be admitted. The time interval between cutting off the gas supply and reading the critical voltage was also several minutes. Since the

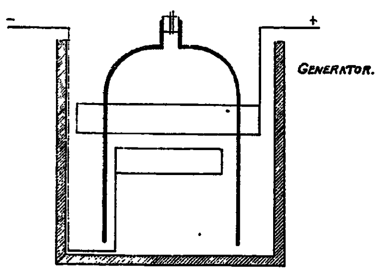

FIG. 2

volume of gas admitted was small and the heat capacity of the corona tube very large, no initial temperature fluctuation of as much as ½ degree cent. was ever observed; and with the time allowance given, temperature variations from point to point in the gas were certainly very much less than 0.5 deg. cent. In taking observations of the critical intensity, the heating effects due to corona were carefully avoided. The error introduced by temperature variations was always less than the other experimental errors.

5. RESULTS

a. Appearance of the Corona. With the wires used, all of which had a diameter of approximately 0.0665 cm., the appearance of light was simultaneous with the first galvanometer deflection due to ionization. This was true for all conditions of pressure and temperature within the range of these experiments, and for all the gases examined.

With the wire positively charged, the corona glow was uniform around and along the wire, and was confined to a region very close to its surface. The extent of this region did not vary with pressure or current density, except that above a certain voltage there was a small brilliant intermittent discharge across from some point near the middle of the wire, to the tube. The rest of the wire retained its uniform glow.

With the fire negatively charged, and a pressure of 2-5 mm., the first corona deflection was sometimes jerky, and the corresponding flow was a faint flicker which was not restricted to any point or points, but was uniform along the wire. A slight increase in voltage brought out the steady continuous glow of negative corona. This was much more diffuse than the positive glow. A further increase of 50 to 100 volts brought out the first faint beads. These were localized discharges on the wire, and were always fairly evenly spaced, especially at higher currents. They increased in number with pressure and current density, and also varied from one gas to another. At approximately equal pressures and current densities the beads were most numerous in the gas for which the corona voltage was lowest. It was frequently observed that when the current density was increased, each new bead appeared approximately midway between two old ones. If the corona was run for a few minutes with the beads in any one position, marks were found on the surface of the wire, which looked as if they might be the result of local heating. The negative beads have been investigated in great detail by Farwell,[3] Davis and Breese,[8] and Crooker.[11] Their results have been found to be confirmed, in general, for all the gases investigated.

The color of the corona discharge was the same for the continuous negative and positive glow; whereas the beads were much more brilliant, and sometimes different in color. The glow was generally a pale red or purple, the color depending on the gas used, but not on the pressure or current density. In helium the discharge was yellowish; and examination with a small grating showed the presence of a yellow line, probably 5875. In hydrogen the glow was generally reddish, while in air and nitrogen the nitrogen bands predominated.

In all cases the critical voltage as well as the appearance of the corona glow was found to be independent of the material of the wire.

b. Variation of corona current with E and P. The initial corona current was always much greater and much more sharply defined for low pressures than for high pressures, and was greater for negative than for positive corona. At low pressure, (1-3 mm.), the first corona deflection was always accompanied by a sharp drop in the voltage across the tube, due to the high rate of discharge of the condensers.

Fig. 3 shows the variation of current with increasing and decreasing voltage, for various pressures and gases. The curves have the same general character in all gases, the negative curve being much steeper than the positive curve. The positive curve for decreasing voltage follows closely that for increasing voltage; in negative corona this is not the case. The negative

increasing curve has a double break, while the decreasing curve has only one, which is very well defined. The double break in the increasing curve may be due to a change in the sign of the space charge near the wire.

The actual values of the total corona current can

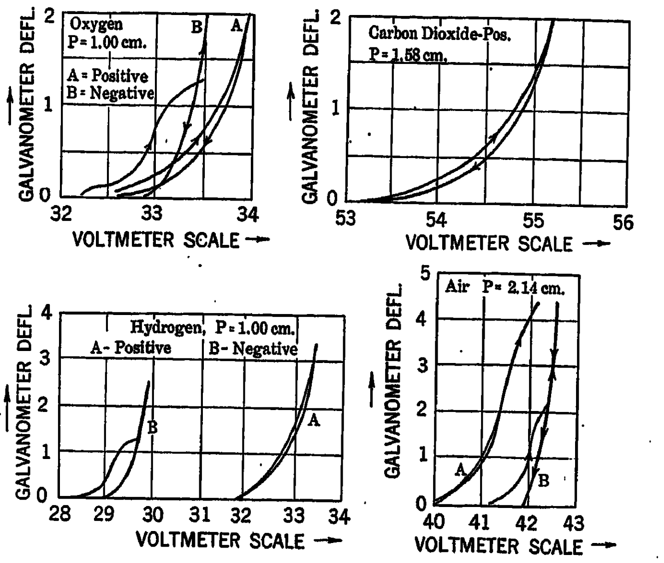

FIG. 3

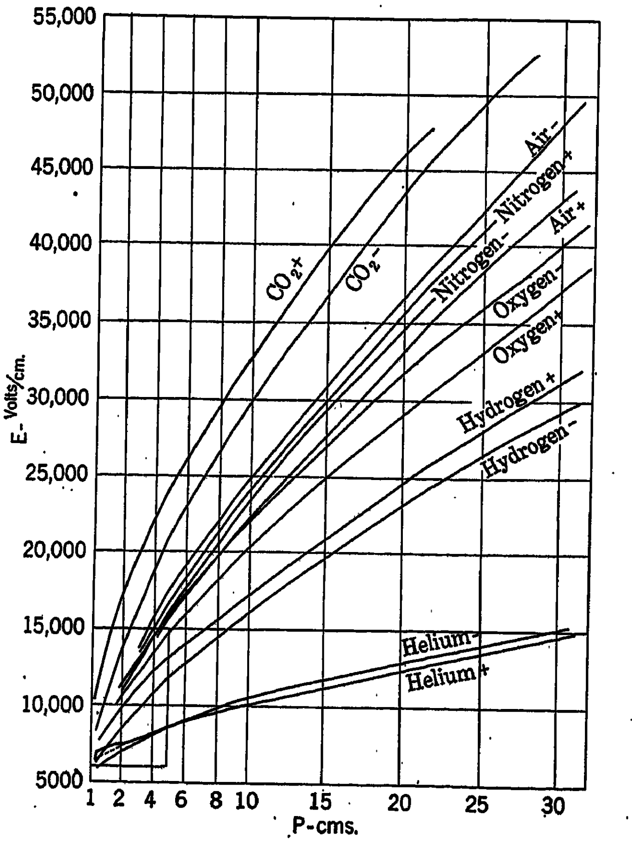

FIG. 4

only be estimated; they are of the order of 10^{-5} ampere per cm. scale deflection. The heating effect of the corona current was always negligible, because the current was never left on for a long time, No changes in pressure were to be expected from this cause. The volume of the corona tube was much too large to give any evidence of the other types of pressure change found by Kunz and his co-workers[12].

c. Variation of critical intensity with temperature. Some preliminary experiments were made with hydrogen at a temperature of 42 deg. cent. When the values of the critical voltage obtained in these experiments were reduced to room temperature (22.5 deg. cent). by applying the density factor $\frac{315}{295.5}$, they agreed well with the values found in experiments at 22.5 deg. cent.,

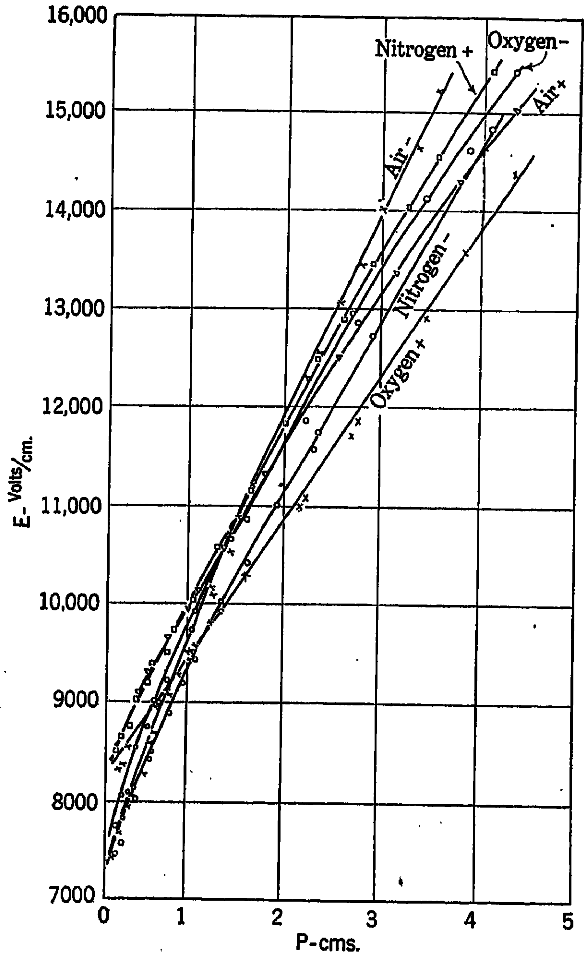

FIG. 5

except for a few points in negative corona at very low pressures. Since the same result was found by Lee for air, no further experiments with higher temperatures were made.

d. Variation of critical intensity with pressure. Figs. 4 and 5 show the variation of critical intensity with the pressure graphically. It will be seen at once that the general shape of each type of curve is the same for all gases and for both polarities.

The curves showing the relation of E to P, (Figs. 4 and 5), are not especially interesting theoretically, but they bring out several important facts. For all but the lowest pressures, $P < 1$ cm., the values of the

critical intensities are in the same order as the values of the mean free path of a gas molecule in the different gases. For all gases, when $P < 1$ cm., positive corona occurs at higher voltages than negative corona. At the higher pressures this is still true for hydrogen, nitrogen, and carbon dioxide; but in helium, air, and oxygen a crossing takes place at intermediate pressures,

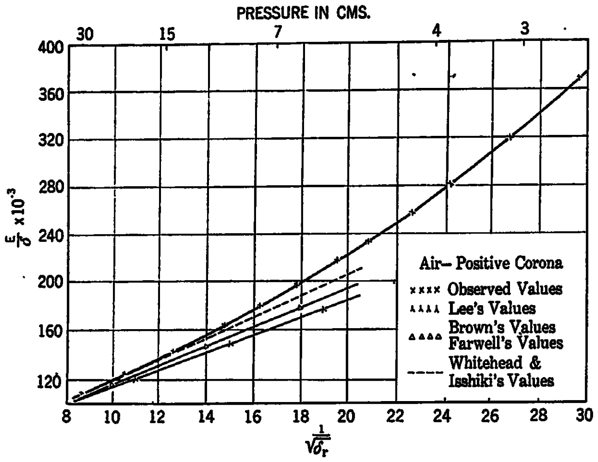

Fig. 6

so that the negative critical intensities become higher than the positive values. For $P > 10$ cms. the positive and negative curves for each gas are approximately parallel.

Fig. 5 shows, on a large scale, the E-P curves at low pressures in oxygen, nitrogen, and air. At the lowest pressures the air curves coincide with the nitrogen

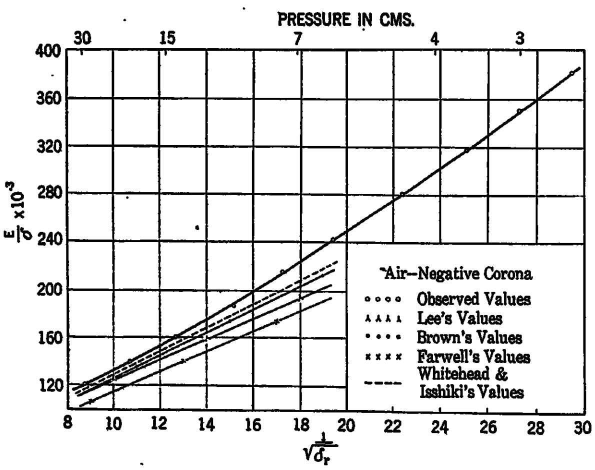

Fig. 7

curves. The positive curve for air remains intermediate between $N_2{}^+$ and $O_2{}^+$, which might be expected; whereas the negative air curve becomes higher than the negative curve for either oxygen or nitrogen; which is difficult to understand.

The curves which are most interesting historically and theoretically are those which express the relation of $\frac{E}{P}$ to $\frac{1}{\sqrt{P}}$. The law of corona formation which was established empirically by the work of Whitehead[1] and Peek[2] on air, states that this relation is a linear one:

$$E = A + \frac{B}{\sqrt{\delta\, r}}, \text{ where } \delta = \frac{3.92\ P\ \text{cm}}{T_{abs}}.$$

This question has been confirmed experimentally by Farwell[3], Whitehead and Brown,[2] and others for variations in r. A number of investigators has confirmed the law for variations in P, while Lee has recently confirmed it for variations in T. All this work was done on air. Davis and Breese[8] conclude that the law is approximately true for hydrogen, departures occurring for low values of P and r.

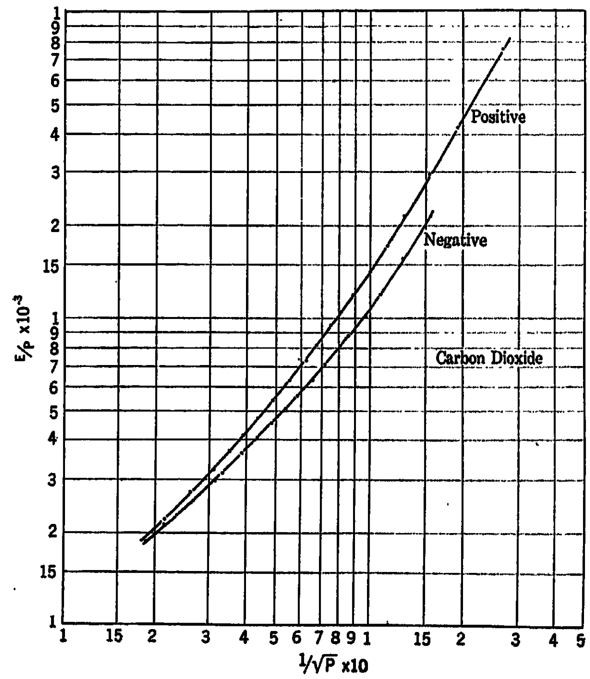

Fig. 8

Figs. 6 and 7 show how the values obtained in the present investigation compare with those obtained by Whitehead and Lee,[6] Whitehead and Brown,[4] Whitehead and Isshiki,[5] and Farwell.[3] The agreement with the values obtained by Whitehead and Isshiki is especially good, in view of the fact that 30 cm. was both the lower limit of their range of pressure and the upper limit of the range of this investigation.

Figs. 8-13 show the relation between $\frac{E}{P}$ and $\frac{1}{P}$, plotted on logarithmic coordinate paper. In this way any power function $\frac{E}{P} = a\left(\frac{1}{\sqrt{P}}\right)^n$ appears as a straight line whose slope is n. An inspection of the curves shows that they all have two regions of small

curvature, one at the lower pressures, where the slope is very near tan 60 deg., or $\sqrt{3}$, and one at the higher pressures, where the slope is near tan 45 deg., or 1. (However, for still higher and still lower pressures, the slope passes beyond these values in some cases.) This means that there is a considerable range of pres-

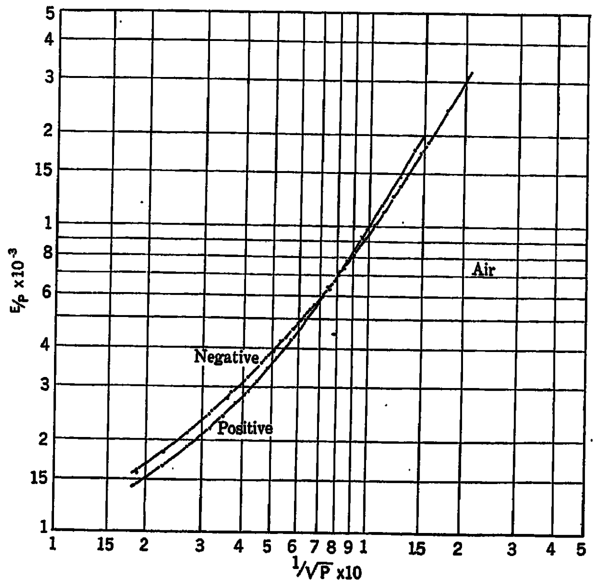

FIG. 9

sures where the relation between $\frac{E}{P}$ and $\frac{1}{\sqrt{P}}$ is very nearly linear; this happens to be the range covered by previous investigations.

All attempts to find a simple formula expressing

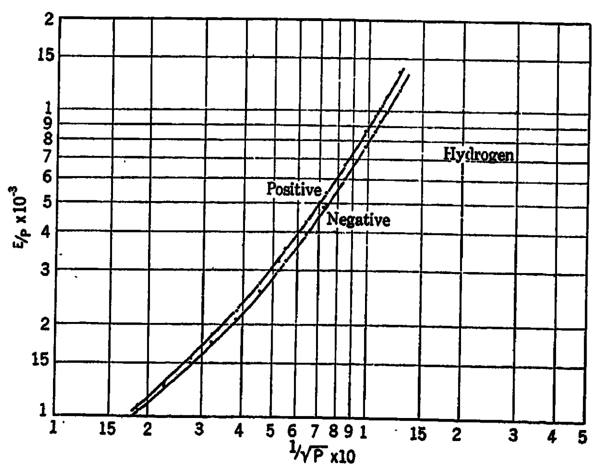

FIG. 10

the relation of $\frac{E}{P}$ to $\frac{1}{P}$ over the whole range of pressures have been unsuccessful. The most satisfactory formula seems to be $\epsilon \frac{A}{P}\frac{E}{} = \frac{B}{P}$, where A and B are constants. The agreement is not close enough, however, to warrant a calculation of the values of A and B for the different gases.

The relation $\frac{E}{P} = f\left(\frac{1}{P}\right)$ is of theoretical in-

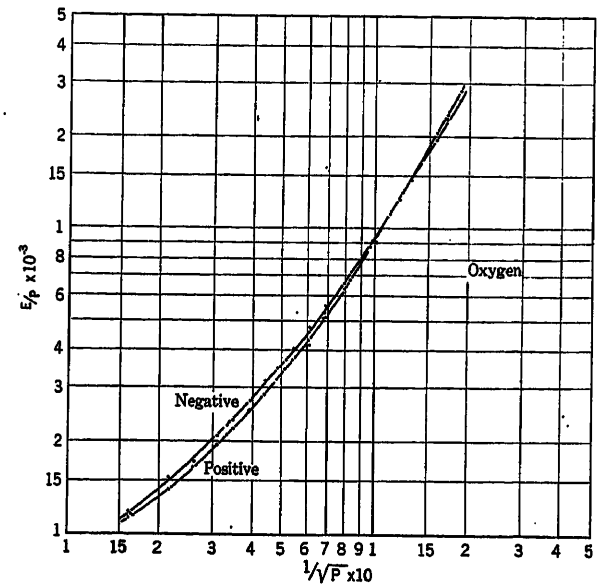

FIG. 11

terest because of the physical significance of $\frac{E}{P}$. This quantity is proportional to the energy which an ion would acquire in traversing its mean free path at

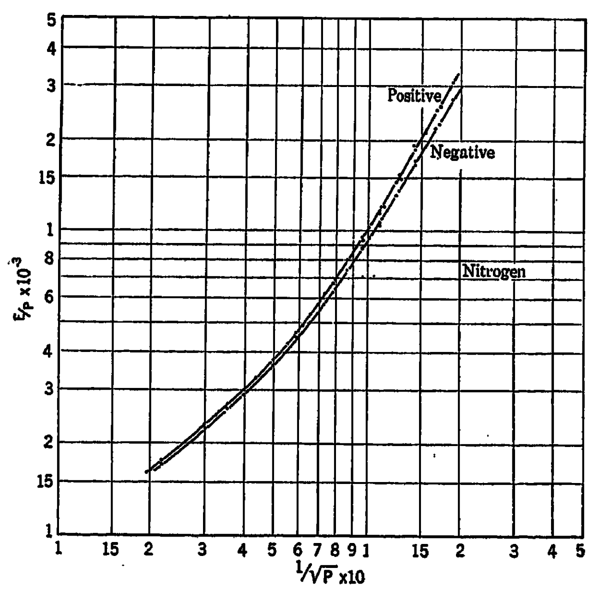

FIG. 12

pressure P in a field of intensity E. If it is assumed that corona is a process of ionization by collision, $\frac{E}{P}$ represents the energy required to ionize a gas molecule. This energy has been shown to be constant

at very low pressures; whereas within the range of this investigation $\frac{E}{P}$ is a function of the pressure. There are several possible explanations: 1. The corona process may not be one of ionization by collision. 2. The ionizing potential of a gas molecule may vary with the pressure. 3. The mean free path of the ionizing agent may not be a function of P only. 4. The E of the theoretical formula may not be equal to the actual electric intensity at the surface of the wire. 5. The ion in question may not acquire its maximum energy in the region of maximum intensity. These possible explanations will be considered in order:

1. The fact that the appearance of a glow in the gas is simultaneous with the first corona current, is strong evidence that corona is an ionization process.

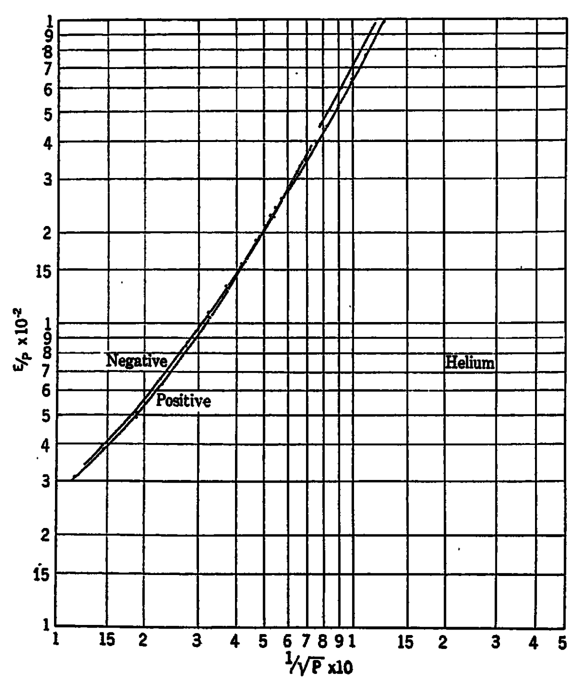

FIG. 13

2. At very low pressures the ionization energy of a gas molecule is known to be independent of the pressure. The present conceptions of atomic structure demand that this be true at all pressures.

3. For the case when the ionizing agent is an electron, the mean free path is almost certainly dependent on E as well as on P, so that this factor must be taken into consideration.

4. When there is an appreciable excess of ions of one sign over those of the opposite sign, the potential gradient at any point in space will differ from the theoretical value because of space charge effects. Before corona begins, the space charge is merely the residual ionization, and the effect is probably insignificant, so that it will not serve to explain the variation of $\frac{E}{P}$ with P. After the corona current is established, however, such an effect will certainly be present, and its amount will depend on the polarity of the wire. When this is negative, the predominating space charge may be negative at first; but eventually, on account of the great difference in the velocities of electrons and positive ions, there will be an accumulation of positive ions; which means that after a period of comparatively slow rise, the potential gradient at the wire will suddenly begin to increase very rapidly, with a corresponding increase in current, until the discharge becomes arc-like. With the positive wire the effect will be much smaller, and may amount to a decrease in potential gradient at the surface. This explains, at least qualitatively, the shape of the current curves.

5. K. T. Compton[13] has shown that electrons moving from a negative wire out to a positive cylinder acquire their maximum energy $\frac{E}{a}$ at a point $r = x$ in the space between the electrodes. x depends upon E, P and the dimensions of the wire and the cylinder. $a = k f^{1/2} p$, where k is a constant characteristic of the gas, f represents the average fraction of the average energy lost in any collision. E_x is the theoretical electric intensity at the surface of the cylinder $r = x$. The case when the wire is positive was not considered by Compton; but a simple calculation shows that the electrons attain their maximum energy at the surface of the wire. The maximum energy is still given by $\frac{E}{a}$ where $x = r_\omega$, the radius of the wire; so that the electrons acquire greater energy when the wire is positive; conversely positive ions attain their greatest maximum energy (at the surface of the wire) when the latter is negative. No matter which we assume to be the ionizing agent, it is necessary to assume a variation of f with electric intensity; in order to give the right effect, f must increase with decreasing E. This is to be expected from the definition of f. This probably involves a departure from theory in the values of the mean free path. Thus the effects considered in Sections 3 and 5 may be used qualitatively to explain the relation of $\frac{E}{P}$ to P.

The curves of $\frac{E}{P}$ against $\frac{1}{\sqrt{P}}$ for + and − corona approach each other very closely at the higher pressures, but diverge more and more when the pressure decreases. This is explained by the equations and curves of Comption (l .c. p. 342). The point x at which the ion when going out from the wire attains its maximum energy, approaches the surface of the wire

as the pressure increases; therefore $\frac{E_x}{a}$ approaches $\frac{E_\omega}{a}$; *i. e.*, the maximum energies of positive and negative corona become more nearly equal as the pressure increases. This is true no matter which type of ion we assume to be the effective ionizing agent.

The mechanism of the corona has been assumed to be a process of ionization by collision. From the general similarity of the positive and negative curves it may be assumed that the mechanism is the same, or at least very similar, for both polarities. Positive glow appears exclusively at the surface of the wire, while negative glow extends considerable distances out into the body of the gas. This points to ionization by electrons. Unfortunately, this assumption leads us to the conclusion (from section 5) that positive corona should appear at lower surface intensities; which in the majority of cases is not true. The maximum energies calculated from Compton's formulas are about 100 times the ionization potentials of the negative gases. Similar difficulties arise if we assume that positive ions are the ionizing agent. It is hard to see how this can be possible, because the energy of the electrons present must be greater than that of the positive ions. Neither assumption can be made to explain the crossing of the two curves in air, oxygen, and helium, without some auxiliary assumptions; and of what nature these assumptions would have to be in order to explain the observed effects, is an open question.

In this discussion, nothing has been said about the possible role of the photoelectric effect in the corona, because visible glow was always found to be simultaneous with the beginning of ionization. Whitehead[1] has shown that there is ultra-violet light present in the corona glow at all stages. It might be supposed that before ionization, ultra-violet light is emitted by excited atoms. This might cause photoelectric emission of electrons from the metal surfaces, thus causing the corona deflections.

There are two possible ways of determining whether photoelectric action is the cause of corona; (a) to study photographically the intensity variation of the ultra-violet light emitted in various stages of the corona. (This is not a conclusive test, because the time interval between the emission of the first traces of light and the beginning of "corona" might be very small.) (b) To bring the critical intensity up to a point just below the critical value, and illuminate the tube with a strong source of ultra-violet light.

The test (b) was tried in air at atmospheric pressure for both polarities. The source of ultra-violet light, an iron arc, was held a few inches from the open end of the tube. The voltage was raised successively to within 200, 50 and 20 volts of the critical voltage, (about 12,000 volts). In each case no effect was produced by starting the arc. As there should have been abundant photoelectric emission at the brass cylinder, this suggests that the emission of ultra-violet light is a consequence and not a cause of corona. Whitehead[1] using x-rays as the ionizing agent also found that artificial ionization produced no change in the critical intensity.

Nevertheless, a spectroscopic investigation of the different stages of the corona glow might prove valuable. This might be conducted in two different ways: (a) by determining whether the different spectral lines appear in any definite order as the voltage is raised up to and beyond the critical value; and (b) by investigating the Stark effect at different points in space in the corona tube. The effect of a longitudinal magnetic field upon the critical intensity is also under investigation. The effect to be expected is a change in the speeds of the ions due to the spiraling around the wire, as in the case of the "magnetron" described by Hull.[14]

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Discussion

J. B. Whitehead: In the first studies of corona that were made in our laboratory, one of the questions that came up was whether or not the separate constituents of air played any part in the determination of the laws by which the corona was formed. And the original question which Dr. Lee started out to investigate was whether the oxygen of the air or the nitrogen of the air would show any remarkably different properties as regards breakdown.

For the engineer there are, then, two matters in this paper which I think are of immediate interest. The first is the answer to the question that I have already indicated, namely: that the properties of oxygen and nitrogen are not markedly different as regards the formation of corona. If they had been different, it would have been very easy to imagine conditions under which advantage might have been taken of any such differences.

The other matter has to do with the values of voltage at which corona forms around wires in the range in which we use high-voltage conductors; that is to say, pressures that are not greatly different from atmospheric pressure.

A matter of interest to me directly is the fact that Dr. Lee and Mr. Kurrelmeyer have linked up the behavior of the several gases investigated at low pressures with that at high pressures. Those who remember my corona-voltmeter paper will recall that there the pressure was carried down to about 30 cm. only. We stopped at 30 cm. I think largely because it became increasingly difficult to make the corona tube tight at lower pressures. Dr. Lee has picked up the air curve below 30 cm. and carried it on well down below 1 cm. He has found that his curves link up with those brought out by Mr. Isshiki and myself, but he shows also that the empirical law which we show for the corona-voltmeter does not hold in his low pressures, that is, in the range of very low pressures.

Turning to the physical discussions in the paper, Dr. Lee has indicated the main results of their investigation. I regard it as a very distinct contribution to the field of physical investigation in the matter of the ionization of gases. The authors have shown very clearly indeed the difficulties of coordinating their results with the only important theory we have, namely: the theory of ionization by collision. And I feel quite sure that this paper will suggest to a number of physicists who are interested in the phenomena of the ionization of gases, the importance of carrying further the study of these interesting questions.

F. W. Peek, Jr.: In establishing the visual corona relation for variation of radius of wire, temperature and pressure, my experiments were carried down to fairly low pressures. The relation holds very well down to 5 cm. This means, of course, that the transmission-line range is more than covered by the equation.

For a given conductor the visual corona gradient decreases with decreasing pressure until a minimum gradient is reached. The gradient then increases with decreasing pressure. At the very low pressures the molecular separation may be fairly large compared to the diameter of the outer tube. A different phenomena thus occurs and the above relation could not be expected to hold.

The interesting part of the paper is that a number of different gases have been investigated. So far the data are limited in that the authors have only used one size of conductor. I hope that the investigation will be carried over a greater pressure range and that different sizes of conductors will be investigated.

NOTE. See also discussion of this subject by Joseph Slepian, page 202.

F. W. Lee: I can say only this in regard to Dr. Slepian's discussion: that so far as using the value E over P as an indication of the energy given in the mean free path of a gas electron is concerned, that was given only with the idea that the values so used came within the range of magnitude that had been worked out at pressures very, very low, such as are used in vacuum tubes. In other words, the phenomenon that we are observing is that the observed results checked in the order of magnitude, although not within the absolute value. The agitation velocity under a charged field, where the transfer of energy goes, you might say, from the very great velocity of the electron and imparts the velocity to the molecules, will give other mean free paths and other distributions of velocity from those given with gas under normal conditions, which I used as the indication of the results in the beginning of the paper.

With regard to Mr. Peek's remarks, I will say that the work is being extended as fast as we can extend it, with the limiting conditions under which we are working and with the material available.

Dielectric Properties of Fibrous Insulation
As Affected by Repeated Voltage Application

BY F. M. CLARK[1]
Associate, A. I. E. E.

Synopsis.—*It is believed by many that the most plausible explanation of fibrous insulation failure is the pyro-electric theory. Under this theory as elaborated by Steinmetz, Wagner and others, insulation under stress is heated by the transformation of electric energy to thermal energy. Insulations are considered as of the nature of poor conductors and subject to the same characteristics. The transformation of electric to thermal emergy is therefore dependent upon the inherent electrical and thermal conducting properties of the material. This transformation proceeds at a rate proportional to the stress applied, until such a voltage value is reached where the heat is generated in the insulation at a rate faster than it is dissipated to the surrounding medium. Further increase in voltage leads to a rapidly mounting temperature with ultimate insulation failure. A strict interpretation of this theory would indicate that insulation failure is a matter of the insulation resistance—temperature relation. Therefore, as the heat stored in a dielectric during stress is allowed to dissipate, care being taken to prevent injury from the testing electrodes, etc., the orignial properties of the material should be restored. This has been found to be true only to a limited extent.*

The present paper deals largely with the effect of relatively high-voltage applications on sheet insulation tested between parallel plate electrodes. It is shown that the question of the mechanism of insulation failure can be separated into two parts. First, failure caused by short-time voltage applications as determined by rapidly applied tests, and secondly, failure of insulation under longer periods of stress as determined by the minute or endurance test methods.

Peek has found that voltages greatly in excess of the "rapidly applied" 60-cycle puncture voltage may be applied to the insulation without rupture, if the time of application be sufficiently short. All such over voltages injure the insulation, "probably by a mechanical tearing and the effect is cumulative." In these experiments of short duration, the problem of heat storage in the insulation is eliminated.

Raynor has found that the rapidly applied strength of insulation is greatly lowered by the previous application of a high voltage. This decrease in strength, however, is lost, given sufficient rest period between the initial and final voltage application. These results are substantiated in the work of the present paper. The length of rest period is shown to be proportional to the initial test voltage applied.

For tests of long duration, the work of this paper has been divided into two parts,—those voltages producing failure with an arbitarry time limit (15 minutes) and those voltages which are able to be applied for an indefinite time without a puncture. According to the pyro-electric theory, the first class deals with those voltages of such value as to produce a slowly mounting temperature rise in the insulation which ultimately reaches a value leading to rapidly decreasing insulation resistance and total loss of dielectric strength. The second class includes those voltages of such value that the rate of transformation of electric to thermal energy is equaled by the rate of dissipation of the heat so formed; thus preventing heat storage in the insulation. According to the thermal theory of breakdown, neither of these voltages should produce permanent injury to the dielectric,—the first, if removed before the stage indicating rapid loss of insulation resistance is reached, and the second, even if applied indefinitely. The present paper shows that the application of voltages of either of the above types leads to deterioration in dielectric strength of fibrous insulation, even aside from such effects that might be traced to corona or mechanical injury to the material. The effect for voltages of the first class is cumulative and expressed by the formula $R \times T = K$ *where* R = *number of repetitions of voltages,* T *the time of each voltage application, and* K *the time needed to puncture for the same voltage continuously applied. The application of this formula to various insulations, its limitations, etc., are discussed.*

The work detailed herein concerns the application of a-c. voltages at room temperature. The investigation is being extended to cover both low and high a-c. and d-c. voltages at and above room temperatures.

Object

It has generally been assumed that when a fibrous insulation is subjected to a potential stress, unless the critical point is reached and exceeded (that is, the point where the resistance suddenly decreases thus allowing the current to run away), the structure of the insulation although temporarily weakened dielectrically, is not permanently damaged and will recover if given a sufficient rest period. Some tests made in a preliminary way indicated that the above assumption was not correct. An extended study of the effect of repeated application of voltage on the dielectric strength of fibrous insulations has proven the validity of these preliminary experiments.

Up to the present time the tests have been confined to 60-cycle voltages and with the materials at room temperatures. Further work is being carried on in this field of research covering the effect of intermittently applied stress under various conditions using d-c. as well as a-c. voltages.

1. Physicist, General Electric Company, Pittsfield, Mass.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925 and at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

Conclusions and Results

The work done under the above stated conditions shows that the breakdown point is generally decreased by a previously applied voltage, this effect being greater as the actual point of failure is approached during the previous test. The exception to the case occurs with rapidly applied tests after a rest period following a previous voltage application. In this instance the material breaks down at approximately the same value with or without the previous application of stress.

However, the intermittent application of a voltage of such a value as will cause breakdown within 15 minutes, if continuously applied, results in a progressive weakening in the strength of unvarnished oil treated fibrous material, the effect being roughly additive.

With the application of a very low voltage producing

no puncture for indefinite time, a deterioration in the strength of oil treated paper has been observed. This in part, however, is able to be traced to a change accompanying oil immersion over long periods.

For varnished insulation, the repeated application of voltage leads to increased long time strength, probably due to a further drying out process taking place under the influence of the applied stress.

With mica or with fibrous materials heated in oil between voltage applications, the effect of intermittently applied stress is diminished and an increase in dielectric life is observed.

For low-voltage application, no permanent change in percentage power factor determination for oil treated fibrous materials has been noted. With high voltages, the per cent power factor values are extremely erratic, but in general show an increase up to the point of failure.

Discussion

It has been shown in an unpublished paper[1] that the strength-thickness relation characteristic of fibrous insulation is greatly affected by voltage application even at low stresses. Thus for example, the relation expressed in the formula $K\,V = A\,T^{0.72}$ as determined for oil impregnated 0.0005-in. linen paper from 0.002-in. to 0.008-in. in thickness is changed to $K\,V = A\,T^{0.56}$ for a ten-day application of 3300 volts. At the end of 51 days the relation has become $K\,V = A\,T^{0.52}$ which is practically unchanged at the end of 77 days of voltage application.

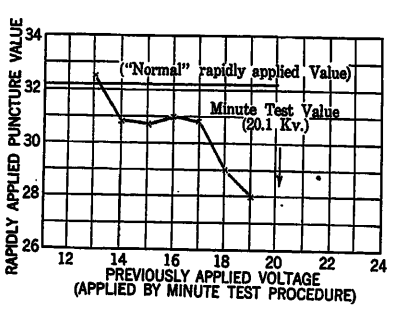

Fig. 1—Effects of Previously Applied Voltages on the Rapidly Applied Puncture Value of Three Layers Black Varnished Bond Paper (0.005 In.)

Test Medium—Transil Oil at Room Temperature
Electrodes—1½-in. Brass with Edges Rounded to 3/64-in. Radius
Test Voltage—60-cycle

Continued study has been made of the effect of voltage upon the breakdown values of a variety of fibrous insulating materials. Experimental data has shown that the important factors involved are the voltage itself, the rest period allowed between applications of stress, the duration of voltage application, and the treatment of the insulation during the rest interval between stress periods.

1. Dielectric strength-thickness relation in fibrous insulation by F. M. Clark and V. M. Montsinger.

The Voltage Factor with Reference to the Rapidly Applied Test Values

It has been found that the rapidly applied breakdown value for black varnished bond paper tested under oil at room temperature is affected by the previous application of electric stress, the effect being greater the nearer the applied voltage is brought to the actual breakdown strength of the insulation. This is illustrated in the following data, all values of which are the average of at least ten tests, the individual tests for the different points being run alternately on the same samples of insulation.

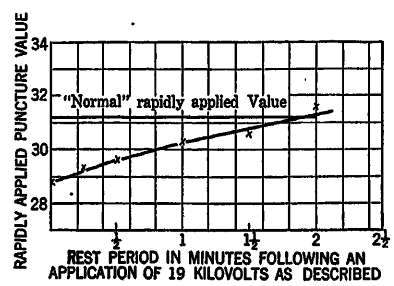

Fig. 2—Effect of a Previously Applied Voltage on the Rapidly Applied Puncture Value of Three Layers of Black Varnished Bond Paper (0.005 In.) as a Function of the Rest Period Involved

Test Voltage—60-cycle
Test Medium—Transil Oil at Room Temperature
Electrodes—1½-in. Brass with Edges Rounded to 3/64-in. Radius

Insulation—3 layers black varnished bond paper (0.005 in.)

Testing voltage—60 cycles.

Testing medium—transil oil at room temperature.

Electrodes—1½ in. brass, with edges rounded to 3/64 in. radius.

Voltage Applied	Breakdown
Rapidly applied (original)	32.2 kv.
Minute test (initial voltage 12 kv. with 1 kv. step-up each 30 seconds) (Original)	20.1 kv.
Rapidly applied preceded by voltage application according to the above minute test procedure up to and including 19 kv.	28 kv.
Rapidly applied preceded as above by the minute test voltage application up to and including 18 kv.	29 kv.
Rapidly applied preceded as above by the minute test voltage application up to and including 17 kv.	30.8 kv.
Rapidly applied preceded as above by the minute test voltage application up to and including 16 kv.	31.0 kv.
Rapidly applied preceded by voltage application by the minute test procedure up to and including 15 kv.	30.7 kv.
Rapidly applied preceded by voltage application by the minute test procedure up to and including 14 kv.	30.8 kv.
Rapidly applied preceded by voltage application by the minute test procedure up to and including 13 kv.	32.5 kv.

These results are illustrated in Fig. 1. Similar effects have been noted using 3-mil. oil-impregnated kraft paper.

The Effect of Rest Period Between Voltage Applications

The extent of the deterioration in the rapidly applied test strength of fibrous insulation is dependent upon the rest period allowed between applications of stress.

Insulation—3 layers black varnished bond paper (0.005 in.)

Test medium—transil oil at room temperature.

Electrodes—1½ in. brass electrodes, edges rounded to a radius of 3/32 in.

Averages based on ten tests. The individual tests for each point were run alternately upon the same samples of insulation.

CASE I (FIG. 2)

Original rapidly applied strength....................31.2 kv.
Minute test strength (Initial voltage 12 kv. followed by 1 kv. increments each 30 seconds)..(original)..20 kv.

Preliminary treatment—voltage applied according to the above minute test procedure up to and including 19 kv.

Rest Period	Final Rapidly Applied Strength
No rest	28.8 kv.
¼-min. rest	29.3 "
½-min. rest	29.6 "
1-min. rest	30.3 "
1½-min. rest	30.6 "
2-min. rest	31.6 "
3-min. rest	31.8 "

Similar results have been obtained using 0.003 in. oil impregnated kraft paper.

CASE II (FIG. 3)

Original rapidly applied strength....................30.7 kv.
Minute test strength (initial voltage 12 kv. followed by one kv. stepup each 30 seconds)....(original)......20 kv.

Preliminary treatment—voltage applied according to the above minute test procedure up to and including 17 kv.

Rest Period	Final Rapidly Applied Strength
No rest	29.5 kv.
¼-min.	29.8 "
½-min.	30.2 "
1-min.	30.5 "
1½-min.	30.6 "
2-min.	30.5 "
3-min.	30.8 "

Similar results have been obtained using 0.003 in. oil impregnated kraft paper.

Effect of Previously Applied Electric Stress Upon the Time-Voltage Relation

The effect of stress application upon the time-voltage relation in fibrous insulation is of interest since it not only involves tests of short duration but also long time tests. In this work the time factor involved has been limited to 15-minute intervals.

In investigating this phase of voltage effect, the stress applications fall into two classes,—first those voltages which will puncture the insulation within an arbitrary time (15 min.) and secondly, those voltages which will give no puncture independent of the time for which applied.

Case I: Those voltages producing a puncture within 15 minutes. The effect of voltages of this type has been investigated with the use of oil impregnated 0.003 in. kraft paper and 0.005 in. black varnished bond paper. The results are shown in Figs. 4 and 5. In these figures the points shown are the average values of at least ten tests. The individual tests upon which the averages of the curves are based have been obtained alternately.

The following facts in Fig. 4 are to be noted:

First, the rapidly applied test value of oil treated kraft paper is greatly affected by the previous voltage application, which effect is partly eliminated with the the intervention of a rest period between the preliminary voltage application and the final test value;

Secondly, the application of stress leads to a permanent injury as determined by the long time test value. That is, the curves with and without a rest period following the initial stress application become identical when the time to puncture factor equals one minute or more.

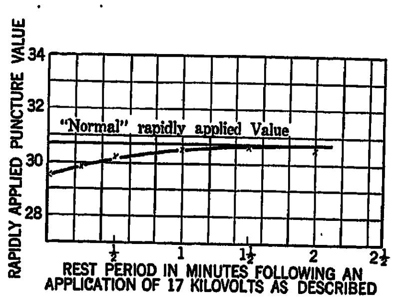

Fig. 3—Effect of a Previously Applied Voltage on the Rapidly Applied Puncture Value of Three Layers of Black Varnished Bond Paper (0.005 In.) as a Function of the Rest Period Involved

Test Voltage—60-cycle
Test Medium—Transil Oil at Room Temperature
Electrodes—1¼-in. Brass with Edges Rounded to 3/64-in. Radius

In Fig. 5 for the varnished paper, although similar effects are noted for short time tests, as observed in Fig. 4 for kraft paper, nevertheless considerable difference is observed in voltage strength for tests occupying more than one minute, where an apparent increase in dielectric strength is obtained following the intervention of a rest period between the initial and final voltage applications. In view of the fact that it is impossible to thoroughly dry varnished materials without injury before test, the increase in strength always noted with low voltage applications has been attributed to a further drying out process.

Case II: Those voltages which do not puncture at indefinite time. The effects of very low stress application involving as it does a period of weeks and months has necessitated the use of a slightly different test method. In this procedure aluminum foil electrodes (1½ in. by 2½ in.) were prepared between which was clamped the insulation under test. The dielectric used was linen paper, 0.0005 in. per layer and made up to a thickness of 0.004 in. The individual units so prepared were carefully dried under vacuum at 115 deg. cent. and impregnated. Part were then tested for the

time voltage relation. The remainder were then submitted to 3300 volts in oil at room temperature. No attempt was made to protect the oil in which the units

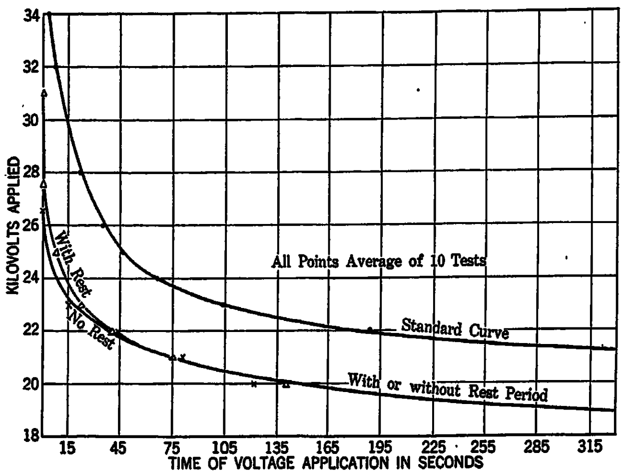

FIG. 4—EFFECT OF A PREVIOUSLY APPLIED VOLTAGE ON THE TIME-VOLTAGE RELATION

Insulation—9 Layers Oil Treated Kraft Paper (0.003 in.)
Test Voltage—60-cycle
Test Medium—Transil Oil at Room Temperature
Electrodes—1½-in. Brass with 3/64-in. Edge Radius
"Standard Curve"—Original Curve for Insulation
"No Rest"—Relation obtained after Application of 22 kv. for 2½ min. without an Intervening Rest Period
"With Rest"—Relation obtained after Application of 22 kv. for 2½ min. followed by a Rest Period of 5 min.

were placed from the effects of the room conditions, aside from the fact that the tests were made in a large "voltage box," covered but not air-tight. The results are shown in Fig. 6. Here again it will be noted that

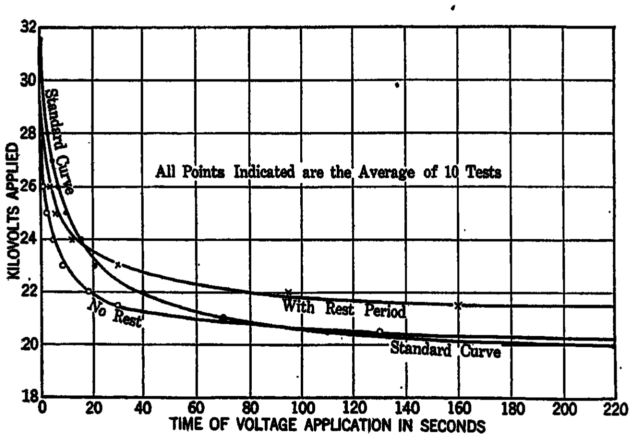

FIG. 5—EFFECT OF A PREVIOUSLY APPLIED VOLTAGE ON THE TIME-VOLTAGE RELATION

Insulation—Two Layers Blank Varnished Paper (0.012 in.)
Test Voltage—60-cycle
Test Medium—Transil Oil at Room Temperature
Electrodes—1½-in. Brass Edges Rounded at 3/64-in. Radius
"Standard Curve"—Original Curve for Insulation
"No Rest"—Relation obtained after Application of 20 kv. for 150 sec. without an Intervening Rest Period
"With Rest Period"—Relation obtained after Application of 20 kv. for 150 sec. followed by a Rest Period of 5 min.

permanent deterioration has occurred in the insulation except in the case of rapidly applied test values.

The effect of long time voltage application at low stress has been investigated by means of the effect noted upon the one minute test values of 0.0005 in. linen paper in thicknesses from 0.002 in. to 0.008 in. In this connection, units similar to those described above were carefully dried under vacuum and oil impregnated. Part were tested by minute test procedure and the remainder placed under 3300 volts in oil at room temperature for varying periods, after which the minute test values were again determined. The results are shown in Fig. 7. It should be remembered that in this case the units tested being of varying thicknesses, the actual electric stress (volts per mil) was not constant. It is to be noted, however, that the minute test value for the 4-mil thickness is affected in almost equal amount in Fig. 6 and Fig. 7, in both of which cases the voltage has been applied for equal intervals, 77 days.

During the course of these experiments, no attempt

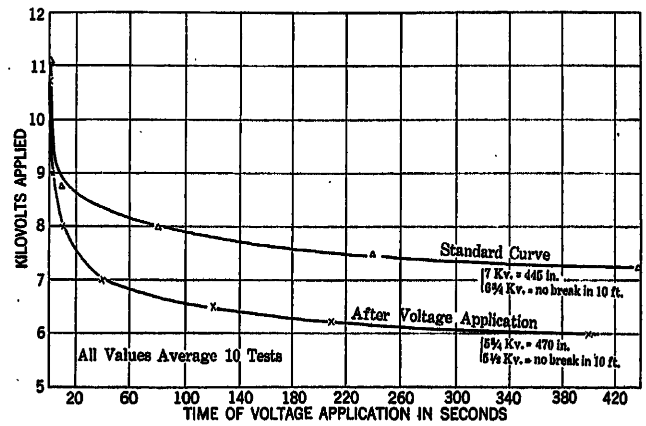

FIG. 6—EFFECT OF A PREVIOUSLY APPLIED VOLTAGE ON THE TIME-VOLTAGE RELATION

Insulation—8 Layers of Oil-Treated Linen Paper (0.0005 in.)
Test Voltage—60-cycle
Test Medium—Transil Oil at Room Temperature
Electrodes—1½-in. by 2½-in. Aluminum (0.001 in.) Foil
"Standard Curve"—Original Curve for Insulation
"After-Voltage Application"—Time-Voltage Relation after the Application of 3300 Volts for 77 Days

has been made to protect the insulation from the effects traceable to atmospheric conditions, the samples being merely immersed under oil whose surface was in contact with the air, of the "voltage box." Fig. 7 also shows the effects which are traceable to factors aside from the applied voltage.

It will be noted in Fig. 4 that the total time to produce a puncture with oil impregnated kraft paper at 22 kv. was originally 188 seconds. In the case when 22 kv. was applied for 2.5 minutes followed by a rest period of 5 minutes, the insulation punctured on the second application of voltage in 42 seconds, the total time for which the 22 kv. was applied before and after the rest period being 192 seconds. Thus the breaking-up of the voltage application into two steps separated by a rest period produced no material increase in the original voltage life of the insulation. A permanent injury of an additional type was apparently produced by each application of voltage.

Repeated Applications of Voltage Stress

In investigating the effects of repeated applications of electric stress, the voltages used were in general limited to those producing a puncture within a time interval of 15 minutes. In order to express the results of the various individual experiments in an easily comparable manner, the following scheme was used. A definite voltage was selected which would produce a puncture continuously applied within the time interval desired. Using this voltage, the average time to puncture on at least 10 tests was determined. This time was accepted as the 100 per cent time factor, *i. e.*, the total time needed to produce a puncture for only one application of the voltage used. With this factor as a basis, the constant voltage accepted was applied for various intervals such as 90 per cent, 80 per cent, 70 per cent, etc., of the 100 per cent time value. With a period of rest allowed between voltage applications, the number of permissible repetitions of stress for each time interval was determined.

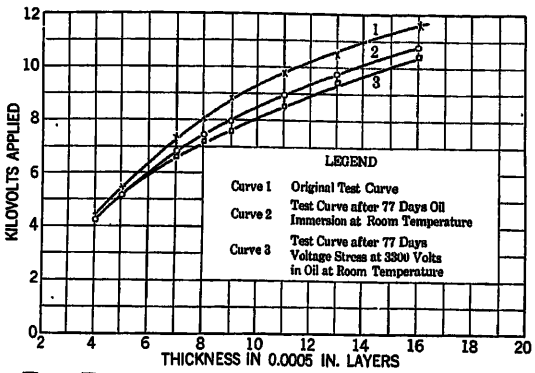

Fig. 7—The Effect of Low-Voltage Stresses upon the Minute-Test

Puncture Value of Fibrous Insulation
Insulation Used—Linen Paper, 0.0005 in. per Layer (2 in. by 3 in.)
Electrodes—1 mil Aluminum (1½ in. by 2½ in.) held in Place by Means of Glass Plates and Spring Clamps
Test Medium—Transil Oil at Room Temperature
Test Voltage—60-cycle
All Values Average of 8 Tests

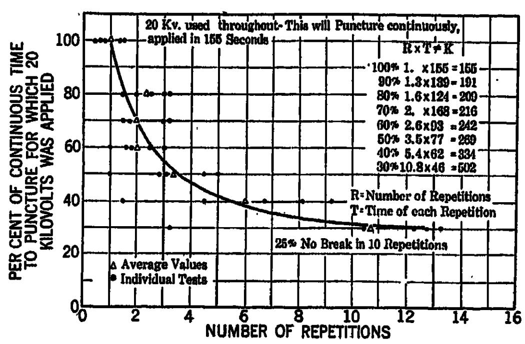

Fig. 8—Effect of Repeated High-Voltage Application upon Varnished Cambric

Insulation—Two Layers of 0.012 in. Black Varnished Cambric
Test Medium—Transil Oil at Room Temperature
Test Voltage—20-kv. 60-cycle
Electrodes—1½-in Brass with Edges Rounded to 3/64-in. Radius
Rest Period—Five min. between Tests

Case I: Figs. 8, 9, 10, 11, 12 and 13 illustrate the typical results produced by repeated voltage applications upon black varnished cambric (0.012 in.), black varnished paper (0.005 in.), kraft paper (0.003 in.), cable paper (0.005 in.), and pressboard (1/32 in. and 3/32 in.) tested under oil at room temperature. In these tests the total number of voltage applications was

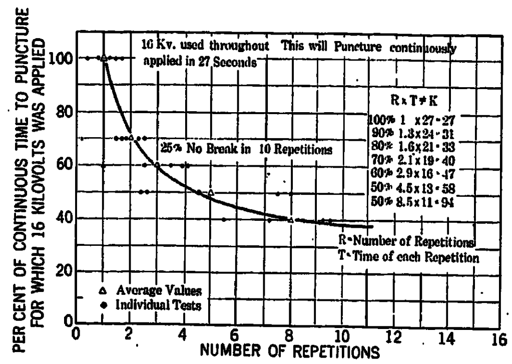

Fig. 9—Effect of Repeated High-Voltage Application upon Varnished Bond Paper

Insulation—Two Layers of 0.005 in. Black Varnished Bond Paper
Test Medium—Transil Oil at Room Temperature
Test Voltage—16-kv. 60-cycle
Electrodes—1½-in. Brass with Edges Rounded to 3/64-in. Radius
Rest Period—Five min. between Tests

limited to from 10 to 15. During the rest period, the insulation was allowed to remain in place between the electrodes immersed in cold transil oil. The varnished cambric and varnished paper were dried in air at 100 deg. cent. for four hours, after which they were immersed in transil oil for 24 hours before test. The unvarnished insulation was subjected to a vacuum drying process and oil impregnation at 110 deg. cent.

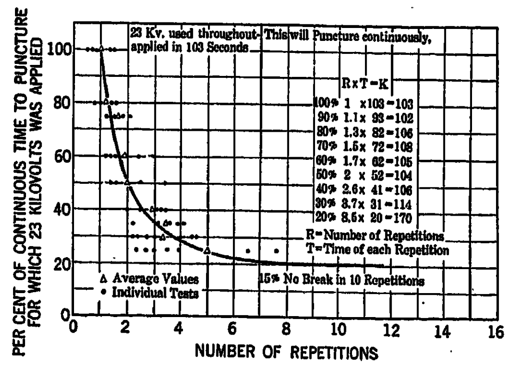

Fig. 10—Effect of Repeated Applications of High Voltage upon Kraft Paper

Insulation—9 Layers of 0.003-in. Oil-Treated Kraft Paper
Test Medium—Transil Oil at Room Temperature
Test Voltage—23-kv. 60-cycle
Electrodes—1½-in. Brass Electrodes Edges Rounded to 3/64-in. Radius
Rest Period—Five min. between Tests

It will be noted in Figs. 8 and 9 that with the varnished materials, the actual voltage life continually increases as the time of each application is diminished. This bears out the results of Fig. 5 showing the effect of previously applied stresses upon the time-voltage relation of black varnished paper. In cases where the breakdown value of varnished insulations, dried as

described, is not closely approached either with respect to the voltage or time factors, an apparent increase in voltage life results. In the case of fibrous, unvarnished but vacuum dried and oil impregnated materials, the effect of each application of voltage seems to be additional and permanent. This effect is expressed in the formula $R \times T = K$ where R is the number of

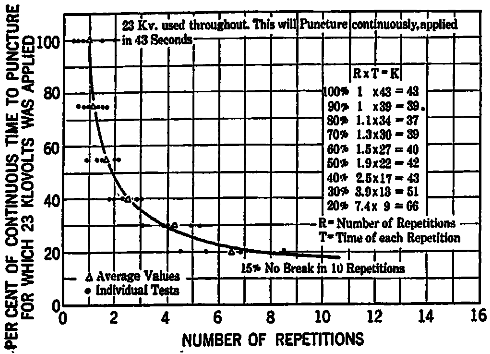

FIG. 11—EFFECT OF REPEATED APPLICATIONS OF 23 KV. ON CABLE PAPER

Insulation—Six Layers of 0.0045-in. Oil-Treated Cable Paper
Test Medium—Transil Oil at Room Temperature
Test Voltage—23-kv. 60-cycle
Electrodes—1½-in. Brass with 3/64-in. Edge Radius
Rest Period—Five min. between Tests

repetitions possible for the time T of each application. The value of K is the time to puncture for the voltage used when continuously applied. The application of this formula is given in each of the figures showing the effect of repeated high-voltage stresses. Increased dielectric life does not result until the time factor of each voltage application has been reduced to less than

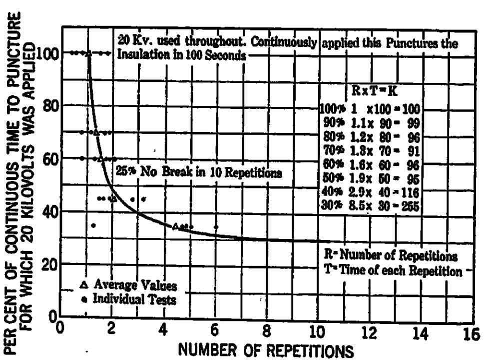

FIG. 12—EFFECT OF REPEATED HIGH-VOLTAGE APPLICATIONS ON PRESSBOARD

Insulation—One Layer, 1/32 in. grade AA Oil Treated Pressboard
Test Medium—Transil Oil at Room Temperature
Test Voltage—20-kv. 60-cycle
Electrodes—1½-in. Brass with 3/64-in. Edge Radius
Rest Period—15 min. between Tests

40 per cent of the continuously applied time to puncture value for the same voltage. This effect is shown in the case of kraft paper, cable paper and pressboard of Figs. 10, 11, 12 and 13.

Case II: In a previous article[2] it has been shown that the dielectric strength-thickness relation for fibrous insulation approaches the first power with the use of impregnated wooden electrodes. Fig. 14 shows the effect of intermittently applied voltage on 0.003 in., oil impregnated kraft paper, tested under transil oil at room temperature with 3-inch round edge impregnated wooden terminals. The effect of voltage application with these electrodes is roughly additional and in no wise different from the results illustrated in the previous figures using metallic electrodes.

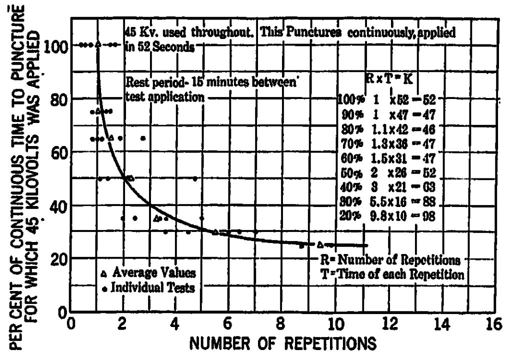

FIG. 13—EFFECT OF REPEATED APPLICATIONS OF HIGH VOLTAGE ON PRESSBOARD

Insulation—One Layer 3/32-in. Grade A Oil-Treated Pressboard
Test Medium—Transil Oil at Room Temperature
Test Voltage—45-kv. 60-cycle
Electrodes—1½-in. Brass with 3/64-in. Edge Radius
Rest Period—15 min. between Tests

Case III: In Figs. 8 to 13 inclusive 1½ in. brass electrodes with edges rounded to a radius of 3/64 of an inch have been used. It has already been shown[2] that in cases where such electrodes are replaced with terminals identical, with the exception that an albarene stone collar has closely fitted around the electrode edge

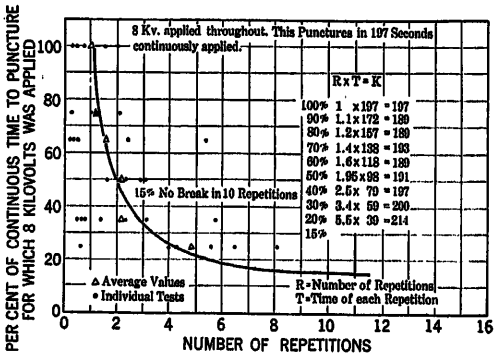

FIG. 14—EFFECT OF REPEATED APPLICATIONS OF VOLTAGE ON KRAFT PAPER

Insulation—4 Layers of Oil-Treated 0.003-in. Kraft Paper
Test Medium—Transil Oil at Room Temperature
Test Voltage—8 kv. 60-cycle
Electrodes—Three-inch, Round Edge, Impregnated Wooden Electrodes
Rest Period—20 min. between Tests

forming a smooth surface with the testing face, the dielectric strength-thickness relation is changed from $Kv = A\,T^{0.70}$ to $K\,v = A\,T^{0.86}$. The effect of such "protected" electrodes on fibrous insulation under repeated applications of stress is shown in Fig. 15 for ⅛ in. pressboard in oil at room temperature. The

2. F. M. Clark and V. M. Montsinger—Ibid

results show a deteriorating effect additional in nature accompanying each application of voltage comparable to that illustrated in the previous figures for "unprotected" electrodes.

Case IV: In the results shown in Cases I, II and III the insulation between test applications has been allowed to rest untouched in oil at room temperature.

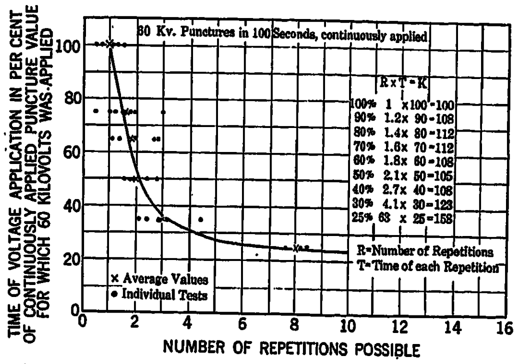

FIG. 15—EFFECT OF REPEATED HIGH-VOLTAGE APPLICATION ON PRESSBOARD

Insulation—One Layer 1/8-in. Oil-Treated Pressboard
Test Medium—Transil Oil at Room Temperature
Test Voltage—60-kv. 60-cycle, Sine Wave
Rest Period between Voltage Applications—10 min.
Electrodes—1½-in. Brass with Albarene Stone Edge Protection

Fig. 16 shows the relationship obtained, using the same procedure as in the previous cases, except that during the rest period the insulation has been heated in oil for a predetermined period. In running these tests, account had to be taken of the aging effect which would result from the intermittent hot oil immersions.

The change in dielectric strength following hot oil

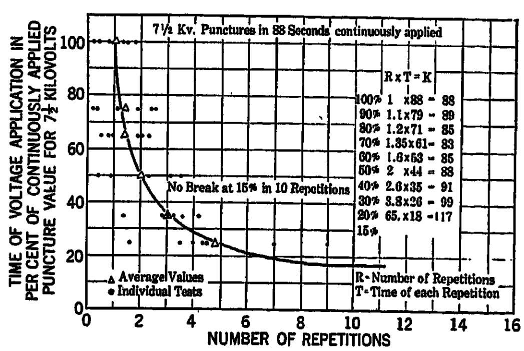

FIG. 16—EFFECT OF REPEATED HIGH-VOLTAGE APPLICATION ON LINEN PAPER

Insulation—8-Layers of 0.0005-in. Linen Oil-Impregnated Paper
Test Medium—Transil Oil at Room Temperature
Test Voltage—7½-kv. 60-cycle
Rest Period between Voltage Applications—One Hour
Electrodes—1½-in. by 2½-in. Aluminum Foil, held in Place between Glass Plates by Means of Spring Clamps

immersion has been found to be rapid in the first stages. In order to avoid such changes during test, the insulation after drying was therefore placed in 80 deg. cent. oil for one week before submitting to voltage stress. Furthermore, the tests, involving long rest periods between voltage applications, eliminated the possibility of using large 1½ in. brass electrodes as done in the previous cases. The following method was therefore adopted. Aluminum foil was cut 1½ in. by 2½ in. for use as electrodes. The insulation tested (0.0005 in. linen paper) was cut 2 in. by 3 in. and held in place between the aluminum terminals by means of glass plates and spring clamps. The miniature condensers so pre-

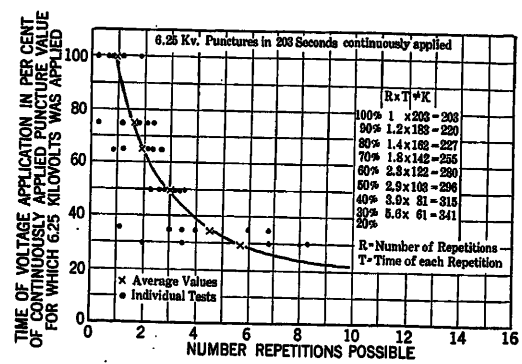

FIG. 17—EFFECT OF REPEATED HIGH VOLTAGES ON LINEN PAPER, HEATED BETWEEN APPLICATIONS

Insulation—8 Layers 0.0005-in. Oil-Treated Linen Paper
Test Medium—Transil Oil at Room Temperature
Test Voltage—6.25-kv. 60-cycle Sine Wave
Rest Period between Tests—at least one hour
Treatment between Voltage Tests—Placed in 80 deg. cent. Oil for 20 Minutes and in 20 deg. cent. Oil for at least 40 minutes
Electrodes—1½-in. by 2½-in. Aluminum Foil, held in Place between Glass Plates by Means of Spring Clamps

pared were vacuum dried at 115 deg. cent. and oil impregnated. They were then placed in transil oil at 80 deg. cent. for a period of one week, whereupon they were cooled to room temperature and the voltage tests made. The voltage repetitions in each case were followed by oil immersion at 80 deg. cent. for 20 to 30

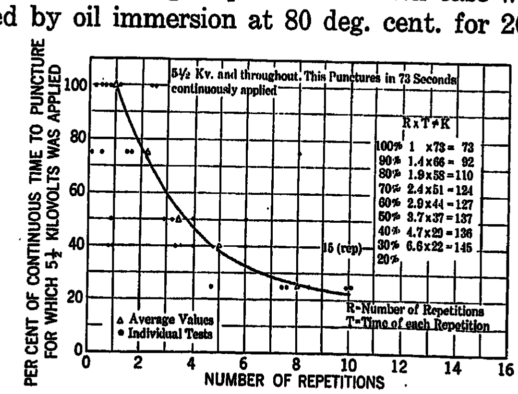

FIG. 18—EFFECT OF REPEATED HIGH-VOLTAGE APPLICATIONS ON MICA

Insulation—2 in. by 3 in. Sheets of Yellow Mica Built to Thickness of 0.004 in.
Test Medium—Transil Oil at Room Temperature
Test Voltage—5½-kv. 60-cycle
Electrodes—1½ in. by 2½-in. Aluminum Foil, held in Place between Glass Plates by Spring Clamps
Rest Period—One hour between Voltage Tests

minutes. The condensers were then quickly transferred to a cold oil tank and brought to room temperature before further voltage was applied. The period between tests was approximately one hour in all cases. Typical results are given in Fig. 16. As will be noted, the application of voltage in steps by this procedure leads to increased voltage life of the insulation as the

time of each voltage application is diminished. This is shown in the values of $R \times T$ given in Fig. 16. These results are directly comparable to those of Fig. 17. In the latter, using the same type of insulation and testing methods, except that the insulation was not heated between voltage periods, the effect of each stress application was found to be additional.

As contrasted to the results obtained with oil treated fibrous insulation, miniature condensers, similar to those used in Case IV above, were made up using mica as insulation. The typical results are shown in Fig. 18. As will be noted, with voltage application in steps, an apparent increase in dielectric life is obtained with or

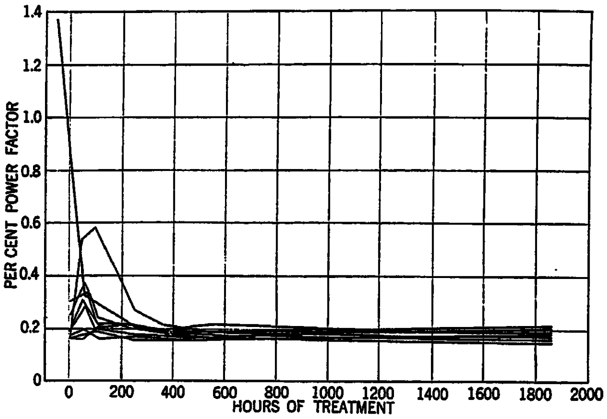

FIG. 19—EFFECT OF LOW-VOLTAGE APPLICATION ON POWER FACTOR

Insulation—8-Layers of 0.0005-in. Linen Oil-Treated Paper

Test Medium—Transil Oil at Room Temperature

Test Voltage—3300-volts 60-cycle

Electrodes—1½-in. by 2½-in. aluminum Foil, held in Place by Spring Clamps between Glass Plates

without heating in oil between tests, *i. e.*, the product of $R \times T$ in Fig. 16 is not constant, but increases as the time of each voltage application is diminished.

EFFECT OF VOLTAGE APPLICATION UPON POWER FACTOR VALUES

The effect of voltage application upon per cent power factor values was investigated using both low and high stresses. In all cases the investigation was carried out by means of miniature condensers as described above, using 1½ in. by 2½ in. aluminum foil electrodes and 0.0005 in. linen paper made up to a thickness of 0.004 in., vacuum dried and oil impregnated at 115 deg. cent.

Case I: Applied voltages producing no puncture for indefinite time.

The results shown in Fig. 19 indicate power factor determinations at 1000 cycles (bridge method) made between voltage applications of 3300 volts on linen paper insulation prepared as described above. The voltage gradient, 825 volts per mil, produced no puncture of the insulation within the limit of the experiment, 77 days. Aside from the power factor variations during the initial stages of the experiment, no change was noticed.

Case II: Applied voltages of relatively high stresses producing puncture when continuously applied within a period of 15 minutes.

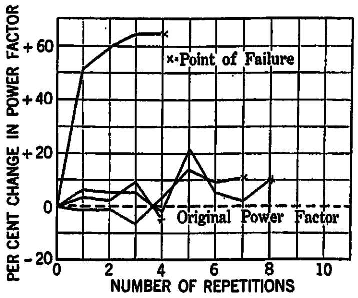

FIG. 20—EFFECT OF HIGH-VOLTAGE APPLICATIONS ON POWER FACTOR

Insulation—8-Layers of 0.0005-in. Oil-Treated Linen Paper (2 in. by 3 in.)

Test Medium—Transil Oil at Room Temperature

Test Voltage—8-kv. 60-cycle, held for 20 sec. which was 40 deg. of the Time to Puncture when continuously Applied

Electrodes—1½-in. by 2½-in. Aluminum Foil, held in Place between Glass Plates by Spring Clamps

The effect of relatively high stresses intermittently applied upon the per cent power factor at 1000 cycles (bridge method) leads to extremely erratic results. In general, the effect shown in Fig. 20 is obtained.

Discussion

J. B. Whitehead: I think the striking result of this paper is the indication that sometimes insulation that has been strained will recover and sometimes it will not. It is easy, from our present knowledge of the properties of insulation, to see how both of those things may occur. Composite insulation is distinguished principally by the property of dielectric absorption, sometimes called viscosity, sometimes spoken of as the property of having residual charge.

If you take a dielectric showing residual charge or absorption and carry it up in temperature, you will find that the absorption will merge into a pure conductivity at high temperatures. If the dielectric is sufficiently homogeneous and uniform and controlled, it may be brought down again from those regions of high temperature, and show all of its original properties. In other words, absorption generally merges into conductivity at high temperatures. It is easy to see, then, how a non-homogeneous dielectric may have this property in excess or preponderance over other properties, and so recover.

The case of non-recovery it seems to me is also equally easy to understand. We very rarely have homogeneous composite dielectrics. Particularly in the case of fibrous dielectrics, we have a variety of other phenomena entering. We have the question of moisture. We also have undoubtedly the phenomena of electrolytic conduction. We have fibers present. We have every possible suggestion of localized structural paths. It is quite easy to see that if we apply voltage to such a structure for a limited time, it is possible that electrolytic conduction, to fix our attention on only one of them, may take place along certain

limited paths to their ultimate destruction. This electrolytic conduction can be a cause of local heating; it can be a cause of the disintegration of the structure of the material itself, without resulting in breakdown.

The conditions of experiment in Mr. Clark's observations were not sufficiently controlled, I think, really to offer us any definite information on the subject of Dr. Wagner's hypothesis. I think we have assumed a wider field for the application of this hypothesis than is warranted. I believe that if we talked to Dr. Wagner himself he would immediately limit his proposal to the range in which the absorptive properties of dielectrics are merging into those of conduction.

H. W. Fisher: Mr. Clark is to be congratulated on his ability to prepare a number of samples in the tests of which he gets such remarkably consistent results. I know from experience that this is quite a difficult undertaking.

I was interested in the fact that some of the samples showed that the material was injured electrically by the application of very high voltages. I have often had reason for believing that the regular run of cables may be permanently injured if too high test voltages are applied. Cables, no matter how well saturated, are changed in their structure, due to the winding and rewinding in the lead covering and installation processes, whereas, the samples prepared by Mr. Clark, were not subjected to mechanical stress, after being prepared for test.

A number of years ago, we had a customer who never specified any high-voltage tests on cables purchased from us. We always made a routine factory test on these cables of at least double working voltage. For years and years no burn-outs were ever reported on cables furnished this customer and our vice-president, Mr. W. A. Conner (who was well known by many of the older members of the A. I. E. E.) often used to remark that the operators who did not specify very high voltage tests seemed to have the least operating troubles so far as the cables were concerned.

I shall cite the case of an experimental cable made about 15 years ago which, when repeatedly tested, did not show this deteriorating effect by the application of high voltages. In the construction of the cable, an extremely viscous and elastic compound was used. The cable withstood voltage tests which were very remarkable at that time and which, in fact, were higher than any required at the present time. Still more remarkable was the fact that although the cable was broken down, a great many times, the fault removed and new tests made, the breakdown tests were within 1000 or 2000 volts of each other every time, the tests being in the neighborhood of 50,000 volts.

Good as this cable appears to be, it would not have withstood the N. E. L. A. bending tests at minus 10 deg. cent. The compound used was a vegetable material, no mineral compound entering into its composition. This does not necessarily mean that cables made with vegetable-base compound do not deteriorate so much from high-voltage tests, as those made with mineral-base compound.

M. F. Skinker: In the synopsis of his paper, Mr. Clark mentions the pyroelectric theory of breakdown as developed by Steinmetz, Wagner, and others. The possibilities offered by thermal theories have been admirably discussed by Dr. E. Dreyfus in Bulletins No. 7 and No. 12 of the *Schweitz Elektrotechnischer Verein*, 1924. His paper deserves to be carefully studied by all who are interested in problems of this nature, particularly from a designer's standpoint. Mr. Clark states that the transformation to thermal energy proceeds "at a rate proportional to the stress applied." Obviously, it should be "at a rate proportional to the *square* of the stress applied."

In order that the original properties of the dielectric after stress might be restored, it must be assumed that no electrical strain or chemical action has taken place until the stress actually reaches the point of failure. Since none of our dielectrics are perfectly homogeneous, we would expect partial deterioration of stresses even far below the average breakdown value, the partial deterioration being nothing but the premature breakdown of weak spots or filaments. It is on the presence of such weak filaments that Wagner builds his thermal breakdown theory, and it has very justly been criticized on account of the arbitrary assumption that must be made relative to their size and physical constants.

"Rapidly applied 60-cycle voltage" is not a definite reproducible quantity unless the exact phase of the voltage is known at the time of its application. There may well be transients and reflections in the dielectric under certain conditions. Therefore, when studying the laws governing breakdown under prolonged stress, the voltage should never be applied abruptly but always should be allowed to start from zero. The time to reach full value need be only a fraction of a second, provided it is large compared to the time taken for an electromagnetic wave to cross the dielectric.

Further, before taking fifteen minutes as an arbitrary time limit, it would be well to investigate thoroughly the effect of long-time application of lower voltages. If we are making tests

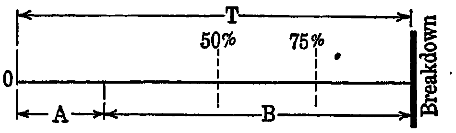

FIG. 1

in the light of the pyroelectric theory, we must take into account all the thermal properties of the system we are studying. When we get a failure due to a continuously applied voltage of several minutes duration, it may be satisfactory to neglect the specific heat of the insulation and that of the electrodes, but certainly when we are considering intermittent application of voltage we must take this into account.

It is possible to make a very simple picture of the reason for the apparent discrepancy in the law $R \times T = K$. When we apply a voltage that will cause a breakdown in an interval, some time will be spent in heating up the system before the critical point is reached. In Fig. 1 let the total interval be T, B the time to reach the danger zone of breakdown and A the rest of the interval.

Now each time we apply the same voltage for a shorter interval (that is, various percentages of the time T as indicated in Mr. Clark's paper) we see the quantity B enters in and should be corrected for. By making simple calculations, considering each of the indicated points on the graph, a mean value of B can be found. For instance, in Fig. 15, B came out 10.2 per cent of the continuous time to puncture; in Fig. 17, 15 per cent; in Fig. 9, 33 per cent, etc.

Now obviously, if we only apply the stress for these percentages of the time, we should not expect to get a breakdown of the insulation. This is faithfully indicated in the curves by the fact that they apparently have asymptotes at these values. If the equation were $R \times T = K$ the horizontal axis should be the asymptote.

The thermal conditions do not permit definite calculations but I cannot see anything in the data that definitely disproves the pyroelectric theory, but they seem to substantiate it.

G. E. Luke: These tests show without doubt that fibrous insulation has the characteristic which may be called "dielectric fatigue." The author stated that this fatigue or deterioration is permanent under the conditions given except for the case when a rapidly applied test is made after a rest period following a pre-

vious voltage application. In this instance the material breaks down at approximately the same value with or without the previous application of stress.

The above author's conclusion will evidently hold true only under the particular conditions tested, since Rayner[2] found that a prevoltage application followed by a rest will materially change the rapidly applied breakdown. Thus Rayner found that a certain oiled cloth will break down in 81 min. on the application of 5500 volts on a green sample, and 6500 volts will break down in about 29 min. on a similar sample. Now when the material is given a prevoltage test of 5500 volts for 40.5 min. (one-half the breakdown time) and allowed to recover over night, its breakdown time on the application of 6500 volts is only 0.8 sec. instead of 29 min. as found on the green material.

The intermittent application of voltage as shown on Figs. 9 to 18 show that the insulation has deteriorated and would not recover since many of the tests were made with 60 min. between tests. The author says, "with mica or with fibrous materials

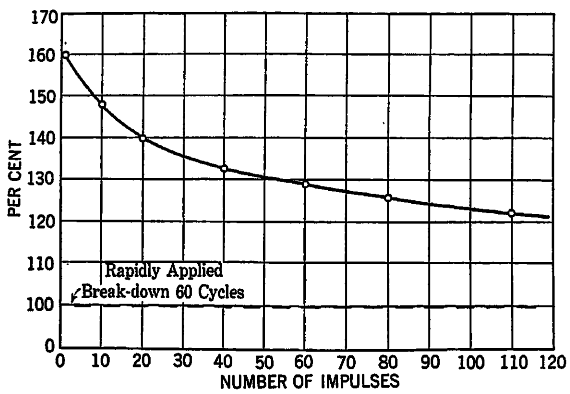

Fig. 2

heated in oil between voltage applications the effect of intermittently applied stress is diminished and an increase in dielectric life is observed." Complete data for such a conclusion could not be found in the paper since the only tests given for mica were on Fig. 18 taken at room temperature. Also due to the erratic variations in the test points it is believed that no definite conclusions can be drawn from the slight differences of curves on Figs. 16 and 17.

In this paper the repeated voltage applications were made with 60-cycle voltage over periods of time of about a minute in order of magnitude, while the time of voltage strain in service due to lightning surges would be measured in microseconds. It will probably be of interest to show the effects of repeated voltage impulses of such short-time waves. Fig. 2 shown here gives the average results of a great number of tests made in the Westinghouse Research Laboratory. The material tested was 0.015-inch sheet fullerboard under oil. The rapidly applied breakdown strength (5 to 10 sec.) with 60-cycle voltage is called 100 per cent. The curve gives the dielectric strength compared to the 60-cycle strength for various number of impulses. The impulse was obtained by charging a condenser until the voltage discharged through a spark-gap. The voltage impulse given to the insulation could be controlled by the constants of the circuit. The total time of the damped single wave of impulse for the tests shown was 4.0×10^{-6} sec. so that on breakdown with one impulse the time of the voltage application was about 2.0×10^{-6} sec. The time interval between impulses was about 5 sec., hence the energy was too small to give any noticeable heating. With 500 to 1000 impulses the voltage breakdown strength was about the same as the 60-cycle breakdown (rapidly applied). The fact that this strength with an impulse of 2×10^{-6} sec. is only 60 per cent higher than the 60-cycle, 5 to 10 seconds, breakdown seems remarkable. Such facts are also confirmed by Peek[3] of the General Electric Company.

This discussion leads up to the questions, what causes the "dielectric fatigue" such as described in Clark's paper, and in general, what causes insulation failure? It is known that under certain conditions insulation may fail due to accumulative heating of small fibers in accordance with the Wagner's pyroelectric theory. This may account for the recovery in insulation strength with a 2-minute rest as shown in Fig. 2 of the paper. In this case the localized heating was non-sufficient to change the material. The results shown with the repeated voltage applications in the author's Figs. 8 to 18 show a deterioration in the insulation, since the time interval between tests was generally sufficient to remove such localized heating, it is evident that some of the insulation fibers were permanently injured by each voltage application. However, on the basis of this pyroelectric theory one would naturally expect mica to stand up much better than fibrous insulation, but the curve on mica, Fig. 18, does not differ much from similar curves on fibrous materials. The data given on Fig. 2 with 4-microsecond impulses can hardly be explained on the basis of the pyroelectric theory since the time of application is so infinitesimal compared to the time used in the other tests. It appears that the fibrous insulation might have a critical voltage similar to the breakdown voltage of air. Such a breakdown might be explained on the basis of corona or excessive internal ionization. Whether dielectrics have free ions and can form corona due to critical voltage stresses is problematical and little information is available. The difficulties of researches along this line are due to the heterogeneous composition of the common dielectrics.

Corona may form in built-up insulation due to occluded air pockets, the results of which may form a chemical action on the insulation. Such action would cause permanent deterioration. This also does not explain the tests shown, since many of the samples were vacuum-dried and impregnated in oil. Also mica, which deteriorated about as fast as the fibrous materials, is especially resistant to this chemical action.

Anderson[4] has shown that if a powerful condenser is discharged through a small wire the wire can be completely exploded into a gaseous residue. The time of the discharge being only a few microseconds, the heating effect is too small even to scorch the cotton covering on the wire. The breakdown of a dielectric sheet with such a voltage impulse seems to be of the same nature, as a small thread through the material may be completely removed without showing signs of burning. The explanation of such a phenomenon does not seem possible on the basis of electrostatic repulsion nor on the basis of electromagnetic forces due to the displacement current; it is a complete disassociation of the molecular structure.

Joseph Slepian: The three papers on *Study of Direct-Current Corona in Various Gases, Effect of Repeated Voltage Application on Fibrous Insulation and Corona in Oil* illustrate two kinds of cumulative effects in insulation, a space-cumulative effect, and a time-cumulative effect. The space-cumulative effect arises from the fact that contiguous layers of insulation mutually influence one another so that the conductivity of one layer is communicated to the next layer, with a resultant cumulative mutual weakening of these layers under stress.

For example, 1 cm. of air will break down at 30,000 volts per cm. gradient, but 0.001 cm. of air withstands 300,000 volts per cm. gradient. Thus, if a thousand layers of air, each 0.001 cm.

2. "High Voltage Tests and Energy Losses in Insulating Materials," by E. H. Rayner, National Physical Lab. 1913.

3. The Effect of Transient Voltages on Dielectrics," by F. W. Peek, Jr. Journal A. I. E. E., 1915.

4. "Spectrum of Electrically Exploded Wires," by J. A. Anderson. *Astrophysics*, January, 1920.

thick, are placed next to one another, they will exert a mutual cumulative effect upon one another in such a manner that each layer is respectively weakened by effects coming from other layers, so that the breakdown gradient is reduced from 300,000 volts per cm. to 30,000 volts per cm.

Similar results are obtained with liquid dielectrics, or solid dielectrics. In Peek's "High Voltage Engineering," I find in one table that 1 cm. of a certain grade of oil would break down at 166,000 volts per cm., whereas ⅛ cm. of that same oil was able to withstand 364,000 volts per cm., a gradient more than twice as great. Evidently by taking eight of these ⅛-cm. layers and building up a whole centimeter, the mutual cumulative effect of these layers upon each other was such as to reduce the strength of each individual layer.

In solid insulation, the breakdown voltage is not proportional to the thickness of the insulation, but to some fractional power of the thickness, which again demonstrates a space-cumulative effect.

The mechanism of this cumulative effect is probably to be found in the nature of conduction in dielectrics. The conductivity is undoubtedly due to charged particles capable of motion in the material, and the effect of high voltage somehow is to increase the number of these charged particles. With insulation of any appreciable thickness, the conductivity produced by the high-voltage gradient at any one portion of the insulation may be communicated by direct motion of these particles to other portions, and vice versa, so that there results a cumulative effect of one layer upon the other, the increase in conductivity of one portion stimulating the increase of conductivity in other portions.

In the paper by Crago and Hodnette, this mutual influence of portions of dielectrics upon each other is very clearly shown. Only the oil next to the wire was very highly stressed, but Crago and Hodnette observe and actually measure the decrease in dielectric strength and increase in conductivity produced at points rather remote in the oil. That this effect exists is not new, but I don't remember seeing any definite quantitative information with respect to this for commercial transformer oil under ordinary operating conditions, so that the results they give are of very considerable interest and in some respects of considerable value.

The other type of effect with which Mr. Clark is principally concerned is the time-cumulative effect. We consider now contiguous elements of time. An element of time in which the insulation is highly stressed will influence the conductivity or dielectric strength of the insulation in the following period of time, usually causing the conductivity to increase and dielectric strength to decrease; a period of low stress may also influence the period immediately following, usually by causing the conductivity to decrease and the dielectric strength to increase.

This time-cumulative effect is not limited to solid insulation. In the case of gaseous insulation, it is a phenomenon long known, but described ordinarily as spark-lag. High voltage may be applied to a spark-gap for a very brief time, with only a very small current flow. If the voltage is kept on, this current flow increases at a very rapid but nevertheless finite rate, until full breakdown takes place. Complete breakdown of a sphere-gap requires a few hundreths of a microsecond. The successive moments of stress all contribute to the breakdown.

That a period of low stress tends to improve the dielectric properties of air is well known to the operating man. When an insulator flashes over, and the circuit breaker trips out, he closes it in again with the reasonable expectation that the rest given to the broken-down air will have permitted it completely to recover its dielectric properties.

In oil we also have this effect and Crago and Hodnette give quantitative measurements on the time involved. In their curves they show the time it takes for abnormal conductivity to disappear during a period of low stress.

Of course, as one would expect, in the case of the gaseous insulation, the time-cumulative effect is exceedingly rapid, in the case of liquids much less rapid, and in the case of solids still less rapid.

The mechanism of this time-cumulative effect is fairly well understood for gases. A high enough gradient in air will cause the production of charged particles which can serve as carriers of current, but the rate at which new charged particles are produced is also proportional to the number of charged particles present. Thus the number of new charged particles produced in any time interval depends on the number of particles left by the preceding time interval. The theory of ionization by collision gives a very vivid picture of this process.

In liquids and solids in all probability, similar phenomena take place, but in these cases another effect may contribute to the time-cumulative effect, namely, the heating of the insulation with resulting increase in conductivity. This aspect has been particularly stressed by K. W. Wagner in his pyroelectric theory of insulation breakdown. There is no doubt that in very many cases breakdown is caused by cumulative heating, but some of the data presented here today, and data given by others, notably Rogowski in the *Archiv der Elektrotechik* show that the pyroelectric theory does not adequately explain dielectric breakdown. If the breakdown voltage for various insulating materials is calculated according to Wagner's theory, using the electrical conductivity and thermal constants of the material as ordinarily measured, values are obtained very much larger than those obtained by experiment.

With all these effects in mind, there is a point that I would like to bring out very emphatically, and that is that we have no right to speak of a breakdown gradient characterizing a dielectric material.

The idea of a definite breakdown gradient probably rose out of the analogy with failure under mechanical stress. Any particular point of a body will yield mechanically if and only if the mechanical stress at that point reaches a certain definite value characteristic of the material, and independent of the stresses which may exist elsewhere. How natural to suppose that under electric stress also, the failure of any point in a body is determined entirely by the electric stress at that point. But it isn't true! There is no definite breakdown gradient. Because of the cumulative space effect, and the cumulative time effect, the gradient which may exist in any dielectric system when breakdown occurs will have no simple relation to the nature of that dielectric. This idea of there being a definite breakdown gradient characterizing materials is one that has colored our engineering thought on insulation for a long time: I believe it has done a great deal of harm and it is time that it should be discarded.

Only a few years ago I remember there was a very considerable discussion as to whether the breakdown in cables could be determined by the maximum stress in the cable or the minimum stress. In the light of the phenomenon that has been described in these papers such discussions seem to be meaningless. Even for a perfectly definite reproducible material, the gradient at breakdown will not be the same for this material with different thicknesses and with different arrangements of conductors. There is no reason why it should be, according to the theory of breakdown in dielectrics and in the light of these effects that have been described.

Lee and Kurrelmeyer, in their paper, take some time to discuss the adequacy of the theory of ionization by collision in explaining their results, but I do not believe that this theory was quite fairly treated in their discussion and some of the conclusions which they say will follow from the theory of ionization by collision do not seem to me to be entirely warranted.

One of the conclusions which they draw from this theory is that E divided by P should be a constant for any particular

gas. E is the gradient at the wire surface when corona begins and P, the pressure of the gas used. They state that E/P is proportional to E times the mean free path, and therefore is proportional to the energy acquired by an ion moving through the mean free path, and that this energy should determine whether ionization by collision takes place or not.

If this were true, then the idea that there is a characteristic breakdown gradient would be justified, because then the breakdown would be determined entirely by whether or not a certain definite stress had been reached. Now I have been trying to bring out the point that breakdown is not determined only by some definite stress being reached.

The reason why this conclusion of Lee and Kurrelmeyer is not warranted is that the mean free path is not a real physical quantity. The mean free path in a gas is only a statistical quantity, an average quantity. If a large number of actual free paths are considered, it will be found that there are many very short ones, and a few very long ones. The average of them all is the mean free path, but there are always a few with much greater lengths. Ionization by collision will take place even at very low gradients due to these few very long free paths. Thus ionization by collision does not set in at a definite gradient, but it occurs weakly for small gradients and then more and more strongly as the gradients increase.

What is it that sets in at some definite voltage on the system? What sets in is an instability due to the mutual action of various portions of the ionized gas upon each other. Given a gas between definite electrodes ionization by collision will occur at all voltages; there is, however, a certain voltage where the ionization produced in one portion of the gas so multiplies the ionization in the next portion, that working back on the first portion, there is produced a cumulative instability and a breakdown.

The proper relation which should be expected, and which I venture to predict will be fairly accurately confirmed, is not that E/P shall be a constant, but another relation given by Paschen's law. Paschen's law connects the breakdown voltage for plane electrodes, the electrode separation and the pressure. Paschen's law has been generalized by Townsend to cover the case of any shape of electrode, and Townsend's generalization may be put in this way: if P is varied, and if at the same time the dimensions of the electrodes are varied so as to keep the ratio of the mean free path to the dimensions of the electrodes constant, then E/P will be constant. Hence in this case E/P will be a constant if the dimensions (here the radius of the wire) multiplied by the pressure is kept constant. It is only if the wire radius is varied in this manner that E/P will be constant, according to the theory of ionization by collision.

It is rather interesting to note that the formula of Peek is consistent with this condition.

D. Bratt: I believe that it is justifiable in this connection to emphasize some general consequences of the purely thermal idea of breakdown that seem to be less realized than warranted by their importance, and which have a direct bearing on the subject of Mr. Clark's paper.

First, it should be noted that breakdown is caused by excessive current, and that voltage as such has nothing whatsoever to do with the so-called "pyroelectric theory." We must therefore carefully distinguish between instantaneous breakdown and thermal breakdown, the former being caused probably by some sort of molecular yielding due to purely Newtonian forces. It does not appear that Mr. Clark makes this distinction in his voltage applications.

The development of heat in a plate, subjected to voltage, not only leads to higher temperatures in the interior, but also to a re-distribution of voltage gradient, even though there be no internal free charges by assumption.

The relation between current density and temperature should therefore be determined, and the subsequent numerical evaluation of the obtained formulas should be based on tests of dielectric losses as a function of current density and temperature.

The maximum stable current density can then be determined. We will get an expression of the following form:

$$\Delta_{max} = \frac{1.84}{d} \sqrt{\frac{\lambda}{\rho_0\, k}}$$

where d = thickness of the plate
λ = thermal conductivity
ρ_0 = initial value of dielectric loss per cm³
k = the logarithmic increment of temperature
and the dielectric loss has been assumed to obey the law

$$\rho = \rho_0 \Delta^2 \epsilon^{k\Theta} \frac{\text{watts}}{\text{cm}^3}$$

The corresponding temperature rise inside the plate

$$\theta_{max} = \frac{1.10}{k}$$

From these equations we obtain the maximum strength

$$E_{max} = \frac{1.84}{f} \sqrt{\frac{\lambda}{\rho_0\, k\, \mu^2}}$$

Where f = frequency, u = the average dielectric constant estimated from the known temperature distribution inside the plate and previous d-c. tests of dielectric constant as a function of temperature.

We have assumed here the simplest possible case, when the surface temperature of the plate is taken as reference level. This temperature is generally not known; therefore, the heat dissipation from the electrodes must be taken into account.

It should be carefully noted that E_{max} is independent at the plate thickness d, assuming as we have done, a homogeneous dielectric.

The strength of the insulation is, therefore, determined by neither $\rho_0\, \lambda\, \mu$ or k alone, but on the factor $q = \dfrac{A}{\rho_0\, k \cdot \mu^2}$.

During a prolonged stress, the strength of most materials has been found to decrease. Thus, were it possible to keep our plate under maximum voltage, we would have to decrease this voltage gradually in order to prevent breakdown.

It is a very reasonable assumption that the cause for this decrease in strength, aside from the natural aging of the insulation, is nothing but the applied voltage itself, or still better, the energy developed. Any structural change in a material requires energy.

For some time, T_0 onward, the change in q becomes very gradual, and we could write, tentatively

$$q = q_0 - a \int_{T_0}^{T} E_{max}^2\, d\, t$$

differentiating

$$\frac{d\, q}{d\, T} = -\, a\, E_{max}^2$$

and according to the expressions found for E_{max}

$$\frac{d\, q}{d\, T} = -\, A\, q$$

(where A is a constant)

Integrating, then, gives $q = q_0\, \epsilon^{-A\,(T - T_0)}$

E_{max} will, therefore, according to the hypothesis made, asymptotically approach zero, and this result indicates in my opinion very definitely the danger of prolonged stresses near the breakdown point.

Everything leads us to believe that this change in strength is an irreversible process. Therefore, if the material has been once so stressed, and the voltage then removed, nothing should lead us to expect a return to the initial state. This does not,

of course, apply to moderate stresses, which have been found to improve the strength in many cases, usually due to some drying-out process.

When studying the effect of prolonged or intermittent stresses on a solid dielectric, we believe it would be profitable to work along some hypothesis similar to the above. If correctly understood, the pyroelectric theory might yield results of great suggestiveness, and in this case, as always, a theoretical analysis prior to any experimental work offers the best possible guarantee against excessive laboratory expenses and results that have no general applicability.

Incidentally, I might add that for a crucial test on the pyroelectric theory as such, direct current ought to be used instead of alternating current. This would eliminate internal charges and some other unknown factors. If this theory failed for direct current, it would certainly not be expected to be valid for alternating current.

A. C. Crago: The most variable quantity which I have run into in engineering work has been the time of breakdown of dielectrics. So far as I know, it is the most variable quantity which we try to assign a definite value. We see in Mr. Clark's paper that the time of breakdown is a decidedly variable quantity.

I shall confine my discussion to the group of curves numbered 8 to 18 in Mr. Clark's paper. I shall review briefly the conclusions or the results which Mr. Clark has obtained from these eleven curves as I understand them.

First, that with oil-impregnated or oil-immersed fibrous materials, the cumulative effect of an intermittently applied voltage is directly additive; second, that with varnished materials, the cumulative effect is not additive; third, that the cumulative effect is not directly additive in any case for shorter times of application than 30 to 15 per cent of the 100 per cent time.

Using these results as evidence, Mr. Clark's general conclusion is, I believe, that there is some permanent deteriorating effect on the insulation in all cases due to this voltage application which is not removed by the rest period between tests.

After a rather detailed study of the data presented in Figs. 8 to 18, I believe that although the above conclusion may be correct, it is not warranted on the basis of the data presented, because of the wide variation in the time required for breakdown with a voltage continuously applied.

The following table presents the range of values obtained in determining the "100 per cent" points of the group of curves:

Fig. No.	Lowest	Highest	Ratio
8	0.40	1.60	4–1
9	0.40	1.7	4.2–1
10	0.50	1.55	3.1–1
10	0.35	2.0	5.7–1
11	0.35	1.8	5.1–1
13	0.35	1.75	5.0–1
14	0.25	2.0	8–1
15	0.5	1.5	3–1
16	0.2	1.85	9.2–1
17	0.2	1.9	9.5–1
18	0.2	2.5	12.5–1
Average Range 6.3 to 1			

Obviously, if we use the average of five or six values which vary over such a wide range, the mean value is subject to a rather large variation. A calculation with an average variation of 50 per cent from the mean, (which is found in some cases) shows that the probable error of this average is over 20 per cent.

However, although the reasoning given above will account for errors in the values of results, it presents no evidence to show that the relation $R\ T = K$ is not necessarily due to a permanent weakening which lasts over the rest period.

I shall now present evidence for this. First, let me say that not all observers have found the effect Mr. Clark gives. E. H. Rayner gives the following table for two thicknesses of Number 12 cloth immersed in oil, which it seems is comparable with the data of Clark:

Recovery in varying periods of rest after application of 9000 volts for 1 minute (breakdown at 9000 volts—1½ to 2 minutes)

Period to rest	Time to break at 11,000 volts
0	2.6 seconds
1 min.	9.5 seconds
2 min.	11.9 seconds
fresh material	12.0 seconds

It will be seen that after a rest period of two minutes, the insulation stood the test voltage for practically as long as unstressed material. I cannot tell you how these results were obtained or the laboratory conditions under which they were obtained, but give them to you as a result which was published by E. H. Rayner under the title of "High Voltage Tests and Energy Losses in Insulating Materials" in the *Proceedings* of the I. E. E. of 1912, Volume 49, page 214.

Instead of assuming that there is a permanent effect on the insulations used in Mr. Clark's tests due to high voltage stress, let us assume that when voltage is removed, the area under test returns, not to its original condition or to a poorer condition, but to the condition of a random point. That is, let us assume that it has the same chance to break down in 10 sec. or 20 or 30, sec., as though we had chosen a fresh point on the insulation. What will be the result?

Let us analyze one of Mr. Clark's curves on this basis, taking as an example Fig. 8.

Referring to this curve, we find, that there is one breakdown obtained at about 40 per cent of the time referred to as the mean value or the 100 per cent time, and that five tests have been used to obtain this average. This means that if we choose new points on the dielectric and apply voltage, approximately one time in five breakdown will occur in 40 per cent of the 100 per cent time. Of course, that cannot be a very definite quantity, because only five points are given here.

Also, if we apply the voltage for 40 per cent of the 100 per cent time, remove it, move the electrodes, and apply it again for the 40 per cent time, a breakdown will occur one time in five.

Let us see now what happens when we apply voltage for periods of 40 per cent of the 100 per cent time and do not move the electrodes. Referring again to Fig. 8, we come down to the 40 per cent time and across till we strike the curve, and we find that that corresponds to a number of applications of 5½, which I would consider an excellent check. That is, this insulation, whatever the cause of the variation, has acted in such a way that the curve could be explained on the basis which I have stated and will state again: that an insulation stressed under the conditions given does not return to its original condition, nor to a poorer condition, but to the condition of a random point.

I have analyzed the eleven curves given, attempting to explain their shape on this basis, and I find that nine of the eleven give a reasonably close check and that two of these nine are more than explained by this assumption.

After reading Mr. Clark's paper I made some tests with oil, using a standard Westinghouse test cup, which consists of two square-edged cylindrical electrodes 1 in. in diameter, spaced 0.1 in. apart. The oil was stirred after each application of voltage. The procedure described by Mr. Clark was used. Under these conditions, no permanent deterioration resulted, the oil on each application of voltage being fresh oil.

I have prepared three illustrations showing the results. Fig. 3 shows the effect of applying 30 kilovolts to oil under the conditions given. I have shown the individual points obtained in getting an average 100 per cent point. The variation is very wide; from 8 per cent to 250 per cent of the average.

The time was reduced to 70 per cent of the average time and new points were determined, getting a new distribution of break-

down time. That is, we applied the voltage for 70 per cent of this 71 seconds, or attempted to apply it for that long, and then cut off the voltage, stirred the oil, applying the voltage again, obtaining an average point which lies out at the value of 2. The time of application was then reduced to a little less than 40 per cent of the average time, obtaining another average point.

The testing voltage was reduced to 27 kv. (see Fig. 4). It was found that the average time for breakdown was 326 seconds.

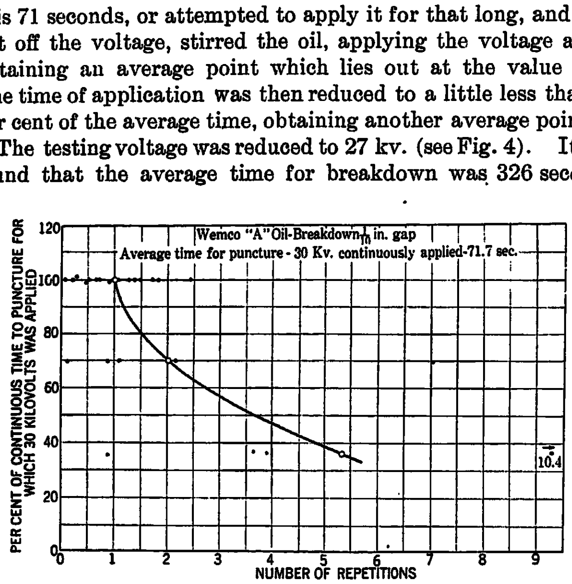

FIG. 3

Following a similar plan, we get a somewhat steeper curve than the one given for 30 kv. You understand, this is the same material, and that no permanent deterioration could occur, and yet we get a curve of this new shape. If we had had all these points lying right on the average point, instead of spread over a wide range as they are, then as soon as we had reduced the time to 70 per cent of the average time, we would have obtained no breakdowns. I realize that the number of points given here is not enough for complete proof, but it checks very well with the theory given.

I have taken the two curves shown before, and plotted them together in Fig. 5, showing the differences obtained, and also plotting a third curve which has the equation $R\ T$ = a constant. The object of this curve is to show that we can obtain almost any law by the conditions of test used; that is, even with the same

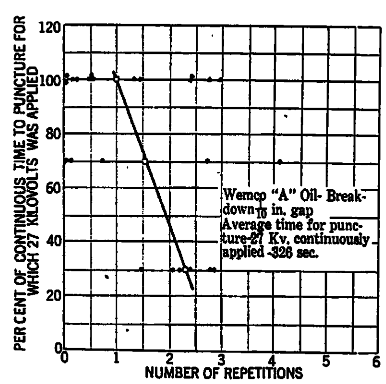

FIG. 4

dielectric, we can get $R\ T$ with a decreasing value, or with an increasing value.

Let me repeat then, in a few words, the ideas I have tried bring out in this discussion of the paper.

First, Mr. Clark's paper explains the shape of the curves in Fig. 8 to 18 by the presence of a permanent deterioration in the dielectric. I have presented evidence attempting to show that it is not necessary to assume permanent deterioriation to explain the shape of the curves, the evidence being, first, an actual experiment by Mr. Rayner in which no permanent deterioration occurred with similar dielectrics; second, an analysis of Mr. Clark's curves on another assumption; third, similar experimental curves where no permanent deterioriation did occur.

It seems probable to the speaker that in actual tests the curve shape is the result of a combination of two factors, variation and deterioration. Only more consistent results for identical tests can determine this point.

The most likely criticism of this discussion is that we are not dealing with the same thing in solid as in liquid dielectrics; that when we put a pair of electrodes down at a certain point on the solid dielectric, that point has certain properties which we will determine. Different points in the dielectric, it may be said, will vary widely, but a given point has rather definite properties, depending on the location of the electrodes.

Of course this is a rather good point, but we can see that because these variations are so very wide there must be something other than just a difference in the quality of the material to explain these wide variations. Perhaps a very slight variation in test voltage may cause it. Perhaps the distribution of charges in the dielectric may cause the variation. Perhaps where we have it immersed in oil, actual movement of the oil due to osmosis in the fibers of the insulation may cause it. Perhaps a slight shift or change in pressure or distribution of pressure of the electrode occurs. I do not offer a complete physical explanation based on

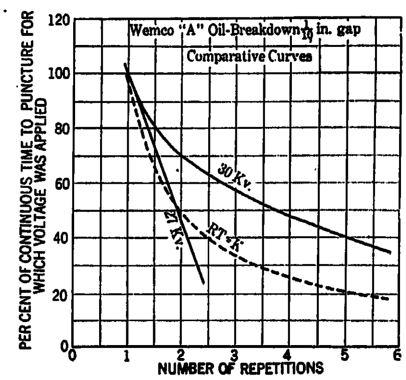

FIG. 5

this other assumption, but I do feel that we are in danger in interpreting results based on averages determined from a few points which vary as widely as those shown in Mr. Clark's tests.

Herman Halperin: It appears to me that the results of the tests in Mr. Clark's paper need not alarm the makers and users of high-tension, impregnated-paper insulated cables.

Figs. 2 and 3 of the paper show that a preliminary application of voltage, equal to about 60 per cent of the normal rapidly applied breakdown voltage, results in deterioration of only about 5 per cent and that after a rest period of three minutes the original dielectric strength is entirely recovered.

For the 12-kv., three-conductor cables purchased by the Commonwealth Edison Company in 1924, the normal rapidly applied breakdown voltage on the new cable at the factory would be about 100-kv. to 150-kv. The full-reel, high-voltage tests which were applied to this same cable was 32½ kv. for five minutes and this voltage corresponds to only about 25 per cent of the normal rapidly applied breakdown voltage of the cable. The acceptance test on the cable after installation and the subsequent proof tests are made at voltages considerably lower than those of the full-reel, high-voltage tests at the factory. The transient voltages, as found at several substations by means of needle gaps, are seldom as high as 32½ kv., when operating the cable. Therefore, the ratio of the extra high voltages that as applied to cables to their normal rapidly applied breakdown voltage is considerably below the ratio of 60 per cent which Mr. Clark used as his previous application of voltage, as pointed out in the previous paragraph.

In Fig. 4 the effect of applying for 2½ minutes a voltage of

about 65 per cent of the normal rapidly applied breakdown strength, is shown to cause a decrease of about 20 per cent in the instantaneous breakdown value of the insulation and the decrease in the voltage time curve at three minutes is only 12 per cent. So one wonders whether or not the effect of this previous application of a high voltage would be further decreased if the voltage-time curve were extended to a matter of hours, months, or years. As I have previously pointed out, in testing high-voltage cables, instead of using 60 per cent or 65 per cent of their normal rapidly applied breakdown voltage, the tests are at about 25 to 30 per cent, so the effect of those tests are further minimized to a very small amount, as may be deduced from Mr. Clark's work.

In Fig. 7, the difference between Curves 2 and 3 show the effect of the long-time application of voltage at a somewhat lower stress. The difference between the two curves is only 3 per cent or 4 per cent; hence it appears that nothing conclusive can be drawn

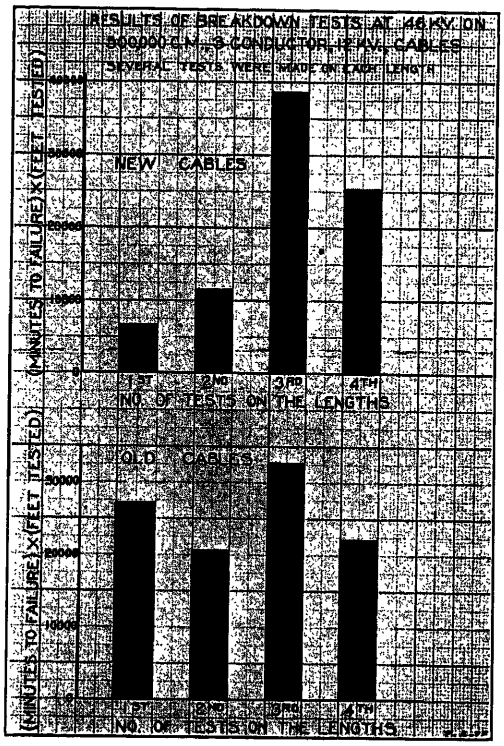

Fig. 6

as to the effect of this application of 3300 volts for seven days, for any one familiar with breakdown tests knows that test results, which show only a slight difference in dielectric strength, do not definitely prove any point, because the individual test results are liable to vary about 10 or 25 per cent. In connection with this curve, it is gratifying to learn that Mr. Clark expects to continue this work of the effect of long-time applications at lower voltages. The stresses which he is using for long-time application correspond to about the stresses used in full-reel high-voltage tests at the factory, after correcting for the thickness and form of the insulation.

Last year the Commonwealth Edison Company tested several 200-ft. lengths of three-conductor, 12-kv. cable. Some of these lengths were new cable as received from the factory and others were used cable which was removed from the system due to failures. The tests were made at 46 kv. three-phase with neutral grounded, and were continued until breakdown occurred. When a given length failed, the portion near one end was removed in case that portion was short; that is, if we had a 200-ft. length and the failure occurred at the 175-ft. mark, the 25 ft. were removed, a pothead was attached at that end and the test was repeated. If the failure occurred near the center of the length, a short section of cable including the failure was cut out and a joint made. Then the length was again ready for test. This was repeated on the various lengths, with from three to six tests on each length.

In the accompanying Fig. 6, the results of the tests on new cables are shown in the upper portion and those on old cables in the lower portion. In order to take into account the fact that the length of a given section of cable decreased due to cutting off portions, we plotted the product of the minutes to failure and the feet tested, against the number of the test on the length. For instance, on the second test, we took the results of all of the second tests on the various lengths, multiplied the minutes to failure of each section by the length of that section in feet, averaged all of these products, and obtained an ordinant of about 11,000. In general, the increase in the life of the new cable was very marked, while on the old cable the product of the minutes to failure times feet tested remained practically constant. If the item of feet tested had not been considered, then the increase of minutes to failure would have been greater for both old and new cables than shown in the chart.

Similar tests at higher voltages have been made on three-conductor, 33-kv. cable and the results are similar to those shown in the figure. That is, on the 33-kv. cable the average time that the section withstood the second and third tests was greater than the time which it withstood the first and second test, respectively.

These tests, which were made at nearly four times the operating voltage, indicate that one application of voltage of five to fifteen minutes at three or four times the operating voltage should produce practically no deterioration of the cable and should not affect its life in service.

E. S. Lee: In connection with Dr. Slepian's remarks regarding the effect of small laminations of the material as opposed to a greater thickness of the material, I want to bring to you something that we have noted, to see whether other folks have noted it because I think it is of great interest.

In Fig. 7 herewith results of dielectric strength tests on some black varnished cloth with both direct and alternating current are shown. It is interesting to see that while the relation between breakdown voltage and thickness for both alternating

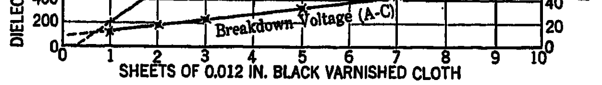

Fig. 7

current and direct current is approximately linear, the relation between dielectric strength and thickness for alternating current and direct current are quite different. This follows because the breakdown voltage-thickness curve with direct-current (extrapolated) cuts the x-axis, while the breakdown voltage-thickness curve with alternating current (extrapolated) cuts the y-axis.

The point I want to bring to you is this: While the a-c. curve says we have a given voltage with no thickness, and the d-c. curve says we have a certain thickness with no voltage, the point I want to make is that they probably both go through zero. Whatever is happening there, then, appears to

be happening within the thickness of one sheet of material. This bears out what Dr. Steinmetz has said: that we will have to study very small distances between electrodes in our study of insulation.

Just one other point: I did just as Mr. Halperin did in taking Mr. Clark's results and referring them to rated cable voltage and test voltage, and I came to the same conclusion that he did: that we were not in that range. But I noticed that when voltages occur on the system within 200 or 300 per cent of normal rated voltage, they come within the range of Mr. Clark's results. So it may be that in cable operation the results shown by Mr. Clark do enter.

R. W. Atkinson: As in the case of the discussion of Mr. Clark's paper authors of papers presented before the Institute are often criticized by physicists and engineers because not enough data are given, and authors are criticized by the Institute management because the papers are too long. These viewpoints are not always mutually exclusive, but very often they are, and an author always has quite a problem to meet both viewpoints, which are both very well taken. But usually a compromise must be arrived at, and usually both sides have some remaining good grounds for criticism.

I am very glad that Dr. Whitehead and Dr. Slepian have emphasized the fact that the pyroelectric theory of breakdown brought out by Dr. Wagner and Dr. Steinmetz is limited in application. Unquestionably this theory can explain breakdown in many cases and unquestionably in many cases breakdown is of that character, but I am convinced that in many cases it is not.

There is one conclusion in Mr. Clark's paper that is very directly applicable to cable insulation in general. I do not know that it makes very much difference whether the explanation of the conclusion is deterioration, or whether the explanation is the random effect suggested by Mr. Crago, but this conclusion is that the total breakdown time for a given voltage is often or usually somewhere nearly independent of whether one test is made on a cable or several. If insulation will break down at a given voltage on an average in one hour, it does not make a great deal of difference whether it is ten 6-min. tests or one 1-hr. test.

That is given with qualifications in Mr. Clark's paper, and I think the qualifications apply. But in general it has seemed in tests on samples of cable insulation and in reel lengths that that general conclusion applies.

F. W. Peek, Jr.: It is a well established fact that when voltages above a certain value are applied to insulation, damage is done. If a succession of over-voltages is applied, the effect is accumulative. This is perhaps more simply shown by the following experiment with impulses of short duration than by Mr. Clark's data. A voltage is readily found at which a single impulse will cause failure. If this voltage is lowered somewhat it may require ten impulses to cause failure; at a still lower voltage one hundred, etc. Finally a voltage is reached where there is no failure on any number of applications. The cumulative effect of the successive impulses of over-voltage is not a pyro effect; it is not due to an accumulation of the heat from each application because it does not depend upon the time between applications. It is caused by a disruption or tearing apart of the material without burning. Of course, this disruptive effect is quite different with different types of insulations. With certain electrically brittle insulations the effect is marked. With insulation built up of laminations filled with oil the effect is probably a minimum because damage is repaired by the flow of the oil. I think that Mr. Fisher's discussion illustrates this.

In addition to the above disruptive effect, there is without doubt a pyro effect. The pyro and conduction effect becomes predominant in certain insulations for moderate "over-voltages" and comparatively long times of application. The theory, now generally known as the Wagner theory, fails, I believe, in that it does not consider all of the variables. The mistake must not be made in neglecting either the disruptive or the pyro and conducting effects. Whether one or the other is the predominant cause of failure depends upon the conditions.

D. W. Roper: In the test described by Mr. Halperin the lead sheath of the cable was very thoroughly grounded so the operators could go along and feel the sheath with their hands and determine the location of the hot spots which developed through local heating. These hot spots were marked, for the purpose of orienting the subsequent failures. The failure occurred in the maximum hot spot in about 40 per cent of the cases. In 25 per cent additional, the failure occurred not in the maximum hot spot, but in some other spot where the heating was appreciable. About 35 per cent of the failures occurred where the temperature was normal, that is, where there was no hot spot. So that the number of cases in which there was no pyroelectric effect was about equal to the number of cases in which the heating was apparently due to the pyroelectric effect described by K. W. Wagner.

C. E. Skinner: I have been very much interested in Mr. Clark's conclusions, particularly in view of my paper entitled, "Energy Loss of Commercial Insulating Materials when Subjected to High Potential Stress," published in the 1902 TRANSACTIONS of the Institute, page 1047. I would especially refer to the numbered paragraphs, 1 to 8, inclusive, pages 1050 and 1051. My tests were made with the view of determining the effect of continued stress of various intensities and frequencies, but many of the conclusions from these tests anticipate those given in Mr. Clark's paper. For example, this statement appears, "The final breakdown in fibrous materials usually results from the burning of the material and not from mechanical rupture." My paper discusses the general question under the headings, "Variation of Temperature due to Variation of Stress," "Variation of Loss due to Variation of Temperature," "Variation of Loss due to Variation of Voltage," and "Variation of Loss due to Variation of Frequency." Mr. Clark has given the results of experiments made from a somewhat different viewpoint and it is naturally gratifying to find that where parallel conclusions are reached, they are substantially the same as those arrived at by the very crude instruments and methods available to us twenty-two years ago.

V. M. Montsinger (by letter): Mr. Lee has mentioned one very interesting point and that is the variation of dielectric strength with thickness. I cannot quite agree with Mr. Lee in that if the first layer is neglected, the points fall on a straight line on coordinate paper, or in other words that the dielectric strength is proportional to the thickness.

An examination of a large number of tests made on various kinds of fibrous insulation shows that in almost every case the points fall on a straight line on log-log paper. This means that the strength vs. thickness can be expressed by the expression:

$K.\ V. = A\ T^n$, where A is a constant T the thickness and n a numerical value depending mostly on the treatment of the material.

If the material is untreated the value of n is close to unity. If the material is well oil-soaked or varnished, the value is around $^2/_3$ or perhaps $3/4$. However, if the layers are very thin in the order of a few mils it is often found that the strength of the first layer is higher than the line indicates it should be. The effect is perhaps the same as what we find for air and oil, namely for very small distances the volts per mil goes way up and departs entirely from the law that holds for the larger thicknesses.

As stated in Mr. Clark's paper, there is scheduled for publication in the near future, an article by Mr. Clark and myself on the subject of dielectric strength vs. thickness and in which the subject is discussed more thoroughly than is possible here.

In reference to Mr. Bratt's formula, which states that the maximum strength is inversely proportional to the frequency of the applied voltage, I would like to point out that while strength

decreases with an increase in frequency, it is not in any way inversely proportional as indicated by the formula.

The results of a large number of tests covering a range of frequency from 25 to 420 cycles show that the factor F should have an exponent M whose value is approximately 0.137 for solid insulations. The difference in strength depending on whether the value of the exponent is 1 or 0.137, is of such a large magnitude that it cannot be neglected. For example, if the strength was inversely proportional to the frequency, the dielectric strength at 420 cycles would be $^1/_7$ or approximately 14 per cent of the strength at 60 cycles. As a matter of fact for solid insulation the 420-cycle strength is approximately 75 per cent of the 60-cycle strength.

Full details covering this subject are given in my paper on "Effects of Time and Frequency on Insulation Tests of Transformers" appearing in the 1924 TRANSACTIONS A. I. E. E. p. 337.

F. M. Clark: I would like to put one thing over and that is that I do not believe that I am attacking the pyroelectric theory of insulation failure. For that reason, I have been glad to hear Dr. Whitehead and others rise to the defense of the pyro theory in statements with which I can heartily agree.

Any discussion of the heat theory of insulation failure, generally involves, sooner or later, a discussion of "weak threads," conducting particles and the like, and such a talk usually ends in a thorough condemnation of the paper manufacturer. I can not agree that the paper or cloth, or whatever may be the solid concerned, is the sole seat of the weak spots involved. There are all sorts of ways in which a weak spot may be formed not even remotely involving the characteristics of the solid itself. From work which I have done, enumerated in this paper and otherwise, it seems as if a weak spot might actually be formed as a result of the voltage application. For example, we know that some molecules are unsymmetrical and polar. They thus possess the property of attracting other molecules. The water molecule has such a property and if present in oil it would become attached with its positive pole or its negative pole as the case may be toward the second molecule involved. This in effect would lead to a molecular polarization in the liquid dielectric possessing all of the characteristics necessary for a weak-spot formation. A thorough study of Fig. 17 will reveal the possibilities of this idea, in connection with the apparently cumulative effect of voltage upon insulating materials.

If we assume this explanation, it will explain a large number of the questions raised in the discussion, I cannot take up all of these but there are several to which I desire to reply.

Mr. Fisher and others have pointed out that apparently with viscuous materials such as impregnating agents the effect of voltage application on cables is negligible. In view of the fact that the cable cited by Mr. Fisher was apparently very well treated and voids eliminated no objection can be raised on that score. But does Mr. Fisher not consider the fact that it is somewhat remarkable that every time his cable broke down and he repaired it, the next break was at about the same voltage? If we assume that each time a break appeared, we are removing a weak spot, the next breakdown ought to be higher. In other words, by removing the weak spots of the cable, the actual breakdown ought to rise gradually with each succeeding test. The fact that it does not, might be explained from the standpoint of a cumulative voltage effect.

I can only remind Mr. Skinker that it is realized that further study must be given to the effect of voltage application involving longer times that an arbitrary limit of 15 min. As stated already, however, this phase of the research is at present being investigated and will be reported upon at the appropriate time. Furthermore, as far as can be detected with the oscillograph, the effects of surges cannot be used to explain the voltage effects set forth. The possibility of surges has been considered and eliminated. Mr. Luke has referred to the researches of Rayner on oiled cloth in which a long-time voltage application produced an effect which did not disappear during a rest period. Although no oiled cloth has been investigated in these researches, there is no apparent reason why its behavior should not correspond with that cited for oiled paper. Certainly its behavior must not be confused with that given for varnished cloth which Mr. Luke has apparently done. If we accept the case of oiled cloth as paralleling that for oiled paper then there should be no recovery from the voltage effect. This is clearly illustrated in Fig. 4 of the paper.

Mr. Luke furthermore points out the mechanical effect of impulses. These results are in agreement with work of similar nature with which I am familiar. Apparently Mr. Luke has misunderstood the statement on the second page with reference to the behavior of fibrous oil-treated material heated between voltage applications and to mica. (Figs. 17 and 18). All actual voltage application has been carried out at room temperature. When the insulation has been heated between voltage stress, it has been cooled to room temperature before the next application of voltage. We have never been able to obtain evidence of an additive voltage effect under such conditions. The effect is cumulative and somewhat erratic as shown in the figures mentioned but in no sense strictly additive. Mica has always been found rather erractic in tests of this sort but the results are in general of the same character.

Mr. Bratt has brought out the necessity of carefully distinguishing between an instantaneous breakdown and a thermal breakdown. This necessity has been recognized throughout and it is for that reason that the paper has been carefully separated into three parts. The very short-time test results are shown in Figs. 1, 2 and 3. Figs. 8 et seq. deal with breakdown involving thermal considerations. These two groups are connected by Figs. 4, 5, 6 and 7 showing the time-voltage relation involving a breakdown from those of "instantaneous" character to those involving thermal considerations.

Mr. Crago has cited the work of Rayner in which a stress application of 11,000 volts produced a break in 12 sec. which was unaffected by a previous voltage application followed by sufficient rest period, the time of rest demanded being about 2 min. A break involving only 12 sec. is of the same category as those designated in this paper as rapidly applied tests, involving roughly 10 sec. For breaks of this character, it is shown that a decided recovery of the insulation occurs during a rest period after a voltage application. Furthermore, as given in Figs. 2 and 3 the rest period demanded is approximately 2 min. although influenced somewhat by the value of the initial voltage stress.

Mr. Crago has claimed that the insulation after being stressed, returns not to the original value nor to a poorer condition but to a random point, which if I understand him correctly may be even better than the original. I can merely say that we have never observed that the strength of insulation when once subjected to high-voltage stress has returned to a value better or even as good as the original when such strength is estimated by tests involving thermal considerations, except in cases where the material has specifically been given a poor drying and oil-treating process. Here, the results have obviously involved factors such as a further drying-out process, and the like, whose effect has overshadowed the effect of the applied voltage.

Mr. Crago has objected to the conclusions of this paper on the ground that the results are based on tests involving wide variations which he calculated as amounting in some cases to as much as 50 per cent from the mean. He cites researches to support his contention using oil as the insulation investigated and draws conclusions based on the data illustrated in his Figs. 4 and 5. And yet the results upon which his conclusions are based show such wide variations that it is surprising that they should be submitted as scientific data. Thus he stated that the average relation of maximum to minimum of the 100 per cent points in the data which I have submitted is as 6.3 to 1. The relation existing for the corresponding values of his research are as 33 to

1 for Fig. 4 and as 118 to 1 for Fig. 5, an average ratio of maximum to minimum of 75.5 to 1. Stated otherwise, from Mr. Crago's calculations of the variations in the data which I have submitted the deviations for the fibrous materials investigated ranges from as low as 20 per cent of the mean to values as high 185 per cent of the mean. In Mr. Crago's work he states that the variations of his Fig. 4 range from 8 per cent to 250 per cent of the mean value. For his Fig. 5, a study will show that the values range from about 2 per cent up to approximately 297 per cent of the mean value. Obviously, the factors involved in Mr. Crago's work have not been sufficiently controlled and his results are condemned by his own contentions.

But suppose we consider Mr. Crago's results further. Why should his data suffer such a wide variation? Mr. Crago bases his experiments upon a time-voltage relation in oil. A study of pure liquid dielectrics and clean, dry oil in particular will reveal the fact that although the time lag is more pronounced than in air and other gaseous dielectrics, yet in no way is it comparable to that which has been found by Mr. Crago. It is only in the case of wet oil or oil containing contaminating dust or fibers that such a marked time-voltage relation can exist. Thus obviously what Mr. Crago is measuring is not the dielectric strength of the oil and therefore not the effect of voltage upon oil but rather the time factor needed to line up these contaminating materials in the 0.1-in. test gap. Thus the time factor involved will entirely depend upon the chance arrangement of these materials at the time of the voltage application together with the resistance which they offer to being sucked into the dielectric field. This explanation gives a reason why Mr. Crago should get two entirely differently shaped curves when he uses 30 kv. as compared to 27 kv. in his experiments. The 30 kv. will rupture, he finds, in 71.7 seconds as opposed to 326 seconds when 27 kv. is used. The lower voltage should obviously take longer to line up the fibres, dust particles, etc., in the test gap. This, however, is not the whole story. In his Fig. 4 the minimum time to puncture is 8 per cent of the 100 per cent time value or 5 sec. In Fig. 5 the minimum time to puncture is 2 per cent of the 100 per cent time value or about 6.5 seconds. Unless the experiment was very carefully controlled, this means that the oil broke down almost immediately when the full voltage was impressed across the gap. Such a breakdown could be explained by the chance "bridge" arrangement of a contamination in the gap at the moment of voltage application. One would not expect under these conditions that the number of repetitions of stress times the time of each repetition to break down would be constant. Rather would one expect that with high voltage and therefore a very short time of repeated voltage application, the product of the factors mentioned might possibly be greater than the original continuously applied (100 per cent) value, since in this case the breakdown would almost entirely depend upon the chance arrangement of a conducting bridge across the test gap at the time the voltage was impressed. With lower voltages and longer time for the repeated voltage application, obviously the second factor, the sucking of the contaminating material into the field would play a much larger part. The product of $R \times T$ might even be smaller than the original 100 per cent time value. This is because of the fact that although the importance of the "sucking actions" of the field has not been diminished to any great extent, nevertheless the possibility of the chance formation of a bridge has been materially increased by breaking the voltage application up into steps and the stirring up of the oil between each stress application. I can thoroughly agree with Mr. Crago that under the conditions of his experiments, he could obtain almost any law by the conditions of the test used.

The case of a 33-kv. cable has been cited by Mr. Halperin in which the average time a section withstood the second and third test was greater than the time it withstood the first. It must be remembered that in cable work failure may be due to a cause entirely foreign to the phenomenon investigated in this paper. For example, cable engineers recognize that a fundamental problem, in the manufacture of their product is the preparation and impregnation of the insulation in order to secure thorough drying and the absence of voids. When a cable fails prematurely it is generally assumed poor, not because of the cumulative effect of voltage, but because of poor treatment, drying and other mechanical factors in its manufacture. Thus in the case of Mr. Halperin's 33-kv. cable, is it not possible that the first failure, the second failure, etc., can be traced directly to some such cause? The removal of each successive section ought therefore to lead to higher and higher breakdown stress until only that insulation is left which contains a minimum of mechanically weak spots and thus allows the manifestation of the deteriorating effects due to the applied voltage. In the experiments of this paper, the use of a small vacuum-dried and well impregnated test specimen together with parallel-plane electrodes has prevented all such treating phenomena from being of major value and has thus allowed the effect of each voltage application an important place in the ultimate breakdown.

Corona in Oil

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Non-member

and

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Non-member

Synopsis.—Experiments were made to determine certain effects of high local voltage stresses in transformer oil. The resistivity of the oil was measured by a special method immediately following a period of voltage stress. The results were:

1. A greatly reduced resistivity when the stressing voltage was greater than that producing visible corona.

2. A gradual increase in resistivity following removal of stress.

The "rapidly applied" breakdown voltage was found for samples of oil which had been previously subjected to high voltage stresses. The dielectric strength varied in a manner similar to the resistivity, but showed an actual improvement in dielectric strength when the oil was given a rest period of 15 minutes.

Definite conclusions are given at the end of the paper.

* * * * *

THE recent increasing investigation of the mechanism of conduction and breakdown in dielectrics makes any additional data bearing on this subject worth while[1]. This paper discusses certain effects on electrical, physical and chemical properties of transformer oil when subjected to high local voltage stresses, dealing chiefly with certain temporary changes in conductivity and dielectric strength resulting from this treatment.

In the latter part of 1923, Professor H. B. Smith of Worcester Polytechnic Institute raised the question of the possibility of the temporary lowering of dielectric strength of oil due to high local voltage stresses, such as might occur at sharp corners in a transformer, although visible corona might not be present. Investigations were undertaken to determine this effect at potentials both below and above corona voltage.

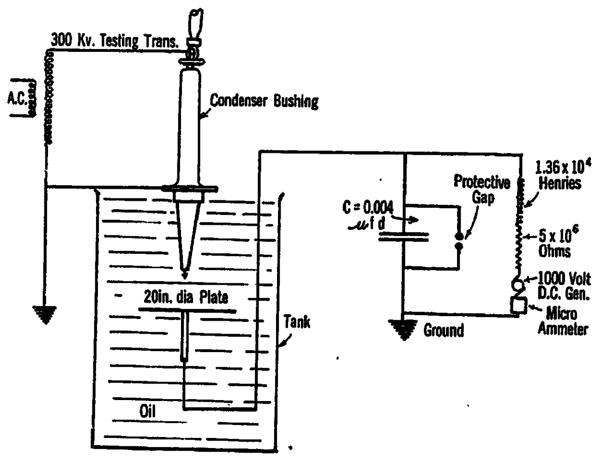

FIG. 1—APPARATUS FOR PRODUCING CORONA AND MEASURING SERIES CONDUCTIVITY

RESISTANCE MEASUREMENTS

A needle gap was used as a source of the high-voltage gradients. This gap consisted of a sharpened 0.05 in. brass rod spaced 12 in. from a ground plate. It gave visible corona at 55 kv., r. m. s.

It was decided to try the resistivity of the oil as an indicator of its quality. It seemed probable that any reduction in the dielectric strength of oil would be accompanied by a reduced resistivity. Various methods of measurement were tried in an effort to find one suitable. A microammeter with a sensitivity of 10^{-7} amperes per scale division was placed in the circuit in series with the high voltage line (see Fig. 1) and properly protected, the 60-cycle alternating current passing through a condenser to ground and the direct-current passing through a high inductance and the meter. A direct-current bias of 1000 volts was used, and the direct-current across the needle gap measured. The sensitivity of the meter was not enough for quantitative results, but deflections were found and increased with the voltage. No effect was noted at potentials lower than that producing visible corona.

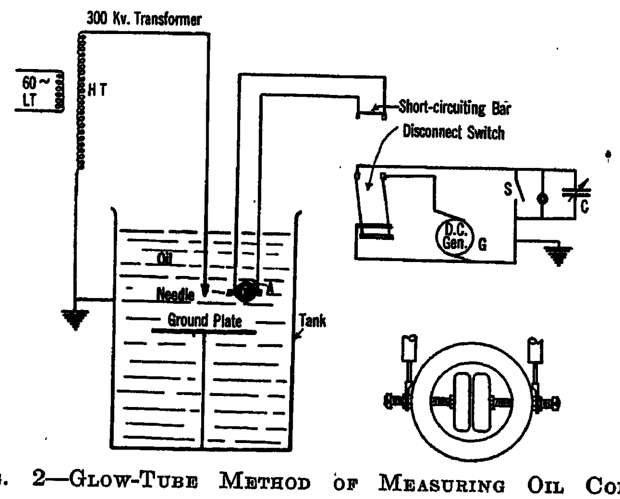

FIG. 2—GLOW-TUBE METHOD OF MEASURING OIL CONDUCTIVITY

Fig. 2A is a detailed sketch of the electrodes used in measuring conductivity

Attempts to measure conductivity between a pair of electrodes (Fig. 2) were more successful. These electrodes were brass disks with rounded corners. They were 1½ in. in diameter, and ⅜ in. thick, spaced 0.10 in. apart. The supporting ring was made of hard rubber, as were the tubes carrying the conductors to the electrodes. The assembled gap was suspended in the oil near the needle with the faces of the electrodes

* Both of Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

1. See Bibliography.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

in the same vertical plane with it, and at approximately the same level as its point.

The conductivity of the oil was found by measuring the current flowing when 1000 volts d-c. was applied across the gap. This current was too small to be measured by available galvanometers. A novel type of meter was suggested by Doctor J. Slepian. This proved to be satisfactory for the service required. The circuit for it is shown in Fig. 2. It consists of a condenser C in series with a 1000 volt generator G and the measuring gap. This condenser is shunted by a neon glow tube and a switch S.

When switch D is closed and the generator is running, a small current passes across the gap, through the shunting switch and to ground. To measure this current, the switch S is opened, and the time required for the condenser to charge up to the glow voltage of the tube is measured with a stop-watch. The tube glows momentarily. The elapsed time is recorded. This operation is repeated as often as desired.

The approximate conductivity is found as follows: The generator was excited to give 1165 volts at the terminals. The glow voltage of the tube is 325 volts. The exact equation for the resistance, assuming it constant is:

$$r = \frac{t}{c} \cdot \frac{1}{\log_e \left(\dfrac{E_g}{E_g - E_t}\right)}$$

where

E_g is the generator voltage

E_t is the glow voltage of the tube

r is the resistance of the oil across the gap

C is the capacity of condenser C in farads

t is elapsed time in seconds.

As a first approximation, when E_t is small compared to E_g, the current equation is,

$$\text{Average current} = \frac{\text{final condenser volts} \times \text{capacity}}{\text{time}}$$

$$i = E_t \frac{C}{t} \text{ and}$$

$$r = \frac{\text{average gap voltage}}{\text{average current}}$$

$$r = \frac{E_g - 1/2 E_t}{i} = \frac{E_g - 1/2 E_t}{E_t} \cdot \frac{t}{C}$$

but $E_g - 1/2 E_t = 1165 - 162.5 = 1002.5$ volts. Using 1000 volts the resistance is,

$$\frac{1000}{325} \cdot \frac{t}{C}$$

This differs from the exact equation results by 1.1 per cent. For a capacity of 0.000115 microfarads and a time of one second, the resistance would be approximately 2.67×10^{10} ohms. Converting this to resistivity with the assumption that Ohm's law holds, the resistivity per inch cube with the above conditions would be,

$$\frac{(1.5)^2}{4 \times 0.1} \pi \times 2.67 \times 10^{10} = 4.73 \times 10^{11} \text{ ohms.}$$

This is resistance measured in one second. Sixty seconds seemed to be about the limit of time for good results. This corresponds to a resistivity of 2.8×10^{13} ohms per inch cube. This method was simple and dependable, and gave the required degree of accuracy.

In operation, the disconnect switch (Fig. 2) was opened. The desired stressing voltage was impressed on the needle-gap for a period of time. Immediately after removing this voltage, switch D was closed, and measurements of resistivity taken as indicated above.

Dielectric Tests

The actual dielectric strength of oil subjected to high local stresses was tested with a Westinghouse standard test-cup (sharp-cornered disk electrodes, one inch in diameter, spaced 0.10 inches apart.)

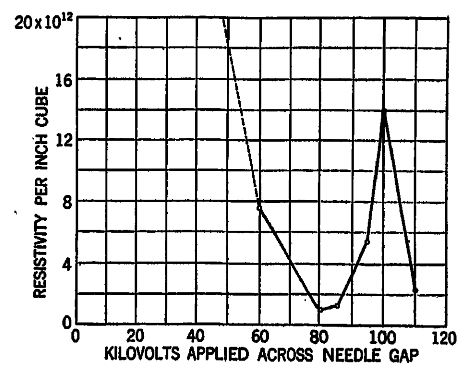

Fig. 3—Variation of Resistivity with Stressing Voltage

Resistivity measured with 1000 volts across 1/10 inch gap

A sample of oil was drawn from the vicinity of the needle immediately after the stressing voltage was removed, and tested for breakdown, an average of five or more breakdowns being used.

Experimental Results

A series of tests was made on Wemco A transformer oil. This oil had been used for miscellaneous testing purposes previous to these tests, but still had a dielectric strength of about 33-kv -r. m. s., which is an indication of good oil.

Fig. 3 shows the variation of resistivity with the stressing voltage, the voltage being applied for 15 min. Note the dip at 80 kv. and the lack of effects below 55 kv., the corona potential.

Fig. 4 indicates the way in which the resistance varies with time after the stressing voltage is removed. The shapes of these curves vary considerably with individual tests. However, they all show: 1. That the low resistance is only temporary: 2. that a resistance meas-

ured about a minute after stressing voltage is removed, is about as accurate as a resistance measured during stressing, since in general, the time-resistance curve approaches the zero-time axis nearly at right angles to it.

Fig. 5 shows the extent of the affected region around the needle. The curve shows that the effect of corona is very decided to a distance of one foot under the conditions of test. Since the effects observed last over

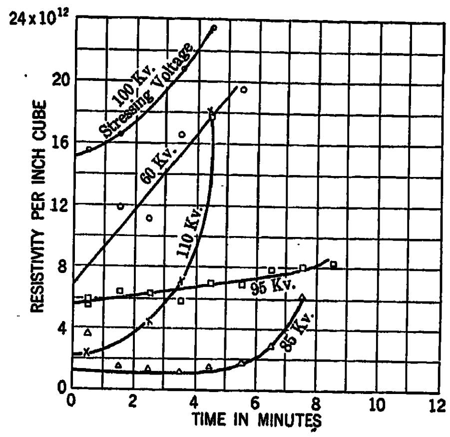

FIG. 4—VARIATION OF RESISTIVITY WITH TIME AFTER STRESSING VOLTAGE IS DISCONNECTED

a period of minutes, it would be very unlikely for them to be highly localized, as there would be a spreading due to diffusion.

Fig. 6 indicates the effect of time of application of the high gradient on the dielectric strength of the oil, showing that most of the lowering of the dielectric strength occurs very quickly.

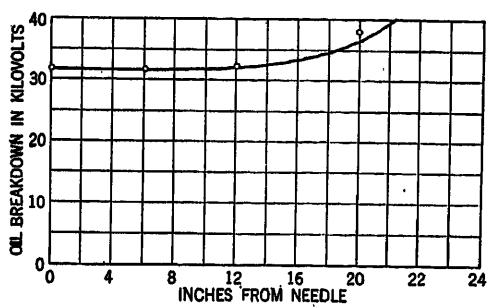

FIG. 5—EXTENT OF THE AFFECTED REGION IN THE TANK

The variation of dielectric strength with stressing voltage is shown in Fig. 7. The effects are not noticeable at voltages which do not produce visible corona, but become pronounced at higher voltages. The hump in the curve at 100 kv. is discussed later.

The following table brings out very emphatically the temporary nature of the corona effects. For each of the three tests, the strength of a sample was measured in the test cup. Then a stressing voltage was applied to the needle gap for the indicated time. Two samples were then taken from the region near the needle. One was given a dielectric test immediately. The other was tested after 15 min. In each test, the lowering of the dielectric strength disappears entirely, and an actual improvement occurs.

Initial breakdown	Stressing kv. r. m. s.	Time applied	Resulting breakdown	Breakdown after 15 min.
32.8	120	15 min.	29.4	39.6
29.6	100	15 min.	26.4	30.8
29.6	100	1 hr.	25.0	30.2

Attempts to find permanent effects on the oil due to the stressing voltage were unsuccessful. The conductivity of samples taken before and after stressing, was measured accurately several days afterwards.

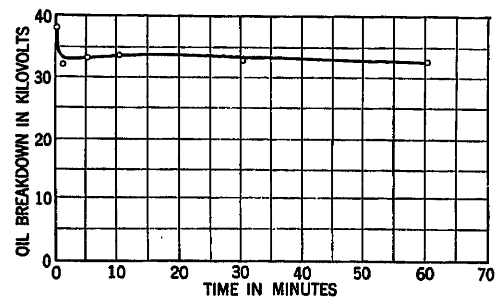

FIG. 6—THE EFFECT OF DURATION OF STRESS ON THE DIELECTRIC STRENGTH OF TRANSFORMER OIL

100 kv. applied on needle

The percentage of unsaturates was also determined. The only variations in these quantities were such as could be accounted for by ordinary experimental error. The water content seemed to gradually decrease as the tests progressed, but varied with the number of tests rather than as any function of the stressing voltage.

ACCURACY OF RESULTS

It is not the desire of the writers to give an impression of great accuracy in the quantitative results obtained. As in most tests with dielectrics, the variations were wide. To a certain extent, tests used in producing the

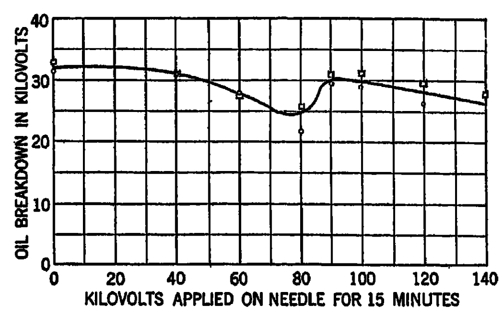

FIG. 7—THE EFFECT OF HIGH LOCAL VOLTAGE STRESSES ON THE DIELECTRIC STRENGTH OF TRANSFORMER OIL

curves are selected tests. However, the tendencies and shapes of the curves are definite and are found in practically all of the tests made. It is this rather than the quantitative results which should be emphasized.

DISCUSSION

Certain parallels may be drawn between the effects of corona in oil and the phenomena of ionization in air.

The conductivity in both cases must result from the presence of charges capable of moving. The normal conductivity in air is due to a very few ionized particles which are always present. The normal conductivity of oil might be due either to the transfer of electrons from one metal electrode to the other across conducting particles, or to the presence of positively and negatively charged ions in the oil or of the oil, migrating due to the electric field. As the voltage stress becomes very great, molecules may be torn apart into plus and minus ions. Due to the relatively high viscosity, and the high molecular weights, the diffusion rate, and consequently the rate of recombination of the charged particles and their rate of migration in an electric field, is very slow. This would account for the persistance over a period of minutes of the effects noted.

As to the effects on the dielectric strength: In air, the dielectric strength is hardly affected unless the ionization is intense enough to produce space charge and disturb the voltage gradient. In oil, the ionization necessary to produce a space charge will be much less since the mobility of the ions is so low. No attempt has been made up to the present to determine mathematically the possibility of space charge with the degree of ionization found in the experimental results. It should be possible to do this, and it will be attempted.

The "hump" mentioned in the curves of kv. vs. resistivity and kv. vs. dielectric strength (Fig 3 and Fig 7) was unexpected. When the first curve of the dielectric strength was determined, a smooth curve was drawn with a downward curvature, assuming the 100-kv. value as high and the 80-kv. value as low. However, when the run was repeated several days later, a curve identical to the previous one was obtained. The same hump was later found to exist in the resistance values.

One plausible explanation of this hump, and one which is presented because no better one appears, follows.

Tests made with oil of low water content did not show the decided dip at 80-kv. It is possible that the size of water particles are such that they migrate rather rapidly at the 80-kv. value, so that they exist in the concentrated electric field in rather large numbers, thus affecting the breakdown. At the 100-kv. point, the migration may be so rapid near the needle that the water content is reduced. Unfortunately, the water content was not determined for the tests referred to, and the oil was later destroyed.

Conclusions

The above tests allow certain definite conclusions to be drawn:

1. That, with the type of needle gap used, no appreciable effect on the electrical characteristics of the oil occurs below the point where visible corona starts.

2. That above this potential, a phenomenon similar in some respects to ionization in air, occurs with oil.

3. That with the sample of oil used, the resistivity and breakdown do not continually decrease as the stressing voltage is increased.

4. That the effects noted are produced in a very few minutes and are temporary in nature.

5. That, since the effects described in this paper all occur only at voltage stresses which produce visible corona, they do not exist in properly designed commercial apparatus.

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Discussion

J. B. Whitehead: It has been known for a long time that liquid dielectrics of even the best type are subject to a moderate electrolytic dissociation, which means that the ultimate parts of the dielectric are at all times, some of them, in a state of separation as regards their component charges—not many, but some of them.

The laws of conduction in liquid dielectrics (and all liquids have some conduction) are in every way consistent with the theory of dissociation that has been built up for gases. It is possible to ionize liquid dielectrics. It is possible to subject them to a regular ionizing agency, and it is possible to measure the rate of recombination of the ions so formed. We have plenty of evidence to prove that liquid dielectrics may be ionized.

So I think it was a great pity that Messrs. Crago and Hodnette should have gone to such great trouble to show that under certain circumstances oil may have an appreciable conductivity. The fact has long been known.

Also, I would like to ask them whether they considered the use of the corona-tube as a method for investigating conductivity. It appears to me that if they had used a small tube and a small wire, the tube being filled with many holes, and immediately outside of it an insulated electrode, that they would have had not nearly the trouble in finding a method for measuring the current caused by the conductivity of oil.

M. F. Skinker: While Mr. Crago and Mr. Hodnette were interested primarily in securing an answer to an engineering problem, it is to be regretted that they did not add a few refinements which would have insured freedom from errror.

Let us see what is required for an accurate scientific investigation and try to outline what should have been done in these experiments.

It is necessary that all conceivable variable factors be kept constant except those which you are attempting to correlate by the investigation. It sometimes happens that a factor cannot be kept constant and a correction must be made in the final result. This, however, should only be done as a last resort.

In the experiments on corona in oil, we have to consider the electrical measuring devices, the oil, the electrodes, and the oil in contact with the electrodes. Fortunately, nearly every one takes care to see that the electrical measurements have a high degree of accuracy.

With regard to the oil, the authors only imply that it was clean and dry by saying the dielectric strength indicated good oil. Why did they not attempt to purify the oil first? I might suggest that if some fine pieces of metallic sodium, or a little aluminum chloride were mixed in the oil and then filtered, this would have removed *all* the water and almost all other impurities.

By an appropriate cover for the testing vessel impurities could have been kept out during the experiment.

Care must be used with parallel plate electrodes which are close together for the electric stress may well be high enough to alter the spacing by several per cent.

It should also be well known that catalytic action of the brass or common metals in the oil can materially affect the results of any experiment. The simplest and most effective way of preventing this would have been to gold-plate all metallic surfaces in contact with the oil, as done by Lee and Kurrelmeyer in their experiments on gases.

With these simple precautions the authors could have collected data that was really worthy of definite conclusions.

The test of accurate data is in the following question: "Can this experiment be reproduced in another laboratory and exactly the same results obtained?"

The test of properly recorded work is in the question: "Have all the conditions under which this experiment was performed been recorded, so that others may draw their own conclusions from the data?"

If these two questions cannot be answered in the affirmative, the experiment is hardly worth publishing.

I make this plea for more scientific work because I believe it is only in this way that we may eventually solve the insulation problems.

J. A. Duncan: There are two significant statements in the paper by Messrs. Crago and Hodnette, namely: "This oil had been used for miscellaneous testing purposes previous to these tests, but still had a dielectric strength of about 33-kv. r. m. s., which is an indication of good oil," and, "Unfortunately, the water content was not determined for the tests referred to, and the oil was later destroyed."

F. W. Peek showed in 1915 that a sample of dry transil oil had a dielectric strength of 62.3 kv., and that the addition of moisture to the extent of one part in twenty thousand caused a 50 per cent decrease in dielectric strength, while twenty times this amount of moisture only reduced the dielectric strength sixty per cent. This certainly shows that it is the first trace of moisture which causes the greatest decrease in strength and indicates the importance of the removal of this trace from oil which is to be used for any scientific purpose. Of course, I do not know how "Wemco" oil compares with ordinary transil oil, but whatever the relation may be, it is safe to say that comparison would probably be more favorable to a recently and carefully dried sample of Wemco than to one with a long and unknown past.

I should like to suggest with Drager in the very paper cited by the authors that it may be the presence of moisture which provides the conducting particles mentioned by the authors in the first paragraph on the fourth page of the paper. On the other hand, attention was called to the fact that "as the voltage stress becomes very great, molecules may be torn apart into plus and minus ions." Evidently they mean to consider the possibility of ions being formed by the actual disruption of the hydrocarbon chains into parts, each containing one or more carbon atoms, this in addition to those formed by the simple removal of an electron from the molecule. Long carbon chains such as we have to deal with here are so easily broken up by the action of heat that it does seem reasonable to suppose that they might also be broken up by the action of the electric field. If such ionization does take place, and Dr. Whitehead has just told us that some ionization is known to occur, it could be identified by its effects on certain properties of the oil such as the temperatures at which various components of the oil would distill off from the remainder. When these long molecular chains do break it is not generally at the center. We have the possibility of ions containing anywhere from one carbon atom to one less than the normal number in the original oil molecule. The products of recombination will then vary anywhere from neutral molecules of two carbon atoms, which would be ethane gas, to one of two less than twice the normal number of carbon atoms. There, abnormally long molecules might again ionize and some of the ions recombine to form still longer and therefore heavier molecules. Such breaking up of the molecules and recombination into molecules both lighter and heavier than normal is known to take place in oils when heated to sufficiently high temperatures. In fact, this action is the very basis of one of our well-known commercial processes for the manufacture of gasoline from crude oils.

I am sorry that the experiment on corona in oils was not continued far enough and long enough to give these recombinations a chance to occur in detectable amounts. The products are easily detected when present. The abnormally heavy ones will settle to the bottom of the oil as a sludge. The abnormally light components will distill off even at near room temperatures and are detectable by various means. There is one method of detecting the formation of the gases which although known long before my time was discovered independently by me in a very striking manner. It was by the simple process of bringing a lighted gas jet a little too near the mouth of a flask half filled with transil oil which had just been heated to slightly above the cracking point. In this case I am forced to join in the conclusion that unfortunately that sample of oil is no longer available for further tests.

One of the conclusions is "That the effects noted are produced in a very few minutes and are temporary in nature," and a curve, Fig. 7, is drawn which shows that the dielectric strength passes through a cycle. In this connection it seems well to emphasize the fact, and this applies not only to the experiment under discussion but to all others as well, that because one property of a substance passes through a cycle all other properties pass through a cycle and return to their original value. One could name an endless number of cases in which this is not so, and Dr. Hans Staeger mentions one which is particularly applicable here.[1] Under the influence of metals as catalyst, the acidity of an oil, for example, may pass through repeated cycles, while the sludge formation and contamination of the oil proceeds cumulatively with the total time.

In that connection, I only want to refer to the conclusions which we have considered most important, and see whether they have not been drawn from the data presented. Probably the most important is a negative conclusion: "That since the effects described in this paper all occur only at voltage stresses which produce visible corona, they do not exist in properly designed commercial apparatus."

This was the main object of the tests and was, I believe, proved rather definitely.

The other conclusions you have already heard and they are in the paper. We feel that they are not extremely definite conclusions, but have been definitely based on the work, the results of which have been given.

I might say in connection with this catalytic action of brass electrodes that Mr. J. E. Shrader of the Westinghouse Company several years ago made a large number of tests on conductivity of oil using gold, copper, silver and brass electrodes, with results so nearly identical that differences were within the limits of experimental error.

I wish to say in conclusion that I do feel that the paper presents data which should be useful as a guide for future work, that we get these effects at rather large distances from the electrodes, and that they do not occur below corona voltages.

A. C. Crago: I realize that those discussing our paper, "Corona in Oil," have been at a disadvantage because the paper was printed within the last week. I believe that the scope of the tests has been somewhat misunderstood. For that reason I wish to give in more detail the reasons the tests were made and the

1. *Schweitz Elektrotechniser Verein Bulletin*, March 1924.

conditions under which they were conducted, reading two paragraphs from the paper.

The tests as they were originally planned were intended to discover whether there were deteriorating effects in oil below a voltage which would give visible corona with certain type of electrodes in the oil. We, therefore, first made an attempt to find these effects. This was naturally an engineering problem, which we expected to solve in a short time. Since we did not find any effects below corona voltage, we continued the work further with higher voltages. The conditions were not greatly changed in extending the work.

Quoting the paper: "In the latter part of 1923, Professor H. B. Smith of Worcester Polytechnic Institute raised the question of the possibility of the temporary lowering of dielectric strength of oil due to high local voltage stresses, such as might occur at sharp corners in a transformer, although visible corona might not be present. Investigations were undertaken to determine this effect at potentials both below and above corona voltage." And:

"It is not the desire of the writers to give an impression of great accuracy in the quantitative results obtained. As in most cases with dielectrics, the variations were wide. To a certain extent, tests used in producing the curves are selected tests. However, the tendencies and shapes of the curves are definite and are found in practically all of the tests made. It is this rather than the quantitative results which should be emphasized."

Let us take the discussions in the order in which they were presented. Professor Whitehead has expressed regret that we have gone to the trouble to show that there is electrical conductivity in oil, and points out that this conductivity is influenced by radioactive or other radiating sources. If the sole purpose of the paper had been to demonstrate the existence of these two well-known phenomena, we agree that the paper would be without value. I believe that Professor Whitehead has misunderstood the purpose of the paper, which was not to show that there is a conductivity, but more to show the effects of a particular source of ionization, which may be found in practise. The use of the "corona tube" was not considered.

Mr. Skinker asks us two questions, which I will quote: "Can this experiment be reproduced in another laboratory and *exactly* the same results produced?" Certainly and emphatically no. The word "exactly" cannot be used properly with respect to experimental results.

"Have *all* the conditions under which this experiment was performed been recorded, so that others may draw their own conclusions from the data?" *All* conditions have, of course, not been recorded. This is a physical impossibility. Whether conditions *necessary for the conclusions drawn* have been given must be judged by those who read the paper; not by the authors.

I might say in connection with the catalytic action of brass electrodes, mentioned by Mr. Skinker, that Mr. J. E. Shrader of the Westinghouse Research Laboratory points out in an unpublished paper that the conductivity of oil using gold, silver, brass, copper and nickel electrodes show no greater differences than the experimental errors.

Mr. Duncan states that "dry transil oil had a dielectric strength of 62.3 kv." omitting to state; 1. whether the value is r. m. s. or peak; 2. the shape and spacing of electrodes. This figure thus cannot be compared with that of 33 kv. r. m. s. given for Wemco "*A*" tested in the present Westinghouse standard-test cup. Most of Mr. Duncan's discussion, although raising interesting possibilities and explanations of certain phenomena, is outside the scope of the paper and does not affect the conclusions drawn.

NOTE See also discussion of this subject by Joseph Slepian, page 202.

Mechanical Stresses in Busbar Supports During Short Circuits

BY O. R. SCHURIG*
Member, A. I. E. E.
and
M. F. SAYRE†
Non-member

Synopsis.—*With the increased magnitudes of short-circuit current obtained in modern busbar circuits, it becomes a matter of great importance to determine the mechanical stresses imposed on busbar supports during short circuits. The stresses are due to the electromagnetic force produced by the current flowing in adjacent conductors in the bus structure. When usefully applied, the electromagnetic force propels electric motors, or sends the pointer of a voltmeter along the scale; but the same electromagnetic force when destructively applied may bring about rupture of bus supports, distortion of conductors or even shut-down of stations. In the days when currents were low, or when large spacings between conductors were maintained, a support sufficiently strong to carry the weight of the conductors often was sufficient to withstand the stresses, due to the electromagnetic forces, but with large short-circuit currents and with reduced spacing between conductors, a more accurate knowledge of the stresses due to the electromagnetic forces is absolutely essential to the designer.*

The electromagnetic force has been the subject of investigations in the past not only for the case of busbars but also for transformers, current-limiting reactors, and disconnecting switches. In the case of busbars, as well as in some of the other cases mentioned, the stresses resulting from the action of the forces applied are very materially affected by the mechanical vibrations produced. The vibrations of the busbar for example have been carefully analyzed by Biermanns‡ who showed that a busbar rigidly supported could be considered as a single-element vibratory system similar to a system comprising a mass suspended from an elastic spring. In the customary busbar structures, however, the supports also are appreciably resilient, so that the busbar-insulator structure must be regarded as a system having two vibratory elements joined together, i. e., a composite structure resembling, for instance, the system comprising a turbine rotor keyed to a flexible shaft, the shaft bearings being attached to an elastic base.

Hence, the determination of short-circuit stresses in busbar supports involves a study of the transient vibrations occurring in the busbar structure which is essentially a damped mechanical system composed of two vibratory elements and having two interrelated natural frequencies, the system being impelled during short circuits by a decreasing electromagnetic force having a unidirectional component and two harmonic components, one at current frequency and another at twice the current frequency. The electromagnetic force upon the busbar itself is given by a well-known formula involving only the current magnitude, the conductor spacing, and the length of busbar span. This force, however, acts upon the busbar rather than on the insulator, and the insulator is stressed only indirectly, after a time lag, as a result of the bending of the busbar. Therefore, the likelihood of failure of the support hinges not only on the initial electromagnetic force but also on the rate of decrement of that force and upon the vibratory characteristics of the busbar-insulator system.

This paper gives an analytical method of maximum stress calculation based on all of the factors mentioned, for the basic case of a bus structure having long, straight, uniform, parallel busbars with equidistant, rigidly mounted supports of uniform characteristics. The method of calculation is a general one applicable to all cases of elastic busbars intermittently supported, irrespective of whether the conductors are placed face-on or edge-on with respect to the supports and irrespective of whether the supports themselves are stressed in bending tension or compression.

The actual bus structure is, of course, often quite complex involving, for instance, the effects of tap connections, bends unequal spans, non-uniform current division. Special consideration may have to be given to these factors in many cases. Moreover, in structures with long spans where the forces so act as to cause relatively large busbar deflections, the tension set up along the busbar may cause large secondary bending stresses in the supports, which stresses act in a direction at right angles to the stresses covered by the formulas developed in the paper. For this type of structure, the secondary stresses have often proved to be the controlling factors in design.

The method of calculation presented here differs from those employed in the past, since the calculations used heretofore have only partially taken into account, or totally neglected, the oscillatory characteristics of busbars and of supports.

Since the natural frequencies of busbar structures range from, say, 10 to 300 cycles per second, mechanical resonance is within common possibility in buses of 25-cycle circuits as well as in buses of 60-cycle circuits. While not all cases of resonance give rise to stresses greatly in excess of those at non-resonance, resonance in some cases multiplies the stresses five or even ten times. It will often be possible by proper methods of design so to change the natural frequencies as to avoid the cases of resonance giving extra heavy stresses.

The analytical equations developed for stress calculation are inherently complex, but their results may be expressed with sufficient accuracy in the form of curves, each group of curves covering a wide range of cases. A representative curve is given in this paper in Fig. 1. The development and use of any practical set of working curves depends in a large measure upon a detailed knowledge of the mechanical characteristics of the structural elements involved.

The following paper contains a discussion of the factors affecting support stresses and an outline of the practical procedure for maximum support stress calculations, including illustrative examples. The appendix contains the development of the equations used in the calculations.

The equations have been confirmed by tests on miniature and on full scale structures. Typical test results are presented.

INTRODUCTION

THE design of busbar layouts calls for the determination of the stresses occurring in the bus supports during short circuits, because the stresses in question may become sufficiently large to weaken, or even rupture, the supports. Moreover, an understanding of the factors affecting the stresses will often permit the designer to modify the layout so as to reduce the stresses, and thus save material or prevent failure. Hence investigations have been directed to-

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‡Bibliography No. 6.

wards the calculation of support stresses due to electromagnetic forces caused by short-circuit currents in busbar structures.

The investigations involved are:

Tests to determine the mechanical characteristice of busbar structures; measurements of the support stresses due to short-circuit currents in assembled bus structures; an analysis of the electromagnetic forces during short circuits; an analysis of the vibrations of bus structures; establishing a practical formula for approximate calculations of maximum stresses.

The progress made in the investigation is, in a large measure, due to D. Basch, for his helpful counsel and practical suggestions; C. W. Frick, for effectively conducting tests and making calculations; and L. F. Woodruff for his valuable cooperation in the experimental and theoretical studies in the early part of the work.

Definitions

The electromagnetic force, as considered within the scope of this paper, is the mechanical push or pull which is caused by the short-circuit current and its magnetic field, and which is exerted on the busbars; *i. e.*, it is the force tending to displace the busbars from their normal position.

The support stress is the mechanical reaction of the support, the reaction being due to the elastic restoring forces set up in the support when deflected; *i. e.*, the tendency of the support to push back towards its position of zero deflection in so far as the tendency is caused entirely by the resilience stored in the support. Hence the stress in a support is a direct measure of its approach to failure. For an elastic support, deflected within its elastic limit, the stress is proportional to the deflection regardless of whether the support is at rest or in motion.

Vibrations of Bus Structure Must Be Considered

While it has been fairly well known how to predict the *electromagnetic forces* acting on busbars during short circuits, the problem of determining the *stresses imposed on the bus supports*, as a result of the electromagnetic forces, has not been fully understood, as indicated by the fact that calculations have frequently been made on the assumption that the maximum stress imposed on a bus support is equal to the maximum electromagnetic force calculated for one span of the busbar. Actually, the support stress is ordinarily not of the same magnitude as the electromagnetic force per busbar span, on account of the vibratory characteristics of the busbars and of the supports. The peak stress on the supports may be far above or well below the maximum value of the electromagnetic force per span, depending on the natural frequencies of the busbar structure. Hence, stress calculations employing simply the peak value of electromagnetic force, or any other value of electromagnetic force, without proper allowance for the motional characteristics of the bus structure can ordinarily be expected to give but a rough approximation to the correct values. Since the methods of stress calculation employed in the past have only partially taken into account, or totally ignored, the oscillatory character of busbars and of supports, a method of calculation which includes the effects of the mechanical oscillations in the structure has been developed.

Object

It is the object of this paper to present

1. A brief review of the factors affecting support stresses.
2. An outline of the basis for stress calculations.
3. A practical procedure for approximate support stress calculations.

Review of Principal Factors Affecting Support Stresses

In order to arrive at a basis for stress calculations, the factors affecting the stresses must be considered. The principal ones are: The electromagnetic force, the current decrement, the mechanical characteristics of busbar structures, their motional resistance, and mechanical resonance.

The Short-Circuit Current and the Electromagnetic Force

The electromagnetic force acting on a pair of parallel, straight, current-carrying conductors (in the absence of magnetic bodies), is proportional to the product of the currents. If, for example, the currents are symmetrical sine currents of constant effective values, the force will consist of a harmonic (alternating), component at double the current frequency superposed on a constant, (or direct) force component, the amplitude of the harmonic component being equal to the magnitude of the direct component.

The maximum forces will result from short-circuit currents whose waves are displaced from the zero value of current. For example, in the case of a *two-wire short circuit* giving an initial sine-wave current fully displaced, *i. e.*, having the largest value of direct component of current, the electromagnetic force is composed of three parts: A harmonic component at current frequency, a harmonic component at twice the current frequency, and a direct component (of constant value if the decrement is neglected). The relative amplitudes of the two harmonic force components are, as is well known,* 1.33 for the first harmonic component and 0.33 for the second harmonic component, if the direct component is equal to 1.0. The proportions given are not maintained throughout the short-circuit transient, because the current decrement does not affect all the force components equally, as is indicated by the fact that the first-harmonic force component vanishes completely, while the second-harmonic amplitude and

*Bibliography No. 19.

the direct component are reduced to finite sustained values of equal magnitudes. Since the maximum stresses are reached during, say, the first fifth of a second of the short circuit, the first harmonic component of electromagnetic force will always be a vital factor in stress calculations for displaced short-circuit currents.

The foregoing considerations indicate, at once, the importance of the direct component of the short-circuit current on the stresses. If we compare the electromagnetic forces and their components at the beginning of the short circuit (two-wire short circuits) in the two cases, *a*, for an undisplaced (symmetrical) sine-wave current, and *b*, for a fully displaced sine-wave current having the same peak value of alternating component as that in case *a*, we find, in accordance with Table I, neglecting the current decrements, that

1. The fully displaced current has an average value (or direct component) of electromagnetic force three times as great as that of the symmetrical current.
2. The peak value of the electromagnetic force for the totally displaced current is four times as large as that of the symmetrical current.
3. The symmetrical current has no first-harmonic component of electromagnetic force, while the totally displaced current has a first-harmonic component of electromagnetic force four times as large as the second-harmonic force component of either case.

In the case of *three-phase short-circuits,* the currents are not in phase, nor are the amounts of displacement the same for the three currents. Moreover, each conductor is acted upon at any instant by two electromagnetic forces, one from each of the other two conductors. Examination of typical three-phase short-circuit currents has indicated that the total electromagnetic force acting on any of the three conductors has direct, first-harmonic, and second-harmonic components similar in character to, but differing in relative magnitudes from, those of the two-wire short circuit. As a rule, however, the electromagnetic forces for two-wire short circuits in a given bus system are greater than those for three-phase short circuits, so that the stresses calculated for two-wire short circuits are generally the ones on which to base the design. In some cases of second-harmonic resonance (or near-resonance) consideration should be given to the three-phase short-circuit stresses.

TABLE I

Comparison of Electromagnetic-Force Components for Fully Displaced and for Symmetrical Short-Circuit Currents, for Two-Wire Short Circuit

The a-c. component of the fully displaced short-circuit current has the same amplitude as the symmetrical current. Wave-shape sinusoidal in both cases. Current decrement neglected.

Current	Components of electromagnetic force			Maximum Electromagnetic Force
	Direct component*	1st Harmonic	2nd Harmonic	
Symmetrical sine current..................	0.333	0	0.333	0.667
Totally displaced sine current.............	1.0	1.333	0.333	2.667

*Average value of total electromagnetic force.

THE CURRENT DECREMENT

The current decrement, as already mentioned, causes a reduction, with time, of each of the electromagnetic force components, each component having its own rate of decrement. Obviously, the decrements must be considered in stress calculations, particularly in view of the fact that the peak stress frequently does not occur until several cycles after the beginning of the short circuit.

THE MECHANICAL CHARACTERISTICS OF BUSBAR STRUCTURES

The relative magnitude of the electromagnetic force components discussed would be of little practical significance in the determination of maximum support stresses were it not for the resilient and oscillatory characteristics of the mechanical elements of bus structures. It is well known for instance, that, a hard-drawn copper busbar, when deflected through the customary range of amplitudes occurring in practise, has substantially elastic properties, and is capable of oscillatory motion when suddenly impelled by an external force, such as a short-circuit electromagnetic force. Thus a copper bus one-quarter inch thick by four inches wide, rigidly clamped to supports forty-eight inches apart, has a natural frequency of 16 cycles per second, when suddenly impelled in the direction at right angles to its faces. The plain 16-cycle motion will occur, for example, when a direct-current short circuit is suddenly applied to a pair of busses of the kind described, if the two bars are placed face to face. Likewise, when the current is suddenly switched off, the bars will tend to spring back to their normal positions, and similar vibrations at 16 cycles per second will be set up. The mechanical damping, which, in the presence of the magnetic field set up by the current, includes the damping due to eddy currents, will gradually reduce the amplitude of the free vibrations.

The important points to be emphasized by this elementary example are; 1, that the reaction of the supports against the bar—a force which for the case in question is proportional to the deflection of the bar, if elastic and deflected through small amplitudes—will be a pulsating reaction (or stress) varying harmonically with gradually diminishing amplitudes, although the suddenly applied electromagnetic force is of constant value; 2, that the maximum support stress will be reached roughly 0.03 sec. after the application of the electromagnetic force; 3, that the value of the maximum support stress for all supports but the terminal ones will materially exceed (by nearly 100 per cent) the electromagnetic force per busbar span; and 4, that stress pulsations will continue for a considerable time after the electromagnetic force is removed. The fourth point has no direct bearing on the maximum

support stress during short circuits, but is brought out to indicate the most decided difference between the support stress and the electromagnetic force as observed in this simplest of illustrations.

If instead of the direct-current short circuit, an alternating-current 60-cycle short-circuit is suddenly applied to the busbars under discussion, there will be not only the 16-cycle free vibrations at the beginning but also forced vibrations at 60-cycles if the initial current is displaced, and 120-cycle vibrations of much smaller (in this case negligible) amplitude, all oscillations being superposed upon a direct component of deflection, the latter corresponding to the direct component of electromagnetic force. To put it in other words, the support stress has a natural-frequency component not present in the electromagnetic force and two forced-frequency components of relative magnitudes which differ from the corresponding components of electromagnetic force because the busbar responds rather selectively to impressed forces of various frequencies, being most partial to vibrations near its natural frequency.

In the preceding simple illustration, the resilient and vibratory characteristics of the support have been totally left out of consideration by assuming the busbar rigidly clamped at its points of support. Actually, however, most bus supports are appreciably flexible in the sense that a force applied at the point of support of the busbar, as under short-circuit conditions, will materially deflect the support. Moreover, most supports possess sufficient resilience, if deflected within the range of safe deflections, to have oscillatory characteristics similar to those of busbar spans, as has been demonstrated by numerous tests. For example, a typical porcelain busbar support, tested without a busbar, had a natural frequency of 45 cycles per second, the frequency being measureably modified by the mass of the busbar when clamped to the support.

When busbars are mounted on flexible supports, in the customary manner, the motion of the busbar span, as well as the motion of the support, shows *two dominant natural frequencies* which occur in combination in each of the two structural members when electromagnetic forces are suddenly applied. Accordingly, the support stress at the beginning of a short circuit contains two natural-frequency components in addition to the forced-frequency components. Furthermore, the forced-frequency support-stress components are related to the corresponding electromagnetic-force components by a modulating factor involving *both* of the natural frequencies.

To summarize:—the support stress during short circuits is, as a rule, materially affected by two natural frequencies due to the flexible and resilient characteristics of the busbar spans and of the supports. In view of the two natural frequencies mentioned, bus supports respond selectively to impressed electromagnetic forces at different frequencies; hence, the magnitude of the electromagnetic force and its mode of variation with time are not usually representative of the magnitude and mode of variation of the support stresses. The peak stresses depend most decidedly upon the natural frequencies of the bus structure.

Damping Due to Motional Resistance of Elements of Busbar Structure

In view of the importance of the natural-frequency components of support stress, the effect of damping on their amplitudes must be considered. The damping is easily determined by test and is readily incorporated in the stress calculations. The tendency of some types of bus supports not to return completely to their original positions, after a given stress has been applied and removed, has been shown to affect the amount of damping during vibrations. As a result of the various types of mechanical construction employed in bus supports, the magnitude of the damping and its effect on the peak stress differ for different types of supports, as demonstrated by test data. While the amount of damping is sufficiently appreciable to require consideration in ordinary stress calculations, there is not enough present in the customary structures to prevent, in some cases of resonance, stresses several times as great as those occurring at non-resonance, under otherwise similar conditions.

In connection with the foregoing, attention is called to the case of bus supports which do not clamp the busbar, but merely guide it loosely, so that a sudden thrust upon the busbar may cause it to be slapped with considerable impact against one side of the guides. The slapping effect is not generally desirable because of possible excess stresses, and when present must be allowed for in calculations.

The conclusion is therefore reached that, in a great many cases, both the busbar span and the support may be regarded as substantially elastic bodies for stress calculations, but the mechanical damping during oscillatory motion must be properly taken into account. The natural frequencies, as well as the damping effect, are readily determined by tests and have been obtained for a variety of structural members, not only when tested separately but also when fully assembled as in practise.

Resonance

Resonance occurs when one of the electromagnetic-force components has a frequency equal to one of the natural frequencies of the bus structure. Since the natural frequencies of busbar structures range from, say, 10 to 300 cycles per second, resonance is within common possibility in buses of 25-cycle circuits as well as in busses of 60-cycle circuits. While not all cases of resonance give rise to stresses greatly in excess of those at nonresonance, the stresses may, in some cases, be multiplied five or even ten times, because the effect of resonance on the maximum stress depends not only on the damping due to motional resistance of the struct-

ural elements, but also, and to an even greater extent,—on the magnitude of the non-resonant natural frequency. It is best, of course, to avoid the cases of resonance causing extra-heavy stresses by suitable design of bus structures.

Basis for Stress Determinations

An exact calculation of stresses taking into account the particular values of all the variables involved in any one problem would be extremely laborious, even if at all feasible. On the other hand, a method of approximately estimating maximum stresses on the basis of selected conditions and average representative constants has been established and is presented in this paper. The results so obtained are sufficiently accurate to be applicable to a large number of cases and have been found to be in reasonably good agreement with stresses observed in tests. Obviously, it would be a lengthy and costly procedure to make tests for the purpose of observing the stresses under the large variety of conditions met in practice.

The technical analysis of test data and the application of theoretical considerations gave the following results:

1. An analytical procedure for determining the support stress and its components in terms of the electrical and mechanical factors involved.

2. A simple routine formula, which, with the aid of practical stress curves, gives the maximum support-stress. The curves are calculated by the method indicated in No. 1, and with the aid of experimentally observed constants.

The analytical procedure for stress determinations involves an analysis of the mechanical forces reacting against the impelling electromagnetic forces during the vibrations of busbar and of support. In the stress calculations presented here elastic busbars and elastic supports were assumed and the two natural frequencies of the busbar structure were taken into account as well as the motional damping and current decrement. The specific circuit conditions selected for the stress calculations are a sine-wave short-circuit current,* the maximum amount of displacement of current from the symmetrical position at the beginning of the short circuit, and separate rates of current decrement for each of the short-circuit current components. The theory upon which the stress calculations are based is outlined in the Appendix.

Practical Formula for Maximum Stress Calculations

The stress formula and the stress curve presented here are based on the following selected conditions:

Two-wire short circuits with the maximum amount of initial current displacement.

Average initial rates of current decrement applicable without material error to maximum stress determinations for short circuits in systems for which the short-circuit reactance does not exceed, say, 30 per cent.

An average value of mechanical damping applicable to a variety of bus structures having copper bars and porcelain insulators.

The application of the formula and of practical stress curves calls for the following data:

1. Initial value of r. m. s. total short-circuit current I_0, as obtained from accepted short-circuit current decrement curves,† I_0 ampere.

2. Length of busbar span between centers of supports, l in.

3. Dimension of individual busbar lamination (measured in the direction parallel to plane of deflection) a in.

4. Dimension of busbar perpendicular to a, b in.

5. Center spacing between bars (or groups of bars) mounted on separate insulators, s in.

6. The two natural frequencies f_1 and f_2 of the busbar structure; they are most readily obtained by a simple test.

7. The frequency f of the current.

The procedure is to calculate F_0, the electromagnetic force exerted on a length of busbar span by the initial r. m. s. value of total short-circuit current, from the formula‡

$$F_0 = 4.5 \times 10^{-8} \times k \times I_0^2 \frac{l}{s} \text{ lb. per span§}$$

and to obtain the maximum support stress P from the formula

$$P = p\,F_0 \quad \text{lb. per support}$$

where p is the stress factor obtained from stress curves selecting the value of p corresponding to the two natural frequencies of the bus structure. Fig. 1 gives a sample stress curve for 60-cycle currents.

Additional working stress curves, similar to that of Fig. 1, but covering the full range of natural frequencies, or other ranges of conditions, may, of course, be prepared by the same procedure.

Examples of Stress Calculations

Example No. 1

Two-wire short circuit between phases A and B, in Fig. 3. Each bar 4 in. by ¼ in. copper.

I_0 = 15,000 amperes
l = 48 in.
s = 6 in.

*While the analysis, outlined in the Appendix, has been applied to the basic case of sine-wave currents, the procedure is, of course, readily applicable to currents having a number of harmonic components, such as, a two-wire short-circuit current having a decided second-harmonic component.

†Bibliography No. 14.

‡Assuming uniform current distribution in the conductors.

§The factor k is a shape correction factor. It depends on the width, thickness and spacing of the bars, and allows for the well-known fact that the electromagnetic forces between bars are not always the same as those obtained on the assumption that the entire current is concentrated at the center of each bus. The value of k frequently differs from 1.0, especially in cases of closely spaced bars. The necessary values of k have been calculated and plotted by H. B. Dwight, Bibliography No. 11, and are reproduced here in Fig. 2.

$a = \frac{1}{4}$ in.
$b = 4$ in.
$f_1 = 15$ cycle per sec.
$f_2 = 45$ cycle per sec.
$f = 60$ cycle per sec.

$$F_0 = 4.5 \times 10^{-8}\, k\, I_0^2 \frac{l}{s}$$

To get the shape-correction factor, k

$a' = a = 0.25$ in.
$b' = b = 4$

$$\frac{s - a'}{a' + b'} = \frac{5.75}{4.25} = 1.35 \quad \frac{a'}{b'} = \frac{0.25}{4} = 0.063$$

Then $k = 0.94$ from Fig. 2. Hence

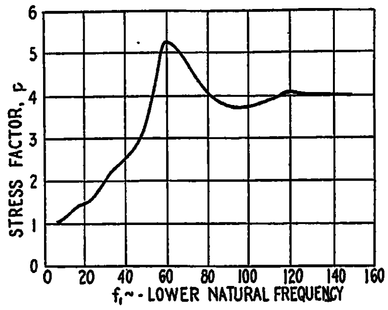

FIG. 1—CURVE OF STRESS FACTOR p

For the calculation of maximum support stress during 60-cycle short circuits, for values of f_1, lower natural frequency, ranging from 5 to 150 cyc. per sec. and for the value of f_2, higher natural frequency, equal to 180 cyc. per sec. The maximum support stress is pF_0, where F_0 is the electromagnetic force per busbar span for the initial r. m. s. (total) short-circuit current.

The curve of p is calculated for the following specific constants (see Appendix).

$$n_1 = n_2 = 5$$
$$g = 12.6$$
$$h = 2.2$$

Attention is called to the fact that the two natural frequencies f_1 and f_2 of the assembled busbar-insulator structure often differ very materially from the values of *busbar natural frequency for rigid supports* and of *support natural frequency without the busbar.* The differences in question are due to the support flexibility affecting busbar deflections and due to the busbar mass affecting support vibrations (compare, for example, equations (1) and (14) in the Appendix).

A useful formula and a simple chart for the determination of the natural frequency of busbars with rigid supports are given by L. F. Woodruff (Bibliography No. 34), but do not apply to busbar-insulator structures with flexible supports.

$$F_0 = 4.5 \times 10^{-8} \times 0.94 \times 15000^2 \times \frac{48}{6} = 76. \text{ lb. per span}$$

$P = \text{p}\, F_0$

$p = 1.6$ calculated by equation (33),* or from curves of stress factor similar to that in Fig. 1 plotted in accordance with (33).

Then the maximum stress is

$$P = 1.6 \times 76 = 122 \text{ lb. per support.}$$

The support must be designed to withstand this stress. The maximum electromagnetic force, however, is $\frac{8}{3} \times 76 \times 0.9^2 = 164$ lb. per span, where the factor $\frac{8}{3}$ is due to the relation between peak and average values of electromagnetic forces for totally displaced currents (see Table I), and the factor 0.9^2 allows for a 10 per cent current decrement during the first half-cycle of short-circuit current. It is seen that, in the case under discussion, the peak electromagnetic force is more than 30 per cent larger than the maximum support stress.

*Damping and current-decrement constants used are those given at the end of the appendix.

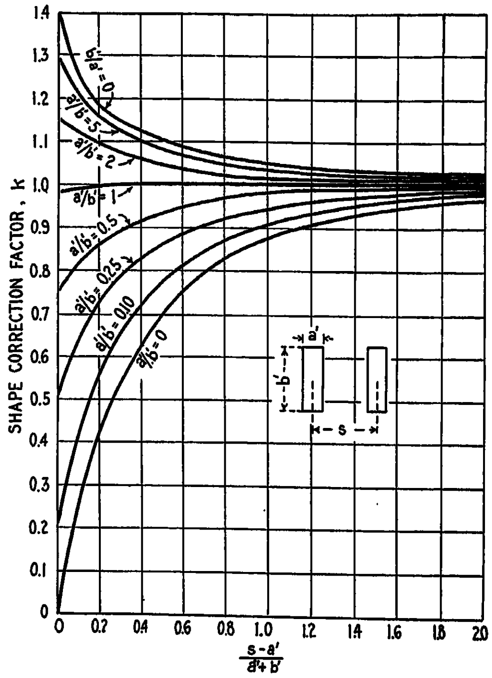

FIG. 2—SHAPE CORRECTION FACTOR k FOR THE CALCULATION OF ELECTROMAGNETIC FORCE

Prepared by H. B. Dwight and Published in the *Electrical World*, Vol. 70, p. 523

The electromagnetic force between parallel, rectangular bars is given by the formula

$$F = 4.5\, k\, I^2 \frac{l}{s} \cdot 10^{8-} \text{ lb. per span}$$

where l is the length of span in inches, s the spacing between conductors in inches, and I the current in amperes.

When the electromagnetic force is to be calculated between a going and a return conductor, each being a single rectangular bar of dimensions a by b, the values of a' and b' for the calculation of the shape correction factor k are:

$$a' = a \text{ and } b' = b$$

When several bars are clamped together in a group, the shape correction factor k may be computed approximately by considering the group to be replaced by a single solid conductor having the same outside dimensions as the group. Thus let the outside dimensions of the group be a' by b', and obtain k from Fig. 2. The approximation is usually very close when uniform current division between the laminations is provided—as, for instance, by proper transpositions—but may be less close when the current division is non-uniform.

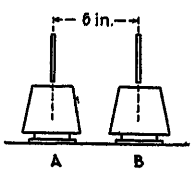

FIG. 3—THE BUS STRUCTURE CONSIDERED IN EXAMPLE 1

Example No. 2

Two-wire short circuit between phases A and B in Fig. 4, each phase having two copper bars 4 in. by ¼ in., with ¼ in. air space between bars.

$$I_0 = 16{,}200 \text{ amperes}$$
$$l = 56 \text{ in.}$$
$$s = 7.25 \text{ in.}$$
$$a = 4 \text{ in.}$$
$$b = \tfrac{1}{4} \text{ in.}$$
$$f_1 = 26.5 \text{ cycle per sec.}$$
$$f_2 = 280 \text{ cycle per sec.}$$
$$f = 60 \text{ cycle per sec.}$$

Then

$$F_0 = 4.5 \times 10^{-8} \times k \times I_0^2 \frac{l}{s}$$

where k is found from Fig. 2 with the aid of

$$a' = a = 4 \text{ in.}$$
$$b' = 0.75 \text{ in.}$$

In accordance with Fig. 2

$$\frac{s - a'}{a' + b'} = \frac{3.25}{4.75} = 0.69$$

$$\frac{a'}{b'} = \frac{4.0}{0.75} = 5.33$$

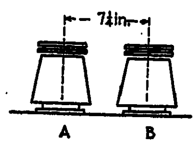

Fig. 4—The Bus Structure Considered in Example 2

and

$$k = 1.05 \text{ from Fig. 2}$$

Hence

$$F_0 = 4.5 \times 10^{-8} \times 1.05 \times 16{,}200^2 \times \frac{56}{7.25} = 96 \text{ lb. per span}$$

and the stress factor p, obtained from a curve similar to that of Fig. 1 (or by calculation from (33)) is 1.8

Thus the maximum stress per support is

$$P = 1.8 \times 96 = 173 \text{ lb.}$$

A test was made on the bus in question with a 25-cycle current, rather than a 60-cycle current. The test at 25-cycle per sec., gave a measured peak stress of 305 lb., for the same value of initial r. m. s. short-circuit current, *i. e.*, 16,200 amp., as indicated in Fig. 8. The greatly increased stress at 25 cycles is due to nearly perfect resonance, the natural frequency $f_1 = 26.5$ cycle per sec. being practically equal to the frequency of the fundamental component of electromagnetic force. It is therefore seen that in a given bus a short-circuit current of one frequency, say 25 cycle per sec., may produce a much greater stress than the same current of another frequency, say 60 cycles per sec.

Example 3

Busbar structure similar to that shown in Fig. 3, except that two 4 in. by ¼ in. copper bars are mounted on each support and that the length of the busbar spans is 28 in. The two bars per phase are separated by an air space of ¼ in. Calculate the maximum stress for a two-wire phase-to-phase short circuit.

$$I_0 = 25{,}000 \text{ ampere}$$
$$l = 28 \text{ in.}$$
$$s = 6 \text{ in.}$$
$$a = \tfrac{1}{4} \text{ in.}$$
$$b = 4 \text{ in.}$$
$$f_1 = 20 \text{ cycle per sec.}$$
$$f_2 = 60 \text{ cycle per sec.}$$
$$f = 60 \text{ cycle per sec.}$$

k is found from Fig. 2 with the aid of

$$a' = 0.75 \text{ in.}$$
$$b' = b = 4 \text{ in.}$$

In accordance with Fig. 2,

$$\frac{s - a'}{a' + b'} = \frac{5.25}{4.75} = 1.11$$

$$\frac{a'}{b'} = \frac{0.75}{4} = 0.188$$

and

$$k = 0.94 \text{ from Fig. 2}$$

Hence

$$F_0 = 4.5 \times 10^{-8} \times 0.94 \times 25{,}000^2 \times \frac{48}{6} = 212 \text{ lb. per span}$$

The calculated stress factor p is 1.9. Thus the maximum stress is

$$P = 1.9 \times 212 = 403 \text{ lb. per support.}$$

It is seen that the maximum stress is not excessive although resonance occurs, because the non-resonant natural frequency is well below the resonant frequency, as already indicated.

Results of Tests

A large number of tests were made to check the stress calculations: 1, on small-scale structures of definitely known and readily adjustable mechanical characteristics and 2, on full-size typical busbar-insulator structures with short-circuit currents up to 25,000 amperes. The results of the tests were in good agreement with the calculations. In the following will be given some of the results obtained in the full-size tests.

The support deflection curve (with time) as well as the busbar deflection curve (with time) were recorded by oscillograph, with the aid of specially designed displacement recorders. Moreover, the deflections of the insulators were measured for slowly applied constant loads, so that the support stresses could be calculated from the oscillographic support deflection records. The bus structures used had five or more uniform busbar spans per phase.

Test No. 1

The object of this test was to compare test results of support stress with calculated results for a bus structure employing porcelain insulators with a long busbar span (56 in.) in which the bars were mounted face-down on the supports, as shown in Fig. 4. The center spacing between phases was 7.25 in., two 4 in. by ¼ in. bars

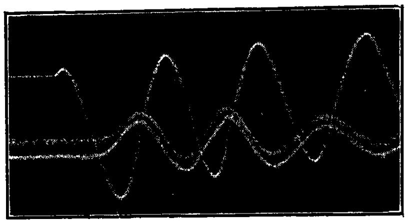

FIG. 5—OSCILLOGRAM PERTAINING TO TEST NO. 1

Upper curve: 25-cycle current, initial r. m. s. value 14900 amperes
Middle curve: Center deflection of busbar span 0.19 in. maximum
Lower Curve: Deflection of support at level of busbar center 0.15 in. maximum

being used per phase. The two natural frequencies of the structure were 26.5 cycle per sec. and 280 cycle per sec.

In the oscillogram, Fig. 5, are shown in the upper curve the 25-cycle short-circuit current of 14,900 amp. initial r. m. s. value, in the middle curve the center deflection of a busbar span, in the lower curve the sup-

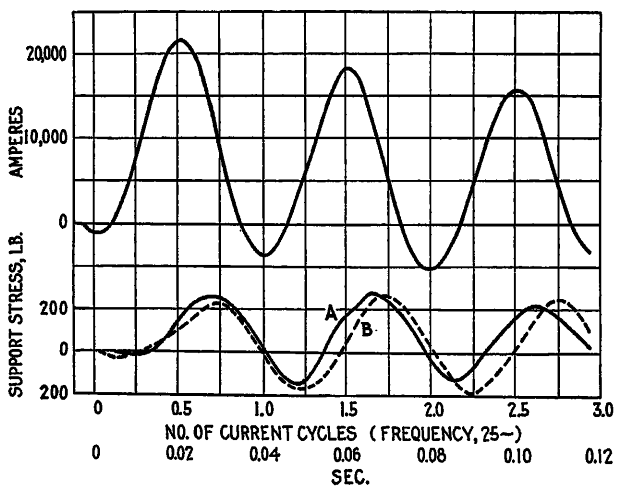

FIG. 6—CURRENT AND STRESS CURVES FOR TEST NO. 1

Upper: Current from Fig. 5
Curve *A*: Support-stress curve by test, from Figs. 5 and 7
Curve B: Calculated support-stress curve, by equation (**40**) and on the basis of the following constants: $f_1 = 25$ cyc. per sec., $f_2 = 280$ cyc. per sec., $f_c = 25$ cyc. per sec., $n_1 = n_2 = 5$, $g = 12.6$, $h = 2.2$

port deflection occurring at the level of the busbar. Fig. 6 shows, in the lower part, the support-stress curve, *A*, by test (from Fig. 5) together with a calculated support-stress curve, *B*, calculated for a lower natural frequency of 25 cycle per sec. (by equation (**40**) of the Appendix). In view of the almost perfect resonance obtained in the test, the stress factor for maximum support stress is 3.5, meaning that the maximum support stress is 3.5 times the initial average electromagnetic force per span. Moreover, the maximum support stress is 75 per cent greater than the electromagnetic force calculated from the maximum value of short-circuit current (21,500 amperes) measured at its first peak.

The load-deflection curve for the insulator appears in Fig. 7.

In Fig. 8 reasonably good agreement is shown between test points and a calculated curve of peak stress for short-circuit current values ranging from 8000 to 19,000 amperes, under the conditions of Test No. 1.

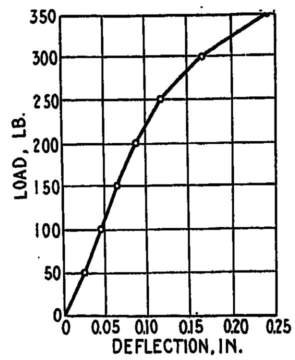

FIG. 7—LOAD-DEFLECTION CURVE FOR SUPPORT USED IN TEST NO. 1

Loads constant and slowly applied at the level of the busbar. Deflection measured at level of busbar

Center spacing between phases 3.75 in.

One 4 in. by ¼ in. bar per phase

Natural frequencies 26 cycle per sec. and 130 cycle per sec.

Test No. 2.

The object of Test No. 2 was to compare test results of support stress with calculated results for a bus structure employing porcelain insulators with a short busbar span (20 in.) in which the bars were mounted edgewise on the supports but face-to-face with respect to each other, as shown in Fig. 3.

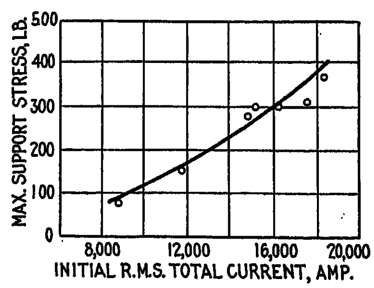

FIG. 8—COMPARISON OF MAXIMUM STRESSES BY TEST AND BY CALCULATION

For conditions of Test No. 1
Points obtained by test, curve by calculation

Current frequency 25 cycle per sec.

Initial r. m. s. current 19,900 amperes.

In Fig. 9, curve *A* shows the experimental stress-time curve. Curve *B* was calculated for perfect tuning by equation (**40**) in the appendix. The early dropping

off of the stress oscillations observed by test is probably due to the somewhat non-elastic behavior of the support at the heavier loads, as indicated in Fig. 10.

In Fig. 11, the calculated peak stress at the higher current values is materially above the test points—a result consistent with the larger values of damping to

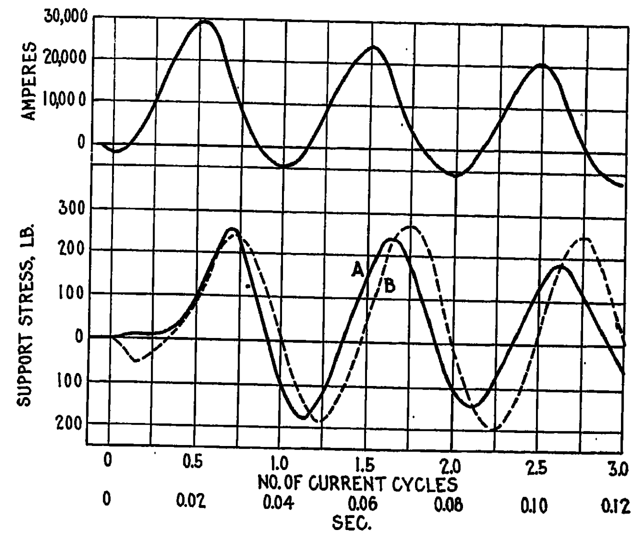

FIG. 9—CURRENT AND STRESS CURVES FOR TEST NO. 2

Upper curve: 25-cycle current, initial r. m. s. value 19900 amperes
Curve A: Support-stress curve by test
Curve B: Calculated support-stress curve, by equation (40) and on the basis of the following constants: $f_1 = 25$ cyc. per sec., $f_2 = 130$ cyc. per sec., $f_c = 25$ cyc. per sec., $n_1 = n_2 = 5$, $g = 12.6$, $h = 2.2$

be expected. At the lower currents, however, test results and calculations are in fairly good agreement.

Test No. 3.

The object of this test was to measure the stresses occurring in a structure having supports of the insulated-rod type, where the damping resistance is low.

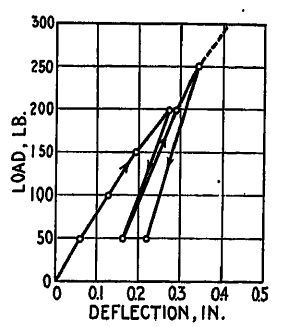

FIG. 10—LOAD-DEFLECTION CURVE FOR SUPPORT USED IN TEST NO. 2

Loads constant and slowly applied at the level of the busbar. Deflection measured at level of busbar. Where dips are shown, the load was reduced and subsequently increased.

Accordingly the bus structure of Fig. 12 was employed. Each conductor of the two-conductor loop was a 4 in. by 3/8 in. copper bus, the center spacing between conductors being 7.25 in. and the longitudinal spacing of supports being 45.5 in. The lower natural frequency of the structure was 27 cycle per sec., while the higher one was 60 cycle per sec.

Curve A of Fig. 13 shows the support stress variation, by test, from the beginning of the short circuit for an initial current of 22,800 amperes. The maximum stress peak did not occur until after the fourth current peak, on account of the low damping rate. Examination of free-vibration records for the structure of Fig. 12 showed

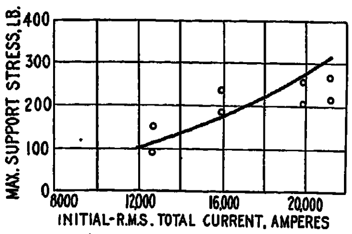

FIG. 11—COMPARISON OF MAXIMUM STRESSES BY TEST AND BY CALCULATION

For conditions of Test No. 2
Points obtained by test, curve by calculation.

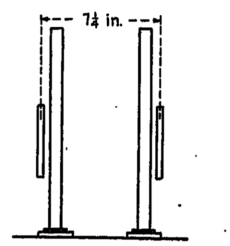

FIG. 12—THE BUS STRUCTURE USED FOR TEST NO. 3

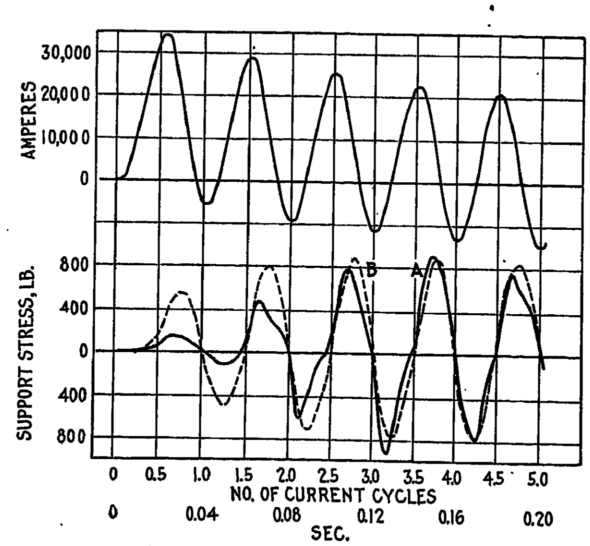

FIG. 13—CURRENT AND STRESS CURVES FOR TEST NO. 3

Upper curve: 25-cycle current, initial r. m. s. value 22800 amperes
Curve A: Support-stress curve by test
Curve B: Calculated support-stress curve, by equation (40) and on the basis of the following constants: $f_1 = 25$ cyc. per sec., $f_2 = 60$ cyc. per sec., $f_c = 25$ cyc. per sec., $n_1 = n_2 = 12$, $g = 9.7$, $h = 2.2$

that the resonance sharpness* (diminishing with increased damping) was more than double that found in Tests No. 1 and No. 2. With the aid of the higher value of resonance sharpness the calculated Curve B of Fig. 13 was obtained. The maximum experimental

*Defined by equations No. (30) and No. (31) in the appendix.

support stress by test was 6.3 times the initial average electromagnetic force, on account of (1) the nearly perfect resonance and (2) the reduced damping resistance.

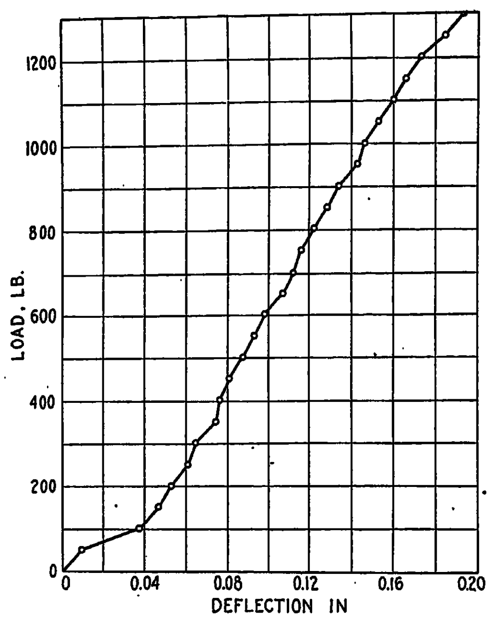

Fig. 14—Load-Deflection Curve for Support Used in Test No. 3

Loads constant and slowly applied at the level of the busbar. Deflection measured at the level of the busbar.

Attention is called to an insulator calibration curve given in Fig. 14.

Test results and a calculated curve of peak stresses for the structure of Fig. 12 are presented in Fig. 15,

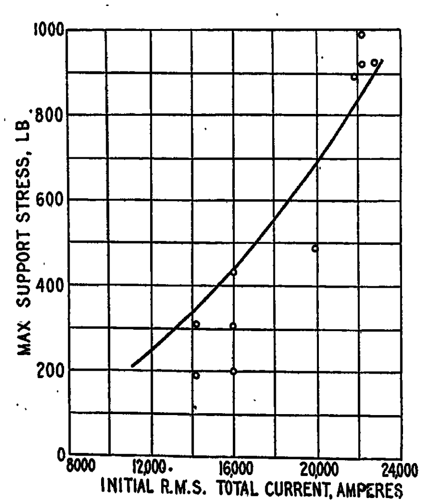

Fig. 15—Comparison of Maximum Stress by Test and by Calculation

For the conditions of Test No. 3
Points obtained by test, curve by calculation

for short-circuit current values ranging from 14,000 to 23,000 amperes.

Test No. 4.

In this test the elastic limit of a porcelain insulator, similar to those used in tests 1 and 2, was considerably exceeded.

The conductor consisted of two 4 in. by ¼ in. copper bars mounted in the same manner as the one bar shown in Fig. 3.

The bus structure when vibrating through small amplitudes had natural frequencies of 10 and 28 cycle per sec.

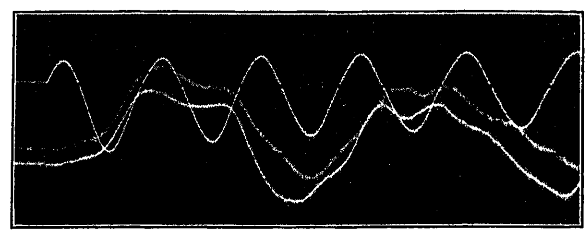

Fig. 16—Oscillogram Pertaining to Test No. 4

Upper curve: 25-cycle current, initial r. m. s. value 18200 amperes
Middle curve: Deflection of support A at level of busbar. 0.40 in. maximum
Lower curve: Deflection of support B at level of busbar. 0.36 in. maximum
Note: Supports A and B were adjacent insulators in the middle of the bus structure

Fig. 16 is a test record showing a 25-cycle short-circuit current of 18,200 initial r. m. s. amperes and two support-deflection curves (lower curves). The corresponding calculated stress curve, calculated for the two natural frequencies of 10 and 25 cycle per sec., is that

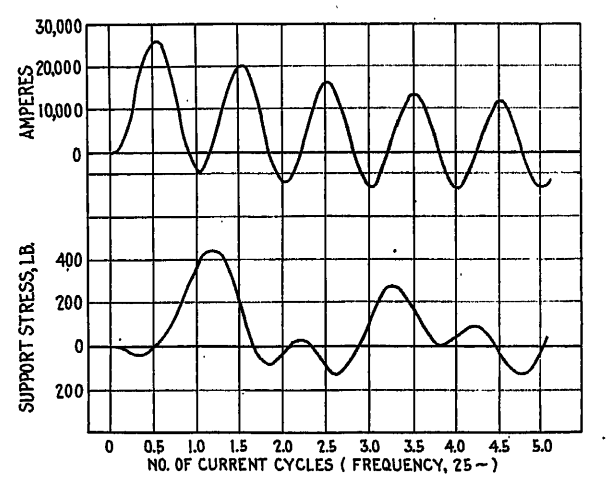

Fig. 17—Current and Stress Curves Relating to Test No. 4

Upper curve: 25-cycle current, initial r. m. s. value 18200 amperes; (assumed fully displaced); decrement factors: $g = 12.6$, $h = 2.2$
Lower curve: Calculated support-stress curve, for the current plotted in upper curve and on the basis of the following constants: $f_1 = 10$ cyc. per sec., $f_2 = 25$ cyc. per sec., $f_c = 25$ cyc. per sec., $n_1 = n_2 = 5$, $g = 12.6$, $h = 2.2$

of Fig. 17. The support load-deflection curve is given in Fig. 18. While the experimental deflections from Fig. 16 clearly show the 10-cycle natural-frequency component, the 25-cycle forced-frequency component of stress is present but less pronounced than that predicted from the calculated stress curve (Fig. 17). The discrepancy is due to the non-elastic behavior of the

support at the larger deflections which reached values as large as 0.4 in.

Attention is called to the deflections and stresses of considerable magnitudes occurring in a direction opposite to that of the electromagnetic force—an observation made in a number of tests and consistent with predictions by calculation. Bus supports must, therefore, be designed with reference to stresses of either direction. The maximum stress will be in the direction of the average electromagnetic force.

SUMMARY

1. A method of calculating the mechanical stresses in busbar supports during short circuits is developed and presented.

2. The method of stress calculation given here takes into account the following factors: The short-circuit current, its decrement rate, the electromagnetic force due to the current, the natural frequencies of busbar structures and their motional resistance.

3. The method of calculation differs from those employed in the past since the calculations used heretofore have only partially taken into account, or totally neglected, the oscillatory character of busbars and of supports.

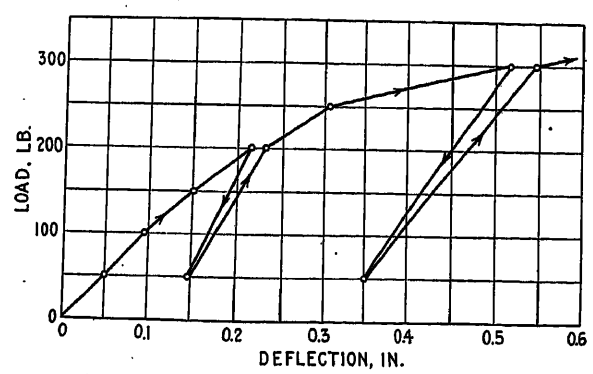

FIG. 18—LOAD-DEFLECTION CURVE FOR SUPPORT USED IN TEST NO. 4

Loads constant and slowly applied at the level of the busbar. Deflection measured at the level of the busbar. Where dips are shown, the load was reduced and subsequently increased.

4. Tests have demonstrated that not only the busbars, but also most types of supports are flexible and may be treated, for the purpose of estimating stresses, as having elastic properties.

5. When busbars are mounted on flexible supports, in the customary manner, the motion of the busbar span as well as the motion of the support show two dominant natural frequencies which occur in combination in each of the two structural members when electromagnetic forces are suddenly applied. The natural frequencies are determined by a simple test, when feasible, or by calculation from the mechanical constants of busbar and of support.

6. In view of the two natural frequencies mentioned, bus supports respond selectively to impressed electromagnetic forces at different frequencies; so that the magnitude of the electromagnetic force and its mode of variation with time are not usually representative of the magnitude and mode of variation of the support stresses.

7. Since the natural frequencies of busbar structures range from, say, 10 to 300 cycles per sec., mechanical resonance is within common possibility in buses of 25-cycle circuits as well as in buses of 60-cycle circuits. While not all cases of resonance give rise to stresses greatly in excess of those at non-resonance, resonance in some cases multiplies the stresses five or even ten times. It will often be possible by proper methods of design so to change the natural frequencies as to avoid the cases of resonance giving extra heavy stresses.

8. In the analysis of stresses presented the bus structure is treated as a two-element vibratory system, *i. e.*, as a mechanical system having two elastic oscillatory elements joined together—a composite structure resembling, for instance, the system comprising a turbine rotor keyed to a flexible shaft, the shaft bearings being attached to an elastic base.

The analysis of bus-support stresses presented here is applied to the basic case of a bus structure having long, straight, uniform, parallel busbars with equidistant, rigidly mounted supports of uniform characteristics. The method of calculation is a general one applicable to all cases of elastic busbars intermittently supported, irrespective of whether the busbars are placed face-on or edge-on with respect to the supports, and irrespective of whether the supports themselves are stressed in bending, in tension or in compression.

9. The transient motion and stresses obtained by tests on full-size structures were found in most cases to be in substantial agreement with the results calculated, proper allowance for damping and current decrement being made in the calculations. Special calculations, not covered by No. 8 will, however, be required in the following cases:

a. When a third pronounced natural frequency is introduced, due for example to a flexible foundation to which the insulators are bolted. A flexible foundation not only introduces additional possibilities of resonance, due to the third natural frequency, but also tends to alter the other two natural frequencies.

b. When the bus supports do not clamp the busbar but merely guide it loosely. Excess stresses may occur and must be allowed for in the calculation.

c. When the elastic limit of the supports or of the busbar is materially exceeded. As a rule the limit of safe stresses is not materially above the elastic limit of the structural members.

Moreover, consideration will sometimes have to be given to additional factors not covered in this paper, such as the effects, upon the stresses, of tap-connections, bends, unequal spans, non-uniform current division, and longitudinal tension set up along the busbars and stressing the insulators in a direction at right angles to the stresses covered by the formulas developed in this paper.

10. Approximate estimates of maximum support stresses in practical design may be made by multiplying the initial average electromagnetic force per busbar span, which force is that due to the initial r. m. s. (total) short-circuit current, by a "stress factor," equal to the maximum stress per support when the initial average electromagnetic force is unity. Representative curves of stress factor calculated from the equations derived may be plotted with respect to the natural frequencies. The curves in question are prepared on the basis of selected conditions and average representative constants so that one set of curves will be applicable with sufficient accuracy to a large number of cases. A sample curve of stress factor is presented in Fig. 1.

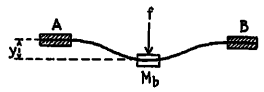

FIG. 19—BUSBAR REPRESENTED WITH CONCENTRATED MASS M AND MOUNTED ON RIGID SUPPORTS A, B

11. The maximum support stress may occur before the end of the first cycle of short-circuit current, or as late as 0.15 sec.

12. Attention is called to the deflections and stresses of considerable magnitudes occurring in a direction opposite to that of the electromagnetic force—an observation made in a number of tests and consistent with predictions by calculation. Bus supports must, therefore, be designed with reference to stresses of either direction. The maximum stress will be in the direction of the average electromagnetic force.

Appendix

Calculation of Stresses in Bus Supports

It is the object of this appendix to establish a procedure for estimating the maximum short-circuit stresses in busbar supports.

THE BUSBAR

With some restrictions and exceptions, the busbar with its distributed mass, may be treated, in regard to its vibratory behavior and support stresses, as a simple elastic beam of concentrated rigid mass, constant stiffness, and motional resistance.

a. Deflection of Support Neglected.

Theoretically, a uniform bar, clamped to a pair of rigid supports, may be expected to have a composite set of free vibrations having a fundamental frequency and a number of overtone frequencies. Practically, however, test records of vibrations of several kinds of busbars show the vibrations at overtone frequencies to be hardly noticeable in the free vibrations occurring when loads are suddenly applied or suddenly removed; *i. e.*, the free vibrations are practically the same as those of a concentrated mass supported by an elastic spring. If, then, M_b is the equivalent busbar mass and S_b its (constant) stiffness* an external force F, slowly applied, as in Fig. 19, to the mass originally at rest, will cause a deflection y of the mass M_b, so that $S_b y = F$. Thus, the stress upon each support will be $(\frac{1}{2}) S_b y$. If there are adjacent spans, similarly loaded, on each side of the first one, the stress on each support, except the end ones, will be $S_b y$. Similarly, during vibrations of an amplitude y, as will occur, for example, when the forces F on each span are suddenly and simultaneously removed (damping neglected), the maximum pressure on the supports will be $S_b y$ and the natural frequency f_b of the vibrations will be defined by

$$(2 \pi f_b)^2 = \omega_b^2 = \frac{S_b}{M_b} \tag{1}$$

For any given busbar, the values of S_b and M_b must satisfy (1), when f_b is the natural frequency of the actual busbar.

Assuming the busbar to be a vibratory element having a single natural frequency, as considered in the foregoing, Biermanns* calculated the busbar deflections in terms of the natural frequency of the busbar span for short-circuit currents on the assumption that the bus is rigidly clamped to non-flexible supports. Flexible supports were not considered.

b. Treatment of Busbars with Flexible Supports.

Busbar structures with flexible supports may be treated as a mechanical system comprising a busbar span of concentrated mass M_b, constant stiffness S_b, and motional resistance, the busbar being held by supports which also are represented by a concentrated mass M_s, a constant stiffness S_s, and motional resistance. The electromagnetic force f per span will be considered as applied to the equivalent busbar mass M_b; while the driving force acting on the support will be $S_b y_b$ where y_b is the deflection of the mass M_b with respect to the supports as indicated in Fig. 20. The system described has two inter-related natural frequencies. Both frequencies are present in the motion of the busbar as well as in the motion of the support, *i. e.*, neither frequency can be definitely assigned to either one of the elements alone.

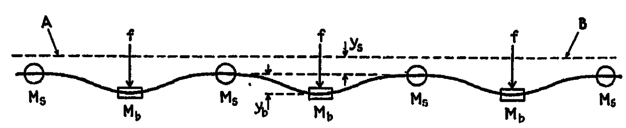

FIG. 20—BUSBAR-INSULATOR STRUCTURE WITH FLEXIBLE SUPPORTS

The busbar is represented with concentrated mass M_b and the support with concentrated mass M_s. Line A-B indicates the rest position of the busbar and of the supports before the electromagnetic forces are applied.

*The stiffness is the force per unit deflection, the force being slowly applied to the mass originally at rest.

*Bibliography No. 6.

The approximations involved in Fig. 20 are the following:

A. The portion of the electromagnetic force which acts directly on the support is neglected; *i. e.*, the entire force f is considered to be directly applied to the busbar span, since the width of the support, measured along the direction of the busbar, is less than 10 per cent of the spacing between supports in the majority of bus structures.

B. The overtone natural-frequency components of the busbar are neglected since they are rapidly damped out, and since the support stresses will be far less affected by any short-wave overtone busbar vibrations than by the fundamental vibrations involving the busbar as a whole.

C. There is a further approximation due to the effect of using the equivalent busbar mass, rather than the actual distributed mass, in a bus system having flexible supports. To investigate this point test values of the two natural frequencies obtained with actual bus structures were compared with values calculated on the basis of the concentrated busbar mass. The comparison showed good agreement, the discrepancies found being well within the limits of accuracy of the observed support constants themselves. It is concluded, therefore, that the approximation in question is allowable in stress calculations, particularly if test values of natural frequencies are used.

In view of the foregoing, the support-stress calculations will be based on the following conditions:

1. Straight, uniform, parallel busbars having equidistant supports of uniform characteristics.
2. All supports, are considered equally stressed, the stresses per support being those due to the electromagnetic force for a full busbar span. A number of similar spans are assumed to be acted on by the same electromagnetic force.
3. Busbar and support deflect when stressed and have elastic properties.
4. Two-wire short circuit, the two conductors carrying equal and opposite currents.
5. Busbars at rest and unstressed before application of the short circuit.
6. The deflection of the busbars is considered to be small in comparison with the spacing between conductors; thus the electromagnetic force is regarded as independent of the deflection of the bars. The longitudinal tension in the bars is neglected.
7. The base of each support is considered to be tightly bolted to a rigid foundation.
8. The assembled bus structure is assumed to have not more than two natural frequencies.
9. In the first analysis, the motional resistance of the structural elements as well as the current decrement will be neglected. The principles of motion and of stresses can adequately and most conveniently be derived from an analysis of the motional behavior of a (hypothetical) system of negligible resistance, in the absence of current decrement. Subsequently, suitable allowance will be made for current decrement and damping.

Support Deflections and Stresses without Damping and Without Current Decrement*

Since the driving electromagnetic force f per span is opposed during vibratory motion by a force due to the acceleration of the busbar mass, whose displacement at any instant is $y_b + y_s$, and by a restoring force due to the resilience of the busbar span, the following equation of busbar motion neglecting resistance is obtained, in accordance with Fig. 20.

$$M_b \left(\frac{d^2 y_b}{d t^2} + \frac{d^2 y_s}{d t^2} \right) + S_b y_b = f \qquad (2)$$

All forces are parallel and in one plane. The corresponding forces acting on the support, according to Fig. 20, define its motion as given in equation (3)†. Resistance is again neglected.

$$M_s \frac{d^2 y_s}{d t^2} + S_s y_s = S_b y_b \qquad (3)$$

In order to obtain the support stress, equations (2) and (3) will be solved for y_s in terms of the structural constants and the electromagnetic force. If equation (3) is differentiated twice, with respect to time, and the equation so found is then combined with (2) and (3) for eliminating y_b and the derivatives of y_b, equation (4), the differential equation for y_s, is obtained.

$$\frac{d^4 y_s}{d t^4} + \left(\frac{S_b}{M_b} + \frac{S_s}{M_s} + \frac{S_b}{M_s} \right) \frac{d^2 y_s}{d t^2} + \frac{S_b S_s}{M_b M_s} y_s = \frac{S_b}{M_b M_s} f \qquad (4)$$

The solution of (4) depends on the form of the electromagnetic force f. A symmetrical sine current of the form

$$I_m \sin \omega t$$

becomes, when totally displaced,

$$i = I_m (1 - \cos \omega t)$$

Thus the instantaneous electromagnetic force between two parallel bars each of which carries current i is, neglecting current decrement,

$$f = A I_m^2 (1 - \cos \omega t)^2$$

$$= 1.5 A I_m^2 \left(1 - \frac{4}{3} \cos \omega t + \frac{1}{3} \cos 2 \omega t \right) \qquad (5)$$

Equation (4) may then be written:

$$\frac{d^4 y_s}{d t^4} + g_0 \frac{d^2 y_s}{d t^2} + h_0^2 y_s = B F_0 \left(1 - \frac{4}{3} \cos \omega t + \frac{1}{3} \cos 2 \omega t \right) \qquad (6)$$

*C. g. s. absolute units are employed, unless otherwise mentioned.

†Equations similar to No. 2 and No. 3 were set up by Hort (Bibliography No. 15) for the vibrations of a system comprising a rotating machine on a flexible shaft held by two bearings which are resting on a yielding foundation.

where B is substituted for $\frac{S_b}{M_b M_s}$

Then y_s is of the form:*

$$y_s = y_1 + y_2 + y_3 + y_4$$

where

y_1 is the particular solution of (6) with the right-hand side equal to $B F_0$

y_2 is the particular solution of (6) with the right-hand side equal to $\left(-\frac{4}{3} B F_0\right) \cos \omega t$

y_3 is the particular solution of (6) with the right-hand side equal to $\left(+\frac{1}{3} B F_0\right) \cos 2 \omega t$

y_4 is the complementary function, with the right-hand side of (6) equal to zero.

Thus the solution for y_1 is

$$y_1 = \frac{B F_0}{h_0^2} \quad (7)$$

since the derivatives of equation (7) are zero. The solution for y_2† is

$$y_2 = \frac{-\frac{4}{3} B F_0 \cos \omega t}{\omega^4 - g_0 \omega^2 + h_0^2} \quad (8)$$

Similarly

$$y_3 = \frac{\frac{1}{3} B F_0 \cos 2 \omega t}{(2 \omega)^4 - g_o (2 \omega)^2 + h_0^2} \quad (9)$$

Finally, y_4 is found from

$$\frac{d^4 y_4}{d t^4} + g_0 \frac{d^2 y_4}{d t^2} + h_0^2 y_4 = 0 \quad (10)$$

The solution is of the form

$$y_4 = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} + c_3 e^{\alpha_3 t} + c_4 e^{\alpha_4 t} \quad (11)$$

where c_1, c_2, c_3 and c_4 are constants dependent on the starting conditions, and α_1, α_2, α_3, α_4 are the four roots of the equation

$$\alpha^4 + g_0 \alpha^2 + h_0^2 = 0 \quad (12)$$

Thus all values of α are pure imaginaries:

$$\left.\begin{aligned} \alpha_1 &= j q_1 \\ \alpha_2 &= -j q_1 \\ \alpha_3 &= j q_2 \\ \alpha_4 &= -j q_2 \end{aligned}\right\} \quad (13)$$

where

$$\left.\begin{aligned} q_1 &= \sqrt{\frac{1}{2}\left(g_0 - \sqrt{g_0^2 - 4 h_0^2}\right)} \\ q_2 &= \sqrt{\frac{1}{2}\left(g_0 + \sqrt{g_0^2 - 4 h_0^2}\right)} \end{aligned}\right\} \quad (14)$$

q_1 and q_2 are the angular velocities of the two natural frequencies of the structure.

*See Bibliography 33, p. 214-218 and p. 240.

†See Bibliography 33, p. 215, equation (17‴).

Dividing q_1 and q_2, in equation (14), by 2π the two natural frequencies f_1 and f_2 of the busbar structure are obtained.

It is seen that q_1 cannot be larger than q_2.

Thus the solution for y_s is

$$y_s = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t} + c_3 e^{\alpha_3 t} + c_4 e^{\alpha_4 t} + \frac{B F_0}{h_0^2} - \frac{\frac{4}{3} B F_0 \cos \omega t}{\omega^4 - g_0 \omega^2 + h_0^2} + \frac{\frac{1}{3} B F_0 \cos 2 \omega t}{(2 \omega)^4 - (2 \omega)^2 g_0 + h_0^2} \quad (15)$$

Now the constants c_1, c_2, c_3 and c_4 in (15) will be determined from the initial conditions of motion. The constants will establish the amplitudes of the free-vibration components of motion. The initial condition to be considered is that the bus structure be at rest and unloaded at a time just before the occurrence of the short circuit or that, in mathematical terms,

$$\text{at the time } t = 0 \left\{\begin{aligned} y_b &= 0 && (16a) \\ \frac{d y_b}{d t} &= 0 && (16b) \\ y_s &= 0 && (16c) \\ \frac{d y_s}{d t} &= 0 && (16d) \end{aligned}\right.$$

The last two conditions will be applied directly to equation (15), while (16a) and (16b) will be applied to equation (3). In the use of (16a) and (16b) with equation (3) the expressions for $\frac{d^2 y_s}{d t^2}$ and $\frac{d^3 y_s}{d t^3}$ at the time $t = 0$ must be found from equation (15) by successive differentiations of (15). The procedure thus outlined gives the following constants:

$$\left.\begin{aligned} c_1 = c_2 &= \frac{y_0 u_2^2}{2 (u_2^2 - u_1^2)} \left[-1 - \frac{4}{3} \frac{u_1^2}{(1 - u_1^2)} + \frac{1}{3} \frac{u_1^2}{(4 - u_1^2)}\right] \\ c_3 = c_4 &= \frac{y_0 u_1^2}{2 (u_2^2 - u_1^2)} \left[1 + \frac{4}{3} \frac{u_2^2}{(1 - u_2^2)} - \frac{1}{3} \frac{u_2^2}{(4 - u_2^2)}\right] \end{aligned}\right\} \quad (17)$$

where

$$y_0 = \frac{B F_0}{h_0^2} = \frac{F_0}{S_s} \quad (18)$$

$$\left.\begin{aligned} u_1 &= \frac{q_1}{\omega} \\ u_2 &= \frac{q_2}{\omega} \end{aligned}\right\} \quad (19)$$

Then we obtain for y_4 from equation (11) with the aid of equation (13) and equation (17)

$$y_4 = 2 c_1 \cos q_1 t + 2 c_3 \cos q_2 t \quad (20)$$

The sum of $y_1 + y_2 + y_3 + y_4$ in accordance with (7), (8), (9) and (20) gives the full solution for y_s.

In practical stress calculations the ratio $\frac{y_s}{y_0} = p$ is convenient to use. The significance of p is due to the fact that y_0 is the support deflection due to a dead load equal to F_0, the electromagnetic force exerted on a busbar span by the r. m. s. initial asymmetrical short-circuit current. Hence, p multiplied by F_0 gives the support stress, and the maximum value of p is the maximum stress obtained when the initial average electromagnetic force per span is one. Accordingly, p will be called the stress factor. Thus the support stress factor, neglecting resistance and current decrement is

$$p = \frac{y_s}{y_o} = 1 + C_1 \cos q_1 t + C_2 \cos q_2 t + C' \cos \omega t + C'' \cos 2 \omega t \qquad (21)$$

where the coefficients C_1, C_2, C' and C'' are defined by

$$\left.\begin{aligned}
C_1 &= \frac{u_2^2}{u_2^2 - u_1^2} \left(-1 - \frac{4}{3} \frac{u_1^2}{(1 - u_1^2)} + \frac{1}{3} \frac{u_1^2}{(4 - u_1^2)} \right) \\
C_2 &= \frac{- u_1^2}{u_2^2 - u_1^2} \left(-1 - \frac{4}{3} \frac{u_2^2}{(1 - u_2^2)} + \frac{1}{3} \frac{u_2^2}{(4 - u_2^2)} \right) \\
C' &= - \frac{4}{3} \frac{u_1^2}{(1 - u_1^2)} \frac{u_2^2}{(1 - u_2^2)} \\
C'' &= \frac{1}{3} \frac{u_1^2}{(4 - u_1^2)} \frac{u_2^2}{(4 - u_2^2)}
\end{aligned}\right\} \qquad (22)$$

It is seen that all constants in equation **21** depend solely on the frequency ratios

$$u_1 = \frac{q_1}{\omega} = \frac{2 \pi f_1}{\omega} \text{ and } u_2 = \frac{q_2}{\omega} = \frac{2 \pi f_2}{\omega}$$

so that, if the two natural frequencies

$$\left.\begin{aligned}
f_1 &= \frac{q_1}{2 \pi} \text{ cycles per sec.} \\
f_2 &= \frac{q_2}{2 \pi} \text{ cycles per sec.}
\end{aligned}\right\} \qquad (23)$$

are known, the stress factor is readily calculated by (21) and (22), neglecting resistance and current decrement.

One of the questions of greatest practical importance is that of resonance. Resonance* occurs when an impressed force has a frequency equal to one of the two natural frequencies f_1 and f_2.

Since there are two natural frequencies and (for displaced short-circuit currents) harmonic electromagnetic force components at two frequencies, there are four possibilities of resonance, *i. e.*, either one of the two natural frequencies may be at resonance with either of the force frequencies. Thus the four conditions of resonance are:

$$u_1 = 1 \qquad \textbf{(24a)}$$

$$u_2 = 1 \qquad \textbf{(24b)}$$

$$u_1 = 2 \qquad \textbf{(24c)}$$

$$u_2 = 2 \qquad \textbf{(24d)}$$

To summarize the principal results derived so far:

Support stresses may be most conveniently expressed in terms of (1) the *initial average electromagnetic force* per busbar span, which force is that due to the r. m. s. initial value of asymmetrical short-circuit current, and (2) the *stress factor*, a value equal to the stress per support when the initial average electromagnetic force is unity.

Thus the maximum stress is obtained by multiplying the initial average electromagnetic force by the maximum stress factor.

For busbar and supports of elastic properties the stress factor, and hence the maximum stress, depend primarily on the two natural frequencies and on the current frequency except for the effects of damping and current decrement, to be considered below.

The two natural frequencies may be determined by calculation from the mechanical constants of busbar and of insulator, by equation (14), or preferably, where feasible, by direct measurement on the assembled bus structure.

Effects of Damping and of Current Decrement at Non-Resonance

Equation (21), derived for zero motional resistance, and for a totally displaced current without decrement, does not give the final values for practical stress calculations, because the damping of the structural elements and the current decrement do materially affect the maximum stresses during short circuits. Structures at non-resonance will be considered first. For the present purpose, "non-resonance" is to include all those cases in which the natural frequencies differ by 25% or more from the frequencies of the impressed electromagnetic force. The calculation of stresses at resonance is discussed later.

For practical calculations of maximum stresses at non-resonance, the following approximations have been made:

a. The electromagnetic force due to short-circuit currents is expressed in the form:

$$f = F_0 \left(\frac{2}{3} e^{-2gt} + \frac{1}{3} e^{-2ht} - \frac{4}{3} e^{(-g-h)t} \cos \omega t + \frac{1}{3} e^{-2ht} \cos 2 \omega t \right) \qquad (25)$$

which is approximately equal to the electromagnetic

*See Bibliography No. 31, pp. 152-155 and p. 175; also Bibliography No. 15, pp. 54-57.

force due to a two-wire short-circuit when the current is*

$$i = I_m e^{-gt} - [(I_m - I_s) e^{-ht} + I_s] \cos \omega t \qquad \mathbf{(26)}$$

where I_m and I_s are the maximum values of initial a-c. component and of sustained current, respectively, and g and h are the decrement constants of d-c. component and of a-c. component, respectively. It is seen that the electromagnetic force of equation (**25**) is obtained from a form of short-circuit current in which the value of I_s in equation (**26**) is neglected.

b. The non-harmonic component of the stress factor, which component is equal to unity without current decrement, is approximately

$$p_1 = \frac{2}{3} e^{-2gt} + \frac{1}{3} e^{-2h} \qquad \mathbf{(27)}$$

when the electromagnetic force defined by equation (**25**) is applied.

The approximation involved in equation (**27**) is very close at non-resonance in a single-element vibrating system having mass M, motional resistance R, and stiffness S, and moving in accordance with

$$M \frac{d^2 y}{d t^2} + R \frac{d y}{d t} + S y = f \qquad \mathbf{(28)}$$

c. The natural-frequency components of the stress factor are given in the following form when the electromagnetic force of equation (**25**) is applied to busbar structures for which the mechanical damping is known.

$$\left.\begin{aligned} p_2 &= C_1 e^{-0.5\alpha' t} \cos q_1 t \qquad \mathbf{(29a)} \\ p_3 &= C_2 e^{-0.5\alpha'' t} \cos q_2 t \qquad \mathbf{(29b)} \end{aligned}\right\}$$

where C_1 and C_2 are the constants of equation (**22**) while α' and α'' are damping decrement constants obtained from free-vibration tests. Observed values of the successive amplitudes of free vibrations may be used to solve numerically α' and α'' from equation (**29**). In calculations it is often convenient to use the term "sharpness of resonance" defined† as

$$\left.\begin{aligned} n_1 &= \frac{q_1}{\alpha'} \\ n_2 &= \frac{q_2}{\alpha''} \end{aligned}\right\} \qquad \mathbf{(30)}$$

Physically, the significance of the sharpness of resonance may be seen by considering, for example, a single-element vibratory system, such as that defined by (**28**), when acted on by a symmetrical harmonic force of amplitude F. The maximum resonant amplitude of vibration‡ is then, very nearly, n times the deflection obtained when a steady force F is slowly applied, where n is the sharpness of resonance defined by

$$n = \frac{\sqrt{\dfrac{S}{M}}}{\dfrac{R}{M}} \qquad \mathbf{(31)}$$

*See Bibliography No. 9 p. 1248.
†See Bibliography No. 17, p. 452.
‡Bibliography No. 31, p. 154.

the constants being those of equation **28.**

d. The forced-frequency components of the stress factor are approximated in the following form at non-resonance when the electromagnetic force of equation (**25**) is applied:

$$p_4 = C' e^{(-g-h)t} \cos \omega t \qquad \mathbf{(32\,a)}$$

$$p_5 = C'' e^{-2ht} \cos 2 \omega t \qquad \mathbf{(32\,b)}$$

Damping, such as encountered in busbar structures, affects the amplitudes in question only slightly at non-resonance.

e. The phases of the components of the stress factor are assumed to be those obtained without current decrement and for zero motional resistance, *i. e.*, in accordance with (**21**). The mechanical constants of busbar structures are such that the phase angles are sensitive to changes of motional resistance, or to the effect of current decrement, at impressed frequencies in the vicinity of resonance only.*

On the above basis, an approximate working formula, taking into account current decrement and damping, for estimating maximum short-circuit stresses in bus supports at non-resonance is then the following:

$$p = \frac{y_s}{y_0} = \frac{2}{3} e^{-2gt} + \frac{1}{3} e^{-2ht} + C_1 e^{-0.5\alpha' t} \cos q_1 t + C_2 e^{-0.5\alpha'' t} \cos q_2 t + C' e^{(-g-h)t} \cos \omega t + C'' e^{2ht} \cos 2 \omega t \qquad \mathbf{(33)}$$

where the constants C are those of (**22**). Equation (**33**) is not applicable to resonant cases in the form given, as already indicated. The practical use of (**33**) calls for the following data:

- the two natural frequencies f_1 and f_2
- the current frequency f_c
- the current decrement constants g and h
- the motional resistance decrement constants α' and α''

Stresses at Resonance

Definition of Resonance. Resonance will be defined as the condition of cyclically increasing stress amplitudes (or vibration amplitudes) occurring when a force of a frequency equal to one of the natural frequencies (f_1 and f_2) is impressed on the bus structure. The four cases of resonance for bus structures were stated in equation (**24**).

Condition for Maximum Stresses. It is well known that, strictly speaking, the maximum deflection due to sustained forced vibrations occurs at an impressed frequency slightly different from the resonant frequency above defined,† the difference depending on the damping constant. The magnitude of the difference, however, is less than one per cent in frequency and less than one per cent in deflection, for the constants coming into question in bus structures. For practical calculations, therefore, the differences mentioned may be ignored.

*Bibliography No. 26, p. 42.
†See Bibliography No. 17, p. 463; also Bibliography No. 26, pp. 41, 42.

(A) Resonance Without Resistance and Without Current Decrement

The four cases of resonance defined in (**24**) are fundamentally similar to one another. As an illustration of the theory, the case of $u_1 = 1$ will be worked out, which is the case of resonance between ω and q_1 in equation (**21**).

As ω approaches q_1 in equation (**21**) the amplitudes C_1 and C' approach infinity. The sum of the terms $C_1 \cos q_1 t + C' \cos \omega t$ must, however, be finite for finite values of time, even if the resistance is negligibly small, since the rate of increase of stress is limited depending on the magnitude of the impressed force at resonant frequency. Mathematically, we may determine the form of the resonant component of stress factor for the case when ω and q_1 become equal, by determining from equation (**21**)

$$\lim_{q_1 \doteq \omega} \left[C_1 \cos q_1 t + C' \cos \omega t \right] = \frac{y_r'}{y_0} \tag{34}$$

where y_r' is the component of support deflection at resonant angular velocity $\omega = q_1$. By evaluating the indeterminate form contained in equation (**34**), in the customary manner, it is found that

$$\frac{y_r'}{y_0} = \frac{4\, u_2^2\, (5 - 2\, u_2^2)}{9\, (1 - u_2^2)^2} \cos \omega t + \frac{2\, u_2^2}{3\, (1 - u_2^2)}\, \omega t \sin \omega t \tag{35}$$

In equation (**35**) the sine term, containing the factor ωt, is the one expressing the progressive increase of vibration amplitude characteristic of all cases of resonance. The expression shows that the amplitude of the resonant component of stress increases at the following rate:

$$\frac{2\, \pi\, u_2^2}{3\, (1 - u_2^2)} F_0 \quad \text{for each half cycle}$$

neglecting the effect of the cosine term which ceases to be of importance after a few cycles. The increase per half cycle may be written in the form

$$\frac{\pi}{2} \left(\frac{4}{3} \, \frac{u_2^2}{1 - u_2^2} \right) F_0 \tag{36}$$

indicating that the increase per half cycle of the support stress component at resonant frequency is equal to that which would be obtained if the support were impelled directly by a harmonic driving force of the amplitude*

$$\frac{4}{3} \left(\frac{u_2^2}{1 - u_2^2} \right) F_0 \tag{37}$$

where the ratio in brackets is a modulating ratio applied to the amplitude of the resonant component of the impressed electromagnetic force. The expression (**37**) may then be regarded as the amplitude of an equivalent driving force impelling the support.

*Obtained from (36) by analogy with the corresponding term for the single-element vibrating system whose motion is derived defined by 28, as indicated in Table II, item 4.

For resonance between q_2 and ω, expressions analogous to (**36**) and (**37**) are obtained with u_1 substituted for u_2.

Since it is desirable to keep the magnitude of the resonant stress as low as possible, the non-resonant natural frequency should be as far from resonance at possible, and preferably of lower value than the resonant natural frequency.

A special case of the equivalent driving force of equation (**37**) is that occurring when the frequency $\frac{q_2}{2\pi}$ becomes very large approaching infinity. Then (**37**) will reduce to

$$\frac{4}{3} F_0 \tag{38}$$

which is nothing else but the amplitude of the impressed electromagnetic force component at resonant frequency. The result (**38**) shows that the double-element system then behaves the same as a single-element system having a frequency equal to the lower natural frequency of the double-element system, as would be expected physically.

To summarize: The maximum rate of resonant stress increase per half cycle is given in (**36**) and is approximately $\frac{\pi}{2}$ times the equivalent driving force amplitude at resonant frequency, according to (**37**). This equivalent driving force is related to the amplitude of the resonant component of electromagnetic force by a simple modulating ratio dependent on the ratio of the non-resonant natural frequency to the current frequency. An expression is given in (**35**) for the stress component at resonant frequency and shows the growth of stress amplitude due to resonance from the instant at which the electromagnetic force is suddenly applied.

(B) Effect of Resistance on Resonant Stress. Current Decrement Neglected

The essential factors defining the resonant stress transient are the amplitude of the equivalent driving force at resonant frequency; the decrement rate due to damping and the current decrement. The equivalent driving force, given in (**37**) for the case $q_1 = \omega$, is practically independent of the resistance when q_2 is non-resonant, but the rate of resonant stress increase is dminished by resistance, so that the final amplitude of resonant stress component (neglecting the electromagnetic force decrement) is n times, rather than ∞ times, the amplitude of the equivalent driving force component at resonant frequency. The significance of n, the sharpness of resonance, has already been pointed out. Current decrement will be considered separately.

The resonant stress formula for the two-element vibratory system—one of the natural frequencies only

being at resonance, the other being outside of the resonance range—will then be worked out on the following basis, when damping resistance is included:

A. That the amplitude of the equivalent driving force component at resonant frequency has the same magnitude when damping is present as for the case when damping resistance is negligible. The damping will, however, materially affect the resonant stress component, and is taken into account in accordance with *B* and *C* which follow.

B. That the final amplitude of resonant stress component is n times the amplitude of the equivalent driving force component at resonant frequency, in accordance with the behavior of a single-element system.

C. That damping resistance reduces the resonant stress increase per half cycle at the same rate for the two-element system as for the single-element system.

A summary of the terms pertaining to resonance, in accordance with *A*, *B* and *C* above, is given in Table II. The principal object of the table is to point out the analogy existing between the behavior of the single-element system (column I), and the two-element system, (column II). It is seen, for instance, in items 3, 4 and 5, that the results previously derived for the two-element system, neglecting damping resistance, differ from the corresponding results for the single-element system by simple modulating factors depending on u_2. In items 6 and 7, column II, the analogy existing between I and II is further applied to the resonant stresses in bus supports when damping is present. Thus the resistance decrement terms

$$n_1 \left(1 - e^{-\frac{\alpha'}{2} t}\right) \text{ for the sine term, item 6,}$$

and

$$e^{-\frac{\alpha'}{2} t} \text{ for the cosine term, item 6,}$$

were adopted for the two-element system by analogy with the single-element system of column I. Consequently for either system the maximum amplitude of resonant stress component is n times the amplitude of the equivalent driving force component at resonant frequency neglecting current decrement, in accordance with proposition *B* above.

TABLE II.
SUMMARY OF TERMS PERTAINING TO RESONANCE
Current decrement not considered

	I Single-element M-R-S system. See Note	II Double-element system; resonance frequency is $\frac{q_1}{2\pi} = \frac{\omega}{2\pi}$
1. Electromagnetic force	$\frac{F_m}{2}(1 - \cos x\,t)$	$F_0\left(1 - \frac{4}{3}\cos\omega t + \frac{1}{3}\cos 2\,\omega t\right)$ (5)
2. Amplitude of harmonic electromagnetic force component at resonant frequency	$\frac{F_m}{2}$	$\frac{4}{3}F_0$ (5)
3. Harmonic stress component at resonant frequency without damping	$-\frac{F_m}{2}\left(\frac{1}{2}\,x\,t\sin x\,t + \cos x\,t\right)$	$\frac{4}{3}\frac{u_2^2}{(1-u_2^2)}F_0\left[\frac{1}{2}\,\omega\,t\sin\omega\,t + \frac{5-2\,u_2^2}{3\,(1-u_2^2)}\cos\omega\,t\right]$ (35)
4. Maximum increase of resonant stress amplitude per half cycle, no damping	$\frac{\pi}{2}\left(\frac{F_m}{2}\right)$	$\frac{\pi}{2}\left(\frac{4}{3}\frac{u_2^2}{(1-u_2^2)}F_0\right)$ (36)
5. Amplitude of equivalent driving force at resonant frequency	$\frac{F_m}{2}$	$\frac{4}{3}\frac{u_2^2}{(1-u_2^2)}$ (37)
6. Harmonic stress component at resonant frequency, damping considered; approximate values	$-\frac{F_m}{2}\left[n\left(1-e^{-\frac{\alpha}{2}t}\right)\sin x\,t + e^{-\frac{\alpha}{2}t}\cos x\,t\right]$	$\frac{4}{3}\frac{u_2^2}{(1-u_2^2)}F_0\left[n_1\left(1-e^{-\frac{\alpha'}{2}t}\right)\sin\omega\,t + \frac{5-2\,u_2^2}{3\,(1-u_2^2)}e^{-\frac{\alpha'}{2}t}\cos\omega\,t\right]^*$
7. Final amplitude of resonant stress component, damping considered; *i. e.* at $t = \infty$	$n\left(\frac{F_m}{2}\right)$	$n_1\left(\frac{4}{3}\frac{u_2^2}{(1-u_2^2)}F_0\right)$ †

*Derives from (35) by analogy with item 6, column I.
†Derived from (37) by analogy with items 4, 5, and 6 in column I.
NOTE: The single-element system dealt with in column I is that defined by (28) when impelled by a driving force F_m $(0.5 - 0.5\cos x\,t)$ at resonance; *i. e.* when $x^2 = \frac{S}{M}$.

(C) Resonant Stresses Including Resistance and Current Decrement

The current decrement will be applied, as in the case of non-resonant stresses, separately to each component of resonant stress, the decrement of each stress component being the same as that of the corresponding electromagnetic force component. In the case of the stress component at resonant frequency, when $q_1 = \omega$,

the decrement term at any time t is then approximately the average of

$$e^{(-g-h)t}$$

during the time interval from zero to t, which average is

$$\frac{1}{(g+h)\,t}\left(1 - e^{(-g-h)t}\right) \qquad (39)$$

Then the stress factor at resonance, including resistance and current decrements is obtained by adding the stress factor components from (27), (29b), (32b), and from Table II, column II item 6, the latter multiplied by the current decrement term (39):

$$p = \frac{y_s}{y_0} = \frac{2}{3}\,e^{-2gt} + \frac{1}{3}\,e^{-2ht} + C_2\,e^{-\frac{1}{2}\alpha'' t}\cos q_2 t$$

$$+ C''\,e^{-2ht}\cos 2\,\omega\,t + \frac{y_r'}{y_0} \qquad (40)$$

where

$$\frac{y_r'}{y_0} = \frac{4}{3}\,\frac{u_2^2}{(1-u_2^2)}\cdot\frac{(1-e^{-(g+h)t})}{(g+h)\,t}$$

$$\left[n_1\left(1-e^{-\frac{\alpha'}{2}t}\right)\sin\omega t + \frac{5-2\,u_2^2}{3\,(1-u_2^2)}\,e^{-\frac{\alpha'}{2}t}\cos\omega t\right] \qquad (41)$$

and C_2 and C'' are the factors defined in (22). Expressions (40) and (41) apply, of course, to resonance between q_1 and ω only, the expressions for the other three cases of resonance being similar.

To summarize: An approximate expression for the stress transient at resonance is worked out including effects of resistance and current decrements. Practical estimates of peak stresses occurring at resonance may thus be obtained when the two natural frequencies, the current frequency, the initial average electromagnetic force, and the decrement constants due to current decrement and due to motional resistance of the bus structure are known.

Stresses at Near Resonance

Two kinds of stress calculations have been considered so far, namely those at non-resonance and those at resonance proper. Non-resonance covers all the cases in which the working natural frequencies differ by 25 per cent or more from the frequencies of the electromagnetic force. There is thus a gap of frequencies 25 per cent above and 25 per cent below each resonance frequency, which gap is not bridged as far as the near-resonance component of stress is concerned. It is believed that in most cases coming within the gap, rough estimates of maximum stresses may be made on the basis of the cases already solved, by interpolation. Analytical calculation of the near-resonance component of stress will not be attempted. The omission mentioned is further justified by the fact that the resonant-frequency stress component is of relatively minor importance in the total stress for those cases in which the non-resonant natural frequency is kept low, as already stated.

Damping and Decrement Constants

A single value of the mechanical damping constant—a value applicable to all cases of busbar stress calculations—can, of course, not be given. Likewise, rates of short-circuit current decrement vary according to the circumstances. For approximate calculations, however, average constants applicable without serious error to a number of cases can be selected.

Damping Constant

As already mentioned, the effect of damping on resonant stresses may be taken into account by the use of the "sharpness of resonance" defined by (30) and (31). Obviously, an increased amount of damping gives a reduced value of resonance sharpness n. Tests performed on a variety of bus structures employing copper bars and porcelain insulators gave $n_1 = n_2 = 5$, on an average. This value was used in the preparation of the stress curve of Fig. 1. Other types of bus structures require, of course, different values of resonance sharpness, in some cases well above 5.

Current Decrement Constants

In (26), exponential rates of current decrement were assumed, different rates being applied to the direct component and to the alternating component. For approximate calculations of maximum short-circuit stresses in busbars, g and h in (26) may be given the values 12.6 and 2.2, respectively, provided 1, short-circuit currents with a maximum amount of initial displacement are considered; 2, the system reactance is within the range from, say five per cent to 30 per cent; and 3, the time of the maximum stress is not later than, say, 0.1 or 0.15 sec. after the beginning of the short-circuit. Tests have shown that the maximum in exceptional cases only, occurs later than 0.15 sec. after the beginning of the short circuit.

Symbols

$\alpha = \dfrac{R}{M}$

α', α'' resistance decrement constants in free vibrations

α_1, α_2, α_3, α_4 roots defined by (12) and (13)

A electromagnetic force per busbar span per unit current

a (in.) dimension of individual busbar lamination, measured in the direction of its deflection due to short-circuit electromagnetic force.

a' (in.) overall dimension of group of busbar laminations, measured in the direction of a.

B substitution for $\dfrac{S_b}{M_b\,M_s}$

b (in.) dimension of individual busbar lamination, measured in the direction perpendicular to its deflection due to short-circuit electromagnetic forces.

b' (in) over-all dimension of group of busbar laminations, measured in the direction of b.

c_1, c_2, c_3, c_4 constants of integration.

C_1, C_2, C', C'' amplitudes of harmonic components of stress factor for zero resistance and no decrement; see (**21**) and (**22**).

e $= 2.718$

l length of busbar span, measured between centers of supports.

F_0 electromagnetic force per span for a current of I_0 per conductor $= A\, I_0^2 = 1.5\, A\, I_m^2$.

F_m maximum value of a suddenly applied pulsating force $F_m \sin^2 \omega t$

f instantaneous value of electromagnetic force per busbar span.

f_c (cyc. per sec.) frequency of current

f_1 (cyc. per sec.) lower natural frequency of bus structure

f_2 (cyc. per sec.) higher natural frequency of bus structure.

f_b (cyc. per sec.) natural frequency of busbar when supports are totally rigid.

$$g_0 = \frac{S_b}{M_b} + \frac{S_s}{M_s} + \frac{S_b}{M_s}$$

g decrement constant of d-c. component of short-circuit current

$$h_0^2 = \frac{S_b}{M_b}\frac{S_s}{M_s}$$

h decrement constant of a-c. component of short-circuit current.

k shape correction factor for the calculation of electromagnetic force.

i instantaneous short-circuit current

I_0 r. m. s. value of initial asymmetrical short-circuit current at time $t = 0$; also $I_0 = 1.225 I_m$.

I_m maximum value of a-c. component of short-circuit current at time $t = 0$.

I_s maximum value of sustained current

j $= \sqrt{-1}$

M_b equivalent busbar mass, concentrated at the center of the busbar span.

M mass of a single-element vibratory system.

M_s equivalent support mass considered concentrated at center line of busbar over the center of the insulator; M_s includes the part of the busbar mass which follows with the support in its motion.

n, n_1, n_2 sharpness of resonance, defined by (**30**) and (**31**)

ω $= 2\pi f_c$ (radians per sec.)

ω_b $= 2\pi f_b$ (radians per sec.)

P (lb. per support) maximum support stress

p stress factor for calculation of support stress

p_1, p_2, p_3, p_4, p_5 components of the stress factor p

q_1, q_2 angular velocities of natural frequency vibrations, defined by (**14**); q_1 is smaller than q_2

R motional resistance of a vibratory system

S stiffness of a single-element vibratory system

S_b stiffness of equivalent busbar in direction parallel to deflection due to electromagnetic force

S_s stiffness of support

s center spacing of parallel busbars

t (sec.) time, measured from beginning of short circuit

$u_1 = \dfrac{q_1}{\omega}$ = ratio of lower natural frequency to current frequency

$u_2 = \dfrac{q_2}{\omega}$ = ratio of higher natural frequency to current frequency

x distance measured along the busbar from the support

x (also) substitution for 2ω

y deflection of a single-element vibratory system

y_b deflection of equivalent busbar span relative to supports.

y_s deflection of support in a direction parallel to the direction of the electromagnetic force

y_1, y_2, y_3, y_4 components of y_s

y_0 average support deflection $= \dfrac{B\, F_0}{h_0^2} = \dfrac{F_0}{S_s}$

y_r' component of support deflection at resonant angular velocity $q_1 = \omega$.

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Electrical Measurement of Physical Values

The Determination by Electrical and Magnetic Means of Quantities not in Themselves of an Electrical Nature

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Member, A. I. E. E.

Synopsis.—This paper was prepared under instructions from the Committee on Instruments and Measurements, with a view to forming a basis for the Committee's work in connection with electrical measurements as applied outside of the strictly electrical field. The system of classification used is in accordance with the nature of the quantity under measurement, rather than with the principle involved; and the subject is treated under nine headings.

Measurement of Temperature: There are recognized two principal methods of electrically measuring temperature; the thermoelectric circuit, and the effect of temperature upon the resistance of many conducting materials. Outlines are given of the elementary applications of these, and a number of practical instruments embodying one or the other of the principles are described. Reference is made to several types of calorimeters embodying electrical methods of temperature measurement, and to hygrometers or humidity meters, in which the principle of the thermocouple has been applied. A brief description is also given of the optical pyrometer.

Measurement of Stress, Strain or Small Changes in Physical Dimensions: General reference is made to the changes which take place in both the electrical and magnetic characteristics of materials when exposed to mechanical stresses, with particular attention given to resistance effects. Several applications of the carbon pile in strain determination are mentioned. Recent developments in the use of the piezo-electric effect of certain crystals are described; and the use of the thermionic tube and its associated circuits for measuring very small dimensional changes is briefly treated. A description of the hot-wire micrometer is given, and a note on the Haigh alternating stress-testing machine. The use of both inductive and resistance circuits in conjunction with the oscillograph for study of vibrations is described: and a reference made to the use of electrical principles in the manometer and engine indicator.

Measurement of Flow: Flow meters are described, making use of the electrical conductivity, the heat capacity and the velocity-head of the fluid under investigation. Prof. Allen's salt-water-velocity method of determining speed of flow in penstocks and conduits is treated. Descriptions are given of a number of flow meters, both for gases and liquids, in which the principle of cooling of a heated conductor is employed, and reference is made to experiments which have been carried out with a view to establishing a definite relationship between the electrical resistance of an electrolyte and its linear velocity past electrodes. Electrical methods of measuring and recording the pressure head due to velocity in a pipe are found in the "Republic" and the General Electric flow meters, both of which are briefly described. Mention is made of a recently developed method of plotting stream-flow diagrams, by reproducing the physical conditions by an electrical system and locating equipotential lines with an alternating current potentiometer.

Measurement of Velocities: The stroboscopic methods of measuring velocities, while not fundamentally electrical, are so closely associated with electrical practise as to receive considerable attention, and the "oscilloscope" for visualizing reciprocating motions is touched upon. The magnetic tachometer and the squirrel-cage speed-indicator are mentioned, as well as the ordinary electrical speed indicators, wherein measurement is made upon the current of a small generator driven by the shaft under measurement. The condenser method of determining linear velocities, and the several inductive methods used in ballistics are treated.

Measurement of Work: No fine distinction is made between work force and energy: and the standard methods of measuring electrical energy with the wattmeter are omitted, as being outside the scope of the paper. Six types of electrical transmission and absorption dynamometers are described, and a reference made to recent tests, in which the characteristics of a steam locomotive were determined by causing it to pull an electric locomotive, measurement being performed upon the regenerated current of the latter. The Gilson device for determining over-all efficiency of a power plant, by balancing in an instrument two electrical forces, one representing the fuel input, and the other the electrical output is briefly treated.

The Measurement of Radiant Energy: Many measurements of radiant energy are made by determining the heat generated when that energy impinges upon a surface; and these would strictly come under the head of temperature measurement. It is realized that it is difficult to apply the term "non-electrical" or "non-magnetic" to any radiant phenomenon: and this section of the paper confines itself to a bare mention of a number of the best known photoelectric cells, used principally in photometry and closely allied work.

Chemical Measurement: The Electrical determination of concentration of the hydrogen ion is treated in some detail. The various electrical and magnetic methods of studying and treating the ferrous metals are described, with particular reference to the "hump" method of steel treating. The application of the cooling properties of various gases upon heated wires, is mentioned, and descriptions are given of the CO_2 and CO recorders. Note is made of the fire-damp detector, and the dionic water tester.

Navigational Measurements: Coupled with measurements used in navigation are found those used in the detection of hidden conditions, such as the locating of ore bodies and water pockets. The mariner's compass is given as the oldest application of magnetic principles to physical measurements, and mention is made of the very recent application of the earth-inductor both as a compass and as an inclimometer. Special attention is given to those electrical aids to navigation developed under the stimulus of the great war, including the leader cable, radio direction-finding and methods for locating mines, submarines and icebergs, as well as sound-ranging and acoustic methods of measuring ocean depths.

Among the devices for detecting unseen conditions are mentioned the water-vapour detector, the magnetic method of locating flaws in steel, and systems which have been employed in the detection of theft or of the presence of unauthorized persons in certain places. A number of methods, more or less sound, for locating ore, oil or water deposits, are described in some detail, and under the same head are mentioned such devices as egg testers and sex detectors.

Physiological Measurements: Attention is called to the rapid advances which have lately been made in the application of electrical measurements to the diagnosis of pathological conditions. The method of diagnosing disease by the so-called "method of electronic reactions" is dismissed as not meriting the attention of electrical men until its proponents are prepared to submit a description couched in intelligible terms. Note is made of the value of electrical measurements of temperature in diagnostic work: and the electrocardiograph is treated fully, both in its application to actual studies of heart performance and in the investigation of psychological conditions. Brief descriptions are given of the "Stethophone" and the "Audiometer," both of which seem destined to fill an import-

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ant place in diagnosis and medical investigations in general.

In conclusion it is pointed out that the art of applying electrical methods of measurement to work outside of the electrical field is advancing so rapidly as to make it practically impossible that one could keep absolutely up-to-date in his knowledge of all the branches. The general effect is one of bringing together technicians in varied branches of scientific pursuit and finding for them a common ground of thought. Wide and diversified as are the practises of measurement in these branches, it is felt that the Institute forms the natural clearing house for such information; and it is to be hoped now that the ground is fairly well cleared, that the Committee on Instruments and Measurements will be kept cognizant of as many as possible of the new developments which will be made in the application of electrical principles to the measurement of non-electrical quantities.

Acknowledgment is made of the valuable help which has been rendered the author by men both within and without the electrical sphere: and a bibliography of over 300 references is appended.

* * * * *

Introduction

While the duties of the Institute's Committee on Instruments and Measurements have to do primarily with the measurement of electrical quantities used in the work of the electrical engineer, they also cover the cognizance of all electrical measurements, including the determination by electrical methods of quantities which in themselves may be far removed from the field of electrical engineering. Technicians in all branches of science and engineering have been quick to realize the advantages obtainable in performing their quantitative measurements by electrical means; and, more or less independently, these workers have developed and adopted such methods and have announced their successes through such channels as would most readily reach their interested confreres.

Electrical men have, for some time, felt that a number of important applications of their principles of measurement were thus escaping the notice of those whom they should most concern; and with a view to bringing these diverse practises under one head, the author was instructed to collect all possible information in respect to the work which had been done, and to compile the material as a basis of reference, so that the Committee might henceforth fulfil its several duties with a fair knowledge of the use which was being made of the art of electrical measurement by specialists in other fields of scientific pursuit.

The principal published sources from which information has been drawn appear in the appended Bibliography; and while this cannot, in any sense, be considered as complete, an endeavour has been made to collect such references as are available—mostly in the English language—to enable those interested to gain a working knowledge of the use which has been made of electrical principles in the developments described.

General

It is doubtful if there is any physical quantity subject to measurement, upon which that measurement has not been performed or attempted by electrical or magnetic means. Aside from all determinations of purely electrical quantities such as voltage, current and power, the electric current, by virtue of its ease of transmission, its infinite divisibility, lack of inertia, and the facility with which it may be measured, recorded or integrated, furnishes a medium almost ideal for the quantitative expression of other physical values. It is to be expected, then, that we should have a host of counting devices, time-measuring mechanisms, position indicators and kindred appliances for adding convenience to measurements already performed by other means. The principal purpose of this paper, however, is not to discuss these, but rather to consider those methods in which some inherent property of the electrical circuit is made use of to actually perform the quantitative measurement and translate the measured value into definite terms. While the greatest number of these are of an electrical nature, there are some which embody the principles of the magnetic circuit; and, because of the close association, it is only logical that they should here be classed along with the electrical methods.

The system of classification used is in accordance with the nature of the quantity under measurement, rather than with the principle involved; but the overlapping which must necessarily follow, in cases where the same principle is applied in work of differing types, has, as far as possible, been reduced to a minimum.

The types of measurement which are performed by electrical means have been divided into nine classes; and while it may seem that this system has resulted in the association of some strange neighbors, it is felt that fundamentally the grouping is that of methods basically kindred in their nature.

1. Measurement of Temperature.
2. Measurement of Stress, Strain or Small Changes in Physical Dimensions.
3. Measurement of Flow of Liquids and Gases.
4. Measurement of Angular and Linear Velocity.
5. Measurement of Work.
6. Measurement of Radiant Energy.
7. Determination of Chemical and Molecular Condition.
8. Navigational Measurement, and Detection of Hidden Conditions.
9. Physiological and Allied Measurements.

I. Measurement of Temperature

While one of the earliest applications of the electric current to measurement outside of the electrical field lies in determination of heat effects, it probably still remains the branch in which the greatest advances and developments have been made within recent years. Since temperature effects play an important

part in other electrical measurements it is fitting that this association of electrical quantities should receive the first attention. Whether the problem be one of detecting temperature changes of a microscopic part of a degree or of controling and recording the performance of industrial furnaces and boilers, the electrical method usually furnishes the most workable solution, and usually at a minimum of cost.

Until late years, the importance of accurate temperature measurement seems to have been overlooked by many branches of industry. For instance, in the treatment of steels, much was left to the judgment of the operator, with a consequent lack of uniformity in the product. And what may be said for the metal industry may be applied to almost every branch of activities wherein temperatures other than atmospheric were concerned. Recently, however, there seems to have been an awakening to the importance of accurate temperature determination, with the result that there have appeared a large number of excellent temperature measuring devices, giving not only rapid and accurate results, but making possible the reading and recording at a central location of temperature at a large number of different points.[1, 2, 3]

Most electrical thermometers are based upon one or the other of two principles, either the potential difference at a junction of two dissimilar metals maintained at a temperature differing from that of the rest of the circuit, a property discovered by Seebeck in 1821; or upon the change in electrical resistance accompanying change in the temperature of any pure metal. A very recent development depends in its operation upon the change with temperature of the magnetic permeability of certain ferrous alloys.[4]

In 1833 the thermopile was combined with the galvanometer by Nobili and Melloni to produce a temperature-measuring instrument of great sensitivity. In 1887, Boys greatly increased the sensitivity of this apparatus by combining the two elements in one instrument, so that the thermo-junction formed a part of the galvanometer loop, and swung with it in the field of a magnet. In 1881 Langley contrived another instrument of very great delicacy for measuring radiations, which he called a *bolometer*. This consisted of four very fine platinum or iron wires, forming the arms of a Wheatstone bridge connection. Alternate strips were exposed to radiation, and a measurement made of the unbalance of the bridge due to changes in the temperature, (and consequently of the resistance) of the arms. The thermo-galvanometer of Boys was estimated to indicate the one hundred-millionth of a degree, while the bolometer was capable of about one-tenth of that sensitivity.

In measurements embodying the use of the thermoelectric principle, it is usual to place one couple in as close association as possible with the body under measurement, and the "cold junction," necessary to complete the circuit, in a location where its temperature can be conveniently determined or regulated. The voltage set up in the circuit is then a measure of the temperature difference between the junctions, and therefore of the temperature of the "hot junction." Measurement of this voltage may be directly made with an instrument of the d'Arsonval type, or it may be performed with a potentiometer, which may be either indicating or recording. Where the direct deflecting instrument is used, a current in proportion to the measured voltage will flow through the circuit; and this, by the Peltier effect, will tend to equalize the temperature of the junctions, thus crowding the scale in its upper ranges. The selection of metals employed will vary with the conditions under which the measurement is made, depending upon the precision and permanency required, upon the temperature range through which the couple may be carried, and upon the possibility of corrosive effects in the atmosphere surrounding the thermo-junction.[5, 6, 7, 8, 9, 10, 11]

In one form of temperature-measuring instrument operating upon this principle, known as the radiation pyrometer, developed by C. Fery, the junction is not placed in direct contact with the heated body, but is used in combination with an optical focusing device,

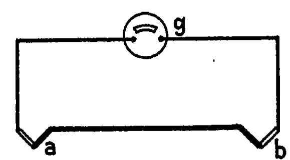

Fig. 1—Elementary Circuit of Thermo-Electric Pyrometer

(a) "Cold" couple, at standard temperature
(b) "Hot" couple, exposed to temperature under measurement
(g) Galvanometer

so that when a certain image of the incandescent surface is obtained in the telescope field, the exposure of the couple is such that the indication of an associated galvanometer is a measure of the temperature. Fery has also produced a bomb calorimeter of interesting construction, in which the steel body of the bomb is plated with copper and supported by two constantin disks. The change of temperature attendant upon combustion of the charge produces a thermo-electromotive force in the structure; and the value of this voltage is read upon a suitable galvanometer directly graduated in calories. An amplification of this principle is found in an isothermal calorimeter exhibited in 1924 by the Research Department of the Arsenal at Woolwich, England. The bomb or container in which combustion takes place is associated with a double set of thermo-junctions, one serving as a source of measurement and regulation, and the other, using the Peltier effect, to neutralize the temperature rise due to the combustion under measurement. The current in the Peltier junctions is regulated, through the agency of the thermocouples, to maintain the tem-

1. For all references see bibliography.

perature of the bomb constant within 0.0001 deg. cent. and a measurement of this current furnishes a direct indication of the rate of heat evolution.[12, 13, 14, 15, 16]

It is characteristic of all pure metals that they change their electrical resistance with changes in temperature; and this property has been widely used in measurement of temperatures. As a rule the detector employed in this class of work consists in a small coil of wire (usually copper, platinum or iron), forming one arm of a Wheatstone bridge, the remainder of the bridge circuit being at or near the point where the indication is required. (see Fig. 2). As in the case of the thermo-

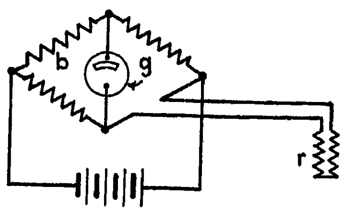

FIG. 2—ELEMENTARY CIRCUIT OF RESISTANCE THERMOMETER
(b) Wheatstone bridge circuit
(r) Resistance coil, exposed to temperature under measurement
(g) Galvanometer

electric system, the indicator may be either direct reading, in which the galvanometer in the bridge circuit is graduated in thermal units, or the galvanometer may be incorporated in a zero-reading device, either indicating or recording. In either case the simplest arrangement is that embodying the direct reading instrument; but far greater accuracy and higher precision are obtained when zero methods are used. In determination of temperatures of small areas the thermo-electric method is usually the more satisfactory; while if the

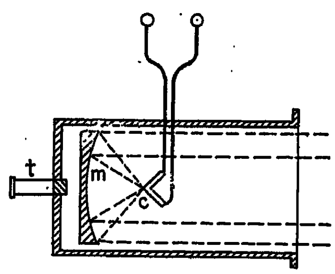

FIG. 3—RADIATION PYROMETER
(m) Concave mirror
(c) Thermocouple
(t) Focusing telescope

average temperature of a considerable mass or area is required, the resistance method is often the better. This is particularly evident in the determination of the temperatures of the windings of electrical apparatus. "Hot spots" are better located by thermocouples placed in proximity to the windings, while average temperatures of coils may be measured by closely associated resistance units or by direct measurement of the resistance of the windings themselves.[17, 18, 20, 21, 22, 23]

Another application of the electric current in temperature measurement is found in the optical pyrometer used in the determination of temperatures in the luminous range (Fig. 4). Here the heated body or surface is viewed in contrast with the filament of an incandescent electric lamp, and the current through the latter adjusted until the filament diasppears against the heated background. A reading is then made of the current, from which the temperature of the filament and consequently of the heated body may be determined (Fig. 5).[24, 25, 26, 27]

Since both the thermo-electric effect and the resistance method are particularly suited to the measurement of temperature difference, use has been made of them in determining quantities which depend primarily

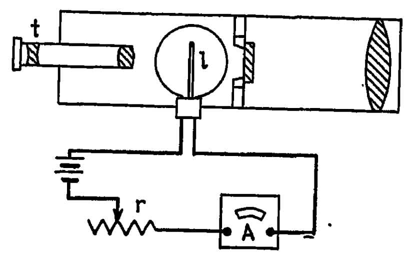

FIG. 4—OPTICAL, OR DISAPPEARING-FILAMENT PYROMETER
(l) Incandescent lamp
(r) Adjusting rheostat
(t) Viewing telescope
(a) Ammeter in lamp circuit

upon this difference. A heat-flow meter dependent upon temperature gradient in a known mass, has been developed for study of boiler-room conditions; and a humidity recorder, depending for its action upon an electrical measurement of the difference in temperature of a wet and a dry-bulb thermometer, has recently made its appearance in several forms. Another form of

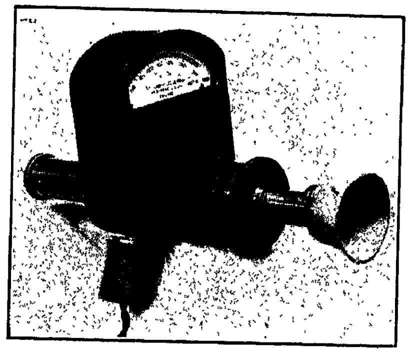

FIG. 5—OPTICAL, OR DISAPPEARING-FILAMENT PYROMETER

hygrometer, lately exhibited in England, directly determines the percentage of moisture in the air by measurement of the dew point. Radiant heat from a small lamp falls upon a polished metal surface, whose temperature is regulated by a brine coil, whence it is reflected and focussed upon a small thermo-junction in the circuit of a galvanometer. When dew forms upon the polished surface the thermo-electric current is reduced; and a distinct disturbance of the galvanometer indication takes place. The sharpness of indication and the essential simplicity of this apparatus make its use highly desirable in hygrometric work, particularly

at low temperatures. Many other applications of electrical methods of measuring temperature and temperature difference will be found in the subsequent sections of this paper.[28, 29, 30, 31, 32, 33, 34]

II. Measurement of Stress, Strain or Small Changes in Physical Dimensions

While it has long been known that molecular strain has the effect of producing measurable changes in the electrical and magnetic characteristics of many materials, the principal uses which have so far been made of these properties in measurement of the stress responsible for distortion are associated more with contact surfaces and air-gaps than with the masses of the stressed materials. The change in contact resistance accompanying change of pressure is well known in its application to the carbon pile, long used for rheostats, where smooth regulation is required. It is only recently, however, that this property has been used in precise quantitative measurement; and in a "Telemeter," lately developed in the Bureau of Standards, the inherent objections to the use of the carbon pile for quantitative work seems to have been removed by a simple system of compensation, and the principle made available for measurements requiring a considerable degree of precision. The basic idea of the tele-

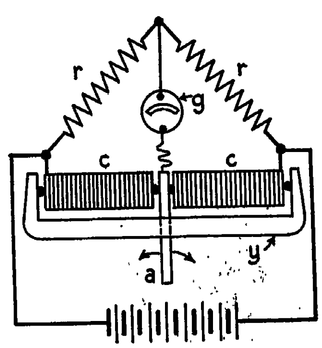

Fig. 6—Carbon Pile Telemeter

(c, c) Carbon Stacks
(r, r) Balancing resistances
(a) Moving element
(y) Supporting yoke
(g) Galvanometer

meter lies in the use of two similar carbon stacks, built up of rings, carefully selected and ground, placed under a predetermined stress, and connected as adjacent arms of a Wheatstone bridge circuit (Fig. 6). With this arrangement, changes in temperature, or over-all pressure and other disturbing factors tend to balance out; and the force under investigation, producing a difference in pressure of the two piles, is the only external influence reflected in the electrical circuit. This principle has been used in the study of not only slowly changing stresses, but, in conjunction with the oscillograph, of vibratory forces of considerable frequency. During the last few months, experiments have been carried out by the Bureau of Standards, in which carbon piles are permanently cast into concrete structures, so that stress determinations may be made at any future time desired.[35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45]

The piezo-electric effect in cyrstals of quartz and certain salts has been employed with great success in many studies of stresses and mechanical vibrations. The best exemplification of this property is probably that found in Rochelle salt. Though its characteristic of exhibiting certain electrical potentials when under stress has been known for some time, it remained for engineers in the laboratories of the Western Electric Company in 1917 to bring this property to practical application in engineering work. By proper prepara-

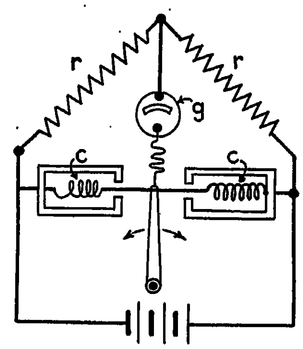

Fig. 7—Hot-Wire Micrometer

(c, c) Helical coils
(r, r) Balancing resistances
(a) Moving element
(g) Galvanometer

tion and treatment of the crystal it was possible to greatly magnify the weak effects which had been observed; and, in general, it was found that the best effects are obtained when the crystal is submitted to torsional forces, when, under proper conditions, a potential difference of several volts may be obtained between points on the crystal before the elastic limit of the material is reached. While the earlier experiments with the piezo-electric effect were directed principally toward the development of sound transmitting and reproducing apparatus, considerable recent work has been done in its use for the measurement of stresses and vibrations in machinery and mechanical structures. The freedom from frictional or inertia effects makes the principle particularly desirable in the study of rapidly changing stresses, and the property has been used with great success in obtaining indicator diagrams of explosion pressures in heavy ordnance.[46, 47, 48, 49, 50, 51]

In the measurement of very small distances or changes of length, there has been found a valuable application of the thermionic tube and its associated circuits. In general the two points between which the measurement is to be made are attached to, or themselves become, the plates of a condenser which is a part of a tuned circuit. A microscopic change in the distance between the points is enough to materially change the capacity of the condenser, and by disturbing the tuning of the system, to introduce beat sounds or other

effects by which the change in dimensions may be gaged. [52, 53, 54, 55, 56, 57]

A novel application of combined temperature and resistance effects to the measurement of small deflections is found in a hot-wire micrometer recently exhibited in England. Two small enclosing cells each contain a loosely wound helix of copper wire. These coils are connected mechanically and electrically by a fine straight wire which holds them in a slight degree of tension. This wire forms the point of attachment of the member the motion of which it is desired to detect, and incidentally of the galvanometer in a Wheatstone bridge circuit. The two coils thus form adjacent arms of the bridge, which is balanced by two other arms in the ordinary way. Any movement of the connecting wire between the coils, in the direction of its length will lengthen one coil and shorten the other, thus changing the ratio between radiating surface and heat generated by the bridge current, and altering the relative temperature of the coils. (See Fig. 7). This, in turn, produces a change in the resistance ratio, with a subsequent unbalancing of the bridge; which unbalance may either be read directly on the galvanometer or compensated by one of the standard types of recording instruments. This arrangement has been devised particularly for application to the measurement of small deflections, such as are produced in specimens under mechanical stress and for application to bourdon tubes. It is claimed for the device, that there may be obtained a magnification of one thousand times between the original motion and the deflection of the indicator. An entirely different application of the convection of heat from small wires to the measurement of pressures is found in a thermal vacuum gage employed by some investigators. A fine electrically heated wire is placed in the evacuated space, and observations made of its resistance, as affected by temperature. As this temperature is a function of the generation and dissipation of heat, it is thus made possible to determine with great accuracy the density of the gaseous medium. [58, 59, 60]

A flexible system of studying small deflections has recently been developed by a German engineer. The points between which measurement is to be made are connected by fine stretched wires, forming a bridge circuit. This circuit feeds an oscillograph, and enables records to be obtained of movements both sustained and oscillatory. Oscillographic records of vibrations in machinery have also been obtained by connecting inductive circuits to the vibrating parts and supplying to the instrument the small currents set up in these circuits by vibratory action. [61, 62, 63, 64, 65]

A unique application of the two-phase current to mechanical testing is found in the alternating stress-testing machine, developed by Professor Haigh of the Royal Naval College, Greenwich, England. This device is used in the performance of fatigue tests on specimens which are required to withstand a great number of successive stresses. The vibrating element of the machine is excited by two-phase currents having a periodicity of about 1000 cycles per minute, and the stresses in opposite directions are equalized by balancing these currents in a differential ammeter. The armature to which the vibratory forces are transmitted carries a set of small search coils; and the average value of the stress is determined by measurements made of the voltage induced in these coils. [66]

In the "Low" oscillograph manograph, use is made of a microphone and a special very heavy design of telephone receiver, in the determination of phenomena attending the performance of internal combustion engines; and with it, it is possible to obtain indicator diagrams, as well as to determine general operation conditions, under very exacting and severe circumstances. [67, 68, 69, 70]

III. Measurement of Flow of Liquids and Gases

The flow of both liquids and gases can be determined by a number of electrical methods; and several of these have become standardized in modern engineering practice. In liquid flow meters use is made of either the electrical conductivity or heat capacity of the fluid, or electrical measuring devices may be employed as auxiliaries in the determination of the pressure head developed by the flow of the liquid through an orifice or a Venturi tube.

A highly satisfactory means of determining the velocity of flow of water, particularly applicable to large conduits, such as penstocks, is found in Prof. C. M. Allen's recently developed "salt-velocity" method. There is injected into the penstock a quantity of saline solution, the presence of which has a marked effect upon the electrical conductivity of the water. The passage of the solution is detected by means of an electrode consisting of a pair of parallel plates projecting well into the liquid and connected in series with a detector circuit. A number of schemes have been used for determining, from the current flow across the electrodes, the velocity of the water in the pipe. One is to measure the time lapse between the injection of the salt solution and the time of maximum conductivity between the electrodes situated a measured distance along the conduit. Another employs two similar pairs of electrodes, and the time interval is taken between the instants of maximum conductivity at the respective stations. While the electrical readings may be obtained with ordinary ammeters, there has been developed an ingenious recording arrangement, wherein the record appears on a graphic chart as two series of dots from which the time interval may be scaled off at leisure. [71, 72, 73]

Experiments have been carried out with a view to establishing a definite relationship between the resistance to passage of an electric current between two electrodes and the velocity of motion of the water in which they are immersed; but in the present stage of development, this relationship, while it unquestionably

exists, has been found to be of too erratic a nature to justify its employment for flow determination.[74, 75, 76]

A device designed particularly for measuring oil-flow in transformers consists in a very small ball of copper wire which is inserted in the pipe. This ball forms one arm of a bridge circuit, the other arms being varied in such a way as to maintain a condition of balance. By this means a determination is made of the number of watts necessary to keep the

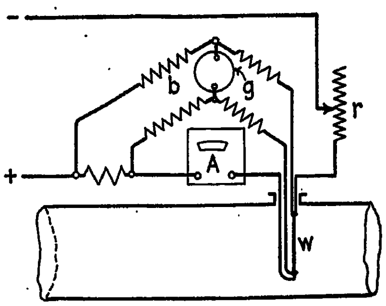

FIG. 8—KING GAS METER
(w) Heated wire
(b) Kelvin bridge circuit
(r) Adjusting resistance
(g) Galvanometer in bridge
(a) Ammeter in current circuit

little ball at a constant temperature. Referring these values to a calibration curve, it is possible to read off the flow of oil in the transformer cooling system. A somewhat similar application is found in the Griffith's depth gage, used for measuring the amount of liquid in tanks. Here the radiating conductor takes the form of a straight slender wire reaching down into the liquid

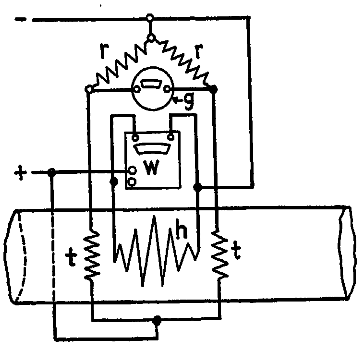

FIG. 9—THOMAS GAS METER
(h) Heating elements
(t, t) Measuring resistances
(r, r) Balancing resistances
(g) Galvanometer in bridge circuit
(w) Wattmeter in heater circuit

from above. A similar wire, parallel to the other, but shielded by a tube from actual contact with the liquid, serves to compensate for changes in the temperature of the liquid. The cooling of the exposed conductor is proportional to its immersion, and the unbalance so introduced is read on a galvanometer or recorded by a graphic instrument. A device of this nature, used in connection with a calibrated weir forms a very convenient flow meter.[77, 78, 79]

In the measurement of gas-flow, a wide use has been made of the principle known as the hot-wire anemometer, in which a heated wire is exposed to the flow, and an electrical determination made of the cooling effect upon the wire. For most practical conditions the indications of the anemometer depend upon the product of density and velocity. In the meter developed by Dr. L. V. King, an electrically heated wire reaches across the pipe in which the measurement is to be made and means are provided for adjusting the current so as to maintain the wire at a fixed value of resistance (Fig. 8). A measurement of this current gives the instantaneous value of the flow of gas, and the results may be obtained in an indicating, graphic or integrated form. A very similar device has been used to measure the air-intake on internal combustion engines.[80, 81, 82, 83, 84]

The cooling effect of the gas stream is made use of in a somewhat different way in the Thomas gas meter, which is based upon the measurement of the amount of heat required to raise the temperature of the gas

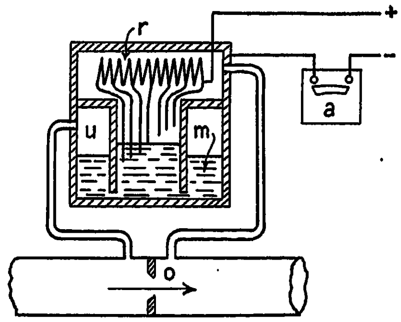

FIG. 10—REPUBLIC FLOW-METER
(o) Orifice or Venturi tube
(u) Modified U-tube
(r) Calibrated rheostat
(m) Mercury
(a) Ammeter in current circuit

through a known range. Two similar resistance thermometers, forming two arms of a bridge are inserted in the main, and between these is placed a heating element. Both the heater and the resistance thermometers are well distributed over the cross section of the pipe (Fig. 9). While this meter has a number of forms, its commercial arrangement is such that a constant temperature difference is maintained between the two resistance thermometers by means of adjustment of the current flowing in the heater, and measurement is made of the power represented by this heating current. The meter is made in a recording form, and may be made to read either in pounds or in cubic feet at any desired pressure and temperature.[85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97]

The importance of knowing the amount of steam consumed in power plants has led to the development of a number of practical flow meters, which have now become standard equipment in all large steam-driven stations. The pressures, temperatures and velocities here met with prohibit the methods which are appli-

cable to ordinary gas-flow measurements and recourse is had to the orifice or the Venturi tube. The pressure-head, so developed, may then be determined by electrical means. One of the best known is the Republic flow-meter, in which the U-tube normally found as an accessory of the Venturi meter is developed into a reservoir containing mercury, which, as it rises, short-circuits the ends of a number of vertical rods of different lengths forming the terminals of resistances (see Fig. 10). Changes in the mercury level thus serve to operate a rheostat, this rheostat forming a part of a circuit supplied from a source of constant voltage. Thus, the value of current flowing in the circuit depends upon the flow through the pipe, and by properly proportioning the resistance between rods, it may be made directly proportional to the flow. The current may be made to actuate an indicator or a recording instrument; or, if desired, it may be passed through an integrating meter to produce a record of the volume of fluid which has passed through the pipe. [98, 99, 100, 101]

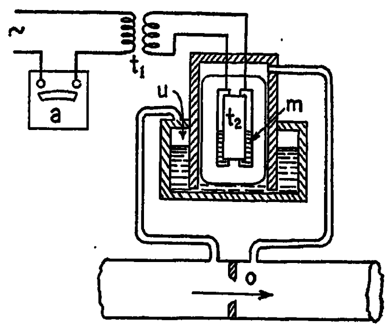

Fig. 11—G. E. Flow-Meter

(o) Orifice or Venturi tube
(u) Modified U-tube
(t_1) Step-down transformer
(t_2) Measuring transformer
(m) Mercury, forming secondary of measuring transformer
(a) Ammeter in A. C. supply circuit

A development somewhat similar to the above is found in the flow-meter recently announced by the General Electric Company, shown in Fig. 11. In this, the column of mercury, instead of bridging the points of a rheostat, forms a short-circuited secondary to a small transformer. As the height of the mercury changes, the cross-section of the ring varies, and with it, the secondary conductivity. This produces a variation in the primary current which may then be measured by any of the standard methods for metering alternating current quantities. The same apparatus has been applied in the indication of vacuum and water level. [102, 103, 104]

As an illustration of the ease with which the properties of the electrical circuit lend themselves to the investigation of problems in other branches of science, there may be mentioned a method of tracing the steam-lines of flow about a ship or other diverting body placed in the path of a fluid. Use is made of the fact that the stream lines for flow past an obstacle are coincident with the equipotentials for a conductor of the same shape in an electrostatic field. A model of the figure to be investigated is placed in a rectangular tank full of water, and containing two sheets of aluminum mounted on its longer sides. Alternating-current of audible frequency is supplied to the sheets, and two electrodes, one fixed and the other movable and attached to a pantagraph which allows it's position to be recorded on a sheet of paper, dip into the water. These electrodes are connected to a low frequency amplifier and a telephone. By moving one electrode to balance out the sound, it is possible to establish a series of equipotential lines corresponding to various settings of the fixed electrode, and thus to very faithfully duplicate the conformation of stream lines in a fluid body. [105]

IV. Measurement of Angular and Linear Velocities

Where alternating-current generators are attached to rotating shafts the electrical measurement of angular velocity consists simply in determination of the frequency of the generated electromotive force which may be accomplished with one of the many forms of electrical frequency meter. In cases where the frequency meter does not possess the requisite accuracy or precision, use may be made of the alternating-voltage as an element in some more precise type of measurement. Where the problem is one of determining the relationship between two speeds, as in the measurement of slip in an induction motor, there are available a large number of stroboscopic methods either partially or wholly electrical. Some part of the revolving part of one machine may be viewed, either through a slit operated synchronously with the other, or by illumination from a source which flickers at a speed definitely related to the velocity of the other. In this conncetion the neon lamp has proved particularly valuable. Or one machine may be caused to operate a synchronous contactor in the circuit of a current which is either generated or commutated by the other. In any of these stroboscopic methods the result is a beating effect, whose period represents the difference in speeds of the machines, and which may be determined with far greater precision than could either speed alone. [106, 107, 108, 109, 110, 111, 112, 113]

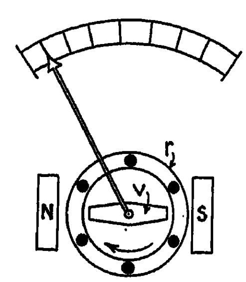

Fig. 12—Squirrel-Cage Tachometer

(n, s,) Permanent magnet
(v) Pivoted vane
(r) Rotor

Where absolute measurement of the speed of one machine is required, a stroboscopic method may still be used, the standard velocity being obtained from a pendulum or tuning fork. When comparatively low angular velocities, such as those of large generators, are to be determined, the measurement may conveniently be made by having a contactor on the shaft operate one pen on a chronograph, in comparison with another operated from a standard pendulum. An interesting application of the stroboscopic principle lies in the Elverson "Oscilloscope," a device designed for visualizing rapidly reciprocating parts of machinery.[114, 115, 116, 117]

A speed indicator depending for its action upon the modification of the main field by the cross-magnetizing effect of the armature current in a small generator is that known as the squirrel-cage type (Fig. 12). The field is produced by a permanent magnet, between the pole-pieces of which rotates a squirrel-cage very similar to the electrical circuit of an induction motor secondary. Within this cage, pivoted upon the main axis of rotation, floats on a small soft iron vane, to which is attached the pointer of the instrument. With the rotor at rest,

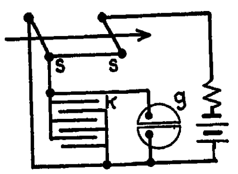

FIG. 13—CONDENSER METHOD OF DETERMINING VELOCITY

(s, s) Strips successively broken by projectile
(k) Calibrated condenser
(g) Ballistic galvanometer

the vane lies along the magnetic axis, and the pointer indicates zero. As the rotor revolves eddy currents flow in the bars, tending to set up a cross-field in the air-gap of the magnet, thus modifying the resultant direction of the field, and causing the vane to take up a new position of equilibrium. The extent of deflection of the vane depends upon the amount of field distortion, and hence is representative of the velocity of the rotor.[118]

Not unlike the squirrel-cage indicator is the well-known magnetic tachometer widely used on automobiles. Here a small permanent magnet rotates within a metal cylinder, inducing eddy-currents and dragging the cylinder around, against the counter-torque of a spring, so that the deflection is proportional to the velocity of rotation. The graduations are marked upon the outer surface of the drum, and are observed through a small window in the casing of the instrument.

The electrical tachometer, in which speed measurement is obtained by means of a small generator driven from the shaft, is standard equipment supplied by a number of manufacturers. In some forms of this type the generator produces alternating-current, and measurement is made on the frequency; but the most common form generates a direct-current the pressure of which is measured with an ordinary voltmeter graduated in revolutions per minute.

There have been developed a number of essentially electrical methods of measuring linear velocities, particularly at very high speeds such as are found in studies of projectile flight. In one of these the projectile is caused to pass through two solenoids in succession, the time interval being determined by timing the impulses set up in a current flowing in the coils. A more precise method employs the discharge of a condenser whose circuits are successively interrupted by the passage of the bullet. (see Fig. 13.) Two strips of tinfoil are placed in the line of flight, and connected in such a way that the rupture of the first removes a short-circuit from the condenser, while the breakage of the second open-circuits the source of supply. The charge accumulated in the condenser in the interval is then measured by an electrometer or otherwise; and from the characteristic curve of charge the value of the time interval may be determined with great precision. It is claimed for this method that the velocity of an ordinary rifle bullet may be determined when the strips are only an inch apart. The method has also been applied to measuring the duration of very short intervals of contact; and a similar device has been adapted for the determination of angular velocities.[119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130]

In an apparatus developed by Mr. R. Ferguson of the Research Department, Woolwich Arsenal, for determining flash velocity and pressure factors of ignitory detonators, the detonator is placed on the top of a vertical tube. The hot ionized gases rushing down the tube close successively two circuits of a battery, condenser valve and one string of an Einthoven galvanometer, from which records of the speed can be obtained. Between the tube and the steel block upon which it rests is placed a turmaline crystal in circuit with a valve and a ballistic galvanometer to measure the piezo-electric current generated by the pressure of the explosion.

V. MEASUREMENT OF WORK

In a general study of the measurement of work, it is not necessary that hair-splitting distinctions be made between *work, force* and *energy*; and some of the methods particularly applicable to the one quantity, are so principally because of their adaptability to the other. Since energy may, itself, be an electrical quantity, the wattmeter and the watthour-meter may be looked upon as instruments performing measurements of work; but, leaving these, which lie outside the scope of this study, there remain a number of devices for measuring physical work by electrical means.

The most common device for direct measurement of work is the dynamometer; and both the transmission and absorption dynamometer have appeared in electrical forms. The transmission type usually takes the form of a torsion meter, measurement being performed

either upon a portion of the main shaft of the unit under test or upon a prepared member of known resiliency.

In the Moore dynamometer, a special spring is inserted in the shaft upon each element of which is mounted a small alternating-current generator. The two generators are identical in design, and are excited with their fields in series from a common source of direct-current. When there is no torque in the shaft, and consequently no twist in the spring, the adjustment of the machines is such that they are of exactly opposite polarity. However, as soon as any angular displacement occurs, a resultant voltage appears, the two generated voltages being no longer in opposition; and since this voltage is a function of torque multiplied by speed, a properly calibrated voltmeter will give a direct measurement of the energy being transmitted by the dynamometer. By measuring either of the component voltages alone, a direct determination of the velocity of the shaft may be obtained.[131]

An interesting form of dynamometer, developed by Messrs. Johnson and Denny, operates on the induction principle, using a differentially wound telephone as a detector. On the shaft in which the torsion is to be measured there are fixed at a suitable distance apart, two light non-magnetic wheels, each carrying a permanent magnet with a rather sharp point. Fixed near the periphery of these wheels are inductor units consisting of sets of little coils arranged about a small angle of the arc of rotation. Each coil of the inductor may be separately connected to the detector circuit by means of dial switches, so that it is possible to oppose any coil in one unit to any coil in the other, through the telephone detector. By using different numbers of coils on the respective inductors a vernier arrangement is obtained, so that by setting the dial switches, coils varying by small angular increments on the two wheels may be opposed. In operation the dials are manipulated until a condition of least noise in the telephone is obtained, when the angle represented by the setting is that of torsion of the shaft, from which value the transmitted power may be computed.[132]

Another electrical type of torsion meter developed by the same investigators depends upon the variation of the constants of a transformer with change in the air-gap. Two iron sleeves, carried on the two ends of the shaft under test, form a part of the magnetic circuit of a small transformer. As the torsion of the shaft varies, the air-gap between these sleeves changes, with the consequent result, that owing to the redistribution of flux, there is a change in secondary voltage. The torque is then determined from the indications of a voltmeter in the secondary circuit. A modification of this arrangement, due to Moullin, includes a generator driven from the same shaft, so that, the frequency changing with the velocity, the current through the magnetizing coil is independent of the speed of rotation and gives a direct measure of the twist or torque in the shaft. This current, bearing a linear relationship to the torque, may be measured either directly in an ammeter or recorded by an oscillograph. This form of the instrument has been found particularly useful in measuring the torque in Diesel engines and other prime movers.[133, 134, 135, 136, 137]

In the absorption dynamometer, the energy transmitted from the shaft is all converted into heat or electrical energy, the measurement performed upon the factors controlling the conversion. There are two principal electrical forms of this dynamometer, known respectively as the eddy-current, and the generator types. In the eddy-current dynamometer the power under measurement is delivered to a disk or other form of short-circuited conductor rotating in an adjustable magnetic field. The torque is opposed by weights, as in the prony brake, but the retardation is adjusted by regulating the field current to a value where a condition of equilibrium exists. The actual quantitative measurement is made upon the weights, and the electro-magnetic effects may be said to serve only as a convenient means of absorbing the energy. The generator type of dynamometer has provision for making a direct measurement of either the mechanical torque or the electrical energy produced. The best-known form of this apparatus consists in a direct-current generator, having its field structure carried on knife-edges, allowing it to rotate through a small angle against a measured force. The output of the generator may be regulated and also metered, thus providing two independent means of determining the value of the power. In many instances an added advantage of this scheme lies in the possible regenerative use of the dynamometer, the power being returned to the main circuits of the plant or employed for other useful purposes. Perhaps the most recent development in electrical dynamometers is that in which an electric locomotive is made use of in testing a steam locomotive. The latter pulls the former along the track; and load is supplied by making use of the regenerative feature of the electric unit, electrical power being back-fed into the trolley and measured with ordinary electrical instruments.[138, 139, 140, 141, 142, 143, 144]

An interesting though hardly precise form of power plant instrument recently developed in England is the Gilson "Efficiency meter," devised for the purpose of giving, at any time, an indication of the efficiency at which a steam-driven plant is operating. The metering element consists in a movement similar to that used in the power-factor meter, wherein a pointer takes up a position representative of the ratio of two currents flowing in separate windings. One of these windings is supplied with current whose value is controlled by the rate of fuel supply to the boiler, while the other receives its current from a transformer or shunt on the main bus. The ratio of these two quantities, as shown on the indicator, will then be a rough measure of the over-all efficiency of the unit under consideration, or of the whole plant, according to the arrangement of the

electrical circuits. The limitation of this type of instrument would appear to lie in the precision with which the input energy may be measured; and it would seem that in hydroelectric plants or in stations using powdered or liquid fuel, where a fairly accurate and continuous measure of the input may be obtained, it might find considerable application, particularly if a differential instrument instead of a ratio type of indicator were employed, the final measurement being one of over-all losses rather than of efficiency.[145]

VI. The Measurement of Radiant Energy

The close association which exists between radiant and electrical effects, as we now understand the terms, gives rise to a host of points of contact between the two classes of measurable quantities. But because of the difficulty of definitely applying the term "non-electrical" or "non-magnetic" to any radiant phenomenon, we are limited in this study to a few of the more outstanding measurements where we may safely feel that we are not too far trespassing upon the preserves of the physicist.

The effect of light upon the electrical resistance of selenium and similar metals is familiar to all; and while many uses have been made of this property, it can not be said that any great progress has been made in the quantitative measurement of illumination by this means. This is due principally to the selective quality of these materials for various wave-lengths differing from that of the retina of the eye, to such an extent as to render impracticable, the replacement of the eye in photometry. A lack of constancy which further characterizes selenium precludes its use for measurements where there is need for any degree of maintained accuracy. Recent researches have placed before us a mass of valuable material in regard to the relations between radiant energy and electric-current phenomena; and the new developments in photo-electric cells already in evidence point to a great expansion in the application of electrical principles to the practice of photometry and allied measurements. The photoelectrical properties of the materials which have been investigated may manifest themselves in a change of electrical resistance or of thermo-electro-motive forces, a change in the electron discharge in an evacuated tube or by a definite electrical potential difference between parts of the specimen under exposure to the radiant energy. A photo-electric cell developed in 1923 by the General Electric Laboratories utilizes the principle of electronic discharge, and has already found wide application in the sorting of materials according to colour.[146.........179 incl.]

The best-known methods of measuring radiant energy are, however, those which employ the thermal effect due to the impinging of the energy upon a surface to be heated; but these are more properly discussed under the head of "Temperature Measurement," and as such have been briefly touched upon in their elementary forms, as the thermo-galvanometer and the bolometer, and will be further referred to in the discussion of iceberg detectors under the heading of "Navigational Measurements."

VII. Determination of Chemical and Molecular Conditions

The rapidly closing gap between the phenomena of chemistry and those of electricity has been bridged be electrical measurement so often as to make it probable that the writer of a few years hence will not be able to class measurements in the field of chemistry as applying to "non-electrical" quantities. Electrical measurements have so intruded themselves into the work of the chemist as to make it appear that in the very near future his voltmeter, his potentiometer, and his oscillograph will be as much a part of his standard equipment as are his present-day reagents and balances.[180]

An electrical measurement which has found very wide application in both organic and inorganic chemistry is the determination of hydrogen ion concentration. The essential characteristic of an acid is its replaceable hydrogen atom, or, in terms of the theory of electrolytic dissociation, its ability to give off hydrogen ions in water solution. In general it may be said that the amount and the speed of the action of an acid is dependent only upon the concentration of the hydrogen ions. When a metal is dipped into an acid solution there at once takes place an interchange of ions, resulting in the establishment of a definite electrical potential between the metal and the solution. If two similar electrodes are dipped into two solutions of different concentration, and these solutions be electrically connected, the potentials will be different; and if an external circuit be provided there will be a tendency for current to flow between electrodes. The potential difference will be proportional to the concentration of the ions of the electrodes in the respective solutions. If, then, an electrode of hydrogen is used, there will be developed, in such a cell, a potential representative in its value of the concentration of hydrogen ions in the electrolytes. Such an electrode may be prepared by depositing a film of finely divided platinum, palladium or iridium upon an inert metal and saturating it with hydrogen gas at a definite pressure. The metal is inert as regards hydrogen ions; hence any electrical potential developed between electrodes is due to the respective concentrations of hydrogen ions in the solutions.[181, 182, 183, 184, 185, 186, 187, 188]

As any flow of current at the time of measurement will have an effect sufficiently disturbing to vitiate the results it is necessary that the measurement of potential be made by some method wherein no current is drawn at the time of reading. There are three general methods whereby this may be done, embodying the use respectively of the quadrant electrometer, the condenser and the potentiometer. Of these the last named is capable

of giving the greatest accuracy, and is probably more convenient in its operation than either of the others. Any of the standard laboratory types of potentiometer may be used; and, if required, the apparatus can be obtained in a continuously recording form. The electrometric method has been successfully applied in determining the acid content of lubricating and transformer oils.

An interesting use of the oscillograph in chemical work is found in a recently published study of the rate of diffusion of certain metals in solutions of their salts. This work, carried on by Professors Miller and Burt-Gerrans, of Toronto University, consists in a detailed study of the intensity of concentration of electrolytes during short time intervals after the application. While it is essentially a fundamental investigation in pure science, it is one whose results should have immediate application in such work as battery charging from rectified currents, and electrolytic processes in general.[189, 190, 191, 192, 193]

Much has been accomplished in the adaptation of electrical measurements to studies of the chemical and molecular properties of steel and ferrous metals in general. A rather indirect method of determining

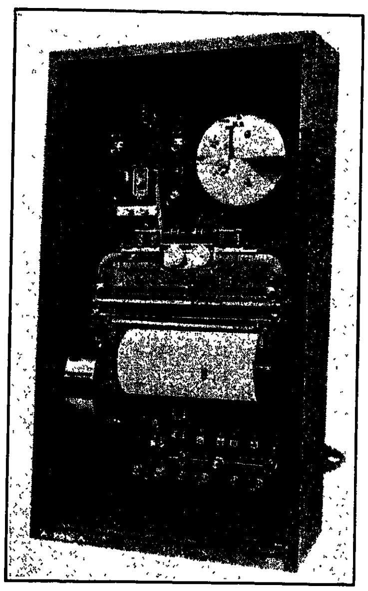

FIG. 14—CALLENDAR (CAMBRIDGE) RECORDER

the carbon content of a steel consists in passing the carbon dioxide produced by direct combustion of the metal into a solution of barium-hydroxide of known resistance and basing the results upon a determination of the increased resistance of the solution due to the precipitation of barium ions. In a practical development of this method, as worked out by Messrs. Cain and Maxwell, of the Bureau of Standards, great accuracy has been obtained, and results can be produced in a very short time. The products of combustion are passed through a special absorption vessel, and the electrical resistance measured by means of an alternating-current bridge operating on commercial frequency. The resistance values are then applied in a special nomographic chart which compensates for temperature conditions and gives a direct reading of the percentage of carbon in the steel.[194]

Through a system of magnetic analysis, the percentage of carbon in steel is determined in the "Carbometer" developed by Messrs. Malmberg and Holmstroem. The method requires a test piece cast in a special mold to fit a space provided in the apparatus so that it be-

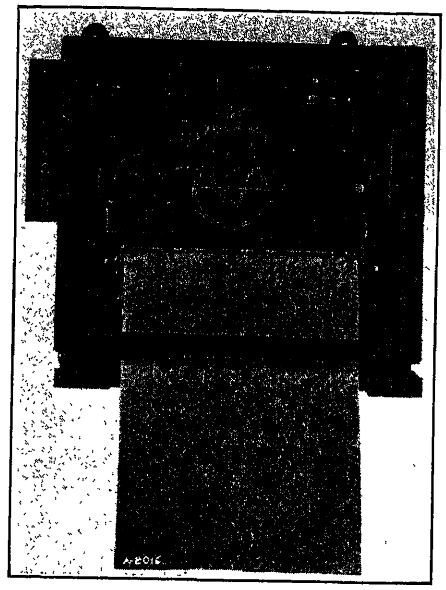

FIG. 15—LEEDS & NORTHRUP RECORDER

comes a part of the magnetic circuit of a special type of generator. The action of the generator, which is operated by clockwork, is to expose the specimen to the alternate action of two magnetic fields of different intensity. After a few cycles of magnetization the relation between the greater and the lesser induction becomes practically constant, and the armature of the generator comes to rest in the position which gives the stronger field through the specimen. By depressing a button, the armature is then caused to rotate to the position of weaker field and the change of flux determined by a coil linked with the magnetic circuit and connected to a ballistic galvanometer. Since the reading is representative of the difference of the two fluxes in the one specimen it has been found that slight flaws, cracks or roughnesses have but little effect upon the final results. With the same treatment it is found that a very definite relationship exists between the readings of the galvanometer and the percentage of carbon in the specimen, enabling determinations of carbon to be made with an accuracy equivalent to that obtaining with the most careful chemical analysis. The carbometer has also been applied in the determination of the amount of other metals such as aluminum, silicon, manganese and chromium in the iron.[195, 196, 197, 198]

Certain operations in the treatment of steel and ferrous alloys in general should take place at definite temperatures which are closely related to what are known as the "transformation points" of the metal. At these critical points there occur important changes in the internal characteristics of the steel; and these, being reflected in a number of electrical and magnetic phenomena, may be determined with great accuracy by electrical means. The critical points have been studied by observations of the thermo-electric effect, the electrical resistance, the magnetic permeability and the heating and cooling curves; and while there does not as yet seem to obtain a perfect agreement among the several methods, there has been collected a vast amount of data, enabling heat treatment to be carried out with an accuracy and precision quite sufficient for the most exacting requirements of modern industry.[199]

The thermo-electric method of determining the transformation points of iron-carbon alloys consists in making up a combination of thermo-couples in which the iron under investigation in the form of a fine wire, forms a part. To the two ends of the wire specimen are welded thermo-junctions, usually platinum, platinum-rhodium; and the wire is placed in a furnace in such a way that there is a slight temperature gradient between its ends. The thermo-couples give the temperatures at the ends of the wire, while the voltage between the platinum leads represents the thermo-electromotive force of the iron against platinum for the temperature and temperature range indicated. The results of these measurements, when plotted graphically, show a distinct discontinuity in the relationship of the respective curves as the critical ranges of the alloys are reached, and make possible the accurate determination of the temperature at which the transformations take place.[200, 201, 202]

In studying the critical ranges of iron by the resistance method, a curve is plotted of electrical resistance against temperature, when it is found that distinct points of inflection make their appearance at temperatures corresponding to the transformation points. When the magnetic method is employed curves of permeability are plotted against temperature, giving very precise results until the magnetic characteristic of the iron almost disappears, at a temperature of about 780 deg. cent.[203]

The method which seems to have found the greatest use as a basis of heat treatment in industry is that based upon a study of the heating or cooling curves of the material. As the changes which occur at the critical points are accompanied by an absorption or evolution of heat, it follows that under the condition of heat being supplied to or abstracted from the specimen at a uniform rate, the temperature of the metal will not follow a uniform law of change, but will pause at a more or less constant value until the change in the structure of the material is completed, when it will resume its change in proportion to the transfer of heat. As a result of this characteristic a curve of temperature of the metal will exhibit a "hump," which indicates the transformation point of the material, and thus gives a very definite point of reference for the control of tempering or hardening operations. A practical apparatus utilizing this principle for the treatment of steel employs a standard type of potentiometer-pyrometer, making its record upon a strip of coordinate paper. The record takes the form of a series of temperature cycles as the furnace is heated and cooled, and as each batch approaches its maximum temperature, there is noted a distinct "hump" in the curve, as the transformation point is reached. (Fig. 16) By carrying the heating to a specified amount beyond this point, and then quenching, (or annealing, as the case may require) a very high degree of uniformity of product in various batches is virtually assured. This method in the past few years has found a wide application both in the heat treatment of tools and dies, and in production work, such as the hardening of gears, ball-races and small parts for automobiles and other machinery.[204, 205, 206, 207]

Certain phases of the chemical analysis of gases may be performed by electrical methods based upon thermal conductivity. Some of the earliest work on this principle was done in England by Mr. G. A. Shakespear; and his method has since made its appearance in a practical form of carbon dioxide recorder, produced both in England and America. The fundamental action of the device is in reality that of the hot-wire anemometer. In practise there are usually two similar heated wires, enclosed in adjacent cavities in a metal block, forming two arms of a Wheatstone bridge. One of the wires is exposed to air or some standard gas, while the other operates in an atmosphere of the gas to be measured. (Fig. 17.) Any difference of temperature, as indicated by an unbalance of the bridge, must then be due to a difference in the thermal conductivity of the gases, from which the characteristics of the gas under investigation may be determined. This apparatus is available in a graphic form, so that a continuous record may be obtained where desired, as in the case of flue gas analysis in power plants. The principle of this device has also been successfully applied in the following measurements: Hydrogen in electrolytic oxygen, Oxygen in electrolytic hydrogen, SO_2 in contact gases and the permeability of fabrics. Another electrical method of determining the percentage of CO_2 in a flue gas bases its results upon the change of resistance of solution through which a sample of the gas is passed. This has already been discussed among the methods for determining the carbon content of steel.[208, 209, 210]

As an accessory to the carbon dioxide recorder, there is sometimes furnished a carbon monoxide recorder, which, while of very similar construction to the former, operates upon a somewhat different prin-

ciple. (Fig. 18.) As in the CO_2 recorder, there are found the two wires in adjacent chambers, forming a part of a similar circuit. Here, however, the wires are kept at a much higher temperature than in the CO_2 recorder, so that the conductor in the active chamber serves as a catalyzer and ignites any CO, which may be present. The rise in temperature due to the combustion is much greater than that which is produced in the CO_2 meter by the change in heat conductivity of the

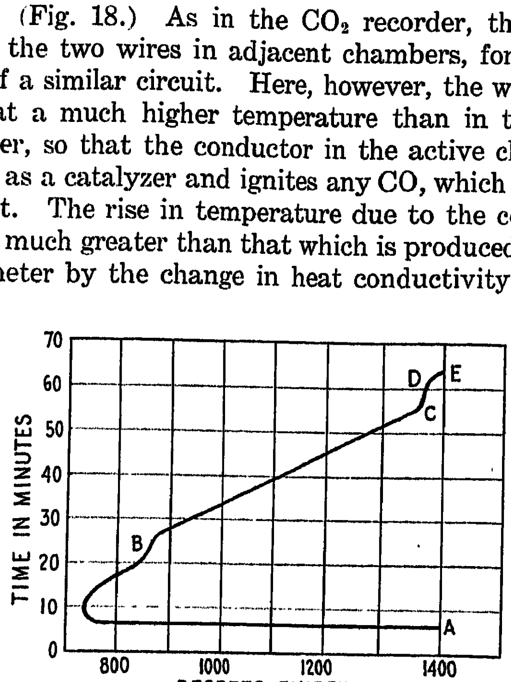

Fig. 16—Chart Showing the "Hump" Method of Steel Treatment

gases, completely overshadowing any change due to the presence of incombustible components and furnishing an almost perfect indication of the presence of CO, or, to be more precise, of ($CO + H_2$), in which terms the instrument is sometimes calibrated. Or, as is sometimes done, a determination of CO may be made by

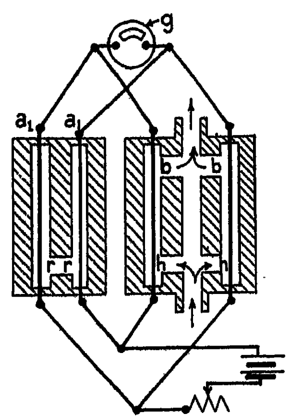

Fig. 17—Electrical CO_2 Meter

(a, a) Chambers containing inert gas or air
(b, b) Chambers containing gas to be measured
(r, r) Balance resistances
(h, h) Variable resistances
(g) Galvanometer in bridge circuit

converting it into CO_2 and passing the gas a second time through a CO_2 meter. Another form of CO recorder uses the electric current as a catalyzing agent only, leaving the final determination to a measurement of the change in volume between the incoming and outgoing gases.[211, 212]

The catalytic property of heated platinum in the presence of certain combustible gases has found an important use in the fire-damp detector used in coal mines. The principle of the several forms is essentially that of a Wheatstone bridge containing in its circuit one or more arms of platinum wire. In an atmosphere containing fire-damp local combustion raises the temperature, and consequently the resistance of the platinum arms, and produces an unbalance which may be read quantitatively on a galvanometer, or may be made to operate a relay, giving an alarm of the presence of the combustible gas.[213, 214, 215, 216, 217]

In the Dionic water tester of Messrs. Digby and Biggs, produced by Evershed, we have a very interesting application of the use of the electrical conductivity of water as a basis in the determination of the percentage of impurities. The instrument consists in a form of direct reading ohm-meter, similar to the well-known Megger, supplied with a specially designed water cell and graduated upon its scale in units of conductivity. By means of tables furnished by the makers

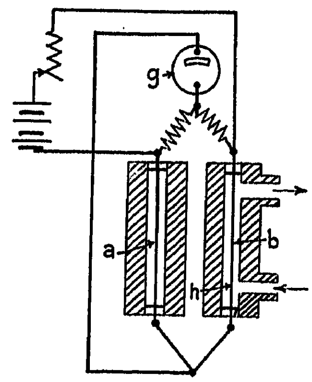

Fig. 18—Electrical C O Meter

(a) Chamber containing inert gas
(b) Chamber containing gas to be measured
(r) Fixed resistance
(h) Variable resistance and ignition
(g) Galvanometer in bridge circuit

of the instrument, it is possible to determine and measure almost infinitesimal quantities of dissolved substances. The principal uses of this instrument are found in the testing of water supplies for cities and for use in power plants, as well as in determination of leakage in surface condensers. Similar applications have very recently been made of electrical conductivity measurement in the determination of the characteristics of organic solutions; and probably the most recent lies in the electrical measurement of the percentage of butter fat in cream.[218, 219, 220, 221, 222, 223, 224, 225]

VIII. Navigational Measurements and Detection of Hidden Conditions

The earliest application of magnetic principles to physical measurement lies undoubtedly in the mariner's compass. The fact that the earth is surrounded by a field, so that a magnetic needle takes up an approximately north and south position seems to have been known to the Chinese at least a thousand years before

the beginning of the Christian era, and two thousand years before this property was known in Europe. Indeed as early as the third or fourth century "Chinese vessels navigated the Indian Ocean under the direction of floating magnetic needles pointing to the south." (Humboldt.) For centuries the magnetic compass has changed but little in form; and until very late years there was found no substitute for the simple structure of a small magnet carried on a frictionless bearing and kept in a reasonably level plane. The prime cause of recent improvements in the magnetic compass is the extremely disturbing condition to which aviation instruments are exposed. This has led to the utilization of the earth-inductor, in which a coil of wire is caused to rotate in the earth's field, the magnitude of the voltage so generated being a function of the orientation of the coil with respect to the earth. The earth inductor has been used thus, not only as a compass but as an inclinometer.[226, 227, 228]

Like many other branches of science, the use of electrical and magnetic aids to navigation received a powerful stimulus in the great war, and for many months the rivalry, not only between contending forces but among friendly navies in their efforts to outstrip one another in the development of these methods of guiding themselves through unchartered waters, knew a keeness which has never been equalled in times of peace. While many of the results of such work have been strangled with red tape, a goodly proportion actually found their way into actual use, and rendered great service; and some have survived to the extent of becoming standard practise, not only in our navies but with the merchant marine of the present day. Leaving aside the gyroscopic compass, a pre-war development employing electromagnetic principles in only an auxiliary sense, there may be mentioned under this head the leader gear, the several systems of radio direction-finding and beacons, as well as electrical applications in depth-sounding and the location of icebergs and other obstructions.[229]

The leader-cable, now permanently installed in several important waterways, was originally developed by Drs. L. A. Herdt and R. B. Owens of McGill University, and a trial installation successfully operated near Sorel, on the St. Lawrence River in 1903 and 1904. The cable is usually laid in the navigable channel; and the ships which make use of it are equipped with sensitive detecting apparatus whereby they may be kept in a position directly over the course of the cable. The particular use of the leader-gear lies in enabling ships to find and enter an estuary in foggy weather, and when usual aids to navigation, such as buoys, light-ships and landmarks, are obscured. The scheme embodies an inversion of some of the systems which have been used for locating faults in defective circuits, in that an alternating current carried in the cable induces effects in the detector circuit at the point of observation. A signaling key is inserted in the cable circuit at the transmitting station; and this enables the current to be interrupted, so that messages may be transmitted if desired. The distortion of magnetic field from the cable, due to the iron in the construction of the ship has a favorable effect, in that it greatly increases the closeness with which the ship may be kept over the line of the cable. With receiving coils mounted on the side of the ship, signals are good up to about 400 yards; and with an auxiliary system of towed electrodes, the cable may be detected at more than 1200 yards.[230, 231, 232, 233, 234, 235]

Quite apart from its sphere of communication, the principle of the wireless telegraph has been employed in a number of systems of direction-finding by radio; and these are proving of inestimable value, particularly in the navigation of coastal waters. There are at present three principal methods by which radio is applied in direction-finding. The best-known of these, the Bellini-Tosi, system employs two land stations or "beacons," sending more or less continuous signals of prearranged characteristics. The ship, equipped with a rotatable loop-antenna, is able to determine the directions from which the respective signals emanate, as well as the angle subtended by them. It is then possible, either by communication with these stations or by direct computation, to quickly and precisely determine the position and course of the vessel. Several modifications of the rotatable-loop receiving antenna are in use.[236, 237, 238, 239, 240, 241]

In the short-wave Marconi system, transmission is made from a coil instead of from the usual aerial and the coil is maintained in rotation at a constant rate about a vertical axis. A maximum amount of energy is radiated in the plane of the coil; and in a perfect system no signals can be obtained in a plane perpendicular to this. The result is a rotating radio beacon. The ship picks up the signals on its ordinary aerial system; and as the transmitting coil rotates, the intensity of the signal varies from a maximum to zero twice for each revolution. When the intensity is zero, the navigating officer knows that the plane of the coil is at right angles to him. A system of distinguishing signals transmitted by the coil when it is due east and west, and when due north and south serves to establish a time upon which the officer may base his calculations and determine the direction from the transmitting station. Two such beacons determine the position of the ship.[229]

Another type of beacon carries two coil-aerials with their planes displaced through an angle of about 135 degrees, a signal being sent alternately from one and the other coil. A ship located on the bisector of the angle between the coils will receive the two signals with equal intensity. Thus a definite course may be held simply by navigating so that the signal strength from the two coils remains balanced.[242, 243, 244]

During the war there were developed numerous electrical and magnetic methods of locating submarines and floating mines; and while, for some of these, most extravagant claims were made, and others, owing to

the sudden cessation of hostilities, proved abortive, there yet remain a number which rendered yeoman service under actual warfare conditions. Apparatus employing the piezo-electric effect of crystals was developed, depending for its operation upon the response of the crystals to vibrations set up by the propeller of the submarine; but the best known, and probably the most useful were those schemes using the alternating magnetic field, and operating on a principle basically that of the Hughes induction balance of 1879.

As an outstanding example of this application, may be mentioned the work done by Messrs. J. B. Whitehead and L. O. Grondahl, first at Johns Hopkins University and later, on vessels afloat, under the jurisdiction of the U. S. Navy. The elementary construction of the device used consists in a long alternating electromagnet, having within its field, but some distance away, a detecting coil, so placed that normally no electromotive force is set up by the field of the magnet. If a considerable mass of magnetic material comes within the range of the magnet, the disturbance of the field is sufficient to upset the balance, causing in the detector circuit a voltage, which not only reveals the presence of the distrubing element, but may also give an indication of its direction.[245]

In measuring the depth of navigable water, as well as in detecting the presence of icebergs or other obstructions, in foggy weather, advantage has been taken of the known speed of propagation of sound waves through the medium. A characteristic sound wave is produced, and the distance determined by the time elapsing until its echo is received. While this is not primarily an electrical method, the success which has attended its use has been due almost entirely to the use of electrical means both for the production of the sound and for the sensitive and accurate determination of the instant of its return. Employing the linear induction motor developed by Professor Fessenden for this purpose, it has been possible to establish communication between moving ships by telegraph, and even for short distances by telephone.[246, 247, 248, 249]

Considerable work has been done in various quarters in the detection of invisible objects by heat radiation. In general, a sensitive thermo-electric cell is used in the focus of a parabolic reflector and the presence of any object of a temperature differing from that of the background of the field of the reflector sets up electromotive forces which are detected by a sensitive galvanometer in the circuit of the cell. During the war this device was made use of in such work as detecting the presence of reconnoitering and working parties in "No-man's land." A similar development is that of A. Larigaldi of Paris for detecting icebergs at sea. In this he makes use of a tellurium cell, very sensitive to a band of heat waves in the infra-red, which have a peculiar property of traveling through fog with little loss of intensity. The cell is protected from air currents and stray radiation by being mounted in a small glass housing fitted with a window of sylvinite, (a crystal transparent to this particular band of radiation), the whole being mounted at the focus of a searchlight mirror about three feet in diameter. The resistance of the tellurium element is balanced in the usual way; but an innovation is introduced in the use of currents of telephonic frequency, and the detection of a condition of unbalance by the use of thermionic amplifiers and a telephone. The mirror is mounted so as to sweep the horizon, and an adjustment is made for minimum disturbance in the telephone. If, for some reason, radiation from the horizon is interrupted by an iceberg (or other obstacle), a sound will be heard in the telephone every time the searchlight is turned toward that particular direction. By this means, large icebergs have been detected as far as six miles away.[250, 251]

An interesting application of the electrical resistance thermometer to navigational purposes is found in a method of iceberg detection developed by L. V. King and H. T. Barnes of McGill University. A continuous record, to a very small part of a degree, being obtained of the temperature of the water through which the vessel is passing; the presence of an iceberg manifests itself by a lowering of this temperature due to the surface layer of recently melted water which has not yet mixed with that of the ocean. While in conditions of calm weather this method has much to recommend it, it has been found that disturbing conditions, such as introduced by ocean currents, are too great to make it infallible in its results. Experiments are still in progress in connection with the applications of this method of recording ocean temperatures to various practical applications, such as meterology.[252, 253]

Much valuable work was done during the great war in the practise of sound-ranging on land. In general there were placed at predetermined locations two or more sound detectors, circuits being carried from these to a central point, where records were made by an Einthoven string-galvanometer or an oscillograph. One form of detector was made up of a bolometer placed behind a very thin rubber diaphragm. A sound wave impinging upon this film caused a momentary condensation and rarefaction of the air about the bolometer, with a consequent impulse in the electrical circuit. By scaling the spacing of the several impulses as recorded by the galvanometer, it was possible to triangulate for the location of the origin of the actuating sound. In this way enemy guns were located with a high degree of accuracy.

The principle of the hot-wire anemometer has been applied in a form of inclinometer for use in aviation. The heated conductor is enclosed within a tight tube, and its cooling depends upon the convection currents in the enclosed air, which currents are much modified by the position of the tube with respect to the horizontal. It is thus possible to obtain from the instruments in the bridge circuit of the anemometer a direct indication of the inclination of the machine. The earth-inductor

referred to in an earlier paragraph, has also been used as an inclinometer.[254, 255, 256]

A very sensitive device for the detection of water vapour in closed pipes has been developed by Messrs. Weaver and Ledig of the Bureau of Standards. A small glass cell is coated on the outside with a platinized surface having its electrical conductivity broken by an etched line. The two areas of metal are connected to separate leads sealed into the glass. The line of separation is painted over with some hygroscopic electrolyte, and the outside of the cell is exposed to the gas whose purity it is desired to observe. The leads are connected into the circuit of an alternating-current bridge; and measurements of the resistance of the cell furnish a very precise indication of the quantity of water vapour which may be present in the gas.[257]

A method originally developed in the Bureau of Standards for the determination of the uniformity of specimens for magnetic tests was turned to a very practical application during the war, in the location of flaws in steel for rifle barrels. In this apparatus the iron under investigation forms a part of the magnetic circuit of a form of continuously operating permeameter; and as the specimen passes slowly through the instrument, irregularities in its structure make their presence very evident by marked changes in the reluctance of the material. It is possible that this method may find application in the testing of such materials as steel rails and cables.[258 ...... 278 incl.]

Many ways have been proposed for the location of ore bodies beneath the surface of the ground, of water pockets, and of buried treasures in general; and these, like methods of producing rain, may be classed as either religious, magical or scientific; and, as in rainmaking, while a study of the first two does not come within the sphere of this paper, an analysis of the so-called scientific methods is generally productive of the most disappointing results. Most of the electrical methods which have been suggested embody either an application of the Hughes induction balance or some most delicate system of electrostatic measurements; and it is very difficult to establish authentic records of metal-bearing ores or other valuable material having been definitely located by such means.[279]

One scheme which has received some note in the technical press is that of H. Lewis, which embodies an application of Kirchoff's laws of potential distribution. An electric field is generated in the area to be investigated; and, with a telephone, a series of level lines is determined on the surface. By observing the disturbances in these lines, it is claimed that the presence and location of deep-lying ores of conducting material or of water may be determined.[280, 281]

Devices purporting to make use of magnetic disturbances due to earth currents in the vicinity of deposits of ore, oil or water have from time to time made their appearance; and some of them show great ingenuity in their design and construction. Probably the best known of these are the instruments of Schmidt, and of Mansfield, which are essentially similar in their arrangement. The general construction, as described in one of the patents, is as follows:

> A horizontal hollow glass cylinder is surrounded by a layer of paper impregnated with paraffin. Around this layer of paper a well annealed iron wire is spirally wound, the successive windings of the same spiral line not being in contact, and the successive layers of wire being separated from one another by a layer of paper impregnated with paraffin. Here and there a layer of paper treated with paraffin is covered with tin-foil over which the wire is wound. The outside layer of wire of the coil thus formed is covered with paper. The coil is open and completely insulated, that is to say, not connected with any source of current. Above this coil a glass plate is placed, in the middle of which is a fine point which supports a needle. The needle is feebly magnetized and turns easily on the point.

It is stated that the instrument will

> Indicate certain atmospheric changes, the nature and cause of which are not yet understood but which manifest themselves in a peculiar way in the neighborhood of the source and course of subterranean waters by rapid oscillations of the pointer of the device.

It is claimed that when the instrument is in the vicinity of a source or a stream of subterranean water the needle will after a time oscillate rapidly. Whether designed for ores, water or oil, there is little structural difference in the instruments.[282, 283]

It was the writer's privilege to take an examination of one of these devices, known as a "water finder," and to study in detail the functioning of the various parts. Structurally the instrument agreed closely with the description given above, and the workmanship was of the highest order. The suspension of the needle was so delicate that slightly magnetized tools some distance away would produce a noticeable disturbance. As no known laws of electricity or magnetism are applicable to the coil beneath, it would appear that any movement of the needle which may take place must be due to changes in the external magnetic field, modified perhaps by the presence of the iron coil, which would represent a magnetic cylinder of comparatively low permeability.

While the instructions accompanying the device very particularly state that it should be protected from the direct rays of sunlight and that it is not effective in locating streams that have "sprung up to daylight," no mention is made of the necessity of shielding the delicately poised needle from nearby magnetic bodies, such as tools, other electrical instruments, or telephone receivers, or from electric currents due either to neighboring conductors or to ground returns. In view of the numerous vitiating influences admitted by the makers of the instrument and of the fact that systematic magnetic surveys of the earth have not shown any local earth or air currents upon which its operation could be based, it can only be concluded that the invention is unsound in principle, and to be classified along with magnetic egg testers, sex detectors, and other devices calculated to prey upon the credulity of those who are

susceptible to impression by high-sounding terms.[284, 285, 286, 287]

Cases are on record where principles of electrical measurement have been made use of in the detection of theft or the presence of unauthorized persons in certain places. In one instance, an electrical manufacturer had noted the disappearance of coils of magnet wire from his stock, and undertook to detect the thief by establishment of a circuit similar to that of the induction balance about a passageway through which all employees were required to pass on leaving the premises. The spools of wire in the stock shelves having been carefully short-circuited, it became an easy matter to detect the presence of the guilty person by observing the unbalance set up by eddy currents in the short-circuited coils concealed beneath his clothing as he left for home in the evening. According to a recently published article it has been proposed to make an electrical investigation of the continuity of the iron chain which lies embedded in the massonry around the base of the dome of St. Paul's Cathedral, London.[288]

IX. Physiological and Allied Measurements

While the attitude of the medical profession toward exact quantitative methods of measurement in diagnosis would to the layman appear characterized by the utmost conservatism, it may, without question, be said that there is at present no field of scientific work wherein there are being made more rapid expansions of the use of electrical measurement than in that of the investigation of physiological and psychological phenomena. In studying the functions of living matter it has been difficult to identify processes which would lend themselves to quantitative expression; and, had it not been for the startling advances but lately made in applying electrical measurements to the diagnosis of disease, the expression "When doctors disagree" might yet have been due for a long lease of life. Recently, however, electrical methods of measurement have made an intrusion into the work of the medical profession which bids fair to place the findings and decisions of this group of technicians upon a basis of precision comparable with that long enjoyed by the devotees of the more exact sciences.[289, 290]

The popular press has of late contained considerable reference to a method of diagnosis wherein it is claimed that perfectly definite results may be obtained upon the basis of measurements of an electrical nature performed upon electronic emissions of the patient under investigation, and that a complete diagnosis of his affliction may be made in absentia by the electronic reactions of a drop of blood. If there be any merit to this system it is one which should most intimately concern the electrical engineer; but until the proponents of the method are in a position to expound their procedure openly, and in terms intelligible to electrical men, it can hardly be said that it need be recognized as an added application of electrical methods of measurement.[291, 292, 293, 294]

Electrical determination of the concentration of the hydrogen ion has found a wide use in the study of biology, but since this has already been referred to in an earlier part of this paper its repetition here will not be necessary. The convenience and precision with which temperature may electrically be determined has proved a great boon in the accurate measurement of body temperatures in pathological work. Both the thermocouple and the resistance coil have been employed in these measurements, the former where extreme precision or observation of rapidly varying values is required, and the latter for ordinary clinical work. The thermocouple can be made to give an accuracy of 0.02 deg. cent. while the resistance thermometer can be relied upon to 0.1 deg. cent. The detector is made in several forms, suited either for contact with the skin or for insertion into the natural cavities of the body. It is usually employed with some form of automatic recorder, so that continuous charts may be obtained. While these temperature measurements find their basic application in the observance of pathological conditions, researches are now under way with a view to correlating the patient's emotional condition with rapid changes of skin temperature, and the results as far as these tests have been carried, go to show that there may be thus obtained much valuable material in psychological investigations.[295, 296, 297, 298]

The outstanding example of electrical measurement of physiological conditions lies in the electrocardiograph, an instrument applying the Einthoven string galvanometer to the direct measurement of electromotive forces set up in the human body, particularly by the functioning of the heart. It is well known that all muscle movements are accompanied by changes in electric potential of the active part, relative to the passive part, the former becoming negative. If, then, a record of the electrical phenomena attending the action of the heart can be obtained, it follows that this must give reliable information regarding the various phases in the performance of this organ. While the theoretical possibility of this had long been appreciated, it was not until Einthoven in 1903 developed the string galvanometer, that it became an accomplished fact. In practise it is customary to make the electrical connections to the patient through non-polarizing electrodes attached to his limbs; and it has been found that the most complete information is obtained from three electrocardiograms taken with the following circuits:

I. Right arm and left arm:
II. Right arm and left leg:
III. Left arm and left leg. (Fig. 19.)

In the use of this device it is necessary not only to guard against polarization at the electrodes but to balance out the potential difference caused by glandular activities of the skin, and usually known as "skin current." An interesting and important feature of the

electrocardiograph as applied to diagnosis lies in the fact that the patient need not be brought into the presence of the instrument, but may have records taken at a remote point, perhaps several miles away, and, in fact, need not know that a study is being made of the performance of his heart.[299, 300, 301, 302, 303, 304, 305, 306, 307]

Studies of heart action are greatly facilitated by the use of the "Stethophone," lately developed by the Western Electric Laboratories, not only replacing the long-familiar stethoscope, but far surpassing it in the volume of information obtained. By the combination of an electromagnetic transmitter with a selected group of filter circuits it is possible not only to faithfully reproduce heart and other body sounds, but to mask out those to which it is not necessary to listen. The sound upon which it is desired to concentrate may then be heard in one telephone or in several hundred. An interesting feature of the device, in connection with its application to lecture room work lies in the fact that where the lecturer wishes to communicate with his audience he need only direct his remarks toward the

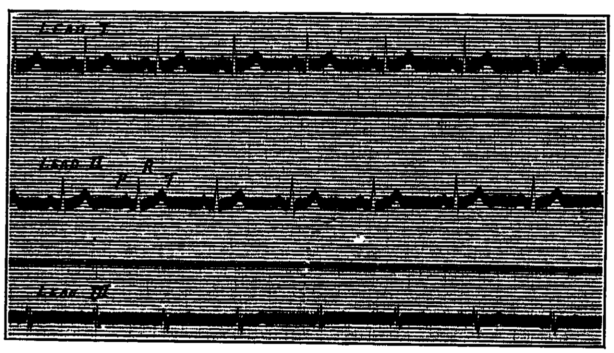

FIG. 19—ELECTROCARDIOGRAM

body of the patient, when the sound vibrations will be picked up and carried to the listeners. Like the cardiograph, the stethophone may be used at a point remote from the patient; and the electrical circuit may, as in that instrument, be associated with the recording features of the string galvanometer.[308, 309, 310]

Another Western Electric development is found in the "Audiometer" a precision instrument capable of measuring the acuity and quality of hearing. While the original purpose of this device was investigation connected with telephone work, it has been adapted to the medical field, and should without question prove a most valuable addition to the equipment of that profession. By means of a special vaccum tube there are produced continuously pure tones, which may be controlled and measured, both as to pitch and intensity; and these are delivered through a telephone receiver directly to the ear. By applying in a standard diagram the values representing the limits for a particular patient, there is obtained an exact index of the condition of his hearing; and this, in the hands of a specialist, furnishes material upon which a very complete diagnosis may be based. This instrument should prove of great usefulness, not only in the study of defective ears, but in all cases where complete physical examinations are required.[311 .... 323 incl.]

Amazing results have been obtained in the association of measurable electrical phenomena with the emotional condition of the human subject. While the "psycho-galvanic" action has been known, though rather imperfectly, for a number of years, it is only lately that there has been made any systematized and definite collection of measurements of the electrical effects which appear in conjunction with emotional conditions. For some time there has been in use an instrument known as the "emotometer," (Gambrell, London), which is a Wheatstone bridge, in whose circuit a portion of the patient's anatomy,—the hand, for instance,—is included. It is found that sudden changes of emotion, resultant upon predetermined stimuli, such as a sensation of falling, produce marked disturbances in the electrical balance of the circuit; and investigations have been carried out with a view to establishing a series of "standard emotions." While the possibilities of the device in its simple form have in no way been exhausted, its field has been immensely widened by the association of the string galvanometer, either as a detector in the bridge circuit, or independently, as in the cardiograph. This development is too recent to have made available any conclusive data; but such records as have been obtained from experimental work at present in progress go to show that, awaiting only the skill of the interpreter, there is at hand a mass of information on the functions of the human mind which will go far toward standardizing the study of psychologic processes, and the host of dependent manifestations of mental activity.[324, 325, 326, 327, 328, 329, 330]

CONCLUSION

Little that bespeaks finality can be found in the application of electrical principles to the innumerable measurements confronting engineers, scientists, navigators or medical men. In some branches of activity there have been adopted fairly well standardized systems for the performance of measurement by electrical means; but in general it may be said that there is apparent a state of development so rapid that in the few weeks which must necessarily elapse between the preparation and the presentation of this material, methods which have been described as modern will have been rendered obsolete and superseded by still more efficient electrical devices for making the required measurements. The phenomenal expansion of electrical practise to include, or to bear most closely upon almost every other field of human endeavor,—the continual breaking down of the barriers between electrical manifestations and the supposedly remote processes of other sciences, is having the effect of narrowing the meaning of the term "non-electrical quantities"

to the extent that the problem of their measurement, while today of interest to the electrical engineer as an appreciation of his working tools by other craftsmen, will very shortly be his problem by right of inheritance.

In collecting material used in this work it has been the privilege of the writer to come into contact with many specialists in fields where the electrical engineer seldom dares to tread. The cheerful cooperation shown, whether by the chemist, the physicist, the physiologist, the psychologist or the brother engineer, has been most encouraging; and the general spirit has been one of realization of the mutual benefit which must result when the engineer compares notes with his fellows in other spheres of technical activity. Space limitations preclude individual acknowledgement to the many who have assisted in the rounding out of the work; and the only tribute here practicable is seen in the liberal use of the results of their studies, as described in the text, and as referred to in the bibliography, which, may perhaps, like imitation, be looked upon as the sincerest form of flattery.

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Use of an Oscillograph in Mechanical Measurements[1]

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I. Introduction

THE oscillograph is an instrument which has been extensively employed by electrical engineers in electrical measurements. It has been used not only in connection with electrical oscillations, such as alternating electromotive forces and alternating currents, but also in connection with acyclic electrical phenomena, such as the current required to blow a fuse, or the discharge current of a condenser. This paper describes methods by which the oscillograph may be used in mechanical measurements. This requires that the mechanical phenomena must affect an electric current which is being continuously recorded on the oscillograph.

II. The Oscillograph

The oscillograph can be briefly described as a galvanometer having a short period and critical damping, which is so arranged that its deflections can be recorded on a moving photographic film. Several models have been brought out by different manufacturers. For the work described in this paper a General Electric oscillograph has been modified to meet the requirements of the special investigations. The most important of these modifications are: (1) A special arc lamp; (2) a large drum which will take a five-foot film; (3) a special table on which is mounted a switchboard and suitable control instruments; (4) a new shutter and shutter control mechanism; (5) a rotating mirror; (6) a tuning fork for measuring the speed of the film.

1. *Special Arc Lamp.* Experiments have been made with several different sources of light, but for the highest film speeds it is necessary to use an arc lamp. For this purpose it is essential that the end of the positive carbon shall lie in the axis of the lens which is used to direct the light beam onto the oscillograph mirrors and very near to the focal point of the lens. It must be possible to advance this carbon without disturbing its adjustment. To accomplish this a special arc lamp was constructed in which the carbon holders slide on metallic ways. The carbons are held in V-shaped grooves which are parallel to the direction of the ways. The carbon holders are advanced by means of racks and pinions. The carbons can be maintained in a desired position by keeping their images, formed by a pinhole camera, at definite points on a screen which is provided for this purpose.

2. *Large Drum which Will Take a Five-Foot Film.* It is often desirable to use films of considerable length and whose motion is uniform during the time that the record is being made. For this purpose drums have been constructed which will take a five-foot film. This length was selected as it corresponds with standard films which can be universally purchased. The rotating portion has a large moment of inertia which insures that the speed will not change rapidly. The opening of the shutter does not appreciably change the speed of the drum.

3. *Special Table on Which is Mounted a Switchboard and Suitable Control Instruments.* Another useful modification was the design of a special table on which there is mounted not only the oscillograph itself but also a switchboard and suitable measuring instruments. This table was especially constructed for field use. The legs are made of two-inch pipe which screw into floor flanges that are mounted on the underside of the table top. The switchboard is hinged to the table apron in such a way that when the legs are removed it will swing into the bottom of the table, where it can be fastened for shipment. The instruments are mounted at a convenient point at the back of the table. These instruments consist of a 25-ampere ammeter for reading the current through the arc, a one-ampere ammeter for reading the current in the field magnet, and three milliammeters, range 150-0-150, which are connected in series with the oscillograph elements. These latter instruments have proven to be especially useful, since most of the difficulties in oscillograph operation occur in the element circuits. These difficulties can readily be traced by means of these milliammeters. In addition, there is a small resistance box in each element circuit, variable from 1 to 1000 ohms in steps of one ohm.

In Fig. 1 is shown an illustration of a complete oscillograph with arc lamp, large drum, instruments, and switchboard. At the right in this picture is the table with switchboard as prepared for shown shipment.

4. *New Shutter and Shutter Control Mechanism.* There are many advantages to be gained by placing the shutter mechanism as near the film as possible. By doing this all of the optical parts can be continuously illuminated so that one can readily see whether they are in proper adjustment. Moreover, by placing the shutter at this position, the time required to open it is reduced to a minimum. Another advantage of placing the shutter in this position will be discussed in connection with the rotating mirror which will be

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described in the next section. A diagram showing the location of the shutter and its method of operation is shown in Fig. 2.

It is important to have a mechanism which will close the shutter when the film drum has made a complete revolution, and it is desirable that it shall do this regardless of the speed of the drum. It should also be as nearly automatic as possible so that it will require little or no attention from the operator. This has been accomplished by the mechanism shown in Fig. 3. A brass gear is mounted either directly on the shaft of the drum or connected therewith through a train of gears so that no slipping can occur. A fiber gear is mounted on an arm in such a way that it is normally held out of mesh from this brass gear by means of a spring, but can be pulled into mesh by means of an electromagnet. This fiber gear has a small portion cut away. When it is rotated to the point where this portion is cut away, it presses against the brass gear and stops rotating, remaining in this position until the operator releases the key. Then as the fiber gear is drawn back by the spring to its normal position, a ratchet attached thereto (not shown in the drawing) turns it through a small angle so that no attention is required to make it ready for the next operation.

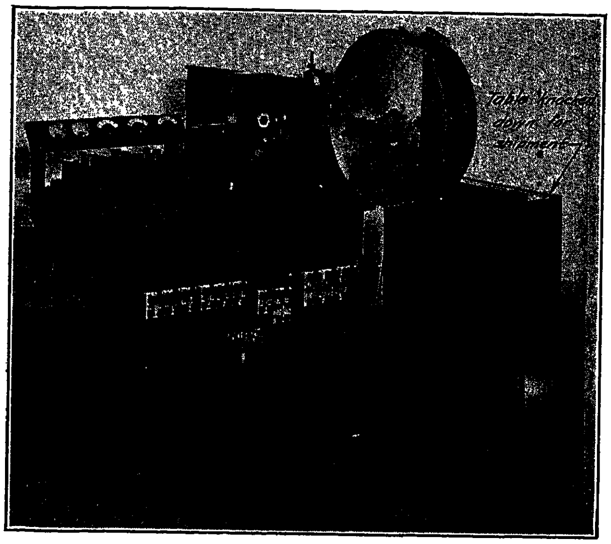

Fig. 1
The oscillograph with large drum, arc lamp, ammeters, resistances, and switchboard mounted on a special table. To prepare the table for shipment, the legs are removed and the switchboard, which is hinged to the apron of the table, is folded against the bottom of the table top without disturbing the wiring.

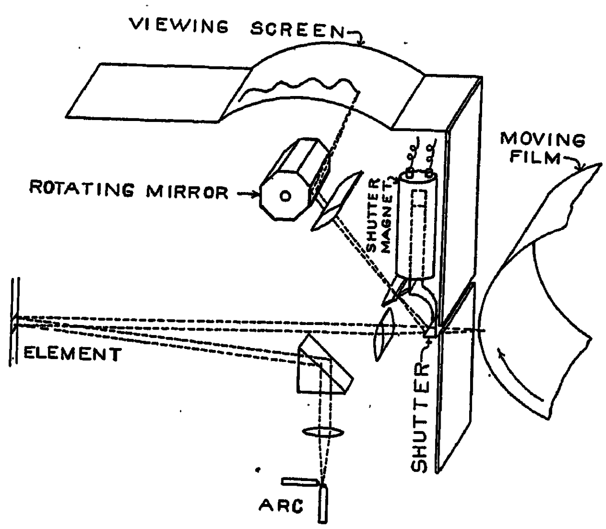

Fig. 2
Diagram to show the optical system when the shutter is placed near the film. A mirror is mounted on the shutter so that the light from an element can be seen on the viewing screen whenever the shutter is closed.

Attached to the fiber gear there is an adjustable cam which opens a contact in the circuit of the shutter magnet. This contact is closed when the fiber gear is in its normal position, but is opened after the gear has rotated a definite amount. By proper adjustment of the cam, the shutter can, for any given speed of the drum, be made to close when the drum has made one revolution.

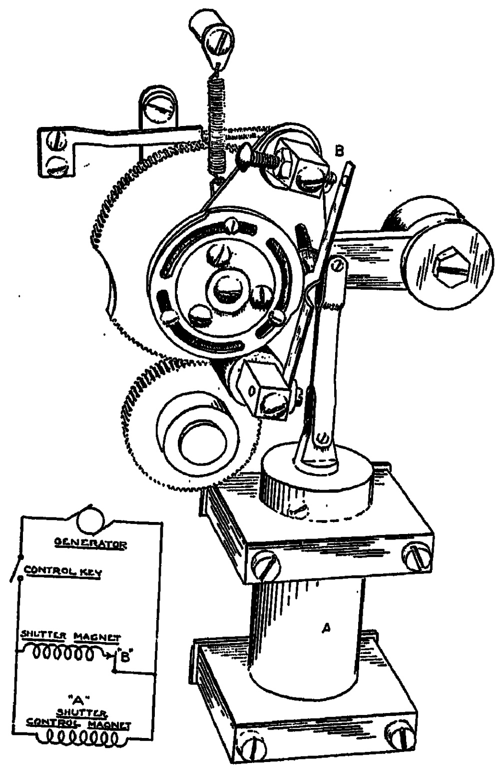

Fig. 3
The shutter-control mechanism for closing the shutter when the drum has made a complete revolution. The figure shows the gears held in mesh by the electromagnet A. The fiber gear (upper in the figure) has made about three-quarters of a revolution. The contact B is open, having been closed during the first part of the revolution.

The adjustment of the shutter mechanism will, in general, depend on the speed of the drum, and is more difficult for high speeds than low speeds. When the drum is rotating rapidly, the time required for the shutter to open, the time required for it to close, and the time required for the fiber gear to be drawn into mesh, are each a large fraction of the time of a revo-

lution of the drum. That the adjustment of the shutter mechanism will depend on each of these can be seen by considering a simple case. Suppose that the gear is drawn into mesh at the same instant that the shutter is opened. Then the shutter circuit will be broken when the drum has made a definite part of a revolution, at which time the shutter starts to close. In the time required for the shutter to close, the drum will rotate through an angle which depends upon its speed of rotation. Hence with this arrangement, if the shutter mechanism was adjusted to work correctly at low speeds, there would be overlapping at high speeds.

The adjustment of the shutter mechanism will be independent of the speed of the drum if the time of closing the shutter is shorter than the time of opening the shutter by the time required for the shutter-closing mechanism to operate. Another way of stating this is that the shutter must open so slowly that it will be closed for a time after the fiber gear comes into mesh, and this time must equal the time required to close the shutter. Up to the present time, it has been found necessary, in order to accomplish the above results, to make the time of opening the shutter about five-hundredths of a second. As this is not satisfactory for some kinds of work, it is frequently more desirable to adjust the shutter to open as rapidly as possible, and then adjust the shutter closing time for a definite speed of the drum. When the speed of the drum is materially changed, it is then necessary to readjust the time of closing the shutter.

5. *Rotating Mirror.* As indicated in Fig. 2, a mirror set at an angle is mounted directly on the shutter so that whenever the shutter is closed the light is reflected through a slit and a second cylindrical lens onto one of the faces of a hexagonal rotating mirror. This mirror reflects the light onto a translucent viewing screen. When used with acyclic phenomena, this hexagonal mirror is not rotated but can be turned to bring the spots of light at a convenient place on the viewing screen. Its principal value is that it enables the operator to see that the spots of light reflected from the element mirrors are in adjustment and are sufficiently brilliant. With cyclic phenomena the rotating mirror can be driven by any variable speed motor, the speed being adjusted so that suitable waves appear on the viewing screen. It should be observed that these waves are slightly distorted and that they do not have quite the same amplitude as the waves which will be photographed on the moving film. In any case, the spots of light which are about to be photographed can be observed up to the instant that the shutter is opened, and they come into view again as soon as the shutter is closed.

6. *Tuning Fork for Measuring the Speed of the Film.* In many experiments it is important to measure the speed of the film with greater accuracy than can be done by the use of a 60-cycle wave. This has been accomplished by means of a tuning fork which is installed in the oscillograph. The principle involved in the use of the tuning fork is shown in Fig. 4. On each prong of the tuning fork there is mounted a vane, these vanes being of such length that they overlap when the tuning fork is at rest. Slits are then cut through these vanes so that, when the tuning fork is at rest, the slits in the two vanes coincide. When the tuning fork

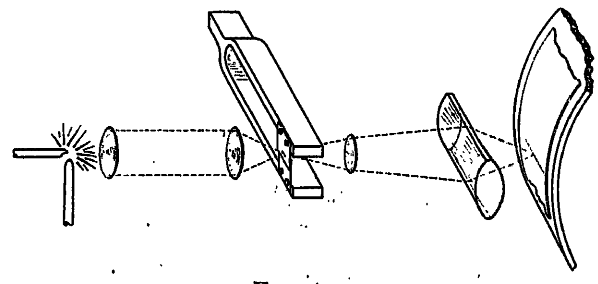

FIG. 4

Diagram showing optical system by which a tuning fork can produce timing lines on a moving film.

vibrates, the slits coincide at the center of the swing, giving momentarily the effect of a single slit. When this slit is brilliantly illuminated, its image, formed by a suitable optical system, can be photographed on a moving film. This gives lines on the film,[2] the distance

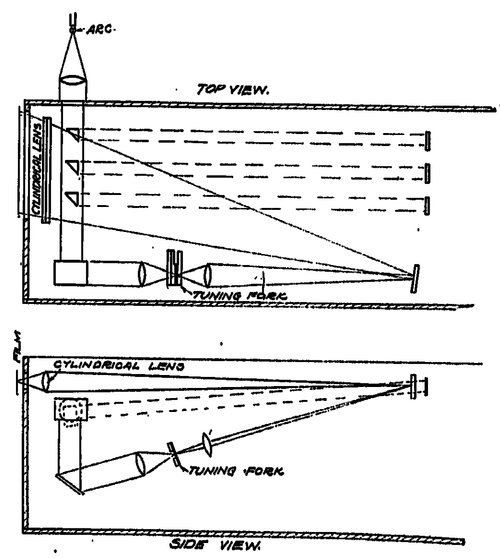

FIG. 5

Diagram showing method of installing a tuning fork in an oscillograph. By means of the tuning fork, the velocity of the oscillograph film can be accurately determined.

between them being the distance that the film travels in the time of a half vibration of the fork. A suitable size for the slits is 0.003 in. width and 0.2 in. length.

For rapidly moving films a 500-cycle fork giving lines one one-thousandth of a second apart has been found

2. A more complete description of this method of timing a moving photograph film is given in Scientific Paper 470 of the Bureau of Standards by H. L. Curtis and R. C. Duncan, entitled "A Method for the Accurate Measurement of Short-Time Intervals."

satisfactory. For slower film speeds a 50-cycle fork giving lines one one-hundredth of a second apart has been useful. A diagram showing the optical system used is given in Fig. 5. Examples of timing lines will be found in Fig. 8.

III. Measurement of Short Time Intervals

When it is desired to study the time of occurrence of a number of events, where order of occurrence is known, it is generally possible to arrange electrical circuits in such a way as to record all of these events with one oscillograph element. An example of this is the recording of the events which take place in a gun at the time it is fired. The electric circuits by which it is possible to record six different events with a single element are shown in Fig. 6.

As shown in this figure, two batteries, one of 6 and one of 12 volts, are connected in opposition so that a continuous current is sent through the resistance, R, and the oscillograph element, E. The series of events is started by the closing of the firing key, S. This ener-

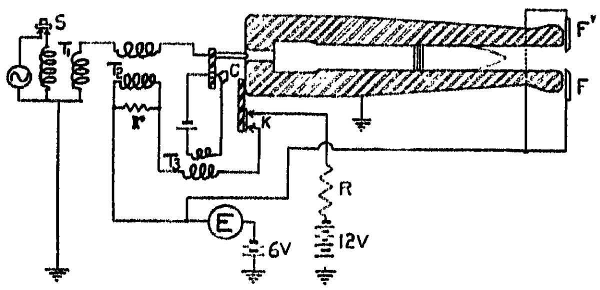

Fig. 6

Diagram showing the electrical connections for recording the important events in the firing of a gun. By circuits not shown, the oscillograph shutter is opened a few hundredths of a second before the firing circuit is closed at S. A description of the sequence of events after the closing of the firing key, and of the consequent variations in current through the element E of the oscillograph, is given in the text.

gizes the firing circuit, sending an alternating current through a fine wire inside of the primer. This wire soon fuses, thus opening the firing circuit. By means of a small current transformer, T_2, called the primer transformer, an alternating current is superposed on the direct current which is already flowing through the oscillograph element. The amount of the alternating current which flows through the oscillograph element is is regulated by a resistance, r , in parallel with the secondary of the transformer. This resistance also serves as a non-inductive path when other current changes are produced in the circuit. By means of this alternating current the time of the closing of the firing circuit and the time at which it opens are recorded on the oscillograph film. Soon after the explosive in the primer is ignited, enough pressure is generated inside the primer to cause it to explode, sending a flame into the powder chamber, and at the same time causing a sudden kick of the firing pin. As this firing pin reacts suddenly, it opens the contacts on the explosion indicator, C, causing the current through the primary of the explosion indicator transformer, T_3, to become zero. This produces in the secondary circuit of T_3 a momentary current which is registered on the oscillograph film, thus giving the time at which the explosion occurred. When the gun starts to recoil, it opens the start-of-recoil contacts, K, thus breaking the continuous current which up to this time has been flowing through the oscillograph element. When the projectile reaches the muzzle, it makes contact with the ogive finger, F', causing a relatively large current to flow through the oscillograph element. This ogive finger is swept away

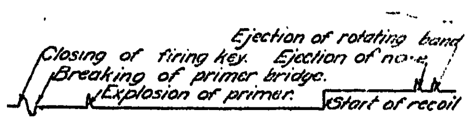

Fig. 7

Diagram showing type of record given by the circuits of Fig. 6.

when the circuit has been closed for only a few ten-thousandths of a second, thus again opening the circuit. This same process is repeated when the rotating band makes contact on the rotating band finger, F.

As these events must occur in a definite order, there is no possibility of one record interfering with any other. Moreover, since all the deflections are of a different type, if one apparatus should fail to record, no uncertainty is introduced regarding the other events. The only exception is in the case of the ogive finger and rotating band finger, whose records are practically

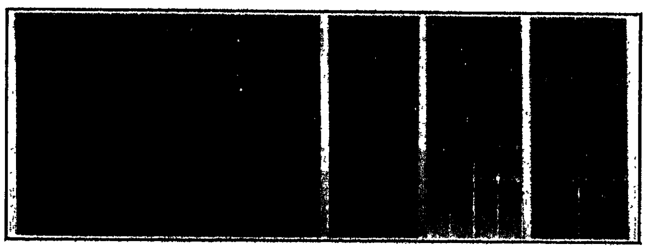

Fig. 8

Four portions of a record taken during the firing of a gun, with the circuits arranged as in Fig. 6. In the first portion is shown the alternating-current wave: in the second portion, the current change caused by the explosion indicator: in the third portion, the displacement of the record caused by the breaking of the start-of-recoil contacts: in the fourth portion, the current changes caused by the grounding of the ogive and rotating band fingers. The distance between timing lines corresponds to one one-thousandth of a second.

identical. As pointed out later, this is necessary for other reasons. It would be very simple to introduce some resistance in one of the circuits so that they could be distinguished.

The record obtained by such an arrangement is so long that it is not feasible to reproduce it. However, the kind of record which this apparatus gives is shown in Fig. 7. Portions of an actual record are shown in Fig. 8. These portions were selected to show the deflections at the times of the different events. In this record the events are shown as recorded by the element

whose trace appears at the top of the figure. The other two elements were used for other purposes. The timing lines are one one-thousandth of a second apart.

When it is desired to obtain the time of occurrence of several events, the order of the occurrence of which cannot be predetermined, it is sometimes feasible to arrange a series of circuits like that described above, so that each event gives a distinctive record on the film. Then the order of occurrence and time of occurrence can be determined from a single record. However, there is always the possibility of two events occurring approximately simultaneously so that the time at which these events occur cannot be accurately determined. When it is important to avoid such a possibility of confusion, it is necessary to record each one of the events on a separate element. If it is necessary to use more than one oscillograph, then some common event must be recorded on all films, so that the time of each event can be determined.

It is sometimes possible, when it is reasonably sure that the events will not all occur simultaneously, to so arrange the electric circuits that at least two independent values of the time interval can be obtained on two different elements, and yet have only a very slight probability that any event will fail to be recorded. As an example, suppose that it is desired to determine with accuracy the time between the contact on the ogive finger and that on the rotating band finger in each of three guns which are fired simultaneously. As explained later, this time difference can be used to get the velocity of the projectile. But since the experimental errors are relatively large, it is important to get as many independent determinations as possible. By using three elements for three guns and connecting them so that the records from gun No. 1 are recorded by both elements 1 and 2, from gun No. 2 are recorded by elements 2 and 3, and gun No. 3 by elements 3 and 1, we are able to obtain two values for the velocity of each projectile, and there can be no uncertainty regarding the value for each gun. Should the times at which two projectiles reach the muzzles differ by less than one one-thousandth of a second, then the record of one element would be of no value, but the records of the other two would be sufficient to give one value for two guns and two values for one gun. Should all three guns react simultaneously, then the records would be valueless, but the probability of such a contingency is very remote.

IV. Measurement of Velocity

If at definite points in its path a moving object changes the electric current in a circuit, then by recording the changes in current on an oscillograph film whose speed is known, it is possible to determine the velocity of the moving object. Since the oscillograph can measure short time intervals, this method has been used chiefly in measurements on objects moving at very high velocities. Perhaps the highest velocities with which engineers at present have to deal are those of projectiles fired from modern guns. Their velocity is of the order of 3000 ft. per second or 3 ft. in one one-thousandth of a second. With an oscillograph film running at 50 ft. per second, the film moves approximately one-half inch in one one-thousandth of a second. With a suitable record, this distance can be measured by means of a comparator with an accuracy of one-fifth of one per cent. However, to obtain this accuracy in the time measurement it is necessary to record both events on one element and to arrange the circuits for recording the two events so that the electrical constants are practically identical in the two cases.

The necessity for using a single element arises from two causes; first, the spots from two elements cannot be adjusted so that they lie exactly on a line parallel to the axis of the drum, and second, the rotational constants of the two elements cannot be made identical. Neither of these causes will introduce errors of more than one or two hundred-thousandths of a second, but in the short interval under consideration, this might produce an error of one or two per cent in the final result.

The necessity for the identity of the electrical constants of the circuits for recording the two events arises from the necessity of a time correlation of the element deflection with the change of current in the circuit. When the projectile closes an electric circuit, the current starts at zero and rises to its maximum by means of an exponential curve, the shape of which depends on the resistance and inductance of the circuit as well as on the applied electromotive force. The element deflects slowly at first, but with a rapidly increasing velocity, partly because the current is increasing and partly because of its own inertia. Hence it is difficult to determine on the film the exact time that a deflection starts. The time difference between two events can, however, be determined with accuracy by taking as the time of the first event the time that the element has deflected one centimeter, say, and an equal deflection for the time of the second event. However, this will accurately determine the time interval only when the electric circuits for the two events have the same constants, and when the rotational constants of the two recording elements are the same. This last is completely met by recording both events on one element. The desirability of measuring from a deflected position is apparent from the center trace of Fig. 8, but it is not to be expected that this will be so apparent in a reproduction as in the original film.

The measurement of the distance between contacts generally introduces a greater error in the velocity when measured by an oscillograph than does the measurement of time. Two methods of obtaining a measurable distance have been used. In the first, each contact consists of two insulated metal plates or screens, so that the projectiles short-circuits them. It is

desirable to put a quick acting fuse in the circuits with the first contact so that if a short circuit persists the fuse will melt and the element will have returned to its zero position before the second contact is closed. In the practical application of this method, it is not generally feasible to measure the distance between the contacts with an accuracy as great as one-tenth per cent. As they are generally thirty or more feet apart, so that the time interval is of the order of a hundredth of a second, little difficulty is experienced in measuring the time to a greater accuracy than is possible in measuring the distance.

In the second method, two contacts are mounted in

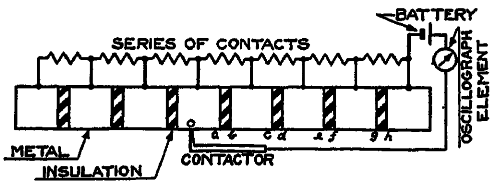

Fig. 9
Diagram to illustrate the step-by-step method.

the same plane to make contact at definite points on the projectile. This is illustrated in Fig. 6 where the ogive finger and rotating band finger are in the same plane, the one making contact on the ogive of the projectile and the other on the rotating band. This method, which is applicable only to projectiles of large size, requires that the distance on the projectile shall be measured before the projectile is loaded and that the path of the projectile shall be known so that the contacts will be made at the measured positions. This last difficulty may introduce errors as great as one per cent. Hence this method should be used only when no other method is available.

V. Measurement of Displacement by the Step-by-Step Method

It is sometimes desirable to determine the time-displacement curve of a moving body. This is readily accomplished by means of the oscillograph by so arranging an electric circuit that the current is changed as the body passes certain definite points. This step-by-step method is shown diagrammatically in Fig. 9, which shows the principles involved in a displaceometer. This instrument is designed to obtain time-displacement curves of the motion of a body in one direction. The series of contacts is mounted on a fixed support while the contactor is attached to the body whose displacement is to be measured. When the body moves, the contact point is drawn over the contacts, thus altering the resistance of the circuit and producing a definite record on the oscillograph film. When the contact point is on an insulating segment, the current goes to zero but rises to a definite value as it comes onto one of the contacts. A record obtained with such an instrument is shown in Fig. 10.

One advantage of this method is the great accuracy which can be obtained by its use. Both time and distances can be measured with high precision. Also, the method is very flexible. For example, in determining the recoil of a gun, the motion is slow at first but changes rapidly. At the beginning, contacts may be made to advantage every one-tenth inch. However, with increasing velocity, these could not be recorded satisfactorily, so that after the first inch it is desirable to have the contacts made every one-half inch, and later when the velocity becomes more uniform, contacts which are made every two and one-half inches have been found satisfactory. This permits greater accuracy in those parts of the curve where accuracy is needed.

The method has, however, certain disadvantages. It is obvious that one can never tell the exact instant that the motion starts. However, if the apparatus is well made so that the first contact occurs one one-thousandth of an inch or less after the motion begins, this is not a serious difficulty. In fact, by such an arrangement it may be possible to determine the time at which the motion begins with greater accuracy than can be done by some method which records the motion directly. Another disadvantage of the instrument is the fact that, if the time-displacement curve has a maximum value, this maximum value is not recorded by this method but must be determined from the plotted curve. The error in the maximum value can be diminished by arranging the apparatus so that contacts occur at frequent intervals near this point. However, when the maximum value of the occurrence is a matter of importance, care must be exercised in designing the apparatus in order that it will give the necessary accuracy.

Fig. 10
Portion of a record obtained by a recoilmeter using the step-by-step method. The distance between timing lines corresponds to one one-thousandth of a second. Contacts were one-tenth of an inch apart.

The fact that the time-displacement curve can be be obtained only by plotting the data which is read from the oscillograph film is, in some cases, a disadvantage. This becomes, however, an advantage when great accuracy is desired, since the films can be read with such accuracy that a very large sized plot can be made. This is very necessary if the time-displacement curve is to be graphically differentiated to obtain a time-velocity curve. Moreover, the time-acceleration curve, which can be obtained by a graphical differentiation of the time-velocity curves is of value only if the time-displacement curve is constructed with

the highest accuracy. In Fig. 11 is shown the time-displacement curve of a gun during the first seven inches of recoil, together with the velocity-time curve and the acceleration-time curve. The last two curves were obtained from the first by graphical differentiation.

In analyzing the motion of a moving body, it is

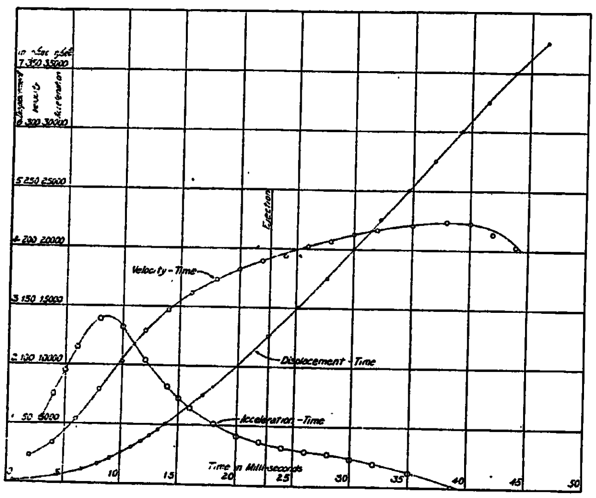

Fig. 11

Curves obtained by means of a recoilmeter during the firing of a large gun. The displacement-time curve is plotted from data obtained from an oscillograph record, together with the calibration data of the recoilmeter. By differentiating this curve graphically, the velocity-time curve is obtained. The acceleration-time curve is obtained by graphical differentiation of the velocity-time curve.

sometimes desirable to obtain time-displacement curves of two different parts of the mechanism from which the displacement of one part relative to the other can be determined. An interesting application of this was in determining the instantaneous torsion of the shaft of

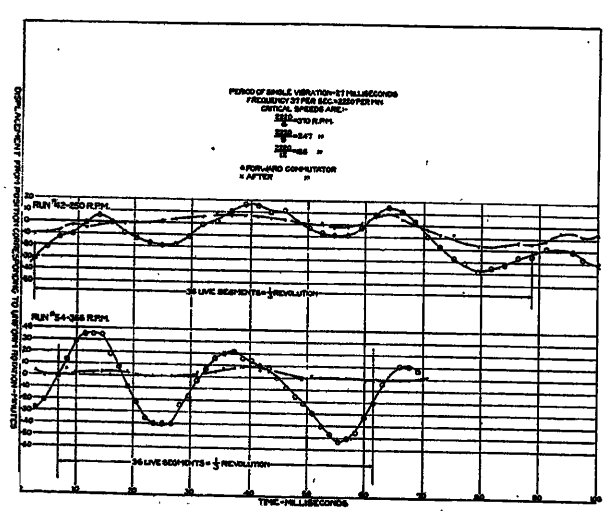

Fig. 12

Torsional vibrations of an engine shaft at two critical speeds.

a large Diesel engine. At certain speeds the shaft of the engine showed large torsional vibrations. By placing accurately made commutators on the two ends of the shaft and recording on the oscillograph the times when the segments passed a definite position, it was possible to plot curves which showed the torsion of the shaft at any instant. Two such curves are shown in Fig. 12. It will be noted that one end of the shaft showed almost uniform rotation, whereas there is a very pronounced sine wave at the opposite end.

The same principle can be applied in the simultaneous measurement of several displacements. If these are recorded by one or more oscillographs so that the relative times can be determined, a very complete knowledge of the motion of the body can be obtained. This has been applied to the study of the motions of the turret and turret structures on a battleship when the

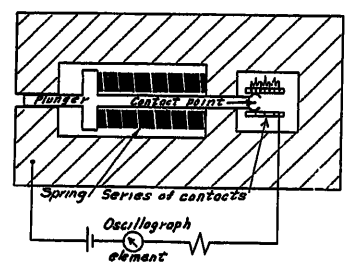

Fig. 13

Diagram of a pressure gage by means of which a pressure-time curve can be obtained when the pressure on the plunger is changing rapidly.

guns of this turret are fired. In this case eight oscillographs were used, giving twenty-four separate records. These showed not only the displacement of the turret in three perpendicular directions but also the rotation about three perpendicular axes. They likewise gave the distortions of the structures which support the turret. This complete analysis was possible not only

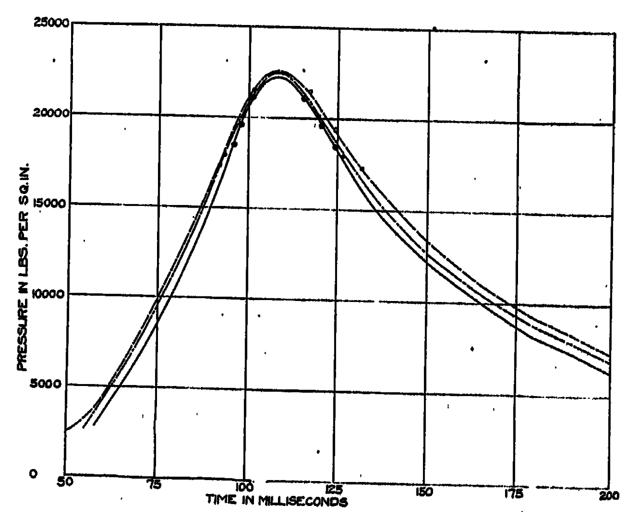

Fig. 14

Curves of pressure obtained simultaneously by three independent gages, mounted in a bomb which had a small vent. The pressure was produced by igniting a charge of powder.

because of the accurate timing which could be obtained through the use of the oscillograph, but also because all times could be referred to the same zero.

VI. Application of Step-by-Step Method to a Phenomenon Which Can be Registered as a Displacement

There are many phenomena whose magnitude can be determined by means of a displacement, and hence they can be recorded on an oscillograph by the method described above. We shall indicate only one application, namely, the measurement of a rapidly varying pressure. A diagram showing the principles involved in such an instrument is given in Fig. 13. The pressure which is applied to the plunger compresses the spring. This causes the contact point to move over a series of contacts, producing a record which is identical in form with that of the displaceometer described above. In Fig. 14 are shown curves obtained by three separate pressure gages which were placed in a bomb. The pressure rose to over 20,000 lb. per square inch in approximately one-tenth of a second. A satisfactory time-pressure curve was obtained.

The illustrations in this article have been drawn from the experience of the author and details are given only in so far as it is thought necessary to make clear the principles involved. There are many other ways in which the oscillograph can be used in measuring mechanical phenomena of short duration. It should be considered whenever it is necessary to investigate a phenomenon whose duration lies between one second and one millisecond.

Several physicists and engineers have been associated with the author in redesigning the oscillograph and in developing the experimental methods herein presented. Particular mention should be made of Dr. R. C. Duncan and Mr. H. H. Moore.

Discussion

J. R. Craighead: Mr. Curtis has given a great deal of attention to pushing the use of the oscillograph into fields decidedly beyond that for which the device was originally designed. The fundamentals of the oscillograph (that is, the natural frequency of the vibrator, and the photographic ability of light which can be placed upon the film) are such that values of the order that Mr. Curtis speaks of can be attained, but the general mechanical construction, and details, have been laid out with a view to practical operation in the shop and in power stations, rather than reaching the extreme values which Mr. Curtis has attained.

I made a few comparisons from the figures given in Mr. Curtis' paper. The width of the beam actually photographed on the film is usually of the order of $^1/_{16}$ in. It can be brought down considerably below that, but in the same oscillogram, the actual width of the beam photographed is dependent on the rate at which the beam is displaced; in other words, the same spot of light displaced somewhat slowly will produce a much wider beam than it will when displaced with extreme rapidity, because of the varying photographic effect proceeding from the center of the beam outward.

Mr. Curtis is measuring, according to the text of the paper, by means of a comparator, distances of approximately 0.001 in. and these distances are being measured on a beam such as I have described which ordinarily would be $^1/_{16}$ in. wide, but which as his oscillogram shows he has been able to get down to much less width. Will he give us a word or two regarding the type of comparator he uses which enables him to place these beams in such a way that distances as small as 0.001 in., or less, as implied in some of his statements, can be correctly measured.

There is no difficulty, of course, in reading that distance on the comparator. What I am trying to get at is the question as to how he can apply the comparator to a somewhat indefinite line and get the accuracy mentioned. It is, of course, essential to place on the film the timing lines to which he refers.

In regard to the actual correctness of the observation, after the photograph has been made, one further consideration of importance comes in. The ordinary kodak film expands when you put it in the developer and contracts when you dry it, after taking it out. The amount of expansion and of contraction and the exact size of the film at the time the photograph is taken are all quantities which depend upon the exact atmospheric conditions and the exact method of treatment. Because of conditions of this kind, timing by lines very closely in the vicinity in which measurement is to be made becomes essential. These lines which Mr. Curtis has placed on his films, it seems to me, should be extended all the way across the film, if he is going to get correct measurements for some of the values which he has indicated as going to the extreme opposite side of the film.

F. V. Magalhaes: Mr. Curtis makes the statement: "Experiments have been made with different sources of light, but for the highest film speeds it is necessary to use an arc lamp."

No question is raised with regard to that particular statement, but it would be of interest to know what measure of success he may have obtained with other sources of light, presumably incandescent lamps, for use with the oscillograph.

In the laboratory, where the particular function which is being measured can be repeated until a proper record is obtained, it is quite possible to use the arc which provides an intensity and a quality of light which is very satisfactory for high-speed films. In the field it is sometimes necessary to use an oscillograph under conditions where it is difficult, if not impossible to repeat the function. The arc under these conditions introduces variables such that consistently good oscillograms are hard to obtain. It is an accomplishment, to my mind, to operate an oscillograph continuously and without failure in the field. It means a careful watch on about a half dozen different factors or parts of the oscillograph, including the arc. When you get the results with the arc, they are very satisfactory, but it is difficult to handle, and it would be valuable indeed to have a satisfactory source of light such as an incandescent lamp, free from the mechanical and electrical variabilities that exist in the arc.

C. E. Skinner: Mr. Magalhaes has suggested the desirability of a light source easier to operate than the arc lamp. Mr. J. W. Legg's work has been done with an incandescent lamp, usually flashed at the instant of the taking of the picture, and I think his discussion will show that he has accomplished considerable in the way of speed in connection with the use of this device.

J. W. Legg: Practically all of the mechanical measurements made by Mr. Curtis have been made by electrical engineers in connection with electrical testing but have not been heretofore so fully described, and in some cases have not been so carefully worked out.

The writer's article in the Transactions A. I. E. E., 1923, Vol. 42, p. 381 on "Expansion of Oscillography," states: "Many purely mechanical movements can be detected and studied only through the medium of electricity and the oscillograph. Undesirable vibrations in machinery, noises, minute movements, momentary pressures, disturbances in the atmosphere, properties of materials and many other non-electrical functions have been studied with the oscillograph. Force or movement has to be transformed into a change in electric current through the medium of the carbon microphone, a special generator, a varying capacity or reactance, or the action of the piezo-electric crystal. The resulting current may be recorded directly by the oscillograph, or first amplified to suitable strength by the

three-electrode vacuum tube." These various mechanical measurements have been made by electrical-apparatus manufacturers in conjunction with electrical tests on quick-acting circuit breakers, electric locomotives, synchronous-converter sets, and the like, but heretofore, have not been taken up by non-electrical apparatus manufacturers, except where the manufacturer is in close touch with the electrical industry, as is the case with some steam-turbine manufacturers.

The value of the various attachments which were made by Mr. Curtis for his particular oscillograph was realized by the writer when designing the oscillograph described in the July 1920 JOURNAL (Vol. XXXIX page 674). An improved optical efficiency and an incandescent lamp were substituted for the old arc lamp; all control apparatus was included in the main unit; a mechanical shutter and control mechanism were designed to cause transients to appear on desired parts of the film, and a fourth optical system was provided for the timing lines. A simultaneous-viewing scheme was proposed but not adopted at that time. The oscillograph described in the February 1923 JOURNAL has been furnished, for particular non-electrical work, with a simultaneous-viewing attachment which has one great advantage over that described by Mr. Curtis, in that viewing continues during the photographic exposure as well as just previous to it. The incandescent lamp is operated on abnormal voltage only during the photographic exposure, hence if the particular transient required does not appear during the extra-bright wave period, then the operator can be sure that it was not successfully photographed and may repeat the test without waiting to develop the film to find the failure.

The increased optical efficiency of the new portable oscillographs, including the six-element instrument, would have been a help to Mr. Curtis with his high-speed films. By arranging the film on the drum to come closer to the oscillograph optical slot, it was possible to use a much wider angle of convergence from the cylindrical condensing lens than was utilized in older forms, thus increasing the intensity of the record on the film in the same way that a larger diaphragm opening in any camera increases the exposure without enlarging the image.

The large-diameter drum, used by Mr. Curtis, is a necessity for extremely high-speed work. Such a large drum has not been required heretofore for electrical phenomena, a compact daylight-loading one being used for the same length films. Such a film holder has been planned with a differential shutter which gives a small fraction of a revolution exposure and then a complete revolution without exposure, followed by another fraction of a revolution exposure, and so on, until the exposures cover the whole film. Thus, several high-speed records may be taken of some transient phenomena which are repeated several times with intervals between each transient longer than the duration of each transient. In the electrical industry such a film holder would be ideal for showing repeated short-circuit phenomena on oil circuit breakers, and switches. For mechanical measurements, such a film holder with differential shutter, might be used to give several high-speed records of the effect on a material of a terrific blow repeated at definite intervals until the material is shattered. The transient and the oscillograph lamp may be controlled by the shutter shaft, while the film rotates at a slightly different speed to get the intermittent exposures in the proper sequence. If the film speed is to be great and the time interval between exposures appreciably long, several revolutions of the film may occur before the lamp is lighted again in time to take another exposure when the shutter opens the next time, etc.

Schemes for recording several mechanical and electrical values with a single vibrator element have often saved much time and expense in testing. Before the development of the new portable oscillographs, the writer frequently recorded five distinct mechanical occurrences and times with one vibrator, leaving the two other vibrators for electrical measurements. The inductive kick from the actuating coil of an electrically driven tuning fork gave sharp kicks to the wave at the rate of 200 per second (or whatever rate was desired) while a potentiometer was used to record the displacement-time curve of the plunger of a quick-acting trip magnet. Contacts with resistances were arranged for showing when the main circuit breaker contacts parted, when the arcing tips parted, when the contact separation reached ½ in., and when the contacts were fully separated. The other vibrators showed the current and the voltage characteristics of the trip circuit.

The use of the double carbon pile for recording pressures, stresses, accelerations, etc., was found to be quite limited in its application, since it is affected by sideways shock. To avoid this error, electromagnetic schemes were resorted to, which were so planned that the converters were not affected by any disturbance except the particular values which were to be recorded.

The step-by-step methods for measuring displacements, including an accurately made commutator with one tooth to every ten degrees (except for the last which was omitted) were found to be altogether too crude for most vibration studies. Special apparatus, operating on the electromagnetic principle, was designed which made it possible to record torsional vibrations of less than one one-hundredth of a degree (0.01 deg.) and less than ten-millionth of an inch (0.000010 in.), at frequencies as high as 2200 cycles per second. Furthermore, the calibration of these devices was constant, and was unaffected by centrifugal force or high-speed rotation.

The writer feels certain that there is a greater field for the oscillograph for observing and recording mechanical and other initially non-electrical properties, than there ever has been in the electrical industry. The advent of the six-element oscillograph and the extremely portable single-element oscillograph described in the December, 1924 "*Electric Journal*" together with Mr. Curtis' article, should open the way for mechanical engineers to study their own problems as never before.

C. H. Sharp: The question of the source of light has been raised by Mr. Magalhaes, and that is an extremely pertinent question. I should like to ask whether an attempt has been made (and if so, whether it has been successful) to apply the little tungsten arc in vacuum, a lamp that is known in England as the Pointolite lamp, to this kind of work. That gives a very small source of light and I think one of very high intrinsic brightness.

Another thing that I wish to point out is this: that the oscillograph as it is at present constructed is essentially an ammeter. It utilizes the magnetic field produced by one turn, and that is all. Its sensitivity is limited by the fact that whatever field operates it is a field produced by a single turn in another field.

It would be an extremely useful thing if we could have an oscillograph which would set up a field from many turns, instead of only one. It would then become a high-resistance instrument, sensitive to extremely small currents, and, of course, requiring higher voltage to operate it. In other words, it would be a voltmeter rather than an ammeter.

We should not lose sight of the fact that the original oscillograph as proposed by Blondel a good many years ago was capable of doing this sort of thing. I refer to the oscillograph in which the moving element was like a Thomson galvanometer, in that it consisted of many little, soft iron needles on a quartz rod, suspended by a quartz fiber, in a magnetic field, and then actuated by another magnetic field at right angles, set up by coils which were in circuit with the current which was to be measured.

These coils evidently could either be of the small number of turns low-resistance type, making the instrument an ammeter, or of a large number of turns of the high-resistance type, making it a voltmeter. Thus, its theoretical applicability was considerably greater than the applicability of the present day oscillograph, which is made on the principle of the d'Arsonval galvanometer rather than the Thomson galvanometer.

I am quite aware of the fact that there are very serious difficulties with the original Blondel oscillograph, and on account of

those practical difficulties, that form of instrument was set aside in favor of the one which is now universally considered. But I want to point out that somebody in the light of present knowledge and of present developments might be able to develop an oscillograph of the multiple-turn type which would still further enlarge the field of this already very powerful instrument.

The cathode-ray oscillograph, of which a very interesting type with a hot cathode has recently come out, will do this thing, because it can use separate coils, and it is also capable of operating on the principle of electrostatic deviation. But, as we all know, this type of oscillograph suffers from certain other disadvantages which are of a great deal of importance in many applications. So that the cathode-ray oscillograph cannot be said to be in a position to displace the electromagnetic oscillograph, but rather to supplement it. As we know it is capable of portraying currents of extremely high frequencies such as are beyond the range of the electromagnetic oscillograph.

To repeat, one thing that we ought to have, if we can have it, is the electromagnetic oscillograph developed in such form that it will be sensitive to currents of very much smaller value than is the present one-turn oscillograph.

Edward Bennett: Dr. Sharp has stated that there is a need for an oscillograph of high current sensibility. It may be well to point out that if the oscillograph is to be used to obtain oscillograms of alternating currents or of rapidly varying currents its current sensibility may be increased a hundred fold or more by the use of a current transformer. Such a transformer is described in the TRANSACTIONS for 1914 in a paper entitled "A Milliampere Current Transformer." By the use of this transformer, extremely accurate oscillograms can be obtained of alternating currents having a r. m. s. value as low as 0.0002 ampere, provided this current is supplied under such conditions that a voltage drop of 4 volts across the current transformer is not objectionable. The limitations of the existing oscillographs as regards sensibility are best thought of and expressed in terms of the watt sensibility of the instruments. Thus, it requires a minimum watts expenditure of 0.2 milliwatts in the vibrator to obtain an oscillogram of reasonable amplitude.

With the advent of permalloy, it should be possible to construct current transformers to obtain oscillograms of currents as small as 20 microamperes.

C. H. Sharp: What about direct-current?

Mr. Bennett: If the current which is to be recorded is varying rapidly enough, that is, if it is not an extremely long-period phenomena the current transformer is satisfactory, although you have to make some interpretation then for the steady drift that occurs under those conditions.

L. T. Robinson: A very interesting point was raised by Dr. Sharp, with reference to the cathode-ray oscillograph, which certainly will do lots of things that nothing else will do but it sometimes fails to operate satisfactorily. In that respect, it differs from the other type of oscillograph, which is about twenty-five years ahead of it, perhaps, in commercial development.

With regard to the sensitivity of the oscillograph, it is seldom the case that Dr. Sharp is wrong, but I think in this instance he is. That is, I don't think that the earlier Blondel type is sensitive, when you think of the watt sensitivity. To be sure, it will go on a very small current, but in watts, I feel sure, that the other type is much more sensitive.

Professor Bennett's contribution about the current-transformer is all right, too, but it puts it into a less sensitive class rather than into a more sensitive class, because, however good the transformer is it must take some energy to run it.

The real way is to make use of the vacuum-tube amplifier. It is very sensitive, very effective, very easy to use, and at once you are in the microwatt class. That is, it consumes very little energy in relation to the quantities to be measured. With a properly designed amplifier it can even be made reasonably successful on direct-current.

F. G. Baum: I am encouraged to tell of the application of the oscillograph to a specific mechanical problem, because I think it will suggest the application to other problems.

A few years ago we started a water turbine under a 425-ft. head, 40,000 h. p., and when the load came on the turbine, a distinct vibration was set up in the penstock, the pipe line being about 1000 ft. long and 8 to 10 ft. in diameter. The vibration was so distinct that you could hear it several hundred feet away, and near the turbine it would break off the small pipes that were connected to the instruments. It was rather a disturbing thing to us and it occurred largely as the load came on. At light load, it occurred very little.

Mr. Roy Wilkins applied the oscillograph to this problem. I think you will find it described either in the Institute records or in the *Electrical World*, I am not certain which. He merely used a diaphragm taken from a phonograph, with a carbon element going to the oscillograph, and the vibration was found, I believe, to be about 80 cycles per second. Changes were made in the runner which changed the vibration to such an extent that the turbine was all right; it was perfectly safe as an operating problem although a slight vibration was still left.

However, it was decided that new runners should be designed and here is an application of the oscillograph to the design of a 45,000-h. p. runner of a water-wheel.

There are other applications for which we might use it. For example, on certain long spans where we pull up the wire tight, the wire at the tower is thrown into vibration. In one of our steel spans crossing the Bay of San Francisco, a span 4400 feet long, there was a decided vibration as the span approached the fixed support. That was corrected by adding weights, beginning with small weights away from the support and gradually increasing the weights as we approach the tower. But I think we could have made a better solution of the problem with the oscillograph.

I. M. Stein: The outstanding feature of an oscillograph is that it records what happens during very short intervals of time. Progress with these new mechanical and physical applications of the oscillograph will be more thorough and more rapid, if we can work with still shorter intervals, and this means a shorter period in the vibrating element. A short period is something which has to be built into the oscillograph. In other words, if the development of the oscillograph is along the line of putting into the oscillograph something which you can't put outside, leaving to vacuum-tube amplifiers the matter of getting proper sensitivity, we will get farther than by trying to get higher sensitivity inside of the instrument.

Direct-current amplification as mentioned by Dr. Sharp is very important. It would be a real service to humanity if one could improve the d-c. amplification methods; I have in mind particularly the need for a device to measure X-ray dosage in the treatment of cancerous growths. The problem involves the measurement of a minute direct current through an ionization chamber. It is only recently that d'Arsonval galvanometers have been developed to be sufficiently sensitive for this work. Such instruments are very satisfactory for investigators, but the X-ray physician needs a more rugged instrument with a good d-c. amplifier.

W. H. Pratt: One point that Dr. Sharp raised in connection with Mr. Curtis' paper (and it has been touched on also by Mr. Stein) is the sensitivity and the periods of operation. I think that Dr. Stein had in mind the Einthoven galvanometer when he made those remarks. Dr. Einthoven told me not long ago that in some of his recent instruments he was able to obtain a natural period of oscillation of the order of a million cycles per second; and the sensitivity to current is correspondingly great.

H. H. Moore: With reference to the method of measuring the speed of the film, I wish to call attention to the fact, that in

addition to the greater accuracy of this method over the use of a 60-cycle a-c. wave, it is also decidedly more convenient, and in the case of a three-element oscillograph it makes available an additional element thus increasing the usefulness 50 per cent, and obviating in many cases the use of an additional oscillograph.

Recently the Ballistic Section of the Naval Research Laboratory made an investigation of the hydraulic steering gear of the *U. S. S. West Virginia* and used extensively the oscillograph and the step-by-step method of recording. This work involved the measurement of hydraulic pressures of approximately 1000 lb. per sq. in. maximum at six principal points of the system; rudder angle, ship's heading, and power input to the steering motor, all relative to time. Each individual run required about 8 min. to complete. The pressure, rudder angle and power input intervals which were approximately equal required about 40 sec.

In connection with this work it was necessary to develop considerably new equipment for the oscillograph. A film holder to take 200 feet of film and give it a uniform rate of travel at about $^1/_2$ in. per second was developed. This compatarively slow film speed required a different timing mechanism and an apparatus was devised to give flashes of light each second. Incandescent lamps were used as the source of illumination. All of the apparatus functioned satisfactorily, and it is not believed that apparatus other than the oscillograph and the step-by-step method of recording are now available which would have given the desired results.

V. Karapetoff: In Fig. 11 curves are shown obtained by a graphical differentiation of an experimental curve. It may be of interest to call attention to my integraph, based on parallel-double tongs, which permits one to draw such derived curves directly, by simply following a given curve with a stylus. The instrument is described in the *Journal* of the Optical Society of America, 6:978, 1922.

G. W. Vinal (communicated after adjournment): Difficulties arising from the amount of current required to operate the oscillograph have been overcome in connection with a study of potential measurements and polarization at the Bureau of Standards, by the use of a resistance-coupled amplifier. The cell under examination was made a part of the potential applied to the grid of a vacuum tube. The oscillograph element was connected with the plate circuit of the second amplifying tube. With such an arrangement no current is required from the cell which is under investigation. By using a half cell, a variation in the potential of single electrodes may also be observed. A discussion of this method of using the oscillograph will appear in a forthcoming paper to be published by the Bureau of Standards.

A. Naeter (Communicated after adjournment): In the discussion following the presentation of the paper by Mr. Curtis it was stated that small transient currents might be stepped up by current transformers to make them sufficiently large for recording by means of an oscillograph. The writer desires to point out that in case the primary current contains transient terms the transformer itself introduces certain terms into the secondary current.

In a paper on "Transient and Permanent Phenomena in Electric Series Transformers" presented before the Royal Society of Canada in Ottawa on May 28, 1913, Andrew McNaughton determined from mathematical considerations and experimental data that in case of transients the secondary current is of one degree higher order than the primary current. Expressing this in physical terms, it means that the secondary transient current is distorted. In a bulletin on the "Characteristics and Limitations of the Series Transformer" issued by the University of Illinois Engineering Experiment Station, A. R. Anderson and H. R. Woodrow show as a result of their investigations that the series transformer, especially with an iron core is unreliable for recording transient or unsymmetrical currents. This bulletin agrees with McNaughton that transient currents are not reproduced exactly in the secondary of a current transformer.

H. L. Curtis: Mr. Craighead has asked concerning the type of comparator which was used in measuring the films. We have used a comparator in which a microscope is moved by a micrometer screw. The operation consists in setting the microscope on two adjacent events and reading the distance from the micrometer screw. With good quality films, it is possible to set the cross hair of a microscope on the center of a line with an accuracy of about 0.001 in. A discussion of the errors in such measurements is given in a paper referred to in Footnote 2 of this paper.

The question of film shrinkage is an important one. If the film shrinks uniformly, no error is introduced since the distance between timing lines shrinks to the same extent as the distance between the events measured. However, if the film shrinks unevenly, errors will be introduced. Some values of the shrinkage of films are given in "Notes of the Shrinkage of Photographic Films (*Journal* of the Optical Society of America and *Review of Scientific Instruments*, Vol. VII, No. 3, March, 1923) and a more complete discussion is given by Frank E. Ross, "Mensurational Characteristics of Photographic Film," *Astrophysical Journal*, pp. 181-191, April, 1924. This last paper was prepared at the Research Laboratories of the Eastmen Kodak Company and indicates that a very satisfactory film can be produced.

Mr. Craighead suggests that it would be desirable to have the timing lines reach the full width of the film. We have used such an arrangement but find that occasionally a timing line will occur at the same time as an event which is to be measured. This makes the accurate measurement of the event very difficult.

Mr. Magalhaes inquires concerning the relative speed of film that can be obtained with an arc lamp and with an incandescent lamp. With an arc lamp, we have successfully worked with a film speed of 100 ft. per sec. With incandescent lamps, we have made satisfactory records with a film speed of 10 ft. per sec. provided the lamp was used at over voltage during the time the film was exposed. The point-o-lite lamp, which is a tungsten arc in vacuo, gives about the same result as an incandescent lamp. Another source of light has been tried which is produced by heating, by means of an electric current, a tungsten wire in air until it vaporizes. The current is thrown onto the wire a few tenths of a second before the shutter is opened. This produces a very intense light but the resulting deposit of tungsten oxide is objectionable. Perhaps this could be avoided by using a carbon filament in place of the tungsten wire.

Much of the trouble with the arc lamp is due to the poor designs which are in common use. We have had very little difficulty with the hand-feed arc lamp here described. Satisfactory automatic arc lamps have been manufactured but they are not now on the market.

Mr. Legg spoke of using a short-focus cylindrical lens. This is, of course, desirable, and should be carried out as far as possible.

When an arc lamp is used the film speed could probably be considerably increased by using a fused-quartz optical system, together with a damping oil which is transparent to ultra-violet light. Such a system has been designed and partially constructed but has not been tested.

Temperature Errors in Induction Watthour Meters

An Analysis and the Development of a Temperature-Sensitive Magnetic Material Suitable for Compensation

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and

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Synopsis.—*This paper is presented in two principal divisions, Part I and Part II. The first deals with the general problem of small errors due to temperature changes in watthour meters and describes methods of segregating the various components of these errors. It is shown that they may be divided into two principal groups, termed Class 1 and Class 2. Class 1 errors are operative at all power factors and are of the greater importance: Class 2 errors are important only at low power factors, and methods of eliminating these are pointed out. Both of these classes are further subdivided into their component parts.*

In Part II of the paper is described a method of compensating for Class 1 errors by means of magnetic shunts made of thermalloy. The term "Thermalloy" is applied to a series of copper-nickel-iron alloys having a large negative temperature coefficient of permeability and other unusual properties. These alloys are discussed in some detail, their manner of preparation and application being considered original.

In the appendix, a novel magnetic thermometer utilizing thermalloy is described.

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Part I

Introduction

AN electromagnetic device, such as an induction watthour meter, the operation of which depends upon the exact magnitude and phase relation of the various fluxes, is more or less susceptible to changes due to variations in temperature. This is to be expected when it is considered that almost all of the properties of ordinary electrical and magnetic materials change to some extent with temperature and that in some of the essential materials in a watthour meter, such as copper and magnet steel, this change is very marked. It is true that this type of watthour meter is, to a large extent, inherently self-compensating and that the variation in speed of a well-designed meter, when a change in temperature occurs, has generally been considered too small to have any serious effect on accurate metering. In keeping with the constant tendency toward more precise measurement of power, however, it is expedient that this problem should be given careful consideration and steps taken to reduce the small effect of ambient temperature and self-heating.

There has been very little material published on the problem of temperature errors in induction meters, as far as the authors have been able to determine. Fawsett[2] has described a precision watthour meter in which he has used a manganin lag plate and a braking magnet mounted upon a bi-metal support which moved in and out in such a manner as to effect a temperature compensation. There has been at least one other meter designed utilizing the bi-metal principle in an endeavor to correct temperature errors by mounting the magnet adjustment-disk so that it is free to move closer to or farther from the magnet depending upon the ambient temperature. These schemes are dependent upon a mechanical motion about which there is always more or less uncertainty, and the magnitude of the compensation depends upon the position of the full load adjustment on the meters; so they are at best only approximations.

In order to arrive at a proper understanding of the nature of this problem, it will be well to review the causes of temperature errors in some detail.

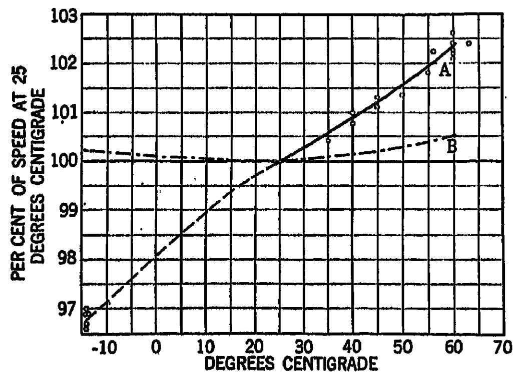

Fig. 1—Temperature-Speed Curves for Induction Watthour Meters
A—Power Factor = 1.0
B—Power Factor = 0.5

Analysis of Temperature Errors

It is generally known that induction-type watthour meters increase slightly in speed when a temperature rise occurs. Fig. 1 is typical of the relationship existing between percentage change in watthour meter speed and temperature for common types of meters at unity power factor and at 50 per cent power factor. It will be seen that a considerable difference in meter speed may be occasioned by a change from outside zero

1. Both of Engineering Department, West Lynn Works' General Electric Company.

2. New Precision Watthour Meter—*The Electrician*, March 7, 1924.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925 and at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

weather to a heated room, which is of course an extreme condition of use. The effect of temperature at low power factors current lagging is much less than at unity, which indicates a greater tendency toward self-compensation under the former conditions. Different makes of watthour meters exhibit considerable difference in temperature characteristics, and some have a greater speed increase with rising temperature than that shown in Fig. 1; but in general they all follow the same laws.

In order to analyze the phenomena of temperature errors, we adopted the experimental method of selecting one possible variable and by holding it constant while the temperature was changed noting the result on the meter's performance.

There are two main classes into which the various factors governing changes due to thermal effects may be divided:

1. Factors governing the magnitude of either potential or current fluxes, or both, or the magnitude of the braking flux.

2. Factors governing the phase relation between potential and current fluxes.

Due to the fact that the torque is proportional to the sine of the angle between the useful potential and current fluxes, and that in a properly adjusted meter these are exactly at right angles for unity power factor, it follows that any small variations in flux phase relations are not important at high power factors. For example, a shifting of three degrees at 100 per cent power factor changes the torque only 0.14 per cent, while the same change in angle if the meter were operating at 50 per cent. power factor causes a change in torque of nearly 9 per cent. It might, therefore, be stated that errors under Class 1, are equally operative at all power factors and that those under Class 2, while negligible at unity power factor, are increasingly operative as the power factor decreases.

It has long been suspected that the control magnets were responsible for a large share of the observed temperature coefficient falling under Class 1. Several writers[3] have pointed out that permanent magnets have a decided temperature coefficient and that their strength decreases with an increase of temperature.

In order to ascertain to just what extent the magnet was responsible for the errors observed at unity power factor, it was necessary to devise some method whereby the braking torque could be held constant while the meter was raised to various temperatures. This was attempted in several ways, one of which was evidently quite successful as will be described below. Tests of this nature are inherently difficult and tedious, but great pains have been taken in obtaining the results herein described and it is believed that the information is reliable.

The first method tried for eliminating the uncertainty in the magnitude of the braking torque when the meter undergoes variations in temperature was the substitution, for the control magnet, of an air fan attached to an extended shaft and enclosed in a glass chamber. Various shapes and sizes of fans were tried without much success. The chief difficulty was in obtaining sufficient retarding torque, by this method, to work with any considerable driving torque or load on the meter; hence, friction played too prominent a part in the result.

It was then decided to endeavor to keep the braking flux constant by keeping the magnet at a constant temperature while the rest of the meter was heated. The meter was mounted in a glass case, which contained a resistance coil for heating and a fan for circulating the air and thus keeping the temperature uniform throughout. It was provided with a long shaft which extended through the top of the case and supported a manganin damping disk. The driving disk was also of manganin, so that any variation in speed of the meter when its temperature was raised should be due only to a change in driving torque, since the temperature of the magnet outside was kept constant as indicated by a thermometer held in close contact.

A difficulty with this arrangement was that the weight of the two disks and the long shaft was so great as to cause excessive friction. This, combined with the light driving torque due to the use of manganin disks, served to make the experiment extremely difficult, although for some time it was thought that we were obtaining quite satisfactory results. Sufficient data were obtained to indicate that the large part of the errors at unity power factor were to be found in the control magnets; but the test was finally abandoned in favor of the one next described.

The method that we found to be most successful for holding the braking flux constant was by the use of a soft iron magnet having the same shape as the standard type I-14, with which we were working. Magnetizing windings, having sufficient turns to provide the required braking flux by the use of a very small magnetizing current, were placed around each half of this magnet. In order to measure accurately just how much braking flux was cutting the disk, an exploring coil wound in the general form of the figure 8 and shaped to conform to the outline of the meter disk was wound on a celluloid form and placed in close proximity to the bottom of the disk. Substantially, all of the braking flux that cut the disk was linked with this exploring coil. By connecting the latter to a ballistic galvanometer and reversing the primary current in the electromagnet, a means of measuring the total flux ballistically was provided. In order to get the required accuracy of one-tenth of one per cent which we desired in this experimental work, a null method of measurement was adopted rather than a deflection method. This consisted essentially of simultaneously reversing the currents in the primary of a standard mutual inductance and in the magnetizing coils on the magnets. The respect-

3. The Effects of Changes of Temperature on Permanent Magnets—*Am. Jour. Sci.*, Vol. XV, No. 87, March, 1903.

ive impulses in the secondaries were passed differentially through a ballistic galvanometer, and when the point of balance was obtained the value of the current in the mutual inductance was the measure of the total effective flux of the control magnets.

A diagram of connections is shown in Fig. 2. The reversing switches S_1 and S_2 were connected together

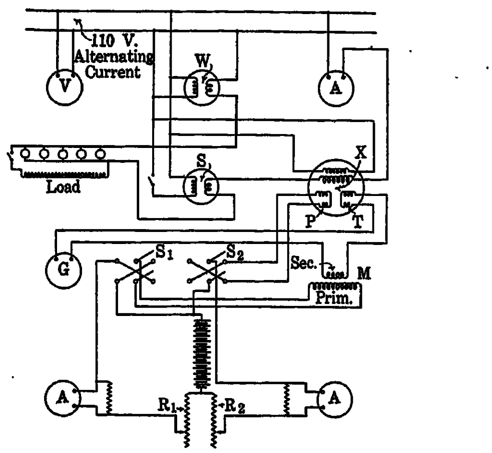

FIG. 2—DIAGRAM OF CONNECTIONS OF APPARATUS FOR DETERMINATION OF TEMPERATURE ERRORS DUE TO MAGNETS

P—Damping Magnet Winding
X—Meter Under Test
T—Exploring Coil
W—Wattmeter
G—Galvanometer
V—Voltmeter
A—Ammeters
M—Mutual Inductance
S—Standard Watthour Meter.

mechanically so that they could both be reversed at the same instant. The procedure in making the test was to adjust the current in the braking electromagnet by means of the rheostat R_2 until the meter ran at the desired speed. The flux was then measured by adjusting the current in the primary of the mutual inductance until the galvanometer gave no deflection when both switches were reversed. The value of the flux passing through the disk and exploring coil could be readily calculated from well-known magnetic formulas such as are used for ordinary ballistic testing and need not be given here. After the meter speed at room temperature had been measured, the temperature of the meter under test was raised to the desired value, the braking flux was adjusted to its original strength, and the speed was again determined by comparison with a standard test meter which was kept at room temperature.

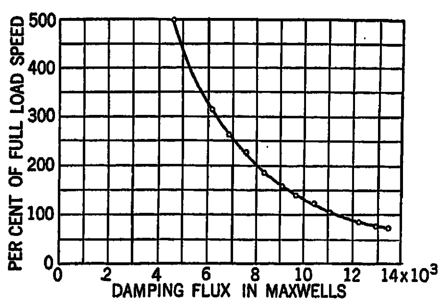

FIG. 3—RELATION OF BRAKING EFFORT TO TOTAL FLUX IN MAGNETS

Meter run under constant load conditions and braking flux varied.

This test was reasonably satisfactory and gave results which were quickly checked to within one-tenth of one per cent. One great advantage which it possessed over all others tried was that the meters under test need not be changed in any other respect; and, therefore, the results obtained were directly applicable to a standard induction watthour meter. This apparatus was first useful to determine the relationship existing between braking or retarding effort and total flux in a magnet. Fig. 3 shows one of the curves obtained on a meter running under full load conditions at unity power factor, the braking flux being varied in such a manner that the speed changed from approximately 75 to 500 per cent of full-load speed.

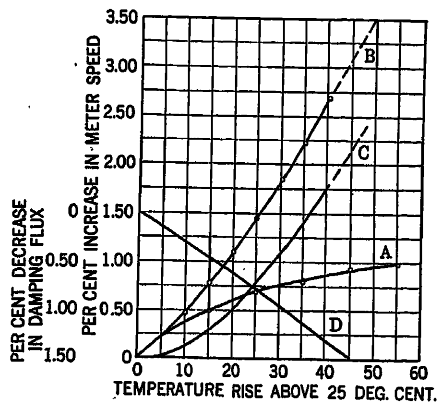

FIG. 4—SEGREGATION OF TEMPERATURE ERRORS IN INDUCTION WATTHOUR METER AT UNITY POWER FACTOR

A—Change in Speed with Constant Braking Flux
B—Change in Speed with Standard Control Magnet
C—Change in Speed Due to Standard Control Magnet
D—Variation in Braking Flux of Standard Control Magnet

If plotted on double logarithmic paper, the speed will be seen to vary inversely as the 1.8 power of the flux. Theoretically this should be a squared relation, and experimental results fit quite closely with this value when it is considered that the data were taken over a very large range of speed and that it is probable the value of friction changed to some extent, as well as other factors not taken into consideration in the theoretical value.

Meter speeds were next measured, at various temperatures, and constant braking torque at unity power factor and 50 per cent power factor both at full load and light load. Part of these tests were made with the lag plate completely removed from the meter, thus determining just what effect this had on the temperature coefficient. Complete curves were taken in each case, and sufficient points were obtained and check tests made to minimize observational or accidental errors.

The results of tests at 100 per cent power factor are shown by Curve A in Fig. 4. It will be seen that the

meter speed still increases with increase in temperature. This must be due to an actual increase of driving flux cutting the disk, as it cannot be explained by a change in phase relation of the flux as noted previously. It is evident that the most plausible reason for this increase

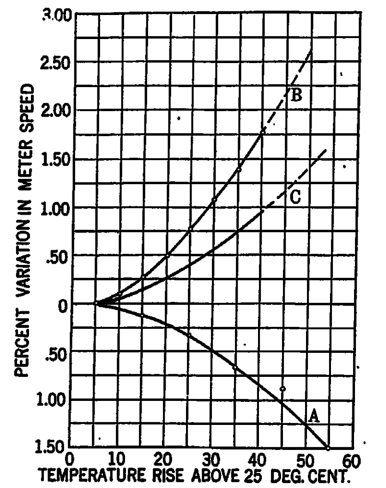

FIG. 5—SEGREGATION OF TEMPERATURE ERRORS IN INDUCTION WATTHOUR METER AT 50 PER CENT POWER FACTOR

A—Change in Speed with Constant Braking Flux
B—Change in Speed Due to Standard Control Magnet
C—Change in Speed with Standard Control Magnet

in driving torque is that the permeability of the silicon steel laminations in the driving elements changes sufficiently with temperature to cause this effect. Changes in iron loss also have a small influence on the magnitude of the induced voltage or flux, as will be

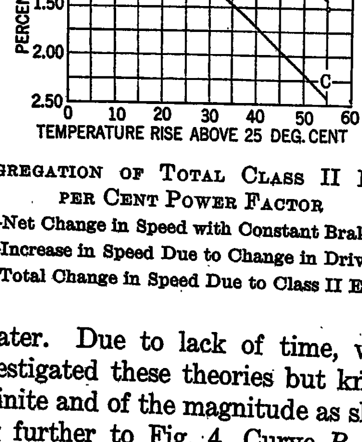

FIG. 6—SEGREGATION OF TOTAL CLASS II ERRORS AT 50 PER CENT POWER FACTOR

A—Net Change in Speed with Constant Braking Flux
B—Increase in Speed Due to Change in Driving Torque
C—Total Change in Speed Due to Class II Errors

explained later. Due to lack of time, we have not further investigated these theories but know that the effect is definite and of the magnitude as shown.

Referring further to Fig. 4, Curve *B* represents a temperature-speed curve taken with the retarding torque supplied by a standard control magnet. The difference between these two curves is represented by Curve *C* and is the effect of the change in flux of the control magnet on the meter speed. Curve *C*, then, is the actual change due to the temperature coefficient of one particular magnet; and, although we have reason to believe that magnets vary somewhat in regard to the magnitude of this effect, this curve is fairly representative. The diagonal Curve *D* in Fig. 4 represents the actual change in total flux of the magnet corresponding to this change in meter speed as obtained from the flux-speed curve shown in Fig. 3. This shows a nearly linear relation and indicates that the temperature coefficient of the magnet is approximately 0.0003 per degree centigrade.

Tests were also made at unity power factor with the lag plate removed from the meter, and it was found that there was a small difference both in shape and in magnitude of Curve *A*. It was not of sufficient magnitude, however, to be considered of much importance, and we

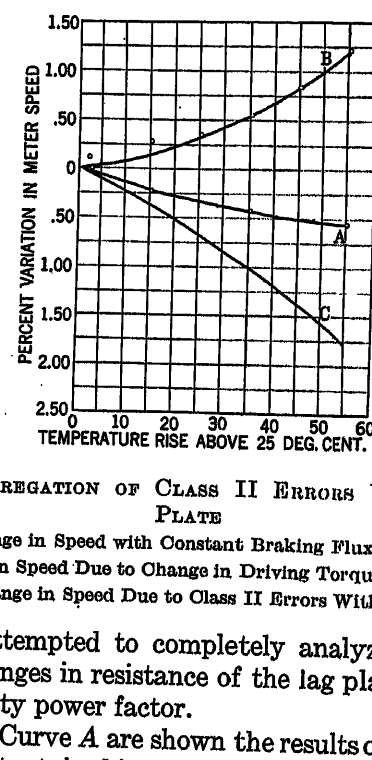

FIG. 7—SEGREGATION OF CLASS II ERRORS WITHOUT LAG PLATE

A—Net Change in Speed with Constant Braking Flux
B—Increase in Speed Due to Change in Driving Torque
C—Total Change in Speed Due to Class II Errors Without Lag Plate

have not attempted to completely analyze the small effect of changes in resistance of the lag plate on meter speed at unity power factor.

In Fig. 5, Curve *A* are shown the results of tests made with a constant braking flux at 50 per cent power factor. In this case, the meter slows down with increasing temperature and this effect is undoubtedly due to the shifting of phase relations noted under Class 2 errors less the increased driving torque, as shown in Fig. 4, Curve *A*. By adding to the former curve, the change in speed due to the permanent magnet Fig. 4 Curve *C*, we arrive at Curve *B*, Fig. 5, which is the usual increase in speed noted with a standard meter at 50 per cent power factor.

Considering Fig. 6, Curve *A* is the same as shown in Fig. 5 and is the net negative error observed at 50 per cent power factor with braking flux held constant. If we now substract from this the error observed at 100 per

cent power factor under the same conditions, which is constantly present in the opposite direction, we arrive at Curve C, the total change in driving torque due to a shift in phase angle with temperature. It will be noted that this is nearly a straight line, and the reason for it will be described later.

The lag plate was then removed and the 50 per cent power factor test repeated. The results are plotted in

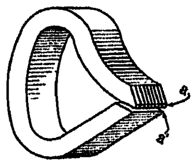

Fig. 8—Standard Type of Single Watthour Meter Magnet

Fig. 7, Curve A. By subtracting from this the results of a similar test at unity power factor, we obtain the total change due to shifting of phase angle, as shown by Curve C. By comparing these results with Fig. 6, it will be seen that the lag plate has a very decided part in the meter's temperature coefficient at low power factors.

In order to check these results, some direct tests (which it is hoped will be described in more detail in a later paper) were made on permanent magnets; and, in so far as horseshoe magnets of the general types found in Fig. 8 are concerned, it has been conclusively shown that the relation between strength and temperature is such as is illustrated by the curve in Fig. 9. In a properly treated magnet, there is a definite strength corresponding to a given temperature, and heating and cooling curves are substantially the same for temperatures under 100 deg. cent. providing the magnet is held at any one temperature for a sufficient length of time to reach equilibrium.

It has been suggested by some of our colleagues that the jaws of the magnet at aa', Fig. 8, might open slightly with a temperature rise, thus causing an increase in the length of air-gap and a consequent diminution of flux. Thorough investigation has been made of this effect and it has been found negligible in relation to the others; for although a very slight effect of this nature does exist, it amounts only to a small percentage of the total observable increase in meter speed with temperature. A very extended investigation of this phase of the problem was completed nearly two years ago by the author, and the method adopted for detecting changes in the gap with temperature was to insert a skeleton meter with magnet attached in an oil bath suitably controlled with heating and cooling coils. Any change in the gap was amplified by means of a scissors-arm arrangement and the motion was accurately measured with a cathetometer. In order to get the effect of changes in the length of gap on the meter speed, the magnets were forced apart by means of a special clamp and the actual change in gap measured with an optical device which reflected a beam of light on a scale at a distance. The principal conclusions derived from this work were as follows:

1. All magnets tested showed a definite tendency to open their gaps under temperature, so that the length of air gap was increased by amounts varying from 0.1 to 0.35 per cent for a 60 degree (Centigrade) rise. This expansion is undoubtedly due to the relieving of internal strains and is partly dependent on the exact heat treatment to which the magnets have been previously subjected.

2. The meter speed was affected by amounts ranging from 0.08 to 0.26 per cent by changes in the dimensions of the air gap due to 60 degrees (centigrade) rise in temperature. The maximum change noted accounts for less than 7 per cent of the total temperature error in the meter, and on the average the increase in the length of the air gap might be expected to account for approximately 5 per cent of this error.

As a result of this work, the factors causing temperature errors may be further subdivided as follows:

Class 1:

1. Internal changes in permanent magnet steel.
2. Change in length of air-gap of magnets.
3. Change in permeability of magnetic circuits of the potential and current elements.
4. Small change in magnitude of potential flux, due to shifting in phase of the magnetizing current (see vector diagram in Fig. 10).

Class 2:

1. Change in resistance of potential windings.
2. Change in iron losses in potential element.
3. Change in resistance of lag plate.

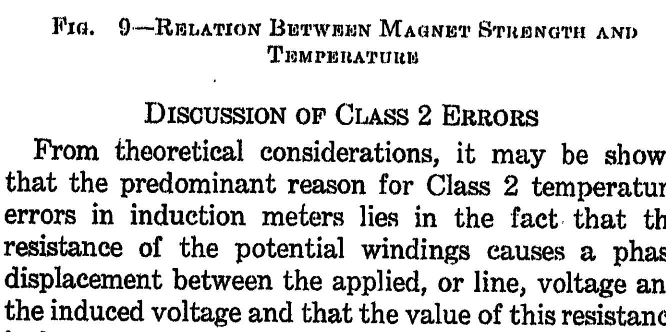

Fig. 9—Relation Between Magnet Strength and Temperature

Discussion of Class 2 Errors

From theoretical considerations, it may be shown that the predominant reason for Class 2 temperature errors in induction meters lies in the fact that the resistance of the potential windings causes a phase displacement between the applied, or line, voltage and the induced voltage and that the value of this resistance is dependent upon temperature.

Without reviewing the general theory, we may state that the torque is proportional to the product of the current and potential fluxes and the sine of the angle

between them. We have already shown (page 12) that the lag plate accounts for about 30 per cent of the Class 2 temperature errors by virtue of its change in resistance; hence we shall now leave it out of the discussion. Since the power used to drive the disk is only about 0.1 per cent of the loss in the potential element, when discussing vector relations between the main fluxes and potential drops this small induced current may be entirely neglected.

Considering the potential element, then, we may represent it by the simple equivalent circuit as shown in Fig. 10A; the line voltage V is balanced by the induced voltage E_1 and the $I_o\, r_1$ drop in the windings. The exciting current I_o may be divided into the usual

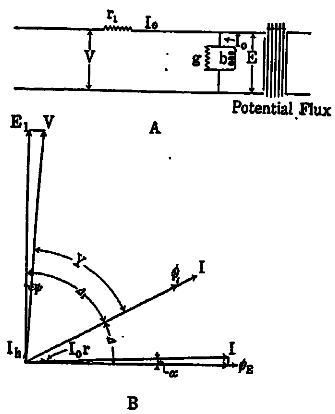

FIG. 10

A. Equivalent Circuit Diagram of Potential Element of Watthour Meter, Secondary Circuit Neglected.

B. Vector Diagram of Watthour Meter Omitting Secondary (Lag Plate and Disk) Currents

V = Line voltage = 110
r_1 = Resistance of potential windings = 63 ohms
E_1 = Induced voltage in potential windings = 109.8 volts
I_0 = Exciting current = 0.0982 ampere
I_h = Power component of $I_0 = gE_1$ = 0.00437 ampere
I_m = Magnetizing component of $I_0 = jbE_1$ = 0.0981 ampere
α = Hysteretic angle = 2.55 deg.
I = Line current
ϕE = Flux in potential element
ϕI = Flux in current element
Cosine λ = Line power factor = 0.50
ψ = Angle of lag

components $E\,g$ and $E_1\,jb$ representing the power and magnetizing portions respectively. If we now draw the vector diagram for this circuit as shown in Fig. 10B, using the actual measured values of the various constants of an induction watthour meter, the relative importance of iron losses and ohmic resistance as they affect the phase relations is seen at a glance. The torque T may be expressed by the following equation as noted above:

$$T \propto \phi_E\, \phi_I \sin \Delta;$$

or, since $\phi_E \propto E_1$ and $\phi_I \propto I$,

$$T \propto E_1 I \cos \Delta_1.$$

Now $\Delta_1 = \lambda + \psi$, where $\cos \lambda$ is the line power factor and ψ is the angle between the line voltage and the induced voltage. Therefore, to get a change in torque due to change in angle ψ, either r_1 or I_0 must change.

It is evident that the angle ψ is approximately directly proportional to the resistance r_1, and, therefore, at low line power factors any change in r_1 means an appreciable change in torque.

If, however, the iron losses are eliminated entirely, thus bringing I_0 in phase and equal to I_m, the magnitude of $I_0\, r_1$ remains practically unchanged with no resulting change in torque. This is not strictly true, because the theory of the meter assumes E_1 to be proportional to V and a shifting of the angle of the vector $I_0\, r_1$ with varying iron losses will result in a small variation in this proportionality. This variation affects Class 1 errors slightly and may be grouped with them.

From the above discussion, it will be seen that if the resistance r_1 be either held constant or reduced to a negligible value, the principal cause of Class 2 temperature errors will be removed. In precision meters it is preferable to reduce the ohmic resistance by enlarging the winding space available in the potential element.

As a matter of interest, we have been able to compensate exactly for change in resistance with temperature by embedding in the potential windings a carbon filament sealed in a glass tube. The negative temperature coefficient of the carbon serves to keep the total resistance of the circuit constant over quite wide ranges. From other considerations, however, this method has its limitations.

In order to check the above analysis experimentally, an induction meter was tested without a lag plate at 50 per cent power factor. By starting with an additional resistance inserted in the potential circuit, it was possible to keep the total resistance constant as the meter was heated to 60 deg. cent. by gradually cutting out the former. It was found that the meter then had substantially the same characteristic speed-temperature curve as that shown in Fig. 1, thus indicating that change in resistance was entirely responsible for Class 2 errors.

Discussion of Class I Errors

These errors are operative at all power factors and are the most important to be considered. They arise from causes that, in general, cannot be removed and hence must be compensated for. In the great majority of cases, single phase meters are operated at power factors above 90 per cent; and, hence, if Class 1 errors can be eliminated, a substantial advance in the art of accurate metering has been effected.

In the case of two-element ployphase meters, one of the elements is frequently operating at a quite low power factor; and Class 2 errors are usually more prominent. In spite of this, Class 1 errors are still of greater importance and it is just as necessary that they be eliminated.

A very satisfactory method of accomplishing this has been worked out and will be described in Part II.

Part II

Thermalloy, a Temperature-Sensitive Magnetic Material

We have recently succeeded in developing a series of magnetic copper-nickel-iron alloys having low points of magnetic transformation combined with a linear temperature-permeability relation. The particular series of alloys that have been found satisfactory for our purposes have been given the distinguishing name of "Thermalloy" and a letter is used to designate the

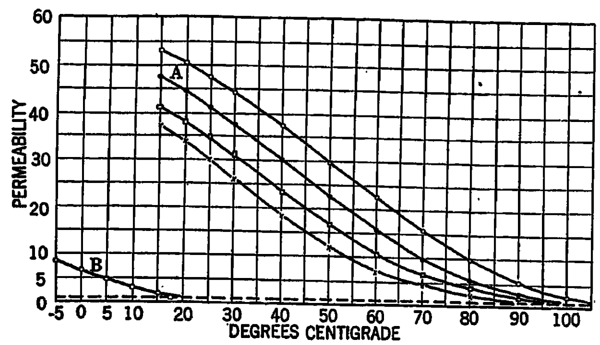

Fig. 11—Temperature-Permeability Curves for Five Samples of Thermalloy $H = 15$

exact composition in each case. At present chief use is made of two particular alloys belonging to this group which are designated as thermalloy *A* and thermalloy *B*. In Fig. 11 is shown a family of curves representing the relationship existing between permeability and temperature for samples of this material in a constant field of 15 gausses.

In order to test these alloys for uniformity, a simple device is made use of as illustrated in Fig. 12. A

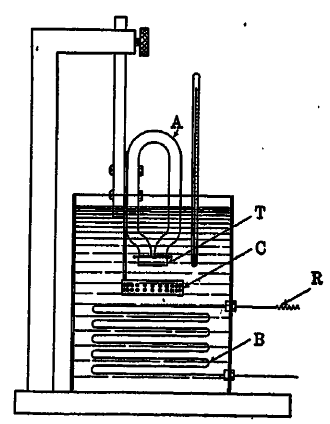

Fig. 12—Device for Control Tests on Thermalloy

standard test piece, *T*, weighing five grams is suspended by a permanent magnet, *A*, in a water bath which can be gradually heated by the small heater *B*. When the test piece has reached the temperature at which the magnet can no longer retain it due to the fact that it has become practically non-magnetic, it falls into the retaining basket, *C*. The temperature of the bath is read by a thermometer and this gives a value that is proportional to the point of magnetic transformation of the material under test. Although this reading is somewhat below the true transformation point, when the test piece falls from the magnet its permeability is very low indeed and for most purposes can be considered non-magnetic at that particular temperature, which we shall term its release point.

In obtaining release points we usually make use of a sample 5/32 inch in thickness, and it has been found from experiment that there is practically no difference in results if samples varying from 1/16 in. to 3/16 in. are used. This simple test, therefore, gives a very ready means of comparing different melts of the same material and also of obtaining the relationship existing between apparent point of transformation and percentage composition of the alloy.

By referring to the premeability-temperature curves in Fig. 11 it will be noted that the permeability rapidly decreases at a nearly constant rate until it approaches that of air when its rate of decrease diminishes. These curves were taken on cast rings of thermalloy, which were carefully machined to size. They were wound with 100 turns secondary and about 500

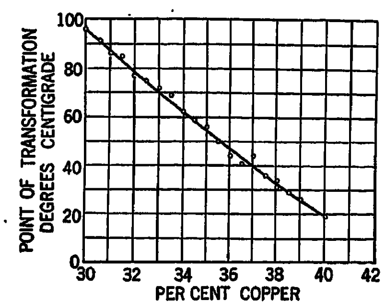

Fig. 13—Curve Showing Relation Between, Copper Content and Apparent Point of Magnetic Transformation of Thermalloy

turns primary so that a minimum heating would result from the magnetizing current. An ordinary ballistic test was employed, but the samples were placed in a tank of oil the temperature of which could be suitably controlled. Base metal thermo-couples were utilized to obtain the exact temperature of each sample, the hot junction of the couple being inserted beneath the insulation on the ring. It might be noted at this time that the so-called "release point" as measured by our testing device agrees very well with the temperature at which these curves appear to reach the permeability of air. For example, Curve *A* is taken on a material that has a release point varying from 92 to 98 deg. cent., whereas the curve indicates approximately 100 deg. cent.

Fig. 13 shows release points plotted against per cent copper, and it is interesting to note that the relationship is practically linear. Small irregularities in the case of some of the samples are probably due to slight variations in the pouring temperatures and cooling rates. The samples were melted in an Ajax high frequency induction furnace, poured, and allowed to cool in sand moulds. Alloys of this nature vary mark-

edly in magnetic properties with composition and heat treatment,[4] hence conditions must be carefully controlled in order to produce uniform results. Gans and Fonseca[5] have published an interesting article showing how the point of transformation from a ferro-magnetic to para-magnetic state in pure copper-nickel alloys varies with the composition. The values of transformation points obtained by them check those shown in Fig. 13 very closely, although they worked

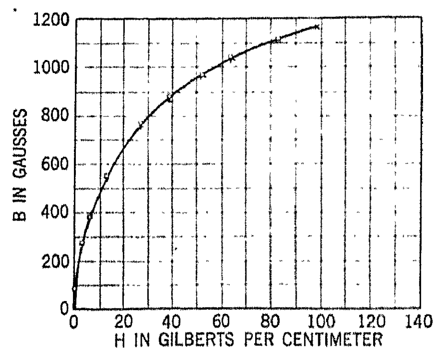

FIG. 14—HYSTERESIS LOOP FOR THERMALLOY A
Temperature = 29.5 deg. cent.

with pure nickel. This close agreement is probably largely accidental, however, because methods of test differ widely and they do not specify heat treatment which is very important. These investigators found a linear relation which checks our results, but so far as we have been able to learn no one has investigated the character of permeability-temperature curves below the transformation point for these alloys.

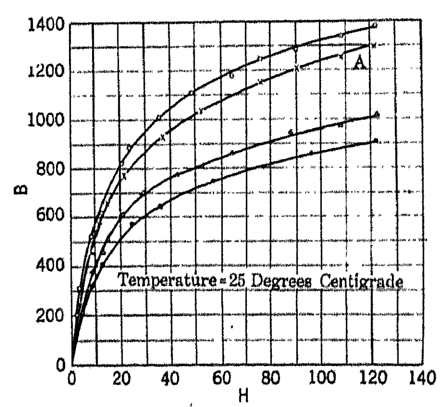

FIG. 15—NORMAL INDUCTION CURVES FOR FOUR SAMPLES OF THERMALLOY
Temperature = 25 deg. cent.

Thermalloy *A* contains approximately:
Cu 30.0 per cent
Ni 66.5 per cent
Fe 2.2 per cent
Impurities 1.3 per cent

One effect of adding iron to the alloy is to raise the point of magnetic transformation. The addition of 2.3 per cent of iron to an alloy of 70 per cent pure nickel and 30 per cent electrolytic copper raised the release point about 45 deg. cent. Since most commercial nickels contain some iron we simply add sufficient additional to bring the total percentage up to the desired value. Among other advantages it is possible to obtain the required results at a much lower cost in this manner.

Heat treatment of thermalloy has important effects on its magnetic properties. Grade *A*, for instance, gives an average reading of 95 deg. cent. as its release point in our test when cooled slowly in a sand mould; if it is cast in graphite moulds it is practically non-magnetic at 20 deg. cent. The same metal when cast in a mould of zerconium silicate released at 60 deg. cent. The castings in both of these cases were one-half inch square and seven inches in length. In the case of the bar having the 60 deg. release point heating for two hours at 700 deg. cent. and cooling in air raised this point to 98 deg. A sample having a release point of 95 degrees was heated to 900 deg. and quenched in cold water. This treatment lowered its release point to

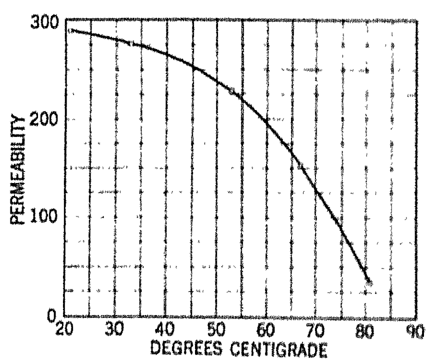

FIG. 16—TEMPERATURE-PERMEABILITY CURVE FOR MONEL METAL

75 deg. It will be seen, therefore, that in order to obtain uniformity in results it is necessary to carefully control the conditions under which the alloys are made.

The heating and cooling premeability-temperature curves are practically identical for thermalloy, providing it is held at any one temperature for a sufficient length of time to reach equilibrium. This is on account of the extremely small hysteresis loss in this material. In Fig. 14 are shown the points obtained on a hysteresis loop taken at a maximum magnetizing force of 100 *H*, which serves to bring it well past the knee of the normal induction curve. Within the limits of accuracy of the apparatus used in obtaining these data very little loss can be detected. Another unusual characteristic is that the retentivity is only about 8 per cent of the maximum induction so that with an air gap in the magnetic circuit the remanence should be inappreciable.

Of course, the permeability of these alloys is quite

4. Nickel and its Alloys—*Bureau of Standards Circular*, No. 100.
5. Die Magnetischen Eigenschaften von Nickel-Kupfer Legierungen—*Ann. Physik*, 61, p.742.

low as indicated by the normal induction curves in Fig. 15. For certain applications, however, high permeability is not essential or even desirable as is the case with the one herein described. The specific resistance of thermalloy A is about 49 michroms per centimeter cube:

The chemical composition of thermalloy A is similar to that of monel metal but by referring to Fig. 16 it will be seen that the characteristic permeability-temperature curve is very much different. The permeability of monel falls off very rapidly as the point of transformation is approached, as is the case with iron and other ferro-magnetic materials. It is believed that this sudden decrease in permeability on approaching the point of magnetic transformation is a characteristic of nickel-copper alloys in which the two metals form a homogeneous mixture.

The Heusler alloys[6] are similar from a metallurgical point of view to the alloys just discussed since both are solid solutions with copper as the solvent. Also the Heusler alloys have low points of magnetic transformation and increasing the copper content lowers this critical point. In general, they are more difficult to work with, both in regard to casting and machining, than the copper-nickel series.

From the foregoing it will be seen that we now have available an almost perfect means of compensating for variations in watthour meter speed with temperature.

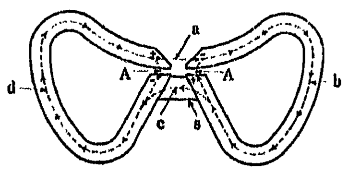

FIG. 17—SCHEMATIC DIAGRAM OF MAGNET COMPENSATION WITH THERMALLOY

Fig. 17 shows a magnetic shunt made of thermalloy inserted between the control magnets, of a watthour meter. In this arrangement two single magnets are paired together and the polarity is such that there is a small amount of flux circulating around the path a, b, c, d. We may think of this part of the flux as being shunted from the main path across the gaps and it is increased by the insertion of the small bridge, S, of thermalloy, which may be termed a magnetic shunt. With an increase of temperature the total flux of a magnet decreases as has previously been described, whereas in order to effect a compensation for Class 1 errors it is necessary that the total damping flux actually increase somewhat. In the device shown in Fig. 17, this condition does exist because as the temperature rises, the permeability of the magnetic shunt decreases linearly and a larger percentage of the total flux crosses the air gaps, $A.A.$ By selecting a shunt of the correct dimensions this effect will exactly compensate for the overall temperature error of the meter.

An average speed-temperature curve for a watthour meter has been shown in Fig. 1. It will be seen that the rate of decrease of meter speed with temperature increases rather rapidly just below room temperature, whereas the rate of increase in permeability of thermalloy A below room temperature has a tendency to decrease. This would result in under-compensation at low temperatures, and in order to overcome this difficulty we have made use of a compound magnetic shunt; that is, we insert a piece of thermalloy B in parallel with the first. This has a release point of 15 deg. cent. and hence is non-magnetic at ordinary room temperatures. As the temperature falls, however, it serves to correct the under compensation as above noted due to its increasing permeability, thus giving results that are correct at very low temperatures. The cross sections of these two shunts are so adjusted that the meter runs at correct speed at −14 deg., +25 deg. and 55 deg. cent., and the compensation holds good over considerably larger ranges.

In order to determine whether uniform compensation could be effected by this means, hundreds of magnets were built up and tested on stock meters, and Table I shows the results obtained on a typical group of 20 magnets. The maximum error due to temperature at unity power factor was 0.4 per cent for a 30 deg. cent. rise. The chief reason exact compensation cannot be effected on a large scale is that individual magnets vary somewhat in temperature coefficient, but the variation is small and for practical purposes can be neglected. Table II shows the results of similar tests on a group of uncompensated magnets. It will be noted that variations occur between magnets of the same order as is present in Table I.

TABLE I.

Temperature Errors at Unity Power Factor Using Compensated Magnets

Magnet	Per Cent Change in Meter Speed, 25 deg. to 55 deg. Cent.	Per Cent Change in Meter Speed, 25 deg. to −14 deg. Cent.
A	−0.02	−0.07
B	−0.07	+0.2
C	0.0	−0.2
D	+0.1	+0.12
E	−0.05	+0.24
F	+0.1	−0.03
G	−0.07	+0.09
H	−0.02	+0.34
I	+0.15	+0.25
J	+0.07	+0.3
K	+0.07	+0.35
L	+0.37	......
M	+0.1	+0.17
N	+0.35	......
O	+0.23	−0.07
P	+0.2	−0.19
Q	+0.22	−0.23
R	+0.09	+0.22
S	+0.4	......
T	−0.07	−0.07
Average	+0.11	+0.08

6. Verhandlungen der Physikalischen Gesellschaft, 5, p. 219; 1903.

TABLE II.
Temperature Errors at Unity Power Factor Using Uncompensated Magnets

Magnet	Per Cent Change in Meter Speed, 25 deg. to 55 deg. Cent.	Per Cent Change in Meter Speed, 25 deg. to −14 deg. Cent.
1	+2.6	−3.0
2	+2.23	−3.07
3	+2.4	−3.3
4	+2.1	−3.06
Average	+2.3	−3.1

Conclusions

Temperature errors in induction watthour meters can be divided into two general classes,—1. Those affecting the magnitude of the driving flux or braking flux, 2. Those affecting the phase relation between the line voltage and induced voltage in the potential element.

The permanent magnets used for braking are responsible for the greater portion of the errors under Class 1, and this being due to a natural characteristic of magnet steel cannot be eliminated.

Ordinarily, Class 2 errors are not of great importance, but they can be reduced by proper design of the potential element. They are caused almost entirely by changes in resistance of the potential windings and lag plate.

Class 1 errors may be neutralized by a single compensation, consisting of a device whereby a small portion of the flux is shunted through a special alloy—(thermalloy)—the permeability of which is very sensitive to temperature.

Induction watthour meters compensated with thermalloy magnetic shunts, even without any further modifications, are practically independent of temperature changes over very wide ranges providing the power factor is reasonably high. By suitable modifications, the compensation may be made independent of power factor.

The application of compensated magnets to meters on a production basis gives a reasonable degree of uniformity in results, and very rarely will errors as high as 0.013 per cent per deg. cent. rise be encountered.

This method of compensation has the distinct advantage of being independent of any necessary adjustment of the meter, and is extremely simple and positive in its action.

By preparing certain copper-nickel-iron alloys in the form of castings suitably heat treated, a linear relationship is obtained between permeability and temperature. By controlling the copper content the point of transformation of these alloys may be made to occur at almost any desired temperature below that of pure nickel.

The hysteresis loss of thermalloy is extremely low; hence the heating and cooling curves in a constant field are practically identical, although there is a small time lag before the metal reaches equilibrium at any one temperature.

It is believed that some of the characteristics of these alloys, such as perfectly reversible straight line permeability-temperature relationship, combined with negligible hysteresis, has escaped previous notice and that they are an important contribution to engineering materials.

It is probable that the linear temperature-permeability relationship of thermalloy is due to the non-homogeneous manner in which the copper is held in solution, which gives the effect of the summation of a large number of alloys, each having a different transformation point.

In addition to temperature compensation, thermalloy may be used for a variety of purposes one of which is a direct reading low temperature thermometer as described in the appendix.

Appendix

Description of Magnetic Thermometer Utilizing Thermalloy

The approximately linear temperature-permeability relation of thermalloy may be utilized in the construction of magnetic temperature indicators. An instrument of this type is shown in Fig. 18. In this figure, *A -A.* is a vane made of thermalloy and *B -B.* is a vane made of iron or of some other magnetic material having a low temperature-permeability coefficient. It is evident that the armature will take up a position in which the sum of the magnetic moments acting on *A.-A.* is equal and opposite to the sum of the magnetic moments acting on *B -B.* This position will vary with a variation in temperature as the moments on *A -A.* vary with its change in permeability. The upper limit of the scale is reached when the temperature is equal to the magnetic transformation temperature of the thermalloy. The lower limit is not sharply defined, as this would require either that the thermalloy reach a value of permeability compared with which that of the iron would be neglibible, or that the temperature-permeability coefficient of the thermalloy would have

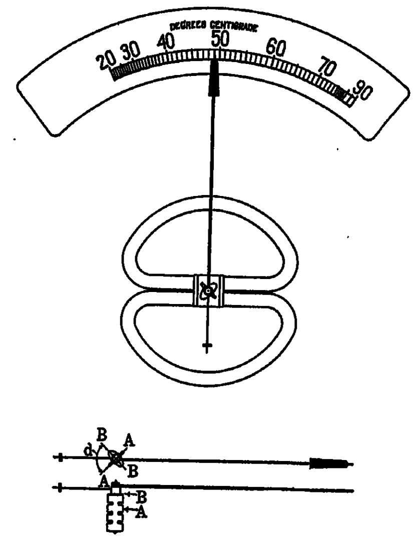

Fig. 18—Diagram of Magnetic Thermometer

to become equal to that of the iron. However, with the instrument shown in Fig. 18 the congested region extends over only a small part of the scale.

It is also possible to construct magnetic thermometers using a thermalloy vane or an iron vane in a magnetic field acting against the torque of a hairspring. In the case of the iron vane instrument, variation of torque with temperature is obtained by means of a thermalloy shunt in parallel with the air-gap in which the iron vane is placed.

Discussion

H. B. Brooks: The paper by Kinnard and Faus is important and valuable, not only for its definite data and clear reasoning, but also because it is a tangible indication of the interest which is being taken in the question of the accuracy of metering. This interest reveals itself in many ways; for example, in the increasing attention which is being given to the ratio and phase-angle performance of instrument transformers. Among reasons for this interest may be mentioned the greater attention which must be given to plant operation, because of higher costs of fuel and other essential supplies; the growing tendency toward interconnection, with the consequent need of accurately measuring interchanged energy; and the necessity for accurate electrical measurements in acceptance tests of turbo-generator units, where an error of a fraction of one per cent may mean an error of thousands of dollars in penalty or bonus. Such tests must necessarily extend over a period of hours, and the load cannot be kept as constant as might be done with small machines in the laboratory. It is only natural, therefore, that those having to make such tests should turn to the watthour meter as a means of measuring the energy and hence the electrical power output. However, from information which has come to us, it appears that such use of watthour meters does not give results as consistent as can be had by the use of indicating wattmeters read at sufficiently frequent intervals. Nevertheless, the large amount of labor required for the latter procedure makes it desirable to bring up the performance of the watthour meter, if possible, to such a stage that it will be good enough for plant acceptance tests. The work done by Kinnard and Faus is an important step in this direction, and I hope they will continue their analysis of sources of meter errors and means for overcoming them.

There is an astonishing lack of accurate information on temperature errors in watthour meters. Published papers exist, but they either relate to meters of obsolete types, or lack necessary data. For example, one paper on temperature errors of American watthour meters leaves it to be inferred that the meters were of the alternating-current type, and while giving many curves of performance, fail to state the power factor of the loads. Text books also are often at fault; for example, an English book published in 1923 states that "In an induction meter both the operating and the braking torques are developed by the medium of eddy currents, often in the self-same disk, so that temperature effects cancel out completely." Similar inaccurate and superficial statements may be found in some other reputable treatises on meters.

Taking up the paper in detail, I would first inquire why a watthour meter was used as a standard by which to determine the performance of the meter under examination. Even though this standard meter was kept at room temperature, it seems probable that errors resulting from self heating and consequent temperature inequalities within the standard meter might affect the observed results undesirably, and that a wattmeter would have been preferable.

Referring to the tests at unity power factor with the lag plate removed, I would suggest that by replacing the lag plate with a lag *coil* of similar form, closed through an adjustable external rheostat, the effect of change of resistance could be readily determined.

A comparison of Fig. 6, Curve *C*, with Fig. 7, Curve *C*, shows that the high temperature coefficient of the copper lag plate is responsible for a fair share of the Class 2 error in this particular type of meter, and suggests the use of some material of lower temperature coefficient.

It is gratifying to have a definite answer, such as the authors give us, in regard to the idea that the length of the gap of meter magnets varies with temperature. The value given for the temperature coefficient of the gap flux presumably refers to the kind of steel in regular use at this time. Fitch and Huber, at the Bureau of Standards, found values ranging from -0.0001 to -0.0003 per deg. cent., with a mean value of -0.0002. At the time this work was done (1913) tungsten steel was used, as far as we were aware.

I think the authors are to be congratulated on their neat and simple solution of the matter of Class 1 errors, especially in view of the peculiar form of Curve *A*, Fig. 1, which they take care of by using two bridge pieces of different grades of thermalloy in parallel.

The statement that Class 2 errors are more prominent in two-element polyphase meters is evidently made with the idea that such meters usually measure loads of power factors less than unity, for on the usual assumption of a balanced three-phase load a polyphase meter working under a given phase-angle error (alike in the two elements) measures the total load on it with exactly the same error as would occur in measuring the load with an otherwise identical single-phase meter with the same phase-angle error. In other words, with balanced loads we do not need to reason differently concerning polyphase meters and single-phase meters as regards either Class 1 or Class 2 errors.

The next step is evidently to get rid of Class 2 errors, and if possible in a manner as simple and elegant as the one given in the paper for Class 1 errors. I wish to mention a method which has occurred to me recently, which has apparently been shown to be sound in principle by a simple experiment. According to Schmiedel, the flux Φ_I from the current electromagnet lags behind the current I (see Fig. 10B) by a small angle, which we may call γ, and which for simplicity the authors of the paper have not considered; the effect of this angle is that the lag plate must be adjusted to lag the voltage flux Φ_E by an angle δ which is not equal to ψ but to $\psi + \gamma$, at standard temperature. When the temperature increases, ψ increases while δ decreases, so that the compensation is no longer correct. In this lies the entire reason for Class 2 errors. I propose that we put a lag coil on the current electromagnet, thus lagging its flux Φ_I by an additional angle which may be called β. Hence at standard temperature we must now adjust δ to be equal to $\psi + \gamma + \beta$, in order that the meter may be correctly lagged. Let us give the lag coil on the current electromagnet a high temperature coefficient, for example by closing it through a resistor of very pure iron wire, and also give the lag coil on the voltage electromagnet a suitably low temperature coefficient. We may thus make the small angle β decrease with temperature at a sufficiently rapid rate to compensate for the increase in ψ and the decrease in δ; in other words, to maintain the relation

$$\delta = \psi + \gamma + \beta$$

at all temperatures, and thus eliminate Class 2 errors. It is obvious that β must also compensate for any change of γ, concerning which at present no numerical data seem to be available. It is of interest to note that the iron-wire resistor for the current lag coil may be embedded, wholly or in part, in the voltage winding, in order to respond to changes in temperature of the latter, just as the authors of the paper did with their carbon filament.

In making this suggestion, I realize that we cannot lose sight of the variables other than temperature to which a meter is

exposed, such as variations of voltage, frequency, and wave form, and if my proposal for eliminating Class 2 errors should lead to an unavoidable increase in the errors from these causes, another and unobjectionable solution must be found. The suggestion is offered for what it is worth, in the hope that it may assist in the eventual solution of the problem of temperature compensation of watthour meters.

J. R. Craighead: With respect to Mr. Kinnard's paper, there is only one point I should like to call attention to. Under Conclusions, he has a conclusion referring to driving torque and damping torque as being only one side of the thing which may cause the watthour meter to go wrong. It seems to me that if the driving torque and the damping torque, in the broad sense, are both correct, you have a correct watthour meter. In other words, errors in driving torque or damping torque constitute the whole error. I would suggest that he use the word "flux" perhaps instead of "torque" in that place as expressing more accurately what he means.

F. B. Magalhaes: I wish to place on record a statement which is not intended to be complimentary on a personal basis to the authors but which would be made quite impersonally to any one presenting the same development. Progress in any field can be considered as going forward by more or less slow steps, and after a step forward has been taken there is a consequent search or analysis made on that step, to develop all of the possibilities that may be obtained from the single step forward.

The authors of this paper in using the methods which they have described have introduced a new means for correcting certain errors that have to the present time been considered more or less accepted or inherent in the design of a watthour meter. Without any doubt, this step forward will therefore be followed by a considerable amount of lateral or parallel research to develop all the possibilities that the step forward may offer. On this basis, it should be recorded that the idea of using a small piece of alloy near the jaws of the magnet as an automatic means for correcting the temperature errors that have heretofore had to be accepted, is distinctly to be commended as an important step forward.

One specific question I would like to ask is in reference to the permanence of the alloys which they have used. They have outlined in some detail the necessity of watching carefully the mixture of the alloy and their heat treatment. It leaves, therefore, a question in the reader's mind, I believe, as to the ultimate permanency of the alloy. The results that have been obtained can be seen on the curves. However, the permanent value of the results that have been obtained will be based largely on the permanence of the correction that has been made. If the alloy is not a stable alloy and may change with the passing of time or under temperature conditions that may be encountered in the field, it will detract very greatly from the value of the results obtained. It would be interesting to know, therefore, what results the authors may have to establish this point of permanence.

Joseph Slepian: There are just a few consequences of the phenomenon that Messrs. Kinnard and Faus use that I think will be interesting.

Some time ago there was a patent issued to Thomas A. Edison disclosing the rather ambitious project of making a heat engine using this very phenomenon. Edison proposed to allow a large mass of iron to be pulled into a magnetic field, there to be heated until it became non-magnetic, and then to be released, thus getting work at the expense of heat energy.

The fact that it is possible to get work out of this phenomenon at the expense of heat makes it possible to apply the principles of thermodynamics and get relations connecting the properties of these materials.

One need only apply the first and second laws of thermodynamics. It will necessarily follow that a piece of magnetic material having a temperature coefficient must change its temperature as it moves from a weak field into a strong field. That is, its specific heat must change. I believe it will have to rise in temperature as it moves into a strong field and fall in temperature as it moves into a weak field.

I would like to ask Messrs. Kinnard and Faus just what are the special merits of the alloys which they have chosen. There are, of course, many alloys that have transition points at temperatures in the ordinary range. Some of them have been known for a long time. Those of most interest were the Heusler alloys which contain no elements which are ordinarily magnetic, and the question occurs to me as to whether these other materials are wholly unusable, or whether it is simply some point of refinement that causes one to use these particular alloys rather than the others that are known.

W. H. Pratt: The method of correcting for Class 1 errors is peculiarly direct and effective. It corrects the errors where they occur and when they occur. I mean by this that the thermalloy shunt is in such intimate contact with the controlling magnets that it partakes uniformly of the temperature changes of the latter, and on account of the absence of hysteresis renders the correction particularly effective.

The course suggested for minimizing Class 2 errors is also directly aimed at securing excellent results, for by the reduction of the resistance of the potential coils not only is the desired result obtained, but it is a change that is in the right direction to improve still further the behavior of the meter in regard to frequency and wave-form errors.

I think this last remark has a bearing on some points that Mr. Brooks raised in his discussion. The arrangement that he proposed for compensating for phase displacement is such as to expose the windings to the rather extreme changes of temperature which comes from the current coils. The coils which he suggested would be naturally exposed to these rather extreme changes, and it would not accomplish that further refinement of the meter that is accomplished by a reduction in resistance.

C. M. Jansky (by letter): This paper is of considerable scientific and practical importance as it is the best analysis of the influence of temperature that I have seen. There is no mention made, however, of the study of this problem that was made at the University of Wisconsin several years ago by Mr. B. E. Miller. The results of his investigations of the effect of temperature on the registration of watthour meters at 10, 50, 100 and 150 per cent loads were published in the September 18, 1915 issue of the *Electrical World.* Mr. Miller's results show that the error is not the same with increasing as with decreasing temperatures. The per cent of registration when plotted against temperature gave a decided "hysteretic" loop. Messrs. Kinnard and Faus do not refer to this paper, but if they have seen it I would be pleased to have their explanation of this difference in the accuracy with increasing and decreasing temperatures.

I. F. Kinnard: In Dr. Brooks' discussion, he asked why we made use of a watthour meter rather than a wattmeter in obtaining our readings. We believe a watthour meter may be advantageously used as a standard, in order to study the effects of varying temperatures on another watthour meter if care is taken to insure that the standard watthour meter is kept at a constant ambient temperature, and if, before readings are made, it is allowed to run a sufficient length of time to reach its maximum self-heating value. Very accurate results are obtainable, with a minimum of trouble due to other possible variations such as frequency.

In connection with the point raised in regard to studying the effects of changes in temperature of the lag plate by the use of a lag coil with an external resistance, this can undoubtedly be done and should give interesting information, although for our particular purposes we did not consider it necessary. We do not feel that the lag plate causes temperature errors of sufficient magnitude to call for the use of brass or manganin as the situa-

tion now stands. It has been pointed out, however, in the paper that for certain applications Class 2 errors can and should be eliminated, or at least reduced by modifications of the potential element. When this is done the copper lag plate will undoubtedly be dispensed with.

It is true, as pointed out by Dr. Brooks, that the same reasoning applies to polyphase meters as discussed for single-phase. Class 2 errors are, however, of more importance in the polyphase meter due to the fact that the line power factor is frequently quite low. This can be fairly satisfactorily taken care of by using but one compensated and one uncompensated magnet on a two-element meter. As I previously stated (and it has been brought out in the discussion), the better way is to eliminate Class 2 errors by reducing resistance of the potential winding.

Mr. Magalhaes raises the question of the permanency of thermalloy used as a magnetic shunt. I wish to say that we have had magnets compensated in this manner under observation for over a year and a half and no aging effects of any magnitude have been observed.

We might also bring to your attention the fact that copper-nickel alloys have been continuously used for a large number of years in connection with resistance materials, Monel metal and other applications, and it is known to be a stable combination.

Mr. Craighead points out a statement in the first part of the conclusions that might be open to some question, and I believe his point is well taken. I meant to refer to the driving flux in classifying the errors that do not depend upon power factor.

Dr. Slepian points out a very interesting fact, that there are other alloys such as the Heusler alloys, which probably have some characteristics similar to those we have described for the copper-nickel series. As a matter of fact, we have mentioned in the paper that the Heusler alloys have a possible application in this connection, although we believe the copper-nickel series can be better controlled, has a somewhat higher permeability, and is easier to machine, outside of the fact that it is of lower cost.

Prof. Jansky asked whether or not there is a hysteresis lag in the heating and cooling curves of a watthour meter. We have not observed any such effect, in the large number of tests made with these meters, although there is a distinct time lag. That is, if a watthour meter is held at any one temperature for a sufficient length of time to reach equilibrium, its cooling curve will be identical with its heating curve.

C. H. Ingalls: For the past twelve years, the Meter Division of the Boston Edison Company has corrected for the temperature errors of its portable test meters through thermometers permanently installed in the meters. A recent occurrence illustrates how the accuracy of an uncompensated meter may be affected by certain abnormal temperature conditions.

Two testers were working in the same locality, one of whom went to work directly from home, the other starting from the office. The former kept his test meter in his automobile over night, while the latter's meter was stored in a warm place. This resulted in a difference in temperature of 20 deg. cent. between the two meters at the start of the day's work, corresponding to a difference in the corrections to be applied to the test meters of nearly two per cent. Subsequent tests proved that the thermometers and the applied corrections were correct. It is particularly gratifying that such a simple and positive means of correction for temperature errors can be applied to test meters. The correction of temperature errors at low power factors is not quite as important as the correction at unity power factor, because, with comparatively few exceptions, watthour meters are tested on the system only at unity power factor.

A. E. Knowlton: It might be well to mention that reference to the temperature errors of watthour meters in this paper gives no occasion to the multitude of users of electric service to be disturbed about the situation. I think it is fairly safe to say that they are having that service metered to them with a degree of accuracy not matched by any other commodity service, whether sold by the pound, by the cubic foot or by count. We are dealing here with the minutiae and any layman who chances to read papers of this kind has no reason to be disturbed by our activities in improving electric metering.

I. F. Kinnard: The point brought out by Mr. Ingalls serves to illustrate how a simple and reliable temperature compensation, such as described in this paper, will aid in obtaining precision results in the calibration of watthour meters. As he has indicated, it was hitherto necessary to be sure that test meters were not at abnormally high or low temperatures before using them, in order to obtain the best results. The automatic compensating shunt, however, insures a constant performance over very wide temperature ranges and makes it entirely unnecessary to use thermometers.

As Mr. Knowlton has intimated, the fact that a distinct refinement has been made in the art of watthour-meter manufacture should not in the least shake our confidence in existing meters. If the paper be read carefully it will be seen that errors or variations due to temperature are inherently small in this class of apparatus. It is, however, a distinct advantage to be able to state that temperature will in no way affect the results obtained irrespective of any extreme conditions of use, and it is to that end we are working.

Storage Battery Electrolytes*

BY G. W. VINAL† and G. N. SCHRAMM†
Member, A. I. E. E. Non-member

Synopsis.—Experiments have been in progress at the Bureau of Standards to determine quantitatively the effect produced by a wide variety of impurities on the rate of sulphation of storage battery plates. The method for making the determinations involves measuring the changing weight of the plates suspended in the solution. The various impurities are classified according as they affect the negative plates, the positive plates or both. A discussion is given also of certain combinations of impurities. Sodium and magnesium sulphates which are sometimes added to the electrolyte to "improve" the behavior of the battery are without effect on the rate of sulphation. It is important that some generally recognized specifications for storage battery electrolytes should be established.

Introduction

The satisfactory operation of a storage battery is in a large measure dependent upon the physical and chemical properties of the electrolyte which it contains. Within recent years storage batteries have come to be used under widely varying conditions of service, and millions of them annually pass into the hands of people who have no technical knowledge of their construction or adequate information with regard to the care which they should receive. Some of them are used at extremely low temperatures in airplanes and signal service while others are subjected to abnormally high temperatures in the tropics. Considering the diversity of the service which they have to perform, it is increasingly important that the physical and chemical properties of the electrolyte should receive more extended study.

For a number of months experiments have been in progress at the Bureau of Standards to determine quantitatively the effect produced by various impurities on the rate of sulphation of storage battery plates. The method which involves a determination of the weight of the plates while suspended in solution has been described in a previous publication.[1]

The results of experiments on a few of the impurities have been previously published in the JOURNAL of the American Institute of Electrical Engineers.[2]

TABLE I
ACCURACY OF MEASUREMENTS ON NEGATIVE PLATES
(Average of 14 experiments, pure electrolytes, 1.250 sp. gr. 25 deg. cent.)

Time (hours)	Average Gain in Weight (grams)	Probable Error of Average Value (grams)	Probable Error of Single Observation (grams)
50	0.69	0.03	0.15
100	1.44	0.06	0.24
200	2.68	0.09	0.33
300	3.87	0.13	0.41
400	5.03	0.16	0.50
500	6.08	0.21	0.60

In the present paper the results of experiments covering a wide range of impurities are given. The tables that follow give the quantitative relation between the amount of the various impurities which were added to the pure electrolyte 1.250 specific gravity, and the effects produced as judged by the change in weight of the plates.

Accuracy of Measurements

In the course of the experiments a number of determinations have been made of the rate of sulphation of storage battery plates in pure electrolytes having a specific gravity of 1.250 at 25 deg. cent. These may be considered as control experiments by which the effects produced by the impurities are to be judged. One control experiment was made with each group of five measurements. An analysis of these permits the computation of the probable error of the average value and the probable error of a single observation.

It is important to establish the range of the probable error in these computations. There are a number of impurities which produce little or no effect, and any definite statement that they do or do not produce an effect must be based upon a comparison with the normal values for plates in pure solutions to within the limits of the probable error.

Table I, for negative plates, and Table II, for positive plates, give the average gain in weight in pure electrolyte for these plates together with the probable error. In classifying the impurities according to the effects produced, the average value of the control experiments is given for comparison, except in Table VII where the measurements all belong to one group. For these, the value of the control experiment of this group is given.

Impurities Affecting the Negative Plates Only

The impurities which affect the negative plates only (Table III) may be divided into two classes:

a. Impurities which are deposited quickly in the metallic state upon the negative plates and produce appreciable gassing. These include platinum, copper and silver. A closed circuit exists between the underlying lead of the plate and the impurity which is deposited upon it. Lead sulphate is formed in proportion to the quantity of electricity flowing and the plate

*Approved by the Director of the Bureau of Standards.

†Both of The U. S. Bureau of Standards

1. Vinal and Ritchie. A new method for determining the rate of sulphation of storage battery plates. Bureau of Standards Technologic Paper No. 225 (1922).

2. Vinal and Altrup. *The Effect of Certain Impurities in Storage Battery Electrolytes.* TRANSACTIONS A. I. E. E., Vol. 43, p. 709, 1924.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

gains in weight. Hydrogen is liberated at the surface of the impurity. The local action produced by these impurities proceeds at a fairly rapid rate until the ultimate capacity of the plate is exhausted. A considerable part of silver and copper, which was deposited on the plates as a spongy or tree-like mass, subsequently fell off so that the gain in weight of the plates represents chiefly lead sulphate. These impurities cannot be eliminated by changing the electrolyte in the battery, but their effect may in some cases be mitigated as Gillette[3] has shown. The results which are given in Table III show that platinum is one of the most deleterious of impurities. Extremely small amounts are sufficient to produce rapid sulphation of the plates. Copper produces less effect. These results have been abstracted from the previous paper by Vinal and Altrup, and in the case of copper a correction has been made for a misplaced decimal point in the former publication.

TABLE II
ACCURACY OF MEASUREMENTS ON POSITIVE PLATES
(Average of 17 experiments, pure electrolytes, 1.250 sp. gr., 25 deg. cent.)

Time (hours)	Average Gain in Weight (grams)	Probable Error of Average Value (grams)	Probable Error of Single Observation (grams)
50	0.34	0.03	0.12
100	0.51	0.04	0.17
200	0.66	0.06	0.22
300	0.86	0.08	0.30
400	1.05	0.10	0.35
500	1.15	0.10	0.32

TABLE III
LOCAL ACTION PRODUCED BY IMPURITIES AFFECTING ONLY THE NAGATIVE PLATES
(Results are expressed as the gain in weight of a single plate in grams, at intervals from 50 to 500 hours)

Impurity	Material Added	Percentage Impurity	Time in Hours					
			50	100	200	300	400	500
None	(Control experiments)	..	0.69	1.44	2.68	3.87	5.03	6.08
Platinum	$PtCl_4$	0.00001	0.7	1.4	3.0	4.8	6.8	8.4
Platinum	do	0.00003	13.2	19.3	26.1	29.9	32.4	34.2
Platinum	do	0.00005	27.4	28.1	28.8	29.2	29.3	29.5
Copper	$CuSO_4$	0.008	1.1	2.1	4.0	6.1	8.1	..
Copper	$CuSO_4$	0.04	7.3	10.7	15.6	19.5	23.5	..
Silver	Ag_2SO_4	0.1	13.5	18.6	24.0	..	..	..
Tin	$SnSO_4$	0.1	4.6	7.0	9.4	11.0	12.5	13.9
Tungsten	WO_3	0.003	0.3	1.7	5.3	10.0	15.0	20.0
Bismuth	Bi_2O_3	0.2	4.5	5.8	8.2	..	..	..
Sulphurous Acid	H_2SO_3	0.05	5.1	6.4	8.3	10.2	11.8	13.6
Sodium Bichromate	$Na_2Cr_2O_7$	0.05	3.3	5.2	8.4	11.4	14.1	..
Arsenic*	As_2O_3	0.001	1.3	2.6	4.8	6.9	8.8	10.9
Arsenic	As_2O_3	0.10	0.8†	..	..	..	..	..
Antimony	$Sb_2(SO_4)_3$	0.001	3.8	8.8	16.3	..	..	..
Nitrates	HNO_3	0.001	1.3	2.0	3.6	..	..	..
do	do	0.004	3.1	4.0	5.2	..	..	..
do	do	0.008	5.3	6.4	7.7	..	..	..
do	do	0.035	23.0	25.3	27.3	..	..	..

*These results are not as reliable as the others.
†At 55 minutes, plates gassing and solution turned brown, test abandoned.

3. *Trans.* Amer. Electrochem. Soc., 41, p. 217 (1922).

b. The second class of these impurities includes those chemical compounds which are reduced more slowly at the negative plates and result in little, if any, perceptible liberation of hydrogen. In some cases these impurities can be eliminated from a battery by replacing the electrolyte. For some of these, a quantitative comparison may be made between the calculated and observed changes in weight of the plate.

Bismuth presents an interesting example of the reduction produced at the negative plates accompanied by the deposit of the bismuth itself as a brown powder on the plate. Bismuth trioxide reacts with sulphuric acid to form bismuth sulphate and this, in turn, is reduced at the negative plate to bismuth with the formation of an equivalent amount of lead sulphate. For the 12.5 grams of bismuth trioxide, which were added to the solution, 24.4 grams of lead sulphate should be formed, and to this must be added the weight of the bismuth, 11.2 grams, deposited on the two plates in the solution making 35.6 grams as the calculated increase in weight of the plates. The amount actually observed was 33.6 grams. No appreciable effect was produced by the bismuth on the positive plates. The local action produced by the bismuth which is deposited in the pores of the negative plates is relatively slow, but the diffusion of the electrolyte into the plates is doubtless impeded with a consequent loss in the available capacity.

Antimony and arsenic, like bismuth, affect the negative plates, but are without apparent action at the positive plates. Antimony in particular produces a rapid discharge of the negative plates. The reactions of antimony and arsenic are probably analogous to those of bismuth, as the reduced material becomes visible after a short time. There is, in addition, a marked accelerating effect in the case of antimony, since the sulphation of the negative plates as measured by the increase in weight is much greater than would be calculated for the equivalent reduction of the antimony sulphate. This effect is less in the case of arsenic. The presence of either antimony or arsenic in the electrolyte is also detrimental because of the possible formation of stibine or arsine in the presence of hydrogen. These poisonous gases, escaping from the cells, become a serious hazard to those using the battery.

Nitrates which were added to the solution as nitric acid were reduced at the negative plate and produced a marked increase in the rate of sulphation. Even so small a quantity as 0.001 per cent produced a measurable result. Small quantities of nitrates probably do little permanent damage as they are gradually eliminated from the cell as oxides of nitrogen or reduced to ammonia. Nitrates were without effect in our experiments on pasted positive plates, but their use as corrosive agent in forming planté plates from sheet lead is, of course, well understood.

TABLE IV

LOCAL ACTION PRODUCED BY IMPURITIES WHICH AFFECT BOTH THE POSITIVE AND NEGATIVE PLATES

(Results are expressed as the gain in weight of a single plate in grams at intervals from 50 to 500 hours)

Impurity	Material Added	Percentage Impurity	Time in Hours 50	100	200	300	400	500
		(Positive Plates)						
None	Control experiments	..	0.34	0.51	0.66	0.86	1.05	1.15
Iron	$FeSO_4$	0.012	0.7	0.9	1.1	1.3	..	..
do	do	0.08	1.5	1.8	2.2	2.3	..	..
do	do	0.4	5.6	6.8	7.2	7.5	..	..
Manganese	$MnSO_4$	0.08	2.9	3.7	4.4	5.0	..	..
do	do	0.4	8.4	10.6	14.4	18.4	..	..
Chlorine	HCl	0.05	5.5	7.1	8.2	11.9	13.5	..
do	NaCl	1.00	23.7	25.4	26.0	26.2	..	..
		(Negative Plates)						
None	Control experiments	..	0.69	1.44	2.68	3.87	5.03	6.08
Iron	$Fe_2(SO_4)_3$	0.012	1.2	2.0	3.4	4.6	5.8	7.0
do	do	0.08	6.3	8.0	9.5	10.8	12.2	13.6
Manganese	$KMnO_4$	0.04	2.2	3.0	4.3	5.6	..	..
do	do	0.40	3.1	4.0	5.3	6.7	..	..
Chlorine	HCl	0.02	0.6	1.3	2.7	..	..	..
do	NaCl	1.00	22.0	27.1	30.2	32.7	..	..

Impurities Affecting both Positive and Negative Plates

In the previous paper,[4] details of the reactions of iron and manganese have been given. Iron is perhaps the most common impurity in storage battery electrolytes. It is oxidized at the positive plate and reduced at the negative plate without depositing on either *ad infinitum.* Since it stays in solution it can be eliminated by changing the electrolyte. Manganese is more detrimental to the positive plates than to the negatives. Its reactions are complicated and will not be repeated here.

Chlorine is a detrimental impurity for both the positive and negative plates, although in small quantities its effects are more pronounced on the positives. Chlorine is liberated at least in part from the cell, but others have stated that a portion of it is oxidized to perchloric acid and this remains in the electrolyte. The addition of sodium chloride to the electrolyte was tried because of the reactions which occur when sea-water may accidentally find its way into storage batteries used on shipboard.

Impurities Affecting Positive Plates Only

Impurities of this class are organic compounds. In our experiments with acetic acid, the effects produced were smaller than were anticipated, and several additional portions of acetic acid were added to the same jar at intervals without much effect as shown in the table.

Following the experiments with acetic acid a small group of separators which had not been previously used in batteries were extracted with sulphuric acid (1.250 specific gravity), and this solution was tried on both positive and negative plates. No appreciable effect on the changing weight of the negative plates was observed. However, the effect on the positive plates was striking. Separators which had been previously treated preparatory to their use in storage batteries produced smaller effects than similar separators which had not been so treated. These results led us to try dextrose, sucrose, invert sugar, and starch which produced effects of about the same amount. Tannic acid, however, produced only a very small effect as shown in the table. These results show that the separators have important effects which warrant further investigation.

Local Action Produced by Combinations of Impurities

Table VI shows the effects produced on negative plates by combinations of impurities which, taken singly, produced less effect than when present in combination.

Kugel[5] showed that the action of such combination as tungsten and copper exceeded materially the effects of either of these materials singly. The extraordinary effect of adding a small percentage of copper to a solution containing a small percentage of tungsten is shown graphically in Fig. 1.

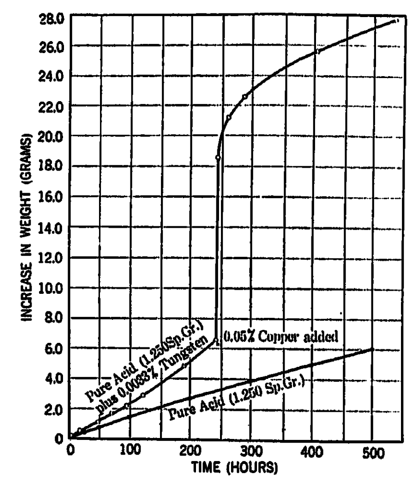

Fig. 1—Effect of a Combination of Copper and Tungsten on Negative Plates

In addition to the experiments on tungsten and copper, combinations of copper with other impurities such as mercury, molybdenum, zinc, arsenic, and antimony were tried. In all of these cases it was found that the rate of sulphation of the negative plates was greatly accelerated. These results are significant in showing that it is important to limit the percentage of copper which may be present in the electrolyte to the smallest reasonable figure.

In explanation of this effect, Kugel has suggested that the polarization on the copper is decreased by the presence of tungsten. Scarpa[6] has advanced a similar explanation that the presence of tungsten lowers the overvoltage necessary for the evolution of hydrogen

4. Vinal and Altrup, *loc. cit.*

5. *Electrotech. Zeit.* 13, pp. 8, 19, (1892).

6. *L'Elettrotecnica 6*, p. 317 (1919).

on the surface of the copper. For this reason the local currents flowing between the copper and the underlying lead of the plate are greatly increased. This appears the most probable explanation and our results show that in addition to tungsten, which is an unusual impurity to find in storage battery electrolytes, there are a number of others much more likely to be present which may produce the same effects.

TABLE V

LOCAL ACTION PRODUCED BY IMPURITIES AFFECTING ONLY THE POSITIVE PLATES

(Results are expressed as the gain in weight of a single plate at intervals from 50 to 500 hours).

Impurity	Percentage Impurity	Time in Hours					
		50	100	200	300	400	500
None	(Control experiments)	0.34	0.51	0.66	0.86	1.05	1.15
Acetic acid	0.1	0.4	..	..	..	..	..
do	1.0	..	0.9	1.6	..	..	..
do	3.0	..	..	..	2.4	3.3	..
Separator extracts (treated)	..	3.2	5.2	7.7	9.4	..	..
Separator extracts (untreated)	..	8.9	13.5	18.6	21.1	..	..
Dextrose	1.0	23.2	26.3	27.2	..	..	..
Sucrose	1.0	23.6	26.7	27.6	..	..	..
Invert Sugar	2.0	23.6	26.4	26.8	..	..	..
Starch	0.5	11.5	20.3	25.1	..	..	..
Tannic Acid	0.10	0.6	1.1	1.9	2.6	..	..

In the course of our experiments combinations of tungsten with arsenic, antimony, and several other impurities were tried, but the tungsten produced no unusual effect except when in combination with copper.

Impurities Producing Little or No Effect

In the course of our experiments a number of impurities were tried which produced little or no effect on either the positive or negative plates. These included sodium, calcium, magnesium, aluminum, zinc, cadmium and mercury.

At various times sodium and magnesium sulphates have been suggested as an addition to sulphuric acid of the ordinary electrolyte to decrease the sulphation of the plates. Table 7 gives in detail the results which were obtained when these materials were added to 1.250 specific gravity electrolyte, and it will be seen that the agreement between the solutions containing these materials and the control experiment is within the limits of experimental error for both positive and negative plates. These results indicate that these substances are without effect on the rate of sulphation in concentrations up to 5 per cent, and no benefit is to be derived by adding them to ordinary solutions. Others have claimed that the presence of sodium sulphate and similar substances is harmful, resulting in the disintegration of the negative plates.

TABLE VI

LOCAL ACTION PRODUCED BY COMBINATIONS OF IMPURITIES AFFECTING THE NEGATIVE PLATES

(Each experiment was started with the first named impurity and then copper was added at the time shown)

Combination	Material Added	Percentage Impurity	Time of Adding Cu (hours)	Time in Hours					
				50	100	200	300	400	500
None (Control experiments)				0.69	1.44	2.68	3.87	5.03	6.08
{ Tungsten	WO_3	0.003	246	1.1	2.4	5.4	23.1	25.6	26.7
{ Copper	$CuSO_4$	0.05							
{ Mercury	Hg_2SO_4	0.01	145	0.8	1.7	31.6	31.8	31.0	31.2
{ Copper	$CuSO_4$	0.05							
{ Molybdenum	MoO_3	0.01	145	1.1	2.0	30.5	24.7	32.4	40.0
{ Copper	$CuSO_4$	0.05							
{ Zinc	ZnO	0.01	145	1.2	1.6	24.0	33.7	33.2	..
{ Copper	$CuSO_4$	0.05							
{ Arsenic	As_2O_3	0.001	145	1.0	2.0	23.2	25.8	28.0	29.2
{ Copper	$CuSO_4$	0.05							
{ Antimony	$Sb_2(SO_4)_3$	0.001	145	3.6	8.8	28.0	38.5	38.0	36.0
{ Copper	$CuSO_4$	0.05							

TABLE VII

EFFECT OF SODIUM AND MAGNESIUM SULPHATES

(Results are expressed as the gain in weight of a single plate in grams at intervals from 50 to 500 hours)

Impurity	Material Added	Percentage	Time in Hours					
			50	100	200	300	400	500
			(Positive Plates)					
None (Control experiment)	..	..	0.05	0.14	0.35	0.52	0.72	0.90
Sodium sulphate	$Na_2SO_4.10H_2O$	2.0	0.05	0.1	0.3	0.4	0.6	0.7
do	do	5.0	0.2	0.3	0.5	0.7	0.8	1.0
Magnesium sulphate	$MgSO_4.7H_2O$	2.0	0.1	0.2	0.3	0.5	0.7	0.8
do	do	4.0	0.2	0.3	0.5	0.6	0.7	0.9
			(Negative Plates)					
None (Control experiment)	..	..	0.80	1.52	2.97	4.25	5.53	6.82
Sodium sulphate	$Na_2SO_4.10H_2O$	2.0	0.7	1.3	2.6	3.8	4.9	5.9
do	do	5.0	0.8	1.4	2.6	3.5	4.5	5.4
Magnesium sulphate	$MgSO_4.7H_2O$	2.0	0.9	1.6	3.1	4.5	5.8	7.1
do	do	4.0	1.0	1.8	3.2	4.6	6.0	7.4

Suggested Specifications for Sulphuric Acid

Specifications for battery acid have been issued by many different agencies and a comparison of them shows wide divergence in the impurities which are listed and the maximum amounts which may be considered permissible. If suitable specifications for sulphuric acid for use in batteries can be formulated and receive general recognition, it is likely that they will find use also in other industries. The amount of concentrated sulphuric acid used per year for batteries in this country is probably in excess of 30,000,000 pounds.

If the amount of impurities allowed by the specifications is made too small, serious difficulty may be encountered in finding acid sufficiently pure to meet them. On the other hand the specifications must limit the impurities to amounts that are within the range for satisfactory battery operation.

Specifications are usually drawn to apply to the concentrated acid. If pure water is used to dilute this to the proper concentration for battery use the percentage of the impurity will be proportionately reduced. It is desirable that water of a high degree of purity be used, and no general statement as to the permissible use of natural water can be made for the reason that its purity varies from place to place and from one time of the year to another.

The following Table VIII, on the purity of sulphuric acid and solutions for battery use, has been taken from a recent book on storage batteries.[7] It is believed that this table is consistent with the results of this investigation and represents acid that can be readily procured. It may be desirable, however, to limit the amount of copper to a smaller percentage in view of the large local action which it produces when present together with certain other impurities shown in Table VI.

TABLE VIII
PURITY OF SULPHURIC ACID AND SOLUTIONS FOR BATTERY USE

	Concentrated Acid	Unused Electrolyte	Used Electrolyte
Specific gravity 60°F	1.835	1.280	1.280
Per cent H_2SO_4	93.19%	36.8%	36.8%
Color	Colorless	Colorless	Colorless
Suspended matter	None	None	Lead compounds only
Platinum	None	None	None
Arsenic and antimony	Traces	Traces	Traces
Manganese	Trace	Trace	Trace
Iron	0.010%	0.004%	0.015%
Copper	0.005%	0.002%	0.005%
Nitrates and nitrites	Traces	Traces	Traces
Chlorides calculated as Cl	Trace	Trace	Trace
Organic matter	Trace	Trace	Trace
Sulphurous acid	Trace	Trace	None

The term "trace" is often used, but ill-defined. In general this term in these specifications should be regarded as meaning less than 0.001 per cent. Any standard specification of this character should include also a statement of the tests to be employed.

Our experiments have shown that there are other impurities, in addition to those included in Table VIII, which produce effects in the storage battery. However, they are rarely present in sulphuric acid. It does not seem necessary to name them all, as a statement can be added calling attention to the fact that other impurities are absent.

The limits for impurities given in Table 8 are presented for discussion, and it is hoped that they may serve as a basis for specifications that will receive general recognition.

7. Storage Batteries, by Vinal, page 120 (1924).

Discussion

J. L. Woodbridge: Mr. Vinal has given us the results of a new method of studying the subject of local action in the storage-battery cell. As he states, in his paper, considerable work has been done on this problem by other methods, one of them being the direct measurement of the capacity of the cell at different times, and another being an indirect method, by measuring the evolution of gases that are given off from the plates.

Each of these methods has certain disadvantages, and probably the true picture can best be obtained by a combination of all three. The direct measurement of the capacity of the cell is subject to error and can be made only at the beginning and the end of the standing test and not at any intermediate points. Furthermore, the capacity of the cell is a very variable quantity influenced by many factors, and it is very difficult to insure that an observed change of capacity is due to the particular factor that you are studying. The measurement of gas evolution is also subject to serious handicap in getting the exact results desired. This new method, therefore, is bound to throw considerable light on the problem.

There are several factors which affect the local action in a cell; for example, the temperature of the cell, the density of the electrolyte, and the history and physical condition of the plates themselves. Mr. Vinal's work has thus far been confined, as represented in his paper, to measurement at constant temperature and at one particular density of the electrolyte. We hope, therefore, that he will be in position to continue this work and to take into account these other factors.

There is another factor which would also have a very marked effect on the local action, and that is the presence or absence of electrolytic action. Mr. Vinal's work has been done so far on the cells standing on open circuit. If current is passed through the cell, a very different set of data will undoubtedly be obtained. For example, the effects of forming agents in the electrolyte will be very different when current is passing through the plates from what they are when the cell is standing on open circuit. Also, the results of the presence of metallic impurities will be varied by the effect of electric current in depositing these impurities on the negative plate.

There is one point in Mr. Vinal's paper in connection with which I would like to ask for a little additional information. For example, in Table III, and I think in other tables, two columns are given, one showing the material added, the other showing the percentage of the impurity used. It isn't entirely clear as to whether this percentage refers to the material in the form in which it is shown in the first column or whether it refers to one element. For example, copper sulphate is given in the first column, and it is not quite clear whether the percentage of copper is intended, or the percentage of the sulphate. The same might apply to nitric acid—I presume the percentage refers to the total percentage of nitric acid.

It is to be hoped that this investigation may be carried further because the problem is a large one and the amount of useful investigation that can be done is almost unlimited.

R. L. Young: Though, not a chemist, I have found the paper interesting, in that it discloses the reasons for some commonly accepted battery practises and points the way toward safe operation.

Impurities: It is of interest to note, for example, that iron, one of the most frequent and active impurities, is not deposited on the battery plates but stays in solution, so a change in electrolyte should reasonably eliminate it from the cell. I assume it would also be necessary to replace wood separators and wash the plates.

The tests showing that certain elements are very active in the presence of copper but are practically harmless otherwise, emphasize the importance of keeping brass or copper terminals and connections free from corrosion, I would enter a plea for the more general adoption by the manufacturers of terminals made exclusively of lead alloy, or where conductivity demands copper, a more complete protection of the copper portions.

Among the harmless impurities it is very fortunate that we find sodium. The foaming action of baking-soda solution during neutralization, and its non-corrosive nature, make it ideal for general use and particularly for washing down bus-bars and connections, also when necessary, walls and ceilings of battery rooms. Apparently little harm would result if small quantities of solution should accidentally drip into the cells and it might be satisfactory to leave bus-bars unwashed by clear water if it is not intended to repaint them at the time. I should like the sanction of the authorities to follow this practise.

Gases: I find a reference to the possible liberation of poisonous gas, stibine or arsine, when antimony or arsenic impurities are present in the electrolyte. A company, with whose practises I am familiar, has some thousands of batteries in operation, some of them having 50 or more cells of large capacities similar to those used in central-station standby service. Practically all of these batteries use antimony-lead alloy grids on both positive and negative plates and it may be expected that antimony gets into the electrolyte. We have never observed any indication of poison or other harmful effects on attendants and I would question whether this danger is not more theoretical than real. Is not the ordinary ventilation provided for reasonable comfort and to prevent the formation of explosive mixtures, sufficient to reduce the antimony hazard to a negligible amount?

Specifications: Efforts toward the establishment of recognized standard specifications for electrolyte are commendable, as they will probably tend toward the development of better quality, larger supply and possibly reduced cost. They may have to be somewhat flexible, or perhaps a series of specifications will be necessary, as I understand that the amount of any one impurity permissible may be affected by the percentages of others present. This at least is more or less the case with water for battery use and I would like to see specifications on this as well. Perhaps radio fans would be interested in knowing that New York City water is usually safe, whereas the very excellent artesian well water furnished in many suburban towns is not recommended for batteries.

In Table VIII, I would suggest that two columns be added to cover values for 1.210 electrolyte, this being the standard for many batteries in stationary service.

G. M. Howard: I would like to say a word with regard to the question of antimony and arsenic in the electrolyte.

To a man who has worked with storage batteries for years, it is rather startling to read a statement like this: "These poisonous gases escaping from the cells become a serious hazard to those using the battery."

Mr. Young has touched on that from the commercial and practical end. I think all users of batteries will agree with him that there have been no cases of poisoning. Certainly we have never heard of any.

As Mr. Vinal has said, the use of antimony in the grid is almost universal. Practically all lead storage batteries today contain antimony, and that means that there is nearly always a trace in the electrolyte. Certainly there is likely to be.

We have made careful tests in our laboratories when antimony was known to be present in the electrolyte, and we have never been able to detect any in the gas. On the other hand, arsenic, if present in the electrolytic may give off arsine, but, of course, arsenic is only an accidental impurity, and is never present in sufficient quantity to constitute a hazard. The point I want to emphasize is that the normal constituent, antimony, does not form stibine.

G. W. Vinal: I would like to discuss first the point which was raised by both Mr. Young and Mr. Howard regarding the possible formation of stibine.

The statement in the paper about antimony and arsenic relates primarily to these materials in solution. It should not be construed as meaning that the use of antimony in the grids is detrimental to the battery, nor liable to be a source of danger. A careful analysis of the electrolytes taken from fifteen batteries of six different makes showed that only very small traces of antimony were present in the solution. In all cases the amount observed was less than one part in 100,000. This clearly shows that under the ordinary operating conditions the antimony used in making grids does not pass into the solution. In rare instances, however, appreciable amounts of antimony have been found in the solutions, and it is to guard against such cases that it seems well to include antimony and arsenic in the proposed specification. Manufacturers of batteries endeavor to use materials which are free from arsenic and which might give rise to the liberation of arsine. Authentic cases of the liberation of stibine appear to be lacking, although we know from chemical reasoning that the conditions do make the formation possible, at least theoretically.

In answer to Mr. Young's question as to whether the ventilation in a battery room is sufficient to take care of such gases, I think the answer is quite clearly "yes."

As to the effect of sodium in the electrolyte, our experiments show that it did not affect the rate of sulphation appreciably.

A word of caution is perhaps desirable and Mr. Woodbridge has very well pointed out the fact that several methods of experimentation are desirable to reach final conclusions. I had some correspondence, with reference to sodium, with engineers of the Prest-O-Lite Company before coming to this meeting. They are of the opinion that traces of sodium may produce deleterious effects when batteries are continuously charged and discharged. That is somewhat contradictory to results which we have obtained, and yet I think that it is a point which may very well be investigated further.

As to the matter of using natural water, I know that water in New York City is commonly used in large batteries. I think New York City is blessed. It has a very fine water supply. I have been very careful, however, not to make any definite statement about the use of natural water, because natural water varies so much from place to place, and while it may be perfectly all right to use it in one location it may be unwise to do so in another. The Bureau of Standards has often been quoted as saying that the use of natural water is permissible. As a matter of fact, we have not issued any such statement.

I believe that nitrates in small amounts, such as are considered here, gradually tend to eliminate themselves from the batteries, some passing off as gas and some being reduced to ammonia.

I want to mention again Mr. Woodbridge's comments about the combination of methods. Of course, the work which we have done gives an indication of the local action which is produced within the cells, but it is quite desirable that these results should be supported, by measurements made on the batteries under normal operating conditions. We have made a few measurements on weighing the plates, with the passage of current both charging and discharging, but that work has not progressed very far. How far we shall be able to go we cannot say at the present time, but there is a very wide field of work.

One more point about the percentage of impurities,—particularly in the case of copper: the percentage which is given in the table is the percentage of copper. The impurity was added in the form of copper sulphate, but allowance was made for the SO_4 radical and for the water of crystallization.

The Theory of Probability
and Some Applications to Engineering Problems

BY E. C. MOLINA[1]
Member, A. I. E. E.

Synopsis.—The purpose of this paper is to encourage a wider recognition by engineers of a body of principles which in its mathematical form is a powerful instrument for the solution of practical problems. Some subjects in which the theory of probability has been used are recalled, the fundamental principles are stated and applied to three problems chosen from the field of telephone engineering.

* * * * *

"THE subject to which I now invite attention has high claims to consideration on account of the subtle problems which it involves, its important practical applications and the eminence of those who have cultivated it." (Todhunter: History of the Mathematical Theory of Probability, 1865).

You are all familiar with the importance of the theory of probability in its applications to life assurance, biology, radioactivity and other branches of pure and applied science. In telephony the theory has been utilized for over a quarter of a century. This is not surprising. The calls to be handled during a busy hour fall at random with reference to a given instant. Hence, a knowledge of total and average loads does not suffice for determining the quantities of equipment required for rendering efficient and economical service. The mathematical theory of probability enables one to evaluate the frequencies of different deviations from known average conditions. With this information the solution of trunking problems becomes precise, the quantities of equipment required for existing systems is determined and as changes in the art occur the merits of proposed systems can be weighed, thereby assuring that the public's demands for service be met satisfactorily.

The theory of probability is of immediate aid in the planning of inspection programs which must be carried out in order that apparatus may leave the manufacturer in fit condition to perform its functions. The interpretation of empirical data is facilitated and often the application of the theory to hypothetical conditions makes unnecessary the carrying out of costly statistical investigations.

The application of the theory of probability to engineering problems is a subject of so much importance that it is believed the engineering societies could well afford to give more consideration to it than has been given in the past. While there have been one or two instances of Institute papers discussing probability matters, such, for instance, as "A Method of Determining Resultant Input from Individual Duty Cycles and of Determining Temperature Rating" by Mr. Bassett Jones, in general the Institute papers have paid little attention to this matter. It is the purpose of this paper to present the fundamentals of the theory of probability in a form which it is hoped will appeal particularly to practical engineers, and to discuss specifically the application of these fundamental principles to a number of practical problems.

The fundamental principles of our subject, first formulated in a comprehensive manner by Laplace in his classic "Théorie Analytique des Probabilités" may be stated as follows:

First Principle. The probability that an event may happen in a specified manner is the ratio of the number of ways it can happen as specified to the total number of ways in which it can happen. For example, the probability that an ordinary six-face die will, when cast up in the air, give a number greater than 4 is 2/6 since there are only two faces marked with numbers greater than 4; whereas, the total possible number of ways in which the die can turn up is obviously 6.

This first principle is really nothing more or less than a definition of probability. It is implicitly assumed that all the possible cases are equally likely or probable. This implicit assumption has been severely criticised by some philosophers. Moreover, Poincaré, Bertrand and others have claimed that a logically satisfactory definition is impossible. However, this statement should not be disturbing any more than being reminded that there is no precise answer to the question "what is matter?"—should disturb a group of chemists.

Second Principle or Theorem of Total Probability. When a complex event can be reduced to a group of *mutually exclusive* simple events, the probability for the complex event is the *sum* of the probabilities corresponding to each simple event. Or thus—the probability of the happening of any one of several events, no two of which can concur, is the *sum* of their separate probabilities. As an example, suppose that one of our men has plans to return from Chicago next summer either by the Pennsylvania R. R., the New York Central R. R. or else by boat through the Great Lakes. If p_1, p_2, p_3, are the probabilities for these three routes respectively, the probability that he returns by rail is

$$p = p_1 + p_2$$

1. American Telephone and Telegraph Company, New York, N. Y.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1926.

Third Principle. If there be given any number of *independent* events, the probability that they will all happen is the *product* of their respective probabilities. Suppose we have in front of us, three bags filled with red, white and blue balls. If 7 per cent of the balls in bag No. 1 are white, 19 per cent of those in bag No. 2 are white and 40 per cent of those in bag No. 3 are white and we draw one ball from each of the bags, then $P = (7/100)\,(19/100)\,(40/100)$ is the compound probability that all three of the drawn balls will be white.

Fourth Principle. The probability of the concurrence of two *dependent* events is the product of the probability of the first times the probability that when that has happened, the second will follow.

Fifth Principle. This fifth principle may be considered as a corollary to the fourth. Suppose we know the *a priori* probability in favor of an event which *has* happened and that we also know the *a priori* probability for a compound event consisting of the event which has occurred followed by another event which has not yet occurred. Then the probability that the second event *will* occur is equal to the *a priori* compound probability divided by the *a priori* probability of the event which has occurred. Example—Bids are open for the construction of a city subway. Let P be the compound probability that a certain construction company bids *and* submits satisfactory plans. Let p_1 be the probability that said company bids. Then, if the company *has* made a bid, the probability that the plans submitted will be satisfactory is $p_2 = P/p_1$.

We now come to a principle which is of fundamental importance for most if not all fields of engineering. The so-called "Theory of Sampling," which is of immediate interest to every engineer, seems to be inextricably tied up with this principle in spite of efforts made to separate them.

The five principles given above relate to the theory of *a priori* probability. The next principle has reference to *a posteriori* probability or probability of *causes.* The essential difference between *a priori* and *a posteriori* probability may be indicated as follows: Consider a bag containing 1000 balls some of which are white. We are dealing with the *a priori* probability when, *knowing the ratio of white to total balls* we put the question, what is the probability that 100 drawings will give 7 whites? It is assumed that a drawn ball is replaced in the bag before the next ball is drawn. We are dealing with *a posteriori* probability when, *knowing that* 100 *drawings did give* 7 *whites* we put the question, what is the probability that the ratio of white to total balls has a specified value, (say 11/1000 for instance)?

As before, we will follow closely in the footsteps of Laplace. His classic generalization of Bayes' theorem laid the corner stone for the edifice erected by mathematicians and statisticians since the publication of the *Théorie Analytique.* Two preliminary definitions will help us to understand Laplace.

Consider again the bag containing 1000 balls from which 100 drawings gave 7 whites. Note that the unknown ratio of white to total balls is a *hypothesis* or *cause* leading to the observed result. We may consider any one of 999 possible hypotheses:

1—ratio is		p_1	=	1/1000
2—	"	" p_2	=	2/1000
3—	"	" p_3	=	3/1000
K—	"	" p_k	=	K/1000
997—	"	" p_{997}	=	997/1000
998—	"	" p_{998}	=	998/1000
999—	"	" p_{999}	=	999/1000

The *a posteriori* theory assumes that there is a known probability for the *Kth* hypothesis before the results of the drawings are disclosed. Call this the *existence probability for the Kth hypothesis.* If the *Kth hypothesis* exists there is a definite probability that it will give the observed result. Call this the *productive probability for the Kth hypothesis.*

Sixth Principle. The *a posteriori* probability in favor of a specified cause is a fraction whose numerator is the product of the existence and productive probabilities of that specific cause while the denominator is the sum of like products for all the causes.

Seventh Principle. If two events are governed by the same set of mutually exclusive causes and one event *has* happened, the probability that the second event *will* happen is equal to the sum of all the products obtained by multiplying the *a posteriori* probability of each cause (as determined from the observed event) by the probability that the cause, if acting, will produce the second event.

Applications. "The applications of the principles which we have just expounded to the various questions of probability requires methods whose investigation has given birth to several methods of analysis and especially to the theory of combinations and to the calculus of finite differences."[2]

As a telephone engineer the author need not apologize for being more familiar with the application of the theory of probability to telephone problems than to problems relating to other fields of engineering. In what follows, it is not intended to discuss at length any major telephone problem. Such a discussion would be worthy of at least as much space as has been allotted to this paper. Moreover, the inherent interest of a major problem would distract attention from the probability principles made use of in its solution.

As an immediate application of the first principle or definition of probability consider the following problem:

n calls fall at random in an interval of time A. As the calls fall they are automatically counted by a meter. This meter, however, cannot function properly if the time interval between two consecutive calls is less than a small interval a.

2. Laplace Théorie Analytique.

What is the probability that a correct count of the n calls will be obtained? In other words, what is the probability that no two consecutive calls will be separated by a distance less than a?

The mathematical analysis contained in the appendix to this paper gives

$$P = [1 - (n - 1)\ (a/A)\ ]^n$$

The curves of Fig. 1 show the numerical values of the probability P for various values of n for the two

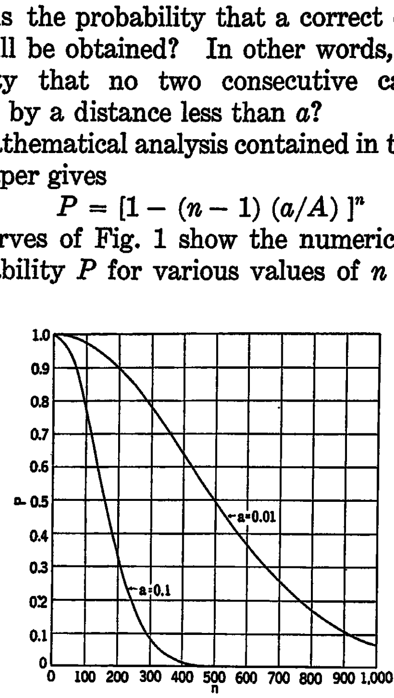

FIG. 1

cases $a = 0.1$ sec. and $a = 0.01$ sec. The large interval A is assumed to be one hour.

For an illustration of the applicability of the second and third principles consider the following telephone trunking problem.

Referring to Fig. 2 consider a group of 269 subscribers, each equipped with a 20-point line switch. The line switches have common access to a group of 20-trunk lines. When a subscriber removes his receiver

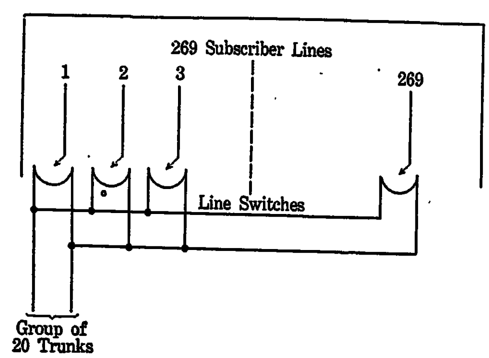

FIG. 2

from the hook, his line switch revolves and connects him to an idle trunk, if one exists.

What is the probability that when a particular subscriber, X, calls he fails to obtain a trunk at once?

It will first be shown that this trunking problem transforms into a simple dice-throwing problem. The answer to the dice problem is then obtained at once by recourse to the *binomial* expansion which was used so effectively by Mr. Bassett Jones in the solution of his problem. To facilitate the transformation, let us make some simplifying assumptions.

A—During the period of time under consideration, the busy hour of the day, each subscriber's line makes one call which is as likely to fall at any one instant as at any other instant during the period.

B—If a call, when initiated, obtains a trunk immediately, it retains possession of that trunk for exactly two minutes. In other words, a constant holding time of two minutes' duration will be assumed.

C—If a trunk is not obtained immediately, the calling subscriber waits for two minutes and then withdraws his call. If, while waiting, a trunk becomes idle, he takes it and converses for the interval of time remaining before his two minutes are up.

Referring to Fig. 3, let point P represent the unknown instant within the hour at which X calls. Consider the two minutes immediately preceding the instant P. Evidently, by assumption C, calls falling outside of this particular two-minute interval cannot prevent X from obtaining a trunk.

If, however, at least 20 of the remaining 268 subscribers initiate their calls within the particular two minutes under consideration, there will be no trunk line immediately available for X. This follows from assumptions B and C.

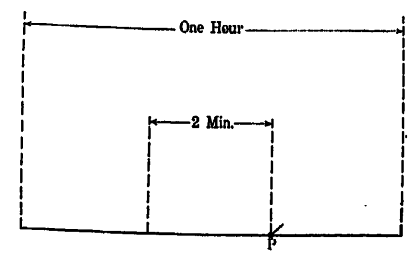

FIG. 3

Consider some one of these 268 other subscribers, for example Y. The probability that Y calls in the two minutes under consideration is, by assumption A, the ratio of 2 minutes to 60 minutes, or 1/30, which is exactly the same as the probability that he would throw an ace if he were to make a single throw with a 30-face die. Likewise, the probability that still another subscriber calls in the two minutes under consideration is exactly the same as the probability that this other subscriber should throw the ace in a single throw with a 30-face die.

It is evident then, that the probability that X fails to get a trunk immediately is the same as the probability of throwing at least 20 aces if 268 throws are made with a 30-face die. To facilitate the determination of this probability and the solution of similar problems, probability tables of the type shown in Table I have been computed. In the table, the average number of times an event may be expected is represented by a. The probability that the event occurs at least a greater number of times $c = a + d$ is represented by P. In the problem under consideration, the average

number of aces expected is $8.96 = \frac{268}{30}$ Likewise in the present problem $c = 20$. Turning to the table, we find that corresponding to $c = 20$ and $a = 8.96$, the value of the probability P is 0.001. In the particular telephone problem under consideration this means that once in a thousand times when X calls, at least 20 of the other subscribers will have called in the two minutes immediately preceding, and therefore X fails to get a trunk immediately. In other words, we may consider that on the average, one in every thousand calls is delayed.

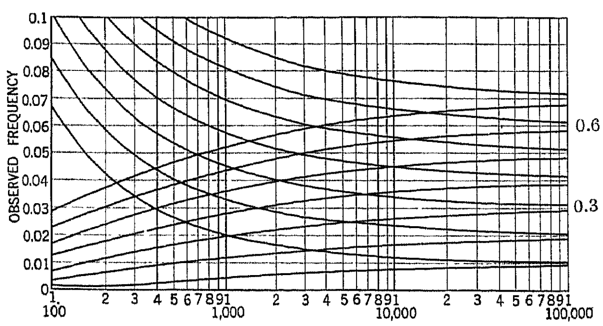

Fig. 4—Curves Showing Probable Range of True Frequency Vs. Number of Observations for Observed Frequencies of 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0 07. Weight = 0.98

Finally, as an application of that most far-reaching but much debated sixth principle, consider this telephone traffic problem:

A group of 50,000 calls originated in an exchange area. An unknown number of them were delayed more than 10 seconds. Observations were made on 300 of the calls and of these 9, or 3 per cent, were delayed more than 10 seconds. With this information is it a safe bet that the unknown percentage for the entire 50,000 calls is below 5? Or better yet, are we justified in betting 99 in 100 that the unknown percentage for the 50,000 calls is below 5? Or again, may we bet 8 in 10 that the unknown percentage is between 0.5 and 5? It is taken for granted that the observer is justified in believing that the calls under consideration fulfill the conditions of random sampling such as that each call is independent of every other call, or that an appreciable number of the calls is not due to the occurrence of some unusual event,—the Wall Street explosion, for example.

Obviously the telephone problem is analogous to the problem of the bag containing an unknown ratio of white balls. The corresponding elements in the two problems may be tabulated as follows:

1st. 1000 balls in bag vs. 50,000 calls originated.

2nd. 100 balls drawn vs. 300 calls observed.

3rd. 7 white balls drawn vs. 9 calls delayed more than 10 seconds (*i. e.*, defective with reference to a particular characteristic).

4th. To the 999 possible, hypotheses with reference to the unknown per cent of white balls correspond 49,999 possible hypotheses with reference to the unknown per cent of calls delayed more than 10 seconds.

The problems differ in that a ball drawn from the bag is returned before another drawing is made, whereas an observed call is comparable to a ball being drawn and not returned. However, with the numbers involved this discrepancy may be ignored.

The attached curves, Fig. 4 show graphically the conclusions to be drawn from the mathematical analysis. A glance at the right-hand end of the curves will show that they are associated in pairs. The upper curve of a pair slopes downward from left to right while its mate slopes upward.

Consider the pair of curves marked 0.03. For the abscissa 300, they give as ordinates the values 0.0625 and 0.014. The interpretation of these figures is as follows: if 300 observations gave 3 per cent of calls delayed then we may bet

1st. 99 in 100 that the unknown percentage of calls delayed is not greater than 6.25.

2nd. 99 in 100 that it is not less than 1.4 per cent.

3rd. 98 in 100 that it lies between 1.4 per cent. and 6.25 per cent.

Likewise, considering the curves marked 0.06 if 1000 observations gave 6 per cent of calls delayed, then we may bet

1st. 99 in 100 that the unknown percentage of calls delayed is not greater than 8.05.

2nd. 99 in 100 that it is not less than 4.4 per cent.

3rd. 98 in 100 that it lies between 4.4 per cent and 8.05 per cent.

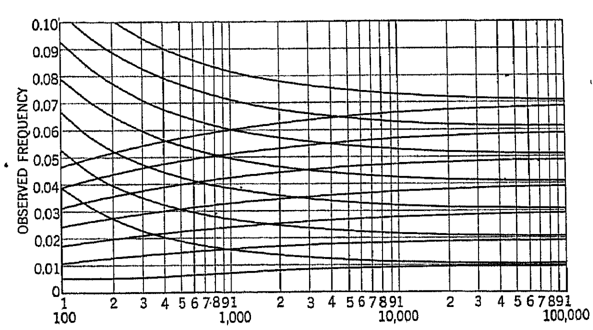

Fig. 5—Curves Showing Probable Range of True Frequency Vs. Number of Observations for Observed Frequencies of 0.01, 0.02, 0 03, 0.04, 0.05, 0.06, 0.07. Weight = 0.08

It is obvious from the shape of the curves that a few hundred observations do not give more than a vague idea as to the unknown per cent of calls delayed. On the other hand, the gain in accuracy obtained by making more than 10,000 observations would hardly justify the expense involved. The number of observations which safety requires in any particular problem must be determined by the conditions of the problem itself. If we are willing to take a chance of 9 in 10 or 8 in 10 instead of 99 in 100 or 98 in 100, respectively, the curves of Fig. 5 will give us an idea of the range

within which the unknown percentage of defectives lies.

Appendix

Counting Meter Problem

n calls fall at random in an interval of time A. As the calls fall, they are automatically counted by a meter. This meter, however, cannot function properly if the time interval between any two consecutive calls is less than a small interval a.

What is the probability that a correct count of the n calls will be made?

Consider the two diagrams of Fig. 6. The lower diagram indicates any one of the many different number of ways in which the calls may fall. Evidently this number is the same as the number of different sets of values which we can give to the set of variables $y_1, y_2, y_3 \ldots y_n$, so that each variable is equal or greater than zero, but their sum not greater than A. Using the language of the integral calculus, the number of possible sets of values for $y_1, y_2 \ldots y_n$ is

$$\int_0 \int_0 \int_0 \ldots \int_0 dy_1\, dy_2\, dy_3 \ldots dy_n$$

where the upper limits of the integrals must be such as to satisfy the condition

$$O < \Sigma y < A$$

On the other hand, the upper diagram indicates any

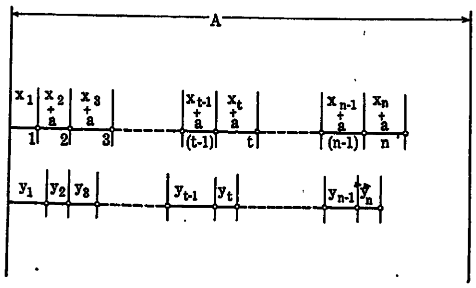

Fig. 6

distribution of the calls consistent with the condition that the distance between consecutive calls is not less than a. Note, that since there are n calls, there are $(n-1)$ intervals, each of which must not be less than a.

The number of distributions satisfying the desired condition is evidently the same as the number of sets of values which can be given to the set of variables $x_1, x_2, \ldots x_n$ so that each variable is equal or greater than zero, but their sum is not greater than $A - (n - 1)\,a$. The number of possible sets is given by the multiple integral

$$\int_0 \int_0 \int_0 \ldots \int_0 dx_1\, dx_2\, dx_3 \ldots dx_n$$

where the upper limits of the integrals must be such as to satisfy the condition.

$$O < \Sigma X < [A - (n - 1)\,a]$$

Therefore, the desired probability is

$$P = \frac{\text{(favorable cases)}}{\text{(total possible cases)}}$$

$$= \frac{\int_0 \int_0 \int_0 \ldots \int_0 dx_1\, dx_2\, dx_3 \ldots dx_n}{\int_0 \int_0 \int_0 \ldots \int_0 dy_1\, dy_2\, dy_3 \ldots dy_n}$$

Transform the integrals by the substitutions

$$y = A\,t, \quad x = [A - (n - 1)\,a]\,s$$
$$dy = A\,dt,\ dx = [A - (n - 1)\,a]\,ds$$

Then

$$P = \frac{[A - (n-1)\,a]^n \int_0 \int_0 \int_0 \ldots \int_0 ds_1\, ds_2\, ds_3 \ldots ds_n}{A^n \int_0 \int_0 \int_0 \ldots \int_0 dt_1\, dt_2\, dt_3 \ldots dt_n}$$

The conditions to be satisfied by the limits of integration in both numerator and denominator are now both the same, that is,

$$O < \Sigma t < 1, \quad O < \Sigma s < 1$$

The integrals, therefore, cancel out, saving us the trouble of evaluating them, and we obtain

$$P = \left[\frac{A - (n - 1)\,a}{A}\right]^n = [1 - (n - 1)\,(a/A)\,]^n$$

TABLE I

AVERAGES (a) CORRESPONDING TO DEVIATION (d) PLUS AVERAGE (a) TO BE EXPECTED WITH DIFFERENT PROBABILITIES

Deviation Plus Average, $c = a + d$	P: 0.001	0.002	0.004	0.006	0.008	0.010	Deviation Plus Average, $c = a + d$
	Average = a						
1	0.001	0.002	0.004	0.006	0.008	0.010	1
2	0.045	0.065	0.092	0.114	0.133	0.149	2
3	0.191	0.243	0.312	0.361	0.402	0.436	3
4	0.429	0.518	0.630	0.709	0.771	0.823	4
5	0.739	0.867	1.02	1.13	1.21	1.28	5
6	1.11	1.27	1.47	1.60	1.70	1.79	6
7	1.52	1.72	1.95	2.11	2.23	2.33	7
8	1.97	2.20	2.47	2.65	2.79	2.91	8
9	2.45	2.72	3.02	3.22	3.38	3.51	9
10	2.96	3.26	3.60	3.82	3.99	4.13	10
11	3.49	3.82	4.19	4.43	4.62	4.77	11
12	4.04	4.40	4.80	5.06	5.26	5.43	12
13	4.61	5.00	5.43	5.71	5.92	6.10	13
14	5.20	5.61	6.07	6.37	6.60	6.78	14
15	5.79	6.23	6.72	7.04	7.28	7.48	15
16	6.41	6.87	7.39	7.72	7.97	8.18	16
17	7.03	7.52	8.06	8.41	8.68	8.90	17
18	7.66	8.17	8.75	9.11	9.39	9.62	18
19	8.31	8.84	9.44	9.82	10.11	10.35	19
20	8.96	9.52	10.14	10.54	10.84	11.08	20
21	9.62	10.20	10.84	11.26	11.57	11.83	21
22	10.29	10.89	11.56	11.99	12.31	12.57	22
23	10.97	11.59	12.28	12.73	13.06	13.33	23
24	11.65	12.29	13.01	13.47	13.81	14.09	24
25	12.34	13.00	13.74	14.21	14.57	14.85	25

Sampling Problem. As stated in the body of the paper, when the total number of calls under consideration is large as compared with the number of calls observed, the problem is essentially identical with the problem of drawing balls from a bag, the ball taken at each drawing being returned before the next drawing is made.

Assume, then, that n drawings from a bag containing an unknown ratio of white to total balls resulted in c white drawings and $(n - c)$ not white. In other words, assume that the observed frequency of white balls was (c/n). Considering the unknown ratio as a cause, let $W(x)$ be the *a priori* existence probability for the value x. The productive probability for the

value x, that is, the probability of obtaining c white and $n - c$ not white balls if the unknown ratio were x, is

$$\binom{n}{c} x^c (1-x)^{n-c}$$

where $\binom{n}{c}$ is a symbol for the combinations of n things c at a time.

By the sixth principle the *a posteriori* probability in favor of the unknown ratio having the value x is

$$P(x) = \frac{W(x)\binom{n}{c} x^c (1-x)^{n-c}}{\sum\limits_{x=0}^{1} W(x)\binom{n}{c} x^c (1-x)^{n-c}} = \frac{W(x)\, x^c (1-x)^{n-c}}{\sum\limits_{x=0}^{1} W(x)\, x^c (1-x)^{n-c}}$$

Therefore, the *a posteriori* probability that the ratio x does not exceed the value p_1 is

$$P(x > p_1) = \frac{\sum\limits_{x=0}^{p_1} W(x)\, x^c (1-x)^{n-c}}{\sum\limits_{x=0}^{1} W(x)\, x^c (1-x)^{n-c}}$$

When the total number of balls in the bag is large so that the difference between any two consecutive possible values for x is small, we may substitute integrals for the summations; giving

$$P = \frac{\int_0^{p_1} W(x)\, x^c (1-x)^{n-c} dx}{\int_0^1 W(x)\, x^c (1-x)^{n-c} dx} \qquad (1)$$

Assume first that $W(x)$ is a constant b for $o < x < g$, where $g > p_1$. Then

$$P = \frac{\int_0^{p_1} x^c (1-x)^{n-c} dx}{\int_0^{g} x^c (1-x)^{n-c} dx + \int_g^1 \frac{W(x)}{b} x^c (1-x)^{n-c} dx}. \qquad (2)$$

Now assume that

$$\int_g^1 \frac{W(x)}{b} x^c (1-x)^{n-c} dx,$$

is negligible compared with

$$\int_0^g x^c (1-x)^{n-c} dx,$$

and also assume that g, c and $(n-c)$ are such that approximately

$$\int_0^g x^c (1-x)^{n-c} dx = \int_0^1 x^c (1-x)^{n-c} dx$$

Then, finally,

$$P = \frac{\int_0^{p_1} x^c (1-x)^{n-c} dx}{\int_0^1 x^c (1-x)^{n-c} dx} = \frac{(n+1)!}{c!\,(n-c)!}\int_0^{p_1} x^c (1-x)^{n-c} dx, \qquad (3)$$

This well-known formula might have been obtained by assuming *a b* initio that $W(x)$ is independent of x. Particularly should it be noted that this independence is not identical with the assumptions made above. In the applications which are here contemplated the values p_1, c and n are such that g need be but a small fraction of the range 0 to 1.

In the "Theorie Analytique" Laplace transforms (3) so that it can be evaluated in terms of the Laplace-Bernoulli integral

$$\frac{2}{\sqrt{\pi}}\int_0^k e^{-t^2} dt$$

where k is a function of p_1, c and n. This transformation is most valuable when p_1 is in the neighborhood o ½. For small values of p_1 the transformation which converts the binomial expansion to Poisson's exponential binomial limit is more appropriate and gives, writing $(n p_1) = a_1$,

$$P = \frac{1}{c!}\int_0^{a_1} y^c e^{-y} dy = P(c+1, a_1) \qquad (4)$$

Discussion

B. Jones: In the introduction to his paper Mr. Molina draws attention to the apparently little recognized value of the Calculus of Probabilities in engineering studies. Permit me to draw your attention to the extraordinary importance this calculus has acquired in modern mathematical physics as a means of developing the necessarily statistical expressions for the so-called laws of nature.

Since the time of Laplace the calculus of probabilities has been the foundation of Astronomical research. It is the basis of the kinetic theory of gases. Maxwell shared with Gibbs the work of developing dynamics as a statistical science. As Reiche puts it "Planck has turned the problem of radiation into a problem of probabilities." Eddington points out that Einstein's grand summary is to the effect that "the present state of the world is that which is statistically the most probable." Thus, a branch of mathematical science born in a gambling dive, is found capable of dealing with the most varied classes of more or less random phenomena which, from craps to atomic structure, make up the detail of this world. It assumes nothing as fixed given or absolute, and, in so far, the formulas of probabilities are very descriptive of the world's behavior. Perhaps our's *is* the most probable world.[1]

Mr. Molina has given us an outline of a few important theorems in the probability calculus. I wish to present the outline of a further very elementary and very useful method of dealing with the probable character of a sample drawn from a mixture when the probable frequencies of the several components of the mixture are known, and when we are interested in the failure of the sample to be thoroughly representative of the mixture. The method has wide engineering application extending from the sampling of crushings, screenings, and concrete mixtures, to the determination of probable duty cycles as a means of selecting apparatus having suitable characteristics. It may also be used to determine the demand factor imposed on hydraulic and electrical distributing systems serving a number of devices operating intermittently, or subjected to varying demands.[2]

Let the total number of elements composing the mixture be P, of which one element has the frequency a, a second element has the frequency b, a third element has the frequency c, and so on. That is, $P = a + b + c + \ldots\ldots$

Let n be the number of different kinds of elements in the mixture, irrespective of the number of elements of each kind.

Let a sample having a total of N elements be drawn.

1. The Most Probable World, Jones, *Tech. Engineering News*, Vol. IV, No. 9, March, 1924.

2. The application of the method to the determination of water supply and waste in buildings is given in a Bureau of Standards *Bulletin*.

Then the probability that any one of these elements is an a, is a/P; that it is a b is b/P; that it is a c, is c/P; etc.

The probability that any one of these elements is not an a, is $1 - a/P$; that is it not a b, is $1 - b/P$; that it is not a c, is $1 - c/P$; etc.

The probability that none of the N elements drawn is an a is $(1 - a/P)^N$; and, so on, for the other kinds. Therefore the average probability that any element in the sample *will not* be any one of the n kinds is

$$1/n \left\{ (1 - a/P)^N + (1 - b/P)^N + \dots \right\} \quad (1)$$

This is also the probability that the sample will not be representative of the mixture.

The probability that the sample will be representative of the mixture is

$$1 - \left[\frac{1}{n} \left\{ (1 - a/P)^N + (1 - b/P)^N + \dots \right\} \right] \quad (2)$$

Therefore the probable number of kinds of elements contained in any sample of N elements is given by

$$S = n\left(1 - \left[\frac{1}{n} \left\{ (1 - a/P)^N + (1 - b/P)^N + \dots \right\} \right]\right) \quad (3)$$

Should the sample be so constrained that any one element, say the p^{th} is certainly present, the corresponding term $(1 - p/P)^N$ is to be omitted from (1), giving, in place of (3),

$$S_{(n-1)} = (n-1)\left(1 - \left[\frac{1}{n-1} \left\{ (1 - a/P)^N + (1 - b/P)^N + \dots \right\} \right]\right) \quad (4)$$

as the probable number of different kinds of elements *other than* p contained in the sample.

Should it happen that all the elements are equally distributed in the original mixture, so that $a = b = c = \dots = P/n$ where, as before, $P = a + b + c + \dots$, and n is the number of kinds of elements in the mixture, then (3) reduces to

$$S = n \left\{ 1 - (1 - 1/n)^N \right\}, \quad (5)$$

a well-known form that may be used in many cases where a b, c, etc., are not materially different from one another.

As a case, let it be required to find the probable number of stops that will be made by an elevator car containing N passengers drawn from a crowd of P people composed of a people bound for the first floor above the bottom terminal stop, b people bound for the second floor, c people bound for the third floor, ... , and x people bound for the top or nth floor.

Since, barring accidents, the top terminal or nth floor is a certain stop, the term $(1 - x/P)^N$ is to be omitted from (1) and the probable number of intermediate stops is given by (4). The total number of stops, including the top terminal, is $S_{n-1} + 1 = S$.

If H be the travel between the bottom terminal and the top terminal, the average distance covered between start and stop is H/S. From this, and the time-velocity characteristics of the equipment, the probable average duty cycle of the hoisting equipment can be determined, which, when converted to equivalent r. m. s. input, gives the equivalent continuous output.

If a number of such equipments are operating simultaneously, the probability method given in my paper on "A Method of Determining Resultant Input from Individual Duty Cycles", TRANS. A. I. E. E., 1922, p. 457, may be used to determine the equivalent continuous rating of the feeders to the hoisting-engine group. In spite of the fact that this paper has been characterized as "school-boy mathematics," the method has been in successful use for a number of years and has resulted in considerable savings in cost of installation, and particularly so in just such cases to which it has been said the method is inapplicable.

The most probable time distribution of load on generating plants has been determined by this method and given to manufacturers as a basis for the submission of apparatus suited both in type and rating to meet such probable actual conditions and not suited to wholly non-existent conditions resulting from purely empirical guesses. In recent cases we have found that the resulting temperature rise of the apparatus in service is extraordinarily close to the expected rise, indicating that the actual r. m. s. value of the load is fairly close to the calculated r. m. s. value, and that the owner has paid for machines neither too small nor too large. I have many records of cases where generating capacity and rated motor output, both determined on a wholly empirical basis, are considerably in excess of the actual demand—in some cases this excess is 50 per cent and more. Feeder sizes often run 100 per cent above the actual necessary rating.

In closing I wish to draw attention to the fact that the method of probabilities does not start from any pre-conceived mathematical notions from which, by purely logical processes and almost mechanical dexterity in the manipulation of mathematical symbols in accordance with fixed rules, an answer is turned out willy-nilly. On the other hand, the method of probabilities is one of continuous cut and try based on no evidence other than the given data. It is no substitute for thought, but rather requires very clear reasoning at all stages of the process. Every step must be carefully visualized. This is its extraordinary advantage as a mental training. It has somewhat the same pedagogical value as the study of legal procedures. The correctness of the results obtained are wholly dependent on the common sense used in its application.

A. G. Chapman: Applications of the theory of probability have long been useful to telephone engineers in studying the problem of limiting cross-talk between telephone circuits to tolerable amounts. In considering cross-talk, two paralleling telephone circuits may be conveniently divided into a succession of short finite elements. The two circuits will be coupled in each element. The coupling is usually due to electric and magnetic induction, although in the case of a phantom circuit and its side circuits, resistance coupling may be of some importance. The speech power at a terminal of the disturbed circuit depends upon the resultant effect of all these couplings and upon the power impressed upon the disturbing circuit. In a well-designed telephone plant the couplings are due to small deviations from perfect construction rather than to dissymetries in design. It is impossible, therefore, to predict accurately the cross-talk between any two well designed and maintained telephone circuits. Estimates may be made, however, of the most probable cross-talk between two circuits and the chance of exceeding the most probable value by various percentages. In case there are a number of circuit combinations having the same probability of a given value of cross-talk, it is useful to estimate the average cross-talk and also a value of cross-talk which there is but a small chance of exceeding.

Such estimates are very useful in studying the importance of deviations from perfect construction in loading coils, in sections of telephone cable and in any apparatus occurring repeatedly in a long telephone circuit. Estimates of probable cross-talk are also essential in studying the design of loading systems and systems of arranging repeaters in telephone circuits as well as in considering the matter of permissible deviations from the theoretical values in the spacing of wires and transposition poles of open-wire lines.

The computations of probable values of cross-talk must, of course, be coordinated with an extensive program of testing to serve as a check on the necessary approximations in the computations and as an indication that the circuits have been installed in the proper manner.

As a formal mathematical problem, applications of the theory of probability to computations of cross-talk offer considerable difficulty and the solutions are necessarily approximate. The problem involves determining the probable resultant of a large

number of deviations, that is, the successive couplings between the elements of the telephone circuits. The law followed by the individual deviations must be approximately determined from experimental data. It is usually satisfactory to assume the normal law of error for the individual deviations but the law of combination of the various deviations is usually complex. Two adjacent deviations of the same magnitudes may not add or subtract algebraically since the resultant cross-talk currents of any particular frequency may be out of phase due to a difference in the nature of the two couplings. In long telephone circuits, the attenuation of the speech waves and the finite speed of propagation with its resultant retarding of phase must be taken into consideration, and also the large number of frequencies of which a speech wave may be assumed to be composed. As a result of the propagation phenomena, two similar but non-adjacent deviations will produce cross-talk currents at a circuit terminal differing in phase and magnitude, depending upon the distance between them and the frequency. In addition, the separation between two telephone circuits may not be uniform. For example, in a telephone cable, two circuits may be adjacent for a short distance, say 500 ft., and then occupy a large number of non-adjacent positions before coming together again. In spite of all these complexities, however, it has been found practicable to obtain approximate solutions of probability cross-talk problems which have been very useful in engineering the telephone plant.

R. S. Hoyt: Mr. Molina has chosen his principle illustrative examples from the extenisve field of traffic and trunking problems.

I believe that the Institute will be interested at this time in an outline of applications of probability theory to certain problems in two other fields of telephone engineering—namely, in telephone transmission engineering, and in radio engineering.

Of the three problems which I propose now to outline, the first two arose in the applications of two-way telephone repeaters to long telephone lines or cables, and the third relates to the use of selective circuits for reducing "static" interference in radio reception.

By way of preface to the first two problems it may be recalled from the theory of repeaters that the practicable amplification obtainable from a two-way repeater is limited by impedance unbalance between the two lines involved, or between the two lines and their balancing networks. The theory of probability finds here a natural field of application because the impedance unbalance mentioned arises mainly, or at least to a considerable extent, from numerous random internal irregularities in the structure of the lines, as will appear more fully in the following outlines of the two problems themselves.

The first problem relates to periodically loaded lines or cables. These contain numerous small random irregularities consisting in slight inequalities in the inductances and spacings of the loading coils. The individual departures are effectively unknown; but their arithmetic averages are known from coil manufacturing data and line construction data, and the probability laws to which they conform are known or else can be reasonably assumed. For such a loaded line, when already constructed, the problem is to calculate the probability that the line has an impedance departure less than a specified value. On the other hand, in the designing of such a loaded line, the probability problem is to predetermine the allowable average coil-inductance departures and spacing departures to meet the requirement consisting in a specified probability that the impedance departure will not exceed a specified value. Such problems arose as long ago as the construction of the first transcontinental telephone line.

The second problem relates to continuously loaded cables. Such a cable may comprise numerous short sections which are slightly unequal in their linear constants, owing to manufacturing variations. If these sections are connected in a random sequence the probability problems are analogous to those outlined in the preceding example. On the other hand, from a study of the measured constants of the individual sections it is possible to determine the best sequence of the sections to minimize the resultant impedance departure of the final cable. However, the securing of this optimum sequence involves additional expense besides delay. To decide whether these are justified, probability may be invoked in order to determine the probable impedance departure that would result if the sections were connected in a mere random sequence. Such a problem arose several years ago in the design and construction of the submarine cable between Key West and Havana.

The two problems just outlined illustrate the fact that in a physical system so extensive as the Bell System there exists many classes of numerous physical elements such that the elements in any one class, while all nominally alike, actually differ from each other by small random amounts in a statistical manner. Usually it is impracticable to know the numerous departures individually, and, even if they were known, it would usually be impracticable to compute their resultant effects of direct summation. But from the mere knowledge of a statistical index (such as the arithmetic average, for instance) and the statistical or probability law to which the departures conform, the probable departure of the system can often be calculated with relatively little labor.

The third problem, which is quite different from the two preceding, arises in radio transmission, and relates to the possibilities and limitations of selective circuits when employed for reducing "static" interference. Owing to the unknown, irregular, and random character of this type of interference, the problem is essentially a statistical and probability problem leading to an expression of results in terms of mean values. Such a treatment has yielded general deductions having practical significance, notwithstanding the meagerness of knowledge regarding the character and the frequency-distribution of static. Reference may be made to a paper by John R. Carson entitled "Selective Circuits and Static Interference" which was presented to the June meeting of this Institute in Chicago. (TRANS. A. I. E. E., 1924, page 789).

Design of Non-Distorting Power Amplifiers

BY E. W. KELLOGG[1]

Associate, A. I. E E.

Synopsis.—The paper deals with the problem of obtaining the maximum output possible from a given amplifier tube, while keeping the distortion down to a negligible amount. A tube can be rated for this purpose in terms of the watts output obtainable, when a sine wave voltage of as great an amplitude as can be advantageously utilized, is applied to the grid. This maximum sine wave output is very much less than the rating of the same tube for oscillator purposes. Starting with a set of static characteristics for a given tube, the dynamic characteristic for any resistance load is readily plotted, and the power output and distortion can be read from the dynamic characteristic. A simple rule has been given by Mr. W. J. Brown for determining the best conditions of load resistance and grid bias for a given plate supply voltage. The best load resistance is shown to be twice the internal plate resistance of the tube. If the supply voltage exceeds a certain value, the application of the rules just mentioned would lead to excessive heating of the anode, and therefore a different procedure is followed, calling for greater grid bias and higher load resistance. There is an advantage in using low impedance tubes. The balanced or push-pull circuit, while reducing distortion, will not make up for failure to operate the tubes under proper conditions, nor will it greatly increase the permissible output per tube. The dynamic characteristic for a reactive load is not readily plotted, but for design purposes it is sufficient to determine the best operating conditions for a resistance load, and then make the impedance of the reactive load high enough to keep the plate current variations within the same limits as for the resistance load. An important application of the principles outlined here, is the design of radio telephone transmitters where serious distortion results from overworking the modulator tubes. For moderately deep modulation there should be from two to four modulating tubes for each oscillator tube. Certain details of design are discussed in the closing paragraphs.

Problem Encountered in Power Amplifiers

In the design of an amplifier whose function is to bring voice or radio signals from bare audibility to comfortable head-phone intensity, the problem of distortion due to curvature of the tube characteristics is hardly a factor, or at least presents no difficulties. The principal concern of the designer is to obtain a large ratio of amplification per stage, and to avoid serious inequalities in the amplification for different frequencies within the required range. When we design an amplifier to operate a loud speaker or to modulate a radio transmitting set the problem takes on an entirely different aspect. A sensitive loud speaker in a small room needs something like a hundred times as much power as a pair of head phones, and here we encounter serious distortion if we attempt to use small tubes or low plate voltage, especially if we push the intensity above a very moderate value. We have reached the power limit of the last tube. By raising the plate voltage and increasing the negative bias on the grid we can raise the power limit and get louder speech or music of good quality, provided we do not exceed the limit of allowable plate voltage, or plate watts and provided there is ample emission from the filament. If we need still more power, we must go to a larger tube, or use several tubes in multiple. Amplifiers the design of which is concerned chiefly with the amount of power obtainable from the final stage, are of the class which we are here designating as "power amplifiers."

Sine Wave Ratings

The relative power obtainable from various vacuum tubes may be estimated in terms of their power outputs of sine-wave alternating current, a sine-wave alternating voltage being impressed on the grid in each case. The magnitude of the grid voltage required is usually of secondary consideration, since it represents no power consumption and is a small factor in determining the total size and cost of the amplifier.

Comparison with Oscillator Ratings

Power tubes as sold are usually given a rating in watts. This rating represents the high frequency output when the tube is used as an oscillator. The power available from the same tube with the same plate voltage is much less when distortionless amplification is required, for the reason that in the amplifier, operation

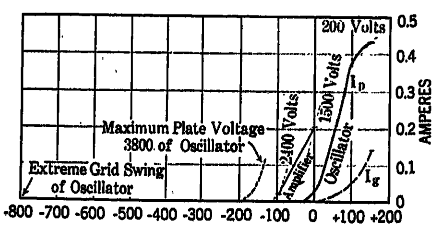

Fig. 1—Comparison of Working Range of Oscillator and Amplifier

must be confined to a portion of the characteristic which is straight, or substantially so, while the oscillator has no such limitation. Thus we find that a tube rated at 250 watts output as an oscillator can supply only about 22 watts of sine-wave output as a straight-line amplifier at the same average plate voltage. Fig. 1 shows the relative working range on a small scale. It will be noticed that the amplifier range is shown as limited to negative-grid voltages. If the grid is allowed to become sufficiently positive (with respect to the negative end of the filament) to take an appreciable electron current, it imposes an irregular load on the preceding

1. Research Laboratory, General Electric Co., Schenectady, N. Y.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

tube which causes wave form distortion. In other words, it is necessary not only that the plate-current vs. grid-voltage characteristic be a straight line but the grid current vs. grid voltage characteristic must be straight over the working range as well, and the only straight part of any considerable length is where the grid current is zero.

Necessary Measurements

The first concern in the design of a power amplifier is the choice of the tube for the power stage, and the determination of operating conditions. This involves a study of the straight line ratings of the tubes under consideration. Except for certain precautions mentioned in a later paragraph we may safely ignore the effects of electrode capacities in an audio frequency amplifier. We can then calculate the performance of a tube from its "static characteristics," or the voltage-current relations obtained point by point by means of meters. The characteristics required are a set of curves showing the plate current as a function of grid voltage, each curve corresponding to a designated plate voltage. Fig. 2 shows the static characteristics for a certain tube having an oscillator rating of 250 watts. The series of plate voltages chosen should preferably be at uniform intervals, and sufficiently near each other to give ten or more curves, ranging from about one-fourth to double the average voltage at which the plate is to be worked. The curves for high voltage need not be carried to high-current values. Some of the points will have to be taken by closing the switch and taking the readings before the plate has had time to become excessively heated.

Dynamic Characteristic

The simplest circuit to analyze is that shown in Fig. 3, in which the alternating current output of the tube

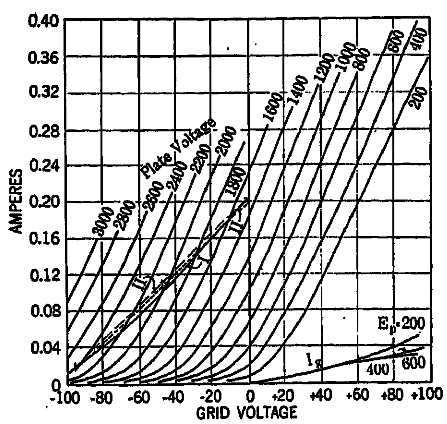

Fig. 2—Static and Dynamic Characteristics of Three Electrode Tube

is used up in the resistance through which the direct current is fed to the plate. While this does not appear to represent the conditions of a practical circuit, we shall find that the conclusions reached in this case are applicable to the commonly employed circuits shown in Fig. 9. and 10. The only essential difference is that in the latter circuits the average plate voltage is practically the same as the supplied voltage.

For the present, considering that the circuit is as shown in Fig. 3, let us find from Fig. 2 the operating characteristic or "dynamic characteristic" for some assumed value of external resistance R, and supply

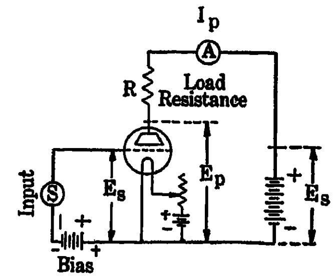

Fig. 3—Simple Circuit for Determining Rating of Amplifier Tubes

voltage E_s; for example, $R = 5000$ ohms and $E_s = 2500$. If the grid voltage is given such a value that the plate current is 0.1 ampere, the voltage at the plate will be $2500 - 0.1 \times 5000 = 2000$. Similarly, 2200 volts at the plate corresponds to

$$\frac{E_s - E^p}{R} = \frac{2500 - 2200}{5000} = 0.06 \text{ amperes,}$$

2400 volts corresponds to 0.02 amperes and 1800 volts to

$$\frac{700}{5000} = 0.140 \text{ amperes.}$$

Points are plotted for these values of current on the corresponding voltage curves, and a curve through the points as shown in Fig. 2, is the dynamic characteristic for the conditions assumed. It will be noticed that while the dynamic characteristic or working curve is straighter than the curves for constant plate voltage, it begins to bend decidedly at low current values, where the other curves turn most sharply. Thus, to keep the wave form distortion within proper limits, the minimum current must not fall below a certain value. In the present case we will take 0.02 amperes as the minimum current. This corresponds to 2400 volts on the plate and − 96 volts on the grid. The other end of the working range is zero grid volts, which, from the curve, is seen to correspond to 1500 volts at the plate and 0.2 amperes. A current varying from 0.02 to 0.2 ampere is equivalent to a direct current of 0.11 amperes, with a superimposed alternating current of 0.09 ampere maximum or $\frac{0.09}{\sqrt{2}}$ ampere effective value. Similarly the voltage varying from 1500 to 2400 is equivalent to a direct voltage of $\frac{1500 + 2400}{2} = 1950$, and an alternat-

ing voltage of $\frac{2400 - 1500}{2} = 450$ volts maximum or $450\sqrt{2}$ volts effective. And the a-c. power consumed in the resistance R is $\frac{0.09}{\sqrt{2}}$ by $\frac{450}{\sqrt{2}} = \frac{0.09 \times 450}{2}$ $= 20.25$ watts. The calculation of power may be abbreviated to

$$\frac{(E_{max} - E_{min})(I_{max} - I_{min})}{8}$$

$$= \frac{(2400 - 1500)\,(0.2 - 0.02)}{8} = 20.25$$

Estimate of Distortion

In calculating the power, no allowance has been made for possible errors introduced by the fact that the characteristic is not truly a straight line. Fig. 4, Curve I, shows the current wave resulting from impressing a sine wave voltage on the grid, varying from 0 to − 96 volts, or in other words, biasing the grid − 48 volts and swinging its potential 48 sine ωt. volts. If we draw a straight line II, on Fig. 2, falling as much below Curve

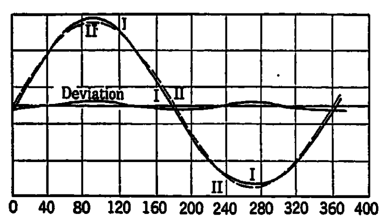

Fig. 4—Output Current Wave Compared with True Sine Wave

I at the ends as it is above it at the middle, and draw the corresponding wave form on Fig. 4, we have a true sine wave II. The difference between the two curves is almost totally a double frequency component with an amplitude 0.056 of the fundamental. Thus Curve II is the fundamental component in Curve I, and its amplitude is 0.09 amperes, or just what was assumed in calculating the power. In general, where the curvature is all in one direction as is true in nearly all cases in the present problem, the wave-form distortion consists principally in the production of even harmonics, and if only even harmonics are present, the difference between the maximum and minimum current is twice the amplitude of the fundamental. In addition to the 20.25 watts of fundamental-frequency power supplied by the tube to the resistance R, there is a small amount, 0.0006 watts, of higher frequencies.

If the working characteristic, Curve I, Fig. 2, has a practically uniform rate of change of slope, or, in other words, if it can be represented by a piece of a parabola, the only harmonic produced is the second or double frequency, and its amplitude is given by the amount by which the Curve I falls below the straight line II, at the center and rises above it at the ends. For simplicity, it seems desirable to compare curvatures on the basis of the maximum deviation of the working curve from a straight line, and since a very close approximation to the actual curve can usually be made by a section of parabola, this deviation may be expressed as the percentage of second harmonic produced, taking the amplitude of the fundamental wave as 100 per cent. In Fig. 2, Curve I is 0.005 amperes below Curve II at the center, and the same amount above it at the ends, so that the second harmonic has an amplitude of 0.005 amperes, as against 0.09 amperes for the fundamental. The ratio is $\frac{0.005}{0.09} = 0.056$ or 5.6 per cent. If the straight line were drawn between the ends of Curve I, it would be 0.01 amperes above Curve I at the middle, and the ratio of this to the total current range is $\frac{0.01}{0.18}$ or again 5.6 per cent. It is unnecessary to draw the straight line. The ratio of second harmonic to the fundamental is

$$\frac{\frac{1}{2}\,(I_{max} + I_{min}) - I_0}{I_{max} - I_{min}} = \frac{0.11 - 0.10}{0.18} = 0.056$$

in which I_0 is the plate current corresponding to the mean-grid voltage, which is $-\frac{0 + 96}{2} = 48$ volts in this case.

It is sometimes more convenient to find the deviation of the curve from the straight line connecting its ends, by a horizontal instead of a vertical measurement at the middle. This horizontal deviation (expressed in grid volts) divided by the total grid swing should give the same result as the vertical deviation divided by the total current swing, provided the curve is parallel to the straight line at the middle. In this case the middle value for plate current is 0.11 amperes which, on Curve I, corresponds to − 43 volts grid, and the relative magnitude of the second harmonic by the grid voltage measurement is $\frac{48 - 43}{96} = 0.052$. The discrepancy is within the limits of accuracy in reading the curves.

If the wave form distortion just calculated is considered more than allowable, it may be reduced by shortening the range, especially by raising the minimum plate current. For example, if the grid voltage varies only between zero and − 88 volts the other conditions remaining the same, we shall have

E_g	E_p	I_p	
0	1500	0.2	I_{max}
− 88	2350	0.03	I_{min}
− 40	1925	0.115	$\frac{1}{2}\,(I_{max} + I_{min})$
− 44	1962	0.1075	

Power $= \frac{1}{8}(850 \times 0.17) = 18$ watts

Second harmonic ratio $= \frac{0.0075}{0.17} = 0.044$ by current values, or

$\frac{4}{88} = 0.0455$ by grid voltages.

Effect of Higher Load Resistance

Let us next try a higher resistance load, 10,000 ohms, raising the supply voltage to 2800, so that with 0.08

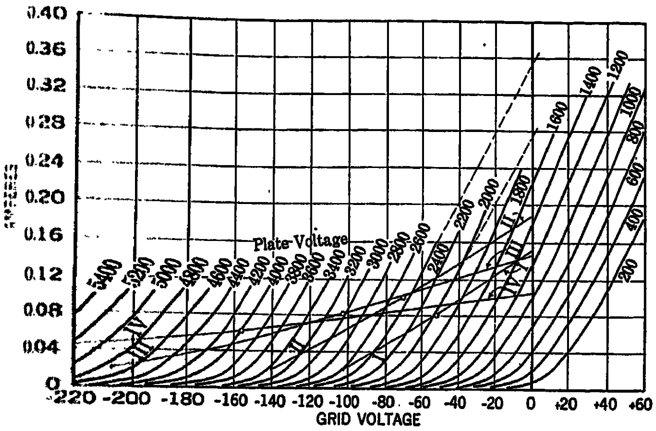

Fig. 5—Dynamic Characteristics with Various Values of Load Resistance

ampere flowing, the plate voltage will be 2000. This gives a point on Curve I of Fig. 5, where the characteristics of the same tube have been redrawn. A change of 200 volts at the plate corresponds to a current difference of $\frac{200}{10,000} = 0.02$ amperes. Starting with the 2000 volt and 0.08 ampere point we may plot the new dynamic characteristic by marking a series of points on the successive voltage curves, each point differing from the preceding one by 0.02 amperes. Again taking 0.02 amperes as the minimum plate current, we find we can swing the grid between zero and − 105 volts, giving

E_g	E_p	I_p	
0	1280	0.152	I_{max}
− 105	2600	0.02	I_{min}
− 49	1940	0.086	$\frac{1}{2}(I_{max} + I_{min})$
− 52.5	1980	0.082	I_o (for mean E_g)

Total current range 0.132 amperes
Total voltage range 1320 volts.
Power $\frac{1}{8}(.132 \times 1320) = 21.8$ watts

Second harmonic ratio $\frac{0.086 - 0.082}{0.132} = 0.03$ by current values

$\frac{52.5 - 49}{105} = 0.033$ by grid voltage

The above figures are comparable with those taken from Curve I Fig. 2, on the basis of substantially the same plate voltage when the grid is at its mean potential. Two results of going to a higher load resistance should be noted; there is more power output, and, for the same value of minimum plate current, the wave form distortion is less. The internal resistance of the tube is about 5000 ohms, and that more power should be delivered to a 10,000-ohm load than to a 5000-ohm load may seem at first surprising to those who are accustomed to think in terms of impedance fit. With a fixed grid swing the 5000-ohm load would receive the greater power, but with the high impedance load, the grid may be swung farther before distortion becomes serious. The common practise of applying a voltage to the grid (from an oscillator, microphone and amplifier, or radio receiving set)—and then varying the load impedance by trying various transformer ratios and selecting the tap which gives the loudest response,—is misleading. A tap which gives weaker response than the maximum may, when the input is readjusted, enable the amplifier to put out more power without distortion.

Best Conditions When Voltage is Limited

Mr. W. J. Brown[1] has given a simple rule for selecting the best load impedance and working range for the case where the mean voltage on the plate is limited to a certain value, and also a proof that the maximum power will be received by a load resistance equal to twice the internal resistance of the tube. In Fig. 6, $F D$ is the minimum value of plate current, below which the curvature becomes excessive; $A B$ is the plate current curve for a constant plate voltage equal to the mean plate voltage assumed in the problem, and $E C$ is the working curve. The constant voltage, curve $G C$, corresponds

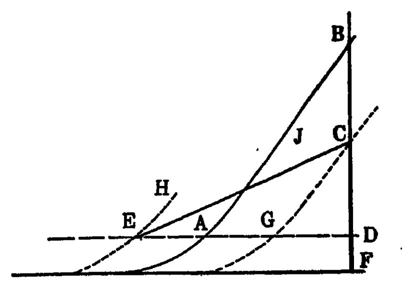

Fig. 6—Diagram Illustrating Method of Determining Best Operating Conditions

to the minimum voltage, and $E H$ to the maximum. The voltage amplitude is $r (B C)$ where r is the internal tube resistance, and the root-mean-square alternating voltage is $\frac{1}{\sqrt{2}} r (B C)$. The measurement $C D$ is equal to twice the current amplitudes, or the root-mean-square alternating current is $\frac{1}{2\sqrt{2}} (C D)$. The pro-

1. Discussion, Symposium on Loud Speakers, *Proceedings of London Physical Society*, Vol. 36, Part 3. April 1, 1924.

duct of the r. m. s. voltage and current, or the power, is equal to $\frac{1}{\sqrt{2}} r\,(B\,C) \frac{1}{2\sqrt{2}} (C\,D) = \frac{r}{4} (B\,C)\,(C\,D)$

Since the measurement $B\,D$ is fixed, the product $(B\,C)\,(C\,D)$ is a maximum when $B\,C = C\,D$ or C is located half way between D and B. The value of the external resistance is equal to $\frac{\text{voltage amplitude}}{\text{current amplitude}}$

$= \frac{r\,(B\,C)}{\frac{1}{2}(C\,D)} = 2\,r$ when $B\,C = C\,D$. The location of the point C and the value of the load resistance determine the working characteristic $E\,C$, and its intersection with $A\,B$ determines the proper value of grid bias.

In the foregoing it is assumed that r is constant, but since this is not the case the result is only an approximation. For larger currents r is less than for small currents, and the value of r in the relation,—voltage amplitude $= r\,(B\,C)$, changes with the location of C. With r a function of the position of C, the condition for maximum product $r\,(B\,C) \times (C\,D)$ is no longer that $B\,C$ shall equal $C\,D$. The shape of the characteristics above the level of the point C can have no bearing on the operation. It would appear logical to use a value of r corresponding to currents between $F\,D$ and $F\,C$ and to locate the point B as if the same value of r held for the higher currents. This means simply to draw the upper part of the line $A\,B$ straight, maintaining a slope equal to that in the working range $A\,J$. This point is brought out not because the error in the method described by Mr. Brown would, in general, be serious, but because in many cases the upper parts of the curves $A\,B$ are more difficult to obtain experimentally or data to plot them may be lacking. As in many other problems involving maxima, precision is not required here, the output remaining nearly the same through a wide range in load resistance.

Limitation Set by Anode Loss

We have so far considered the case where the mean plate voltage is limited, for example by the available supply, but where no other limitation is imposed. If the supply voltage can be raised sufficiently to enable the tube to put out all the power of which it is capable the limit of output may be set either by the power which can safely be dissipated from the plate, or by a combination of plate dissipation and insulation. For voltages below a certain value the rule illustrated in Fig. 6 is applicable. Applying this rule, the output goes up very fast with increase of voltage, and the average power supplied to the plate also increases rapidly. For example, Curve I of Fig. 5 is for a mean-plate voltage of 2000 and gives 21.8 watts output and 162 watts, plate loss. Curve II is for 2400 volts and gives 40.8 watts output and 250 watts plate loss. This is the maximum plate loss which we may safely allow for the tube in question. Up to this point the mean plate current has been increasing with increase of voltage. For higher voltages the current must be limited to

$$\frac{\text{Permissible plate loss}}{\text{Mean plate voltage}}.$$

The mean plate voltage and plate current determine the grid bias. Since the grid voltage may swing to zero in the positive direction, the extreme negative swing will be twice the bias. The minimum value of plate current and the extreme negative grid potential fix the lower end of the working characteristic and the maximum plate voltage. Thus we have the voltage amplitude and the current amplitude, from which we find the load resistance and the power output. Referring again to Fig. 5, if the mean plate voltage is 3000, the mean current is limited to $\frac{250 \text{ watts}}{3000 \text{ volts}} = 0.0833$ amperes. On the 3000 volt curve we find that the grid bias corresponding to 0.0833 amperes is $-$ 103 volts. The grid may then swing between zero and $-$ 206 volts. The plate voltage corresponding to 0.02 amperes, assumed minimum current, and $-$ 206 grid volts is 4700. Then the voltage amplitude is $4700 - 3000 = 1700$, the current amplitude $0.0833 - 0.02 = 0.0633$, the load resistance $\frac{1700}{0.0633} = 26{,}800$ and the power output $\frac{1700 \times 0.0633}{2} = 54$ watts. With the load resistance known the working curve may be drawn. If its intersection with the zero grid volts axis shows that the current and voltage swing during this half cycle is greater than during the other, there will be a correction to the power calculation. In this case the maximum current is 0.148 amperes and the minimum voltage is 1250. The revised power calculation is then $\frac{(4700 - 1250)\,(0.148 - 0.02)}{8} = 55$. If the curvature is small as in this case it is hardly necessary to draw the curve, the first calculation being sufficiently close.

If experimental characteristics at the extreme high voltages are not available, substantially the same result may be obtained if the curvature is slight, by locating the upper end of the working characteristic, which will be on the zero grid volts line, and with a plate current value as much above the average, as the minimum is below. If there is considerable curvature it is better to follow the first method, drawing in additional constant voltage curves by extrapolation if necessary. Fortunately for the ease of extrapolation the constant voltage characteristics are substantially alike in shape, and each is displaced horizontally from the next by the voltage interval divided by the amplification constant of the tube.

That the method just described will give the greatest possible power output for the voltage and mean plate current assumed, may be seen from the following. In swinging the grid from zero to twice the bias, we are using the largest grid swing allowable, increasing the bias without raising the plate voltage would give only a small increase in grid range, but a large reduction in current amplitude. Any lower load resistance would cause the lower end of the working curve to fall below the permitted minimum plate current, and therefore give distortion. Any higher load resistance with the same grid swing would decrease output, because we are already using a resistance much higher than the internal resistance of the tube, and it is a general principle that with a given grid swing, the more nearly the load resistance can be made to approach the tube resistance the greater will be the tube output.

The following table shows how the output of the tube with characteristics given in Fig. 5, varies with the voltage of the plate supply. The voltage E_0 given in the first column is the plate voltage when the grid is at its bias potential, and I_0 is the corresponding plate current.

Plate Volts E_0	Plate Amperes I_0	Grid Bias	Plate Loss	Output			Load Resist	Second Harmonic Ratio
				Amperes Max. A. C.	Volts Max. A. C.	Watts		
1980	0.082	−52.5	162	0.066	660	21.8	10,000	0.03
2460	0.10	− 71	246	0.085	960	40.7	11,250	0.03
3000	0.0833	−103	250	0.064	1725	55	26,800	0.0052
4000	0.0625	−157	250	0.0425	2970	63	70,000	
5000	0.05	−210	250	0.03	4120	62	137,000	

Since the tube is not designed to withstand more than approximately 2500 volts, the maximum output which can be realized is 41 watts. The purpose of carrying the calculations to higher voltages is merely to illustrate the method of determining load impedance and output when plate loss rather than plate voltage is the limiting factor, and to bring out certain relations. In the above table the minimum current has been taken throughout as 0.02 amperes, although in view of the fact that distortion is less with the high resistance loads, the lower limit of current might properly have been reduced. Had this been done the maximum output would have occurred at a still higher voltage. If we permitted the minimum current to go to zero it would have appeared that the output increased indefinitely with increase of voltage. This is because the voltage also cannot go to zero. The power output of the tube is $\frac{1}{2}(I_0 - I_{min})(E_0 - E_{min})$ which becomes $\frac{1}{2} I_0 (E_0 - E_{min}) = \frac{1}{2} I_0 E_0 - \frac{1}{2} I_0 E_{min}$ if I_{min} is zero. As the voltage is increased, the supplied watts $E_0 I_0$ being maintained constant, I_0 becomes less and also E_{min}, so that the output approaches $\frac{1}{2} I_0 E_0$.

Limiting Value of Output

If the tube impedance is lowered by using coarser grid mesh or closer electrode spacings, E_0 can be lowered and a greater output obtained. If we imagine a tube of such low impedance that the voltage could go to zero, as well as the current, the output would be $\frac{1}{2} E_0 I_0$, or one-half the permissible plate loss; and if the latter is kept constant, the output would be independent of the voltage chosen. In practical cases, it appears in general that, for a given value of plate loss, the lower the tube impedance and the higher the supplied voltage, the greater will be the output, and for most efficient use, a power amplifier tube should be worked at the highest voltage compatible with good life.

Output Subtracts from Plate Loss

The average power $E_0 I_0$, supplied to a straight line amplifier tube, is the same whether an alternating voltage is impressed on the grid, or the grid potential remains stationary at the bias value. When the tube is operating into a resistance load, the plate loss is reduced by the amount of the useful output. However, this does not make it permissible to make $E_0 I_0$ greater than the allowable plate loss, for amplifiers do not operate with any fixed grid swing; and even if an amplifier should be used for constant tone production the alternating voltage might go off at any time. Nor should it be inferred that in all amplifiers the plate loss will go down when an alternating voltage is applied to the grid. If there is much curvature or the swing is excessive, the average current will generally increase, and if the load is of low resistance there will be little power absorbed in the load to offset the increase in the supplied watts. Under such conditions the plate loss may increase when alternating voltage is applied to the grid.

Permissible Distortion

In comparing various tubes for a given purpose or in assigning power ratings to tubes, it would be logical to set a limiting value of distortion. For scientific or measurement work the distortion permitted would depend on the nature of the work. On the other hand for the reproduction of speech or music it does not seem possible to assign any general rule as to how much distortion should be permitted, nor to give any simple criterion for setting a limit. There is good reason to believe that the response of the ear is not linear, or, in other words, that harmonics are produced in the ear

itself[2]. This being the case, the ear would not be critical toward the production of overtones in the amplifiers. On the other hand, if the working characteristic is fairly straight over a certain range and then turns quite sharply, an impairment of quality is quickly noticed if the grid swing exceeds that which corresponds to the straight part of the characteristic. It would appear that much greater deviation from a straight line can be allowed if the curvature is practically uniform, than if the characteristic turns abruptly. While it does not seem possible to set a limit to distortion, applicable to all tubes, the practical limit in a particular case does not seem difficult to decide upon. For example, in the case of the tube whose characteristics are shown in Fig. 5; if we assume E_0 to be 2500 volts and the plate loss not to exceed 250 watts, we can assign various values to the minimum current and find the maximum power obtainable for each case. A low minimum current means a considerable curvature allowed, but permits more power to be obtained. Fig. 7 shows the power output and the distortion as functions of I_{min}. It will be noticed that below about 0.02 amperes, the gain in output is slow and the increase in distortion is rapid. Analysis of a number of practical tubes indicates that it is rarely necessary to permit more than about 5 per cent distortion in order to realize about all the useful output of which the tube is capable.

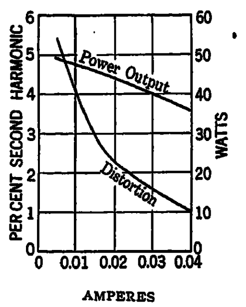

FIG. 7—RELATION BETWEEN DISTORTION AND MINIMUM CURRENT

PUSH-PULL CIRCUIT

In this connection, the question naturally arises whether the working range can be increased by use of a "push-pull" circuit, like that shown in Fig. 8. Such a connection balances out the even harmonics if the tube characteristics are alike, and is, therefore, a possible means of reducing distortion. The gain in output per tube, however, is small. Referring to Fig. 5, Curve II, we see that, for the range shown the curvature is very slight, but we cannot extend the curve on the lower end without its turning horizontal, nor on the upper end without driving the grid positive. Trying to drive the grid positive causes a distortion of the grid voltage wave which has the same effect on the plate current as if the grid voltage wave were not affected, but the working characteristic turned toward the horizontal to the right of the zero-grid-voltage point. A working characteristic which bends toward the horizontal at both ends, or in other words, has a reversed curvature, causes odd harmonics in the output current wave, and the balanced, or push-pull circuit, does not neutralize odd harmonics. While the push-pull circuit does not make it permissible to increase the working range of the tubes appreciably, there are cases in which it is advantageous. For some scientific work, it is important to reduce all distortion to the minimum, and if the tubes are worked over a very moderate range so that only the double frequency harmonics are produced, the balanced circuit can be made to practically eliminate the distortion. It possesses a practical advantage in that the direct currents in both halves of the output transformer winding, balance each other magnetically, thus reducing the tendency to saturate the core, and making a lighter, smaller design of transformer possible. As against these advantages, the push-pull circuit calls for an interstage transformer which introduces some distortion.

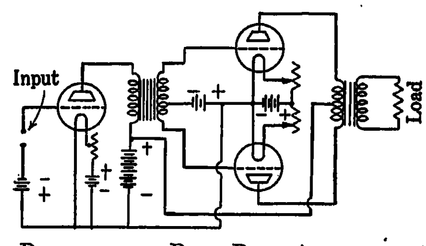

FIG. 8—BALANCED OR PUSH-PULL AMPLIFIER CIRCUIT

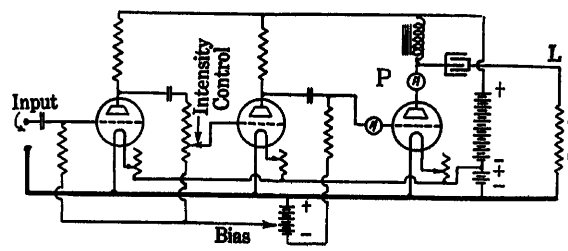

FIG. 9—RESISTANCE CAPACITY COUPLED AMPLIFIER WITH CHOKE COIL-CONDENSER FEED TO LOAD

LOAD CIRCUITS

Figs. 9 and 10 show connections frequently used for the output of power amplifiers. In Fig. 9, the choke coil must have a high reactance compared with the tube and load resistance, and the stopping condenser must have a low reactance compared with the load resistance, for all frequencies in the working range. Otherwise, distortion will result, the low frequency

2. "Sensation of Tone" by H. L. F. Helmholtz (Edition of 1877), Chapter VII. Quantitative measurements supporting this theory are given by R. L. Wegel and C. E. Lane, *Physical Rev.*, Feb. 1923. "Auditory Masking of One Pure Tone by Another."

currents being partially suppressed either by excessive reactance in the condenser, or too much leakage through the choke. With a properly designed circuit, we may think of the choke as maintaining a constant direct current equal to the average plate current, I_0, while the condenser offers a practically constant counter-electromotive force equal to the average plate voltage E_0. Now, suppose that the plate current changes momentarily to $I_0 - i$. Since the current through the choke remains constant, the difference or i amperes must flow through the load. The potential at L then becomes $+ i R$ which, added to the strain E_0 on the condenser, brings the potential at P to $E_0 + i R$. Similarly, when the plate current is $I_0 + i$ the load current is reversed, the potential at L is $- i R$ and at P is $E_0 - i R$. It will be recognized that from the standpoint of the tube, this condition duplicates that illustrated in Fig. 3, the only difference being

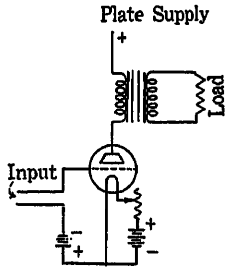

FIG. 10—TRANSFORMER COUPLED LOAD

that in Fig. 3, the supply voltage would have to be $E_0 + I_0 R$ in order to maintain the average voltage E_0 at the plate. The conditions met in the transformer connection of Fig. 10 are similar. An ideal transformer with a resistance R connected to its secondary terminals introduces into the primary circuit, an effective resistance for alternating currents equal to R multiplied by the square of the ratio of primary to secondary turns, but for direct current, it introduces only the ohmic resistance of the primary winding. The practical transformer, if properly designed to avoid distortion, gives a close approximation to the relations just stated. The magnetizing current must be small compared with the with the load current at the lowest frequencies with which we are concerned, and the series reactance due to leakage flux must be small compared with the load impedance for the highest frequencies considered.

REACTIVE LOADS

We have so far considered only pure resistance loads. The dynamic characteristic for a tube with a reactive load is not readily plotted, but its general form can be shown and the conditions for avoiding distortion as found for the case of resistance load are applicable to the reactive load. If there is reactance in the load, the dynamic characteristic becomes elliptical in form; a true ellipse if the constant voltage curves are straight over the range between maximum and minimum current, and a distorted ellipse if there is appreciable curvature within this range. It is generally possible to adjust the load impedance to any desired value, either by means of a transformer, or by changing the number of turns in the windings of the instrument which constitutes the load. Such changes do not materially alter the power factor. In fitting a reactive load to the amplifier tube, the procedure recommended is first to determine for the case of a resistance load, the load resistance, plate voltage, grid bias, and grid swing for maximum output; then using the same plate voltage and grid swing, make the load impedance such that the maximum and minimum values of plate current will be the same as in the case of the resistance load. This means making the vector sum of load impedance and tube plate resistance the same for the two cases[3]. If these conditions are complied with, the principal sources of distortion are avoided, namely, swinging the grid positive, and working with too low minimum currents.

Since load impedances are functions of frequency and audio frequency amplifiers must operate properly for a wide range of frequencies, the question arises at what frequency should the load impedance have the value which has been determined as suitable? The only safe rule is to make the calculation for the frequency at which the load impedance is lowest. Exception might be made in case currents of the frequency in question were known to be of low intensity compared with those of other frequencies. For example, most telephone receivers and loud speakers have lowest impedance at low frequency. A case might arise in which it was known that the low frequency components in the currents to be amplified were weak as compared with those of higher frequency. It would then be permissible to determine the load impedance at a higher frequency. But such a situation is difficult to imagine. When pains are taken to build a distortionless amplifier, it is only reasonable to assume that the remainder of the system would be designed with a view to minimizing distortion, and since the greater part of the energy in both voice and most music, is carried by the lower tones, the amplifier would be called on to handle the largest amplitudes at low frequency.

MODULATION OF RADIO TRANSMITTERS

Fig. 11 shows the modulator and oscillator tubes of a radio telephone transmitter. There are many possible arrangements of the oscillating circuits, but the means

3. The alternating current in the plate circuit is equal to

$$\frac{\text{(alternating grid voltage) (amplification constant of tube)}}{\text{(vector sum of load impedance and tube resistance)}}$$

For derivation of this relation see "The Thermionic Vacuum Tube" by H. J. Van der Bijl, pages 157 and 177. See also pages 175 and 176 for discussion of form of dynamic characteristic with reactive load.

for varying the intensity of the oscillations or "modulating" them is, in most cases, like that shown in Fig. 11. The large inductance, L_1, keeps the total current constant, while the function of the small inductance, L_2, prevents absorption of radio frequency power by the modulator tube. When the grid of the modulator tube swings toward positive the plate takes more current, leaving less current available for the oscillator, and the voltage drops, causing a reduction in the amplitude of the oscillations. Similarly, when the grid of the modulator swings negative, the voltage rises and the oscillation amplitude increases.

If the voltage supplied to an oscillator is varied and the current read and plotted, the relation in general is found to be represented by a straight line over the range for which the oscillations are stable. Therefore, the oscillator is equivalent to a resistance load on the modulator tube. If the time of discharge of the grid condenser is appreciable compared with an audio cycle,

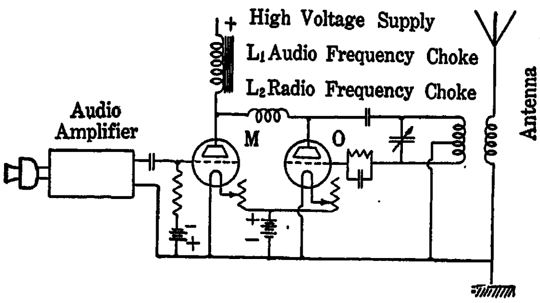

FIG. 11—RADIO TELEPHONE TRANSMITTER SHOWING METHOD OF MODULATION

a reactive component is introduced and the effective resistance of the oscillator will be less for high than for low frequencies, but with properly proportioned grid leak and condenser, the oscillator is practically a pure resistance. From the standpoint of the modulator tube the circuit then becomes equivalent to that shown in Fig. 9, with the blocking condenser omitted and the current through the choke increased by the amount of the average direct current taken by the oscillator.

The type of oscillator and supply voltage having been chosen, the design of the modulating system involves first the determination of the effective resistance of the oscillator. This may be found by varying the voltage and measuring the change in supplied current corresponding to a given change in voltage. The effective resistance depends, among other things, upon the grid leak, antenna resistance, antenna coupling, and filament current; and if any of these factors are changed, a new measurement of resistance will be necessary.

The modulator tube or tubes must work at the same mean-plate voltage as the oscillator. The load resistance can be controlled within limits by changing the antenna adjustments, but if the full oscillator output is desired, the principal means for obtaining a suitable load resistance must be to employ the proper number of modulator tubes compared with the number of oscillator tubes. Thus, doubling the number of modulator tubes, doubles the load resistance into which each modulator tube works. The load resistance per tube determines the slope of the dynamic characteristic, and its position, which for a given supply voltage depends on the grid bias, should be so chosen that when the grid reaches a negative potential equal to twice the bias, the plate current will have the minimum value permissible in view of curvature of the tube characteristics. The voltage range covered by the dynamic characteristic shows the percentage modulation of the oscillations which can be accomplished without exceeding the proper range of the modulator tubes. For illustration, let us suppose that we wish to modulate a 250-watt oscillator, operating at 2000 volts, and showing an effective input resistance of 8000 ohms. First, we may try a single modulator tube of the type represented in Fig. 5, taking 0.02 amperes as the minimum plate current. To find the proper grid bias, we might draw several parallel curves, each corresponding to an 8000-ohm load resistance, and choose the one which gave equal voltage swings above and below 2000, but it is simpler to try several values of grid bias, in each case, assuming that at double the bias voltage, the plate current is cut down to 0.02 amperes. Find what load resistance this would mean, and pick the grid bias which corresponds most nearly to the actual load resistance. For example, referring to Fig. 5, a bias of − 50 volts with 2000 volts on the plate gives 0.095 amperes plate current. A grid voltage of − 100 and plate current of 0.02 amperes corresponds to 2500 volts on the plate. A dynamic characteristic through these

two points would correspond to $\dfrac{2500 - 2000}{0.095 - 0.02} = 6660$

ohms. The trials may be shown in tabular form.

E_g	E_p	I_p	R
− 50	2000	0.095	6,660
− 100	2500	0.02	
− 55	2000	0.079	11,800
− 110	2700	0.02	
− 52	2000	0.09	8,300
− 104	2580	0.02	

The last value is close enough. An 8000-ohm dynamic characteristic passing through the point $E_p = 2000$, $I_p = 0.09$, would intersect the 1400 volt line at $I_p = 0.165$, and the 1200 volt line at $I_p = 0.19$. Connecting these two points gives the intersection with the vertical line over $E_g = 0$ as $I_p = 0.17$ and $E_p = 1360$. Therefore with one modulator tube, we could control the oscillator voltage down to 1360 and up to 2580. Owing to the slight curvature, equal grid swings do not give exactly

equal plate swings in both directions. Taking the average, which is 610 volts, we may say that the modulation is $\frac{610}{2000}$ or 30.5 per cent. Heavier modulation should not be attempted with the one modulator tube. Characteristics of oscillating tubes show in general that through most of the stable range, the antenna current is a straight line function of the supplied voltage. This being the case, there is no distortion introduced at this stage except that due to the curvature of the modulator tube dynamic characteristic. This distortion is offset by the characteristics of the detectors used in receiving sets which rectify strong currents more efficiently than weak ones. Suppose, for example, that the rectified current is proportional to the square of the amplitude of the incoming waves, which law would represent fairly well the action of most detectors on weak signals. Then the relation between modulator grid potential and rectified receiver current would be as follows:

Modulation grid voltage..............	0	−52	−104
Voltage supplied to oscillator (proportional to oscillation amplitude)......	1360	2000	2580
Square of oscillator voltage ÷ 10^6 (proportional to rectified receiver current).	1.85	4	6.65

This shows a swing of 2.15 in one direction and 2.65 in the other. Had there been no curvature in the modulator tube characteristic and the plate voltage swung 610 volts in each direction, the voltage square factors would have been 1.94, 4.0, 6.8, or a swing of 2.06 in one direction and 2.8 in the other,—a greater wave form distortion than with the curved modulator characteristic.

The next question is how much can we increase the modulation by adding a second modulator tube. With two modulator tubes, the load resistance for each tube is 16,000 ohms and we again find the proper grid bias by trial.

E_g	E_p	I_p	R
− 60	2000	0.06	22,500
− 120	2900	0.02	
− 57	2000	0.07	15,800
− 114	2790	0.02	

The 16,000-ohm dynamic characteristic through the point $E_g = -57$, $I_p = 0.07$, intersects the $E_g = 0$ axis at $I_p = 0.124$, $E_p = 1120$. With a voltage swing of 790 volts in one direction, and 880 volts in the other, the mean modulation is $\frac{845}{2000} = 0.422$ or 42 per cent as compared with 30.5 per cent. with one modulator tube.

With reduced supply voltage, the percentage modulation which can be obtained with a given number of modulator tubes is less. For example, if the supplied voltage is 1600 and the oscillator effective resistance again taken as 8000 ohms, we find that with one modulator tube and a bias of − 38 volts we can obtain a modulation of 27 per cent and with two modulators and a bias of − 42 volts, we can modulate 40 per cent. In both cases, 1600 and 2000 volts supplied, the addition of a modulator tube about doubles the audio frequency power which would be delivered by a receiving set.

The question of how much modulation should be attempted is one which has been much discussed. It should be borne in mind that the 40 per cent modulasion just calculated applies to the extreme peak values of the audio frequency waves, and with most speech or music it would represent an average modulation of less than 5 per cent, which is small compared with most radio telephone practise. In most receiving sets the rectified current varies as some function between the first and second powers of the amplitude of the incoming high frequency waves, being approximately proportional to the square for small amplitudes and more nearly a linear function with stronger signals. With the type of transmitter circuit shown in Fig. 11, the aim is to make the envelope of the oscillation amplitude show the exact wave form of the original audio-frequency currents. This would result in distortionless reproduction by a receiver having a straight line detector, or one giving a rectified current directly proportional to the amplitude of the incoming radio frequency waves. Since practically no receivers have this characteristic, distortion results, but as the percentage modulation is reduced, the distortion becomes less, until at 10 per cent modulation, it is of practically no consequence whether the receiver is a linear or a square law detector. This has been used as an argument for limiting the modulation to a very small value,—perhaps ten or twenty per cent on the peaks. But there are arguments in favor of larger modulation, particularly if the purpose is to obtain the best possible reception with a given average radiated power. In the first place, the distortion which results when, for example, a square law detector is used to receive fairly deeply modulated waves, does not appear to be of an especially objectionable kind, and as already explained is partially offset by the distortion due to the curvature of the modulator characteristic. In the second place, there is a positive advantage in the use of moderately heavy modulation, in that the ratio of the desired sound to static and stray noises is better with large than with small modulation. People who have operated receiving sets will probably agree that the soft passages in musical numbers are rarely so satisfactory to listen to as the louder parts. The "blasting" that frequently occurs in the loudest parts is due to exceeding the proper working range of the modulator tubes, or to overworking some other part of the system, rather than to large modulation itself.

It would appear, then, that a modulator capacity of two, or even four times the oscillator capacity would be desirable in the case of the tubes used in our illustrations. The degree of modulation which can be accomplished is increased by use of low impedance tubes and

increases as the voltage is raised, up to the point where it is necessary to reduce the mean plate current of the modulator tubes below the value that would otherwise be assigned, in order not to overheat the plate.

Plate and Grid Ammeters

It is desirable to design the earlier stages of a power amplifier with ample margin so that if any tube is overworked, it will be the power stage. A very satisfactory indicator, to show when the intensity exceeds the straight line capacity of the amplifier, consists in a milliammeter in the grid circuit and a meter in the plate circuit. If the grid swings positive with respect to the filament, the grid meter will "kick" and perhaps the plate meter also, while too great a negative swing will cause a kick on the plate meter. Careful listening will reveal an impairment of quality which disappears as soon as the intensity is reduced to a point where the meters cease to show disturbance. Listening alone is not a satisfactory substitute since the distortion is not so quickly nor surely noticed, particularly in view of fatigue of attention, and if distortion is noticed, its origin might be elsewhere in the system.

Tubes in Multiple

The increased output from a power amplifier obtained by adding power tubes in multiple, is often disappointing, the gain being slow compared with that which can be accomplished with increased voltage or with tubes of higher rating. However, there are cases in which it is desirable to use several tubes in multiple. With the load impedance already several times the tube resistance, which is the proper relation for maximum output, the addition of a second tube in multiple with the first would not give a perceptible increase in output, but if the load impedance is readjusted to half the previous value, twice the power output can be obtained, corresponding to about 40 per cent increase in sound amplitude, which is a noticeable, but not a striking difference.

When several tubes are connected in multiple, they may form an oscillating system and fill the tubes with high frequency oscillations. This is less likely to occur if the connecting wires are made very short, but with high power tubes, it is frequently necessary to employ some means of stopping the parasitic oscillations, such as an individual resistance or choke connected in series close to the grid of each tube. A few turns of wire on a small solid iron core will suffice, since such a choke introduces a high effective resistance at the extremely high frequencies concerned.

Interstage Connections

With a supply of high voltage available, such as is necessary for the power tubes, the design of the preceding amplifier stages is simple. Resistance-capacity coupling can be used with sufficiently high plate resistance to obtain three-fourths or four-fifths of the full amplification of the tube. Under these circumstances, and with high amplification tubes available, there would be little argument for transformer coupling. In designing a resistance-capacity coupled amplifier, the following points must be kept in mind:

1. The reactance of the coupling condenser at the lowest frequency to be passed, must be less than the grid leak resistance.

2. No grid should swing positive with respect to the negative end of the filament, or, in other words, the grid bias must be greater than the extreme grid swing required. This does not apply to detector tubes.

3. If the expected swing in plate potential of any tube is more than about 20 per cent of the average voltage on the plate or if the plate current is very small, the dynamic characteristic should be worked out to make sure of constant proportionality between plate and grid voltage swings. The load resistance is that of the plate-feed resistance and the grid leak of the next tube in multiple.

4. If a battery of power tubes in multiple is employed, so that the grids constitute a considerable capacity load, or if any of the earlier tubes have very high internal resistance, or are fed through very high plate resistances, calculation should be made of the magnitude of the capacity load at the highest frequency which the amplifier must handle. Owing to the simultaneous swinging of the plate potential the effective capacity of a tube grid may be several times the grid capacity as measured with plate and filament grounded. The capacity reactance of the grid must be high compared with both the internal and external plate resistance of the previous tube; otherwise distortion may result, either because of reduced amplification of the high frequencies, or because the actual dynamic characteristic is steeper than estimated (lower impedance load) with resulting curvature.

With a common source of plate voltage for all of the tubes of a multi-stage amplifier, back coupling with consequent oscillations may occur through the plate supply line, if there is enough resistance in the supply so that the power-stage plate current affects the voltage of the supply line materially. If such back coupling occurs, the cure may be to secure a lower resistance supply, or to reduce the variations in the voltage fed to the earlier tubes, particularly the second tube preceding the power tubes, by filters or potentiometer connections. The filters, if of series resistance and shunt capacity, must be effective at as low a frequency as the lowest at which appreciable amplification takes place. Therefore, the filtering is simplified by designing the amplifier not to pass any frequencies lower than really required.

To obtain a given voltage swing on the grid of the power tube may be easier with resistance connection than with an interstage transformer, in spite of the step-up ratio of the transformer. This is because even the best designed transformers drop to an impedance comparable with the tube resistance at high frequency owing to capacity on the secondary side, and at low frequency owing to magnetizing current. This low

impedance load on the tube may greatly reduce the plate voltage swing, obtainable without distortion.

OUTPUT TRANSFORMERS

If the output of the power stage goes through a transformer, the design of the transformer follows the general principles of audio transformer design, of which the following is a summary:

1. The impedance at the terminals of the tube is equal approximately to the load impedance multiplied by the square of the ratio of primary to secondary turns, and the turn ratio should be chosen to give the desired impedance at the tube.

2. With the secondary open-circuited, the primary reactance should be at least equal to the effective load impedance on the primary side, at the lowest frequencies to be passed.

3. The leakage reactance, found by measuring the reactance of one winding with the other short-circuited, should be low compared with the effective load impedance at the highest frequencies in the working range.

4. The winding resistance loss should be low compared with the power supplied to the load.

5. If the direct current component is sufficient to saturate the core, it will frequently be found that the same number of turns gives more inductance with an air-gap in the magnetic circuit than without. The air-gap should be just sufficient to prevent saturation. The inductance for a given number of turns is then practically proportional to the core cross section, and a heavy transformer is the price of efficiency. If the required air-gap is very short, 0.002 in. or 0.005 cm. or less, there may be an advantage in using special high permeability core material, while with longer gaps, 0.010 in. or 0.025 cm. or more, ordinary transformer steel is satisfactory.

Discussion

G. L. Bayley: I would like to ask the author if it is possible to make an amplifier self-exciting, very much like a d-c. generator. Is it possible to introduce resistance in the plate circuit to give a voltage drop, and to apply that drop to the grid voltage so that the total effect would be much greater amplification.

A. V. Loughren: I wonder if the members of the Institute have ever had called to their attention just how important these limits in operating a tube really are.

At a recent radio show a well-known company which sells resistance-coupled amplifiers, advertising them as panaceas for all disturbances, had a set-up in which there were two amplifiers, one transformer-coupled and the other resistance-coupled. The transformer-coupled amplifier was so arranged that when an audio-frequency input was applied to the grid of the first tube, the mean plate current increased, and the obvious inference which was brought out very strongly by the salesman was that a transformer-coupled amplifier for that reason was very hard on the battery. There was altogether too low a plate voltage in the set, of course, for the grid bias used, and the trouble was that the plate current was decreasing almost to zero on the negative half of the wave, so that when the grid was made to oscillate, the mean plate current rose. Naturally, there was distortion there; but the striking point was that at the same time they had a resistance-coupled amplifier alongside it that was operated without any grid bias, so that when a speech input was put on, the grids all drew current, and for that reason on the positive half of the wave the plate current did not increase as much as it decreased on the negative half. The mean plate current accordingly decreased and the result was that the resistance-coupled amplifier was fine for the battery; but nothing was said about distortion.

As far as experimental results are concerned, the theory is absolutely correct. Last summer, Mr. Kellogg was working on this problem, and I had some work to do on it from a different angle. I found it necessary to get some experimental checks. Working on tubes of somewhat lower rating than the 250-watt tube that he describes, I got experimental verifications of the predictions of power based on the characteristic curves which were good to better than 5 per cent, and the amount of distortion produced by operating under various conditions could be checked to probably 10 per cent. The method of measuring distortion, though, isn't as accurate from an experimental point of view, because it means measuring small quantitites.

G. D. Robinson: Mr. Kellogg speaks of, I believe, "negligible" distortion. It is my recollection (I fail to place it precisely) that either one year ago or two years ago at the Midwinter Convention the statement was made that two tubes working in a push-pull circuit were good for 18 times the output of one tube singly, which, of course, does not check with Mr. Kellogg's statement about the small increase in value.

Another point: There are substantially no broadcast transmitters working with four modulator tubes per oscillator and probably there are substantially no transmitters working with two modulator tubes per oscillator.

In connection with that same thing, we are assured by the Westinghouse Company that it is permissible to run a certain quite observable grid current on the modulator tubes without serious distortion. That probably goes back again to: What do we mean by serious distortion or negligible distortion?

P. W. Gumaer: I wish to emphasize the importance of this subject as affecting future improvements in the quality of broadcasting and its reception. One of the limiting features in perfection of the quality of radio reception has been in the acoustic end, particularly in the connection with loud speakers. Recent developments in loud speakers have overcome that difficulty. There is now a loud speaker that has no distortion that can be detected by the human ear.

The next step in the development of perfect radio reception is now up to the electrical end. In order to reach perfection in the quality of radio reception we need, I believe, a unit of measurement of distortion and a less laborious method of measuring distortion. I do not refer to any particular frequency but to the whole range of audio-frequencies, and it is indeed a difficult problem.

D. F. Whiting: One point occurs in the first paragraph of Mr. Kellogg's paper, where he speaks about the relatively greater energy required to operate a loud speaker satisfactorily in comparison with the energy required to actuate a telephone headset. Considering as a basis for comparison the relative energy requirements necessary to produce approximately the same response in the ears for the two cases, I believe that Mr. Kellogg is somewhat more conservative in his figures than is warranted by the conditions that obtain. Instead of an energy increase of one hundred times, as he has stated, I should be inclined to further emphasize the importance of this paper by favoring an increase in the vicinity of ten thousand times in the energy that is required by the loud speaker in comparison with that necessary for the headset. This value is borne out by the following figures, expressed in transmission units, such that twenty-five transmission units below an arbitrary standard, which we call "zero level" in telephone parlance, represents about the right energy to use on a headset for persons of normal hearing, whereas an energy level of

fifteen transmission units above this same standard is required for good loud-speaker reception. The difference between these two figures is forty transmission units, a voltage and current increase of one hundred times, and an energy increase of ten thousand times. Some people may be satisfied with less from their loud speakers than these figures represent, but it is my feeling with respect to them that they are easily satisfied.

Incidentally, the values which I have quoted represent the amplification derived from two stages of a good-quality amplifier, or about one and one-half stages of amplification in which less attention is paid to frequency discrimination.

In regard to the dynamic characteristics of the vacuum tubes, Fig. 3 of Mr. Kellogg's paper shows a circuit by means of which certain characteristics of the tubes may be secured, but the results obtained from such a circuit are not the true dynamic characteristics of the tubes. The dynamic characteristic is affected by the impedance through which the grid receives its charge, especially when the grid runs positive, but Mr. Kellogg has limited his considerations of the dynamic characteristic, to the effects which take place when the grid potential is negative with respect to the filament, thereby neglecting the determination of the characteristic when the grid is positive. However, the dynamic characteristic covering the whole range in which we are interested may be determined by including a resistance in the circuit of Fig. 3 between the grid and the upper point where the grid supply voltage is measured. If this resistance is equal to the impedance of the circuit normally connected to the grid, the resulting dynamic characteristic will represent the effects which occur under normal operating conditions; and, if this resistance is great in magnitude, the dynamic characteristic will change its slope as the grid runs positive, tending to run in a more horizontal direction, the degree of departure from the static characteristic being dependent on the magnitude of the included resistance. The resulting dynamic characteristic represents a more accurate depiction of the effects which actually take place in the tube when in service. Although Mr. Kellogg has taken care to point out the occurrence of this effect, the curves given in his paper do not show it and the cause of the limitation is not pictured very clearly to the minds of those unfamiliar with the subject.

In reference to the discussion centering around the push-pull form of circuit, I wish to state that this type of circuit displays its principal advantages in comparison with the more simple arrangement when conditions surrounding the tube such as those discussed in Mr. Kellogg's paper—that is, grid and plate voltages, filament currents, impedances connected to grid and plate—are not optimum. However, under optimum or nearly optimum conditions, the push-pull circuit provides an increase in energy-handling capacity of approximately 25 per cent. Relative to what may be accomplished in other ways, this does not represent a very great increase. On the other hand, this increase in power output is gained very cheaply; for no additional apparatus or power is required and, while the amplification of the final stage may be lowered slightly thereby, this usually can be restored more cheaply in the preceding stages.

E. W. Kellogg: May I ask a question at this point? On what basis are you making comparison, on the basis of an equal amount of distortion?

Mr. Whiting: Yes.

Mr. Kellogg: Total energy of harmonics the same in the cases?

Mr. Whiting: Yes. The amount by which the energy capacity increases due to the push-pull circuit feature will vary, of course, with each individual test, with the criterion assumed for the state of full load and with the accuracy with which the conditions imposed on the tubes during the test simulate the optimum conditions for maximum output. The increase I have quoted is based on the results of a rather large number of tests obtained from many tubes of various kinds and under conditions which, to the best of my knowledge, were optimum for those tubes.

In the last sentence under the paragraph entitled "Push-Pull Circuit," Mr. Kellogg states, "As against these advantages, the push-pull circuit calls for an interstage transformer which introduces some distortion." I think this sentence should be modified to state that this comparison is made with reference to a resistance-coupled amplifier, whereas, of course, the same comment would not apply if the amplifier which he was considering in relation to the push-pull amplifier was of the transformer-coupled type already.

The second paragraph under the subject of "Reactive Loads" is exceptionally good, and I want to emphasize especially what it contains. If an amplifier is called upon to transmit good-quality speech or music currents, the maximum energy which the amplifier is required to deliver occurs ordinarily at very low frequencies; and, if the amplifier is to operate without overloading, it should be capable of carrying its maximum load at the frequencies that require the greatest energy capacity. This means that the amplifier should be designed in such a manner that the output impedances of the tubes bear the right relationship to the impedance of the load at the low frequencies where the larger proportion of the energy is likely to occur. This practise is not generally followed in the case of telephone repeaters because matters concerning line impedance termination and impedance balance are of greater importance to the operation of repeaters than is the matter of maximum power capacity.

S. Stern: I have found that, very often the number of amplifying stages that are found necessary, must be decreased in order to prevent feed-back. This feed-back is very rarely due to the inductive coupling between transformers, if the proper shielding precautions are taken. The cause when traced is usually found in the tube itself in which the grid-plate capacity is often of the proper value to act as a coupling capacity between the matched audio-tuned coils of the transformers in the grid and plate circuits. A direct remedy would therefore be to reduce this capacity but this cannot be conveniently done. However, adding capacity to the grid-plate capacity, will decrease the frequency at which feed back can occur to that below the audio range. If the capacities used for this purpose have very little resistance, no decrease in amplification will be experienced.

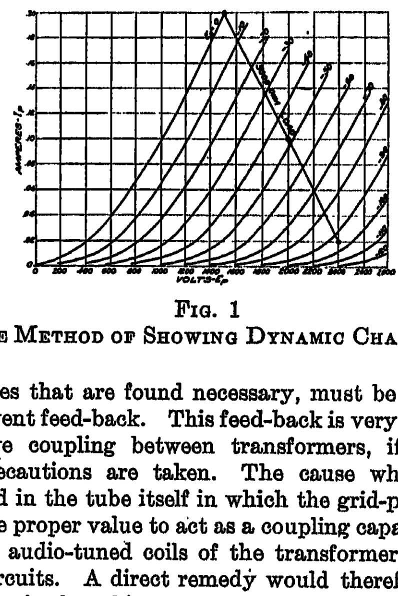

FIG. 1

ALTERNATIVE METHOD OF SHOWING DYNAMIC CHARACTERISTICS

A detailed description of this method and also the construction of a four-stage transformer amplifier, can be found on page 460 of May 1925 number of *The Experimenter* published by the Experimenter Publishing Co. of New York City.

E. W. Kellogg: Since the paper was written, my attention has been called by Mr. John C. Warner of General Electric Research Laboratory, to advantages in using a set of plate current vs. plate voltage curves to show the tube characteristics, instead of plate current vs. grid voltage as used in the paper. The two sets of curves give the same information. Thus the curves of Fig. 1 herewith are derived from Fig. 2. of the paper. When plate voltage is used instead of grid voltage for the abscissas, the dynamic characteristic for a resistance load becomes a

straight line whose slope corresponds to the load resistance. The 5000 ohm dynamic characteristic shown on Fig. 1 herewith is the equivalent of Curve I of Fig. 2. Different values of load resistance, plate voltage and grid bias can be tried and compared very quickly on such a chart as Fig. 1. If the mean plate voltage, E_o, the load resistance, and minimum current are given, we first find the slope from the value of load resistance, then slide a straight edge, maintaining this slope, until we find a position at which the grid voltage for the minimum current is twice the grid voltage corresponding to the intersection with the mean plate voltage line. Thus the dynamic characteristic shown in Fig. 1. crosses the horizontal .02 ampere line at $Eg = -96$ volts (interpolation) and it crosses the vertical 2000 volt line at $Eg = -48$ volts. Using the approximate method given in the paper for estimating distortion, the information is conveyed readily by Fig. 1, distortion in the case of Fig. 1 being shown by the fact that the point on the dynamic characteristic corresponding to mean grid voltage is not at the middle of the characteristic.

While the use of plate voltages as abscissas makes the resistance load dynamic characteristic a straight line, it does not make a true ellipse of the characteristic for a reactive load. For a general understanding of what takes place and for estimating both the kind and amount of distortion, the form of curves used in the paper is to be preferred, since the evidence of distortion is presented to the eye in much more striking form and it is at once evident whether the bend in the characteristic is approximately uniform or is abrupt. On the other hand, for rapid calculations, the second method has a great advantage.

There were a few questions brought up on my paper. The first was whether you could make an amplifier work something like a d-c. generator, self-excited. If I understand the question, it was whether we could use feed-back to get more amplification without distortion (I am putting this into language that is more familiar to radio fans). I have never tried in any thorough way to do it and I don't know what the possibilities are. I can say, certainly, that it is pretty difficult and it is questionable whether it is really desirable. It is so easy to get all the amplification you need in the regular way that it hardly seems worth while to go to the difficulties that you encounter with feed-back when you try to make the feed-back aperiodic.

The question was brought up as to the number of modulators. I was somewhat surprised when I began making calculations on the subject to find how the case stood. I think it is no doubt true that a good many stations do not employ more modulators than they have oscillators, and I think they ought to. Of course, that is tied up with the question of how heavy modulation is being attempted. That subject is discussed in the paper. It is a debatable question how deep modulation is desirable. But the point I wanted to bring out was that if you are going to attempt to use as heavy modulation as is very commonly employed, and considered by many to be desirable, then you need plenty of modulator capacity. It isn't my purpose to say how many modulator tubes are needed, but just to speak of an excess, for the purpose of calling attention to the fact that you need to study the tube characteristics and the oscillator impedance and to find out just how many you do need.

As to working the grids positive, there are possibly cases where it can be done to advantage, by suitable circuits you might reduce the distortion introduced in that way to as low a limit as is necessary, but the task is not simple. In Fig. 2, for example, I have plotted the grid current to the same scale as the plate current, and if you figure out the resistance corresponding to the slope, you will find that by the time you have the grid 20 volts positive, which won't extend your range very much, you are down to a grid impedance of 2500 ohms, which would be a heavy load on the plate of the preceding tube, with which you are trying to push the grid. The curves as I have plotted them don't turn over, because my abscissas are grid volts, regardless of how much current I may have to supply to get the grid voltages. Mr. Whiting has suggested a way of plotting the curves, which will take into account the effect of the grid load, and make apparent the distortion which results from trying to push the grid positive.

I accept Mr. Whiting's correction of my figure for the ratio of input to telephone and loud speaker. It was merely an illustrative figure and I probably put it very mildly.

Metallic Polar-Duplex Telegraph System
for Long Small-Gage Cables

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Synopsis.—*In connection with carrying out the toll-cable program of the Bell System, a metallic-circuit polar-duplex telegraph system was developed. The metallic-return type of circuit lends itself readily to the cable conditions, its freedom from interference allowing the use of low potentials and currents so that the telegraph may be superposed on telephone circuits. The new system represents an unusual refinement in d-c. telegraph circuits, the operating current being of the same order of magnitude as that of the telephone circuits on which the telegraph is superposed.*

The following are some of the outstanding features of the present system. Sensitive relays with closely balanced windings are employed in the metallic circuit, and "vibrating circuits" are provided for minimizing distortion of signals. Repeaters are usually spaced about 100 miles apart. Thirty-four-volt line batteries are used and the line current is four or five milli-amperes on representative circuits. Superposition is accomplished by the compositing method which depends upon frequency discrimination, the telegraph occupying the frequency range below that of the telephone. New local-circuit arrangements have been designed, employing polar relays for repetition of the signals; these arrangements are suitable for use in making up circuits in combination with carrier-current and ground-return polar-duplex telegraph sections. New forms of mounting are employed in which a repeater is either built as a compact unit or is made up of several units which are mounted on I-beams, and subsequently interconnected. In the latter case the usual arrangements for sending and receiving from the repeater are omitted, and a separate "monitoring" unit provided for connection to any one of a group of repeaters.

The metallic system is suitable for providing circuits up to 1000 miles or more in length, the grade of service being better than that usually obtained from ground-return circuits on open-wire lines for such distances. About 55,000 miles of this type of telegraph circuit are in service at present.

* * * * *

Introduction

THERE has been developed recently by the Bell System a low-current metallic telegraph system, of the polar-duplex type, which is suitable for superposition on telephone circuits in long small-gage cables. In certain sections where long-distance toll traffic is heavy, it becomes desirable, from the standpoints of economy and continuity of service, to employ such cables to replace existing open-wire lines and to provide for future growth. The new telegraph system is being applied on a considerable scale in connection with the toll cable system, the general features and telephone arrangements of which have been described in previous papers.[3] The present paper outlines the general features of the metallic telegraph system and the method of superposing telegraph circuits of this type upon "two-wire" and "four-wire" telephone circuits in small-gage cables.

The metallic-return or two-wire type of telegraph circuit was chosen in preference to the ground-return type because it appeared to offer a more straightforward solution of the technical problem and to be more economical, sufficient cable conductors being available as a result of the telephone requirements. On a long telephone circuit in a small-gage cable it is necessary to employ a number of repeaters with comparatively large amplification and also to insert loading coils in the line at short intervals. As a result, the interference from superposed telegraph would be excessive unless the telegraph voltages and currents were kept far below the values ordinarily employed for ground-return telegraph. To allow the use of small currents and potentials with ground-return telegraph would require the development of arrangements for neutralizing difference in earth potential and inductive interference from telegraph circuits in the same cable as well as from power circuits. It will be evident that a metallic telegraph circuit possesses certain transmission advantages over a ground-return telegraph circuit in the same way that a metallic telephone circuit possesses advantages over a ground-return telephone circuit.

This development resulted in a telegraph system which in some ways is unique in its refinement. The telegraph line currents are of the same order of magnitude as those of the telephone circuits which use the same wires. Although cable is fundamentally much less favorable to telegraph transmission than open wire, one mile of small-gage cable having as much effect as many miles of open wire, the present system affords satisfactory operation on each pair of the cable for distances up to 1000 miles (1600 km.) or more.

Two improved forms of mounting are employed; in one of these a repeater is built as a single self-contained unit and in the other a repeater consists of several units mounted on upright I-beams. The relays are quiet in operation and sounders are normally made inoperative mechanically as they are seldom used. Altogether, a metallic repeater office bears little resemblance to the older type of office with apparatus mounted on tables and hundreds of sounders in operation.

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1. Bell Telephone Laboratories, Inc.
2. American Telephone and Telegraph Co.
3. Philadelphia-Pittsburgh Section of the New York-Chicago Cable, J. J. Pilliod, TRANSACTIONS A.I.E.E., 1922, p. 446.

Telephone Transmission Over Long Cable Circuits, A. B. Clark, TRANSACTIONS A.I.E.E., 1923, p. 86.

Telephone Equipment for Long Cable Circuits, C. S. Demarest, TRANSACTIONS A.I.E.E., 1923, p. 742.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

Principles of Operation

In describing the general principles upon which the present telegraph system operates, it will be convenient to evolve it from the familiar ground-return polar-duplex system, the essential features of which are illustrated in Fig. 1. It will be seen that at each end of the line circuit there are provided a transmitter and a receiving relay. The operation of the transmitter sends current into the line and the artificial line, one polarity being used for "marking" and the other for "spacing." If the artificial line has the same impedance as the real line, there will be no effect upon the receiving relay, since the latter is connected differentially. Currents received from the transmitter at the distant station will, however, cause the receiving relay to operate. The arrangement, therefore, makes it possible to send telegraph signals in either direction, or in both directions simultaneously.

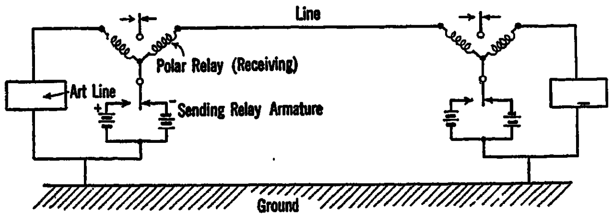


Fig. 1—Differential Duplex on Grounded Circuit

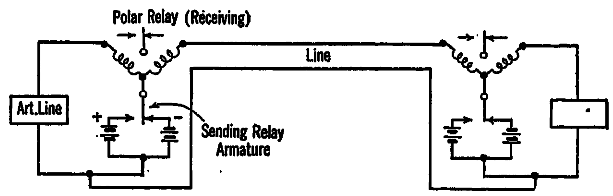


Fig. 2—Differential Duplex on Metallic Circuit

In Fig. 2 the ground-return is replaced by a second line wire so that the circuit is now a metallic circuit.

Fig. 3 differs from Fig. 2 only in that each receiving relay has its windings divided into four parts instead of two, making the circuit symmetrical.

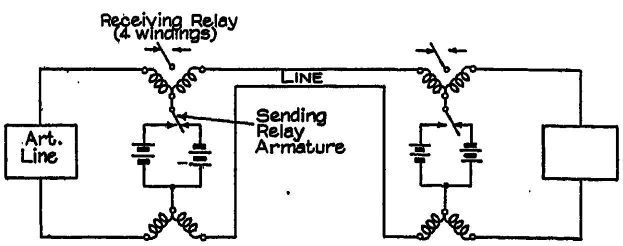


Fig. 3—Symmetrical Differential Duplex on Metallic Circuit

For actual operation involving the working of a number of circuits in a given office from the same set of batteries, it is desirable to make a connection to ground at each station at the point *G* as shown in Fig. 4. These connections stabilize the system and facilitate the clearing of accidental grounds. Although this results in unbalancing the currents in the circuit, there is substantially no effective change in the metallic or two-wire operating currents if the line and apparatus are well balanced, and this arrangement has the essential characteristics of an actual metallic telegraph circuit. It may be helpful, however, to consider that the upper wire is employed for the transmission of signals and the lower wire is used to carry only neutralizing current to offset the effect of currents in the upper wire which are due to earth-potential differences and voltages to ground caused by induction from power or telegraph circuits. Since each pair in the cable is closely balanced, encloses a small loop, and is frequently transposed by twisting, it will be apparent that the currents due to interference are practically equal in the two wires, flowing in the same geographical direction and therefore do not affect the balanced relays.

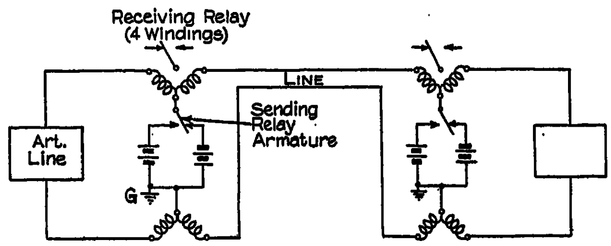


Fig. 4—Metallic Duplex Circuit—Single Commutation

Fig. 5 shows another arrangement of a metallic telegraph circuit in which the transmitter comprises two tongues, reversing the connections to a single battery instead of switching between two different batteries as in the case of Fig. 4. The ground connection at the midpoint of the battery at each station is for the purpose of stabilizing the system and facilitating the clearing of trouble.

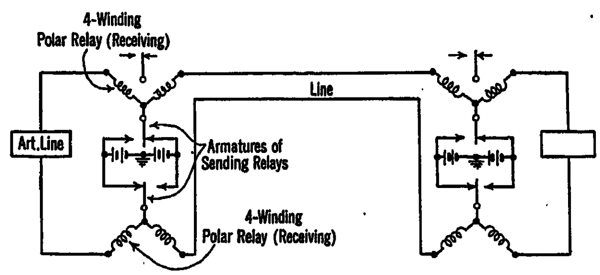


Fig. 5—Metallic Duplex Circuit—Double Commutation

Circuits of the type shown in Fig. 5 were first developed and put into extensive use in preference to the type shown in Fig. 4, largely for the reason that it was not at first practicable to obtain sufficiently close balance of relay windings. With improved relays, telegraph repeaters have been designed to operate on the basis of Fig. 4, effecting certain economies. These two arrangements, which are known respectively, as "double commutation" and "single commutation," may be

operated one against the other in a telegraph repeater section.

The local circuits of the repeaters are arranged so that they may be conveniently set up either for simultaneous operation in both directions (known as full-duplex) or for operation in only one direction at a time (called half-duplex), the latter giving the same communication facilities as a simple open-and-close Morse telegraph circuit.

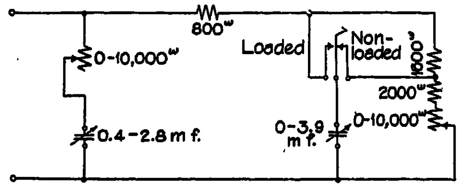


Fig. 6—Duplex Artificial Line

General Features

As in the case of other telegraph systems it is necessary to subdivide a long circuit into sections by means of repeaters to avoid the use of excessive potentials and to limit the distortion of signals. For repetition between two metallic cable circuits a simple arrangement called a "through repeater" is employed. The equipment used at the end of a metallic telegraph circuit is known as a "terminal repeater."

The metallic polar-duplex system operates with a potential of 34 volts, requiring one 34-volt battery for double-commutation and two such batteries for single-commutation. Where both are used in the same office, one of the single-commutation batteries may be used for double-commutation, this being equivalent to the

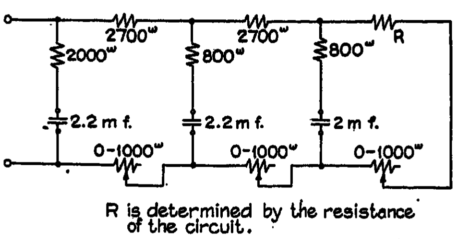


Fig. 7—Artificial Line for 19-Gauge Circuit Loaded with 0.174 h. Coils at 6000-Foot Intervals

regular arrangement with a ground potential of 17 volts in addition. The batteries are ordinarily "floated." Tungar rectifiers are generally used, without causing any noise in the telephone circuits.

The telegraph current in the cable circuit, with the batteries at the two ends aiding, is from about 3 to 15 milliamperes depending on the resistance of the line circuits. With the batteries opposing, the current is, of course, practically zero.

The small-gage cables are made up of No. 16 and No. 19 B. & S. gage (1.29 and 0.91 mm. respectively) copper conductors, and the metallic telegraph system may be operated over conductors of either gage, or over the derived phantom circuits when the latter are not in use for telephone service. The maximum distance between two consecutive repeaters is about 120 miles (195 km.) on 19-gage, composited pairs, or 160 miles (260 km.) on 16-gage. For non-composited circuits the corresponding distances are about 140 miles (225 km.) and 190 miles (305 km.) respectively. The

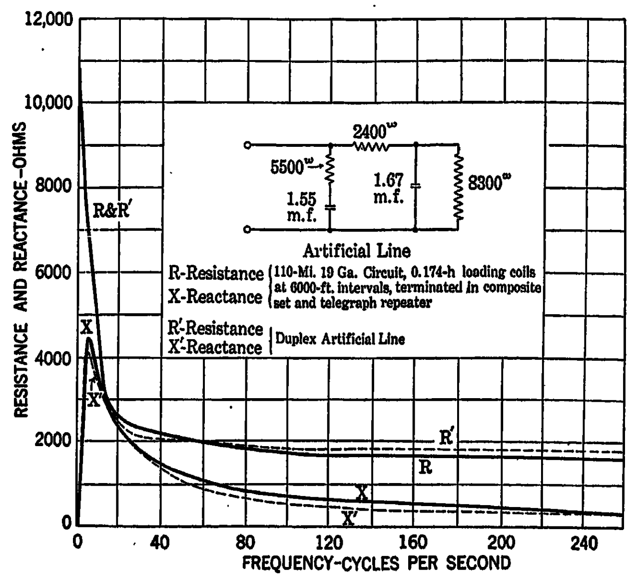


Fig. 8—Impedance of Line and Artificial Line

average telegraph repeater section is about 100 miles (160 km.) in length as a result of the telephone requirements in connection with locating repeater stations. In some cases the telegraph is operated over non-

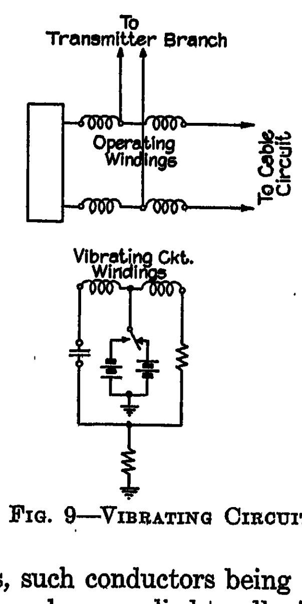


Fig. 9—Vibrating Circuit

loaded circuits, such conductors being available before loading coils have been applied to all wires of the cable. The telegraph transmission is practically the same on non-loaded and loaded circuits.

For maintaining an impedance balance, which, as brought out previously, is essential for polar-duplex operation, two different types of artificial line are used:

a flexible line with adjustable resistances and capacities adapted to balance any type of small-gage cable circuit, and a less flexible line having resistances as its only variable members and designed to balance accurately only 19-gage circuits with a certain type of loading. The first type of artificial line is shown schematically in Fig. 6 and the second in Fig. 7. The former balances with sufficient accuracy for full-duplex operation any circuit which does not contain intermediate compositing equipment. It also balances, well enough for half-duplex operation, circuits containing intermediate compositing equipment. The second type of line can be used for full-duplex service with only the type of circuit for which it was designed and for half-duplex with a limited variety of circuits. It is not so flexible therefore, as the other type. However, it is considerably cheaper and is somewhat easier to adjust, since to obtain a balance it is necessary only to secure a correct d-c. or resistance balance with the three adjustable resistances approximately equal. This line is built in H sections so the structure is similar to that of the real line; the effect of the loading coils on the impedance is simulated however, by the resistances in the three bridged members. Since in cable circuits leakage is negligible and the only effect of temperature changes is variation in resistance, the only adjustable members required in the latter type of artificial line are the three series resistances.

Curves of resistance and reactance versus frequency are shown in Fig. 8, for a representative metallic line section and the corresponding artificial line. It will be noted that there are large variations in these impedance components in the frequency range from zero to about 30 cycles per second, and they tend to become constant as the frequency is further increased. At the lower frequencies the effect of the distant terminal apparatus is, of course, large. Curves for non-loaded lines are similar except that at the higher frequencies the resistance is lower and the reactance higher.

A feature which has an important effect on the quality of the received telegraph signals is the "vibrating circuit" which was devised originally by Gulstad. This circuit comprises two auxiliary windings on the receiving relay, a condenser and two resistances as illustrated in Fig. 9. A current through the resistance branch of the vibrating circuit moves the relay armature to the opposite contact when the effective operating current, in reversing, approaches zero value. While the armature is passing between contacts, the condenser in the other branch partially discharges through both windings in series, the discharge current accelerating the armature. As soon as the armature touches the other contact, a transient current completing the discharge of the condenser and charging it in the opposite direction holds the armature firmly against this contact until the operating current has had time to become large enough to assume control. The vibrating circuit therefore increases the sensitivity, reduces the time of armature travel, lessens chatter of the armature contacts and makes the operation of the relay more positive. Furthermore, the constants of the vibrating circuit are so proportioned as to minimize distortion of signals, the relay being caused to operate near the steepest part of the received current wave.

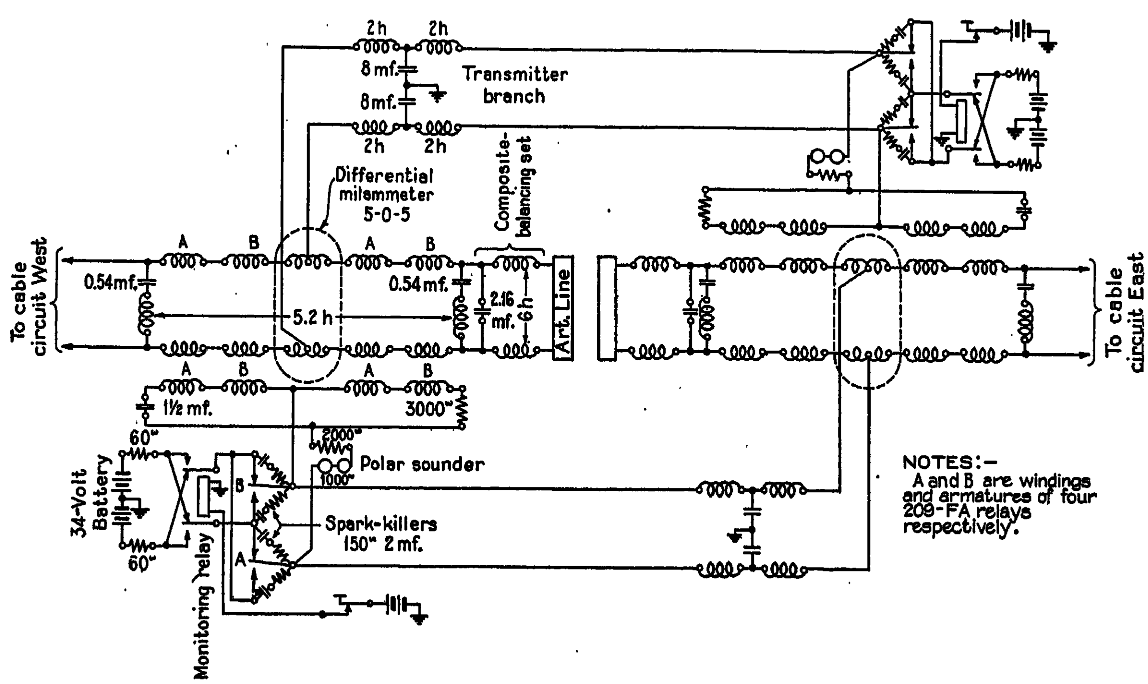


FIG. 10—THROUGH REPEATER

The receiving and transmitting relays used in metallic telegraph repeaters are the 209-F A and 215-A relays, respectively, which are being described in a separate paper. The former is a highly sensitive polarized

relay, furnished with vibrating windings, whereas the latter is of the same general construction but less sensitive and has no vibrating windings. The 215-*A* relay is also used in the arrangements provided for facilitating "breaking." In cases where a terminal repeater is operated between a ground-return circuit and a metallic circuit, relays of this type function as receiving relays for the ground-return section.

local circuit. Polarized sounders and other monitoring features similar to those in the through-type set are provided. The local circuit arrangements are described in detail in the next section.

Local Circuits

To avoid supplying battery at outlying points and to facilitate setting up and changing circuits which have a number of stations in the same locality or have

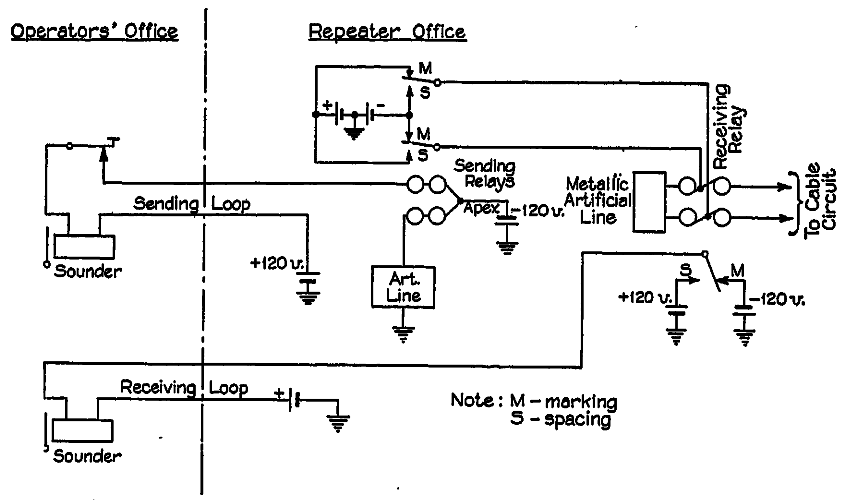


Fig. 11—Terminal Repeater—Full-Duplex Local Circuits

The through-type repeater is a direct-point repeater; the armatures of sensitive polar relays, operated by the line current from one direction, repeat the signal (differentially through the windings of the opposite receiving relays) into the other line in the opposite direction. A simplified diagram of this repeater is shown in Fig.10. This repeater is a full-duplex repeater but is used on half-duplex circuits without change. As shown,

branches, a two-wire circuit or "loop" is extended from the repeater office to each operator's station. For the marking or closed condition the current is approximately 60 milliamperes and for spacing it is zero.

For full-duplex service the arrangement is simple, involving the use of a receiving loop and a sending loop as shown in Fig. 11. In the receiving loop the batteries are aiding when the line relay tongue is on mark-

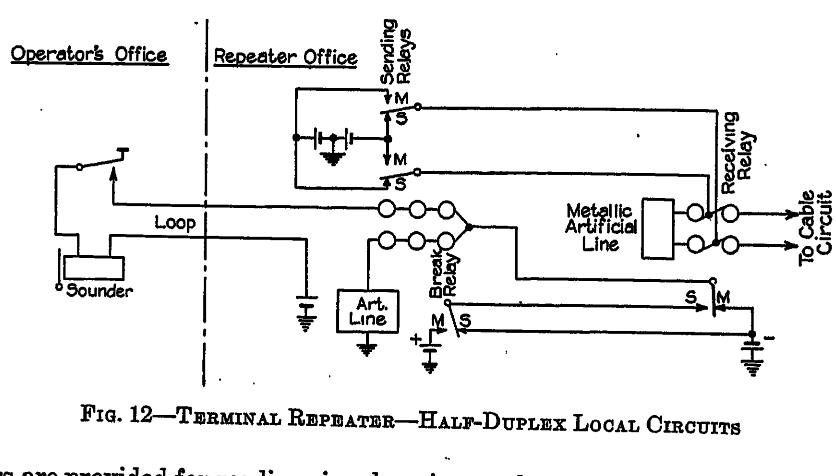


Fig. 12—Terminal Repeater—Half-Duplex Local Circuits

two polarized sounders are provided for reading signals, and a telegraph key controls the operation of local neutral relays, designated monitoring relays, making it possible to send into either line independently, or in both directions at once.

The terminal-type repeater is also a direct-point repeater and is used to repeat signals between a metallic cable section and either a ground-return circuit or a

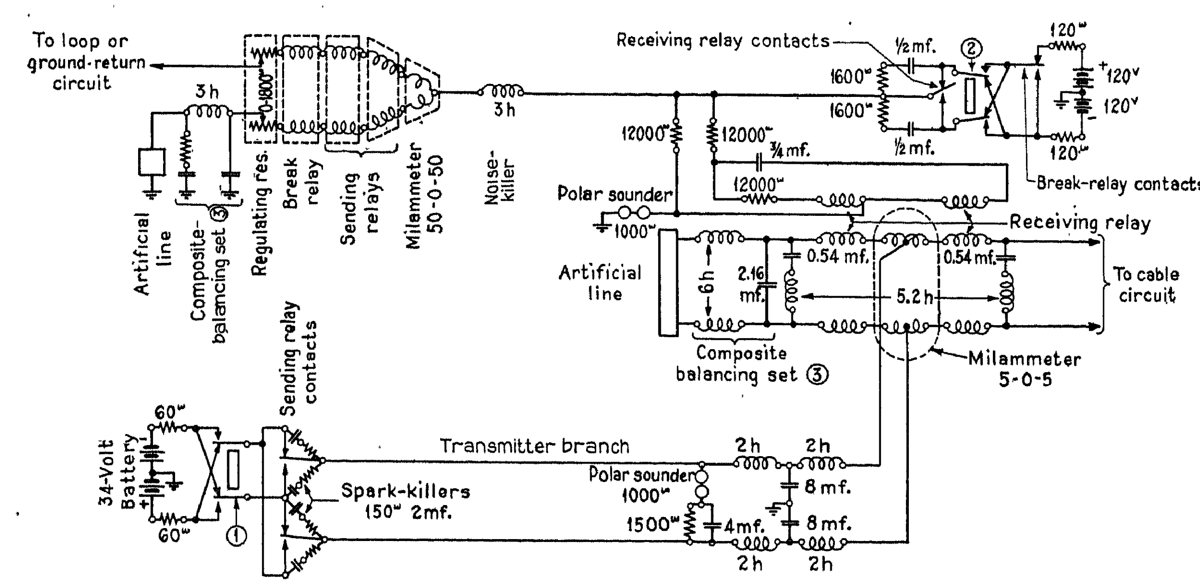
ing, and opposing when it is on spacing. Signals may, therefore, be received by the operator by means of an ordinary Morse (neutral) relay or main-line sounder. The sending loop is opened and closed by the operator's key in sending out signals. The sending relays are of the polar type and may be considered to have a biasing circuit which includes the battery connected to the apex point, the lower windings and the artificial

line. When the key is closed the effect of the biasing current is overpowered by the loop current, as the latter is twice as great. When the key is opened the biasing current moves the relay armatures from marking to spacing.

For half-duplex service, a single loop is used for both sending and receiving as depicted in Fig. 12. Signals are sent out in precisely the same manner as in full-duplex and do not affect the metallic line relay on account of the balanced duplex connection. The sending relays, although connected in the loop, are unaffected by received signals as they are differential as regards current from the receiving relay tongue. This is in fact a duplex connection, and it allows the working of the grounded side of the terminal repeater directly into a long circuit having a standard ground-return peated back into the line, reversed. This would result in a slow and uncertain break.

In using two terminal repeaters to connect an operator's office at an intermediate point to a metallic circuit, the loop or loops are connected in tandem between the two repeaters. In full-duplex the sending leg of one repeater is connected to the receiving leg of the other, and vice versa, with a loop in series with each connection. In half-duplex the two local circuits are connected together with the loop in series, and at one repeater, batteries on the receiving relay contacts are reversed and the connections to the loop and biasing windings of the sending and break relays are interchanged.

Local circuit arrangements of the type just described make the metallic repeaters suitable for use in



NOTES:—
1. Monitoring relay for sending on metallic circuit, controlled by a telegraph key.
2. Monitoring relay for sending on ground-return circuit, controlled by a telegraph key.
3. Composite-balancing set removed by key for loop or non-composited operation.

FIG. 13—TERMINAL REPEATER

polar-duplex repeater at the distant station. A suitable artificial line is provided for this purpose.

To facilitate interruption of the sending operator by the receiving operator a "break relay" is also provided, operating simultaneously with the sending relays. To understand its function, assume the key in the loop to be opened; as soon as a marking signal is received from the line, the sending relay armatures will be moved to spacing due to the current in the biasing windings and the absence of current in the loop. The break relay at the same time connects marking battery to the spacing contact of the receiving relay so that no matter what signal impulses are subsequently received from the line the sending relays will be unaffected. If the break relay were not used, incoming signals would operate the sending relays and be re- combination with the carrier-current and ground-return polar-duplex repeaters used in the plant. This flexibility has been secured by designing the loop circuits to operate with 60 milliamperes current for marking and zero for spacing. Briefly, the flexibility necessary to permit of setting up long circuits with branches is in no wise sacrificed by the use of the several systems.

The essential features of the circuit of the terminal-repeater are shown schematically in Fig. 13.

For convenience in testing and in patching circuits the loop is connected to the telegraph repeater through a series of jacks at the "Morse board" called a "Morse line terminal." The latter consists of a number of jacks for inserting loops in series and testing the batteries and circuit in case of trouble.

Superposition on Telephone Circuits by Compositing

In superposing the metallic telegraph on telephone circuits, the well-known "compositing" method is used. This is based on frequency discrimination, the telegraph occupying the range below that of the telephone.

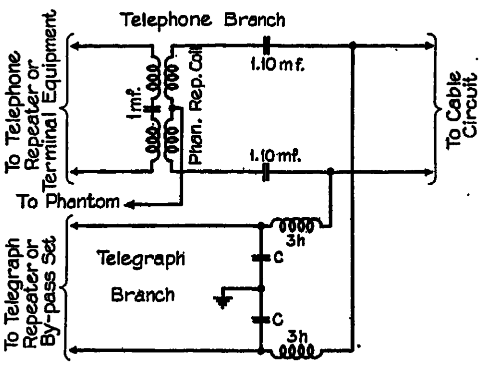


Fig. 14—Composite Set

For satisfactory results, the telegraph and the telephone arrangements, including signaling, as well as the composite sets, must be designed in conjunction with the line circuits so as to avoid serious interference between telegraph and telephone. Furthermore, the compositing means employed should have but little detrimental effect on the transmission of the three forms of communication operating separately, and must not upset the symmetrical circuit arrangement upon which freedom from external interference depends.

Interference from telegraph and telephone manifests itself in two ways. The first of these is telegraph "thump" which is the name given to a low-pitched noise in the telephone due to a small part of the telegraph current passing through the telephone branch of the composite set and entering the telephone apparatus. The thump, in addition to being audible, may effect the telephone signaling equipment to the extent of causing false rings. In addition to the thump at the transmitting end of the circuit, thump is produced at the receiving end by the vibrating circuit through transformer action of the relay windings. In providing protection from thump, both phantom and side circuits have to be considered. The second kind of interference is the flutter effect[4] due to the fact that rapid changes in the telegraph currents momentarily increase the effective resistance of the loading coils, thereby varying the attenuation of the circuit at telephone frequencies.

The telegraph branch of the composite set (see Fig. 14) consists of series inductance and shunt capacity and therefore offers to line currents of telephonic frequencies high impedance and attenuation. It has little effect upon the low frequencies required for satisfactory telegraph transmission, and at the same time sufficiently attenuates the higher frequency components of the telegraph waves to avoid excessive thump. In order that the telegraph branch may be effective in reducing thump voltages in the phantom circuit, the two windings of the retardation coil are made with a negligible mutual inductance, and the bridged capacity consists of two balanced condensers with the midpoint grounded. It has been found necessary to make this retardation coil of very stable inductance by using a comparatively large amount of iron, since a coil with less stable characteristics would cause excessive thump, due to the generation of harmonics.

The telephone branch consists of series condensers and a low-inductance repeating coil or transformer and has high impedance and attenuation for line currents of telegraph frequencies, but has little effect upon telephone transmission. It supplements the telegraph branch in reducing thump and also serves to limit mutual interference between telephone signaling and telegraph. The repeating coil is also used for deriving the phantom circuit in the usual manner.

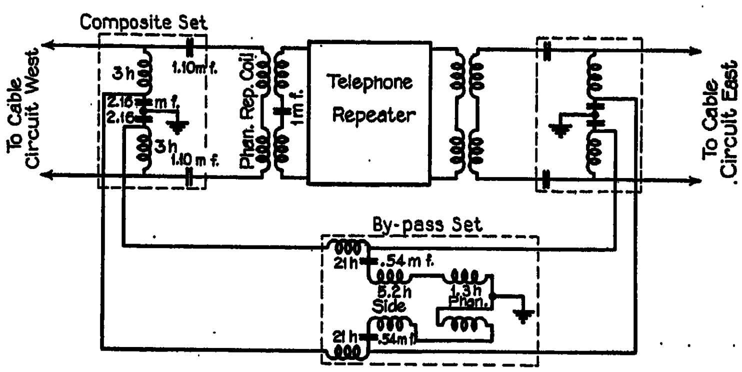


Fig. 15—Intermediate Compositing Arrangements

The composite set is sufficient to limit receiving-end thump to a harmless amount, but greater protection is necessary against sending-end thump. In order that the additional equipment for this purpose may have the minimum effect on telegraph transmission, it is placed in the transmitter branch where it affects outgoing signals only. It consists of series inductances and bridged capacities to suppress the high-frequency

4. Paper by Martin & Fondiller, Transactions A. I. E. E., February 1921, page 553.

components of the telegraph impulses as in the composite set; the mutual inductance of the coils is made small so that they may be effective in reducing thump in the phantom circuit. In single-commutation repeaters, another coil is necessary in the transmitter branch to prevent excessive phantom circuit thump. This coil is connected with its windings parallel-aiding as regards the phantom circuit and therefore is series-

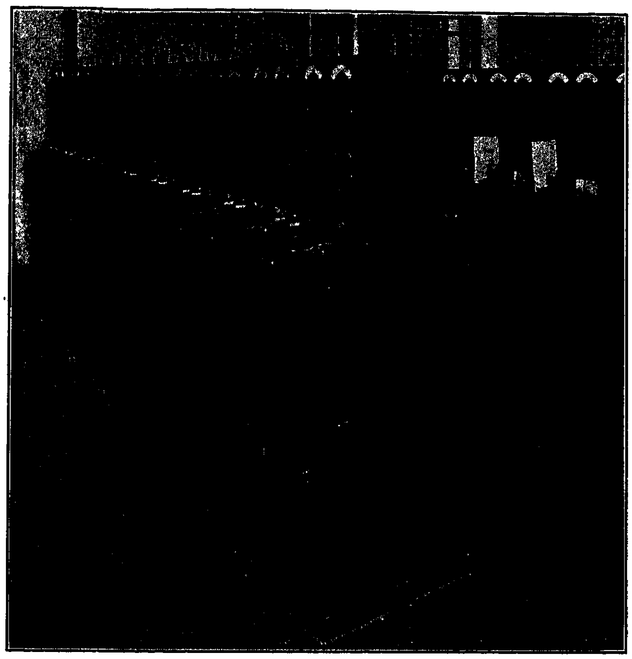


Fig. 16—Installation of Metallic Telegraph Repeaters—Terminal Type

opposed or non-inductive for the metallic telegraph operating currents. An examination of the circuits will show that in double-commutation, operation of the telegraph impresses voltage on the phantom circuit only if the two transmitting tongues fail to operate in exact synchronism; in single-commutation, voltage is impressed on the phantom circuit by the normal operation of the transmitter, since the telegraph current, being unbalanced, has a large longitudinal component.

To preserve the duplex balance when using a composited line, a composite balancing set, consisting of a series coil and a bridged condenser, is provided for insertion in the artificial line branch as shown in Figs. 10 and 13.

To protect the receiving relay from interference from the 135-cycle current used for telephone signaling, a resonant shunt is bridged across the telegraph set on the line side of the receiving relay and a balancing shunt is bridged across the set on the artificial-line side. A single coil is made to serve for both of these shunts, one winding being placed in the line side and the other in the artificial-line side.

Twenty-cycle ringing current, which is used for signaling in the local terminal equipment of the telephone circuit, and operation of the telephone receiver switch-hook, give rise to transient currents which tend to harm telegraph transmission. To minimize this effect, a condenser is connected between windings of the repeating coil.

Since metallic telegraph repeaters are spaced about 100 miles (160 km.) apart and telephone repeaters on many circuits about 50 miles (80 km.), means must be provided for passing the telegraph currents around the intermediate telephone repeaters. This is done by inserting an "intermediate" composite set on each side of the telephone repeater and connecting the telegraph branches together through a "by-pass" set. This arrangement is shown in Fig. 15. The intermediate composite set is very similar to the terminal composite set. The by-pass set consists of a retardation coil of high inductance and little mutual inductance between windings, with or without a resonant shunt. The purpose of this by-pass set is to keep the amplification characteristic of the telephone repeater from being affected by currents feeding back through the telegraph branches of the composite sets from the output

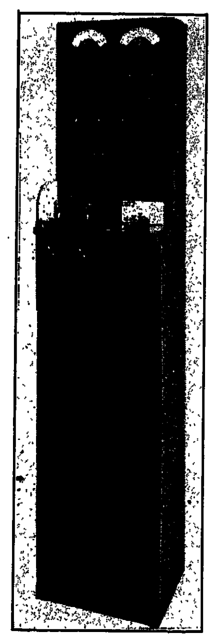


Fig. 17—Metallic Telegraph Repeater—Terminal Type

into the input of the telephone repeater. For four-wire telephone circuits, on which repeaters work with comparatively high amplification, it is necessary to bridge a shunt, resonant at about 135 cycles per second, at one end of the by-pass to prevent excessive feedback at 135 cycles per second and neighboring frequencies. It is grounded in the middle and two coils are provided, connected so that one will be effective for the side circuit and the other for the phantom circuit. For two-wire telephone circuits the resonant shunt is unnecessary.

Equipment Arrangements

The terminal-type repeater is assembled as a complete unit at the time of manufacture and therefore the installation work consists only in arranging the repeaters in rows and connecting the line conductors, loops and batteries to the terminal strips. A typical installation is shown in Fig. 16. A terminal and a through repeater are shown in Fig. 17 and Fig. 18, respectively.

The artificial-line equipment is mounted in the upper section of the repeater. Condenser switches, dial-type resistance switches, milammeters and miscellaneous keys are mounted on a hinged panel of insulating material. On the back of the panel, immediately behind the dial switches are the associated resistance units. The condensers which form part of the artificial line are stacked up in the space immediately behind the panel. The apparatus in the artificial line section is divided, so that the equipment which balances the cable pair is on the right side and that associated with the loop or ground-return section is on the left side.

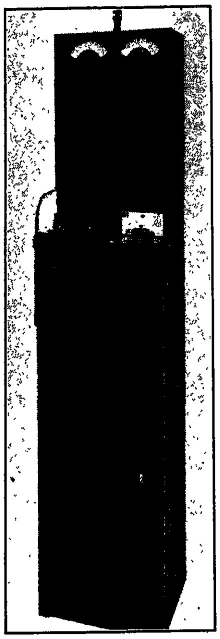


Fig. 18—Metallic Telegraph Repeater—Through Type

Below the hinged panel is the keyshelf, on which are mounted the loop and line sounders and the monitoring telegraph key. At the rear of the keyshelf and fastened perpendicularly to it is a panel on which are mounted the switching keys for controlling the battery connections and for arranging the repeater to work under various circuit conditions. Underneath the keyshelf is a section for the condensers in the transmitter branch, the spark-killers and the vibrating circuit.

In the lower section of the repeater is a small mounting plate carrying the relays and the resistance units associated with the spark-killers and vibrating circuit. The lower end of this mounting plate is hinged so that it may be swung forward, thereby giving access to connections of apparatus mounted on it. Below this is a terminal strip for the lines, batteries and the sending and receiving legs. Below the terminal strip and just above the floor are the retardation coils used in the transmitter branch.

A terminal repeater stands 62 in. (1.57 m.) high and occupies a space 14 in. (36 cm.) wide and 12 in. (30 cm.) deep and weighs about 220 lbs. (100 kg.). The keyshelf is about 40 in. (1 m.) above the floor. On the top of the repeater is mounted the operator's "calling-in" lamp.

The floor-mounted type of through repeater has the same equipment assembly for both the east and west sides and these are practically the same as the portion of the terminal repeater which operates on the cable section. The equipment in the right-hand section of the through panel is for repeating signals from the east line to the west line, and the left vice versa. This repeater weighs about 230 pounds (105 kg.) and occupies the same space as a terminal repeater.

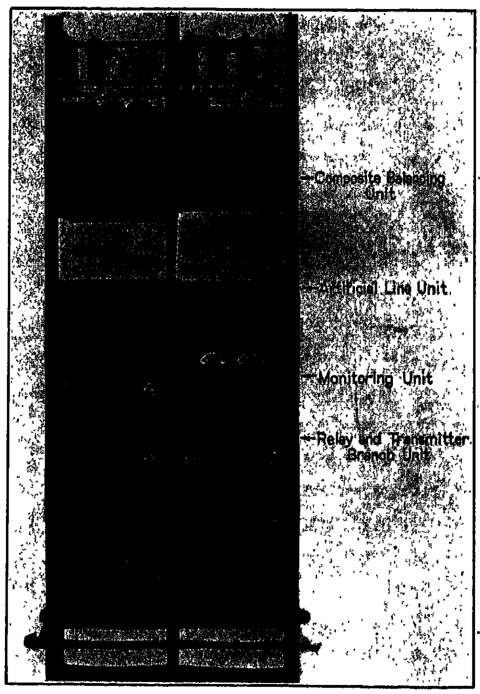


Fig. 19—Rack-Mounted Metallic Telegraph Repeaters—Through Type

The rack-mounted through repeater was developed after experience with the floor-type had shown how little monitoring attention was required. For that reason the repeater was simplified by the elimination of the line meters and monitoring apparatus. A unit termed a "monitoring unit" is provided for a group of about seven repeaters, and it can be connected into any one repeater by means of cords and plugs. A rack-type repeater consists of three units, the relay and transmitter-branch unit, the balancing-composite unit,

and the artificial-line unit. Each of these units consists of a steel panel with necessary apparatus, arranged for mounting on two upright standard I-beams, thus forming a "bay." Generally there are four repeaters, or three repeaters and a monitoring panel per bay. Fig. 19 shows an arrangement of repeaters on racks having a height of about 90 in. (2.3 m.). This type of repeater is supplied for single-commutation operation only, whereas both forms of "floor-mounted" repeaters are supplied for double-commutation operation. Considerable economy in first cost and maintenance is secured by the use of this rack-mounted equipment.

Operation and Maintenance

The metallic telegraph repeaters require comparatively little attention on the part of repeater attendants. Under normal operating conditions one man takes charge of about 24 terminal repeaters or 40 through repeaters. The duties of the repeater attendants consist mostly in maintaining satisfactory impedance balance of the artificial lines against the real lines. This balance is, of course, more exacting for full-duplex operation than for half-duplex. The capacity balance varies only a slight amount. Variations in resistance balance are caused by temperature changes, the average daily variation being about 6 per cent. The differential milammeter is used as an indicator in determining the resistance and capacity values required to obtain a balance.

The equipment maintenance work required for these repeaters is exceedingly small. For a typical installation of 200 repeaters, the adjustment of relays and general maintenance will necessitate not more than four or five man-hours per day.

The maintenance schedule for adjusting the relays is somewhat variable, depending upon the type of circuit in which they are operating. In general, a 209-*FA* relay in a terminal repeater will give uninterrupted service for two to three months and in a through repeater for four to six months. The 215-*A* relays are adjusted about every three weeks when used as "break" relays and every six months operating as pole-changing relays.

With proper maintenance the transmission of the metallic telegraph system is such as to furnish high-grade half-duplex manual service for distances up to 2000 miles (3200 km.) or more. For the longer distances, the signal propagation time is increased to an amount which makes the time required to "break" appreciable, but not objectionable. For half-duplex printer operation the metallic circuits are satisfactory for speeds up to about 19 dots per second, which corresponds to about 300 characters per minute for the start-stop type of printer.

For full-duplex service, the metallic system affords very good transmission with manual operation for distances up to 1000 miles or more. With careful maintenance of duplex balances, such a circuit is satisfactory for full-duplex printer operation at speeds up to about 16 dots per second, corresponding to about 260 characters per minute for start-stop printers and 385 for multiplex printers.

It is of interest to note that metallic circuits in cable are much more dependable and less subject to interruption than open-wire circuits. Such data as are available indicate that the annual lost time on a long metallic cable circuit is only about one-tenth as great as that on a ground-return polar-duplex circuit of the same length over open wire.

Commercial Use

At the present time there are in operation in the Bell System about 55,000 miles (89,000 km.) of metallic telegraph circuits of this type of which 30,000 miles (48,000 km.) are worked on a composited basis. Approximately 20 per cent of the total mileage is operated full-duplex. There are now installed in the plant about 430 through repeaters and 1050 terminal repeaters.

Discussion

R. N. Nicely: In connection with the extensive toll cable program of the Bell System, it has of course, been highly desirable that satisfactory methods be made available for utilizing the cable facilities for telegraph as well as for telephone service. This has been accomplished by the development of the metallic polar-duplex telegraph system and the voice-frequency carrier telegraph system. Each of these systems is adaptable to operation under widely varying cable circuit layout arrangements and each has certain characteristics which make it particularly suitable for use in connection with different specific requirements. In actual practise, each of the systems is a valuable supplement to the other.

The metallic polar-duplex system, in addition to being extensively used on long trunk-cable routes, is well adapted to telegraph-circuit distribution from large centers to outlying towns or cities reached by cable as well as to telegraph stations at the center itself or at outlying points reached by aerial wire circuits.

The metallic system is of value also during the construction of long toll cables which, as in the case of the New York-Chicago cable, may be extended over a period of several years. As the cable is extended it is of course, desirable that heavily-loaded aerial wire lines on or near the cable route be lightened by transferring at least a part of the circuits to completed sections of the cable. The metallic telegraph repeater being adapted for operation on metallic cable circuits in one direction and on aerial wire or grounded cable circuits in the other, is particularly well suited for use at points where the two types of circuits are joined. Furthermore, the metallic repeater being a practically self-contained unit, may be readily moved from point to point if desired, as the cable construction advances.

With the metallic telegraph system, intermediate repeaters are in general, required at more frequent intervals than is the case on aerial wire telegraph circuits but, owing to the greater stability of the cable circuits, this apparent disadvantage is offset by the very small amount of equipment adjustment required by the metallic apparatus during operation.

J. H. Bell: To an old-time telegraph man used to the noises in a large telegraph office, a visit to one of the small-gage cable telegraph repeater stations where practically all the repeaters are of the type described in this paper would be somewhat of a sur-

prise. The quietness of the room is broken only by the click of a sounder during periodic tests by the repeater attendants listening for a moment or two to passing signals. I believe at first it was thought the repeater attendants who had been accustomed for so long to the clatter of sounders would take unkindly to working in a nice quiet office, but in that regard the psychology was wrong. I am safe in saying, I think, that the repeater men much prefer the new conditions.

There is one apparatus unit in the sets to which I would like to refer, namely, the dial-type switch for enabling the value of capacity in the artificial line to be varied. To one accustomed to juggle with five plugs or levers to secure some one of the thirty-one possible values of capacity, the convenience of stepping round a dial, adding, say, 0.1 micro-farad at each step and getting values of between 0.1 and 3.9 microfarads, is immediately noticeable and appreciated. One can strike a duplex balance in a fraction of the time taken with the older apparatus.

In regard to the operation of the system, it should perhaps be pointed out that the speeds of operation given in the paper do not necessarily represent the maximum speeds obtainable. In designing the system the aim was to provide for high-grade service at fast hand speed; that is, about 15 dots per second. The vibrating circuits are consequently permanently set for operation at about that speed so that at higher speeds the vibrating feature would probably be detrimental rather than beneficial.

The most important unit of a telegraph system is the relay. One might say it is the heart of the system, so that it is difficult to discuss a new system without making some reference to its heart. The sensitive polarized relay described in the paper by Messrs. Fry and Gardner was designed primarily for the metallic system, although it has been found particularly applicable to the voice-frequency carrier system and the high-frequency carrier system which was described in a paper presented at an Institute meeting some years ago. With the improvements which have recently been incorporated in it, namely, the accurately balanced windings, permalloy magnetic material, the new contact material, the anti-chatter armature, I think we have in it now a high-grade polar relay which is likely to be our standard for several years to come.

In the discussion of a telegraph paper before the British Institute of Electrical Engineers recently, one of the speakers very cleverly compared the art of communication with the art of transportation. He said the telephone was like the passenger service, a very personal affair demanding quick service, whereas the telegraph was like the express service, where one hands over a message to be transmitted or transported and the operating company does the rest. Of course, this simile does not apply very aptly to the telegraph service of the Bell System. The Bell telegraph service furnishes to each customer or patron a single-track or double-track private express railroad for his exclusive use, and by the ingenious compositing scheme the same lines are made use of for a public passenger service, without any fear of collisions.

Perhaps I may be permitted to borrow the simile and to enlarge upon it by saying that in the metallic cable system and the voice-frequency system we have provided for these private express railroad services a much improved rolling stock.

NOTE: See also discussion on Voice-Frequency Carrier Telegraph Systems for Cables, by B. P. Hamilton, H. Nyquist, M. B. Long and W. A. Phelps, and Polarized Telegraph Relays, by J. R. Fry and L. A. Gardner. (Page 339.)

Voice-Frequency Carrier Telegraph System for Cables

BY B. P. HAMILTON,[1] Associate, A. I. E. E.
N. NYQUIST,[1] Member, A. I. E. E.
M. B. LONG,[2] Associate, A. I. E. E.
and W. A. PHELPS[2] Associate A. I. E. E.

Synopsis.—*Carrier telegraph systems using frequencies above the voice range have been in use for a number of years on open-wire lines. These systems, however, are not suitable for long toll cable operation because cable circuits greatly attenuate currents of high frequencies. The system described in this paper uses frequencies in the voice range and is specially adapted for operation on long four-wire cable circuits, ten or more telegraph circuits being obtainable from one four-wire circuit. The same carrier frequencies are used in both directions and are spaced 170 cycles apart. The carrier currents are supplied at each terminal station by means of a single multi-frequency generator.*

* * * * *

A TELEGRAPH system has recently been developed which utilizes the range of frequencies ordinarily confined to telephonic communication. It represents a special application of the carrier method of multiplexing telephone and telegraph circuits, which has already been described.[3]

The new system has been designed particularly for application to four-wire telephone circuits. Installations have been made at New York and Pittsburgh, by means of which ten telegraph circuits are derived from one four-wire telephone circuit extending between these cities. Additional installations are planned and under way in which it is expected that a greater number of telegraph circuits will be obtained from each four-wire telephone circuit.

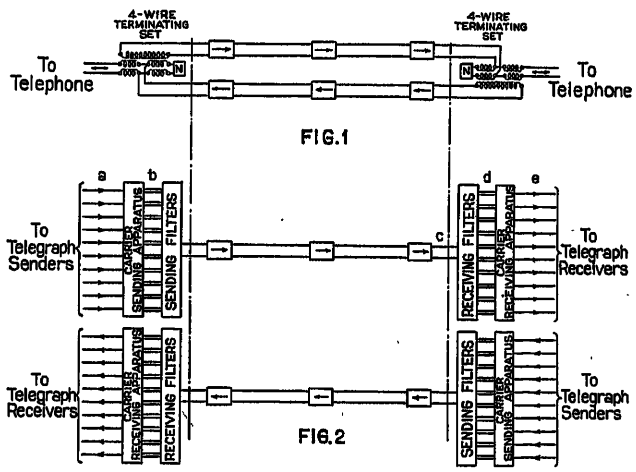
Experience in commercial service extending over a considerable period has fully demonstrated the effectiveness of this system.

General Features

In a general way, the voice-frequency system resembles the high-frequency carrier system for open-wire lines, which has been described in the paper referred to above. The most important differences are that the voice-frequency system uses (1) a four-wire cable circuit instead of a two-wire open-wire circuit, (2) the same frequencies for transmission in both directions, (3) frequencies of the voice range rather than the higher frequencies used in open-wire carrier telegraph systems, (4) a multi-frequency generator instead of vacuum tube oscillators to supply the carrier currents and (5) fixed band pass filters instead of adjustable tuned circuits for segregating the several telegraph circuits.

Fig. 2 shows in a simplified manner the essentials of the telegraph system under discussion. Reference to Fig. 1, which shows a four-wire telephone circuit,[4] will make clear how the line portion of the telegraph system is derived from such a telephone circuit. As indicated in Fig. 1, the four-wire cable circuit uses two pairs of wires, one pair for transmission in each direction. When a voice-frequency telegraph system is applied to a telephone circuit the four-wire terminating sets, which normally terminate the circuit when used for telephone purposes, are removed and voice-frequency carrier telegraph equipment is substituted.

Signal Traced Through System. A general layout of the system is shown in Fig. 3 and, in describing the operation, reference is made to this figure. The path of a signal from the sending operator to the receiving



Figs. 1 and 2

operator, on one of the ten two-way circuits will be considered. To produce a spacing signal the sender opens his key (shown at the left of the figure) which causes the sending relay to operate so as to short-circuit the source of alternating current. To produce a marking signal the key is closed, which causes the sending relay to operate and to remove the short circuit. This permits the alternating current from the generator to flow freely into the filter. This sending filter is so constructed as to permit relatively free passage of current of frequency near the particular carrier frequency for which it is designed. For other frequencies the filter practically shuts off the current.

1. American Telephone & Telegraph Co., New York, N. Y.
2. Bell Telephone Laboratories, Inc., New York, N. Y.
3. *Carrier-Current Telephony and Telegraphy*, E. H. Colpitts and O. B. Blackwell, Transactions, A.I.E.E., 1921, page 205.
4. *Telephone Transmission Over Long Cable Circuits*, A. B. Clark, Transactions, A.I.E.E., Vol. XLII, 1923, page 86.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., February 9-12, 1925.

After passing through the filter, the current mingles with currents from other channels and all are transmitted over the line as a resultant composite current. After flowing through the line in this mixed-up condition, the currents encounter the receiving filters which resemble the sending filters in that each transmits a relatively narrow range of frequencies in the neighborhood of the carrier frequency for which it is designed, and in that it acts substantially as an open circuit to other frequencies. By means of these receiving filters the currents are separated and each flows freely into its own channel. After passing through the receiving filter the current enters the detector whose function is to convert the alternating-current signals into direct-current signals which are capable of actuating the receiving relay. The receiving relay in turn transmits direct-current signals to the receiving operator's sounder or local relay.

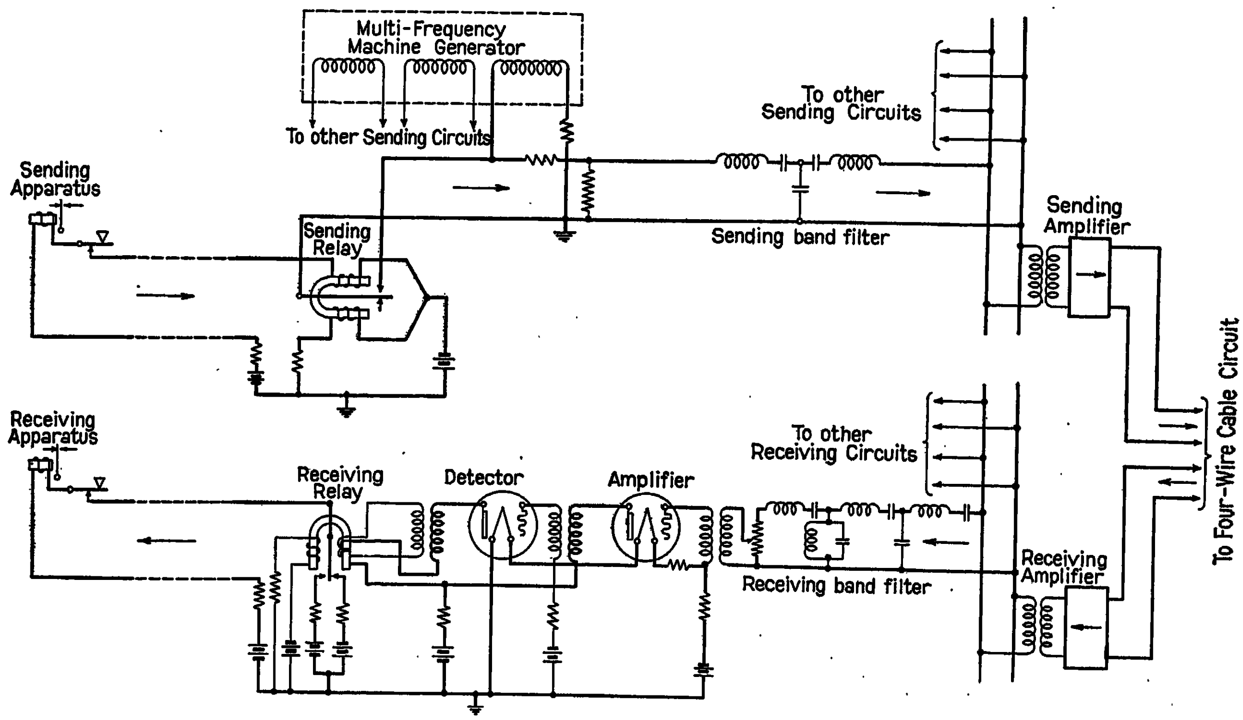


Fig. 3

This sequence of events is illustrated in the series of oscillograms of Fig. 4, which shows the different forms of a group of telegraph signals in the 425-cycle channel from the time, when as d-c. impulses, they flow through the sending relay windings, to the time when again, as d-c. impulses, they flow through the receiving relay and sounder circuit. It shows (a) their form in the sending relay and telegraph key circuit, (b) their translation into alternating current prior to passing into the sending filter, (c) their mingling with other similar impulses of different carrier frequencies after passing through the sending filter and on to the line as a single resulting wave flowing through the four-wire circuit, (d) their form after separation from the other channels by the receiving filter and (e) their final form in the receiving sounder circuit. The points where the oscillograms were taken are shown in Fig. 2 at *a*, *b*, *c*, *d*, and *e*, the cases being correspondingly denoted on the oscillograms.

Carrier Frequencies. The carrier frequencies are so chosen as to be odd multiples of a basic frequency of 85 cycles per second. The lowest frequency used is the fifth multiple of 85 cycles, that is, 425 cycles per second. Starting with this frequency, the carriers are spaced at 170-cycle intervals from their nearest neighbors, so that in the ten-channel system the uppermost frequency is 1955 cycles per second. Each channel has assigned to it a range of frequencies 85 cycles above and below its own frequency. For example, the chan-

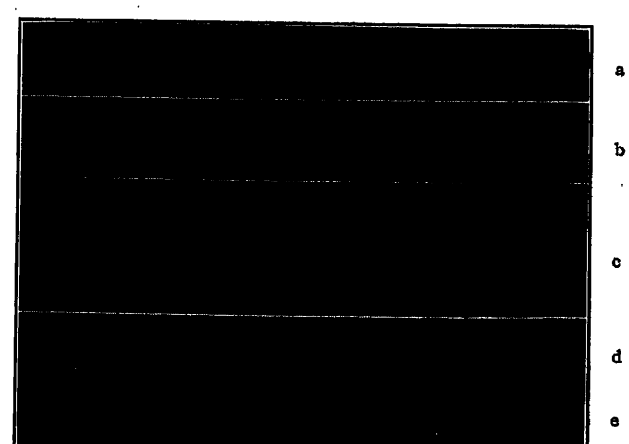


Fig. 4

nel using a carrier frequency of 1105 cycles has assigned to it the range between 1020 to 1190 cycles. Choosing the carrier frequencies in this manner and placing each carrier midway in the band of frequencies assigned to it, has the effect of giving maximum discrimination against interfering frequencies generated in the various vacuum tube repeaters. As is well known, when a number of frequencies are transmitted simultaneously through a vacuum tube, currents which cause interference are generated due to small departures from linearity on the part of the tube characteristic. Some of the most important of these currents have frequencies equal to the sum and difference of the frequencies of the transmitted currents taken in pairs. Since the carrier frequencies are all odd multiples of the common frequency, 85 cycles, it follows that the sum and difference of the frequencies are even multiples of 85 cycles and therefore are located midway between the carrier frequencies. This permits obtaining the maximum discrimination against these interfering frequencies by means of the filters, of which the characteristics are set forth below.

The number of carrier telegraph circuits which can be derived from a single four-wire cable circuit depends on the type of loading and, to a less extent, on the length of the circuit. It has been mentioned above that at the present time ten two-way carrier telegraph circuits are operated simultaneously over a four-wire circuit between New York and Pittsburgh, a distance of about 400 miles (644 km.). This is not, however, the maximum possible number of telegraph circuits which can be derived from the type of circuit used with this installation. Four-wire circuits which are loaded with coils of small inductance transmit a wider range of frequencies and are already in use for telephone purposes. If such circuits were used instead of the type employed with the present installation, at least fifteen two-way carrier telegraph circuits could be obtained.

Description of Apparatus

Carrier Current Generator. Vacuum-tube oscillators are the source of the carrier current in carrier systems previously developed. In this system, however, all the carrier currents for the ten channels are obtained from a compact multi-frequency generator driven by a motor built into the same housing with the generator.

The generator is an inductor-alternator designed to generate currents of ten different frequencies in ten different magnetic circuits electrically independent of each other. The machine has two field coils common to all the stators. The exciting current for these two windings is supplied by a storage battery. On the pole arc of each stator opposite each of the narrow disk-like rotors, mounted in a row on the shaft, are cut a number of slots, the number per unit length depending on the frequency to be generated. The stator windings for each circuit are placed in these slots. The rotor belonging with each stator has a corresponding group of slots cut in it but no windings are placed in these rotor slots. The result is equivalent to ten separate alternators except that the field excitation is common to all. The flux in any stator tooth is greatest when a rotor tooth is opposite it and least when a rotor slot is opposite it. This variation in flux in the stator teeth as the rotor moves induces the voltage in the windings on these teeth. All the windings of a given stator are connected in series, so the total voltage generated in each stator is the sum of the separate voltages in the several windings.

A comparatively small generator is able to supply carrier currents to several ten-channel systems because, by using terminal repeaters or amplifiers (Fig. 3) the amount of energy required to operate each telegraph channel is very small, and no channel produces any noticeable interference in another drawing current from the same stator winding. The terminal voltage

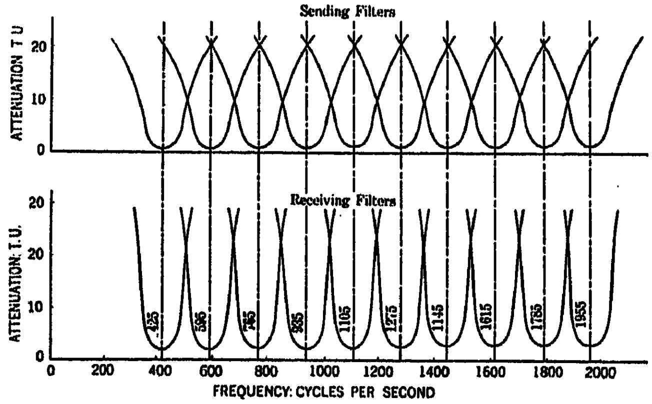


Fig. 5—Characteristics of Filters

of each stator is 0.7 volt and a current of 40 mils may be drawn from it without producing change a in terminal voltage sufficient to cause interference in any telegraph circuit drawing current from the same set of windings.

The driving motor is a small shunt-wound machine which receives its energy from a 24-volt storage battery. The speed of the motor is maintained accurately at 1700 rev. per min. by means of a centrifugal type of governor which controls the amount of current flowing through the shunt field winding. As the stability of the carrier frequencies depends on the constancy of the motor speed, it is necessary that the governor control the speed within narrow limits.

As a means of checking the speed of the generator an electrical frequency indicator is provided. This device is connected to and indicates the frequency of one of the generator circuits. As the frequency of an alternator is directly proportional to the speed it gives an indication of the correctness of the speed and also of all frequencies produced by the generator.

Filters. Fig. 5 shows the transmission characteristics of the transmitting and receiving filters. These filters are designed to transmit as wide a range in the neighborhood of the carrier frequencies as is necessary

to secure the desired quality of transmission and at the same time exclude interfering currents, whether they be caused by foreign interference, direct transmission from other channels, or distortion in the repeater tubes. The principal interfering currents due to the latter are located 85 cycles on either side of the carrier frequencies. The receiving filters have been designed to reduce these interfering currents to about 10 per cent of their original value.

In addition to screening out any undesired frequencies produced in the generator windings, the sending filters have the following more important functions. Each sending filter presents a high and comparatively non-dissipative impedance to the currents issuing from the other sending filters and also "rounds off" the impulses of the modulated carrier wave passing through it. The modulation of the carrier current by the sender's key produces what is called a "square" wave, that is, a wave containing not only the carrier plus and minus the frequency at which the key is operated but also the carrier plus and minus a large number of multiples of the frequency. Some of the component frequencies of this transmitted wave not only are found unnecessary in reproducing the transmitted signal at the receiving end but also lie within the range of adjacent channels and produce interference in them unless screened out by the sending filter in the channel in question.

The effect of the sending and receiving filters in "rounding off" the modulated carrier wave, that is, in screening out the objectionable components of the signal wave, is shown by the oscillograms of Fig. 4. The combined effect of the two filters on the shape of the modulated carrier may be seen by comparing oscillograms (*b*) and (*d*) of this figure, which show respectively the appearance of the modulated wave before it enters the sending filter and after passing through the sending filter, over the line and through the receiving filter. Another interesting point in connection with these oscillograms is the time lag due to the circuit which is shown by the relative differences in position of the two waves referred to above. Owing to the limitations imposed by the ordinary oscillograph all of the traces shown in Fig. 4 were not taken simultaneously. This accounts for minor inconsistencies which are revealed by a careful inspection.

Detector. The detector receives alternating-current signals from the line after the signals belonging to that particular channel have been selected by the receiving filter. It consists of two vacuum tubes in tandem, the first tube (Fig. 3) amplifying the received signals, and the second converting them into direct-current pulses which operate the receiving relay. The receiving relay then repeats these telegraph signals into the receiving direct-current circuit which contains the receiving sounder.

To improve the operation of the receiving relay a device called an accelerating circuit or "kick" circuit, such as is used in open-wire carrier-telegraph systems, is interposed between the detector tube and the receiving relay. This circuit is obtained by introducing a transformer whose high-voltage side is connected in series with the detector tube and a winding of the receiving relay and whose low-voltage side is connected to another winding of the relay. When the current in the high-voltage side is constant, there is no current in the low-voltage side, but if the former current suddenly changes, as at the beginning or end of a marking signal, there is a sudden rush of current in the low-voltage circuit which has the effect of causing the relay to operate promptly and positively.

Relays. As shown in Fig. 3, the sending and receiving relays are of the polar type. These relays are identical and interchangeable with those used in the metallic and open-wire carrier-telegraph systems. They are described in the paper on telegraph relays which has been prepared for presentation at this meeting.

Power and Testing Equipment. In the development of the voice-frequency carrier telegraph system, the central thought was the desirability of designing a system which would fit into the existing cable telephone and telegraph plant. It has been possible to use the standard voltages obtainable from the storage batteries in such plants without exception.

In line with the policy of simplifying this new system as far as possible, the amount of auxiliary testing apparatus was reduced to a minimum. This policy has been assisted by the stability of the cable circuits and the use of a multi-frequency generator as a source of carrier currents. Only two pieces of special testing apparatus are used at each station, namely, the frequency indicator, and a thermo couple voltmeter for checking the alternating voltage in each generator circuit.

Line and Repeaters

As has been pointed out elsewhere in this paper, the voice-frequency carrier telegraph system was designed primarily for use on small-gage, four-wire cable circuits. These circuits are loaded and provided with vacuum tube repeaters at 50 to 100-mile (80.5 to 161 km.) intervals, depending on the weight of loading used. The repeaters used in long toll circuits are similar to those described at an earlier date.[5] The characteristics of the long cable circuits used in voice-frequency carrier telegraph transmission have also been described in a more recent paper.[6]

Equipment Features and Arrangements for Giving Service

The apparatus which is associated with each of the ten two-way circuits in this system has been segregated according to function and each group of apparatus per-

5. *Telephone Repeaters*, by Bancroft Gherardi and Frank B. Jewett, TRANSACTIONS, A.I.E.E., 1919, page 1287.

6. Clark, Loc. cit.

form'ng the same function, such as the detector, has been mounted on a separate steel panel. Each one of these panels forms a unit in itself. This type of construction allows the substitution of new apparatus performing some particular function in the system without an expensive redesign. Thus, it is possible to

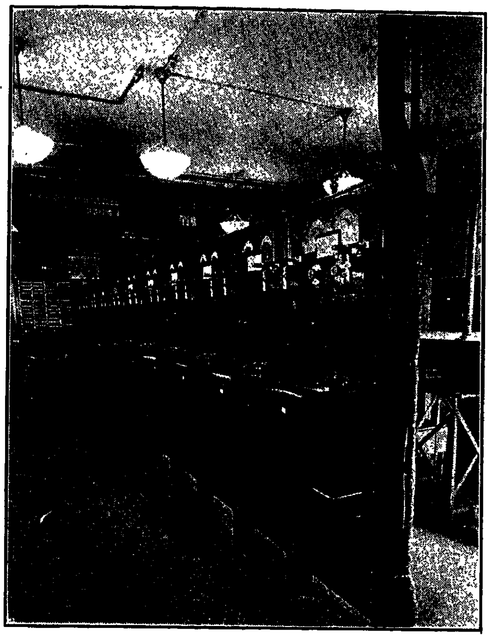


FIG. 6

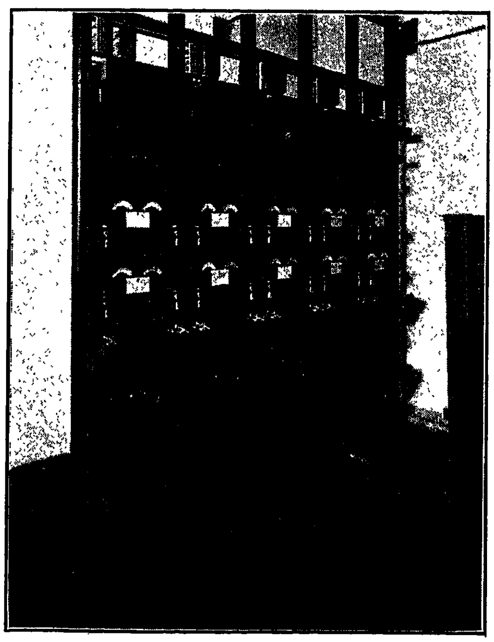


FIG. 7

install future improvements in the several circuits of the system in an economical manner.

These unit panels are mounted on pairs of vertical I-beams and the combination is termed a "bay." The bays are of different heights, depending on the requirements of the office in which they are installed. Fig. 6 shows a line-up of so-called low-type bays (about five feet high) in the Pittsburgh office. Each bay in this line-up contains sufficient equipment to provide for the transmission and reception of signals at the Pittsburgh terminal of one of the ten two-way telegraph circuits. Fig. 7 shows a line-up of similar equipment in the New York office, this layout differing from the one in Pittsburgh in that it uses high instead of low-bays. Each bay in this line-up contains sufficient terminal equipment for two of the ten two-way telegraph circuits.

In addition to the bays described above there are three bays, carrying auxiliary equipment. This auxiliary equipment consists primarily of control and testing apparatus for batteries and carrier supply. Two of these bays, namely, the generator and carrier supply bays are shown in Fig. 8. This figure shows two of the multi-frequency generators (one a spare machine) described above, and the carrier testing equipment. The control

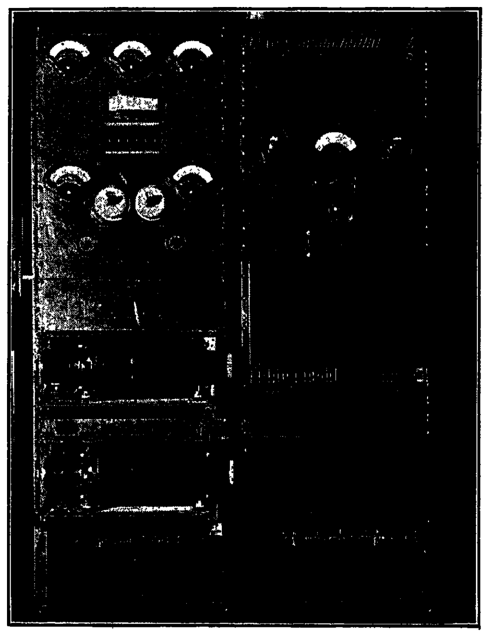


FIG. 8

equipment associated with these machines is mounted on the panels above the generator and the frequency indicator is mounted on the panel to the right of this control apparatus.

Switching and Monitoring Arrangements

The monitoring arrangements, which enable the attendant to check the quality of signals passing over a circuit or to trace trouble quickly and easily, are similar to those now in use in the open-wire carrier and metallic telegraph systems. These arrangements are described in the paper on the metallic telegraph system which has been prepared for presentation at this meeting and, therefore, will not be given in detail here. In a general way it may be said that switches and meters are provided to connect the telegraph batteries to local

apparatus, to provide either one-way or two-way service and to facilitate repeating to other telegraph systems.

Capabilities of System

Field tests over the New York-Pittsburgh system have shown that each telegraph circuit derived therefrom is of high grade, allowing signal speeds of 35 to 40 cycles per second. That is, with machine sending, it is possible to transmit 140 to 160 words per minute (five letters and a space per word) each way over each telegraph circuit. Considerably higher speeds may of course be obtained by widening the frequency range assigned to each telegraph circuit.

The New York-Pittsburgh system may be used in connection with a multiplex printing telegraph system and three printer messages may then be sent simultaneously in either direction on each carrier circuit. Assuming 50 words per minute as the working speed for each of the three printers a total of 1500 words per minute could be transmitted simultaneously in either direction over the ten circuits.

A simple numerical example will indicate what is technically possible by the application of this type of telegraph system to toll cables. A toll cable 2 5/8 inches (6.7 cm.) in diameter contains about 300 pairs of No. 19 B. & S. gage (0.91 mm.) conductors. Utilizing the phantom circuits this gives a total of 225 four-wire circuits. Counting 30 messages in each direction per four-wire circuit it is evident that it is technically possible to transmit 6750 messages in each direction simultaneously.

The "break" feature of this system is satisfactory. It functions in a manner similar to that used with the metallic telegraph system. It takes about 0.1 second to transmit a "break" signal over a 1000-mile (1610 km.) circuit.

Fields of Application

It will be evident that while the foregoing description assumes that this system is applied to four-wire circuits, it could be readily applied to two-wire circuits by transmitting half of the carrier frequencies in one direction and the other half in the opposite direction. Furthermore, if the impedance characteristic of the line could be reproduced with sufficient accuracy in networks to balance the line at the repeaters, the same frequencies could be transmitted in both directions and as many of them could be so transmitted as the natural "cut-off" of the line would permit.

While the voice-frequency carrier telegraph system has been designed primarily for use on an ordinary telephone circuit, the system may be applied to carrier telephone or radio telephone channels without involving radical changes in either the telegraph system or the telephone circuit to which it is applied.

Discussion

For discussion of this paper see page 339.

Polarized Telegraph Relays

BY J. R. FRY[1] and L. A. GARDNER[2]
Non-member Associate, A. I. E. E.

Synopsis.—This paper is to discuss two forms of polarized telegraph relays which have been developed by the Bell System and applied originally to the metallic telegraph system. One of these relays was designed primarily for operation under severe and exacting circuit conditions and the other for application generally. Both relays are of the same general construction except that the former is more sensitive than the latter and is furnished with an auxiliary accelerating winding. Furthermore, this relay has incorporated in it certain refinements in construction and materials, which cause it to be extremely sensitive and to give reliable service for a comparatively long period without attention. The construction is of special interest in that the polarizing force of the permanent magnet is neutralized by the mechanical restraining force of the armature. A magnetic material having characteristics well suited to relay design and a new contact material are employed, which greatly improve the operating characteristics of the relay.

INTRODUCTION

THE development of new types of sensitive polarized relays, for use in the Bell System, was undertaken as a result of a demand which arose in connection with the development of the metallic polar-duplex telegraph system. A paper on this system is being presented at this session.[3] Later these relays were applied to the carrier telegraph systems,[4] one of which is being described in a paper at this meeting.

A number of polarized relays, manufactured in this country and abroad, were experimented with and found inadequate, in certain respects, for application to these systems without extensive modifications. For satisfactory performance, it was found that the relay should meet the following general requirements: 1. Have a high degree of sensitivity, 2. Retain adjustment for a long period, 3. Faithfully and accurately repeat signals with a small amount of maintenance.

From the standpoint of sensitivity, the relay is required to operate on reversals of line current of a minimum of one milliampere and at the same time obtain proper impedance characteristics without exceeding practical limits in the dimensions of the windings. Over small-gage cable conductors having large mutual capacity and high resistance, the wave-shape of the received current is badly distorted. This wave-shape, in addition to the magnitude of the current available, makes more drastic the requirements for sensitivity. It is interesting to note that, under average conditions, the ratio of power controlled by the contacts in the local circuit to that required by the line windings, is about five thousand to one.

In the case of main-line telegraph relays, it is essential that no considerable changes in length of signals take place as such effects are generally cumulative over long circuits, where several relays are employed in repeating signals. As contrasted with relays used for circuit-switching purposes, telegraph relays are called upon to function continuously under severe conditions, and to perform successively hundreds of thousands of times in a few hours.

In order to furnish a good quality of telegraph service, it is essential that very few relay failures occur during service periods. If this is not accomplished, considerable valuable circuit-time is lost and the relay maintenance expense becomes an excessive factor in the cost of giving service.

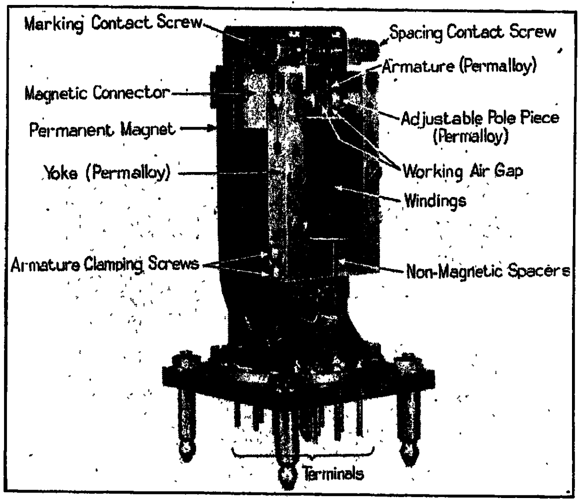


FIG. 1—No. 209-FA POLARIZED RELAY (COVER REMOVED)

DESIGN FEATURES

The relay which has been designed to meet the foregoing requirements is shown in Fig. 1 and is known as the 209-*FA* relay.

A polarized relay is fundamentally different in its operation from the ordinary neutral relay, in that it is selective to the polarity of the operating current and is more sensitive in its operation. A type of magnetic circuit was chosen which, it was felt, would best lend itself to obtaining a relay which would meet, as completely as possible, all of the requirements above outlined.

1. Bell Telephone Laboratories, Incorporated, New York, N.Y.
2. American Telephone and Telegraph Company, New York, N. Y.
3. Metallic Polar-Duplex Telegraph System for Long Small-Gage cables. J. H. Bell, R. B. Shanck and D. E. Branson.
4. "Carrier Current Telephony and Telegraphy" by E. H. Colpitts and O. B. Blackwell, TRANSACTIONS of the A. I. E. E., 1921, page 205.

Presented at the Midwinter Convention of the A. I. E. E., New York, N. Y., Feb. 9-12, 1925.

Fig. 2 shows the magnetic circuit and the arrangement of the relay elements. It is seen to consist of a Wheatstone-bridge type of magnetic circuit in which the four equivalent air-gaps, two upper and two lower, may be considered as the four arms of the bridge with the permanent magnet placed across two opposite corners of the bridge and the armature and windings in the position customarily occupied by the galvanometer. The operating windings are placed over the armature in the form of a single stationary spool, sufficient clearance being provided to allow the armature to move within the spool. The paths of the polarizing and operating fluxes are shown by the solid and dash lines, respectively, for the armature in its midway or neutral position. When the armature is in the midway position there is no polarizing flux through it, since it connects two points of the circuit of equal magnetic potential; as it moves toward the left pole-piece, the

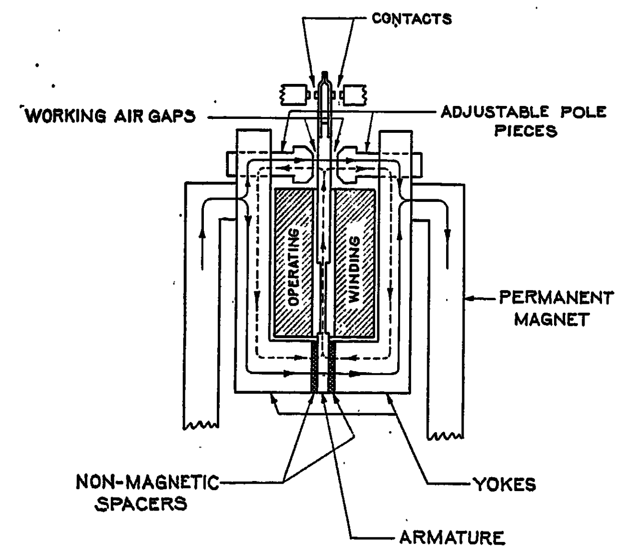


Fig. 2—Diagram Showing Magnetic Circuit and Arrangement of Relay Elements

polarizing flux flows through it in a direction assumed to be positive and when it moves toward the right pole-piece, the polarizing flux flows in the opposite or negative direction. The operating flux, in addition to the two return paths shown through the yokes, has a third path through the permanent magnet, but, substantially, none of this flux returns by this path due to the high reluctance of the permanent magnet as compared to that of the yokes. It is seen that these fluxes aid and oppose each other in the four gaps in such a manner as to produce a torque on the armature about a point midway between the upper and lower sets of air-gaps. With the direction of fluxes shown, the armature would tend to move in a clockwise direction. The forces established in all four of the gaps can be utilized in moving the armature only when the armature is movably supported at a point midway between the upper and lower gaps. In the design of this relay, the armature is not supported at this point, but is firmly clamped and supported at the lower gaps between the yokes, non-magnetic spacers forming proper reluctances to prevent short-circuiting the permanent magnet. This method of suspension, of course, prevents the forces established in the lower gaps from doing useful work upon the armature, but the overall advantages gained by this arrangement more than compensate for the apparent loss in sensitivity. This method of armature support permits of a more simple design of the relay; it eliminates the use of pivots; it permits the use of a single-spool construction instead of a two-spool arrangement; it allows a more practicable adjustment of the relay and more stable operation, and it affords a simple arrangement whereby the stiffness of the armature can be utilized to neutralize the polarizing force acting on the armature, thereby increasing the sensitivity of the relay as will be explained later in greater detail.

The pull acting between the pole-piece and the armature in each gap is expressed, according to Maxwell's law, by the equation:

$$P = \frac{(\phi_p + \phi_i)^2}{8\,\pi\,S} = \frac{1}{8\,\pi\,S}\,(\phi_p^2 + 2\,\phi_p\,\phi_i + \phi_i^2)$$

where P is the force in dynes, ϕ_p is the polarizing flux in the air-gap set up by the permanent magnet, ϕ_i is the operating flux generated in the air-gap by the winding (both expressed in maxwells), and S is the effective area of the air-gap in sq. cm.

This shows that the pull acting on the armature in each air-gap is made up of three components: The first term is the pull due to the permanent magnet and is the force acting on the armature with no current on the relay and is commonly spoken of as the polarizing force. The second is the important component, as it is the force which operates the relay; it is this component which makes the relay selective to the polarity of the operating current because its direction is dependent upon the sign of ϕ_i. Since its magnitude is proportional to the product of the polarizing flux and the flux generated by the operating current it shows why a high polarizing intensity is effective in obtaining a highly sensitive relay. The third term is a pull on the armature which would be the only one to appear if the relay were not polarized and is of twice the frequency of the second term.

As explained above there are two magnetic air-gaps in which forces are effective to produce motion of the armature, and it is the resultant of these component forces in the two air-gaps that controls the relay. In Fig. 3 is shown how the resultant of the polarizing components of force in the two air-gaps varies as the armature is displaced in the air-gap. In the midway or neutral position of the armature the resultant polarizing force on the armature is zero, but as the armature is displaced in either direction the force is one of attraction of increasing intensity between the armature and pole-piece toward which the armature is displaced as

shown by the solid line. The armature, being supported in cantilever beam fashion, will be resistant to displacement from its midway position in the air-gaps due to its natural stiffness. This force of restitution of the armature is opposite in direction or opposing to the polarizing force as the armature is displaced from the midway position and is proportional to the displacement. If the armature is designed so that its force of restitution is approximately equal to the polarizing force acting on it for all positions of the armature

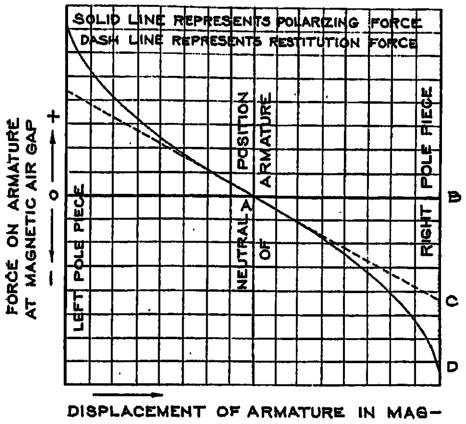


FIG 3—DIAGRAM SHOWING HOW POLARIZING AND RESTITUTION FORCES VARY WITH POSITION OF ARMATURE.

the sensitivity of the relay will be greatly increased. Thus, in Fig. 3, if the polarizing force on the armature were not in some manner neutralized, the work required to move the armature across the air-gap would be represented by the area *ABD*. By balancing the force of restitution of the armature against the polarizing force on the armature in the manner described, the work necessary to move the armature across the air-gap is reduced to the area represented by *ACD*. In actual practise, the motion of the armature is limited by the contacts rather than by the pole-pieces, and the displacement of the armature between the contacts is made small in comparison with the length of the magnetic air-gaps. Hence, the working range of the armature displacement in the magnetic gaps is limited to a short distance on either side of the neutral position shown in Fig. 3, where a very close balance between the polarizing force and restitution force of the armature, can be realized. The armature is practically in a floating condition and can be controlled by the application of a small force. The advantage of neutralizing the polarizing force on the armature by an external force and thus increasing the sensitivity of the relay was pointed out by Mr. D. D. Miller of the Bell Telephone Laboratories, Incorporated.

Fig. 4 shows how the resultant of the operating forces, or the component represented by the second term of the above discussed equation varies in the two gaps as the armature is displaced in the air-gap. The condition of constant strength and direction of current through the relay winding is assumed. It is minimum when the armature is in the midway position and increases as the armature moves toward either pole-piece. Its direction may be either to aid or oppose the direction of the polarizing component shown in Fig. 3, since it is dependent upon the direction of the operating current. It is this component of force which controls the operation of the relay under influence of the operating current and it can be of small magnitude since it need be only of sufficient value to upset the equilibrium of forces shown in Fig. 3. The third component of force, acting on the armature represented by ϕ_i^2 is of small magnitude compared to the first two terms, but its effect is to decrease the sensitivity of the relay and increase the distortion and residual characteristics, since its direction opposes the operating component when the armature moves toward the neutral position and aids it when the armature moves away from the neutral position.

Thus it is seen that when the relay operates under the influence of reversals of current through the relay winding, both the operating and polarizing fluxes reverse in direction through the armature, which causes the relay to be highly sensitive to the magnetic properties of the armature. The parts entering into the magnetic circuit of this relay, except—the permanent magnet—are made of an improved magnetic material known as permalloy[5] recently developed in the laboratories of the Bell System. This material has a very low coercive force compared to customary magnetic materials used in relays, resulting in almost complete collapse of the operating flux when the magnetizing force is removed. As a result, distortion of the re-

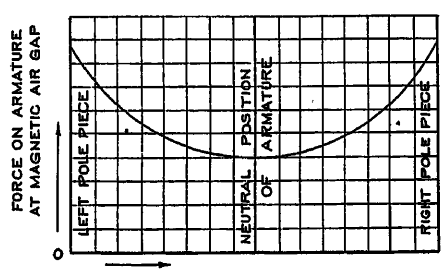


FIG. 4—DIAGRAM SHOWING HOW OPERATING FORCE VARIES WITH POSITION OF ARMATURE

peated signal is less, since the relay is less affected by the previous state of magnetization to which it may have been subjected by the preceding signal. On account of the higher permeability and the smaller residual effects of the material, the sensitivity of the relay is increased. It is evident from Fig. 3 that if the relay retains an appreciable amount of residual mag-

5. "Permalloy, a New Magnetic Material of Very High Permeability" H. D. Arnold and G. W. Elmen. The Bell System Technical *Journal* for July 1923. Page 101. "Electrical Communication", April 1924.

netism after the subsidence of the signal current, it will cause the armature to become biased by upsetting the balance between the polarizing and restitution force and decrease the sensitivity of the relay on the succeeding signal. Thus the use of this material results in marked improvements in the operating characteristics of the relay.

The relay is provided with six separate windings; four are employed as line windings, and two as auxiliary windings connected in a local circuit. The requirements of the metallic polar-duplex telegraph system are such that the four-line windings should be mutually balanced to a high degree both with respect to their impedance characteristics and their operating effects upon the relay. In order to meet these requirements it is necessary that each of the four windings have the same number of turns and resistances and be symmetrically located with respect to the armature and working air-gaps of the relay. This is obtained in practise by spirally twisting four well-insulated copper wires together and then winding as a single conductor on the relay spool in the usual manner. On top of and concentric with the line windings are two conductors wound in parallel. In the metallic telegraph system, these two windings are used in a circuit to form what is known as the *vibrating* or *accelerating* circuit, and this circuit is controlled from the contacts of the same relay. This feature serves to reduce distortion of signals by causing the relay to operate under the influence of the current in a local circuit before the operating current from the line at the time of reversal becomes sufficient to move the armature. It also reduces the armature travel-time and chatter and tends to make the relay more sensitive in its operation.

The relay contacts are required to make and break fairly large currents in circuits having large values of inductance and capacity. On relays used in terminal

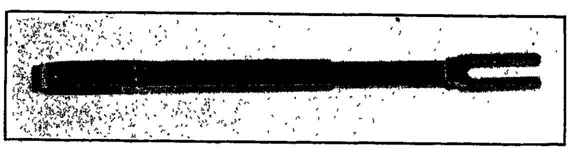


FIG. 5—ARMATURE OF NO. 209-FA RELAY

type repeaters, the voltage between one stationary contact and the corresponding armature contact is 240 volts. Due to the feeble currents which operate the relay, the contact-gap must necessarily be small, which, in practise, is adjusted to about 0.004 in. (0.10 mm.). The work of obtaining an adjustment of this gap is aided by marks on the capstan head of the contact screws. It is highly desirable to have a relay the contacts of which do not rebound or chatter as the armature breaks and makes contact. By eliminating contact-chatter the relay will be capable of transmitting signals at greater speeds and with less distortion, and the life of the contacts will be increased. An extended study of the dynamic action of the armature at the contacts was made with a view of diminishing or eliminating contact rebound. This is an old problem of telegraph relay design and many efforts have been made during the past to solve this problem satisfactorily, but most solutions proposed resulted in devices or designs which were complicated and not adapted for commercial use.

A practical design of armature was developed, which is easy to manufacture and is effective in eliminating contact rebound. This design is shown schematically in Fig. 2 and by a photograph in Fig. 5. The magnetic part of the armature extends through the relay coil and is

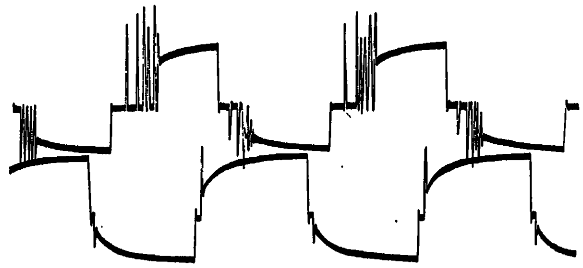


FIG. 6—OSCILLOGRAM SHOWING CURRENT FROM ARMATURE CONTACTS

Upper curve—Armature of ordinary design
Lower curve—Armature of standard design

just long enough to reach through the working air-gaps. The armature is extended to the stationary contact screws of the relay by riveting on to it two nickel-silver springs carrying the armature contacts. These two springs are bent at the free end and tensioned so that they touch and rest upon one another with a fixed pressure between them. By this construction the mass of the moving end of the armature is kept as small as possible so that at the moment of impact the tendency to rebound is thereby reduced. When the armature contact strikes the fixed contact one armature contact spring is displaced with respect to the other spring causing the ends of the springs to slide upon each other. This rubbing action tends to damp out the rebound as the energy of impact is absorbed by friction between the two springs. Other advantages gained by keeping the effective mass of the armature small, are reduction in the travel time of the armature and quicker time of response. In order to prevent the armature from adhering to either pole-piece, small nickel-silver disks are welded to each side of the armature directly opposite to the pole-pieces. Fig. 6 shows oscillograms of the current from the armature contacts with the relay in a terminal type metallic telegraph repeater, operating full duplex. The top wave shows the current from the armature contacts with the relay equipped with an armature of ordinary design, that is, having the contacts mounted directly on the magnetic material which extends the entire length of the armature. The lower wave shows the same relay operating under identical conditions as

above except that it is equipped with the standard design armature shown in Fig. 5. The improvement in the repeated signal of the relay is apparent both with respect to contact chatter and travel-time of the relay.

On account of the high voltage, the energy content of the circuits which the relay controls, the small contact-gaps and the feeble forces available to operate the relay, it was found that contacts made of material usually found on telegraph apparatus would not withstand the large number of operations daily required without excessive maintenance. This problem was studied with reference to the contact requirements of relays operating under the conditions imposed by the various repeaters, and an alloy was developed which is a decided improvement. The use of this improved contact material extends the service period of the relay, without contact failure, to approximately thirty times over that with alloys previously used. Fig. 7 shows photo-micrographs of armature contacts of a relay which had been in service for 8½ months without being removed from service. This amount of service is equivalent to each contact making and breaking the circuit approximately 45,000,000 times.

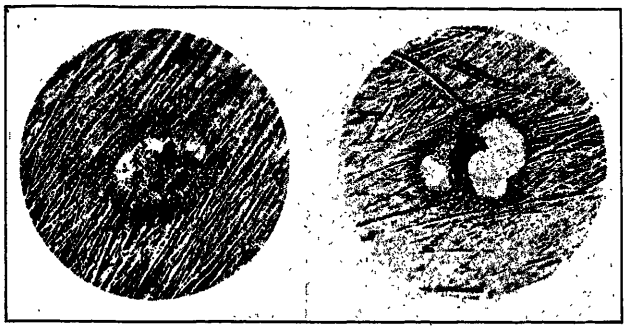


FIG. 7—PHOTO-MICROGRAPHS OF NO. 209-FA RELAY ARMATURE CONTACTS
Right—Marking contact
Left—Spacing contact

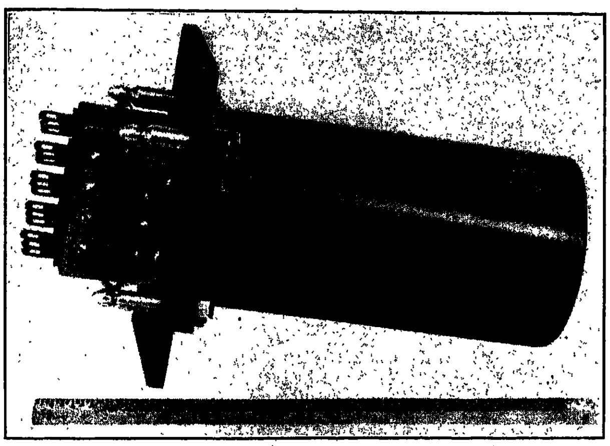


FIG. 8—NO. 209-FA RELAY WITH MOUNTING PLATE AND ASSOCIATED CONNECTING BLOCK

One of the design features of this relay is the arrangement which permits of the ready removal of the relay from the circuit. This feature is illustrated in Fig. 8, which shows the relay with its cover attached to a mounting plate and connecting block. The square phenol-fibre base of the relay is provided with four guide pins which definitely locate the relay with respect to the mounting plate and the connecting block, and is also furnished with 15 terminals on which electrical connections to the windings and contacts of the relay are terminated as shown in detail in Fig. 1. The connecting block is the medium by which electrical connection is established between the circuit and the relay. It is fastened to the rear of the mounting plate, has the same number of terminals as the relay, and the circuit is permanently wired to these terminals. When the relay is inserted into position on the mounting plate, electrical contact between the relay and connecting block terminals is automatically established. Resilient members fastened on the rear of the mounting plate, engage into recesses in the four guide-pins and hold the relay rigidly in position on the mounting plate. In case of relay trouble, the relay may be quickly removed and a spare relay inserted, thus keeping the time of circuit interruptions to a minimum.

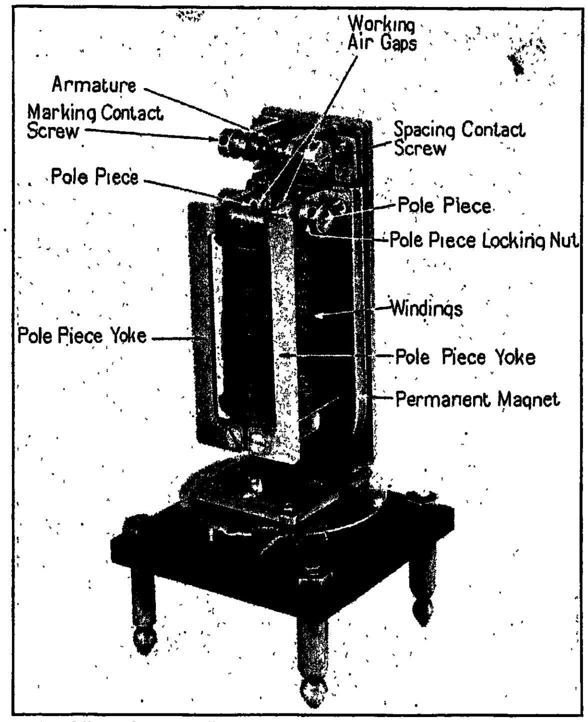


FIG. 9—NO. 215-A POLARIZED RELAY (COVER REMOVED)

There are a number of places in these systems where polarized relays are required, but which do not have to operate under as exacting conditions as the relay just described. For this purpose, a cheaper relay has been developed known as the 215-*A* relay, and this is shown in Fig. 9. It employs the same type of magnetic circuit as the 209-*F A* relay, and the same design features of mounting and chatterless armature springs are also incorporated. It does not have as refined adjusting features, nor will it operate on as small currents. It is provided with two balanced line windings.

Application and Maintenance

At the present time there are in operation about 2500 of the 209-type relays. Of this number, about 2000 are operating in the metallic telegraph system and the remainder in the carrier telegraph systems. These carrier telegraph systems make use of about 1000 relays of the 215-type and in the metallic telegraph system, about 2400 relays of this type are used.

As a result of field experience, it has been found that the maintenance schedules for these relays can be arranged to cover long periods with practically no interruptions occurring in service. In terminal-type repeaters and through-type metallic repeaters, this period is for satisfactory relay operation, approximately three months and six months, respectively.

Means are specially provided whereby these relays may be checked or adjusted quickly and accurately external to the telegraph repeaters; that is, the relays

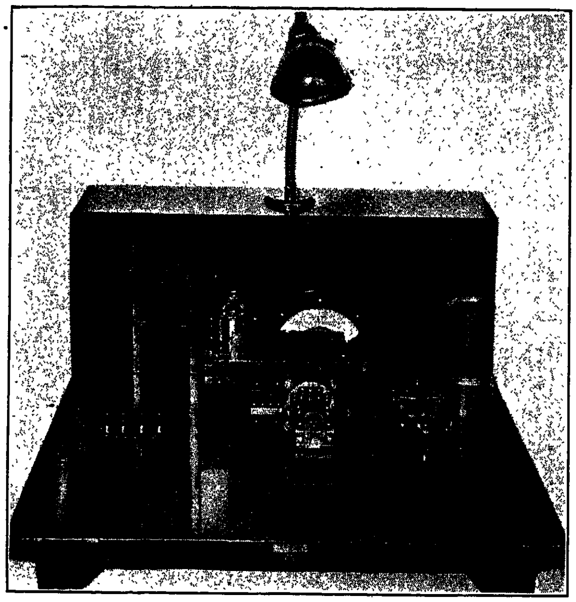


Fig. 10—Relay Test Table

are adjusted to the best operating condition without consideration of the telegraph circuit to which they are to be applied. This arrangement, known as a relay-test table, is shown in Fig. 10. These tests consist of obtaining an inspection and adjustment for sensitivity, differentiality, and freedom from bias. A local vibrating circuit is also provided for testing the 209-type relay so that it will function properly under the influence of the vibrating circuit in the telegraph repeaters.

The essential steps in adjusting a relay consist in withdrawing the contact screws sufficiently to permit inspection and cleaning. The pole-pieces are then withdrawn so that the effect of the permanent magnet upon the armature is made small, thus permitting the armature to stand in its neutral position. Any foreign material may be readily removed from the working air-gaps. Each contact-screw is then adjusted independently so as to have a separation of 0.002 in. (0.05 mm.) from the corresponding armature contact, and this results in a contact travel of 0.004 in. (0.10 mm.). Following this, the pole-pieces are adjusted so that the relay will respond without bias to 20-cycle a-c. ringing current of a magnitude about equal to that available in the telegraph repeaters. In order to have a margin of operation in the repeater, a d-c. sensitivity test is given with less current than above. In addition to the above a vibrating-circuit test on the 209-type is made. In general, the time required to adjust and test a relay is about six minutes.

Appendix

This appendix will consider briefly the advantages derived by the application of permalloy magnetic material to the polarized telegraph relay developed for use under comparatively severe and exacting circuit conditions. As pointed out in the body of the paper, this new magnetic material has a lower coercive force and higher permeability than magnetic materials ordinarily employed in relay construction.

The small amount of residual magnetism existing in this new magnetic material on account of its low coercive force as compared with a commercial grade of silicon steel, reduces appreciably the effect of bias, *i. e.*, lengthening or shortening of the telegraph signals. Also, the sensitivity, the stability and service life of the relay are increased. This increase in service life is on an average of about 50 per cent. A comparative value of the residual effects of the magnetic material used in the armature, yokes and pole pieces of two relays which are otherwise identical in their construction, can be realized from the following:

If a relay is magnetized with 120 ampere-turns, it is found that the contact pressure as a result of the residual properties of the magnetic material is in the case of a relay equipped with the permalloy material about one-seventh of that obtained when ordinary magnetic material is used. This value of 120 ampere-turns is approximately fifteen times that employed under normal telegraph circuit conditions.

Furthermore, the use of this new material causes an increase in contact pressure under average working conditions of about 50 per cent., which is in general a factor contributing towards minimizing contact chatter.

The minimum operating current for satisfactory telegraph signals is decreased on relays equipped with this permalloy material from about two milliamperes to one milliampere. Of course, in telegraph practise, in order to introduce a certain margin of operation, a somewhat larger current than one milliampere is often used.

Discussion

VOICE-FREQUENCY CARRIER TELEGRAPH SYSTEM FOR CABLES

(Hamilton, Nyquist, Long and Phelps)

POLARIZED TELEGRAPH RELAYS

(Fry and Gardner)

New York, N. Y., February 12, 1925

J. J. Pilloid: The three papers, *Metallic Polar-Duplex Telegraph System for Long Small-Gap Cables*, *Voice-Frequency Carrier Telegraph Systems for Cables* and *Polarized Telegraph Relays*, describe two telegraph systems particularly adapted for use on long toll cables, and a relay which is an important feature of the two systems. By long toll cables we mean here cables several hundred, or a thousand or more, miles long, used for telephone or telegraph purposes.

Such cables, as it has been reported to the Institute before, can each take the place of about ten heavily loaded open-wire lines, and they are used in sections where traffic is heavy. That means, of course, that there will be sections of the country where cables will not be placed for a good many years and so the older and existing telegraph systems (that is, the ground-return system) will be used for many years. One of the prime requisites, therefore, of these new systems, as brought out in these papers, is that they shall be adaptable for working with the existing systems.

There is just a mention of this idea on the fourth page of the voice-frequency carrier telegraph system paper, and if I do nothing else, I would like to emphasize that one statement. It is tucked away where you can hardly see it. It reads: "In the development of the voice-frequency carrier telegraph system, the central thought was the desirability of designing a system which would fit into the existing cable telephone and telegraph plant." As I pointed out before, it is also advisable that it fit into the existing open-wire telephone and telegraph plant.

The extent to which these systems will be used depends, of course, upon the development of this long toll cable plant and possibly a word as to the present development of that might be of interest. The cables at the present time are in service, as you know, from Boston to Washington and from New York through to Cleveland and to a point about half-way between Cleveland and Toledo. We expect to finish the cable into Toledo in a few weeks, where it will connect with a cable which is now in service between Toledo and Detroit. Cable is also in service for a distance of about 100 mi. east of Chicago, as far as South Bend, and work is under way on the remaining gap between Toledo and South Bend. It is expected that that section will be completed about October of this year. Also plans have been approved for a cable from St. Louis to Peoria and it is expected that this section will be completed to Chicago in 1926, so that we already have several long toll cables to operate these systems on, and are going to have more.

As the cables reach successive cities along the way toward their ultimate destinations the new telegraph systems and also the telephone systems join the older systems, and it is advantageous that both are easily adaptable and flexible enough for this purpose.

The voice-frequency telegraph system paper refers to the speed and message-carrying capacity of these systems. On the last page something is said about 6750 messages simultaneously each way on one full-sized cable. If such a cable, fully equipped with these systems, were devoted entirely to telegraph service, one could transmit all of the text of the five papers presented in this communication session and all of the discussions, in a fraction of a minute.

R. N. Nicely: The voice-frequency carrier telegraph system, providing as it does, a large number of telegraph circuits with a comparatively small amount of intermediate repeater apparatus, is well suited to use on long trunk routes between widely separated large cities from which distribution to outlying points is made by other telegraph systems. It will satisfactorily transmit signals at considerably higher speeds than is the case with aerial wire grounded polar duplex circuits used in the telephone plant and is therefore well suited to use with start-stop or multiplex printing telegraph apparatus.

It may be of interest to point out that the metallic polar duplex telegraph system and the voice-frequency carrier telegraph system may, if desired, be operated simultaneously on the same cable conductors.

The Rotating Magnetic Field Theory of A-C. Motors

BY K. L. HANSEN[1]

Associate, A. I. E. E.

***Synopsis.**—The predetermination of the performance of a polyphase a-c. machine is greatly facilitated by the fact that at constant voltage and frequency its magnetic field is of constant intensity and rotating with uniform velocity. It is easy to form a mental picture of lines of force moving in space and being cut by conductors, which may be moving or stationary. Furthermore, the rate of cutting, and therefore the generated voltages, which form the basis for quantitative analysis, are readily determined by the relative motion of the flux and the conductors.*

Because of the ease with which a physical conception can be formed of a rotating magnetic field, the idea of considering a single-phase alternating field as made up of two oppositely rotating fields has been found very useful. In a paper entitled "A Physical Conception of the Operation of the Single-phase Induction Motor" TRANSACTIONS A. I. E. E., 1918, p. 627), Mr. B. G. Lamme has given an excellent description of single-phase induction motor operation based on a conception of two oppositely rotating magnetic fields.

From the discussion of Mr. Lamme's paper, it appears to be the concensus of opinion that the method he uses furnishes the simplest and clearest physical conception of the single-phase motor. However, this is not the method usually employed in the quantitative analysis. Reference to text books will show that the mathematical treatment is usually based on the so-called "cross field" theory. In this method the secondary induced voltage is considered made up of two components, one the voltage induced by transformer action of the alternating field and the other the voltage generated by rotation of the secondary conductors in the magnetic field.

It has been argued against the method based on two oppositely rotating fields, also known as the "Rotating Field" theory, that it is more apt to lead to erroneous results, requires more expert handling and that it is an indirect method, being based on the previously determined performance of the polyphase motor. However, the main argument against it seems to be its limitation to induction motors only, and that it must be abandoned when we come to motors of the commutator type. Even those who otherwise favor the method appear to agree that it is not applicable to commutator motors as we are then no longer dealing with induction machines, but with shunt or series motors, as the case may be.

The objection to the rotating field theory, that it is applicable to induction motors only, would be a serious one if it were valid. However, it will be shown in this paper that the theory can be readily applied to commutator machines also, and that so far from being more apt than other methods to lead to erroneous results, it undoubtedly furnishes the simplest and most direct means for mathematical deductions in the more complicated problems where three or more circuits are inductively related and moving with respect to one another.

GENERAL DISCUSSION

IN its general form, the a-c. motor comprises one or more stationary circuits and one or more rotating electric circuits inductively related to the stationary circuits. Energy is transferred from the stationary to the rotating circuits through the medium of a common magnetic field. In this discussion the magnetic field is in general considered the resultant of two fields rotating in opposite directions. These two components are not necessarily equal, in fact one of them may vanish and the resultant is then a uniformly rotating field, as for example in a polyphase machine. Only motors in which the magnetic reluctance is uniform in all directions will be considered, that is, motors with projecting poles are not included.

There are then two main groups of motors to be discussed; the induction type, in which the rotor circuits are short-circuited upon themselves; and the commutator type, in which the rotor circuits are either short-circuited or connected to an external circuit through a commutator. A number of essential features are common to both types. Thus, at any speed, the m. m. fs. of the rotor rotate with velocities which, combined with the velocity of mechanical rotation, are always equal to the velocities of the stator fields. The common magnetic field is produced by an m. m. f., which is the vector sum of the stator and rotor m. m. fs.

When running at any slip, s, there are in general two voltages induced in the rotor circuits of frequencies $s\,f$ and $(2 - s)\,f$, according to whether it is induced by the field rotating in the same or in opposite direction to the rotation of the rotor, where f is the stator frequency. In magnitude the induced voltages are, of course, proportional to their respective frequencies. As this viewpoint obviously eliminates the necessity of considering the rotational and transformer voltages separately, the solution of many problems are greatly simplified thereby. The torque developed by either field is also readily determined, it being in all cases the product of the field, the m. m. f. of the rotor circuits revolving in the same direction as the field and the sine of the angle between them.

As already stated, the voltages induced in the rotor circuits are of frequencies $s\,f$ and $(2 - s)\,f$. In the induction type the rotor currents resulting from these voltages are likewise of frequencies $s\,f$ and $(2 - s)\,f$. However, in the commutator type the resulting rotor currents are at all speeds converted by the commutator into line frequency. In their magnetic reactions the rotor currents become fixed in space by the commutator and the rotor circuits can therefore be considered as remaining stationary in every respect, except as regards the magnitudes of the induced voltages, which are determined by the frequencies of slip, as pointed out.

The rotor currents becoming fixed in space by the commutator is the cause of some essential differences

1. Consulting Engineer, Milwaukee, Wis.

in the operation of the induction and commutator types. Thus, in the induction machine, the reactance of the rotor circuit changes with the slip, while in the commutator type it is the same at all speeds. Also, by shifting the brushes on the commutator, the relative position of the stator and rotor m. m. fs. is changed so that the voltages that are induced in the stator circuits by the rotor currents may be advanced or retarded in time. This becomes of importance when an external e. m. f. is impressed on the rotor as it introduces the possibility of power-factor correction.

The subject of power-factor correction has lately been the object of considerable discussion and the readiness with which the rotating field theory lends itself to the analysis of just such problems, involving a displacement angle between the stator and rotor currents is another marked advantage of this method. Application of the method to the analysis of some proposed schemes for power-factor correction will be taken up in the mathematical section.

However, before taking up the mathematical discussion, it will be shown by way of illustration how readily the method can be applied to the much discussed problem of calculating the performance of a single-phase induction motor.

Consider a single-phase motor and a two-phase motor of the same constants per circuit as the single-phase machine. Then if E be the impressed voltage and I_p the current per phase of the two-phase motor at slip s, the apparent impedance per phase is $Z_a = \frac{E}{I_p}$. Similarly, if I_b is the current per phase when running backwards at the same speed, the apparent impedance at slip $(2 - s)$ is $Z_b = \frac{E}{I_b}$. It is then shown in the appendix that the apparent impedance of the single-phase motor at slip s is $Z_s = \frac{Z_a + Z_b}{2}$ and the current of the single-phase motor is $I_s = \frac{E}{Z_s}$. Let T_a = torque of two-phase motor at slip s and T_b = torque at slip $(2 - s)$. The torque developed in the single-phase motor at slip s by the field revolving in the same direction as the rotor then is $T_1 = \left(\frac{e_1}{E}\right)^2 \times T_a$, where $e_1 = \frac{Z_a I_s}{2}$, and the torque developed by the oppositely rotating field $T_2 = \left(\frac{e_1}{E}\right)^2 \times T_b$, where $e_2 = \frac{Z_b I_s}{2}$. The resultant torque of the single-phase motor at slip s, $T_s = T_1 - T_2$. Herefrom the remaining quantities, power, efficiency and power factor can be determined directly. Thus, by extremely simple calculations the performance of a single-phase motor is derived from the performance of a two-phase motor of same constants per circuit. Which one of the numerous methods that have been devised for calculating the performance of a two-phase motor to use is, of course, a matter of choice.

When it is desired to calculate the performance from the running and locked test readings, the locked single-phase readings can be used directly in calculating the two-phase performance. The single-phase no-load reading with the rotor short-circuited, that is, running light, can not be so used, because the exciting admittance per phase of the two-phase motor is then a little more than one half the single-phase admittance, or, what amounts to the same thing, the two-phase no-load impedance per phase is almost twice the single-phase no-load impedance. The amount by which the two-phase impedance falls short of being exactly twice the single-phase no-load impedance is obviously the apparent impedance per phase when running backwards at full speed. Since this latter impedance is almost independent of the exciting current it can readily be determined to a high degree of accuracy. On the basis of the modified single-phase no-load reading the diagram of the two-phase motor can be constructed and the single-phase performance calculated therefrom.

As a numerical example, the following readings have been taken on a single-phase motor;

Running Light		Rotor locked
110	Volts	110
3.2	Amperes	14.8
78	Watts	982
$\cos \phi_0 = 0.221$	P. F.	$0.603 = \cos \phi_L$
Primary resistance		2.36 Ohms

The no load impedance per phase $Z_0 = \frac{110}{3.2} = 34.4$.

$$R_0 = Z_0 \cos \phi_0 = 7.6 \quad X_0 = Z_0 \sin \phi_0 = 33.6$$

It is known that the two-phase no-load impedance per phase is slightly less than $2\,Z_0$, or the magnetizing current a little more than one half of 3.2 amperes. To determine how much $2\,Z_0$ should be reduced, construct a diagram, using the observed readings, except that the magnetizing current is reduced to approximately one half, say 1.7 amperes.

No refinements are necessary in the construction of the diagram to modify the no-load readings and the simple diagram as shown in Fig. 1 is sufficient. From the current triangle $O\,P\,R$ in this diagram the apparent impedance at slip = 2 is found to be $Z_b = \frac{110}{16.5} = 6.67$.

$R_b = Z_b \cos \phi_b = 3.2$ and $X_b = Z_b \sin \phi_b = 5.85$. Subtracting R_b from $2\,R_0$ leaves 12, and X_b from $2\,X_0$ leaves 61.35, hence the corrected no-load impedance per phase is $\sqrt{12^2 + 61.32^2} = 62.5$. The no-load

values to use in calculating the two-phase performance are therefore 1.76 amperes and 0.192 power factor. Fig. 2 shows the curves of the two-phase motor. Then at some speed, for example 1600 rev. per min., we find,

Forward rotation 4.67 amperes, 0.872 power factor
Backward rotation 16 amperes, 0.49 power factor.

$$Z_a = \frac{110}{4.67} = 23.55 \quad R_a = 23.55 \times 0.872 \doteq 20.55 \quad X_a = 23.55 \times 0.488 = 11.50$$

$$Z_b = \frac{110}{16} = 6.87 \quad R_b = 6.87 \times 0.49 = \frac{3.37}{23.92} \quad X_b = 6.87 \times 0.87 = \frac{5.97}{17.47}$$

Adding

Dividing by 2 $\quad 11.96 \quad\quad 8.735$

Single-phase current $I_s = \dfrac{110}{\sqrt{11.96^2 + 8.735^2}} = 7.43$ amperes at 0.81 power factor.

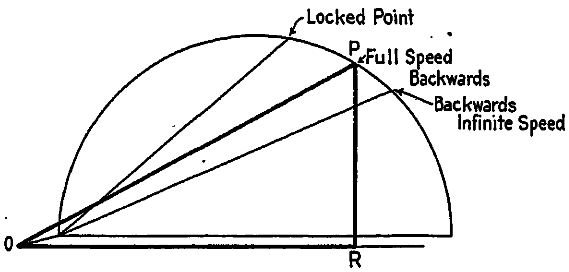


FIG. 1

$$e_1 = \frac{23.55 \times 7.43}{2} = 87.4 \quad e_2 = \frac{6.87 \times 7.43}{2} = 25.5$$

$$T_1 = \left(\frac{87.4}{110}\right)^2 \times 2.875 = 1.808 \text{ lb. at 1 ft. radius.}$$

$$T_2 = \left(\frac{25.5}{110}\right)^2 \times 2.05 = 0.110 \text{ lb. at 1 ft. radius.}$$

$$T_s = T_1 - T_2 = 1.698 \text{ lb. at 1 ft. radius}$$

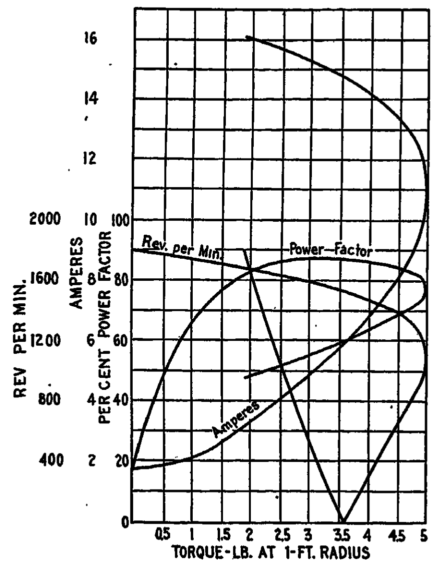


FIG. 2

equals resultant single-phase torque at 1600 rev. per min The complete single-phase performance is shown in Fig. 3.

In this simple method it may be of interest to note the absence of complicated geometrical figures, trigonometric transformations, inversions, equivalent circuits, empirical coefficients, etc., in which the quantitative analysis of the single-phase motor usually abounds. Furthermore, it will be seen in the appendix that a conception of two oppositely rotating fields immediately furnishes sufficient data to write down one set of equations, the solution of which forms the basis of the method described.

To further illustrate the usefulness of the conception of rotating magnetic fields the mathematical analysis will be extended to the somewhat more involved problems of phase conversion and power-factor correction.

Appendix

Mathematical Expression for Two Oppositely Rotating Fields

A current with maximum value I and varying periodically according to the cosine law is usually

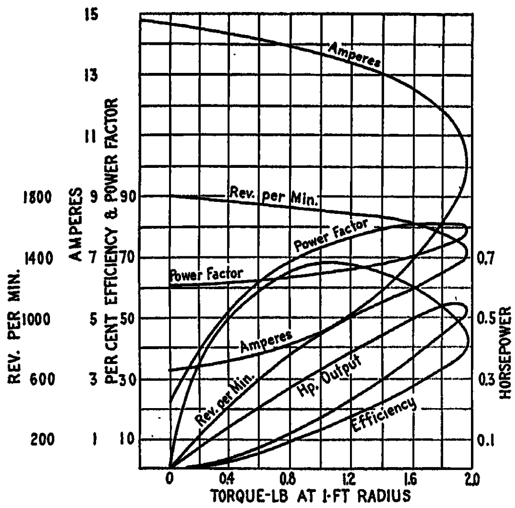


FIG. 3

represented by the expression $I \cos \omega t$, where $\omega = 2\pi f$, f being the frequency. Using the exponential form of the cosine, a sinusoidal current may be represented by a pair of rotating vectors, that is $I \cos \omega t$ may be written $\frac{I}{2}\epsilon^{j\omega t} + \frac{I}{2}\epsilon^{-j\omega t}$. Many cumbersome trigonometric expressions and transformations are frequently avoided by the use of this notation (see for example "Radio Communication," by John Mills). That the expression $\frac{I}{2}\epsilon^{j\omega t}$ represents a vector revolving in counter-clockwise direction with angular velocity ω is readily seen by writing

$$\frac{I}{2}\epsilon^{j\omega t} = \frac{I}{2}(\cos \omega t + j \sin \omega t)$$

and assigning to t a series of increasing positive values. The expression

$$\frac{I}{2}\,\epsilon^{-j\omega t} = \frac{I}{2}\,(\cos\omega t - j\sin\omega t)$$

is likewise seen to represent a vector of same length and revolving in clockwise direction. In Fig. 4 let a current $I\cos\omega t$ be flowing in a coil $N\,N$ of n turns. The instantaneous value of the m. m. f. of the coil then is $n\,I\cos\omega t$ or in the above notation

$$\frac{n\,I}{2}\,\epsilon^{j\omega t} + \frac{n\,I}{2}\,\epsilon^{-j\omega t}$$

As the m. m. f. is a directed quantity in space the first term of this expression represents a m. m. f. of constant intensity $\frac{n\,I}{2}$ rotating in positive direction, and the second term represents a m. m. f. of same intensity rotating in opposite direction. Let the coil

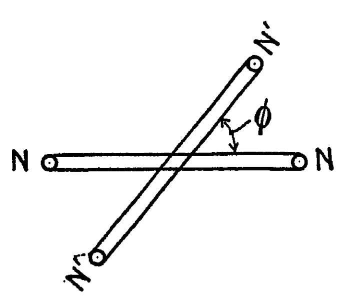


FIG. 4

$N - N$ be turned through an angle of ϕ radians in positive direction and its m. m. f. becomes

$$\frac{n\,I}{2}\,(\epsilon^{j\omega t} + \epsilon^{-j\omega t})\,\epsilon^{j\phi} = \frac{n\,I}{2}\,\epsilon^{j(\omega t+\phi)} + \frac{n\,I}{2}\,\epsilon^{-j(\omega t-\phi)}$$

that is, the m. m. f. revolving in positive direction has been advanced ϕ radians and the m. m. f. revolving in negative direction has been retarded ϕ radians.

The most striking feature of the polyphase system, that it can produce a rotating magnetic field of constant intensity, is most readily deduced by use of this notation. To illustrate, assume m coils, symmetrically based on a cylindrical core, the space angle being $\frac{2\pi}{m}$ radians. The coils are excited by currents I, differing in phase by $\frac{2\pi}{m}$ radians. The m. m. f. of the k^{th} coil then is

$$\frac{n\,I}{2}\left[\epsilon^{j(\omega t+\frac{2\pi k}{m})} + \epsilon^{-j(\omega t+\frac{2\pi k}{m})}\right]\epsilon^{j\frac{2\pi k}{m}}$$

and the total m. m. f. of all the coils is

$$\sum_{0}^{m-1} k\,\frac{n\,I}{2}\left[\epsilon^{j(\omega t+\frac{2\pi k}{m})} + \epsilon^{-j(\omega t+\frac{2\pi k}{m})}\right]\epsilon^{j\frac{2\pi k}{m}}$$

$$= \sum_{0}^{m-1} k\,\frac{n\,I}{2}\,\epsilon^{j(\omega t+\frac{4\pi k}{m})} + \sum_{0}^{m\;1} k\,\frac{n\,I}{2}\,\epsilon^{-j\omega t}\,\epsilon^{0k}$$

$$= \frac{m\,n\,I}{2}\,\epsilon^{-j\omega t} \text{ since } \sum_{0}^{m-1} k\,\frac{n\,I}{2}\,\epsilon^{j(\omega t+\frac{4\pi k}{m})} = 0$$

That is, the resultant m. m. f. is of constant intensity $\frac{m\,n\,I}{2}$ and rotating in negative direction. If the sign of either time angle or space angle be changed the resultant m. m. f. is seen to rotate in opposite direction.

In Fig. 4, let Z_m be the mutual impedance between coils $N\,N$ and $N'\,N'$ when in coaxial position. The voltage induced in the coil $N'\,N'$ by the current flowing in $N\,N$ then is $Z_m I$. When $N'N'$ is turned ϕ radians in positive direction the voltage induced by the positively rotating field of $N\,N$ is retarded ϕ radians and becomes $\frac{Z_m I}{2}\,\epsilon^{-j\phi}$ and the voltage induced by the negatively rotating field is advanced ϕ radians and becomes $\frac{Z_m I}{2}\,\epsilon^{j\phi}$. The voltage induced by both fields is

$$\frac{Z_m I}{2}\,\epsilon^{-j\phi} + \frac{Z_m I}{2}\,\epsilon^{j\phi} = Z_m I\cos\phi$$

SINGLE-PHASE INDUCTION MOTOR

The rotor circuits of the single-phase induction motor being short-circuited upon themselves in all directions constitute a polyphase system and for convenience will be considered two-phase with constants determined accordingly. Assume the rotor to be revolving at slip s and consider its direction of rotation positive. Using the customary notation let $Z_m = r_m + j\,x_m$ = Mutual inductive impedance between stator and rotor circuits.

$Z_0 = r_0 + j\,x_0$ = Self inductive impedance of primary

$Z_1 = r_1 + j\,s\,x_1$ = Self inductive impedance to rotor current of frequency $s\,f$. x_1. x_1 = secondary reactance at standstill in terms of primary

$Z_2 = r_1 + j\,(2 - s)\,x_1$ = Self inductive impedance to rotor current of frequency $(2 - s)\,f$

E = volts impressed on primary

I_s = Primary current

I_1 = Positively rotating component of rotor current

I_2 = Negatively rotating component of rotor current

The voltages induced in the primary then are:
by the primary current $I_s = (Z_m + Z_0)\,I_s$
by the positively rotating rotor current $Z_m I_1$
by the negatively rotating rotor current $Z_m I_2$
the currents I_1 and I_2 being two-phase have full inductive effect. The sum of these voltages equals the impressed volts, hence,

$$(Z_m + Z_0)\,I_s + Z_m I_1 + Z_m I_2 = E$$

The voltages induced in the rotor are:
by the positively rotating component of primary current

$$\frac{s\, Z_m\, I_s}{2}$$

by the positively rotating component of rotor current

$$(s\, Z_m + Z_1)\, I_1$$

by the negatively rotating component of primary current

$$\frac{(2 - s)\,(Z_m)\, I_s}{2}$$

by the negatively rotating component of rotor current

$$[\,(2 - s)\, Z_m + Z_2]\, I_2$$

The sums of voltages of frequencies $s\,f$ and $(2 - s)\,f$ are separately equal to zero, hence

$$\frac{s\, Z_m\, I_s}{2} + (s\, Z_m + Z_1)\, I_1 = 0 \text{ or } s\, Z_m\, I_s + 2\,(s\, Z_m + Z_1)\, I_1 = 0 \text{ and}$$

$$\frac{(2 - s)\, Z_m\, I_s}{2} + [\,(2 - s)\, Z_m + Z_2]\, I_2 = 0 \text{ or}$$

$$(2 - s)\, Z_m\, I_s + [\,(2 - s)\, Z_m + Z_2]\, I_2 = 0$$

The e. m. f. equations of primary and secondary are then

$$\left.\begin{array}{ll} (Z_m + Z_0)\, I_s + Z_m\, I_1 + Z_m\, I_2 & = E \\ s\, Z_m\, I_s + 2\,(s\, Z_m + Z_1)\, I_1 & = 0 \\ (2 - s)\, Z_m\, I_s + 2\,[\,(2 - s)\, Z_m + Z_2]\, I_2 & = 0 \end{array}\right\} \quad (1)$$

Solving these equations

$$I_s = \frac{E\, 2\,\{\, s\,(2 - s)\, Z_m^2 + (2 - s)\, Z_m\, Z_1 + s\, Z_m\, Z_2 + Z_1\, Z_2\}}{2\, Z_0\{s(2 - s)Z_m^2 + (2 - s)Z_m\, Z_1 + s\, Z_m\, Z_2 + Z_1\, Z_2\} + Z_m\,\{\,(2 - s)\, Z_m\, Z_1 + s\, Z_m\, Z_2 + 2\, Z_1\, Z_2\}} \quad (2)$$

$$I_1 = \frac{-\, E\, s\, Z_m\,[(2 - s)\, Z_m + Z_2]}{2\, Z_0\{s(2 - s)Z_m^2 + (2 - s)Z_m\, Z_1 + s\, Z_m\, Z_2 + Z_1\, Z_2\} + Z_m\{\,(2 - s)\, Z_m Z_1 + s\, Z_m\, Z_2 + 2\, Z_1\, Z_2\}} \quad (3)$$

$$I_2 = \frac{-\, E\,(2 - s)\, Z_m\,(s\, Z_m + Z_1)}{2\, Z_0\{s(2 - s)Z_m^2 + (2 - s)Z_m\, Z_1 + s\, Z_m\, Z_2 + Z_1\, Z_2\} + Z_m\{\,(2 - s)\, Z_m\, Z_1 + s\, Z_m\, Z_2 + 2 Z_1\, Z_2\}} \quad (4)$$

The equations of a two-phase motor of same constants per phase are easily found to be

$$(Z_m + Z_0)\, I_0 + Z_m\, I_1' = E$$
$$s\, Z_m\, I_0 + (s\, Z_m + Z_1)\, I_1' = 0$$

where I_0 = primary current per phase and I_1' = secondary current.
Here from

$$I_0 = \frac{E\,(s\, Z_m + Z_1)}{s\, Z_m\, Z_0 + Z_m\, Z_1 + Z_0\, Z_1} \quad (5)$$

$$I_1' = \frac{-\, E\, s\, Z_m}{s\, Z_m\, Z_0 + Z_m\, Z_1 + Z_0\, Z_1} \quad (6)$$

The apparent impedance per phase of the two-phase motor at slip s then is

$$Z_a = \frac{E}{I_0} = \frac{s\, Z_m\, Z_0 + Z_m\, Z_1 + Z_0\, Z_1}{s\, Z_m + Z_1}$$

The apparent impedance at same speed rotating backwards is

$$Z_b = \frac{E}{I_0} = \frac{(2 - s)\, Z_m\, Z_0 + Z_m\, Z_2 + Z_0\, Z_2}{(2 - s)\, Z_m + Z_2}$$

Adding the apparent impedances of both rotations

$$Z_a + Z_b = \frac{s\, Z_m\, Z_0 + Z_m\, Z_1 + Z_0\, Z_1}{s\, Z_m + Z_1} + \frac{(2 - s)\, Z_m\, Z_0 + Z_m Z_2 + Z_0\, Z_2}{(2 - s)\, Z_m + Z_2} =$$

$$\frac{2\, Z_0\{s(2 - s)Z_m^2 + (2 - s)Z_m\, Z_1 + s\, Z_m\, Z_2 + Z_1\, Z_2\} + Z_m\{\,(2 - s)Z_m Z_1 + s\, Z_m\, Z_2 + 2\, Z_1\, Z_2\}}{s\,(2 - s)\, Z_m^2 + (2 - s)\, Z_m\, Z_1 + s\, Z_m\, Z_2 + Z_1\, Z_2}$$

Comparing with (2) it will be seen that the apparent impedance of a single-phase motor at slip s is one half the sum of the apparent impedances of a two-phase motor of same constants at slips s and $(2 - s)$. Furthermore, if the impedance drop $\frac{Z_a\, I_s}{2}$ be substituted for E in (6) the result is identical with (3), showing that the low frequency component of secondary current is equal to the two-phase secondary current at this reduced voltage. Similarly, the high frequency component of secondary current is seen to be equal to the two-phase secondary current at slip $(2 - s)$ and the voltage reduced to $\frac{Z_b\, I_s}{2}$. The torque components of the single-phase motor in positive and negative directions are likewise seen to be equal to the torques of the two-phase machine at corresponding slips and reduced voltages.

It follows therefore that in calculating the performance by the graphical method there is no need of a special diagram for the single-phase motor, the same diagram being applicable to both polyphase and single-phase motors.

Induction Phase Converter

In applications where it is essential to obtain polyphase power from a single phase supply an induction phase converter is often used to effect the desired transformation. The converter consists essentially of a single-phase induction motor with a tertiary circuit on the stator displaced at a certain angle from the primary circuit. Thus, in Fig. 5, R is the rotor of a single-phase motor, P_1 the primary winding and P_2 the tertiary winding displaced 90 deg. in positive

direction from P_1. P_2' is one phase of a load circuit, on which it is desired to impress a voltage in quadrature to the voltage on the other phase, P_1', and as nearly as possible equal to it in magnitude. In Fig. 6 is shown a phase converter with the primary connected in series with one of the load phases, as distinguished from the shunt connection of Fig. 5.

Using the same notation as before, the mutual and self inductive impedances of the tertiary circuit are respectively equal to the mutual and self inductive impedances of the primary Z_m and Z_0. Let Z_a be the

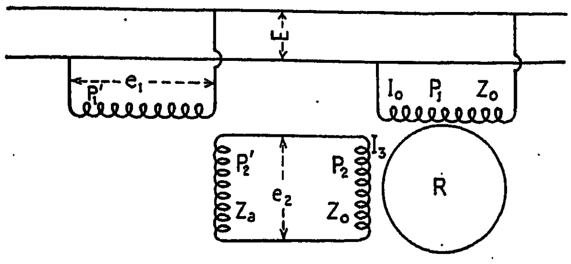


FIG. 5

impedance per phase of the load circuit and let $Z_0' = Z_a + Z_0$. The speed being very close to synchronous, the forwardly rotating component of secondary current can be neglected, that is $I_1 = 0$. The voltage generated in the tertiary circuit by the backwardly rotating component of secondary current is 90 deg. ahead of the voltage generated by this current in the primary circuit and is, therefore, $Z_m I_2 \epsilon^{j\frac{\pi}{2}} = j\, Z_m I_2$.

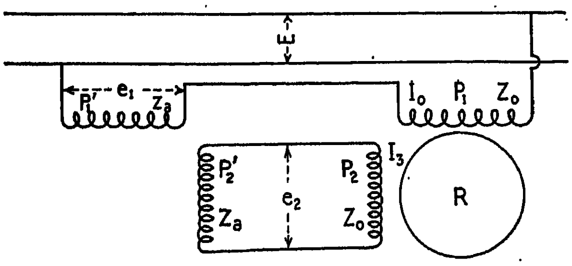


FIG. 6

The double-frequency voltage generated in the secondary by the current in the tertiary circuit is likewise seen to be $Z_m I_3 \epsilon^{-j\frac{\pi}{2}} = -j\, Z_m I_3$.

From Fig. 5 is then readily obtained

$$\left.\begin{aligned} (Z_m + Z_0)\, I_0 + Z_m I_2 \quad\quad\quad &= E \\ Z_m I_0 + (2\, Z_m + Z_2)\, I_2 - j\, Z_m I_3 &= 0 \\ j\, Z_m I_2 + (Z_m + Z_0')\, I_3 \quad &= 0 \end{aligned}\right\} \quad (7)$$

Solving

$$I_0 = \frac{E\,[Z_m^2 + Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0']}{(Z_m+Z_0)[Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0'] - Z_m^2 Z_a} \quad (8)$$

$$I_2 = \frac{-E\,(Z_m^2 + Z_m Z_0')}{(Z_m+Z_0)[Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0'] - Z_m^2 Z_a} \quad (9)$$

$$I_3 = \frac{j\, E\, Z_m^2}{(Z_m+Z_0)[Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0'] - Z_m^2 Z_a} \quad (10)$$

$$e_2 = Z_a I_3$$

Denoting the denominator by D, it is interesting to note that the first term of I_0 is $\frac{E\, Z_m^2}{D}$, which, combined with $I_3 = j\, \frac{E\, Z_m^2}{D}$ produces a m. m. f. rotating backwards, which is balanced by the first term of I_2, which is equal to $\frac{-E\, Z_m^2}{D}$. It is, therefore, the double

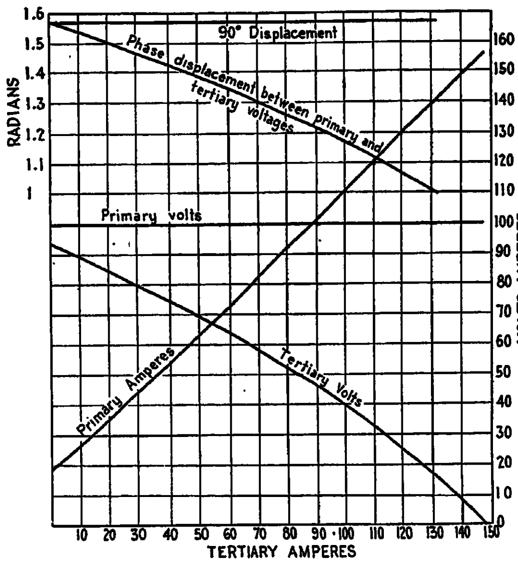


FIG. 7

frequency component of secondary current which increases with increasing load on the tertiary circuit. In Fig. 7 are shown the curves of a shunt connected converter of the following constants: $Z_0 = 0.05 + j\, 0.15$ $Z_2 = 0.05 + j\, 0.03$ $Z_m = 0.975 + j\, 9.9$ $E = 100$. Z_a is assumed to vary so as to maintain a constant power factor of 80 per cent.

For the series-connected converter is obtained the following equations from Fig. 6:

$$\left.\begin{aligned} (Z_m + Z_0')\, I_0 + Z_m I_2 \quad\quad\quad &= E \\ Z_m I_0 + (2\, Z_m + Z_2)\, I_2 - j\, Z_m I_3 &= 0 \\ j\, Z_m I_2 + (Z_m + Z_0')\, I_3 \quad &= 0 \end{aligned}\right\} \quad (11)$$

and here from

$$I_0 = \frac{E\,[Z_m^2 + Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0']}{(Z_m + Z_0')\,[Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0']} \quad (12)$$

$$I_2 = \frac{-E\,(Z_m^2 + Z_m Z_0')}{(Z_m + Z_0')\,[Z_m Z_2 + 2\, Z_m Z_0' + Z_2 Z_0']} \quad (13)$$

$$\frac{j\,E\,Z_m^2}{(Z_m + Z_0)\,[Z_m Z_2 + 2\,Z_m Z_0' + Z_m Z_0']} \qquad (14)$$

$$e_1 = Z_a I_0 \quad e_2 = Z_a I_3$$

In Fig. 8 are shown the curves of the same converter shown in Fig. 7, except that it is connected in series. It will be noted that with the series connection the

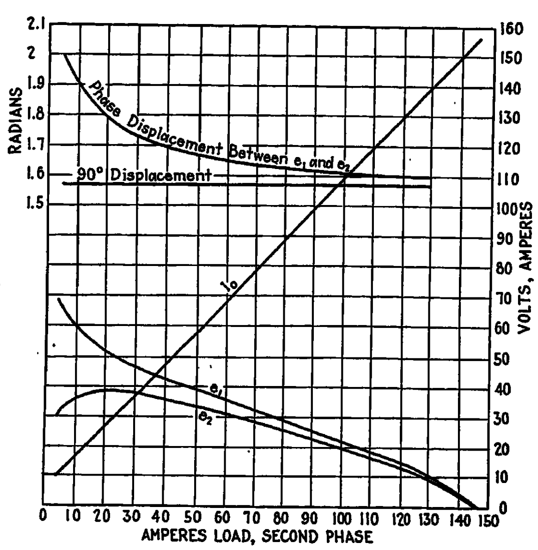


Fig. 8

phase voltages of the load circuit tend towards equality in magnitude and quadrature relation in phase as the load increases, while the reverse is true for the shunt connection.

Repulsion Motor

As illustration of the application of the method to commutator motors, consider the repulsion motor shown diagramatically in Fig. 9. It being at the present merely intended to show that the rotating field theory is applicable to commutator motors also, only the plain repulsion motor will be considered, although the method can be readily extended to the various forms of compensated motors, and to include the phenomena of brush short-circuit currents.

In Fig. 9 let the brushes be shifted λ radians in positive direction from the line $A\,B$, which is the position of maximum mutual inductive effect between stator and rotor circuits. As in the single-phase induction motor, there are two voltages generated in the secondary of frequencies s and $(2 - s)$. However, the currents resulting from these voltages are converted by the commutator into line frequency at all speeds, and consequently combine into one secondary current. The voltage induced in the primary by the positively rotating component of the secondary current is advanced λ radians in phase, and the voltage generated by the negatively rotating component is retarded λ radians. The voltage equation of the primary is, therefore,

$$(Z_m + Z_0)\,I_0 + \frac{Z_m I_1}{2}\,\epsilon^{j\lambda} + \frac{Z_m I_1}{2}\,\epsilon^{-j\lambda} = E$$

Similarly the voltage equation of the secondary is seen to be

$$Z_m\left[\frac{s\,\epsilon^{-j\lambda}}{2} + \frac{(2-s)\,\epsilon^{j\lambda}}{2}\right] I_0 + \left[\frac{s\,Z_m}{2} + \frac{(2-s)\,Z_m}{2} + Z_1\right] I_1 = 0$$

Substituting

$$\epsilon^{j\lambda} = \cos\lambda + j\sin\lambda$$
$$\epsilon^{-j\lambda} = \cos\lambda - j\sin\lambda$$

and the voltage equations reduce to

$$\left.\begin{aligned} (Z_m + Z_0)\,I_0 + Z_m\cos\lambda\, I_1 &= E \\ Z_m[\cos\lambda + j(1-s)\sin\lambda]I_0 + (Z_m + Z_1)I_1 &= 0 \end{aligned}\right\} \qquad (15)$$

Solving

$$I_0 = \frac{E\,(Z_m + Z_1)}{[Z_m^2 + Z_m Z_0 + Z_m Z_1 + Z_0 Z_1] - Z_m^2\cos\lambda\,[\cos\lambda + j\,(1-s)\sin\lambda]} \qquad (16)$$

$$I_1 = \frac{-E\,Z_m\,[\cos\lambda + j\,(1-s)\sin\lambda]}{[Z_m^2 + Z_m Z_0 + Z_m Z_1 + Z_0 Z_1] - Z_m^2\cos\lambda\,[\cos\lambda + j\,(1-s)\sin\lambda]} \qquad (17)$$

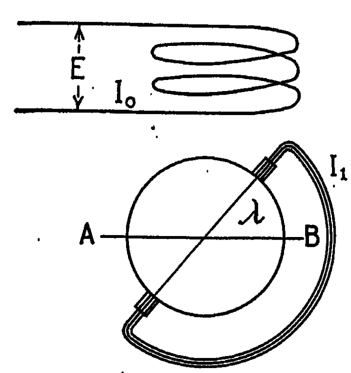


Fig. 9

The exciting current of the positively rotating field

$$I_0' = \frac{I_0\,\epsilon^{-j\lambda} + I_1}{2}$$

$$= \frac{Z_1\,\epsilon^{-j\lambda} - j\,(2-s)\,Z_m\sin\lambda}{2\,\{[Z_m^2 + Z_m Z_0 + Z_m Z_1 + Z_0 Z_1] - Z_m^2\cos\lambda\,[\cos\lambda + j\,(1-s)\sin\lambda]\,\}} \qquad (18)$$

The exciting current of the negatively rotating field

$$I_0'' = \frac{I_0\,\epsilon^{j\lambda} + I_1}{2}$$

$$= \frac{Z_1 \epsilon^{j\lambda}}{2\{[Z_m^2 + Z_m Z_0 + Z_m Z_1 + Z_0 Z_1]}$$

$$\frac{+ j\, s\, Z_m \sin\lambda}{- Z_m^2 \cos\lambda\,[\cos\lambda + j\,(1 - s)\sin\lambda]\,\}} \tag{19}$$

The voltage induced by the positively rotating field

$$e_1 = I_0^1 X_m \epsilon^{-j\left(\frac{\pi}{2} + \psi\right)}$$

and the voltage induced by the negatively rotating field

$$e_2 = I_0'' X_m \epsilon^{j\left(\frac{\pi}{2} - \psi\right)},$$

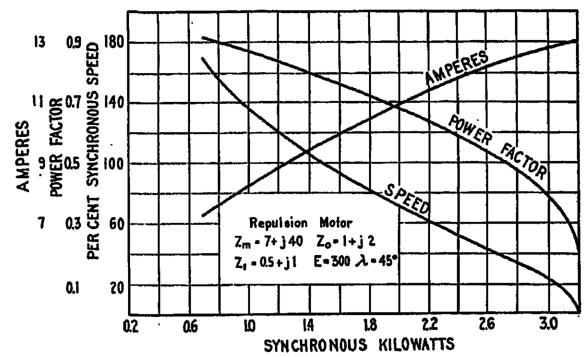


FIG. 10

where ψ is the angle by which the exciting current leads the true magnetizing current

The torque developed by the positively rotating field $T_1 = e_1 I_1 \cos\omega_1$, ω_1 = phase angle between e_1 and I_1.

The torque developed by the negatively rotating field $T_2 = e_2 I_2 \cos\omega_2$, ω_2 = phase angle between e_2 and I_2.

Resultant torque $= T_1 + T_2$.

Fig. 10 shows the curves of a motor of constants $Z_m = 7 + j\,40$ $Z_0 = 1 + j\,2$ $Z_1 = 0.5 + j\,1$ $E = 300$ $\lambda = 45$ deg.

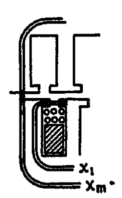


FIG. 11

POWER-FACTOR COMPENSATION

To further illustrate the application of the above method to commutator motors, it may be of interest to consider the case where one of the rotating fields vanishes, that is the symmetrical polyphase motor. There is, apparently, an increasing demand for means of correcting power factor on inductive loads, and the use of commutators in connection with induction motors may, therefore, become of considerable importance in the near future. In Steinmetz's "Theory and Calculation of Electrical Apparatus" (Pages 52-92 and page 379) are discussed a number of methods of power-factor compensation.

The simplest and most economical of the various methods proposed appears to be the so-called Heyland motor. In addition to the ordinary squirrel-cage, the rotor of the Heyland motor is supplied with a compensating winding connected to a commutator and usually placed in the same slots as the rotor bars. By means of brushes bearing on the commutator, a voltage of suitable magnitude and phase is impressed on the compensating winding.

With the compensating winding placed in the same slots as the rotor bars, practically all of the cage leakage reactance becomes a part of the mutual reactance between the cage and the compensating winding. The impedance of the cage Z_1 can, therefore, be considered as consisting of the resistance, r_1, only. The mutual impedance between the cage and the compensating winding, $Z_m' = Z_m + j\,x_1$, where Z_m is the mutual impedance between the primary winding and the cage, and also the mutual impedance between

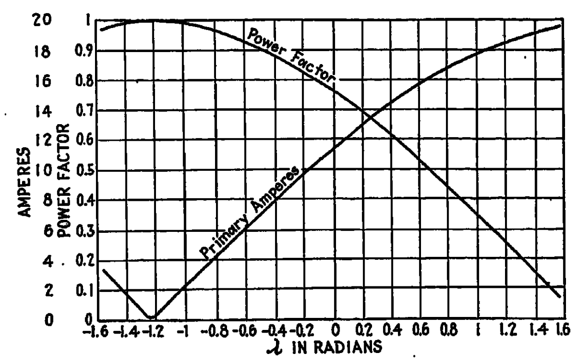


FIG. 12

the primary and the compensating winding reduced to primary terms. $Z_2 = r_2 + j\,x_2$ is the impedance of the compensating winding in terms of primary, x_2, which is the reactance of the leakage flux around the end windings, remains constant at all speeds.

Let I_0, I_1 and I_2 be the current in the primary, the the squirrel cage and the compensating winding, respectively, all in terms of primary. With the brushes shifted λ radians in positive direction, and a voltage $c\,E$ impressed on the commutator, the voltage equation of the three circuits are

$$\left.\begin{aligned} (Z_m + Z_0)\, I_0 + Z_m I_1 + Z_m \epsilon^{j\lambda} I_2 &= E \\ s\, Z_m I_0 + (s\, Z_m' + Z_1)\, I_1 + s\, Z_m' \epsilon^{j\lambda} I_2 &= 0 \\ s\, Z_m \epsilon^{-j\lambda} I_0 + s\, Z_m' \epsilon^{-j\lambda} I_1 + (s\, Z_m' + Z_2)\, I_2 &= c\,E \end{aligned}\right\} \tag{20}$$

solving

$$I_0 = \frac{E\,\{\, s\, Z_m' (Z_1 + Z_2)}{s(Z_1 + Z_2)\,[Z_m' (Z_m + Z_0) - Z_m^2]}$$

$$\frac{+ Z_1 Z_2 - c\, Z_m Z_1 \epsilon^{j\lambda}\,\}}{+ Z_1 Z_2 (Z_m + Z_0)} \tag{21}$$

$$I_1 = \frac{-\,s\,E\,\{\,Z_m\,Z_2 + c\,[Z_m'\,(Z_m + Z_0) - Z_m^2]\,\epsilon^{j\lambda}\}}{s\,(Z_1 + Z_2)\,[Z_m^1\,(Z_m + Z_0) - Z_m^2] + Z_1\,Z_2\,(Z_m + Z_0)} \qquad (22)$$

$$I_2 = \frac{s\,E\,\{\,c\,[\,Z_m'\,(Z_m + Z_0) - Z_m^2\,] - Z_m'\,Z_1\epsilon^{-j\lambda}\} + c\,E\,Z_1(Z_m + Z_0)}{s\,(Z_1 + Z_2)\,[\,Z_m'\,(Z_m + Z_0) - Z_m^2] + Z_1\,Z_2\,(Z_m + Z_0)} \qquad (23)$$

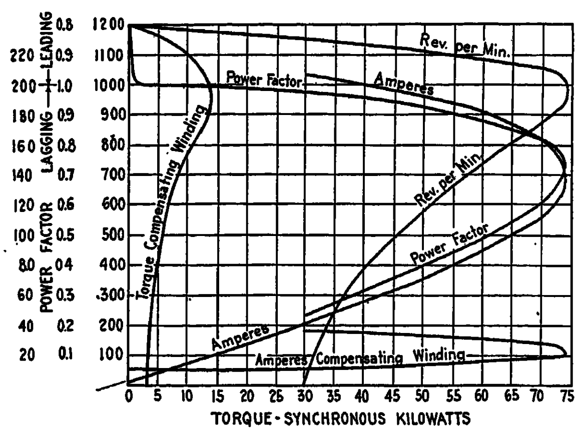


FIG. 13

The exciting current

$$I_0' = I_0 + I_1 + I_2\,\epsilon^{j\lambda} =$$

$$\frac{E\,\{\,s\,(Z_1 + Z_2)\,(Z_m' - Z_m) + Z_1\,(Z_2 + c\,Z_0\,\epsilon^{j\lambda}\,\}}{s\,(Z_1 + Z_2)[Z_m'(Z_m + Z_0) - Z_m^2] + Z_1\,Z_2(Z_m + Z_0)} \qquad (24)$$

$$e_1 = X_m\,I_0'\,\epsilon^{-j\,(\frac{2}{\pi} + \phi)} \qquad e_2 = x_m\,I_0'\,\epsilon^{-j\,(\frac{\pi}{2} + \phi + \lambda)}$$

$$T_1 = e_1\,I_1 \cos\,\omega_1 \qquad T_2 = e_2\,I_2 \cos\,\omega_2$$

Resultant torque $T = T_1 + T_2$

At synchronism $s = 0 \quad I_1 = 0$ and

$$I_0 = \frac{E\,(Z_2 - c\,Z_m\,\epsilon^{j\lambda})}{Z_2\,(Z_m + Z_0)} \qquad (25)$$

$$I_2 = \frac{c\,E}{Z_2} \qquad (26)$$

From formula (25) it will be seen that I_0 becomes equal to zero when c and λ are so chosen that $Z_2 = c\,Z_m\,\epsilon^{j\lambda}$. However, to get complete compensation when loaded the motor must be somewhat over compensated at no load.

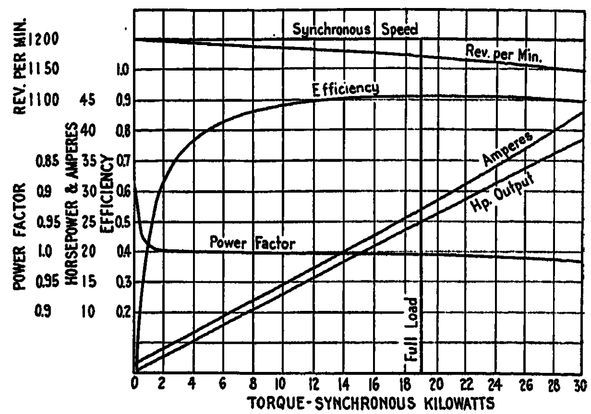


FIG. 14

As illustration a 25 h. p. 3-phase 440-volts star connected motor has the following constants

$Z_m = 2.5 + j\,25 \quad Z_m' = 2.5 + j\,25.6 \quad Z_0 = 0.2 + j0.56$

$Z_1 = 0.3 \qquad Z_2 = 1 + j\,0.25$

Letting $c = 0.0415$, Fig. 12 shows the no-load values for different brush positions. Figs. 13 and 14 show the speed-torque curves and performance over the operating range when c is increased to 0.0435 and λ fixed at 1.3 — radians.

On a Design for a Bifilar Type of Non-Reactive Resistance Coil

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Associate, A. I. E. E. Non-member

Synopsis.—*A bifilar non-reactive coil usually gives a condensive reactance at high frequency. The present paper analyses this frequency error observed over the audio-frequency range. An impedance characteristic curve is recognized, resembling that of an electric cable. From this experimental fact, a cable theory of a bifilar non-reactive coil is deduced. Employing an experimental constant, which the writers call the apparent dielectric constant K, a method is given for designing a 100,000-ohm coil, which gives a phase angle less than 5 deg., at a frequency of 5000 cycles per second, assuming the apparent dielectric constant to be K = 3.*

* * * * *

I—Experiment to Determine the Frequency Characteristic of a Bifilar Type of Non-Reactive Coil

A B. w. g., No. 36 silk covered manganin wire was used, which has the following constants.

d = diameter (bare) = 0.01015 cm.
D = diameter (silk covered) = 0.0198 cm.
R = 46.5 ohms per wire meter.

Two parallel lengths of this wire, as shown in Fig. 1, were put lightly in contact, resulting in a bifilar resistance. This bifilar loop was arranged as shown in Fig. 2, in order to measure its impedance. In Fig. 2, T, is about 5.75 meters and U is about 6.5 cm.

The wire was stretched in a vertical plane, parallel to the wooden wall of the room, and the distance between the parallels was about 13 cm. By the arrangement described above, the mutual electrostatic

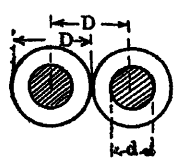

FIG. 1—CROSS SECTION OF THE BIFILAR RESISTANCE

and magnetic effects in the lengths of wire and those between them and earth, which must exist in a practical bifilar coil were minimized. This made an ideal bifilar resistance, which is identical with a simple uniform cable line. The length from A to B (one half of the total wire length) was 215 meters, and 20,000 ohms was selected as the direct-current resistance of this bifilar coil, when B was short-circuited. In order to observe the variation of the impedance of the ideal bifilar resistance with the frequency, the impedance measurement was taken over the audio-frequency range. The measuring arrangement is shown in Fig. 3 where the plieotron oscillator is used as the alternating-current source, and the frequency is changed by adjusting the condenser C_1. By connecting the A end of the bifilar coil (Fig. 2) to the arm A of the impedance bridge, the impedance of the bifilar coil can be measured by adjusting R and C. Curtis coil resistances were used as the non-reactive resistances represented by R_1, R_2, R. These Curtis coils were previously ascertained to be practically non-reactive, compared with the non-reactive resistances of a mannit solution and a graphite-bar. The results of measurement are recorded in Table I. In this Table, Z_s is the impedance when the B end is short-circuited and

$$Z_s = r_s - j\frac{1}{\omega c_s} \qquad \text{Ohms} \angle$$

In the same Table, Z_f is the impedance when the B end is open-circuited and

$$Z_f = r_f - j\frac{1}{\omega c_f} \qquad \text{Ohms} \angle$$

The short-circuit impedance Z_s is vectorially represented in Fig. 4. As can be seen in this figure, the size of the short-circuit impedance Z_s is almost constant up to 1000 cycles per second; but the condensive reactance increases rapidly at first, as the frequency becomes larger. The increasing rate of the condensive reactance becomes gradually smaller, and the decreasing rate of the effective resistance becomes larger. Near the frequency of 2500 cycles per second, the condensive reactance becomes a maximum.

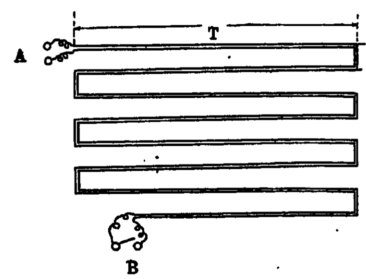

FIG. 2—IDEAL ARRANGEMENT OF THE BIFILAR RESISTANCE

Next, the arrangement of the bifilar resistance was left unchanged, as shown in Fig. 2, but the wire was coated with a paraffin solution over the silk insulation, as shown in Fig. 5. The result of the measurement of this bifilar resistance is shown in Table II and Fig. 6.

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The impedance characteristic curve is similar to that of the previous case, but the curve is elongated on the same locus.

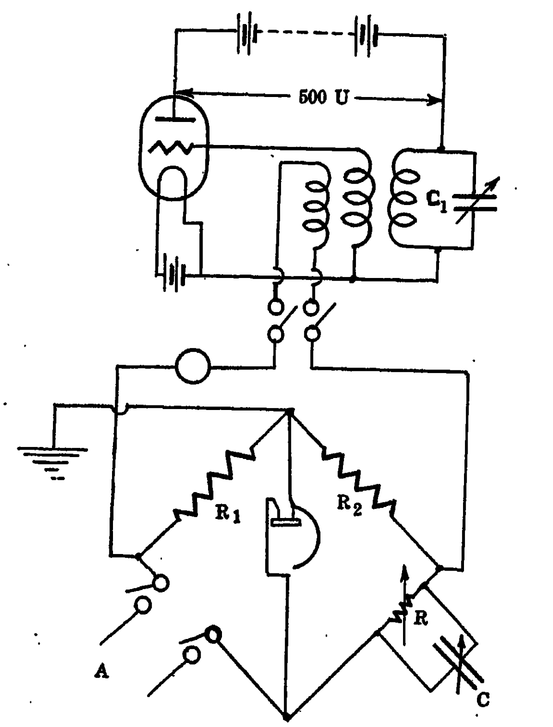

FIG. 3—DIAGRAM OF CONNECTIONS FOR MEASURING THE IMPEDANCE OF THE BIFILAR RESISTANCE

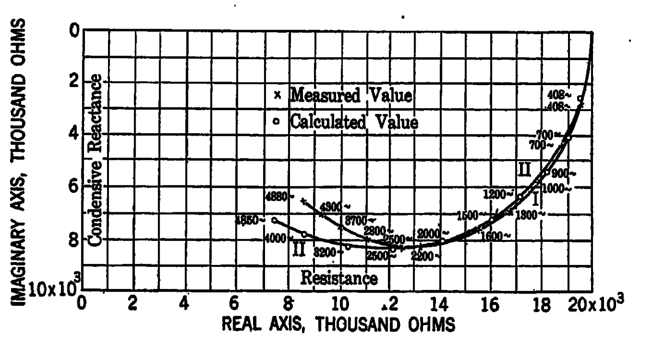

FIG. 4—IMPEDANCE CHARACTERISTICS OF THE BIFILAR RESISTANCE IN THE IDEAL ARRANGEMENT (NOT IMPREGNATED WITH PARAFFIN)

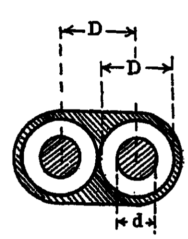

FIG. 5—CROSS SECTION OF THE BIFILAR RESISTANCE (PARAFFINED)

The arrangement shown in Fig. 2 was then taken down, and wound progressively along the axis of a bobbin as shown in Fig. 7. By winding in this way, we reduce the effect of stored electrostatic energy, *i. e.*, the condensive reactance effect, as the turns which have a greater potential difference are thus not contiguous. The impedance of this wound bifilar coil, was then measured. The result is shown in Table III and Fig. 8. In this case, the locus is almost identical with the other two already mentioned, but points of same frequency shift forward on the same locus.

II—Theory of a Bifilar Coil as an Electric Cable

As can be seen from the experimental results stated above, the impedance characteristic of the bifilar resistance is very similar to that of an electric cable. The idea of this similarity can be realized from the construction of the bifilar coil, which has a smaller linear self-inductance and larger linear capacity, as it is formed by wires closely in contact. With this idea, we treated the bifilar resistance as a uniform electric line and found its constants from the short-

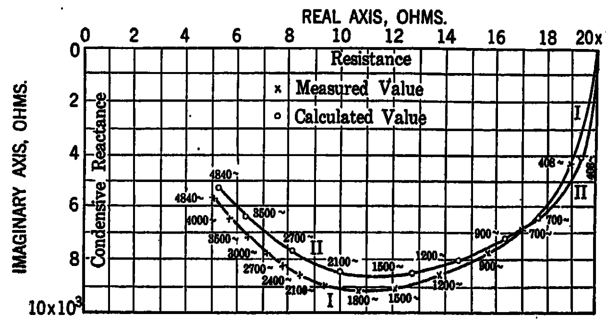

FIG. 6—IMPEDANCE CHARACTERISTICS OF THE BIFILAR RESISTANCE IN THE IDEAL ARRANGEMENT (IMPREGNATED WITH PARAFFIN)

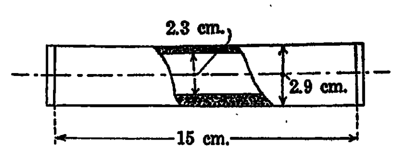

FIG. 7—ACTUAL ARRANGEMENT OF THE BIFILAR RESISTANCE COIL

circuit impedance Z_s and open-circuit[1] impedance Z_f

The surge impedance

$$z_0 = \sqrt{Z_s Z_f} \qquad \text{Ohms} \angle \quad (1)$$

The linear hyperbolic angle

$$\alpha = \frac{1}{x} \tanh^{-1} \sqrt{\frac{Z_s}{Z_f}} \qquad \text{hyps.} \angle \quad (2)$$

where x is the total length of the bifilar winding.

The linear impedance

$$z = z_0 \alpha = \frac{1}{x} \sqrt{Z_s Z_f} \tanh^{-1} \sqrt{\frac{Z_s}{Z_f}} \qquad \text{Ohms} \angle \quad (3)$$

The linear admittance

1. Bibliography 1.

$$y = \frac{\alpha}{z_0} = \frac{1}{x}\ \frac{1}{\sqrt{Z_s Z_f}} \tanh^{-1} \sqrt{\frac{Z_s}{Z_f}}$$

Mhos ∠ (4)

Columns 8, 9, 10 and 11 in Tables I, II and III represent the surge impedance z_0, total hyperbolic angle $\theta = x\,\alpha$, total impedance $Z = x\,z$, and total admittance $Y = x\,y$, which are calculated by the above equations from the measured values of Z_s and Z_f. In all these results, the slope of the total impedance Z

TABLE I

TABLE OF CONSTANTS OF THE BIFILAR RESISTANCE WHEN THE WIRE WAS COVERED WITH SILK ONLY

f ~	r_s ohm	c_s μf	Z_s ohm	r_f ohm	c_f μf	Z_f ohm	z_0 ohm	$\theta = \alpha x$	Z ohm	Y ∠φ mho	$c_1 = \frac{\gamma \sin \varphi}{\omega x}$ μf
0	20000		20000	1.9×10^7		1.9×10^7					
408	19570	0.14500	19570−j2690	7540	0.00761	7540−j51400	32100 ⦣44°44′	0.620 ∠44°30′	19800 ⦣0°14′	1.93×10^{-5} ∠89°14′	0.0350
700	18850	0.05230	18850−j4350	7470	0.00754	7470−j30100	24730 ⦣44°32′	0.83 ∠44°15′	20550 ⦣0°17′	3.36×10^{-5} ∠88°47′	0.0354
900	18250	0.03310	18250−j5350	7400	0.00746	7400−j23700	21720 ⦣44°31′	0.940 ∠44°	20450 ⦣0°31′	4.335×10^{-5} ∠88°31′	0.0356
1300	16800	0.01810	16800−j6720	7340	0.00734	7340−j16700	18170 ⦣44° 4′	1.125 ∠43°40′	20420 ⦣0°23′	6.20×10^{-5} ∠87°43′	0.0352
1600	15700	0.01335	15700−j7450	7270	0.00716	7270−j13900	16500 ⦣43°54′	1.250 ∠43°50′	20600 ⦣0° 4′	7.68×10^{-5} ∠87°44′	0.0356
1900	14550	0.01055	14550−j7950	7210	0.00703	7210−j11910	15170 ⦣43°42′	1.37 ∠43°50′	20800 ⦣0° 7′	9.40×10^{-5} ∠87°32′	0.0352
2200	13550	0.00880	13550−j8230	7120	0.00683	7120−j10630	14250 ⦣43°45′	1.47 ∠43°45′	20900 ⦣0° 0′	10.35×10^{-5} ∠87°30′	0.0348
2500	12630	0.00780	12630−j8160	7060	0.00669	7060−j 9510	13550 ⦣43° 8′	1.56 ∠45°	21100 ∠1°51′	11.47×10^{-5} ∠88°8′	0.0340
2800	11920	0.00705	11920−j8060	7020	0.00655	7020−j 8690	12680 ⦣42°34′	1.635 ∠44°50′	20700 ∠2°15′	12.90×10^{-5} ∠87°24′	0.0341
3100	11190	0.00645	11190−j7960	6960	0.00640	6960−j 8000	12050 ⦣42°12′	1.72 ∠44°50′	20700 ∠2°37′	14.23×10^{-5} ∠87° 2′	0.0340
3400	10600	0.00605	10600−j7750	6900	0.00625	6900−j 7500	11590 ⦣41°48′	1.78 ∠45°	20600 ∠3°11′	15.38×10^{-5} ∠86°48′	0.0336
3700	10100	0.00575	10100−j7500	6830	0.00604	6830−j 7130	11220 ⦣41°25′	1.82 ∠44°50′	20400 ∠3°25′	16.2×10^{-5} ∠86°15′	0.0324
4000	9700	0.00545	9700−j7310	6800	0.00591	6800−j 6750	10790 ⦣40°54′	1.90 ∠45°	20500 ∠4° 5′	17.6×10^{-5} ∠85°54′	0.0324
4300	9300	0.00523	9300−j7100	6720	0.00580	6720−j 6390	10425 ⦣40°27′	1.93 ∠45°	20100 ∠4°33′	18.48×10^{-5} ∠85°27′	0.0316
4600	9000	0.00505	9000−j6850	6700	0.00571	6700−j 6070	10110 ⦣39°42′	1.89 ∠45°	20000 ∠5°18′	19.55×10^{-5} ∠84°42′	0.0313
4880	8750	0.00502	8750−j6500	6680	0.00575	6680−j 5700	9780 ⦣38°34′	2.02 ∠45°	19800 ∠6°26′	20.60×10^{-5} ∠83°34′	0.0310

TABLE II

TABLE OF CONSTANTS OF THE BIFILAR RESISTANCE WHEN THE WIRE WAS COATED WITH PARAFFIN OVER THE SILK INSULATION

f ~	r_s ohm	c_s μf	Z_s ohm	r_f ohm	c_f μf	Z_f ohm	z_0 ohm	$\theta = \alpha x$	Z ohm	Y ∠φ mho	$c_2 = \frac{\gamma \sin \varphi}{\omega x}$ μf
0	19975		19975	6.3×10^6		6.3×10^6					
408	18900	0.08790	18900−j4435	6880	0.01220	6880−j32000	25100 ⦣45°32′	0.794 ∠43°50′	19940 ⦣1°42′	3.163×10^{-5} ∠89°22′	0.0575
700	17160	0.03340	17160−j6800	6700	0.01175	6700−j19320	19450 ⦣46°15′	1.048 ∠43°30′	20380 ⦣2°45′	3.590×10^{-5} ∠89°45′	0.0565
900	15800	0.02270	15800−j7790	6590	0.01147	6590−j15410	17180 ⦣46°25′	1.182 ∠43°25′	20310 ⦣3° 8′	6.890×10^{-5} ∠89°58′	0.0567
1200	13820	0.01535	13820−j8650	6460	0.01074	6460−j12350	15080 ⦣47°14′	1.360 ∠43°15′	20500 ⦣3°59′	9.020×10^{-5} ∠90°29′	0.0559
1500	12070	0.01167	12070−j9100	6330	0.01025	6330−j10360	13560 ⦣47°48′	1.538 ∠43°10′	20710 ⦣4°38′	1.126×10^{-4} ∠90°58′	0.0559
1800	10650	0.00960	10650−j9220	6190	0.00971	6190−j 9110	12460 ⦣48°21′	1.675 ∠44° 0′	20870 ⦣4°21′	1.344×10^{-4} ∠92°22′	0.0554
2100	9500	0.00842	9500−j9000	6050	0.00916	6050−j 8265	11585 ⦣48°38′	1.800 ∠44° 7′	20850 ⦣4°31′	1.554×10^{-4} ∠92°45′	0.0546
2400	8510	0.00766	8510−j8660	5900	0.00870	5900−j 7620	11015 ⦣48°53′	1.912 ∠44°16′	21070 ⦣4°37′	1.735×10^{-4} ∠93° 9′	0.0535
2700	7810	0.00716	7810−j8220	5750	0.00846	5750−j 6960	10110 ⦣48°27′	1.998 ∠45°10′	20200 ⦣3°17′	1.976×10^{-4} ∠93°37′	0.0541
3000	7230	0.00682	7230−j7850	5650	0.00803	5650−j 6600	9620 ⦣48° 1′	2.075 ∠44°36′	19970 ⦣3°25′	2.159×10^{-4} ∠92°37′	0.0532
3500	6360	0.00631	6360−j7200	5430	0.00738	5430−j 6150	8875 ⦣48°33′	2.256 ∠44°10′	20010 ⦣4°23′	2.546×10^{-4} ∠92°46′	0.0552
4000	5700	0.00611	5700−j6520	5260	0.00710	5260−j 5610	8160 ⦣47°50′	2.440 ∠44°30′	19900 ⦣3°20′	2.980×10^{-4} ∠92°20′	0.0552
4840	5050	0.00572	5050−j5750	5000	0.00644	5000−j 5100	7395 ⦣47° 8′	2.700 ∠44°48′	19950 ⦣2°20′	3.653×10^{-4} ∠91°56′	0.0551

TABLE III

TABLE OF CONSTANTS OF THE BIFILAR RESISTANCE WHEN THE WIRE WAS WOUND ON THE BOBBIN

f ~	r_s ohm	c_s μf	Z_s ohm	r_f ohm	c_f μf	Z_f ohm	z_0 ohm	$\theta = \alpha x$	Z ohm	Y ∠φ mho	$c_3 = \frac{\gamma \sin \varphi}{\omega x}$ μf
0	19998		19998	1.35×10^7		1.35×10^7					
408	18600	0.08100	18600−j4815	7020	0.01394	7020−j28000	23550 ⦣45°14′	0.855 ∠44° 0′	20100 ⦣1° 4′	3.63×10^{-5} ∠89°14′	0.0671
700	16570	0.03190	16570−j7150	6950	0.01350	6950−j16720	18300 ⦣45°23′	1.145 ∠43°50′	21000 ⦣1°33′	6.24×10^{-5} ∠89°13′	0.067
900	15000	0.02183	15000−j8100	6900	0.01325	6900−j13360	16240 ⦣45°32′	1.290 ∠43°25′	21000 ⦣2° 9′	7.95×10^{-5} ∠88°57′	0.0666
1200	12920	0.01500	12920−j8850	6800	0.01265	6800−j10500	14000 ⦣45°42′	1.50 ∠45°42′	21000 ⦣2°12′	1.07×10^{-4} ∠89°12′	0.066
1500	11020	0.01200	11020−j8850	6690	0.01205	6690−j 8820	12510 ⦣45°48′	1.67 ∠43°30′	20900 ⦣2°18′	1.335×10^{-4} ∠89°18′	0.0669
1800	9900	0.01010	9900−j8760	6610	0.01153	6610−j 7670	11550 ⦣45°22′	1.85 ∠45° 0′	21300 ⦣0°22′	1.6×10^{-4} ∠90°22′	0.066
2100	9000	0.00915	9000−j8280	6510	0.01047	6510−j 7240	10920 ⦣45°19′	1.96 ∠45° 0′	21400 ⦣0°19′	1.79×10^{-4} ∠90°19′	0.0643
2400	8080	0.00835	8080−j7950	6410	0.00998	6410−j 6650	10200 ⦣45°19′	2.11 ∠45°30′	21500 ∠0°11′	2.07×10^{-4} ∠90°49′	0.0640
2700	7710	0.00798	7710−j7400	6290	0.00943	6290−j 6250	9740 ⦣44°19′	2.15 ∠45°30′	21000 ∠1°11′	2.2×10^{-4} ∠89°49′	0.0618
3000	7250	0.00765	7250−j6950	6170	0.00909	6170−j 5840	9230 ⦣43°39′	2.21 ∠45°30′	20400 ∠1°51′	2.39×10^{-4} ∠89° 9′	0.0590
3500	6710	0.00711	6710−j6400	5980	0.00840	5980−j 5410	8640 ⦣42°54′	2.35 ∠44° 0′	20400 ∠1° 6′	2.72×10^{-4} ∠86°54′	0.0555
4000	6260	0.00684	6260−j5820	5900	0.00790	5900−j 5040	8130 ⦣41°42′	2.51 ∠43°30′	20400 ∠1°48′	3.1×10^{-4} ∠85°12′	0.0560
4840	5840	0.00652	5840−j5050	5700	0.00729	5700−j 4510	7490 ⦣39°36′	2.70 ∠43° 0′	20200 ∠1°48′	3.61×10^{-4} ∠82°36′	0.0481

is nearly zero, and that of the total admittance Y is nearly 90 deg. From these facts, we can see that linear impedances are approximately pure resistances, and linear admittances are approximately pure susceptances. The quotient of the imaginary part of the linear admittance $y = g + j\,\omega\,c$, divided by ω, is

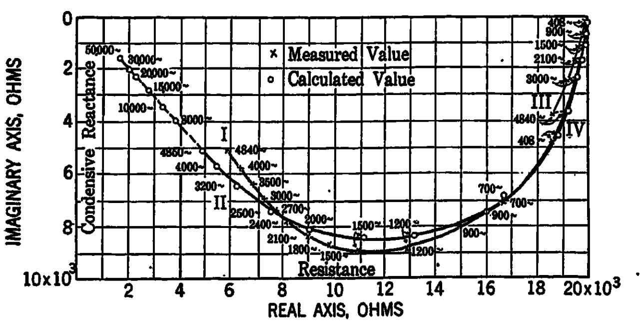

FIG. 8—IMPEDANCE CHARACTERISTICS OF THE BIFILAR RESISTANCE WHEN WOUND ON A BOBBIN

the linear capacity. In Tables I, II and III, column 12 represents these linear capacities in micro-farads per km. The value of these capacities slightly decreases as the frequency increases; but can be considered to be practically constant. Now we take the average value of these linear capacities and derive the equation

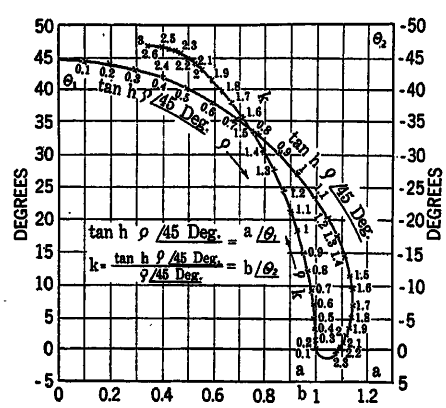

FIG. 9—CHART SHOWING THE VALUE OF CORRECTING FACTOR k

of a uniform line which has the linear impedance of pure resistance r and linear admittance of pure condensive susceptance $\omega\,c$. So $z = r$, $y = j\,\omega\,c$.

The surge impedance and the linear hyperbolic angle are

$$z_0 = \sqrt{\frac{z}{y}} = \sqrt{\frac{r}{j\,\omega\,c}} = \sqrt{\frac{r}{\omega\,c}}\;\backslash 45^\circ \qquad \text{Ohms} \angle \quad (5)$$

$$\alpha = \sqrt{z\,y} = \sqrt{j\,\omega\,c\,r} = \sqrt{\omega\,c\,r}\;/45^\circ \qquad \text{Hyps.} \angle \quad (6)$$

The impedance Z_s of the bifilar coil is

$$Z_s = z_0 \tanh \alpha\,x = \sqrt{\frac{r}{\omega\,c}}\;\backslash 45^\circ \tanh\,(x\,\sqrt{\omega\,c\,r}\;/45^\circ) \qquad \text{Ohms} \angle \quad (7)$$

From these equations, the impedance of the bifilar coil is easily calculated, by the use of a table of semi-imaginary hyperbolic tangents. Fig. 9 shows diagrammatically the semi-imaginary hyperbolic tangent (Bibliography 4, 5). The letters along the curve show the size of the angle, the abscissa shows that of the tanh, and the ordinate the slope of the tanh.

In the case of Table I, $x = 0.215$ km.

$r = 93{,}000$ ohms/km. $c_1 = 0.035\ \mu f$/km.

In the case of Table II, $x = 0.215$ km.

$r = 93{,}000$ ohms/km. $c_1 = 0.0558\ \mu f$/km.

In the case of Table III, $x = 0.215$ km.

$r = 93{,}000$ ohms/km. $c_1 = 0.065\ \mu f$/km.

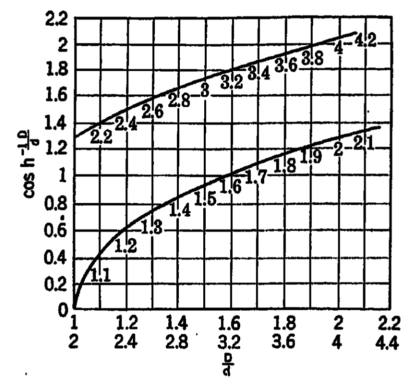

FIG. 10—VALUE OF $\text{COSH}^{-1}\frac{D}{d}$

The value of theoretical impedance which has been calculated from Eq. (7), through the use of Fig. 9, is indicated by Curve II in Figs. 6 and 9. As can be seen in Eq. (7), $\omega\,c$ is always combined as one term, so when r is constant, the change in the value of c does not change the impedance locus, but the point corresponding to a given frequency shifts forward on the same locus by the increase of c comparing these results with the results of measurement, identity can

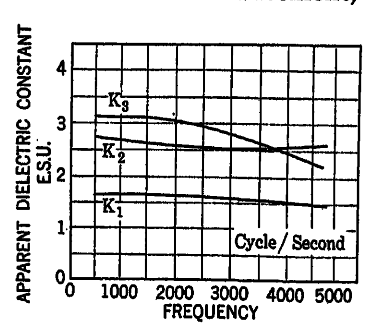

FIG. 11—APPARENT DIELECTRIC CONSTANT

be found in both cases. We can thus treat the problem of the bifilar coil by the theory of the ideal electric cable.

III—Apparent Dielectric Constant

We may compare the linear capacity obtained by experiment with the linear capacity obtained by theory.

When the line is constructed as shown in Fig. 1, and the insulating medium is air, the theoretical linear capacity c_0 is

$$c_0 = \frac{0.02778}{\cosh^{-1}\dfrac{D}{d}} = 0.02135\ \mu f/\text{km}. \qquad (8)$$

Where $d = 0.01015 \quad D = 0.0198$ cm.

Fig. 10 shows the curve of $\cosh^{-1}\dfrac{D}{d}$ for the smaller values of $\dfrac{D}{d}$, for aid in design. In the ideal case of the bifilar coil, the linear resistance is constant for a given resistance wire, and the linear capacity only changes with the arrangement of the wire. From these considerations, we call the ratio of the linear capacity c which is obtained by experiment, with a theoretical linear capacity c_0 as an apparent dielectric constant of a given line arrangement. The calculation of the design will be conveniently made on the basis of this dielectric constant. The apparent dielectric constant

$$K_1 = \frac{c_1}{c_0}, \; K_2 = \frac{c_2}{c_0}, \; K_3 = \frac{c_3}{c_0} \qquad (9)$$

These data in Tables I, II and III are shown diagrammatically in Fig. 11 as a function of the frequency.

In the case of Table I, the apparent dielectric constant K_1 has the value of nearly 1.6, and is independent of the frequency. This value is satisfactory, as silk is used for insulation. As can be seen in Table II, when paraffin is used for the insulation, the apparent dielectric constant K_2 is approximately 2.6, and independent of the frequency. In Fig. 5 the cross-hatched parts are filled with paraffin, which has a dielectric constant nearly 2, so the increase of apparent dielectric constant to the value of 2.6 is reasonable. Next in the case of Table III, when the wire is wound on the bobbin, the apparent dielectric constant K_3 increases to 3.1, and is almost constant in the frequency range up to 2000 ~; but in the higher frequencies decreases remarkably. In this case, turns of the wire at different potentials are in contact with each other, so that the apparent dielectric constant may increase to some extent (in this example approximately 20 per cent) and owing to relative electrical conditions between the wires, complicated electrical phenomena may occur in the winding, consequently the apparent dielectric constant changes with the frequency. But as in this case, if the apparent dielectric constant does not vary beyond a reasonable range, it may be conveniently used for calculation in the design. The inductive effects of the current will probably cancel as the bifilar is wound at random on the bobbin. In the practical design, we should have more experimental constants, but the writers have not sufficient data, so we assume the value of K to be large enough to make allowance for the error.

IV—Method of Reducing the Error in the Bifilar Resistance due to Frequency

The simplest method of reducing the error due to frequency, is to sectionalize the coil into many parts of shorter wire lengths. This can be ascertained from Eq. (7). From Eq. (7)

$$Z_s = x\,r\,\frac{\tanh\left(x\sqrt{\omega\, c\, r}\,\underline{/45^\circ}\right)}{x\sqrt{\omega\, c\, r}\,\underline{/45^\circ}} \quad \text{Ohms} \angle \qquad (10)$$

$z = x\,r$ shows the ideal case where there is no error.

so
$$k = \frac{\tanh\left(x\sqrt{\omega\, c\, r}\,\underline{/45^\circ}\right)}{x\sqrt{\omega\, c\, r}\,\underline{/45^\circ}} \quad \text{numeric} \angle \qquad (11)$$

is the error factor.

As can be seen in Fig. 9, the error factor k is approximately 1 when the total hyperbolic angle $x\sqrt{\omega\, c\, r}\,\underline{/45^\circ}$ of a bifilar coil is very small, so for given ω, C, R, we can reduce the error by sectionalizing the total length of coil into many shorter parts. As shown in Fig. 12,

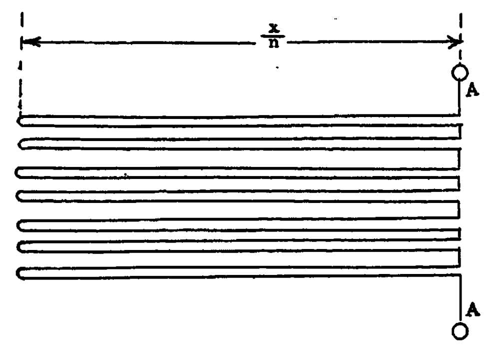

Fig. 12—Sectionalized Bifilar Resistance

if the coil be divided into n sections and connected in series. The impedance Z between both ends A A is

$$Z = n\left\{\frac{x}{n}\,r\,\frac{\tanh\left(\frac{x}{n}\sqrt{\omega\, c\, r}\,\underline{/45^\circ}\right)}{\frac{x}{n}\sqrt{\omega\, c\, r}\,\underline{/45^\circ}}\right\}$$

$$= x\,r\,\frac{\tanh\left(\frac{x}{n}\sqrt{\omega\, c\, r}\,\underline{/45^\circ}\right)}{\frac{x}{n}\sqrt{\omega\, c\, r}\,\underline{/45^\circ}} \quad \text{Ohms} \angle \qquad (12)$$

The error factor

$$\frac{\tanh\left(\frac{x}{n}\sqrt{\omega\, c\, r}\,\underline{/45^\circ}\right)}{\frac{x}{n}\sqrt{\omega\, c\, r}\,\underline{/45^\circ}}$$

will become smaller by increasing the number of sections n.

When the total resistance $x\,r = R$ is given,

$$\frac{x}{n}\sqrt{\omega\,c\,r}\;\underline{/45^\circ} = \frac{R}{n}\sqrt{\frac{\omega\,c}{r}}\;\underline{/45^\circ}$$

As can be seen from the above equations, the error can be reduced by the use of material of larger linear resistance r; but this method will be practically restricted. On the other hand, the method of sectionalizing is simple and efficient, and is not difficult in practise. We divided the bifilar coil which had been used in previous experiment, into four sections. To obtain the value of k when $\omega = 20{,}000$, ($f = 3180 \sim$), $K = 3.1$, $c_0 = 0.02135\ \mu f/\text{km.}$, $c = c_0 = 0.0662$ $\mu f\,/km.$

The linear hyperbolic angle α is

$$\alpha = \sqrt{20{,}000 \times 0.0662 \times 10^{-6} \times 93{,}000}\;\underline{/45^\circ}$$
$$= 11.09\;\underline{/45^\circ} \quad \text{Hyps. } \angle$$

The total hyperbolic angle θ of one section is

$$\theta = \frac{0.215}{4}\,\alpha = 0.597\;\underline{/45^\circ} \qquad \text{Hyps. } \angle$$

From Fig. 9,

$$k = \frac{\tanh 0.597\;\underline{/45^\circ}}{0.597\;\underline{/45^\circ}} = 0.99\;\overline{\backslash 6^\circ}\,5$$

When the coil is sectionalized into four sections, the error is 1 per cent in size, and 6 deg. 30 min. in slope, at $\omega = 20{,}000$. In order to test the theory, we sectionalized the coil which has been shown in Fig. 7 into 20

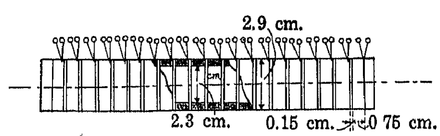

FIG. 13—ACTUAL ARRANGEMENT OF THE SECTIONALIZED BUFILAR RESISTANCE

sections, as shown in Fig. 13. It was first connected as the 4-section coil, and the frequency character of the impedance was measured. The result is shown in

TABLE IV
TABLE OF CONSTANTS WHEN THE BIFILAR RESISTANCE WAS DIVIDED INTO 4 SECTIONS

$f \sim$	r_s ohm	$c_s\ \mu f$	Z_s ohm
408	19960	1.025	19960$-j$ 382
900	19780	0.231	19780$-j$ 765
1500	19620	0.0858	19620$-j$1240
2100	19410	0.0445	19410$-j$1700
3000	19130	0.022	19130$-j$2410
4500	18600	0.0087	18600$-j$3770

Table IV and Fig. 8 by Curve III. Curve IV in Fig. 8 is that calculated by Eq. (**12**), and Fig. 9, and the differences between these curves are very small. Next we made the coil into 20 sections, by suitably connecting the same coils. We could detect no reactance in this sectionalized coil.

V—Calculation Example

Using a resistance material of the following constants. B. w. g., No. 44 silk covered manganine wire.

d = diameter (bare) = 3 mils = 0.00761 cm.

D = diameter (silk covered) = 7 mils = 0.01778 cm.

r = 1.424×10^5 ohms per loop km. ($\frac{1}{2} \times 1.424 \times 10^5$ ohms per wire km.)

The number of bifilar sections is required such that the phase-angle error for a 100,000-ohm coil is less than 5 deg. at $f = 5000 \sim$. From Fig. 10,

$$c_0 = \frac{0.02778}{\cosh^{-1}\dfrac{D}{d}} \times 10^6 = \frac{0.02778}{1.493} \times 10^{-6}$$
$$= 1.86 \times 10^{-8} \text{ farads per km.}$$

Taking the apparent dielectric constant to be 5

$c = 5\,c_0 = 9.3 \times 10^{-8}$ farads per km.

Therefore the linear hyperbolic angle at $f = 5000 \sim$, is

$$\alpha = \sqrt{2\,\pi \times 5000 \times 9.3 \times 10^{-8} \times 1.424 \times 10^5}\;\underline{/45^\circ}$$
$$= 20.4\;\underline{/45^\circ}$$

The total length x of the bifilar of 100,000 ohms is

$$x = \frac{100{,}000}{1.424 \times 10^5} = 0.701 \text{ km.}$$

Therefore total hyperbolic angle is

$$\theta = \alpha\,x = 14.3\;\underline{/45^\circ}$$

By Fig. 9, $\rho = 0.5$ when $\theta_2 = \overline{\backslash 5^\circ}$.

Therefore the number of sections required is

$$n = \frac{14.3}{0.5} = 28.6 \cong 30$$

In Fig. 9, $k = 0.99$ when $\rho = 0.5$. In this case the error in the size of the impedance is 1 per cent. This value is permissible for practical purposes, so three sections per 10,000 ohms would be sufficient.

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The Trenton Channel Plant
of the Detroit Edison Company

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Synopsis.—*This plant is the second to be built by this Company well outside the corporate limits of Detroit, the two being connected by a 120-kv. tower line arranged for supplying suburban areas and the outer part of the city. The plant has a planned ultimate capacity of 300,000 kw. It contains both d-c. and a-c. house service, turbine-driven units and nearly all important variable speed auxiliaries are driven by d-c. motors. Coal is used in pulverized form, the paper giving the conclusions which led to its adoption.*

THE Trenton Channel Plant is one of several interconnected plants serving Detroit and surrounding territory. Its location and many of its characteristics are determined by this fact and a brief statement of some of the characteristics of the territory and system is therefore desirable.

Detroit is located on the Detroit River and has a roughly semicircular shape with the river bank serving as the diameter. This can be seen in Fig. 1, in which the territory served by The Detroit Edison Company is indicated. It will be seen that the City of Detroit is on what might be called one edge of this area.

Steam plants generating power for the supply of Detroit and surrounding territory must logically be located on the water front provided by the Great Lakes System since no rivers in this region are large enough to supply the circulating water requirements of stations of modern size. This brings about a situation in which the power is necessarily generated along the easterly boundary of the area, that is externally, except in so far as a few small hydroelectric plants located on the Huron River give what might be called an internal supply.

The first two large steam plants of the company are located within the city boundaries and are known respectively as the Delray and the Connors Creek Stations. The power generated by these stations is carried out underground and most of it at 23,000 volts. The territory external to Detroit was thus necessarily supplied by lines radiating from Detroit and fed by the underground system.

This arrangement came about quite naturally and was quite satisfactory for some years. However, as the density of load in Detroit and in the surrounding territory increased it brought about several very illogical, undesirable and costly consequences. The most obvious were:

1. Power generated in plants located on costly land within the city and subject to high city taxes was transmitted over costly underground structure, also subject to city taxes, for the purpose of supplying overhead transmission lines serving small suburban towns and cities and country areas.

2. The power required for service to the least saturated territory had to be passed through the most nearly saturated territory thus unnaturally increasing the difficulties brought about by congested streets, high property values and the like.

3. The point of power generation was located some distance from a few of the larger suburban loads, as at Pontiac and Port Huron, making regulation difficult or exceedingly complicated and costly.

4. The coal supply for these plants comes in by rail from the South and must be handled through congested city yards and over congested terminal railroads while much of the power generated from this coal was again shipped out of the congested territory.

In addition to these obvious disadvantages it became evident that the total capacity of the combined plants would shortly be required to supply the city alone and that it would not be long before even that capacity would be inadequate for the city's needs. The natural thing to do was to locate future plants outside the city and in such positions as to be of maximum value to both the surrounding territory and the municipal area.

The first of these newer plants was constructed at Marysville near Port Huron. This location was near one end of one of the longest transmission lines and very close to one of the largest concentrated loads outside the city of Detroit. The second of these plants was built almost at the other end of the available water front, namely, just below the city of Trenton and is known as the Trenton Channel Plant.

This location is such that the coal carrying roads run close to the plant on their way to Detroit, so that coal can be delivered to the plant without having to enter the congested Detroit district. The location also has the advantage of placing the plant in a section of the territory which is rapidly developing to a dense industrial district and all indications point to this section as the one which will continue to develop most rapidly in this way. Other advantages of the location will become apparent from subsequent paragraphs. The locations of the two older and the two newer plants are indicated on Fig. 1 and the air-line distances between them are also given.

With these two new plants in existence, an entirely different system of transmitting energy becomes available. A 120,000-volt line which might be de-

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scribed as a trunk has been run between them and it has been given such a course that it swings out into the country areas to the north and west of Detroit. The location of this trunk is shown on Fig. 2 by a heavy line. The lighter lines indicate existing 23,000- and 45,000-volt lines which originally transmitted energy from the Detroit plants.

These lines will ultimately be fed from step-down stations located along the high-tension line and some of them will not only supply the smaller towns and country

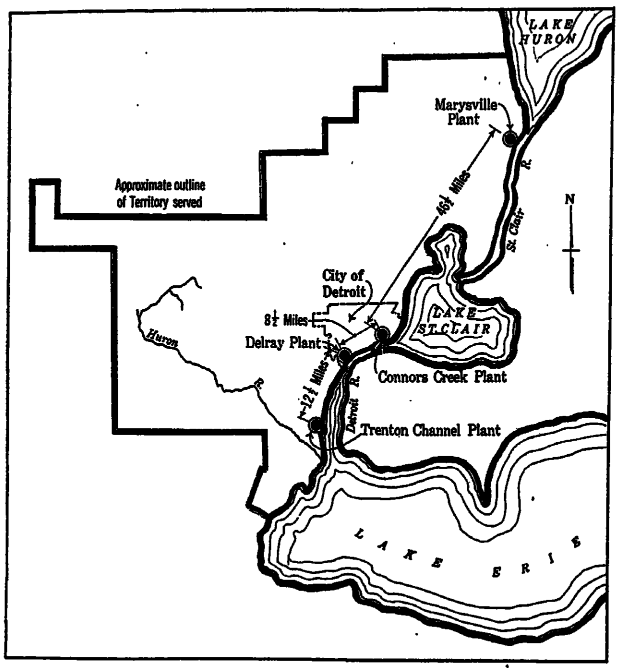

FIG. 1—TERRITORY SERVED BY THE DETROIT EDISON COMPANY

areas but will also feed back into Detroit. This gives a more logical arrangement since large amounts of energy can be brought in through the least congested areas to feed the most congested.

It will be seen in Fig. 2 that the high-tension trunk runs through a place named Brownstown near Trenton Channel and also that high tension lines run roughly north and south from Brownstown. Power generated at Trenton Channel is transmitted to a switching station at Brownstown and is there routed over such circuits of the three radiating lines as may be desired. In addition, some of the power received at Brownstown is there stepped down to 23,000 volts and distributed for the supply of adjacent territory. Power routed to the north is used to supply the industrial area southwest of Detroit and also to supply the west side of Detroit, thus partly relieving the old Delray Station. Power routed to the south now supplies a rather large load in Monroe at the extreme southern end of the territory and will ultimately also supply a heavy industrial development which is expected to follow down along the water front. Power routed to the west supplies the country areas and suburban municipalities and some of it will ultimately be fed back into the northern part of Detroit.

When the lines running out from Trenton Channel are completed there will be three steel-tower lines, each carrying two three-phase circuits of No. 3/0 copper, thus giving six circuits or transmission lines. Four of these will run direct to Brownstown and two will run to one of the outdoor stations lying on the western edge of Detroit. Connections between this station and another of similar character will serve to tie this line from Trenton Channel to the short line running north from Brownstown when desired, thus giving a looped supply to these two important substations.

Arrangements have been made at Trenton Channel to step up to voltages lying between 110,000 as a lower limit and 120,000 as an upper limit. Arrangements are being made for two separate high-tension busses. It is obvious that when completed this will make it possible to supply the loop just referred to at a low voltage and still use the higher voltage on the trunk line if this is required to meet existing conditions of load.

The use of two separate busses also makes it possible to continue and extend a practise which has been a striking characteristic of the operating methods of the company for several years. This practise may be described as a loose linkage or a limited linkage between generating units, using the latter term to mean a group

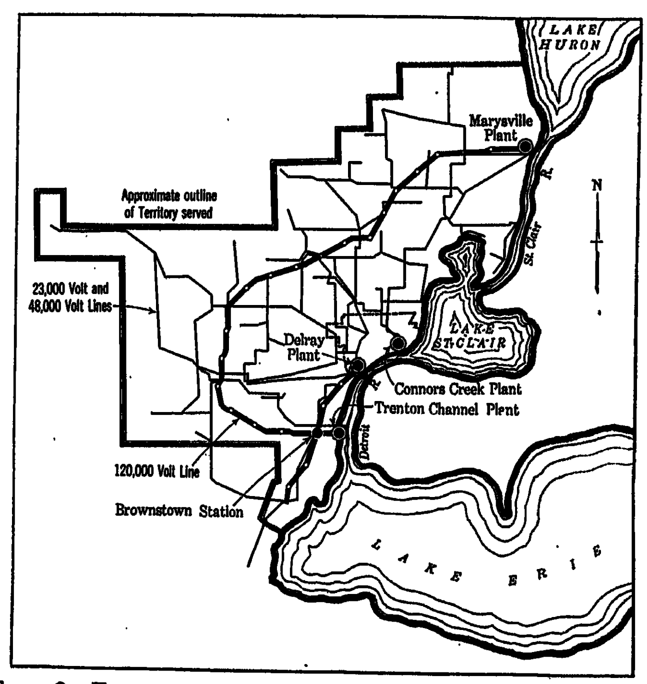

FIG. 2—TRANSMISSION LINES OF THE DETROIT EDISON COMPANY

of turbo generators such as a large section of a single generating plant or an entire plant. The practise was started by dividing up the territory in such a way that each plant supplied the area nearest to it and then connecting the separate areas by the minimum possible amount of copper. As a result, all plants remained in step under normal conditions and even under slightly abnormal conditions such as the loss of a generator at one of the plants. However, a serious fault such as a

bad ground would create sufficient disturbance to break the areas apart, thus confining any subsequent trouble to the affected area. This practise was brought about by an effort to reduce the severity of the service on oil switches.

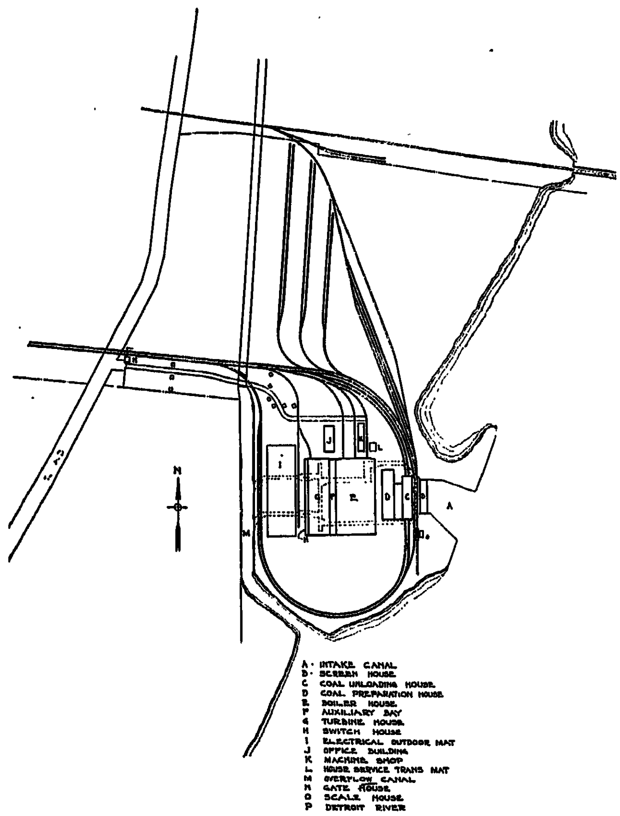

Fig. 3—Ground Plan, Trenton Channel Plant and Yard

The Trenton Channel Plant is located on the southern end of an island in the Detroit River. This island is known as Slocums Island and is or was merely a piece of high ground separated from the high mainland by a narrow marsh. A channel has been dredged through this marsh so that the so-called island is now an island in fact. The plant site is underlain with a good grade of limestone rock at a depth of about 18 to 20 feet beneath the original surface of the ground so that ideal foundation conditions exist.

The plant is so located on the site that its major axes run approximately north and south and the flow of circulating water through it is approximately from the east toward the west. This water is taken from the Trenton Channel of the Detroit River on the east of the island and is discharged into the dredged channel to the west previously mentioned. The general location and arrangement are shown in Figs. 3 and 4.

Starting at the river, the circulating water enters a forebay protected by deflecting piers, booms and skimming aprons and flows through the screen house on its way to the intake tunnel under the turbine house. The usual arrangement of inclined traveling screens protected by stationary rough screens or grizzlies is used.

Next to the screen house is located the coal unloading house. This contains two through tracks located above track hoppers into which the coal is dropped from gondola type cars. Crane rails, supported on the building columns in the usual way, carry a coal spudding device similar to those used for breaking up and unloading coal in the other plants of this company.

All coal entering the plant or going into storage is

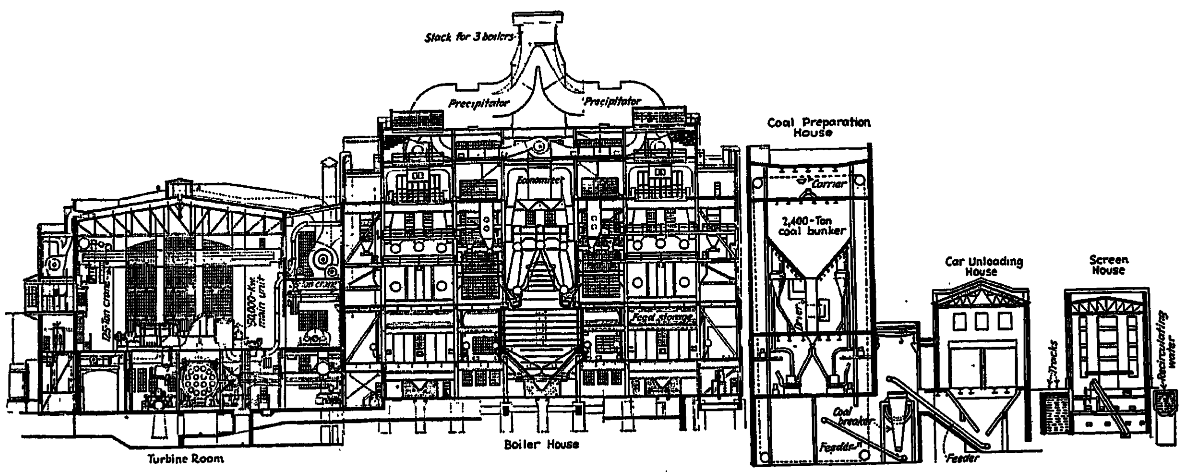

Fig. 4—Cross Section, Trenton Channel Plant

The same practise was adopted with respect to the individual plant. When conditions are sufficiently severe the plant bus is split and the feeders and networks so handled that there is minimum possibility of a serious fault on one part of the bus affecting service from the other part. It will be seen that the double bus arrangement provided at Trenton Channel fits into this method of operating.

weighed on railroad track scales located on a track which runs between the screen house and the unloading house. These scales are indicated in Fig. 3.

Coal dropped into the track hoppers is carried away and elevated by means of pan conveyers which discharge it into the feed end of a Bradford breaker. This arrangement together with subsequent coal handling equipment is shown in Fig. 5. From the breaker

the coal is elevated to the green coal bunker at the top of the coal preparation house. From there it gravitates through steam-heated, air-circulating driers into the pulverizing mills. The pulverized fuel is carried out of the mills by air, separated from the air in Cyclone separators and delivered from these by screw conveyers to the hoppers of Fuller-Kinyon pumps. These pumps are coupled to a duplicate piping system which distributes the pulverized fuel to the bunkers in the boiler house.

Pulverized fuel firing was chosen for this plant after elaborate studies which were based on the best attainable data. It was realized that recent developments in stokers combined with the meagerness of information regarding actual operating costs with pulverized fuel made a study of this kind exceedingly difficult and tended to throw doubt on the correctness of the conclusions. However, the following conclusions seemed to be justified:

1. The pulverized fuel method had been developed to such an extent that continuity of service approximately equal to that obtained with stoker firing could be expected.

2. The total cost of steam, including all operating and capital charges, would probably not be any higher with the pulverized fuel method than with stokers and possibly a little lower when using the same number of boilers with either method of firing.

3. It appeared that it would be possible to obtain a flatter efficiency curve over a wide range with pulverized fuel than with stokers and to obtain high boiler ratings with greater facility, thus making it possible to use a smaller number of boilers advantageously and to reduce capital charges by a corresponding amount.

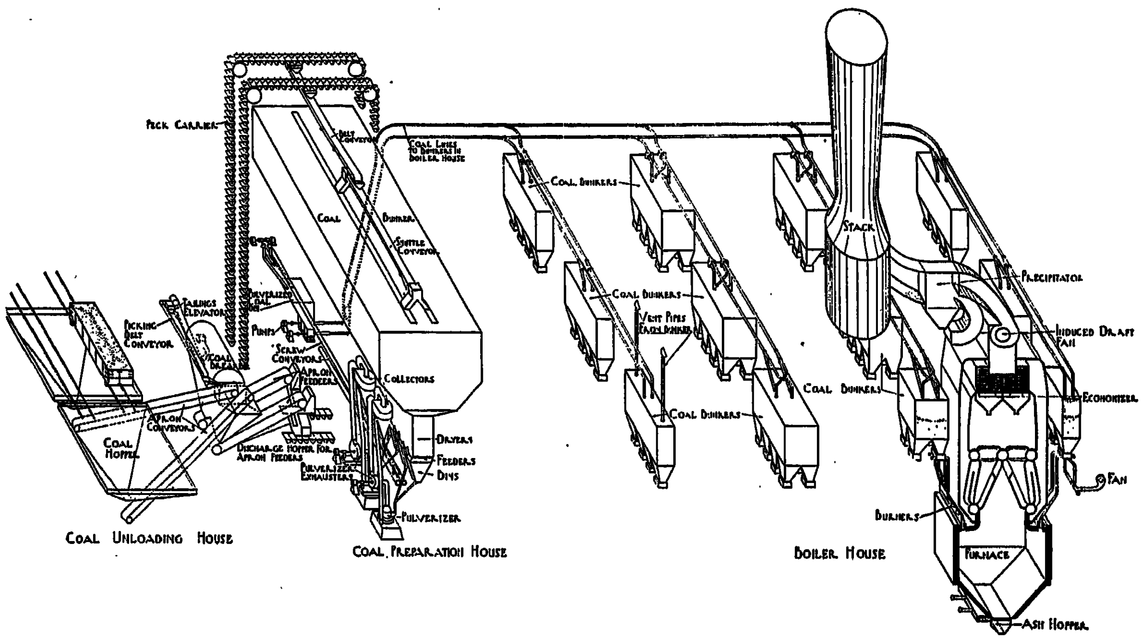

Fig. 5—Coal Handling Preparation, and Firing Equipment

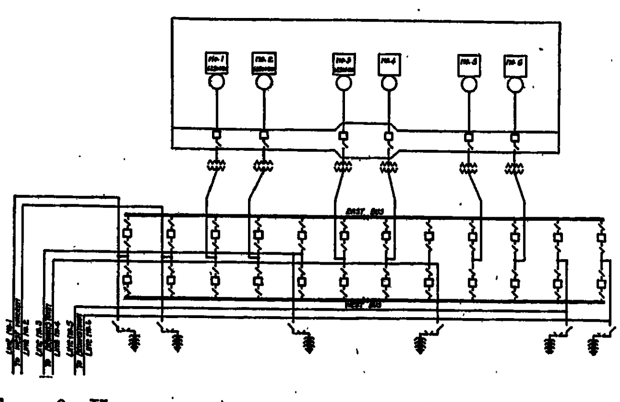

Fig. 6—Ultimate Arrangement of Busses and Lines

4. It seemed certain that a pulverized fuel plant would be less dependent on the supply of a particular quality of coal. This is not intended to mean that it was expected that such a plant could burn all sorts of coal with equal facility but that such variations as must naturally be expected to occur in the deliveries from a given field would not have as great an effect upon capacity and efficiency as had proved to be the case with the older, stoker fired plants. It was also hoped that poorer grades of coal than had been found best suited for stokers of the type previously used might be found to give commercially satisfactory results, thus increasing the flexibility of coal purchasing in several respects.

The decision in favor of pulverized fuel was based very largely upon the fourth conclusion, namely greater flexibility with respect to character of coal. This was particularly important in this case because the plant is so located that cars of poor or questionable coal can easily be dropped there before entry to Detroit. It is thus possible to skim off the best coal, figuratively speaking, for delivery to the Detroit plants and to drop the poorer varieties on the way into the city.

The plant was laid out for an ultimate installed generating capacity of 300,000 kw. The design is based on the use of six 50,000-kw. units. Three have been installed and the fourth will be installed in the near future.

The generators of these units are rated at 62,500 kv-a. at generator voltage corresponding to 120,000 volts on the transmission line. The nominal generator voltage is 12,200. The turbines are designed for best efficiency at a generator output near 43,000 kw. with steam at 370 lb. and 700 deg. fahr., but can carry something over 50,000 kw. at reduced efficiency. Under ordinary conditions the generators will be operated well below their maximum capacity and therefore at low temperatures which should result in long life of insulation. The field circuit has been designed very liberally and the direct-connected exciter is generously large so that occasional operation with high current, as with poor power factor or maximum output at low voltage, is provided for. Great flexibility is thus obtained with respect to generating conditions and full advantage may be taken of the double bus already described and the variation of voltage desirable in feeding from the same plant concentrated loads at short distances and scattered loads at great distances.

Each generator is tied solid to a 25,000-volt, 4000-ampere oil switch which is located in a bay which runs along one side of the turbine room. Beyond this oil switch there is located a disconnecting switch and then the line runs outdoors to the step-up transformers. There are three water-cooled, outdoor-type transformers per generator and they step up through delta-Y connections to bus voltage, that is, to a value between 110,000 and 120,000 as desired. The transformer ratio is fixed, the variation of voltage being obtained at the generators.

The step-up transformers are connected to the outdoor busses through 132,000-volt, 400-ampere, outdoor-type oil circuit breakers with the usual disconnecting switches.

Thus, each generator with its own set of transformers and its switches is a separate unit right up to the high-tension, or transmission line bus. The lower-voltage, indoor switch is used for normal operations such as connecting a generator to the bus after synchronizing. Both the indoor and the outdoor switches, however, are automatic under relay operation.

The neutral point of each generator and the neutral point of each step-up transformer Y is connected solidly to ground. Differential relay protection is provided for each generator and bank of step-up transformers as a unit and differential and overload relay protection is provided on out-going lines, the pairs of lines normally being operated in parallel.

A single line diagram of generator connections, outdoor yard and out-going lines is shown in Fig. 6 in the form which will be assumed when the plant is completed. A cross-section of the electrical end of the house and the outdoor yard is shown in Fig. 7.

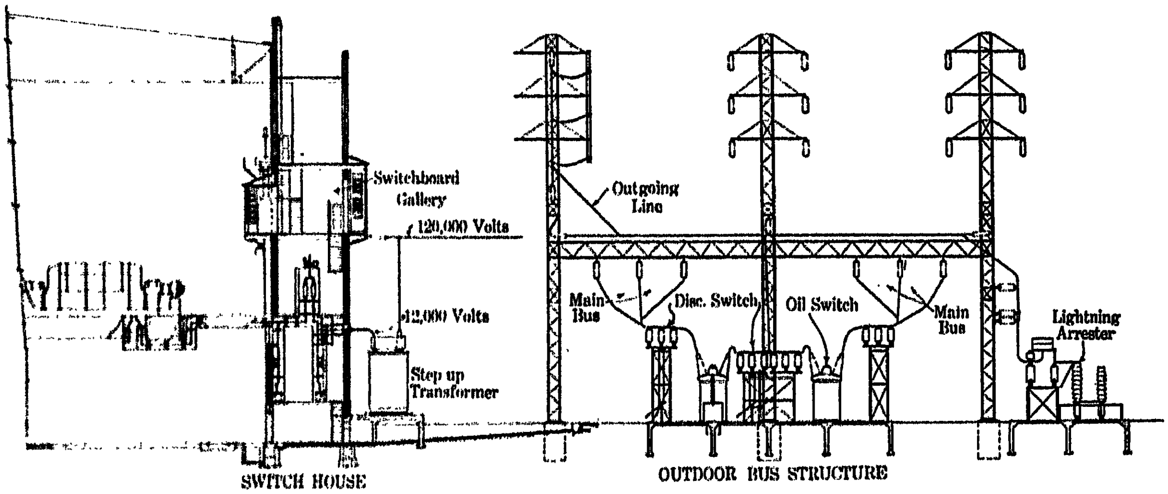

FIG. 7—ARRANGEMENT OF ELECTRICAL END OF PLANT

The uncertainty with respect to what limitations might be discovered in the use of pulverized fuel dictated a rather generous design of boiler room. It seemed probable that eighteen boilers of the size chosen should be sufficient for a 300,000-kilowatt station but the design was so arranged that three more could be added if necessary. To date operation has indicated that eighteen will certainlv be sufficient and it is possible that a still smaller number will prove satisfactory.

The boilers chosen are a double-ended, five drum, curved tube type similar to those used in the older plants of the company. They have, however, been redesigned in the light of experience and tests, with the result that greater capacity is obtained within a given floor area and height with an increase in the boiler efficiency. The boilers installed contain about 29,000 sq. ft. of saturated surface designed for an opera-

ting pressure of 416 lb. per sq. in. Hearth screens add about 1200 ft. of saturated surface. The superheaters give a steam temperature of about 700 deg. fahr. Each boiler is surmounted by two steel tube economizers built into a single housing, the gases flowing from the dampers through flues and economizers toward the center of width of the economizers and thence through induced draft fans to the stacks.

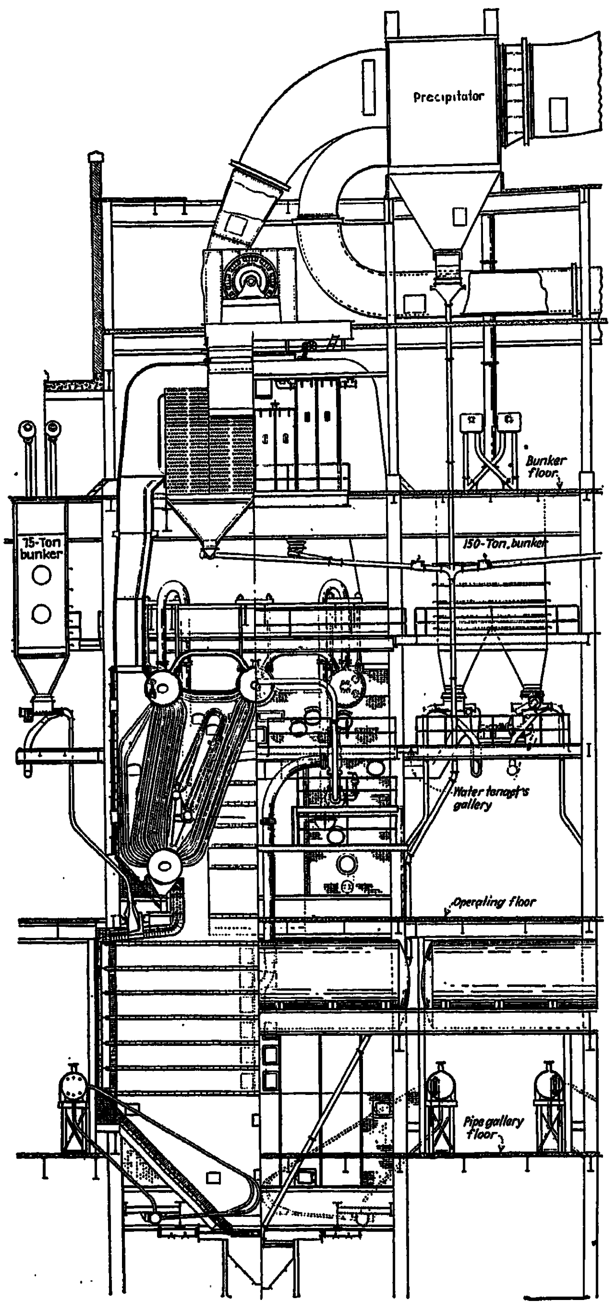

FIG. 8—ELEVATION AND PART SECTION, ONE COMPLETE BOILER UNIT

A vertical, cross section of a complete boiler and furnace unit is shown in Fig. 4 and to a larger scale in Fig. 8. It will be observed that the furnace is of the common air-cooled type and that it is fired from two sides. The ash which collects in the furnace and that which collects in boiler and economizer is all dropped into a sluiceway under the boiler and carried to a settling basin outside the plant. At the present time this material is being used for fill on the property, being transported by means of a dredge pump and moveable pipes.

The Trenton Channel Plant happens to be located within a very short distance of a fine residential community. For this reason it was felt that it would be desirable to determine early in the life of the plant the best means of preventing excessive discharge of ash from the stacks. At the time that the plant was designed the Cottrel precipitator appeared to be the only commercially available device which had been used extensively for separating fine dust from hot gases and such precipitators were therefore installed on two stacks serving six boilers for the purpose of obtaining actual operating experience with them under power plant conditions. The limitations set by dimensions and by cost resulted in the purchase of precipitators intended to remove not much more than 90 per cent of the dust carried toward the stack. Experience to date is not sufficient to justify the drawing of sweeping conclusions but seems to indicate that it is possible by these means to eliminate from 80 to over 90 per cent of the dust without incurring prohibitive capital or operating charges.

The Trenton Channel Plant operates on what is commonly known as the regenerative cycle. That is, the main unit is bled at several points and the steam thus obtained is used for heating that unit's condensate in a series of closed heaters. However, the system as used in this plant is complicated by the use of certain auxiliary generators and other distinctive features and some of these should logically be considered before taking up the flow of steam and of condensate.

The experience of the company which built, owns and operates this plant leads it to believe that auxiliary energy supply should be reasonably independent of the main station bus, that is of the main generators. This is particularly true with respect to the supply for what are commonly known as essential auxiliaries. This experience also leads the engineers of this company to believe that when all things are considered the most satisfactory results are obtained when variable speed auxiliaries are driven by direct-current motors.

A supply of auxiliary energy independent of the main generators of the plant could have been obtained by feed back from the other stations of the company, by the use of auxiliary generators coupled to the shafts of the main units or by the use of independent, steam-driven, house-service generators. The first of these possibilities was eliminated except as will be noted later because of the distance from other stations, the possibility of having to operate the plant isolated from the others at times, and the complications and costs involved in bringing power back from outside in such a way as to make the auxiliary system independent of the

voltage variations planned for the station busses and the outgoing lines.

The use of direct-coupled, house-service units was given serious consideration. It has the great advantages of simplicity and low cost but certain undesirable features when applied to this particular plant. The desire to use both alternating and direct-current auxiliary supply and at the same time to make the auxiliary energy supply independent of other plants would have involved the use of alternating-current house service machines together with converting equipment for direct-current supply. Further, the starting of a single generator with its main auxiliaries driven by direct-current motors would have been practically impossible without an initial supply of energy from an external source. When all of the possible solutions along this line were considered it was felt that the third possibility represented the best solution in this case.

The station as finally constructed may be said to contain three different sets of generating units or to consist of three different generating stations. When completed the six main generating units will constitute one set; eight or ten direct-current generators driven in pairs through gearing by condensing steam turbines will constitute another set; and three or four 2300-volt alternators driven by condensing steam turbines will constitute the third set.

The direct-current units are 2000-kw., 250-volt machines with two armatures mounted side by side on a single shaft which is driven at 360 rev. per min. The turbine driving this double unit with a total capacity of 4000 kw. is a cross-compound affair with the high-pressure unit operating at 4000 rev. per min. and the low-pressure unit at 3000 rev. per min. These turbines drive the generators through pinions on opposite sides of a single large gear, the generator shaft being coupled to the shaft of this gear.

These turbines are arranged for bleeding at a pressure of about 10 lb., gage. The bled steam is used for building heating and for coal drying.

The steam which passes through these turbines is condensed in small surface condensers suspended beneath the individual turbines and each equipped with its own circulating pump, air pump and condensate pump. The condensate is delivered into the main feed water system as will be described later.

The alternating-current auxiliary units are rated at 2000 kw. each at 2300 volts. They are direct-connected to single barrel, condensing steam turbines which are arranged for bleeding but are not bled at present. The condensing equipment is similar to that just described and the condensate also enters the main feed water system.

A list of motor-driven auxiliary equipment with certain important data such as type of motor, speed, etc., is given in Table I. It will be observed that practically all motor-driven equipment in the boiler house is equipped with adjustable-speed, direct-current

TABLE I MOTOR SCHEDULE

Motors for	No.	H.P.	Voltage	Type	Rev. per Min.
		A-C. Motors			
Ash removal pumps....	3	60	2200	Squirrel Cage	1200
Fuller-Kinyon coal pumps..............	3	60	2200	Squirrel Cage	1200
Pulverizer mill exhausters.................	14	60	2200	Squirrel Cage	1200
Aux. Air compressor, general service.......	1	60	2200	Squirrel Cage	1200
North and south long apron coal conveyor..	2	25	2200	Slip Ring	600–1200
North and south short apron coal conveyor..	2	25	2200	Slip Ring	1200
East and west pulverized coal screw conveyor.	4	25	2200	Slip Ring	600–1200
South, middle, raw coal bucket conveyor.....	2	25	2200	Squirrel Cage	900
Coal breaker...........	1	150	2200	Slip Ring	520–720
Belt and shuttle coal conveyor............	2	10	230	Slip Ring	900
Coal pulverizer mills....	14	100	2200	Squirrel Cage	450
Picking belt conveyor.	1	7½	220	Slip Ring	750–1200
2000-kw. house alternator circulating pump.	2	60	220	Slip Ring	720–450
2000-kw. house alternator hot well pump....	2	15	230	Squirrel Cage	1800
Air compressor for coal transportation.......	1	185	2200	Synchronous..	225
Main air compressor, general service.......	1	185	2200	Synchronous	225
Crane over coal breaker.	2	30 Total	220	Slip Ring	620
Screen house general service pump........	1	240	2200	Squirrel Cage	1700
Coal unloader..........	6	20	230	Slip Ring	900
2000-kw. house alternator vacuum pump....	2	25	220	Squirrel Cage	1800
Coal dryer............	1	25	2200	Slip Ring	600–1200
Fan for crusher and breaker............	1	25	220	Slip Ring	600–1200
		D-C. Motors			
4000 kw. d-c. house service turbo-generator hot well pump.......	3	25	240	Compound	1200–1600
Primary feeder blowers for boilers...........	16	30	240	Compound	1200–1600
Boiler coal feeders......	64	2	240	Shunt	300–1200
Boiler damper..........	8	⅛	230	Series	575
50,000-kw. turbo-generator main circulating pumps..............	6	325/74	240	Shunt	255–175
General service pumps..	2	175/240	240	Compound	1200–1700
50,000-kw. turbo-generator main dry vacuum pumps..............	3	66/14	240	Shunt	120–60
4000-kw. house service d-c. turbo-generator auxiliary dry vacuum pumps..............	3	25/12	240	Shunt	125–60
4000-kw. house service d-c. turbo-generator auxiliary circulating pump...............	3	60/37	240	Shunt	435–215
Boiler induced draft fans	8	350/12	240	Shunt	500–166
125- and 25-ton turbine room crane..........	4	170 total	230	Compound	725–800
Crane over Turbo-generators.............	3	47½ total	240	Compound	700–800
Blower for ventilating d-c. house service generators.............	3	20	240	Compound	460
Turbineroom exhaustfan	2	25/8	230	Shunt	1150–800
East roof ventilation fan	1	10/2.5	230	Shunt	420–290
West roof and window ventilation fan.......	1	15/4.5	230	Shunt	300–230
North window ventilating fan...........	1	10/2	230	Shunt	600–440
Hot drip return pump..	2	25	230	Compound	1750
Boiler feed and hot well pump..............	3	700/400	240	Compound	1200–1000

motors and that the larger turbine house auxiliaries are similarly fitted. On the other hand, certain pieces of equipment located in the coal handling and coal preparation buildings and which require adjustable-speed motors are equipped with alternating-current drive. This was done because of the practical impossibility of preventing the generation and settling of coal dust in such places. It was felt that the greater freedom from danger of sparking together with the lower maintenance cost to be expected under such conditions of operation justified the use of adjustable-speed, alternating-current motors for such service. Constant speed motors are of the alternating-current type throughout.

operate a 500-kw. motor-generator set to supply the necessary direct current for starting a boiler in addition to operating such alternating-current motors as must be used during the starting up of the plant from cold.

The arrangement of auxiliary busses together with sources of supply and driven equipment is shown in Fig. 9. The direct-current bus is operated throughout at a nominal 240 volts and takes the form of several rings so arranged as to give maximum assurance of continuity of service. Structurally, this bus consists of copper bars supported by steel frames hung from the ceiling, the copper being insulated from the frame by blocks of alberene stone which are notched to receive the bars and are shaped to fit properly in clamps which

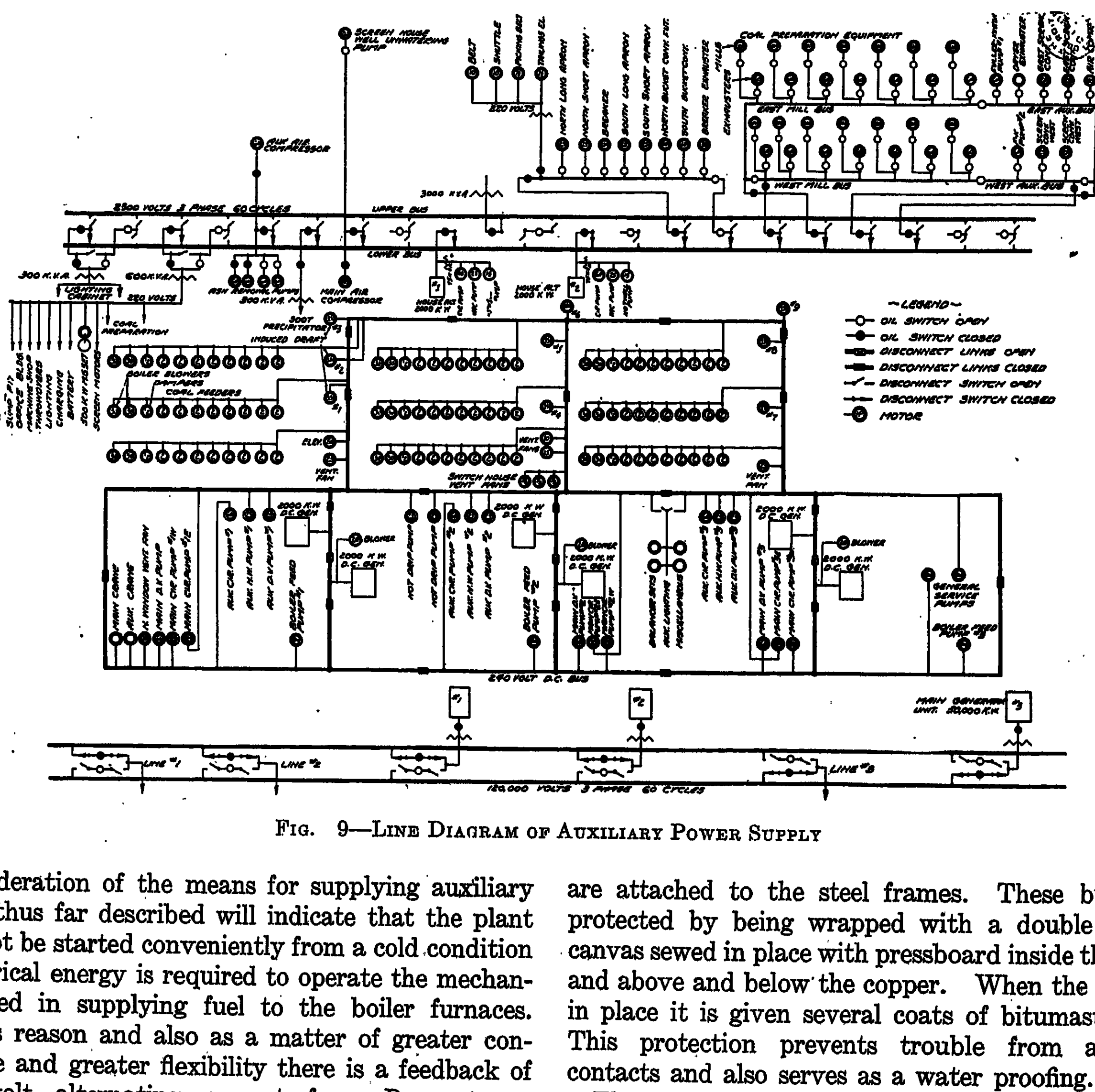

Fig. 9—Line Diagram of Auxiliary Power Supply

Consideration of the means for supplying auxiliary energy thus far described will indicate that the plant could not be started conveniently from a cold condition as electrical energy is required to operate the mechanisms used in supplying fuel to the boiler furnaces. For this reason and also as a matter of greater convenience and greater flexibility there is a feedback of 23,000-volt alternating current from Brownstown. This supply is stepped down through a 3000 kv-a. transformer and may be connected to the 2300-volt alternating-current bus. Ultimately this feedback connection will merely be a tap on the 23,000-volt distribution system in this territory.

This supply from outside the plant can be used to

are attached to the steel frames. These busses are protected by being wrapped with a double layer of canvas sewed in place with pressboard inside the canvas and above and below the copper. When the canvas is in place it is given several coats of bitumastic paint. This protection prevents trouble from accidental contacts and also serves as a water proofing.

The alternating-current bus is merely a duplicate bus of conventional type operating at 2300 volts. Throw-over switches for the larger motors or groups of motors together with provisions for connecting the two busses together give ample flexibility.

The plant and yard lighting is carried on the 2300-volt bus as shown at the extreme left hand end of Fig. 9.

Certain emergency lights throughout the power plant buildings have an automatic throw-over to the battery. Special outlets are provided throughout the plant for extension-cord lights at 28 volts alternating current. The use of such low voltage for extension cord work has been common in the plants of this company for many years and has been found exceedingly desirable because of the safety against severe shock. The lamp used for extension cords is the standard railroad coach light and is exceptionally rugged.

Returning now to the feed water circuit, it has been stated above that this is arranged for regenerative heating. A diagrammatic representation of the entire system is shown in Fig. 10. The normal water circuit may be traced by starting at the hot well of the main condenser in the lower left hand corner of the illustration. Water flows through the lower line from the hot well to the motor-driven condensate pump and then rises vertically to the 19th stage heater. It flows successively through the 19th and 17th stage heaters, the evaporator vapor condenser and the 14th stage heater and then vertically downward to the suction of the motor-driven boiler feed pump. This pump forwards it to the feed water headers from which the water flows through the various economizers to the feed drums of the boilers. The lower horizontal line in the diagram represents a bypass around all feed water heaters which makes it possible to pass water direct from the discharge of the hotwell pump to the suction of the boiler feed pump.

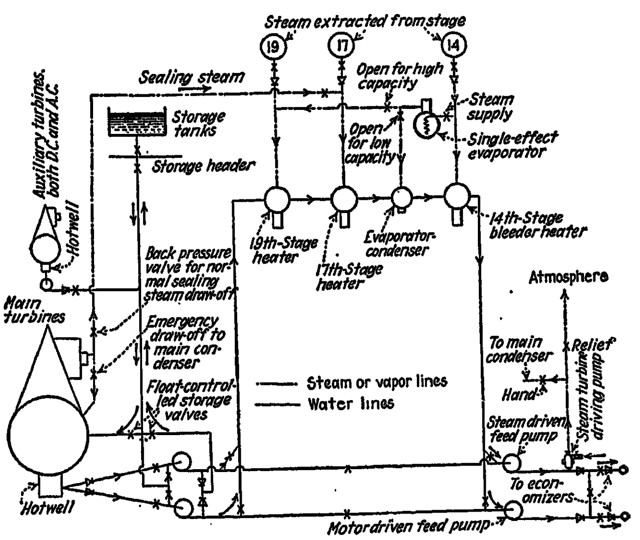

Fig. 10—Diagrammatic Representation of Condensate System

The motor driven hotwell pump and the motor driven boiler feed pump are coupled together and driven by the same motor. They are shown separated in Fig. 10 merely as a convenience in drawing the lines of flow. In effect these two pumps are equivalent to a single, multistage pump with provision for taking the water out of the pump at the discharge of one intermediate runner and returning it again to the suction of the next runner. Each main unit is supplied with one of these combination pumps driven by a variable speed, direct-current motor. The motor speeds are controlled by hand to regulate feed water pressure and distribution of condensate flow between main units.

It is quite obvious that the speeds at which these pumps are operated depend upon the demands of the boilers and that some other provision must therefore be made to maintain a proper rate of condensate removal from the condenser. Also, the total quantity of water held by the boilers varies greatly with load so that a certain amount of flexibility is required with respect to the water content of the system. This is provided in two ways. First, the lower part of the condenser shell is built in the form of a large, deep box or "bath tub" giving a hot well of large storage capacity and capable of permitting a relatively great variation of condensate level. Second, further flexibility is secured by introducing large storage tanks.

Floats operated by high and by low water in the condenser hot well put these storage tanks into and out of use in the following way. If the water level rises too far it indicates that the condensate pump is not removing water fast enough, that is, that under existing conditions, the boiler feed pump is not taking water as fast as the condenser is making it. If the water level rises far enough, the high level float opens the right hand valve of the two marked "Float Controlled Storage Valves" in Fig. 10, thus permitting the condensate pump to discharge into the storage tank in addition to discharging into the suction of the boiler feed pump. On the other hand, if the water in the hot well drops to the limiting lower level the low level float opens the left hand valve of the two indicated and permits water to flow from storage to the condenser, thus giving the condensate pump a supply commensurate with the demands of the boiler feed pump. To prevent excessive oxygen content and resultant rapid corrosion of the steel tube economizers, provision is made for steam sealing of the storage tanks and, in addition, the water discharged from them into the condenser is admitted in such a way and place as to ensure maximum deaeration.

The condensate discharged by the hotwell pumps of the condensers on the auxiliary turbines already described is discharged into the lines connecting with the storage tanks as shown in Fig. 10. This arrangement results in the deaeration, in the main condenser, of all water coming from the small units which are more apt to develop air leaks at low pressure turbine shaft seals and at hotwell pump shaft seals.

Each main unit is provided with a duplicate steam driven combination pump. This is exactly like the motor-driven outfit just described, except that it is driven by a single-stage steam turbine instead of by a motor. This steam driven unit will be used only in emergency and then it will be started without prelimin-

ary warming up, if necessary, and will exhaust to the atmosphere as shown in Fig. 10. If the conditions require this pump to operate for any length of time the valve shown to the left of its exhaust line will be opened so that the turbine driving it will discharge into the main condenser. Such operation will be resorted to merely for the purpose of saving the steam which is all made from distilled water.

It will be noted from the diagram in Fig. 10 that the make-up for the station is supplied by a single effect evaporator taking its steam from the 14th stage bleeder connection and normally discharging its vapor to a condenser located on the high side of the 17th stage bleeder heater. When operating in this way the evaporator is capable of producing about 1¼ per cent make-up. A greater production can be obtained by increasing the temperature head and provision is therefore made for discharge of evaporator vapor to the 19th stage bleeder heater connection as shown.

Under normal operating conditions the temperature of feed water entering economizers will vary between about 200 and 250 deg. fahr., depending upon the vacuum carried in the main condensers and the loads on the main turbines. Higher values could have been obtained easily but the values chosen seemed to give the best commercial solution when cost and performance of economizers were taken into consideration. Higher values could have been used if air heaters had been installed instead of, or even possibly in addition to, economizers. However, at the time this plant was designed the air heater was considered to be very decidedly experimental. In addition, the furnace walls were to be air cooled and nothing was known about maximum permissible temperatures for air used for this purpose nor maximum permissible temperature for air entering the furnace interior.

The summary at the end of this paper gives essential data with respect to all major equipment and the more important minor equipment. Taken in combination with the motor data given in Table I it gives in convenient form most of the important information in connection with the apparatus installed. Further details of the ideas and theory underlying the design of the plant and of the equipment can be obtained by referring to the following three articles which have been published in the technical press:

1. The Trenton Channel Plant of the Detroit Edison Company; C. H. Berry; *Power*, May 27, 1924; Page 848.
2. Design Features of Trenton Channel, P. W. Thompson; *Electrical World*; May 31, 1924; Page 1115.
3. Unusual Electrical Features Found in Trenton Channel Station; *Electrical World*; January 31, 1925; Page 247.

SUMMARIZED STATEMENT OF MAJOR EQUIPMENT

Main Generating Units. Three 50,000-kw. single-barrel horizontal turbines driving three 62,500-kv-a., three-phase, 60-cycle generators with direct-connected exciters. Turbines operate at 1200 rev. per min., have 21 stages, a water rate (at 42,500 kw. without bleeding) of 9.4 lb. per kw-hr. when supplied with steam at 375 lb. gage and 700 deg. fahr. Turbines are bled at 19th, 17th and 14th stages. The nominal generator voltage is 12,200.

Generator Air Coolers. Horizontal, finned-tube cooler contained in housings within generator foundations. Coolers arranged for circulation of condensate or water from intake tunnel of plant.

Main Condensers. Three 47,300-sq. ft., single-pass condensers arranged with steam belt. Tubes are made of 70 per cent Cu, 28 per cent Zn and 1 per cent Sn alloy, 1 in., No. 18 Stubbs gage and are arranged in single lines converging from a maximum spacing at top of condenser of 4 in. to the minimum possible with 1 in. tubes at entrance to air cooler. Each condenser is equipped with two 60,000-gal. per min. circulating pumps driven by direct-connected variable speed d-c. motors; two condensate pumps one of which is mounted on shaft of motor-driven boiler feed pump and one on shaft of steam-driven boiler feed pump; one vertical reciprocating dry vacuum pump.

Main Hotwell and Boiler Feed Pumps. Hotwell and boiler feed pumps are mounted on same shaft. There are two for each main unit, one driven by 700-h. p. adjustable-speed d-c. motor and the other by a 750-h. p. steam turbine. The steam-driven pump has a capacity of 1300 gal. per min. at 1700 rev. per min. and the motor-driven pump has a capacity of 1300 gal. per min. at 1200 rev. per min.

Main Generator Switches (12,200-volt). Three switches one for each main unit. Each switch is a 4000-ampere, 25,000-volt oil switch with motor-operated mechanism.

Main Step-Up Transformers. Each main unit has three, single-phase, 21,000-kv-a., 12,000, 120,000-volt, oil-insulated, water-cooled, outdoor-type transformers.

Main Generator Outdoor Switch (132,000-volt). Each main generator has two three-phase, 400-ampere, 132,000-volt outdoor-type oil switches, each phase in separate oil tank. Each switch is motor operated by centrifugal operating mechanism.

Direct-Current House Service Machines. Three 4000-kw. units, each unit consisting of cross-compound steam turbines driving through a reduction gear, two shunt-wound, interpole, 2000-kw., d-c. generators mounted on a single shaft. High-pressure element operates at 4000 rev. per min., low pressure at 3000 rev. per min. and generators at 360 rev. per min.

Condensers for Direct-Current House Service Machines. Three 7500-sq. ft., two-pass condensers completely equipped with circulating pumps, hotwell pumps and air pumps.

Alternating-Current House Service Machines. Two single-barrel turbines each driving one 2000-kw. three-phase, 60-cycle, 2300-volt generator with direct-connected exciter, at 3600 rev. per min.

Condensers for Alternating-Current House Service Machines. Two 4100-sq. ft. two-pass condensers equipped with circulating pumps, hotwell pumps and air pumps. These auxiliaries are driven by a-c. motors and can be brought up with machine if desired, priming being taken care of by priming header running the length of the plant.

Main Steam Piping. Van Stone joints with gaskets.

Main Turbine Room Crane. One 125-ton crane with span of 97 ft. 6 in. Can handle circulating pumps and other auxiliaries on condenser room floor as well as main unit parts.

Boilers. Eight 29,085-sq. ft., five-drum, curved-tube boilers built for operating pressure of 416 lb. on drums and equipped with two superheaters per boiler which give total steam temperature of 700 to 725 deg. fahr. A hearth screen is used at the bottom of furnace under each boiler but is entirely separate from the boiler in so far as circulation is concerned. Each hearth screen consists of two separate boilers with independent feed. Steam made in these boilers joins steam made in main boiler and the combination passes through superheater of main boiler. Boilers are arranged in groups of three across main axis of boiler house and are fired from both ends thus giving four firing aisles running lengthwise.

Pulverized Fuel Preparation and Handling Equipment. Fourteen steam-heated air-circulating driers feeding coal to 14 pulverizing mills. Pulverized coal raised from mill to cyclone separator by No. 12 mill exhauster, one exhauster per mill. Exhausters driven by 60-h. p. a-c. motors. Mills driven by 100-h. p. a-c. motors. Coal from cyclone separators forwarded by four screw-conveyors running lengthwise of house to hopper over three 10 in. pulverized coal pumps. These pumps driven by 60-h. p. a-c. motors.

Pulverized Fuel Burning Equipment. Sixteen burners per boiler, eight on each end, fed by duplex feeders driven by two-h. p. adjustable-speed d-c. motors. Primary air for each boiler supplied by two 55-in. blowers driven by 30-h. p. variable speed d-c. motors.

Boiler Furnaces. Air-cooled, refractory-wall type with eight burners at opposite ends. Distance, arch to hearth screen, 20 ft; distance between opposing rows of burners 29 ft; distance center line of burners to adjacent wall 2½ ft.; distance between burners 2 ft. 4 in.; distance end burner to side wall 3 ft. 4 in.; inside width furnace 23 ft.; total volume from hearth screen to peak of furnace 25,140 cu. ft.

Ash Handling. All ash collecting in furnace and in boiler passes, flues and economizer hoppers, is discharged to hydraulic sluiceway running longitudinally in boiler house basement and is sluiced to settling basin. Material removed from settling basin by centrifugal dredge-type pump and used for filling on property.

Economizers. Two, steel-tube economizers per boiler with 9492 sq. ft. of surface per economizer. Tubes are plain, 2-in. tubes rolled into rectangular forged headers.

Induced Draft Fans. One double-inlet, 193,000-cu. ft. per min., at 375 deg. fahr., induced draft fan per boiler, capable of producing maximum draft of 7.15 in. water at fan. Each fan driven by 350-h. p. adjustable-speed d-c. motor. Three fans serving one row of three boilers discharge through curved flues into single stack.

Stacks. One stack for every three boilers. Stacks have Venturi shape with inside diameter of 15 ft. ½ in. at throat and 21 ft. at top. Top of stack is 192 ft. above burners. Stack is self-supporting steel, carried on building steel and is brick lined.

Discussion

H. R. Woodrow: The Brooklyn Edison generating plants are laid out for a different method of operation from that discussed by Mr. Hirshfeld. The three generating plants are operating continuously in parallel with heavy tie lines between stations. One of these stations is 25-cycle and the other two 60-cycle and a 35,000-kv-a. frequency changer ties the 25-cycle station to the 60-cycle system.

The base load of the entire system is carried on the most economical units in the most economical plant which is, at the present time, Hudson Avenue Station. It is intended that these stations should hold in step during the major number of troubles and sectionalizing reactors are very generally used to segregate the trouble and limit the value of the short-circuit current.

For a detailed description of the Hudson Avenue plant and system connections of the Brooklyn Edison Company, I would refer to the April 1925 issue of the *Electric Journal.*

In all cases second-line defenses are provided in the form of automatic switches segregating certain groups of feeders in case of the main feeder switch failing to operate and each feeder is provided with reactors and reactors are installed between sections to maintain voltage for high continuity of service under disturbance conditions.

The auxiliaries in the power house are all electrically driven with the exception of reserve boiler feed pumps and all auxiliaries are provided in duplicate for the so-called essential duties, each half of which is supplied from different sources although tied together through reactors.

F. A. Scheffler: I want to compliment Mr. Alex Dow on his wonderful "sticktoitiveness," you might say, in adhering to the design of the type W Boiler that he selected about fifteen or twenty years ago to use in the Delray plant of the Detroit Edison Company.

I had the opportunity and great pleasure of working out that design of boiler with Mr. Dow, and the results were eminently satisfactory in producing a type of boiler at that time which was four times the size of any other boiler in the country. I think Mr. Dow deserves a great deal of credit for introducing, in this country, the large unit type of boiler for public-service stations.

He has continued to use that type in all of his other plants, and, today, I believe they have thirty-five to forty boilers of that type operating at from 300 per cent to 400 per cent of rating on peaks.

H. W. Brooks: Many writers on combustion in the past have stressed the importance of turbulent flow, agitation and mixing in pulverized-fuel furnaces as an aid to the natural diffusivity of the burning gases upon which speed of combustion, length of flame travel and hence combustion volume depends. A practical means of accomplishing this mixing has heretofore remained undiscovered.

Several years ago it occurred to one of the engineers of the Fuller-Lehigh Company that one of the most intense manifestations in Nature of turbulent flow and agitation of gases and particles in suspension was in the tornado, where it had been repeatedly demonstrated that materials of considerable tensile strength had actually been torn apart, disintegrated and reduced to fine pieces and sometimes to powder by the intense centrifugal action of the air. Experiments were started at Fullerton on a small furnace 18 in. square by 3 ft. deep in which the jets were placed to throw the flame tangent to a tornado of fire within the furnace, the flame of the first jet being deflected before it reached the refractory walls by the impingement with equal velocity at right angles of the flame from the second jet, the third jet again changing the direction 90 degrees, and the fourth jet completing the tornado within the pot. Fig. 1 shows in plan, the application both to the circular and the square-pot furnaces.

It was well known that the external walls of the tornado of nature had been observed to be rather sharply defined from the surrounding air. Thus it was decided to adapt this principle by placing the pulverized fuel jets in such relation to the refractory walls that there was little, if any, impingement of the fuel tornado on the furnace walls. As was anticipated, therefore, the refractory walls showed little damage due to erosion, and thermocouple temperatures taken on the inner refractory surface of the furnace showed that the inner face of the brick never exceeded 2700 to 2900 deg. fahr.

It has also long been recognized that the transmission of sensible heat from the hot gases from the furnace through the tubes of the boilers to the water on the other side was limited by the existence at the boundary surface of a "dead film," generally assumed to be about 1/40 in. in thickness, of relatively cool gas in which convection currents apparently did not take place, and which owing to its low conductivity offered a high resistance to the passage of heat (other than radiant) through it. This "dead film" is the main and most potent obstacle to rapid heat transmission in the boiler, and it follows that its complete or even partial destruction will greatly assist the flow of heat and hence the efficiency of the boiler. Prof. J. T. Nicolson of the Manchester School of Technology demonstrated a formula proving that the heat transfer was a direct function of the velocity of molecular travel of gases passing the tubes. Thus it was anticipated and tests subsequently proved that with the higher scouring action of the swirling gases in meeting the boiler tubes there would not only be a higher rate of heat transfer and higher

boiler efficiency, but also a cleansing action which would keep the tubes clear of deposited ash.

Early during the experiments the nature of the ignition and combustion taking place in the well, proved quite unlike anything experienced pulverized-fuel engineers had hitherto seen in powdered-coal combustion. The flame itself resembled that of a blow torch, virtually a "ball of fire," combustion apparently taking place in a limited zone within the well. By regulating air admission and air pressure it was possible to move the hottest zone into the well itself or to a point just in front of the well. Appreciating further the gains due to the more efficient transfer of heat by radiant energy as a result of this "ball of fire," the inventor had still further reason to expect efficiencies higher than had been before demonstrated in pulverized-fuel combustion, as well as a flatter over-all performance curve at high ratings.

These experiments were kept secret for several years; until finally a few of the larger operators were shown the experimental furnace at Fullerton. Several immediately volunteered to install a furnace of the new type, but the results which indicated heat releases many times greater per cubic feet of combustion volume than had ever been accomplished before were so revolutionary in character that the company felt it best to withhold a commercial-scale installation until they had finally and thoroughly convinced themselves that no practical operating difficulties would intervene.

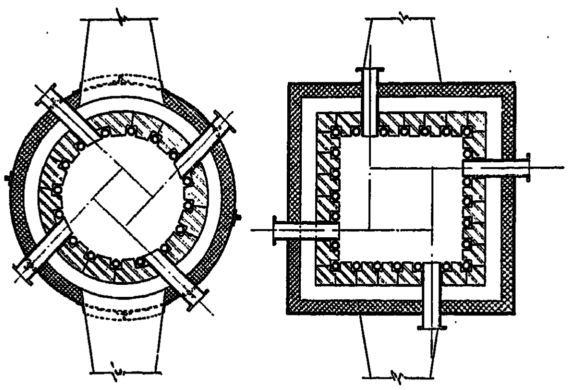

Fig. 1

Finally on February 5, 1924 an agreement was entered into with the United Electric Light and Power Company of New York City by which a commercial-scale experimental installation of the new furnace would be made at their Sherman Creek Station alongside five other boilers fired by the older pulverized-fuel firing methods.

The operating company expressed the desire to make the tests entirely with their own personnel. Today it is understood these tests have been completed. Although full details cannot be made public we are permitted to state that the efficiencies have actually proved approximately 3 per cent higher than with any other method of firing, and the curve of efficiencies at various ratings has been much flatter than anything heretofore demonstrated.

Fig. 2 shows one of the more recent applications of this furnace. It will be noted that above the well there is provided a dispersion chamber in which the hot gases after burning are allowed to expand before reaching the boiler tubes, thus reducing the tornado effect to a value which can cause no possible erosive effect on the boiler tubes. The theory of this chamber is to allow just sufficient velocity to scour off the "dead film" from the boiler tubes without going any further in attacking the tubes themselves. It is not possible to state at this time the minimum size to which this chamber may eventually be reduced. It has definitely been established, however, that the combined volume of well and dispersion chamber can be made substantially less than the combustion space provided in the old-fashioned stoker setting. Knowing that we can convert present stoker-fired settings by this means, and attain the highest boiler ratings, we have at this time no occasion to go further until the application is extended to other services, such as transportation and marine. Fig. 3 shows a plan of the water-cooled well as actually applied on the later designs.

Simultaneous with the tests at New York, research has been continued at Fullerton on a furnace consisting of an 8-ft. cube. In this furnace the coal equivalent of a 1500-h. p. boiler at 632 per cent of rating has been burned in the volume of an 8-ft. cube.

It is felt that the "well-type" furnace is a demonstrated success

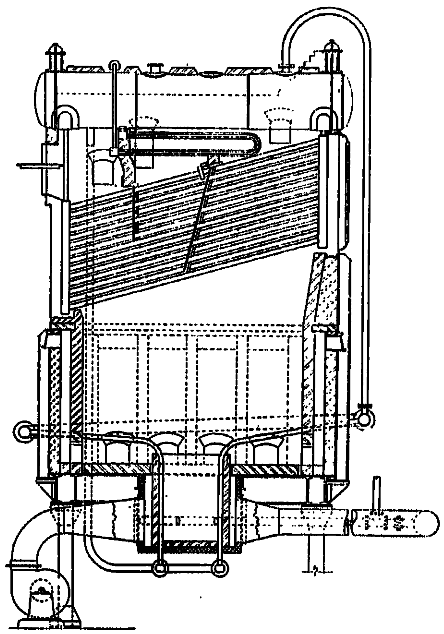

Fig. 2

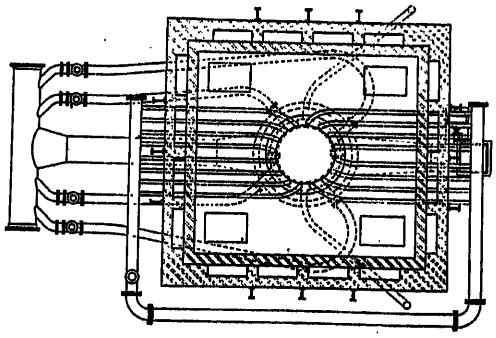

Fig. 3

and that for the first time in the history of pulverized coal the problem of turbulent flow and hence the problem of combustion volume has been solved. Not only may pulverized coal be efficiently burned in combustion volumes as small as those previously employed for stokers but actually smaller furnaces may be installed where necessary. For the first time, completely successful application of pulverized coal to transportation and marine service becomes possible, as does also the conversion of present stoker-fired furnaces to the advantages of pulverized coal without the necessity in many cases of more than slightly modifying the present furnace construction.

H. R. Summerhayes: The voltage selected for the Trenton Channel Station is 120,000, and 132,000-volt apparatus has been used. That, we might say, is commendable conservatism, but as

manufacturers, we think it is unnecessarily conservative. We have evidence from experience that the Institute tests are about right. We think the equipment would stand 132 kv. and if it is to be used at 120 kv. it would seem to us unnecessary to have so large a factor of safety. On the Western Coast, we have used for 220-kv. circuits apparatus designed for high-potential tests corresponding to 187 kv., which has given good service.

The next point that I want to comment on is along the lines Mr. Woodrow mentioned, namely, that the plants of Detroit system are tied together rather loosely, with weak linkage, whereas, it is the practise in other large systems to tie the plants together rather solidly, using reactors, and I am inclined to think that as the plans of the Detroit Company progress they may find it advantageous to have a tighter linkage and links capable of transferring more power.

The other point is the old question of direct-current auxiliaries and I suppose that will never be settled. It appears that means of obtaining direct current, perhaps, cost more and auxiliary motors may cost more, but you do obtain very perfect control, and I think that the question has yet to be settled as to whether the perfection of the control obtained is worth the money it costs.

K. M. Irwin: It is brought out in this paper that theDetroit Edison Co. ties its stations together loosely. This must mean that the Delray, Connors Creek and Trenton Channel plants take the loads of their particular districts, and therefore, the load factors of their particular districts. As the Trenton Channel plant does not operate as a base-load plant, but operates on the load factor of its district, it must have been more difficult to justify the cost of equipment than it would have been if this plant had operated on the base of the load, and in that way had a higher load factor and a greater divisor to divide its investment costs.

In the reasons given for the choice of pulverized coal, the statement is made that a reduced number of boilers could be used with pulverized coal than could have been used with a stoker installation. I wonder whether Mr. Hirshfeld has found that the limitations in boiler capacity are in the fuel-burning equipment. With the type of boiler adopted at Trenton Channel, I would believe that the limitation would be found in the boilers and not in the fuel-burning equipment. Tests have been run on a stoker installation in which ratings have been obtained in the neighborhood of 500 per cent without serious falling-off in efficiency. I do not believe that these ratings could be exceeded in Detroit even with pulverized coal.

In the list of motors, the 2000-kw. house-generator sets have their auxiliaries a-c. driven, and the 4000-kw. d-c. house sets have their auxiliaries d-c. driven. It would be interesting to know the reasons for the changes of the current for the auxiliaries on these house sets.

Louis Elliott: The recent putting into operation of the Philo and Crawford Avenue stations, with the low fuel consumption reported, makes of particular interest a decision such as that made for Trenton Channel,—to install 375-lb. turbines working on the regenerative cycle, as against a higher pressure plant designed for reheat.

A recent estimate for a moderate-sized station indicated an excess cost for a 650/550-lb. reheat plant, as compared with 425/375-lb. of about 7 per cent. With a capacity factor of 60 per cent annual, this additional investment would entail an increased fixed charge around 0.3 mills per kw-hr. If it be assumed that the 550-lb. station would permit a saving in production cost equivalent to 10 per cent of the fuel cost, this saving with 10,000 B. t. u. coal at about 1.1 mills per lb. in the pulverized bins would amount to between 0.15 and 0.2 mills per kw-hr. output.

For this situation therefore the recommendation was made that 425-lb. boilers be installed, with turbines good for 375 to 400 lb. at the throttle. On the above basis coal would have to cost nearly double its present price in order to give an equivalent cost of energy for the two pressures. Different assumptions as to load factor, coal cost, or other factors would, of course, modify the result. In general an expensive high-efficiency installation should justify itself for the first few years of operation, for as the years go on the reduction in load factor will tend to counterbalance any increase in fuel cost.

Aside from the estimated saving in combined fixed and production cost of energy, the lower-pressure equipment is more thoroughly developed and therefore probably more reliable, and the 375-lb. regenerative-cycle plant would be simpler to operate than the 550-lb. reheat station. The 375-lb. pressure is now standard, and it is uncertain as to what the next higher standard will become,—whether 550, 750 or higher. It is believed that in general there should be a considerable saving assured before newly developed equipment and designs are adopted.

It would be of great interest if the author would give the expected economy in B. t. u. per kw-hr. output,—with fuel available and with load factor as anticipated. A statement also as to any estimate of comparative plant cost for the pressure adopted as against a 550-lb. reheat plant, and the estimated comparative economies would be of great value.

C. F. Hirshfeld: When I spoke of pulverized fuel as being advantageous in that it would enable us to take advantage of variable qualities of coal, I did not mean that we could use West Virginia coal one day and Illinois coal the next day. Buying as carefully as we know how with men who live in the coal fields, it is impossible for us to bring to Detroit one grade of coal.

Some of the coal we get is poorer than the rest. Our stoker-fired plants in the past have proven very touchy with respect to the grade supplied. If we can get the grade of coal for which the plant was designed, it does very nicely. If we get a greater quantity of ash, we lose both in capacity and efficiency, and it was in that respect that we had considered variable qualities of coal in making this decision.

Improvements in the manufacture of stokers since the time this decision was made (which is several years ago) have greatly extended the capabilities of the stoker in this direction. At least, some varieties of stoker are not now limited to the same extent as they then were.

In connection with the use of d-c. auxiliaries, I stated quite flatly that it was a case of personal prejudice on the part of the engineers responsible for the designing of this plant.

It may be of interest to you to know that the cost of obtaining our d-c. auxiliary power, with the separately driven d-c. generators, is practically the same as though we took a-c. power from our main units and converted it. There is, however, an advantage in our d-c. power supply not being tied to the operation or performance of our main units. We admit that our investment charges are higher but we prefer our system nevertheless.

With respect to Mr. Sheffler's comment on boilers, we now have forty-four, varying in size from 1250 h. p. to approximately 3000 h. p. boilers. He was not correct in his statement regarding the ratings at which those boilers are operated. The old stoker-fired boilers reached limitations at about 200 per cent in some cases and 250 per cent in others. The boilers at Trenton Channel Station, which are pulverized-fuel-fired have been driven to about 350 per cent of rating. In no case in which the double-ended or Type W boiler was used has the limit been found in the boiler.

Referring to Mr. Erwin's discussion, the matter is something like this: There is no question but that you could get a stoker to give ratings equally as high or higher than obtained in Trenton Channel Station. Parenthetically, it is also true that we could drive the ratings at Trenton Channel very much higher than those obtained. As we saw the problem at the time we designed this plant, the rating to which a stoker-fired boiler could be driven was dependent only upon the grate area or the area of the stoker and the ability to maintain the furnace wall.

However, we do not believe and we have not yet seen any proof that a stoker can be built which is capable of driving a boiler to ratings in the neighborhood of 400 per cent or higher,

which will give a reasonably high efficiency at those ratings and maintain a reasonable efficiency at very much lower ratings. In other words, you sacrifice considerably in what you might call the combination of flexibility and efficiency when you attempt to get a stoker-fired equipment to carry over a wide range of rating. I do not know if the stoker manufacturer is going to be able to remove that limitation or not. So far as I have been able to obtain proof he has not yet done so.

Mr. Elliott asked about the performance of this station. The station was designed to give a net kilowatt hour, that is a kilowatt hour of output, for about 16,000 B. t. u. when operating with an annual load factor of the order of 50 per cent. The station has now been in operation, in one way or another, for something over six months, possibly eight months, and the performance has been improving monthly. Some weekly figures have already dropped below 16,000 B. t. u., the value which we assumed when we designed the station. We are not able to say whether those figures are correct, but the figures that we are willing to give you hit close to the 16,000 B. t. u., which was the design point and we may do better.

The cost of preparing pulverized coal is always one of the contentious points. I have here the figures for three months, January, February and March, the March figures being incomplete. In these figures, we have included every item of operating cost for coal handling and preparation from the time when the coal finds itself in the track hopper and until it finds itself in the pulverized-fuel bunker, ready to be fired into the boiler furnace. We have included all labor costs, both operating and maintenance. We have included all power, we have included all steam, both the steam used in drying the coal and the steam used for heating the preparation house and you will notice that this was during the two severe months of the year. We have included all operating material and all maintenance material. The figure runs at approximately 30 c., pulverized, during January and February. In January, it was 29.14c. In February, it was 30.36c.—the increase in February being due to the fact that maintenance was quite a bit heavier in that month than the preceding month.

To make these figures of real significance, I should say this: They are obtained from a plant which is using small pulverizing mills, nominally six-ton mills. They are obtained on a preparation house which is designed to handle exactly twice the amount of material that is now being handled. The labor used will be the same when handling twice the material as it is now. The maintenance will, of course, go up. As near as we can estimate, if this plant were used to capacity and all the charges were made as they have been outlined, the figure for preparation would be a little under 22c. per ton. In charging steam and in charging power in these figures, we have used cost of steam and cost of power as actually shown on our books for steam sent out from the boiler room to the turbine room, and for power sent out over the lines to our customers, so that we haven't either overloaded or underloaded those items.

With respect to Mr. Woodrow's and Mr. Hays' discussions, in which they refer to our, let us say, loosely linked system, I believe that we may ultimately come to a more tightly linked system. We have followed the other method principally for three reasons. In the first place, it fitted our conditions ideally. In the second place, we were fairly sure that it would work, and experience has shown that it does work moderately well. Third, we did not like to contemplate the cost of reactors and places in which to place them and all the trimmings that go with them.

I believe it was Mr. Erwin who worked around from that loosely linked system to the conclusion that our Trenton Channel plant, which is our most efficient, could not be used to the best advantage. To a certain extent, that is true, but only to a very limited extent. Immediately adjacent the Connors Plant and the Delray Plant there is sufficient load to load the plant properly. Trenton Channel can obtain quite enough load to operate as near base load conditions as we care to approach by supplying the so-called down-river industrial load, that part of the industrial load that it can pick off from the Western side of the City of Detroit, such load, very largely industrial, which can be obtained in the outlying district, combined with such industrial load as can be obtained through feeding back into the city from the 120-kv. bus or trunk referred to in the paper. So our loose linkage has very little effect on our efficiency. On the other hand, it is not the policy of our company to load up its new plants at the expense of its older plants. It pays us, or at least it has paid us in the past to go a certain distance in that direction, but not as great a distance as has been proven advisable in other cases.

As Mr. Brooks was describing his new furnace, it passed through my mind that if one could use a small furnace but had to add to it a large mixing chamber, it did not make a lot of difference since you ultimately used the same number of cubic feet as you do with the older designs. I can't see that there is much choice. Mr. Brooks rather implied that one could do away with that mixing chamber if forced to do so, but I am wondering if that furnace were brought too close to the boiler tubes, whether you would not carry up so much fused ash as to put the equipment out of commission very quickly. I do not believe the manufacturer has had sufficient experience with the device to be able to answer that question. I followed the tests at Sherman Creek very closely, but they were not planned in such a way that that question could be answered by them.

Mr. Woodrow's discussion reverts to tight linkage as against loose linkage. He refers to safety devices intended to function serially. No matter how you plan you may never be sure that this thing or that thing will really work. On that account we have preferred to cut out all such trimmings and limit ourselves to loose ties which would break loose or burn themselves up.

Mr. Erwin asked me why we used a-c. auxiliaries on the a-c. house-service machines. That is due to another one of our "hobbies." If we have steam, we can start the a-c. machine and with the a-c. machine we can start everything else. We felt that if we faced a situation in which we wanted to start that little house-service unit in a hurry, we wanted to start with the minimum of manipulation. The a-c. driven auxiliaries are so connected that they come up with the machine and we feel that this is the simplest starting scheme we could get. That is why we used the a-c. driven auxiliaries with the a-c. house-service units. It is not the first time we have used that scheme and it has worked very well in all cases so far.

With respect to deaeration, we have used in this plant steel-tube converters. The natural conclusion would be that we should be very careful not to have any more than the minimum feasible oxygen content in the water. Our experience in Detroit has been that our water is of such a character that we can stand more oxygen than is commonly given as the maximum limit consistent with reasonable life of steel tubes. Our experience with steel tubes at the old Delray plants leads us to believe what I have stated.

The system at Trenton is built so that there is a surge tank. The surge tank is arranged so that it can be sealed with steam if necessary, but it is not so sealed now. The water passes into the condensers where it is reasonably well deaerated, although we have not provided separate deaerating devices. It is too early to say whether this system works or not. Four or five years hence we will be able to tell if it doesn't work.

With respect to the pressure on the suction of the boiler-feed pumps; when we purchased these pumps, we had all the characteristic curves modified until we got what appeared to be a combined pump unit, which would be safe against flashing at the boiler-feed pump suction. Up to date, we have had no trouble from that source. We have had a little trouble due to hammering of the hot-well pump. The reason for that is that the pump does not have a sufficient head of water above it. We are now attempting to rectify that and expect to succeed in doing so.

Use of Frequency Changers For Interconnection of Power Systems

BY H. R. WOODROW[1]
Fellow, A. I. E. E.

FOLLOWING an extensive study into the cost of power as delivered to the customer, the Brooklyn Edison Company adopted the policy of supplying all the increase in business at 60-cycle alternating-current and curtailing the direct-current load within the capacity of the existing substations. With the 25-cycle generation confined primarily to the supply of power to the direct-current system all new prime movers are being installed with 60-cycle generators. The 25-cycle system is therefore left in a position of barely holding its own.

The new 60-cycle developments being considerably more economical than the old 25-cycle, there is a large economy factor in carrying the maximum amount of the combined system load on the 60-cycle generators. The installation of frequency changers between the 25- and 60-cycle systems made it possible to carry the base load of the 25-cycle system on the new 60-cycle developments and in addition thereto made it possible to pool

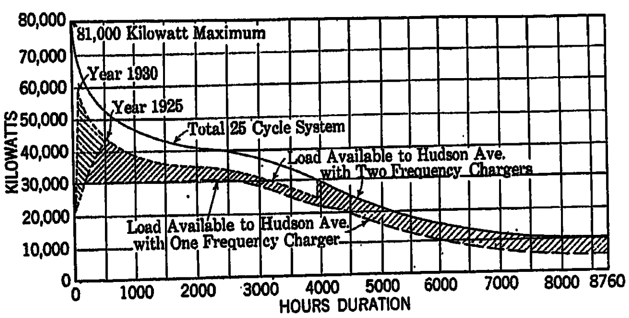

FIG. 1—LOAD-DURATION CURVE OF 25-CYCLE SYSTEM

the reverses on the two systems thereby giving greater utilization and availability of the generating capacity.

In Fig. 1 is shown the load-duration curve of the 25-cycle system with amount of load convertible to the 60-cycle prime movers by the installation of first, one 35,000-kv-a. frequency changer; and second, two 35,000-kv-a. frequency changers. The area in the first block represents 160,000,000 kw-hr. per year with a unit cost saving, which for this feature alone, more than pays for the carrying charges and losses of the first frequency changer installation.

The 25-cycle generating capacity amounts to 125,000 kw. with a possible peak load of 85,000 kw. and in addition there is available 15,000 to 20,000 kw. in 25-cycle tie capacity with other companies. The installation of the first frequency changer added 30,000 kw. in reserve generating capacity to the 60-cycle system, and the second unit would add an additional 25,000 to 30,000 kw. This 50,000 to 60,000 kw. of reserve made available to the 60-cycle system is of considerable value with the new 60-cycle generating developments using 50,000-kw. units. This feature alone can easily be said to more than justify the cost of the first frequency changers since the completed installation costs less than $20.00 per kw. while generating station capacity costs upwards of $100.00 per kw., and there was a deficit in 60-cycle generating capacity in 1923.

The frequency changers are of additional value in providing reserve to the 25-cycle plant for a case of unusually bad luck of machine outages in the station.

The metropolitan companies have for a number of years followed the policy of operating at all times with sufficient generating capacity to be capable of carrying the load should the largest generator be dropped from the system. It is, therefore, evident that the continuous parallel operation of the 25- and 60-cycle plants saves the operation of one turbo-generator unit. This permits an economical loading on the units and especially at light load period effects a real saving in production cost.

The installation of the second frequency changer would make it possible to shut down the 25-cycle station for one watch or nearly 3000 hours a year.

The interconnection of systems through frequency changers is of the same general character with similar problems and possible economies as inter-connection of systems through tie lines. The total savings under either condition run into large figures if all concerned cooperate in the operation of the more economical units for base load and shut down the plants and units of less efficiency for use only on peak.

Where two or more companies are concerned in the interconnection, the rate for interchanged power complicates the situation, and it is essential than an equitable rate be established in order to allow operation for maximum economy.

It may be unnecessary to say that the only way to use interconnecting tie lines for obtaining the maximum over-all economies is to operate at all times with the systems in parallel. This allows full advantage to be taken of the three separate points previously mentioned, namely:

1. Greatest utilization of the most economical plants and units.

2. Maxumum availability of spare capacity.

1. Brooklyn Edison Co., Brooklyn, N. Y.

Presented at the Spring Convention of the A. I. E. E., St. Louis, Mo., April 13-17, 1925.

3. Reduction in number and amount of idle or lightly loaded turbo-generator units at all times.

The parallel operation of large systems through frequency changers or tie lines necessitates that these ties have the proper characteristics to hold the systems in step for the major number of operating disturbances. It may not be possible, or even desirable, to hold the systems in step during all disturbances, but it must be a rare case when they are allowed to fall out of step and provisions must be made for the systems to break apart through automatic oil circuit breakers on these rare occasions. Although the Brooklyn Edison Company

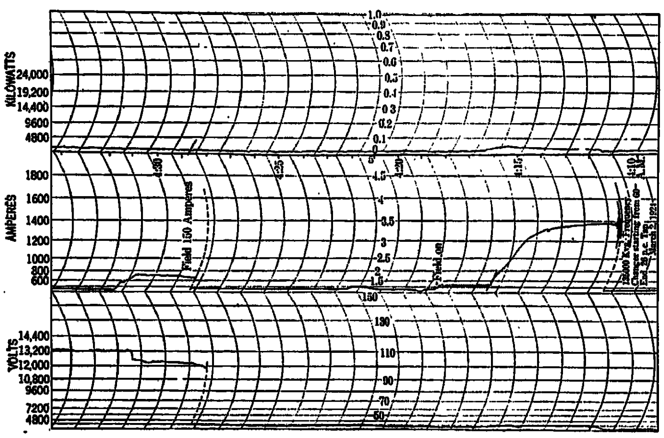

Fig. 2

had a number of smaller frequency changers in operation, converting from 25- to 62½-cycles they were discarded for the larger more efficient unit which would permit parallel operation.

The character of the normal fluctuations of load or disturbances is, therefore, the determining feature of the characteristics of the ties. The loads on both the 25- and 60-cycle systems of the Brooklyn Edison Company are of a rather steady character and the maximum

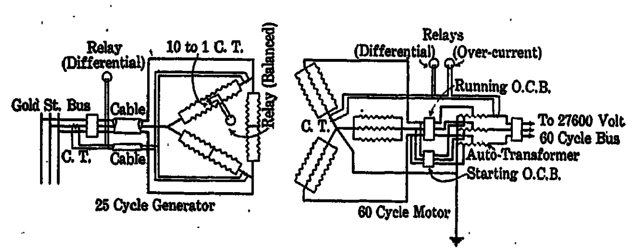

Fig. 3—Relays for Induction Type Motor Shown for One-Phase Only, the Other Two Phases Being the Same

amount of load which can be thrown from one system to the other may be considered as the capacity of the largest 25-cycle unit. This condition may occur were this unit to be automatically tripped from the system, and, as previously stated, this operation is considered a normal function in the metropolitan territory, and, therefore, the two systems should remain in step under these conditions.

When a sudden load is thrown from one system A to another system B, there is necessarily brought about a reduction in the speed of system A to give the phase-angle displacement for transfer of power. It therefore follows, that at the instant when the phase-

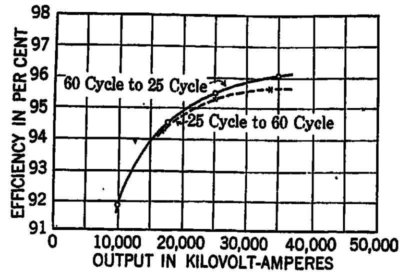

Fig. 4—Efficiency Curves, 35,000-Kv-a. Frequency Changer

angle is reached for the transfer of this amount of power the speed of system A is lower than system B and overshoots the mark. For purely elastic systems, this would require a maximum power transfer of double this value when the oscillations are not accompanied by an impetus at each swing, such as caused by improper governor operation. Tie connections having a maximum synchronizing capacity of double the permissible fluctuation of load should hold the systems in stable operation. Under usual conditions the load on each system provides a dampening characteristic which makes the factor two a rather safe figure to use.

The largest generator on the 25-cycle system of the Brooklyn Edison Company is 30,000 kw. and therefore,

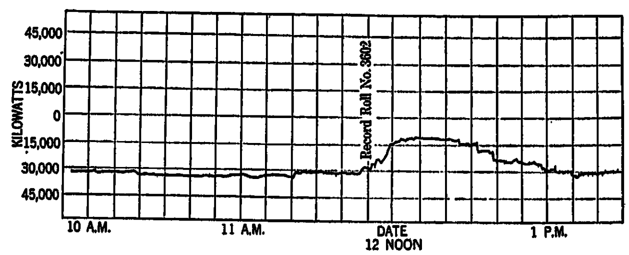

Fig. 5

a frequency changer having a pull-out point above 60,000 kw. was chosen. The 35,000 kw. unit selected has a pull-out point of a little over 70,000 kw. and although a number of system disturbances have occurred while the systems were in parallel through this unit, the two systems have never pulled apart. These disturbances have at times reduced the voltage and as the synchronizing power is a function approaching the square of the line voltage, the synchronizing power at such times was reduced considerably below 70,000 kw. In one case, a direct 3-phase short occurred on the 25-cycle generating station bus with three generators and the frequency changer operating, and although two of the turbine tripped off, the frequency changer and the other generator held in and carried the entire load.

The fluctuation in load through the frequency changer very seldom reaches more than 5000 kilowatts, and in Fig. 2 is shown a typical record of a graphic kilowatt meter in this tie connection.

A one line diagram showing the relay connections on this 35,000-kv-a. frequency changer is given in Fig. 3. The balanced and differential relays are for the protection of the system and the frequency changer in case of short circuits within the machine. The overload relay is for the purpose of automatically tripping out the tie connecting oil circuit breaker in case the two systems fall out of step and the relay is set for sufficient current and time to accomplish this result without tripping out on momentary short circuits.

The synchronous-synchronous type of frequency converter was selected as, for our purpose, it gave the following advantages:

Greater flexibility in voltage control on either system.

Elimination of voltage disturbances passing from one system to the other under short circuit conditions. With somewhat different operating conditions some engineers have felt the electrical tie is an asset under disturbance condition so that a voltage dip will be reflected from one system to the other.

Slightly higher guaranteed efficiency.

The efficiency of the first 35,000-kv-a. unit exceeded the guarantees throughout the full range of operation with the results as shown on Fig. 4.

Amortisseur windings are provided on the 60-cycle end for starting with quarter voltage tap on the transformer whereas the unit when started from the 25-cycle end is connected directly to a turbine with both units started from standstill. In Fig. 5 is shown the current, voltage and kilowatts at the time of starting the frequency changer from the 60-cycle end. The ease with which this machine comes up to full speed is rather remarkable and you will note in the curve at the various points the speed relations of the revolving field with the rotating armature flux as it passes the critical points.

Discussion

R. W. Wieseman: I should like to ask Mr. Woodrow about the pull-out torque of this frequency-converter set; this is given as a little over 70,000 kw., or over twice the normal torque of the set. Is this pull-out torque the actual synchronous torque which the set can furnish for an appreciable time or is it the momentary torque of this set which includes the inertia torque of the rotors? The inertia torque is useful in a synchronous motor for a small fraction of a second only. The energy stored in the rotors of this frequency-converter set could furnish over twenty times normal load for but a few cycles. However, the limiting feature of the power output of the set is the ability of the generator to transmit power to its system. The power output of the generator, for a fraction of a second, is limited by the leakage impedance and not by the synchronous impedance because the generator magnetic flux can not change quickly and therefore it cannot be influenced by the opposing armature magnetomotive force. Thus, only two or three times normal load can be delivered by the generator for a small fraction of a second.

Therefore, to say that a frequency-converter set has a high pull-out torque without fully explaining the conditions, is misleading. The rational basis upon which synchronous machines can be compared is the constant or steady-state overload for an appreciable time, such as thirty seconds. Unity-power-factor machines of this type are usually designed with a short-circuit ration of unity and, consequently, the *true* synchronous pull-out torque is only 150 per cent of the normal torque.

Mr. Woodrow states that the synchronizing power is a function approaching the square of the voltage. In any simple circuit with constant impedance the power varies exactly as the square of the voltage. Two duplicate generators operating in parallel through a transmission line have a synchronizing power which also varies as the square of the line voltage assuming that both generators have approximately the same voltage. With a frequency-converter set operating between two systems the capacities of which are much greater than the capacity of the set, the maximum power that can be transferred varies approximately as the first power of the line voltage because, Maximum instantaneous power = (line voltage × machine virtual voltage) ÷ (transient impedance) or Maximum steady-state power = (line voltage × machine nominal voltage) ÷ (synchronous impedance). Since the virtual voltage (flux) or the nominal voltage (excitation) are constant, respectively, the maximum power varies approximately as the first power of the line voltage.

It follows that a frequency-converter set which has a steady-state maximum power of only 50 per cent above normal can carry 100 per cent overloads with a reasonable drop in line voltage for a small fraction of a second.

H. R. Woodrow: The 35,000-kv-a. frequency changer has a pull-out point of 70,000 kw. for the first few cycles covering the first oscillatory swing of surging between the systems. The sustained pull-out point is about 55,000 kw.; that is, after the magnetizing action of the motor field comes into play.

The equation for maximum instantaneous power given by Mr. Wieseman is approximately correct where the resistance of the circuits is small compared with the reactance and when his term "transient impedance" represents self inductive reactance and resistance and "machine virtual voltage," the generated voltage in the motor. The maximum steady-state power is represented by exactly the same formula, but Mr. Wieseman has used an imaginary expression, "machine nominal voltage" which represents the motor generated voltage multiplied by the ratio of another imaginary term synchronous impedance and real impedance.

Under normal operations the real impedance drop through the machine is small and therefore the voltage generated in the motor is approximately the same as the line voltage and therefore the maximum instantaneous power is equal to the line voltage squared, divided by the real impedance.

The exact expression for maximum instantaneous power between two synchronous machines having the same generated voltage is

$V^2/Z \times (1- R/Z)$ where V = voltage, Z = impedance and R = resistance.

Eight Years Experience with Protective Reactors

BY JAMES LYMAN[1] LESLIE L. PERRY[1] and A. M. ROSSMAN[1]
Fellow, A. I. E. E. Fellow, A. I. E. E. Associate, A. I. E. E.

Synopsis:—*In 1914 the authors presented before the Institute two analytical papers on Protective Reactors. During 1917, 1918 and 1919 three large power stations embodying what seemed to be the best system of bus and feeder reactors were placed in operation. The present paper gives brief accounts of sixteen accidents on the busses or feeders of these power stations. In each case the reactors effectually kept the power concentration within such limits that the damage was localized, and no troubles were experienced from mechanical displacement of conductors and insulators elsewhere.*

In concluding, the authors state that their experience with reactors leads them to believe that short circuits in the largest power stations can be so controlled by a properly designed system of reactors that their destructive effects will be confined to the immediate vicinity of the arc, and that it will be possible so to tie together power stations and power systems of any magnitude that a short circuit will be of merely local significance and will not jeopardize the service of other parts of the system.

IN February, 1914, the authors presented before the A. I. E. E. a paper entitled "Protective Reactances in Large Power Stations." In October of the same year they presented a second paper entitled "Protective Reactances for Feeder Circuits of Large City Power Systems." These studies were made for the purpose of determining the best reactor system for two large steam power stations on which preliminary design studies were then being made.

In the second paper it was shown by a series of curves[2] (Curve Sheet No. 6) that after a certain number of generators are connected to a ring bus, with reactors between adjacent generators, the number may then be increased to infinity without appreciably increasing the amount of current flowing into a short circuit on the bus. Based on the reactance values of generators and bus bar reactors which at that time seemed most desirable and which experience has changed but little, the following figures derived from the curves[2] will be of interest:

1. After three generators are installed, the number of generators can be increased to infinity without increasing by more than 50 per cent the amount of current that can flow into a short circuit on the bus.
2. With an infinite number of generators, the maximum amount of current that can flow into a short circuit on the bus is about 3½ times the short-circuit current than can be obtained from a single generator.

The system of connections finally adopted was a ring bus with five per cent bus reactors between adjacent units, based on turbo-generator units of the order of 25,000 kw. to 30,000 kw. at 0.8 power-factor, and a radial system of feeders with three per cent feeder reactors, based on feeders of 6000 kv-a. to 7500 kv-a. at 11 kv. to 13.2 kv. The bus reactors are shunted by oil circuit breakers which are so controlled that when one or more generators are disconnected from the bus, the corresponding set of adjacent bus reactors is automatically shunted, thereby preventing more than one set of bus reactors from being in circuit between any two adjacent running generators. This scheme of connections with eight generators in service is shown diagrammatically in Fig. 1. In Fig. 2 and Fig. 3 are shown the connections when but one-half the generators are in service. This system of connections limits the flow of current into a short circuit on a generator or on any section of the bus after a lapse of 0.1 sec. to a value of approximately 750,000 kv-a. The flow of current into a feeder short circuit is limited to a value of approximately 250,000 kv-a.

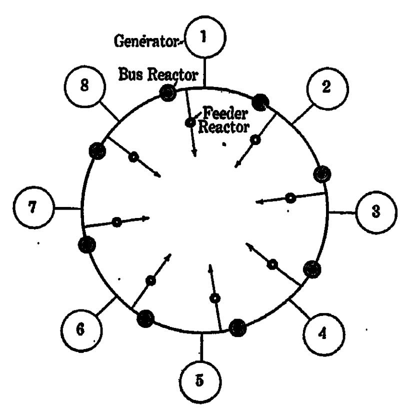

FIG. 1—SCHEME OF CONNECTIONS—ALL GENERATORS IN SERVICE

It is the object of this paper to recount some of the actual operating experiences in power stations where this reactor system has been in service for several years.

In 1916 construction work was started on the Windsor (West Virginia) Power Station of the American Gas & Electric Company and the West Penn Power Company. This station was placed in operation during 1917 with one 30,000-kv-a. turbo-generator. The following table shows its growth:

Year	Units Started Operation	Total kv-a. Capacity
1917	One—30,000-kv-a.	30,000-kv-a.
1918	One—30,000-kv-a.	60,000-kv-a.
1919	Two—33,333-kv-a.	126,666-kv-a.
1923	Two—33,333-kv-a.	193,333-kv-a.

During the years that this station has been in operation, there have been several short circuits of sufficient

1. All of Sargent and Lundy, Inc., Chicago.
2. These curves give current values which correspond approximately to the 0.1 sec. values on the latest decrement curves.

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severity to test the effectiveness of this scheme of reactors, and to demonstrate its ability to keep down the flow of short circuit current to a value well within the interrupting capacity of moderately heavy duty oil circuit breakers and to localize the damage to the immediate vicinity of the arc. Below are given brief narratives of several of these short circuits:

1. On August 15, 1919, with three generators running, an employe accidentally placed a ladder against a high tension lead

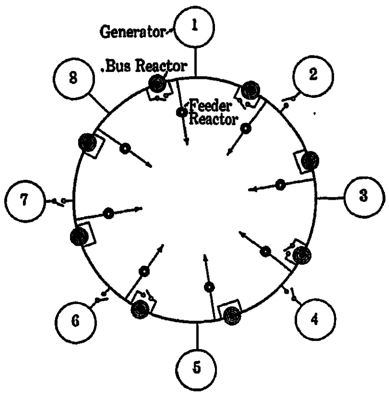

FIG. 2—SCHEME OF CONNECTIONS—GENERATORS NOS. 2, 4, 6 AND 7 SHUT DOWN

in the 66-kv. switchyard. The circuit breaker on the low-tension side of the transformer bank, feeding these bus bars, tripped and cleared the trouble. A few minutes later the low-tension breaker was again closed. Immediately a quantity of oil squirted out of one of the transformers between the tank and cover. The 11-kv. circuit breaker again tripped out and cleared the circuit. An examination of the damaged transformer, which was one unit of a 20,000-kv-a. bank, showed that the lower turns of the 11-kv. coils were badly damaged both by burning and by the destructive forces of heavy currents. The high-tension coils, although un-

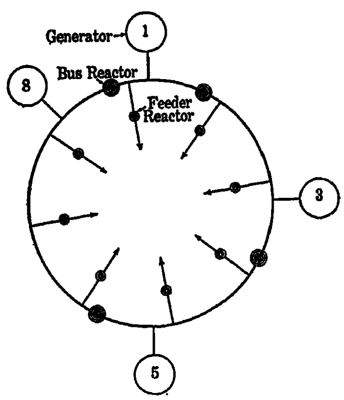

FIG. 3—SIMPLIFIED DIAGRAM OF CONNECTIONS—GENERATORS NOS. 2, 4, 6 AND 7 SHUT DOWN

damaged, were somewhat displaced. The total damage was confined to the one transformer.

2. On May 17, 1922, with four generators running, one of five 3-conductor 500,000-cir. mils 11-kv. cables, feeding a 30,000-kv-a., 132-kv. transformer bank, failed under emergency overload, taking with it several of the adjacent cables. The 11-kv. circuit breaker cleared the trouble from the bus. The following morning a cable on a duplicate transformer bank of the same capacity and voltage failed in a similar manner. In this case also the 11-kv. oil circuit breaker cleared the trouble from the bus bars. These cables were connected directly to the 11-kv. bus bars without series reactors. In neither case did the damage extend beyond the cables and their enclosing conduits.

3. On September 1, 1922, lightning broke down a bushing on one of the 66-kv. oil circuit breakers, causing failure to ground. The overload relay on the 11-kv. side operated properly, but the tripping relay on the 11-kv. oil circuit breaker was out of adjustment and failed to operate. Four generators were connected to the bus at the time, and the flow of current was sufficient to burn out the windings of two transformers of a 20,000-kv-a. bank feeding the 66-kv. bus. The pressure inside of the tanks was so great that the manhole covers were blown off in quick succession, and each transformer sent up a column of flaming oil higher than the stacks of the power-station building. It was several minutes before the switchboard operators discovered the location of the trouble and cleared the transformer bank from the bus, and some time before the oil fire in and around the transformers could be extinguished. Here again, the damage was limited to the two transformers and the area over which the flaming oil had spread.

4. On November 13, 1923, there occurred the most serious electrical trouble which had developed since the station was placed in operation. With all six generators and a seventh duplicate machine operating as a synchronous condenser,—a total of 223,333-kv-a., connected to the bus,—an arc was started by the breakdown of an insulator. The subsequent arcs and flames established a short circuit between the three phases of the main 11-kv. bus. More than a minute elapsed before the switchboard operators located the trouble and killed the power-station bus. A subsequent examination showed that the damage was confined entirely to the short section of bus across which the arc had played. A lighting fixture on the ceiling of the room directly above the arcing bus was not damaged; neither was the reserve bus structure which was separated from the main structure by an aisle space of eight feet. The circuits were, accordingly, transferred to the reserve bus and service was resumed within 48 min.

The West End Power Station of the Union Gas & Electric Company of Cincinnati was placed in operation in 1918, with two 30,000-kw. (31,250-kv-a.) turbo-generator units. In 1920, a third unit, and in 1921, a fourth unit of the same capacity were placed in service making a total present capacity of 120,000 kw. (125,000 kv.a).

All faults on West End Power Station have occurred on the feeders beyond the feeder reactors. Thus, the maximum fault kv-a. has been limited, in all cases, by the feeder reactors which are rated at 0.76 ohm on a basis of 13.2 kv. (three per cent on 300-amperes). Three of the most severe faults are listed below.

1. On July 18, 1921, at 11:42 a. m., a three-phase short circuit developed in a 400,000-cir. mil cable approximately 1½ miles from West End Power Station. At the time, there were two generators running with a total load of 45,000 kw., and two bus reactors in service. The feeder-oil circuit breaker opened to clear the fault without signs of any great disturbance on the rest of the system.

2. On December 15, 1923, a short circuit developed in a 400,000-cir. mil cable approximately ½ mile from West End Power Station. There were three generators running at the time and three bus reactors were in service. The fault current was 6000 amperes in each phase, and was broken by the opening of the feeder-oil circuit breaker. However, there was a sufficient force developed between two of one set of feeder reactors to pull them together with a resultant breaking of the concrete side slabs. These reactors were spaced horizontally one foot apart and were not fastened to the floor. The voltage on the directly affected section of bus was maintained at 70 per

cent of normal, and the rest of the system did not suffer any serious disturbance.

3. The most severe fault that the system has suffered occurred on October 31, 1924, at 9:57 a. m. This was a three-phase short circuit on two adjacent 400,000-cir. mil cables at a point where they changed from underground cables to overhead open lines. At the time there were four generators running and four bus reactors were in service. The two cables were connected to different sections of the bus, so that two of the generators were connected directly to the faulty cables without interposing bus reactors. The fault current was 4300 amperes per phase in each cable (8600 amperes from the system). The bus voltage dropped to 77.5 per cent of normal. One hurling pump motor which was connected to the same section of bus as one of the faulty cables, was tripped out by an undervoltage relay. No other auxiliary motors were affected. The relays operated correctly to open the oil circuit breakers on the two faulty cables.

In 1925, Miami Fort Power Station, with an initial installation of two 38,000-kw. (44,705-kv-a.) turbo-generators will go into service about twenty miles west of West End Power Station. This station will have the same system of bus and feeder reactors that has worked out so successfully at West End Power Station.

In the winter of 1919-1920 the Northeast Power Station of the Kansas City (Missouri) Power & Light Company was placed in operation with two 20,000-kw. (25,000-kv-a.) turbo-generator units. The following table shows the growth by years of this station:

1919—One 20,000-kw. 25,000-kv-a.
1920—Two 20,000-kw. 25,000-kv-a.
1922—One 30,000-kw. 37,500-kv-a.
1925—One 30,000-kw. 37,500-kv-a.

Total present capacity 120,000 kw. (150,000-kv-a.)

The Northeast Power Station is located northeast of the main business district of Kansas City on the low bottom lands which border the Missouri River. As there was initially no practical way of carrying underground cables in ducts across these bottoms, the 13.2-kv. outgoing feeders were carried a distance of one-half to one mile on pole lines with conductors supported on 25-kv. porcelain insulators. Lightning is particularly severe in this district and although the circuits, where they changed from overhead to underground construction, were all protected by oxide-film lightning arresters and some of them also had lightning arresters at the power station end, there were, during the five years from 1920 to 1924 inclusive, a total of six cases where lightning entered the power station over these circuits and damaged equipment in the building. There was also one case of a cable breakdown near the feeder exit bushing and another case where an operator pulled a wrong disconnecting switch near the exit bushing, both of which resulted in practically the same kind and amount of damage as that caused by the lightning.

In the majority of these cases the damage consisted of the breakdown of one or more current transformers followed by an arc which burned one or more feeder reactors and destroyed some of the disconnecting switches, primary and secondary wiring, insulating supports and compartment doors. Each of these cases of trouble caused the temporary loss of a feeder circuit and was accompanied by a momentary drop in voltage. In not one of the eight cases, however, did any of the auxiliary motors drop out of step.

This power station has also had one case of trouble in which the bus bars were directly involved. On August 4, 1922, during a period of construction work, an operator carelessly closed a disconnecting switch on a grounded oil circuit breaker which grounded the *C*-phase of the main bus bars. Three 20,000-kw. units were in service at the time, and the neutral of No. 3 unit was grounded through a 3-ohm resistor.

When the accident occurred, generator No. 3 was tripped out by its balanced relays. The operator then tripped off generators Nos. 1 and 2 thereby shutting down the entire system. A subsequent examination disclosed the following damage: The disconnecting switch, with which the operator established the ground, was burned up. The neutral resistor developed a ground and shunted out 5/7 of the resistance, permitting a current flow calculated at 8800 amperes as against a flow of 2500 amperes had all the resistance been in circuit. This excess current burned out and open-circuited that part of the resistor remaining in circuit. Two jumper connections between coils on the *C* phase of unit No. 3 burned open. Within 12 minutes the station was again in operation, and within 30 minutes the load was back to normal.

At no time in any one of the three Power Stations, even under the worst of short-circuit conditions, has there been any trouble experienced from the shifting of cables and bus bars, or the breaking of bus bar and insulator supports by the mechanical forces which are usually characteristic of a heavy flow of uncontrolled short-circuit current.

Based on the experience given above, the authors feel that short circuits in the largest power stations can be so controlled by a properly designed system of bus reactors that their destructive effects will be confined to the immediate vicinity of the arc. There will be some disturbance in voltage beyond the bus reactors, but in only the most severe cases will this be of sufficient magnitude to cause auxiliary motors to drop out of step. Furthermore, they feel that with properly chosen reactors, it will be possible to tie together power stations and power systems of any magnitude so that a short circuit will be of merely local significance, and will not jeopardize the service of the other parts of the system.

For assistance in compiling the information on short circuits, the authors are indebted to E. H. McFarland of the American Gas & Electric Company, Windsor, West Virginia, C. W. DeForest of the Union Gas & Electric Company, Cincinnati, Ohio, and Edwin Jowett of the Kansas City Power & Light Company, Kansas City, Missouri.

Discussion

J. Lyman: To obtain the minimum concentration of power is an everpresent problem in the design of large power stations, substations, and distribution systems. A circuit breaker must be built, capable of opening its circuit, under any short-circuit conditions, but more serious than the problem of designing adequate circuit breakers are the electrical and mechanical shocks that may be transmitted to the entire electrical system from a modern bulk-power station, or to a power distribution and transmission system, tying-in several such great stations. At least 0.3 sec. elapses before a circuit breaker can operate. During this time the full force of the shock is imparted to the entire system.

As the units of power to be controlled are continually increasing, the design of circuit breakers capable of controlling them is becoming more and more important. The combination of current-limiting reactors with the circuit breakers in such a way that a minimum concentration of power is secured without materially affecting voltage regulation should, therefore, be the aim in the design of power stations and distribution systems. Thus electrical and mechanical impacts due to electrical disturbances of whatever nature, will be kept down to a minimum consistent with successful operation and the greatest reliability of service secured.

Reliable and efficient power stations, transmissions and distributions are rapidly bringing to a reality universal 60-cycle power throughout the country. Clearly in any electric power system the latest, most economical power station should be operated as a strictly base-load plant. In this way the lowest possible cost per kw-hr. is obtained and advantage taken of the remarkable advances in power-station design.

By judicious location of synchronous condensers and reactors, voltage regulation can be maintained and power drawn from any power station over the entire system.

In general, the energy losses in the synchronous condensers are compensated for by the improved efficiency of transmission lines, transformers and generators, supplying power at the higher power factor, while the capital costs of the synchronous condensers are more than offset by the increase in capacity of the system at the higher power factor. Thus, it is possible to operate with reliability the latest power plant on the system as a base-load plant with the resultant system economies. By so doing, the electric power can be supplied over a wide radius to the large industrial plant or to an electrified steam railroad at a lower price than that at which these companies are able to make power because they have a comparatively fixed power demand at a comparatively low load factor; thus when their power stations become obsolete, they cannot write them off their books and discard them. They cannot take advantage of the improved efficiencies of the latest power-station design and of the high load factor that is obtained on an extended improved power system.

H. W. Eales: Mr. Rossman has called attention to the remarkable fact that it is possible for short circuits to occur within power stations and for the operating staff to be unaware of the fact. Mr. Rossman mentions a short circuit of this nature at Windsor Station.

About twelve years ago, one of the 11-kv. busses burned in two in the Keokuk Station. For some time the operators there were unaware that they were delivering single-phase power over the 143-mi. transmission line to St. Louis.

In December 1924 a short circuit occurred in one of the bus chambers at Cahokia Station and although the operator knew that a short circuit existed on the system he did not ascertain for several minutes that it was within his own station although it later developed that it was located on the same floor on which the operating benchboard is situated.

As a result of that and other experiences we have recently concluded to install in Cahokia Station an indicating and ground-current protective system.

We believe that, with the isolated-phase arrangement of oil circuit breakers and other accessory equipment as now completely installed at Cahokia Station, a three-phase short circuit is virtually an impossibility. In designing the indicating and protective system about to be installed, it is necessary to consider only protection from phase to ground.

A review of all cases of trouble which have involved the station bus revealed that in order to clear the short circuit it was finally necessary, by manual operation, to isolate the bus section in trouble and disconnect its generator and other sources of power connected to it. This consumed valuable time during which system service suffered. As long as it is necessary to do this ultimately, it has been decided to arrange for its accomplishment automatically and instantly.

The method to be employed will consist of introducing current transformers in the apparatus ground bus ahead of the connection to earth. The secondary leads from these current transformers will be brought to the benchboard operating room to relays whose operation will cause the lighting of telltale lamps indicating the bus chamber in which the short circuit occurs, and simultaneously cause the opening of the bus-section circuit breakers, at both ends of the bus section involved, open the generator breaker or this bus section, and also tie feeders, if any, from other stations or systems.

E. C. Stone: The determination of the arrangement of bus reactors in a power plant must take into account both the protection of equipment and the safeguarding of service. Of the five cases of bus-reactor operations reported in this paper, three apparently resulted in complete shut-down of the power plant and consequently serious interruption to service. In my opinion the bus reactor problem is not solved until service as well as equipment is completely protected.

H. R. Summerhayes: I have had occasion to review and analyze most of the major bus short circuits that have occurred in large power stations, and it appears in nearly all of them that if the bus section could be segregated instantly from the rest of the station, matters would be better than they are. That is to say, some service might have been lost, but not nearly so much service as has been lost in most of these short circuits. Therefore, I am heartily in favor of some system of protection, which will sectionalize the bus. It has been accomplished by the differential protection used in the Brooklyn Edison Company's system and in the hydroelectric plant at Queenston, Ontario. In the latter, I believe the differential protection has actually worked successfully several times. It is also accomplished by the ground protection system mentioned by Mr. Eales.

The ground protection system is simple and it is based on the probability that nearly all bus troubles on the isolated-phase system, or on the group-phase system, involve a ground, and there is some ground current flowing.

It is very simple, then, to connect all of the non-current-carrying metal in the vicinity of a bus section and the switches pertaining thereto through a current transformer to ground. Instead of connecting them all to the common station ground, we run all the ground connections from switch bases, operating mechanisms, etc., of one bus section through a current transformer to the station ground, and that current transformer is made to trip the section switches, feeder switches and the generator switches on that bus section, and do so instantly. In this manner the service can be resumed with the least amount of interruption.

While reactors are very effective in limiting short-circuit currents, there is one trouble that has been encountered with the use of bus reactors, namely, when a large amount of current is being passed over a bus reactor from one section to another, the bus voltage differs on the two sections, or rather there is a phase

difference which unbalances distribution of load in feeders from different bus sections to the same substation. To prevent this unbalance it is desired to make a reactor which will be of low reactance at normal load, and high reactance at short circuit; this has been the aim of inventors for a good many years.

I hope that that problem will be solved; that some form of linkage will be found which will enable a large amount of power to be exchanged between bus sections or generating stations under normal conditions and will interpose a high reactance instantly before one cycle is passed when short circuit occurs.

F. H. Kierstead: This record of eight years' experience with a variety of reactors and only one instance of a reactor failure (and in this instance, the fault appears to have been with the installation and not with the reactor) is, I think, a positive testimonial of the reliability of modern current-limiting reactors. However, I, personally, am not satisfied with even the fine record that reactors have made, but feel that we must press on toward a still higher degree of the perfection of this device upon which we depend to protect everything else in great power systems. This greater perfection must come not alone from increased efforts on the part of the manufacturers but also through a better understanding of the characteristics of reactors by the operators and from more consideration being given by them to the installation and upkeep of the reactors.

Turning to the record in this paper, it is to be noted that in the one case of a reactor failure the reactors pulled together because they had not been bolted down to the floor. Now I am not bringing up this point in order to excuse a reactor failure, (the reactors were not of our manufacture, but the results would doubtless have been the same had they been our reactors, if the means which we provided for bolting the reactors to the floor had not been used); nor do I bring it up in order to point out a faulty installation, but I am bringing it up for consideration because it illustrates my point that a better understanding of the characteristics of reactors is required and more consideration must be given to their installation in order to obtain a higher degree of perfection in the service that they render. It should be clearly appreciated that there is a large magnetic force between adjacent reactors when carrying short-circuit current and that for this reason reactors should always be bolted to the floor and, in many cases, must be further braced to withstand this force.

There is another source of danger to reactors which I think is worthy of consideration at this time. It is one which we all should realize but one which many of us overlook. I refer now to the failure of a reactor which may be caused by foreign conducting material accidentally dropping into a reactor and lodging in the winding. We have proven, by careful tests, that if a piece of metal, such as a screw, bolt, nut, or washer becomes lodged in the winding and escapes notice during inspection, no indication of its presence in the reactor will usually be given during the normal operation of the circuit because of low voltage between turns, but at the instant a short circuit occurs, when the voltage between the different sections of the winding jumps to many times its previous value, incipient arcs shoot out where the metal bridges across a section of the winding and are followed instantly by a complete flash-over of the reactor. Our tests have further proven that thin insulation on the conductor will not prevent such failures, even though the insulation has ample dielectric strength to withstand the voltage placed across it by a short circuit for the reason that the magnetic force exerted on iron and steel objects will cause them to break through thin insulation.

This danger may be eliminated by a very careful inspection of the reactor before it is placed in service in order to be sure that no such foreign material is lodged in the winding. The inconvenience of making such an inspection should not deter one from making it for it is in those places most difficult to inspect that foreign material is most likely to lodge.

Attention is further called to the fact that the magnetic field of a reactor carrying a short circuit will reach out to a distance at least equal to the diameter of the reactor and draw loose magnetic material into its winding. Therefore, great care must be taken to keep the passageway where reactors are installed clear of such material. Doors or screens across the openings of the compartments in which reactors are installed are recommended as a means of preventing any such material which may be dropped in the passage-way from getting into the reactors.

For those installations where it is very difficult to be sure that foreign material will not enter into the reactors and for those operators who require a safeguard from it in addition to that afforded by inspection, The General Electric Company has developed a thick asbestos insulation which is closely and firmly woven on the reactor conductor and is treated with flame-proof compound which makes the covering very strong and able to resist cutting and tearing very tenaciously. This insulation is used solely to prevent foreign material causing a short circuit between turns. The special grade of Portland cement concrete supports so successfully used for years is still the major insulation. Reactors with this insulation have been thoroughly tested by making many short-circuit tests upon them with a 26,700-kv-a., short-circuit testing generator and these tests have proven that this insulation affords adequate protection to the conductor from foreign material and that the insulation is so free from any inflammable material as to be properly classed as fireproof.

H. W. Osgood: This timely paper is a valuable confirmation of the effectiveness of reactors placed in a ring bus in large generating stations. Reactors placed in a bus between generating units are more effective for limiting short-circuit current than reactors of greater reactive voltage drop placed in series with the generators and in effect paralleled on a bus. Reactors of moderate size are also necessary in the generator leads as these give to the generator circuit and the busses protection equivalent to that provided by feeder reactors.

In looking up earlier information on use of current-limiting reactors, we find that in June, 1909, Mr. Junkersfeld presented a paper at Atlantic City, before the National Electric Light Association on "The Use of Reactance Coils in Generating Stations." In this paper, mention was made of the first application of reactors in the Cos Cob Station of the New York, New Haven and Hartford Railroad, an installation in the Central Station at Baltimore and in the Fish Street Station at Chicago. Reactors of the choke-coil type had, however, been used on many overhead and a few underground circuits prior to these installations in Cos Cob, Baltimore, and Chicago.

Dr. Steinmetz, in his discussion of that paper in 1909, said among other things, "When you come, however, to still larger stations—and some of the largest stations in the country are rapidly approaching that condition—then we meet the condition where even with this generator reactance, limiting the momentary short-circuit current to about ten times full-load current, the rush of current at the bus bars is altogether too much to permit a switch to be designed economically to take care of it, and then we shall be obliged to carry the limitations still further in the manner suggested by Mr. Junkersfeld, in sectionalizing the bus bars by reactance. This suggestion is therefore not an alternative to the generator reactance but is in addition thereto, for those cases of huge powers—200,000 kw. or more—where the generator reactances are no longer sufficient to limit the momentary short-circuit current at the bus bars to a reasonable value; that is, to half a million or a million kilovolt-amperes."

During these 16 years since this discussion by Dr. Steinmetz, the rupturing capacity of oil circuit breakers has been increased, in some cases, to 1,500,000 kv-a., but the ultimate capacity of generating stations has been increased in some cases to a projected total of 600,000 kw. Current-limiting reactors have therefore become indispensable between bus sections, as well as

in the generator leads and feeders to limit the short-circuit current, within the rupturing capacity of circuit breakers.

The isolated-phase arrangement with vertical separation of phases is peculiarly adapted to the installation of current-limiting reactors. Space is afforded in the switchhouse where the generator leads come up or the feeders drop down to the cable tunnel, the bus structures and oil-circuit-breaker compartments being grouped in the center with an operating aisle between structures. With the reactors isolated in this manner there is less likelihood of a phase-to-phase short circuit and if one occurs the reactors intervene to limit the current.

Experience indicates that in addition to a proper scheme of connections for reactors, the following points among others should be given careful consideration:

Terminals should be placed at opposite ends of a reactor to prevent danger from flashover across terminals and with the vertical-phase separation one terminal can be placed at the top and one at the bottom of the coil.

Non-combustible material should be used in their construction.

Reactors should be satisfactorily insulated from ground and enclosed in a compartment arranged for ventilation.

Reasonable clearances should be provided to all magnetic material.

Insulation of conductors, if used, should be of heat-resisting material.

As the reactor is installed as a protective feature, it should be carefully constructed and installed in every detail so that the chances of the reactor, or its connections, being the source of trouble or the means for spreading trouble will be reduced to an absolute minimum.

Mention is made in this paper that when one or more generators are disconnected from the bus, adjacent sets of reactors are automatically shunted by short-circuiting breakers, thereby preventing more than one set of bus reactors being connected between two adjacent running generators. A description of the means for automatic shunting of the reactors would be of interest.

S. I. Oesterreicher: In this paper there is one set of records of particular interest to me, namely, the short-circuit disturbances in the West End Power Station of the Union Gas & Electric Company of Cincinnati.

It is stated in the paper that in one case the magnetic attraction between two adjacent co-axially arranged reactors was of sufficient magnitude to damage the two reactor housings. With a separation of 12 in. between the two concrete housings, the magnetic centers of the two coils are 33 in. apart. At this distance and with 6000 amperes flowing across each reactor the magnetic attractive forces between the reactors was about 14,500 lb. The weight of the reactor was only 13 per cent of the attractive forces. Thus the damage done to the concrete housings due to their tender contact is excusable, if it is known that no interbracings between the two reactors were used.

The other set of records about which I desire to comment refer to the eight disturbances of the North East Power Station of the Kansas City Power & Light Company. Of these eight disturbances, six were caused by lightning troubles. The paper states that: "In the majority of those cases the damage consisted of the breakdown of one or more current transformers followed by an arc which burned one or more feeder reactors, etc." I believe it to be unfair to record in this paper reactor failures caused by lightning disturbances.

In my humble opinion, a current-limiting reactor will give no protection against lightning regardless of its type of construction. It would be of interest to know whether or not these reactors had shunted resistors. This would throw light to some extent upon a very live topic among many engineers.

A. M. Rossman: We are indebted to Mr. Eales for the information he has given us on the recent short circuit in the Cahokia Switch House, and the protective measures he has decided to install to limit the extent of the damage in the event of another short circuit. I believe the ground relaying system which he describes to be a step in the right direction.

Mr. Stone emphasizes the fact that in the two cases which are cited, where the bus bars were directly involved in a short circuit, the power station was completely shut down. Our paper states that in each of these instances the bus bars were cleared, as a safety measure, by the switchboard operators themselves, before they could definitely determine the exact location of the short circuit. Had they opened the bus-section oil circuit breakers instead of the generator breakers, or had there been a relaying system similar to the one described by Mr. Eales, I doubt if there would have been any interruption to service outside of the bus section which was actually involved in the arc.

Mr. Kierstead, Mr. Oesterreicher, and others lay stress on the need for reliability of the reactors themselves. I might say that of all the cases of trouble given, not one originated in the reactor itself, nor did any of the reactors develop weaknesses under short-circuit conditions. Several of the Kansas City feeder reactors were damaged by arcs which started in current transformers when lightning came in over the overhead 13.2-kv. feeder circuits and broke them down. This source of trouble has since been eliminated by placing all of the outgoing 13.2-kv. feeders under ground and by relocating the current transformers.

Mr. Summerhayes has discussed the effect of busbar reactors on the voltage of the different busbar sections. We have found that with the reactors we have used, the ratings of which were carefully determined by considering both voltage regulation under normal conditions and current flow under abnormal conditions, this has given us very little concern. With a radial system of feeders, which is the system followed in the three power stations under discussion, this effect is negligible.

Mr. Osgood has asked how the busbar reactors are automatically shunted in and out of service. This we have done by providing either an auxiliary contact on the control switch of the main generator oil circuit breaker, or an auxiliary switch on the generator oil circuit breaker itself. When the operator closes the generator circuit breaker, the circuit breaker shunting the corresponding busbar reactor opens; when he opens the generator circuit breaker, the breaker shunting the busbar reactor closes.

In concluding, I might state that our experience with power-station reactors has fully justified our expectations and has strengthened our confidence in them as a means of successfully keeping within controllable limits, the flow of current during short circuits.

Mississippi River Crossing of Crystal City Transmission Line

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Synopsis.—*The description and illustrations which follow refer to overhead wire crossing of the Mississippi River near Crystal City, Mo., of a double circuit, 132,000-volt, three-phase, 60-cycle transmission line now in the process of construction. The terminal points of this line at the present time are the Cahokia steam power station of the Union Electric Light & Power Company and the glass manufacturing plant of the Pittsburgh Plate Glass Company at Crystal City, Mo. The general location of this line is shown in the accompanying map, Fig. 1. Its length is 30.8 miles, (49.6 km.), of which 28.4 miles (45.7 km.) are in Illinois and the remainder in Missouri and in the river crossing. The purpose of this article is primarily to describe the problems involved in the river crossing.*

At the point of crossing, the Mississippi River is approximately 4000 ft. (1220 m.) wide, bank to bank. On the Missouri side there is a high limestone bluff the top of which is at elevation 745 with reference to Memphis, Tenn. datum. The average ground level on the Illinois bank is at elevation 395 or 350 ft. (106.8 m.) lower than the Missouri bluff, opposite.

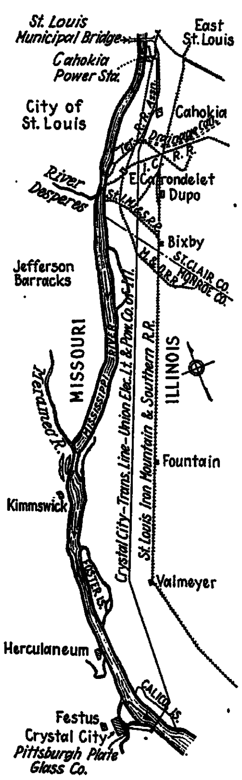

FIG. 1—MAP SHOWING LOCATION OF TRANSMISSION LINE AND MISSISSIPPI RIVER CROSSING

The hydrographic records of the Mississippi River indicate that extreme high water in 1844 was at elevation 410.5 and high water in 1903, at elevation 406. A levee protecting the Illinois low lands parallels the river for a distance of approximately 6500 ft. (1983 m.) from the Missouri bluff. The Illinois shore at, this point between the river's edge and the levee, is known as Calico Island.

It is observed from the foregoing figures that any foundation between the levee and the Missouri bluff would be in an area subject to overflow and in addition would be subject to the characteristic scouring action of the Mississippi River during each such overflow period.

In order to obtain data for the design of foundations for towers two test borings were made to rock on the Illinois bank, the first at a distance of approximately 400 ft. (122 m.) from the river's edge and the second about 50 ft. (15.3 m.) from the edge. Both of these borings showed that the ground was typical river bed deposit consisting of fine silt on the top strata, the lower strata consisting of coarse sand and gravel, the gravel increasing in size to stones several inches in diameter as rock was approached. Since these data checked closely with numerous other borings of the Illinois bed of the river between this location and East St. Louis, Ill., it was not considered necessary to make more than the two borings taken. Government records (House document No. 762, 63rd Congress, 2nd Session) contain reports of test borings taken over a distance of four miles east and west beginning at Steins Street, St. Louis, on the Missouri bluff to the Illinois bluffs. The borings on Calico Island showed deposits very similar to the government borings. Other boring data available were those made at the time of construction of the four bridges spanning the Mississippi at St. Louis and those for the Cahokia power station below East St. Louis, Ill.

The Federal requirements with respect to river clearance at this point called for a mechanical clearance of 64 ft. (19.5 m.) from the lowest point of the sag to high water elevation on the basis of 1903 high water. It was considered advisable to add 10 feet (3.05 m.) for electrical clearance making total clearance 74 feet (22.6 m.)

The preceding description outlines the physical problem; the solution of the problem was made as follows:

It is apparent from the data given that to support a

1. Both of Union Electric Light & Power Co. of Illinois, St. Louis, Mo.

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crossing span on towers outside the area of overflow would involve a span distance of 7000 feet (2135 m.) It was determined, therefore, to construct the crossing in two spans supported by three towers, the western tower to be placed on top of the Missouri bluff, the intermediate tower approximately 300 ft. (91.5 m.) from the existing water's edge on the Illinois bank, and the third tower a short distance east of the levee. It was further decided to arrange the Missouri tower and the Illinois levee tower as anchor structures, both designed to support the unbalanced stress of all six conductors under worst conditions of ice and wind loading, and to arrange the intermediate tower so as to support the conductors on insulators in suspension position.

Sketch, Fig. 2, shows all these data in graphical form. From this sketch it will be observed that the horizontal distance between the Missouri tower and the intermediate tower is 4279 ft., (1305 m.) and the horizontal distance between the intermediate tower and the Illinois terminal tower, 2681 ft., (818 m.) The profile

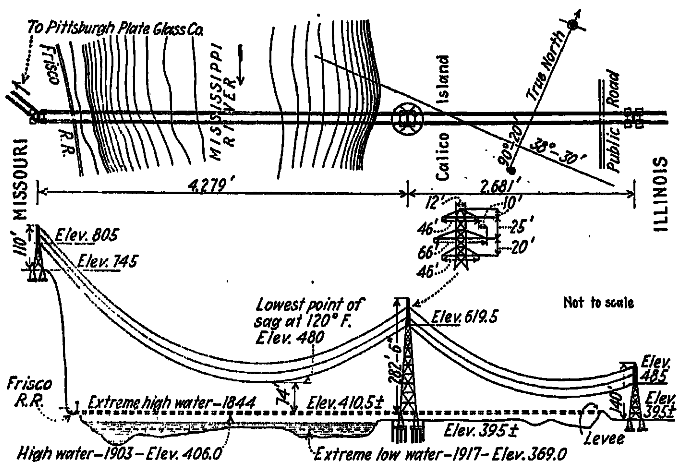

Fig. 2—Plan and Profile of Mississippi River Crossing

shows clearly the different elevations involved. The clearance to river level is shown on the basis of conductor sag at 120 deg. fahr. (48.8 deg. cent.) and under extreme assumption that such sag will occur with the river at an elevation duplicating that of 1903.

The lowest crossarm of the Missouri tower will be 60 ft. (18.3 m.) above its base. The lowest crossarm of the intermediate tower will be 227½ ft. (69.5 m.) above its base, and the lowest arm of the Illinois anchor tower will be 90 ft. (27.5 m.) above its base.

The three conductors of each circuit are to be carried across the river in vertical configuration. From the tower details shown in Fig. 2 it will be noted that there will be a vertical separation of 20 ft. (6.1 m.) between the conductors of each circuit at the two anchor towers, but a vertical separation of 25 ft. (7.6 m.) between the top conductor and the middle conductor at the intermediate tower, and a vertical separation of 20 ft. (6.1 m.) from the middle conductor to the lowest conductor at the intermediate tower. It will be further noted that the center conductors are to be offset outwardly from the towers 10 ft. (3.m.) with respect to the top and bottom conductors, both of the latter being at the same distance from all towers. The length of the crossarms are based upon—

1. The foregoing vertical and horizontal separations of the conductors
2. The length of the assembly of insulating supports, hardware, etc., as will be discussed later, and
3. A minimum clearance to steel structure of 4½ ft. (1.37 m.) with conductor swung at an angle of 45 deg. from the vertical toward the tower

It will be observed that this has resulted in top and bottom crossarm length of 46 ft. (14.03 m.) and middle crossarm length of 66 ft. (20.2 m.)

It will be found upon a detailed analysis of a problem of this character that the final solution of;

a. The strength of crossing conductors
b. The insulating supports at the towers
c. The towers themselves
d. The tower foundations

are interdependent one upon the other and that a joint solution of all four phases of the problem must be made more or less simultaneously. The resulting composite solution should be the one which results in allowing ample factors of safety and the lowest over-all total cost of the crossing.

The specific economical study of this particular problem has resulted in the selection of a steel-reinforced aluminum conductor to be employed. The design tension for the conductors and for the other three factors of the major problem was taken as 33,000 lb. (15,000 kg.) which will occur under conditions of ½-in. (1.27 cm.) ice loading at 0 deg. fahr. (−17.7 deg. cent.) and 12 lb. per square foot wind pressure (58.7 kg. per sq. m.).

The specific requirements which each of the four major elements should meet were then chosen as follows:

1. Towers

a. Anchor Towers. The anchor towers are to be capable of withstanding the unbalanced pull of six conductors with 33,000-lb. (15,000 kg.) tension in each. The design of the tower steel members specifies unit stress requirements to be in accordance with the National Electrical Safety Code for the above loading with the tower steel computed on a basis of 50 per cent overload on the above specific loading. The tower foundation specification requires these to be designed for 75 per cent overload with respect to the above specific tower loading. In addition to the ice and wind loading on the conductors, the tower is required to meet an ice weight and wind loading produced by the weight of ½ inch ice covering on its members and with a wind pressure equivalent to twice the area of one side on the basis of 25 lb. per square foot pressure (122.2 kg. per m^2).

b. Intermediate Tower. The specification for the intermediate tower gives the maximum loading as three

broken conductors with a tension of 33,000 lb. per conductor. The tower steel is to be designed on the basis of 50 per cent overload on this specific loading and with unit stresses on the design basis in compliance with the stresses of the National Electrical Safety Code. In addition to the ice and wind loading on the conductors the tower steel design includes the additional load produced by the weight of ½-in. ice covering on its members and with a wind pressure equivalent to twice the area of one side on the basis of 25 lb. per square foot pressure (122.2 kg. per sq. m.)

2. FOUNDATIONS

General. Since the three towers are of different heights and impose different loadings on their foundations, and since these in turn have different soil conditions, separate foundation computations were required for each tower. For the purpose of foundation data for the anchor towers, the shoreward conductors were assumed broken. It was considered that the unbalanced pull of the conductors on these towers would always be in a direction toward the river and that in consequence the piers on the river side would always be under compression at time of maximum load and those on the land side under uplift conditions. Advantage was taken of this in reducing the size of the two compression piers as compared to the uplift piers.

For the intermediate tower it was concluded that all four piers should be of similar design.

For the two anchor towers, which are at locations not subject to overflow, the full weight of the backfill earth and of the concrete piers is taken into account in resisting the uplift of the base.

In the case of the intermediate tower the weight of the earth backfill was not considered in calculating the uplift quantities and the concrete was figured for its uplift value when submerged in water.

The total reaction and uplift figures for all foundation calculations are indicated in the accompanying Table I.

These conditions have resulted in foundations of the following specific dimensions:

1. MISSOURI TOWER

a. Shoreward Piers. Each of these two piers consists of a reinforced concrete base pad 20 ft. (6.1 m.) square and 2 ft. (0.6 m.) thick from which rises a pier 12 ft. (3.66 m.) square at the base, tapering to 4 ft. (1.22 m.) square at the top, which is 9 ft. (2.74 m.) above the top of the base pad. The entire block is liberally reinforced with formed steel rods. The foundation bolts 2¼ in. (5.72 cm.) in diameter and six in number for each pier, extend 10 ft. (3 m.) into the concrete. Pairs of bolts are tied together with angle plates and reinforcing rods, in turn, are looped over the top of these angles.

b. Riverward Piers. These piers are the same as the shoreward piers with the exception that the base pad is 14 ft. (4.17 m.) square instead of 20 ft. (6.1 m.) square.

TABLE I

Reactions ...	*Wire Pull	Wind	Dead Load	Total
110 Ft. (33.5 m.) Tower—Illinois Bank				
Piers toward river.......	313,100 lb. (142200 kg.)	15,600 lb. (7090 kg.)	25,500 lb. (11600 kg.)	354,200 lb. (161200 kg.)
Uplifts				
Piers toward bank.......	285,700 lb. (129700 kg.)	15,600 lb. (7090 kg.)	25,500 lb. (11600 kg.)	275,800 lb. (125200 kg.)
Tower weight.				132,000 lb. (60000 kg.)
140Ft. (42.8 m.) Tower-Missouri Bank				
Reactions				
Piers toward river.........	326,200 lb. (148400 kg.)	20,100 lb. (9140 kg.)	31,500 lb. (14320 kg.)	377,800 lb. (171500 kg.)
Uplifts				
Piers toward Bank	298,800 lb. (136200 kg.)	20,100 lb. (9140 kg.)	31,500 lb. (14320 kg.)	287,400 lb. (130700 kg.)
Tower weight.				160,000 lbs. (72727 kg.)
285 Ft. (87. m.) Tower-Suspension Type				
Reactions ...	258,450 lb. (117500 kg.)	47,300 lb. (21480 kg.)	66,500 lb. (30200 kg.)	372,250 lb. (169400 kg.)
Uplifts.......	225,050 lb. (102080 kg.)	47,300 lb. (21480 kg.)	66,500 lb. (30200 kg.)	205,850 lb. (93700 lb.)
Tower weight.				288,000 lb. (131000 kg.)

*Weight of wire has been added to reactions and subtracted from uplifts. 16700 lb. (7600 kg.) for 285 ft. (86.8 m.) tower; 13,700 lb. (6230 kg.) 110 (33.5 m.) and 140 ft. (42.8 m.) towers.

All of these foundations are located in hard clay soil on top of a limestone rock bluff. The centers of the piers form a square 32 ft. (9.77 m.) on sides.

2. ILLINOIS ANCHOR TOWER

a. Shoreward Piers. These piers are the same as the shoreward piers of the Missouri anchor tower.

b. Riverward Piers. The riverward piers are the same as the shoreward piers except that the base mat is 17 ft. (5.18 m.) square instead of 20 ft. (6.1 m.) square.

All of these foundations are in silt and sand river bottom land.

The foundation bolts are the same in size and number and arranged in similar manner to those for the Missouri tower. The centers of the piers form a square 42 ft. (12.8 m.) on a side.

3. INTERMEDIATE TOWER

From the boring data mentioned in the preceding part of this article, it was evident that the foundations for the intermediate tower would be required to rest upon piles. From a study of the boring data and the tower uplift and reaction figures given in the preceding tabulation, the arrangement shown in Figs. 3 and 4 was selected. From Fig. 3 it will be noted that each of the four piers is to be supported upon a cluster of sixteen 35 ft. (10.8 m.) reinforced concrete piles driven so that their points will be at elevation approximately 339, *i. e.*, to a depth 47 ft. (14.35 m.) below the present ground level. Each of these piles is to be 15 inches (38.1 cm.) square and the steel reinforcing is so arranged as to connect the pile head with the base of the foundation pier. The base of the foundation pier is approximately square in shape, 17 ft. (5.18 m.) on a side and 6

ft. (1.83 m.) thick. From the top of this base the pier proper tapers from a dimension 11 ft. (3.4 m.) square at its base to 5 ft. (1.52 m.) square at the top. The top is at elevation 407, or 1 ft. (0.3 m.) higher than 1903 high water elevation. Six foundation bolts, 1⅞ inch (4.76 cm.) in diameter, are buried in this concrete pier to a depth of 10 ft. (3.05 m.) The bottoms of pairs of bolts

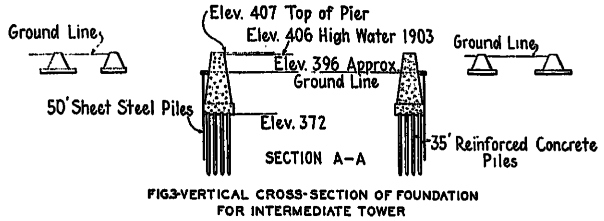

Fig. 3—Vertical Cross-Section of Foundation for Intermediate Tower

are tied together by substantial angle plates and hairpin shaped reinforcing rods in turn extend from the bottom of the pier over the tops of these angle plates. Thus, it will be seen that through the arrangement of the reinforcing employed, tension stresses of the pier are transmitted from the top of the pier to the bottom of the concrete piles. From Fig. 4 it will be observed that the centers of the four piers form a square 80 ft. (24.4 m.) on sides.

To protect these foundations against scour action of the river, it was decided to employ an envelope of steel sheet piling surrounding all four foundations. From Fig. 4 it will be observed that this piling is to be driven in the form of a circle 130 ft. (39.8 m.) in diam-

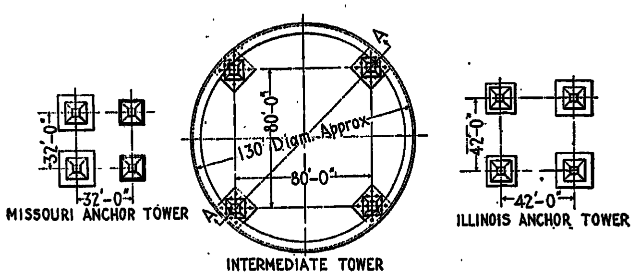

Fig. 4—Plan View All Foundations and Scour Apron for Intermediate Tower

eter, the outer edges of which are in contact with the outer edges of the foundation bases. The sheet steel piling consists of 50-ft. (15.25 m.) lengths made up with 2 per cent copper. From Fig. 3 it will be noted that a circular, reinforced-concrete girder, 1½ft. by 6 ft. (0.46 m. by 1.83 m.) cross section is to extend around the entire top perimeter of the steel piling and to be bonded to the outer edges of each of the four foundations. Thus, this girder will act as a stiffener to the piling and at the same time will reinforce the four base cords of the steel tower. The concrete will also protect the steel piling against rust at the ground line.

Fig. 5 illustrates the process of driving this steel sheet piling envelope. The steel sheet piling was driven by means of a steam hammer suspended from the end of derrick boom and supplemented by 300-lb. per sq. in. pressure water jet. (21.5 kg. per cm².)

Fig. 6 shows the process of driving steel sheet piling for the excavation for the cofferdam for an individual foundation pier, and the method of driving concrete piling in one of these excavations. Note that the steel sheet piling envelope shown in the foreground has been driven to ground level.

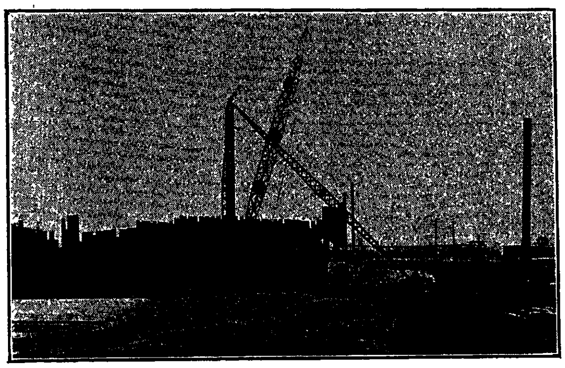

Fig. 5—Construction Progress Driving Steel Sheet Piling Apron Intermediate Tower

Fig. 7 shows the interior view of steel sheet piling cofferdam for one of the foundation piers for the intermediate tower. One of the concrete piles is shown in position ready to be driven.

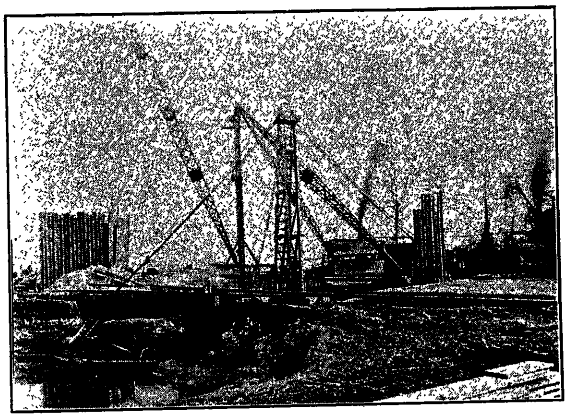

Fig. 6—Construction Progress Driving Sheet Steel Piling for Foundation Pier Excavation and Driving Concrete Piles in one of These Excavations

The concrete piles were driven by the combined efforts of standard pile driver hammer supplemented by 300-lb. per sq. in. (21.5 kg. per cm.²) pressure water jet. In driving the piles the ground was first partially excavated, the sides of the excavation held in place by steel sheet piles and the concrete piles then driven as above to proper cut-off.

A few special features of the river crossing towers may be of interest. Access to the top of each tower will be by means of a ladder the steps of which will be steel bars 4 by ⅜ by 11 in. (10.2 by .95 by 27.9 cm.) bolted to two angles carried up the center of one face of the tower.

From the top of the first panel of the tower, or 15 ft. (4.57 m.) above the base, these steps will be enclosed in a safety basket extending to the top of the tower and with rest platforms at convenient intervals. The bottom of this safety ladder will be provided with gate and padlock. The width of each crossarm will be 12 ft. (3.66 m.). To provide working space for men, each of these crossarms will be constructed as a platform employing subway type grating for flooring and with hand-railings 3 ft. (0.9 m.) high on each side. The space within the basket at the top of each tower will be used for storage boxes to store spare insulators and hardware, construction tools and tackle for the tower. All of these towers will be hot galvanized and will be bolted.

4. Conductors

The conductors for the river crossing consist of 318,000-cir. mil steel reinforced aluminum cables for 250 amperes maximum current requirements and each cable was purchased as one 7500-ft. (2290-meter) continuous length, one spare conductor being supplied.

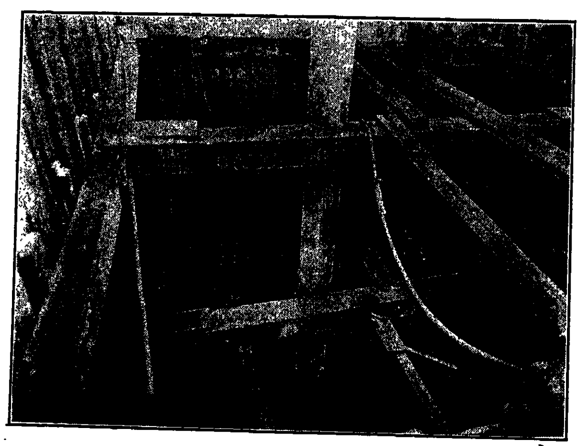

Fig. 7—Interior View of Steel Sheet Piling Cofferdam for Foundation Pier for Intermediate Tower

The steel core consists of 43 strands of various size galvanized steel wires so woven that all wires are woven in the same direction without the crossing of one strand over another. Long experience elsewhere with a number of long crossing spans has indicated the desirability of this arrangement for the purpose of reducing the possibility of breakage of strands due to gradual wearing of one strand into another at points of crossing. Surrounding the steel core are 24 strands of aluminum wires approximately 0.1151 inches (2.92 mm.) in diameter. The aluminum strands are woven on the steel core in a direction the reverse of that of the steel strands. The steel core is to be made up of strands each of which is to be a continuous length without weld, braze, or splice of any character. The above results in a cable the outside diameter of which is approximately 1.036 in. (2.55 cm.). The over-all diameter of the steel core will be 0.807 in. (2.05 cm.) with a breaking strength of 64,000 lb. (29,100 kg.). The ultimate strength of the completed cable is 67,600 lb. (30,700 kg.) and its elastic limit 53,500 lb. (24,300 kg.). The weight of completed bare cable will be 1.684 lb. (0.765 kg.) per foot.

The disposition of towers, cross-arms, etc., as above described will result in the following elevations at various locations of lowest conductor:

a. Crossarm attachment on the Missouri end at elevation 805

b. Lowest point of sag at elevation 480 at temperature 120 deg. fahr. (48.9 deg. cent.)

c. Conductors at the intermediate tower at elevation 620

d. Crossarm attachment at the Illinois terminal tower at elevation 485

Under these conditions the vertical sag at 120 deg. fahr. (48.9 deg. cent.) without wind will be 325 ft. (99 m.) below the Missouri tower support.

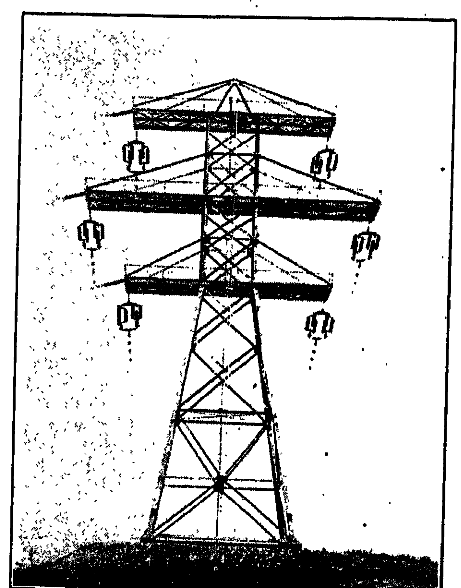

Fig. 7—(a) Missouri Strain Tower, Showing Method of Attaching Conductors

The sag and tension figures for other conditions of ice and wind loading, temperature, etc., are given in Table II below. As an appendix to this article are given the

TABLE II
STRINGING SAGS AND TENSIONS

Temperature	Loading	Sag below Upper Support	Sag below Lower Support	Tension
0° F. (−17.8° C.)	½ in. ice 12 lb. wind (1.27 cm. 58.5 kg/sq. m.)	318.3′ (Res.) (97. m.)	171.5′ (Res.) (52.3 m.)	33,500 lb. (15000 kg.)
32° F. 0°(C.)	½in. ice no wind (1.27 cm.)	328.3 ft. (100. m.)	142.8 ft. (45.1 m.)	27,370 lb. (12400 kg.)
−20° F. (−28.9°C.)	no ice, no wind	312.8′ (95.1 m.)	127.3′ (38.8 m.)	18,770 lb. (8500 kg.)
32° F. (0° C.)	no ice, no wind	317.5′ (96.7 m.)	132.′ 40.3 m.)	18,380 lb. (8320 kg.)
60° F. (15.6° C.)	no ice, no wind	320.′ (97.5 m.)	134.5′ 41.0 m.)	18,160 lb. (8250 kg.)
120° F. (48.9° C.)	no ice, no wind	325.′ (99.1 m.)	139.5′ (42.6 m.)	17,710 lb. (8050 kg.)

complete calculations of sags and tensions from which the values in the tabulation were derived.

5. Insulated Supports

The requirements to be met by the insulated supports for the conductors have been suggested above in the discussion with respect to the conductors. For the purpose of selecting the insulator linkages the maximum tension of the conductor was taken as 35,000 lb. (15,900 kg.) and a mechanical factor of safety of two was selected on the total linkage to be employed. The insulators

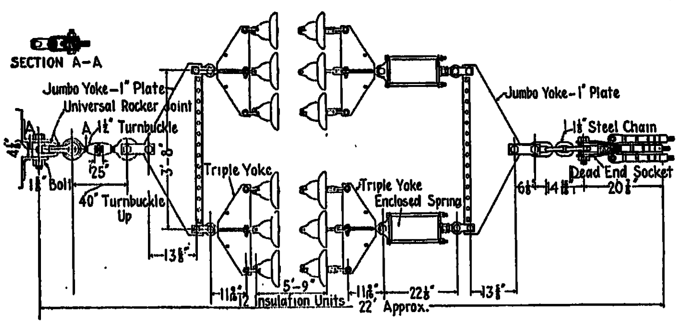

Fig. 8—Assembly of Conductor Insulating Supports for Anchor Tower

are each to be proof-tested at 12,000 lb. (5450 kg.) and to have a maximum ultimate strength of 18,000 lb. (8200 kg.) each.

a. Supports at Anchor Towers. To meet the above conditions six strings of insulator disks of this description are required per conductor at each anchor tower, these to be arranged in two groups of three strings each. The three strings of 12 disks are to be assembled with triple yokes top and bottom and two sets of triple yokes to be combined by one jumbo yoke at top and bottom. Details of this assembly are shown in Fig. 8 from which it will be noted that the over-all distance from the cross-arm to the conductors is approximately 22 ft. (6.7 m.)

The following features of interest are pointed out with respect to this arrangement. The attachment at the crossarm consists in effect of a universal joint made of two steel forgings. Motion in a vertical plane is obtained about the 1⅝-inch (4.13 cm.) diameter steel pin mounted in the crossarm clips and motion in a horizontal plane is obtained by rocker shaped surfaces on the two forgings. A turn-buckle is next provided which provides two feet of take-up. The jumbo yoke is long enough to permit proper equilization of tensions on the six strings of insulators. The holes shown in the triple yokes are for the purpose of attaching outriggers for pulling up the strings of insulators to permit of replacements. Next to the outer triple yoke is a double car spring which will be calibrated so as to show the tension on the insulator strings at any time. It is believed that these springs will also act as shock absorbers in the case of sudden movement of the conductors due to ice falling from them, etc. Next to the outer jumbo yoke are shackles and four links of heavy dredge chain. The entire structure has been designed for a maximum of flexibility and equalization of stresses over the individual insulator strings. With the exception of the steel casting covers of the car springs all metal parts even the insulator caps are made of steel forgings or plates.

The specification calls for a test of the completed assembly of three strings of insulators of 35,000 lb. (15,900 kg.) and of the completed assembly of six strings of insulators at 70,000 lb. (31,800 kg.)

The form of clamp employed at the end of the linkage for attaching to the conductors and the method of handling the conductors are also of special interest. This clamp consists of an aluminum body clamp which is compressed about the exterior of the complete steel reinforced aluminum cable by means of three compressions. The aluminum conductors are then cut off and the steel core continued into a steel Roebling bridge socket into which they are sweated with zinc. The aluminum body clamp and the steel core socket clamp are both carried by a common steel bolt 1⅝ inch (4.13 cm.) in diameter.

The details of this dead-end socket clamp are shown in Fig. 9. The aluminum body clamp contains three projections, the purposes of which are as follows. To one projection will be bolted two small aluminum body clamps attached by compression joints to two lengths of 300,000-cir. mil A. C. S. R. conductors of the type employed in the land transmission line. These two cables arranged in parallel horizontally will then be carried as the loop connection under the crossarm from

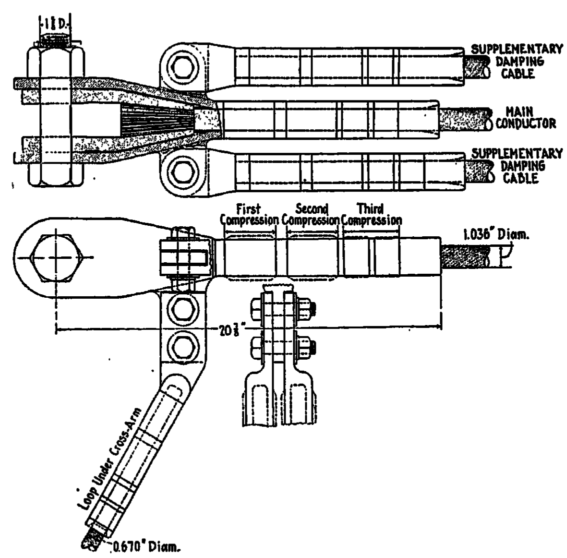

Fig. 9—Detail of Conductor Cable Socket Clamp for Anchor Towers

the river span dead end to the land span dead end. They will be clamped at five-foot intervals with three-bolt, parallel-groove aluminum clamps. The purpose of this latter arrangement is to stiffen the loop and hold to a minimum its swinging with the wind toward the tower. This precaution is of importance on account of the extreme length of these loops which will be about 45 feet (13.75 m.).

As shown in Figs. 8 and 9, there will be bolted to each of the other two projections on the main aluminum body clamp another aluminum body clamp which will be attached by compression joints to a length of supplementary 318,000-cir. mil A. C. S. R. cable which will parallel the main cable for a distance of 20 ft. at the dead end in one case and 15 ft. in the other, and be clamped to it by means of parallel-groove aluminum clamps.

As is well known, long transmission line spans involving high wire tensions are peculiarly susceptible to vibration, the laws of which are not so well known. The vibration, however, has a definite nodal point at the junction with the cable clamp resulting in a tendency for the strands of the cable to break at the point of attachment. The result of study by others of vibration in existing spans indicates that the probability of damage will be materially reduced by spreading the vibration over a number of clamps arranged in parallel, and increasing in mass as above described.

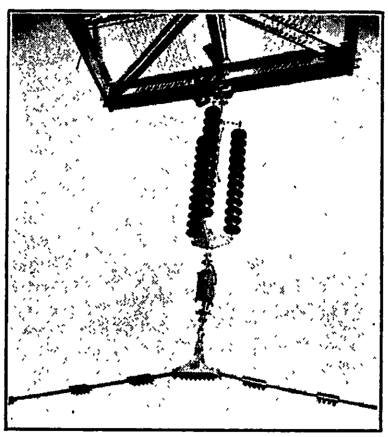

Fig. 9—(a) Conductors in Suspension Position to Intermediate Tower

b. Supports at the Intermediate Tower. As indicated in the description of the intermediate tower itself, the conductors are to be supported by insulators arranged in suspension. This arrangement is in conformity with that followed throughout the rest of the line where every effort has been made to reduce to an absolute

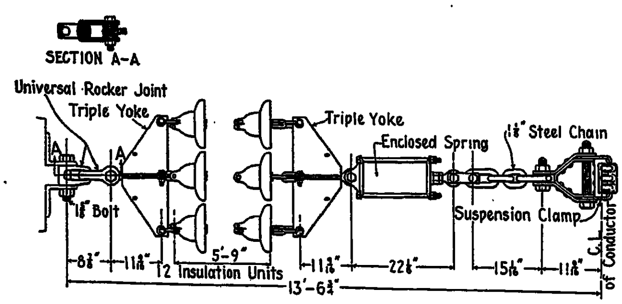

Fig. 10—Assembly of Conductor Insulating Supports for Intermediate Tower

minimum the number of dead-end insulator assemblies. In the river crossing the dead-ending of the conductors on the two terminal towers was a physical necessity. The dead-ending of these conductors was not necessary on the intermediate tower, and accordingly is not employed. In case of breakage of either span the linkage at the intermediate tower is strong enough under its normal strength characteristics to hold the remaining span without slippage. The reasoning behind this arrangement was as follows:

All records indicate that this crossing is in a region of heavy sleet loading. It was considered possible that the span between the intermediate tower and the Missouri anchor tower might be coated with ice while the east span from the intermediate tower to the Illinois anchor tower might not be so coated. This is not an unusual condition in transmission line work and follows usually from the sleet having melted and fallen from one conductor span before doing so in the adjacent span. Under such condition it was considered that if the conductors were carried on rollers or sheave wheels on the intermediate tower, the unbalanced loading of ice on the river span would cause an excessive sag in this span. To prevent just such occurrence or the reverse occurrence of the pulling down of the long land span between the intermediate tower and the Illinois terminal tower it was considered that the conductor should be definitely clamped to the insulator string at the intermediate tower.

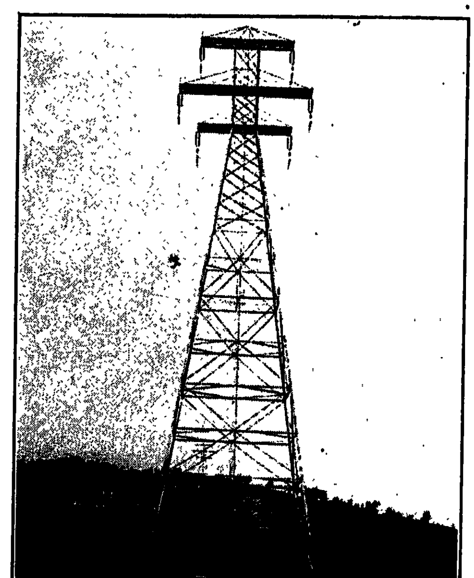

Fig. 10—(a) Intermediate Tower in Foreground; Missouri Tower in Background

Figs. 10 and 11 show the insulator and conductor clamp assembly selected. The order of assembly of the insulator strings, hardware, conductor clamp, etc., are similar to those employed on the dead-end towers, except that a single group of three strings of insulators in parallel is employed. The insulators and hardware exclusive of conductor clamp are the same as those used on the anchor towers and need no further description. The following specific features of the conductor clamp are of interest:

The conductor clamp consists of two aluminum castings bolted together about the conductor and supported

from the insulator assembly in a substantial steel saddle. The cable grooves in the aluminum clamp approximate the position the wire will take due to its normal sag. Paralleling the main conductor will be two supplementary lengths of cable of the same character as the main cable and bolted to it by means of parallel-groove clamps for distances of approximately 20 ft. and 15 ft. respectively on each side of the main supporting clamp. The use of the supplementary cables and clamps constitutes the treatment of the vibration problem in a manner similar to that used on the dead ends on the anchor towers.

For the reasons outlined above it is also considered necessary that this clamp shall prevent excessive slippage of the conductor during any unbalanced loading due either to ice or a break in the adjacent span. This is of particular importance from the standpoint of river navigation as well as of operation of the circuit. The specification requires that this clamp shall hold the cable against slippage at 35,000 lb. (15,900 kg.) tension in the event of breakage of one of the spans.

The foregoing description applies to the design of the river crossing only.

Land Transmission Line

The fundamental design factors included in the specification for the steel tower performance may be summarized as follows:

1. Arrangement of conductors for each circuit—vertical configuration with middle conductor off-set

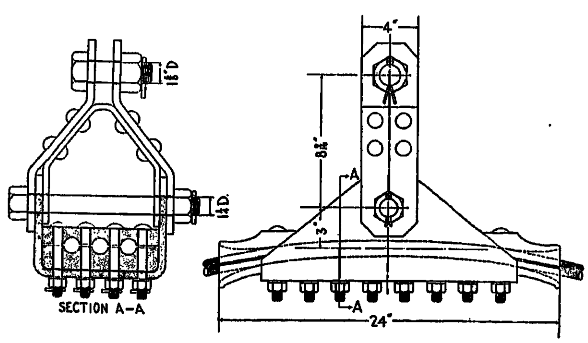

Fig. 11—Detail of Conductor Clamp for Intermediate Tower

from the tower horizontally 3 ft. (.9 m.) with respect to top and bottom conductors.

2. Minimum vertical separation between crossarms 12 ft. (3.7 m.)

3. Minimum clearance conductor to tower steel with conductor swung at angle 60 deg. from the vertical 3 ft. 6 in. (1.06 m.)

4. Number of insulators—10 per string (maximum length 4 ft. 6 5/16 in. (1.37 m.) for suspension tower; strain tower two parallel strings of 10 insulators (maximum length 6 ft. 3¾ in. (1.9 m.); anchor tower two parallel strings of 12 insulators (maximum length 7 ft. 1¼ in. (2.14 m.)

5. Average span 800 ft. (244 m.)

6. Minimum height to lowest crossarm suspension and strain towers 50 ft. (15.3 m.); anchor tower 45 ft. (13.7 m.).

7. Conductor to be employed—300,000-cir. mil A. C. S. R. 30 per cent steel.

8. Maximum tension in conductor for 800 ft. (244 m.) span under conditions 0 deg. fahr. ½ inch (1.27 cm.) ice loading and 8 lb. per sq. ft. (39 kg. per sq. m.) transverse wind pressure—6000 lb. (2725 kg.)

9. Towers to be computed on basis of 13 lb. per sq. ft. (62.5 kg. per sq. m.) wind pressure on 1½ times the exposed area of one side.

10. Maximum unit stresses in tower steel as prescribed by National Electrical Safety Code.

11. Design loading for towers—25 per cent overload over conditions stipulated for each type of tower as given in the following summary, except that crossarms are to be designed for 50 per cent overload and anchors for 50 per cent overload against uplift.

Type of Tower	Loading Conditions to be Met
I—Suspension tower 2 deg. angle	Any one wire broken with 2 deg. angle in line with wire loading as in (8) above.
II—Strain tower 15 deg. angle	Any three wires broken with 15 deg. angle in line with wire loading as in (8) above.
III—Anchor tower 30 deg. angle	Any three wires broken with 30 deg. angle in line with wire loading as in (8) above.
IV—Anchor tower no angle.	All six wires broken on one side with wire loading as in (8) above.
V—Anchor tower 90 deg. angle	All six wires broken with 90 deg. angle in line with wire loading as in (8) above.

The above requirements have resulted in four types of towers. Type *A* suspension tower will meet requirements I; type *B* strain tower will meet requirements II; type *C* anchor tower to meet requirements III and IV; Type *D* 90 deg. anchor tower to meet conditions V. The following tabulation gives a summary

TABLE III

Type	Length Top & Bottom Crossarms	Length Middle Crossarm	Height of Lowest Crossarm	Vertical Separation between Crossarms	Cross Section of Anchor	Depth Anchor Setting	Wt. of Tower Including Anchors	Spacing of Anchors
A	21 ft.	28 ft.	51 ft. 5½ in.	12 ft.	3 ft. 2 in. by 3 ft. 3⅜ in.	8 ft.	8900	16 ft.
B	26 ft. 6 in.	33 ft. 6 in.	51 ft.	13 ft. middle to top 12 ft. middle to bottom	4 ft. 1 in. by 4 ft 8 in.	9 ft. 9 in.	13475	18 ft.
C	20 ft. 6 in.	27 ft. 6 in.	45 ft.	12 ft.	5 ft. by 5 ft. 7⅛ in.	9 ft. 9 in.	13460	18 ft.
D	20 ft. 6 in.	27 ft. 6 in.	45 ft.	12 ft.	5 ft. 2 in. by 5 ft. 9 in.	11 ft. 6 in.	18610	18 ft.

of the physical dimensions and weights of these towers:

To supplement the four types of towers described, 8-, 16- and 32-foot extensions were employed to be used in some cases where obstructions required towers higher than the standard towers to obtain clearance or where the ground conditions required spans longer than the average span.

Type *A* suspension towers were used for all tangent

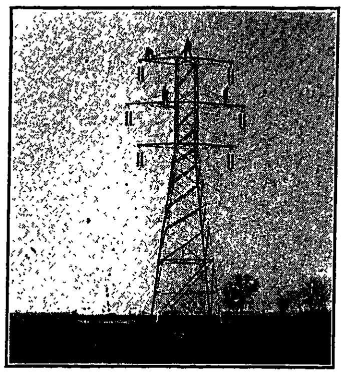

FIG. 12—PHOTOGRAPH OF STANDARD STRAIN TOWER

towers and angles not exceeding 2 deg. Type *B* strain towers were in general employed for every eighth tower position in a straight run and also as corner towers for angles up to 15 deg. Type *C* towers were used at the two terminal towers on the land line and also as corner towers for angles between 15 and 30 deg. For two 90 deg. angles in the line, type *D* towers were employed.

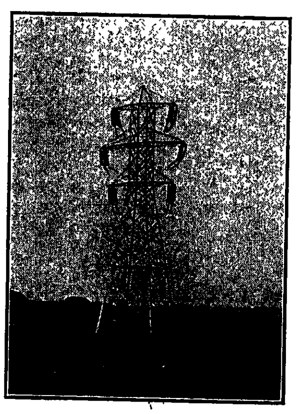

FIG. 13—PHOTOGRAPH OF STANDARD ANCHOR TOWER

Fig. 12 shows the arrangement of a standard strain tower, type *B*, as given in preceding tabulation. From the illustration it will be observed that two parallel strings of 10 insulators, each with equalizing yoke top and bottom, are used in suspension position. These two strings of insulators, together with the conductor clamp, are capable of holding the conductor in case of break in the adjacent span under a tension of 6000 lb.

Fig. 13 shows a standard anchor tower, type *C*. The foundations employed for the transmission line are the so-called earth anchors made of galvanized steel. These anchors consist of a pyramid-shaped frame work of angle iron with their bases completely closed by means of substantial angle iron. The closure was considered desirable on account of the fact that the towers were to be located on sand and silt river bottom land. The use of this type of anchor eliminates the need of transporting concrete materials and plant over the right-of-way. Bolted steel templets were designed for each type of tower and extension to facilitate the leveling of the anchors.

Fig. 14 shows, (1) A group of anchor holes where quicksand was encountered, and (2) the general nature of the anchors themselves as described above.

Fig. 15 shows a close-up view of one of these excavations. The method of handling these wet holes con-

FIG. 14—EXCAVATIONS FOR STANDARD TOWER ANCHORS SHOWING TYPE OF ANCHORS

sisted in shoring the sides of the hole with an open bottom wooden box. The hole was then pumped out by means of a portable gasoline driven pump, the boxing being driven down as the excavation progressed. The bottom was then filled with crushed rock tamped sufficiently to permit of leveling the anchors.

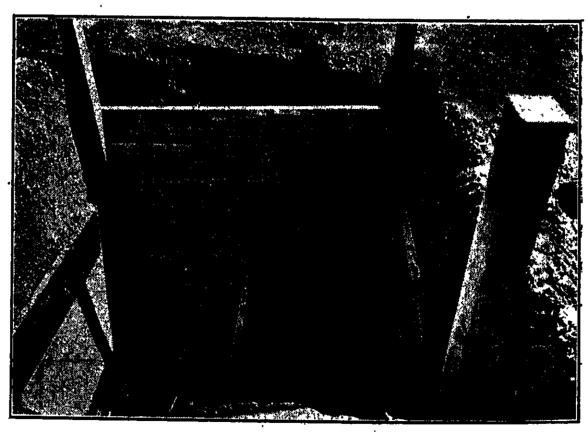

FIG. 15—EXCAVATION PROCEDURE FOR WET ANCHOR HOLES

Fig. 16 shows one of the earth anchors in place. Most of the transmission line as shown in the foregoing illustrations passes through flat bottom land. Fig. 17, however, suggests some difficult clearing work through densely wooded swamp land.

Grateful appreciation is here expressed for the suggestions and assistance received from Messrs. A. O.

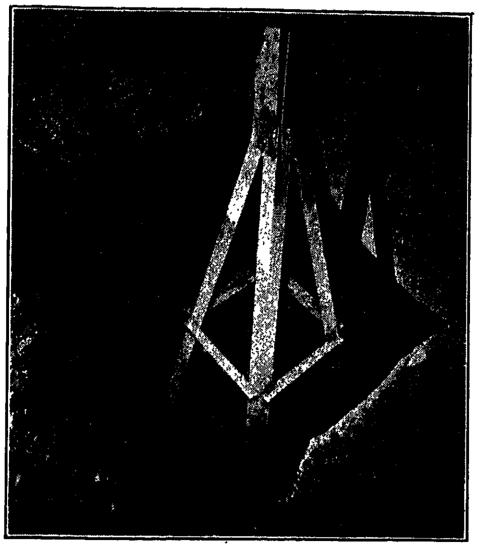

Fig. 16—Close-up View of Standard Tower Anchor in Place

Fig. 17—Clearing Right-of-Way Through Dense Woods

Austin, S. W. Bowen, H. O. Hill, J. S. Lapp, Prof. G. O. James, S. Stokes, T. Varney and J. P. Jollyman.

Appendix

The calculation of the performance of flexible cables and wires in equilibrium for supports at equal elevations and for short spans may be approximated to the extent of employing the parabolic form of equations or any one of a number of sets of prepared tables or charts. The ordinary tranmission line range of span seldom exceeds 1200 ft. and for such spans the values calculated by such methods serve all practical purposes.

However, when the specific problem arises of a long span and the supports are so arranged as to be at unequal elevations, and when it is important that certain clearances over navigable waters must be maintained and a certain stress is not to be exceeded, the economy of design and assurance of performance require some more rigorous calculation.

The calculation of long spans with supports at unequal elevations arises infrequently, and but little has

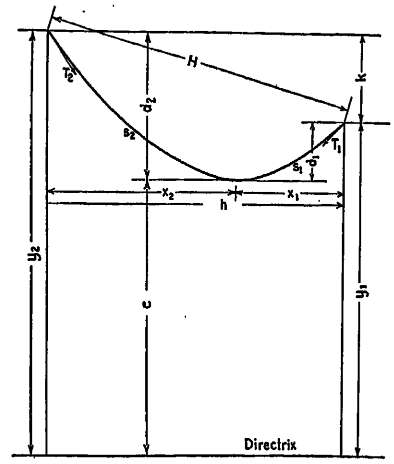

Fig. 18—Graph for Appendix

SYMBOLS

h = horizontal distance between supports.
x_1 = horizontal distance from lower support to the vertex of the catenary
x_2 = horizontal distance from the upper support to the vertex of the catenary
H = air line distance between the upper and lower supports
y_1 = the height of the lower support above the "directrix" or reference line of the catenary
y_2 = height of the upper support above the "directrix" or reference line of the catenary
c = height of the vertex of the catenary above the "directrix"
s_1 = length of the suspended wire from the lower support to the vertex of the catenary
s_2 = length of the suspended wire from the upper support to the vertex of the catenary
l = total length of the suspended wire
T_1 = the tension in the wire at the lower support
T_2 = tension in the wire at the upper support
d_1 = sag below the lower support
d_2 = sag below the upper support
k = the difference in elevation between the lower and upper supports
p = pressure per unit length of wire of wind on the ice covered wire
w_i = weight per unit length of wire of the ice on the bare wire
w_c = weight per unit length of bare wire
E = modulus of elasticity of the material carrying the stress
A = cross-section area of the material carrying the stress
α = coefficient of linear expansion fahr. of the material carrying the stress

been published to indicate the method of solving the performance of such spans.

Calculation of Sag and Tension Performance for Long Spans with Supports at Unequal Elevations

The mathematical curve in which a stretched flexible cable or wire hangs in equilibrium is the well-known catenary of the form,

$$y = c \cosh \frac{x}{c}$$

$$s = c \sinh \frac{x}{c}$$

$$T = w\,y$$
$$y^2 = s^2 + c^2$$
$$d = y - c$$

It can readily be demonstrated that the curve has the same general form whether the supports are at equal elevations or at unequal elevations.

When the supports are at unequal elevations, y_1 and y_2, these formulas may be written as

$$y_1 = c \cosh \frac{x_1}{c}, \qquad y_2 = c \cosh \frac{x_2}{c}$$

$$s_1 = c \sinh \frac{x_1}{c}, \qquad s_2 = c \sinh \frac{x_2}{c}$$

$$T_1 = w\,y_1, \qquad T_2 = w\,y_2$$
$$y_1^2 = s_1^2 + c^2, \qquad y_2^2 = s_2^2 + c^2$$
$$d_1 = y_1 - c, \qquad d_2 = y_2 - c$$
$$h = x_1 + x_2$$
$$l = s_1 + s_2$$
$$k = y_2 - y_1$$

From these equations we may then by properly combining derive the following formulas:

$$\sqrt{l^2 - k^2} = 2\,c \sinh \frac{h}{c} \qquad (1)$$

which determines the length of cable between supports, and when

T_2, w, h and k are known

$$y_2 = \frac{T_2}{w}, \quad y_1 = y_2 - k$$

$$c^2 \cosh \frac{h}{c} - y_1\,y_2 - \sqrt{(y_1^2 - c^2)\,(y_2^2 - c^2)} = 0 \qquad (2)$$

h = horizontal length of span, or its equivalent
k = difference in elevation of supports, or its equivalent

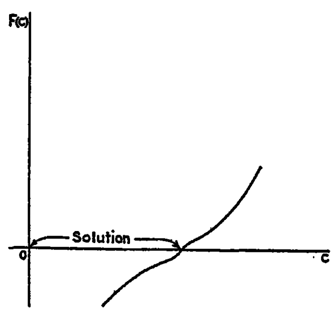

FIG. 19—GRAPH FOR APPENDIX

Then from equation (2) c may be determined by the graphical or trial method, plotting the equation (2) as $F(c)$ as ordinate versus c as abscissa.

It is readily seen from the conditions of the problem that $o < c < y_1$, and in general it is found that $0.9\,y_1 < c < y_1$. After the first two trials of c the actual value of c for the accuracy desired can be quickly arrived at by exterpolation or interpolation and the range of c materially narrowed.

I—INITIAL DETERMINATION OF CHARACTERISTICS OF THE CATENARY UNDER ICE AND WIND LOAD

The first assumption in calculating any span performance is usually the one regarding maximum tension in the cable when certain ice and wind loads prevail at a certain temperature. In the particular case considered it was assumed that the maximum tension shall not exceed 33,000 pounds at 0 deg. fahr., ½ inch ice and a 12-lb. per square foot wind on the ice-covered cable. Under this condition the catenary is hanging in a plane whose angle, θ, from the vertical is determined by the ratio of wind load to weight of bare cable plus ice. The values of h and k for the catenary in this plane are no longer the same as the physical values of h and k in the vertical plane as determined by survey. The one physical dimension, however, which does remain constant is H, the air line distance between the upper and lower supports,

where

$$H^2 = h^2 + k^2$$

The new value of k is the projection of the vertical value of k on the inclined plane established by wind pressure. The new value of k is

$$k_0 = k \cos \theta$$

$$\theta = \tan^{-1}\left(\frac{p}{w_c + w_i}\right)$$

the change in k, $\delta\,k$, may then be expressed as

$$k + \delta\,k = k_0$$
$$\delta\,k = k \cos \theta - k = k\,(\cos \theta - 1) \qquad (3)$$

From

$$H^2 = h^2 + k^2$$

$$\delta\,h = -\frac{2\,k\,\delta\,k + (\delta\,k)^2}{2\,h} \qquad (4)$$

The new value of h then becomes

$$h_0 = h + \delta\,h = h - \frac{2\,k\,\delta\,k + (\delta\,k)^2}{2\,h}$$

These values h_0 and k_0 must then be employed in making the initial determination of c when the maximum tension for a given temperature and ice and wind loading are assumed. The equation for the initial determination of c then becomes,

$$c^2 \cosh \frac{h_0}{c} - y_1\,y_2 - \sqrt{(y_1^2 - c^2)\,(y_2^2 - c^2)} = 0$$

where

T_2, w, h_0 and k_0 are known and

$$y_2 = \frac{T_2}{w}$$

$$y_1 = y_2 - k_0$$

With the value of c thus determined by trial the complete characteristics of the suspended cable at 0 deg. fahr., ½ inch ice and 12 lb. wind are readily tabulated. See example.

II—Determination of Characteristics of the Catenary Without Wind Load

Using as the basis for the determination of the characteristics of the catenary when hanging in a vertical plane (without wind load) the characteristics of the catenary in the inclined plane (with wind load), the following differential equations may be set up.

It is obvious from the previous determination that the change in k_0 and h_0 must be of the same magnitude, but with opposite algebraic sign in order to re-establish the plane of the catenary in the vertical when the wind pressure is removed.

Therefore, $\delta\, k_0$ and $\delta\, h_0$ are known from (3) and (4). From

$$y_2 = y_1 + k_0$$
$$\delta\, y_2 = \delta\, y_1 + \delta\, k_0 \qquad (5)$$

and from

$$y_2^2 = s_2^2 + c^2 \text{ and } y_1^2 = s_1^2 + c^2$$

$$\delta\, y_2 = \frac{\delta\, l + \left(\dfrac{y_1}{s_1}\right)\delta\, k_0 + \left\{\dfrac{c}{s_1} + \dfrac{c}{s_2}\right\}\delta\, c}{\dfrac{y_1}{s_1} + \dfrac{y_2}{s_2}} \qquad (6)$$

and from

$$\sqrt{l^2 - k_0^2} = 2\, c \sinh \frac{h_0}{2\, c}$$

$$\delta\, l = \frac{k_0\, \delta\, k_0}{l} + \sqrt{\frac{l^2 - k_0^2}{l}}\Bigg[\left(2\sqrt{\frac{y_1\, y_2 + s_1\, s_2 - c^2}{2\, c^2}} - \frac{h_0}{c}\sqrt{\frac{y_1\, y_2 + s_1\, s_2 + c^2}{2\, c^2}}\right)\delta\, c + \sqrt{\frac{y_1 y_2 + s_1\, s_2 + c^2}{2\, c^2}}\,\delta\, h_0\Bigg] \qquad (7)$$

This equation (7) is $\delta\, l$ expressed as a function of $\delta\, c$ where $\delta\, k_0$ and $\delta\, h_0$ are known, and equation (6), in which $\delta\, y_2$ is expressed as a function of $\delta\, l$ and $\delta\, c$, may now be expressed as a function of $\delta\, c$.

It now remains to express $\delta\, c$ in some known quantities so that a solution for the change in y_2, l, etc., may be had.

In changing from the condition of ice and wind load to the condition of ice load alone a change in w, the unit weight takes place. This change may be expressed as $\delta\, w$.

$$\delta\, w = (w_c + w_i) - \sqrt{(w_c + w_i)^2 + p^2} \qquad (8)$$

From

$$T_2 = w\, y_2$$
$$\delta\, T_2 = (w + \delta\, w)\, \delta\, y_2 + y_2\, \delta\, w \qquad (9)$$

The equation of the modulus of elasticity of the material carrying the stress may be expressed as

$$E = \frac{\delta\, T_2 \,.\, l}{A \,.\, \delta\, l} \qquad (10)$$

Combining equation (9) and (10)

$$\delta\, l = \frac{l}{E\, A}\left[(w + \delta\, w)\, \delta\, y_2 + y_2\, \delta\, w\right] \qquad (11)$$

Equation (11), expressed in terms of functions of $\delta\, l$ and $\delta\, y_2$, which in turn are functions of $\delta\, c$, is an equation which will give a solution of $\delta\, c$, $\delta\, w$ being known.

Having arrived at a numerical value of $\delta\, c$ the numerical values of $\delta\, y_2$, $\delta\, y_1$, δ 1 and $\delta\, T_2$ may now be established. The new values y_2', y_1' l', c', and T_2' for a given temperature, ice load, but without wind may now be tabulated. See example and detailed mathematical development.

III—Determination of Characteristics of the Catenary for Change of Temperature—Ice Load on the Cable to 32 Deg. Fahr.

Having resolved the catenary into the vertical plane, the values of h and k will be constant for any variations of the catenary in the vertical plane. The calculation of the characteristics of the catenary for change of temperature is introduced at this point in order to be able to determine the characteristics of the catenary at 32 deg. fahr. with ice, which may be found as the condition of maximum sag. This condition of sag, occurring as it does during cold months may however not be of interest as controlling the clearance over water, since during such months ice in the river itself usually is cause for abandoning navigation.

The first equation which may be established for change of temperature is the one expressing the physical phenomenon of change of length, $\delta\, l'$, due to change of temperature, $\delta\, t$,

$$\delta\, l' = l' \,.\, \alpha \,.\, \delta\, t \qquad (12)$$

$\delta\, t$ is predetermined by the condition which is to be investigated and the known condition used as the basis, therefore is a known quantity. (In going from 0 deg. fahr. to 32 deg. fahr. $\delta\, t = 32$). Therefore, we have $\delta\, l$ as a known quantity.

From equation (1)

$$\delta\, c' = \frac{\dfrac{l'}{\sqrt{(l')^2 - (k')^2}}}{2\sqrt{\dfrac{y_1' y_2' + s_1' s_2' - (c')^2}{2\, c^2}} - \dfrac{h'}{c'}\sqrt{\dfrac{y_1' y_2' + s_1' s_2' + (c')^2}{2\, (c')^2}}}\,\delta\, l' \qquad (13)$$

From

$$(y_1')^2 = (s_1')^2 + (c')^2 \text{ and } (y_2')^2 = (s_2')^2 + (c')^2$$

$$\delta\, s_2' = \frac{y_2'}{s_2'} \,.\, \delta\, y_2' + \frac{c'}{s_2'} \,.\, \delta\, c' \qquad (14)$$

$$\delta\, s_1' = \delta\, l' - \delta\, s_2' \qquad (15)$$

$$\delta y_2' = \frac{\delta l' + \left(\frac{c'}{s_1'} + \frac{c'}{s_2'}\right)\delta c'}{\frac{y_1'}{s_1'} + \frac{y_2'}{s_2'}} \tag{16}$$

$$\delta y_1' = \delta y_2' \tag{17}$$

From

$$T_2' = w' y_2'$$
$$\delta T_2' = w' \delta y_2' \tag{18}$$

The characteristics for the catenary at the new temperature may now be tabulated. See example and detailed mathematical development.

IV—Determination of Characteristics of the Catenary for the Bare Cable, Ice Load Removed

At any given temperature when ice prevails the characteristics of the catenary without ice load may be determined by applying the equations which follows:

The plane of the catenary under this condition is in the vertical so that h and k are constants.

The removal of the ice load is a change in weight, $\delta w'$; so that we have as known

$$\delta w'' = w_c - (w_c + w_i) = -w_i \tag{19}$$

Equation (16) is repeated, holding true under the conditions in this section, as

$$\delta y_2'' = \frac{\delta l'' + \left(\frac{c''}{s_1''} + \frac{c''}{s_2''}\right)\delta c''}{\frac{y_1''}{s_1''} + \frac{y_2''}{s_2''}} \tag{20}$$

Equation (9) is repeated holding true under the conditions in this section

$$\delta T_2'' \doteq (w'' + \delta w'') \delta y_2'' + y_2'' \delta w'' \tag{21}$$

and Equation (11) may be repeated

$$\delta l'' = \frac{l''}{E A}\left[(w'' + \delta w'') \delta y_2'' + y_2'' \delta w''\right] \tag{22}$$

Similarly equations (14), (15) and (17) may be employed in this section by properly noting the change from the use of the prime values to the use of the second values.

The characteristics of the catenary at the temperature considered for the bare cable may now be tabulated. See example and detailed mathematical development.

V—Calculation of the Characteristics of the Catenary for the Bare Cable at Various Temperatures

Having determined in Section IV the characteristics of the catenary for the bare cable at a particular temperature, it now remains to determine the characteristics of the catenary at various conditions of temperature in order to have the complete performance of the catenary so that the stringing sag may be governed during installation in order that the clearance at maximum temperature, which will occur during navigation season may be maintained and maximum tension not be exceeded when the ice and wind load come on the cable.

These characteristics may be determined using the characteristics of the catenary for the particular temperature as determined under section IV, and applying to them for various values of δt the equation indicated in section III. See example and detailed mathematical development.

Remarks. It is to be noted in the development as indicated in detail that in most cases differentials of higher than the first order have been neglected. In two instances differentials of the second order have been included where their values were appreciable. The first and most important point at which the differential of the second order must be preserved is in the cases of changing load.

From

$$T_2 = w y_2$$
$$T_2 + \delta T_2 = (w + \delta w)(y_2 + \delta y_2)$$
$$T_2 + \delta T_2 = w y_2 + w \delta y_2 + y_2 \delta w + \delta w \,.\, \delta y_2$$

By subtraction

$$\delta T_2 = w \delta y_2 + \delta w \delta y_2 + y_2 \delta w$$
$$= (w + \delta w) \delta y_2 + y_2 \delta w$$

$\delta w \,.\, \delta y_2$ is of the second order and is not negligible, for although δy_2 is relatively of small value compared to y_2, δw we know to be relatively large in comparison with w. When changing from ice and wind load (3.322 lb. per ft.) to ice load (2.623) δw is -0.699 lb. per ft. and represents approximately 21 per cent of w. Similarly when removing the ice load δw is -0.939 lb. per ft. and represents approximately 35 per cent of w.

In determining h_0 and k_0 for the catenary in the inclined plane $(\delta k)^2$ was preserved while $(\delta h)^2$ was dropped. In this case δk is approximately 21 per cent of k so that $(\delta k)^2$ has appreciable value.

It is to be noted that after the initial solution by trial for c that the formulas have been resolved into forms which do not contain hyperbolic functions. Although tables of hyperbolic functions are available it has been found that where eight place accuracy is required the use of hyperbolic functions from tables is about as cumbersome, due to interpolation, as arriving at the functions by means of the necessary number of terms from the series expressing the desired function. Eight place accuracy is required in order to preserve reasonable accuracy at certain points where differences are taken. Where hyperbolic functions have been replaced by an algebraic expression in these formulas the expression is exact for the conditions which exist.

At any point in the calculations the following check may be applied, which may be taken as a check on the accuracy of the method developed.

$$T_2'' = \delta T_2' + T_2' = w'' y_2''$$

This check applied in the example given shows that the error is less than 0.001 of 1 per cent for the tension.

It is to be noted that in the calculated example that

the change of temperature to 32 deg. fahr. with ice and wind was performed before the wind load was removed. This was done as a matter of convenience in order to find the condition of maximum sag early in the arithmetical calculations. The order of applying the various sections in determining span performance may be arranged to suit the convenience of the calculator in order that any particular point of interest may be arrived at quickly.

Experience indicates that the calculation of the span performance by this method involves little or no more work than by more approximate methods.

Complete Mathematical Treatment

General. Formulas for the symmetrical catenary (supports at equal elevations).

$$y = c \cosh \frac{x}{c}$$

$$s = c \sinh \frac{x}{c}$$

$$T = w\,c \cosh \frac{x}{c} = w\,y$$

$$y^2 = c^2 \cosh^2 \frac{x}{c}$$

$$s^2 = c^2 \sinh^2 \frac{x}{c}$$

$$y^2 - s^2 = c^2 \left(\cosh^2 \frac{x}{c} - \sinh^2 \frac{x}{c} \right) = c^2$$

$$y^2 = s^2 + c^2$$
$$d = y - c$$

The formulas for the unsymmetrical catenary (supports at unequal elevations) may be written as follows,

$$y_1 = c \cosh \frac{x_1}{c}, \qquad y_2 = c \cosh \frac{x_2}{c} \qquad (1)$$

$$s_1 = c \sinh \frac{x_1}{c}, \qquad s_2 = c \sinh \frac{x_2}{c} \qquad (2)$$

$$T_1 = w\,y_1 \quad , \qquad T_2 = w\,y_2 \qquad (3)$$
$$y_1^2 = s_1^2 + c^2 \quad , \qquad y_2^2 = s_2^2 + c^2 \qquad (4)$$
$$d_1 = y_1 - c \quad , \qquad d_2 = y_2 - c \qquad (5)$$
$$h = x_1 + x_2 \qquad (6)$$
$$l = s_1 + s_2 \qquad (7)$$
$$y_2 = y_1 + k \qquad (8)$$
$$d_2 = d_1 + k \qquad (9)$$

$$l^2 = s_1^2 + s_2^2 + 2\,s_1\,s_2$$

$$= y_1^2 - c^2 + y_2^2 - c^2 + 2\,c^2 \,.\, \sinh \frac{x_1}{c} \,.\, \sinh \frac{x_2}{c} \qquad (10)$$

$$\cosh \frac{h}{c} = \cosh \frac{x_1 + x_2}{c}$$

$$= \cosh \frac{x_1}{c} \,.\, \cosh \frac{x_2}{c} + \sinh \frac{x_1}{c} \,.\, \sinh \frac{x_2}{c} \qquad (11)$$

$$\sinh \frac{x_1}{c} \,.\, \sinh \frac{x_2}{c} = \cosh \frac{h}{c} - \cosh \frac{x_1}{c} \,.\, \cosh \frac{x_2}{c} \qquad (12)$$

$$l^2 = y_2^2 + y_1^2 - 2\,c^2 + 2\,c^2 \,.\, \cosh \frac{h}{c}$$

$$- 2\,c \,.\, \cosh \frac{x_1}{c} \,.\, c \,.\, \cosh \frac{x_2}{c}$$

$$= y_2^2 - 2\,y_1\,y_2 + y_1^2 + 2\,c^2 \left(\cosh \frac{h}{c} - 1 \right)$$

$$= (y_2 - y_1)^2 + 2\,c^2 \left(\cosh \frac{h}{c} - 1 \right)$$

$$= k^2 + 2\,c^2 \left(\cosh \frac{h}{c} - 1 \right)$$

$$l^2 - k^2 = 2\,c^2 \left(\cosh \frac{h}{c} - 1 \right) \qquad (13)$$

$$\cosh \frac{h}{c} = \cosh 2 \left(\frac{h}{2\,c} \right) = 1 + 2 \sinh^2 \frac{h}{2\,c} \qquad (14)$$

$$l^2 - k^2 = 2\,c^2 \left(1 + 2 \sinh^2 \frac{h}{2\,c} - 1 \right)$$

$$= 4\,c^2 \,.\, \sinh^2 \frac{h}{2\,c}$$

$$\sqrt{l^2 - k^2} = 2\,c \,.\, \sinh \frac{h}{2\,c} \qquad (15)$$

I. Determination of Characteristics of the Catenary Under Maximum Ice and Wind Load

T_2, w, h and k are known.

$$\theta = \tan^{-1} \left(\frac{p}{w_o + w_i} \right)$$

$$k_0 = k \,.\, \cos \theta$$
$$k + \delta\,k = k_0$$
$$\delta\,k = k\,(\cos \theta - 1) \qquad (16)$$
$$H^2 = h^2 + k^2$$

$$\delta\,(H^2) = 0 = 2\,h \,.\, \delta\,h + 2\,k\,\delta\,k + (\delta\,h)^2 + (\delta\,k)^2$$

$(\delta\,h)^2$ is negligible

$$\delta\,h = - \frac{2\,k\,\delta\,k + (\delta\,k)^2}{2\,h} \qquad (17)$$

$$h_0 = h + \delta\,h$$

$$y_2 = \frac{T_2}{w}, \qquad y_1 = y_2 - k_0$$

From (11)

$$c^2 . \cosh \frac{h_0}{c}$$

$$= c . \cosh \frac{x_1}{c} . c . \cosh \frac{x_2}{c} + c . \sinh \frac{x_1}{c} . c . \sinh \frac{x_2}{c}$$

$$c^2 . \cosh \frac{h_0}{c} = y_1 y_2 + s_1 s_2$$

$$= y_1 y_2 + \sqrt{(y_1^2 - c^2)(y_2^2 - c^2)}$$

$$c^2 \cosh \frac{h_0}{c} - \sqrt{(y_1 - c)(y_1 + c)(y_2 - c)(y_2 + c)} - y^1 y^2 = 0 \qquad (18)$$

Solve by trial for c

Obviously

$$0 < c < y_1$$

In general

$$.9\, y_1 < c < y_1$$

At maximum ice and wind load (Maximum Tension)

$$d_2 = y_2 - c$$
$$d_1 = d_2 - k_0$$
$$s_1 = \sqrt{(y_1 - c)(y_1 + c)}$$
$$s_2 = \sqrt{(y_2 - c)(y_2 + c)}$$
$$l = s_1 + s_2$$

II. Determination of Characteristics of the Catenary with Ice Load, Wind Load Removed

$$\delta k_0 = - \delta k$$
$$\delta h_0 = - \delta h$$

From (8)

$$\delta y_2 = \delta y_1 + \delta k_0 \qquad (19)$$

From (4)

$$\begin{cases} 2\, y_2 \delta y_2 = 2 s_2 \delta s_2 + 2 c \delta c \\ 2\, y_1 \delta y_1 = 2 s_1 \delta s_1 + 2 c \delta c \end{cases}$$

$$y_2 \delta y_2 = s_2 \delta s_2 + c \delta c \qquad (20)$$

$$y_1 \delta y_1 = s_1 \delta s_1 + c \delta c \qquad (21)$$

Substituting (19) in (21)

$$y_1 (\delta y_2 - \delta k_0) = s_1 \delta s_1 + c \delta c$$

$$y_1 \delta y_2 = s_1 \delta s_1 + c \delta c + y_1 \delta k_0 \qquad (22)$$

From (20)

$$\frac{y_2}{s_2} \delta y_2 = \delta s_2 + \frac{c}{s_2} \delta c \qquad (23)$$

From (22)

$$\frac{y_1}{s_1} \delta y_2 = \delta s_1 + \frac{c}{s_1} \delta c + \frac{y_1}{s_1} \delta k_0 \qquad (24)$$

Adding (23) and (24)

$$\left(\frac{y_1}{s_1} + \frac{y_2}{s_2} \right) \delta y_2$$

$$= \delta s_1 + \delta s_2 + \left(\frac{c}{s_1} + \frac{c}{s_2} \right) \delta c + \frac{y_1}{s_1} \delta k_0$$

From (7)

$$\delta l = \delta s_1 + \delta s_2$$

$$\delta y_2 = \frac{\delta l + \frac{y_1}{s_1} \delta k_0 + \left(\frac{c}{s_1} + \frac{c}{s_2} \right) \delta c}{\frac{y_1}{s_1} + \frac{y_2}{s_2}} \qquad (25)$$

From (15)

$$\frac{l \delta l - k_0 \delta k_0}{\sqrt{l^2 - k_0^2}}$$

$$= \left(2 \sinh \frac{h_0}{2c} - \frac{h_0}{c} \cosh \frac{h_0}{2c} \right) \delta c + \cosh \frac{h_0}{2c} \delta h_0$$

$$\delta l = \frac{k_0 \delta k_0}{l} + \frac{\sqrt{l^2 - k_0^2}}{l}$$

$$\left[\left(2 \sinh \frac{h_0}{2c} - \frac{h_0}{c} \cosh \frac{h_0}{2c} \right) \delta c + \cosh \frac{h_0}{2c} \delta h_0 \right]$$

$$\sinh \frac{h_0}{2c} = \sinh \frac{1}{2} \left(\frac{x_1 + x_2}{c} \right) = \sqrt{\frac{1}{2} \left(\cosh \frac{x_1 + x_2}{c} - 1 \right)}$$

$$= \sqrt{\frac{1}{2} \left(\cosh \frac{x_1}{c} . \cosh \frac{x_2}{c} + \sinh \frac{x_1}{c} . \sinh \frac{x_2}{c} - 1 \right)}$$

$$= \sqrt{\frac{1}{2c^2} \left(c . \cosh \frac{x_1}{c} . c . \cosh \frac{x_2}{c} + c . \sinh \frac{x_1}{c} . c . \sinh \frac{x_2}{c} - c^2 \right)}$$

$$\sinh \frac{h_0}{2c} = \sqrt{\frac{y_1 y_2 + s_1 s_2 - c^2}{2c^2}}$$

By similar expansion

$$\cosh \frac{h_0}{2c} = \sqrt{\frac{y_1 y_2 + s_1 s_2 + c^2}{2c^2}}$$

$$\delta l = \frac{k_0 \delta k_0}{l} + \frac{\sqrt{l^2 - k_0^2}}{l}$$

$$\left[\left(2 \sqrt{\frac{y_1 y_2 + s_1 s_2 - c^2}{2c^2}} - \frac{h_0}{c} \sqrt{\frac{y_1 y_2 + s_1 s_2 + c^2}{2c^2}} \right) \delta c + \sqrt{\frac{y_1 y_2 + s_1 s_2 + c^2}{2c^2}} \delta h_0 \right] \qquad (26)$$

$$E = \frac{\delta T_2 l}{A \delta l} \qquad (27)$$

From (3)

$$\delta T_2 = w \delta y_2 + y_2 \delta w + \delta w \delta y_2$$

$$= (w + \delta w) \delta y_2 + y_2 \delta w \qquad (28)$$

$$w = \sqrt{(w_c + w_i)^2 + p^2}$$

$$w + \delta w = w_c + w_i$$

$$\delta w = w_c + w_i - \sqrt{(w_c + w_i)^2 + p^2} \qquad (29)$$

(25) may be expressed as

$$\delta y_2 = a + b\,\delta c \qquad (30)$$

and (26) may be expressed as

$$\delta l = e + f\,\delta c \qquad (31)$$

Substituting (28), (30) and (31) in (27)

$$E = \frac{l\,[(w + \delta w)\,(a + b\,\delta c) + y_2\,\delta w]}{A\,(e + f\,\delta c)} \qquad (32)$$

$$E\,A\,(e + f\,\delta c)$$
$$= l\,a\,w + l\,b\,w\,\delta c + l\,a\,\delta w + l\,b\,\delta w\,\delta c + l\,y_2\,\delta w$$
$$(E\,A\,f - l\,b\,w - l\,b\,\delta w)\,\delta c$$
$$= l\,a\,w + l\,a\,\delta w + l\,y_2\,\delta w - E\,A\,e$$

$$\delta c = \frac{l\,a\,(w + \delta w) + l\,y_2\,\delta w - E\,A\,e}{E\,A\,f - l\,b\,(w + \delta w)} \qquad (33)$$

The value of δc derived from (33) when substituted in (23), (25), (26) and (28) will give the values of δs_2, δy_2, δl and δT_2.

The new values of y_2, y_1, c, l, s_2, s_1, T_2, h and k may now be tabulated.

$$\begin{aligned}
y_2' &= y_2 + \delta y_2 \\
k' &= k_0 + \delta k_0 \\
y_1' &= y_2' - k' \\
l' &= l + \delta l \\
s_2' &= s_2 + \delta s_2 \\
s_1' &= l' - s_2' \\
c' &= c + \delta c \\
T_2' &= T_2 + \delta T_2 \\
h' &= h_0 + \delta h_0 \\
d_2' &= y_2' - c' \\
d_1' &= y_1' - c' \\
w' &= w + \delta w = w_c + w_i
\end{aligned}$$

III. Determination of Characteristics of the Catenary for Change of Temperature. Ice Load on the Cable to 32 Deg. Fahr.

w', h', and k' are constant in this section.

$\therefore\ \delta h' = 0$ and $\delta k' = 0$ and $\delta w' = 0$

$$\delta l' = l'\,\alpha\,\delta t \qquad (34)$$

From (26)

$$\delta l' = \frac{\sqrt{(l')^2 - (k')^2}}{l'}\left(2\sqrt{\frac{y_1'\,y_2' + s_1'\,s_2' - (c')^2}{2\,(c')^2}} - \frac{h'}{c'}\sqrt{\frac{y_1'\,y_2' + s_1'\,s_2' + (c')^2}{2\,(c')^2}}\right)\delta c' \qquad (35)$$

$$\delta c' = \frac{\dfrac{l'}{\sqrt{(l')^2 - (k')^2}}}{2\sqrt{\dfrac{y_1'\,y_2' + s_1'\,s_2' - (c')^2}{2\,(c')^2}} - \dfrac{h'}{\ '}\sqrt{\dfrac{y_1'\,y_2' + s_1'\,s_2' + (c')^2}{2\,(c')^2}}}\,\delta l' \qquad (36)$$

From (25)

$$\delta y_2' = \frac{\delta l' + \left(\dfrac{c'}{s_1'} + \dfrac{c'}{s_2'}\right)\delta c'}{\dfrac{y_1'}{s_1'} + \dfrac{y_2'}{s_2'}} \qquad (37)$$

From (3)

$$\delta T_2' = w'\,\delta y_2' \qquad (38)$$

From (23)

$$\delta s_2' = \frac{y_2'}{s_2'}\,\delta y_2' - \frac{c'}{s_2'}\,\delta c' \qquad (39)$$

By the successive operations indicated in (34), (35), (36), (37), (38) and (39) the values of $\delta l'$, $\delta c'$, $\delta y_2'$, $\delta T_2'$ and $\delta s_2'$ are established.

The new values of y_2, y_1, c, l, s_2, s_1 and T_2 may now be tabulated.

$$\begin{aligned}
y_2'' &= y_2' + \delta y_2' \\
y_1'' &= y_2'' - k' \\
l'' &= l' + \delta l' \\
s_2'' &= s_2' + \delta s_2' \\
s_1'' &= l'' - s_2'' \\
c'' &= c' + \delta c' \\
T_2'' &= T_2' + \delta T_2' \\
d_2'' &= y_2'' - c'' \\
d_1'' &= y_1'' - c'' \\
h'' &= h' \\
k'' &= k' \\
w'' &= w' = w_c + w_i
\end{aligned}$$

IV. Determination of Characteristics of the Catenary for the Bare Cable, Ice Load Removed

h'' and k'' are constant in this section.

$$\therefore\ \delta h'' = 0, \qquad \delta k'' = 0$$

$$\begin{aligned}
w'' + \delta w'' &= w_c \\
\delta w'' &= w_c - w_c - w_i = -\,w_i
\end{aligned}$$

From (25)

$$\delta y_2'' = \frac{\delta l'' + \left(\dfrac{c''}{s_1''} + \dfrac{c''}{s_2''}\right)\delta c''}{\dfrac{y_1''}{s_1''} + \dfrac{y_2''}{s_2''}} \qquad (40)$$

From (26)

$$\delta l'' = \frac{\sqrt{(l'')^2 - (k')^2}\left(2\sqrt{\dfrac{y_1''\,y_2'' + s_1''\,s_2'' - (c'')^2}{2\,(c'')^2}} - \dfrac{h'}{c''}\sqrt{\dfrac{y_1''\,y_2'' + s_1''\,s_2'' + (c'')^2}{2\,(c'')}}\right)}{l''}\,\delta c'' \qquad (41)$$

$$\delta c'' = \frac{\dfrac{l''}{\sqrt{(l'')^2 - (k')^2}}}{2\sqrt{\dfrac{y_1'' y_2'' + s_1'' s_2'' - (c'')^2}{2\,(c'')^2}} - \dfrac{h'}{c''}\sqrt{\dfrac{y_1'' y_2'' + s_1'' s_2'' + (c'')^2}{2\,(c'')^2}}}\,\delta l'' \qquad (42)$$

From (28)

$$\delta T_2'' = (w'' + \delta w'')\,\delta y_2'' + y_2''\,\delta w'' \qquad (43)$$

From (33)

$$\delta c'' = \frac{l''\, y_2''\,\delta w''}{E\,A\,f'' - l''\,b''\,(w'' + \delta w'')} \qquad (44)$$

From (23)

$$\delta s_2'' = \frac{y_2''}{s_2''}\,\delta y_2'' - \frac{c''}{s_2''}\,\delta c'' \qquad (45)$$

The value of $\delta c''$ derived from (44) when substituted in (40), (41), (43) and (45) will give the values of $\delta y_2''$, $\delta l_1''$, $\delta T_2''$ and $\delta s_2''$. The new values of y_2, y_1, c, l, s_2, s_1, and T_2 may now be tabulated.

$$\begin{aligned}
y_2''' &= y_2'' + \delta y_2'' \\
y_1''' &= y_2''' - k' \\
l''' &= l'' + \delta l'' \\
s_2''' &= s_2'' + \delta s_2'' \\
s_1''' &= l''' - s_2''' \\
c''' &= c'' + \delta c''' \\
T_2''' &= T_2''' + \delta T_2'' \\
d_2''' &= y_2''' - c''' \\
d_1''' &= y_1''' - c''' \\
h''' &= h' \\
k''' &= k' \\
w''' &= w'' + \delta w'' = w_c
\end{aligned}$$

V. Determination of Characteristics of the Catenary for the Bare Cable at Various Temperatures

This section is the same as section III, except that the weight of the bare cable is employed, the ice load having been removed in the preceding section. The following formulas are given for the sake of completeness.

w''', h''', and k''' are constant in this section.

$$\because \delta w''' = 0, \quad \delta h''' = 0, \text{ and } \delta k''' = 0$$

From (34)

$$\delta l''' = l'''\,\alpha\,\delta t \qquad (46)$$

From (36)

$$\delta c''' = \frac{\dfrac{l'''}{\sqrt{(l''')^2 - (k')^2}}}{2\sqrt{\dfrac{y_1''' y_2''' + s_1''' s_2''' - (c''')^2}{2\,(c''')^2}} - \dfrac{h'}{c'''}\sqrt{\dfrac{y_1''' y_2''' + s_1''' s_2''' + (c''')^2}{2\,(c''')^2}}}\,\delta l''' \qquad (47)$$

$$\delta y_2''' = \frac{\delta l''' + \left(\dfrac{c'''}{s_1'''} + \dfrac{c'''}{s_2'''}\right)\delta c'''}{\dfrac{y_1'''}{s_1'''} + \dfrac{y_2'''}{s_2'''}} \qquad (48)$$

From (38)

$$\delta T_2''' = w'''\,\delta y_2''' \qquad (49)$$

From (39)

$$\delta s_2''' = \frac{y_2'''}{s_2'''}\,\delta y_2''' - \frac{c'''}{s_2'''}\,\delta_c''' \qquad (50)$$

By the successive operations indicated in (46), (47), (48), (49), and (50) the values of $\delta l'''$, $\delta c'''$, $\delta y_2'''$, $\delta T_2'''$ and $\delta s_2'''$ are established.

The new values of y_2, y_1, c, l, s_2, s_1 and T_2 may now be tabulated.

$$\begin{aligned}
y_2'''' &= y_2''' + \delta y_2''' \\
y_1'''' &= y_2'''' - k' \\
l'''' &= l''' + \delta l''' \\
s_2'''' &= s_2''' + \delta s_2''' \\
s_1'''' &= l'''' - s_2'''' \\
c'''' &= c''' + \delta c''' \\
T_2'''' &= T_2''' + \delta T_2''' \\
d_2'''' &= y_2'''' - c'''' \\
d_1'''' &= y_1'''' - c'''' \\
h'''' &= h' \\
k'''' &= k' \\
w'''' &= w''' = w_c
\end{aligned}$$

Calculation of River Crossing

Span 4279 Ft. Maximum Tension 33,000 Lb. at 0 Deg. Fahr. ½ In. Ice and 12 Lb. per Sq. Ft. Wind

Cable Data

Size......................318,000-cir. mil, A. C. S. R.
Stranding................24 x 1151 Aluminum
43 strands various size steel

Steel core stranding

Number of Strands	Diameter
1	0.127 in.
6	0.120 in.
6	0.052 in.
12	0.115 in.
18	0.112 in.
Over-all diam.	0.804 to 0.810 in.
Total sectional area	(A) 0.3952 sq. in.
Wt. per ft.	1.386 lb.
Breaking Strength	64,000 lb.
Diam. of complete cable-overall	1.036 in.
Elastic limit of complete cable	53,500 lb.
Breaking strength of complete cable...	67,600 in.
Modulus of elasticity (E) for steel	3 x 10⁷ lb./in.²
For steel core alone E A...	11,856,000 lb.

	Vertical		Horizontal
	Dead	Dead + ½ in. Ice	12 lb. Wind
Weight, lb. per ft.	1.684	2.623	2.036

	Resultant
	Dead + ½ in. Ice + 12 lb. Wind
Weight, lb. per ft....	3.322

The calculations which are to follow will be based upon the assumption that all the stress will be carried on the steel core only.

At 0 deg. fahr., ½ in. ice + 12 lb. wind $\quad k = 185.5, \quad h = 4279.$

$$T_2 = 33{,}000 \qquad \delta\ k = k(\cos\ \theta - 1) = -38.64$$
$$w = 3.322$$
$$h_0 = 4280.50$$
$$k_0 = 146.86 \qquad h = -\frac{2\ k\ \delta\ k + (\delta\ k)^2}{2\ h}$$
$$y_2 = 9933.77$$
$$y_1 = 9786.91 \qquad = 1.50$$

Determine c from

$$c^2 \cdot \cosh\frac{h_0}{c} - \sqrt{(y_1^2 - c^2)\ (y_2^2 - c^2)} - y_1 \cdot y_2 = 0$$

Let c =	9620.00	9615.00	9610.00	9615.44
(1) $c^2 \cdot \cosh\frac{h_0}{c}$	101,858,646	101,762,630	101,781,830	101,771,077
(2) $\sqrt{(y_1^2 - c^2)\ (y_2^2 - c^2)}$	4,458,003	4,559,017	4,538,836	4,550,139
(3) $y_1 \cdot y_2$........	97,220,913	97,220,913	97,220,913	97,220,913
(4) (2) + (3)......	101,678,916	101,779,930	101,759,749	101,771,052
(5) (1) − (4)......	179,730	−17,300	22,081	25

$$\text{The value of } c = 9615.44$$
$$\text{for } c = 9615.44$$
$$s_1 = 1923.99$$
$$s_2 = 2494.61$$
$$l = 4318.60$$

At 32 deg. fahr. ½ in. ice and 12 lb. wind

$$\delta\ l = l\ .\ \alpha\ .\ \delta\ t$$
$$\alpha = 0.00000662$$
$$\delta\ t = 32$$
$$\delta\ l = 0.91485222$$

$$\delta\ c = \frac{\dfrac{l}{\sqrt{l^2 - k_0^2}}}{2\sqrt{\dfrac{y_1\ y_2 + s_1\ \ s_2 - c^2}{2\ c^2}} - \dfrac{h_0}{c}\sqrt{\dfrac{y_1\ \ y_2 + s_1\ \ s_2 + c^2}{2\ c^2}}}\ \delta l$$

$$\frac{l}{\sqrt{l^2 - k_0^2}} = \frac{4318.60}{4316.102} = 1.0005787$$

$$2\sqrt{\frac{y_1\,y_2 + s_1\,s_2 - c^2}{2\ c^2}} = 0.44887198$$

$$\frac{h_0}{c}\sqrt{\frac{y_1\,y_2 + s_1\,s_2 + c^2}{2\ c^2}} = (0.44518711)\ (1.0248763)$$
$$= 0.45626173$$

$$\delta\ c = \frac{(1.0005787)\ \delta\ l}{0.44887198 - 0.45626173} = -\ 123.87180$$

$$\delta\ y_2 = \frac{\delta\ l + \left(\dfrac{c}{s_1} + \dfrac{c}{s_2}\right)\delta\ c}{\left(\dfrac{y_1}{s_1} + \dfrac{y_2}{s_2}\right)}$$

$$= \frac{0.91485222 + (5.2716517 + 3.8544863)(-123.87180)}{(5.3656599 + 3.9820934)}$$

$$\delta\ y_2 = -\ 120.81580$$
$$\delta\ T_2 = w\ \delta\ y_2 = -\ 401.35$$

$$T_2' = 32598.65$$
$$y_2' = 9812.96$$
$$y_1' = 9666.10$$
$$k' = k_0 = 146.86$$
$$h' = h_0 = 4280.50$$
$$c' = 9491.57$$
$$l' = 4319.51$$
$$s_1' = 1828.87$$
$$s_2' = 2490.64$$
$$w' = 3.322$$

At 32 deg. fahr. ½ in. ice, no wind

$$\delta\ w' = -.699$$
$$\delta\ k' = 38.64$$
$$\delta\ h' = -1.50$$

$$\delta\ l' = \frac{k'\ \delta\ k'}{l'} + \frac{\sqrt{(l')^2 - (k')^2}}{l'}\Bigg[\left(2\sqrt{\frac{y_1'\ y_2' + s_1'\ s_2' - (c')^2}{2\ (c')^2}}\right.$$
$$\left. - \frac{h'}{c'}\sqrt{\frac{y_1'\ y_2' + s_1'\ s_2' + (c')^2}{2\ (c')^2}}\right)\delta\ c$$
$$+ \left(\sqrt{\frac{y_1'\ y_2' + s_1'\ s_2' + (c')^2}{2\ (c')^2}}\right)\delta\ h'\Bigg]$$

$$= 1.3137301 + (0.99942192)$$
$$[(0.45482378 - 0.46249362)\ \delta\ c' + 1.5382983]$$

$$\delta\ l' = 0.2245682 - 0.007665406\ \delta\ c'$$

$$\delta\ y_2' = \frac{\delta\ l + \dfrac{y_1'}{s_1'}\ \delta\ k' + \left(\dfrac{c'}{s_1'} + \dfrac{c'}{s_2'}\right)\delta\ c'}{\dfrac{y_1'}{s_1'} + \dfrac{y_2'}{s_2'}}$$

$$\delta\ y_2' = \frac{-.2245682 - .007665406\ \delta\ c' + 204.22343 + 9.000750\ \delta\ c'}{9.2252205}$$

$$\delta\ y_2' = 22.113169 + 0.97483691\ \delta\ c'$$
$$\delta\ y_1' = \delta\ y_2' - \delta\ k'$$
$$\delta\ y_1' = -\ 16.526831 + 0.97483691\ \delta\ c'$$

$$\delta\ l' = \frac{l'}{E\ A}\ .\ \delta\ T_2'$$

$$\delta\ T_2' = (w' + \delta\ w')\ .\ \delta\ y_2' + y_2'\ .\ \delta\ w'$$

$$\delta l' = \frac{l'}{EA}\,[(w' + \delta w')\,\delta y_2' + y_2' . \delta w)]$$

$$-0.2245682 - 0.007665406\,\delta c' = \frac{4319.51}{11856000}$$

$[(2.623)(22.113169+0.97483691\,\delta c')+(9812.96)(-.699)]$

$=0.00036433114(58.002482+2.5569972\,\delta c'-6859.20)$

$-0.2245682-0.007665406\,\delta c'$

$=-2.4779094+0.0009315837\,\delta c'-0.0085969897\,\delta c'$

$=-2.2533412$

$$\delta c' = 262.10816$$
$$\delta y_2' = 277.63$$
$$\delta y_1' = 238.99$$
$$\delta l' = -2.23$$

$$\delta s_1' = \frac{y_1'}{s_1'}\,\delta y_1' - \frac{c'}{s_1'}\,\delta c = -97.19$$

$$\delta s_2' = \delta l' - \delta s_1' = 94.96$$

$\delta T_2' = (w' + \delta w')\,\delta y_2' + y_2'\,\delta w'$

$=(2.623)(277.62588)+(9812.96)(-0.699)=-6131.50$

$T_2'' = 26467.62$

$y_2'' = 10090.59$ Sag below upper support

$y_1'' = 9905.09$ $\quad d_2'' = y_2'' - c'' = 336.91$

$k'' = 185.50$ Sag below lower support

$h'' = 4279.00$ $\quad d_1'' = y_1'' - c'' = 151.41$

$c'' = 9753.68$

$l'' = 4317.28$

$s_1'' = 1731.68$

$s_2'' = 2585.60$

$w'' = 2.623$

At 32 deg. fahr. no ice, no wind

$\delta w'' = -0.939$

$$\delta l'' = \frac{\sqrt{(l'')^2-(k'')^2}}{l''}\left(2\sqrt{\frac{y_1''\,y_2'' + s_1''\,s_2'' - (c'')^2}{2(c'')^2}} - \frac{h''}{c''}\sqrt{\frac{y_1''\,y_2'' + s_1''\,s_2'' + (c'')^2}{2\,(c'')^2}}\right)\delta c''$$

$\delta l'' = -0.00722691\,\delta c''$

$$\delta y_2'' = \frac{\delta l'' + \left(\dfrac{c''}{s_1''} + \dfrac{c''}{s_1''}\right)\delta c''}{\dfrac{y_1''}{s_1''} + \dfrac{y_2''}{s_2''}}$$

$\delta y_2'' = 0.97662108\,\delta c''$

$\delta T_2'' = (w'' + \delta w'')\,\delta y_2'' + y_2'' . \delta w''$

$= 1.6446299\,\delta c'' - 9475.0640$

$$\delta l'' = \frac{l''}{EA} . \delta T_2''$$

$$-0.00722691\delta c'' = \frac{4317.28}{11856000}(1.6446299\,\delta c'' - 9475.0640)$$

$= 0.00059888\,\delta c'' - 3.4502787$

$\delta c'' = 440.88562$

$\delta y_2'' = 430.58$

$\delta 1'' = -3.19$

$\delta s_1'' = -20.40$

$\delta s_2'' = 17.21$

$\delta T_2'' = 8749.97$

$T_2''' = 17717.65$

$y_2''' = 10521.17$

$y_1''' = 10355.67$ Sag below upper support

$k''' = 185.50$ $\quad d_2''' = y_2''' - c''' = 326.60$

$c''' = 10194.57$

$h''' = 4279.00$ Sag below lower support

$l''' = 4314.09$ $\quad d_1''' = y_1''' - c''' = 141.10$

$s_1''' = 1711.28$

$s_2''' = 2602.81$

Calculation for various temperatures with no ice or wind loading. Basis 32 deg. fahr. no ice, no wind.

$\delta l''' = l'''\,\alpha\,\delta t$

$$\delta c''' = \frac{\dfrac{l'''}{\sqrt{(l''')^2 - (k''')^2}}}{2\sqrt{\dfrac{y_1'''\,y_2''' + s_1'''\,s_2''' - (c''')^2}{2\,(c''')}} - \dfrac{h'''}{c'''}\sqrt{\dfrac{y_1'''\,y_2''' + s_1'''\,s_2''' + (c''')^2}{(2\,c''')^2}}}\;\delta l'''$$

$\delta c''' = -150.14441 . \delta l'''\;\delta l''' = -.006660255 . \delta c'''$

$$\delta y_2''' = \frac{\delta l''' + \left(\dfrac{c'''}{s_1'''} + \dfrac{c'''}{s_2'''}\right)\delta c'''}{\dfrac{y_1'''}{s_1'''} + \dfrac{y_2'''}{s_2'''}}$$

$\delta y_2''' = 0.97872241 . \delta c'''$

$$\delta s_2''' = \frac{y_2'''}{s_2'''}\,\delta y_2''' - \frac{c'''}{s_2'''}\,\delta c'''$$

$\delta s_2''' = 0.0394556\,\delta c'''$

$\delta s_1''' = \delta 1''' - \delta s_2'''$

$\delta T_2''' = w'''\,\delta y_2'''$

$\delta T_2''' = 1.6481685\,\delta c'''$

$\alpha = 0.00000662$

$l . \alpha = 0.02855927$

TEMPERATURE FAHR.

	32 deg.	−20 deg.	0 deg.	20 deg.	40 deg.	60 deg.	80 deg.	100 deg.	120 deg.
δt	0	−52	−32	−12	8	28	48	68	88
$\delta l'''$	0	−1.485082	−.913897	−.342711	.228474	.799659	1.370845	1.942030	2.513216
l''''	4314.09	4312.60	4313.18	4313.75	4314.32	4314.89	4315.46	4316.03	4316.60
$\delta c'''$	0	222.976	137.066	51.456	−34.304	−120.064	−205.825	−291.585	−377.345
c''''	10194.57	10417.55	10331.64	10246.03	10160.27	10074.51	9988.74	9902.98	9717.22
$\delta y_2'''$	0	218.23	134.15	50.36	−33.57	−117.51	−201.44	−285.38	−369.32
y_2''''	10521.17	10739.40	10655.32	10571.53	10487.60	10403.66	10319.73	10235.79	10151.85
y_1''''	10335.67	10553.90	10469.82	10386.03	10302.10	10218.16	10143.23	10050.29	9966.35
$\delta s_2'''$	0	8.80	5.41	2.03	−1.35	−4.74	−8.12	−11.50	−14.89
s_2''''	2602.81	2611.61	2608.22	2604.84	2601.46	2598.07	2594.69	2591.31	2587.92
$\delta s_1'''$	0	−10.29	−6.32	−2.37	1.58	5.54	9.49	13.44	17.40
s_1''''	1711.28	1700.99	1704.96	1708.91	1712.86	1716.82	1720.77	1724.72	1728.68
$\delta T_2'''$	0	267.50	225.91	84.81	−56.54	−197.89	−339.23	−480.58	−621.93
T_2''''	17717.65	18085.15	17943.56	17802.46	17661.11	17519.76	17378.42	17237.07	17095.72
d_2''''	326.60	321.85	323.68	325.50	327.33	329.15	330.99	332.81	334.63
d_1''''	141.10	136.35	138.18	140.00	141.83	143.65	145.49	147.31	149.13

$d_2'''' = y_2'''' - c''''$ Sag below upper support
$d_1'''' = y_1'''' - c''''$ Sag below lower support

Discussion

J. S. Martin (communicated after adjournment): I have been especially interested in the authors' methods of calculation of the sags required in the wire, as this is a subject of which I have made considerable study. In the proceedings of the Engineering Society of Western Pennsylvania for November 1922, I published a tabular method of calculating sag including a set of tables giving the functions of the catenary in the same manner that the ordinary trigonometrical tables give the functions of the circle. By means of these tables the sag required for any span and any wire can be quickly and accurately determined when the span is level. For the calculation of spans on the slope, the writer has resorted to an approximate method which gives results as close as the work of sagging can be done in the field and in nearly all cases the slight error is on the safe side.

The accurate calculation of the sag of wire in a span where

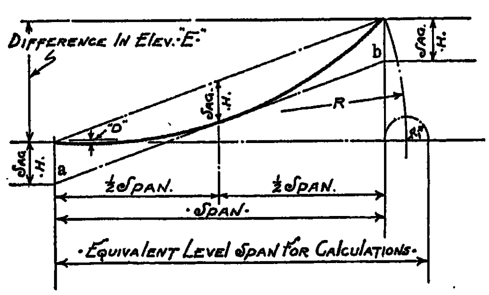

FIG. 1

the supports are not on the same level is a long and tedious process for an expert mathematician even when assisted by these tables, so that for general use in transmission-line work the accurate method is impractical. The computations presented by Messrs. Eales and Ettlinger have afforded the writer an opportunity to compare results with his method.

The method used by the writer for general calculation of sag in sloping spans is to take the difference between the level measurement of span between the points of support, and the actual measurement between these points of support. This difference equals R_1 in the accompanying diagram. Then add this difference to the actual distance between points of support, and use this value as an equivalent level span.

A marking point is then placed on each tower at a distance from the point of support equal to the sag H, of the equivalent level span. See a and b in the diagram. A line of sight $a - b$ is taken between these two marking points, this line of sight being parallel to a line joining the points of support. The wire is then sagged till it strikes this line of sight.

If the wire would hang in a parabola, a similar method could be used, using the level measurement between the points of support as the length of span and the results would be accurate, but since we assume that the wire is hanging in a catenary, there must be a correction, and the method mentioned above is the one used by the writer to obtain this correction. No mathematical demonstration can be given by the writer for this method, except that it is the result of a long series of test problems.

The calculations for the sag of the wire in the span under consideration can be obtained by the tabular method in an hour or less of work and the computation can be entrusted to an apprentice or office boy. With the method presented by the authors of the paper under consideration, the calculations will take a long time and must be made by an expert mathematician.

The writer has compared the results obtained by the two methods and it was found that they differ in giving the elevation of the low point of the wire by about 2.2 ft. In order to check the accuracy of these results the writer calculated the exact location of the low point of the wire at zero temperature. It was found that while for zero temperature there is a difference of 2.19 ft. between the results obtained by the two methods, the exact method gives a result between the two, giving an elevation of 0.73 ft. above the elevation given by the method presented in the paper and 1.46 ft. below the elevation given by the writer's method. This would show that the error of the writer's method is about 0.45 per cent while that of the method given in the paper is 0.30 per cent. This percentage is based on the maximum tension in the wire as strung according to the two methods. This happens to be one of the few cases where the writer's approximate method for sloping spans would give a stress slightly in excess of the theoretical stress desired, but the results are well within the limits of the probable errors in field work and in the assumptions with regard to the quality and characteristics of the wire used, so that this error is negligible.

Where it is necessary to know the elevation of the low point of the wire for determining clearances, the writer has found the following parabolic formula sufficiently accurate for practical purposes.

Let, H = sag as determined for the equivalent level span as previously explained
E = difference in elevation of the points of support
D = deflection of low point of wire below lower support

Then,

$$D = (1/H)\,(H - E/4)^2$$

In our office, a large part of our routine calculations for sags in wires are entrusted to an office boy to do in his spare moments.

The tabular method is so well adapted for the use of a calculating machine, such as the Marchant or Monroe, that it is no trouble at all for any person who is ordinarily careful to determine quickly the proper sag in the wire even for important spans.

E. Ettlinger: The solution of the specific engineering problem of this river crossing, due to the nature of the profile, produced two adjacent long spans. The desire to avoid dead-end construction, influenced the use of insulating supports in suspension on the intermediate tower. Initially, consideration was given to a roller-cradle type of support which would allow the cable to roll from one side to the other with changes of temperature and loading. Numerous objections were found to this type of support, the most serious of which was the danger incident to the dropping off of ice in one span while the ice remained on the adjacent span, resulting in the decision to clamp the cable definitely.

The clamping of the cable at the intermediate tower and supporting it by suspension assembly of insulators introduced the problem of the stress in this support due to temperature and loading variations. Calculations indicated that the suspension assembly would not vary from the vertical by more than approximately 5 deg. Although the suspension assemblies were designed for one-half the ultimate strength of the dead-end assemblies, the actual component of force in them would not reach an excessive value.

The authors are aware of the fact that there are numerous noteworthy tabular and chart methods of calculating spans with supports at equal elevation. Also that various approximate methods are in use by which spans with supports at unequal elevations may be calculated.

The importance of the problem involved in crossing the largest and most important navigable river in this country, it is believed, warranted the development of a direct method of calculations for spans with supports at unequal elevations. The expense and time required to make these calculations are really insignificant when it is realized that the design and construction of a crossing of the Mississippi River with the rigid requirements of clearance as specified by the War Department involves considerable responsibility.

Problems of this magnitude arise but infrequently and are hardly to be classified as a matter of routine calculation. The method was developed for the specific purpose of assurance of span performance and as a check on less accurate and empirical methods. The method of calculation illustrated in the paper should not consume more than three or four hours to complete after one is familiar with the procedure.

Automatic Control for Substation Apparatus

BY WALTER H. MILLAN[1]

Member, A. I. E. E.

Synopsis.—The use of automatically operated stations has become quite general over the country, but as the development has been very rapid, and the engineers involved so busy with their individual problems, very little has been accomplished toward the pooling of ideas and experiences.

This paper outlines some of the more important problems being encountered in automatic development such as the need for automatic fire protection in the stations, the necessary future development of thermal protective devices, voltage regulating devices, etc. Particular stress is laid on the fact that, while we have some problems which we presume have been worked out, some of the solutions are far from perfect.

This paper is prepared in a manner which, it is hoped, will bring out many valuable ideas in its discussion.

THE engineering profession has been presented with many papers and reports covering the field of automatic operation, but in almost every case an attempt has been made to describe some particular installation and, as is most natural, the author, whether manufacturer or operator, brings out all of the clever and agreeable points that he can find. This is very creditable but it is felt that the development has reached a stage where the injection of some pessimism would seem to inspire faster progress. The highest developed is the railway class, being the first to be attempted. This class rapidly came to its present high state of development, principally because of its simplicity. In railway operation, very low load factors are encountered thus avoiding, at least to some extent, temperature troubles with their attending ventilating problems. Also, voltage regulation is more or less rough and distribution system resistance high, lowering the precision required in regulating devices and making parallel operation of machines and stations fairly easy.

1. Union Electric Light & Power Co., St. Louis, Mo.

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The next highest developed is the alternating-current transforming and distributing substation. Again we have simplicity, in that there are comparatively few functions, the operation of which actually had to be automatized; the balance being only a matter of finding sufficient courage to close and lock the doors on equipment which had previously been operated with an attendant to watch it. We have had the automatic induction regulator for a good many years but always with someone close by, and while there is no reason why it should not be trusted to operate without a guardian, the author confesses that he had some very anxious hours during the first few weeks of such operation of equipment for which he was responsible. There are only three other automatic operations of a major nature in the average station of this class: First, the automatic switching of supply circuits, not hard to accomplish unless automatic synchronizing is required: Second, the automatic reclosing of distribution circuits on trouble, which detail varies in complexity with the class of service. At the outside, the problem requires the application of only four or five relays or devices per three-phase feeder over and above the standard

apparatus which would be present if the station had an attendant. The third is water control and is present only where transformers and similar equipment is operated "water cooled" over peak-load periods. This is a simple function and is usually given a passing glance only notwithstanding the fact that it is of the highest importance.

The third and least developed class is that of supply stations to heavy Edison system networks where extremely low system resistance is encountered, close precision is called for in voltage regulation and an interruption to service is an unpardonable offense. The author will not attempt to discuss weak points in the automatic railway station field as he is not qualified to do so from the standpoint of experience. Therefore, the following discussion will cover the other two classes only.

Class 2—Automatic A-C. Distribution and Transformer Stations

There are a great many of these stations in successful operation and almost every one of them has some specific points, good as well as unique. Usually these points are emphasized almost to the exclusion of others of equal importance; for example, we may find a station in which a great deal of effort and money has been concentrated on the automatic operation of its supply circuits to prevent the possible loss of supply power due to failure of supply circuit. This is accomplished by automatic transfer devices to keep power on the station bus as long as one of its supply circuits is alive. In the same station we may find only a single transformer unit supplying several important distribution feeders, no provision having been made to save the day if, by any chance, the transformer should fail. Frequently this transformer depends on cooling water to carry it over the peak and we find that there is but one source of water supply; or, if there are two, they are not automatically "changed" in event of failure. The conventional way to protect such a transformer against sustained overload is by means of the well-known thermal relay which is supposed to possess the same relative heating and radiating characteristics as the transformer, and so adjusted that full load amperes will permit the unit to stay on continuously but approximately 25 per cent in excess of this amount tripping the transformer off the line in thirty minutes. The exact principle of the operation of this device is usually only partially understood by the man who actually locates it in the station and upon this point depends its real utility. To be consistent, this thermal relay must undergo exactly the same abuse and benefits regarding ventilation that the transformer suffers and it can be seen that, should the thermal relay be so located that its heat is dissipated by a draught of cold air from a ventilator or by conduction to a wall on which it may be mounted, it may fail to protect the transformer. The manufacturers have recognized this liability and generally provide a thermostat submerged in the transformer oil, which trips off and locks out the transformer if the oil exceeds about 85 deg. cent. This would probably save the transformer (but not the service) if the overload was approached slowly so that too great a drop in temperature between the "hot spot" and the oil was not in evidence, but, if a sudden overload appears after many hours of very light load, when the oil has cooled to a very low temperature, the oil with which the "hot oil" thermostat is in contact may be 50 deg. cooler than the "hot spot" and the thermostat quite useless should the thermal relay fail to function. The above would indicate that we should attempt to develop a means of direct relaying from the actual, measured, hot spot temperature rather than try, by means of a proportional auxiliary circuit, to imitate the performance of the transformer. Much may be said about thermostats. The average operating engineer notes in the specifications that a thermostat will stand guard over his transformer and he accepts this face value. He may, in fact, for years possess many of these devices embedded in his apparatus without ever having actually seen one of them. Without injustice to the electrical manufacturers, it may be said that we should not expect them to know as much about building these devices as do the manufacturers of heating and ventilating apparatus. The author has found that these latter manufacturers possess equipment of this nature directly applicable to our needs which has been developed and in successful operation for years. When applied to transformers, thermostats are usually mounted on some projection on the head block or core, and this means that the transformer must be taken out of service and the oil lowered to get at them for test purposes. With "conservator" or "inertaire" types of transformers, where the case lid is tightly sealed, this becomes an expensive matter and as a result the thermostats once installed are not tested. We attacked this problem some months ago and, as a result, we are installing in many St. Louis station transformers, thermostats made by a well-known manufacturer of heating apparatus who knew nothing of the possible applications to this field. These thermostats are no closer to the "hot spot" than their predecessors—(that being another problem)—but they may be removed and tested without even taking the transformer off the line. This applies to the "sealed" types of transformers as well as others.

The control of cooling water, so far as the thermostat is concerned, is also partly solved by the above, but, automatically operated water valves have not been given sufficient attention. The author has had experience with two types of valves which are furnished by the electrical manufacturers,—the motor operated type and the type which is operated by the expansion and contraction of a gas or fluid in a capillary tube. In his opinion, the latter type is dangerous, as a leak in the pressure system renders the device inoperative without visible indication of trouble until it fails to operate—

and perhaps the load lost. The motor-operated valve, of course, depends upon the continuity of the wiring and the 100 per cent behavior of the motor, which is usually small and operates through a reduction gear, the friction of which is a large percentage of the total power required. The author's experiences with both of the above types have been unhappy, in that both types have failed to function on many occasions, resulting (fortunately in one case only) in the loss of 5000 kw. of distribution load for about one hour. A water valve has been made up of standard materials; this by means of current flowing through a solenoid, holds the water off. The opening of the circuit to the solenoid by the thermostat allows the law of gravity to turn the water on. This device is in effect a closed circuit one and accidents (electrical or mechanical), can result only in wasted water. Some of these devices have been in operation in St. Louis for two years without a failure and almost all station transformers, under the writer's jurisdiction, either have been or are being equipped with this type of valve.

No attempt has been made to protect against failure of induction regulators for distribution circuits, other than to depend on the overload relays to clear them from the line when failure actually occurs. The induction regulator is admittedly a weak piece of apparatus because of its very nature; and it would seem advisable to take steps to give them at least temperature protection. There is a tendency on the part of the operating companies to allow their distribution feeders to become overloaded and, with regulators designed for a 55-deg. rise, this abuses the regulator. The failure of an induction regulator in a manual station may not be serious when an operator who can operate a fire extinguisher is present, but here, again, in the automatic station, we have in most cases failed to provide any protection against fire except faith. Automatic equipment possesses only the intelligence which is incorporated in its design and this falls short of human intelligence in that we have not as yet been able to give it all five of the human senses. It can only *feel* but does not possess the senses of smell, taste, hearing or sight. In manual stations many serious accidents have been forestalled by the operator's ability to "smell something getting hot," *hear* the growl of an arc or *see* smoke issuing from a piece of apparatus the protective devices for which have not *felt* the necessity of removing it from the line; that necessity not being felt until the trouble has developed into a short-circuit, which usually means fire. In general, while this class of station may be said to be satisfactory, it is obvious that there is still a very extensive field in which to progress.

Class 3—Automatic D-C. Substations Feeding Heavy Edison Networks

This class has been developed mostly along the lines of single-unit stations consisting of a motor-generator set or synchronous converter located here and there throughout systems, the major portion of the system being manually operated. This means that the automatic stations are merely followers supplying power to parts of the net-work which grew out of the original manual capacity. In systems of over 30,000 or 40,000 kw. cases where automatic stations are the predominating factors are extremely rare; if, in fact, there are any at all. Where the system is so heavy, it is usually of high density and this in turn means not only a large number of stations but stations of such capacity that several large units per station are necessary. With these large stations close together, parallel operation begins to present a problem. To begin with, the demand upon the operating attendants in the manual stations of such a system is that they hold the "standard feeder-end pressure" to within one volt (0.5 volt per side). There is a reason for this other than the required candle power of customer's lamps, and it is a fact that by lowering or raising the voltage in a station one volt from standard, upwards of 1000 kw. of the load will be shifted to and from adjacent stations. The writer has been unable to find on the market an automatic regulating device which could be applied to this class of service and which might be adjusted to give closer than 1.5 volts, plus or minus (a zone of three volts) on the 240-volt basis. It is obvious then that the stations would not only not give sufficiently close regulation but would be continually "stealing" load from each other. It might be possible, of course, by means of costly pilot circuits to prevent the load from shifting, but we would still lack the precision in voltage regulation at the customer's lamp. On voltage regulators it would seem that some intensive development is necessary. The voltage regulator being the keynote to this situation, too much stress cannot be placed on this point. The question of parallel operation of any number of machines in the same station is being worked upon and there seems to be no doubt that all obstacles will ultimately be overcome; not however, without doing some things that have never been done before. It is always possible to make each machine literally an isolated station by feeding directly into the system over its own feeders, but it means that all copper is not working on light loads unless some extra money is invested in heavy bus tie breakers and, after all, it is dodging the main problem. While the apparent "intelligence" of some of the d-c. automatics now in service, is remarkable, we are still relying too much upon pilot wires and periodical inspection to approach the intelligence of an operating attendant. We must remember that automatic equipment will do just the things we build it to do and, incidentally, keep on doing them even if dangerous to life or property. In other words, it cannot use judgment, change its mind nor respond to the dictates of fear. This means that the equipment comprising an automatic station must be rugged in every detail. For example, it would not be desirable to protect all d-c. circuit-breakers with temperature devices and they

should therefore be selected so as to allow ample margin. Based on actual experience, the author feels that no d-c. circuit-breaker in this class of service should be operated at a contact brush density of more than 350 amperes per sq. in. If heavier duty is imposed it is possible for a breaker to heat up so quickly that its bridges will be annealed before this is discovered. The author advocates the use of "maximum" thermometers, the fluid columns of which remain at the highest point reached unless shaken down intentionally. One of these thermometers on the contact block of each breaker will divulge, to the inspector, the value of the highest temperature reached during his absence and an oxidized contact can be caught before the critical point (annealing) is reached. This same idea may be applied to bearings, as the only protection they now have is through the medium of a gas type thermostat by which the machine is locked out when a dangerous temperature is reached on any of the bearings. Ventilating equipment is another factor that should be fortified with ample margin and ruggedness, as in most cases the station cannot operate unless its ventilating equipment is delivering. Again the "maximum" thermometer is useful, as it can be applied to the machine and outside temperatures in conjunction with recording load-indicating devices to determine if the equipment has been properly cooled in the absence of the inspector and a brewing case of trouble arrested. The method of tripping oil and air circuit-breakers and contactors is one that should have close attention. There is a tendency to employ, so far as possible, standard breakers which almost always are held in by a mechanical latch or toggle. This must be struck sharply by the tripping solenoid to actually open the main device. Breakers furnished for standard manual application have been known to fail to trip on the first impulse of the tripping solenoid and as there is no attendant in an automatic station to follow up and trip it manually, this breaker remains closed. The author feels that a holding-in type of latch should be used in these applications, as it does not depend on the continuity of an electrical circuit for its operation. In some cases this is not possible, as in the case of automatizing an existing station already equipped. We have partly solved this problem by the application of two relays in the tripping circuit, so connected that if the main device does not open at the first impulse, it is followed up by a succession of hammer blows which will either open the toggle or break it, in which latter case the main device, although broken, would be opened. Too much stress cannot be brought to bear on the matter of auxiliary switches on the mechanisms of all types of breakers. Originally these switches were for the sole purpose of operating indicating lamps and an occasional interlock. In an automatic station there is hardly an operating circuit that does not interlock through the auxiliary switch of some breaker or contactor and it follows that the failure of almost any of these auxiliary switches will cripple the station. The author has had some bad experiences with this equipment, both where existing devices were automatized and where the equipment was furnished specifically for automatic operation.

General

While the author feels that progress is being made, there is still too much of a tendency to be satisfied with some questionable equipment as its exists. It is necessary that we look further into the future and in solving the problems which we encounter as we go along, bear in mind the importance to always leave sufficient margin so that that particular thing will be forever placed behind us.

Discussion

E. C. Stone: I want to second vigorously Mr. Millan's conclusion that the auxiliary equipment, relays, etc., which are used in automatic substations must be designed with a very great degree of reliability.

Automatic equipment adds cost to the station but we often forget that it saves operators' salaries. In any event the automatic equipment is only a small percentage of the cost of the stations and we should not hesitate to spend even 50 per cent more on it if, by doing so, we could be assured of obtaining perfect operation.

Chester Lichtenberg: One point which might be emphasized is the rapid development of automatic-station control equipment during the past three years. A survey of the situation indicates that during this period a great deal more attention has been paid to the details of design not only of the complete equipments but also of the individual devices so as to make them of maximum reliability with minimum maintenance.

For feeder-voltage regulators Mr. Millan suggests two types of protection.

1. Grounding protective relay.
2. Temperature protective relay.

Both of these devices are available and have been in automatic station service for several years. The grounding protective relay (device function No. 64) is quite well known in certain parts of the country where it has proven exceptionally valuable in giving the impulse for disconnecting apparatus from service when such apparatus developed faults to ground. The same device slightly modified last year has been successfully applied to feeder-voltage regulators and other similar devices. It will operate on any fault to ground in excess of 30 amperes and in combination with suitable oil circuit breakers affords protection against extensive damage.

Temperature protective relays (device function No. 49) for feeder-voltage regulators are available. They have not been sold in general, however, because the purchasers of the feeder-voltage regulators do not care to pay the small additional expense which these devices would add.

Most of the a-c. automatic stations which have been installed to date are of the relatively simple type described by Mr. Millan. There are outstanding examples, however, of very complete a-c. automatic stations which are very much more extensive and which have been in successful operation for three or more years. For example, the Kansas City Power & Light Company has had in operation since 1921 two a-c. automatic substations. They each have two or more incoming lines with two or more banks of transformers and a dozen or more outgoing feeders. The stations are designed so as to have only sufficient transformers connected in service to supply power to the load. Consequently, in times

of light load a minimum transformer capacity is excited. The stations are also arranged so that in case of trouble to any transformer bank or incoming line the bank or line is automatically switched out of service and replaced by an emergency bank or line.

The successful operation of these automatic stations emphasizes another point. It is the requirement that relaying devices for automatic substations be considered on quite a different basis from those for manual substations. In the manual station there is an attendant. It is his ordinary duty to watch the instruments provided for him so that he may check the operation of the various relays, etc. In case of trouble he is expected to substitute himself for the relay devices should they fail. In an automatic station there is no attendant. If a relay is called upon to perform its function it must not only be ready to perform that function but must perform it successfully. If there is dust on the contacts the contacts must be so designed that when operating the dust must either be wiped off or contact be made notwithstanding the dust. If the operation is to occur at 180 volts then it must occur at 180 volts and not wait until the pressure is at some other value. This has meant a new and very rigid standard with regard to the performance of relaying devices for automatic stations and the successful operation of several thousand of these during the past ten years evidences what can be done.

In connection with automatic stations there have been developed a number of supervisory systems. Some of these use automatic telephone relays. Such relays are suited for automatic telephone-central service but are not suited for automatic power-substation service. For example, the usual automatic telephone exchange has reasonably clean rooms kept at a relatively constant temperature. Besides, the relays are usually inspected and tested about once a day or at the most once a week. In automatic station service, however, supervisory system relays are frequently called upon to operate without inspection or test excepting at intervals of one month or more. The ones in the outlaying stations are subjected to temperatures varying from 40 deg. cent. below zero to 80 deg. cent. or more above zero. The conditions, therefore, are quite different and it is easy to see why a totally different class of relaying device is required for a supervisory system in automatic power-substation service than for automatic telephone-exchange work. This analysis has been proven correct by actual experience. So much so that one of the manufacturers furnishing supervisory systems has discarded all automatic telephone relays and uses relays developed exclusively for railway train-dispatching service where inspections are made not oftener than once a year. These relays represent almost the last word in relay development because experience has indicated that when correctly installed they require no maintenance and practically no inspection.

E. K. Huntington: In our system in Rochester, New York, we have recently installed two a-c. and one Edison automatic substations. The reference that various devices, which have been taken from the manual station and applied to the automatic station, should be more reliable is certainly one that needs our attention. Among these might be mentioned bearing temperature of relays and devices which cannot be easily tested after their installation. We have found that most devices are subject to change in calibration after installation and unless tested periodically are unreliable for that reason. In one automatic station in which there is a vertical hydro-generator, we are using a recording bearing-temperature device which not only records the bearing temperature and shows that it is working, but also has contacts on it for tripping the machine in case the temperature reaches the value at which the contacts are set to operate. Such a device seems very desirable where its cost is warranted.

We have found that constant attention is necessary on all devices in automatic stations. At the present time we are making a practise of putting each automatic operation through its complete cycle at least once every month, as a check upon the operation of various devices which may stand for a considerable time without operating in actual service.

In our Edison system, we have about a 20,000-kw. load supplied over an area of about one mile radius. The voltage-regulating devices, which were supplied with our Edison automatic substation equipment hold bus voltage within about two volts either way, or a four-volt swing on the 250-volt system. In our particular case, this has worked out very satisfactorily and no objectionable exchange of load has been noticed. More sensitive voltage-regulating devices are undoubtedly necessary, in supplying the highly concentrated loads of the larger cities but not in the average Edison system.

H. O. Stephens (communicated after adjournment): Mr. Millan points out a number of details in the automatic equipment which may give trouble on account of failure to operate. In particular, he mentions the possibility of trouble developing with thermostats and water-control valves on water-cooled transformers. The remedy is obvious and should not be passed over without mention.

When the type of transformer for automatic sub-stations is being selected, there are usually three choices available:

First: Water-cooled transformers.

Second: Combination self-cooled, water-cooled transformers capable of carrying light loads without water, but with automatic thermostats and valves for turning on the water supply for water cooling during peak loads.

Third: Self-cooled transformers capable of carrying the maximum peak load.

The cost of a self-cooled transformer will range from zero to fifty per cent greater than the cost of a water-cooled transformer, depending upon the size, complications, and voltage; while the cost of a combination self-cooled, water-cooled transformer will be approximately midway between the cost of a water-cooled transformer and a self-cooled transformer.

Transformers of all three types have been used in automatic substations but it is my opinion that while more or less practical control devices are on the market for controlling the water for water-cooled and combination self-cooled water-cooled transformers, they involve altogether too "clever" designing. The simple self-cooled transformer capable of carrying the maximum peak load is the obvious solution as all of the auxiliary thermostats and valves for controlling the water supply are eliminated and the increase in cost of the self-cooled transformer can be justified when it is considered that the efficiency of the self-cooled transformer is also usually higher. The self-cooled transformer has another decided advantage since the oil temperature is a much better indication of the load than it is on a water-cooled transformer; while the oil temperature of combination self-cooled water-cooled transformer fluctuates so widely that it is of little or no value in indicating the load. A reliable thermostat for tripping off the load in case the transformer reaches a dangerous temperature can readily be installed in the top oil of a self-cooled transformer. These facts should be very carefully considered before deciding on the use of anything but self-cooled transformers in an *automatic* substation.

Initial and Sustained Short-Circuits in Synchronous Machines

Analytical and Graphical Treatment of General Cases of Armature Windings Displaced by Arbitrary Angles, with Applications to One-, Two- and Three-Phase Machines

BY VLADIMIR KARAPETOFF[1]

Fellow A. I. E. E.

Synopsis.—A knowledge of the instantaneous values of armature and field currents, when an alternator is short-circuited, is becoming of increasing practical importance. These currents determine the rating of protective apparatus, mechanical stresses in the machine itself, possible damage to other equipment, etc. During the first few cycles, immediately after a short-circuit, the currents are usually much larger than those on sustained short-circuit, and gradually approach the latter values over a number of cycles. It is, therefore, necessary to distinguish between the initial and sustained values of currents, and formulas are deduced in this paper for both. The novel feature of the treatment consists in starting with a generalized unsymmetrical three-phase winding, also containing an external inductance in one of the phases. The Kirchoff equations are written and solved for this general case, and it is then shown how the formulas for the usual one-, two-, and three-phase machines can be directly derived from the general expressions, without considering the magnetic linkages in detail in each case. A graphical interpretation of the equations is also given, in the form of space-vector diagrams, in which the m. m. f s. vary according to the sine law in space but not in time.

Index to Appendices

INTRODUCTION

THE purpose of the paper is to deduce general formulas for the field and armature currents in a synchronous machine on initial and sustained short-circuit. While the subject is by no means new, the present treatment is perhaps more general than those published heretofore (See Appendix XII).

In a companion paper by R. F. Franklin, entitled "Short-Circuit Currents of Synchronous Machines," emphasis is laid upon the important practical cases, and the formulas are illustrated by calculated curves of currents. In the present paper, emphasis is laid upon the general method of derivation, and specific cases are carried out only far enough to show that the results check with Franklin's work.

In accordance with the preference of the Institute readers, the paper, itself, is made short and non-mathematical, while the details of derivation of the formulas are placed in the appendices to which references are made in the text.

THE DIAGRAM OF CONNECTIONS

Figs. 1 and 2 show a three-phase winding of a synchronous machine in which, for the sake of generality, the electrical angles between the phases are assumed to be different from 120 deg. Moreover, each phase winding is assumed to possess a different number of turns and therefore a different inductance (for notation, see Appendix XIII). An external inductance, L, is shown in series with one of the armature windings, and equations are derived with this inductance in the circuit. By putting $L = 0$, an ordinary three-phase short-circuit is obtained; by putting $L = \infty$. the phase C is opened and a single-phase short-circuit is obtained between the phases A and B.

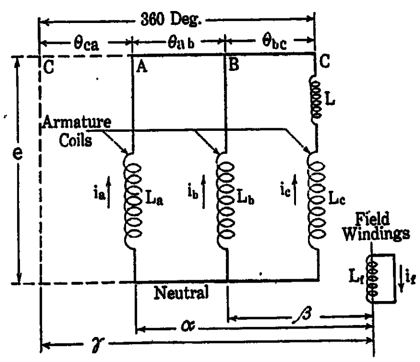

FIG. 1—GENERALIZED UNSYMMETRICAL THREE-PHASE WINDING

The field winding is assumed to be placed on a cylindrical rotor, so that the self and mutual inductances of the armature windings may be considered to remain constant throughout a cycle. Moreover, the mutual

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inductances between the field and the armature windings have been assumed to vary harmonically (Appendix I).

The resistances of all the windings are neglected altogether. For this reason, the field winding is shown in Fig. 1 short-circuited upon itself, since the excitation voltage is only necessary for overcoming the resistance of the winding. A certain field current is assumed to exist at the instant of short-circuit, and then to vary only under the influence of the armature currents so as to keep the flux linkages constant; equations (4) and (25).

The assumption of zero resistances is justified during the first half cycle or so after the instant of short-circuit, when the magnetic fluxes essentially determine the currents. With large alternators, this assumption is also justified for the armature windings on sustained short-circuit, the ohmic drop being practically negligible. The resistances of the windings enter as a factor in the gradual adjustment of the currents from the initial values to those on sustained short-circuit. This transitional period is not considered in the paper.

By neglecting the resistances, it becomes possible to integrate directly the Kirchoff equations (1) to (4). With resistance terms, the corresponding simultaneous differential equations become practically unsolvable. A few attempts to solve them even approximately in the simplest cases have led to expressions which are too complicated for practical use. The method of successive approximations, although quite tedious, may possibly be applied with more success.

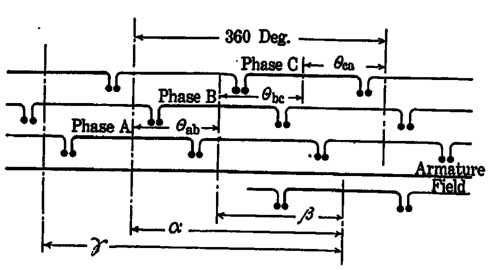

Fig. 2—The Relative Position of the Armature and Field Windings

Having obtained the fundamental equations, (1) to (5), for the general case shown in Fig. 1, and integrated them in the form of equations (22) to (25), various special cases are considered in the subsequent appendices, as indicated below.

Single-Phase Short-Circuit

The most general case of single-phase short-circuit considered in this paper is that through an external inductance (Appendix V). The angle, θ_{ab}, is put equal to zero and the windings in the phases A and B are assumed to coincide, so as to form but one winding. The current flows through this combined winding, through the inductance L, and through the phase winding C. Since the angle, θ_{bc}, is not necessarily equal to 180 deg., this case covers not only a short-circuit of a single-phase machine, but a single-phase short-circuit of a polyphase machine as well.

More specific cases are then considered in detail in Appendix VII, with the assumption $L = 0$. For the single-phase machine $L_c = 0$; for a two-phase machine the inductance L_c is equal to the parallel combination of L_a and L_b, and $\theta_{bc} = 90$ deg.; for a three-phase machine $\theta_{bc} = 120$ deg. The final formulas are shown to check with those in Franklin's paper. Finally, a graphical interpretation of the single-phase short-circuit is given in Appendix XI.

Polyphase Short-Circuit

The general expressions for the armature and field currents, equations (22) to (25), are first interpreted graphically, as shown in Fig. 4 (Appendix III). This figure gives a much clearer idea of the inter-relationship of the currents than the equations themselves. It is then shown in Appendix IV that when $L = 0$ and $L_c > 0$, the voltage e (Fig. 1) between the terminals and the neutral, is always equal to zero. This leads to a simplification of Fig. 4 to Fig. 6, and the corresponding equations are derived in Appendix VI. These equations are then applied to the usual two-phase and three-phase machines in Appendix VIII, and the results are shown to check with Franklin's formulas.

Individual vs. Common Short-Circuit

With two-phase and three-phase short-circuits, several cases have to be considered, as shown in Figs. 7 and 8. In the Appendix IX, it is shown that the cases 7 a and 7 b are electrically equivalent, and that identical currents may be expected in both, at least within the limits of the fundamental assumptions made in the paper. Similarly, the cases 8 a and 8 b are identical, but the case 8 c presents some additional features. The influence of the ohmic resistance and of the character of the mutual inductance between the phases is discussed, and the conclusion is reached that at least in usual alternators the currents in case 8 c may be expected to be approximately equal to those computed for the cases 8 a and 8 b.

Conclusion

The determination of short-circuit currents in synchronous machines is of considerable importance to the designer as well as to the operating engineer. These currents may cause considerable mechanical stresses in the machine, influence the selection of the protective equipment, determine the transient conditions in the connected lines, etc.

The fundamental equations with which this investigation begins (Appendix I), are of quite general application, and it is hoped that the mathematical portions of the paper, especially the general method, may be of service not only in cases in which the equations or their solutions actually apply, but also in other cases, (for example, in an induction machine), in which simi-

lar equations may be established and a similar method of solution used.

Mr. R. E. Doherty, of the General Electric Company, suggested this investigation as a sequel to his own well-known researches in the subject, and the author wishes to acknowledge, gratefully, his encouragement and assistance in the preparation of the paper. Mr. K. C. Mobarry read and corrected the manuscript and made several valuable suggestions, for which the author wishes to express to him his gratitude.

Appendix I

The Fundamental Equations

Referring to Fig. 1, let at an instant (t) the voltage between the neutral points be (e). Then, equating to (e) the total e. m. f. induced in phase (A), we have,

$$L_a\, d\, i_a/d\, t + M_{ab}\, d\, i_b/d\, t + M_{ca}\, d\, i_c/d\, t + d\,(M_{fa}\, i_f)/d\, t = e \qquad (1)$$

For notation, see Appendix XIII. The coefficients of mutual induction, M_{ab} and M_{ca}, do not vary with the time and therefore, are left outside the sign of the derivative. The coefficient of mutual induction, M_{fa}, between the field winding and the armature phase (A), is a function of time and, therefore, a derivative must be taken of the linkages, $M_{fa}\, i_f$.

By analogy, we can write for phases (B) and (C), respectively, with a cyclic substitution of the subscripts (a), (b), (c):

$$L_b\, d\, i_b/d\, t + M_{bc}\, d\, i_c/d\, t + M_{ab}\, d\, i_a/d\, t + d\,(M_{fb}\, i_f)/d\, t = e \qquad (2)$$

$$(L_c + L)\, d\, i_c/d\, t + M_{ca}\, d\, i_a/d\, t + M_{bc}\, d\, i_b/d\, t + d\,(M_{fc}\, i_f)/d\, t = e \qquad (3)$$

In equation (3), $(L_c + L)$ is used instead of (L_c), where (L) is the external inductance. (L) is not interlinked magnetically with any of the phases, and therefore does not enter in the other equations. For the field circuit, we have;

$$L_f\, d\, i_f/d\, t + d\,(M_{fa}\, i_a)/d\, t + d\,(M_{fb}\, i_b)/d\, t + d\,(M_{fc}\, i_c)/d\, t = 0 \qquad (4)$$

The first Kirchoff law, applied to the armature winding, gives,

$$i_a + i_b + i_c = 0 \qquad (5)$$

Equations (1) to (5) contain five unknown functions of time (t); namely, i_a, i_b, i_c, i_f, e; by solving the equations, all these functions can be determined.

The foregoing equations can be somewhat simplified by expressing the inductances through the corresponding equivalent permeances and the numbers of turns[2]. Moreover, with the fluxes and m. m. f's. assumed to be distributed sinusoidally along the air-gap, the various mutual inductances are simple cosine functions of the angles of separation between the two coils.

Let the permeance of the useful (or common) magnetic path through the field and the armature be $(\mathcal{P})$ henries. Let τ be the magnetic leakage coefficient of the field circuit, that is, let,

$$\tau = \Phi_{useful}/\Phi_{total} \qquad (6)$$

when the field alone is excited. Similarly, let (σ) be the magnetic leakage coefficient of the armature winding. Both (σ) and (τ) are less than unity. The four self-inductances can be expressed as follows:

$$L_a = \sigma^{-1}\, \mathcal{P}\, N_a^2;\; L_b = \sigma^{-1}\, \mathcal{P}\, N_b^2;\; L_c = \sigma^{-1}\, \mathcal{P}\, N_c^2 \qquad (7)$$

$$L_f = \tau^{-1}\, \mathcal{P}\, N_f^2 \qquad (8)$$

It is convenient to assume the external inductance coil (L) to have the same number of turns as one of the phase windings, for example phase (A), and to represent its equivalent permeance in the form, $k\, \sigma^{-1}\, \mathcal{P}$, so that,

$$L = k\, \sigma^{-1}\, \mathcal{P}\, N_a^2 \qquad (9)$$

The factor (k) may have any positive value between zero and infinity, so that equation (9) in no way limits the value which may be assigned to (L).

The simplest assumption which can be made in regard to the coefficients of mutual inductance is that they are harmonic functions of the space angles between the coils. This approximately holds true for machines in which the armature m. m. f. is distributed in space in accordance with the sine law. Let the coils in phase (B) be shifted until they completely coincide with the coils in phase (A). Then, the coefficient of mutual inductance between the two is equal to $\mathcal{P}_m\, N_a\, N_b$, where $\mathcal{P}_m$ is the mutual or common permeance of the magnetic circuit embraced by the two groups of coils. When the coils completely coincide, $\mathcal{P}_m = \sigma^{-1}\, \mathcal{P}$. Now, as the coil (B) is moved back to its true position, we may assume that the part of the flux due to (A) and linking with (B) varies as the cosine of the angle of shift. The mutual permeance varies in the same ratio. With these assumptions, we obtain the following expressions for the various M' s;

$$M_{ab} = \sigma^{-1}\, \mathcal{P}\, N_a\, N_b \cos\theta_{ab} \qquad (10a)$$

$$M_{bc} = \sigma^{-1}\, \mathcal{P}\, N_b\, N_c \cos\theta_{bc} \qquad (10b)$$

$$M_{ca} = \sigma^{-1}\mathcal{P}\, N_c\, N_a \cos\theta_{ca} \qquad (10c)$$

$$M_{fa} = \mathcal{P}\, N_a\, N_f \cos\alpha \qquad (11a)$$

$$M_{fb} = \mathcal{P}\, N_b\, N_f \cos\beta \qquad (11b)$$

$$M_{fc} = \mathcal{P}\, N_c\, N_f \cos\gamma \qquad (11c)$$

The coefficient of mutual inductance, M_{fa}, reaches its maximum when $\alpha = 0$. By definition, the coefficient of magnetic coupling, K, is determined from the relationship,

$$K^2 = (\text{max. } M_{fa})^2/(L_f\, L_a) \qquad (12)$$

Substituting the values from equations (7), (8), and (11a), we get, after reduction,

$$K^2 = \sigma\, \tau \qquad (13)$$

The variable angles α, β, γ, differ from each other by constant amounts; namely,

$$\beta = \alpha - \theta_{ab} \qquad (14)$$

$$\gamma = \alpha + \theta_{ca} \qquad (15)$$

2. V. Karapetoff, "The Magnetic Circuit," p. 184.

Instead of t, we shall take α for the independent variable, where

$$\alpha = \omega\, t = 2\,\pi\, f_0\, t \tag{16}$$

We then have,

$$d/d\,t = \omega\,.\,d/d\,\alpha = \omega\,.\,d/d\,\beta = \omega\,.\,d/d\,\gamma \tag{17}$$

It is convenient to represent (e) in equation (1) as a derivative of some function with respect to time, because then the equation can be integrated directly. We, therefore, put,

$$e = \sigma^{-1}\, \mathcal{P}\, N_a^2\, d\,s/d\,t \tag{18}$$

where (s) is a function of time, containing no constant term. However, any desired constant term may be added to (s) without changing the value of e in equation (18). We shall arbitrarily assume that this constant term is zero so that $s = 0$ when $e = 0$. The factors σ, N_a^2, and $\mathcal{P}$ are added in order later to cancel them on both sides of certain equations.

The following assumptions are further made:

(a) The coils (A) and (B) have the same number of turns (N), so that,

$$N_a = N_b = N \tag{19}$$

(b) The number of turns in coil (C) is (c) times that in (A); in other words,

$$N_c = c\,N \tag{20}$$

The factor (c) may have any value between zero and infinity.

(c) The number of turns in a field coil is (f) times that in the coil (A), so that,

$$N_f = f\,N \tag{21}$$

Substituting the foregoing expressions in equations (1) to (4), we obtain, after integration and simplification:

$$i_a + i_b \cos\theta_{ab} + c\, i_c \cos\theta_{ca} + f\,\sigma\, i_f \cos\alpha = s + A \tag{22}$$

$$i_b + c\, i_c \cos\theta_{bc} + i_a \cos\theta_{ab} + f\,\sigma\, i_f \cos\beta = s + B \tag{23}$$

$$(c^2 + k)\, i_c + c\, i_a \cos\theta_{ca} + c\, i_b \cos\theta_{bc} + c\, f\,\sigma\, i_f \cos\gamma = s + c\,C + [k\, I_c] \tag{24}$$

$$i_a \cos\alpha + i_b \cos\beta + c\, i_c \cos\gamma + f\,\tau^{-1}\, i_f = F\, f\, \tau^{-1}\, I_f \tag{25}$$

Here, (A), (B), (C), (F), are constants of integration; for the initial short-circuit conditions, they depend upon the values of the currents and of (s) at the instant of short-circuit. In order to make the initial constant (C) independent of (k), the term $k\,I_c$ is added in equation (24). This term is placed in the brackets to indicate that it is used only for the initial short-circuit. With a sustained or established short-circuit, the currents are independent of the initial values, such as I_c, and the term $k\,I_c$ is simply omitted.

Equations (22) to (25), together with equation (5), contain five unknown functions of the time-angle (α), namely i_a, i_b, i_c, i_f, s, and can be solved for these as simultaneous equations.

Appendix II

Constants of Integration

1.—*Armature Constants for the Initial Short-Circuit.*

For the instant of short-circuit, equations (22) to (24) become

$$U \cos\xi = S + A \tag{26}$$

$$U \cos(\xi - \theta_{ab}) = S + B \tag{27}$$

$$c\, U \cos(\xi + \theta_{ca}) = S + c\,C \tag{28}$$

where

$$U \cos\xi = I_a + I_b \cos\theta_{ab} + c\, I_c \cos\theta_{ca} + f\,\sigma\, I_f \cos\alpha_0 \tag{29}$$

$$U \cos(\xi - \theta_{ab}) = I_b + c\, I_c \cos\theta_{bc} + I_a \cos\theta_{ab} + f\,\sigma\, I_f \cos\beta_0 \tag{30}$$

$$U \cos(\xi + \theta_{ca}) = c\, I_c + I_a \cos\theta_{ca} + I_b \cos\theta_{bc} + f\,\sigma\, I_f \cos\gamma_0 \tag{31}$$

In equation (29), the initial linkages on the right-hand side are arbitrarily denoted by $U \cos\xi$. Granting this notation, the other two equations can be deduced as follows:

Let I_a, I_b, I_c, I_f and U be thought of as vectors (in space, *not* in time.) Then equation (29) may be thought of as the real part of the expression

$$U\,\epsilon^{j\xi} = I_a\,\epsilon^{j0} + I_b\,\epsilon^{j\theta_{ab}} + c\, I_c\,\epsilon^{j\theta_{ca}} + f\,\sigma\, I_f\,\epsilon^{j\alpha_0} \tag{32}$$

Multiplying this equation throughout by $\epsilon^{-j\theta_{ab}}$, and equating the real parts, equation (30) is obtained. Multiplying both sides of equation (32) by $\epsilon^{j\theta_{ca}}$ and equating the real parts, gives equation (31).

To solve equations (29) to (31) for (U) and (ξ), use equation (32), since it gives, directly, the values of $U \cos\xi$ and $U \sin\xi$. Dividing the second value by the first will give $\tan\xi$ and consequently $\cos\xi$. (U) is then found by dividing the expression for $U \cos\xi$ by $\cos\xi$.

In the actual solution of equations (22) to (25), the first step is to eliminate (s) by subtraction. Consequently, it is also convenient to eliminate (S) from equations (26) to (28). We then get[3]

$$B - A = 2\,U \sin 0.5\,\theta_{ab} \sin(\xi - 0.5\,\theta_{ab}) \tag{33}$$

$$A - c\,C = U\, b \sin(\xi + \theta_{ca}') \tag{34}$$

where

$$b \sin\theta_{ca}' = 1 - c\cos\theta_{ca} \tag{35}$$

$$b \cos\theta_{ca}' = c \sin\theta_{ca} \tag{36}$$

Equation (34) is deduced as follows: Subtracting equation (28) from equation (26), we get

$$U\,[\cos\xi - c\cos(\xi + \theta_{ca})] = A - c\,C \tag{37}$$

Expanding the expression in the parentheses and introducing the quantities (b) and θ_{ca}', according to the defining equations (35) and (36), gives:

$$U\,[b\cos\xi \sin\theta_{ca}' + b\sin\xi\cos\theta_{ca}'] = A - c\,C \tag{38}$$

3. The symbol (b) appearing in equations (35) and (36), and later in equations (66) and (67), has nothing to do with phase (B), but is an auxiliary constant defined by these equations.

From this expression, equation (34) follows directly. A geometric interpretation of the auxiliary quantities (b) and θ_{ca}' is shown in Fig. 3. ($A\,C$) represents the phase (A), with the number of turns arbitrarily assumed to be equal to unity. To the same scale, ($A\,B$) represents the winding of the phase (C), with the number of turns (c) times that of phase (A). The space angle between the windings is θ_{ca}. The closing line of the triangle ($A\,B\,C$) represents (b), and the angle which it forms with the normal to ($A\,C$) is equal to θ_{ca}'. Equations (35) and (36) are then satisfied. We also have

$$b^2 = 1 + c^2 - 2\,c\cos\theta_{ca} \qquad \textbf{(38a)}$$

$$\tan\theta_{ca}' = (1 - c\cos\theta_{ca})/(c\sin\theta_{ca}) \qquad \textbf{(39)}$$

When $$I_a = I_b = I_c = 0 \qquad \textbf{(40)}$$

equations (29) to (31) simply become

$$U = f\,\sigma\,I_f; \quad \xi = \alpha_0 \qquad \textbf{(41)}$$

with the corresponding simplification of equations (33) and (34).

Because of the form (41) to which the expression for (U) is reduced in the simplest case, it is convenient for some purposes to put generally:

$$U = u\,f\,\sigma\,I_f \qquad \textbf{(42)}$$

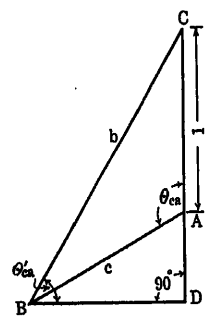

FIG. 3—DEFINITION OF THE AUXILIARY QUANTITIES B AND θ_{ca}

where (u) is a proper factor to make equation (42) agree with equations (29) to (31).

It is shown in Appendix IV that when $k = 0$, (s) is equal to zero at all instants, so that (S) is also equal to zero. Equations (26) to (28) are then simplified accordingly.

2.—Armature constants for the permanent short-circuit.

In this case, omitting the term [$k\,I_c$] in equation (24), we simply have:

$$A = B = U = 0 \qquad \textbf{(43)}$$

and $$C = 0 \qquad \textbf{(44)}$$

for the following reasons: The armature currents i_a, i_b, i_c in equation (22), can have no d-c. component, because the phenomenon now consists in an established operation of the alternator, all transients having died out. Hence, the average value of each current over a cycle is equal to zero; since

$$\text{average } i = (2\,\pi)^{-1}\int_0^{2\pi} i\,d\,\alpha$$

we have, $$\int_0^{2\pi} i\,d\,\alpha = 0 \qquad \textbf{(44a)}$$

The field current, i_f, has a d-c. component and (we may provisionally assume) has some sinusoidal harmonics. However, a sine-wave component of fundamental frequency is absent because it could be induced only by a stationary armature flux or by one moving at twice the synchronous speed. Therefore, expanding the field current into a Fourier series, we have,

$$i_f = i_0 + i_2\sin(2\,\alpha + \mu_2) + i_3\sin(3\,\alpha + \mu_3) + \text{etc.}$$

and consequently

$$i_f\cos\alpha = i_0\cos\alpha + i_2\cos\alpha\sin(2\,\alpha + \mu_2) + i_3\cos\alpha\sin(3\,\alpha + \mu_3) + \ldots + i_n\cos\alpha\sin(n\,\alpha + \mu_n) \qquad \textbf{(44b)}$$

But,

$$\cos\alpha\sin(n\,\alpha + \mu_n) = 0.5\sin[(n+1)\,\alpha + \mu_n] + 0.5\sin[(n-1]\,\alpha + \mu_n].$$

Multiplying by $d\,\alpha$ and integrating over a cycle, we get,

$$\int_0^{2\pi}\cos\alpha\sin(n\,\alpha + \mu_n)\,d\,\alpha = 0$$

Hence, if we multiply equation (44b) by $d\,\alpha$ and integrate over a cycle, each term on the right-hand side is separately equal to zero, and consequently

$$\int_0^{2\pi} i_f\cos\alpha\,d\,\alpha = 0 \qquad \textbf{(44c)}$$

Therefore, from equations (44a) and (44c) we see that if equation (22) is multiplied by $d\,\alpha$ and integrated over a cycle, each term on the left-hand side is separately equal to zero, so that the right-hand side is also equal to zero.

But, by assumption, (s) has no constant term and consists of sinusoidal terms only, so that its integral over a cycle is equal to zero. Consequently $2\,\pi\,A = 0$ or $A = 0$. By a similar reasoning it can be shown from equation (23) that $B = 0$; equation (44) can be proved from equation (24). The result $U = 0$ follows from equations (33) and (34).

3.—Field Constant for the Instant of Short-circuit.

The constant of integration (F), in equation (25), is determined from the condition

$$(F - 1)\,f\,\tau^{-1}\,I_f = I_a\cos\alpha_0 + I_b\cos\beta_0 + c\,I_c\cos\gamma_0 \qquad \textbf{(45)}$$

When the condition (40) is satisfied,

$$F = 1 \qquad \textbf{(46)}$$

4.—Field Constant for Permanent Short-Circuit.

Permanent armature currents can induce in the field winding only alternating voltages, without any d-c. component. Hence, the average value of i_f over a cycle is equal to the actual value I_f of the field current at no-load. In other words

$$\int_0^{2\pi} i_f\,d\,\alpha = 2\,\pi\,I_f \qquad \textbf{(47)}$$

Under the assumed conditions, the field current is a periodic function of α; so that, in general, we may write,

$$i_{fp} = m\, F\, I_f\, \phi\,(\alpha) \tag{47a}$$

where the subscript (p) signifies "permanent," (m) is a known constant, and $\phi\,(\alpha)$ is a certain function of the time-angle α. Therefore, equation (47) becomes:

$$m\, F \int_0^{2\pi} \phi\,(\alpha)\, d\,\alpha = 2\,\pi \tag{47b}$$

Knowing (m) and $\phi\,(\alpha)$, the factor (F) can be determined from this expression.

In this investigation, only two particular forms of $\phi\,(\alpha)$ occur. Each of these forms permits definite integration and, hence, a solution for (F). For a two-phase or three-phase short-circuit, with $k = 0$, $i_{fp} = I_f$ (see equation **134**), so that it is permissible to put $F = m = \phi\,(\alpha) = 1$. With a single-phase short-circuit, equation (**106**), the function ϕ is of the form:

$$\phi\,(\alpha) = [n - 2\,q \sin^2(\alpha - \theta')]^{-1} \tag{48}$$

where n, q, θ', are known constants. Equation (**48**) may also be written as

$$\phi\,(\alpha) = [(n - q) + q \cos 2\,(\alpha - \theta')]^{-1} \tag{48a}$$

Integrating in this latter form, we get[4]

$$\int_0^{2\pi} \phi\,(\alpha)\, d\,\alpha = 2\,\pi / \sqrt{(n - q)^2 - q^2} \tag{49}$$

so that, from equation (**47b**),

$$F_{1p} = m^{-1} \sqrt{(n - q)^2 - q^2} \tag{49a}$$

The subscript (p 1) signifies "permanent single-phase short-circuit."

Appendix III

Graphical Representation of the Linkage Equations (22) to (25) in a Polyphase Short-Circuit[5]

Equations (**22**) to (**25**) are represented in Fig. 4 by means of a polygon of *space vectors*. The currents being non-sinusoidal in time, no ordinary time-vector diagrams can be used. However, the distribution of all the m. m. f's in space being by assumption sinusoidal, these m. m. f's, for a particular instant of time, can be represented by space vectors. For another instant of time, the lengths of the vectors i_a, i_b, i_c, i_f, and the angle (α), are different, but the angles θ_{ab}, θ_{bc}, θ_{ca}, ξ, and the lengths $O\,K$ and $H\,N$ remain constant.

Taking first equation (**22**) and substituting for (A) its value from equation (**26**), we get:

$$\begin{aligned} i_a + i_b \cos \theta_{ab} + c\, i_c \cos \theta_{ca} + f\, \sigma\, i_f \cos \alpha \\ = U \cos \xi + (s - S) \end{aligned} \tag{50}$$

In Fig. 4, $O\,A = i_a$ and $A\,B = i_b$, the space angle (*not* the time angle) between these two m. m. f. vectors being θ_{ab}. Since $i_a + i_b + i_c = 0$, instead of laying off the vector $c\, i_c$ in the direction of i_c, we draw the vector $c\,(i_a + i_b) = B\,C$ in the opposite direction. Let, at the instant shown in Fig. 4, the angle (α) be equal to $A\,O\,P$. In other words, if $O\,A$ represents the axis of an armature coil in phase A, $O\,P$ is the axis of the field coil, revolving clockwise.[6] The vector $C\,G = f\,\sigma\, i_f$ is drawn parallel to $O\,P$ and consequently at an angle α to $O\,A$. The vector $O\,K = U$ makes an angle ξ with $O\,A$. Finally, the vector $K\,A' = s - S$ is drawn parallel to $O\,A$. $K\,A'$ is shown as a chord of a circle drawn on $K\,G$ as a diameter. This is to indicate that $G\,A'$ is perpendicular to $O\,A$. The direction $G\,K$ is extended to L, and a circle is drawn on $K\,L$ as a diameter. The length of this diameter is such that the chord $K\,A'' = S$, where A lies on the same straight line with K and A'. Since $K\,A' = s - S$, we have that $A''\,A' = s$. Thus, by means of the two circles both s and S can be represented separately.

In the polygon $O\,A\,B\,C\,G\,A'\,K$, all the sides, except $A'\,G$, have been expressed through the physical quantities which enter into our problem. If, however, all the sides of the polygon be projected on $O\,A$, the side $A'G$ is eliminated, and we may write that

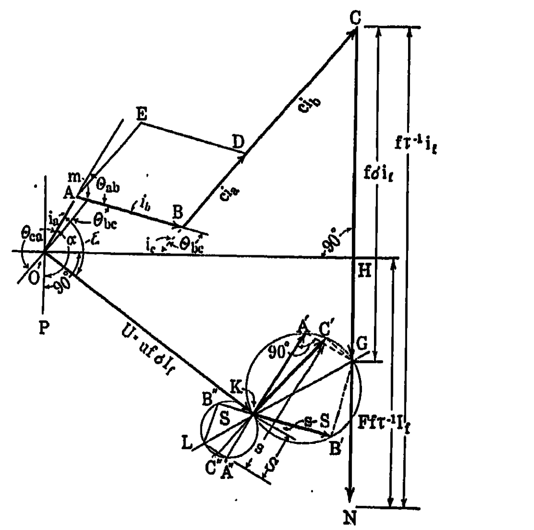

Fig. 4—Space Diagram of the Armature and Field m. m. f's.

$$\begin{aligned} &\text{Sum of projections on } O\,A \text{ of } (O\,A + A\,B + B\,C + C\,G) = \\ &\text{Sum of projections on } O\,A \text{ of } (O\,K + K\,A') \end{aligned} \tag{51}$$

A comparison of this equation with equation (**50**) shows the two to be identical, and we conclude that the above-mentioned polygon represents equation (**22**).

By analogy with equation (**50**), equation (**23**) may be written in the form:

$$\begin{aligned} i_b + c\, i_c \cos \theta_{bc} + i_a \cos \theta_{ab} + f\, \sigma\, i_f \cos \beta \\ = U \cos (\xi - \theta_{ab}) + (s - S) \end{aligned} \tag{52}$$

Thus, to represent equation (**23**), it is only necessary to replace $K\,A'$ by $K\,B'$ parallel to $A\,B$. When projecting the new polygon on the direction $A\,B$, the unknown vector $B'\,G$ is eliminated. Since both $K\,A'$ and $K\,B'$ are equal to $s - S$, the angles $A'\,K\,G$ and $B'\,K\,G$ must in reality be equal.

4. See, for example, Peirce's "Short Table of Integrals," p. 41, Equation (**300**), the last line.

5. For a similar representation in the single-phase case see Appendix XI.

6. The counter-clockwise rotation has been standardized for time vectors only, and there is no objection to using the clockwise rotation for space vectors.

To represent equation (**24**), we first eliminate $c\,C$ by using equation (**28**). The result is:

$$c^2 i_c + c\, i_a \cos \theta_{ca} + c\, i_b \cos \theta_{bc} + c\, f\, \sigma\, i_f \cos \gamma$$
$$= c\, U \cos (\xi + \theta_{ca}) + (s - S) - k\, i_c + [k\, I_c] \quad \textbf{(53)}$$

Here we have to distinguish between the case when $c > 0$ and when $c = 0$. When $c > 0$, both sides of equation (**53**) can be divided by c. The result corresponds to the polygon $O\,A\,B\,C\,G\,C'\,K$, where $K\,C' = (s - S)\, c^{-1} - k\, i_c\, c^{-1} + [k\, I_c/c]$ and is parallel to $B\,C$. When $c = 0$ equation (**53**) is reduced to:

$$s - S = k\, i_c - [k\, I_c] \quad \textbf{(53a)}$$

and permits to eliminate $s - S$ in a simple manner.

The polygon which corresponds to equation (**25**) is $O\,A\,B\,C\,H$, where:

$$C\,H = f\, \tau^{-1} (i_f - F\, I_f) \quad \textbf{(54)}$$

The sum of the projections of $O\,A + A\,B + B\,C + C\,H$ upon $O\,P$ is always equal to zero, for any value of angle α. In other words,

$$C N + \text{Sum of projections of } (O A + A\,B + B\,C) = H N \quad \textbf{(55)}$$

This is identical with equation (**25**).

As angle α varies, the directions $O\,A$, $A\,B$, $B\,C$ remain the same, only the values of the currents, and consequently the positions of the points A, B, C, vary. The vector $O\,K$ remains constant in magnitude and in direction. The vector $C\,N$ turns so as to remain parallel to $O\,P$, and its part $H\,N$ remains of constant length equal to $F\,f\,\tau^{-1}\,I_f$. The diameters $K\,G$ and $K\,L$ vary in magnitude and in direction.

When there is no external reactive coil (that is, when $k = O$), $S = s = O$ at all instants, and the voltage between the neutral points remains equal to zero (see proof below). Consequently, both circles in Fig. 4 shrink to zero and the diagram is considerably simplified; see Appendix VI.

Appendix IV

PROOF THAT $s = O$ WHEN $k = O$ AND $c > O$

Multiply equation (**50**) by $c \sin \theta_{bc}$, equation (**52**) by $c \sin \theta_{ca}$ and equation (**53**) by $\sin \theta_{ab}$, and add them together. In the result, the factor by which $c\, i_a$ is multiplied is

$$\sin \theta_{bc} + \sin \theta_{ca} \cos \theta_{ab} + \sin \theta_{ab} \cos \theta_{ca}$$
$$= \sin \theta_{bc} + \sin (\theta_{ca} + \theta_{ab}) \quad \textbf{(56)}$$

But, according to Figs. 1 and 2,

$$\theta_{ab} + \theta_{bc} + \theta_{ca} = 360 \text{ deg.} \quad \textbf{(57)}$$

so that $\sin (\theta_{ca} + \theta_{ab}) = - \sin (\theta_{bc})$ **(58)**

Therefore, expression (**56**) is identically equal to zero. Similarly, it can be proved that the resulting equation does not contain i_b and i_c.

One of the factors by which $c\, i_f$ is multiplied in the result is:

$$\text{Cos}\, \alpha \sin \theta_{bc} + \cos (\alpha - \theta_{ab}) \sin \theta_{ca} + \cos (\alpha + \theta_{ca}) \sin \theta_{ab}$$
$$= \cos \alpha (\sin \theta_{bc} + \sin \theta_{ca} \cos \theta_{ab} + \sin \theta_{ab} \cos \theta_{ca})$$
$$+ \sin \alpha (\sin \theta_{ab} \sin \theta_{ca} - \sin \theta_{ca} \sin \theta_{ab}) \quad \textbf{(59)}$$

In this expression, the term by which $\sin \alpha$ is multiplied is identically equal to zero, and the term by which $\cos \alpha$ is multiplied is the same as expression (**56**) which we have shown before to be equal to zero. Thus, the resultant equation does not contain i_f. For the same reason the quantity (U) is eliminated. Thus, the result is:

$$(s - S)\,(c \sin \theta_{bc} + c \sin \theta_{ca} + \sin \theta_{ab})$$
$$= k\, i_c \sin \theta_{ab} - [k\, I_c \sin \theta_{ab}] \quad \textbf{(60)}$$

If k is not equal to zero then, since i_c is a function of time, (s) is also a function of time. But when $k = O$, s must be equal to (S), where (S) is a constant; consequently, (s) must also be constant. According to equation (**18**), this means that when $k = O$, (e) is also equal to zero, no matter how unbalanaced the phases may be. But, by assumption, (s) contains no constant term; hence, when $k = O$,

$$s = S = O \quad \textbf{(61)}$$

The foregoing deduction, being based on equations (**56**) and (**59**), presupposes that the coefficients of mutual inductance are harmonic functions of space angles; in other words, that equations (**10a**) to (**11c**) hold true. If the winding is such that these relations are not satisfied, or satisfied only approximately, (s) may depart from zero and be a function of time. When equation (**61**) is satisfied, only two out of the three equations, (**22**), (**23**) and (**24**), are independent of each other. The third one can be obtained by properly combining the other two. The same is true of equations (**50**), (**52**), and (**53**). This may be seen directly from Fig. 4. When $S = s = O$, point K coincides with G (Fig. 6) and the closed polygon $O\,A\,B\,C\,G\,K$ is the same for each of the three aforementioned equations. But the condition that a polygon is closed is expressed by stating that the sum of its projections on any two axes is equal to zero. It is superfluous to equate to zero the sum of its projections on any third axis, because the equations so obtained can be written by properly combining the other two equations. This follows from the fact that if the projections of a vector on two given directions are known, its projection on any third direction is also known.

Thus, in this case we have only four equations instead of five, but we also have only four unknown functions; namely, the armature currents and the field current. This case is considered in detail in Appendix VI.

The foregoing deduction, and the condition (**61**), do not necessarily hold true when one of the angles θ is equal to zero. Let, for example θ_{ab} be equal to zero, so that $\theta_{bc} = 360$ deg. $- \theta_{ca}$. Then equations (**50**) and (**52**) become identical and can no more be considered as independent equations. Moreover, both sides of equation (**53**) cannot be multiplied by $\sin \theta_{ab}$, because $\sin \theta_{ab} = O$. From the physical point of view, when any two of the windings,—say A and B,—coincide in space

they form a single phase, with the winding C as the return phase. We thus have a single-phase circuit, the sum of the currents i_a and i_b being considered as one current. The voltage between the neutral and the terminals (Fig. 1) is then not equal to zero, unless $c = O$, that is, unless the phase C is simply a jumper of negligible impedance. This case is further considered in Appendices V and X.

Appendix V

Single-Phase Short-Circuit Through an Inductance

Let, in Fig. 1 and 2, the angle θ_{ab} be equal to zero, so that the windings of the phases (A) and (B) coincide. Let the angle θ_{bc} now be simply denoted by θ; then, $\theta_{ca} = 360$ deg. $- \theta$. The currents (i_a) and (i_b) do not exist singly, but their sum is equal to $- i_c$. Equation (**50**) becomes,

$$- i_c (1 - c \cos \theta) + f \sigma i_f \cos \alpha = U \cos \xi + (s - S) \qquad (62)$$

Equation (**52**) becomes identical with equation (**62**) and cannot be used. Equation (**53**) becomes,

$$i_c (c^2 - c \cos \theta) + c f \sigma i_f \cos (\alpha - \theta) = c\, U \cos (\xi - \theta) + (s - S) - k\, i_c + [k\, I_c] \qquad (63)$$

Equation (**25**) becomes,

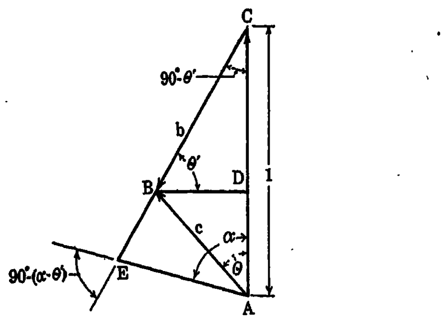

Fig. 5—Definition of the Auxiliary Quantities B and θ^1

$$f \tau^{-1} i_f + i_c [c \cos (\alpha - \theta) - \cos \alpha] = F f \tau^{-1} I_f \qquad (64)$$

To eliminate $(s - S)$, we subtract equation (**62**) from equation (**63**). The result is,

$$i_c [c^2 - 2\, c \cos \theta + 1 + k] + f \sigma i_f [c \cos (\alpha - \theta) - \cos \alpha] = U [c \cos (\xi - \theta) - \cos \xi] + [k\, I_c] \qquad (65)$$

In this equation, the expressions within all the brackets can be simplified by introducing auxiliary quantities (b) and (θ'), shown in Fig. 5. ($A\,C$) is a unity vector; ($A\,B$) is equal to (c) and is drawn at an angle (θ) to ($A\,C$). Then, (b) is the closing side of the triangle and (θ') is the angle between (b) and the perpendicular $B\,D$ to $A\,C$. Let the direction $A\,E$ be drawn at an angle α to $A\,C$. Then, from the geometry of the figure, the angle at E is equal to 90 deg. $+ \theta' - \alpha$, and we have,

$$1 - c \cos \theta = b \sin \theta' \qquad (66)$$

$$c \sin \theta = b \cos \theta' \qquad (67)$$

$$b^2 = c^2 + 1 - 2\, c \cos \theta \qquad (68)$$

Multiply equation (**66**) by ($\cos \alpha$) and equation (**67**) by ($\sin \alpha$); subtracting the first result from the second, gives,

$$c \cos (\alpha - \theta) - \cos \alpha = b \sin (\alpha - \theta') \qquad (69)$$

and by analogy,

$$c \cos (\xi - \theta) - \cos \xi = b \sin (\xi - \theta') \qquad (70)$$

Therefore, equation (**65**) becomes,

$$i_c (b^2 + k) + f \sigma b\, i_f \sin (\alpha - \theta') = U\, b \sin (\xi - \theta') + [k\, I_c] \qquad (71)$$

while equation (**64**) is reduced to,

$$b\, i_c \sin (\alpha - \theta') + f \tau^{-1} i_f = F f \tau^{-1} I_f \qquad (72)$$

Solving equations (**71**) and (**72**) for i_c and i_f, we get,

$$i_{f1} = \frac{F\, I_f [1 + (k/b^2)] - [k\, I_c]\, \tau \sin(\alpha = \theta')/(b f) - (U/f)\, \tau \sin (\xi - \theta') \sin (\alpha - \theta')}{[1 + (k/b^2)] - K^2 \sin^2 (\alpha - \theta')} \qquad (73)$$

$$i_{c1} = \frac{- (F/b)\, I_f f \sigma \sin (\alpha - \theta') + (U/b) \sin (\xi - \theta') + [k\, I_c]/b^2}{[1 + (k/b^2)] - K^2 \sin^2 (\alpha - \theta')} \qquad (74)$$

where K^2 is defined by equations (**12**) and (**13**), and the subscript (1) stands for "single-phase." For the determination of the constants, U, ξ, and F, see Appendix II. In particular, for a permanent short-circuit, $U = O$, $I_c = O$, and comparing equation (**73**) with equations (**47 a**) and (**48**), we find that,

$$m = n = 1 + (k/b^2); \quad q = 0.5\, K^2$$

Hence, equation (**49a**) will give,

$$F_{p1} = [1 + (k/b^2)]^{-1} \cdot \{ [1 + (k/b^2) - 0.5\, K^2]^2 - (0.5\, K^2)^2 \}^{0.5} \qquad (75)$$

For a graphical interpretation of equations (**71**) and (**72**), see Appendix XI.

Appendix VI

Three-Phase Short-Circuit Without External Inductance

It is shown in Appendix IV that $s = O$ when there is no external inductance in the phase C ($k = O$). In this case, Fig. 4 is simplified to Fig. 6, because the diameter $K\,G$ of the circle shrinks to zero.[7] The armature currents are shown in Fig. 6 in a somewhat different order; namely, the vectors $O\,A$ and $B\,D$ ($= A\,E$), both of which are proportional to (i_a), are placed in succession. Similarly, the vectors $A\,B$ ($= E\,D$) and $D\,C$, both proportional to (i_b), are drawn together. While the resultant vector between O and C is not changed thereby, it becomes possible to combine the vectors (i_a) and ($c\, i_a$) into one vector, $O\,E = l\, i_a$, where (l) is a numerical factor; the direction of $O\,E$ is characterized by the angle λ which it forms with $O\,A$. The sum of the vectors $E\,D$ and $D\,C$ is represented by the vector $E\,C = h\, i_b$, where (h) is a constant factor; $E\,C$ is

7. Fig. 6 does not hold true when $\theta_{ab} = 0$, even though (s) may be equal to zero. We then have a single-phase short-circuit, considered graphically in Appendix XI.

characterized by the position angle (η) which it forms with $O\,A$. In this manner, the number of armature current vectors between O and C is reduced from four to two, shown by the heavy lines.

Equations (50), (52), and (53), together with the conditions $s = S = k = O$, represent the sums of the projections of the sides of the closed polygon $O\,A\,B\,C\,G\,O$ (Fig. 6) on three different directions. But, the sum of the projections of this polygon on any axis is equal to zero. Hence, these three equations can be combined into one vectorial equation of the form:

$$i_a\,\epsilon^{j\delta} + i_b\,\epsilon^{j(\theta_{ab}+\delta)} + c\,i_c\,\epsilon^{j(-\theta_{ca}+\delta)} + f\,\sigma\,i_f\,\epsilon^{j(\alpha+\delta)} = U\,\epsilon^{j(\xi+\delta)} \tag{76}$$

In this expression, (δ) is an arbitrary angle; by giving it a suitable value and equating the real and the imaginary parts of equation (76) separately to zero, algebraic equations of projections on any two desired perpendicular directions can be obtained.

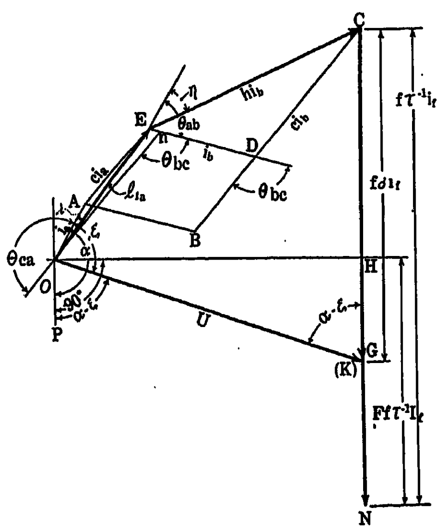

FIG. 6—A PARTICULAR CASE OF FIG. 4, WHEN $E = 0$

Substituting for (i_c) its value from equation (5), equation (76) becomes,

$$i_a\left[\epsilon^{j\delta} - c\,\epsilon^{j(-\theta_{ca}+\delta)}\right] + i_b\left[\epsilon^{j(\theta_{ab}+\delta)} - c\,\epsilon^{j(-\theta_{ca}+\delta)}\right] = U\,\epsilon^{j(\xi+\delta)} - f\,\sigma\,i_f\,\epsilon^{j(\alpha+\delta)} \tag{77}$$

The two vectors in the first brackets represent $O\,A$ and $A\,E$, respectively; the two vectors in the second brackets represent the vectors $E\,D$ and $D\,C$. Using their geometric sums, $O\,E$ and $E\,C$, equation (77) is simplified to:

$$l\,i_a\,\epsilon^{j(\lambda+\delta)} + h\,i_b\,\epsilon^{j(\eta+\delta)} = U\,\epsilon^{j(\xi+\delta)} - f\,\sigma\,i_f\,\epsilon^{j(\alpha+\delta)} \tag{78}$$

To solve this equation for i_a, we put,

$$\eta + \delta = 90 \text{ deg.} \tag{79}$$

Then, the real part of the equation becomes,

$$l\,i_a \cos(\lambda + 90 \text{ deg.} - \eta) = U \cos(\xi + 90 \text{ deg.} - \eta) - f\,\sigma\,i_f \cos(\alpha + 90 \text{ deg.} - \eta) \tag{80}$$

Solving for i_a, we get,

$$i_a = \frac{U \sin(\xi - \eta) - f\,\sigma\,i_f \sin(\alpha - \eta)}{l \sin(\lambda - \eta)} \tag{81}$$

By analogy, putting in equation (78) $\lambda + \delta = 90$ deg., we obtain,

$$i_b = \frac{U \sin(\xi - \lambda) - f\,\sigma\,i_f \sin(\alpha - \lambda)}{h \sin(\eta - \lambda)} \tag{82}$$

The auxiliary quantities (l) and (λ) are found from the equation,

$$l\,\epsilon^{j\lambda} = 1 - c\,\epsilon^{-j\theta_{ca}} \tag{83}$$

Equating separately the real and the imaginary parts, we get,

$$l \cos\lambda = 1 - c\cos\theta_{ca} \tag{84}$$

$$l \sin\lambda = c \sin\theta_{ca} \tag{85}$$

From these equations, (l) and (λ) can be readily computed. Similarly, for (h) and (η) we may write,

$$h\,\epsilon^{j\eta} = \epsilon^{j\theta_{ab}} - c\,\epsilon^{-j\theta_{ca}} \tag{86}$$

from which,

$$h \cos\eta = \cos\theta_{ab} - c\cos\theta_{ca} \tag{87}$$

$$h \sin\eta = \sin\theta_{ab} + c\sin\theta_{ca} \tag{88}$$

The field current (i_f) is determined from the condition,

$$H\,G = U\cos(\alpha - \xi) = H\,N - G\,N \tag{89}$$

Consequently,

$$U\cos(\alpha - \xi) = F\,f\,\tau^{-1}\,I_f - (f\,\tau^{-1}\,i_f - f\,\sigma\,i_f) \tag{90}$$

from which,

$$i_f = \frac{F\,I_f - \tau\,(U/f)\cos(\alpha - \xi)}{1 - K^2} \tag{91}$$

where K^2 is defined by equations (12) and (13). Substituting the value of (i_f) from equation (91) in equations (81) and (82), we get,

$$i_a = [(1 - 0.5\,K^2)\,U\sin(\xi - \eta) - \sigma f F I_f \sin(\alpha - \eta) + 0.5\,K^2\,U\sin(2\,\alpha - \xi - \eta)]/[l\,(1 - K^2)\sin(\lambda - \eta)] \tag{92}$$

$$i_b = [(1 - 0.5\,K^2)\,U\sin(\xi - \lambda) - \sigma f F I_f \sin(\alpha - \lambda) + 0.5\,K^2\,U\sin(2\,\alpha - \xi - \lambda)]/[h\,(1 - K^2)\sin(\eta - \lambda)] \tag{93}$$

In the derivation of these expressions, the following trigonometric transformation was used:

$$\cos(\alpha - \xi)\sin(\alpha - \eta) = 0.5\sin(2\,\alpha - \xi - \eta) + 0.5\sin(\xi - \eta) \tag{94}$$

From equations (92) and (93), using equation (5), we can readily find the value of $i_c = -(i_a + i_b)$.

Equations (91), (92), and (93), are not applicable when one of the space angles, say θ_{ab}, becomes equal to zero. This case is analyzed in Appendix X.

Appendix VII

SPECIAL CASES OF SINGLE-PHASE SHORT-CIRCUIT, EQUATIONS (73) AND (74)

When $k = O$, these equations are simplified as follows:

(a) *For the initial short-circuit,* with equations (40), (41), and (46) satisfied:

$$i_{f1i} = \frac{I_f\,[1 - K^2 \sin(\alpha_0 - \theta') \sin(\alpha - \theta')]}{1 - K^2 \sin^2(\alpha - \theta')} \qquad \text{(94a)}$$

$$i_{c1i} = \frac{(f\,\sigma\,I_f/b)[-\sin(\alpha - \theta') + \sin(\alpha_0 - \theta')]}{1 - K^2 \sin^2(\alpha - \theta')} \qquad \text{(95)}$$

where the subscript (1) stands for "single-phase" and the subscript (i) for "initial." The denominator can also be transformed into a function of 2 α; namely,

$$1 - K^2 \sin^2(\alpha - \theta') = (1 - 0.5\,K^2) + 0.5\,K^2 \cos 2\,(\alpha - \theta') \qquad \text{(96)}$$

If (i_{c1i}) and (i_{f1i}) are to reach their greatest possible values after the short circuit occurs, we must put,

$$\alpha_0 - \theta' = -\,90 \text{ deg.} \qquad \text{(97)}$$

Thus, equations (94) and (95) have their maxima at

$$\alpha - \theta' = 90 \text{ deg.} \qquad \text{(98)}$$

that is, half a cycle after the instant of short-circuit.

(1) *Single-phase machine,* or a polyphase machine with one phase-terminal grounded to the neutral. The winding C then electrically does not exist, so that $c = O$. From equations (66) and (67), $b = 1$ and $\theta' = 90$ deg. Consequently, $\alpha_0 = O$, and,

$$i_{f1i} = I_f\,\frac{1 - K^2 \cos\alpha}{1 - K^2 \cos^2\alpha} \qquad \text{(99)}$$

$$i_{c1i} = -\,f\,\sigma\,I_f\,\frac{1 - \cos\alpha}{1 - K^2 \cos^2\alpha} \qquad \text{(100)}$$

These expressions agree with Franklin's equations (10) and (11), remembering that his (i_a) is the same as our ($-\,i_{c1i}$).

(2) *Two-phase machine,* terminal-to-terminal short-circuit. In this case, $c = 1$ and $\theta = 90$ deg. Equations (66) and (67) are satisfied when $\theta' = 45$ deg. and $b = \sqrt{2}$. Consequently, $\alpha_0 = -\,45$ deg. and

$$i_{f1i} = I_f\,\frac{1 + K^2 \sin(\alpha - 45 \text{ deg.})}{1 - K^2 \sin^2(\alpha - 45 \text{ deg.})} \qquad \text{(101)}$$

$$-i_{c1i} = (f\,\sigma\,I_f/\sqrt{2})\,\frac{1 + \sin(\alpha - 45 \text{ deg.})}{1 - K^2 \sin^2(\alpha - 45 \text{ deg.})} \qquad \text{(102)}$$

These expressions are identical with Franklin's equations (23) and (24), keeping in mind that his current (i_a) in the machine itself is equal and opposite to our current (i_{c1i}) in the external connection.

(3) *Three-phase machine,* terminal-to-terminal short-circuit. In this case, $c = 1$ and $\theta = 120$ deg. Equations (66) and (67) give $\theta' = 60$ deg. and $b = \sqrt{3}$. Hence, $\alpha_0 = -\,30$ deg. and,

$$i_{f1i} = I_f\,\frac{1 + K^2 \sin(\alpha - 60 \text{ deg.})}{1 - K^2 \sin^2(\alpha - 60 \text{ deg.})} \qquad \text{(103)}$$

$$-i_{c1i} = (f\,\sigma\,I_f/\sqrt{3})\,\frac{1 + \sin(\alpha - 60 \text{ deg.})}{1 - K^2 \sin^2(\alpha - 60 \text{ deg.})} \qquad \text{(104)}$$

These expressions agree with Franklin's equations (40) and (41).

(b) *For the permanent short-circuit,* $U = O$. From equation (75), putting $k = 0$, we find

$$F_{p1} = \sqrt{1 - K^2} \qquad \text{(150)}$$

so that equations (73) and (74) become

$$i_{f1p} = I_f\,\frac{\sqrt{1 - K^2}}{1 - K^2 \sin^2(\alpha - \theta')} \qquad \text{(106)}$$

$$-i_{c1p} = [(I_f f \sigma \sqrt{1 - K^2})/b]\,\frac{\sin(\alpha - \theta')}{1 - K^2 \sin^2(\alpha - \theta')} \qquad \text{(107)}$$

In these expressions, the values of (b) and (θ') are the same as those computed above for the initial short-circuit. With these values, equations (106) and (107) will be found to check with Franklin's equations (80) and (81), except for the sign of the armature current, as explained above.

For a graphical representation of the field and armature currents in the single-phase case, see Appendix XI.

Appendix VIII

Some Particular Cases of Polyphase Short-Circuit, Equations (91) to (94)

(a) *Initial Short-Circuit.* Assuming equations (40), (41), and (46) to be satisfied, we get

$$i_{f23i} = I_f\,\frac{1 - K^2 \operatorname{Cos}(\alpha - \alpha_0)}{1 - K^2} \qquad \text{(108)}$$

$$i_{a23i} = f\,\sigma\,I_f\,\frac{(1 - 0.5\,K^2)\operatorname{Sin}(\eta - \alpha_0) + \operatorname{Sin}(\alpha - \eta) - 0.5\,K^2 \operatorname{Sin}(2\,\alpha - \alpha_0 - \eta)}{l\,(1 - K^2)\operatorname{Sin}(\eta - \lambda)} \qquad \text{(109)}$$

$$i_{b23i} = f\,\sigma\,I_f\,\frac{(1 - 0.5\,K^2)\operatorname{Sin}(\alpha_0 - \lambda) - \operatorname{Sin}(\alpha - \lambda) + 0.5\,K^2 \operatorname{Sin}(2\,\alpha - \alpha_0 - \lambda)}{h\,(1 - K^2)\operatorname{Sin}(\eta - \lambda)} \qquad \text{(110)}$$

where the subscript $23i$ stands for "two-phase- or three-phase, initial short-circuit"[8].

The field current reaches a maximum value

$$\max.\, i_{f23i} = I_f\,(1 + K^2)/(1 - K^2) \qquad \text{(111)}$$

at the field position corresponding to

$$\alpha - \alpha_0 = 180 \text{ deg.} \qquad \text{(112)}$$

that is, half a cycle after the instant of short-circuit.

Let it be required to find the values of α_0 and α such that the current i_a reaches its absolute maximum. The numerator of expression (109) becomes a positive maximum when the following three conditions are fulfilled:

$$\sin(\eta - \alpha_0) = 1 \qquad \text{(113)}$$

$$\sin(\alpha - \eta) = 1 \qquad \text{(114)}$$

$$\sin(2\,\alpha - \alpha_0 - \eta) = -\,1 \qquad \text{(115)}$$

These equations are satisfied when

$$\alpha_0 = \eta - 90 \text{ deg.} \qquad \text{(116)}$$

$$\alpha = \eta + 90 \text{ deg.} \qquad \text{(117)}$$

The same result can be obtained by equating to zero the partial derivatives of expression (109) with respect to α and α_0. We see that the armature current in phase

8. The equations deduced in this Appendix do not hold true when $\theta_{ab} = O$. We then have a single-phase short-circuit treated in Appendices V, VII, and XI.

A reaches its maximum half a cycle after the instant of the short-circuit. Substituting the value of α_0 from equation (116) in equations (108), (109), and (110), gives

$$i_{f23i} = I_f \frac{1 - K^2 \sin(\eta - \alpha)}{1 - K^2} \quad (118)$$

$$i_{a23i} = f\,\sigma\,I_f \frac{(1 - 0.5\,K^2) + \sin(\alpha - \eta) - 0.5\,K^2 \cos 2\,(\alpha - \eta)}{l\,(1 - K^2)\sin(\eta - \lambda)} \quad (119)$$

$$i_{b23i} = f\,\sigma\,I_f \frac{-\,(1 - 0.5\,K^2)\cos(\eta - \lambda) - \sin(\alpha - \lambda) + 0.5\,K^2 \cos(2\alpha - \eta - \lambda)}{h\,(1 - K^2)\sin(\eta - \lambda)} \quad (120)$$

(1) *Two-phase machine, two-phase short-circuit.* In this case $c = O$ and $\theta_{ab} = 90$ deg. From equations (84) and (85), $l = 1$ and $\lambda = O$. From equations (87) and (88), $h = 1$, $\eta = 90$ deg. Consequently,

$$\alpha_0 = O \quad (121)$$

$$i_{f2i} = I_f \frac{1 - K^2 \cos\alpha}{1 - K^2} \quad (122)$$

$$i_{a2i} = f\,\sigma\,I_f \frac{(1 - 0.5\,K^2) - \cos\alpha + 0.5\,K^2 \cos 2\,\alpha}{1 - K^2} \quad (123)$$

$$i_{b2i} = f\,\sigma\,I_f \frac{-\sin\alpha + 0.5\,K^2 \sin 2\,\alpha}{1 - K^2} \quad (124)$$

These expressions are identical with Franklin's equations (48) to (50).

(2) *Three-phase machine, two-phase short-circuit.* Here again $c = O$, but $\theta_{ab} = 120$ deg. Consequently $l = 1$; $\lambda = O$; $h = 1$; $\eta = 120$ deg. Therefore,

$$\alpha_0 = 30 \text{ deg.} \quad (125)$$

$$i_{f2(3)i} = I_f \frac{1 - K^2 \cos(\alpha - 30 \text{ deg.})}{1 - K^2} \quad (126)$$

$$i_{a2(3)i} = f\,\sigma\,I_f\,(2/\sqrt{3}) \cdot \frac{(1 - 0.5K^2) - \cos(\alpha - 30\text{deg.}) + 0.5K^2\cos 2\,(\alpha - 30\text{deg.})}{1 - K^2} \quad (127)$$

$$i_{b2(3)i} = f\,\sigma\,I_f\,(2/\sqrt{3}) \cdot \frac{0.5\,(1 - 0.5\,K^2) - \sin\alpha - 0.5\,K^2 \cos 2(\alpha + 30 \text{ deg.})}{1 - K^2} \quad (128)$$

These expressions agree with Franklin's equations (57) to (59).

(3) *Three-phase machine, three-phase short-circuit.* In this case $c = 1$; $\theta_{ab} = \theta_{ca} = 120$ deg. Hence, $l = \sqrt{3}$; $\lambda = 30$ deg.; $h = \sqrt{3}$; $\eta = 90$ deg. We therefore have:

$$\alpha_0 = O \quad (129)$$

$$i_{f3i} = I_f \frac{1 - K^2 \cos\alpha}{1 - K^2} \quad (130)$$

$$i_{a3i} = (2/3)\,f\,\sigma\,I_f \cdot \frac{(1 - 0.5\,K^2) - \cos\alpha + 0.5\,K^2 \cos 2\,\alpha}{1 - K^2} \quad (131)$$

$$i_{b3i} = -\,(2/3)\,f\,\sigma\,I_f \cdot \frac{0.5(1 - 0.5K^2) + \sin(\alpha - 30 \text{ deg.}) + 0.5\,K^2 \cos(2\alpha + 30 \text{ deg.})}{1 - K^2} \quad (132)$$

These equations check with Franklin's formulas (71) to (74).

(b) *Permanent Short-Circuit.* In this case, according to equation (43), $U = O$, so that equation (91) gives

$$i_f = F\,I_f/(1 - K^2) \quad (133)$$

But the armature currents cannot induce a constant e. m. f. in the field circuit, so as to reduce the field current I_f in a constant ratio $F/(1 - K^2)$. Hence,

$$i_f = I_f \quad (134)$$

and

$$F = 1 - K^2 \quad (135)$$

Equations (92) and (93) become

$$i_{a23p} = \frac{\sigma\,f\,I_f \sin(\alpha - \eta)}{l \sin(\eta - \lambda)} \quad (136)$$

$$i_{b23p} = -\,\frac{\sigma\,f\,I_f \sin(\alpha - \lambda)}{h \sin(\eta - \lambda)} \quad (137)$$

In these expressions, the same values of l, λ, h, η, apply as those deduced above for the initial short-circuit. We, therefore, get the following specific results, all of which check with the corresponding expressions deduced by Mr. Franklin.

(1) *Two-phase machine*:

$$i_{a2p} = -\,\sigma\,f\,I_f \cos\alpha \quad (138)$$

$$i_{b2p} = -\,\sigma\,f\,I_f \sin\alpha \quad (139)$$

(2) *Three-phase machine, two-phase short-circuit*:

$$i_{a2(3)p} = -\,(2/\sqrt{3})\,\sigma\,f\,I_f \cos(\alpha - 30 \text{ deg.}) \quad (140)$$

$$i_{b2(3)p} = -\,(2/\sqrt{3})\,\sigma\,f\,I_f \sin\alpha \quad (141)$$

(3) *Three-phase machine, three-phase short-circuit*:

$$i_{a3p} = -\,(2/3)\,\sigma\,f\,I_f \cos\alpha \quad (142)$$

$$i_{b3p} = -\,(2/3)\,\sigma\,f\,I_f \sin(\alpha - 30 \text{ deg.}) \quad (143)$$

Appendix IX

INDIVIDUAL VS. COMMON SHORT-CIRCUIT TO THE NEUTRAL

1. *Two-Phase Short-Circuit.* For a two-phase machine, the general diagram of connections shown in Fig. 1 is reduced to that shown in Fig. 7A, by putting $L = L_c = O$. Therefore, the formulas for two-phase short-circuit currents, deduced in Appendix VIII, apply to Fig. 7A. However, the currents in the case shown in Fig. 7B are identical with those in Fig. 7A, so that the same formulas hold true, no matter whether a common jumper (j) is used for both phases, or individual jumpers, (j_1) and (j_2), for each phase. The connection (j_3) does not affect the values of the currents.

The fact that the diagrams shown in Fig. 7A and

7B are electrically equivalent follows directly from a consideration of the magnetic linkages in both cases. The currents in Fig. 7B at each instant are of such values as to keep the flux in each circuit constant. But in Fig. 7A the points (p) and (q) are at the same potential, because the jumper (j), by assumption, is of zero resistance. Thus, the same two individual circuits exist in Fig. 7A as in 7B, and the first Kirchoff law does not enter in the problem. From a formal mathematical point of view, the same linkage equations can be written for both diagrams; hence, the solutions for the currents are also identical.

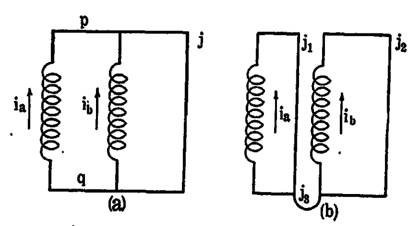

FIG. 7—COMMON AND INDIVIDUAL TWO-PHASE SHORT-CIRCUIT

2. *Three-Phase Short-Circuit.* When in Fig. 1 the external inductance (L) is omitted, the diagram of connections becomes identical with Fig. 8A. Therefore, the formulas deduced in Appendices VI and VIII apply first of all to the short-circuit of the three terminals, as shown in Fig. 8A. However, it may be shown that the same results hold true for the case shown in Fig. 8B, with a jumper connection (j) between the short-circuited terminals and the neutral, and also to the case shown in Fig. 8C where each phase is short-circuited individually.

It is proved in Appendix IV that the difference of potential between the points (p) and (q) in Fig. 8A is zero at all instants. Hence, the addition of the jumper (j), shown in Fig. 8B, does not alter the potential distribution in the phases. Moreover, the sum of the three armature currents in Fig. 8A is equal to zero at all instants, and since the phase currents are the same in Fig. 8B, the return current i_c in the common return conductor (j) theoretically is equal to zero. Of course, in practise, some return current may be expected, partly because the resistances have been neglected, and partly due to higher harmonics, multiple of three. Moreover, (s) has been shown to be equal to zero only when the mutual inductances obey the harmonic law, in accordance with equations (**10a**) to (**11c**); otherwise (s) may be a function of time.

For the case of the individually short-circuited armature phases, Fig. 8C, equations (**1**) to (**4**) apply, with $e = O$. Equation (**5**) does not necessarily hold true. By first transforming equations (**1**), (**2**), and (**3**) into the form (**50**), they can be finally combined into the vectorial equation (**76**). Equation (**4**) can be reduced to the form (**90**) and gives a perfectly definite value of the field current. Equation (**76**) is equivalent to two algebraic equations of projections, for the three unknown quantities i_a, i_b, i_c. Thus, the problem seems to be indefinite and to admit of an infinite number of sets of values of armature currents which satisfy all the equations which apply in this case.

This indefiniteness is due to two simultaneous assumptions made in this investigation; namely, (a) that the resistances of the windings are negligible and (b) that the coefficients of mutual induction are cosine functions of the space angles. If either one of these assumptions be dropped, the problem becomes definite. We shall consider the two cases separately.

(a) *Armature windings possess small resistances.* By assuming the resistances to be small, we can neglect their influence in the foregoing linkage equations, but at the same time add the condition that the heat generated is a minimum[9].

We then have

$$r_a i_a^2 + r_b i_b^2 + r_c i_c^2 = \text{min.} \tag{144}$$

This is a fourth condition which, with the foregoing three, makes it possible to obtain definite values of armature currents. For example, if the three resistances are equal, equation (**144**) may be written in the form

$$(i_{a0} + \Delta i)^2 + (i_{b0} + \Delta i)^2 + (i_{c0} + \Delta i)^2 = \text{min.} \tag{145}$$

where i_{a0}, i_{b0}, i_{c0} are some currents which satisfy Kirchoff's first law, so that

$$i_{a0} + i_{b0} + i_{c0} = O \tag{146}$$

The quantity Δi is the unknown difference between the real currents and the fictitious currents which satisfy equation (**146**). Equation (**145**) may be written in the form

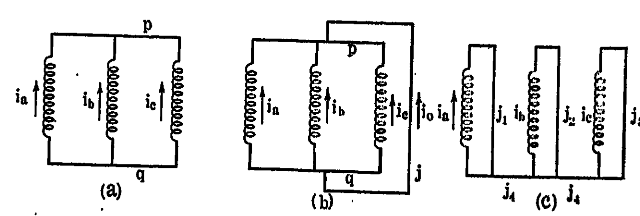

FIG. 8—THREE KINDS OF THREE-PHASE SHORT-CIRCUIT

$$\begin{aligned}(i_{a0}^2 + i_{b0}^2 + i_{c0}^2) + 2\,\Delta i\,(i_{a0} + i_{b0} + i_{c0})\\ + 3\,(\Delta i)^2 = \text{min.}\end{aligned} \tag{147}$$

The second term is equal to zero in view of equation (**146**), and the remainder becomes a minimum when $\Delta i = O$. Thus, the three currents which satisfy Kirchoff's first law produce the least amount of heat loss and are those which would actually flow under the limitations specified above.

In the general case, two of the armature currents, say i_a and i_b, can be expressed through the third, i_c, and

9. J. H. Jeans, The Mathematical Theory of Electricity and Magnetism, Fourth Edition, p. 321. Because of the small magnitude of the resistances and a comparatively low frequency, this law may be assumed to hold true for the variable currents in the case under consideration.

their values substituted in equation (144). Equating to zero the first derivative with respect to i_c, the latter can be determined, and consequently the other two currents evaluated.

(b) *The coefficients of mutual inductance between the armature windings depart from the cosine law.* In this case the equations (1), (2) and (3) still hold true (with $e = O$), but after integration give three independent equations, which cannot be reduced to two equations of projections, as heretofore. These three equations, together with the integrated equation (4), give four conditions which the four currents must satisfy. The problem is then perfectly definite.

As an example, consider three identical windings displaced by 120 electrical degrees, as in an ordinary three-phase alternator. Equations (1), (2), and (3), after integration and evaluation of the constants of integration, become:

$$L\, i_a + M\, i_b + M\, i_c + i_f\, M_f\, \psi\,(\alpha) = I_f\, M_f\, \psi\,(\alpha_0) \qquad (148)$$

$$L\, i_b + M\, i_c + M\, i_a + i_f\, M_f\, \psi\,(\alpha - 120 \text{ deg.}) = I_f\, M_f\, \psi\,(\alpha_0 - 120 \text{ deg.}) \qquad (149)$$

$$L\, i_c + M\, i_a + M\, i_b + i_f\, M_f\, \psi\,(\alpha + 120 \text{ deg.}) = I_f\, M_f\, \psi\,(\alpha_0 + 120 \text{ deg.}) \qquad (150)$$

In these expressions, the subscripts a, b, c, of (L) and (M) have been omitted since these quantities are the same in all the three armature phases, because of the assumed symmetry of the windings. M_f is the maximum value of the mutual inductance between the field winding and one of the armature phases, when the two are in space opposition. The function $\psi\,(\alpha)$ expresses the law according to which the mutual inductance between the field and an armature phase winding varies with the angle α. Adding the three equations term by term, we get:

$$(L + 2\,M)\,(i_a + i_b + i_c) = I_f\, M_f\,[\psi\,(\alpha_0) + \psi\,(\alpha_0 - 120 \text{ deg.}) + \psi\,(\alpha_0 + 120 \text{ deg.})\,] - i_f\, M_f\,[\,\psi\,(\alpha) + \psi\,(\alpha + 120 \text{ deg.}) + \psi\,(\alpha - 120 \text{ deg.})\,] \qquad (151)$$

Everywhere in the text above it is assumed that $\psi\,(\alpha) = \cos \alpha$. With this assumption, both expressions in the brackets on the right-hand side of the foregoing equation are equal to zero. On the left hand side, we then must have

$$\text{either} \qquad L + 2\,M = O \qquad (152)$$

$$\text{or} \qquad i_a + i_b + i_c = O \qquad (153)$$

Assuming equations (7) and (10) to hold true, equation (152) is satisfied, and, therefore, equation (153) does not necessarily have to be satisfied. This leads to the indeterminate case mentioned above. On the other hand, if the mutual inductances of the armature phases do not follow exactly the cosine law, while the mutual inductances between the field and the armature do, equation (152) is not satisfied, and, therefore, equation (153) must hold true. This is equivalent to including equation (5) with equations (1) to (4) and thus making the problem definite.[10]

Summing up the foregoing discussion, it would seem that in the case represented by Fig. 8C, the currents may be expected to satisfy equation (5) at least approximately, and that in machines of the usual types the short-circuit according to Fig. 8C is electrically equivalent and causes nearly the same currents as a short-circuit according to Fig. 8A or 8B.

Delta-connected machine. The conditions as analyzed above for Fig. 8C also apply to a short-circuit of all the three terminals of a delta-connected machine. Here also we have three individual windings, each short-circuited upon itself, with practically no current interchange between the phases. We, therefore, reach the conclusion that the short-circuit currents in the windings themselves have the same values as have been deduced above for Y-connected windings. The instantaneous current in each lead to a terminal is equal to the algebraic difference of the currents in the adjacent phase windings.

Appendix X

THE CRITICAL CASE OF $\theta_{ab} = O$

Let in Fig. 1 and 2, the space angle (θ_{ab}) between the windings in the phases A and B be gradually reduced to zero so that in the limit the axes of both windings coincide in space. With $\theta_{ab} = O$, equations (84) and (85) become identical with equations (87) and (88), so that $\lambda = \eta$. Consequently, from equations (92) and (93),

$$i_a = -\, i_b = \infty \qquad (154)$$

Thus, it would seem that, as the two windings approach a coincidence, a circulating current of ever increasing magnitude takes place between them, while both the current in the third branch and the field current remain of finite magnitude, the latter current being expressed by equation (91). On the other hand, when the phases A and B coincide, they simply form a single-phase winding consisting of two parallel branches (Appendix V), and there is no physical reason for an infinitely large circulating current between the branches. Moreover, equation (91) does not hold true for a single-phase circuit; in Appendix V it is shown that in this case the denominator of the expression for (i_f) is also a function of (α).

We thus have a seeming paradox that the answer to the case $\theta_{ab} = O$ is different, according to whether we gradually reduce this angle to zero and use the formulas for the polyphase circuit, or assume a single-phase circuit beforehand and use the corresponding equations. The following considerations will help to understand the real conditions.

(1) All the equations in Appendix VI are deduced on the supposition that $s = O$ because $k = O$. However, it is shown at the end of Appendix IV that pre-

10. I am indebted to Mr. R. H. Park for having called my attention to the relationship expressed by equation (151) and to the fact that equation (152) must be satisfied when equation (153) does not hold true.

cisely in the case of $\theta_{ab} = O$, this conclusion does not hold true, unless (c) is also equal to zero. Thus, the case under consideration cannot be treated as a particular application of equations (**91**) to (**93**).

(2) Fig. 6 does not hold true when (s) is not equal to zero, and the more general Fig. 4 must be used. Consequently, equation (**90**) does not apply in this case and the field current is not expressed by equation (**91**).

(3) Equations (**81**) and (**82**) are based on equation (**76**). When $\theta_{ab} = O$, equation (**76**) can be solved only for the sum of i_a and i_b, and not for each current separately.

(4) Even when $k = c = s = O$, Fig. 6 does not hold true for $\theta_{ab} = O$. In this case equations (**50**) and (**52**) are identical and both are reduced to the form

$$(i_a + i_b) + f\,\sigma\,i_f \cos\alpha = (I_a + I_b) + f\,\sigma\,I_f \cos\alpha_0 = A \qquad (155)$$

Equation (**53**) becomes $O = O$. The condition (**155**) does not require that the quantities $(i_a + i_b), f\,\sigma\,i_f$, and A, form a closed triangle. The vectorial condition of a closed polygon exists only when equations of projections, such as (**155**), can be written for two directions. But, if Fig. 6 does not apply in this particular case, equation (**90**) cannot be written, and its solution, equation (**91**), does not hold true. A graphical representation of the single-phase short-circuit is given in Appendix XI.

(5) According to equations (**92**) and (**93**), the currents (i_a) and (i_b) increase indefinitely as the angle θ_{ab} approaches zero. On the other hand, equations (**1**) and (**2**) show that the voltage (e) induced in each phase tends to a finite limit. Throughout this investigation, the resistance of the windings has been neglected, because, with finite currents, the ohmic drop is small as compared with the various induced voltages which balance each other magnetically. However, in this particular case, with the currents seemingly tending to infinity, the ohmic drop, if properly considered, would limit the currents to moderate finite values. In other words, the very fact that the armature currents tend to infinite values when θ_{ab} approaches zero, shows the necessity of using, in this particular case, more accurate differential equations, that is, equations of the form (**1**) with the term $(i_a\,r_a)$ added on the left-hand side.

The following three conclusions are therefore reached;

(a) As θ_{ab} becomes smaller, the currents in the phases A and B become larger.

(b) For small values of θ_{ab}, current equations derived without considering the ohmic drop are not reliable and should not be used.

(c) When $\theta_{ab} = O$, the phases A and B become two windings in parallel, and the formulas derived in this investigation for the single-phase short-circuit should be used.

Appendix XI

Graphical Representation of Currents in a Single-Phase Short-Circuit

The relations treated in Appendix VII have a graphical interpretation shown in Fig. 9. We shall limit the analysis to the simplest case of $c = O$. Putting in equation (**22**) $\theta_{ab} = O$, $c = O$, and $s = O$, and using $-\,i_c$ in place of $(i_a + i_b)$, we get,

$$-\,i_c + f\,\sigma\,i_f \cos\alpha = A \qquad (156)$$

where the constant of integration (A) is equal to the value of the expression on the left-hand side of equation (**156**) at the instant of short-circuit; that is,

$$A = -\,I_c + f\,\sigma\,I_f \cos\alpha_0 \qquad (157)$$

Equation (**23**) becomes identical with equation (**156**), and equation (**24**) gives $O = O$. Equation (**25**) becomes,

$$f\,\tau^{-1}\,i_f - i_c \cos\alpha = F\,f\,\tau^{-1}\,I_f \qquad (158)$$

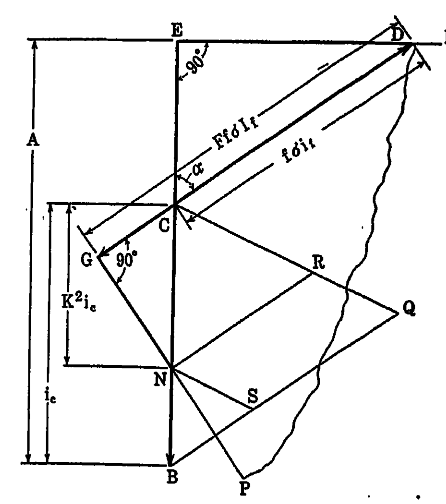

Fig. 9—Graphical Interpretation of Single-Phase Short-Circuit

Multiplying both sides of this expression by K^2, according to equation (**13**), we get,

$$f\,\sigma\,i_f - K^2\,i_c \cos\alpha = F\,f\,\sigma\,I_f \qquad (159)$$

Equations (**156**) and (**159**) are the only ones to be satisfied; they can be combined into a single diagram, as shown in Fig. 9. The current (i_c), at a certain instant corresponding to the angle (α), is represented by the vector $C\,B$.[11] The vector $C\,D = f\,\sigma\,i_f$, represents the field current referred to the armature circuit by means of the factor $f\,\sigma$. The length $B\,E$ represents the constant of integration A. Therefore, equation (**156**) simply becomes,

$$B\,C + C\,E = B\,E \qquad (160)$$

Point N is so chosen that it divides the length $B\,C$ in the ratio of K^2 to $1 - K^2$, and $C\,N = K^2\,i_c$. The length $G\,D$ is constant and equal to $F\,f\,\sigma\,I_f$. Hence, equation (**159**) becomes,

11. Space vectors and not time vectors are, of course, understood. The currents are non-sinusoidal in time and the length of a vector gives only an instantaneous value.

$$C\,D - C\,G = G\,D \tag{161}$$

Here $C\,D$ and $C\,G$ are to be understood as two vectors drawn from point C in the oppos te directions. Their difference, $G\,D$, is represented by a vector connecting their extremities G and D.

To make this diagram represent the conditions at any instant of time, assume $B\,E$ to be constant in direction and magnitude, and draw the direction $E\,E'$ perpendicular to $B\,E$. Out of a sheet of paper or celluloid, cut the figure $D\,G\,P$ in which $D\,G = F\,f\,\sigma\,I_f$, and $G\,P$ is perpendicular to $G\,D$. Imagine a pair of proportional dividers, $B\,C\,Q$, with their points laid along $B\,E$, so that the ratio of $B\,N$ to $N\,C$ always remains the same when the points C and N move along $B\,E$. For a chosen position of C, there is but one possible position of the figure $D\,G\,P$ for which D lies on $E\,E'$, $G\,D$ passes through C, and $G\,P$ passes through N. This determines the angle (α) and the instantaneous value of the field current (i_f) for the chosen value of (i_c). Conversely, by assuming a value of (α), the position of C may be found and hence the instantaneous values of the armature and field currents.

Thus, the periodic variations of i_c and i_f can be visualized, and the instantaneous values actually measured, by assuming $G\,D$ to be rotating in the plane, with the point D always remaining on $E\,E'$ and the sides $G\,D$ and $G\,P$ passing through the points C and N of the proportional dividers. The point C is variable, but the angle $E\,C\,D$ always gives the instant to which the values of the currents refer.

For an initial short-circuit, A is given by equation (**157**). For a sustained short-circuit, $A = O$ (see Appendix II), and the points B and E coincide. The value of F for the initial short-circuit, according to equation (**45**), is determined by the condition,

$$(F - 1)\,f\,\tau^{-1}\,I_f = -\,I_c \cos \alpha_0 \tag{162}$$

When $I_c = O$, $F = 1$. For a permanent short-circuit, F is determined by equation (**105**).

One can thus visualize the change from a transient to a permanent short-circuit by starting in Fig. 9 with the values of A and F corresponding to an initial short-circuit and assuming the lengths $B\,E$ and $G\,D$ to change gradually to their final values, while $G\,D$ is revolving uniformly and the point C is moving periodically up and down.

Notation

(All the quantities are understood to be *per pole*, where such an interpretation is possible.)

A, B, C	Constants of integration referring to the armature currents; equations (**22**) to (**24**).
F	Constant of integration referring to the field current.
I_a, I_b, I_c	The initial values of the armature currents, at the instant of short-circuit.
I_f	Initial or steady value of the field current at the instant of short-circuit.
K	Coefficient of magnetic coupling, equations (**12**) and (**13**).
L	External inductance in phase C, Fig. 1.
L_a, L_b, L_c	Self-inductances of the armature phases.
L_f	Self-inductance of the field current.
M_{ab}, M_{bc}, M_{ca}	Coefficients of mutual induction between the armature phases.
M_{fa}, M_{fb}, M_{fc}	Coefficients of mutual induction between the field winding and the three armature phases, respectively.
N	See equation (**19**)
N_a, N_b, N_c	Numbers of turns in the armature windings.
N_f	Number of turns in the field winding.
$\mathcal{P}$	Permeance of the useful (or common) magnetic path through the field and the armature.
S	Initial value of s at the instant of short-circuit.
U	Constant of integration, defined by equations (**26**) to (**31**).
b	Auxiliary constant, defined by equations (**35**) and (**36**), and by Fig. 3. It is also defined by equations (**66**) and (**67**), and by Fig. 5.
c	Ratio of turns in phases C and A; equation (**20**).
e	Instantaneous voltage between the terminals and the neutral in Fig. 1.
f	Ratio of turns in the field winding and the phase A; equation (**21**).
f_0	Frequency of the armature currents in a steady state.
h	Geometric sum of 1 and c; Fig. 6, triangle $E\,D\,C$; equations (**87**) and (**88**).
i	Subscript signifying "initial short-circuit."
i_a, i_b, i_c	Instantaneous currents in the armature phases, Fig. 1.
i_f	Instantaneous value of field current.
j	$\sqrt{-1}$
k	Numerical factor which characterizes the external inductance L, equation (**9**).
l	Geometric sum of 1 and c; Fig. 6, triangle $O\,A\,E$; equations (**84**) and (**85**).
m	A constant; equation (**47a**).
n	A constant; equation (**48**).
p	Subscript signifying "permanent short-circuit."
q	A constant; equation (**48**).
s	Auxiliary function of time defined by equation (**18**).
t	Time.
u	Coefficient of proportionality, defined by equation (**42**).

1, 2, 3	Used as subscripts, indicate the number of phases.
α, β, γ	Variable electrical angles which determine the instantaneous position of the field winding with respect to the three armature windings; Figs. 1 and 2, and equations (**14**) and (**15**).
$\alpha_0, \beta_0, \gamma_0$	The initial values of α, β, γ, at the instant of short-circuit.
δ	An arbitrary angle; equation (**76**).
ϵ	Base of natural logarithms.
η	Angle at E, Fig. 6; equations (**87**) and (**88**).
θ	The value of the angle θ_{bc} when $\theta_{ab} = 0$; Appendix V.
$\theta_{ab}, \theta_{bc}, \theta_{ca}$	Electrical angles between the armature phases, Figs. 1 and 2; the sum of these angles is equal to 360 deg.
θ'	Auxiliary angle defined by equations (**66**) and (**67**), and in Fig. 5.
θ_{ca}'	Auxiliary constant angle defined by equations (**35**) and (**36**), and in Fig. 3.
λ	Angle at O, Fig. 6; equations (**84**) and (**85**).
ξ	Constant electrical angle defined by equations (**26**) to (**31**).
σ	Magnetic leakage coefficient of the armature circuit, defined in a manner similar to equation (**6**).
τ	Magnetic leakage coefficient of the field circuit, defined by equation (**6**).
ϕ	A function of the time-angle α; equation (**47a**).
ψ	A function of α; equations (**148**) to (**150**).
$\omega = 2\pi f_0$	Electrical angular velocity, or the angular speed of a two-pole machine.

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For discussion of this paper see page 430.

Short-Circuit Currents of Synchronous Machines

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Synopsis.—The importance of short-circuit forces in large alternators has warranted an investigation of them. The first step in this investigation, which is discussed in this paper, is the calculation of the short-circuit currents that produce them. The method of solution used is that of the assumption of zero resistance and constant flux linkages which has proven so useful in the solution of many short-circuit problems. Formulas are calculated in the Appendix for both the initial and permanent short-circuit currents of all circuits involved in the short-circuit. The formulas cover all the common connections of single-phase, two-phase and three-phase alternators. A discussion of some of these formulas, together with a plot of them for assumed alternator constants, is also given.

INTRODUCTION

ONE of the difficult problems encountered in the design of large alternators is that of providing sufficient strength of the different parts to withstand the forces produced during short circuit. The author participated in an investigation of these short-circuit forces which was begun a couple of years ago and recently completed. The results of this investigation will be given in this and future papers before the Institute.

The short-circuit forces depend largely upon the abnormal short-circuit current that flows in the various windings and can be calculated if the character and magnitude of these currents are known. The first step, therefore, in the calculation of these forces is the calculation of the short-circuit currents that produce them.

Since Bucherot's excellent discussion of alternator short-circuit currents in 1911[1], many papers have been published treating of both the phenomena involved and the method of calculation. A method of solution which has proven useful in the investigation of many short-circuit problems is that of the assumption of zero resistance and constant flux linkages. This method was used in 1918 in a paper by Messrs. R. E. Doherty and O. E. Shirley[2], for an explanation of short circuit phenomena in synchronous machines, and again in 1921 in a paper by Messrs. R. E. Doherty and E. T. Williamson[3] for an investigation of the short circuit current of induction motors and generators. In a recent paper[4] Mr. Doherty again emphasized the importance of this method of interpreting short circuit problems and gave a number of applications. In a more recent paper[5] Mr. C. M. Laffoon applied this method to the calculation of several cases of short circuit of an alternator. It is the purpose of this paper to apply this constant-linkage method to the calculation of both the *initial* and *permanent* short-circuit currents of synchronous machines.

METHOD OF SOLUTION

The constant-linkage method is based on the assumption that the electrical resistance of the various circuits is zero. As a result of this assumption the following *constant-linkage theorem* has been proved[6]: "*In a circuit having zero resistance the algebraic sum of the flux linkages of the circuit must remain constant.*" The application of this theorem to the calculation of short-circuit currents is very similar to that of the application of Kirchoff's laws to the solution of networks. First, currents are assumed to flow in the different branches of the circuits. The flux linkages in any branch are those due to the current in that branch and those due to currents in other branches. The flux linkages of each circuit are then summed up and equated to some constant value of flux linkages which is known to exist in the circuit. By solving, simultaneously, these flux-linkage equations expressions for the currents flowing in the different branches of the circuits are obtained.

RESULTS

The various alternator connections and kinds of short-circuit for which formulas are derived are shown by the nine cases in Fig. 1. For each case formulas are derived for both the *initial* and *permanent* short-circuit conditions. A tabulation of the formulas for the various cases of Fig. 1 are given in Table I. These formulas are plotted in Figs. 3 to 14, for assumed alternator constants.

INITIAL SHORT CIRCUIT CURRENT

The initial short-circuit current waves, Figs. 3 to 8, show all peaks of the same height since, resistance being neglected, the transient decay of the current wave actually obtained in practise is not present. The instant of short-circuit in each case is so chosen as to give the maximum possible value of current in phase *a*. The maximum value of current occurs 180 electrical degrees after the instant of short-circuit. During this time the resistance of the circuits, which in the calculations has been neglected, reduces this peak value slightly from that calculated. The formulas thus

*D. C. Engineering Dept., General Electric Co., Jan. 13, 1925.

1. Bibliography 1.
2. Bibliography 3.
3. Bibliography 5.
4. Bibliography 8.
5. Bibliography 9.

Presented at the Spring Convention of the A. I. E. E., St. Louis, Mo., April 13-17, 1925 and at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

6. For a proof of this theorem refer to discussion of Mr. R. E. Doherty's paper, Bibliography 5.

give a peak value a little higher than actually obtained in practise. In Table II are given the peak values of the initial short-circuit current for the various cases in terms of the peak value of the current obtained in the three-phase alternator, case 9, in which all three phases are short-circuited simultaneously. It will be observed from Table II that the short-circuit current obtained for a single-phase short-circuit of one phase of a three-phase alternator from one terminal to ground is 150 per cent of the three-phase value. A single-phase short circuit between terminals gives only 86 per cent of the three-phase value, and a two-phase short-circuit between two terminals and ground, 173 per cent of the three-phase value.

PERMANENT SHORT-CIRCUIT CURRENT

In the calculation of the permanent short-circuit current formulas the flux linkage equations of the armature circuits were equated to zero. The reason for this is, that during the transient period of a short circuit, the

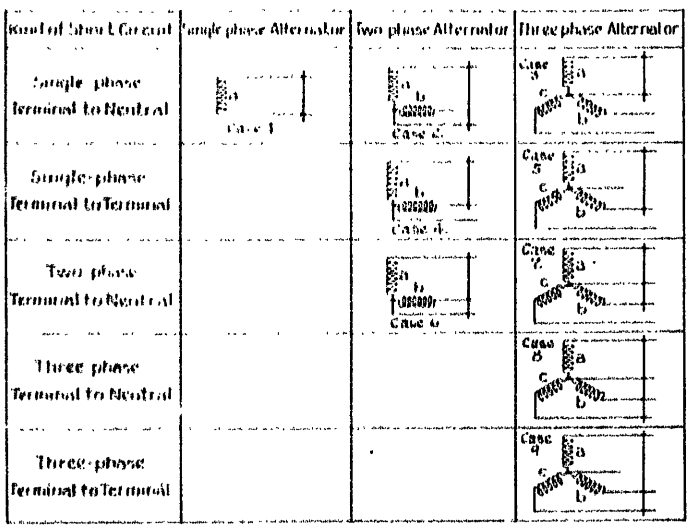

FIG. 1—CASES FOR WHICH INITIAL AND PERMANENT SHORT-CIRCUIT CURRENT FORMULAS ARE DERIVED

flux linkages which are caught by the armature circuits at the instant of short circuit are consumed by the resistance of the circuits. Therefore, when the permanent condition is reached the only flux linkages supplied by the field circuit are those which are being consumed by resistance. If zero resistance is assumed in the permanent condition, then no flux linkages are supplied to the armature circuits by the field circuit and the currents that flow in the armature circuits must be such as to at all times, keep out all field flux linkages. In other words, the flux linkages of each armature circuit must be zero.

The formulas for the permanent condition are plotted in Figs. 9 to 14. The single-phase cases show the characteristic double frequency pulsation in the field current. In Fig. 15 is given an oscillogram of the armature and field current of a three-phase alternator during a permanent single-phase short circuit between terminals in which the shape of the current waves is very similar to those calculated in Fig. 11.

The field current for the three-phase alternator case with two phases short-circuited between terminals and ground, (Case 7, Fig. 13), does not have any alternating component induced in it. This is due to the fact that the coefficient of inductive coupling between the short-circuited phases a and b was assumed in the derivation of the formulas, equal to the cosine of the angle between phases. Thus, for a three-phase alternator, this coupling coefficient was assumed equal to cos 120, or $-$ 0.5. This ideal value of coupling is seldom obtained in practise. The effect of a coupling co-efficient different from $-$ 0.5 on the field current is shown by the oscillogram of Fig. 16. The field current contains an alternating component of double frequency and of about $\pm$ 9 per cent of the d-c. component. This corresponds to a coupling coefficient between phases a and b of about $-$ 0.49. In the study of this phase of the subject formulas were calculated for all cases in which no definite value of coupling coefficient between phases was assumed. These formulas were very complicated compared to the formulas here given. The effect of this coupling coefficient is so slight that it was felt unwarranted to complicate the formulas by making them more general in this respect. For all practical applications the value $-$ 0.5, assumed in the calculation of the formulas, is sufficiently accurate.

TABLE I.
SHORT-CIRCUIT CURRENT FORMULAS

Case (See Fig. 1)	Initial Condition: Formulas	Fig.	Permanent Condition: Formulas	Fig.
1, 2, 3	(10) (11)	3	(80) (81)	9
4	(23) (24)	4	(86) (87)	10
5	(40) (41)	5	(92) (93)	11
6	(48) (49) (50)	6	(101) (102) (103)	12
7	(57) (58) (59)	7	(110) (111) (112)	13
8, 9	(71) (72) (73) (74)	8	(117) (118) (119) (120)	14

TABLE II.
Comparison of Maximum Peak Values of Initial Short-Circuit Currents. The current values are based on constant conductors per inch.

Case (See Fig. 1)	Maximum Peak Value of Current in Phase a as a Ratio of that obtained in Case 9
1	0.75
2	1.06
3	1.50
4	0.75
5	0.866
6	1.06
7	1.732
8	1.00
9	1.00

ASSUMPTIONS

The following assumptions were made in the calculation of the formulas:

a. Zero resistance in all circuits
b. Magnetic saturation neglected
c. Sine wave distribution of field flux
d. Constant coefficient of self-inductance of armature phases.
e. Variation of mutual inductance between armature phases as the cosine of the angle between them.

The calculation of alternator short-circuit currents becomes very involved if too many refinements are attempted. Therefore in order to obtain as simple a solution as possible, the above assumptions are necessary. While some of these assumptions may not appear reasonable a careful study of them and a comparison with results in practise reveals that for very many practical problems they can be tolerated.

The assumption of zero resistance was made necessary by the method of solution of the problem. The predominence of reactance over resistance during the short-circuit of an alternator makes it possible to neglect the resistance in this kind of a problem. Assumption *b* should involve greater error, but any attempt to take saturation into account makes the problem very complicated. The error due to saturation can be minimized by taking it into consideration in the calculation of the inductance coefficients. Assumption *c* is very close to present practise in the design of alternators. Assumption *d* involves the neglect of the salient pole feature of alternators since the coefficient of self inductance of the armature phases is not constant but will vary somewhat with the position of the field poles. An investigation of the affect of this variation upon the short-circuit currents showed that only a small double frequency current was introduced which did not appreciably effect the results obtained. The effect of *e* has already been discussed. Formulas were calculated for all cases in which the coefficient of coupling between armature phases was not assumed constant. The formulas are greatly complicated by this general assumption. The assumption of -0.5 for this coupling coefficient which was made in the derivation of the formulas of this paper greatly simplify the formulas and does not introduce appreciable error.

In concluding the author wishes to express his appreciation of the assistance of Messrs. R. E. Doherty and R. H. Park and Professor V. Karapetoff in the solution of this problem.

Appendix

Derivation of Short-Circuit Current Formulas Initial Condition

1. *Single-Phase Short Circuit.*

(a) Single-phase or polyphase alternator.

One general formula can be obtained for the single-phase short-circuit current of a single-phase, two-phase, or three-phase alternator when only one armature phase is involved in the short circuit. (Cases 1 to 3, Fig. 1). The instant of short circuit will be taken when the armature phase *a* encloses maximum field flux as this is the condition when maximum short circuit current is obtained. This condition occurs when the angle α in Fig. 2A equals zero[7].

There are only two circuits involved; the field circuit and the armature circuit *a*. The flux linkages of these two circuits at the instant of short circuit, *i. e.*, when α equals zero, are respectively[8].

$$\Omega_f = I_f L_f \tag{1}$$

$$\Omega_a = I_f M_0 \tag{2}$$

The flux linkages of these two circuits at any instant after short circuit are,

$$\Omega_f' = i_f L_f + i_a M_{fa} \tag{3}$$

$$\Omega_a = i_a L_a + i_f M_{af} \tag{4}$$

Applying the constant linkage theorem, the flux linkages at any instant after short circuit, (3) and (4),

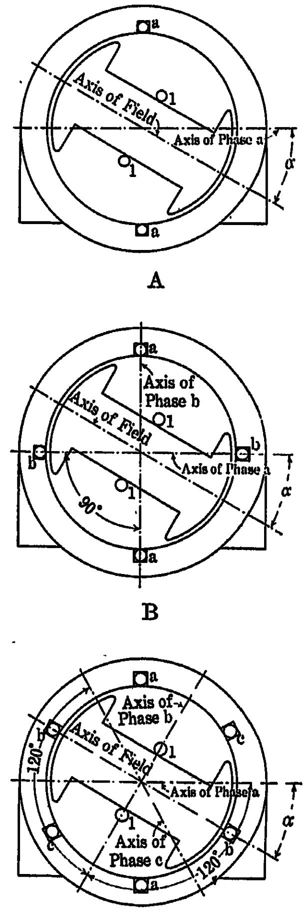

Fig. 2—Schematic Diagrams Showing the Relations of the Various Alternator Circuits

must equal the flux linkages before short circuit, (1) and (2) respectively,

$$i_f L_f + i_a M_{fa} = I_f L_f \tag{5}$$

$$i_a L_a + i_f M_{af} = I_f M_0 \tag{6}$$

7. For simplicity a two pole alternator is shown in which case the electrical angle is equal to the mechanical angle.

8. For definition of symbols see Notation.

Solving these two equations simultaneously for the currents i_f and i_a,

$$i_f = I_f \frac{L_f L_a - M_0 M_{fa}}{L_f L_a - M_{fa}^2} \tag{7}$$

$$i_a = I_f \frac{L_f M_0 - L_f M_{fa}}{L_f L_a - M_{fa}^2} \tag{8}$$

The inductance coefficients L_f and L_a are assumed constant. The coefficients M_{fa} varies with the rotation of the rotor, and may be approximated by a cosine function; thus[9]

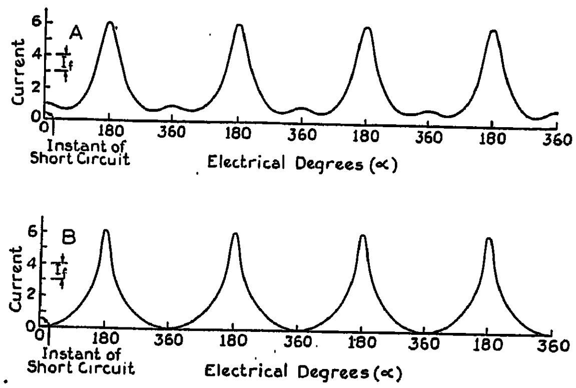

FIG. 3—INITIAL SHORT-CIRCUIT CURRENTS; CASES 1, 2 AND 3

Curve *A*—Field current. Eq.(10).
Curve *B*—Current in phase *a*. (Eq.11).

$$M_{fa} = M_{af} = M_0 \cos \alpha \tag{9}$$

Substituting this relation in (7) and (8) and dividing both numerator and denominator by $L_f L_a$,

$$i_f = I_f \frac{1 - K^2 \cos \alpha}{(1 - 0.5\, K^2) - 0.5\, K^2 \cos 2\,\alpha} \tag{10}$$

$$i_a = I_f \frac{M_0}{L_a} \frac{1 - \cos \alpha}{(1 - 0.5\, K^2) - 0.5\, K^2 \cos 2\,\alpha} \tag{11}$$

where, K = coefficient of magnetic coupling between the two circuits

$$= \frac{M_0}{\sqrt{L_f L_a}} \tag{12}$$

Equations (10) and (11) are plotted in Fig. 3 for

$$K = 0.85 \text{ and } \frac{M_0}{L_a} = 0.85.$$

(b) Two-phase alternators, single-phase terminal to terminal short-circuit, (Case 4, Fig. 1).

In this case there are only two circuits involved since armature phases a and b are connected in series, forming only one circuit $a\,b$. As seen in Fig. 2B the circuit $a\,b$ does not enclose maximum field flux when α equals zero, but when

$$M_{fab} = M_{fa} - M_{fb}$$

is a maximum that is, when

$$\frac{d\, M_{fab}}{d\, \alpha} = 0$$

From (9) and the relation

$$M_{fb} = M_0 \cos (\alpha - 90) = M_0 \sin \alpha \tag{13}$$

$$\frac{d\, M_{fab}}{d\, \alpha} = M_0 (- \sin \alpha - \cos \alpha) = 0$$

$$\text{or } \alpha = -\,45 \text{ deg.}$$

Therefore, for maximum field flux enclosure by the circuit $a\,b$ the short circuit must occur at $\alpha = -\,45$ deg. The flux linkages of the field and armature circuits at this value of α are,

$$\Omega_f = I_f L_f \tag{14}$$

$$\Omega_{ab} = \Omega_a - \Omega_b = I_f M_{af}' - I_f M_{bf}'$$

where, M_{af}' and M_{bf}' are the values of M_{af} and M_{bf} respectively at $\alpha = -\,45$ deg.

Hence from (9) and (13)

$$\Omega_{ab} = \sqrt{2}\, I_f M_0 \tag{15}$$

The linkages at any instant after short circuit are,

$$\Omega_f = i_f L_f + i_a M_{fa} + i_b M_{fb} \tag{16}$$

$$\Omega_{ab} = \Omega_a - \Omega_b = i_f (M_{af} - M_{bf}) + i_a (L_a - M_{ba}) + i_b (M_{ab} - L_b) \tag{17}$$

Due to reverse connection of a and b

$$i_{ab} = i_a = -\, i_b \tag{18}$$

Thus from (14) (15) (16) (17) and (18)

$$i_f L_f + i_{ab} (M_{fa} - M_{fb}) = I_f L_f \tag{19}$$

$$i_f (M_{af} - M_{bf}) + i_{ab} (L_a + L_b) = \sqrt{2}\, I_f M_0 \tag{20}$$

Since, the axes of the phases are at right angles,

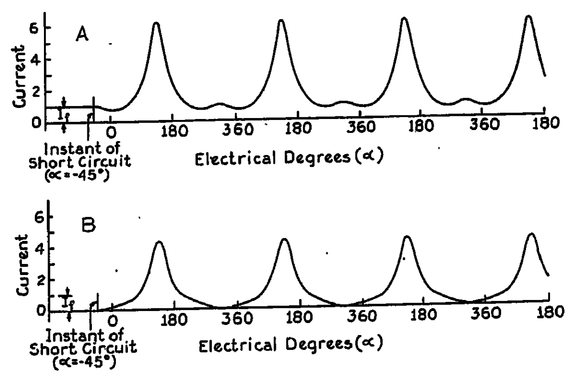

FIG. 4—INITIAL SHORT-CIRCUIT CURRENTS; CASE 4

Curve *A*—Field current. Eq.(23)
Curve *B*—Field current in circuit *a b*. Eq.(24)

$$M_{ab} = M_{ba} = 0 \tag{21}$$

Solving (19) and (20) simultaneously and substituting (9), (12), (13) and

$$L_a = L_b \tag{22}$$

$$i_f = I_f \frac{1 - K^2 \cos (\alpha + 45)}{(1 - 0.5\, K^2) - 0.5\, K^2 \cos 2\,(\alpha + 45)} \tag{23}$$

$$i_{ab} = \frac{1}{\sqrt{2}} I_f \frac{M_0}{L_a} \frac{1 - \cos (\alpha + 45)}{(1 - 0.5\, K^2) - 0.5\, K^2 \cos 2 (\alpha + 45)} \tag{24}$$

9. $M_{fa} = M_{af}$ when saturation is neglected.

These currents are plotted in Fig. 4 for $K = 0.85$ and

$$\frac{M_0}{L_a} = 0.85.$$

(c) Three-phase alternator, single-phase terminal to terminal short circuit, (Case 5, Fig. 1).

There are only two circuits to consider, the field circuit f and one armature circuit composed of phases a and b in series. The armature circuit encloses maximum field flux when M_{fab} is a maximum.

This occurs when

$$\frac{d\, M_{fab}}{d\, \alpha} = 0$$

Now since phases a and b are displaced 120 deg. from each other

$$M_{fb} = M_0 \cos (\alpha - 120) \qquad (25)$$

and

$$M_{fab} = M_{fa} - M_{fb} = M_0 [\cos \alpha - \cos (\alpha - 120)] \qquad (26)$$

Differentiating, equating to zero and solving for α,

$$\alpha = -30 \text{ deg.}$$

Thus the instant of short-circuit should be taken at

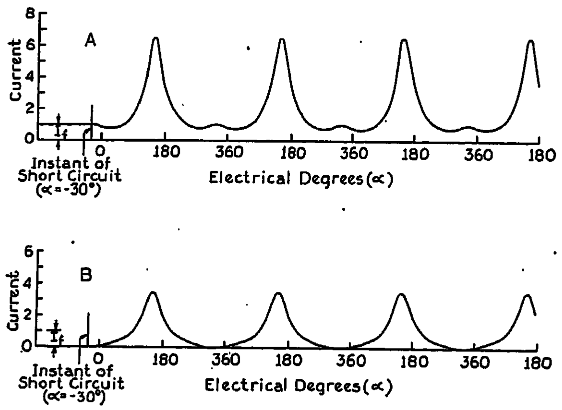

FIG. 5—INITIAL SHORT-CIRCUIT CURRENTS; CASE 5
Curve A—Field current. Eq. (40)
Curve B—Current in Circuit ab Eq. (41)

$\alpha = -30$ deg. The flux linkages of the two circuits at this instant are,

$$\Omega_f = I_f L_f \qquad (27)$$

$$\Omega_{ab} = \Omega_a - \Omega_b = I_f M_{af}'' - I_f M_{bf}'' \qquad (28)$$

where M_{af}'' and M_{bf}'' are the mutual inductances between the field circuit and phases a and b respectively at $\alpha = -30$ deg.

Thus from (9) and (25)

$$\Omega_{ab} = \sqrt{3}\, I_f M_0 \qquad (29)$$

The flux linkages of the two circuits at any instant after short-circuit are,

$$\Omega_f = i_f L_f + i_a M_{fa} + i_b M_{fb} \qquad (30)$$

$$\Omega_{ab} = \Omega_a - \Omega_b \qquad (31)$$

where,

$$\Omega_a = i_a L_a + i_f M_{af} + i_b M_{ab} \qquad (32)$$

$$\Omega_b = i_b L_b + i_f M_{bf} + i_a M_{ba} \qquad (33)$$

But due to reverse connection of a and b

$$i_{ab} = i_a = -i_b \qquad (34)$$

The coefficient M_{ab} is defined as

$$M_{ab} = k \sqrt{L_a L_b}$$

The coefficient of coupling k depends upon the relative positions of the two phases a and b and may be assumed to vary as the cosine of the angle between their axes. Thus in this case where the angle between phases is 120 deg.

$$k = \cos 120 \text{ deg.} = -0.5$$

and,

$$M_{ab} = -0.5\, L_a \qquad (35)$$

Substituting (34) in (30)

$$\Omega_f = i_f L_f + i_{ab} (M_{fa} - M_{fb}) \qquad (36)$$

Substituting (32) (33) (34) and (35) in (31)

$$\Omega_{ab} = i_f (M_{af} - M_{bf}) + i_{ab}\, 3\, L_a \qquad (37)$$

Equating (27) and (36), and (29) and (37)

$$i_f L_f + i_{ab} (M_{fa} - M_{fb}) = I_f L_f \qquad (38)$$

$$i_f (M_{af} - M_{bf}) + i_{ab}\, 3\, L_a = \sqrt{3}\, I_f M_0 \qquad (39)$$

Solving (39) and (40) simultaneously and substituting (9) (12) and (25),

$$i_f = I_f \frac{1 - K^2 \cos (\alpha + 30)}{(1 - 0.5\, K^2) - 0.5\, K^2 \cos 2\, (\alpha + 30)} \qquad (40)$$

$$i_{ab} = \frac{1}{\sqrt{3}} I_f \frac{M_0}{L_a} \frac{1 - \cos (\alpha + 30)}{(1 - 0.5\, K^2) - 0.5\, K^2 \cos 2 (\alpha + 30)} \qquad (41)$$

Equations (40) and (41) are plotted in Fig. 5 for

$$K = 0.85 \text{ and } \frac{M_0}{L_a} = 0.85.$$

2. Two-Phase Short Circuit

(a) Two-phase alternator, (Case 6, Fig. 2).

There are three circuits to consider; the field circuit, phase a and phase b. The flux linkages of these circuits at the instant of short-circuit, *i. e.*, when $\alpha = 0$ are,

$$\Omega_f = I_f L_f \qquad (42)$$

$$\Omega_a = I_f M_0 \qquad (43)$$

$$\Omega_b = 0 \qquad (44)$$

The flux linkages at any other field position are,

$$\Omega_f = i_f L_f + i_a M_{fa} + i_b M_{fb} \qquad (45)$$

$$\Omega_a = i_a L_a + i_f M_{af} \qquad (46)$$

$$\Omega_b = i_b L_b + i_f M_{bf} \qquad (47)$$

since $M_{ab} = 0$ as for the previous two-phase case considered.

Equating (42) and (45), (43) and (46), (44) and (47), solving simultaneously and substituting (9) (12) (13) and (22),

$$i_f = I_f \frac{1 - K^2 \cos \alpha}{1 - K^2} \qquad (48)$$

$$i_a = I_f \frac{M_0}{L_a} \frac{(1 - 0.5\, K^2) - \cos \alpha + 0.5\, K^2 \cos 2\, \alpha}{1 - K^2} \qquad (49)$$

$$i_b = -I_f \frac{M_0}{L_a} \frac{\sin \alpha - 0.5\, K^2 \sin 2\, \alpha}{1 - K^2} \qquad (50)$$

These currents are plotted in Fig. 6 for K = 0.85 and

$$\frac{M_0}{L_a} = 0.85.$$

(b) Three-phase alternator, (Case 7, Fig. 1).

There are three circuits, the field circuit, phase a and phase b. Maximum current is obtained in phase a if the short-circuit occurs at α = 30 deg. since at this armature position there is no field flux enclosed by phase b. The flux linkages of the three circuits at α = 30 deg. are

$$\Omega_f = I_f L_f \tag{51}$$

$$\Omega_a = 0.5 \sqrt{3}\, I_f M_0 \tag{52}$$

$$\Omega_b = 0 \tag{53}$$

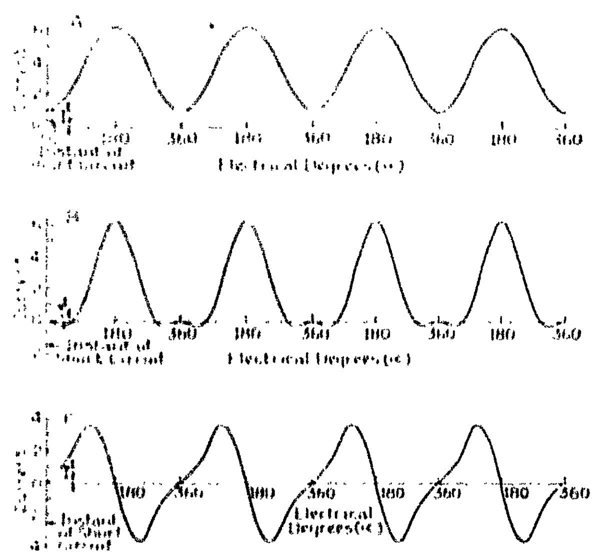

FIG. 6—INITIAL SHORT-CIRCUIT CURRENTS; CASE 6

Curve A—Field current. Eq. (48)
Curve B—Current in phase a. Eq. (49)
Curve C—Current in phase b. Eq. (50)

The flux linkages for any value of α are,

$$\Omega_f = i_f L_f + i_a M_{fa} + i_b M_{fb} \tag{54}$$

$$\Omega_a = i_a L_a + i_f M_{af} + i_b M_{ab} \tag{55}$$

$$\Omega_b = i_b L_b + i_f M_{bf} + i_a M_{ba} \tag{56}$$

Equating (51) and (54), (52) and (55), (53) and (56), solving simultaneously and substituting (9), (12), (22), (25), and (35)

$$i_f = I_f \frac{1 - K^2 \cos(\alpha - 30)}{1 - K^2} \tag{57}$$

$$i_a = \frac{2}{\sqrt{3}} I_f \frac{M_0}{L_a} \frac{(1-0.5\ K^2) - \cos(\alpha - 30) + 0.5\ K^2 \cos 2(\alpha - 30)}{1 - K^2} \tag{58}$$

$$i_b = \frac{2}{\sqrt{3}} I_f \frac{M_0}{L_a} \frac{0.5\ (1 - 0.5\ K^2) - \sin\alpha - 0.5\ K^2 \cos 2(\alpha + 30)}{1 - K^2} \tag{59}$$

These currents are plotted in Fig. 7 for K = 0.85 and

$$\frac{M_0}{L_a} = 0.85.$$

3. Three-Phase Short-Circuit

(a) Three-phase alternator, (Case 8, Fig. 1).

There are four circuits to consider; the field circuit, phase a, phase b and phase c. The flux linkages of these circuits at α = 0 are,

$$\Omega_f = I_f L_f \tag{60}$$

$$\Omega_a = I_f M_0 \tag{61}$$

$$\Omega_b = -\ 0.5\, I_f M_0 \tag{62}$$

$$\Omega_c = -\ 0.5\, I_f M_0 \tag{63}$$

The flux linkages at any value of α are,

$$\Omega_f = i_f L_f + i_a M_{fa} + i_b M_{fb} + i_c M_{fc} \tag{64}$$

$$\Omega_a = i_a L_a + i_f M_{af} + i_b M_{ab} + i_c M_{ac} \tag{65}$$

$$\Omega_b = i_b L_b + i_f M_{bf} + i_a M_{ba} + i_c M_{bc} \tag{66}$$

$$\Omega_c = i_c L_c + i_f M_{cf} + i_a M_{ca} + i_b M_{cb} \tag{67}$$

Equating (60) and (64), (61) and (65), (62) and (66), and (63) and (67), solving simultanelusly and substituting (9), (12), (13), (22), (35), and the three relations

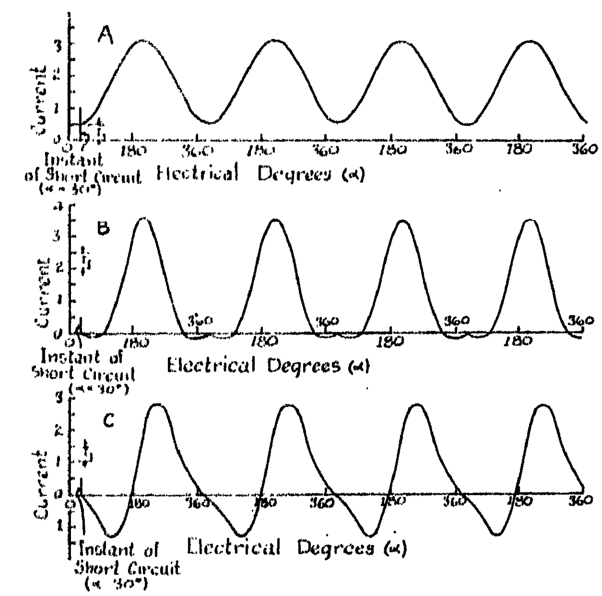

FIG 7—INITIAL SHORT-CIRCUIT CURRENTS; CASE 7

Curve A—Field current. Eq. (57)
Curve B—Current in phase a. Eq. (58)
Curve C—Current in phase b. Eq. (59)

$$L_a = L_c \tag{68}$$

$$M_{ac} = -\ 0.5\, L_a \tag{69}$$

$$M_{cf} = M_0 \cos(\alpha + 120) \tag{70}$$

$$i_f = I_f \frac{1 - K^2 \cos\alpha}{1 - K^2} \tag{71}$$

$$i_a = \frac{2}{3} I_f \frac{M_0}{L_a} \frac{(1 - 0.5\ K^2) - \cos\alpha + 0.5\ K^2 \cos 2\alpha}{1 - K^2} \tag{72}$$

$$i_b = -\frac{1}{3} I_f \frac{M_0}{L_a}$$

$$\frac{(1-0.5\,K^2)+2\cos(\alpha-120)-K^2\cos 2\,(\alpha+120)}{1-K^2} \quad (73)$$

$$i_c = -\frac{1}{3} I_f \frac{M_0}{L_a}$$

$$\frac{(1-0.5\,K^2)+2\cos(\alpha+120)-K^2\cos 2\,(\alpha-120)}{1-K^2} \quad (74)$$

These equations are plotted in Fig. 8 for $K = 0.85$, and

$$\frac{M_0}{L_a} = 0.85.$$

(b) Three-phase alternator (Case 9, Fig. 1)

The current equations obtained for a three-phase

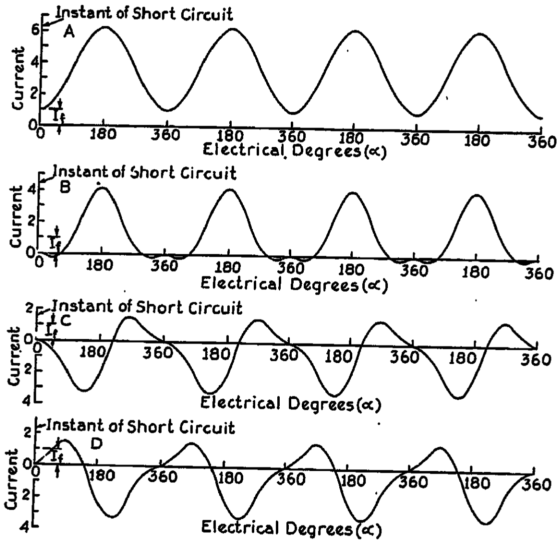

FIG. 8—INITIAL SHORT-CIRCUIT CURRENTS; CASES 8 AND 9

Curve *A*—Field current. Eq. (71)
Curve *B*—Current in phase *a*. Eq. (72)
Curve *C*—Current in phase *b*. Eq. (73)
Curve *D*—Current in phase *c*. Eq. (74)

short-circuit terminal-to-terminal are the same as those for a three-phase short-circuit terminal-to-neutral.

PERMANENT CONDITION

In the permanent condition, the algebraic sum of the flux linkages in each armature circuit must be zero since the flux linkages originally caught in these circuits have been dissipated during the transient period between the initial and permanent states. The flux linkages of the field circuit will be equal to those supplied by the exciter.

1. Single-Phase Short Circuit.

(a) Single-phase or polyphase alternator.

As in the corresponding case for the initial short-circuit condition a general formula can be obtained for the single-phase, short-circuit current of a single-phase, two-phase, or three-phase alternator, when only one armature phase is involved in the short circuit (Cases 1 to 3, Fig. 1.) The flux linkages of the two circuits involved, are,

$$i_f L_f + i_a M_{fa} = I_f' L_f \quad (75)$$

$$i_a L_a + i_f M_{af} = 0 \quad (76)$$

where I_f' is that constant value of field current that would be required on open circuit to produce the same number of flux linkages with the field circuit as exist under the permanent short circuit condition.

Solving these two equations simultaneously and substituting (9) and (12)

$$i_f = I_f' \frac{1}{(1-0.5\,K^2) - 0.5\,K^2 \cos 2\,\alpha} \quad (77)$$

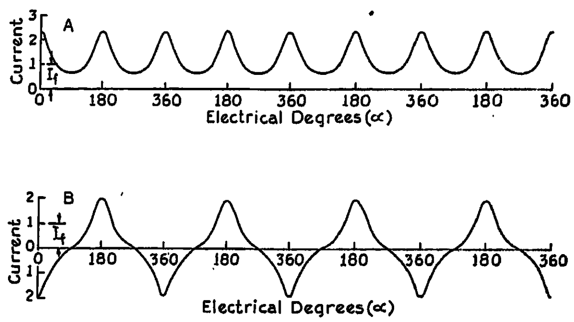

FIG. 9—PERMANENT SHORT-CIRCUIT CURRENTS; CASES 1, 2 AND 3

Curve *A*—Field current. Eq. (80)
Curve *B*—Current in phase *a*. Eq. (81)

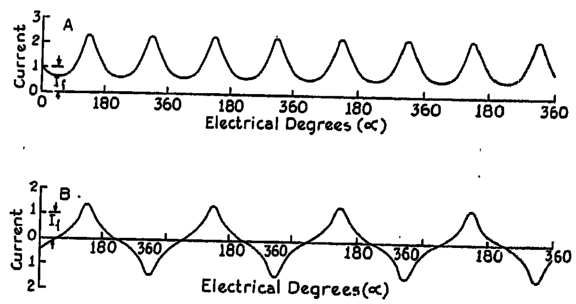

FIG. 10—PERMANENT SHORT-CIRCUIT CURRENTS; CASE 4

Curve *A*—Field Current. Eq. (86)
Current *B*—Current in circuit *a b*. Eq. (87)

$$i_a = -I_f' \frac{\cos\alpha}{(1-0.5\,K^2) - 0.5\,K^2\cos 2\,\alpha} \quad (78)$$

The direct component of the field current (77) is due to the flux linkages supplied by the exciter and is, therefore, equal to the field current I_f which flowed before short-circuit. This direct component is found by integrating (77) between proper limits and dividing by the abscissa. Thus,

$$I_f = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{I_f'}{(1-0.5\,K^2) - 0.5\,K^2\cos 2\,\alpha}\, d\,\alpha$$

$$= \frac{I_f'}{\sqrt{1-K^2}}$$

or

$$I_f' = I_f \sqrt{1 - K^2} \tag{79}$$

Substituting (79) in (77) and (78),

$$i_f = I_f \frac{\sqrt{1 - K^2}}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,\alpha} \tag{80}$$

$$i_a = -\,I_f \frac{M_0}{L_a} \frac{\sqrt{1 - K^2} \cos \alpha}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,\alpha} \tag{81}$$

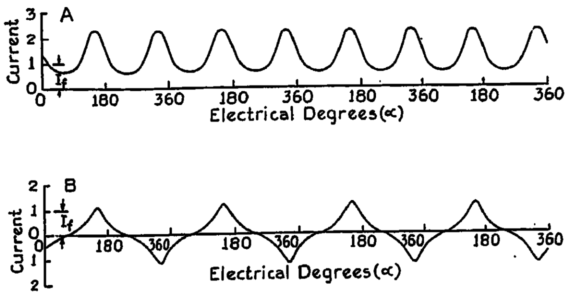

FIG. 11—PERMANENT SHORT-CIRCUIT CURRENTS; CASE 5
Curve A—Field current. Eq. (92)
Curve B—Current in circuit $a\,b$. Eq. (93)

These currents are plotted in Fig. 9 for $K = 0.85$ and

$$\frac{M_0}{L_a} = 0.85.$$

(b) Two-phase alternator, single-phase terminal-to-neutral short-circuit (Case 4, Fig. 1).

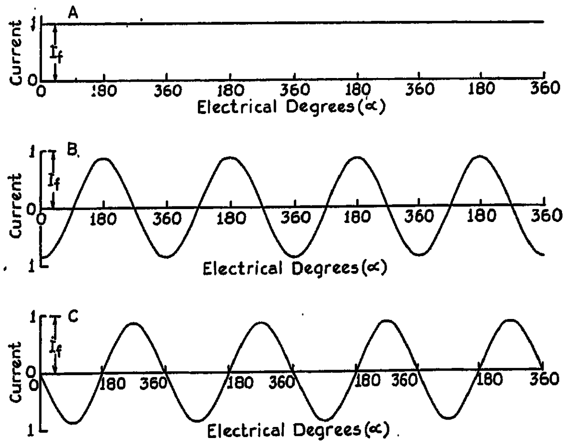

FIG. 12—PERMANENT SHORT-CIRCUIT CURRENTS; CASE 6
Curve A—Field current. Eq. (101)
Curve B—Current in phase a. Eq. (102)
Curve C—Current in phase b. Eq. (103)

The flux linkages of the two circuits involved are: [See (19) and (20)].

$$i_f\,L_f + i_{ab}\,(M_{fa} - M_{fb}) = I_f'\,L_f \tag{82}$$

$$i_f\,(M_{af} - M_{bf}) + i_{ab}\,(L_a + L_b) = 0 \tag{83}$$

Solving simultaneously and substituting (9), (12), (13) and (22).

$$i_f = I_f' \frac{1}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 45)} \tag{84}$$

$$i_{ab} = -\,\frac{1}{\sqrt{2}} I_f' \frac{\cos(\alpha + 45)}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2(\alpha + 45)} \tag{85}$$

Since (84) is of the same form as (77) the value of I_f' for this case will be the same as that given by (79). Substituting (79) in (84) and (85),

$$i_f = I_f \frac{\sqrt{1 - K^2}}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 45)} \tag{86}$$

$$i_{ab} = -\,\frac{1}{\sqrt{2}} I_f \frac{M_0}{L_a} \frac{\sqrt{1 - K^2} \cos(\alpha + 45)}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 45)} \tag{87}$$

These currents are plotted in Fig. 10 for K and $\frac{M_0}{L_a}$ equal to 0.85.

(c) Three-phase alternator, single-phase terminal-to-terminal short-circuit (Case 5, Fig. 1).

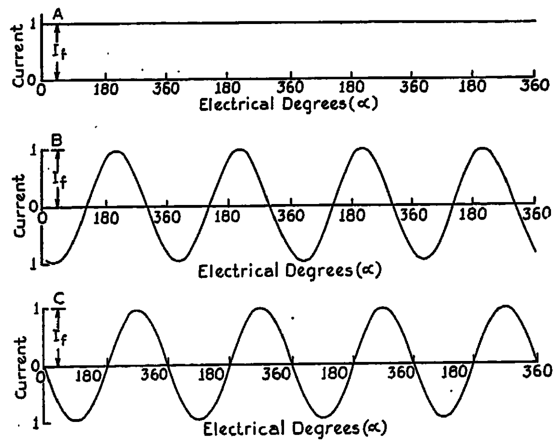

FIG. 13—PERMANENT SHORT-CIRCUIT CURRENTS; CASE 7
Curve A—Field current. Eq. (110)
Curve B—Current in phase a. Eq. (111)
Curve C—Current in phase b. Eq. (112)

The linkages of the circuits involved are from (36) and (37),

$$i_f\,L_f + i_{ab}\,(M_{fa} - M_{fb}) = I_f'\,L_f \tag{88}$$

$$i_f\,(M_{af} - M_{bf}) + i_{ab}\,3\,L_a = 0 \tag{89}$$

Solving simultaneously and substituting (9), (12) and (25),

$$i_f = I_f' \frac{1}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 30)} \tag{90}$$

$$i_{ab} = -\,\frac{1}{\sqrt{3}} I_f' \frac{M_0}{L_a} \frac{\cos(\alpha + 30)}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 30)} \tag{91}$$

Equation (93) is again of the same form as (77) so

that I_f' is given by (79). Substituting for I_f' in (90) and (91);

$$i_f = I_f \frac{\sqrt{1 - K^2}}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 30)} \tag{92}$$

$$i_{ab} = -\frac{1}{\sqrt{3}}\, I_f \frac{M_0}{L_a} \frac{\sqrt{1 - K^2}\cos(\alpha + 30)}{(1 - 0.5\,K^2) - 0.5\,K^2 \cos 2\,(\alpha + 30)} \tag{93}$$

These equations are plotted in Fig. 11 for K and

$$\frac{M_0}{L_a} = 0.85.$$

2. Two-Phase Short Circuit.

(a) Two-phase alternator, (Case 6, Fig. 1).

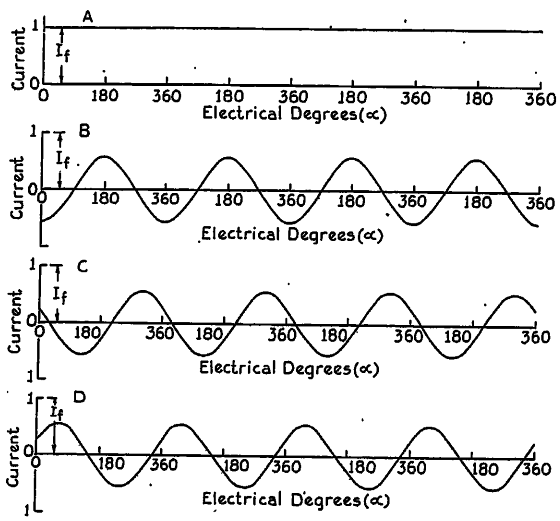

FIG. 14—PERMANENT SHORT-CIRCUIT CURRENTS; CASES 8 AND 9

Curve A—Field current. Eq. (117)
Curve B—Current in phase a. Eq. (118)
Curve C—Current in phase b. Eq. (119)
Curve D—Current in phase c. Eq. (120)

The flux linkages of the circuits involved, from (45), (46) and (47) are,

$$i_f L_f + i_a M_{fa} + i_b M_{fb} = I_f' L_f \tag{94}$$

$$i_a L_a + i_f M_{af} = 0 \tag{95}$$

$$i_b L_b + i_f M_{bf} = 0 \tag{96}$$

Solving simultaneously and substituting (9), (12), (13), and (22),

$$i_f = I_f' \frac{1}{1 - K^2} \tag{97}$$

$$i_a = -I_f' \frac{M_0}{L_a} \frac{\cos\alpha}{1 - K^2} \tag{98}$$

$$i_b = -I_f' \frac{M_0}{L_a} \frac{\sin\alpha}{1 - K^2} \tag{99}$$

The field current (97) has only a direct component, and is therefore equal to I_f.

Hence,

$$I_f' = I_f\,(1 - K^2) \tag{100}$$

Substituting (100) in (97), (98) and (99),

$$i_f = I_f \tag{101}$$

$$i_a = -I_f \frac{M_0}{L_a} \cos\alpha \tag{102}$$

$$i_b = -I_f \frac{M_0}{L_a} \sin\alpha \tag{103}$$

These currents are plotted in Fig. 12 for K and $\frac{M_0}{L_a}$ equal to 0.85.

(b) Three-phase alternator, (Case 7, Fig. 1).

The flux linkages of the circuits involved are from (54), (55) and (56),

$$i_f L_f + i_a M_{fa} + i_b M_{fb} = I_f' L_f \tag{104}$$

$$i_a L_a + i_f M_{af} + i_b M_{ab} = 0 \tag{105}$$

$$i_b L_b + i_f M_{bf} + i_a M_{ba} = 0 \tag{106}$$

Solving simultaneously and substituting (9), (12), (22), (25) and (35),

$$i_f = I_f' \frac{1}{1 - K^2} \tag{107}$$

$$i_a = -\frac{2}{\sqrt{3}} I_f' \frac{M_0}{L_a} \frac{\cos(\alpha - 30)}{1 - K^2} \tag{108}$$

$$i_b = -\frac{2}{\sqrt{3}} I_f' \frac{M_0}{L_a} \frac{\sin\alpha}{1 - K^2} \tag{109}$$

The field current (107) is the same as (94) so that the value of I_f' is given by (100). Substituting for I_f';

$$i_f = I_f \tag{110}$$

$$i_a = -\frac{2}{\sqrt{3}} I_f \frac{M_0}{L_a} \cos(\alpha - 30) \tag{111}$$

$$i_b = -\frac{2}{\sqrt{3}} I_f \frac{M_0}{L_a} \sin\alpha \tag{112}$$

These equations are plotted in Fig. 13 for $K = 0.85$ and

$$\frac{M_0}{L_a} = 0.85.$$

3. Three-Phase Short Circuit.

(a) Three-phase alternator, (Case 8, Fig. 1).

The flux linkage equations of the four circuits involved are,

$$i_f L_f + i_a M_{fa} + i_b M_{fb} + i_c M_{fc} = I_f' L_f \tag{113}$$

$$i_a L_a + i_f M_{af} + i_b M_{ab} + i_c M_{ac} = 0 \tag{114}$$

$$i_b L_b + i_f M_{bf} + i_a M_{ba} + i_c M_{bc} = 0 \tag{115}$$

$$i_c L_c + i_f M_{cf} + i_a M_{ca} + i_b M_{cb} = 0 \tag{116}$$

Solving simultaneously and substituting (9), (12), (22), (25), (35), (68), (69) and (70), and the value of I_f' given by (100) since i_f comes out the same as (97),

$$i_f = I_f \tag{117}$$

$$i_a = -\frac{2}{3} I_f \frac{M_0}{L_a} \cos \alpha \tag{118}$$

$$i_b = -\frac{2}{3} I_f \frac{M_0}{L_a} \cos (\alpha - 120) \tag{119}$$

$$i_c = -\frac{2}{3} I_f \frac{M_0}{L_a} \cos (\alpha + 120) \tag{120}$$

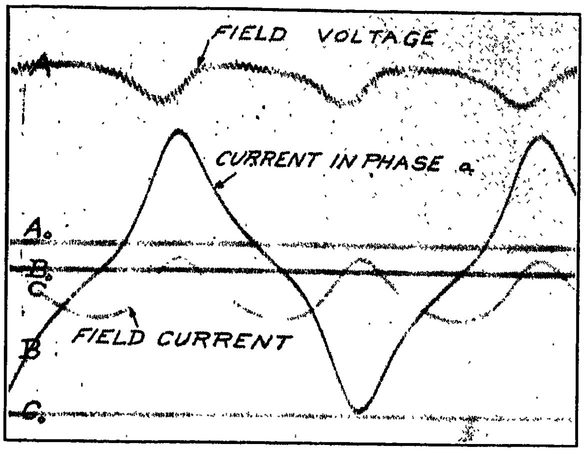

FIG. 15—OSCILLOGRAM OF THE PERMANENT SINGLE-PHASE SHORT-CIRCUIT CURRENTS OF A THREE-PHASE ALTERNATOR SHORT-CIRCUITED BETWEEN TWO TERMINALS

These currents are plotted in Fig. 14 for $K = 0.85$ and

$$\frac{M_0}{L_a} = 0.85.$$

(b) Three-phase alternator, (Case 9, Fig. 1).

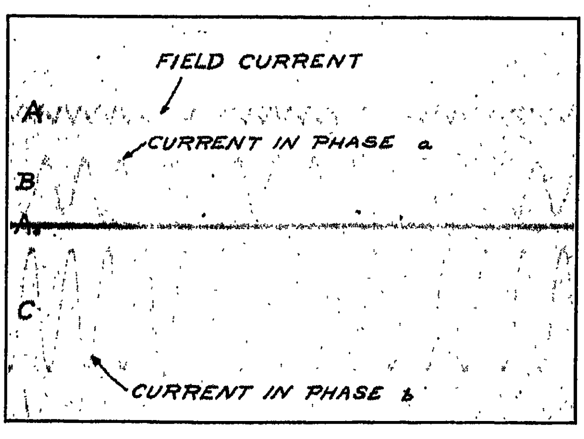

FIG. 16—OSCILLOGRAM OF THE PERMANENT TWO-PHASE SHORT-CIRCUIT CURRENTS OF A THREE-PHASE ALTERNATOR SHORT-CIRCUITED BETWEEN TWO TERMINALS AND NEUTRAL
The two armature currents are of different calibration.

The current equations for this case are the same as those for a three-phase short-circuit terminal-to-neutral.

NOTATION

α electrical angle of rotation of field. (See Fig. 2).

I_f value of field current in amperes before short-circuit.

I_f' the constant value of field current in amperes under the permanent short-circuit condition that would be required on open circuit to produce the same number of flux linkages with the field circuit as exist under the permanent short-circuit condition.

i_a, i_b, i_c instantaneous values of current in amperes in phases a, b and c respectively.

i_{ab} instantaneous values of current in amperes in the single-phase circuit composed of phases a and b in series.

i_f instantaneous value of field current in amperes.

K coefficient of magnetic coupling between any one armature phase and the field circuit. (See Eq. 12).

L_a, L_b, L_c true coefficient of self inductance in henrys of phases a, b and c respectively.

L_f true coefficient of self inductance in henrys of field current.

$M_{ab} = M_{ba}, M_{ac} = M_{ca}, M_{bc} = M_{cb}$ coefficient of mutual inductance in henrys between phases a and b, a and c, and b and c.

$M_{fa} = M_{af}, M_{fb} = M_{bf}, M_{fc} = M_{cf}$ coefficient of mutual inductance in henrys between the field circuit f and phases a, b and c respectively for any value of α

M_{fab} coefficient of mutual inductance in henrys between the field circuit and phases a and b in series for any value of α

M_0 coefficient of mutual inductance in henrys between the field circuit and any one armature phase when in the position of maximum coupling.

$\Omega_a, \Omega_b, \Omega_c$ flux linkages of phases a, b and c respectively for any value of α

Ω_{ab} flux linkages of the single-phase circuit composed of phases a and b in series for any value of α

Ω_f flux linkages of the field circuit for any value of α

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Discussion

INITIAL AND SUSTAINED SHORT CIRCUITS IN SYNCHRONOUS MACHINES

(KARAPETOFF)

SHORT-CIRCUIT CURRENTS OF SYNCHRONOUS MACHINES

(FRANKLIN)

ST. LOUIS, MO., APRIL 14, 1925

R. E. Doherty: Progress in most investigations of the complexity of those taken up in the two papers on short-circuit currents is made step by step.

As Professor Karapetoff brought out, there are still further steps to take; saturation must be taken into account and also the effect of salient poles. But I think that the equations and the results which have been obtained by Mr. Franklin and which Professor Karapetoff's method has completely checked have formed another definite step in advance in this problem.

I wish to say a word about methods. In 1918, I was engaged in some work along these lines and I was confronted, like everybody else who has undertaken such studies, with the practically impossible undertaking of solving short-circuit problems when resistance is taken into account. In the investigation it appeared that sufficiently close approximation could be made in many instances by neglecting resistance.

I proposed, at that time, the constant-linkage theorem which Mr. Franklin has utilized in his paper. It is the simple relation which follows at once from Kirchoff's Law and the assumption of zero resistance. From these two premises, the theorem stated in Mr. Franklin's paper is this: In any closed circuit without resistance the flux linkages must remain constant. It doesn't matter how many secondary circuits there are, or what the network involves, the theorem is rigidly true.

Now, the question which I wish to raise with respect to Prof. Karapetoff's paper is that if this theorem is true (he makes the assumption that the resistance is zero), why is it necessary to go back to Kirchoff's equations in each particular case as he does, performing the integration and finding the integration constant, which in every case, of course, turns out to be the known magnetic linkages existing when the circuit was closed?

Prof. Karapetoff has stated that after making the integration, that is, dropping the derivative symbol, the problem is solved. That is exactly where you start if you apply the constant-leakage theorem. Mr. Franklin has followed the latter method. That is a point more of interest than of importance, but it is a question of ease with which the problem can be solved, if advantage is taken of the facilities afforded by a simple theorem.

R. W. Wieseman: In single-phase generators usually only two-thirds of the coils per pole are used and connected in series. This corresponds to a 120-deg. phase belt and is classified under Case 5 of Mr. Franklin's paper. From Table II we find that the initial single-phase short-circuit current peak value (Case 5) is only 86 per cent of the three-phase short-circuit peak, Case 9. Furthermore, the current wave of Case 5, Fig. 5, is more peaked than that of Case 9, Fig. 8, so that the effective value of the initial single-phase short-circuit current, Case 5, will be less than 86 per cent of the initial three-phase short-circuit current. Therefore, at the initial short circuit an armature coil in a single-phase generator will have only about half as much heat to dissipate as an armature coil in a three-phase generator. However, a study of a large number of oscillograms shows that the initial single-phase short-circuit current is practically the same as the three-phase short-circuit current.

The five assumptions which the author made in deriving the formulas are reasonable and they should not introduce an appreciable error in most cases. These assumptions must be made if a simple and practical solution of such a complicated problem is to be obtained. There are, however, a few cases which might be mentioned as a matter of interest in which different results are obtained in practise from those given by the formulas. I refer to the sustained short-circuit armature current waves of Cases 1, 2, 3, 4 and 5 shown by Figs. 9, 10 and 11 which show symmetrical short-circuit current waves. Fig. 1 accompanying this discussion shows a typical sustained armature current wave corresponding to the author's Case 5, Fig. 11, of a cylindrical-rotor generator without a pole-face damper winding. The five assumptions are realized in this type of a machine more than in any other type. Contrary to Fig. 11, Fig. 1 herewith is not a symmetrical wave and it contains cosine terms as well as sine terms. Fig. 2 herewith shows a typical sustained short-circuit current wave corresponding to Case 5, Fig. 11, of a salient-pole machine without a pole-face damper winding. Fig. 2 herewith is a symmetrical wave, but this machine does not fulfil condition *D* of the paper.

Fig. 14 of the paper shows the same short-circuit current waves for both Cases 8 and 9. For Case 8 the short-circuit armature current wave is rarely a sine wave as shown by Fig. 14. The short-circuit current wave for Case 8 usually contains a negative third harmonic which gives a peaked wave as shown by the accompanying Fig. 3. For Case 9 the sustained short-circuit current wave is practically a sine wave as shown by the accompanying Fig. 4. It should be noted that this Fig. 4 agrees with Fig. 14 in the paper.

In addition to the five assumptions made in deriving the formulas of this paper, there is another assumption which the author made but apparently neglected to mention, namely, that the machines have no pole-face damper windings. All synchronous motors are furnished with pole-face starting windings and many generators have pole-face damper windings. If the machines whose sustained short-circuit armature current waves are shown by Figs. 1 and 2 herewith are equipped with pole-face windings, the short-circuit current waves will now be as shown by Figs. 5 and 6 herewith respectively. It is apparent that these waves are as nearly true sine waves as can be obtained. The addition of a good damper winding short circuits the double-frequency armature-reaction component which induces (by transformer action) a triple-frequency voltage in the armature winding. Since the phase belt is 120 deg. (Case 5) the third-harmonic voltage induced (by dynamic action) in the armature winding by the third harmonic of the flux wave cannot appear at the terminals of the machine. Consequently, a nearly true sine wave of current results.

Fig. 16 of the paper shows an oscillogram of the sustained two-phase short-circuit currents of a three-phase alternator short-circuited between two terminals and neutral, Case 7. It is pointed out in the paper that the alternating component induced in the field winding in this case is very small and the field-current Curve *A*, Fig. 13, is shown as a straight line. Fig. 16 shows a double-frequency component in the field current of only plus or minus 9 per cent. The generator which was used in this particular case was equipped with a pole-face damper winding which tends to damp the single-phase double-frequency pulsations in the field circuit. Fig. 7 herewith shows an oscillogram of the armature current and field current for the same conditions as shown by the author's Fig. 16 taken from a generator which had no pole-face winding. It should be noted that the variation in the field current is plus or minus 33 per cent. Therefore, Case 7 is in reality a partial single-phase load (the two armature currents being out of phase by 120 deg.) and results in the usual

double-frequency pulsation in the field winding. Of course, Case 7 does not give so great a double-frequency pulsation as Cases 3 and 5, but it is decidedly more than Cases 8 and 9. If the value of the coupling coefficient K is calculated for this particular machine, it is found to be from -0.35 to -0.40 instead of cosine 120 deg. which equals -0.5.

J. F. H. Douglas (by letter): The points that impress one most forcibly in Professor Karapetoff's paper are the mathematical elegance and the generality of the relations discussed,

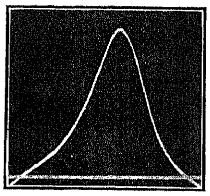


Fig. 1—Sustained Single-phase, Short-Circuit Current, Terminal to Terminal, (Case 5). Cylindrical Rotor Generator without Damper Winding

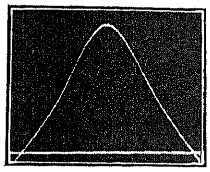


Fig. 2—Sustained Single-Phase Short-Circuit Current Terminal to Terminal (Case 5) Salient Pole Generator without Damper Winding

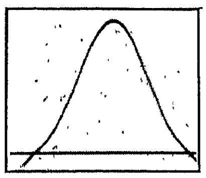


Fig. 3—Sustained Three-Phase, Short-Circuit Current, Neutral Connected (Case 8) Generator without Damper Winding

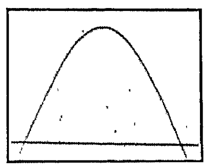


Fig. 4—Sustained Three-Phase, Short-Circuit Current, Neutral not Connected (Case 9) Generator without Damper Winding

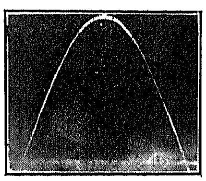


Fig. 5—Sustained Single-Phase, Short-Circuit Current, Terminal to Terminal (Case 5) Cylindrical Rotor Generator with Damper Winding

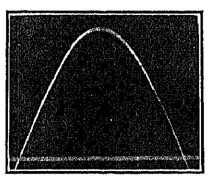


Fig. 6—Sustained Single-Phase, Short-Circuit Current, Terminal to Terminal (Case 5) Salient Pole Generator with Damper Winding

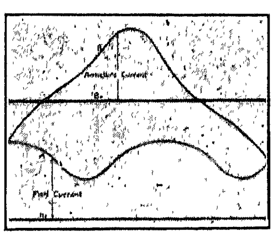


Fig. 7—Sustained Two-Phase, Short-Circuit Current of a Three-Phase Generator between Two Terminals and Neutral, without Damper Winding

together with the space-vector diagrams. There are a few points which I should like to mention.

In the first place it should be noted that the equations (21) to (25) are really of great simplicity. The chief difficulty in the appendices that follow lies in the determination of the constants of integration. Whether one reads the latter appendices or not, the first appendix should be of great general interest, in that it proves the general principle involved.

The physical interpretation of Equations (21) to (25) is that there is continuity and approximate constancy in the linkages existing at the instant of short circuit, so that, the armature drags its linkages around with it. A picture of my interpretation of these equations is shown in Fig. 8 herewith, where a two-phase machine is shown with 50 per cent pitch armature coils. At the top of the picture are shown conditions at instant of short circuit. Phase A links with useful flux, phase B with no flux and the field with the total flux. Phase A has no leakage locally around the slots at this instant. The middle figure shows flux lines 90 deg. later in time phase. Phase A has dragged its linkages around into the interpolar region. Phase B has reflected or warded the pole flux away from entering the armature. The lowest portion of the figure shows flux lines 180 electrical degrees from the start. Phase A is now feeding flux into the

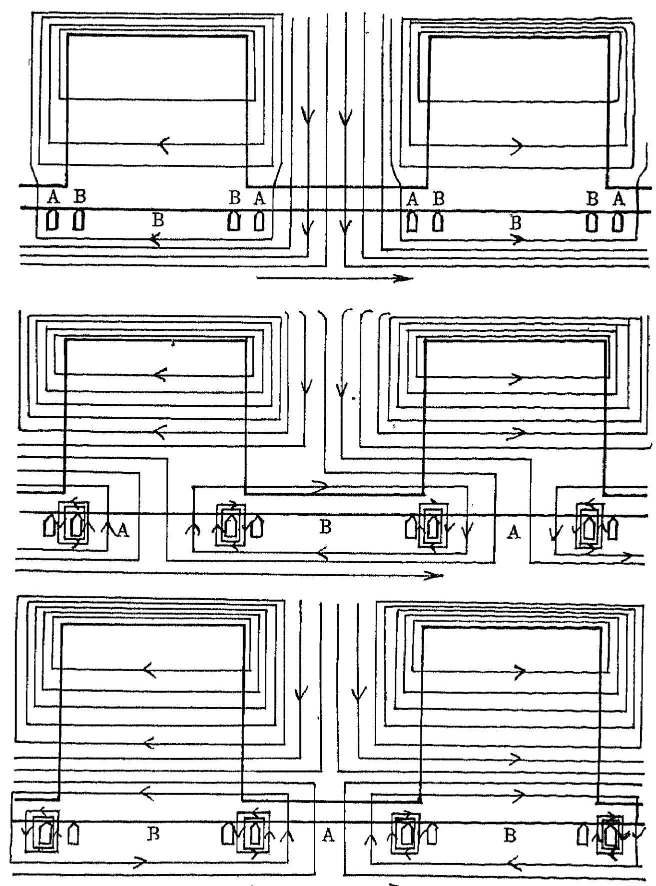


Fig. 8—Diagram Showing Continuity of Flux Leakages During Short Circuit

pole, and the leakage flux is immensely increased, both locally and between the poles. This picture shows perhaps more clearly than any formula the advantage of having the values of σ and τ less than one and a value of K considerably less than unity, if possible. It is readily seen that the values of armature current and field current corresponding to the bottom diagram must be very large to force the flux through paths of such high reluctance. A similar condition obtains in the case of a machine with a distributed field.

In the fifth paragraph of the paper the assumption is made that the field winding is placed upon a cylindrical rotor, so that the self and mutual inductances of the armature windings may be considered to remain constant throughout a cycle. These constants are treated as absolute constants, however, and so it

may be said that saturation is also neglected. These assumptions are the same assumptions upon which the idea of "synchronous impedance" is based. The first of these assumptions is not true in any machine, for normal field currents. The principle of continuity in the linkages existing at short circuit, would tend to perpetuate saturation existing at the instant of short circuit. Nevertheless, the assumption of uniform air-gap permeance, constant inductances, and synchronous impedance, based upon them, all are useful as first approximations.

As a second approximation, in the case of sinusoidal and steady load currents, I have found the idea of synchronous impedance useful even in the case of salient-pole machines, because of its simplicity. But, one must point out that at high saturations the numerical value of this constant is reduced, and that at high power factors the numerical value is also decreased. Whether, this same second assumption of an average and constant L

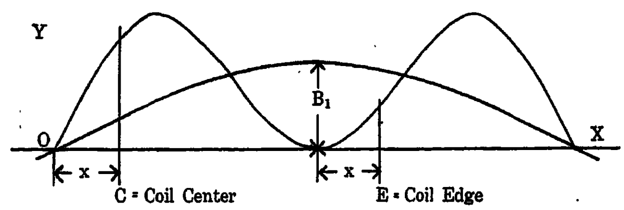


FIG. 9—TRANSVERSE FLUX WAVE SHAPE

is also applicable to the case of the transient condition of short circuit I am not prepared to state. It would certainly be some simplification in the theory if it should prove to be the case.

It occurs to me that a third approximation ought to take in the variation of the permeance of the air-gap with the angle from the pole center. Perhaps this can be best done by considering the flux resolved into two components, along the polar, and transverse to the polar axis. The flux produced by the direct ampere-turns, may be considered substantially as sinusoidally distributed, while that caused by any transverse ampere-turns may be considered as distributed as in Fig. 9 herewith, the equation of which is $Y = B_1 (\sin x + \sin 3x)$.[1] The m. m. fs of the armature are to be thought of as sinusoidally distributed, the direct flux as sinusoidal, but the transverse flux as non-sinusoidal.

Let us assume a two-phase machine, with the windings identical, and displaced 90 deg. on the armature surface. Let, τ, σ, N_a, N_f, I_f, i_f, L_a, L_f, α, have the meaning in Prof. Karapetoff's article. Let the direct flux be ϕ'_d, the transverse ϕ_t'. Let χ be the ratio of the transverse flux set up by 1 ampere-turn to the direct flux (sine-wave component). Let the arrangement of the circuits be as shown in Fig. 10 herewith. Then we should note that equations (1), (2), (3), (10a), (10b) and (10c) are not correct, since although M_{ab} would appear to be zero, there is mutual induction between the phases owing to differing air-gap permeance in different directions.

$$\phi_t = \chi\,(L_a/N_a)\,(i_a \sin\alpha - i_b \cos\alpha) \qquad (1)$$

$\phi_d = (L_a/N_a)\,(i_a \cos\alpha + i_b \sin\alpha),\ + i_f\,\tau\,L_f/N_f$ linking with armature (2)

$\phi_f = (L_a\,\sigma/N_a)\,(i_a \cos\alpha + i_b \sin\alpha) + i_f L_f/N_f$ linking with field (3)

The linkages in phase with a coil in position x is, of course,

$$\phi_x = \phi_d \cos x + \phi_t\,(\sin x - 0.33 \sin 3x) \qquad (4)$$

When x is changed to α equation (4) herewith becomes equation for ϕ_a.

To find ϕ_b we let x = 90 deg. $- \alpha$, and get

$$\phi_b = \phi_d \sin\alpha + \phi_t\,(\cos\alpha + 0.33 \cos 3\alpha) \qquad (5)$$

If the initial value of the field current if I_f and the short circuit occurs when the phase a is in maximum linkage with the pole.

1. Distribution of Φ_t is in accordance to density $(\sin x + \sin 3x)$ and according to linkage $(\sin x - {}^1/_3 \sin 3x)$.

$$\phi_f = I_f L_f/N_f,\quad \phi_b = 0,\quad \phi_a = \tau\,I_f L_f/N_f \qquad (6)$$

The constants in equations (3), (4) and (5) herewith, are given in (6) or in similar equations for other initial conditions. The variables ϕ_t and ϕ_d can be obtained from equations (1) and (2). These equations contain only three unknown quantities i_a, i_b, and i_f, and therefore, permit us to solve simultaneously. Rather, however, pursuing the matter further, I call upon Professor Karapetoff to state, whether in his judgment, this reasoning is valid and if it would, or would not, lead to materially different results.

It is interesting to note that in equations (10a) and (10c) if θ_{ab} and θ_{ac} are each 120 deg. and if $N_a = N_b = N_c$, that $M_{ab} = M_{ca} = -0.5\,L_a$. It is interesting also to note that equation (1) might apply individually to phases even though a short circuit were not present. In equation (1) if we let $L = 0$, and remove the short circuit, the voltages e are replaced by e_a, e_b, e_c. Let us now assume, that i_a, i_b, and i_c, are sinusoidal, equal in magnitude, and displaced by 120 deg. time phase, then

$$(d\,i_b/d\,t) + d\,i_c/d\,t = -\,(d\,i_a/d\,t),$$

$$d\,i/d\,t = j\,2\,\pi f\,\dot{I} \text{ and } d\,(M_{af}\,i_f)\,d\,t = \dot{E}_o$$

and equation (1) reduces to the familiar form

$$\dot{E}_o - j\,2\,\pi f\,(3/2)\,L_a\,\dot{I}_a = \dot{E}_a \qquad (7)$$

Or, stated in words, the terminal voltage in any phase with balanced load, is equal to the voltage induced in that phase at no load, minus a drop in time quadrature with the current, a drop which is usually called the synchronous reactance drop.

Inasmuch as I have been questioned by Professor Karapetoff in discussion of a recent article of the JOURNAL for the use of synchronous impedance even as a second approximation, I should like him to explain clearly for the benefit of the readers of the JOURNAL his position on this subject so far as it is implied in his paper.

R. H. Park (Communicated after adjournment): On the second page of his paper in referring to the coefficient of coupling between the phases of an alternator, Mr. Franklin states,

"Thus, for a three-phase alternator, this coupling coefficient was assumed equal to cos 120, or -0.5. This ideal value of coupling is seldom obtained in practise."

Actually, it may be shown that for a symmetrical three-phase

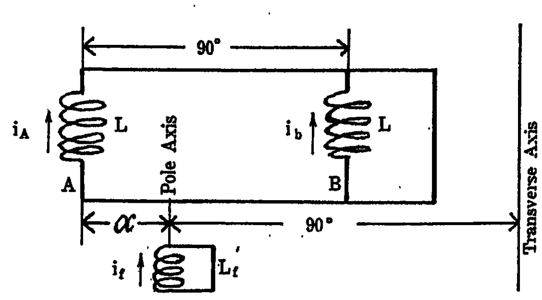


FIG. 10—DIAGRAM OF TWO-PHASE ALTERNATOR

alternator the numerical value of the mutual inductance M between phases must be less than the self inductance L per phase, that is, $L + 2\,M > 0$.

For suppose the alternator of Fig. 11 herewith to be used as a transformer, power being supplied to Phase 1 as the primary and taken from Phase 2 and 3 in parallel as the secondary.

The statement of Kirchoff's law is for the primary,

$$r\,i_1 + L\frac{d\,i_1}{d\,t} + \frac{M\,d\,(i_2 + i_3)}{d\,t} = e_1$$

and for Phase 2 of the secondary,

$$r\,i_2 + L\frac{d\,i_2}{d\,t} + \frac{M\,d\,(i_3 + i_1)}{d\,t} = e_2$$

But by symmetry it follows that the currents in Phases 2 and 3 are equal, that is, $i_2 = i_3$. Substituting and rearranging gives,

$$r\, i_1 + L \frac{d\, i_1}{d\, t} + \frac{M\, d\,(2\, i_2)}{d\, t} = e_1 \qquad \textbf{(c)}$$

$$r\, i_2 + \frac{L + M}{2} \frac{d\,(2\, i_2)}{d\, t} + \frac{M\, d\, i_1}{d\, t} = e_2 \qquad \textbf{(d)}$$

Since in equation (**d**) $2\, i_2$ represents the total current entering the secondary of the transformer, $\frac{L + M}{2}$ becomes the equivalent self-inductance of the secondary of the transformer.

In order that the coefficient of coupling of the transformer shall

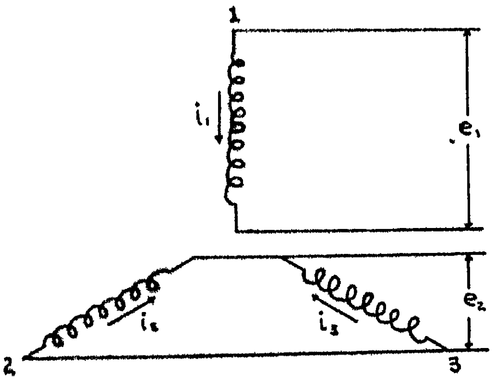


FIG. 11

be less than unity, it is necessary that $L_1 L_2 > M^2$ where L_1 is the primary and L_2 the secondary self-inductance and M is the mutual inductance of transformer. Therefore it follows that

$$L_1 L_2 = \frac{L\,(L + M)}{2} > M^2 \qquad \textbf{(e)}$$

$$L^2 + L\, M - 2\, M^2 = (L - M)\,(L + 2\, M) > 0 \qquad \textbf{(f)}$$

or since M is essentially negative $L + 2\, M > 0$.

N. S. Diamant (by letter): One of the interesting features of these papers is the extensive use of the ideas of mutual and self inductance and fluxes instead of reactances. There was a time when the use of these more elementary notions was not very popular and we had to use reactances and impedances. In this connection Prof. Karapetoff's method of dealing with permeances is very interesting and should prove useful. After a little practise the expressions (**7**) to (**11c**) should prove very simple and useful in general engineering work. One of the great advantages of such expressions is the fact that they keep before us a clear mental picture of the underlying physical phenomena. In this connection it is very important to remember the meaning of $K = M^2/L_f\, L_a$. If $K = 1$ then $M^2 = L_f\, L_a$ and there is no leakage flux between armature and field; this is a condition never realized and thus K is always smaller than 1 and $K^2 = \sigma\, \tau$ where σ and τ are the leakage coefficients for the armature and field respectively.

The fundamental treatment of the subject is the same in both papers although equations (**1**) to (**4**) of Prof. Karapetoff have been written in terms of e. m. f. and Mr. Franklin's equations (**3**) to (**6**) in terms of flux. For example, translating into English equation (**1**) of Prof. Karapetoff we have: e. m. f., e induced in phase A = the e. m. f. of self-inductance of phase A + the e. m. f. of mutual inductance of phases B and C + the e. m. f. of rotation due to the field flux. Similar to this is Mr. Franklin's equation (**113**) for a three-phase alternator or the simple equations (**3**) to (**6**).

Returning to Prof. Karapetoff's fundamental equations, it may be well to call attention to the fact that the total induced e. m. f., e is arbitrarily assumed to be a function of s and to be equal to $S + A$ for phase A; $S + B$ for phase B, etc. Then the value of s is taken as equal to S at the instant of short circuit and a new arbitrary relation involving angle ξ is introduced in equations (**26**) to (**28**).

Translating this statement into mathematical shorthand we obtain:

$$L_a\, i_a + M_{ab}\, i_b + M_{ca}\, i_c + M_{fa}\, i_f = (N_a\, \Phi \sin \omega\, t)\, \epsilon^{-\alpha_a (t - t_1)}$$

for phase A, and similar expressions for the other phases. For the field we have:

$$L_f\, i_f + M_{fa}\, i_a + M_{fb}\, i_b + M_{fc}\, i_c$$
$$= N_f\, [\, \Phi_{fasc} + (\Phi_f - \Phi_{fasc}\,)\, \epsilon^{-\alpha_f (t - t_1)}]$$

I have used the same notation as Prof. Karapetoff and t_1 or θ_1 is the time or the corresponding time angle at which the sudden short circuit occurs.

All this is very good, but an engineer will do well to keep in mind always the physical meaning and interpretation of the fundamental equations. Unlike the pure mathematician, when an engineer introduces arbitrary functions, he will do well to try to understand the physical side of his mathematics. If he does not, he will be merely riding through a tunnel, having left the light behind him and he may emerge again into light or he may not. I have tried to look over Prof. Karapetoff's paper from this point of view of coordinating mathematics with their physical meaning and although I had some difficulty in understanding the physical meaning of the functions introduced, no doubt this was my fault. Prof. Karapetoff with his usual clearness no doubt has presented the subject thoroughly but it may require more study to understand the physical meaning of all the fundamental concepts involved.

There is another very important point to be considered in connection with these papers, namely, that they give expression for instantaneous and sustained short-circuit currents but none for the transient.

It is simple to obtain the fundamental expression whose solution will give the transient currents as well as their initial and final values. Consider one of the phases of a three-phase alternator and suppose that phase No. 1 is at the position indicated in Fig. 12 herewith, when the three phases are suddenly and simultaneously short-circuited. Assuming a sinusoidal flux

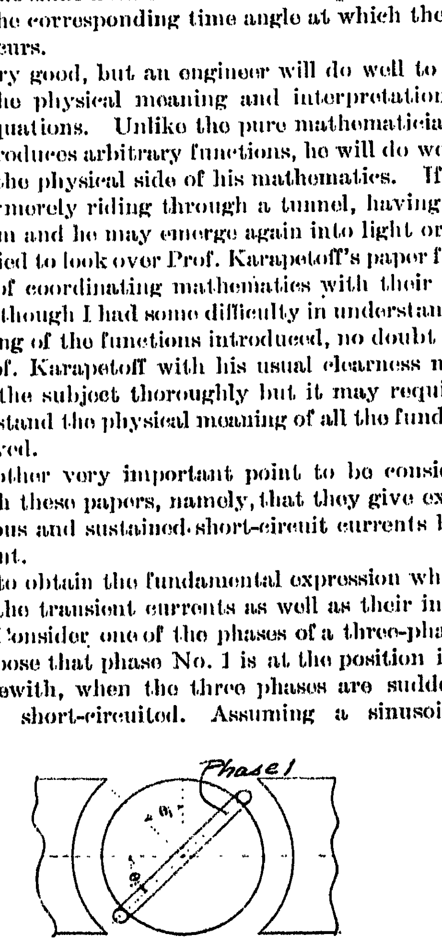


FIG. 12—CONDITION ASSUMED FOR DISCUSSION

distribution, the flux inclosed by phase No. 1 at the instant of the short circuit will be:

$$N_a\, \Phi_a \sin \omega\, t_1 = N_a\, \Phi_a \sin \theta_1$$

where $N_a\, \Phi_a$ = maximum flux inclosed by the N_a turns of any given phase. The flux inclosed by phase No. 2 at the instant of the sudden short circuit will be $N_a\, \Phi_a \sin (\theta_1 + 2\, \pi/3)$, and similarly for the third phase.

As is well known, the flux inclosed by any given phase under sustained short circuit is a very small fraction of $N_a\, \Phi_a$. Therefore, if it is assumed that the sustained short-circuit armature reaction (a) is wholly demagnetizing (b) and sinusoidally distributed like the field flux, and that (c) *the armature resistance drop is negligible compared to the reactance drop*; then a little consideration will show that the armature flux under sustained short circuit will be zero. Thus the flux linking with phase No. 1 is $N_a\, \Phi_a \sin \theta_1$, at the instant of the sudden short circuit when $t = t_1$; it is reduced to zero at the end of the short circuit when $t = \alpha$, and it is $(N_a\, \Phi_a \sin \omega\, t)\, \epsilon^{-\alpha_a (t - t_1)}$ at any time t during

the interval of sudden short circuit, α_a being the attenuation factor of the armature. Under normal or transient conditions the *flux linking phase No. 1 is equal at every instant to the mutually inductive flux coming from the field plus the self-inductive flux of phase No. 1 itself.* The corresponding condition holds true for the other phases. Compare Karapetoff's equations (1) to (4).

The field flux, which is $N_f \Phi_f$ under normal conditions, dies down to $\Phi_{fs\,c}$ at the end of the sudden short circuit or at the beginning of the sustained short circuit; at any time after the sudden short circuit it is equal to (See Fig. 14)

$$\Phi_f = \Phi_{fssc} + (\Phi_f - \Phi_{fssc})\, \epsilon^{-\alpha_f (t - t_1)}$$

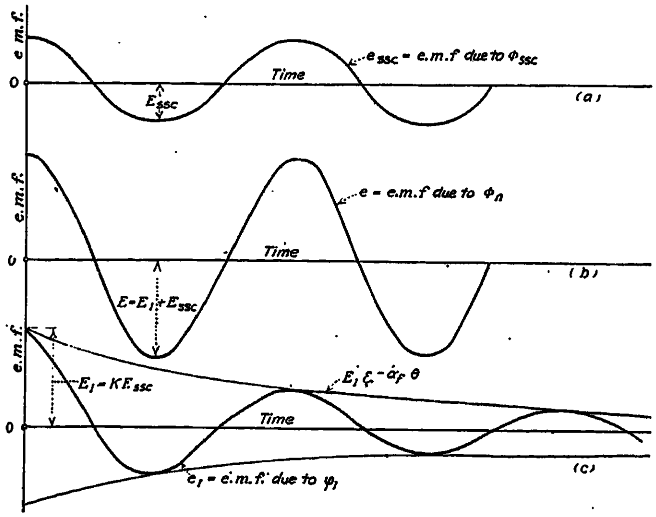


FIG. 13—E.M.FS. PRODUCED BY CONDUCTORS CUTTING ϕ_{ssc}, ϕ_n AND $(\phi_n - \phi_{ssc})$

where α_f is the attenuation factor of the field. Under normal or transient conditions this flux *is equal at every instant to the mutually inductive flux coming from the three phases plus the self-inductive flux of the field itself.*

With reference to Table II in Mr. Franklin's paper it was very gratifying for me to see that Cases 3 and 5 act so differently. In 1915 I had noticed this and in view of my meager data stated: "the case of single-phase short circuits, between two terminals or terminal and neutral, present considerable difficulties; the latter seems to give much higher current rushes than a short circuit between terminals," and as seen from the table the ratio is 1 to 1.732.

The sustained short-circuit phenomena are very interesting and particularly the flux distribution is quite fascinating. As a result of a fairly elaborate investigation I had shown in 1918[2] that the armature current and the e. m. f. waves may be nearly sinusoidal but the resultant flux wave may be extremely distorted for the simple reason that with the very low voltages which obtain under s. s. c. conditions the fundamental is so greatly reduced that the higher harmonics assume a very predominant role. For example, if the field flux consists of: Fundamental, 100 per cent; third harmonic + 10 per cent, and fifth harmonic + 5 per cent, while the armature flux consists of: Fundamental − 95 per cent; third harmonic, + 10 per cent, and fifth harmonic, + 5. per cent; then the resultant will be: Fundamental, + 5 per cent; third harmonic, + 20 per cent and fifth harmonic, + 10 per cent, — which will be a very distorted wave.

C. M. Laffoon (by letter): The general method of solution as used by Mr. Franklin in determining the magnitude of the short-circuit currents during the first cycle after the short-circuit occurs is the same as was used in my paper[3] on the same subject at the 1924 Midwinter Convention of the A. I. E. E. Moreover, the various cases of generator winding combinations and short-circuit conditions are also practically the same.

The expressions for the maximum or peak values of the short-circuit waves as derived by Mr. Franklin are in general, the same as given in my paper. However, there are two features or discrepancies to which it is necessary to call attention. It is indicated in Table II that the peak value of the current in the case of a single-phase short circuit between main terminals is irrespective of the width of the single-phase winding belt. The general expression for the peak value of the armature current in a single-phase short circuit is

$$i_a = \frac{2\, I_f\, M_o}{L_a - \dfrac{M_o^2}{L_f}}$$

Hence in order for the armature current of a single-phase generator with any width of winding belt to have a constant ratio with respect to a terminal-to-terminal three-phase short-circuit current, it is necessary for both numerator and denominator of the above expression to vary in the same way with respect to the width of the winding belt. There can be no question but that the quantity $2\, I_f\, M_o$, varies as the sine of the width of the winding belt. My analysis shows that the quantity $L_a - \dfrac{M_o^2}{L_f}$ varies approximately as the sine-square of the width of the winding belt. These relations are shown in Figs. 18 and 19 of my original paper. The general shape of the curve of $L_a - \dfrac{M_o^2}{L_f}$ as a function of the width of the winding belt has been checked experimentally by (a) locking the rotor so that the axis of the field winding coincides with the axis of the portion of the armature winding under consideration; (b) short-circuiting the field-winding; and (c) then applying single-phase voltage to the armature winding. The test results which were obtained on a 6250-kv-a. turbo generator are shown in Figs. 15 and 16 herewith. On this basis, the general relations between the different armature currents of a single-phase generator with different widths of winding belts are shown in Fig. 5 of my paper.

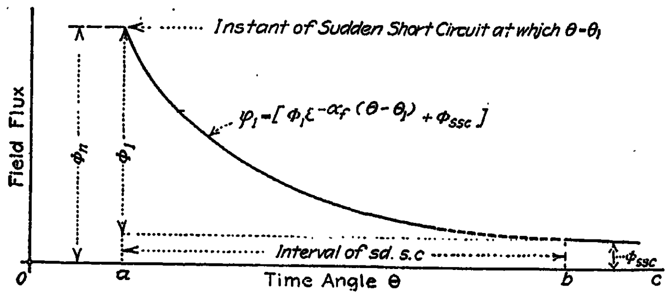


FIG. 14—ATTENUATION OF FIELD FLUX

In my paper, the maximum value of the armature current for a three-phase alternator when the short circuit occurs between terminals as given in Case 10 of Table I is based on the condition that the short circuit occurs when the axis of the field winding coincides with the axis of the portion of the armature winding included between two main terminals. If the short circuit occurs at the instant when the axis of the field winding coincides with the axis of one phase, terminal to neutral, the peak value of the maximum current will be the same as in Cases 7, 8 and 9 of my paper and Cases 8 and 9 of Mr. Franklin's paper.

V. Karapetoff: In my paper I assume that there is no saturation and no salient poles, but, as has been pointed out by Professor Douglas before this problem is finished, we ought to extend the solution to machines with definite poles and

2. Sustained Short-Circuit Phenomena and Flux Distribution of Salient-Pole Alternators, by N. S. Diamant, A. I. E. E. TRANSACTIONS, Vol. XXXVII, page 1141.

3. Short-Circuits Current of Alternating-Current Generators, by C. M. Lafoon, A. I. E. E. JOURNAL, August 1924, page 737.

machines involving saturation. In the initial short circuit, if there is saturation in the beginning and the field persists, that field will also contain saturation, after the instant of short circuiting. Mr. Franklin points out in his paper that this factor may be approximately taken into account by properly modifying the values of self and mutual induction.

Another answer to Professor Douglas is this: When we investigate the operation of a machine under normal conditions, we want to know its characteristics more or less exactly, but in the case of a short circuit, the indefinite resistance of the leads

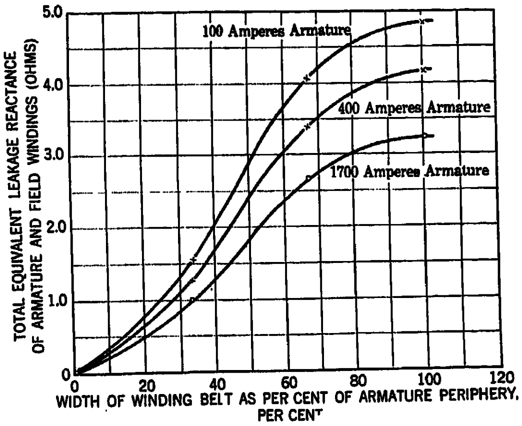


Fig. 15—Total Equivalent Leakage Reactance of Armature and Field Windings Combined. Axes of Two Windings Coincide, Single-Phase Armature Winding

and of the nature of the phenomenon itself are such that we can seek hardly a definite solution, but only a possible maximum. So that no such accuracy is required in problems involving short circuits as in problems referring to operating conditions.

Mr. Doherty wishes to know why I start with differential equations rather than using directly the condition of constant magnetic linkages. One reason is that in most of our problems we have resistance as a factor, and for one problem that involves no resistance, I probably have a dozen which contain resistances. So it seems more natural always to start with Kirchoff's Law. The other reason is this: In a star-connected combination, constant flux linkages refer to a closed circuit, which, in this case, contains two phases in series. Two equations of constant linkages may be written, and these equations will contain, say, phase No. 1 twice; but phases No. 2 and No. 3 only once. Therefore, the equations are not symmetrical. From a mathematical point of view, it is better to have symmetrical equations, and this is done in my paper by introducing an auxiliary voltage e. However, I grant that it is possible to introduce a fictitious flux Φ and in this manner keep the flux equations symmetrical. In other words, instead of writing the flux linkage equations in the form

$$\Sigma \Phi_1 - \Sigma \Phi_2 = \text{Const.}$$
$$\Sigma \Phi_1 - \Sigma \Phi_3 = \text{Const.}$$

they can be written in the symmetrical form

$$\Sigma \Phi_1 = \Phi + A$$
$$\Sigma \Phi_2 = \Phi + B$$
$$\Sigma \Phi_3 = \Phi + C$$

R. F. Franklin: The method of solution is, as stated in the introduction, the same as that used several times by R. E. Doherty in the solution of various kinds of short-circuit problems, and by C. M. Laffoon in his recent paper on alternator short-circuit currents. However, whereas Mr. Laffoon's paper deals principally with formulas for initial values and a physical interpretation of short-circuit phenomena by the constant-linkage method, the present paper gives the derivation of the formulas for the instantaneous values of both the initial and sustained short-circuit currents for all the ordinary alternator connections. These formulas will be found very valuable to the designer in many problems where a knowledge of short-circuit currents or forces is desired.

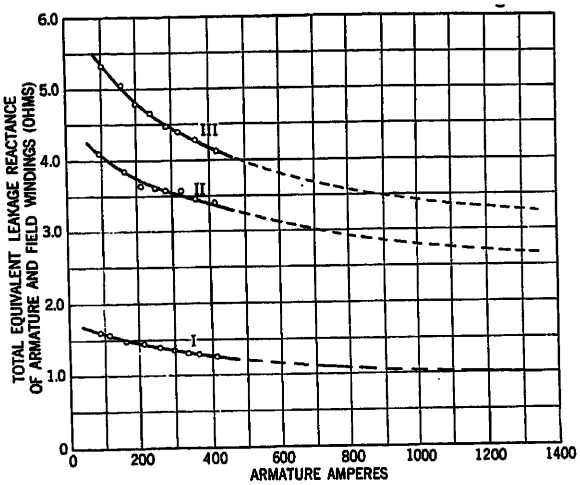


Fig. 16—Total Equivalent Leakage of Armature and Field Windings Combined. Axes of Two Windings Coincide. Single-Phase Armature Winding

I. One-Third of Armature Conductors in Series
II. Two-Thirds of Armature Conductors in Series
III. Three-Thirds of Armature Conductors in Series

I am indebted to Mr. Laffoon for calling attention to several errors in the table of maximum peak values of initial short-circuit currents (Table II). The wrong ratios were those for the single-phase and two-phase alternator cases which should have been divided by 2 and $\sqrt{2}$ respectively in order to reduce them to the bases of constant conductors per inch. This revision has been made in Table II. The value for the single-phase alternator case (case 1) depends upon the width of the phase belt as also pointed out by Mr. Laffoon. The width of the phase belt for the ratio given in Table II is 180 deg.

A Two-Speed Salient-Pole Synchronous Motor

BY ROBERT W. WIESEMAN[1]
Associate, A. I. E. E.

Synopsis.—The design features and performance characteristics of salient-pole synchronous motors are well known and have been thoroughly covered in the technical press. The synchronous motor has been handicapped in the past because it is inherently a single-speed machine, and a change in speed could be obtained only by a change in the frequency of the power supply. Changing the frequency, however, is not practical in most applications.

The special pole described in this paper which allows two-speed operation of a synchronous motor to be obtained at high efficiency is a new feature[2]. The same principle applied to a generator enables two frequencies to be obtained at the same speed or the same frequency at two different speeds. All that is necessary to change the speed (or frequency) is a pole-changing switch for the stator winding and a reversing switch for the rotor winding. A 5000/2500-h. p., 600/300-rev. per min., two-speed synchronous motor was built without having first constructed a model of any kind. This motor proved to be entirely satisfactory and its characteristics obtained by test agreed very closely with the calculated characteristics.

At either speed the two-speed synchronous motor functions exactly as the ordinary synchronous motor. There is nothing special or complicated about its construction, it does not require any more attention than the ordinary synchronous motor, and its expense of maintenance is just the same. The first cost of such a motor is only slightly more than that of the ordinary synchronous motor whose rating is equal to the low-speed rating of the two-speed motor. Therefore, the two-speed synchronous motor is a practical machine and it should open a new field for synchronous motor application.

* * * * *

Purpose

The purpose of this paper is to present the theory of the two-speed (constant torque) salient-pole synchronous motor, to describe a 5000/2500-h. p., 600/300-rev. per min., unity power-factor, 60-cycle, 2300-volt machine of this type. It will also be shown that its characteristics can be predetermined with the same degree of accuracy as those of the ordinary synchronous motor, and that this machine, when driven at constant speed, can be made to function as a two-frequency generator.

Introduction

The ordinary synchronous motor is inherently a one-speed machine because its revolving field, which is excited by direct current, must rotate in exact synchronism with the gliding magnetomotive force of the stator. Induction motors can be built to operate at two, three, and four speeds because the induction motor has a cylindrical rotor, a uniform air-gap, and a distributed rotor winding. These features make it easy to change the synchronous speed by regrouping the stator and the rotor coils. In the synchronous motor with the usual salient-pole construction, the air-gap is not uniform and the field winding is concentrated. Therefore, to operate a salient-pole synchronous motor from a constant-frequency supply at more than one synchronous speed, a special design is necessary. Such a motor is particularly suitable for constant-torque loads because synchronous motors are most efficient when operated at normal magnetic flux and current densities at either speed.

The multi-speed synchronous motor can operate at unity power-factor (or at any desired leading power-factor), whereas the multi-speed induction motor usually has a rather low lagging power-factor at the lower speed, unless a special compensating arrangement is used. It will be seen later that the particular two-speed synchronous motor described has an efficiency of 95.6 per cent when operating at 5000 h. p., 600 rev. per min., and at 2500 h. p., 300 rev. per min. It can thus be seen that the possibility of obtaining two speeds from synchronous motors opens a new field of application which so far has been covered by induction motors.

This particular motor (described later) was built to drive an a-c. generator at two speeds in order to obtain two frequencies. This motor can function equally well as a two-frequency generator when driven at constant speed. Thus a frequency converter set, consisting of two of these machines, could supply three different frequencies.

In some mine fan installations the full capacity of the fan is not required at certain periods. In cases where a speed ratio of one to two is satisfactory, the two-speed synchronous motor should be suitable. Although this load would not be a constant-torque load, the efficiency at both speeds could probably be made higher than that of a corresponding two-speed induction motor. Furthermore, the synchronous motor could be operated at any power factor to obtain power-factor correction. Thus, at the one-half speed condition only about one-eighth power is required and the remaining synchronous motor capacity can be utilized for power-factor correction. One difficulty in this application is that 100 per cent pull-in torque is required which necessitates a heavy starting winding. The ordinary starting winding can be used if a clutch is installed or if the motor is equipped with a rotating frame (supersynchronous motor).

Synchronous motors are now being used for ship propulsion. Changes in motor speed are accomplished by changing the generator frequency by varying the turbine speed. With a two-speed motor half speed can be obtained at normal frequency. The advantage of this scheme lies in the operation of the steam turbine at normal speed where the efficiency is maximum.

1. General Electric Co., Schenectady, New York.
2. U. S. Patent No. 1,491,451 April 22, 1924.

Presented at the Spring Convention of the A. I. E. E., St. Louis, Mo., April 13-17, 1925 and at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

Backwater conditions at certain waterpower plants are such that the effective head is much less (sometimes one-half) during the rainy season than during the dry season. For efficient operation the water turbine speed should be reduced with the head, a condition which gives a much lower turbine speed during the rainy season. In cases where the turbine speed is reduced to

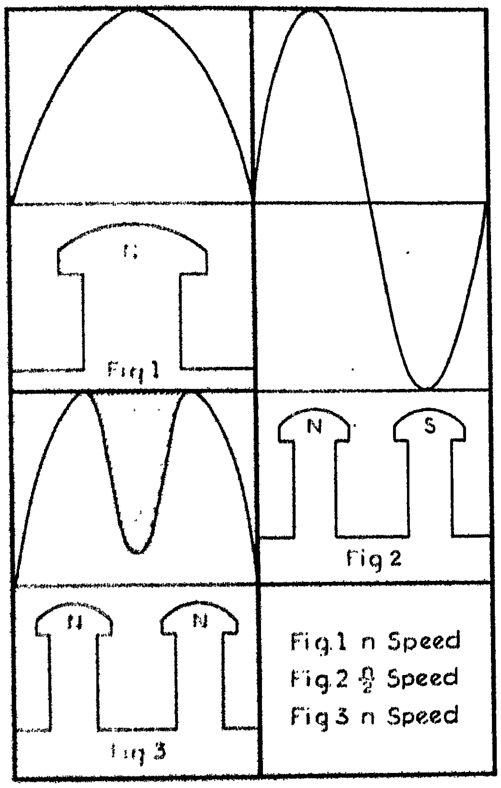


Fig. 1-2—Flux Distribution in the Air-Gap of an Ordinary Salient Pole Machine at no Load

Fig. 3—Flux Distribution in the Air-Gap of an Ordinary Salient Pole Machine at no Load when One Pole is Reversed

one-half, a two-speed generator would give normal frequency at both normal speed and one-half normal speed of the turbine.

It was originally hoped that the two-speed synchronous motor could be used advantageously for refrigeration. In the summer the ammonia compressors are usually worked continuously at full capacity. In the winter the compressors are unloaded (two-cylinder compressor operated with only one cylinder), a condition which requires abnormally large fly-wheels. Otherwise trouble due to hunting and excessive current pulsations will be experienced. This difficulty is now being overcome by using the modern clearance pockets in the cylinder of the compressor. With a two-speed motor both cylinders could be used at one-half speed which would give the same degree of refrigeration as at full speed with only one cylinder; but about five times the normal speed fly-wheel effect is necessary to prevent excessive current pulsations. Thus the fly-wheel for this condition is nearly as large as the one required when the compressor is unloaded, and the advantage of the two-speed synchronous motor in this case is not so marked.

Special Pole Necessary for Two Speed Operation

Fig. 1 shows diagrammatically the flux distribution in the air-gap over a pole-pitch in an ordinary salient-pole synchronous motor. In order to make this machine operate at half of its normal speed, the number of poles must be doubled and the armature winding must be reconnected accordingly. Thus, to have a motor which will operate at normal and one-half normal speed, it is necessary to arrange the armature and field windings so that they can be connected for either normal or twice normal number of poles. Fig. 2 shows the flux distribution in the air-gap over two poles of opposite polarity which are placed in the same space as that shown in Fig. 1. This represents at one-half speed the

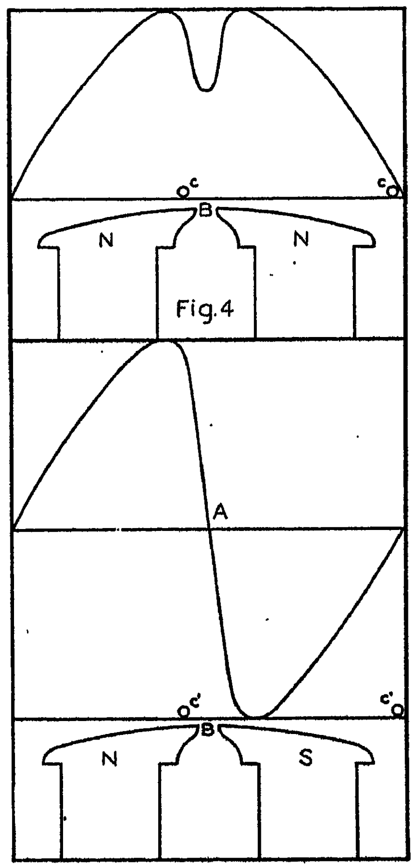


Fig. 4-5—Flux Distribution in the Air-Gap of a Salient Pole Machine at no Load with Special Poles at the High and Low Speeds Respectively

flux distribution, which is the same as in an ordinary machine. Now, to operate this machine (Fig. 2) at normal speed, the polarity of one of the poles must be reversed; the flux distribution will then take the form shown in Fig. 3. This condition gives a flux wave with a pronounced third harmonic which gives a very high core loss due to excessive hysteresis. Furthermore, the flux represented by the shaded area is lost and thus the capacity of the machine is decreased.

To overcome the objectionable condition shown in Fig. 3, the author devised a special pole piece. Figs. 4 and 5 show the flux distribution in the air-gap with this special pole shape for the high and low speeds respectively. Comparing the flux wave of Fig. 4 with that of Fig. 3, it will be apparent that the core loss will be be much less for the flux distribution represented by Fig. 4. Tests have shown that, with the special pole, (Fig. 4), at normal speed, the core loss is only 15 per cent greater than that of the ordinary salient-pole machine (Fig. 1); but this is not objectionable. At half speed, (Fig. 5), this special pole increases the core loss over that shown in Fig. 2. The hysteresis loss is the same in

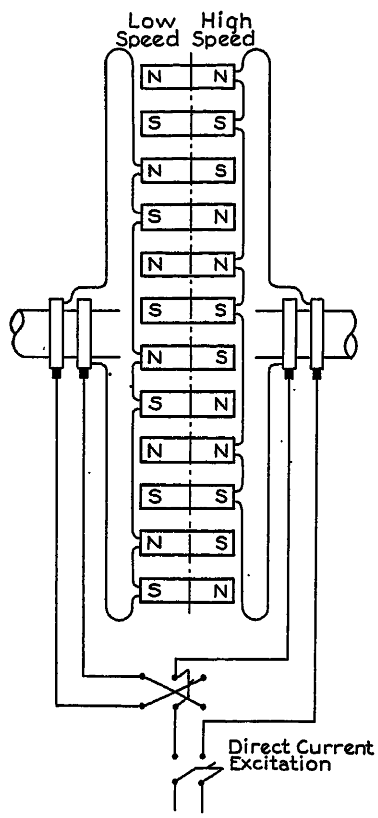


Fig. 6—Method of Changing Poles on the Rotor of a Two-Speed Synchronous Motor

both cases, but the eddy-current loss is greater due to the steep wave front at A. Tests show, however, that the core loss for the half speed condition, (Fig. 5), is also only about 15 per cent greater than that of an ordinary salient-pole machine, (Fig. 2). The distance between the poles B should be made large enough to prevent a high leakage flux for the low speed condition, (Fig. 5). This leakage flux would be high only in a machine with a very large number of narrow poles. The shape of the pole face is a compromise between the best shapes required at the two speeds respectively. If the motor is to operate most of the time at the high speed, the distance between the pole pieces B, (Fig. 4), should be made relatively small. Conversely, if the motor is to operate mostly at the low speed, the distance B should be increased.

Scheme for Changing Number of Rotor Poles

The scheme for changing the number of effective rotor poles in the ratio of two to one is shown by Fig. 6. The polarity of the polar projections for the high speed is indicated by N, S, S, N, etc., to the right of the vertical center line and the polarity for the low speed is indicated

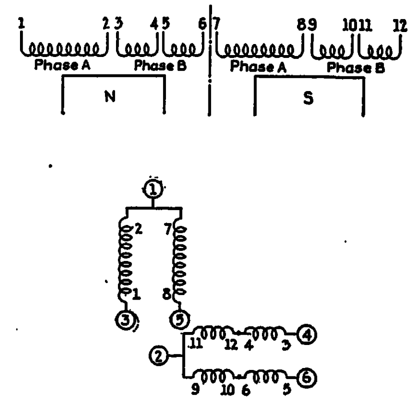


Fig. 7

by N, S, N, S, etc., to the left of the vertical center line. To simplify the method of connecting the field coils, half of the coils have their connections made on one end of the rotor and the other half are connected on the opposite end. It should be noted that every other pair of polar projections have the same polarity for both the high and low speed conditions and the field coils on these polar projections are connected to one set of collector

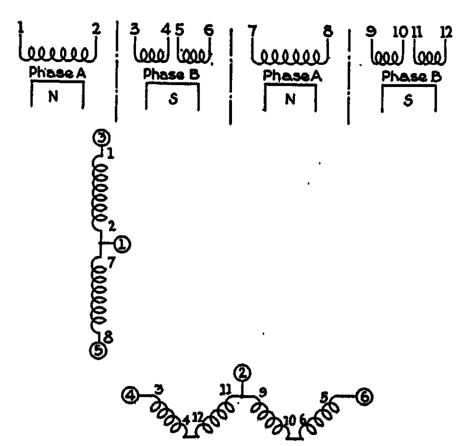


Fig. 8

rings—to the right. The pairs of polar projections whose polarities must be reversed when changing from the high to the low speed, or vice versa, are connected to the other set of collector rings—to the left. The two sets of collector rings are connected in series through a reversing switch. The number of rotor poles is changed in the ratio one to two,—high to low speed, or vice

versa, by throwing the reversing switch to the right or to the left.

Description of Stator

The stator of a two-speed synchronous motor is practically the same as the stator of an ordinary synchronous motor. The stator frame and stator punchings are exactly the same. No special arrangement of the turns or the conductors in the coil is necessary. The only difference is in the stator coils and coil connections. The coils usually have a smaller pitch than those used on ordinary synchronous machines and thus the coil and projection is less. The stator coil pitch can be varied within certain limits to accommodate the conditions of design. At the high-speed connection the coil pitch can be made 50 to 60 per cent (coil $c\,c$, Fig. 4), which makes the pitch 100 to 120 per cent (coil $c'c'$, Fig. 5) at the low speed connection.

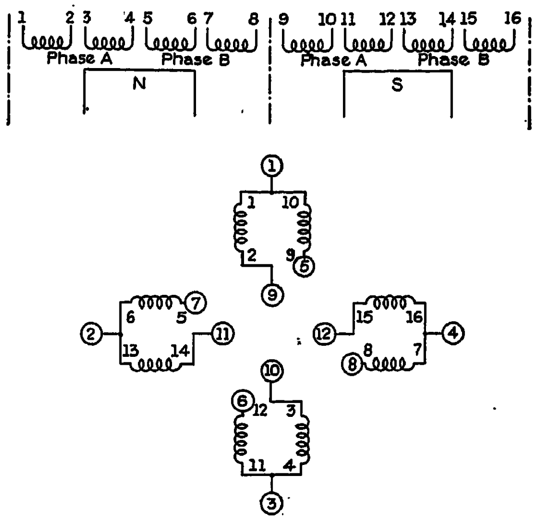


Fig. 9

A-c. motors are usually supplied with electric power from constant potential lines. Since it is desirable to have the magnetic flux density in the air-gap approximately the same at both speeds, it follows that the number of turns in series per phase of the stator winding at low speed must be twice the number in series at high speed. Therefore, changing the number of stator poles in the ratio of one to two consists of changing the number of circuits in the ratio of two to one in such a manner that half of the phase groups are reversed. The method of reconnecting one phase of the stator winding by reversing every other phase group is similar to reconnecting the field coils as shown by Fig. 6.

Two-Phase Stator Windings

Figs. 7 and 8 show the method of connecting a two-phase stator winding for the high and low speeds respectively. This connection requires only six terminals and it can be used for either three- or four-wire two-phase circuits. The advantage of this connection is its simplicity. For this reason it is used on induction motors. Its disadvantage is that the phase belt is 180 deg. at the low-speed condition. A 180-deg. phase belt is not recommended for a two-speed synchronous motor because the short-circuit core loss with a 180-deg. phase belt is nearly twice that obtained when the phase belt is 90 deg. Furthermore, the flux per pole is 41 per cent greater with the 180-deg. phase belt, and thus the core

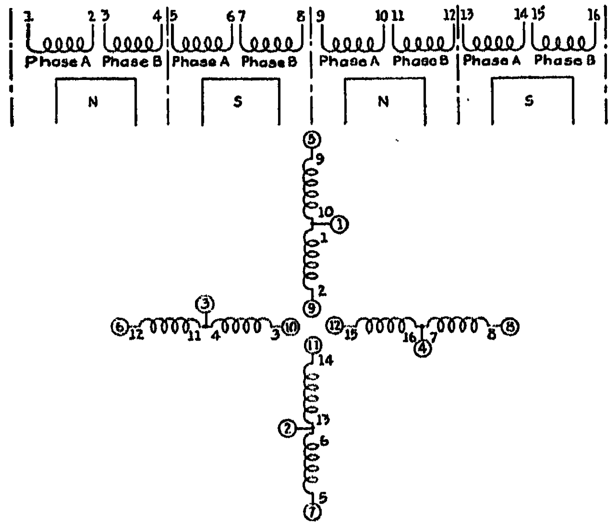


Fig. 10

loss is about 90 per cent greater and the field $R\,I^2$ loss about 250 per cent greater.

To overcome the objectionable features of the 180-deg. phase belt at the low-speed condition, a 90-deg. phase belt should be used and the stator winding connected as shown by Figs. 9 and 10. In Fig. 9, each phase is split into two 45-deg. belts, which become 90

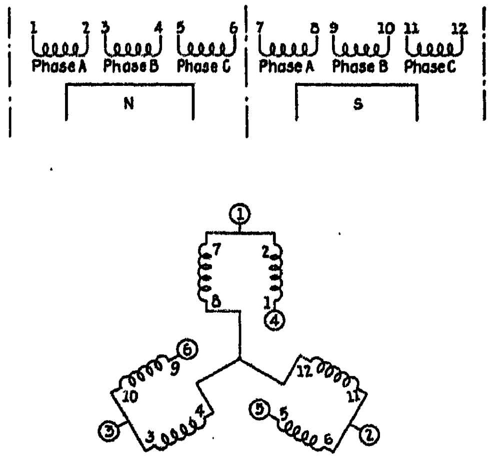


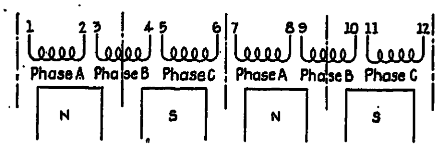
Fig. 11

deg. belts in Fig. 10. Twelve terminals are required when the two-phase power supply is three-wire. If the power supply is four-wire, the leads 9, 10, 11, and 12, (Figs. 9 and 10), can be connected internally and then only eight terminals are necessary. In this case the line terminals for the high speed, Fig. 9, will be 1-3,

2-4, and the leads 5, 6, 7, and 8 are connected. For the low speed, Fig. 10, the lines will be 5-7, 6-8, and no connection will be necessary.

Three-Phase Stator Winding

Figs. 11 and 12 show the wiring diagrams for a three-phase stator winding which requires only six terminals.



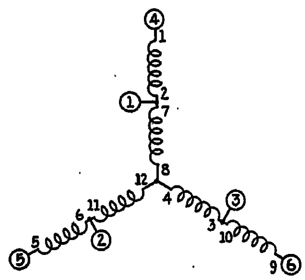


Fig. 12

The phase belt for the high-speed condition is 60 deg. and for the low-speed condition 120 deg. The increase in the phase belt from 60 deg. to 120 deg. helps to counteract the decrease in flux due to the increase of the stator coil pitch when changing from high to low speed.

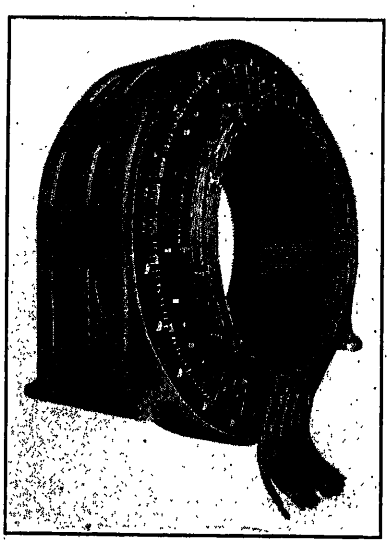


Fig. 13—Stator of a 12/24-Pole, 5000/2500-H. P. 600/300-Rev. per Min. Two-Speed Synchronous Motor

Description of a 5000/2500-H. P., 600/300-Rev. per Min., Two-Speed Synchronous Motor

Fig. 13 shows the stator of a 5000/2500-h. p., 600/300-rev. per min., 12/24-pole, 60-cycle, unity power-factor, two-phase, 2300-volt, two-speed, synchronous motor with the end shields removed. The stator winding is arranged so that a 90-deg. phase belt is obtained at both speeds.

Fig. 14 shows the rotor completely assembled. The poles are equally spaced so that the maximum amount of interpolar space can be utilized for the field coils. This motor is used on a 24,000-kv-a. frequency converter set where it was desirable to place the motor coupling between the rotor and the bearing. In order to reduce the axial length of the set to a minimum, all four collector rings were placed at one end of the rotor. The usual practise is to place the coupling beyond the bearing and then it is desirable to place two collector rings on each side of the rotor.

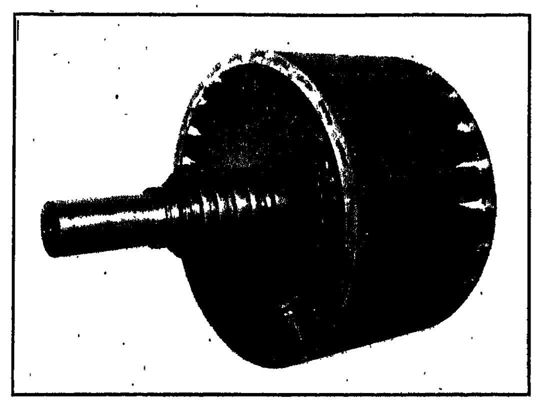


Fig. 14—Rotor of a 12/24-Pole, 5000/2500-H. P. 600/300-Rev. per Min. Two-Speed Synchronous Motor

Fig. 15 gives a section view of the motor showing the unsymmetrical pole tip. The poles are equipped with an amortisseur starting winding, consisting of five bars per pole. Starting tests show that the unsymmetrical pole face has no tendency to cause the motor to lock at a sub-synchronous speed when it is connected for either speed.

An interesting feature in the rotor construction of this two-speed motor is the possibility of arranging the field coils at the high speed connection to form a two-phase winding which would give considerable starting torque when the motor is operated as an induction motor. This can be done by dividing the coils into two independent groups, each group consisting of alternate

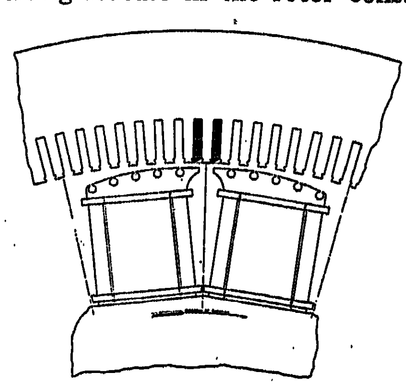


Fig. 15—Section View of Two-Speed Synchronous Motor

coils connected in series, and by short-circuiting the groups separately. The disadvantage of this scheme is that eight collector rings would be required and thus

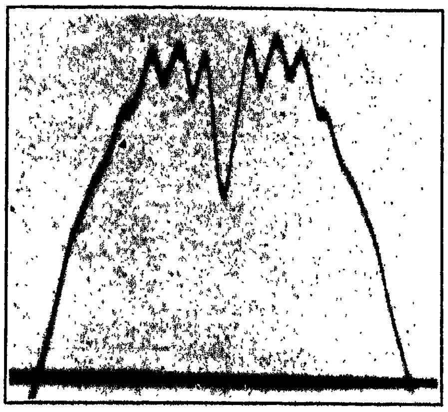


FIG. 16—FLUX WAVE IN AIR-GAP AT HIGH SPEED NO LOAD

the axial length of the motor would be materially increased.

PREDETERMINATION OF MOTOR CHARACTERISTICS

The actual flux distribution in the air-gap is obtained by taking an oscillogram of the voltage induced in an exploring coil placed on the armature face. Figs. 16 and 17 show the flux waves obtained by tests for the high and low speed conditions respectively.

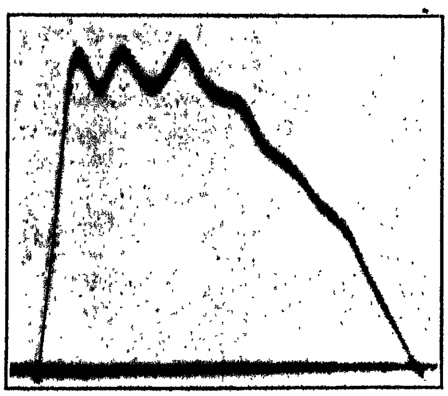


FIG. 17—FLUX WAVE IN AIR-GAP AT LOW SPEED NO LOAD

Fig. 18 shows the outline of the magnetic field structure drawn to scale. The flux distribution in the air-gap is obtained graphically by plotting the equipotential lines of magnetomotive force and the tubes of magnetic flux. The influence of the stator and rotor slots is neglected. The dotted lines in Fig. 18 give the calculated flux distribution in the air-gap and the full lines show the flux distribution obtained by test. In plotting the actual flux waves (Figs. 16 and 17) in Fig. 18 the ripples due to the rotor amortisseur winding slots were neglected. It should be noted that in Fig. 18 the full wave (one-half cycle) of the low-speed flux wave is shown and only half (one-quarter cycle) of the high-speed flux wave is shown, since this is a symmetrical wa e (Fig. 16). The calculated flux waves agree very closely with those obtained by test. Since the predetermination of the flux distribution in the air-gap is the foundation upon which the design calculations are based, it follows that the characteristics of a two-speed synchronous motor can be readily predetermined.

Fig. 19 shows the calculated and test saturation, syn-

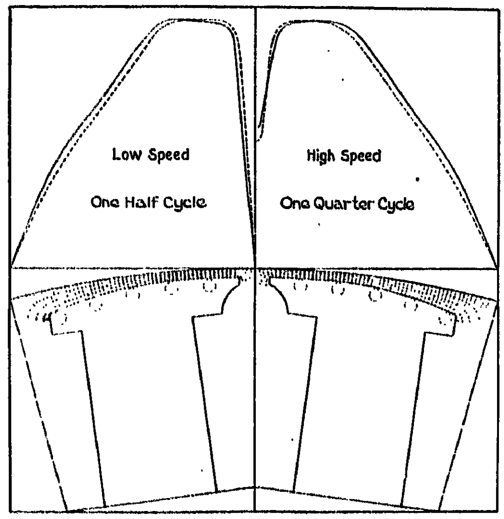


FIG. 18—FLUX DISTRIBUTION IN AIR-GAP AT NO LOAD

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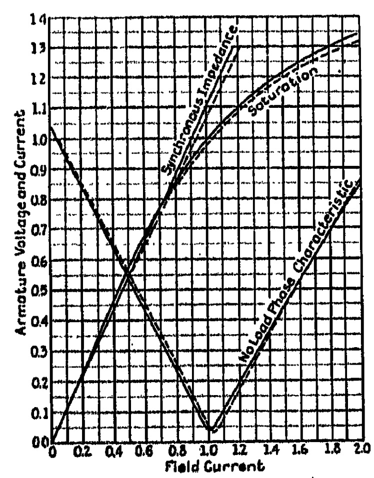

FIG. 19—CHARACTERISTIC CURVES 12/24-POLE 5000/2500-H. P. 600/300-REV. PER MIN. TWO-SPEED SYNCHRONOUS MOTOR AT 600-REV. PER MIN. NORMAL VOLTAGE

——————— TEST
. . . . CALCULATED

chronous impedance, and no-load phase characteristic curves for the high-speed condition; Fig. 20 gives similar low-speed characteristics. Figs. 21 and 22 give the starting torque and starting current for the high and low-speed conditions respectively. A comparison of Figs. 21 and 22 will show that the starting torque developed with the high-speed connection is much higher than that developed under the low-speed con-

dition. The flux density in the air-gap is higher and the reactance is lower for the high-speed condition, and since the starting torque varies approximately as the square

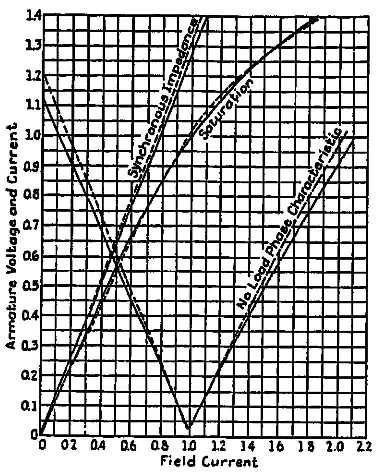

FIG. 20—CHARACTERISTIC CURVES 12/24-POLE 5000/2500-H. P. 600/300-REV. PER MIN. TWO-SPEED SYNCHRONOUS MOTOR AT 300-REV. PER MIN. NORMAL VOLTAGE

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of the flux density and inversely as the reactance, it follows that the high-speed condition should give a higher starting torque.

The efficiency curves of this motor at the high and low

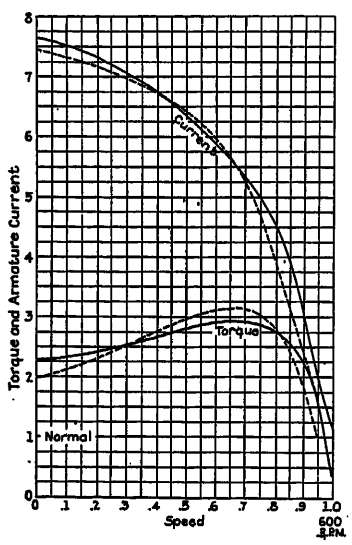

FIG. 21—STARTING TORQUE 12/24 POLE 5000/2500-H. P. 600/300-REV. PER MIN. TWO-SPEED SYNCHRONOUS MOTOR NORMAL VOLTAGE 600-REV. PER MIN. CONNECTION

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speeds are shown in Fig. 23. These efficiencies were obtained experimentally by the segregated-loss method and they include the windage and friction loss, open-circuit core loss, short-circuit core loss, armature and field RI^2 losses. These efficiency curves show that at normal load the efficiency is about the same at both speeds. This refers to 5000 h. p., 600 rev. per min., and

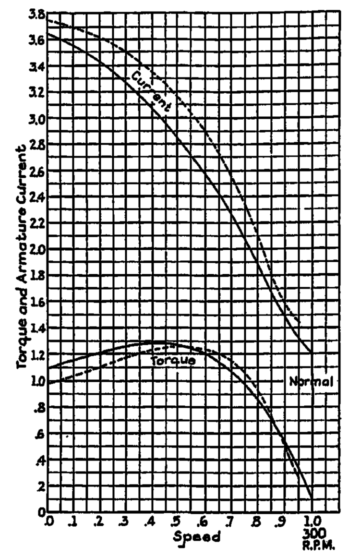

FIG. 22—STARTING TORQUE 12/24 POLE 5000/2500-H. P. 600/300-REV. PER MIN. TWO-SPEED SYNCHRONOUS MOTOR NORMAL VOLTAGE 300-REV. PER MIN. CONNECTION

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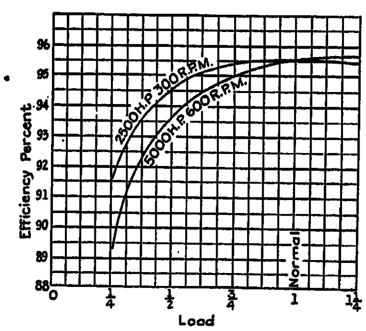

FIG. 23—EFFICIENCY 12/24 POLE 5000/2500-H. P. 600/300 REV. PER MIN. TWO-SPEED SYNCHRONOUS MOTOR

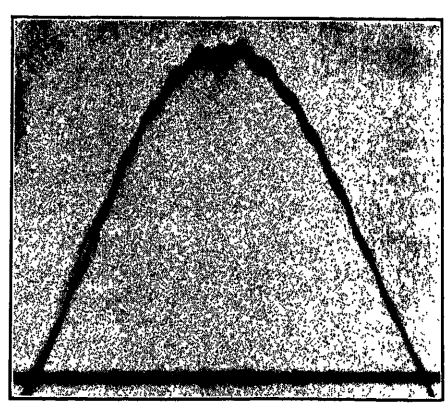

FIG. 24—VOLTAGE WAVE AT HIGH SPEED NO-LOAD

to 2500 h. p., 300 rev. per min. The stator RI^2 losses are about the same in either case, but the windage and core losses are much less at the low speed. This accounts for the high efficiency at the low speed.

Two-Frequency Generator

A two-speed synchronous motor can also function as a two-frequency a-c. generator when driven at constant speed. Although the flux wave at either frequency deviates appreciably from a sine wave, the voltage wave can be made nearly sinusoidal by a suitable choice of the number of slots and of fractional pitch of the arma-

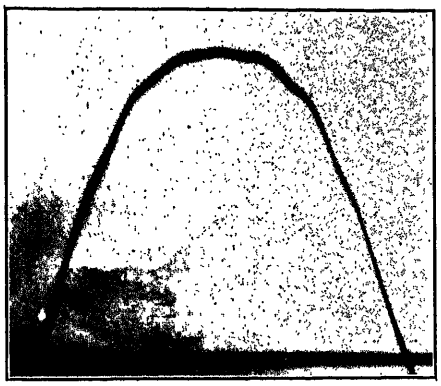

Fig. 25—Voltage Wave at Low Speed No-Load

ture coils. Fig. 24 shows an oscillogram of the no-load voltage wave when the above described machine is connected for the normal number of poles, and Fig. 25 for twice the normal number of poles. These voltage waves are really good waves when we consider that the machine was connected two-phase and had an integral number of armature coils per pole per phase. The voltage waves would be much better in a three-phase machine with a fractional number of stator coils per pole per phase. By comparing the flux wave, Fig. 16, with its voltage wave, Fig. 24, and similarly, Fig. 17 with Fig. 25, it will be seen how much the pitch and distribution of the armature coils reduce the various flux ripples and harmonics.

The efficiency of this machine obtained by test, when used as a multi-frequency generator, is as follows:—

300 Rev. per Min.

12 poles, 30 cycles	24 poles, 60 cycles
2000 kw., 1.0 P. F.	2000 kw., 1.0 P. F.
Efficiency 95.4 per cent	95.6 per cent

600 Rev. per Min.

12 poles, 60 cycles	24 poles, 120 cycles
4000 kw., 1.0 P. F.	4000 kw., 1.0 P. F.
Efficiency 95.7 per cent	95.8 per cent

The efficiency at 300 rev. per min., 60 cycles is slightly higher than at 300 rev. per min., 30 cycles because the field $R\,I^2$ loss is less for the 24-pole connection.

Conclusion

There is nothing special or complicated about the construction of the two-speed synchronous motor. Its performance can be predetermined with the same degree of accuracy as that of the ordinary synchronous motor. It does not require any more attention than an ordinary synchronous machine, and its maintenance expense is just the same. The cost of such a motor is only slightly higher than that of an ordinary synchronous motor whose rating is equal to the low-speed rating of the two-speed motor. Therefore, this new synchronous motor is a practical machine and should open a new field for synchronous motor application.

Discussion

S. H. Mortensen: Mr. Wieseman's paper brings out, clearly, the fact that two-speed synchronous motors can be built economically, and that a machine of this kind properly proportioned will practically maintain all the virtues of the standard salient-pole synchronous motor. The field of application for a motor of this kind is at the present time rather limited. The only application the speaker can think of in addition to the ones Mr. Wieseman has mentioned is that for driving two-speed pumps such as are sometimes used with condenser installations.

This type of drive would, of course, not be a constant-torque proposition at the two speeds any more than the mine fans mentioned by Mr. Wieseman. The horse power required at either of these drives varies approximately with the cube of the speed of operation. A study of the starting characteristics shown in Figs. 21 and 22 indicates that with 12-pole stator connections, this machine can start and synchronize more than full load. With 24-pole connections, its starting characteristics are much inferior. However, very large loads corresponding to the 24-pole operation could be brought into synchronism by starting the motor as a 12-pole machine, bring it up to a speed beyond the 24-pole synchronous speed and then by suitable switching, change its stator connections from a 12-pole to a 24-pole winding. The fields could then be excited and the motor would slow down and lock into synchronism. If this procedure is followed for fans, pumps and similar drives, the motor could be started upon a comparatively low starting voltage and brought into synchronism without causing undue line disturbances.

In connection with the starting-torque curves, Figs. 21 and 22, it would be of interest to know if these curves were obtained with the motor fields short-circuited on themselves, or through a resistance, or, possibly, with the field circuit open at the starting period.

The efficiencies shown in Fig. 23 are at full load from 1 per cent to 1½ per cent lower than the efficiencies that might be expected upon a single-speed machine of this rating. At fractional loads, this condition will be even more favorable to the standard machine. In this connection it would be of interest to know what class of steel is used in the stator of this particular machine. The flux-distribution curves shown suggest that high-silicon steel would have a decided advantage over standard steel to the extent of reducing eddy-current and hysteresis losses. As the flux waves shown in Figs. 16 and 17 have very decided ripples, it would be of interest to know whether this machine was noisy during operation.

Regarding Mr. Wieseman's statement that a machine of this design is only slightly more expensive than a standard machine, it would seem to the speaker that this would apply only to high-speed machines, where the field leakage and heating of the rotor coils is not a limiting feature. Where slow-speed machines are involved, the increased field leakage, in addition to the reduced field ventilation caused by the proximity of the pole tips, would limit the output and make it necessary to supply a considerably larger machine for two-speed operation than would be necessary for a standard one-speed machine.

H. Weichsel: We have been accustomed for many years to hear and talk about multiple-speed induction motors, and, on the other hand, we had a more or less deep-rooted belief that synchronous motors are inherently single-speed machines.

This limitation has been taken more or less as a matter of course without analyzing the underlying reasons why synchronous motors were not built as multiple-speed machines.

During the last few years, the general interest in the synchronous motor has grown enormously. This is largely due to the better understanding of the tremendous losses and engineering difficulties which are created in a-c. systems when energy is transmitted under low power factor. The operating difficulties and economic losses, due to low power factor, hit the operating engineer first, then the consumer, and, finally, reacted upon the manufacturer of the electric machinery.

We are now in the third stage, as is evidenced by the fact that a very large number of operating engineers and power consumers have turned for help to the motor manufacturers, asking them to assist in solving the tremendously important question of power-factor correction. The result of this appeal has been that during the last few years, great efforts have been made to introduce more generally the application of such motors as inherently operate at unity or even leading power factor. It is a well-known fact that the synchronous motor has the desired property of operating with leading or unity power factor. Unfortunately, while the conventional type of synchronous motor possesses the desirable ability of good power factor, yet it is also guilty of several serious shortcomings in comparison with the induction motor so generally used at present.

The designing engineers, as well as the inventors, have worked diligently during the last few years on the problem of freeing the conventional synchronous motor of its shortcomings. It is well-known that during the last few years remarkable progress has been made in approaching the goal of an ideal single-speed synchronous motor, but thereby still leaving the field of multiple-speed motors to the induction machines. This, on the other hand, is particularly undesirable from the power factor viewpoint, as all induction motors, and especially multiple-speed induction motors, have a poor power factor at low speeds.

Mr. Wieseman is, therefore, to be congratulated for having attacked the problem of two-speed synchronous motors and solved it in a remarkably satisfactory manner.

As far as I can see, in analyzing the problems of two-speed synchronous motors, it appears that the real problem lies in the rotating or d-c. member. The regrouping of the d-c. pronounced poles, such as are used in the conventional type of synchronous motor, is extremely simple in principle when a speed ratio of 1-to-2 is desired and has been understood for many years. Unfortunately, such a regrouping of the poles of a standard synchronous motor produces an entirely unsatisfactory field distribution which results in excessive losses and low weight efficiency of the machine. The problem of producing two-speed primary windings, on the other hand, is in principle the same as that which for many years has been solved in connection with two-speed induction motors.

The important question is, therefore, the creation of a d-c. member which can be satisfactorily excited for two different numbers of poles. In order to accomplish this result, Mr. Wieseman found it necessary to reduce the distance between the pairs of poles to a value very materially below the distance between pole horns as found in standard designs. From Fig. 4 of the paper, it appears that the gap between two adjacent pole horns is made so small that the surface of those poles which form one pair approaches, in its appearance, the cylindrical surface of an induction-motor pole. The following question arose in my mind:

Why not go the whole way and construct the rotor entirely on the lines of an induction motor and provide the rotor with a winding which allows the regrouping of poles in the ratio of 2-to-1. It is well known that such an arrangement can readily be obtained, for instance, by winding the rotor exciting winding as per Fig. 1 herewith. This winding consists of two groups of poles which can be reversed against each other, similar to the arrangement shown in Fig. 6 of Mr. Wieseman's paper. Single-speed synchronous motors without pronounced poles, using a kind of induction motor rotor, have been built very successfully for several years, especially by some European concerns. They are known under the name of "synchronous induction motors."

These motors have shown themselves in several respects superior to the conventional type with pronounced or salient poles. It is possible to obtain with this type of synchronous motor starting and synchronizing characteristics which are superior to those usually available in pronounced pole synchronous motors. This advantage is obtained by using the rotor during starting as the wound secondary of an induction motor.

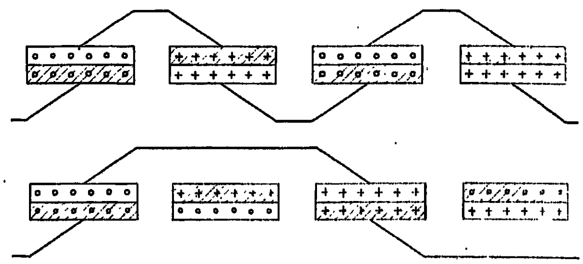

Fig. 1

By introducing resistance in this winding, a high starting torque with low starting current can be obtained, and when machine has come up to speed, the induction motor slip can be held very low, as the winding is completely short-circuited. This low slip, on the other hand, results in a good synchronizing torque.

I have made some very rough calculations for a medium-size machine of moderate speed which seem to indicate that two-speed synchronous induction motors are quite feasible. I would like to ask Mr. Wieseman's opinion on the possibility of such a type of machine. No doubt, Mr. Wieseman has investigated this problem and is in a position to point out the shortcomings of a two-speed synchronous induction motor when compared with a two-speed salient-pole synchronous motor.

Mr. Wieseman makes a statement which is particularly interesting to me, namely, that the stator winding for the high-speed connection should have a coil pitch of about 50 per cent to 60 per cent which makes the coil pitch 100 per cent to 120 per cent for the low-speed connection.

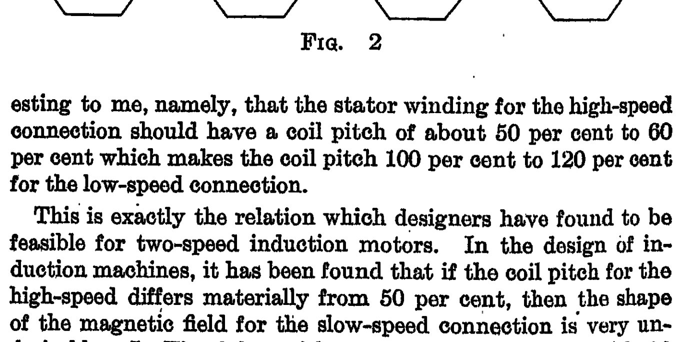

Fig. 2

This is exactly the relation which designers have found to be feasible for two-speed induction motors. In the design of induction machines, it has been found that if the coil pitch for the high-speed differs materially from 50 per cent, then the shape of the magnetic field for the slow-speed connection is very undesirable. In Fig. 2 herewith, a three-phase winding with 66 per cent for high-speed (4 poles) and 132 per cent for low-speed (8 poles) has been shown. The field for the slow speed can be considered as being made up of a symmetrical 8-pole field and superposed over this are fields with a different number of poles than desired. These higher "harmonic" fields are quite detri-

mental for multiple-speed induction motors and are often responsible for subsynchronous speeds and also for abnormally large leakage with consequent reduced output of the machine. I surmise that this is the reason why Mr. Wieseman recommends for two-speed synchronous motors, a coil pitch of 50 per cent to 60 per cent of the high-speed pole pitch.

The starting performance of the two-speed synchronous motor as given on the seventh page of Mr. Wieseman's paper is very satisfactory. It would be of interest, however, if we could hear from Mr. Wieseman regarding the synchronizing torque which these machines are capable of developing, and in this connection, he might enlighten us also on the question as to whether these machines can be switched from low-speed connection to full-

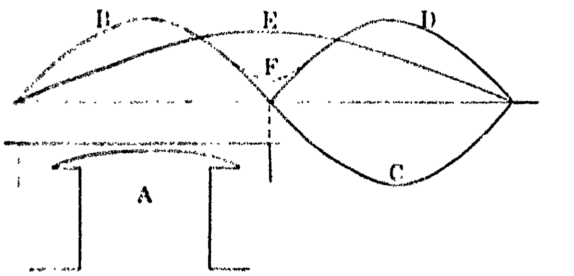

FIG. 3 POLE SHOE GIVING IDEAL WAVE FORM AT HALF SPEED

speed connection, and vice versa when operating under full load. In other words, is or is it not necessary to remove the load before the windings can be switched from one speed to another?

In Figs. 16 and 17 of Mr. Wieseman's paper, oscillographic records are given of the voltage in a search coil. Mr. Wieseman then draws in Fig. 18, the "tested" field shape by using the oscillogram records for the voltage neglecting the ripples in the voltage oscillograms.

In the A. I. E. E. PROCEEDINGS 1912, page 526, I pointed out that the photographic records of the voltage induced in a search coil do not represent the true field distribution, because due to the passing of the teeth, the magnitude as well as the shape of the magnetic field produced by the d-c. winding is not constant but changes rapidly when the teeth pass each other, due to the movement of the rotor.

I presume that this phenomenon induced Mr. Wieseman to draw the tested field curve from the oscillograph record by neglecting the ripples and thereby obtaining a kind of an average.

J. F. H. Douglas and E. W. Kane (by letter): One understands upon reading the section headed "Special Pole Necessary for Two-Speed Operation," that this pole contributes to the efficiency of the motor's operation, and that the core loss is kept within 15 per cent of the value usual in single-speed motors of this size, and with a reasonable leakage. However, it does not appear that the more usual shapes of pole shoe would be inoperative, nor is it claimed that the pole shown is a form giving maximum effectiveness or that the wave form is good. Data on the maximum power factor obtainable with this motor running light would be of interest in this connection.

One understands that the poles are close together to cut down core loss, yet by referring to Figs. 16 and 17, we see pronounced ripples which must increase core loss. These ripples are of the 31st and 33rd order in Fig. 16 where the stator teeth per pole are 16. Fig. 17 has large 15th and 17th harmonics when the stator teeth per pole are 8. Both cases come under the general rule applying to tooth harmonics, namely, if N is the number of teeth per pole (being an integral number) then they may cause harmonics of the $(2\,N \pm 1)$ order. They may be eliminated, of course, by partly closing the slots, but they can be also eliminated by eliminating harmonics of tooth-ripple frequency from the pole shoe. The proof of this proposition was given in an appendix to a paper we read in Chicago, June 1924, before A. I. E. E. on "Potential Gradient and Flux Density."[2] At any rate, a Fourier analysis of curves in Fig. 18 was made, and even without pronounced ripples of flux showing, harmonics of considerable size of the above orders were found.

An ideal wave form for low speed is shown in Fig. 3 herewith by curve $B\ C$ from poles A. When the poles are reversed for high-speed operation the curve $B\ D$ would probably result with the cusp replaced by F. The core loss of this wave would probably be 50 per cent greater than a sine wave, but the flux lost would not be the area between the two peaks B and D but merely the difference between the peak B, and the peak of E the fundamental sine-wave component. The T. I. F. of the wave was computed as 235, and while interference is of no particular interest in this connection, this factor will serve as well as any for relative comparisons.

An ideal wave form for the high speed is shown in Fig. 4 herewith by the curve $B\ C$ produced by the split pole A. When the pole halves are properly reversed for half speed, the wave $B\ D\ E$ results, which has a T. I. F. of 1040, worse than the first wave in Fig. 3. With the pole tips separated to point F, we have the arrangement devised by Mr. Weiseman, with rounded corners H for the low speed, and the dip or dimple G for the high speed. We analyzed the wave forms shown in Mr. Weiseman's paper in Fig. 18, and found that for the low speed the T. I. F. was 140, and for the high speed the T. I. F. was 286.

It is plain then that a perfect wave cannot be found for both speeds, but that a compromise must be made. If it were thought desirable, we have no doubt further improvements could be made. How well this design secures a favorable compromise on all of the questions involved in the design is truly remarkable.

R. W. Wieseman: The twelve-pole starting characteristics, Fig. 21, are much better than the twenty-four-pole characteristics, Fig. 22, as Mr. Mortensen has pointed out. This was mentioned in the paper and the reasons for this difference were given. In a three-phase motor the difference between the two starting characteristics would not be so pronounced because in a three-phase

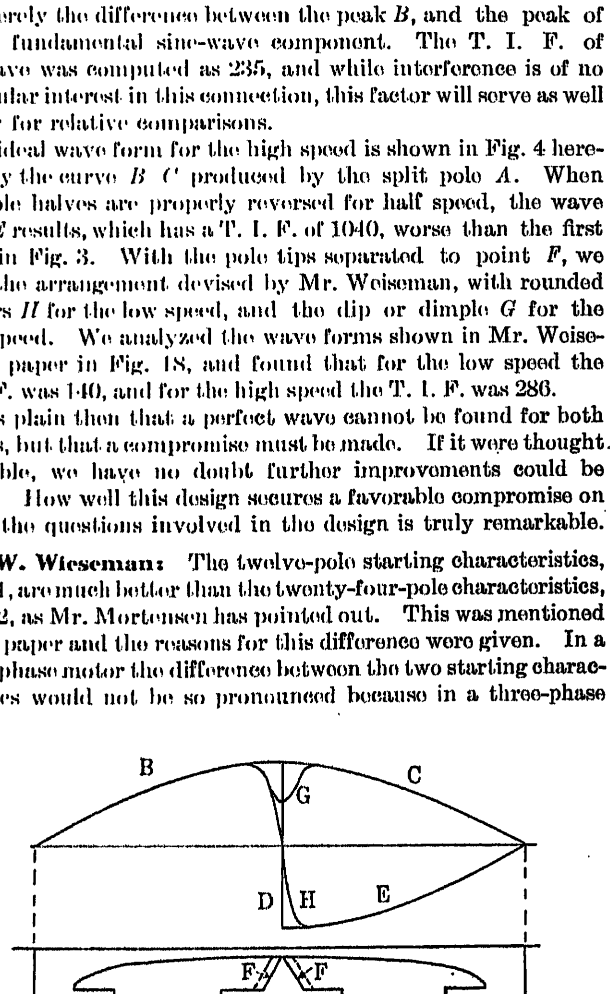

FIG. 4—POLE SHOE GIVING IDEAL WAVE FORM AT HIGH SPEED

machine the air-gap flux density can be more readily made the same for both the high- and the low-speed conditions. The starting characteristics, Figs. 21 and 22, were taken with the field winding open circuited. If the field winding had been short-circuited through a suitable resistance, the starting torque (at zero speed) would have been a little lower, but the pull-in torque (at 95 per cent speed) would have been considerably higher.

Mr. Mortensen stated that the full-load efficiencies are from 1 to 1.5 per cent lower than the efficiencies that might be expected from a single-speed machine of the same rating. Whether an efficiency is high or low is a matter of opinion. The efficiency of a machine depends largely upon the amount and kind of material which is used in its construction. With a better grade of iron and additional copper, there is no question that the effi-

ciency of the motor could be increased. This increased efficiency, however, would increase the cost of the motor and, consequently, it depends upon how much a per cent of efficiency can be capitalized. A machine which is used continuously should naturally have a higher efficiency and cost more than a machine which is used periodically.

Mr. Mortensen also stated that since the flux waves, Figs. 16 and 17, have very decided ripples, it would be of interest to know whether this machine is noisy during operation. At the low speed the motor is very quiet, having no magnetic noise and only a little windage noise. At the high-speed there is no magnetic noise, but the windage noise is much more pronounced.

Mr. Mortensen has called attention to the fact that a low-speed motor of this type would be considerably larger than a standard single-speed motor. I agree that a low-speed machine of this type is larger, and, therefore, more expensive than the standard motor. However, this motor is in reality two synchronous motors in one so that it is natural that the motor should cost more than one standard motor. Just how much more a two-speed motor would cost depends somewhat upon the speed.

The cylindrical rotor with a distributed rotor field winding could be used successfully in a two-speed synchronous motor as described by Mr. Weichsel. This construction would be advantageous only for small machines; for large motors I think the salient-pole and concentrated-field winding would be more economical and more efficient.

The choice of armature coil pitch for a two-speed synchronous motor is determined by the same principles as the case of the two-speed induction motor as Mr. Weichsel has pointed out.

The possibility of changing from low to high speed when operating under full load depends upon the design of the pole-face starting winding and the moment of inertia of the load. Synchronous motors can be built with very heavy starting windings (with a large thermal capacity) which can furnish full-load torque for an appreciable time. This type of machine is naturally more expensive than the ordinary motor. Furthermore, a large pole-face winding requires a deep pole tip which reduces the available space for the rotor field coil and it also increases the field leakage flux. Consequently, a synchronous motor (single-speed or multi-speed) which is designed to accelerate normal torques is not so good a synchronous motor as one which has the usual starting winding. The motor described in this paper is capable of giving full-load torque from 50 to 90 per cent speed at the high-speed connection with 65 per cent armature voltage. By short-circuiting the field winding with a suitable resistance, and then exciting the field winding, the motor should pull into synchronism. If the inertia of the load is such that this operation could be accomplished in less than a minute the motor would not overheat. In changing from the high speed to the low speed under load, the problem is not so difficult. With a little practise, I think an operator could synchronize the motor as it comes down through half-speed with a reduced voltage impressed on the armature winding.

The voltage wave induced in an exploring coil is not strictly the same as the flux wave as pointed out by Mr. Weichsel. My reason for omitting the ripples in the flux waves, Fig. 18, was to check the predetermined flux waves. The usual design calculations will not give very accurate results when they are applied to a machine which has a special field structure as shown in Fig. 15. Therefore, it was necessary to predetermine the flux distribution over the poles and then obtain the various flux-distribution coefficients which are used in the design calculations. In predetermining the flux distribution in the air-gap graphically by plotting the equipotential lines of magnetomotive force and the tubes of magnetic flux, it is convenient to neglect the influence of the stator slots and the rotor pole-face winding bars. It is assumed that the flux waves obtained in this manner will be average waves and that the distribution coefficients obtained from these flux waves will, therefore, represent the average condition. In this way the characteristics of the motor, Figs. 19 and 20, were predicted very closely.

Messrs. Douglas and Kane stated that it did not appear that the more usual shapes of pole shoe would be inoperative. Of course not, and the paper contains no statement to the contrary. However, the usual pole shape would not be so efficient as the one shown in Figs. 4 and 5. The maximum power factor obtainable when this motor is running light at either speed is 100 per cent.

I am at a loss to know why Messrs. Douglas and Kane calculated the telephone interference factor (T. I. F.) of the flux waves and not the voltage waves. The T. I. F. factor is the number of micro-amperes per volt flowing in a tuned network which is weighted so that the current will be a maximum at 1120 cycles. Since the flux wave cannot appear at the terminals of the motor, it cannot affect a telephone circuit, so that the T. I. F. of the flux wave is useless. Furthermore, the T. I. F. reading favors the seventeenth and the nineteenth harmonic in a 60-cycle wave while other harmonics have a reduced effect. Consequently the T. I. F. is only an indication of the relative value of a few harmonics in the wave and not a measure of all of the harmonics. The discussion of the pole shape by Messrs. Douglas and Kane, as shown in their Figs. 3 and 4, is practically the same as that given in the paper.

Self-Excited Synchronous Motors

BY J. K. KOSTKO[1]

Associate, A. I. E. E.

Synopsis:—*The theory and applications of self-excited synchronous motors have been repeatedly discussed in the literature, but the subject has been usually presented as a study of some special variety of this type. It now seems timely to give an outline of the general theory of the subject as a basis of comparison of the proposed types and a starting point for further development work.*

The study of a self-excited motor is mainly a study of its exciting system. The writer shows that any combination of the exciting circuits is equivalent to one comparatively simple type, and studies two problems in connection with this standard type; (1) determination of performance of a given motor; (2) determination of design constants giving a desired performance. In this study stress is laid on the elements peculiar to this type of motor; but no attempt is made to treat fully the elements which the self-excited motor has in common with other, better known, motor types. The current locus of the motor is found to be a circle, and it is shown that any circle in the plane can be obtained by a suitable choice of the exciting system.

The subject of synchronising is treated by a method showing an intimate connection between the synchronising process and the synchronous operation. The discussion of synchronising can thus be limited to the standard type, because the equivalence of the synchronous operation means also the equivalence of the synchronising features. It is shown that very high torque can be obtained during synchronising.

The theory is applied to a brief study of a few types of the self-excited motor.

DURING the last few years the increasing importance of a good power factor gave a fresh impetus to the study of the possibilities of synchronous motors. Naturally enough, the efforts of the inventors and manufacturers were directed primarily against the greatest drawback of synchronous motors—their poor starting characteristics; the result was the appearance of a number of "self-starting" synchronous motors of the Danielson type; these motors are structurally similar to the induction motors with phase-wound secondaries, and act as such during the starting period, but later are converted into synchronous motors by supplying one phase (or a combination of phases) of the secondary with direct current.

This type requires a separate source of excitation and is suitable only for large outputs; but it becomes increasingly evident that efforts are being made on many sides to extend the self-starting principle to the smaller sizes by making them self-excited. For this purpose the primary member carries in addition to the main a-c. winding a small d-c. type winding connected to a commutator; the secondary member carries a field winding and brushes bearing on the commutator and connected to the field winding. At synchronism the magnetic field is at rest with respect to the brushes; a continuous e. m. f. appears at the brushes and supplies the excitation. As in the Danielson motor, the secondary is of the polyphase type, with one or several phases used as the field winding.

In connection with the non-salient pole motors the principle of self-excitation was found to possess remarkable advantages which go far towards compensating for the complication of the commutator and brushes:

1. A synchronous motor develops a field of armature reaction whose direction and magnitude are determined by the load; since this field is stationary with respect to the secondary, *i. e.*, brushes, it is possible to make use of it in order to obtain some desired relation between the voltage at the brushes and the load; usually an increase of excitation with the load.

2. As the motor approaches synchronism, its induction motor torque approaches zero, and the final synchronization is accomplished by the synchronous torque. In a separately excited motor the synchronous torque, considered as a function of the coupling angle, is approximately alternating, with the average value zero; in a well-designed self-excited motor the synchronous torque is pulsating, with the positive part far in excess of the negative part, *i. e.*, with a definite positive (motoring) average value; synchronizing becomes a very simple, entirely automatic process.

It was only natural that the first motors built or theoretically investigated were of the simplest kind: a single axis field winding and a single set of brushes[2]; in the great majority of cases this arrangement is quite satisfactory; but it is not the only possible form; motors with several angularly displaced field windings and brush sets possess some valuable features not obtainable with the single axis field winding. The added complication is not very great because the self-starting feature requires in any case several angularly displaced secondary windings (phases).

The self-excited motor is still in the first stages of development; as in the case of the single-phase commutator motor the experimental research is complicated by the great number of possible combinations of as yet unknown relative merit. The purpose of this paper is to systematize the subject by reducing this apparent variety to a few fundamental types, and to develop for these types a method of theoretical investigation such as should always go hand in hand with the

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2. A brush set will always mean two diametrically opposite brushes in a bipolar motor, or their equivalent in a multipolar motor. The theory, of course, holds good with any angle A between the brushes of a set, because, with sinusoidal fields, the voltage across them is in a constant ratio with the voltage across two diametrically opposite brushes standing on the line perpendicular to the bisector of A.

practical work if the waste of haphazard experimenting is to be avoided.

In the writer's opinion the graphical method is best adapted to give an idea as to the possibilities and limitations of a new motor type; the study is made on the basis of the current locus, and all the performance elements are derived from it. The subject is treated with the usual assumption of sine wave voltages and currents, uniform air-gap, proportionality between the m. m. fs. and the fluxes, and the sinusoidal distribution of the latter.

Since the polyphase feature of the secondary is of importance only at starting, a perfect symmetry is not essential; in the matter of distribution of copper between the phases and the interconnection of windings the designer has a considerable freedom, which can be used to obtain a good wave of m. m. f. and a small loss in the field winding. The main a-c. winding, of any polyphase type, may be either interconnected with the d-c. winding or independent from it. As a rule, the advantages of interconnection are small in comparison with mechanical complications of bringing out a number of taps; in what follows the windings will be assumed independent and, for the sake of simplicity, the connection diagrams will show only the exciting circuits.

At present the practise is to use the rotor as the primary member, connected to the supply by slip rings; the d-c. winding and the commutator are then on the rotor; the field windings and the brushes are stationary; this arrangement is adopted in the figures of the paper. The inverted arrangement, however, has many points in its favor.

The flux F_f due to a field winding is proportional to the e. m. f. at the brushes connected to this winding; this e. m. f. is proportional to the component F_n of the total flux of the motor normal to the line of brushes; therefore, for each exciting circuit consisting of the d-c. armature winding, a set of brushes, and a field winding, there is a constant ratio between F_f and F_n. This ratio will be called "circuit constant;" in a motor with several field windings and sets of brushes each exciting circuit has a circuit constant. The calculation of these constants for given electric and magnetic circuits is very simple, and will not be considered here.

At synchronism the action of a field winding is fully determined by its circuit constant and the position of the brushes; but it must be remembered that in a self-starting motor the field winding is used as the secondary of an induction motor, and must be designed so as to give a safe open-circuit voltage at starting. It is found that even with high open-circuit voltages the proportions of the field winding are such that the d-c. exciting voltage is very low; the variable brush resistance causes a variation of the circuit constant, especially at light loads, when the voltage at the brushes is low. This may cause the observed performance points to deviate from the positions indicated by the theory. Another cause of discrepancy is that the brushes short-circuit coils moving in a magnetic field; the influence of the circulating currents cannot be determined by calculation.

In the most general case the motor may have any number of field windings and brush sets, the angular spacing of windings and sets being entirely arbitrary; but the theory is greatly simplified by the following remarks based on the assumptions stated above:

1. *Two field windings connected to the same brush set are obviously equivalent to a single winding connected to this brush set.*

2. *Any number of coaxial field windings 1, 2, etc., Fig. 1, connected to different brush sets 3, 4, etc., are equivalent to a single winding of the same axis connected to a suitably located brush set.* This will be proved for two coaxial windings, but the demonstration is quite general. Let F be the total flux of the motor, α, β, δ = angles counted from a fixed axis OX, and a, b, the circuit constants of the windings 1 and 2; the fluxes due to 2 and 1 are then $a\,F \sin(\delta - \alpha)$ and $b\,F \sin(\delta - \beta)$. The constant c of the equivalent circuit and the angle γ between OX and its brush line must satisfy the equation:

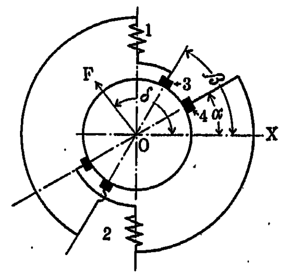

FIG. 1

$a\,F \sin(\delta - \alpha) + b\,F \sin(\delta - \beta) = c\,F \sin(\delta - \gamma)$ for all values of F and δ; this equation has always a solution: $c = \sqrt{a^2 + b^2 + 2ab\cos(\alpha - \beta)}$; $\tan\gamma = (a \sin\alpha + b \sin\beta) \div (a\cos\alpha + b\cos\beta)$.

3. *An exciting system consisting of any number of angularly displaced field windings and brush sets is equivalent to two field windings acting along arbitrarily selected axes and connected to two suitably located brush sets.* For, by paragraph 1, each field winding can be resolved into two windings acting along two arbitrary axes and connected to the original brush set; and by the paragraph 2 each of these two sets of coaxial windings is equivalent to a single winding connected to a suitably located brush set.

The choice of the axes being arbitrary, it is possible to further simplify the problem without restricting its generality by taking a set of rectangular field axes.

To sum up: *The most general exciting system is equivalent to two field windings displaced 90 electrical degrees and connected to two angularly displaced brush sets.*[3]

3. It can also be shown that it is equivalent to two arbitrarily spaced brush sets connected to two suitably proportioned and located field windings.

This system will be studied in the paper. The equivalence of the synchronous performance means the equivalence of the synchronizing features, as will be shown below.

The fundamental difference between a motor with a single set of brushes or its equivalent (paragraph 2) and the general case is the fact that in the former the axis of the d-c. excitation remains fixed with respect to the brushes, while in the latter the resultant axis of the d-c. excitation moves relatively to the brushes. In what follows, these two classes of motors will be denoted as "fixed axis" and "variable axis" motors.

Fig. 2 shows the general form of the exciting system. It is determined by four arbitrary constants: two circuit constants of the field windings 1 and 2 and two constants determining the positions of the brush sets. Since it is impossible to tell beforehand what values of these constants are of practical importance, no restriction will be made in this respect. The geometrical figures are apt to be misleading in such a case, and it is necessary to adopt a convention of signs for the fluxes and the angles. Positive directions are assumed for all axes, and a component of a flux along an axis is always

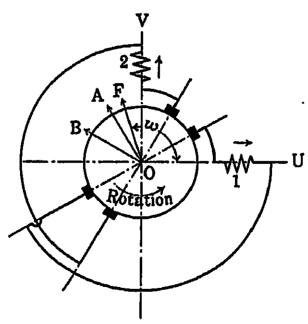

Fig. 2

taken with a sign. The positive directions of the field axes $O\,U$, $O\,V$, marked by the arrows, are denoted by $\overline{U}$, $\overline{V}$. The positive direction of angles and the rotation of the primary relative to the secondary are assumed counterclockwise. The angle between any two vectors $\overline{A}$ and $\overline{B}$ (or between the positive directions of two directed axes) is denoted by $(\overline{A}, \overline{B})$ or simply (A, B); it is positive or negative according as a rotation which makes $\overline{A}$ parallel to and of the same direction as $\overline{B}$ is in the positive or negative direction of angles; thus, $(\overline{U}, \overline{V}) = \dfrac{\pi}{2}$. The angle between any two vectors $\overline{A}$ and $\overline{M}$ of a system of vectors $\overline{A}, \overline{B}, \overline{C}, \ldots. \overline{L}, \overline{M}$, is given by the well-known relation $(\overline{A}, \overline{M}) = (\overline{A}, \overline{B}) + (\overline{B}, \overline{C}) + \ldots. + (\overline{L}, \overline{M})$.

The position of the brushes is determined as follows: let $\overline{A}$ and $\overline{B}$ be arbitrarily selected positive directions of the axes of the brush sets (axes perpendicular to the lines of brushes); the connections between the field windings and the brushes will be assumed such that, when the positive directions of the axes of a brush set and of the winding coincide, the action is as in a true self-excited motor, *i. e.*, an originally existing flux along the axis is then increased by the exciting current. With this convention the fluxes F_f and F_n whose ratio is the circuit constant have always the same sign, *i. e.*, the circuit constant is always positive. The position of the brushes is now fully determined by the angles $\alpha = (\overline{U}, \overline{A})$ and $\beta = (\overline{V}, \overline{B})$.

Let $\overline{F}$ be the vector and F = the numerical value of the flux of the a-c. armature reaction stationary with respect to the secondary; let ω be the angle $(\overline{U}, \overline{F})$; the components F_u and F_v of $\overline{F}$ along $O\,U$ and $O\,V$ are $F_u = F \cos(\overline{U}, \overline{F}) = F \cos \omega$, and $F_v = F \cos(\overline{V}, \overline{F}) = F \cos[(\overline{V}, \overline{U}) + (\overline{U}, \overline{F})] = F \sin \omega$. If F_1 and F_2 are fluxes set up by the windings 1 and 2, then the total fluxes along $O\,U$ and $O\,V$ are $F_u + F_1$ and $F_v + F_2$ (neglecting the small flux due to the currents in the d-c. armature winding), and the total flux along the brush axis $O\,A$ is $(F_u + F_1) \cos(\overline{U}, \overline{A}) + (F_v + F_2) \cos(\overline{V}, \overline{A}) = (F_u + F_1) \cos \alpha + (F_v + F_2) \sin \alpha$. Similarly, the total flux along $O\,B$ is $(F_u + F_1) \cos(\overline{U}, \overline{B}) + (F_v + F_2) \cos(\overline{V}, \overline{B}) = -(F_u + F_1) \sin \beta + (F_v + F_2) \cos \beta$. Let a and b be the circuit constants of the windings 1 and 2; their definition gives the equations: $F_1 = a\,[(F_u + F_1) \cos \alpha + (F_v + F_2) \sin \alpha]$ and $F_2 = b\,[-(F_u + F_1) \sin \beta + (F_v + F_2) \cos \beta]$. These equations, solved for F_1 and F_2, give: $F_1 = m_1 F_u + n_1 F_v$; $F_2 = m_2 F_v - n_2 F_u$, where

$$\left.\begin{aligned}
m_1 &= \frac{a \cos \alpha - a\,b \cos(\alpha - \beta)}{1 - a \cos \alpha - b \cos \beta + a\,b \cos(\alpha - \beta)} \\
m_2 &= \frac{b \cos \beta - a\,b \cos(\alpha - \beta)}{1 - a \cos \alpha - b \cos \beta + a\,b \cos(\alpha - \beta)} \\
n_1 &= \frac{a \sin \alpha}{1 - a \cos \alpha - b \cos \beta + a\,b \cos(\alpha - \beta)} \\
n_2 &= \frac{b \sin \beta}{1 - a \cos \alpha - b \cos \beta + a\,b \cos(\alpha - \beta)}
\end{aligned}\right\} \quad (1)$$

Substitution of the expressions of F_u and F_v found before gives

$$\left.\begin{aligned}
F_1 &= F\,(m_1 \cos \omega + n_1 \sin \omega) \\
F_2 &= F\,(m_2 \sin \omega - n_2 \cos \omega)
\end{aligned}\right\} \quad (2)$$

Let r and x be the primary resistance and leakage reactance per phase, and X = armature reactance corresponding to the flux F. Fig. 3 is the vector diagram of the motor. $O\,X_1$ and $O\,Y_1$ are rectangular axes, $O\,I'$ is the current I' per phase, $O\,A' = r\,I'$, $A'\,B' = x\,I'$, $B'\,C'$ = counter – e. m. f. $X\,I'$ due to the flux F, $C'\,M'$ = e. m. f. E_1 due to the flux F_1, $M'\,N'$ = e. m. f. E_2 due to F_2; $C'\,N'$ is the resultant counter – e. m. f. E, and $N'\,O$ is the applied voltage. The e. m. f.s. E_1 and E_2 are functions of ω; for a given ω their vectors $C'\,M'$ and $M'\,N'$ are found as follows:

it is seen in Fig. 2 that, with the adopted counter-clockwise rotation of the primary, a positive flux along $O\,U$ sets up an e. m. f. whose vector makes with the vector $B'\,C'$ an angle ω, laid off from $B'\,C'$ with due regard to the sign of ω; if the flux along $O\,U$ is negative, the vector obtained by this rule should be reversed. Hence the general rule; the numerical expression of an e. m. f. due to a flux along $O\,U$ in Fig. 2 is given the same sign as the flux; its vector in Fig. 3 is then obtained by laying off the algebraic value of the e. m. f. along a directed axis whose positive direction $\overline{L}$ is defined by the relation $(\overline{X}', \overline{L}) = \omega$. The rule for e. m. fs. due to the fluxes along $O\,V$ is similar, the positive direction $\overline{T}$ of vectors being defined by $(\overline{X}', \overline{T})$

$$= \omega - \frac{\pi}{2}.$$

Since the armature reaction flux F sets up an e. m. f. numerically equal to $X\,I'$, the algebraic values E_1 and E_2 of the e. m. fs. set up by F_1 and F_2 (*eq.* **2**) are

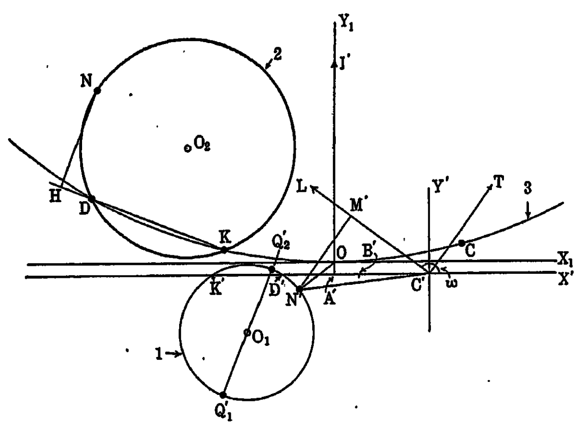

Fig. 3

$$\left.\begin{aligned} E_1 &= X\,I'\,(m_1 \cos\omega + n_1 \sin\omega) \\ E_2 &= X\,I'\,(m_2 \sin\omega - n_2 \cos\omega) \end{aligned}\right\} \quad (3)$$

If I' is kept constant, the point N' describes the locus of the applied voltage at constant current. Let $C'\,X'$ and $C'\,Y'$ be a set of coordinate axes with positive directions $\overline{X}'$ and $\overline{Y}'$ parallel to $\overline{X}_1$ and $\overline{Y}_1$; the projections of the vectors $\overline{E}_1$ and $\overline{E}_2$ on $C'\,X'$ are $E_1 \cos(\overline{X}', \overline{L}) = E_1 \cos\omega$ and $E_2 \cos(\overline{X}', \overline{T}) = E_2 \sin\omega$. Their projections on $C'\,Y'$ are $E_1 \cos(\overline{Y}', \overline{L}) = E_1 \sin\omega$, and $E_2 \cos(\overline{Y}', \overline{T}) = -E_2 \cos\omega$; therefore, the coordinates x' and y' of the point N' are

$$\left.\begin{aligned} x' &= E_1 \cos\omega + E_2 \sin\omega \\ y' &= E_1 \sin\omega - E_2 \cos\omega \end{aligned}\right\} \quad (4)$$

From these two equations E_1 and E_2 can be obtained and substituted in eq. (**3**); this gives:

$$x' \cos\omega + y' \sin\omega = X\,I'\,(m_1 \cos\omega + n_1 \sin\omega)$$
$$x' \sin\omega - y' \cos\omega = X\,I'\,(m_2 \sin\omega - n_2 \cos\omega)$$

The elimination of ω between these equations gives

$$\begin{vmatrix} y' - n_1\,X\,I' & x' - m_1\,X\,I' \\ x' - m_2\,X\,I' & -y' + n_2\,X\,I' \end{vmatrix} = O$$

which can be written:

$$(x' - x_c')^2 + (y' - y_c')^2 = R^2 \quad (5)$$

with

$$x_c' = \frac{X\,I'}{2}\,(m_1 + m_2) \qquad y_c' = \frac{X\,I'}{2}\,(n_1 + n_2),$$

$$R = \frac{X\,I'}{2}\,\sqrt{(m_1 - m_2)^2 + (n_1 - n_2)^2} \quad (6)$$

This shows that the locus of the point N' is a circle 1 of center (x_c', y_c') and radius R. Therefore, the current locus at constant voltage E_0 is another circle 2 of center O_2, obtained from circle 1 by inversion with O as center and $E_0\,I'$ as constant of inversion, followed by a rotation over 180 deg. around $O\,I'$.

In what follows the corresponding points in the constant current (*c. c.*) and constant voltage (*c. v.*) diagrams will be denoted by the same capitals with and without accent, respectively. The same convention will be applied to the applied voltage E_0, current I and e. m. fs. E, E_1, and E_2 at the corresponding points of the two diagrams, so that

$$I = I' \times \frac{E_0}{E_0'} \qquad E = E' \times \frac{E_0}{E_0'} \text{ etc.}$$

Part 1. Determination of Performance of a Motor of Given Constants

The constants of a motor are r, x, X, a, b, α, β. Instead of the last four it is convenient to use m_1, m_2, n_1, n_2, because they have an important geometrical meaning: the points Q_1' of coordinates $X\,I'\,m_1$ and $X\,I'\,n_1$, and Q_2' of coordinates $X\,I'\,m_2$ and $X\,I'\,n_2$ are diametrically opposite points on the circle 1, as can be proved by the substitution in the eq. (**5**), and by observing that x_c' and y_c' are coordinates of the middle point of the segment $Q_1'\,Q_2'$.

Construction of the current locus. Torque. The current locus 2 can be drawn as follows: let $p\,I'$ and $q\,I'$ be coordinates (with reference to $C'\,X'$ and $C'\,Y'$) of any point N' of the *c. c.* diagram: the coordinates with respect to $O\,X_1$ and $O\,Y_1$ are $(p + x + X)\,I'$ and $(q - r)\,I'$; the applied voltage $N'\,O$ is $I' \sqrt{(p + x + X)^2 + (q - r)^2}$ and the corresponding point N of the locus at the applied voltage E_0 can be found by observing that the current vector $O\,N$ is equal to $\dfrac{E_0}{\sqrt{(p + x + X)^2 + (q - r)^2}}$, and is symmetrical to $O\,N'$ with respect to $O\,X_1$, *i. e.*, it makes with $O\,X_1$ an angle δ such that $\tan\delta = \dfrac{q - r}{p + x + X}$.

By this rule the two points Q_1 and Q_2 corresponding to

Q_1' and Q_2' can be obtained. Moreover, the line $O\,O_2$ makes with $O\,X_1$ an angle η such that

$$\tan \eta = \frac{y_c' - r\,I'}{x_c' + (x+X)\,I'} = -\frac{X\,(n_1 + n_2) - 2\,r}{X\,(m_1+m_2+2) + 2\,x};$$

the center O_2 is the intersection of $O\,O_2$ with the perpendicular bisector of $Q_1\,Q_2$.

The circle diagram shows clearly the difference between the motors with fixed and variable axes of excitation. In the former $b = O$, so that $m_2 = n_2 = O$, *i. e.*, the circle 1 passes through the fixed point C', independent of the exciting system. The current locus 2, therefore, always passes through the fixed point C; theoretically C is the no-load point of the induction motor having the same r, x and X. The motor with a variable axis of excitation is free from this limitation; in fact, as will be shown, any circle in the plane can be obtained by a suitable choice of constants.

At D' and K' the torque is zero because the resultant e. m. f., $E = C'\,N'$, is normal to the current $O\,I'$; in the *c. v.* diagram the points of zero torque are D and K, on the inverse of the line $O\,X'$, *i. e.*, on a circle 3 of radius $\frac{E_0}{2\,r}$, tangent to $O\,X_1$ at O and having its center on $O\,I'$.

It is known that in the *c. v.* plane the loci of points of constant power transferred from the primary to the secondary are circles concentric with the circle 3 which corresponds to zero torque; the radius of the circle corresponding to the torque of P synchronous watts is

$$\sqrt{\left(\frac{E_0}{2\,r}\right)^2 - \frac{P}{r}}.$$

These circles are convenient for comparison of polyphase motors having the same r, whether synchronous or induction type, because they represent the torque regardless of the type of the rotor. The torque at any point N is proportional to the distance $N\,H$ from the line $D\,K$. These theorems are well known; their proof can be found in the article 2 of the bibliography.

The torque comprises the friction torque and the torque exerted by the field on the d-c. winding; the latter, expressed in synchronous watts, is the power consumed in the exciting circuits.

The circle 1 intersects $C'\,X'$ only if $R > |\,y_c\,|$, *i. e.*, if $(a \cos \alpha - b \cos \beta)^2 > 4\,a\,b \sin \alpha \sin \beta$. This condition is always satisfied in motors with a fixed axis of excitation ($b = O$); but motors with a variable axis can be designed so that $R < |\,Y_c\,|$ as will be shown; such a motor has not only a maximum torque, but also a minimum torque, below which the synchronous operation is not possible.

Analytical expression of the torque. Maximum torque. Let $\theta = (\bar{L}, N'\,O) = (\bar{L}, \bar{E}_0')$ be the angle between the vector $N'\,O = \bar{E}_0$ of the applied voltage and $\bar{L}$, which may be considered as a directed axis attached to the secondary member; then

$$(\bar{T}, \bar{E}_0') = (\bar{T}, \bar{L}) + (\bar{L}, \bar{E}_0') = \frac{\pi}{2} + \theta;$$

$$(\bar{I}', \bar{L}) = (\bar{I}', \bar{X}') + (\bar{X}', \bar{L}) = -\frac{\pi}{2} + \omega;$$

$$(\bar{I}', \bar{T}) = (\bar{I}', \bar{L}) + (\bar{L}, \bar{T}) = \omega - \pi.$$

Projecting the closed line $O\,A'\,B'\,C'\,M'\,N'\,O$ on the directions $\bar{L}$ and $\bar{T}$ it is found with the aid of the foregoing relations:

On $\bar{L}$: $-r\,I \sin \omega + (x+X)\,I \cos \omega + E_1 = -E_0 \cos \theta$

On $\bar{T}$: $r\,I \cos \omega + (x+X)\,I \sin \omega + E_2 = E_0 \sin \theta$

With E_1 and E_2 from eq. (3) these equations, solved for $I \sin \omega$ and $I \cos \omega$, give:

$$\left.\begin{aligned} I \sin \omega &= E_0\,(A \cos \theta + B \sin \theta) \\ I \cos \omega &= E_0\,(A' \cos \theta + B' \sin \theta) \end{aligned}\right\} \quad (7)$$

where

$$A = -\frac{X\,n_2 - r}{C}, \quad B = \frac{x+X+X\,m_1}{C},$$

$$A' = -\frac{x+X+X\,m_2}{C}, \quad B' = -\frac{X\,n_1 - r}{C}$$

and $C = (X\,n_1 - r)\,(X\,n_2 - r) + (x + X + X\,m_1)\,(x + X + X\,m_2)$.

The torque per phase in synchronous watts (considering a motoring torque as positive) is $-\,I'E_1 \cos(\bar{I}'\,\bar{L}) - I\,E_2 \cos(\bar{I}', \bar{T}) = -\,I\,E_1 \sin \omega + I\,E_2 \cos \omega$. The negative torque acting on the d-c. winding is the power consumed in both exciting circuits; it can be represented by $k_1\,E_1^2 + k_2\,E_2^2$ (k_1 and k_2 = constants referred to one phase, whose numerical values can easily be calculated from the electrical and magnetic data of the motor). This assumption disregards the fact of the superposition of the two exciting currents in the d-c. winding; but the loss in it is relatively very small. The torque per phase T exerted on the shaft is $T = -\,I\,E_1 \sin \omega + I\,E_2 \cos \omega - k_1\,E_1^2 - k_2\,E_2^2$. The substitution of E_1 and E_2 from eq. (3) gives: $T = -X\,I^2\,(f \sin^2 \omega + g \cos^2 \omega + h \sin \omega \cos \omega)$, with $f = n_1 + k_1\,X\,n_1^2 + k_2\,X\,m_2^2$; $g = n_2 + k_2\,X\,n_2^2 + k_1\,X\,m_1^2$, and $h = m_1 - m_2 + 2\,k_1\,X\,m_1\,n_1 - 2\,k_2\,X\,m_2\,n_2$. With $I \sin \omega$ and $I \cos \omega$ from eq. (7) this becomes $T = -\,X\,E_0^2\,(p \sin^2 \theta + q \cos^2 \theta + t \sin \theta \cos \theta)$, where $p = f\,B^2 + g\,B'^2 + h\,B\,B'$; $q = f\,A^2 + g\,A'^2 + h\,A\,A'$; and $t = 2\,f\,A\,B + 2\,g\,A'\,B' + h(A\,B + B\,A')$. Finally, let an angle θ_0 be defined by the relations

$$\sin 2\,\theta_0 = \frac{p - q}{\sqrt{(p-q)^2+t^2}} \text{ and } \cos 2\,\theta_0 = \frac{t}{\sqrt{(p-q)^2+t^2}}$$

then $T =$

$$-\frac{X}{2}E^2_0 \left[(p+q) + \sqrt{(p-q)^2+t^2} \times \sin 2(\theta - \theta_0) \right] \quad (8)^*$$

The extreme values of the torque are

$$-\frac{X}{2}E_0^2 \left[p + q \pm \sqrt{(p-q)^2 + t^2}\right].$$

E. m. fs. E, E_1, and E_2 in the Circle Diagram. E_1 and E_2 determine the loss in the field windings. Fig. 4

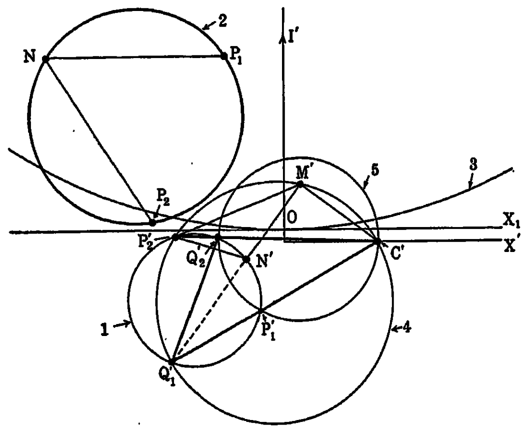

FIG. 4

reproduces some essential parts of Fig. 3. The points Q_1' and Q_2' have for coordinates $(X\,I'\,m_1, X\,I'\,n_1)$ and $(X\,I'\,m_2, X\,I'\,n_2)$. On the basis of eq. (3) it can easily be seen that when ω varies the ends of the vectors $\overline{E}_1'$ and $\overline{E}_2'$ drawn from C' move on the circles 4 and 5 described on $C'\,Q_1'$ and $C'\,Q_2'$ as diameters, and that these diameters pass through the common points P_1' and P_2' of the circles 1 and 5, and 1 and 4. For a given point M' on 4 the vectors $\overline{E}_1'$ and $\overline{E}_2'$ are $C'\,M'$ and $M'\,N'$ respectively; if N' moves on the circle 1, the angles of the triangle $P_2'\,M'\,N'$ remain constant because two of them are subtended by constant arcs $Q_1'\,P_2'$; therefore, there is a constant ratio between the segment $P_2'\,N'$ and the vector $M'\,N'$ of E_2. Let N, P_1 and P_2 be the corresponding points in the *c. v.* diagram. By the well-known property of the inverse points

$$P_2\,N = P_2'\,N' \times \frac{\text{inversion const.}}{O\,P'_2 \times N'\,O}$$

$$= P_2'\,N' \times \frac{E_0\,I'}{O\,P_2' \times N'\,O}.$$

*Starting from this formula the performance can be determined entirely by calculation: it gives θ for a definite output; equations (7) give I and ω; the e. m. fs. E, and E_2 result from eq. (3); the power factor $\cos\varphi$ is given by the relation $(\overline{I}', \overline{N}'\,O) = (\overline{I}', \overline{X}') + (\overline{X}', \overline{L}) + (\overline{L}, N'\,O) = -\frac{\pi}{2} + \omega + \theta$, hence $\cos\varphi = \sin(\omega + \theta)$.

But P_2' is a fixed point, so that $O\,P_2' = I' \times$ constant, where the constant depends only on the constants of the motor; as shown above $P_2'\,N'$ is proportional to the e. m. f. E_2' at voltage E_0', and $N'O = E_0'$; therefore,

$P_2\,N$ is proportional to $\frac{E_2' \times E_0\,I'}{I' \times E_0'}$ = proportional to $E_2' \times \frac{E_0}{E_0'}$ = proportional to E_2. By the same reasoning, the segment $P_1\,N$ is proportional to the e. m. f. E_1. The coefficients of proportionality can be found as follows: the segment $C'\,N'$ represents the resultant e. m. f. E' of E_1' and E_2'; but

$$C\,N = C'\,N' \times \frac{E_0\,I'}{O\,C' \times N'\,O}$$

$$= C'N' \times \frac{E_0\,I'}{I'\sqrt{r^2 + (x+X)^2} \times E_0'}$$

$$= \frac{E}{\sqrt{r^2+(x+X)^2}}$$

i. e., $C\,N$ represents the resultant e. m. f. E. When the point N coincides with P_2, the e. m. f. E_2 vanishes; $C\,P_2$ and $P_1\,P_2$ represent then the same e. m. f. E_1; therefore, for every point N,

$$E_1 = P_1\,N \times \frac{C\,P_2}{P_1\,P_2}\sqrt{r^2 + (x + X)^2};$$

similarly,

$$E_2 = P_2\,N \times \frac{C\,P_1}{P_1\,P_2}\sqrt{r^2 + (x + X)^2}.$$

The expression $E = C\,N\sqrt{r^2 + (x + X)^2}$ shows that for given r, x and X the total e. m. f. due to the field

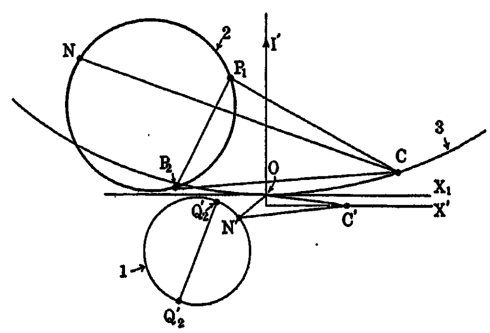

FIG. 5

windings is constant at a given point N, regardless of the locus to which N may belong. This remark should be borne in mind when choosing the working region of the plane for a new motor. It may also be noted that

$O\,C = \frac{E_0}{\sqrt{r^2 + (x + X)^2}}$, *i. e.*, $O\,C$ represents the applied voltage E_0 at the same scale as $C\,N$ represents the resultant e. m. f. E.

Stability. When the load increases the rotor falls back, *i. e.*, the angle θ increases. In general, the operation can be stable only on the part of the circle where an increase of θ corresponds to an increase of the torque. The eq. (3) and (4) give, after elimination of E_1 and E_2 and a few simple transformations: $x' - x_c = R \cos(2\,\omega - \epsilon)$ and $y' - y_c = R \sin(2\,\omega - \epsilon)$, where ϵ is a constant; R and $2\,\omega - \epsilon$ can be considered as the polar coordinates of the circle 1 with its center as origin and $2\,\omega - \epsilon$ as the polar angle. This shows that when the point N' moves on the circle 1 in a definite direction, the sense of the variation of $2\,\omega - \epsilon$ (therefore, of ω) remains the same.

Since the coordinates of C' relative to $O\,X_1$ and $O\,Y_1$ are $(x + X)\,I'$ and $-\,r\,I'$, the expressions of A, B, A' and B' in eq. (7) show that their nominators are proportional to the coordinates of Q_1' and Q_2' relative to $O\,X_1$ and $O\,Y_1$. Let a_1, b_1, and a_2, b_2, denote these coordinates; equations (7) give by division

$$\tan \omega = \frac{b_2 \cos\theta - a_1 \sin\theta}{a_2 \cos\theta + b_1 \sin\theta},$$

hence, after some transformations,

$$\frac{d\,\omega}{d\,\theta} = -\frac{(a_1\,a_2 + b_1\,b_2)\cos^2\omega}{(a_2\cos\theta + b_1\sin\theta)^2}.$$

But $a_1\,a_2 + b_1\,b_2$ is the scalar product of $O\,Q_1'$ and $O\,Q_2'$ and has the sign of $\cos \angle\, Q_1'\,O\,Q_2'$ which is positive or negative according as the point O is outside or inside of the circle 1, because $Q_1'\,Q_2'$ is a diameter. Therefore, when N' moves on the circle 1 in the clockwise direction (decreasing ω), θ increases if O is outside of the circle, and decreases if O is inside. The locus 2 is obtained from 1 by an inversion and a rotation over 180 deg. around $O\,Y_1$; if O is outside of 1, each operation reverses the rotation of N once; if O is inside of 1, only the last operation reverses the rotation. As O is either inside of both circles 1 and 2, or outside of both, it follows that the clockwise motion of N on the locus 2 always corresponds to an increase of θ: *the stable operation corresponds to that part of the current locus on which the point of maximum torque is reached from the point of zero torque by a motion in the clockwise direction.*

PART 2. DETERMINATION OF THE EXCITING SYSTEM FOR A GIVEN CURRENT LOCUS

The constants r, x and X will be assumed as known. Let a circle, such as 2 in Fig. 3, be a locus, presumably within the capacity of the motor; it is desired to check this assumption by determining the exciting system and the excitation losses.

Since a circle is an inverse figure of itself, it is convenient to take for the constant current locus 1 a circle symmetrical of 2 relative to $O\,X_1$. The value of the constant current I' is obtained from the fundamental relation of the inverse points, by drawing any convenient secant. The points A', B' and C' can now be located as in Fig. 3. Let $Q_1'\,Q_2'$ be an *arbitrary* diameter; it follows from what was found in Part I that, if the coordinates (with respect to $C'\,X'$ and $C'Y'$) of the points Q_1' and Q_2' be divided by $B'\,C' = X\,I'$, and the quotients substituted for m_1, m_2, n_1 and n_2 in eq. (1) the numbers a, b, α and β in these equations are the constants giving the locus 2. The solutions of these equations are

$$\left.\begin{aligned}
a &= \frac{\sqrt{n_1^2 + (m_1 + G)^2}}{S},\\
\sin\alpha &= \frac{n_1}{\sqrt{n_1^2 + (m_1 + G)^2}},\\
\cos\alpha &= \frac{m_1 + G}{\sqrt{n_1^2 + (m_1 + G)^2}}\\
b &= \frac{\sqrt{n_2^2 + (m_2 + G)^2}}{S},\\
\sin\beta &= \frac{n_2}{\sqrt{n_2^2 + (m_2 + G)^2}},\\
\cos\beta &= \frac{m_2 + G}{\sqrt{n_2^2 + (m_2 + G)^2}}
\end{aligned}\right\} \quad (9)$$

where $S = 1 + m_1 + m_2 + m_1\,m_2 + n_1\,n_2$; $G = m_1\,m_2 + n_1\,n_2$, and the radicals are taken with the signs giving positive values for a and b. The roots are always real; therefore, theoretically, *there always exists an exciting system giving any desired locus in the plane.* In fact, the problem has an infinity of solutions because the diameter $Q_1'\,Q_2'$ is arbitrary. The study of the constants a, b, α, β, corresponding to different diameters is facilitated by the following remarks: (1) since $Q_1'\,Q_2'$ is a diameter, S and G are independent from the choice of $Q_1'\,Q_2'$ because they are proportional to the scalar products $\overline{B'\,Q_1'} \times \overline{B'\,Q_2'}$ and $\overline{C'\,Q_2'} \times \overline{C'\,Q_1'}$ respectively; (2) the radicals in (9) are proportional to the distances of Q_1' and Q_2' from a fixed point on the axis $C'\,X'$ whose abscissa (with respect to C') is $x' = -\,X\,I' \times G$; the coefficient of proportionality $X\,I'$ is the same as for G and S. For instance, in the case when the circle 1 passes through C' (*i. e.*, $G = O$), let Q_2' coincide with C'; this gives $m_2 = n_2 = O$, $b = O$; the motor has a fixed axis of excitation and a single field winding; if Q_1' and Q_2' are different from C', the condition $G = m_1\,m_2 + n_1\,n_2 = O$ gives $\cos(\alpha - \beta) = O$, $\alpha - \beta = \dfrac{\pi}{2}$, *i. e.*, the brush sets coincide and are equivalent to a single set; the motor has two field windings connected to the same brush set and is equivalent to the first case.

Synchronising. When a motor, carrying a reasonable load as an induction motor, comes up to speed, the slip of the rotor is low and the conditions are similar to the synchronous performance with the rotor gradu-

ally falling back under the influence of the load. The torque is the sum of the induction torque and the synchronous torque, the latter passing through a sequence of values corresponding to a complete cycle of synchronous performance represented by the current locus. In a separately excited motor the torque is nearly alternating, with a small negative average value; in a self-excited motor, however, the choice of the locus is arbitrary; if the exciting system is such as to give a circle similar to Figs. 9 or 11, entirely or for the most part inside of the circle 3, *i. e.*, in the region of positive (motoring) torques, the torque during the synchronising process either remains positive, or is mostly positive, with a small negative interval of short duration. Analytically, the synchronizing torque is expressed by eq. (8).

A commercial synchronous motor must have a synchronous no-load point, *i. e.*, the circles 2 and 3 must have common points; the pulsations of the current and of the torque are inavoidable during the synchronising process, whether the axis of excitation is fixed or variable; but in the latter case it is possible to adjust the constants (for instance, the position of brushes) temporarily so as to throw the locus toward the center of the circle 3 in the region of the high-torque points of the plane, and out of contact with 3; and to reduce its diameter; in this way the synchronising torque may be considerably increased and the pulsations reduced until the motor reaches synchronism, when, with a little care, the change back can be accomplished without causing an undue shock to the system.

PART III. EXAMPLES

It is intended here to pass rapidly in review some of the types of the self-excited motor and to show that, with the aid of the diagrams and formulas of the paper, an idea as to the possibilities and limitations of a type can often be obtained without any extensive calculations. This study is facilitated by observing that, since the angle of two curves remains unchanged by the inversion, the angle γ between the circles 2 and 3 is the same as between the circle 1 and the line $A'X'$; it is given by $\cos \gamma = \dfrac{y_c'}{R}$, eq. (6).

1. *Fixed Axis of Excitation.* In all motors of this class it can be assumed that $b = O$, $m_2 = n_2 = O$, so that $\cos\gamma = \dfrac{n_1}{\sqrt{m_1^2 + n_1^2}} = \sin \alpha$, *i. e.*, $\gamma = \dfrac{\pi}{2} - \alpha$.
All fixed axis motors pass through a fixed point C, the no-load point of the induction motor having the same r, x and X.

Example 1. Brushes on neutral axis, Figs. 6 and 7. In this case $\alpha = O$, $\gamma = \dfrac{\pi}{2}$, the circle 2 is normal to the circle 3. For comparison. let $2'$ be the locus of an induction motor of the same constants r, x and X; $2'$ is also normal to the circle 3 (see, for instance, JOURNAL, A. I. E. E., April 1921, p. 326). If both motors have the same maximum torque, 2 and $2'$ are tangent to the same constant torque circle $3'$ and, if the synchronous motor is designed for leading power factor, as shown, the excitation must be so strong that its no-load current $O\,D$ is of the same order of magnitude as the locked current of the induction motor. More-

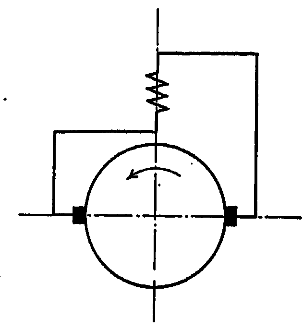

FIG. 6

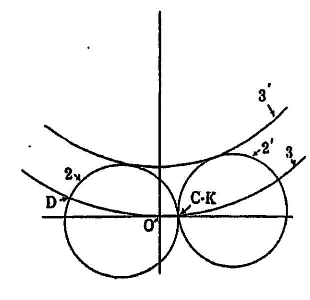

FIG. 7

over, the synchronous torque which is proportional to the distances of the points of the circle to the line K-D is very nearly alternating, so that the synchronising characteristics are very poor.

Example 2. Brushes on the axis of the fieldwinding, Figs. 8 and 9. Here $\gamma = O$; the circles 2 and 3 are tangent. The torque is always positive (motoring); the synchronising features are excellent, but the motor

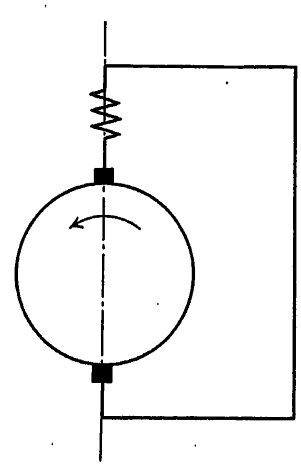

FIG. 8

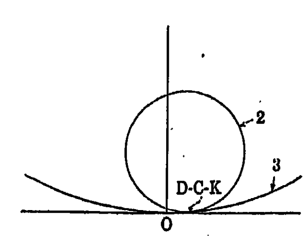

FIG. 9

is very sensitive to the brush position, and the power factor at light loads is not as good as can be expected in a synchronous motor.

The correct setting of the brushes is as in Fig. 10, intermediate between Figs. 6 and 8, but nearer to the latter than to the former. The locus is then as in Fig. 11.

2. *Variable Axis of Excitation.* Example 3: One might be tempted to improve the starting characteristics and to simplify the construction by using two identical exciting circuits displaced 90 deg. against one

another; but in this case $a = b$, $\alpha = \beta$; hence, $m_1 = m_2$, $n_1 = n_2$, therefore $R = O$ eq. (6): the circles 1 and 2 collapse each into a point; the synchronous performance is simply the synchronous point of a more general, variable speed performance of the motor.

Example 4. Fig. 12. The windings are connected to the brushes so that $\alpha = O$, $\beta = \pi$; 1 is the exciting winding proper, while 2 opposes the transverse arma-

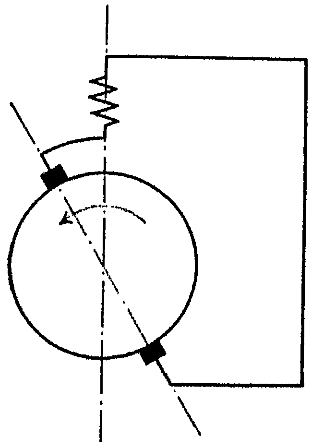

FIG. 10

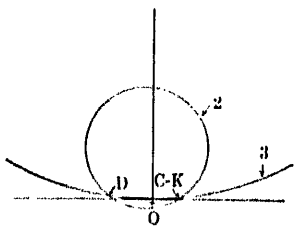

FIG. 11

ture reaction. Equations (1) give $n_1 = n_2 = O$, therefore, $\cos \gamma = O$, $\gamma = \pi/2$; the locus 2 is normal to the circle of zero power 3, as in the example 1, Figs. 6 and 7, but is not restricted to pass through C; the synchronising torque is alternating, therefore, the synchronising characteristics are poor.

Example 5. Fig. 13. Both windings act along the lines of brushes, $\alpha = \beta = \pi/2$; if the windings are

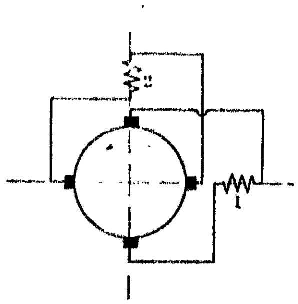

FIG. 12

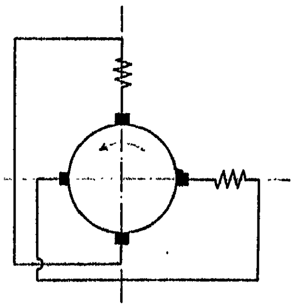

FIG. 13

indentical, the locus is a point, as explained above; if a is not equal to b, the locus is a circle, but, since the condition $(a \cos \alpha - b \cos \beta)^2 > 4\, a\, b \sin \alpha \sin \beta$ cannot be satisfied, this circle has no common points with the circle 3: the motor has no synchronous zero torque points.

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Discussion

H. Weichsel: The self-excited synchronous motor has, during the last few years, forged its way rapidly to the front and promises to take a very important part in the lay-outs and designs of future power plants, distribution systems and consumers' plants.

Mr. Kostko's paper on this type of machine is, therefore, to be welcomed, as it deals with the general theory governing the different machines which belong to this general class of self-excited synchronous motors.

Mr. Kostko has, in an admirable manner, gone to remarkable details in deriving the general locus for the current vector of these machines and in doing so has made full use of the principle of inversion, which is an extremely useful mathematical tool when it is desired to derive the current locus of an electric machine which is connected to a supply of constant potential. It is to be regretted that this method is not more generally known and used, and, therefore, it is to be hoped that Mr. Kostko's elegant application of this principle will help to stimulate the general interest in this method of attack for deriving the current locus of electric machines.

Personally, I have been, for a great number of years, an ardent advocator of graphical methods for the solution of a-c. electrical phenomena and, therefore, welcome heartily Mr. Kostko's method of deriving circle diagrams for this type of machine.

Experience has shown that the circle diagram of a simple induction motor has contributed more than any other factor to the full understanding of the performance and the interaction of the different phenomena which take place in this type of machine. The graphical methods readily give an answer to the behavior of the machine under most any imaginable operating condition.

The induction-motor circle diagram has also largely contributed to the high state of development of the present induction motor. The economic savings which have resulted from the general application of the induction-motor circle diagram must be enormous and no doubt run into millions of dollars.

There are two kinds of circle diagrams for the induction motor; namely, the so-called Heyland or Behrend diagram and the Osanna diagram.

The first-mentioned diagram neglects in the derivation of the current locus the influence of the ohmic resistance. The second-mentioned diagram considers the ohmic resistance and, is, therefore, of particular interest from the theoretical point of view.

The mathematical derivation of the first-mentioned diagram is extremely simple. The contrary is true for the Osanna diagram. Also, the actual construction and application of the Hyland diagram is the simplest possible, while this cannot be said for the Osanna diagram. The additional accuracy obtainable by the use of the more complicated Osanna circle over the extremely simple Heyland circle is small and often even of doubtful value. For this reason, the Heyland circle has found a very wide field of application, while the Osanna circle is used only for special cases.

Another reason for the general preference of the Heyland circle will be found in the fact that both diagrams are based on certain assumptions, such as constant leakage reactance and proportionality between magnetic lines and ampere-turns. It is a well established fact that the assumptions are not completely fulfilled in practical machines. Why, therefore, go into

the complication of considering the influence of the ohmic resistance, which, in the majority of cases, produces a very much smaller effect on the current locus than the variable leakage and saturation which is found in nearly every actual machine. The great popularity of the extremely simple Heyland diagram is, therefore, entirely justified. Especially if we consider that by a very slight addition to the Heyland diagram, it is possible to obtain the exact theoretical performance with due consideration of the resistance.

Similar conditions exist in self-excited synchronous motors. The influence of the leakage reactance and especially ohmic drop on the current locus is small.

In actual machines, neither the assumption that proportionality exists between magnetic lines and ampere-turns nor the assumption that the reactance is constant is fulfilled.

In addition to this, there exists a magnetizing effect due to the currents under the brushes which cannot be readily considered in a diagram. The details of this can be seen in the paper I presented before the Midwinter Convention of the A. I. E. E., 1925.[2]

The great complications introduced in the derivations of the circle diagram due to the influence of ohmic resistance, as well as the complications produced thereby in the construction and application of these diagrams may quite possibly prove themselves to be so large in comparison with the available increased accuracy, that the exact diagram will find only a relatively small field of practical application, but it will always be considered as a very important feature from the strictly theoretical point of view. At least, this would be in agreement with the experience of the Osanna circle diagram for induction motors.

On the other hand, a circle diagram for self-excited synchronous motors which neglects such minor effects as ohmic resistance promises to find a wide field of practical application on account of the extreme simplicity of its derivation and construction. Its simplicity, similar to the Heyland diagram, makes it further extraordinarily easy to understand at a glance the influence of the main factors which enter in the design and operation of these motors.

Furthermore, similar to the Heyland diagram, it is possible to derive from this simple diagram by relatively simple means the performance even under cases where it is desired to consider the factors which have been neglected in the derivation of the diagram. For the last few years, I have successfully used in practical design work, such a simplified circle diagram. Its derivation is based on the fact that the resultant field in any a-c. motor must be constant when constant voltage is impressed on its terminals and the influence of the ohmic resistance and reactance is negligible. With this assumption as a basis, it requires about two lines of mathematical formulas to derive the current locus.

Fig. 1, herewith, shows a current locus derived in such a manner for a machine connected in accordance with Fig. 8 of Mr. Kostko's paper. In this particular case, the ordinates of the circle are positive for all load conditions. Therefore, the very interesting result can immediately be seen from this diagram that a machine connected as per Fig. 8 is unable to operate as a synchronous generator as long as the field and armature are in the motoring connection and the direction of rotation for generating and motoring is assumed to be alike.

Many other extremely interesting conclusions can quickly be drawn from such simplified diagrams. For instance, by constructing the diagram for different angles between brush axis and field axis, it will be seen at a glance that under idle running conditions the voltage across brushes is not zero when the brush axis coincides with the field axis and does not reverse when the brush axis is displaced in either direction from the field axis, but follows a law given in full lines in the Fig. 2 herewith and does not follow the dotted lines, as one is inclined to expect. This simplified method of attack lends itself equally well to any kind of self-excited synchronous motor. A further discussion of the details of the general principle and application involved in these simplified diagrams would lead too far on this occasion.

There are a few statements in Mr. Kostko's paper which might lead a casual reader to wrong conclusions and a few words to prevent such pitfalls may be appropriate.

Mr. Kostko states that any number of coaxial field windings connected to different brush sets are equivalent to a single winding of the same axis connected to a suitably located brush set. This statement might lead to the conclusion that *equal performance* could be obtained by a machine with one field winding and one set of brushes and a machine of *equal dimensions* but possessing two field windings and two sets of brushes. However, a number of electrical, as well as mechanical reasons, exist, which make this impossible and invariably result in a materially poorer performance for the machine with two brush and field sets.

Mr. Kostko's statement regarding the synchronizing torque may leave the impression that the torque existing during the synchronizing period is in every respect equal to the torque of the machine at synchronous operation. This, however, is not the case under all conditions, but is true just at the theoretical dividing point between synchronism and slip. At speeds below synchronism, the voltage induced in the secondary windings due to the speed between resultant motor field and secondary winding produces an additional induction-motor torque, as shown in my paper mentioned before.

There is another very interesting feature which might be mentioned in this connection: If the load on a self-excited synchronous motor is suddenly increased, the motor can momentarily develop more than its so-called maximum synchronous torque, because a sudden load causes the resultant magnetic field of the motor to fall back suddenly and the velocity of the falling-back magnetic field produces, by induction, an e. m. f. on the field winding which strengthens the excitation until the field has assumed its final position in space. If the machine has an auxiliary secondary winding for starting purposes, then, under the conditions described, a current is induced in this winding also during the falling-back period of the resultant field which adds further to the motor torque until such time when the field has taken its final position in space.

In conclusion, may I state that Mr. Kostko's paper offers a tremendous amount of interesting information and my remarks advocating the attack of the problems of these motors on the simplified basis by neglecting ohmic resistance and leakage reactance, are, in no manner, intended to criticize or minimize the excellent work done by him, but are intended to point out that for the purpose of practical application, the simplified method of dealing with the subject will probably find a wider field of usefulness for the same reasons which made the Heyland circle so much more popular than the Osanna circle, but from the truly theoretical viewpoint, the complete diagram will always remain the most important and the most interesting one.

V. Karapetoff: It would be of interest to go over Mr. Kostko's deduction of the circular locus, using the vector analysis method.[3] With this method, the whole statement of the problem is first written down in the form of vectorial equations, and then some of the variables are eliminated by short-cut methods not possible with elementary algebra or geometry.

Let I be a variable current vector whose locus is a circle of diameter D passing through the origin. The vector connecting the ends of the vectors D and I is $D - I$, a geometric subtraction being understood. According to a familiar property of the circle, the vectors I and $(D - I)$ are perpendicular to each other; hence, in the language of Vector Analysis,

2. A New A-C. General-Purpose Motor, H. Weichsel, TRANS. A.I.E.E., 1925, p. 7.

3. See V. Karapetoff, The Use of the Scalar Product of Vectors in Locus Diagrams of Electrical Machinery; A. I. E. E. JOURNAL, 1923, Vol. 42, p. 1181. A good elementary book on Vector Analysis is by J. G. Coffin (Wileys).

$$(D - I) \,.\, I = O \tag{1}$$

Here the dot between the two factors stands for the so-called *scalar product*. By definition, if A and B are two vectors, their dot product is

$$A \,.\, B = A\, B \,\mathrm{Cos}\,(A, B) \tag{2}$$

Since Cos 90 deg. $= O$, the condition that two vectors are perpendicular to each other is

$$A \,.\, B = O \tag{3}$$

Eq. (1) is an application of this relationship to a circle and represents the vector equation of a circle of diameter D passing through the origin.

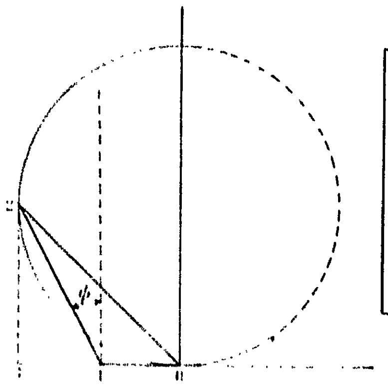

FIG. 1

I shall now show an application of this method to a very simple and well-known a-c. locus problem, namely that of a constant reactance x and a variable resistance r in series across a

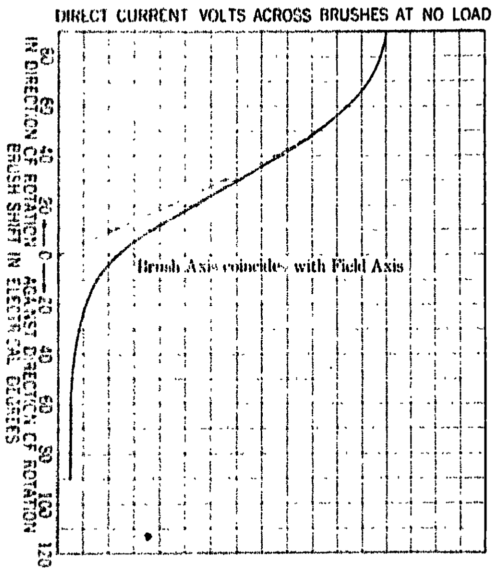

FIG. 2

source of constant sinusoidal voltage E. It is required to find the locus of the current I. We have

$$E = I\, r + x\,(j\, I) \tag{4}$$

where $j\,I$ is a vector numerically equal to I and leading it in phase by 90 deg. Since r is variable, and we want a locus, it is necessary to eliminate $I\,r$ from eq. (4). The general method used in vector analysis is to take a scalar product of both sides of eq. (4) with some vector perpendicular to the vector to be eliminated. The vector $I\,r$ is in phase with I; hence both sides must be multiplied by $(j\,I)$, because $(r\,I)\,.\,(j\,I) = O$. The result is

$$[E - x\,(j\,I)\,]\,.\,(j\,I) = O \tag{5}$$

or, after division by x,

$$[\,(E\; x) - (j\,I)\,]\,.\,(j\,I) = O \tag{6}$$

It will be seen that eq. (6) is of the form (1) so that the locus of $(j\,I)$ is a circle of diameter E/x. If desired, eq. (6) can be also written in terms of I instead of $j\,I$. It is only necessary to multiply the both factors on the left-hand side by $-j$, thus turning them by 90 deg. The result is

$$[\,(-j\;E/x) - I\,]\,.\,I = O \tag{7}$$

It is true that this particular problem can be just as readily solved by elementary geometry, but I purposely selected the simplest possible case to illustrate the general method.

V. A. Fynn: Mr. Kostko's paper is devoted to a very interesting attempt to develop a general theory of the "exciting" system of self-excited synchronous motors so as to secure a basis of comparison for the several already known types and for such others as further development work may disclose. The paper, however, also deals with the subject of synchronization and touches lightly on the starting conditions.

I have been actively interested in this field ever since 1905 and took out my first patent relating to synchronous induction motors in 1906 (See B. P. 11,298 of 1906). In the last few years I have gone over this ground with a fine-tooth comb in an attempt to discover a polyphase motor with controllable power factor which would be generally acceptable. During the process I too have analyzed this type of machine along very general lines. My experiences in this connection have led me to the conclusion that an exhaustive study of the "exciting" system is not sufficient, that the synchronizing torque is of at least equal importance in the case of self-excited, and of even greater importance in the case of separately excited, synchronous induction motors, and that the asynchronous performance of such machines is almost as vital if a proper basis for the comparison of various types is desired.

It has long been the practise to speak of the unidirectional ampere-turns on the secondary of a synchronous motor as "exciting" ampere-turns, Mr. Kostko does so in his paper, I have often done so myself, but the fact is that these ampere-turns do not always contain *real exciting* ampere-turns and do always carry something else. The term "excitation" conveys to one's mind a picture of the usually few ampere-turns necessary to produce the resultant motor magnetization and with this picture in mind one is very apt to form an entirely wrong conception of the dimensions and practical significance of the commutator of a synchronous induction motor and of the difficulties met with by the designer of such machines. The fact is that the unidirectional ampere-turns on the secondary of a synchronous motor are *always at least equal to the load ampere-turns*, sometimes to *load plus exciting* and mostly to *load plus over-exciting ampere-turns*. At a certain value of lagging power factor, the secondary of a synchronous motor carries exactly that number of ampere-turns as is carried by the secondary of a non-synchronous induction motor under like conditions. At such time all the exciting ampere-turns are carried by the primary, as is the case in non-synchronous induction motors. When a synchronous motor operates at unity power factor then its secondary carries the vectorial sum of load and exciting ampere-turns. For leading power factors its secondary carries load plus over-exciting ampere-turns. The result is that the commutator of a self-excited synchronous or synchronous induction motor carries working plus exciting or plus over-exciting current. Such a commutator must necessarily be large and if anything happens to it or to its cooperating circuits, the machine is put out of commission.

We ought not to speak of the secondary ampere-turns of a synchronous motor as "exciting" ampere-turns.

In the fifth paragraph of his paper Mr. Kostko says that synchronization is accomplished by the synchronous torque—strictly speaking, by a torque which later becomes the synchronous torque. This is true of all the earlier forms of synchronous or synchronous induction motors but is not necessarily true of all synchronous motors. Some of the several motors of this type which I have invented depart from this rule as I have shown in a paper read February 27th last before the Columbus Section of the A. I. E. E. and elsewhere.

In the same paragraph Mr. Kostko states that in separately excited synchronous motors the synchronizing torque is alternating. This used to be so but I have devised separately excited motors with strictly unidirectional and pulsating torques and others with even strictly constant synchronizing torques with quite a variety of synchronizing-torque configurations in between these limits to choose from, see "Engineering" February 20, 1925 and a paper presented to the Schenectady Section of the A. I. E. E. on March 27, 1925.

Just for the sake of historical accuracy it should be stated that contrary to Mr. Kostko's impression as voiced in his paragraph 6, the first self-compounding synchronous induction motor is apparently that of Burge, see B. P. 3227 of 1913, which has two secondary windings and *two* displaced sets of brushes.

I should like to stress my absolute agreement with Mr. Kostko's plea, paragraph 7, that a thorough theoretical investigation go hand-in-hand with experimental work and that haphazard experimenting be avoided. I have preached this doctrine for many years, also when connected with a company for which Mr. Kostko was working at the time, and I have successfully practised what I preached. In many cases, including all the synchronous induction motors I have invented, I was able to go even further and to work out the theory quite completely before making a single test, with the result that experimental development work was reduced to practically nothing.

I am bound to disagree with Mr. Kostko as to the opening phrase of his ninth paragraph. This statement of his explains why he has given such scant attention to the asynchronous features of the motors he has discussed as affected by the several arrangements of secondary windings and primary brushes he has referred to. My experience is that the effect of the "exciting" and synchronizing system on the asynchronous characteristics of the machine cannot be so lightly set aside.

The arrangement of secondary windings and primary brushes is now usually used to start, to synchronize and to operate the motor. If they are not so used additional windings are necessary which increase the cost and complicate the manipulations, if they are so used, then the secondary windings act as induction-motor secondaries at starting and their action is modified by the voltage appearing at the commutator brushes included in their circuits. In many cases this results in a considerable unbalancing of the several secondary circuits and a correspondingly poorer starting performance. To illustrate my point, refer to Mr. Kostko's Fig. 12. At the moment of starting, the brush voltage impressed on the secondary 2 leads the voltage generated in 2 by the synchronously revolving motor flux by 90 deg., whereas the brush voltage impressed on No. 1 lags by that amount behind the voltage generated in No. 1.

But of far more consequence is the influence of the configuration of the synchronizing torque on the asynchronous performance of the machine upon a torque demand which exceeds the maximum synchronous torque. As a specific example take a synchronous induction motor provided with the type of "excitation" shown in Fig. 10 and which was disclosed by me in 1916, see U. S. P. 1,337,648. Because of the fact that the unidirectional ampere-turns on the secondary of a synchronous motor are not "exciting" but really "load plus exciting" ampere-turns, also because these must be accommodated in a single-phase as against a polyphase winding as used in non-synchronous motors and finally because in addition to the winding carrying the unidirectional ampere-turns it is necessary to have another closed winding on the secondary to permit of the motor operating asynchronously, it is found that even by using more copper on the secondary than in the corresponding asynchronous motor the maximum synchronous torque can hardly be made to equal more than two-thirds of the maximum asynchronous torque. If the motor is to have the overload capacity which the material used permits of being developed asynchronously, then part of its overload must be taken care of asynchronously. In the case of a motor such as outlined in Fig. 10, the asynchronous overload capacity is not practically available because the alternating synchronizing torque with unequal positive and negative maxima which can be exhibited by such polyphase motors at subsynchronous speeds causes their speed to oscillate rapidly under asynchronous overload conditions. It is clear that only the roughest kind of work will permit of such an irregular motive power and such motors are accordingly so rated that their full-load torque equals about half their maximum synchronous and about one-third of their maximum asynchronous torque. This condition is another reason why such motors are very costly.

In prior publications I have shown how this very undesirable condition can be remedied, thus greatly increasing the weight efficiency and therefore reducing the cost of such machines and need not again go into this question here, but the condition just discussed does show the vital importance of the configuration of the synchronizing torque on the weight efficiency of synchronous induction motors and clearly suggests that it does not suffice simply to study their compounding characteristic if a true picture of the relative merits of different types is desired.

I should further like to point out that Mr. Kostko's assertion in paragraph 14, column two, on page 448, is true enough in so far as synchronizing and synchronous operation is concerned, but not true in regard to starting for the reason that if two displaced windings are used instead of one only and both are connected to the same brush set, the two component windings form a polyphase secondary and make it unnecessary to add a winding on this member which will be active at subsynchronous and inactive at synchronous speed.

That Mr. Kostko's assertion in paragraph 15 is true, so far as it goes, was demonstrated in my paper "Another New Self-Excited Synchronous Induction Motor."[4] It clearly appears from that paper that Mr. Kostko's proposition is not true when it comes to synchronization. Furthermore, for certain angular displacements there is a great practical difference between the two types. The single-brush-set type is so touchy for certain angular displacements as to be useless in practise, whereas quite steady corresponding operation can be had with the two-set type.

In the last paragraph of Part two, Mr. Kostko states that the pulsations of current and torque during the synchronizing period are unavoidable whether the axis of "excitation" is fixed or variable. This is true for the type with fixed but not true for that with variable position of the axis of "excitation." I have shown how such pulsations can be avoided in separately, as well as in self-excited, synchronous motors.

The motor dealt with under "example 1" is my 1906 motor; not only is the synchronizing torque in this machine "very nearly alternating" but it is actually so and its frequency is double the slip frequency as pointed out in detail in my paper "A New Self-Excited Synchronous Induction Motor."[5]

As to the motor discussed under "example 2," I gave the complete circle diagram for this machine in my contribution to the discussion of the papers presented at the 1925 Midwinter Convention and have shown that at light loads the primary current of this motor lags by nearly 90 deg. behind the terminal voltage,

4. A. I. E. E. TRANSACTIONS, 1925, p. 64.

5. A. I. E. E. TRANSACTIONS, 1924, p. 660.

that at a certain load this lag diminishes abruptly and is converted into a lead, after which the lead diminishes steadily with a further increase in load and finally again becomes a lag. It is therefore putting it very mildly to say that the power factor at light loads is not as good as can be expected in synchronous motors.

My investigation of this whole subject has revealed quite a number of interesting conditions not apparent from the Kostko analysis. Thus when using a phase-displaced system of secondary windings connected to a phase-displaced system of brushes on the primary, or more broadly, connected to a plurality of voltages which are phase-displaced when alternating, it is possible to cause the motor to operate synchronously over a wide range of loads and asynchronously at loads below, as well as at loads above, said range. It is further possible to cause the axis of the synchronous unidirectional magnetization on the secondary to travel in the one or the other direction when the axis of the resultant magnetization of the motor moves in a given direction as the load of the motor increases or decreases. This leads to some very interesting combinations to which I may refer in greater detail at some later date. One way to secure the change in question is to reverse the connections between one of the secondary windings and its brushes in Fig. 13.

On the whole, I must admit that my study of the synchronous induction motor has been somewhat disappointing. I feel reasonably certain that I have reached rock bottom, having devised and carefully analyzed some seven or eight different types and worked out a fairly complete general theory which agrees with the results of tests, but I have not found any self-excited synchronous induction motor which I can recommend as a general-purpose motor. My view is that many synchronous motors on any one system of distribution are undesirable; because of their rigid speed characteristic these machines require *instantaneous* response from the generating set and sudden changes in load become so many hammer blows on the system. The fact that self-excited motors require slip-rings as well as a commutator and that both of these are in constant use are undeniably practical disadvantages, particularly in view of the fact that the commutator carries load as well as exciting currents. My conclusion is that synchronous induction machines are much better suited for use as synchronous condensers than as motors. They are much superior to synchronous condensers with defined polar projections now in general use for the reason that no line disturbance can put them out of commission which, so often happens with the pronounced-pole type. Synchronous induction machines used as condensers automatically resume synchronism under any and all conditions and when so used, can be located in sheltered positions where the slip-rings and the commutator will have a chance of operating satisfactorily.

I believe that conditions are much more favorable in the case of larger, separately excited synchronous induction machines and such can no doubt be advantageously used as motors in a number of cases. But when it comes to a general-purpose motor I think that the solution will be found in a compensated non-synchronous machine.

J. K. Kostko: The inversion is undoubtedly the most powerful method of graphical study of circuits yet devised; it originated in this country (F. Bedell and A. C. Crehore, "Resonance in Transformer Circuits," *Physical Review*, Vol. II, 1894-1895, p. 451), but it is much better known and more used abroad. In a circuit containing no independent e. m. f. the current is proportional to the applied voltage; if, for a certain phase angle between the current and the voltage, the value of the former is I' for the voltage E' and I for the voltage E, then the lengths of the vectors $O\,E'$ and $O\,I$ representing E' and I are given by the relation $O\,E' \times O\,I = E\,I'$. If I' and E remain constant, $O\,E'$ describes the locus 1 of the voltage at constant current, while $O\,I$ describes the locus, 2, of the current at constant voltage; the foregoing relation immediately suggests the possibility of passing from one locus to the other by inversion (for details see *Electrical World*, Vol. 75, p. 724, 1920). It is usually much easier to construct the locus 1 than the locus 2, because, if we start from a known current, we can draw a vector diagram with correct linear and angular relations between the vectors. In practise it is seldom necessary to go through the process of inversion; the construction usually can be simplified by making use of the fact that the angle between two curves remains unchanged by the inversion. Thus, in Fig. 3 the line $A'\,X'$ is the constant current locus of a circuit consisting of the primary resistance r and a variable inductance. Its inverse is the circle 3, of diameter $\dfrac{E}{r}$. $A'\,X'$ and this circle are the loci of points at which the power from the line is consumed entirely in the resistance r. Therefore, at points common to the circle 3, and to the locus of the motor no power is transferred from the primary to the secondary. The circle, 3, is zero-power or zero-torque circle. It is usually possible to determine the angle between the line $A'\,X'$ and the constant current locus 1; it is also the angle between the final locus 2 and the circle 3. Since the latter is fully determined by the primary resistance, r, this angular relation is an important element determining the locus 2. For instance, in the polyphase induction motors the locus 2 is normal to the circle 3. In a single-phase induction motor it is very nearly normal to it (the exact angle can be calculated from design or test data). In a polyphase synchronous reaction motor circles 2 and 3 are also normal. In a synchronous motor of the type of Fig. 10 of the paper the angle between 2 and 3 is equal to the angle of brush displacement. If two points A and B are inverse of A' and B' respectively, and the constant of inversion with respect to the origin O is c, then $A\,B = A'B'\,\dfrac{c}{O\,A' \times O\,B'}$. This relation can be used in order to represent an element such as an e. m. f. by means of a segment in the diagram. The e. m. fs. E, E_1 and E_2 of the paper are represented in this way; other examples can be found in various publications of the writer.

Thus, by the use of various geometrical properties of inverse figures we obtain a complete graphical representation of the performance with its elements shown directly in the diagram. It is hardly necessary to insist on the advantages of such a representation, especially in the study of a new motor type, where calculations should be undertaken only after a clear idea as to the useful range of the design constants has been obtained from the study of the diagram.

A diagram in which the zero-torque circle 3 is used is the exact one, taking into account the primary resistance. The transition to the simplified diagram is easy. If r decreases, the diameter of 3 increases; for $r = O$ the circle 3 coincides with the axis of abscissas. The feeling against the exact diagram is probably due to the fact that the old methods of taking the primary resistance into account are extremely complicated in comparison with the inversion method making use of the circle 3. An article in 1921, TRANSACTIONS [6] shows how the problem of constructing the exact locus of a polyphase induction motor from test data is simplified by the use of this method.

Mr. Weichsel's remark as to the meaning of the equivalence is to the point, but a careful reader is not likely to make a mistake in this respect because the writer took good care to point out in the paper (page 11, first column) that the torque in the diagram comprises the friction and the loss in the exciting circuits; the net torque is, therefore, somewhat affected by the choice of the exciting system, but not in any definite way. By suitable choice of mechanical features, such as the weight of copper, any one of the two equivalent motors can be made either better or worse than the other; even in the same motor a change

6. A. I. E. E. TRANSACTIONS, 1921.

of the field copper (leaving the size of conductor unchanged) will affect the net torque without in any way affecting the current locus. The physical change from one exciting system to an equivalent one is affected by the structural features of the motor. No general rules can be given, but each case must be studied individually.

It would be of interest to "go over" my deduction of the current locus using Prof. Karapetoff's method; but the current locus alone is of little value to an engineer. If a comparison of the methods is intended, it would be of greater interest to take up the problem of the graphical study of the motor from the beginning and try to find a solution as general and complete as that obtained in the paper; in my opinion this is the only sensible and fair way of comparing the two methods. The solution of Prof. Karapetoff's example by the inversion method is as follows: let $O\,I'$ be the vector of the current I'; the constant current locus is a line parallel to $O\,I'$ and at a distance $x\,I'$ from it; the current locus at constant voltage E is the inverse of this line, *i. e.*, a circle passing through the origin O, tangent to $O\,I'$ and of diameter $\dfrac{E\,I'}{x\,I'} = \dfrac{E}{x}$.

In his discussion Mr. Fynn continuously refers to the features of his separately excited motor as contradicting my conclusions which are derived for the self-excited motor. My statements as to the latter type are not directly attacked, but expressions such as "true as far as it goes," "true enough" etc., are likely to cause some doubt in the minds of the readers. It is well to point out that there is nothing ambiguous or vague in these statements if applied to the motor type for which they have been developed. For that matter, they apply to Mr. Fynn's motor as well, but, for the present, I shall confine myself to the type studied in my paper, because the experience with this type shows that, with such complicated subjects, convincing arguments and a common ground for a profitable discussion can be found only in a rigorous and impersonal mathematical study; and, while I have done myself some work on this subject, it would clearly be out of the question to take it up now.

Mr. Fynn's remark that the study of the exciting system is not sufficient, is correct; it is for this reason that I developed in the paper not the theory of the exciting system, as he says, but the complete theory of the self-excited motor. The synchronous (and synchronizing) performance is determined by the constants a, b, α, β, r, x, X; the first four determine the exciting system and are investigated in detail; the last four determine the main a-c. winding. They are used in all vector diagrams for the construction of the current locus, but no further study of these elements is undertaken because they occur in many well-known motor types and are quite familiar to the readers.

Mr. Fynn's remark as to the large size of the commutator gives an impression that it is due to the amount of ampere-turns on the secondary. Such is not the case: it should be charged mainly against the self-starting feature of the motor, which involves a very low exciting voltage, as explained in the paper. Mechanically, the commutator must fit the size of the motor, but its output is only a few per cent of that of the motor. If we could use 150-250 volts for excitation, instead of 15-25, the commutator would be as small as structurally possible, regardless of the number of ampere-turns. The danger of sparking and circulating currents in a coil of many turns short-circuited in a strong field also militates against the choice of a high exciting voltage.

In my opinion Mr. Fynn attaches an exaggerated importance to the asynchronous features of the motor as compared to the synchronous performance. His remark as to the voltage appearing at the brushes at starting is true, but he overlooks the order of magnitude of the phenomenon. At starting we may have some 500 induced volts in a field winding, and some 20 volts across the brushes, of the same frequency and added vectorially. An unbalance which may be caused by these 20 volts in the two phases of the secondary is really a small matter, the more so because the winding of a self-excited motor is usually made stronger than that of the induction motor in the same frame, so that there is a margin for a slight reduction of the asynchronous torque. If this unimportant action of the brushes be disregarded (or simply avoided by disconnecting the brushes at starting) then the windings of the secondary should be designed (1) as a part of the exciting system, and (2) as a secondary of an induction motor. Plan (1) is extensively studied in the paper; but, as Mr. Fynn puts it, only a scant attention is given to plan (2), because it was not intended to take up here matters pertaining to the induction-motor design, important as it is.

That synchronization is accomplished by the synchronous torque is admitted by Mr. Fynn, at least for the motors studied in the paper; therefore, all parts of the paper dealing with the synchronous performance, *i. e.*, practically the entire paper, can be considered as dealing with the synchronizing torque. It is rather strange to read Mr. Fynn's admonition that "it does not suffice simply to study their compounding characteristics...." etc.

With reference to Mr. Fynn's remark about a motor with two displaced windings connected to the same brush set: I never said that the equivalence of the synchronous and synchronizing operation also means the equivalence of the starting features.

All the motors shown and described in Mr. Fynn's paper "Another New Self-Excited Synchronous Induction Motor" are fixed-axis motors, equivalent to the type of Fig. 10 of my paper, with respect to synchronizing as well as the synchronous operation. If the constants of a motor are changed, for instance by reconnecting field windings, as described in Mr. Fynn's paper, we have not one, but several distinct motors, and to each of them the principle of equivalence to the motor of Fig. 10 applies in full.

In the example 1, Fig. 6, the locus 2, Fig. 7, is normal to the circle of zero-torque 3, of radius $\dfrac{E}{2\,r}$; the torque at a point of the locus is measured by the distance of this point from the chord $D\,C$ (not shown). Due to the curvature of the circle 3 the positive maximum of the torque is somewhat less than the negative; hence the expression "nearly" alternating. Only if r is neglected, the circle 3 coincides with the axis of abscissas and the torque becomes truly alternating.

The variation of the lag of the current in example 2 can be followed in Fig. 9. In practise the motor would not be as bad as it appears, because, on account of losses, the no-load point will be nearer to the origin than D-C-K, and the transition to the lead will occur at a very light load.

The interesting conditions mentioned by Mr. Fynn can be found in the paper; the one which is really important is given quite a prominent place in the chapter on synchronizing. I refer to the motor which has minimum as well as a maximum torque at synchronism. The analytical conditions are given on the bottom of page 68. The current locus of such a motor has no common points with the zero-torque circle 3. If the constants are such that its radius is small or vanishes, as in examples 3 and 5, the pulsations of the torque and of the current are also small or vanish. The travel of various fluxes can readily be followed on the constant-current locus 1.

Notes on the Development of a New Type of Hornless Loud Speaker

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and

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Synopsis.—*The paper describes a series of tests directed to the evolution of a loud speaker, free from resonance. Various types of sound source were tried. For the most part horns were avoided. Diaphragms, when employed, were either so light and stiff that their natural resonance was above the essential frequency range, or so flexible that their resonance was below the lowest important acoustic frequency. Best results were obtained with the latter type, and it is shown on theoretical grounds that a small diaphragm, the motion of which is controlled by inertia only, and located in an opening in a large flat wall, will give an output sound pressure proportional to the actuating force, independent of frequency. It should be possible to make an ideal sound reproducer on this principle. A practical loud speaker which approximately fulfills the above conditions has now been evolved. It consists of a flexibly-supported paper cone actuated by a coil in a magnetic field and provided with a baffle. As compared with ordinary loud speakers, this instrument radiates much more of the low tones and more of the very high frequencies which makes for clearer articulation.*

The extension of the range of response of the loud speaker to higher and lower frequencies, makes defects in the remainder of the system more noticeable, particularly roughness and blasting due to overworked amplifiers. It is, therefore, important that the amplifier used with the new loud speaker be designed to have ample capacity.

* * * * *

Initial Tests With Non-resonant Types

SEVERAL years ago tests were undertaken in the Research Laboratory of the General Electric Company, to ascertain whether or not it would be possible, by sacrificing sensitivity, if necessary, to produce a loud speaker free from the most objectionable of the distortion which characterizes loud speakers in general.[2]

Amplifiers and amplifier tubes had been developed to a point where there was no difficulty in obtaining voice currents of any required magnitude, practically free from distortion. Aperiodic microphones and condenser transmitters were in use in broadcasting station studios.[3] The availability of these comparatively new tools and the fact that in the matter of distortion the loud speaker was the weakest link in the chain of apparatus involved in transmission and reproduction of speech and music, appeared to justify a renewed attack on the problem. Even though it should be found that a large and expensive amplifying system was necessary, owing to sacrifice of sensitivity, it was felt that there would be many applications of a loud speaker of high quality. Happily in the later designs it was found that the anticipated sacrifice of sensitivity was not necessary.

The worst distortion in the ordinary loud speaker is due to horn-resonance and diaphragm-resonance. To eliminate the horn-resonance, it was proposed to abandon the horn. To avoid the diaphragm-resonance we might, for example, eliminate the diaphragm, by using a "talking arc;" or we might use diaphragms in which the resonance frequencies were above or below the working range.

One of the first undertakings was to build a resistance-capacity-coupled amplifier in which the final stage was a tube having an oscillator rating of 250-watts output. With a 1500-volt plate supply, this amplifier could deliver about 70 milliamperes of sine wave current at 200 volts, with practically no wave-form distortion, and it was possible to test some very insensitive devices. Among the things tried were:

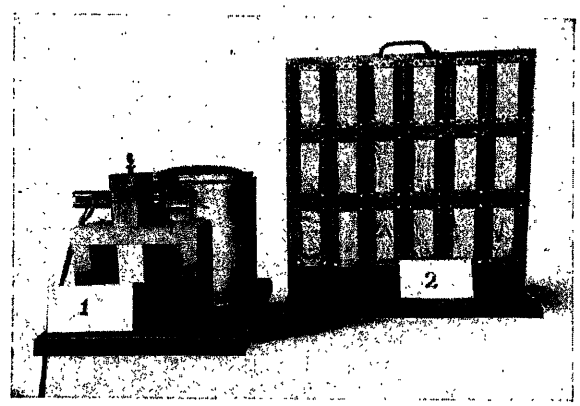


Fig. 1—Siren and Gold-Leaf Thermo Phone

1. A gold leaf thermophone with an area of about one-half square foot, shown in Fig. 1. The voice current superimposed on a direct current causes temperature fluctuations in the gold leaf. The adjacent air expands and contracts and produces sound waves.

2. Various designs of electrostatic loud speakers with large diaphragms: In these the diaphragm is a thin sheet of conducting material, actuated by the electrostatic attraction between it and an electrode placed close to it. Fig. 2 shows a model in which the electrodes were of felt, painted with graphite and separated by two sheets of varnished cambric.

3. A siren, shown in Fig. 1: Instead of moving a diaphragm to set up air waves, the voice currents are

1. Both of the Research Laboratory of the General Electric Co.

2. A general discussion in the form of a symposium on the loud speaker problem is published in *Proceedings* of the Physical Society of London, Vol. 36, Parts 2 and 3, Feb. & Mar., 1924.

3. The construction and calibration of condenser transmitters are described by F. C. Wente in the *Physical Review*, July, 1917, and May, 1922. The theory of air damping as applied to the condenser transmitter is discussed by I. B. Crandall, *Phys. Rev.*, June, 1918.

Presented at the Spring Convention of the A. I. E. E., St. Louis, April 13-17, 1925.

made to operate a delicate throttle-valve which controlled the amount of air issuing from a jet. This principle is employed in the "Creed Stentorphone" evolved by Gaydon and manufactured by Creed of Croyden, England.

4. An agate cylinder machine, depending on varying the frictional force between a rotating drum of polished agate and a piece of metal attached to the diaphragm: This principle was first applied by Edison[4] using chalk cylinders, and later by Johnson and Rahbeck who used agate[5]. A modified form of frictional machine, called the "Frenophone," has recently appeared[6].

5. A talking arc.

6. Multiple unit area devices, made up of a large number of similar magnetic telephones, as shown in Fig. 3.

7. Combinations of several horn instruments, having different characteristics, so that each supplements the others. Fig. 4 shows a photograph of this arrangement.

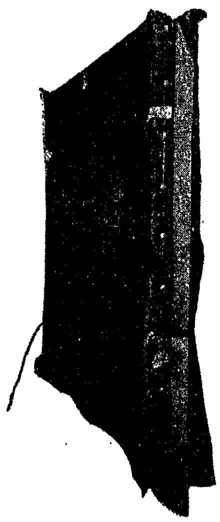


Fig. 2—Electrostatic Loud Speaker

8. The induction phone developed by Dr. C. W. Hewlett[7].

This is illustrated in Fig. 5. The diaphragm is a thin sheet of aluminum loosely supported between two pancake-type coils, wound with suitable venting spaces. Direct current is passed through the coils in such a direction as to give a radial field in the region of the diaphragm, and the voice current circuit is connected so that both coils act as primaries to induce currents in the diaphragm. The resulting force can be made to be almost uniform over the whole diaphragm.

9. Various designs of small diaphragm moving coil instruments.

Fig. 6 shows a number of instruments set up for comparison.

Of the possibilities, some were dropped after one or two experiments, while the more promising types were the subjects of considerable development. The electrostatic phone is capable of giving very fine quality reproduction, but owing to the low breakdown strength of air, only a small force can be applied to the diaphragm and a very large area is required to give a reasonable volume of sound.

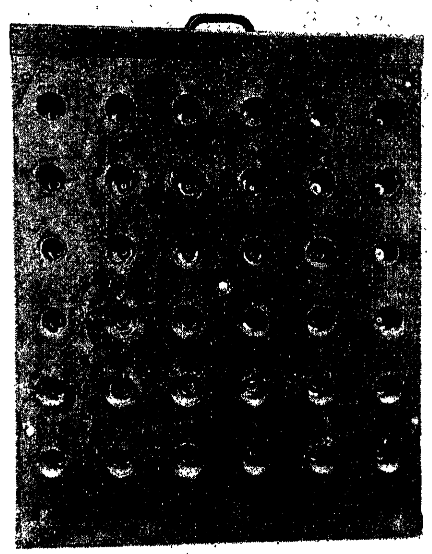


Fig. 3—Multiple-Unit Loud Speaker

Fig. 4—Triple-Horn Loud Speaker

Characteristics of Large Area Diaphragms

One of the first tests was an attempt to obtain the equivalent of a large area diaphragm by placing a number of small units close together. The panel is shown in Fig. 3. A single telephone receiver without a horn, gives entirely inadequate radiation of the low tones. The horn helps to bring out the low tones but introduces resonance. Placing a number of telephones in close proximity with their diaphragms moving in

4. British patent No. 2909,—1877.

5. Described in *Zeitschrift fur Techinsche Physik*, 1921, No. 11, also *Journal* I. E. E., No. 61, July 1923, p. 713.

6. Model exhibited at Liverpool meeting of British Association for the Advancement of Science, Sept. 1923. See *Sci. American*, Jan. 1924.

7. *Phy Rev.*, 17, p. 257, 1921. *Phy. Rev.*, 19, p. 52, 1922. *Jour. Opt. Soc. Am.* 4, p. 1059, 1922.

phase also improves the radiation of lower tones without creating any resonance. The multiple unit instrument was a considerable improvement over a single unit with horn. But as the device was first built, the diaphragms themselves were resonant at about 1000 cycles. The next step was the substitution of telephones with diaphragms of steel, 0.0015 in. thick, stretched so tightly that their natural frequency was above 6000 cycles, or practically out of the voice range. When used as head-phones, these receivers gave very fine quality but as a loud speaker the multiple unit device gave undue prominence to the high frequency components of speech. Voices sounded thin and hard. Only after electrical compensation had been introduced by means of the circuit shown in Fig. 7A did the multiple unit loud speaker give a natural reproduction of voice. In other words, while the effect of the large area diaphragm as compared with a small diaphragm was in the right direction in helping the radiation of low tones, as had been anticipated by Rice in proposing this experiment, there was still an accentuation of the high frequencies when the multiple unit loud speaker was used without the compensating circuit.

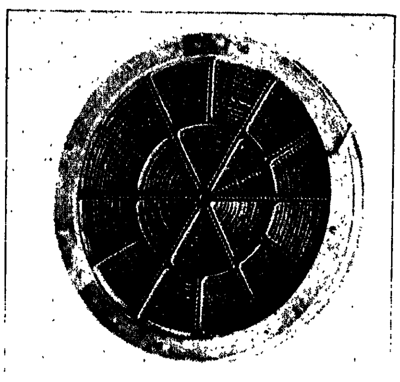


Fig. 5—Hewlett Induction Type Loud Speaker

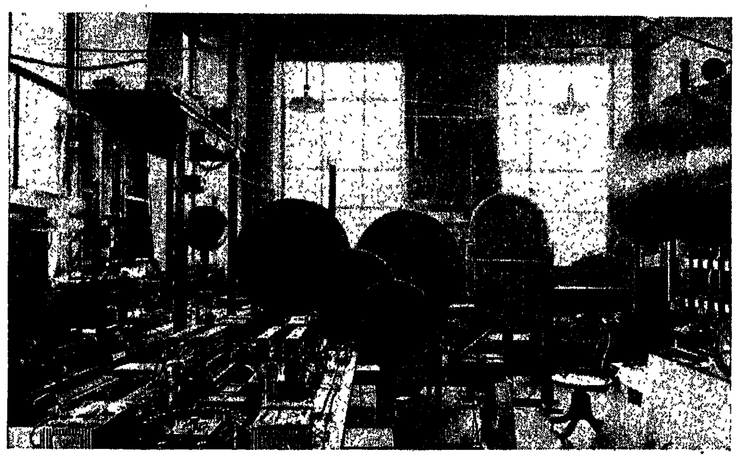


Fig. 6—Loud Speakers Assembled for Comparative Tests

Let us consider from a theoretical standpoint the difference between what makes a satisfactory headphone and what is required for a loud speaker. The ideal sound pick-up, or transmitter, would develop a voltage proportional to the pressure which the sound wave exerts on the diaphragm, whatever the frequency or wave shape. There are in use in broadcasting studios, special microphones and electrostatic transmitters which approximate this ideal characteristic very closely over a wide range of intensity and over a frequency range of from 30 to over 6000 cycles. Amplifiers can be constructed having almost any required amplification ratio, the output currents of which reproduce the input voltage wave with virtually no distortion. The perfect receiver, or loud speaker, must take these amplified voice currents and translate them into air pressures, exactly reversing the function of the pickup or transmitter. The transmitter being equally efficient for all frequencies in the working range, the telephone receiver or loud speaker must convert input current into sound pressure at the listener's ear with equal efficiency at all frequencies. Throughout the remainder of this paper, when a device is spoken of as giving sound radiation independent of frequency, it is to be understood that what is meant is that if a series of pure tones of equal power but varying pitch are produced in a damped room in front of a perfect transmitter with distortionless amplifier, and the output of the amplifier is fed to the loud speaker, the latter will radiate the series of tones with equal power.

In the case of the receiver held to the ear, there is a small cavity between the receiver diaphragm and the ear drum, and, assuming that there is no leakage of air, the pressure which the ear drum feels is proportional to the change in volume of the cavity caused by the deflections of the diaphragm. A diaphragm with natural frequency above the voice range, or, in other words, one in which elasticity and not inertia or damping controls the vibrations, gives a deflection proportional to the magnetic force, or practically proportional to the current through the coils, independent of frequency. Therefore, such a diaphragm gives a sound pressure in the ear proportional to the current supplied to the receiver, provided the receiver is held tight against the ear, and this characteristic is exactly what is wanted for a head telephone. What happens when such an elastic control diaphragm is operated in unconfined air? The simplest case is that of a large area diaphragm radiating plane waves. Here the pressure in the sound wave is proportional not to the deflection of the diaphragm, but to the maximum velocity which it attains, which, for pure tones or sine waves, is proportional to the maximum deflection multiplied by the frequency. Therefore, if we succeeded in obtaining the equivalent of a large area diaphragm by grouping together a number of telephones with tightly stretched

diaphragms, the device would still radiate the high frequencies in undue proportion. With the capacity shunt in the amplifier, as shown in Fig. 7A, this discrepancy is corrected.

Test of Small Inertia Diaphragm

At this point, Kellogg suggested using a coil-driven diaphragm with practically no elastic restoring force, so that at low frequencies very large amplitudes could be attained. In an ordinary telephone the stiffness of the

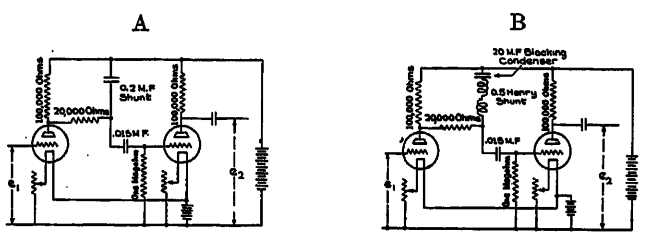


Fig. 7—Compensating Circuits

Type A gives $e_2 = e_1 \times \frac{1}{f} \times$ (Constant)

Type B gives $e_2 = e_1 \times f \times$ (Constant)

f = frequency

diaphragm is depended upon to prevent its sticking to the magnet poles and is, therefore, not suitable where an entire lack of restoring force is desired. On the other hand, the moving coil drive is eminently suited to this purpose, since no stabilizing force is required. Fig. 8 shows the construction of the first model built to try out the free diaphragm principle. Not only did this device produce more of the low tones than any previously tried, but it held up remarkably well for the very high notes, not showing any marked resonance. It did, however, have a rough quality in voice reproduction, which was corrected in the later designs.

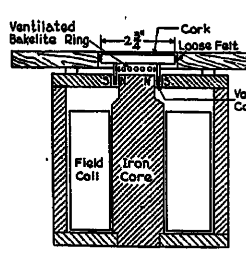


Fig. 8—First Model of Inertia Controlled Diaphragm Loud Speaker

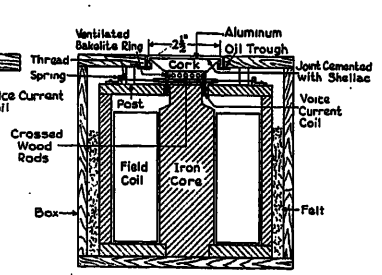


Fig. 9—Improved Design of Inertia Diaphragm, Loud Speaker

Trial of Triple Horn Loud Speaker

The multiple unit horn device, already mentioned and shown in Fig. 4, was first suggested and worked on in our group by Kellogg. It was an attractive possibility, especially in view of the sensitivity obtainable with horns, and the ease with which the balance between high and low tones could be adjusted. For the lower end of the scale, a Baldwin phone with a large exponential horn seemed to be the best combination of available apparatus, while some higher pitched phones with smaller horns took care of the middle and upper ranges. Experiments with this arrangement showed clearly that the three instruments supplemented each other, the combination sounding much better than any one alone, and the consciousness on the part of the listener of the presence of a horn was much less pronounced than in the case of a single horn device. The Baldwin phone would not give sufficiently low frequency output and after a number of attempts to build a suitably low-pitched phone, the conclusion was reached that the most satisfactory low-pitched phone would be one designed along the lines of the moving coil instrument already described. However, when this construction was adopted, no supplementary high pitched instruments were needed.

Improved Designs of Inertia Diaphragm Instruments

Fig. 9 shows the manner of construction of a moving coil instrument designed by Kellogg with a view to avoiding two of the possible causes of the rough quality which had characterized the machine shown in Fig. 8; namely, friction around the edges, and failure of the diaphragm to act as a true piston, or remain flat during vibrations. A maximum of rigidity, combined with light weight, was sought in the diaphragm design, an oil seal was provided around the edge, and the support consisted of four threads at right angles, held in slight tension, so that motion was very free in the axial direction, but practically no sidewise movements could take place. Vibration amplitudes as great as 1/32 inch were frequently observed on this diaphragm. The rough quality was practically eliminated in the new design. Provision was made for boxing in the instrument, and an interesting experience in this connection was that of placing the box over the back, which had the same general effect on sound quality as applying a short horn to the front of the diaphragm. Both helped to bring out the low tones and gave rise to some resonant effects. Bringing out the low tones was due principally to preventing circulation of air between the front and back of the diaphragm. The resonance was in the horn in one case, and in the box in the other. A peculiarity of devices employing very flexibly supported diaphragms is that resonant air chambers behind the diaphragm do about as much harm as resonant cavities in front of the diaphragm, the diaphragm usually taking part in the resonance. Attempts to damp the interior of the box with felt were not entirely successful.

The Use of a Baffle

A happy solution of the problem of preventing circulation was obtained by employing a flat baffle-board, at the suggestion of Rice, who was the first of the group to recognize the importance of the circulation factor in preventing the radiation of low tones. With the flat baffle, no air resonance occurs and both sides of the diaphragm give useful radiation, the total power radiated for a given diaphragm amplitude being nearly four

times as great as that radiated when the back of the diaphragm is enclosed.

Definition of Term "Inertia Controlled"

Subsequent experiments were devoted largely to the development of the free diaphragm type of sound reproducer, in which, throughout the essential frequency range, the electrical driving force is expended in accelerating the mass of the diaphragm. We shall speak of such diaphragms as "inertia controlled." A certain amount of elastic restoring force is unavoidable in the supporting system, and consequently the diaphragm must have a natural frequency. But if the natural frequency is below the important acoustic frequency range, the diaphragm may properly be described as inertia controlled. We have obtained best results with natural frequencies below about 70 cycles per second. Higher natural frequencies can be tolerated if the vibrations are well damped.

Theory for Large and Small Diaphragms

Diaphrams may be classified according to their mechanical properties as:

1. Inertia controlled
2. Damped or resistance controlled
3. Elastic controlled
4. Resonant
5. Diaphragms having wave action or phase differences between the different parts of the surface

Of these, the first three have simple relations between actuating force, frequency, and amplitude of motion, and would, therefore, appear to offer most promise of affcrding a sound source of constant efficiency. The resistance-controlled diaphragm, while difficult to obtain, is included in the list as representing a possible type and is of theoretical interest. It will be assumed in discussing the first three types of diaphragm that all parts of the surface move together, which means that either the diaphragm is small and rigid, or else the actuating force is applied to all parts. The wave action diaphragm is best represented by the large, shallow, paper cones which have been employed with considerable success. Familiar examples of this type are found in the Pathé and new Western Electric 540 AW loud speakers. Some other experiments along these lines were recently reported by Sutton.[8] Here the actuating force is applied at the center, or vertex, and flexural waves radiate toward the outer edge. If there is considerable energy loss so that these waves are attenuated rapidly, the net result is that at high frequencies a small area of the diaphragm near the vertex radiates sound, while at lower frequencies a larger area works. If the attenuation of the flexural waves is small, standing-wave conditions exist and there is a series of frequencies at which resonance occurs.

The relation between amplitude, frequency, and driving force, for diaphragms with pure elastic, resistance, or inertia control is as follows. If the diaphragm vibrates with simple harmonic motion at a frequency f cycles per second, and with an amplitude X centimeters, the deflection may be expressed by

$$x = X \sin 2 \pi f t = X \sin \omega t, \text{ centimeters} \qquad (1)$$

the velocity is

$$u = \frac{d x}{d t} = \omega X \cos \omega t\text{—centimeters per second} \qquad (2)$$

and the acceleration is

$$\alpha = \frac{d u}{d t} = \frac{d^2 x}{d t^2} = - \omega^2 X \sin \omega t \qquad (3)$$

centimeters per second, per second.

With elastic control, the deflection is proportional to the applied force; or to give the deflection shown in (1), we must apply a force

$$F \sin \omega t = K x = K X \sin \omega t \qquad (4)$$

in which F = maximum force in dynes, and K the diaphragm stiffness in dynes per centimeter.

From (4)

$$X = \frac{F}{K} \qquad (5)$$

or the amplitude is proportional to the maximum of the alternating force, independent of frequency.

With resistance control of motion the velocity is proportional to the instantaneous applied force, or to give the motion expressed in equation (2) we must apply a force

$$F \cos \omega t = R u = R \omega X \cos \omega t \qquad (6)$$

in which R is the resistance to motion in dynes per unit velocity.

From (6)

$$X = \frac{F}{\omega R} = \frac{F}{2 \pi f R} \qquad (7)$$

or the amplitude is proportional to the applied force divided by the frequency.

In the case of inertia control, the applied force is equal to the mass M times the acceleration. Hence to obtain the motion expressed in equations (1), (2) and (3) we must apply a force

$$- F \sin \omega t = M \alpha = - M \omega^2 X \sin \omega t \qquad (8)$$

or

$$X = \frac{F}{\omega^2 M} = \frac{F}{4 \pi^2 f^2 M} \qquad (9)$$

In this case the amplitude for a given driving force varies inversely as the square of the frequency.

There are two classes of diaphragms the sound radiation of which are simple functions of amplitude, frequency and diaphragm size.

a. Diaphragms large enough in comparison with the longest waves to give plane-wave radiation for all essential frequencies.

b. Diaphragms small enough in comparison with the shortest waves, to be treated as virtually point

8. *Wireless World and Radio Review*, Nov. 19, 1924.

sources for all essential frequencies, circulation of air between the front and back of the diaphragm being prevented by either a baffle or by enclosing the space on one side of the diaphragm.

The power in ergs per second[9], radiated from one side of the large diaphragm is

$$P = \frac{1}{2} \rho v S \omega^2 X^2 \tag{10}$$

in which

ρ = mean density of air in grams per c. c.

v = velocity of sound in air in cms. per sec.

S = area of diaphragm in sq. cm.

$\omega = 2 \pi f$

X = amplitude of diaphragm motion, assumed to be the same over the entire surface.

In the case of the small diaphragm, the power radiated from one side is[10]

$$P = \rho \frac{S^2 \omega^4 X^2}{2 \beta v} \tag{11}$$

in which β is the solid angle into which the radiation takes place, 4π for complete spherical waves, and 2π for hemispherical waves, or in other words, if a flat baffle is used. With a flat baffle, the radiation from both sides of the diaphragm is

$$P' = \rho \frac{S^2 \omega^4 X^2}{2 \pi v} \tag{12}$$

For these two types of diaphragm we can see from equations (**10**) and (**12**) what would have to be the relation between amplitude and frequency in order to have the sound output the same at all frequencies.

For the large diaphragm, equation (**10**) shows that P becomes independent of ω or of f if X varies as $\frac{1}{f}$, and equation (**7**) shows that applying a force, F, the same at all frequencies to a resistance controlled diaphragm, gives an amplitude which varies as $\frac{1}{f}$

If we have the means for making the force F a direct or inverse function of frequency, a large diaphragm with inertia or elastic control can be made to give the same output at low and high frequencies. For example, in the polarized electrostatic sound source, the force variation is proportional to the alternating voltage between electrodes. If the charging current supplied to the electrodes is fed through a resistance high compared with the capacity reactance of the device, then a constant voltage applied to the resistance and capacity in series will result in a voltage across the capacity inversely proportional to the frequency, and such a voltage will give constant sound radiation from a large area elastic control diaphragm. In like manner in the case of magnetically actuated diaphragms, in which the force is in general proportional to the current, if the inductive reactance of the windings is high compared with their resistance plus the plate resistance of the amplifier tube, the current through the windings will be proportional to the voltage applied to the grid of the tube, divided by the frequency. This will give the desired relation between force and frequency, for constant sound radiation from a large area elastically controlled diaphragm.

If we extend the consideration to the use of pick-up devices or transmitters having direct or inverse frequency characteristics, or include compensating circuits of the type shown in Fig. 7, there are many combinations which give a constant over-all efficiency. Of such combinations, those which throughout the system preserve the normal balance between high and low frequencies, have the advantage that a satisfactory ratio of useful to stray noises is more readily maintained, and the required amplifier capacity is less. We shall, therefore, assume that the goal is to produce a sound source which, when operated by the output of an amplifier tube, will give a sound wave pressure proportional to the voltage applied to the grid of the tube, the proportionality factor being independent of frequency. Such a sound source is the large diaphragm electrostatic unit, with resistance controlled diaphragm motion, and with capacity reactance high compared with the tube plate resistance. Another example is the large diaphragm electrostatic unit with elastic control of the diaphragm and with capacity reactance low compared with the tube plate resistance throughout the essential frequency range. Electromagnetically driven large area diaphragms, with elastic control of motion and inductive control of curent, or with resistance control of motion and resistance control of current, are also possibilities.

Turning to the case of the small diaphragm working with a baffle, equation (**12**) shows that for constant sound output $X^2 \omega^4$ must be constant, or the amplitude must vary inversely as the square of the frequency, and equation (**9**) shows that a constant driving force and inertia controlled motion gives just this required relation between frequency and amplitude. This relationship was pointed out by Kellogg who took especial interest in this type of device and designed all of the models employing small rigid diaphragms. The driving force independent of frequency is available in the form of a coil in a constant magnetic field and with enough resistance in the circuit compared with the reactance to make the current independent of frequency. There are obviously other combinations using small diaphragms, such as resistance control of motion and inductive control of current, which would give constant output, but the inertia controlled small diaphragm with resistance controlled current has all the advantages.

9. Rayleigh, Theory of Sound. Vol. II. Page 16.

10. Rayleigh, Theory of Sound. Vol. II. Page 113.

Effect of Intermediate Diaphragm Size

Formulas for the sound radiation from a flat circular diaphragm situated in a flat wall or baffle of infinite extent have been developed by Rayleigh[11], all parts of the diaphragm being assumed to have the same motion. Fig. 10 shows the relative output at different frequencies of an inertia controlled diaphragm six inches in diameter, actuated by a vibratory force of variable frequency but constant magnitude. For wave lengths greater than about 1.5 feet, (46 cm.), the diaphragm may be treated as a small source, and the sound output is constant. At frequencies for which the wave length is less than the diameter of the diaphragm, the radiation practically follows the law expressed in equation (4) for large area diaphragms. Within this part of the frequency range inertia control gives too rapid a drop in amplitude as the frequency is increased, with the result that the power radiated is less at high frequency. There is, however, a compensating effect. At the same

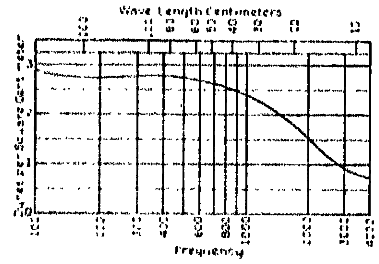


Fig. 10—Radiation Characteristics of Inertia Controlled Diaphragm Six In. (15 Cm.) in Diameter

Ordinates are R. m. s. Pressure at One Meter Distance

$$= \sqrt{\rho v \times \frac{\text{Power radiated from one side}}{\text{Area of hemisphere of 100 cm. radius}}}$$

In which

ρ = Density of Air
= 0.0012 grams per cu. cm.
v = Velocity of Sound
= 3.42 × 10⁴ cm. per sec.
Actuating force assumed = 83,000 dynes. r. m. s. value
Mass = 10 grams

time that the total radiation becomes less, the directivity increases, and the listener in front of the diaphragm receives a larger share of the total power radiated. The result is that with a diaphragm of this size, the listener, if stationed in front of the diaphragm, loses very little of the high frequency components. Even with larger diaphragms, very pleasing results have been obtained with inertia-controlled diaphragms, notably the Hewlett induction phone in which a 24-in. diaphragm has been employed with good effect. In this the tendency to give excessive radiation of low tones is reduced by the fact that no baffle is employed, and there is a diminution in diaphragm currents at low frequency due to imperfect transformer action. The directive properties of this large area diaphragm are very striking.

In order to radiate in accordance with equation (12) or as a small source, up to 5000 cycles, a diaphragm would have to be less than about 1 ½ inches in diameter, but since experiments indicate that much larger inertia-controlled diaphragms give good results, there is no justification for limiting the size to the value mentioned, particularly as sensitivity is gained from the adoption of larger sizes.

Requirements for True Piston Action

Since the inertia-controlled small diaphragm appeared, from both theoretical and experimental evidence, to offer the greatest promise of a practical solution of the loud speaker problem, subsequent experiments were devoted to finding the best form of device embodying this principle. The first models left much to be desired in the way of sensitivity and considerable room for improvement in quality. The condition that the diaphragm must move as a unit meant two alternatives; either the diaphragm must be very rigid for its weight and must be quite small, or the driving force must be applied uniformly over the entire diaphragm surface.

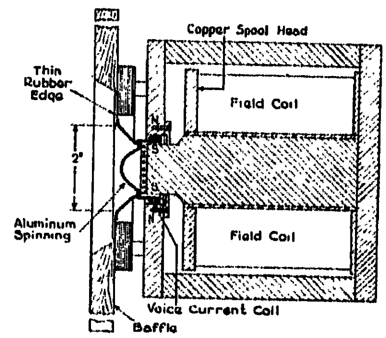


Fig. 11—Baffle-Type Rigid Diaphragm, Loud Speaker

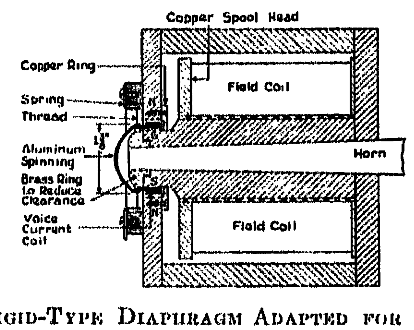


Fig. 12—Rigid-Type Diaphragm Adapted for Use with a Horn

Models with Rigid Diaphragms

Two of the most satisfactory forms of small rigid diaphragms are shown in Figs. 11 and 12. Of available materials, aluminum has as high a ratio of elastic modulus to density as any, combined with light weight. The oil seal of Fig. 4 was considered impractical for any but a laboratory model, and in place of it, either small clearance was depended upon to prevent leakage or a thin rubber membrane was used to bridge the gap between moving and stationary parts. Fig. 12 was de-

11. Theory of Sound. Vol. II. Pages 162-165.

signed for use with a horn[12]. The horn was four and one-half feet long with an exponential expansion from ½ inch to 30 inches diameter. Fig. 13 shows the same horn with the earlier model instrument illustrated in Fig. 9. Such a horn carries down to about 200 cycles satisfactorily and has no very strong resonances[13].

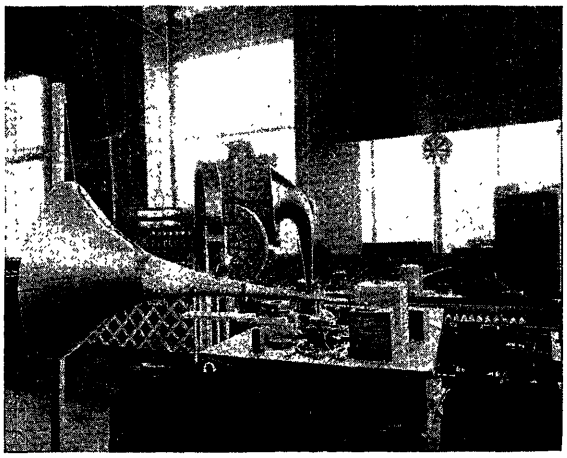


FIG. 13—RIGID-TYPE INERTIA-CONTROLLED DIAPHRAGM WITH EXPONENTIAL HORN

Resistance and not inertia control is the ideal diaphragm characteristic for an instrument employing a slow expanding exponential horn, and that pleasing results should be obtained with the free diaphragm must be ascribed to the fact that the diaphragm never attained the amplitudes at low frequencies corresponding to true inertia control, because of damping by the air column in the horn and strong electromagnetic damping characteristic of the moving coil drive. A

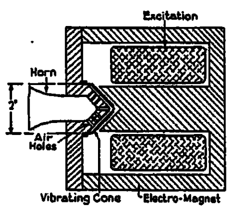


FIG. 14—COMMERCIAL GAUMONT LOUD SPEAKER

horn of adequate dimensions, however, is exceedingly bulky, and better results were subsequently obtained with flat baffles.

MODELS WITH DISTRIBUTED DRIVING FORCE

Of the devices in which the actuating force can be applied over the whole diaphragm area, thus making rigidity unnecessary, the electrostatic phone and the Hewlett induction phone have already been mentioned. In the latter, the air-gap length is the radius of the diaphragm, and a very strong magnetic field is practically out of the question. As a result the efficiency is low and the fine quality of which the instrument is capable is secured at the cost of a high power amplifier and considerable direct-current power for excitation. It is, therefore, not a household device, its field of application being rather in auditoriums.

Rice urged the probable value for the case of the small diaphragm instruments, of distributing the driving

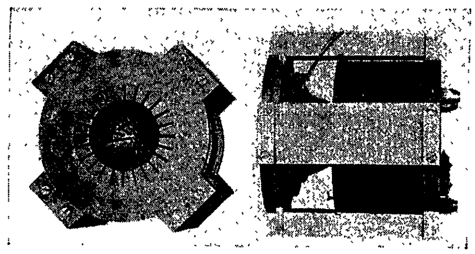


FIG. 15—GAUMONT TYPE LOUD SPEAKER ASSEMBLED

force over the entire diaphragm area, and believed that this could be done without sacrifice of field flux density. At this juncture a letter was received from Dr. W. R. Whitney describing a loud speaker which had been shown him in France by its inventor, Mr. Gaumont, and which fulfilled this condition.

Fig. 14 shows the construction of the commercial loud-speaker built by the Société des Etablissements Gaumont. The diaphragm is a cone of thin silk, to

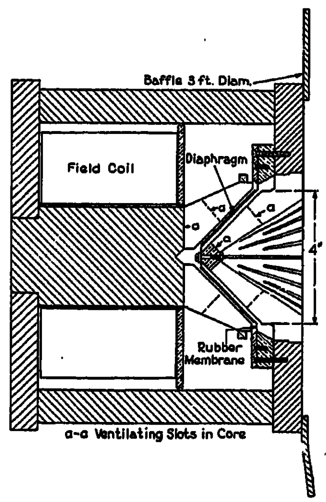


FIG. 16—MODIFIED DESIGN OF GAUMONT TYPE LOUD SPEAKER

which is cemented a single layer spiral of fine aluminum wire. An extremely light diaphragm is thus possible, making for sensitiveness and freedom from resonance. The reaction of the voice currents in the aluminum coil with the radial component of the magnetic field gives the useful driving force.

Figs. 15 and 16 show one of the Gaumont type loud speakers designed by Rice, for use with a baffle. The

12. A description is given in the *Wireless World* and *Radio Review* of Dec. 17, 1924, by Capt. H. T. Round, of a loud speaker resembling that shown in Fig. 12 in some respects.

13. "Function and Design of Horns for Loud Speakers" by C. R. Hanna and J. Slepian, JOUR. A. I. E. E., March, 1924. "The Performance and Theory of Loud Speaker Horns" by A. N. Goldsmith and J. P. Minton, *Proc.* I. R. E., Aug. 1924.

diaphragm support used in the commercial instrument shown in Fig. 14 does not afford sufficient flexibility. The substitution of a flat edge of thin rubber, as illustrated in Fig. 15, gave the necessary flexibility without which adequate radiation of low tones from a small diaphragm had been found impossible. Experiments also indicated that a considerably heavier diaphragm

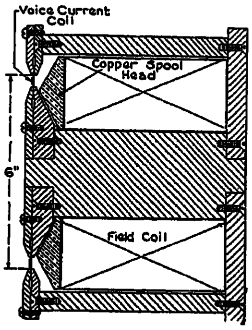


FIG. 17—CONSTRUCTION OF ANNULAR TYPE LOUD SPEAKER

than that employed in the commercial machine was required for good balance between high and low frequencies. The diaphragm which worked best was four inches in diameter and weighed 11 grams. It consisted of a single layer of copper wire embedded in rubber,

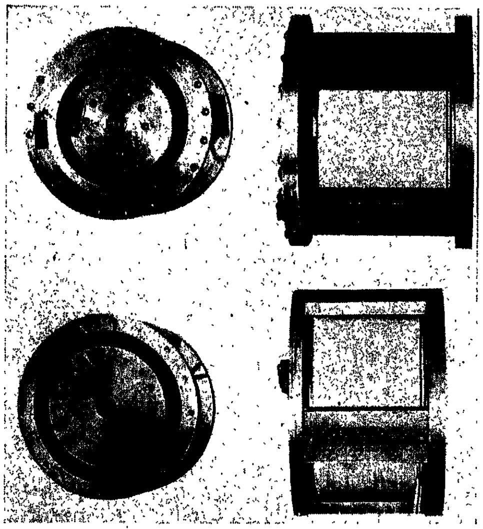


FIG. 18—VIEWS OF TWO ANNULAR TYPE INSTRUMENTS

which gave a soft, non-resonant structure. Adequate venting proved to be of utmost importance in this design in order that air reactions on the diaphragm might not affect its motion adversely and in order to let out the sound without muffling. A baffle 3 ft. in diameter was employed.

An annular diaphragm type of loud speaker having uniformly distributed driving force is shown in Figs.17, 18 and 19. This instrument which was designed by Rice presents no venting difficulties and gives the possibility of considerable diaphragm area. A thin rubber membrane spans the air-gap, and a single layer coil cemented to the membrane provides the driving force. The construction difficulties are much less and the total weight less than in the case of the conical diaphragm type. In the annular design, if the air-gap is kept short enough to give reasonable magnetic field strength; it becomes difficult to make the natural frequency of the diaphragm low enough to fall below the important acoustic range. Electrical correction by

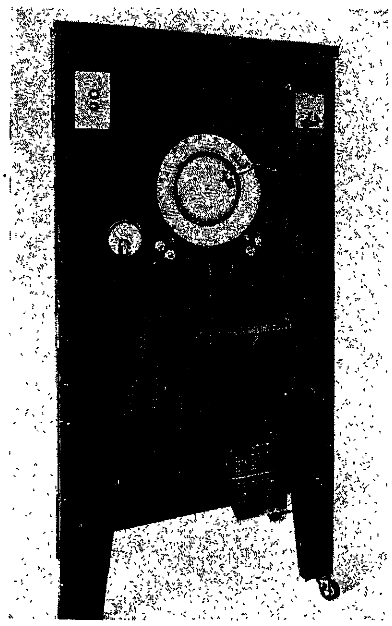


FIG. 19—ANNULAR TYPE LOUD SPEAKER ASSEMBLED IN CABINET WITH AMPLIFIER

means of a series-tuned shunt across the output of the amplifier eliminated the most objectionable results of the diaphragm resonance, and in this form the device was very satisfactory. A somewhat similar instrument used as a transmitter has recently been described by Round[14].

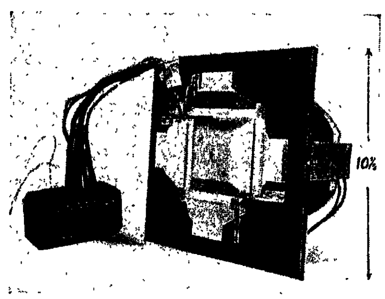


FIG. 20—ALUMINUM FOIL DIAPHRAGM LOUD SPEAKER WITH TRANSFORMER

Efforts to gain sensitivity by the use of extremely light diaphragms have always, in our experience, led to disappointment. The expected gain in sensitivity was not realized, and in most cases the light diaphragm devices were very high pitched; or in other words,

14. *Wireless World and Radio Review*, Nov. 26, 1924.

radiated too much high frequency sound compared with the low. This, for example, was the characteristic of the design shown in Fig. 20 in which a strip of aluminum foil[15] a half mil thick, by a half inch wide, carrying the voice current and vibrating in a gap between magnet poles served as diaphragm. Careful transformer design, very flexible mounting of the foil, and an adequate

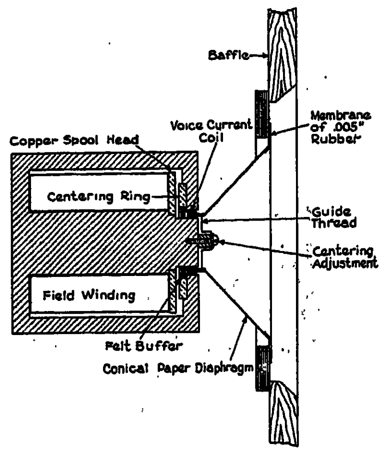


FIG. 21—CONSTRUCTION OF FREE EDGED, COIL-DRIVEN, CONICAL DIAPHRAGM LOUD SPEAKER

baffle made it appear that the result was not due to overlooking any of these factors. We must conclude either that unavoidable leakage was responsible for the failure to radiate the lower tones in due proportion,

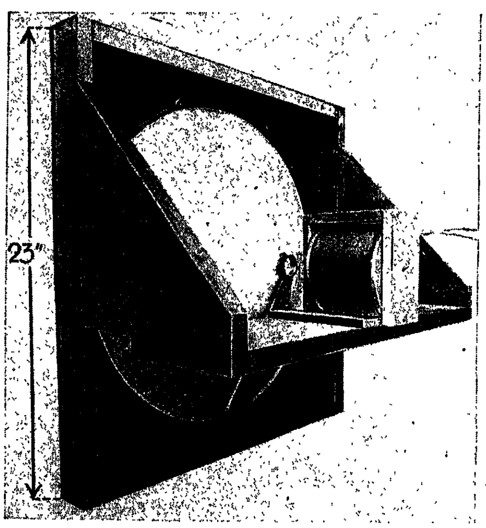


FIG. 22—LARGE FREE-EDGED PAPER CONE WITH COIL DRIVE

or else that air reactions, and electromagnetic damping play such important parts in the diaphragm motion compared with the inertia of the diaphragm itself, that the motion cannot be looked upon as inertia controlled, except in the upper frequency range.

Electromagnetic damping by the reaction of the driving coil with the magnetic field takes the form of a counter electromotive force which reduces the supplied current and driving force. The net result is the same as though the driving force had remained constant and a retarding force had been developed by a counter current generated by the motion of the coil. Thus "motional impedance" and electromagnetic damping are two aspects of the motion of the conductor in a magnetic field. Electromagnetic damping only becomes an important factor when the motional impedance becomes considerable compared with the other resistances and reactances in the circuit. Electromagnetic damping can be controlled in a measure by the ratio of transformation if a transformer is used. Making the load impedance high compared with the tube plate resistance gives maximum damping, while

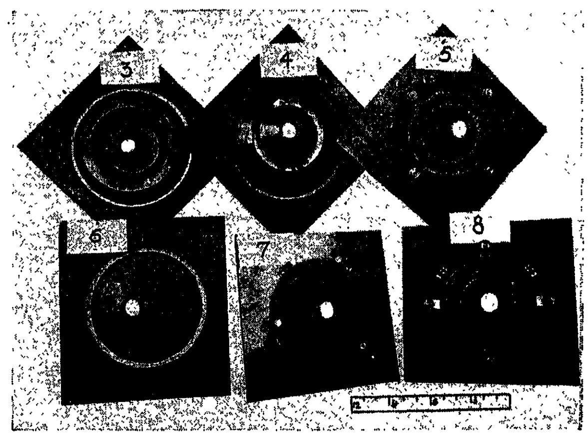


FIG. 23—EXPERIMENTAL DIAPHRAGMS

3. Mechanical filter type, three stages
4. Mechanical filter type, two stages, center for high frequency, whole surface for low
5. Extra supporting rubber membrane, half way out
6. Reinforced center, graded thickness
7. Reversed conical stiffening edge, thread-suspension, no-edge membrane
8. Extra supporting, flat-paper membrane half-way out.

making the load impedance low relative to resistance in the circuit minimizes the magnetic damping.

SEMI-RIGID DIAPHRAGMS

With the rigid type of diaphragm, a question to be settled experimentally was how large could the diaphragm be made before the quality of sound reproduction became impaired through failure of the diaphragm to act as a unit or plunger throughout the entire frequency range. A simple cone is an exceedingly rigid structure for its weight, particularly with respect to vibrations of the type which could be excited by a symmetrically applied force in the axial direction, such as is used in driving a conical diaphragm; or in other words, the cone is rigid with respect to vibrations like the opening and closing of an umbrella. With an angle of 45 deg. between the axis and wall of the cone, the rigidity is practically at its maximum. Paper cones of various sizes were tried with free or flexibly supported outer edges, using baffles to prevent circulation and

15. A loud speaker employing an aluminum strip in a magnetic field, manufactured by Siemens & Halske is described in "The *Wireless World* and *Radio Review*," July 2, 1924.

small coils as shown in Fig. 21 for drive. Diameters from 4 to 24 in. and angles from 45 deg. to 75 deg. were tried. These paper cones gave marked increase in

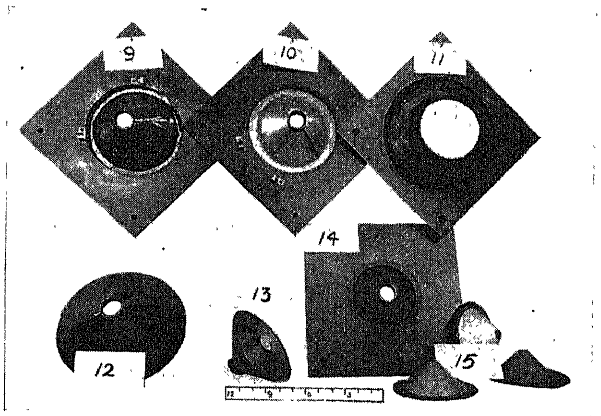


FIG. 24—EXPERIMENTAL DIAPHRAGMS

9. Glacine-paper cone 0.0015 in. thick, flat edge of same material
10. Aluminum Cone 0.002 in. thick, 0.005 in. rubber edge
11. Large coil, 4½ in. diameter, three-ply cone, 0.18 in. blotting Paper between Sheets of half-tone paper
12. Blotting paper, surface hardened with shellac to increase rigidity
13. Internal conical stiffener
14. Rubberized cloth, same material for edge
15. Early experiments with coil-driven cones four in. diameter bond paper, blotting paper and 0.010 in. aluminum

sensitivity over the devices shown in Figs. 11, 12, 15 and 17, the general sensitivity on speech being at least equal to that of a good horn type loud speaker. Size seemed

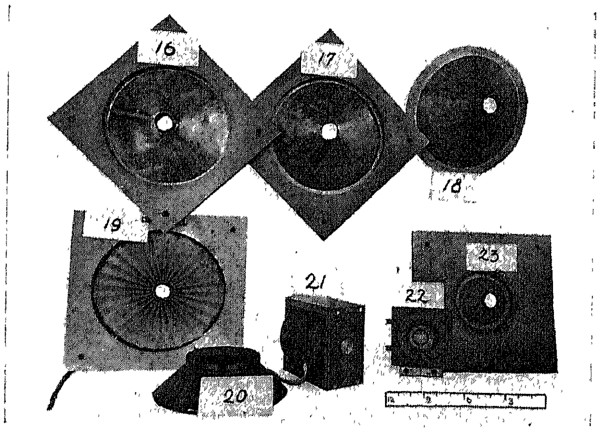


FIG. 25—EXPERIMENTAL DIAPHRAGMS

16. Eight-in. diameter cone, one in. deep to vertex, 0.007-in. paper, rubber edge 0.005 in. thick and ¼-in. wide
17. Same as (16) except two in. deep to vertex
18. Eight-in. diameter, 45-deg. cone, 0.007-in. paper, ¼-in. rubber edge
19. Lumier type diaphragm rocker joint at center, coil-drive, outer edge damped with sponge rubber
20. 4½-in. diameter coil, aluminum wire, cone eight in. diameter, 45 deg. of 0.007 in. paper
21. Field magnet for coil drive.
22. Aluminum spinning diaphragm with rubber edge for coil drive
23. 3¾ in. diameter 45 deg. cone. 0.007 in. paper; flat paper edge 0.0035 in. thick by $^3/_8$ in. wide.

to have little effect on general sensitivity, but did somewhat alter the quality, diameters between four and eight inches giving the best results. The angle did not appear to be critical, but with very shallow cones speech sounded muffled, the high frequencies being lacking.

A rough calculation indicates that a 45 deg. paper cone 4 in. in diameter would begin to depart materially from rigid plunger action, at frequencies of between 3000 and 4000, while with larger diameter cones the change would take place at lower frequencies. The paper-cone diaphragms used in this series of tests must, therefore, be considered as acting substantially as

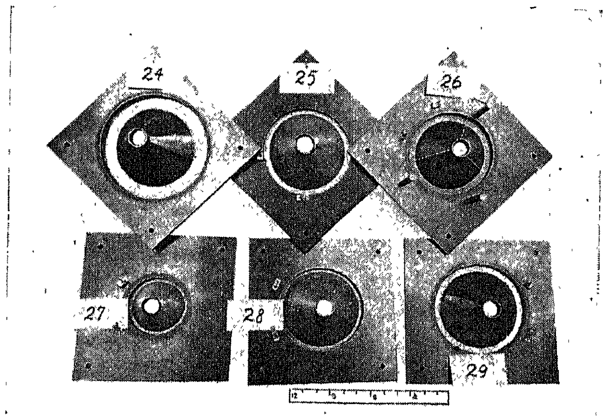


FIG. 26—EXPERIMENTAL DIAPHRAGMS

24. 5¾ in. diameter 45 deg. cone of 0.007 in. paper with one in. flat paper edge
25. Same as (24) with $^3/_8$ in. wide paper edge
26. 5¼ in. diameter 45 deg. cone of 0.007 in. paper with 0.005 in. rubber edge ½ in. wide. Guide threads to center coil
27. 3¼ in. diameter 45 deg. cone of 0.007 in. paper with ¼ in. wide rubber edge
28. 5¾ in. diameter 45 deg. cone of 0.007 in. paper with 0.005 in. rubber edge ¼ in. wide
29. Same as (28) except ½ in. wide silk edge

plungers for the lower frequencies with a gradual transition to wave action or progressive deflection at the higher frequencies. If there was any loss of quality due to the failure of the diaphragm to move as a whole at high frequencies, there was a compensating improvement as compared with the small diaphragm instruments, in a better radiation of the low tones, for the small diaphragms did not give

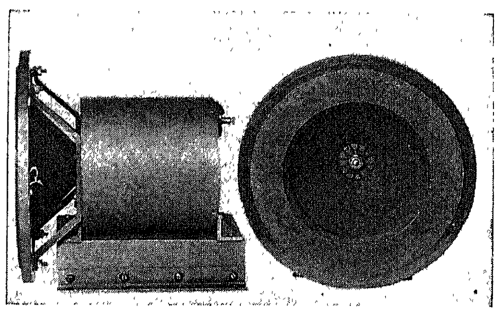


FIG. 27—FREE-EDGED, COIL-DRIVEN, CONICAL PAPER DIAPHRAGM LOUD SPEAKER UNIT

quite enough low frequency radiation. An extended series of tests was made to see whether a further improvement in quality could be obtained, making the

cone of various materials and different thicknesses and by employing stiffening members to reduce the tendency of the cone to break up into vibrations either of the kind with circular nodes or with radial nodes. Figs. 22 to 26 inclusive show some of the forms of diaphragm tried. Nothing better was found, however, than a simple 45-deg. cone of 0.007-in. to 0.010-in. paper, about six in. in diameter, with a flexible support around the outer edge consisting of a membrane of rubber 0.005 in. thick and ¼ in. wide, under very slight tension. Fig. 21 shows the general construction of diaphragm and field, and Figs. 27 and 28 show views of two models of the instrument with baffles omitted. Leaving the center of the cone open as indicated in Fig. 21 simplified construction and avoided the necessity of venting the space in front of the magnet core. A baffle, two feet square, appeared adequate. If the shortest air path between the front and back of the diaphragm is a quarter wave length or more there is no loss of radiation through circulation, although regions of interference between the two sound sources appear. An eight-foot wave length corresponds to a frequency of 135 cycles, and loss of output below this frequency may be attributed, in part, to circulation.

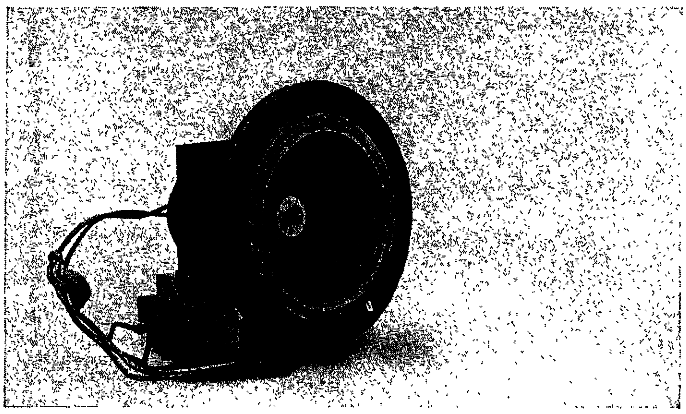


Fig. 28—Coil Driven Conical Paper Diaphragm, Loud Speaker Unit with Flexible Paper Edge

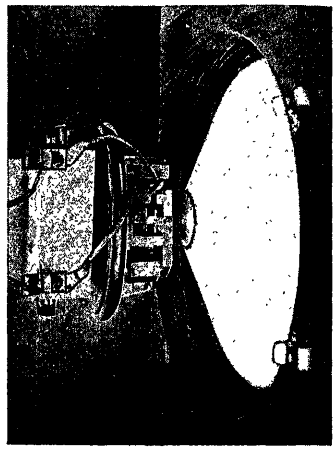


Fig. 29—Conical Paper Diaphragm with Free Outer Edge and Magneto Phone Drive

Summary of Conditions for Good Reproduction

The conclusion from these experiments was to the effect that the best practical solution of the loud speaker problem was a device combining the following features: *a conical diaphragm four inches or more in diameter with a baffle of the order of two feet square to prevent circulation and so supported and actuated that at its fundamental mode of vibration the diaphragm moves as a whole at a frequency preferably well below 100 cycles.*

The larger the diaphragm the less seriously will the baffle be missed if omitted. Thus, if the diaphragm were two feet in diameter, the effect of adding a baffle would be difficult to detect by ear. The use of the baffle gives latitude in the choice of diaphragm size. The baffle need not necessarily be flat, but if concave the solid angle included on the concave side should be at least as great as that in a cone with an angle of 45 deg. between wall and axis. With a smaller solid angle, resonance rapidly becomes noticeable and a change in general pitch level characteristic of horn action is brought about[16].

In order that the diaphragm may vibrate as a whole, the support at the outer edge must be very flexible compared with the diaphragm itself.

Comparison of Driving Systems

A low natural frequency means either a heavy diaphragm which would cost sensitivity, or else that the elastic restoring force supplied by the diaphragm supports plus any elastic restoring force in the driving mechanism must be small. Electromagnetic driving systems with moving iron armatures all require a certain amount of elastic restoring force to maintain stability. The required stiffness may be reduced by (1) lengthening the air-gaps, (2) working with weaker average magnetic field, (3) using a lever system which makes the diaphragm motion greater than the change in air-gap length; all of which mean a sacrifice of driving force. Fig. 29 shows a model employing a free edge cone with iron armature drive. By using a fairly large diaphragm and working close to the limit of stability it is possible to use this type of drive and obtain a low enough natural frequency and moderate sensitivity, but the moving coil drive has the following advantages: 1. The elastic restoring force may be made as low as desired without sacrifice of sensitivity. 2. Very large amplitudes can be allowed. (With a drive using variable length air-gaps, the change in gap length must always be small compared with the average length if distortion is to be avoided). 3. The relation between current and force is strictly linear and there-

16. "Effect of a Horn on the Pitch of a Loud Speaking Telephone," by E. W. Kellogg, *General Electric Review*, August, 1924.

fore distortion due to bends in the magnetization curves of iron is avoided in the moving coil drive. 4. If a strong magnetic field is provided the coil drive gives greater sensitivity than the iron armature drive. 5. No adjustment is upset if the weight of the diaphragm causes it to shift somewhat when the instrument is tilted. The disadvantages of the moving coil drive are the size and weight of the field magnet and the power required for excitation, but these disadvantages are outweighed by the advantages, in the opinion of the writers and their associates, particularly, when a special ampli-

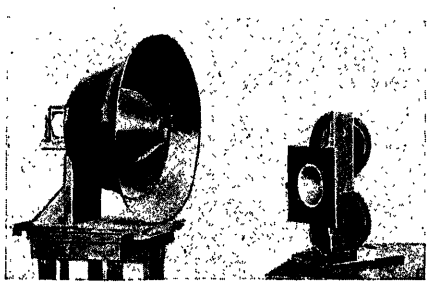


Fig. 30—Moving Coil Type Loud Speaker Unit with Permanent Magnet Field and Three-Pass Exponential Horn

fier is part of the loud speaker equipment. A loud speaker with moving coil drive and permanent magnet field is shown in Fig. 30.

Comparison of Free and Fixed Edge Cones

It may be of interest to compare the action of the flexibly supported conical diaphragm with that of one with comparatively rigid support. The latter has had extensive application in hornless loud speakers. If a vibrating force of constant strength, but variable frequency is applied at the vertex of the rigidly supported cone, there is small motion and little sound radiation at frequencies below the lowest resonance. Additional resonances occur at frequencies of the order of 3, 5, 7 times that of the fundamental. In order that low tones shall not be almost entirely lost in the output of such a device, it must be constructed so that the fundamental resonance occurs at a frequency low in the range of important sound. The diaphragm action may be said to be characterized by a series of resonances. The upper resonances are closer together on the musical scale, and this together with the greater damping makes the radiation more nearly uniform in this part of the range. The very high frequencies, however, are not adequately radiated when the cone is made very flat, or shallow compared with its diameter. The requirement of a low fundamental resonance calls for a shallow cone, unless the diameter is very large. The choice of angle is, therefore, a compromise.

The free-edged cone permits large amplitudes at low frequency, and as already pointed out the relation of amplitude to frequency is such as to maintain practically constant radiation. Since the radiation of the lower frequencies is not critical to either size or cone angle, these factors may be chosen entirely with reference to obtaining the best effects at high frequencies. Since the flexural waves which travel from the vertex toward the edge, are reflected as well by a free as by a fixed edge, only in opposite phase, standing wave conditions with circular nodes can occur with the free edge cone as well as with one having a fixed edge, and hence a series of resonances is possible. If the cone is steep, which means high velocity wave propagation, and small in diameter, the first resonance will occur at a high frequency, and the action characterized by a series of resonances will be confined to a minor part of the essential frequency range. Strong resonances in this region have not been observed, from which fact the conclusion must be drawn that damping in the diaphragm material and perhaps also in the material of the flexible edge is considerable.

The difference in the action of the diaphragms may be illustrated by the analogy of the behavior of two electrical transmission lines when an alternating voltage of variable frequency is applied. The free edged diaphragm is like a line short circuited at the end farthest from the generator, and the fixed edge diaphragm corresponds to an open circuited line. Current in the line is analagous to velocity of motion of the diaphragm. Large currents can be sent through the short-circuited line at low frequencies, the voltage and line inductance determining the current values. No resonance occurs until the frequency reaches a value which makes the line a half wave length long. Additional resonances occur at frequencies two, three, four, five, or more times that of the first. In the case of the open-circuited line little current flows until the first resonance is approached, at which the line is a quarter wave length long. The higher resonances are at frequencies three, five, and all odd multiples of the first. To make the

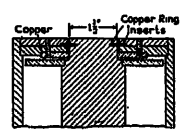


Fig. 31—Copper Ring Inserts to Reduce Impedance of Moving Coil

fundamental resonance occur at a low frequency, the line must be long, and constructed in such a way that the rate of wave propagation is slow.

Copper Rings to Reduce Impedance

In loud speakers employing a moving coil drive the radiation at high frequency can be increased, if desired, by a scheme due to Rice which consists in placing copper rings near the moving coil, so disposed that they form short-circuited secondaries and reduce the impedance of the coil at high frequency. In one coil tested the impedance at 4000 cycles was 57 ohms without the copper rings and 24 ohms with the rings in place. At 500 cycles the ohmic resistance of the coil was the princi-

pal factor, so that the rings had little effect. Fig. 31 shows the disposition of rings used for this purpose.

Special Amplifier

As was pointed out by Martin and Fletcher[17], voices and music do not sound natural unless reproduced at approximately the original level of intensity, even though the reproduction may be free from all wave form distortion. In order, therefore, that the full benefit of a high grade loud speaker may be realized, it is important that the amplifier which goes with it should have sufficient capacity to give a natural volume or intensity.

Fig. 32 shows the circuits of an amplifier which has proved satisfactory for household service. Two U V-216 kenetrons rectify 70 milliamperes at 550 volts. The field of the loud speaker serves as filter choke. In order that pulsations in the rectified current in the exciting coil may not cause changes in the air-gap flux and thereby produce hum and modulation of the sound output, the head of the spool on which the field coil is

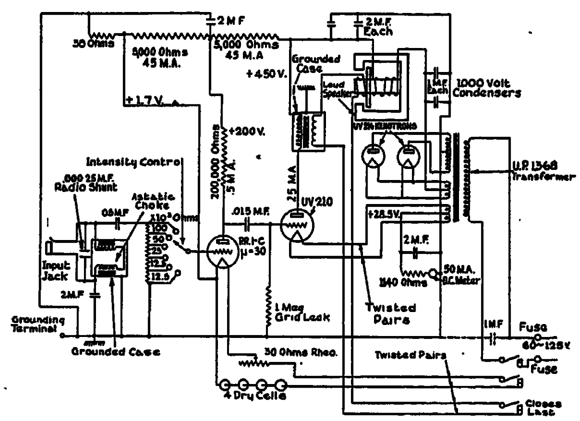


Fig. 32—Rectifier and Amplifier Circuit

wound is made of copper one-fourth inch thick. This expedient suggested by Kellogg steadies the flux so that there is almost no ripple. About 100 volts are dropped in the field coil, leaving 450 volts for the plate supply of the U V-210 radiotron which serves as the power tube. A bias of about 28 volts is required for the grid, and this bias is obtained by dropping 28 volts in a resistance so that the filament runs at a mean potential of + 28 volts with respect to the negative terminal of the rectifier. This makes the net plate voltage across the tube about 422, and the mean plate current is 25 milliamperes. Under these conditions an average U V-210 radiotron can send 10 milliamperes, r. m. s. value, of sine wave current through a 10,000 ohm load without appreciable wave distortion[18]. This represents about thirty times as much power as the same tube could put out with a plate supply of 120 volts.

Cabinet Set

Figs. 33 and 34 are views of a laboratory model of a cabinet set containing rectifier, amplifier and loud speaker. The front of the cabinet acts as baffle. To prevent air resonance in the box, the sides and back are vented by inserting panels of perforated brass. The ammeter shown in the picture is connected to read the plate current of the radiotron. Whenever the peak

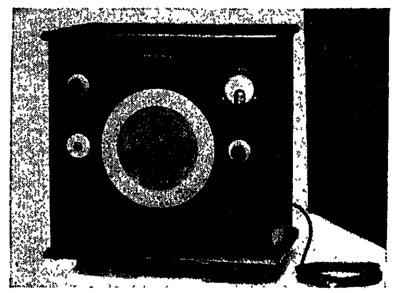


Fig. 33—Laboratory Model of Cabinet Set—Front View

values of the voltage applied to the grid of the tube exceed the value for which distortionless operation is possible the meter needle shows disturbance. If roughness commonly termed "blasting" is noticed in the reproduction, and if at the same time the meter needle kicks, the intensity of the input should be reduced. If the meter needle is steady, the fault is probably not in the amplifier.

Psychological Factors

Certain experiences connected with testing and demonstrating loud speakers of the type we have de-

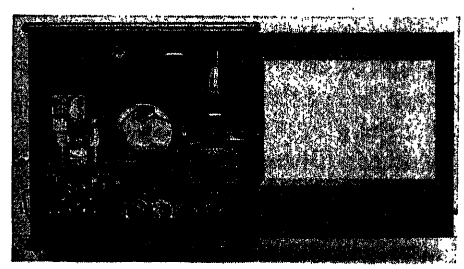


Fig. 34—Laboratory Model of Cabinet Set—Back View

scribed are of interest. The possession of an instrument in which distortion is minimized and whose response covers a wide frequency range, transfers the interest of the broadcast listener from "fishing" for distant stations to that of trying to find the best program among the near-by high grade broadcasting stations, and to enjoying the music or speeches themselves.

17. "High Quality Transmission in Reproduction of Speech and Music," by W. H. Martin and H. Fletcher. Journ. A. I. E. E., March, 1924.

"Physical Measurements of Audilion and their Bearings on the Theory of Hearing," by H. Fletcher. *Jour.* Franklin Inst., Sept., 1923.

18. Design of Distortionless Power Amplifiers, by E. W. Kellogg, A. I. E. E., 1925 Midwinter convention.

There are, on the other hand, conditions when the difference between the loud speaker with a wide frequency range, and one of the ordinary horn type which loses both very high and low frequencies, is not at all striking, and the latter may even sometimes be preferred. Measurements of sound intensity required for audibility[19] show that as intensity is reduced, the low tones will be lost first, since the threshold intensity for example of a 100-cycle tone is of the order of fifty times that for a 1000-cycle tone, intensity being expressed in sound wave pressure. As a result of this, when the reproduction as a whole is very faint, the instrument which produces the low tones does not sound materially different from one which does not, for even if reproduced in the correct relative intensity compared with the higher tones, the low tones are below audibility.

When a radio program is half smothered in static, it may sound better through a loud speaker whose response is mainly between 500 and 2000 cycles, than through one having a greater range. The energy in the incoming static is likely to be almost uniformly distributed over the audio frequency range, provided the receiving set is not responsible for distortion, whereas the range 500 to 2000 cycles includes the major part of the essential voice frequencies. Extending the range above and below would add to clearness and naturalness in the absence of interference, but with heavy static it may often bring in enough additional disturbance to more than offset the gain. Lack of clearness may be less irritating to the listener than disturbing noises. Hence the enjoyment of the wide range loud speaker is largely confined to strong stations or else to times of comparative freedom from static. A similar observation applies to roughness caused by "blasting" from overworked amplifiers or other causes. When any piece of acoustic apparatus is worked beyond the maximum amplitude for which the output bears a linear relation to the input, the resulting wave distortion takes the form of the production of overtones. The rough harsh sounds which result are much less noticeable with an instrument which cuts off the high frequencies. Therefore, if the improved articulation and greater detail in music, which are made possible by response to high frequencies, are to be a real advantage, we must avoid the faults just mentioned in the currents supplied to the loud speaker. The logical place to begin is the amplifier associated with the loud speaker. This must be carefully designed and have ample capacity so that there will be little temptation to overwork it. Few pieces of apparatus are so frequently worked beyond their proper capacity as loud speaker amplifiers. This is natural in view of the initial expense of an adequate amplifier, and the desire for volume of sound from the loud speaker. With the usual type of loud speaker a slight overworking of the amplifier is hardly noticed, and rather than provide greater amplifier power, users of loud speakers have compromised with low volume and some amplifier distortion, and either educated their ears to accept the result as good, or else lost interest.

19. Physical Measurements of Audition and their Bearing on the Theory of Hearing, by H. Fletcher, *Journ.* Franklin Inst., Oct. 1923, Bell System *Tech Jour.* Oct. 1923.

Another factor bears on the question of amplifier capacity. With distortion such as is usual in receiving sets and loud speakers the reproduction sounds best when weak, perhaps because the distortion is similar in some respects to the effects of distance. Use of such equipment results in one's forming the habit of enjoying faint music. With more nearly correct reproduction of the original music, enjoyment is increased by bringing the volume up to normal or the intensity to which one is accustomed when listening directly. In several instances it has been observed that when a loud speaker of the new type has been placed in the home of some one previously accustomed to a loud speaker of the usual construction, at first the listener preferred to keep the intensity very low, but after a few days we find him working with normal volume.

Output Measurements

Sound pressure measurements with constant input and variable frequency have been taken with two samples of the new type loud speaker, and these in general confirm the aural impressions that the instrument covers a wide frequency range without the great inequalities in output at different frequencies which characterizes loud speakers in general. Such measurements, however, are likely to be misleading unless extreme precautions are taken to avoid certain errors. Up to the time of writing the authors have not been able to obtain measurements under conditions with which they were completely satisfied, and it is, therefore, deemed best not to publish any of the output data so far obtained.

Acknowledgments

The writers wish to express their deep appreciation for the never failing interest and encouragement given by Dr. W. R. Whitney throughout the long investigation. We are also greatly indebted to our colleague, Dr. C. W. Hewlett, for many helpful suggestions and for the production of our first really high-grade loud speaker which was constantly used as a standard of comparison. We are further indebted to Mr. E. P. Lawsing for able assistance in the experimental work and to Mr. W. F. Winter in the mechanical construction.

Discussion

H. A. Frederick: In reading this paper one cannot but wish that some definite standard or method of rating might have been employed, so that the results of this investigation of many types of loud speakers might be placed quantitatively in definite positions on some scale of merit. Since the authors, as pointed out in the paper, have not as yet been able to obtain satisfactory quantitative measurements with their best designs, the readers are not in a position to judge the accomplishment.

While these measurements are not simple and are liable to be misleading, unless very carefully made, still the same comment applies to qualitative judgments by the ear alone.

In this work, the authors have primarily stressed the obtaining of a loud speaker giving high quality; that is, one which faithfully and without distortion reproduces the sound whose electrical counterpart has been fed to it. In striving for this result, it is difficult also to obtain efficiency. In the authors' analysis of the problem they have largely sacrificed considerations of efficiency. For example, the "inertia control" used in their design involves a diaphragm 90 deg. out of phase with its driving force or, in other words, low power factor. They have also neglected motional reactions of the mechanical system on the electrical as well as transition losses due to the connection of vibratory systems of different impedances. The analysis as a result is limited in its field of application. It is probable, on the other hand, that improvements to overcome such losses will be effected and that they will make possible loud speakers of high quality which will also be of materially higher efficiency than those now available.

Another phase of the loud-speaker problem which would seem to warrant consideration is the load capacity. This, of course, must be defined in terms of sound-power output. A loud speaker, to be satisfactory for certain very important classes of service, must be capable of giving out a very considerable amount of sound without having the relation between output and input depart from a linear characteristic. In rating the various types studied it would be of interest to know how they were found to compare on this score; also, it would be of interest to know what were the limits of output found for these most promising designs and whether magnetic or vibratory, namely, mechanical limitations were first encountered or whether heating of the coil limited its output.

In the favored design, the size of the baffle, of course, sets a lower frequency limit while the size of the cone and its rigidity set a higher frequency limit. It would be of interest to know where these two limiting frequencies were to be found.

Near the end of the paper, it is stated that the stiffness of a magnetic-type motor system can be decreased by weakening the polarizing magnetic system. As pointed out by Hanna, the polarizing field gives rise to what might be termed a negative stiffness since it acts in opposition to the mechanical stiffness of the system and, therefore, has the opposite effect to that stated.

The authors state that "if a strong magnetic field is provided, the coil drive gives greater sensitivity than the iron-armature drive." I would like to ask if they would explain a little more in detail just what is meant by sensitivity, and how this conclusion was reached. It would seem that the magnitude of the mechanical force and the mass, as well as the electrical impedance and current, would have to enter a satisfactory expression for sensitivity.

In the article appears the statement that "four times the power is radiated with the baffle as with the back enclosed." Is the conclusion based on theoretical grounds or was it determined entirely by observation?

The reference to the resistance-controlled type as being of only "theoretical interest" might perhaps appear to underestimate its importance since any loud speaker to have high efficiency must be largely resistance-controlled, the resistance coming from useful radiation of sound into the air.

B. F. McNamee: The loud speakers which have been on the market using a movable coil system are supplied (as I suppose this one is) with a field current, usually from a storage battery. I believe that such loud speakers have met with a certain amount of sales resistance, due to that fact. Especially where dry-cell tubes are popular, a source of field current is not very readily available. I would like to ask how permanent field magnets would work out in this case, or what other provision has been made.

V. E. Thelin: In endeavoring to get quality of tone, I have adapted a Western Electric unit to a talking machine which has a wooden tone arm, as well as a wooden horn, and I attribute the fact that the so-called horn effect had disappeared to a considerable extent, to this wooden construction. I compared this combination with a new speaker of the parchment-cone type, and an adaptor type unit of another manufacture. I noticed that the Western Electric unit and the parchment type speaker gave the same volume and tone quality and it was difficult to tell them apart when they were operating at the same time. The adaptor type, however, changed the quality considerably and on the low notes of the piano it was very mushy.

It seems to me that there is a large field for the amplifier unit demonstrated here today. With a unit of this kind, it, no doubt, would be possible to take a phonograph record, and using a needle which has practically no scratching, and, therefore, has perfect tone, but whose music is too soft to be heard in the horn of the talking machine, and amplify it to a considerable volume but still retain the perfect tone quality. In this way it would be possible to preserve the music of the present-day artists on the radio and hear this music many years to come.

I would like to ask Mr. Kellogg if the baffle need be of a certain kind of material for obtaining the best results.

R. S. Glasgow: The authors mention the Hewlett type of loud speaker in their paper and point out that one of its disadvantages is the long air-gap that the radial magnetic field has to traverse, with the result that considerable energy is required for excitation purposes.

I would like to know whether the quality of reproduction would be effected by the substitution of a thin iron diaphragm in place of the non-magnetic materials that are usually employed.

A. Nyman (by letter): The conclusion the authors have drawn from physical considerations is that the resistance control is the ideal for loud speakers; meaning, of course, by the resistance control any control which is proportional to the velocity of the movement of the sound-generating surface. From the ordinary physical consideration the loud speaker can be regarded as an ordinary electric motor with certain peculiar load conditions capable of giving resonance at certain frequencies, elastic control below that frequency, and inertia control above that frequency. It is quite evident that a motor of this type would operate most satisfactorily if the elastic control and the inertia control could be so small as to be negligible compared to the actual load output. Considering the loud speaker from this point of view, it is also evident that if a loud speaker can be designed with a large efficiency that the resistance control will naturally follow.

The problem therefore comes down to the design of a sound-producing structure of such a nature that the energy input from the electrical instrument is converted largely into sound energy and only a very small percentage into elastic or inertia energy. It is also evident that a large horn properly designed in such a way as not to have any permanent resonant characteristics would form an ideal load on the loud speaker, but involves an unwieldy mechanical structure. It has to be quite long before it is suitable. The attempts carried out by the experimenters, using large conical diaphragms apparently discloses the fact that it is difficult to construct a large conical diaphragm that will avoid irregular movements and local resonance difficulties.

In future, the development will be probably in this direction; that is, the construction of a mechanical structure for radiating sound in the space.

Even under the best conditions the energy that can be radiated into the air with an average loudness of sound is quite small. From this it follows that the restoring force, due to the elastic control should be small, and the inertia force should be also small. This condition naturally leads to a low-frequency structure, but consisting of light enough parts to be capable of responding to high-frequency currents. The writers of this

paper achieved this object by the construction of a moving-coil unit with a practically floating diaphragm, and a consequent natural frequency of around 100 cycles. It is, however, possible to achieve frequencies as low as this with the ordinary electromagnetic types of loud speakers, both the steel-diaphragm type and the moving-armature type.

In either of these types there are two forces opposing each other under normal operative conditions. These forces are the elastic forces in the diaphragm, and the magnetic pull of the magnets. Now, it is possible to adjust the relation between these two forces in such a way that the difference between them is quite small. The resulting natural frequency of the whole system is consequently also very small. This can be done without any sacrifice on the part of the strength of magnetization. As a matter of fact, it is necessary to choose a rather soft diaphram on the moving-iron type in order to achieve this magnetic balance.

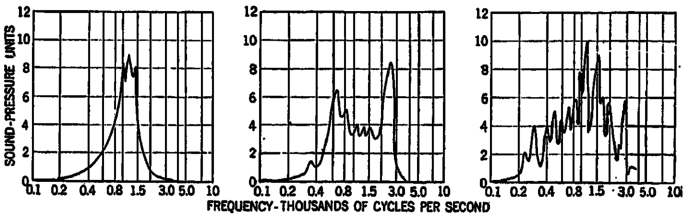


Fig. 1—For the Year 1921 Fig. 2—For the Year 1922 Fig. 3—For the Year 1923

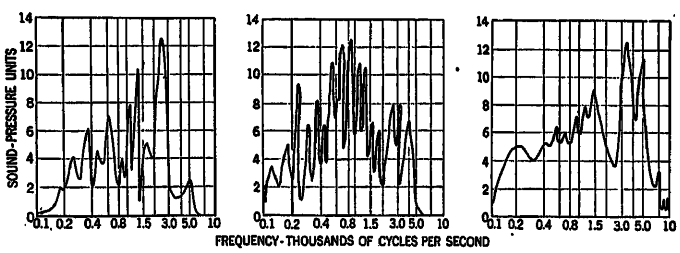
The sound-pressure measurements to which the writers refer at the end of the paper are undoubtedly still in a rather imperfect stage and can only afford comparative information on different types of loud speakers. It should be borne in mind, however, that under the best conditions the sound-pressure measurement will not give the complete information on a loud speaker. It is possible to choose a fairly resonant loud-speaker unit and a fairly resonant sound-diaphragm system, which in combination would give sound pressure measurements of almost constant value at different frequencies. However, this loud speaker will not necessarily give good musical or speech reproduction. There is a phenomenon which may be described as persistance of sound in all musical instruments; it has the same effect as the reverberation in large auditoriums, and is caused by the fact that resonance condition exists. This persistence causes the sound to continue radiating from the sound-distributing structure after the loud speaker has ceased to produce this sound. Of course, if the following note is of a different frequency from the persisting note, and possibly of a frequency causing a musical dissonance, the resulting sound would have a jarring effect in a musical composition. This phenomenon, which is not very well known, and as far as I know, has not been investigated by physical measurements, is however quite pronounced, and if precautions are taken to eliminate it, a considerable improvement in sound quality is possible.

J. P. Minton (communicated after adjournment): In a series of popular articles appearing during the past year in the *Wireless Age* I have pretty well covered the whole field of the ear, the voice and the loud speaker up to the present moment. As a result of this study and experimentation it is clear to me that up to the time, the loud speaker, in spite of improvements, has failed to give entirely satisfactory reproduction. Until the appearance of this new loud speaker, this device has been the weakest link in radio. In this respect radio has suffered in the past the same limitation that the phonograph has existed under. In phonograph reproduction the sound box and its associated horn were the weakest links. Recording was developed to an extent which made it possible to get into the record much more than the sound box and horn were able to give out. In a sense, radio reception has existed under the same impediment. However, radio originally was developed entirely for the transmission of intelligence and not for that of pleasure and entertainment. The requirements and sustaining forces in the two cases, therefore, were quite different. Accordingly, radio was developed to a high degree of perfection while the phonograph has not been so highly developed.

When radio broadcasting and reception came into existence, the already highly developed state of radio made possible the rapid growth of this new form of entertainment and education. The loud speaker had to be injected into this picture. Its use had previously been limited to certain fields of not very great importance. It had not received very serious consideration up to this point. Phonograph quality was quickly attained by use of horns of various sizes and shapes to which were attached units of the usual types. This stage was a temporary one, but it has existed for four or five years. In the meantime, a great deal of fundamental research was undertaken and now as a result of this work the loud speaker has been brought to a point where its performance is such as to make possible the reproduction of the original with an exactness astonishing to all of us.

As the authors have indicated the development has gone through a number of stages and in addition to their own excellent contribution to this work much credit for the gradual evolution of the loud speaker is due to many other workers in this and the allied fields of voice, ear and music analysis. Among the names of those who have contributed most from the fundamental point of view to this work will be found Rapleigh, Lamb, Webster, Stewart, Miller, Foley, from our universities and from our industrial research laboratories will be found such men, in addition to the present authors, as Hewlett, Slepian, Hanna, Fletcher, Wegel, Crandall, Maxfield, Goldsmith, Ringle, Wolff, Kranz and others.

From the scientific point of view it will be interesting to show a group of six curves which may represent the evolution of the loud speaker. The ordinates in these curves are proportional to sound pressures and the abscissas are frequencies divided by 1000. During the early stages (say 1921) of broadcasting, Curve 1 may represent an average loud speaker. Curve 2 may represent the state of the art in 1922. Curve 3 represents what was obtainable for 1923, Curves 4 and 5 for 1924 and Curve



Figs. 4—For the Year 1924 Fig. 5—For the Year 1924 Fig. 6—For the Year 1925

6 for 1925. These curves represent a steady progress in four or five directions. First, extension of the range of response to include both higher and lower frequencies. Second, uniformity of response, or the gradual elimination of the sharp peaks and depressions. Third, more nearly equal response at all frequencies. Fourth, reduction of non-linear distortion. Fifth, introduction of pure low-frequency response.

Curve 6 represents the performance of the new Rice-Kellogg loud speaker. The curve was taken close up to the loud speaker so that the characteristics of the vibrating system itself, actuated by a constant force, would be obtained. The loud speaker covers quite affectively a frequency range extending from 100 to

7500 and perhaps 10,000 cycles. The response is quite uniform compared with all other types of loud speakers and this new speaker gives an exceedingly small amount of non-linear response and therefore small distortion compared with all other loud speakers.

I wish to call attention to the fact that Curve 6 does not indicate complete agreement between the theory based on inertia action and the response at various frequencies. If the cone followed this simple theory then the response as measured by sound pressure should be constant at the various frequencies. Now, in this particular sample tested the response rises abruptly at 100 cycles; it also falls off abruptly above 6000 or 7000 cycles; there is also quite a marked depression in the region of 2000 cycles and a minor one at 4000 cycles. I am quite inclined to the view, therefore, that, in addition to the inertia-controlled motion which the authors seem to favor, there are also present to a marked extent flexural vibrations which, due to circular and diametral nodes of motion, corresponding to a plane membrane, produce the characteristics as indicated by the curve. We have studied these types of motion, for somewhat larger cones than the 6-in. one adopted by the authors, both theoretically and experimentally, and have found very curious nodes and interesting data which will prove of considerable practical value. At a later date we hope to have the opportunity to present these results.

E. W. Kellogg: Mr. Frederick has called attention to the fact that an inertia-controlled diaphragm implies a low-power-factor system, or one in which force and motion are nearly in quadrature, and therefore only a small fraction of the driving force is expended in the useful work of producing sound radiation. I have made a calculation which indicates that the efficiency of the loud speaker described in our paper is of the order of one per cent, which, it must be admitted is low, but compares very well with that of other loud speakers.

It may be of interest to review the possibilities of a sound reproducer in which the diaphragm motion is in phase with the driving force. Such a condition obtains when a diaphragm is in resonance, and efficiencies of 50 per cent or more are probably possible at a single frequency, using a resonant diaphragm. But when we impose the requirement of substantially constant efficiency over a frequency range of 100 to 5000 cycles, we must forego the benefits of resonance. The resistance-controlled diaphragm will have the correct radiation characteristic, 1—if it is large enough compared with the longest waves to give plane wave radiation, or 2—if it is used with a properly designed horn. For efficiency, the force which resists motion must be due to the air reaction on the diaphragm. In free space this air reaction is very small, and if it is to be large compared with diaphragm inertia or elastic forces, an extremely light and flexible diaphragm must be used. Such a diaphragm must be actuated by a uniformly applied force. Electrostatic loud speakers have been built with large-area diaphragms of very light material, and these have probably had quite high efficiency, if we define efficiency as the ratio of sound power output to electrical power input. But unfortunately we have to pay for the total volt-amperes supplied rather than simply for the electrical power, and the electrostatic loud speaker has a very low electrical power factor.

The case for the horn-type loud speaker has been discussed by Messrs. Hanna and Slepian[1]. By means of the horn the air reaction on the diaphragm can be increased to a point where it will effectively damp the motion of as stiff and heavy a diaphragm as is commonly employed in loud speakers. Thus a magnetically driven diaphragm may be used with resistance-controlled motion, or with a unity-power-factor relation between force and velocity. But a magnetic drive has good efficiency only when the motional impedance is a large part of the total impedance, or in other words, when as in the case of an electric motor, most of the impressed voltage is used in overcoming the counter electromotive force due to armature motion. A study of the motional impedance of magnetic telephones shows that only in the neighborhood of a resonance frequency is the motional impedance considerable compared with the resistance and inductive reactance, and if sufficient damping is introduced by the air reaction on the diaphragm to give substantially uniform response over a wide frequency range, the motional impedance becomes very small at all frequencies. This probably explains the fact that no horn-type loud speaker which we have tested shows any greater average efficiency than our inertia-controlled paper cone. I do not despair of considerably greater efficiencies being ultimately obtained in loud speakers, but from the standpoint of present progress in the art of sound reproduction, I do not believe that the adoption of inertia-controlled diaphragms can be construed as a step in the wrong direction.

Mr. Frederick raised the question of load capacity. The moving-coil drive has a distinct advantage over the iron-armature drive on this score. With the cabinet-set amplifier described in the paper, our loud speaker can easily reproduce vocal solos with the original sound volume, or can reproduce a piano selection as it would be played in a drawing room, though perhaps not with the maximum loudness that would be used in a large concert hall. The limit of loudness is set by distortion in the amplifier rather than in the loud speaker. In fact the latter will handle all the output which can be obtained without distortion from a U. V. 211 radiotron (50 watts oscillator rating) with a 1000-volt plate supply, or eight times the power obtainable from the cabinet-set amplifier.

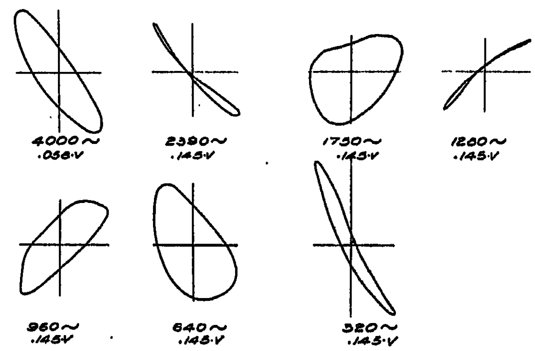


FIG. 7—BRAUN-TUBE RECORDS OF SOUND WAVES

In stating that working with a weak magnetic field makes possible a lower net restoring force in the case of an iron-armature driving element, we made the assumption also made by C. R. Hanna in his January 1925 I. R. E. paper, that the magnetic reduction of stiffness cannot be more than a certain fraction (say 50 per cent) of the spring stiffness.

In comparing two loud speakers, we have rated the one as more sensitive which would produce on the average more total sound output from a given vacuum-tube source, both instruments being equally fitted to the tube impedance. In saying that the moving coil gives greater sensitivity than the iron-armature drive, we are reporting our experience with these types of drive as applied to paper-cone diaphragms.

Mr. McNamee asks about the use of permanent magnets for the field. When the loud speaker is combined with an amplifier, the dynamic field is hardly a drawback since the field winding acts as a necessary filter choke. We have done some experimenting with permanent magnets, but have not, up to the present succeeded in obtaining an adequate field for a moving-coil instrument without a very heavy magnet system.

Mr. Thelin asked about the material of the baffle. It should

1. The Function and Design of Horns for Loud Speakers, by C. R. Hanna and J. Slepian, JOURNAL A. I. E. E., March 1924, page 250.

be stiff and heavy enough so that it will not readily be set in vibration by the air pressure. Wood, pressboard, and similar materials are satisfactory and convenient to use.

The question has been asked whether an iron diaphragm would increase the sensitivity of a Hewlett loud speaker. Dr. Hewlett found no gain from the use of iron, but instead, a distinct loss as compared with copper or aluminum. The increased flux density must be around, rather than in, the conductor in order to increase the force.

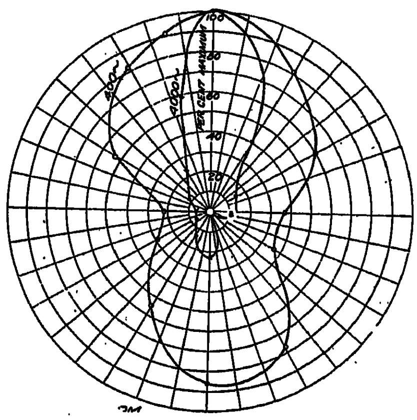


FIG. 8—SHOWING SOUND OUTPUT OF LOUD SPEAKER AT DIFFERENT POSITIONS

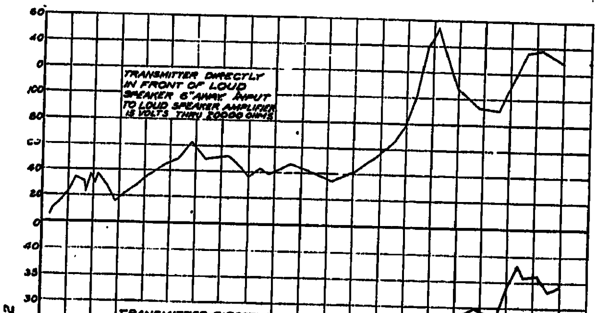


FIG. 9—CONDENSER TRANSMITTER 34 IN. FROM CENTER OF CABINET. HORIZONTAL DIRECTIVITY OF LABORATORY MODEL OF RICE-KELLOGG LOUD SPEAKER, PER CENT MAXIMUM SOUND PRESSURE VS. ANGLE.

I am submitting as part of the discussion some sound-pressure curves which we have recently obtained. The measuring arrangements were still in the developmental stage when these curves were taken, and a high degree of accuracy is not claimed. In fact in acoustic measurements so great are the difficulties encountered that a measurement that can be trusted to be within 50 per cent of the correct value might be regarded as highly satisfactory. In the present case a condenser transmitter, with amplifier, detector, and galvanometer, was used for measuring sound pressure. The amplifier, detector and galvanometer system was calibrated by introducing a measured low voltage in series with the condenser transmitter. The condenser transmitter was similar in construction to that described by F. C. Wente in the *Physical Review*, July 1917, and May 1922. A calibration of the condenser transmitter was made by actuating its diaphragm by means of a special, laminated-pole telephone magnet held 1/16 in. from the diaphragm, the force being assumed to be proportional to the current through the coils. This does not give an absolute calibration, but should show any radical departure from the shape of the curve given in the *Physical Review* article already mentioned.

In loud-speaker tests it is important to guard against the error of crediting sound radiated in harmonics to the fundamental frequency. For example, if an instrument is supplied with a 200-cycle alternating current, and it happens to be one hundred times more sensitive at 400 and 600 cycles, than at 200, then a very small percentage of harmonics in the supplied current, plus harmonics produced in the instrument itself may give rise to a much larger radiation of 400- and 600-cycle sound than of 200-cycle sound. If the sound-measuring apparatus measures total r. m. s. pressure independent of frequency, considerable sound pressure will be indicated, and this would naturally be assumed to be 200-cycle output pressure.

To make sure that no serious error arose from this source, a Braun-tube oscillograph was set up. Between one pair of plates, a voltage was impressed, proportional to the current supplied to the loud speaker while a voltage from the condenser-transmitter amplifier, proportional to the sound pressure, was impressed across the other pair of plates of the Braun tube. If both voltages are sine waves, the figure which appears on the screen is an ellipse or an inclined straight line. Harmonics in one of the voltages result in deformations of the figure. In the present measurements, the oscillograph figure was watched throughout the entire range of frequency, and tracings were made of all the figures which were seriously distorted. In no case was it found that the harmonics carried more than 25 per cent of the energy of the fundamental in the output sound wave. Several samples of Braun-tube figures are shown here in Fig. 7.

The next serious problem in the testing of loud speakers results from the fact that the curve of sound pressure vs. frequency, changes in shape with change of microphone position. Which of the many possible positions will give a curve best representing what listeners on the average will hear? It would seem logical to avoid the irregularities due to standing waves in the room, and give a curve in which the sound pressure shown is a measure of the total sound power output. An approximation to this is obtained by averaging the square of the sound pressure over a considerable space by moving the microphone rapidly to and fro during each reading. Facilities for moving the microphone or transmitter in this manner were not available in our case and as an alternative, several curves are shown in Fig. 8 for different transmitter positions. The high or low regions which are common to all three curves may be interpreted as indicating large or small output from the loud speaker, while the irregularities which are different in the different curves are principally room effects. It will be noticed that the curves taken with the transmitter directly in front of the loud speaker show a marked increase in sound pressure above 2300 cycles. The instrument, however, has rarely been criticised on listening tests as having too much high-frequency output. The curve taken with the transmitter to one side, does not show such an excess of high tones. Evidently then the high sound pressures recorded in the upper frequency range are due in part to the concentration of the sound in a forward beam. The curves of Fig. 9 were taken by moving the transmitter in a circle and recording the sound pressure every 30 deg. They show the

radiation at 4000 cycles to be sharply directed forward whereas at 400 cycles there is only a slight depression at the side due to interference between the waves from the front and back of the diaphragm. In total sound radiated, therefore, the excess of high frequencies is only slight. If high frequencies are lost in the transmitting or receiving systems, the listener prefers to take a place directly in front of the loud speaker, so as to get the full benefit of what is left, while if articulation is good, but there are roughnesses, or high-frequency disturbances present in the currents fed to the loud speaker, the listener will sit to one side.

It is probable that the frequency of 2300 cycles, where the forward projected sound begins to increase, marks the transition between the two modes of action of the cone. Below this frequency it acts as a unit or plunger while at higher frequencies there is wave action with some resonances. The depression in the region of 2300 cycles may correspond to the droop in the calculated curve in Fig. 10. This lends support to the belief that practical plunger action is maintained up to 2300 cycles. In the upper range, irregularities in the response may be expected not only from resonance in the cone, but also from the fact that the cone depth is appreciable compared with the sound wave length, and, therefore, the diaphragm no longer radiates like a flat plate.

The loss of output below 200 cycles is due to a decrease in the driving force. If the current through the moving coil is held constant the output sound pressure is practically the same at 80 as at 200 cycles, but in the curves shown here, it is the voltage supplied to the first stage of the amplifier which is maintained constant. The current through the moving coil then changes with changes in the coil impedance. This impedance rises from about 17 ohms at 200 cycles to over 80 ohms at 80 cycles and the consequent decrease in coil current results in reduced driving force and reduced sound output. The rise in impedance is due to the motion of the coil, which with inertia control becomes very large at low frequency.

Echo Suppressors for Long Telephone Circuits

BY A. B. CLARK[1]
Member, A. I. E. E.

and

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Associate, A. I. E. E.

Synopsis.—*A device has been developed by the Bell System for suppressing "echo" effects which may be encountered under certain conditions in telephone circuits which are electrically very long. This device has been given the name "echo suppressor" and consists of relays in combination with vacuum tubes, which are operated by the voice currents so as to block the echoes without disturbing the main transmission.*

This paper gives a brief description of this device, together with a discussion of its possibilities and limitations. A number of echo suppressors have been operated on commercial telephone circuits for a considerable period so that their practicability has been demonstrated.

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INTRODUCTION

In designing telephone circuits which are electrically very long, an important problem is presented by the necessity of avoiding serious "echo" effects. Echo effects are caused by reflections of voice waves which take place whenever electrical irregularities are encountered in telephone circuits. The effects produced are very similar to echoes of sound waves. Some of the reflected waves return to the receiver of the talker's telephone so that if the effects are severe, he may hear an echo of his own words. Other reflected waves enter the receiver of the listener's telephone and, if severe, cause the listener to hear an echo following the directly received transmission.

Reflections of voice waves occur in all practical telephone circuits. It is only in telephone circuits of such length as to require a number of repeaters, however, that echo effects become serious. The fact that the circuits are electrically very long makes the time lag of the echoes appreciable. At the same time, the telephone repeaters overcome the high attenuation of these long circuits and, consequently, make the echoes louder. The seriousness of the effect is a function both of the time lag and the volume of the echoes relative to the direct transmission.

A brief discussion of these echo effects was given in a paper[3] presented before this Institute about two years ago, and in a later paper[4] some examples of their relative effects in practical telephone circuits were given. In these papers the importance of keeping electrical irregularities within proper limits was pointed out as was also the advantage gained by using circuits having a high velocity of propagation so that the lag of the echoes is reduced.

As a supplement to these methods, a device to which has been given the name "echo suppressor" was developed by the Bell System, along lines suggested by John Mills[5]. In all practical telephone circuits involving more than a single repeater there are points where the transmission in the two directions passes through two separate paths. At these points the direct transmission passes through one path while only reflected currents or echoes pass through the other. The echo suppressor is located at one of these points. In this device, the voice currents, with the help of vacuum tubes, are caused to actuate relays which cut off the echoes in the return path without disturbing the other path through which passes the main transmission.

This paper, after briefly reviewing the nature of echo effects in four-wire circuits, explains, in a general way, how an echo suppressor functions on such a circuit. The four-wire echo suppressor is then described together with some variations in its design for use under special conditions and with other circuits. This is followed by a discussion of the possibilities and limitations of echo suppressors, both on four-wire and other types of telephone circuits.

REVIEW OF NATURE OF ECHO EFFECTS IN FOUR-WIRE CIRCUIT

Fig. 1 illustrates the way echo currents may be set up and circulate in a four-wire circuit. In this figure, *a* shows a four-wire circuit in diagrammatic form. The squares at the extreme right and left are intended to represent the telephone sets used by two subscribers at the terminals *W* and *E*. The squares marked *N* represent electrical networks which simulate or balance, more or less perfectly, the impedances of the two telephone lines terminating in the instruments at *W* and *E*. In the four-wire circuit, the squares with arrows represent one-way repeaters. At each terminal the two separate one-way circuits comprising the four-wire circuit are joined together by means of the familiar balanced transformers. When *W* talks, the transmis-

1. American Telephone and Telegraph Co., New York City.
2. Bell Telephone Laboratories, Inc., New York City.
3. "Telephone Transmission over Long Cable Circuits" by A. B. Clark, TRANS. A. I. E. E., Vol. XLII, 1923, p. 86.
4. "Telephone Transmission over Long Distances" by H. S. Osborne, TRANS. A. I. E. E., Vol. XLII, 1923, p. 984.

Presented at the Spring Convention of the A. I. E. E., St. Louis, Mo., April 13-17, 1925.

5. U. S. Patent No. 1,434,790, John Mills, "Two-Way Transmission with Repeaters." Issued Nov. 7, 1922.

sion passes directly to E over the upper pair of wires in the four-wire circuit, while, when E talks, the direct transmission passes over the lower pair of wires.

Below the diagram of the four-wire circuit is given another diagram, b, showing the path of the direct transmission as well as the paths of the echoes which are set up when W talks to E. The heavy line in the diagram represents the path of the direct transmission through the upper pair of wires in the four-wire circuit.

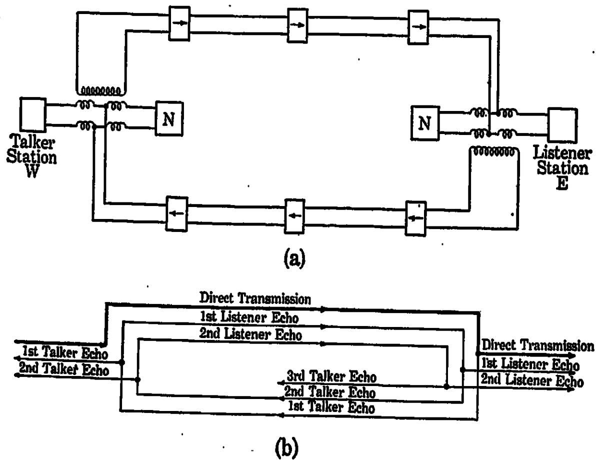

FIG. 1—ECHOES IN FOUR-WIRE CIRCUIT

In a practical four-wire circuit it might require, say, 0.05 second for the voice currents to make this journey. This would be the case if the four-wire circuit were 1000 miles (1600 km.) long in cable with extra-light loading—coils of 0.044 henry spaced 6000 feet (1.8 km.) apart. Cable circuits loaded in this way have a velocity of propagation of about 20,000 miles (32,000 km.) per second.

When the voice currents reach the distant end of the four-wire circuit, the larger part goes to the listener at E. If the balance between the line and network at the distant terminal is not perfect, however, a portion of the currents will travel back over the lower pair of wires toward W as an echo. This echo will, in the case assumed, reach the receiver of the telephone at station W, 0.1 second after the original voice wave is impressed on the line at that station. The path of this echo is labeled "1st Talker Echo." It is evident that if this echo is loud enough it may seriously distract the talker.

If the balance between the line and the network at Station W is also not perfect, part of this first echo will travel back over the upper pair of wires to Station E, the path of this echo being labeled "1st Listener Echo." The listener at E will hear this echo, if strong enough to be audible, 0.1 second after he hears the direct transmission. Evidently, if this echo is sufficiently loud as compared to the direct transmission, it will cause difficulty in understanding.

When the "1st Listener Echo" arrives at the end of the four-wire circuit, there is still another reflection of part of the current which occurs producing the "2nd Talker Echo."

This process is repeated, producing successive echoes which are received at both terminals W and E as indicated, the successive echoes getting weaker and weaker.

ACTION OF ECHO SUPPRESSOR ON FOUR-WIRE CIRCUIT

An echo suppressor will now be applied to the four-wire circuit and consideration given to its action and to its effect on the echoes. Fig. 2 shows a four-wire circuit which it will be assumed is exactly like the one shown in Fig. 1, with the exception that an echo suppressor has been applied to it. As before, the diagram a shows the four-wire circuit, while, below this, another diagram b shows the paths of the direct transmission and of the echo.

In Fig. 2a the echo suppressor is shown in very simple diagrammatic form. It will be described later in detail. For the present it is sufficient to explain that the echo suppressor consists of two similar high-impedance vacuum tube amplifier-detectors bridged across the two sides of the four-wire circuit, each amplifier-detector having associated with it a relay which operates whenever alternating voltage of sufficient strength is impressed across the input. The operation of either relay places a short circuit across the side of the four-wire circuit opposite to the one to which the input of its particular amplifier-detector is connected. This short circuit blocks the transmission flowing in one side of the four-wire circuit and, at the same time, renders the other amplifier-detector inoperative.

Normally, the contacts of the two relays are open,

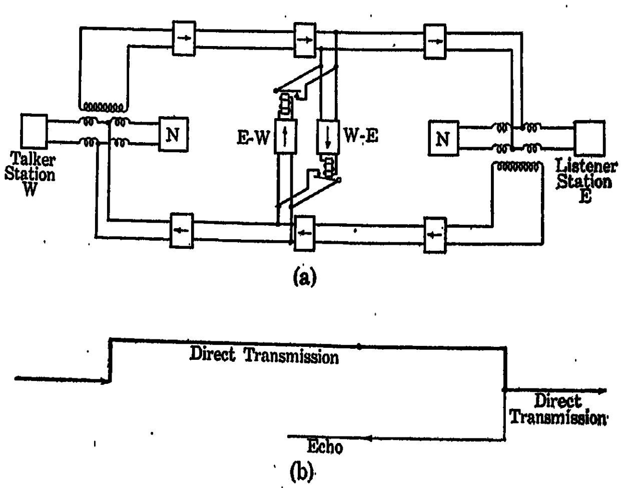

FIG. 2—ECHO SUPPRESSOR CUTTING OFF ECHO IN FOUR-WIRE CIRCUIT

so that talking may be done in either direction over the circuit. When W begins to talk, the condition illustrated in the figure is produced. W's voice currents, when they reach the middle of the circuit, cause the relay associated with amplifier-detector W-E to operate, thus placing a short circuit across the lower pair of wires in the four-wire circuit. The direct transmission from W to E is not affected at all, passing on to Station E where it is heard by the listener. The echo, which

starts back from Station E, travels toward Station W as far as the point where the echo suppressor is connected to the circuit. It is stopped there, however, by the short circuit which the echo suppressor has applied.

In the same way, when E talks, his voice currents actuate the amplifier-detector marked E-W and apply a short circuit to the upper pair of wires, thus preventing the passage of the echo current around the circuit.

The circuit shown in Fig. 2a is one of the more convenient for satisfying the fundamental operating conditions of an echo suppressor. These may be stated as follows: When no one is talking, free paths should exist for transmission in either direction and each suppressing relay should be ready to act at the passage of speech over the side of the circuit with which it is associated. When speech passes in one direction over the circuit the resulting operation of the corresponding half of the suppressor should not only interrupt the continuity of the opposite side of the circuit, but at the same time prevent the other half of the suppressor from functioning. The latter condition is desirable as otherwise the returning echo might have enough energy at times to operate the opposite part of the suppressor circuit and so interrupt the direct transmission. Outside of this restriction the selection of the points from which the echo suppressor input currents are derived and the points at which the relay control functions are applied is governed only by such considerations as economy of apparatus and convenience. In general, it is the more economical arrangement to have a single relay, which interrupts the path through which the echoes return, also remove the speech input from the suppressor by such a relative association of parts as shown in Fig. 2a.

It will be noticed that, as a finite time is needed for the switching operation, there is a possibility, if the two subscribers begin talking simultaneously, of both halves of the suppressor being operated together and remaining operated, with both sides of the circuit cut off, for a time equal to the release time of the relays. However, for the times of operation and release, which are found desirable from other considerations, it has been found that no apparent difficulty has been caused by this effect.

Because of the fact that an appreciable time is required for the voice currents to travel, it will be seen that exceedingly fast operation of the relays is not necessary. In the example given, if it is assumed that the echo suppressor is connected to the circuit at its midpoint, the echo requires 0.05 second to reach the point where the short circuit is applied, after the voice currents reach the input of the amplifier-detector. The echoes will be cut off by the relays, therefore, even if the latter require as long as 0.05 second for operation. If the echo suppressor is nearer to the end of the four-wire circuit this operating time would need to be somewhat shorter. In practical four-wire circuits it is seldom that an operating time shorter than about 0.02 second is required. It is an easy matter to secure this speed of operation with standard telephone relays.

The diagram also shows that, in order to completely cut off the echo, the echo suppressor relay must not open, after talk ceases, until the complete train of echoes has reached the point where the short circuit is applied. In the example given, the length of time required to reach the point where this relay applied the short circuit after the voice currents pass the input of the amplifier-detector is 0.05 second. It is seldom that this lag is greater than 0.1 second in practical four-wire circuits.

It is seen from the two above paragraphs that it is desirable for a four-wire echo suppressor to possess a moderately short operating time and a longer releasing time. How this is accomplished will be described in what follows.

Description of Four-Wire Echo Suppressor

In Fig. 3 is shown a circuit diagram of one-half of the echo suppressor, which is shown complete but in less detail in Fig. 2a. It consists of two vacuum tubes operating in tandem, the first functioning as an amplifier and the second as a combined amplifier and detector.

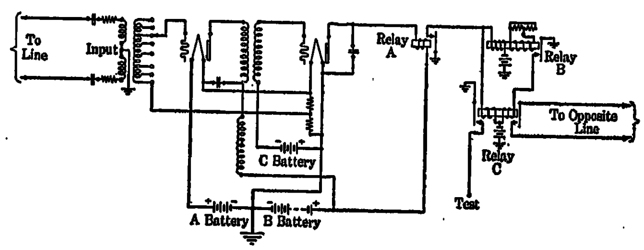

Fig. 3—Circuit Diagram of One-Half of a Four-Wire Echo Suppressor

As was shown in Fig. 2a the voltage impressed on this amplifier-detector combination is derived from speech currents passing over one side of the circuit, while the relay controlled by this combination short-circuits the other half of the circuit.

The voltage input to the amplifier tube is supplied through a transformer which is broadly tuned by series condensers to produce a circuit efficient at the more important voice frequencies but inefficient at other frequencies, particularly below 500 cycles per second. The circuit thus functions to minimize the effect of noise currents on the operation of the relays. Likewise, in the interstage transformer coupling, emphasis has been placed on securing the maximum voltage step-up to the detector grid in this same frequency region. To avoid any harmful reaction upon the transmission characteristics of the main circuit which might result from bridging on an input circuit whose impedance varies so greatly over the speech frequency range, this circuit is arranged to have a high impedance. The input transformer is also provided with a series of taps

on one of the windings, thus affording a simple means of varying the sensitivity of the device.

The detector tube is operated with a sufficiently large negative grid potential to reduce its space current to zero, or nearly so, when no input is applied to the circuit. Accordingly, relay A which is connected in the plate circuit is normally in a released condition. When

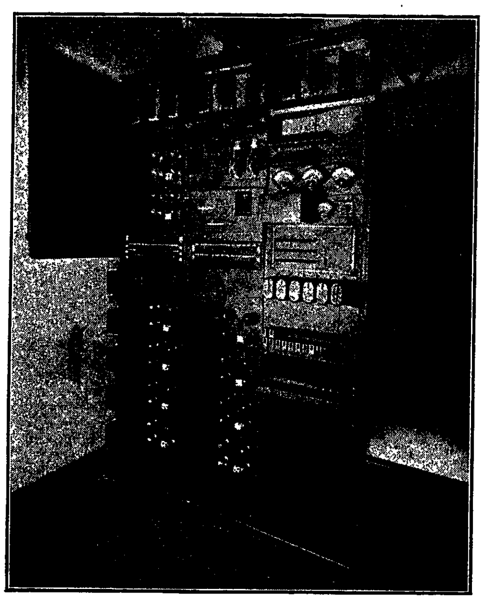

FIG. 4—INSTALLATION AT HARRISBURG, PA.

speech currents are applied to the circuit the voltage on the grid of this tube fluctuates. Those variations which make the grid more negative produce no effect but those which make it more positive allow pulses of current to pass through relay A, tending to operate it. A condenser is bridged from plate to filament of this tube, the purpose of which is to average these rectified half waves of applied speech so as to insure smooth and positive operation of the relay.

When speech is applied to the circuit the resulting operation of relay A does two things. It causes the operation of relay C by connecting a ground to one of its windings, and it likewise operates relay B. The operation of relay C short circuits the opposite line. The time required for the operation of relay C, in response to a sustained alternating e. m. f. suddenly applied to the input of the amplifier detector, is about 0.02 second. As pointed out above, operation in this length of time takes care of conditions in the large majority of cases encountered on four-wire circuits.

The function of relay B is to provide a delay in the release of relay C after speech has ceased to be applied to the suppressor circuit input and relay A has released. Its operation in response to that of A, it will be noticed, connects ground to a second winding on relay C which will then in turn remain operated as long as the relay B maintains this auxiliary current after the relay A has released. Relay B is made slow releasing by an auxiliary winding closed through a low resistance, and its time of release can be adjusted over a considerable range to meet different operating conditions by changing the value of this resistance. Differing adjustments are rarely called for in practise and these relays are normally set for a releasing time of 0.1 second.

A number of echo suppressors have been installed at Harrisburg, Pa., where they are now in service on a group of four-wire circuits. Fig. 4 is an illustration of this installation of four-wire echo suppressors. Fig. 5 shows a close-up view of an individual panel from the front. Both halves of the suppressor working on a single circuit are mounted together on one panel. The method of mounting and the type of equipment in the echo suppressors are in general quite like the standard for the four-wire circuits with which they operate. Although in Fig. 3 the battery supply circuits are shown individual to this set, in the actual installation common batteries are used. The four filaments of the tubes on one panel are operated in series from the 24-volt battery.

The operation and maintenance of these devices involve little that is different from standard repeater equipment. There is one test, however, which is employed in checking the times of functioning that perhaps deserves special mention. This test involves observing the time needed for the suppressor to go through any number of complete cycles of operation and release. To make this test, the short-circuiting contacts of relay C and the input of the suppressor circuit are connected together and to an oscillator as shown in Fig. 6. As soon as the oscillator is connected to the input, the relay train begins operating and the shorting contacts of relay C in turn cut off the applied voltage. This short circuit is maintained across the input for a time by the slowness of release of relay B as previously explained. When it finally releases and in turn

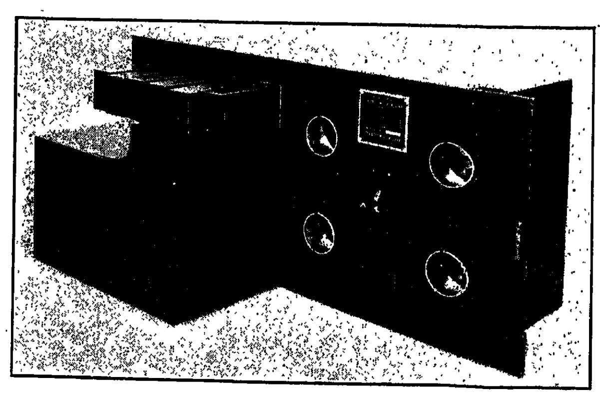

FIG. 5—FRONT VIEW OF ECHO-SUPPRESSOR PANEL

releases relay C, the suppressor again operates and the process is repeated over and over. At each repetition of the cycle the auxiliary contacts of relay C apply a ground to the test terminal which is connected to a counting device. With the aid of a stop-watch the number of cycles in any given time is readily determined and thus the time of a single cycle of operation.

This time is the sum of the time needed for relays A and C to make and the time needed for relays A, B and C to release. By observing the uniformity and smoothness of operation with which this cycle is carried out the tester can check the adjustment of all the relays. If relays A and C are properly adjusted so that their operation is positive and uniform, the operating time

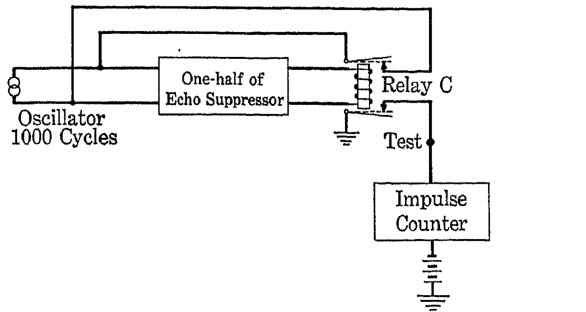

Fig. 6—Circuit for Testing Time of Operation and Release

will vary but slightly from the proper value of about 0.02 second. The test, therefore, gives a good measure of the longer release time which would normally be about 0.1 second.

Some General Considerations

As pointed out above, when an echo suppressor is applied to a telephone circuit, the telephone circuit remains operative in both directions when it is in the normal condition, *i. e.*, when no one is talking. It is only when talking is done over the circuit that the path for transmitting in the reverse direction, which is then useless so far as talking is concerned but which is harmful because it furnishes a path for the echoes, is blocked. The advantages gained by this arrangement are: (1) there is no possibility of cutting off the first part of words owing to the fact that the transmission path actually carrying the speech is unaffected by the switching operations; and (2) if the relays should fail to operate because the voice currents happen to be very weak, the listener at the distant end would still hear the speaker although both he and the talker might also hear some echoes. Weak speech does not, in general, give rise to such serious echoes as does strong speech. Therefore, when the voice currents happen to be so weak that they fail to operate the suppressor, the echoes produced may not be serious.

Now, in order to obtain these advantages it is necessary to face the possibility of "singing," since when no one is talking, the paths for transmission in both directions remain in their normal operative condition. It is evident that if the repeater gains are raised high enough, singing will begin exactly as it would if the circuit contained no echo suppressor. If singing starts in a circuit containing an echo suppressor, the circulating currents will build up until they become strong enough to cause operation of the relay associated with one half or the other of the echo suppressor so that one of the transmission paths will be blocked. This will temporarily stop the singing. It will commence again, however, as soon as the relay falls back to its normal condition. Thus, a chattering condition is produced which, in general, would not be tolerated.

In order to overcome the limitations which may be set on a circuit by the possibility of singing, it is necessary to go back to the old idea of a voice-controlled system in which the transmission is blocked when no one is talking. It is not necessary, however, to block both of the transmission paths since if one path only is blocked, singing evidently cannot occur.

Fig. 7 shows one of the possible arrangements of a voice-operated system in which singing is prevented. It will be seen that this arrangement includes an echo suppressor to which an additional relay D has been added, which keeps the upper transmission path blocked when the circuit is normal, *i. e.*, when no one is talking. Singing is, therefore, not possible when the circuit is normal.

Now, when talking is done at Station W the voice current waves, on arrival at the middle of the circuit, cause operation of the two relays associated with the amplifier-detector W-E. An appreciable length of time is required, of course, to operate relay D. To avoid the possibility of cutting off the initial parts of words during the time before relay D operates, it is desirable to delay the main transmission. What has been called a "delay network" has, therefore, been included as shown in the figure. This delay network may, of course, assume various forms, one of which might be an artificial loaded line or low pass filter. By including such a delay network, the voice currents can be retarded long enough to give the contacts of relay D time to clear the path before the voice currents reach the point in the circuit where the transmission has been blocked.

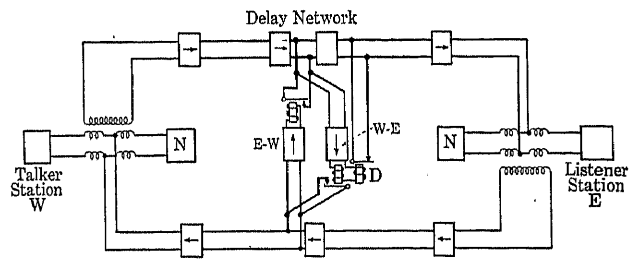

Fig. 7—Four-Wire Circuit with Voice-Operated Device Arranged to Suppress Echoes and Singing

In addition to clearing a path for the main transmission in the direction from W to E, the transmission path from E to W is blocked by the operation of the other relay associated with amplifier-detector W-E. The circuit, therefore, has no chance to sing when in the condition for talking from W to E.

When talking is done at Station E, the relay associated with amplifier-detector E-W is operated. This

prevents the echo returning from Station W from operating the relays associated with amplifier-detector W-E. For talking in this direction, therefore, the upper transmission path remains blocked. There is, therefore, no chance for singing, as was also the case for the other conditions.

By adding the delay network, one of the disadvantages of voice-controlled relay systems which keep transmission normally blocked is overcome in large part. This is the clipping off of the first parts of words, the possibility of which was mentioned above.

There remains, however, an important disadvantage in the fact that it is necessary that the voice currents never fail to operate relay D. If they did fail to operate this relay, the listener at Station E would hear nothing. It is necessary, therefore, that the amplifier-detector-relay system W-E be sensitive enough so that the voice currents which traverse the upper path in the four-wire circuit will never fail to cause operation of its relays.

On the other hand, noise currents which traverse the upper path in the four-wire circuit must never cause operation of the relays associated with the amplifier-detector W-E. Such false operation would, of course, prevent transmission over the lower pair of wires from Station E to W and would, therefore, render the four-wire circuit inoperative.

To overcome the singing limitation, it is thus seen that it has been necessary to produce a device which requires greater sensitivity and is, therefore, more seriously affected by noise currents than is a simple echo suppressor. This is in addition to the further complications involved.

Now, in applying simple echo suppressors to long telephone circuits, it is in general not the possibility of singing, but rather, the necessity of avoiding false operation of the relays by noise currents that constitutes the most serious limitation. This is discussed in more detail in what follows. For the present, it is sufficient to note that in the case of most long-distance telephone circuits the method of avoiding singing, which has been described, appears to offer possibilities of limited application only.

Echo Suppressors Applied to Other Types of Telephone Circuits

It will, of course, be understood that in practise a normal commercial telephone circuit is always two-wire at the two ends where connection is made to the subscribers' instruments. The rest of the circuit may be entirely four-wire or it may be all two-wire, or a combination of both. The application of echo suppressors to circuits which are not all four-wire will now be considered.

One important practical case is that where a four-wire circuit is sandwiched in between two two-wire circuits. Such a case is illustrated in Figs. 8 and 9. Fig. 8 shows conditions without an echo suppressor while Fig. 9 shows conditions with an echo suppressor. In both figures, a diagram of the circuit itself is shown in the upper part a, while in the lower part b are shown the paths of the direct transmission and echoes. These transmission paths illustrate the condition when talking is being done from Station W to Station E. In both figures, for simplicity, the first echoes affecting the talker and listener only are shown, echoes of these echoes being ordinarily of little importance.

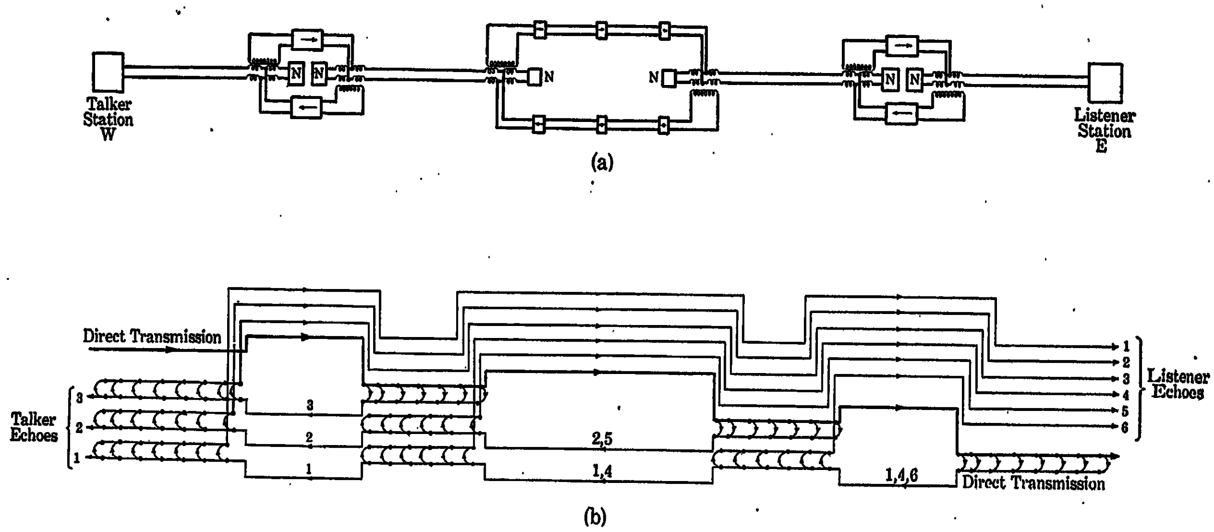

Fig. 8—Echoes in Combination Two-Wire and Four-Wire Circuit

It will be observed in Fig. 8b, which represents the condition of affairs when no echo suppressor is used, that the listener hears echoes coming from as many as six different paths. The talker hears echoes from three paths. Now compare this with Fig. 9b which represents the condition of affairs when an echo suppressor is employed. It will be observed that all of the echoes which return through the four-wire circuit have been suppressed. Echoes from only two paths now reach the listener, while echoes from only one path now reach the talker. Furthermore, the echoes affecting both talker and listener, which remain when the echo suppressor is

employed, are those whose paths are comparatively short. The echoes whose paths are the longest have been cut off by the action of the echo suppressor. These echoes which travel over the long paths have the greatest lags and are usually most serious. Consequently, cutting these echoes off makes a material improvement possible even though the echoes whose paths are short remain.

In order that an echo suppressor may operate satisfactorily on a circuit, such as the one shown in Fig. 9a, it is necessary that the time required for operation of the relays be short enough so that, if there are any serious echoes returning over short paths, the relays will operate before these reach the suppressor. After operation, the suppressor relays must remain operated until the echoes whose paths are the longest have been suppressed.

In the case of telephone circuits worked entirely on a two-wire basis, echoes may also constitute an important limitation when the circuits are electrically long. On such circuits it is also generally true that the most serious echoes are those whose paths are the longest, namely, those which travel back and forth between points at or near the ends of the circuit. The application of an echo suppressor to one of the repeaters in a two-wire circuit, therefore, offers possible advantages.

If it is imagined that the four-wire circuit shown in Fig. 9a is shortened so that the whole four-wire circuit is located at one point, the two-wire condition would be represented. The time lags which were introduced by the lines comprising the four-wire circuit are now absent. It is possible, however, to introduce delay networks into the two sides of the two-wire repeater to which the echo suppressor has been applied, so as to make it effectively a four-wire circuit, although the two ends are not geographically separated. This would evidently allow the four-wire echo suppressor which has already been described to be applied without modification.

By using somewhat higher speed relays and switching systems, however, it has been found possible in tests which have been made, to obtain satisfactory operation on an all two-wire circuit without introducing devices to produce time lags. This is possible because the important echoes in a two-wire circuit generally lag enough to allow time for relays to operate. Only a few of the echoes return to the suppressor with very small time lags. Some of these can be allowed to pass without causing appreciable impairment, provided they are not strong enough to cause false operation of the relays which block the main transmission path.

Possibilities and Limitations of Echo Suppressors

The curves in Fig. 10 show how, when no echo suppressors are employed, the echo effects limit the extent to which the over-all loss of a circuit may be lowered by the application of repeaters.[6] The curves in this figure apply to four-wire circuits of various lengths (without echo suppressors) used to handle terminal business, *i. e.*, connections to subscribers not involving the use of other toll lines in tandem with the four-wire circuit. It is assumed that simple compromise networks giving only a rough degree of simulation of the impedances of the terminal circuits are used. The curves, which are based on experimental data, indicate roughly how the over-all volume efficiency must be limited to keep echo effects small enough so that they are not considered disturbing when ordinary telephone conversations are carried on.

Consider, for example, what are the limitations for

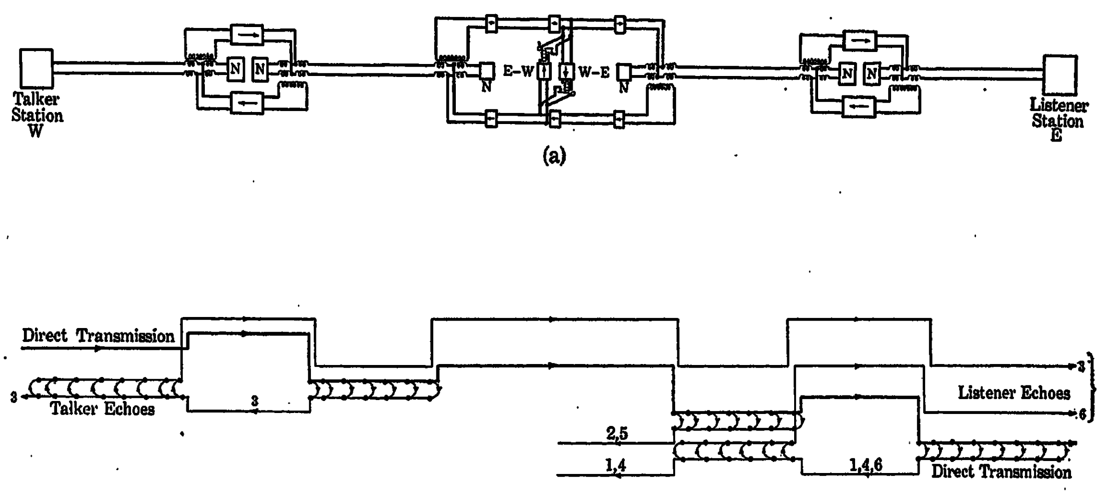

FIG. 9—ECHO SUPPRESSOR CUTTING OFF ECHOES IN COMBINATION TWO-WIRE AND FOUR-WIRE CIRCUIT
(a) Combination two-wire and four-wire circuit with echo suppressors
(b) Paths of direct transmission and echoes

6. The ordinates on this figure are in terms of the new "transmission unit" abbreviated "TU," which is defined in a paper entitled "The Transmission Unit, etc." by W. H. Martin, JOURNAL of the A. I. E. E., June 1924. Also in the article entitled "The Transmission Unit" by R. V. L. Hartley, *Electrical Communication*, July 1924.

a circuit 1500 miles (2400 km.) long, with extra light loading. One of the curves which is marked "Talker" shows that in order to keep the echoes which affect the talker sufficiently low, requires that the over-all loss in the circuit be made no lower than about 14 TU.[7] The other curve marked "Listener" shows that keeping the echoes which affect the listener within proper limits is a less severe limitation, requiring only that the net loss be made no less than about 7 TU. Singing of a circuit such as this would not ordinarily begin until the loss was reduced to zero or even, perhaps, made less than zero, *i. e.*, an over-all gain.

If an echo suppressor were applied to a circuit such as the above, a maximum improvement of the order of 14 TU might be looked for. As a matter of fact, results as good as this have been obtained in tests.

In order to obtain a result as good as this requires, of course, that the echo suppressor be given a sensitive enough adjustment so as to cut off substantially all of the echoes, even when the voice currents are weak. When given such a sensitive adjustment, there will, of course, be a tendency for noise currents to produce false operation. In certain cases, avoiding such false operation may require that the sensitivity of the echo suppressor be reduced to the point where weak voice currents fail to operate the relays. In such cases, results as good as the above will not be obtainable.

In practise, little or no trouble from false operation due to noise within the cable facilities, comprising a four-wire circuit, is experienced. When the connections to the terminals of the four-wire circuit are short, therefore, so that, on these terminal connections, the noise currents are comparatively weak and the voice currents large, it is possible to realize in practise the full theoretical possibilities from an echo suppressor. In other words, it is possible to work a four-wire circuit under these conditions at a very low loss, or even an over-all gain.

When the lines connecting the subscribers with the terminals of the four-wire circuit are long, so that the voice currents may be weaker and, perhaps, the noise currents may also be stronger, results as good as this may not be obtainable. However, even in this case, a material improvement can usually be effected by the echo suppressor.

For the condition in which a four-wire circuit is switched to a variety of different circuits at the terminals, it was shown in Fig. 10 that the requirement that echoes should not disturb the talker is more severe, so far as limiting the minimum loss is concerned, than the requirement that echoes should not affect the listener's transmission. It will, of course, be obvious that talkers connected to either terminal of the four-wire circuit through connections involving small transmission losses will hear louder echoes than will talkers connected through circuits having larger losses. In other words, the minimum net loss of a four-wire circuit used in this way is limited by the requirement that the talkers connected through low losses should not receive too much echo. Now, of course, the relays in the echo suppressor will respond most readily to these talkers. Satisfactory operation of the relays for these talkers will, therefore, be secured even though the echo suppressors be given such an adjustment that the relays will not respond to the voice currents from talkers connected to the circuit through a higher loss. Cutting

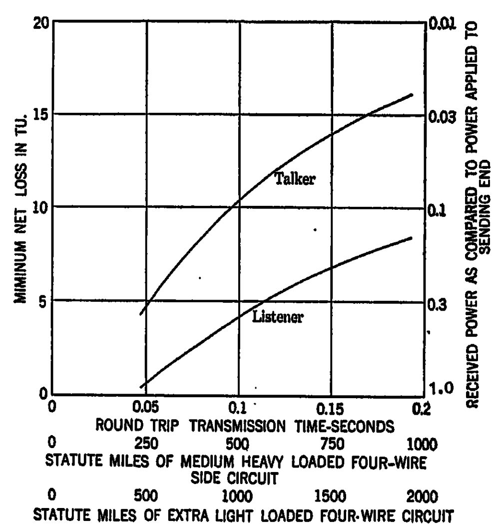

FIG. 10—ECHO LIMITATIONS ON LOSS OF A FOUR-WIRE CIRCUIT

off the talker echoes in the case of the connections involving low losses, therefore, makes it possible to materially lower the loss introduced by the four-wire circuit even though other echoes are not cut off.

In the curves of Fig. 10 it is seen that for a 1500-mile (2400 km.) extra-light loaded circuit the possible improvement which may be secured by cutting off the talker echoes from low loss connections may be as much as 7 TU even though echoes from other connections are not cut off.

In general, for combinations of four-wire and two-wire circuits and for circuits which are all two-wire as well, talker echoes are also more serious than listener echoes provided that the impedance irregularities at intermediate points in the circuit are small, as is usually the case with high grade circuits. Consequently, echo suppressors make it possible to effect improvement in many cases even if the line noise which is present requires reduction of the sensitivity of the echo sup-

7. Due to transmission variations of the different parts comprising long telephone circuits such as these, the over-all loss varies to a certain extent with time. In practise, adjustments of circuits in the Bell System Plant are made often enough to keep the variations within about $\pm$ 2 or 3 TU. The working net loss must, of course, be made high enough so that echo difficulties will not be encountered when the variations combine in such a way as to give the over-all, or net loss, its minimum value. For example, in the case of the 1500-mile (2400 km.) circuit above, if it is assumed that the circuit is limited by echoes to a 14 TU minimum net loss and that it is maintained within limits of variation of $\pm$ 3 TU, the working net loss would be 17 $\pm$ 3 TU.

pressors to the point where weak voice currents fail to operate the relays. If the line noise requirement does not enter as a limitation, a greater improvement is, of course, possible as is also the case with all four-wire circuits.

Conclusion

The echo suppressor, which has been described, offers attractive possibilities in supplementing other methods for obtaining satisfactory transmission over long two-way telephone circuits.

The application of an echo suppressor to a telephone circuit requires no changes in the circuit itself, the echo suppressor being merely attached to the circuit at some convenient point.

For any particular type of circuit, the advantages to be gained by using echo suppressors increase with length. For a given circuit length the advantages to be gained are greater with low-speed than with higher-speed circuits.

Echo suppressors offer the greatest possibility of usefulness on cable circuits, owing to the inherent low-speed and quietness of such circuits. Generally speaking, the application of echo suppressors to cable circuits offers possibilities of effecting savings by allowing the use of heavier weight, lower-speed loadings in place of lighter weight, higher speed loadings, as well as the imposition of less severe requirements as to impedance uniformity of the circuits.

Discussion

S. P. Shackelton: When any device, which differs as fundamentally from the existing order of things as the echo suppressor is introduced into a working system there will arise problems as to its practical application. To a very large extent these problems have been met in the installation at Harrisburg which is referred to in the paper. It may be profitable to review somewhat the experience obtained in that installation.

It will be realized that the proper association of an echo suppressor with a telephone circuit is essential for satisfactory operation. This involves not merely obtaining the correct circuit connections in the office where the suppressor is located but also in securing a suitable circuit layout for the entire length of the telephone circuit. While considerable latitude is possible in the geographical location of the suppressor, still the time intervals introduced by the relays require that it be located at some distance from the ends of the circuit. The introduction of telephone repeaters a number of years ago imposed somewhat similar restrictions as to circuit-layout changes. These factors all tend to eliminate temporary circuit-layout changes and to confine the changes to those authorized after consideration of complete circuit requirements. This is particularly true on toll cable where the need for emergency changes is less frequent than with open-wire lines.

A consideration of the operating conditions to which echo suppressors are subjected suggests certain differences between the relay requirements and those usually met by telephone relays. As a rule relays are given specified adjustments, either mechanical or electrical or both, and then are expected to function in the circuit with a certain margin between the adjusted condition and the normal working condition. In the echo suppressor no such margin is possible, the relay simply operating on voice currents of sufficient magnitude and failing to operate on weaker ones. The margin here required is not possible in the usual sense by means of relay adjustment and hence the provision is made for varying the sensitivity by means of different connections to the input transformer. It will probably not be possible to adjust all circuits for the same sensitivity owing to the different conditions of noise and energy level encountered. Also the margins may differ widely. In fact, there may be no margin in the usual sense, the suppressor being adjusted to operate only on the relatively strong voice currents.

No unusual maintenance requirements are introduced by the use of echo suppressors in the plant. The conditions just outlined call for somewhat different treatment, however, than is usually followed. The paper outlines the operation of a testing circuit for checking the relay adjustment. Such a circuit indicates the combined effect of all the time intervals introduced by the different relays in normal operation modified by the action of the amplifier rectifier in short circuiting its input. Even with the use of such a testing circuit it is necessary to give the individual relays their proper adjustments. Also some experience is necessary in the use of the testing circuit to interpret its results. Provision is made for reading the current in relay *A* Fig. 3, in normal operation and at the same time it is possible to monitor on the suppressor. In this way a check on the operation can be obtained. This is particularly desirable as the relay *A* is subject to severe operating conditions and requires closer maintenance than would be the case for direct-current operation. It is to be noted that the entire operation of the circuit is dependant on relay *A*.

In usual telephone practise, even with the simplest circuits, whenever relays are involved, it is customary before putting a circuit in operation to check the relay adjustment by means of a current flow and give the prescribed current-flow adjustment for those relays. An interesting incident in connection with the installation at Harrisburg, might be brought out. The suppressors were all assembled completely in the Western Electric Company laboratory, tested out and shipped to Harrisburg. In shipment, relays cannot be expected to maintain their adjustment. The men who put the suppressors into service at Harrisburg had had no previous instructions as to relay adjustments, and yet in spite of that fact when they completed the installation, they put one of the suppressors on a telephone circuit which was set up for trial without any check whatsoever on the adjustment, and it worked along very nicely. This would bear out our assumption that in spite of the fact that maintenance conditions may be somewhat different, they are not any more severe than any normal circuit operation.

H. S. Foland: In connection with echo suppressors for long telephone circuits, it occurs to me that some additional discussion of the trial installation at Harrisburg may be of interest.

The paper mentions this installation and shows a picture of the apparatus as installed. Harrisburg was selected as a suitable location for a trial installation of this apparatus for several reasons:

First: There were four-wire cable circuits between New York and Pittsburgh, so loaded that any material extension of them should result in noticeable echo effects. These circuits were of course, provided primarily for New York-Pittsburgh circuits.

Second: It was possible to extend these circuits from Pittsburgh by means of several different types of facilities to points sufficiently distant to produce the desired echo effects.

Third: Harrisburg is suitably located between New York and Pittsburgh to provide the time interval required for the apparatus to function and to permit the operation of the circuits to be observed from the New York end.

Fourth: Harrisburg is a repeater point on these cable circuits, (approximately at the center of the New York-Pittsburgh section) and thus a suitable point for an echo-suppressor installation.

Ten 19-gage, medium heavy loaded four-wire cable circuits between New York and Pittsburgh were taken and the Pittsburgh terminals extended to a number of distant points over a variety of types of facilities:

Four circuits were extended to Chicago by means of four-carrier telephone channels;

One circuit was extended to Chicago by means of an open- wire circuit;

One circuit was extended to Cincinnati by means of an open-wire circuit;

Three circuits were extended to Cleveland by means of cable facilities between Pittsburgh and Cleveland, in part four-wire and in part two-wire;

One circuit was extended to Detroit by means of an open-wire circuit to a point near Toledo and from that point to Detroit in a two-wire cable circuit in the Toledo-Detroit Cable.

It will be appreciated, I believe, that this was a rather comprehensive selection of facilities on which to institute a trial and that the results obtained from these several combinations might reasonably be taken as indicative of the performance of the echo suppressors.

These circuits were then equipped with the echo suppressors at Harrisburg and a close supervision maintained of their performance, both from the standpoint of equipment trouble and from the standpoint of the effects of the echo suppressors on the operation of the circuits. The data collected have indicated that these echo suppressors function in an effective manner and that they have not been subject to an abnormal amount of trouble.

The paper points out that this apparatus operates so as to short circuit the return transmission path. This was an interesting feature from an operating standpoint, since the thought naturally occurs that such an arrangement would give the speaker right of way over the circuit and that the listener at the opposite end of the circuit would be compelled to wait until the speaker had finished or at least had made an appreciable pause before it would be possible to interrupt him. A very considerable number of conversations were carefully observed to determine if there was any indication of this effect, and it was established that the users of these circuits carried on conversations in a perfectly normal manner, interrupting one another in apparently the same manner as on circuits not so equipped. This result may be explained, of course, by the very short intervals of time involved in the operation of this equipment.

From an operating and maintenance standpoint, there was something disturbing in the thought of giving service dependent upon the operation of relay equipment a considerable distance from the speaker and actuated by the speaker's voice, but experience with the Harrisburg installation has indicated that the echo suppressor is a practical device, both effective and reliable in its operation.

A. B. Clark: I do not want to leave the impression that echo suppressors are vitally needed on all cable circuits. As a matter of fact, there are in service today types of cable circuit capable of giving a good grade of commercial telephone transmission for distances of at least 1000 mi. without any echo suppressors at all. However, it is possible that in certain cases echo suppressors may allow the desired transmission results to be secured more cheaply.

The echo suppressors which have been installed at Harrisburg and have already been referred to are working on types of cable circuit which were designed for use up to moderate distances only. When the echo suppressors were placed on these circuits, the circuits were intentionally pushed beyond the limits for which they were originally designed. If the "long-distance type of cable circuit" were available, it would be possible, for the circuit lengths involved in this case, to dispense with the echo suppressors at Harrisburg.

Frequency Multiplication
Principles and Practical Applications of Ferro-Magnetic Methods

BY N. LINDENBLAD[1] and W. W. BROWN[2]
Non-member Non-member

INTRODUCTION

MANY of the principles involved in the multiplication of frequencies by the use of highly saturated iron cores have been established by early investigators in the field of radio engineering. A number of articles on the subject have been published by various investigators, although very little information about the performance of frequency multipliers under load conditions was found.

1. E. F. W. Alexanderson, *Magnetic Properties of Iron at Frequencies up to 200,000 Cycles.* TRANSACTIONS A. I. E. E., November, 1911.
2. Dr. A. N. Goldsmith, Radio Frequency Changers. *Proceedings* I. R. E., March, 1915.
3. J. Zenneck, A Contribution to the Theory of Magnetic Frequency Changers. *Proceedings* I. R. E., December, 1920.
4. T. Minohara, Some Characteristics of the Frequency Doubler as Applied to Radio Transmission. *Proceedings* I. R. E., December, 1920.
5. M. Latour, Static Frequency Multipliers for the Production of Very High Frequencies in Radio Telegraphy. *Revue Generale de l'Electricite*, July, 1922.
6. K. Schmidt, High Frequency Sender for Radio Telephony. *Elektrotechnische Zeitschrift*, October, 1923.

Frequency multipliers were investigated in connection with the development of the 200-kw. Alexanderson radio system; however, it was found entirely feasible to build 200-kw. alternators which would generate directly the frequencies required. During these and other investigations, frequency multipliers were used to generate a wide range of frequencies in a study of transmission phenomena, and very satisfactory performance of the multipliers was obtained. Simplified circuits were developed and communication established between Schenectady, New York, and New York City with very small amounts of power. Both telegraphy and telephony were used.

This paper describes some of the results to date of a joint investigation by the Radio Corporation of America and the General Electric Company, and is intended to present, in a condensed form, physical conceptions of the operation of iron-cored frequency multipliers. The main object of the investigation is to determine the possibilities of increasing the usefulness of the 200-kw. Alexanderson alternators. Results, to date, include a number of improvements in the design of frequency multipliers and arrangement of circuits, and indicate that frequency multipliers may be used advantageously with 200-kw. alternators to meet special requirements. Indications are that frequency multipliers could also be used in industrial applications where relatively large amounts of power at relatively high frequencies are required.

PRINCIPLES OF SINUSOIDAL AND SHOCK EXCITATION

Any periodic curve is the resultant of a number of sinusoidal curves of various amplitudes and frequencies, and these can be separated by mathematical treatments in accordance with the well-known Fourier's theorem. The periodic fluctuations of the distorted magnetic field in frequency multipliers has thus been studied, affording a basis of incomparable value in such work.

In practical work, with iron-cored frequency multipliers, a distinct line may be drawn between two methods of excitation,—the sinusoidal and the shock methods. Considered theoretically, the former represents a condition in which the resultant flux curve consists, in its elements, of only two sine curves, the fundamental and the harmonic desired. The resultant flux curve obtained when using the shock excitation method may be resolved into a greater number of elementary waves.

The sinusoidal method is more and more nearly approached as the desired harmonic is supplied with new energy during the greater part of the duration of its period. In the well-known case of doubling the frequency of the fundamental by means of a double unit arrangement, the sinusoidal method is predominant. The arrangement consists of two separate multipliers, each saturated to the correct degree by direct current and connected in such a way that a distortion of the magnetic flux takes place alternately in each unit for each half-cycle of the fundamental. The magnitudes of direct current and alternating current can be so chosen that the voltage induced by the distorted field is sinusoidal and is the second harmonic of the fundamental frequency.

As the art of the ferro-magnetic method advanced, it was found by various investigators that higher efficiencies could be obtained by the shock excitation method; that is, instead of obtaining an induced e. m. f. of sinusoidal character by the sinusoidal method, it is better to force the device to its maximum efficiency by shock excitation. The output of a given multiplier, when shock excited, has been found in nearly all cases to be larger than when adjusted to induce a sinusoidal e. m. f.

1. Radio Corporation of America, New York, N. Y.
2. General Electric Co., Schenectady, N. Y.

Presented at the Spring Convention of the A. I. E. E., at St. Louis, Mo., April 13-17, 1925.

It was found that, when doubling a fundamental frequency, the difference between the two methods from an efficiency and output standpoint was not very great. However, as the multiplication ratio was increased, the efficiency of the sinusoidal method dropped very rapidly. When using the sinusoidal method at higher multiplication ratios, a number of

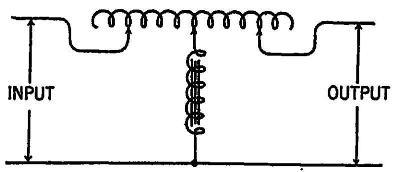

Fig. 1—Frequency-Multiplier Circuit

units are required, combined in cascade, and the over-all efficiency is the product of the efficiencies of the individual stages. On the other hand, when using the shock excitation method, the desired multiplication is accomplished in one stage, and, though the efficiency of this one stage may be lower than the efficiency of one of the stages in cascade, the over-all efficiency will usually be higher.

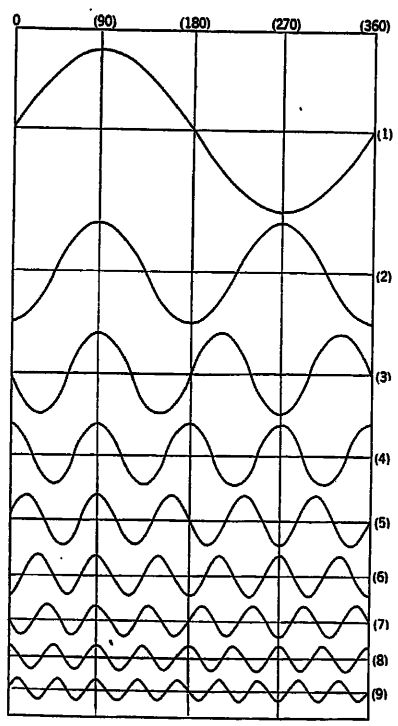

Fig. 2—Phase Relations Between a Fundamental Wave and Various Harmonics

When using the shock excitation method, the energy is supplied to the harmonic during a relatively shorter time period and assumes a transient character. This introduces two problems which become more difficult to solve as the multiplication ratio is increased.

The first problem was manifest in the inability to load the system. The fundamental frequency circuit would not oscillate unless the kv-a. of the circuit was made relatively large, since using relatively low kv-a. in the circuit, little energy is stored and the disturbance at the time of the energy transfer to the harmonic circuit is too great. Also by employing a circuit of reasonably large kv-a., when working with transient impulses, a cushion effect is obtained across the multiplier to allow the induced voltage peaks to develop. This is accomplished by the resonant circuits connected to the multiplier being series circuits of relatively high inductive reactance. A series-tuned circuit has an infinitely high reactance to transients, whereas a

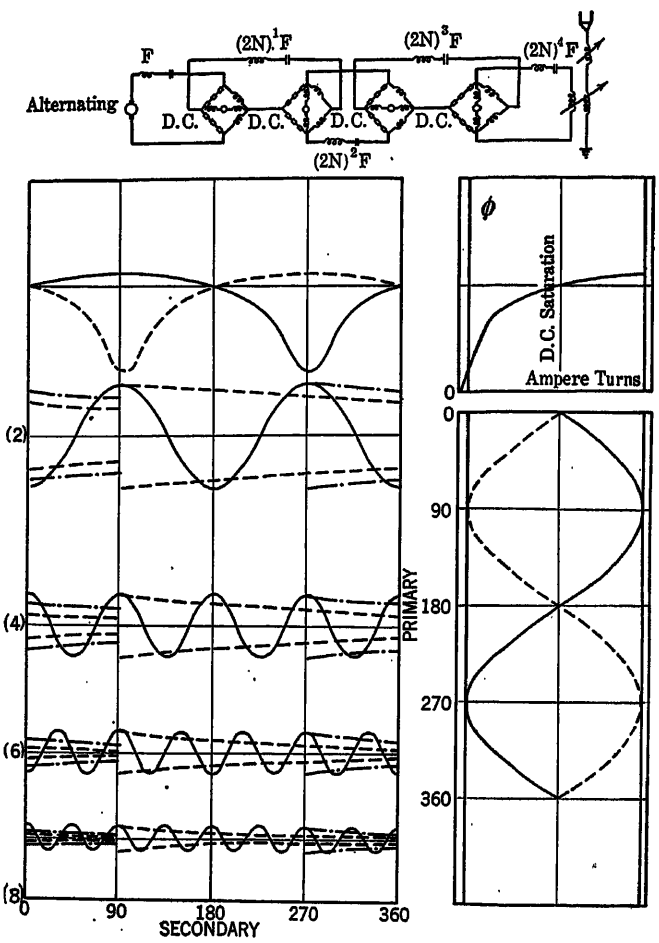

Fig. 3—Half-Cycle Excitation Multistage Arrangements of Frequency Multipliers for the Production of Even Harmonics

parallel-tuned circuit has zero reactance to transients. The reactance in the resonant circuit also prevents the fundamental or some undesirable harmonic that may accidently develop from entering the output circuit. In the case of doubling the frequency, the reactance required to separate the two frequencies needs to be larger than would be required by any of the reasons previously given. To avoid an unduly large reactance for this case a circuit was developed by E. F. W. Alexanderson, which is shown in Fig. 1. The circuit consists of an auto-transformer arrangement, utilized in such a way that the section from one end of the coil

to its intermediate tap comprises the inductance in the resonant circuit for the fundamental frequency. The remaining portion of the coil from the intermediate tap to the opposite end comprises the inductance of the resonant output circuit. The intermediate tap of the coil to which the multiplier is connected is chosen so that the voltage induced in one section from the other section neutralizes the voltage impressed on the intermediate tap from the multiplier. For circuits arranged for higher multiplication ratios, this arrangement ceases to function on account of the pronounced transient character of the energy transfer. This

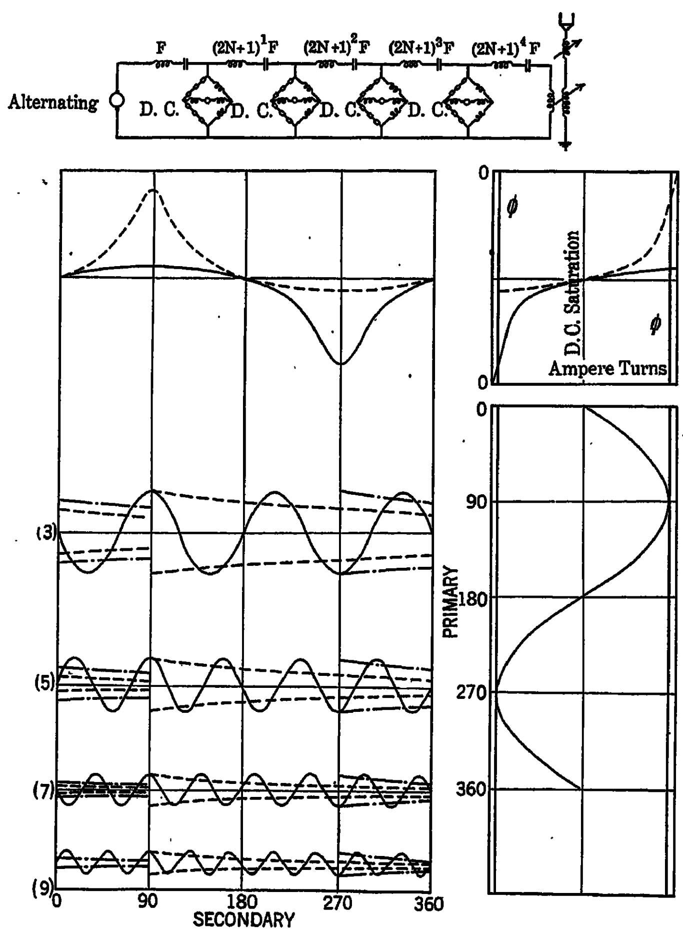

FIG. 4—HALF-CYCLE EXCITATION MULTISTAGE ARRANGEMENT OF FREQUENCY MULTIPLIERS FOR PRODUCTION OF ODD HARMONICS

circuit is not required for higher multiplication ratios, as the difference between the input and output frequencies is so great that ample separation is afforded through simple tuning.

The other problem introduced by the shock excitation method, which increases in difficulty with increased multiplication ratios, is the damping effect in the output circuit. Difficulties due to this phenomenon were first encountered in a transmission test on a wave length of 3200 meters (94,000 cycles) which involved quadrupling a fundamental frequency of 23,500 cycles. The test was very successful in all respects, except that it was difficult to obtain a clear note at the receiving station. The reasons were that the oscillations were damped and declined as a logarithmical function, thus giving a signal not as clear as obtained from a wave of constant amplitude. Side frequencies were also partly responsible for the impurity of the note. These side frequencies constitute a quite serious problem in radio applications, as they are the results of characteristics inherent in the circuit. If a circuit has more than one degree of freedom, the oscillations, when left without a continous guiding e. m. f., degenerate into oscillations corresponding to the various degrees of freedom of the circuit. The conditions are generally not improved by loosening the coupling between the circuits, as this merely tends to bring the various degrees of freedom closer together. It was therefore realized that, in order to improve the arrangement so as to give it real commercial value, the method of energy feed must be greatly improved.

As has been pointed out, the side frequencies and the damping effect were the results of too infrequent energy impulses. Some improvement could be obtained by means of a relatively large energy storage or tank circuit, but such an arrangement would consume relatively large amounts of power. A much better way would be to increase the number of impulses and thereby provide a more frequently occurring guiding voltage for the oscillations.

CIRCUIT IMPROVEMENTS FOR SHOCK EXCITATION

If the multiplication ratio is not too high, it is sufficient to obtain one impulse from each half-cycle of the fundamental. The connection arrangements to obtain the correct phase combinations of the harmonics are indicated in Figs. 2, 3, and 4. It should be noted that the circuits and relations differ for odd and even harmonics. When working with higher multiplication ratios, the one impulse from each half-cycle of the fundamental is not sufficient. One arrangement by which a greater number of energy impulses can be supplied to the harmonic is by the use of polyphase power supply. A similar arrangement has been suggested by Mr. Dornig[3], who also offered a method of combining a harmonic of one fundamental frequency with a harmonic from a different fundamental frequency. The use of polyphase or equivalent excitation is vital in circuits to transfer large amounts of power, as the reactive circuits can be reduced considerably and perhaps eliminated entirely if there is an energy impulse for each half cycle of the harmonic.

ADVANTAGES OF "DIP" METHOD

Regarding the methods of exciting the multiplier and utilizing the bend of the magnetic curve in the most efficient manner, it has been found advantageous

3. Contribution to Frequency Transformation by Means of Iron Core Inductance, *Elektrotechnische Zeitschrift*, Vol. 45, No. 42, October 18, 1924, pp. 1107-1108.

to employ direct-current excitation regardless of whether the harmonic is an odd or an even multiple of the fundamental. While it is true that odd harmonics can be produced without saturating the multiplier with direct current, a considerable increase in efficiency is obtained by using direct-current excitation. The reason is that when using direct-current excitation a much smaller alternating component can be used and a large portion of the iron losses eliminated. This is easy to realize by reference to a *H-B* curve of iron. When the iron is already saturated with direct current to a point above the bend of the curve, only so much alternating current need be superimposed to produce a "dip" in the flux intensity. The dimensions of this "dip" are determined by the harmonic desired. Thus the iron loss only occurs at the same time as the "dip" in the flux. During the rest of the time, the iron is fully saturated and the losses relatively small. On the other hand, when no direct current is used, a much larger amplitude of alternating current is required to reach the desired point of flux deformation. Besides, the iron is being worked in an unsaturated state the greater part of the cycle, which results in considerably higher iron losses. The only apparent advantage of the multiplier without direct current for producing odd harmonics is that, by using a single multiplier unit, an impulse is transferred from each half-cycle of the fundamental, whereas, when direct current is used, two multiplier units are required arranged in such a way that the phases of the impulses are correctly combined. However, this latter arrangement adds very little complication to a system.

Advantages of Short Magnetic Path

Theoretical considerations of the design of frequency multipliers indicated a decided increase in efficiency should be obtained by making the diameter of the core relatively small. The reason is that the inducing power of a magnetic field is due to the cross-section of the field, but not to the length of the path. Substantially the same results are therefore obtained with smaller amounts of iron and the correspondingly smaller core loss. In substantiation of this theory, a circuit containing a multiplier having a magnetic circuit 25 in. in length, see Fig. 5, was found to have an efficiency of 66 per cent when doubling. The same circuit, but containing a multiplier having a magnetic circuit 12 in. long, had an efficiency of 78 per cent. Similar effects have been observed by other investigators, but apparently the principles have not been investigated to determine the limits.

To investigate the possibilities of further improvement in this direction, a series of sample units have been made up in which the length of the magnetic circuit ranges from three inches to 3/8 inch. Measurements made on the unit having the shortest magnetic circuit indicated that a stage was reached at which the copper and iron losses approached equality, thus indicating that optimum dimensions had been reached. In this particular case, in quadrupling a fundamental frequency of 23,500 cycles, an efficiency of 90 per cent was obtained. These dimensions are, of course, dependent upon the application of the device and the materials used. Fig. 6 shows a sample multiplier with multiple-turn conductor around the core, and Fig. 7 shows a sample with single-turn conductor. The single-turn conductor is necessitated by the extremely small dimensions of the core.

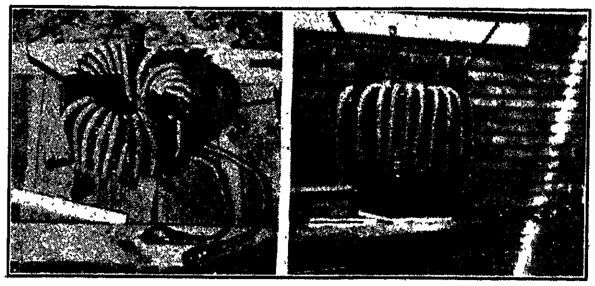

Fig. 5—Two Views of Frequency Multiplier *F M*-3

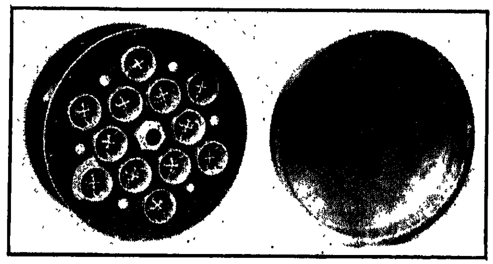

Fig. 6—Frequency Multiplier *F M*-9 (Bottom View With Base Pan Removed

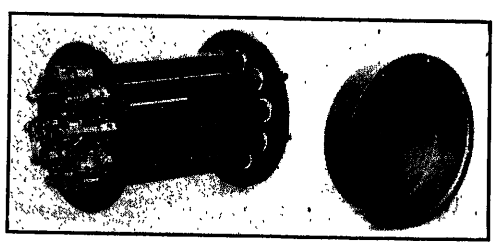

Fig. 7—Frequency Multiplier *F M*-10. 50-kw. Oil Pan Removed and One Section of Core Partly Withdrawn to Show Construction

Core Material

The efficiency and general performance of an iron-cored frequency multiplier is dependent, to a great extent, upon the characteristics of the iron used. In order to make direct comparisons of different grades of iron, samples were prepared in the form of a toroid having the same length of path, same amount of iron,

and with winding of the same conductor and the same number of turns. These samples were immersed in oil for tests. The tests consisted of measuring—

1. The impedance of the various samples at 20,000 cycles which indicates mainly the *B-H* characteristic of the iron.

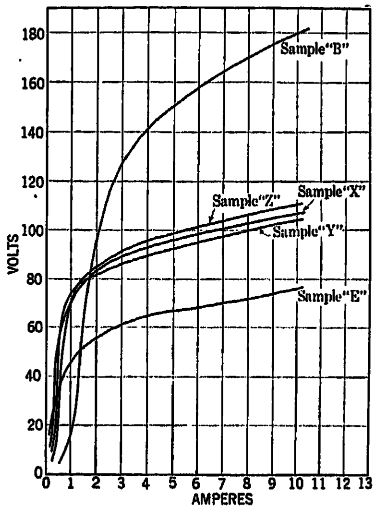

FIG. 8—20,000-CYCLE SATURATION CURVES OF IRON SAMPLES FOR FREQUENCY MULTIPLIERS

2. Measuring the temperature rise of the samples at various values of volt-amperes at 20,000 cycles.

The results of these tests are shown in Figs. 8 and 9. Conclusions from these tests, substantiated by results from other tests in which various grades of iron were used in frequency multiplier circuits, are as follows: The iron which has high *B* for a given *H* above the knee

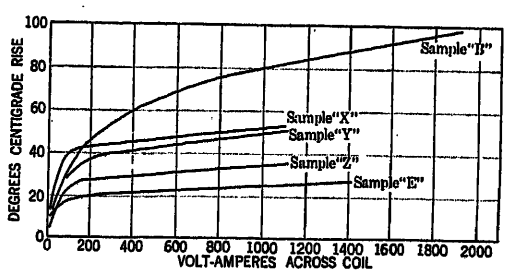

FIG. 9—20,000-CYCLE CHARACTERISTICS OF IRON SAMPLES FOR FREQUENCY MULTIPLIERS

of the curve stores more energy at operating adjustment than the iron which has a lower *B* for a given *H*. Stable operation is obtained with iron having a high *B-H* relation. The iron which has a relatively low *B* for a given *H*, while more efficient in operation, has a tendency to be unstable, due apparently to the relatively small amounts of energy stored. A compromise between stability and efficiency is indicated as a most desirable grade, from a practical standpoint.

The intermediate curves in Fig. 8 show the characteristics of an iron which apparently would be best suited. Another advantage of this iron is the relatively sharp bend in the *B-H* curve which should enable the transfer of a greater proportion of the stored energy of the fundamental frequency to the harmonic frequency circuit. This is particularly true in circuits which operate on a principle of shock excitation.

METHODS OF MEASURING EFFICIENCY

Losses in frequency multipliers are being measured by electrical and calorimeter methods. The results obtained by the two methods are in close agreement and are considered sufficiently accurate for practical purposes.

RADIO APPLICATION

The radio application of frequency multipliers provides a means to greatly increase the usefulness of high-power alternators. A 20-kw. frequency-multiplier installation is being made at the Radio Corporation's transatlantic station at Marion, Mass. This 20-kw. set will be supplied with power from eight armature coils out of a total of sixty-four coils from either of two 200-kw. Alexanderson alternators. This small part of the total capacity of an alternator will reduce the power from the main circuit only 12½ per cent, but will provide a separate transmitter for marine service. This 20-kw. transmitter will multiply a frequency of approximately 25,000 cycles five times in one stage. This set will include the best-known features in accordance with the principles outlined herein.

INDUSTRIAL APPLICATIONS

The industrial application of frequency multipliers appears to be where relatively large amounts of power are required at moderate frequencies. In the design of salient-pole alternators for relatively high frequencies and for relatively large powers, a nominal limit is between 500 and 1000 cycles. Alternators to develop frequencies appreciably higher than this nominal limit are usually of the inductive type, and, being in a special class, are costly to build and have relatively low efficiencies. It appears entirely feasible to use a 500-cycle salient-pole alternator, and, by the use of iron-cored frequency multipliers, produce relatively high frequencies in sufficient power capacities for many industrial applications.

Discussion

E. W. Kellogg: Mr. Brown spoke of getting a wave of 500 cycles and make it into a 1000-cycle wave with 90 per cent efficiency. The story is not complete until we know just what it does to the efficiency of the generator supplying the original wave.

W. Rogers: I should like to ask what use has been made of this principle in multiplying the ordinary ringing frequency which is 16 cycles to 133 cycles for use on composite ringers on telephone circuits.

S. P. Shackleton: I think the answer is that there has been no practical use made of it, and the principal reason is, I believe,

that the frequencies are so low that, to get very great efficiency it would require apparatus which would cost more than other apparatus available for generating 133 cycles.

For a number of years we did use the fundamental ringing frequency of 16 cycles or 20 cycles, whichever it happened to be, to operate an interrupter to produce the higher frequency. That, however, was none too efficient, and recently that has been superseded almost entirely either by a motor-generator set or by an improved type of interrupter operating from direct current.

W. Rogers: I do not suppose that this is a matter of much importance to the telephone companies, but it is of some importance to the railroads. The telephone company is in a position to use the methods described by Mr. Shackleton, because of their larger installations and because of the fact that their long-distance circuits are numerous and centralized. However, on the railroad, we have composite circuits which terminate at points where these things are not available. One of our greatest troubles is with our composite ringers. Most of us are still using the old interrupter, operated from d-c. or 16-cycle ringing current, and it has been suggested, and I believe demonstrated, that it is possible to take two, or, perhaps, three 47A repeating coils and, by saturating the cores of one of them and connecting them in a certain manner, change 16-cycle ringing current to approximately 133 cycles.

S. P. Shackleton: The only case of which I know where the higher frequency, 133 cycles, has been obtained by frequency changers of that type has been in Philadelphia, where our men became very much interested in that problem. They had working for several years, composite ringers supplied by means of such a device. I looked over the layout there and do not remember all the details of it, but I am very sure that if the frequency changer was made commercially available it would cost more than the interrupter which we are now using on our telephone circuits.

W. W. Brown: In reply to Mr. Kellogg's question regarding the efficiency of the power supply for frequency multipliers load: It is possible to adjust the frequency-multiplier circuits so that the load is approximately at unity power factor. This is entirely feasible in radio-frequency circuits, but uneconomical in relatively low-frequency circuits on account of the large kv-a. of the condensers required. Multiphase power supply appears to be the solution for the relatively low-frequency circuits.

Another probable radio application of frequency multipliers will be in connection with necessary changes to increase the possible rate of sending by long waves. In order to improve the efficiency of long-wave antennas, the resistance has been decreased to a remarkably low value. The effect of lowering the resistance is to lengthen the time constant of the antenna. A low time constant of such an antenna tends to cause distortion of signals at high rates of sending. By using a higher frequency, the time constant of a given antenna is shortened, which permits higher rates of sending. The application then will be in multiplying the entire output from a large alternator for high-speed transmission.

N. Lindenblad: In answer to Mr. Kellogg's question, the primary source is not affected as feed-back effects are prevented either by means of tuning or the Alexanderson neutralizing transformer.

In answer to Mr. Roger's question, it must be understood that frequencies obtained by the multiplication method referred to are only whole multiples of the fundamental frequency, not fractional multiples.

I agree with Mr. Shackleton regarding the probable high cost of the frequency changer for increasing the frequency of 16 cycles to 133 cycles, unless the demand for such a device is large enough to justify the cost of the development work.

Electric Propulsion of Ships

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Associate, A. I. E. E.

AND

W. E. THAU[2]
Member, A. I. E. E.

Synopsis.*—This paper deals with the electric propulsion of practically all classes of vessels. The several types of electric drive are treated, in general, with special emphasis on Diesel-electric drive, as it is believed that this type of electric drive will be of most general interest, particularly to the inland section of our country, since it is especially applicable to the smaller size vessels, such as are used on rivers and lakes, to vessels in the coastwise trade and to the usual size of ocean freighter.*

All types of propelling equipment, however, will be referred to for the purpose of comparing the advantages of electric drive with other methods of ship propulsion and to show the natural advantages of each particular system.

It is the intent of this paper to provide data valuable in the selection of the proper type of drive for each application, which must necessarily be determined by the character of the vessel and the service it is to render. Each type referred to has a particular field in which it is best suited.

The authors desire to give credit to the Westinghouse and General Electric Companies for much of the data and material for cuts included in this paper.

Introduction

EVEN a comparatively small ship represents a considerable investment. Capital being traditionally conservative and as ship owners have to consider the lives of passengers and crew as well as property, it follows that the established systems of mechanical transmission will be preferred until it has been established to the entire satisfaction of owners and operators that the electric drive possesses elements of safety, reliability and economy superior to any other type which may be considered.

The object of this paper is to show that electric drive, when compared with any other system, has outstanding qualities of safety and reliability and that the economy is superior except where special conditions favor one of the older types. Electric drive is, of course, still in its infancy and it is to be expected that a longer period of development will still farther extend its field of application.

As will be noted in the bibliography, several papers have already been written on different phases of this subject, but in this paper it is proposed to cover the entire subject in general.

The use of electricity for ship propulsion is no longer an experiment. H. L. Hibbard's paper on *The History of Electricity on Shipboard,* refers, in general, to several installations, all of which have been most successful. The superiority of this type of drive has been apparent for some time and the constituent parts have been well established; but not until comparatively recent years has it been practically applied.

The United States Navy, realizing the advantage of electrical drive, decided to give it a comparative trial with that of reciprocating engines and geared turbines. The success of the electric method of propulsion in the U. S. collier *Jupiter,* (now the aeroplane carrier *Langley*), as compared with that of the engine-driven *Cyclops* and the geared turbine-driven *Neptune* was instrumental in inspiring the U. S. Navy to adopt it for all subsequent capital ships, the first of which was the U. S. S. *New Mexico.*

The justification of the turbine-electric system of propulsion for ships is that it affords a transmission efficiency very nearly equal to that obtainable with gears of suitable speed reduction for economical turbines and propellers, and that it accomplishes this result by a simpler and more flexible means.

The justification of the Diesel-electric system for propulsion of ships is that it affords the fuel economy of the Diesel engine with the added advantages of smaller and simpler engines, flexibility, reliability and superb maneuvering qualities.

Reciprocating Engine Drive

For many years the reciprocating engine has been a virtually completed development, possible improvements being mere matters of detail. In many cases and especially in the larger installations, it is objectionable, due to its weight and space factors. However, this type of drive is reliable and thoroughly understood by operating engineers. It will probably be used indefinitely for ship drives as there are certain types of ships in which it would be inexpedient to apply other methods of propulsion, due to definite economic reasons.

The cost of fuel does not permit the general use of reciprocating engines in land stations, and this same objection exists on board ship.

Turbine Direct Drive

The first radical change in steamship propulsion was brought about by the introduction of the direct-connected turbine. This type of drive, however, is admittedly imperfect because the most efficient, lightest and simplest turbine is essentially a machine of a definite high speed, while the most efficient propeller is a low-speed device, so that the efficiency of both must be sacrificed in compromising the speed. The great majority of direct-connected turbine-driven ships now in operation have turbine speeds entirely too low for

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good economy, with consequent efficiency little better than that of reciprocating engines, whereas, ordinarily, a turbine may be considered appreciably more efficient than a reciprocating engine, due to simple rotation and a much higher ratio of steam expansion. This disadvantage undoubtedly retarded the introduction of turbines for propulsion of ships before the use of gears. Nevertheless, this method of propulsion has been installed in some very fast passenger vessels as well as warships and destroyers. The *Mauretania* is a striking example of compromise in speed necessary, as the propeller and turbine speed is 188 rev. per min., whereas the most economical speed for the turbine would be 1400 rev. per min. and for the propeller, about 150 rev. per min. High-speed turbines of equal capacity in daily use are about 80 per cent efficient, while the *Mauretania* turbines are only 62.75 per cent efficient. This objectionable feature was principally instrumental in the introduction of mechanical reduction gears for ships.

As the turbine is not reversible, it is necessary either to provide reversing wheels in the main unit or a complete separate reversing turbine. A reversing element, when running ahead, introduces losses as great as 1½ per cent, and, being relatively small and inefficient, provides less backing power than that of reciprocating engines. The friction loss of the reversing turbine in the ahead direction is several times that in the astern direction. There is also a grave possibility of dangerously heating the reversing wheels if the vacuum drops. These causes have contributed to many turbine failures.

Turbines with reversing elements must necessarily be designed with larger clearances, thus lowering their efficiences; and the larger diameter necessitates larger radial clearances. Also the losses in the large connecting piping of cross-compound turbines are very appreciable. Separate reversing turbines complicate the control and add to the weight, space and cost. At its best, therefore, this method of drive provides rather poor maneuvering qualities and slow stopping, requiring, in some cases, as long as forty seconds to stop the screws when going full speed ahead.

The direct-connected turbine is further handicapped by its inability to utilize high superheat.

Therefore, the direct-connected turbine, although attractive in its apparent simplicity, has probably progressed as far as can be expected, and cannot be regarded as a satisfactory general method of propulsion.

Turbine Gear Drive

Many of the objectionable features of direct-turbine drive have been overcome by the introduction of mechanical reduction gears. They permit both the turbines and propeller to be designed for their most efficient speeds. Reduction gears are especially applicable to destroyers and light cruisers where the high-speed light turbine and relatively low-speed propellers are desired. Here the weight factor is most important and probably more than offsets the advantages of electric drive in such installations. Gear drive machinery for low-speed cargo vessels is, in general, lighter and the transmission efficiency is probably better than that of electric drive. However, when considering losses in the reversing element, extra oiling requirements, packing losses, extra steam losses due to larger clearances, the transmission efficiency is reduced very nearly to that of turbine electric drive.

The rubbing contacts of gears is of utmost concern and, to be safe, the gears must be liberally designed. With either the turbine direct or gear drive with more than one screw, the disabling of one unit means dragging a propeller. Furthermore, reduced speed conditions introduce inefficient operation as all prime movers must run at lower than designed speeds. Sudden application of load and vibration are hard on gears as is also misalinement due to springing of the hull or inaccurate workmanship. In many cases the noise of gears has been objectionable, especially on passenger ships. Life of gears has been uncertain and there have been many disappointments. However, with proper design and improved methods of manufacture, these troubles are being overcome. Considering the numerous installations, the most of which have proven satisfactory, and the relatively small percentage of failures to this type of drive is due considerable credit. Some cargo ships have run as much as 300,000 miles with the gears still in good condition.

Like the reciprocating-engine drive, gear drive will undoubtedly be used indefinitely as the application of electric drive in certain classes of vessels appears, at the present time, unwarranted.

Diesel Direct Drive

Following the development of the steam turbine came that of the Diesel engine, with its high fuel economy and attendant advantages.

A large percentage of European cargo carriers are equipped with direct-connected Diesel engines, and this type of propulsion is gaining rapid headway in this country. In addition to the many American Diesel driven ships already in commission the Shipping Board has decided to equip eighteen vessels of our Merchant Marine with direct-connected Diesel engines. This type of engine, however, has its limitations in capacity although the size of units is steadily increasing. With the larger sizes the space factor is of great concern as engines rooms must be made larger to accommodate them. Twin screw direct drive has been frequently used in order to provide two units for increasing reliability and permit the higher propeller speeds required by the engines, and also to have the engine come within the present limit in capacities. In many such cases the use of one propeller would permit a much better design.

Reversing difficulties with the Diesel engine, while now not as serious as in the earlier designs, are appreciable as the design is more complicated and extra precaution is necessary with most makes of engines,

always to have available high-pressure air. When maneuvering, this feature is of appreciable concern. The most economical speed of the Diesel engine is much closer to that of the propeller than is the case with the steam turbine, but the difference is great enough to cause considerable losses. These losses are, however, not sufficient to warrant the use of gears with Diesel engine drive.

Electric Propulsion General

While the development of the horizontal steam turbine and the Diesel engine opened new eras in ship propulsion, it remained to develop some speed-reducing connecting link between these prime movers and the propellers that would allow both to be designed and run at their most efficient speeds. The mechanical reduction gear provided a very logical connecting link; however, the advent of the electric drive introduced a fully reliable system with inherent advantages. The following enumerated points, substantiating this statement, are by no means theoretical, but, on the contrary, are the results of actual observations taken over a considerable period of time and with different types of ships. Such reasons are responsible for the rapid increase in the number of electrically propelled ships.

Reliability. The universal use and the indisputable success of electrical apparatus on land is sufficient testimony in behalf of its reliability. Its application to ship propulsion presents no serious difficulty, as the deleterious affect of salt and moisture have long been recognized and are easily surmounted by proper insulation of the windings and other precautions.

Electrically propelled ships now in service have demonstrated their unquestionable reliability and ability to withstand the most rigorous climatic and sea conditions.

The electrical transmission of power from the prime mover to the propeller provides the simplest, most elastic, most flexible and probably the most reliable method of speed reduction.

Economy. When considering fuel, water, lubricants and supplies in economy turbine electric drive compares very favorably with geared turbine drive and greatly exceeds that of reciprocating engine drive.

The fuel economy of the Diesel direct or the Diesel-electric drive is admittedly very much better than that of other types of drives, and on an overall comparison there appears to be little difference between these two types, as the flexibility and intangible advantages of Diesel electric make up for the expected better fuel economy of the limited Diesel direct drive.

While turbine electric drive is apparently less economical than geared turbine drive at full power in the ships which operate at several speeds, there are decidedly superior economic results at reduced power, especially on twin or multiple screw vessels having two or more main generators, as all propellers may be operated from one generator. Also, efficiency of electrical machinery does not change with age.

Commander S. M. Robinson has pointed out that the fuel consumption of the U. S. S. *Idaho* and *Mississippi*, (direct-connected turbine drive), depending on the speed, is from 20 to 47.8 per cent more than the U. S. S. *New Mexico* and at full power is 32 per cent more. Subsequent electrically driven battleships are showing better economy than the *New Mexico.*

Weight. The total machinery weight with electric drive is much lighter than with reciprocating engine drive, but is usually slightly heavier than geared turbine installations, although on certain classes of ships it may be lighter. This is particularly true for Diesel-electric drive. This depends somewhat on the arrangement, design and foundations in the engine room. Weight is an important item, as it directly affects cargo space and displacement. In analyzing comparative weights, a fair allowance must be made for the excess weight incident to overload capacity of electrical apparatus, factor of safety and flexibility of reserve.

Space. The position of the main generator with respect to the propelling motor is not essential. Therefore, a more convenient arrangement is possible than with any non-electric type of drive.

Of course, the Diesel electric shows up most favorably in space requirement due to the absence of boilers and the use of smaller fuel tanks.

With turbine-electric drive the condensers can be located directly under the turbines thereby saving floor space. However, in most cases, turbine electric drive requires more space than geared-turbine drive, although in many cases the flexibility of location with electric machinery permits a more advantageous utilization of space and requires a smaller engine room. The number of propellers and beam of ship have a direct bearing on this feature. In the case of the 7000-ton U. S. collier *Jupiter* the weight of the electric propelling machinery is 156 tons, whereas for her sister geared turbine drive ship *Neptune*, the weight is 150 tons. For 3000-ton freighters with certain makes of double reduction gears the weight is 9 tons less than turbine electric drive, but the saving in shafting by locating the motor aft makes the electric drive lighter.

Cost. The initial cost of the propulsion equipment has a direct effect upon the earning power of the ship because of interest, depreciation and insurance charges and, therefore, is an important item.

The features entering a definite comparison in cost of the various systems are somewhat difficult to determine, but on the basis of equal performance in regard to propeller torque and speed and considering all items relative to the propelling equipment, the costs of electric-drive systems do not appreciably exceed the cost of other types of drive.

When considering that the electric drives, especially the Diesel type with its smaller engines, permit more

or less standardization of prime movers and generators, resulting in a better production and stocking basis, the cost item gives them somewhat of a preference. The cost of the propulsion machinery alone is, however, not the only important factor, as advantages of the electric drives in the saving of piping, shafting, shaft alleys, oiling systems, maintenance, etc., will more than offset the extra cost of electric machinery in many cases where other systems, on the first analysis, appear preferable.

Operation. Ordinarily, as there are no large valves to operate, only one man is required to handle the control. The motor requires little attention and even if located in a separate compartment does not require an extra man. The fear of inability to obtain proper personnel for the operation of electrically driven ships was in the beginning an anticipated obstacle, but since the *Jupiter*, *Kamoi*, *J. H. Senior*, *Poughkeepsie*, *Golden Gate* ferry, *New Mexico* and many other similar ships have been successfully operated with no trouble from this source, this objection no longer exists. It is not necessary that the operator have any more theoretical knowledge of the electrical machinery on shipboard than on land, but simply must know its operating characteristics and general care. In many cases the operators of electric drive ships have had little experience with electrical machinery and principally due to simple control they have had no trouble in properly handling it.

The Westinghouse and General Electric companies have conducted special training schools for such operators and other similar schools will undoubtedly be arranged if found necessary.

The same doubt arose concerning operators for turbine driven ships, but there has been no trouble in providing an adequate supply of them. This is also true of electric locomotives and in practically all cases the operators are so enthusiastic they would hestitate to return to the steam drive.

The omission of the boilers in the case of Diesel-electric drive correspondingly reduces the operating force with an appreciable saving in expense, especially at sea.

Electrically driven vessels will naturally attract competent engineers who will take the best care of the machinery and thus tend to realize better efficiencies. First hand knowledge of the facts will convince the doubtful ones of the value of the electric drive.

Maintenance. Electrical machinery does not wear as does other machinery, and with the absence of wearing parts, except in bearings, there is little to get out of order. Furthermore, the maintenance would naturally be less than for other forms of drive for the reason that load conditions and control operations are less severe on the machinery and propeller. As a matter of fact, the years of experience with electrically driven vessels have proved that the cost of upkeep is surprisingly small.

Performance. The inherent characteristics of electrical machines are particularly well suited to ship propulsion. Their operating characteristics are the same in either direction of rotation, and consequently, except when peculiar conditions make it undesirable to utilize this feature, afford full torque and power for reversal. The reversing provision with geared turbine drive is only from 40 to 60 per cent of the ahead power and although this is claimed to be sufficient for ordinary conditions, emergency cases frequently warrant full power astern. This is particularly true of high-speed ships. An electric drive ship can be stopped in considerably less time than a geared turbine ship. This is not due wholly to the lesser backing power of the latter, but to the inefficiency of the reversing element of the geared turbine which incidentally causes an enormous draft on the boilers, greatly reducing the steam pressure.

The ability of the electric machinery to absorb and dissipate the power delivered by the propeller is of considerable value. Furthermore, the control of the propeller with electric drive is speedier than with any other type of drive. Analysis shows that reversing at full speed is easily accomplished and does not place harmful requirements upon the electrical machinery. These characteristics make electric propulsion ideal for difficult driving, stopping and maneuvering.

Efficient Speed Reduction. Electric drive lends itself to any desired reduction between the high-speed turbine and the low-speed propeller and both may be designed to operate at a speed corresponding to their highest efficiency. While the speed reduction ratio for Diesel electric is not so great as with turbine electric drive, it permits the use of standard speed engines and the best propeller speeds. With two generating units, one will propel the vessel at approximately three-quarters speed with practically no sacrifice in economy. With direct drive, a disabled unit would necessitate an idle propeller.

Electric drive affords means of reversal by a simple change in electrical connections without changing the direction of rotation of the prime mover and any desired reversing torque, up to full torque or more, can be obtained without affecting the efficiency of the equipment in the forward direction.

Both the turbine and Diesel-electric systems are capable of developing large overload torque at reduced speed and installations to date have proved this feature to be sufficient to meet all emergencies.

Measurement of Power. Accurate and constant knowledge of the power output is always obtainable from meters on the control board. In this manner the power requirements under varying operating conditions may be ascertained and studied. Also by recording electrical meters the total power on any trip or selected part of a trip may be obtained. This data with a record of fuel consumption affords simple means of calculating economy. Further, especially with two or more screws, it permits the detection of:

a. Excessive friction in thrusts, propeller shaft bearings or stern tube bearings.

b. Incorrect machinery or wrong pitch of propeller and bent or broken propeller blades,

c. Poor steering or improper rudder action due to mechanical troubles or wave movements on account of storm or wind conditions.

With no other type of drive can such defects as above noted be quickly detected and proper remedies expeditiously applied.

High Pressure and High Temperature. As the turbines used in connection with electric drive have but one direction of rotation they readily accommodate themselves to high pressure and high temperature steam and may, therefore, take advantage of these two very important economic measures.

Flexibility of Machinery. With a plurality of generating units, the failure of one will not disable the vessel. Simply by switching, any number of the generators may be made to supply power to all motors or any may be disconnected with little handicap to the remainder of the outfit. Furthermore, if reduced speed is desired for economy, part of the generators may not be run, thus permitting the active ones to operate economically near their rated and most efficient output. This would not be true with direct-connected units, as they must all operate at the reduced and consequently uneconomical loads. This feature is particularly desirable for ferry-boats and constitutes a reserve power factor; also for long cruising periods, it is of great importance in its affect upon fuel storage. It has a decided advantage in connection with battleships. Furthermore, the plurality of engine sets makes it unnecessary to carry on board as many spare parts as with direct-drive installations.

Main Generators Supply Auxiliaries. A very outstanding feature common to many turbine electric and most Diesel-electric driven ships is the ability to use the main propulsion generators for supplying power to axuliaries used in port. The power required for handling motor-operated cargo machinery is practically always considerably more than that required for the electric auxiliaries used at sea. Therefore in the non-electric-drive ships, the electric plant must necessarily be of sufficient size to provide for port conditions. This, of course, results in appreciable idle generating capacity at sea, or otherwise the sacrifice of motor-operated port auxiliaries. As Diesel-driven ships ordinarily have electric cargo machinery and as the cost of Diesel-driven generators is considerably more than for steam engine or turbine-driven generators this feature becomes quite important. These advantages have constituted a prominent factor in deciding for electric drive in several installations.

Location of Machinery. Electric drive permits the turbines to be most conveniently located, especially with reference to the boilers reducing the steam piping to a minimum, with consequent saving in weight, steam leakage and condensation. The motors may be located near the propellers thus reducing the long and expensive shafts, shaft alley and shaft bearings.

In case of battle craft, the units may be located in the safest place, constituting important naval advantages. With any electric-drive ship the fact that the large and bulky engines or turbines do not have to be in line with the propeller shafts gives great advantage in distributing the machinery and more economically arranging them in the machinery space.

Again electric drive affords a much cleaner engine room than can be obtained with most other types of drive.

Absence of Vibration and Noise. Electrical machinery operates without vibration and practically without noise. These features are highly desirable for all classes and especially passenger vessels.

No Racing of Propellers. As the prime movers for generators have efficient governors, the tendency of the propellers to race in heavy weather is eliminated. Although it is not the present practise, this feature can also be obtained by other types of drives, providing governors are used on the propelling units. In the case of direct and geared turbine drives it is customary to install only speed limit governors.

TURBINE-ELECTRIC DRIVE

With turbine-electric drive alternating current appears most satisfactory, as the generator is inherently suitable for direct-connection to the economical high speed turbine. D-c. turbine-electric installations have, however, been successfully used in ferry-boats, but this system is not suited for very high powers because geared turbines must be used. Also, the use of alternating-current permits high voltages with less current and consequently smaller interconnecting cables.

The electrical machinery is practically the same as used in land installations, except that it is specially treated to withstand marine conditions of moisture and salt. Unlike land installations, however, no power other than that for propulsion is required of the generators and, therefore, their speed and control may be varied to suit the desired combination with the motors. With high-speed turbines and improved electrical apparatus a good transmission efficiency can be obtained. This efficiency compares very favorably with double reduction gears when considering the losses in the reversing turbine, extra shaft bearings, etc.

When starting a turbine electric-drive vessel with a-c. machinery the turbine speed is reduced and excess generator excitation applied for pulling the motor in step, after which the speed of both generator and motor increases together.

Although the use of direct-current power is not so popular with turbine electric drive as alternating-current power, the installation in the ferry-boats of the San Francisco-Oakland Terminal Railways is of considerable interest.

The propulsion machinery consists of a three stage condensing 1100-kw., 3600-rev. per min. horizontal Curtis steam turbine, connected through reduction gears to a d-c. generator rated at 1000 kw., 500 volts, 900 rev. per min., supplying power to two double armature motors rated at 1200 h. p., 500 volts, 100/125 rev. per min. One motor is coupled to the forward propeller and the other motor to the aft propeller.

A 75-kw., 115-volt, 900 rev. per min. exciter is mounted on the end of the generator shaft and supplies excitation current to the generator and motor fields, and control current for operating the contactor solenoids and relays on the control panel.

The propelling machinery is controlled from the engine room. By means of a controlling rheostat, the shunt-field current of the generator can be varied to obtain different propeller speeds or reversed to change from ahead to astern movement, or vice versa. This rheostat also controls the operation of the two main-line contactors and the two contactors, which shunt the speeding-up section of the motor-field resistor. The motor operating as the astern or driving motor has full resistance in its field circuit, while the forward motor has sufficient of its field resistance short-circuited to produce a speed just above the drag speed of the propeller. The two armatures of each motor are connected in series, while the two motors are connected in parallel and in series with the main generator.

Overload protection is provided by means of line relays opening contactors in the motor circuits. In case the overload relays function, it is necessary to move the controller to the "off" position to reset them.

For the purpose of supplying auxiliary power for lighting and various other needs of the ship, two 35-kw., 125-volt, steam-turbine d-c. generator sets are installed.

Both the induction and the synchronous type motors are being used with complete success.

Induction Motors. To date, three general types used are:

a. Form-wound with external resistance

b. Form-wound with an additional high-resistance squirrel-cage winding for starting and maneuvering.

c. Double squirrel-cage winding, one high resistance for starting and maneuvering.

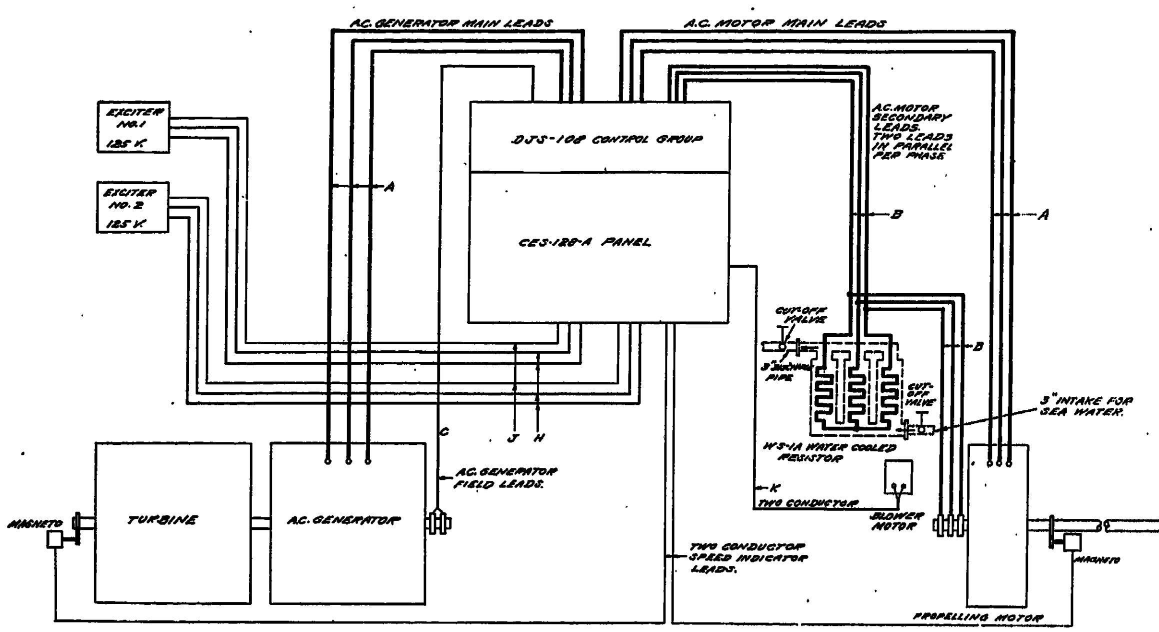

FIG. 1—CABLE CONNECTIONS FOR SINGLE SCREW BOAT PROPELLED BY INDUCTION MOTOR

	Cables	Size	Outside Diam.	Spec.
A	Single Conductor............	1,250,000 cm.	2.5	391–1
B	Two single conductors.........	1,250,000 cm.	2.5 ea.	391–1
C	Two conductor...............	200,000 cm.	1.92	15 c 1 *d*
H	Single conductor exciter.......	300,000 cm.	1.24	15 c 1 *d*
J	Single conductor ex. field......	30,000 cm.	.62	15 c 1 *d*
K	Two conductor to vent. motor	60,000 cm.	1.26	15 c 1 *d*

Type *a* is used mostly for merchant ships and also on the U. S. S. *Jupiter, Tennessee* and *Colorado;* type *b* is used on the U. S. S. *California, Maryland* and *West Virginia;* type *c* is used on the U. S. S. *New Mexico.*

By providing the motors with pole-changing windings, an additional means is afforded for a different speed ratio between the generator and propeller, thus utilizing, in a most economical manner, the full output of one generator at reduced propeller speeds. This feature, to date, has been used only on battleships

where an economical cruising condition is of great importance.

The concatenation of motors, principally where two per shaft are used, is another way of obtaining a further speed ratio between generators and propellers. This method was contemplated on some of the battle cruisers which were cancelled by the Limitation of Arms Conference.

Typical merchant ship installations with induction induction motor, which is directly connected to the propeller shaft. The reduction ratio between the turbine and shaft speed is 30:1.

The propelling motor has a motor-driven blower, while the generator is self-ventilated. The air from both generator and motor is led away through ducts.

Two separate 125-volt exciters supply current for generator field excitation and the motor-driven blower. For operating the motor, in conjunction with the

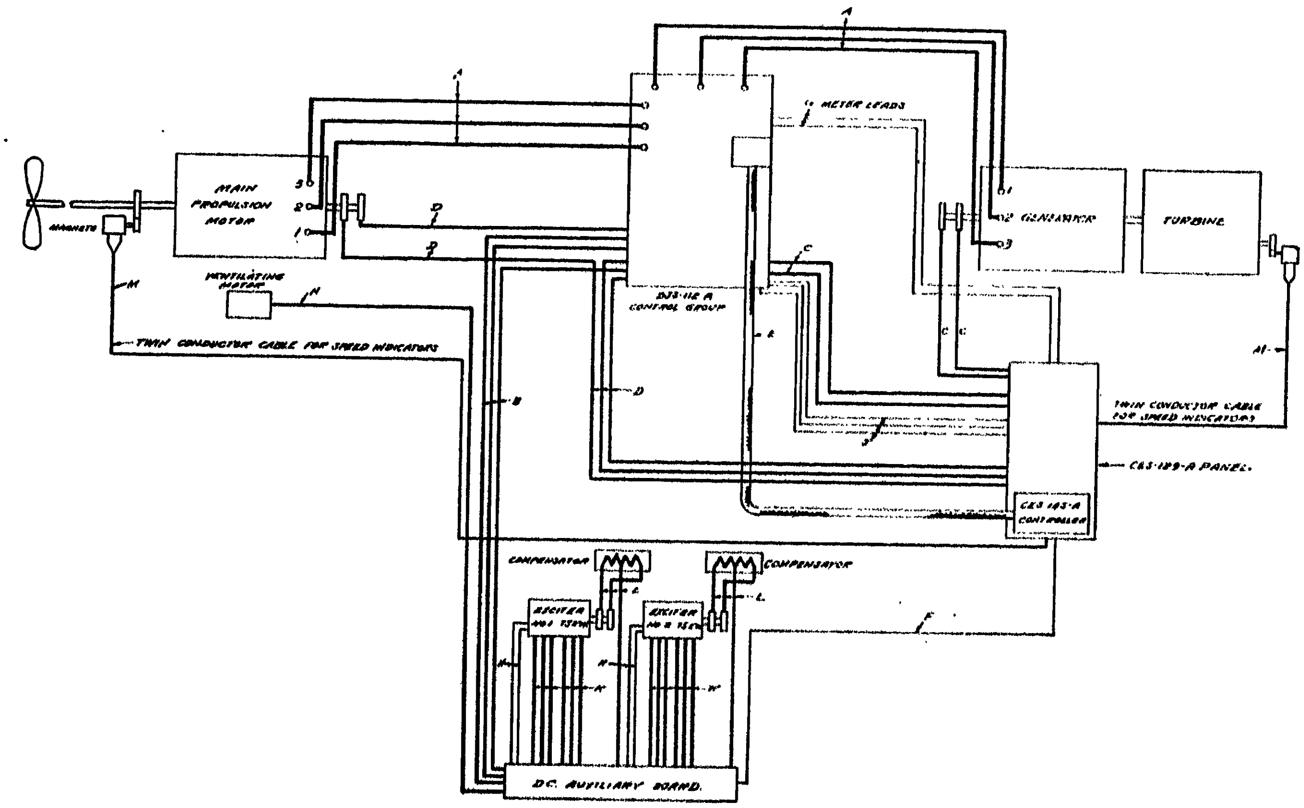

Fig. 2—Cable Connections for Single Screw Ship Propelled by Synchronous Motor

Symbol	Location	Volts	Size	Overall Diam.	Spec
A	M in A. C. cable	2300	650,000 cm.	1.67″	435
B	Main d. c. cable	250	400,000 cm.	1.4″	15 *CID*
C	Gen. field cable	250	200,000 cm.	1.07″	15 *CID*
D	Motor field cable	250	400,000 cm.	1.4″	15 *CID*
E	Control cable	125	2-9con.11,000 cm.	1.32″ea.	15 *CID*
F	Control supply	125	2 cond. 30,000 cm.	1.05″	15 *CID*
G	Meter leads	125	3 cond. 11,000 cm.	.87″	15 *CID*
H	Disch. res. & exc. sh. fld.	125	30,000 cm.	.62″	15 *CID*
J	Motor shunt leads	125	2 cond.	.8″	15 *CID*
K	Exciter pos. & neg.	250	600,000 cm.	1.6″	15 *CID*
L	Neutral	250	200,000 cm.	1.07″	17 *CID*
M	Tachometer				
N	Vent. motor	125	2 cond. 60,000 cm.	1.258″	15 *CID*

Specify varnished cambric insulation, leaded and armored for all conductors except *A* & *M*.

motors will be found in the Shipping Board vessels *Eclipse, Invincible, Archer, Independence* and *Victorious.* The propelling machinery for each of these vessels consists of an eight-stage, 3000 rev. per min., horizontal steam turbine direct-connected to a 2300-volt, two-pole, three-phase, a-c. generator, supplying a 3000-h.p., 2300-volt, 100-rev. per min., three-phase, 60-pole turbine generator, a control equipment is installed consisting of a water-cooled resistor and a combined control group and control panel.

The motor is started, stopped and reversed by means of the high- and low-voltage contactors of the control group. In starting from rest or reversing normally, the water-cooled resistor is inserted in the motor form

wound rotor circuit. When the motor is nearly up to speed in either direction, the resistor is short-circuited.

The motor speed is varied by varying the generator frequency which is accomplished by changing the turbine speed. The turbine main hydraulic operated governor will regulate its speed from 20 per cent to 110 per cent.

The *Eclipse* was the first electrically driven merchant ship to circumnavigate the globe. She made the trip of 29,763 miles at an average speed of 10.3 knots, making twenty ports without one stop at sea.

Synchronous Motors. For ship propulsion, synchronous motors are provided with a high-resistance squirrel-cage winding, (amortisseur), embedded in the pole face of the d-c. field poles. This winding is used for starting and maneuvering and where thus far applied, provides sufficient torque for all conditions.

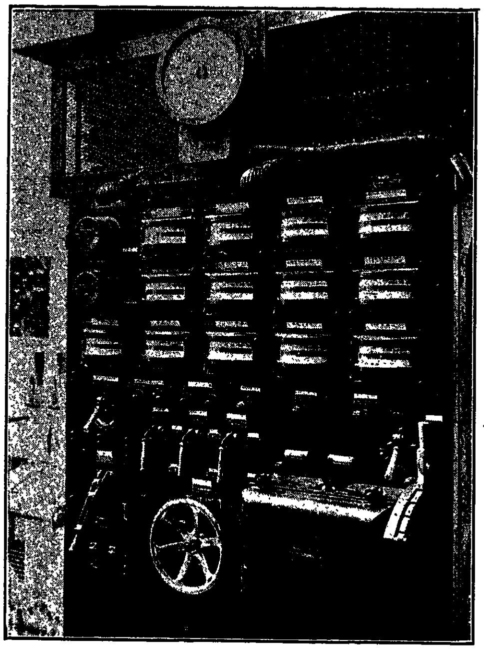

FIG. 3—PROPULSION CONTROL AND PANEL BOARD, JAPANESE NAVY FUEL SHIP *Kamoi*

After being brought up to speed as an induction motor, direct-current field is applied and the motor then runs in synchronism.

The Japanese fuel oil vessel *Kamoi* is a typical installation of synchronous-motor drive. The propulsion machinery consists of a 10-stage, 2400-rev. per min., horizontal Curtis turbine direct-connected to a 6250-kw., 2300-volt, 40-cycle, three-phase a-c. generator supplying power to two 3125-kw., 4000-s. h. p., 2300-volt, 120-rev. per min., three-phase synchoronous motors direct-connected to the propeller shafts.

The motors are open type and have motor-driven exhaust fans for ventilation; the generator is enclosed and self-ventilating. The air from both the generator and motors is led away through ducts. Temperature detecting equipments are provided for the generator and motors.

Two 400-kw., 112/225-volt, 5000/1100-rev. per min. turbine driven, three-wire exciter sets supply the necessary excitation current for the fields of the generator and motors and also power for the blower motors and ships auxiliaries. In addition, a 625 kw., 750-volt, 1100 rev. per min., a-c. generator is placed in line with the outboard exciter and in case of emergency can be coupled to the exciter shaft and used to supply line current to the propelling motors sufficient to bring the ship into port at a speed of about seven knots.

The control equipment consisting of two control groups and a control panel is installed for operating the motors in connection with the main turbine generator or auxiliary a-c. generator.

The motor is started, stopped and reversed by means of levers on the control panel, which electrically operate the line and field contactors in the proper sequence for the desired operating condition. These contactors can also be closed manually by means of levers at the side of the control panel.

In starting from rest, double field current is applied to the generator and the motors are operated as induction motors until nearly up to speed, at which time current is supplied to their d-c. fields, when the motors operate as synchronous motors. The generator field current is then reduced to normal. In reversing, the main operating lever is moved to the "off" position, the reversing lever set to the astern position and the main lever advanced as when starting ahead from rest. The motors are thus operated as induction motors bringing the propeller to rest and starting in the reverse direction until nearly up to speed, when d-c. field is applied for operation as synchronous motors.

The motor and propeller speed is varied by changing the generator frequency, which is accomplished by changing the turbine speed by means of its hydraulic operating governor mechanism.

The motor can be operated continuously at about 50 rev. per. min., in either direction as an induction motor, if necessary.

The Coast Guard Cutters *Modoc*, and others, also have synchronous motor equipments and similar control to that of the *Kamoi*, although they have a slightly different maneuvering arrangement. The *San Benito*, for which the propelling machinery was supplied by the British Thompson Houston Company, is also a good example of synchronous motor drive.

DIESEL-ELECTRIC DRIVE

The inherent advantages of this system indicate remarkable possibilities in the development of ship propelling machinery. The several ships now equipped with machinery of this type have shown most creditable performance and have lived up to every expectation. Although this method is relatively new, the constituent parts have been well established.

Diesel Engines. Any reliable make of engine which operates at a reasonably high rotative speed may be used. Engines for electric drive need not exceed established safe piston speeds for continuous operation. By using several cylinders of short stroke and small bore the heat stresses common to large cylinder slow speed engines are minimized and the result should be an engine of less maintenance and simpler construction. As the engine always starts under no load and runs in thus, in some cases, more than offsetting the added weight of the electrical machinery and resulting in an overall machinery weight probably less than for certain other types of drive.

The field of application of the Diesel engine to ship propulsion is greatly enlarged by electric drive, as several available units may be used in one installation whereas direct drive is somewhat limited by the present limit in sizes of most makes.

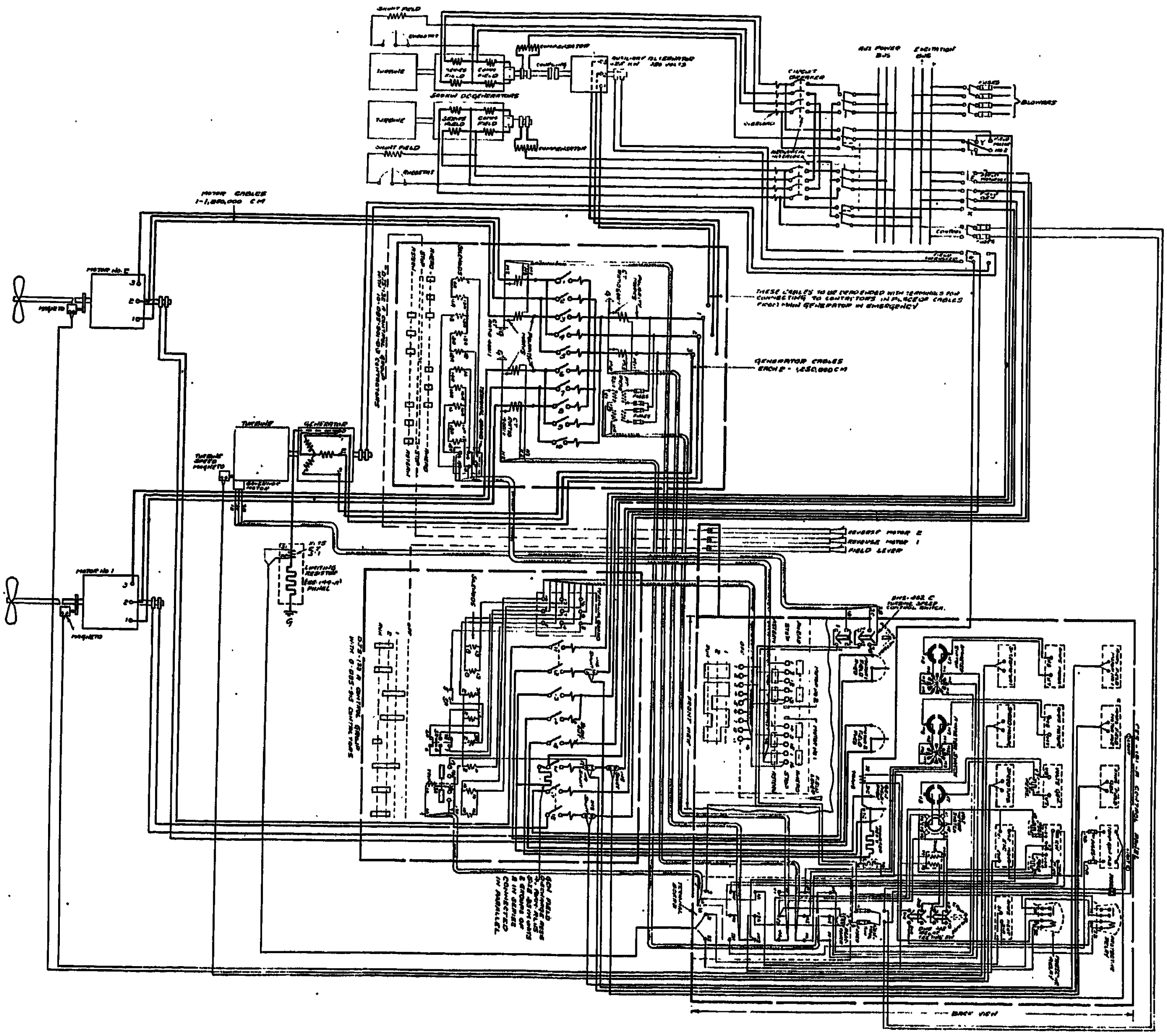

FIG. 4—CONNECTIONS FOR JAPANESE NAVY FUEL OIL SHIP *Kamoi*

the same direction and at constant speed, the design is further simplified by the omission of the reversing gear; also the large supply of high-pressure maneuvering air is reduced to a minimum. If desired, only one air-starting is necessary, as the remaining engines may be started by their connected generators running as motors supplied by power from the first generator.

In permitting high rotative speed engines, Diesel-electric drive rapidly reduces the weight per b. h. p.,

Selection of Power. Direct-current machinery is preferable as it permits of the generators being connected in series. Parallel connection of d-c. generators is practicable, but requires close governing and sacrifices other desired features of series operation.

While a-c. machinery could be used, especially with one generator and one motor, it is objectionable in that the propeller speed must be varied by changing the engine speed. Furthermore, an installation with

more than one generator would require parallel operation and close governing.

Therefore, the direct-current system described is, decidedly simpler, more flexible and more reliable, affording more refined control and providing greater power in case of casualty to a generating unit.

Control. With the d-c. system, motor speed and maneuvering may be controlled by armature rheostatic means. However, this method is wasteful during maneuvering and speed changes and necessitates a complicated controller.

The Ward Leonard System of voltage control is by far the most satisfactory and has been used in practically all installations to date. With this system, both the generators and motors are usually of the shunt type, but in some cases have differential field windings and are separately excited preferably from the same source. The exciters may be direct-connected to the main units, or separate. Also, excitation current may be taken from the auxiliary set, as two sources of excitation should be supplied. The generators and motors may or may not have forced ventilation depending upon the speed and size and the desirability to keep size of machines to a minimum.

The motor fields are excited with constant potential and always in the same direction. The generator field excitation is varied to suit the motor speed and direction. The motor speed under this condition varies in direct proportion to the voltage applied to the motor terminals and may be reversed by reversing the generator excitation; consequently it is only required to vary the generator fields from full excitation in one direction to full excitation in the opposite direction to maneuver from full speed ahead to full speed astern.

With the series connection of generators, like speeds for all engines is unnecessary as a variation simply produces a proportionate difference in the load carried by the generators, provided they are equally excited and have identical performance characteristics. Since it is necessary to handle only the comparatively low generator field excitation currents, for complete control ahead or astern the Ward Leonard system is obviously much simpler and less expensive than the armature rheostatic control. Such simplicity has a further direct affect upon the maintenance of the equipment.

The control board contains suitable switches, instruments, meters, protective relays, circuit breakers, etc., and a special reversing field rheostat and a remote control mechanism for operating the propulsion generator rheostat. This mechanism has an inherent time element for limiting the speed of operation for the safety of the electrical machinery. Such a speed limit, however, does not handicap the operation of the ship as only about five seconds are required from full-speed ahead to the stop position.

The switches for the machines are so arranged that any particular generator or motor may be taken out of service by simply throwing its switch from one position to another and this can be done without interrupting service to the propelling motor. In large installations this is accomplished by a mechanically operated "set-up" device.

With this type of drive provision is usually made for both pilot house and engine room control with a double throw switch on the main switchboard for transferring to the desired station. The control pedestal in the pilot house is arranged for remote generator field control and has the necessary ammeter, voltmeter and speed indicators to provide constant knowledge of the performance of the propelling machinery. Pilot house control is, to date, practical with Diesel-electric drive only, and is of great convenience especially for tug boats and other types of vessels when maneuvering in close waters. No time is lost or mistakes made in transmitting orders.

Overload protection for the generators and motors is provided in the form of line circuit breakers or line relays for opening the generator field in case of severe overload. Necessary interlocks and precautions are taken to insure proper operation and avoid mistakes.

Fuel Economy. A properly designed Diesel-electric propulsive equipment requires about 0.55 lb. of oil per shaft h. p. for all purposes, whereas the average high-grade steam installation of any type requires about 1.0 lb. of oil per shaft h. p. This superiority results in a great reduction in fuel cost and permits additional cargo due to decreased weight or an increased cruising radius for a given amount of fuel. The economy of either type of Diesel drive is far ahead of that of all other types and is, therefore, most attractive for the moderate and medium large size installation.

The Diesel engine system can be made ready for sailing on short notice and stand-by losses are eliminated. This is not true of the steam system which requires that the boilers, turbines, etc., be warmed up for a considerable period before getting underway.

Flexibility and Reliability. By providing a reasonable number of small generating units and using the Ward Leonard system of control with series operation of generators, the reliability of Diesel-electric drive is superior to that of any direct-connected drive, mechanically-geared drive, or any form of electric drive using single generating equipment. The smaller Diesel engines have probably been developed to a more reliable state than the larger sizes. Furthermore as the driving power varies as the cube of the speed a casualty to one of several generating units does not greatly impair the ability of the vessel to proceed. With a three generator installation the disabling of one engine set would reduce the ship speed to 88 per cent and the failure of two would reduce it to approximately 70 per cent of full speed. The only change in connections is the disconnecting of the disabled units and weakening the motor field to take the full load armature current. The faulty unit or units being of the comparatively small size may usually be repaired at sea. No other type of drive

will permit the utilization of the full power of the remaining units without overloading them or increasing the original size and weight. The reserve power thus obtained is far in excess of any other type of drive, including a-c. turbine electric. With a cross compound geared turbine single screw drive the failure of one element would give appreciably less than 50 per cent power from the remaining element and this would be with a sacrifice of considerable speed and economy.

As the generating units are independent of the propeller they may be located to best suit the engine room arrangement and being of the smaller type, lend more readily to the better utilization of available space.

Many Diesel electric installations include a double unit motor consisting of two armatures on the same shaft. This type of motor permits smaller diameter, added reliability and reserve power as one armature may be disconnected and the ship may still proceed with the remaining armature at approximately 70 per cent speed. The superiority of this feature to that of any other single screw drive is obvious. In the case of long cruises at reduced power this flexibility of units permits ideal conditions for economical operation. Any or all generators and motor armatures may be used in any desired combination by simple switching. Furthermore, the main generators may be used for operating cargo oil pumps, winches, etc., thus reducing the size of auxiliary generators to a minimum and only sufficient for port duty when a main generator is not running. For most ships this avoids considerable idle machinery while underway.

The Diesel-electric type of propulsion with the Ward Leonard system of control affords excellent maneuvering characteristics. When the generator field circuit is removed preparatory to reversing, or at any other time, the braking effect of the generator armature connected across the motor terminals brings the propeller almost instantly to a creeping speed, the energy being dissipated partly in friction, due to the motor being driven by the propeller and supplying power to the generator which, in turn, tends to drive the engine, and partly as heat in the motor and generator windings. The performance of the *J. H. Senior* and other Diesel electric vessels shows that the power delivered by the generator is not sufficient to cause any perceptible increase in engine speed and that the demand for maneuvering can be met without exceeding the permissible temperatures of the electrical equipment.

With multiple-screw vessels there is considerable increase in load on the propellers on the inboard side of a turn. This feature naturally tends to overload the prime mover and for turbine electric drive with alternating-current machinery is taken care of by limiting the steam supply to the turbine in order to keep a stable condition between the generators and motors. The inherent characteristics of the d-c. generators and motors used with Diesel-electric drive, however, are such that they will carry this increased torque without the least danger of becoming unstable. No special precaution need therefore be taken as there is always a stable coupling between the generators and motors.

Applications. Because of the attractive advantages previously described Diesel-electric drive is particularly applicable to barges, cable ships, cargo ships, coastwise vessels, dredges, ferry boats, fire boats, fishing boats, lake boats, packets, river boats, tow boats, tug boats, yachts and any other requiring economy and good maneuvering and performance characteristics over a wide range of speed.

The following table indicates those advantages offered by Diesel electric drive which are of particular importance in the various types of ships:

TABLE I
PRINCIPAL APPLICATION ADVANTAGES
MAIN MACHINERY ONLY

Type of Ship	Fuel Consumption	Machinery weight	Refined Control	Cruising Radius	Reliability	Reserve Power and Flexibility	Port Consumption*
Barges	×	×	×	×	×	×	
Cable Ships	×	×	×	×	×		×
Cargo Ships	×	×		×	×		×
Certain Naval Craft	×	×	×	×	×	×	×
Coastwire Vessels		×		×	×		
Dredge-Self propelled	×	×	×	×	×	×	
Ferry Boats	×		×	×	×	×	
Fire Boats		×	×	×	×	×	
Fishing Boats	×			×	×		
Lake Boats	×	×		×	×	×	
Liners	×			×	×		
Light Ships			×	×		×	× †
Packets	×	×	×	×	×	×	
River Boats	×	×	×	×	×	×	
Tow Boats	×		×	×	×	×	
Tug Boats	×		×	×		×	
Yachts	×		×	×	×	×	

*Principally stand-by losses.
†While on duty.

In order to give a fair idea of the machinery pertaining to the propulsion outfit of typical Diesel-electric installations the following brief synopsis is offered.

Yachts

The three-masted schooner yacht *Alcyone* was originally fitted with reciprocating engines, but upon learning of Diesel electric drive with its cleanliness, absence of boilers and smoke, less vibration and numerous other advantages previously described, its owner, Henry W. Putnam, decided to change over to electric propulsion, using the same propeller and shafting. The *Alcyone* is 182 feet long overall, with a beam of 30 feet and draft of 16.2 feet, and is registered 420 gross tons. The propelling plant consists of two six-cylinder, 225-h. p., 250-rev. per min. Winton Diesel engine, each direct-connected to a Westinghouse 140-kw., 125-volt, shunt-wound, d-c. generator and a 15-kw., 125-volt, compound-wound exciter, chain-driven, at 900 rev. per min. from the generator shaft.

The propelling motor is of the double armature,

shunt-wound, self-ventilated type rated 350 h. p., 250 volts, 175 rev. per min. and is located well aft.

The control is arranged for the Ward Leonard system which provides all desirable features of flexibility, simplicity and ease of operation. Thirty economical speeds ahead or astern are afforded by the reversing rheostat located on the engine room switchboard and controlled by a handwheel.

At 8.5 knots the *Alcyone* has a cruising radius of 6450 miles on one engine and at her rated speed of 10.5 knots she has a cruising radius of 3915 miles on two engines.

The sea trials of this yacht more than met expectations and showed considerable reserve power for emergency. During maneuvering she could be stopped from full speed ahead in less than twice her length.

Several other yachts, some of which have control

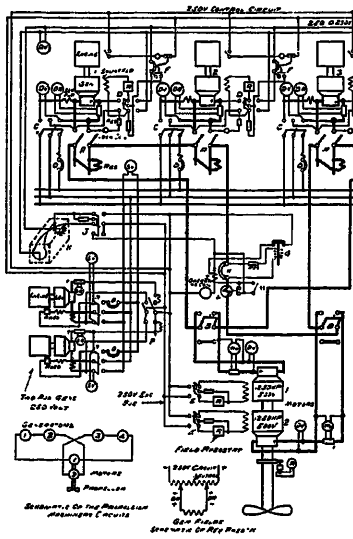

FIG. 5—CONTROL DIAGRAM FOR A 2500-S. H. P. DIESEL-ELECTRIC DRIVE

APPARATUS KEY

A—Generator cut-out switches with electric lock
B—Motor cut-out switches
C—Generator aux. bus switches
D—Generator Field Discharge switches
E—Motor field Discharge switches
F—Engine-failure-trip field circuit breaker
G—Voltage interlock relay for locking-out main circuit breaker
H—Contactor type circuit breaker
J—Field discharge switch for reversing field rheostat
K—Reversing field rheostat
L—Overload Inverse time limit relay
M—Switch for Making O. L. relay inoperative
N—Main switches for the auxiliary Generators
O—Oitl. circuit breakers
P—Transfer switch for Excitation and control circuits
S V and *S A*—Single reading voltmeters and ammeters
D V and *D A*—Double reading voltmeters and ammeters
W H.—Watthour meter
R—Magneto-voltmeter shaft revolution indicator

from the pilot house, have been equipped with electric drive as will be noted from the tabulation of installations. The popularity of this system of propulsion is most certain to grow in the yachting field.

TUGS

The numerous and severe requirements of operation and maneuvering of tug boats presents an attractive field for the application of Diesel-electric drive with its valuable feature of pilot house control.

The Pennsylvania Railroad, after careful study of the advantages afforded, has installed Diesel-electric propelling machinery in one of their many tugs. This action, however, was not taken until they had tried out Diesel direct drive and found that it did not develop the power expected principally due to the small diameter propeller and comparatively high-speed engine.

The *Float* type of tug comprising about 80 per cent of all the railroad tugs was chosen for this installation. This tug is 105 ft. long with a beam of 24 ft. and a draft of 12 ft. 5 in. Observations showed that an average of 566 signals were given and answered on this class of tug every eight hours. Naturally pilot-house control with its safer and speedier operation was selected.

Two Winton, full Diesel, four-cycle engines are each direct-connected to a Westinghouse 235-kw., 250-volt, 260 rev. per min. shunt-wound generator with an attached 25 kw. compound-wound exciter. These exciters also furnish all other electric power needed

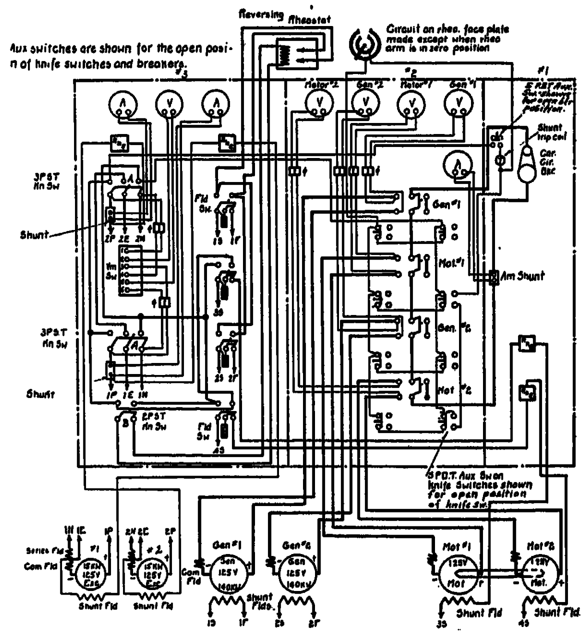

FIG. 6—DIAGRAM OF CONNECTIONS FOR *Alcyone*

on the boat. The propelling motor is of the shunt-wound double armature type rated at 575 h. p., 500 volts, 125 rev. per min. The Ward Leonard system of control and series generator operation is provided.

In addition to the generator voltage-control system, the motor is provided with a certain amount of speed variation by field control, the purpose of which is to enable the motor to operate at a lower speed when towing than when running light, without change of generator voltage. This feature enables the generators to operate at constant voltage when the tug is under full headway, and, in turn, enables the engines to develop full power when the boat is towing and when it

is running light. This is a primary advantage over the Diesel direct system for tug boats and constitutes a marked handicap over any direct connected or geared system and any form of electric drive using other than d-c. machines.

Tests proved the superiority of Diesel electric over Diesel direct drive in acceleration, greater speed and greater tow line stress at the same power. The Diesel direct drive tug with which the electric tug was compared required 15 per cent more power on its propeller shaft when running free and 26 per cent more power when towing than the electric tug. This is probably due to the propeller design. Even with the 16 per cent loss between the engine and shaft, due to the use of generators and motor there is a net gain of 18 per cent over the direct drive on the basis above mentioned. With Diesel direct drive the stress on the tow line was 32 per cent while with electric drive it was 41 per cent of the power delivered to the screw. The electric tug is lighter than an equivalent steam engine driven tug using Scotch boilers by a ratio of 165 to 177 tons and requires one man less to operate it. The fuel cost for eight hours is $12.96 for electric and $23.45 for steam drive. This with the reduced personnel and other advantages results in a saving of 24 per cent in operating expenses over the steam drive.

The further advantages of advancing toward a pier slowly or quickly with assurance of control, elimination of danger of mistakes in receiving orders, smooth taking-up on tow line relieving excessive strains, full power always available, ability to operate on one engine, avoidance of delay caused by condition of fires, quality of coal and inefficiency of firemen, appear to put electric drive far ahead of steam drive for tug boats. Also the time to take on coal and water and remove ashes for a steam tug requires two hours in 24, while the electric tug requires only two hours a week for taking on oil; less physical work is required of the machinery force and a better grade of men is attracted toward it. Navigators are greatly pleased to control propellers which, incidentally, require less thinking and less work than pulling bell signals.

The tug and fire boats *Van Dyke I*, *Van Dyke II* and *Van Dyke III*, equipped with General Electric machinery, are also rendering excellent service and exceeding all expectations.

These tugs are owned by the Atlantic Refining Company. They are 97 ft. long with a beam of 21 ft. and depth of 11 ft. 6 in.

The propelling machinery of each tug boat consists of two three-cylinder, 225-b. h. p., 257-rev. per min., Price Rathbun Airless Injection Diesel engines manufactured by the Ingersoll Rand Company, each direct-connected to a d-c. shunt-wound generator, rated at 155 kw., 125-volts with a 26-kw., 125-volt, compound-wound auxiliary generator mounted on the main generator shaft extension.

Each auxiliary generator is capable of supplying the excitation for the main generators and propelling motor and power for the electrically driven auxiliaries and lights.

The propelling motor is shunt-wound, rated at 375 s. h. p. at 250 volts. By adjustment of the motor field rheostat this motor will deliver 375 h. p. constantly at 240 volts, from 140 rev. per min. down to 107 rev. per min. with a corresponding increase in torque. This means that at reduced speeds the full output of one engine can be utilized on the propelling motor without exceeding the normal rated current thereby increasing the overall efficiency with one engine shut down.

There are two control stands similar in appearance to an engine room telegraph, one located on either side of the pilot house. The levers are interlocked so that either stand may be used.

This same system of Ward Leonard pilot house control was used in the Chicago fire boats, *Joseph Medill* and *Graeme Stewart*, placed in service in 1908.

The *Van Dyke* tugs are also used as fire and power supply boats.

In going to a fire, both main units may be used for propulsion and, after arriving at the fire, one main unit is used for maneuvering the ship and the other main unit is used for operating the fire pump.

Terminals have been brought out on either side of the engine room so that the tug boats may be used to furnish power to barges with motor-operated pumps.

Detail comparative tests have been conducted between one of these Diesel-electric tugs and a Diesel direct drive tug of similar characteristics. The results were very similar to those obtained with the Pennsylvania Diesel-electric tug previously described and show that Diesel-electric drive is superior in every way as compared to the Diesel direct drive for tow boat work, whether it be running light, towing, accelerating a tow or maneuvering. A more detailed report of these tests will probably appear in the press at a later date.

Ferry Boats

A particularly good application for Diesel-electric drive is found in the ordinary ferry boat. The ability to have complete control of the propelling motors from either pilot house is a highly desirable feature for a boat operating in a crowded harbor and docking so frequently.

The *Golden Gate* and *Golden West* ferries are typical of ferry installations to date. The electric propulsion machinery in each of these boats consist of two Pacific Diesel Engine Companies' engines driving General Electric shunt wound generators rated 360 kw., 250 volts, 225 rev. per min., each with a direct-connected, compound-wound 35-kw., 115-volt exciter. These exciters also supply all auxiliary power required for the boat.

For supplying power when the main engines are shut down a 25-kw., 125-volt, 950-rev. per min., Diesel engine-driven auxiliary generator is installed. The two pro-

pelling motors are rated at 750 h. p., 500 volts, 145/180 rev. per min. The higher speed is used on the stern driving motor. The lower speed is used on the forward motor and is just sufficient to relieve any dragging load caused by its connected propeller. The generators are connected in series and the motors in parallel. One generator will drive the after motor at 125 rev. per min.

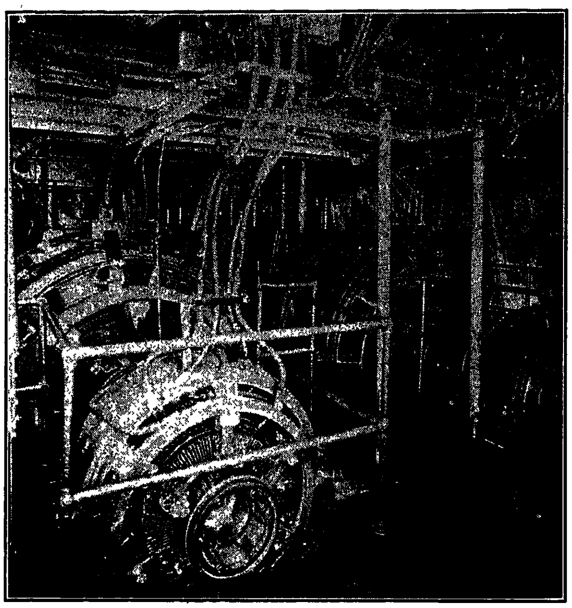

FIG. 7—GENERATORS AND DIRECT CONNECTED EXCITERS ON FERRY-BOAT *Golden Gate*

and this connection is used much of the time when the higher speed is not required to meet schedule.

The voltage of the propelling generator is varied to obtain the several propeller speeds. Sufficient instru-

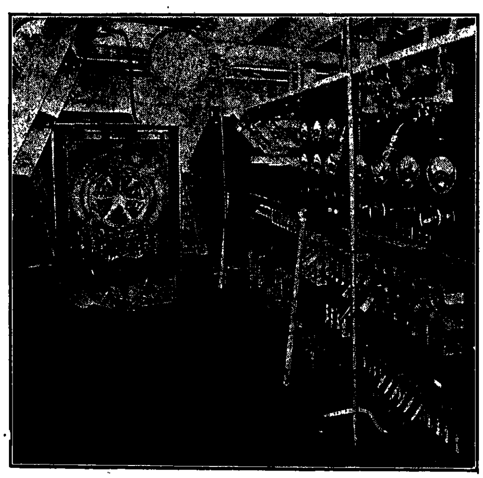

FIG. 8—ENGINE ROOM CONTROLLING RHEOSTAT CONTROL PANEL AND SWITCHBOARD, ON FERRY-BOAT *Golden Gate*

ments are provided with the control pedestal in each pilot house for close observance of the machinery operation.

The control panel in the engine room contains sufficient interlocked switches for disconnecting the main and field circuits and selecting operating stations; also sufficient field rheostats and instruments and the necessary contactors for controlling the armatures and fields in proper sequence. Pilot-house control permits the

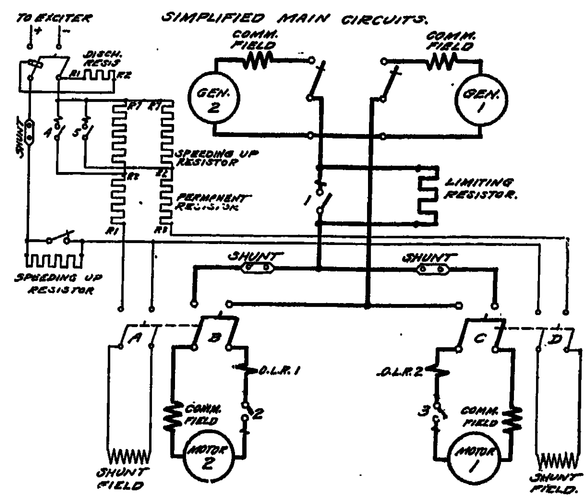

FIG. 9—SIMPLIFIED WIRING DIAGRAM, FERRY-BOAT *Golden Gate*

boat to be steered into the slip instead of being "jockeyed" in. These contactors and the generator field rheostats are controlled from the pilot house. The speed of the forward motor is reduced by short circuiting a step of resistance in its field circuit, the short circuit being removed when the motor becomes the stern motor. These ferries are giving splendid service.

FIG. 10—DIESEL-ELECTRIC MOTOR SHIP *Twin Ports*, MINNESOTA-ATLANTIC TRANSIT CO., STARBOARD SIDE OF ENGINE ROOM LOOKING FORWARD, SHOWING ONE OF THE MAIN GENERATORS CONTROL PANEL AND DISTRIBUTION BOARD

The ferry-boat, *Poughkeepsie*, is equipped with Westinghouse machinery and is also giving good performance. Several other Diesel-electric drive ferries are contemplated and in general this type of drive should show rapid progress for ferry installations.

CARGO BOATS

The application of Diesel-electric drive for the *Twin Cities* and *Twin Ports* will be of particular interest to

those interested in inland water ways. They are owned by the McDougall Terminal Warehouse Company and operate principally on the Great Lakes and New York State Barge Canal between Duluth and New York, but are also designed for coastwise trade from New York to Florida during the winter.

The propelling machinery of each consists of two Lombard Governor Company's Diesel oil engines, each direct-connected to a General Electric 250 kw., 230-volt, 260-rev. per min. compound-wound, self-excited, open-type generator. There is also a 40-kw., 230-volt, 360 rev. per min. auxiliary generator driven by a Diesel engine of the above manufacture. This generator may be operated in parallel with the 250-kw. generators for supplying power to the propelling motors and ships' auxiliaries and lighting when under way, but is normally used for supplying ships' power when the boat is at rest and the other sets are not running.

The two propelling motors per ship are each rated at 250 h. p., 230 volts, 180 rev. per min. and are of the separately ventilated shunt wound type.

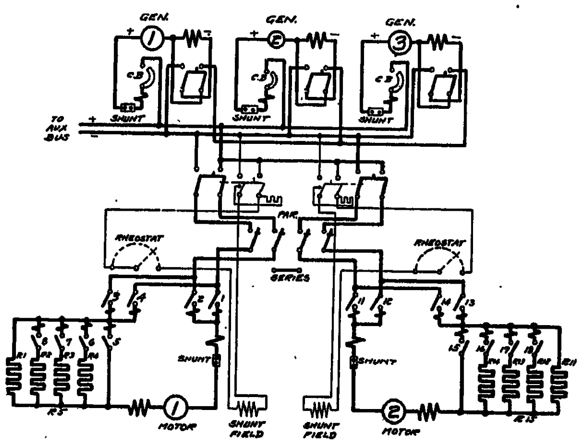

FIG. 11—SIMPLIFIED WIRING DIAGRAM, MAIN CIRCUITS, *Twin Cities* AND *Twin Ports*

The propulsion generators run at constant speed and voltage and the motors are started by the resistance method. Both the generators and motors are run in para lel for normal operation but the motors connected in series across 230 volts give a speed of approximately 135 rev. per min. on each motor. Sufficient flexibility of operation is therefore obtained for a vessel in this class of service. The propellers may be controlled from a deck type controller located aft and another located forward. The master controller is located in the pilot house.

The main control equipment is designed to control the operation of the three Diesel engine driven generators and the two propulsion motors under both normal and emergency conditions. It consists of an engine room control panel, a motor room panel, two motor control groups, two motor starting resistors, and the pilot house and deck controllers previously mentioned. Sufficient instruments, switches, field rheostats, starting and reversing contactors, protective devices, etc., for the complete control of all apparatus and the isolation of units are included.

It will be noted that the control for these boats is unlike that of most Diesel-electric ships, as the Ward Leonard system of generator voltage control is not employed. This simply goes to show the wide range of control of electrical machinery and the need of study for each particular installation. The *Mariner* and *Fordonian* have the same type of control as these two ships.

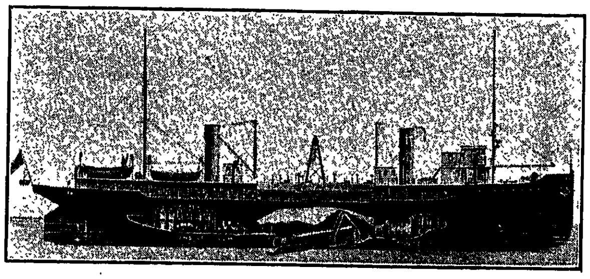

FIG. 12—THE ALL-ELECTRIC HYDRAULIC TYPE, SEA-GOING HOPPER DREDGE *A. acKenzie*

DREDGES

The ability of using the generators interchangeably for propulsion and dredging make Diesel-electric drive economically applicable for dredge propulsion.

U. S. Engineer Corps Hopper Dredges. The four U. S. Engineer Corps' hopper dredges, the first of which completed was the *A. MacKenzie*, are each 268 ft. long with a beam of 46 ft. and depth of 22 ft. They each have a capacity of 1250 cu. yd.

FIG. 13—PILOT HOUSE CONTROL STATION, SEA-GOING HOPPER DREDGES

The machinery installation in these dredges is particularly interesting and includes the largest Diesel electric plant ever installed in a ship. Each vessel is fitted with three main six-cylinder, four-cycle, air-injection, cross-head-piston, 1000-b. h. p., McIntosh & Seymore Diesel engines, each direct-connected to a Westinghouse 700- kw., 500-volt, 150-rev. per min., d-c. generator. Any two of the main generators supply power to the two 800-h. p., 480-volt, 110-rev. per min.,

double-armature propelling motors, which are also capable of developing full power at 90 rev. per min. for dredging speed. The third main generator supplies power to an 800-h. p., 480-volt, 135/160-rev. per min. single-armature motor driving the dredging pump.

generators. A 25-kw., 250-volt, gasoline engine driven set is also installed on the upper deck. Two balancer sets each consisting of two 5.5-kw., 115-volt generators are installed for lights and 115-volt motors.

A very interesting feature in this installation is the

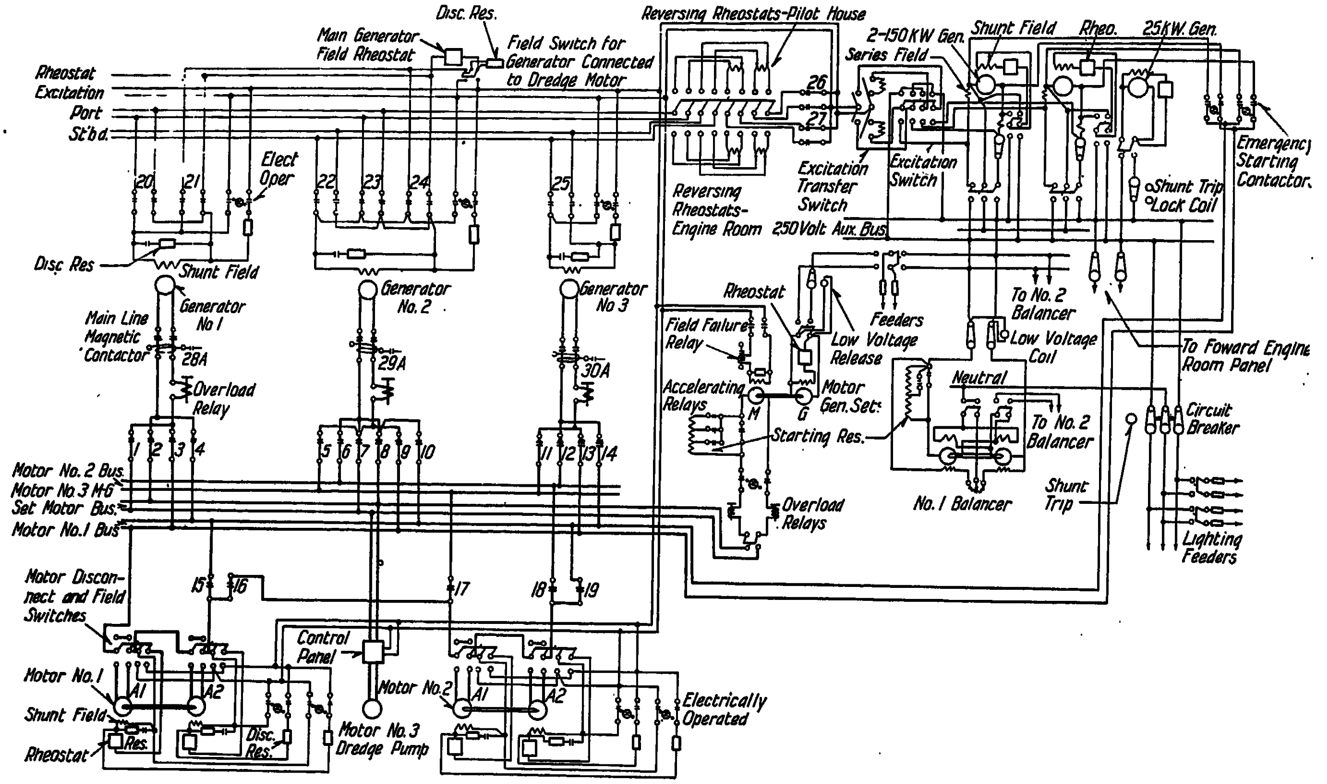

Fig. 14—Schematic Control Diagram of Main Circuits, Hopper Dredges

The generators and motors are totally enclosed and have forced ventilation.

In addition to the main power plant there are two Diesel engine-driven generating sets each consisting of a 225-h. p. four-cylinder, four-cycle, trunk-piston, air-injection type engine direct-connected to a 150-kw., 250-volt, 275-rev. per min. generator. These sets are used for excitation and auxiliaries on the vessel.

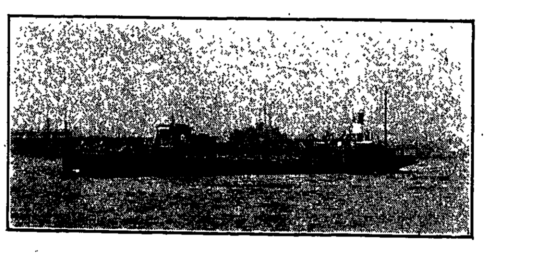

Fig. 15—S. O. Co. of New Jersey, Tanker *J. H. Senior*

There is also a motor generator set consisting of a 150-kw., 250-volt, generator driven by a 225-h. p., 480-volt, 1200-rev. per min. motor supplied from the main generator which supplies the dredge pump motor. This set is used for supplying the 150-h. p. jet-pump motor and other ship auxiliaries when desired, although normally they are supplied from the auxiliary 150-kw. control of the propelling and dredging machinery Eight combinations of connections are arranged in the hand-operated control switch group forming part of the main switchboard in the after engine room. Adequate

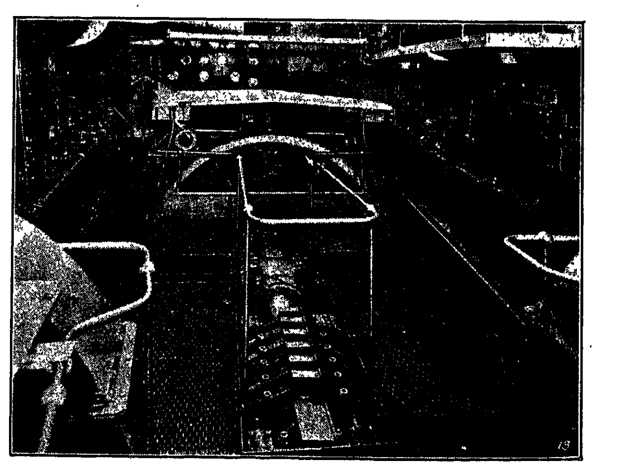

Fig. 16—Main Propelling Motor, *J. H. Senior*

switches, instruments, etc., are also provided on this switchboard for the complete control of the engine room machinery. Interlocks are provided to prevent

improper operation and the opening of main circuits until load is removed.

After the set-up connections are made complete

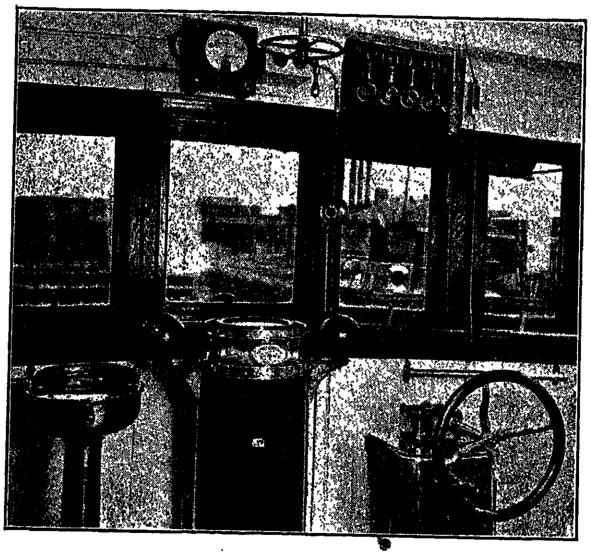

FIG. 17—PILOT-HOUSE CONTROL STATION, *J. H. Senior*

control of the starting, stopping, reversing and speed variation of the propelling motors is carried out by controlling the excitation of the generators supplying

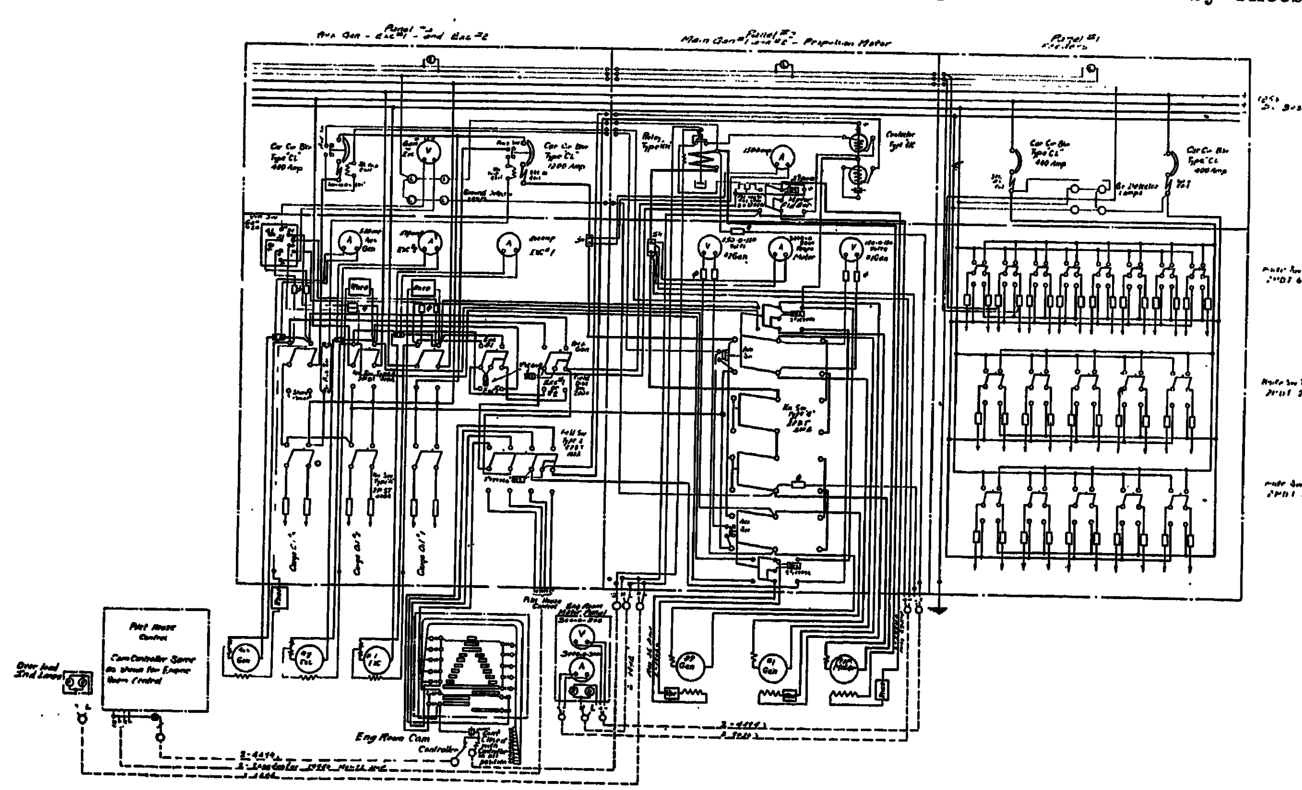

FIG. 18—WIRING DIAGRAM, *J. H. Senior*

them, both generators and motors being separately excited.

The Ward Leonard system of generator voltage control is provided both from the pilot house and the engine room. Sufficient instruments are located in the pilot house for observing the behavior of the electric propelling machinery.

Overload relays and magnetic line contactors pro-

FIG. 19—MAIN CONTROL ROOM, *U. S. S West Virginia*

vide protection against extreme overloads and short-circuits.

The dredge pump motor is started by rheostatic means using a drum-type master controller and a contactor panel.

No steam is used on the vessel for any purpose as all equipment is electrically operated.

It is believed by the Engineers Corps that these

electrically operated dredges will be considerabyl more economical than the steam operated ones.

Dredge for City of Portland. The new 30-in. Diesel electric dredge for the City of Portland, Ore., will be one of the largest and most powerful suction dredges ever constructed. There will be four main Diesel engines, two of 900 h. p. each and two of 800 h. p each. The 900-h. p. engines will be direct-connected to Westinghouse 610-kw., 500-volt, d-c. generators, while the 800-h. p. engines will be direct-connected to 540-kw., 500-volt, d-c. generators giving a total plant capacity of 2300 kw. The main pump will be operated by a 2700-h. p., 500-volt, direct-current motor capable of operating over a speed range from 250 to 360 rev. per min. The use of a d-c. motor with its simplicity and high efficiency was decided upon after careful comparison with an a-c. system.

An interesting feature in this installation will be the use of Ward Leonard system of control for the 250-h. p. cutter motor and the 75-h. p. swinging winch motor. Power for these drives will be furnished by a five unit motor generator set operated from the main 500-volt generator bus. Two of the five units will be 250-h. p. motors for driving the set and to furnish a neutral point for a 500/250-volt, three-wire system, the armatures of these two motors being wound for 250 volts and connected in series across the 500-volt bus. The remaining three units of this set will be a 200-kw. cutter generator, a 60-kw. swing generator and a 60-kw. auxiliary generator. There will be also a motor-operated hoist for the dredge ladder forward and another located near the stern for hoisting the spuds and for miscellaneous duty.

All control equipment on the dredge will be of the remote control type and complete manipulation of it will be possible from the pilot house.

Stern-Wheel Towboat

The success of the Diesel-electric stern wheel towboat *J. B. Battle* provides valuable information to operators of river boats in this country as to the economy and benefits to be derived by the use of this system for vessels of similar size and also for those of larger dimensions and powers.

This complete steel vessel was designed for the U. S. War Department in the office of the Chief of Engineers at Washington, D. C. She is 119 ft. 4 in. long with a beam of 23 ft. and depth of 5 ft. and draft loaded of 3 ft. 6 in. The machinery consists of two 150-h. p. four-cycle Winton engines each direct-connected to a Westinghouse 90-kw., 125-volt, 450-rev. per min., d-c., main generator and a 10-kw., 125-volt exciter. The propelling motor is rated 200 h. p., 240 volts, 600 rev. per min., and is flexibly coupled through a Fawcus 23 to 1 ratio double reduction gear to the lay shaft extending through both sides of the deck house. The two cranks on this shaft are connected by parallel rods to similar cranks on the paddle-wheel shaft thus transmitting power to the stern-wheel which is 15 ft. in diameter and 15 ft. long and has 13 buckets 30 in. deep.

There is also a 7½-kw., 125-volt Winton gasoline engine driven auxiliary generator for supplying auxiliaries and lights when the main sets are not running.

Complete Ward Leonard system of control of the propelling machinery is provided in the wheel house and includes adequate instruments for observing operating conditions at a glance. The excess power of the exciters not required for excitation of the propelling machinery is used for auxiliaries and lights.

The trials of this vessel were conducted in December 1923 and pronounced highly successful and the propelling machinery performed to the satisfaction of all concerned.

It is believed that the advantage of electric drive for this type of boat will induce the building of many more in the near future.

Tanker

One of the very interesting installations of the Diesel electric system of propulsion will be found in the Standard Oil Co. of New Jersey tanker

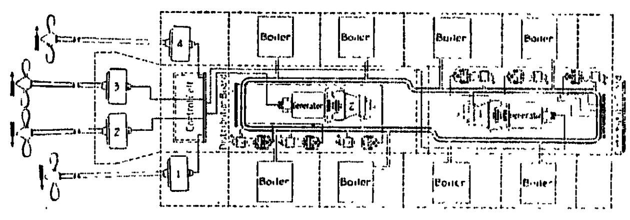

Fig. 20—Diagrammatic Arrangement of Propulsion Machinery, *U. S. S. California, Maryland* and *West Virginia*

J. H. Senior. This vessel is 220 ft. long with a beam of 38 ft. and draft maximum of 13 ft. Her propelling machinery consists of two McIntosh and Seymour 350-b. h. p., 275-rev. per min., heavy-duty Diesel engines, each, direct-connected to a Westinghouse 185-kw., 125-volt, d-c. generator and a 35-kw., 125-volt exciter.

The propelling motor is rated 455 s. h. p., 250 volt, 100 rev. per min.

The exciters provide, in addition to the excitation requirements of the propelling machinery, power for all auxiliaries and lighting. When the main units are not running in port, a 35-kw., 125-volt Diesel-engine-driven auxiliary generator supplies all the ship's power.

A unique feature in this installation is the use of one of the main units for supplying current to the three 35-h. p. cargo oil pump motors as well as other ships power thus keeping the size of the auxiliary plant to a minimum.

The Ward-Leonard system of control is used and two control stations are provided, one in the forward end of the engine room and one in the pilot house. At each station is provided an ammeter, a voltmeter and a propeller speed indicator.

The pilot house station is used almost exclusively and the control of the ship directly by the officer on watch without recourse to bells or other signals is considered by the owners and crew a valuable feature incidental to electric drive.

The main switchboard is located in the after end of the engine room and contains necessary air-break switches, instruments, rheostats, etc., for the complete control of the electrical machinery. Adequate interlocks are provided to prevent wrong operation and the switching system is arranged in a very simple manner for each manipulation.

electric drive and its wisdom in sponsoring it has been admirably substantiated.

The complete success of the collier *Jupiter* completed in 1913 was instrumental in causing the Navy to adopt electric drive for all capital ships beginning with the U. S. S. *New Mexico* in 1915. This act in itself is sufficient testimony of what the builders and users of war vessels think of its merits.

Service operation of several battleships has indisputably proven the prime requisite of reliability and economy of operation.

Electric drive, compared with other drives, shows

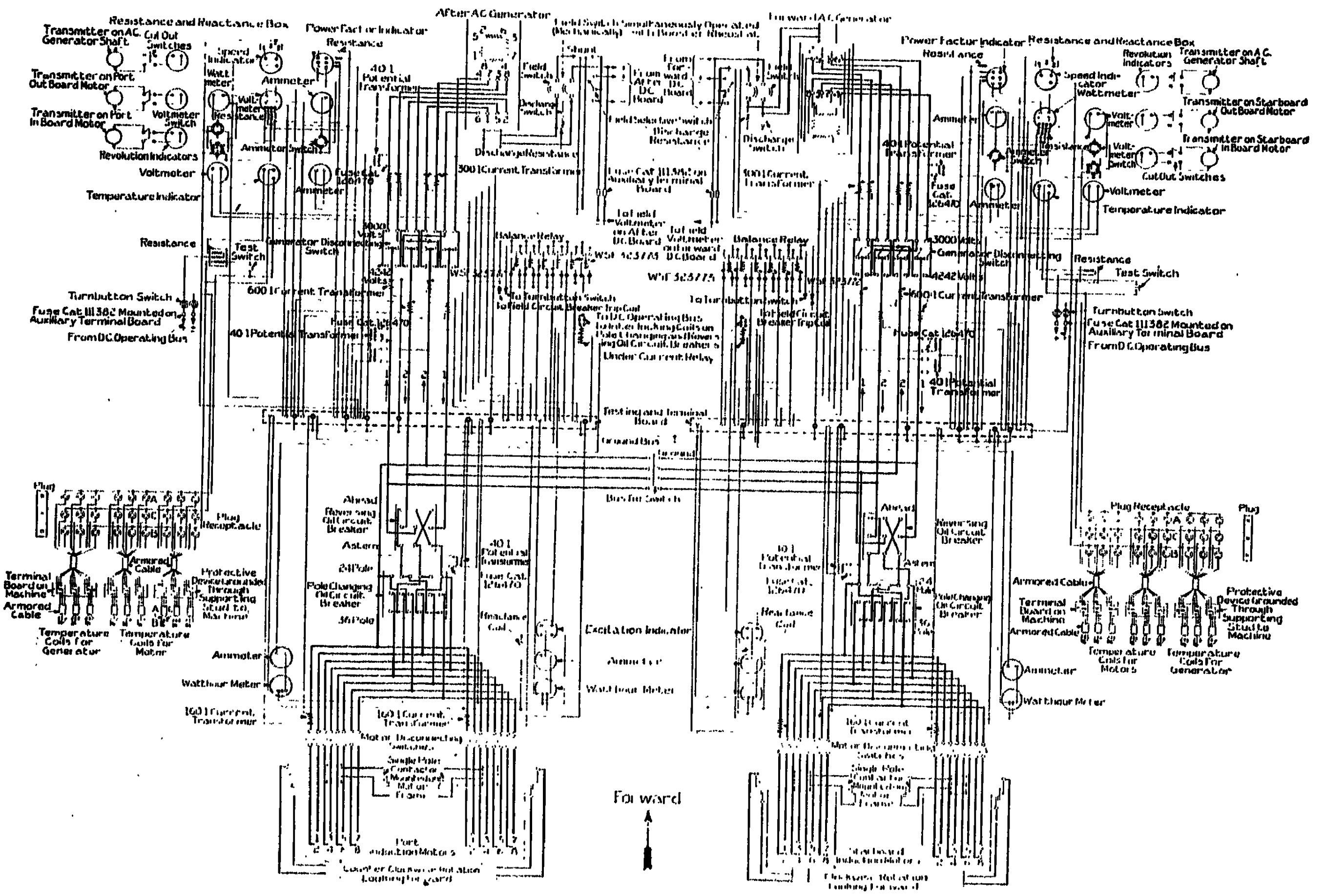

FIG. 21—DIAGRAM OF A.C. CONTROL WIRING, *U. S. S. California, Maryland* AND *West Virginia*

The trials of this vessel proved most satisfactory and met all expectations.

The *Standard Service* equipped with Pacific Diesel and General Electric machinery belongs to the Standard Oil Co. of California and is very similar to the *J. H. Senior*. This tanker has been in service for some time on the West Coast and has performed in a evry satisfactory manner.

U. S. NAVY

While the U. S. Navy is principally responsible for introducing turbine electric drive for ships it is not considered advisable to dwell in detail upon this branch of the marine field in this paper.

The U. S. Navy may be considered the pioneer in

superior protection from torpedo attack upon the machinery by virture of the flexibility of arrangement of the electric units and also provides superior maneuvering qualities.

Because of the large powers involved turbine electric is better suited for propulsion of war vessels than is Diesel electric. The development of larger Diesel engines for electric drive, however, may modify this situation in the near future. The present battleships are rated about 30,000 s. h. p. and the airplane carriers are rated 180,000 s. h. p. To date, therefore, electric-drive installations cover powers from less than 100 to 180,000 s. h. p. showing the extreme range of application.

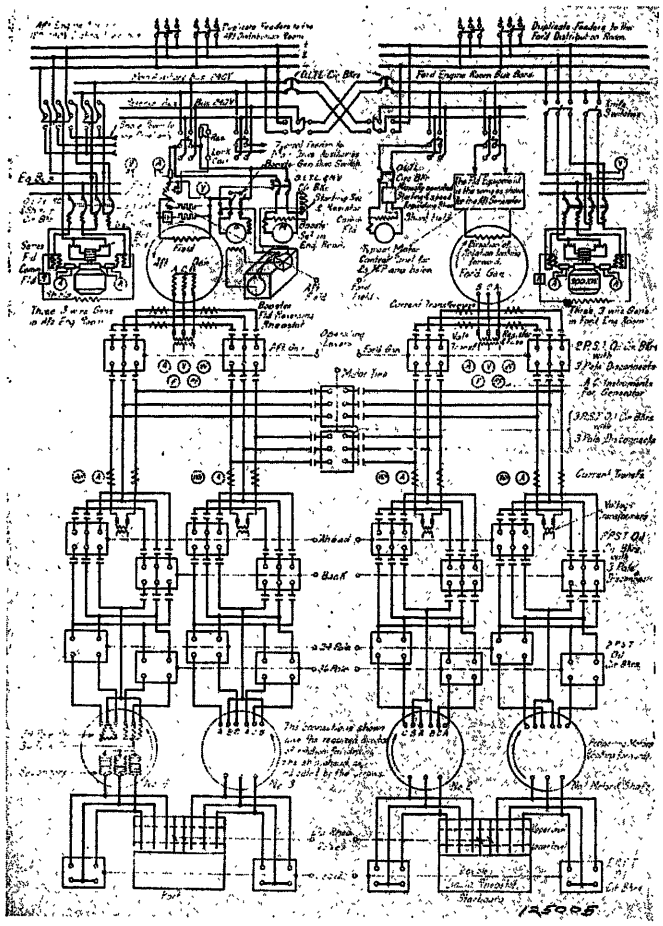

FIG. 22—SCHEMATIC ARRANGEMENT OF PROPULSION CONTROL, *U. S. S. Tennessee*

Induction motors of various rotor winding combinations have been used and all have proven satisfactory. Pole changing arrangements are provided for reduced speed and cruising conditions when using only part of the generating plant. This feature affords marked superior economy over other types of drive where the prime movers must all run at reduced speeds with correspondingly less efficient operation.

With full backing power available the electric drive ship possesses a decided advantage over the turbine drive ship when maneuvering. The ability to stop quickly and the positive control of all propellers are important features to a war vessel.

The control equipment is centralized and located in probably the most protected place in the vessel. The flexibility of control is such that almost any emergency resulting from casualty to any equipment connected with the propulsive machinery proper can be quickly taken care of by disconnecting the disabled unit from the source of power.

The control room is equipped with all necessary instruments, gages, etc., for affording the engineer in charge a complete pulse on the operation of the entire propulsion machinery.

The maintenance of electrically propelled battleships to date has been very slight. While casualties to the electrical machinery at sea appear quite remote, sufficient spare parts are carried for effecting repairs on board ship if circumstances warrant it.

Electric drive for capital ships is probably here to stay and is gaining reputation with every installation. Much credit is due the U. S. Navy for its advance step in the art of electric propulsion.

CONCLUSION

While the growth and adoption of electricity as a means of propelling ships was very slow for sometime, the past six or seven years in particular, have shown definite and rapid progress. Electric drive, after fair and impartial trial, has been well established and those who have been concerned with its development are greatly pleased with its growing popularity.

With the advent of the heavy oil engine the field of application of electric drive has been greatly broadened and the decided impetus already gained will carry this system of propulsion into practically all classes of vessels. The success of electric drive to date is beyond question and the widespread interest is conclusive evidence of its indisputable merits and most promising future.

The art of electrically propelling ships, while conceived many years ago, is comparatively new in application and it is a source of gratification to the pioneers that their early predictions are rapidly being fulfilled. After this system has passed through the lengthy stage of development common to the other types of drives the expectation of the most enthusiastic will undoubtedly be realized.

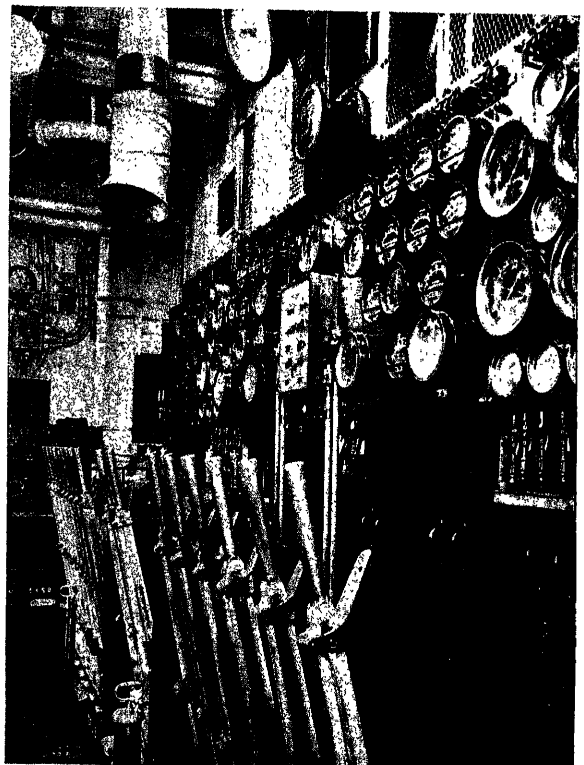

FIG. 23—MAIN CONTROL ROOM, *U. S. S. Colorado*

TABLE II-VESSELS EQUIPPED WITH TURBINE ELECTRIC PROPULSION MACHINERY

	Name	Tonnage Displacement	Type	Year in Service	Turbine-Generator R. P. M.	Turbine-Generator No.	Turbine-Generator K.W.	Voltage	Type Drive	S. H. P.	Propeller No.	Propeller R.P.M.	Knots	Pressure Gauge	Pressure Vac.	Super-heat
1.	S. S. Joseph Medill	500	Fire Boat	1908	1800	2	200	275	TE-DC	500	2	179	11.7 m	160	27	0
2.	S. S. Graeme Stewart	500	Fire Boat	1908	1800	2	200	275	TE-DC	500	2	179	11.7 m	160	27	0
3.	USS Langley ex-Jupiter	19360	Airplane Carrier	1913	1900–2100	1		2300–2420	TE-AC-IM	5400	2	110	14.99	175	27½	0
4.	USS New Mexico	32000	Battleship	1918	2130	2		3000–4242	TE-AC-IF	28000	4	161	21	250	28½	50
5.	SS Cuba	3580	Pass-Cargo	1920	3000	1		1150	TE-AC-S	3000	1	100	17	175	27½	150-250
6.	SS Eclipse	15900	Cargo	1920	3000	1		2300	TE-AC-IM	3000	1	100	11	200	28½	200
7.	SS Invincible	15900	Cargo	1921	3000	1		2300	TE-AC-IM	3000	1	100	11	200	28½	200
8.	SS Archer	15900	Cargo	1921	3000	1		2300	TE-AC-IM	3000	1	100	11	200	28½	200
9.	SS Independence	15900	Cargo	1921	3000	1		2300	TE-AC-IM	3000	1	100	11	200	28½	200
10.	SS Victorious	15900	Cargo	1921	3000	1		2300	TE-AC-IM	3000	1	100	11	200	28½	200
11.	USS.Tamps	1200	C. G. Cutter	1921	3000	1		2300	TE-AC-S	2600	1	130	16	200	28	75
12.	USS Haida	1200	C. G. Cutter	1921	3000	1		2300	TE-AC-S	2600	1	130	16	200	28	75
13.	USS Mojave	1200	C. G. Cutter	1921	3000	1		2300	TE-AC-S	2600	1	130	16	200	28	75
14.	USS Modoc	1200	C. G. Cutter	1921	3000	1		2300	TE-AC-S	2600	1	130	16	200	28	75
15.	SS San Benito (d)	5000	(Pass-Cargo (Fruit Carrier	1921	3000	1		1100	TE-AC-S	3000	1	100	12½	190	27½	200
16.	USS Tennessee	32300	Battleship	1921	2075	2		3270	TE-AC	28000	4	170	21	250	28½	50
17.	USS California	32300	Battleship	1921	2065	2		3000–4242	TE-AC-IFN	28000	4	170	21	250	28½	0
18.	USS Maryland	32600	Battleship	1921	2065	2		3000–4242	TE-AC-IFN	28000	4	170	21	250	28½	0
19.	HIJMS-Kamoi	10200	Fuel Ship	1922	2400	1		2300	TE-AC-S	8000	2	120	15	250	28½	150
20.	USS Colorado	32600	Battleship	1923	2075	2		3270	TE-AC	28000	4	170	21	250	28½	50
21.	USS West Virginia	32600	Battleship	1923	2065	2		3000–4242	TE-AC-IFN	28000	4	170	21	250	28½	0
22.	SS Hayward	1375	Ferryboat	1923	3600/900	1		1100– 500	TE-DC	1200	2	100/125	9	210	28½	50
23.	SS San Leandro	1375	Ferryboat	1923	3600/900	1		1100– 500	TE-DC	1200	2	100/125	9	210	28½	50
24.	SS W. R. Hearst	875	Ferryboat	1923	3240	1		2300	TE-AC-IF	2100/100	2	122/176	16m(b)	250	28½	200
25.	SS Rodman Wanamaker	875	Ferryboat	1923	3240	1		2300	TE-AC-IF	2100/100	2	122/176	16m(b)	250	28½	200
26.	SS Geo. W. Loft	875	Ferryboat	1924	3240	1		2300	TE-AC-IF	2100/100	2	122/176	16m(b)	250	28½	200
27.	USS Lexington	43500	Airplane Carrier	(a)	1800	4		5000	TE-AC-IN	180000	4	317	33	265	28½	100
28.	USS Saratoga	43500	Airplane Carrier	(a)	1800	4		5000	TE-AC-IN	180000	4	317	33	265	28½	100
29.	Bradley Transp. Co. (c)	13000	Limestone Carrier	1925(a)	3450	1		1150	TE-AC-IM	3000	1	100	—	300	28½	200

(a) Under construction
(b) Designed speed miles per hour
(c) Self Unloader type
(d) Turbine-Electric Propulsion Machinery by the British Thomson-Houston Company, Ltd., of Rugby, England.
Propulsion Machinery and control for other vessels by General Electric & Westinghouse Companies

TE Turbine Electric
AC Alternating Current
DC Direct Current
IN Induction Motor Slip Ring Type—Internal Resistance
IM Induction Motor Slip Ring Type—External Resistance
IF Induction Motor Double Squirrel Cage Type
IFN Induction Motor Double Squirrel Cage Type with Slip Rings
S Synchronous

TABLE III—VESSELS EQUIPPED WITH DIESEL ELECTRIC PROPULSION MACHINERY

	Name	Tonnage Displacement	Type	Year in Service	Generators No.	KW ea.	Voltage ea.	R.P.M.	Type Drive	S.H.P.	Propeller No.	R.P.M.	Knots
1.	M. S. Mariner	500	Trawler	1920	2	165	125	350	DE-DC	400	1	70/200	10
2.	Elfay	313 gr.	Sea-going Yacht	1920	1	75	125	425	DE-DC	90	1	360	8-8½
3.	Guinevere	574 gr.	Sea-going Yacht	1921	2	225	125	225	DE-DC	550	1	220	11
4.	Alcyone	453 gr.	Sea-going Yacht	1922	2	140	125	250	DE-DC-DA	350	1	175	10½
5.	M. S. Fordonian	4050	Cargo-Gt.-Lakes	1922	2	350	250	200	DE-DC	850	1	120	8½–9
6.	Valero II	—	Sea-Going Yacht	1922	2	90	125	450	DE-DC-DA	215	1	275	—
7.	Poughkeepsie	400 disp.	Ferry	1922	2	90	250	450	DE-DC	200	2	600/220 gr.	10 (b)
8.	M.S.Golden Gate	1005	Ferryboat	1922	2	360	250	225	DE-DC	750	2	145/180	12 m. p. h.
9.	M.S. Golden West	1005	Ferryboat	1923	2	360	250	225	DE-DC	750	2	145/180	12 m. p. h.
10.	Naldnah II	—	River Yacht	1923	2	25	250	1000	SE-DC	60	2	600	—
11.	M. S. Standard Service	2725	Tanker	1923	2	245	230	265	DE-DC	600	1	130	9
12.	M. S. Alaska Standard	2725	Tanker	1923	2	245	230	265	DE-DC	600	1	130	9
13.	M.S. Twin Cities	3000 DWT	Cargo-Canal-Gt. L.	1923	2	250	230	260	DE-DC	500	2	180	8–10–m (b)
14.	M. S. Twin Ports	3000 DWT	Cargo-Canal-Gt. L.	1923	2	250	230	260	DE-DC	500	2	180	8-10-m (b)
15.	Cutty Sark ex-Ariadne	246 gr.	Sea-going Yacht	1924	2	65	125	600	DE-DC	150	1	240	—
16.	J. B. Battle	3'-6" draft	Stern Wheel River Towboat	1923	2	90	125	450	DE-DC	200	1	600/26 gr.	8–9 m. p. h.
17.	J. H. Senior	2335 disp.	Tanker	1924	2	185	125	275	DE-DC	455	1	100	9
18.	A. Mac Kenzie	2000 DWT	Sea-going Hopper Dredge	1924	3	700	500	150	DE-DC-DA	1600	2	90/110	11½
19.	W. L. Marshall	2000 DWT	Sea-going Hopper Dredge	1924	3	700	500	150	DE-DC-DA	1600	2	90	11½
20.	Dan C. Kingman	2000 DWT	Sea-going Hopper Dredge	1924	3	700	500	150	DE-DC-DA	1600	2	90	11½
21.	William T. Rossell	2000 DWT	Sea-going Hopper Dredge	1924	3	700	500	150	DE-DC-DA	1600	2	90	11½
22.	P.R.R.Tug No. 16	—	Tug	1924	2	235	250	260	DE-DC-DA	575	1	125	—
23.	M.S.Van Dyke I	138 gr.	Tug & Fireboat	1924	2	155	125	257	DE-DC	370	1	120	10
24.	M.S.Van Dyke II	138 gr.	Tug & Fireboat	1924	2	155	125	257	DE-DC	370	1	120	10
25.	M. S. Van Dyke III	138 gr.	Tug & Fireboat	1924	2	155	125	257	DE-DC	370	1	120	10
26.	M. S. Brilliant	863 gr.	Tanker	1924	2	155	125	257	DE-DC	370	1	120	10
27.	M.S. La Playa (c)	3682 gr.	Pass-Cargo	1924	4	500	220	250	DE-DC	2500	1	95	12½
28.	M.S. LaMarea (c)	3689 gr.	Pass-Cargo	1924	4	500	220	250	DE-DC	2500	1	95	12½
29.	J. W. Van Dyke ex-Allentown	7000	Tanker	(a)	3	600	250	225	DE-DC-DA	2300	1	100	11½
30.	Kanawha	—	Riverboat	(a)	1	50	250	600	DE-DC	60	Stern Wheel Reduction Gear		
31.	M.S. Hawaiian Standard	—	Tanker	(a)	2	245	230	225	DE-DC	600	1	130	9
32.	—		Experimental Towboat	(a)	2	7–½	125	1200	SE-DC	18	2	Tunnel-Screw Type Reduction Gear	
33.	M. S. Lake Weir	2018 gr.	Dredge	(a)	2	400	240	200	DE-DC	1000	2	130	10 m
34.	F. M. Coots, Ex-S. S.McChestney,Jr.	700 gr.	Ferryboat	(a)	2	175	230	257	DE-DC	360	2	14	8 m. p. h.

Two War Dept Towboats are under construction, having one 85-Kw., 240-Volt, 450 rev. per. min. generator, 100-h. p. motor, geared to stern paddle wheel, Diesel electric drive.

(a) Under Construction
(b) Designed speed, miles, per hour.
(c) Propulsion Electric Machinery and Control by the British Thomson-Houston Company, Ltd., Rugby, England. Equipment for the other vessels by Westinghouse & General Electric Companies.

DE Diesel-Electric
SE Gasoline Electric
DC Direct Current
DA Double Armature
GR Geared to Motor

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Discussion

V. Karapetoff: The Diesel engine is essentially a constant-speed prime mover, and I should like to ask Mr. Thau about its application to the tug service, which, to me, is essentially a variable speed application, and also one involving a high slip, either in the electrical equipment or between the propeller and the churning water. With a d-c. drive, the Diesel engine can run at a constant speed, and the d-c. motor on the shaft can be controlled either by a series resistance or by a resistance in the field circuit. But I should like to know how this is done with a-c. drive, or is the a-c. drive not suitable for the tug service?

I wish also to mention a specific engineering problem which arises in the turbine-electric marine drive with either induction or synchronous motors. The induction motor has a circle diagram, but this diagram holds true only at a constant current or constant voltage. In marine drive we have neither. If my information is correct, generators are usually operated at a constant field current, in which case the terminal voltage between the motor and the generator varies with the torque on the propeller shaft. As this torque increases, the generator voltage dies down. In computing the performance characteristics of such a set, we have a rather difficult engineering problem, because the internal reactance in the generator and its armature reaction have to be considered, as well as the constants of the motor itself, the voltage between the two machines being at the same time a function of the load.

C. H. Giroux: I shall confine my remarks to river boats, as I believe that many engineers from this vicinity are interested in this subject.

The authors have omitted reference to one vessel which I believe deserves mention, as it is one of the pioneers in its field, namely the *Chenoka*. The *Chenoka* is a stern-wheel towboat; 77 ft. long, 10-ft. beam, and having a displacement of 32 long tons.

During 1920, the Cincinnati district of the U. S. Engineer Department equipped it with electric propulsion, using a 25-kw., gasoline-electric generating set, a 50-h. p., shunt-wound motor, and rheostatic control. For experimental purposes, a producer was installed and the engine operated on coal gas for a short time. The intermittent service in which the vessel was used, however, was not conducive to good results with the producer, and it was finally abandoned. Kerosene was used with a fair degree of success, but the advantage of lower fuel cost was offset by operating difficulties, and the engine is now being run on gasoline only.

While the equipment on the *Chenoka* does not conform to present marine standards in many respects, it has been in successful operation for a sufficient length of time to prove the reliability of electric drive for small river boats where the maintenance is not always the best, and where skilled operators cannot be employed for monetary reasons.

Based on the results obtained on the *Chenoka* and the Diesel-electric towboat, *J. B. Battle*, the U. S. Engineer Department is designing and constructing a number of stern-wheel towboats for use on the Mississippi and Ohio rivers. These boats will have improvements incorporated which should cause them to show even better results than those now in operation.

The most important of these improvements is the adoption of a propulsion motor, the speed of which may be varied over a wide range by field control, keeping the horse-power output constant. Ward Leonard control for slow-speed running, accelerating and reversing is, of course, retained. The variable-speed motor allows full rated output of the prime movers to be applied to the wheel at any speed of the towboat from standstill to light-running speed.

This is a decided advantage over any direct drive system where the horse power varies with the speed of the paddle wheel. From tests on existing towboats it is estimated that the effectiveness will be increased as follows:

Conditions	Variable-Speed, Electric Drive	Other Systems
Towing effort at standstill, per cent....	160	100
Speed towing four barges, per cent.....	110	100
Speed light without tow, per cent......	100	100

The first condition means that the tow can be accelerated more quickly and a heavier tow can be maneuvered.

The second condition shows a substantial saving in time when handling heavy tows.

The third condition shows that there is no sacrifice in running light speed for the gain in towing ability.

As the weight of the tow is often limited by the rapidity of the current when going up-stream, the extra towing effort available may enable the boat to handle heavier tows and to make "double tripping" unnecessary in the more rapid parts of the stream.

Considerable attention has been given to the control equipment, with the idea in mind of simplifying the methods of operation. The next towboats built for the U. S. Engineer Department will be equipped with automatic control from the pilot-house. By means of a single master switch, the boat may be maneuvered from full speed ahead to full speed astern without the necessity of watching electrical instruments or without danger of overloading the prime movers. No adjustment of rheostats in the engine room will be required for handling tows of various weights, as this is accomplished by the automatic devices.

An attempt is being made to place the design of river boats on a more scientific basis than has been possible heretofore, not only by means of complete model tests, but also by securing operating data from vessels in service. Electric drive, with its attendant possibilities for accurate measurement of power under daily service conditions, will do much to aid the investigators in this attempt.

A. A. Coyle: The propulsion of vessels by electric power is yet in its infancy, and like most other inventions, will be improved from time to time as defects in design and mechanism develop. This is the history of mechanical devices, and while there may be exceptions, we know of none in which the original design has not been changed to meet the conditions under which they must operate.

Generally speaking, electric drive for boats has passed the experimental stage, the installations made in the past and the successful operation of the various craft in which the installations were made, demonstrate its practicability and efficiency for boat propulsion.

Boat owners and operators have been slow in adopting new or untried methods for the operation of their boats and the principal reason for their conservatism will be found in the fact that the margin of profit is so slight that experimental work could not be undertaken, or even considered. Operating costs have

more than doubled, and the receipts or advance in rates have not been in the same proportion, resulting in what is termed hand-to-mouth existence. With conditions unfavorable for successful operation of their boats, many have disposed of their interests, while others have continued the operation of their fleet with the hope that means for reducing the operating costs would eventually be provided. The successful operation of boats propelled by Diesel-electric power has inspired boat owners to investigate the advantages of Diesel-electric over the steam drive, the result of which has convinced many that the Diesel-electric power is what is needed if river traffic is to be revived, and the operation of river craft made profitable. Diesel-electric power for river boats has the following points in its favor:

a. Economy in operation (fuel and labor)
b. Efficiency in maneuvering
c. Concentration of responsibility in one man
d. Positive control
e. Insurance reduction
f. Cleanliness
g. Requires less deck space
h. More deck space available, due to storage of fuel in hold
i. Equal torque at all speeds

With the facts above mentioned brought to their attention, and the necessity of reducing operating expenses to the minimum to realize a reasonable return on their investment, many boat owners have decided to install Diesel-electric power on all boats to be constructed by them in future, and in some cases will remove the present steam installation and install Diesel-electric power. This will tend to revive river traffic, and the means by which the operators of river boats may recoup themselves for some of the losses sustained in the past.

As is generally known, there are in use three methods for the application of electric power to stern-wheel boats, *i. e.*,

a. Longitudinal shafts with bevel or worm gears
b. Chains running over sprocket wheels
c. Pitmans, similar to those in use for steam drive

For many reasons, the first and second methods are not satisfactory where applied to stern-wheel boats operating on rivers where silt and drift are prevalent. The construction, and the bracing of the paddle-wheel supporting beams must necessarily be light to insure the desired draft of boats for shoal rivers, and the wheel being subjected to both vertical and longitudinal movement (however slight) tends to throw out of alinement the drive shafts and gears, with a corresponding loss of power. There is also the danger of drift and broken buckets coming in contact with the shafting, which often causes serious damage to the parts and quite frequently disables the boat until such time as the parts can be unshipped, sent to a machine shop, repaired and replaced. As the boat is often some distance from a shop in which repairs might be made, it is evident that delay in such cases is very serious to the owners. For these reasons it has been the aim of designers and owners of stern-wheel boats to eliminate all rods, cams and gears from on or near the wheel. It may be said by those advocating the chain or gear drive that all parts may be housed for protection, but even though this housing may protect the parts so easily affected, they cannot prevent the vertical or longitudinal movement of the wheel, which tends to throw the driving mechanism out of alinement. The housing must necessarily be made very strong to withstand the usage to which they will be subjected, for if this is damaged it is a serious matter.

The third method, (pitmans) to transmit the power from the motor to the wheel shaft is in our opinion the best yet devised. However, the drive as applied, is not entirely satisfactory to many boat owners and it is thought that a much better arrangement can and will be designed, eliminating the objectionable features.

The installation on the U. S. Towboat, *J. B. Battle*, includes a large and very long counter-shaft, extending across and nearly the width of the boat. To this are attached the cranks and a large spur gear. This gear is attached to the counter-shaft at about the center, thus necessitating three or more bearings, a very bad feature when one takes into consideration the light construction of the hull to which the counter-shaft bearings are secured, because it is then impractical to provide foundations with sufficient rigidity to insure proper alinement—an essential feature in fuel economy. And while the shaft alinement is not observable, it is more or less out of alinement, the amount depending upon the rigidity of the hull to which the shaft bearings are attached. A single motor is used for the full power required to rotate the paddle wheel at the desired velocity, and, as all of the power is applied to the large spur gear attached to the counter-shaft, it is evident that when one of the pitmans is on center the full power of the motor is transmitted to the other pitman which, for *steam drive*, would be designed to transmit only half of the actual horse power developed. Under no circumstance could more than half the power be applied to either pitman. Therefore, it is evident that pitmans, cranks and wrists for the electric drive as now applied must be proportionately larger than those in use for steam drive with equal power, to care for the increased thrust. This addition for increase in strength and weight of the pitmans, cranks and wrists occurs where it is most objectionable, as our great problem is to keep the stern of the boat up, in order to obtain maneuvering benefits. With the counter-shaft extending across the boat, and, necessarily, three to five feet above the deck, the space aft of the counter-shaft is not available for other purposes.

We are of the opinion that it is possible to design, and apply a more efficient and better general arrangement than is now in use, and we are working with that end in view. We believe the power required to propel the boat should be in two units, (two motors) each acting independently on its own pitman, with a suitable control to cut out the current gradually as the pitman approaches the center, and again applying the current gradually immediately after the pitman passes the center, the application of power being similar to that now in use with steam drives.

There is a difference of opinion among electrical engineers regarding the practicability of such an arrangement, some contending that motors as now constructed, would not stand up under such usage but, as the motor is not stopped, or even perceptibly slackened, it occurs to us that if motors as now constructed will not meet the requirements, a motor can be built that will.

R. A. Beekman: My comments refer to several points in the paper as indicated in the following notations:

Paragraph beginning "The justification of the turbine electric system, etc." Taking transmission efficiency in its broadest sense, that is, from the boiler to the propeller, the efficiency of the turbine electric system using a-c. machinery is in many cases equal to, and in some cases better than, the corresponding transmission efficiency obtained with geared turbines. Namely—the geared-turbine losses including the losses in the gears, reversing element, cross-over connections, additional bearings, packings and lubricating oil required over and above that required by the turbine electric system, give a total of from 7 to 10.3 per cent of the total shaft horse power.

In the case of the turbine electric drive, the total loss including losses in the motor, generator, control, and cables, and the power required for excitation and for ventilation of the main machinery, is approximately 7.6 per cent of the power delivered to the propeller shaft. Furthermore, where single reduction gears are used, the turbine efficiency on the Rankin cycle should be higher in the case of the turbine for electric drive than the turbine for geared drive, since the former turbine can be run closer to its most economical speed.

Paragraph beginning "Turbines with reversing elements, etc." The larger diameter necessitates larger radial clearances with

reaction-type turbines, but if impulse-type turbines are used, radial clearance is not a factor.

Paragraph beginning "With turbine electric drive alternating current is more satisfactory, etc." Greater emphasis should be given to the advantages of d-c. turbine electric drive for the smaller ships where turbines are suitable. The important advantage is that a constant-speed turbine generator may be used which permits taking the auxiliary power either from the main generator or from the direct-connected exciter. We also have all of the advantages incidental to the flexibility and reliability of the Ward Leonard system of control repeatedly referred to in this paper.

Paragraph beginning "The electrical machinery is practically the same as used in land installations," etc. There is an increasing tendency to run the electric auxiliaries from the main propulsion generator, in order to get auxiliary power at approximately the efficiency of the main unit.

Paragraph beginning "A 75-kw., 115-volt, 900-rev. per min. exciter," etc. This exciter also supplies power for miscellaneous auxiliaries.

Paragraph beginning "The Ward Leonard system of voltage control is by far the most satisfactory," etc. The Ward Leonard system is not necessarily the most satisfactory system, inasmuch as the rheostatic-control system has been used most satisfactorily on at least five ships. The rheostatic control was selected after careful consideration of the special conditions to be met. Where two or more generators are used one generator may be operated at constant potential, furnishing power for excitation and the auxiliaries as well as for propulsion, and the second generator may be operated on the voltage-control basis. This is a combination of rheostatic and Ward Leonard control where all maneuvering of the ship between 50 per cent and 100 per cent propeller speed is obtained on a Ward Leonard, and hence, highly efficient, basis.

Paragraph beginning "With the series connection of generators," etc. The statement regarding Ward Leonard system being less expensive may be somewhat misleading, due to the fact that the necessary exciters should be considered a part of this system, both as regards expense and complication.

Paragraph beginning "The control board contains suitable switches," etc. In the early days of Diesel-electric drive it was thought to be desirable to have a time-delay element for limiting the speed of operation of the control, but experience has shown that this is unnecessary and that we can go from full speed to the stop point without hesitation.

Paragraph beginning "With this type of drive provision is usually made for both pilot-house and engine-room control," etc. Another advantage of pilot-house control of tug boats comes in being able to take slack quickly out of the tow ropes and uniformly accelerate the tow without objectionable strains.

Paragraph beginning "Flexibility and Reliability." The number of generators is not limited by either the rheostatic or Ward Leonard system of control.

Paragraph beginning "It will be noted that the control for these boats," etc. The *Fordonian* has a control which is a combination of Ward Leonard and rheostatic system, as described under Comment No. 6.

Paragraph beginning "The *Standard Service* equipped with Pacific Diesel and General Electric machinery, etc."

The *Standard Service* was put into commission in March, 1923, and similar tankers, the *Alaska Standard* and the *Hawaiian Standard*, were put into commission in December 1923, and February 1925, respectively. This is a good example of an owner having added Diesel-electric ships to his fleet after experience with the first boat.

W. E. Thau: Professor Karapetoff struck on two vital points in connection with electric drive; one relating to the flexibility of direct current for tug boats and the other the unit design of an a-c. propulsion system.

If an attempt is made to use alternating current for tug boats and that class of service, we should have a plant which is extremely inflexible in comparison with direct current; and, furthermore, if we had multiple-unit arrangements of engines, it would be necessary to operate a-c. machines in parallel, which is questionable, regardless of the arrangement; that is, the idea of operating Diesel-driven, a-c. generators parallel on board the ship. In addition, we would lose about 80 per cent of the flexibility which the d-c. system possesses for tugboat work.

Furthermore, by using the d-c. system, say with two generators and a single- or double-armature motor, we are able to operate that tug with one unit in case of failure of one of the engines. In case of failure of a direct-connected engine, the situation is helpless. This particular advantage has proved to be of great practical value in connection with the *P. R. R. Tug No. 16*, operated by the Pennsylvania Railroad in New York. On one occasion they had trouble with an engine and that engine was shut down and the tug operated on the remaining engine, without those for whom the towing work was done knowing whether one or two engines were in operation.

With the d-c. system, the Ward Leonard system of motor speed control is used. This permits the use of simple generator-field rheostats for providing a large number of speed values in each direction in the most economical manner. The engines run at constant speed. The generators are connected in series, thus dispensing with the necessity for parallel operation. The electrical efficiency is maintained near the full-load value throughout the entire speed range from 10 per cent to 100 per cent speed. In other words, the combined electrical efficiency curve is very flat.

In the case of alternating current, the speed control would have to be obtained by engine control or rheostatic control. The former is no improvement over the direct-connected engine and the latter is both complicated and wasteful.

Furthermore, the d-c. system permits full power to be had from a fractional number of engines, whereas with a-c. the engine speed would have to be reduced to suit the load in the case of operation on less than the full number of engines.

In case of turbine electric drive, the question of torque couple between the motor and generator was brought up. This is something that would probably be overlooked by the uninitiated, but the fact is that the whole system from turbines to control must be designed as a unit and the functions of each part correlated.

The pulling out of step of an a-c. drive is not due to the fact that the motor breaks its torque couple. The motor is designed for about one and three-quarters pull-out torque, and that gives it plenty of margin, so long as it is provided with a constant voltage. The real cause of pull-out between the generator and motor is the collapse of the generator voltage due to overload which Professor Karapetoff has mentioned.

In all land installations, we have a relatively unlimited supply of power and the generators are sufficient in number and in size to supply the total demands and take care of peak loads. Such machines are usually designed to provide about 5 per cent to 10 per cent torque margins at the nameplate rating.

In the case of ship drive, when the load varies with sea conditions, maneuvering, and so forth, we must provide an ample torque margin in the generator. That is provided by additional excitation. If we operate with too great excitation, the efficiency of the motor drops considerably. This condition is very pointedly shown in certain tests that are made on board battle ships, and are known as excitation, or drop-out tests. We start with a high generator excitation and gradually decrease it until the motor and generator torque couple is broken. The power input to the motors at the start of the test is appreciably higher than when zero torque margin is approached (approximately 10 per cent) and, therefore, it is uneconomical to run with too much torque margin. That condition has been recognized in the Navy, and steps have been taken to operate vessels with an economical torque margin. That margin is indicated by an

instrument called a stability indicator which shows the torque ratio at any instant.

Mr. Barrister brought out a point in connection with the steering gear on board the ship. This is another case where the complete facts must be known, and the motor, controller, and resistors designed as a unit and not with a motor picked here, a controller there and a resistor some other place. As already mentioned the various units picked at random have proven a failure. It is a definite application problem and where early failures have occurred, they can almost invariably be traced to the fact that the equipment has been taken out of stock and put on board the ship. Those of us who have had experience with the sea conditions know that that stock equipment does not work. When such care is necessary in the design of the equipment, it is no wonder we ask for competent electricians to take care of this electrical equipment.

Mr. Giroux is in a very good position to discuss the river-boat situation, being the electrical engineer for the United States Engineering Department at Washington, and having had considerable to do with practically all of their engineering work in that line, which he has so ably described.

The War Department is investigating and pioneering in that field and so soon as they prove to the private owners that the electrical is the proper system, we shall see private boats driven by the Diesel-electric system. Of course, the War Department is in a better position to make study and investigation of that kind because of its organization and funds. We civilians pay the taxes and they do the pioneer work in that and other fields. The results benefit every citizen in the country. The War Department is doing wonderful work along these lines, even though most of us are not aware of it.

Mr. Coyle has enumerated the advantages of electric drive for river boats from a designer's and builder's point of view, and it will be noted that he has confirmed, to a great degree, the claims for this type of drive in the paper. Mr. Coyle has spent practically his whole life in this work, and so is in a good position to analyze the situation.

We feel that he is correct in his statement, that gears and chain drives are not as applicable to stern-wheel, river-boat propulsion as the pitman drive. It represents a very intricate problem and one which will require much close study before it comes to a high degree of perfection. The fact that the electric motor tends to produce constant torque introduces a problem for consideration in design of pitman. If it were possible to regulate the motor performance similar to that of a steam-engine performance, it would help things along considerably. There has been devised a special form of pitman connection using two pitmans on each side of the wheel, and in each pair the pitmans are placed 180 deg. from one another. With that arrangement, having the other side at 90 deg., we have a system which will develop a constant turning effort on the wheel. That has a further good point, in that the wheel bearings do not have to be anchored solid. That eliminates a lot of trouble that might arise from any other pitman design.

I should like to make just a few remarks in connection with Mr. Beekman's discussion of the paper. I thought the paper recognized the suitability of d-c. equipment with turbine electric drive. We stated that the application for that type of equipment was confined more to the vessel requiring very restrictive maneuvering. For a vessel, such as a tug or a ferry, operating in unrestricted water direct current is better suited than alternating current, and while the cost is more, the installation of the direct current is justified by its increased flexibility.

The question of rheostatic control versus Ward Leonard is one that can probably be discussed for a long time. Some of the present rheostatic-control vessels are handicapped. In small installations there is no question that the rheostatic-control system can be used. I think that was mentioned in the paper and an example cited. Those applications that require constant voltage can be supplied by exciters of suitable capacity driven from the main engines.

The time-delay device, which was referred to, I believe, is absolutely necessary and essential on many classes of ships. So far, the application of Diesel-electric drive has been confined to slow-speed, coarse-line, low-inertia vessels, but when this type of drive is applied to fine-line boats with high powers, requiring considerable torque to stop them, some kind of a device will be required to regulate the current. D-c. machinery cannot be abused to the same extent as a-c. machinery. Mr. Giroux touched on that point in his discussion, and described how the U. S. Engineering Department expects to provide for a current-regulated device on its tow boats.

In the case of direct-connected impulse turbines the large disk area resulting from the large-diameter wheels is a factor contributing to additional losses which are comparable to the radial-clearance losses in the reaction turbine.

Auxiliary power cannot be taken from main units during maneuvering if the Ward Leonard system is used.

In any event, exciters or direct-connected auxiliary generators are very desirable for supplying auxiliary load, and they may just as well be made large enough to supply excitation for the Ward Leonard system. Therefore, they are not an added complication.

Coal-Mine Electrification

BY W. C. ADAMS[1]

Member, A. I. E. E.

Synopsis. *To purchase or generate power for mine requirements is determined on basis of minimum power charge against coal production. Alternating-current transmission results in better efficiencies and lower costs. The choice of coal hoisting systems is based on reliability with cost secondary. Skip hoisting for large mines gives lower costs of installation and operation, with greater reliability. Man-and-material hoists arrange to take care of every possible contingency against shut-down is advisable. Converting stations is the logical method of supplying direct current to underground equipment.*

THE conditions vary so greatly at the different mines that it is impossible to thoroughly cover this subject by discussions as to the many problems to be solved under varying conditions. This paper will include a brief discussion of some of the important problems of coal-mine electrification with descriptions of some concrete installations.

Power supply to the mine is an important factor and must be considered as coming from one of two main sources, either generated by a plant at the mine or purchased from a public utility company. Each has its place and the choice can be made only on the basis of cost as it affects the cost of coal production.

A vital part of a shaft coal mine is the hoisting equipment and reliability must be of first consideration. The economic side is also discussed to show why different types of hoists may be installed to meet special conditions. A general description of the largest coal hoist in the world is here given with a discussion of some of the problems involved in its design.

After the coal is hoisted to the surface it must be properly prepared and suitable features of control have a vital bearing on the success of the preparation.

The mining equipment in the underground workings must be supplied with power and here, the distribution problem is most important, as the life and maintenance of all electrically driven equipment and its ability to produce maximum results is materially affected by power distribution.

SOURCE OF POWER

The Public Service Companies have materially reduced the cost of power for coal mining operations. They have made it possible for the coal operator to purchase his power requirements at a much lower cost than that at which he could produce it with the make-shift uneconomical plants which used to be a feature of every mining operation. In doing this they have set up a bogey unit power cost for each mine. A generating plant owned and operated by the mining company is not warranted unless it can produce the power requirements at a cost at least equal to that of purchased power, which cost must include a reasonable return on the additional capital invested.

The Public Service Company has the advantage of a large output and a relatively high load factor which, for a mining operation, is usually about 14 per cent and rarely exceeds 20 per cent. Both the quantity of output and load factor might be increased if it were possible to serve some outside off-peak load; but this is rarely the case.

Where two or more operations can be served from one plant, the load factor will be improved, reducing the unit investment cost per ton output and cost per unit of power generated. In making a comparison of the cost of power generated and power purchased, the advantage of grouping is often over estimated unless a like grouping with a single metering point is considered for purchased power.

As an illustration, a mining company operating seven properties with purchased power metered for individual mines made the necessary investment for distributing lines from a single metering point. With this system the power bill was reduced 19 per cent, which represented over 20 per cent net annual return on the investment.

A Public Service Company can locate its plant at some advantageous point largely determined by the water supply. At a mine site the quality and quantity of water available for boiler and condenser uses is often inadequate. This factor must be carefully considered before it can be determined that a generating plant for mine requirements is even feasible, for the expense and losses due to a shortage of water supply during one prolonged dry period may offset the accumulated advantages of a mine plant over a period of many years.

As against these advantages, the Public Service Companies must pay freight on their coal, and usually, because of these freight charges, must buy high grade fuel, and then make a profit over their costs.

The individual operator can burn coal which he cannot sell or a grade which has relatively low market value or is hard to move. In the preparation of coal at almost every mine, impurities containing a certain amount of burnable material, are picked out, which if properly crushed and mixed with the low-grade sizes made in coal preparation, should give a fuel containing from 30 to 35 per cent ash.

Every coal-mining generating plant should be equipped to burn this product, thereby reducing materially the largest item of cost in power production by a Public

1. Allen & Garcia Co., Chicago.

Presented at the Spring Convention of the A. I. E. E., to be held at St. Louis, Mo., April 13-17, 1925.

Service Company. The plant must also be properly engineered, designed and equipped for the usual economies found in good Central Station practise, making allowances only for the difference in sizes of equipment.

Reliability of the power source is essential and a Public Utility System, with its vast net work of transmission lines, is often at a disadvantage as compared with a plant located at the mine site. A Public Service Company which cannot show a good record for continuity of service or that reasonably continuous service can be supplied, should not be considered a possible source of power for a coal operation where a power plant at the mine site is practical.

"The coal mine operator in installing a power plant for generation of his power requirements is entering a field for which he is not organized" is an argument very frequently used for the purchase of power. This, however, is not an argument that carries little weight for coal operators can and are now producing their own power requirements at a profit.

This is demonstrated by an existing installation where a power plant was put in for a shaft operation having a daily potential capacity of 7000 tons. This plant is designed to secure maximum economies consistent with capital investment, and is burning the crushed mine pickings, mixed with the lowest grade slack produced.

The estimate on which the decision to install a plant at the mine site was based, showed that the saving of power generating cost as against purchased power cost, would pay 11 2/10 per cent interest on an investment of $460,000.

This estimate of comparative power cost included for the generated power all items of labor, supplies, maintenance, fuel, water, amortization, obsolesence, and overhead items, excepting interest on capital investment. For purchased power, similar items were added to the power bill, and the cost of operating, maintaining, etc., a steam plant necessary for furnishing the steam required for heating, wash house purposes, air humidification and standby equipment was also included.

Based upon operation to date, it is safe to say that the saving estimated will be exceeded. This, together with the fact that the Public Service Company had not been especially reliable, has made the generating station at the mine a profitable investment.

Like estimates for a somewhat similar operation in a field where lower power rates were in force failed to show that any saving could be made by the installation of a generating plant at the mine. The failure to show any saving is partly explained by the fact that the capital investment was excessive due to the natural conditions affecting power plant construction.

An investigation of cost of generating power versus purchased power for a large operating company where two mines could be served from one plant, determined that by an additional expenditure of $760,000, power could be produced at a cost which would earn 7⅜ per cent on the investment in comparison with purchased power costs.

This plant would require some special provision for insuring water supply and this fact, considered with the earnings on the investment, resulted in a decision in favor of purchased power.

Opinions as to the proper returns that such an investment should show varies and is somewhat dependent upon the availability of capital from a company's reserves or the borrowing capacity. It must also be based upon the probable returns that could be secured by an investment by the company in other channels; another coal operation for instance. It should not be less than 10 per cent and might, in many cases reasonably be set as high as 15 per cent.

The last mentioned company is now purchasing power for the complete operation of one plant and partial operation of the second. The power is metered at one point and parallel transmission lines are to be provided from this point to the Power Company's system and to the second mine as an insurance against possible line trouble.

The service is taken from the Power Company's system at a point that is provided with transmission from one station over three routes, from a second station over one route and from a third station over one route, to make possibility of power failure remote. It is, however, not often possible to make such an ideal connection for power service, but wherever feasible, loop transmission should be provided with proper switching equipment placed at the mine site to take advantage of the same.

The practise of purchasing power for part of a mine's power requirements and maintaining a steam plant for the hoist is quite common although is a very expensive practise. This condition is usually the result of an attempt to minimize capital investment, either for supplying additional power requirements or the rehabilitation of the existing plant. The savings to be secured by using purchased power throughout are considerable and often obtainable by a relatively small additional investment.

Power Characteristics

Progressive mining operations are now being equipped with every mechanical mining device which will meet the requirements of the existing condition and reduce the cost of coal production. Motor-driven cutting machines, loading machines, loading conveyers and loading scrapers can be equipped with either d-c. or a-c. motors. However, locomotives have proved satisfactory only with d-c. motor equipment.

All this equipment operates at the mining faces, and it is necessary to provide transmission facilities for power to the extreme points of the area to be developed by a given operation.

A d-c. transmission system for small mines, having a

small assigned acreage, may be designed for efficiency and installed with reasonable expenditure. However, the cost for such a system for the larger mines with large assigned coal acres becomes prohibitive, both because of initial investment and subsequent operation.

The economical installation for such an operation employs alternating current for transmission with converting substations at load centers for the underground a-c. equipment and transformer substations for a-c. equipment.

A capital fund when a large operation is opened, set aside at four per cent interest to cover the cost and maintenance of such a system throughout the life of the property would be from 25 to 40 per cent of the sum required for a complete d-c. system, and the average efficiency of transmission would be at least 15 per cent lower with the direct current. It is impractical to consider a d-c. transmission system on the same basis of efficiency as an a-c. system with converting substation, as the returns from such an increased efficiency would not be adequate for the additional investment required.

An investigation for a large operation with an existing d-c. transmission system determined that to bring the transmission up to reasonable efficiency, additional copper at an estmated cost of $87,000 would be required. There would also be a yearly expense of approximately $300,000 additional for the life of the operation for reinforcement of copper. The initial expenditure for installation of motor-generator sets and substation was estimated at approximately $100,000 with an additional cost of $45,000 for changes of lines, bore holes, substations to keep the latter reasonably near load centers throughout the life of the operation. Therefore installation of motor generator sets showed a saving of $242,000 in capital investment through the life of the mine. The use of the motor-generator sets near the load centers made it also possible to maintain an average of 16 per cent better efficiency.

The advantages derived from a-c. transmission will outweigh any advantages that might be secured by d-c. motor drive for the tipple units, mine fan or shop equipment.

Therefore, the source, whether from a public utility company or a plant at the mine, should provide a-c. power.

Coal Hoisting

In planning a mine, and especially a mine of large capacity, the effect of loads and rope on the size, power costs, and initial investment for the equipment should be considered. The rope speeds that can be obtained in hoisting are limited; hence the maximum capacity of a mine is set by the load carried per trip. Where large capacities are set for mines on a basis of cage hoisting, it is always possible to reduce the hoist initial cost and the hoist operating cost by using skips whereby two or more cars of coal can be hoisted at one trip.

Table No. I and Fig. 1 will give a comparison between the hoisting equipment, the power consumption and the initial cost for two assumed installations, all designed for the same capacity. The first is a self-dumping cage installation and the second an installation using overturning skips.

Comparisons are made on the basis of a 400-ft. shaft and a hoisting capacity of 800 tons per hour. This, of course, is somewhat high for cage hoisting, although it is still within the limits of actual practise. If the comparison had been made on the basis of 1000 tons per hour, or for a deeper shaft, the skip installation would show only a moderate increase of power consumption while the cage installation would increase in such a rapid ratio as to soon exceed the practicable limits of construction and operation.

Obviously comparisons throughout could not be

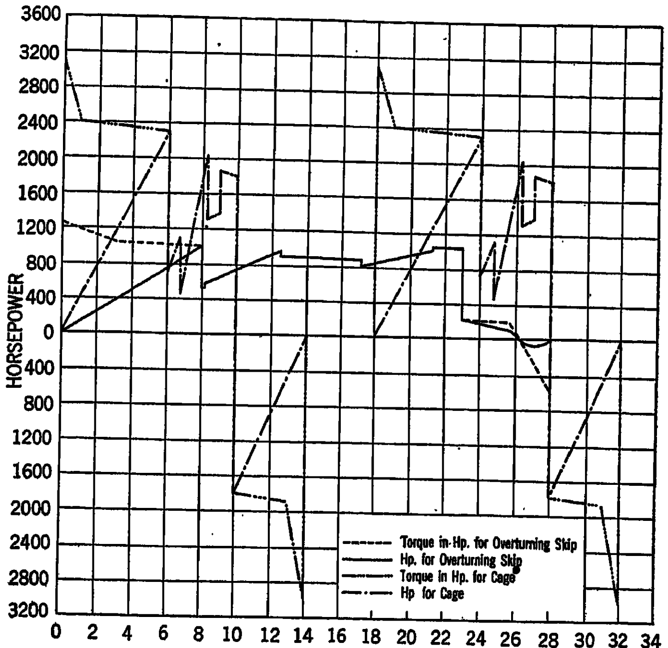

Fig. 1

made for two existing mines of these types, as it would be impossible to obtain an exact paralled. The figures used, however, are based on actual results and are worked out upon the same basis, so as give a true comparison between the different systems and for conditions assumed.

Comparisons are made on the basis of an equalized hoisting system which will limit the peak from the power source to approximately the average. On some power systems the peaks as imposed by the hoist motor used in connection with the skip hoist would be permitted and an induction motor type hoist could be used with a material reduction in first cost. (See Fig. 1.)

It will be noted that the peak of the cage-hoisting cycle is about 3100 h. p. while the skip-hoisting cycle is about 1300 h. p. The skip installation shows a power

economy of nearly 25 per cent as compared with the cage type. There is also a very great difference in cost between the hoisting equipments for cage and skip, this being approximately $37,000 in the examples cited.

Where steam hoists are used, there is not so great a difference in favor of the skip since the size of the hoisting engine is determined by the total load rather than acceleration peaks. However, the steam consumption will be reduced approximately as for the examples shown for electric hoists.

The consideration of steam hoisting is necessary only in case power is generated at the mine. For the Donk Brothers Coal & Coke Company's Mine No. 4, where there was a decision to generate rather than purchase power on the basis of savings that might be affected, the relative economies of a steam-hoisting engine and an electric hoist were carefully considered from every possible angle.

TABLE NO. 1

	Self-Dumping Cage	Overturning Skip
Depth of Shaft	400 ft.	400 ft. Pit 24 ft.
	Tipple 60 ft.	Tipple 46 ft.
Total Hoist	Dumping 6 ft. = 466 ft.	Dumping 19 ft = 489 ft.
Capacity	800 tons per hr.	800 tons per hr.
Hoisting Cycle: Loading or caging	4 sec.	8 sec.
Accelerating	6 sec.	8 sec.
Running	4 sec.	15 sec.
Retarding	4 sec.	5 sec.
	18 sec.	36 sec.
Weight of cage or skip	14,000 lb.	13,000 lb.
Car	4,000 lb.	....
Coal	8,000 lb.	16,000 lb.
	26,000 lb.	29,000 lb.
Size of hoisting rope	1½ in.	1⅝ in.
Hoist during dump	6 ft. 0 in.	13 ft. 0 in.
Net wt. available for OtWt.	12,000 lb.	8,400 lb.
Average rope speed	2,000 ft. per min.	1,050 ft. per min.
Max. rope speed	3,610 ft. per min.	1,480 ft. per min.
Size of cylindro-conical drum	7 ft. to 11 ft. diam. Direct connected	8 ft. to 11 ft. diam. Geared
Size of d-c. hoist motor sq. r. m. s.	2,000 h. p.	850 h. p.
Size of d-c. generator on Illgner set	1,400 kw.	600 kw.
Size of a-c. motor on same	800 h. p.	600 h. p.
A-c. input per hour	656 kw-hr.	525 kw-hr.
A-c. input per ton of coal	0.82 kw-hr.	0.650 kw-hr.
Hoist-overall efficiency based on 466 ft. hoist	43.0 per cent	53.8 per cent
Approx. cost f. o. b. factory 1920 basis) drum	$ 25,000.00	$30,000.00
Electrical Equipment	90,000.00	48,000.00
	$115,000.00	$78,000.00

The coal is hoisted in skips and the data upon which the hoisting equipment was based are:

Total distance of hoist	304 ft.
Maximum weight of coal per trip	18,000 lb.
Average weight of coal per trip	12,000 lb.
Weight of skip and yoke	11,800 lb.
Rest period	9 sec.
Total cycle	24 sec.

An electric hoist for this service must necessarily be of the equalized type, as the generating equipment installed for economical plant operation would not meet the peak demands of an induction-motor hoist superimposed upon the peaks of the mining load.

To secure comparable economy in plant operation when hoisting with steam equipment, the exhaust from the hoisting engines must be utilized. This requires the installation of a mixed pressure turbine with a regenerator to store and regulate the flow to the turbine of low pressure steam in accordance with the demand.

The electric-hoisting system would permit of greater flexibility of plant operation and give greater insurance against shut-down. The economy of the whole operation with steam hoisting depends upon utilizing the hoist exhaust steam for power generation. A failure in the mixed pressure turbine would necessitate wasting this steam and, for lack of sufficient boiler power a probable curtailment of production. Such an emergency might be cared for by the installation of a spare mixed pressure turbine, but this expenditure is hardly warranted. With the electric hoist all of the turbines would be of like type, and spare capacity could be provided for hoisting and mine load by one unit.

Comparative estimates of cost and economies of the two systems determined that the electric hoisting system would require an additional investment of $42,500 for hoisting equipment, buildings, installation, etc. The savings that could be effected by the electric hoist, including operation, supplies, maintenance, amortization, etc., for the two systems, equaled a 5.3 per cent return on the additional investment.

On this basis the steam hoisting system was installed with equipment as follows:

1 Pair 24 in. by 42 in. first motion hoisting engines.

1 937-kv-a. mixed pressure turbo-generator with the prime mover designed to carry 937 kw. with all low pressure steam at 0-lb. gage.

29 ft. dia. by 25 ft. long Rateau regenerators.

14,000-sq. ft. surface condenser.

The hoist equipment employs a double-rope system which so far as is known, is new in operation of this kind. A one-inch hoisting rope is used, both ends of which are fastened to the drum and coiled on in parallel grooves. The tension is equalized by a sheave on the skip-yoke.

This system permits of the use of a smaller diameter drum at a higher speed and makes it possible to use a first motion steam hoist. It will also be an advantage for installations of electric hoists when the single rope requires very low-speed motors or an excessive gear reduction.

When power is purchased, the coal hoisting equipment should be electric, except possibly in the case of changing over an existing mine with but a short life. The practise of purchasing power for the mining load and maintaining a steam plant and engines for hoisting, costs the operator from two to six cents per ton. Completing the electrification of such a plant has shown as high as 22 per cent return on the additional investment required for the installation of an electric hoist.

The system of electric hoisting applicable to any installation can be determined only by an analysis of the requirements to determine comparative efficiencies, operating characteristics, costs, etc., of the several types. The power source, limitations of peaks and peak penalties imposed by the power company serving the installation also have an important bearing on the proper equipment to be used to secure the desired mine output.

In designing the hoisting equipment for the Chicago, Wilmington & Franklin Coal Company, Orient No. 2 Mine, only the Ilgner-Ward Leonard system was considered. The hoist has a potential capacity of 14,400 tons in eight hours, with an expected average of 11,500 tons, making the operation the largest shaft coal mine in the world.

The coal is hoisted in skips and the data on which the hoist was based are:

Total distance of hoist	607 ft.
Maximum weight of coal per trip	26,000 lb.
Average weight of coal per trip	22,000 lb.
Weight of skip and yoke	15,100 lb.
Rest period	9 sec.
Total cycle	26 sec.

To fulfill the cycle, a maximum rope speed of approximately 4000 ft. per min. with a rope tension of 67,000 lb. for about 40 per cent of the travel, is required. This duty necessitates a carefully made rope provision for the fatiguing action which would result from the high-operating speed with the pronounced vibration that would be set up. A factor of safety of 8.1 based on the dead load is used. This factor of safety is provided by a two-in. diameter rope of the modified Seale construction using special strength plow steel wires.

The drum is of the cylindro-conical design having a minimum diameter of 10 ft., a maximum diameter of 17 ft. with a pitch of the grooves on the slope of 12 in. To take care of the side wear of the grooves which is greatest on the small cylindrical portion, the groove pitch for this section is increased to 2¼ in. as compared with 2⅛ in. on the large cylindrical portion. In designing the drum, careful consideration was given to the proper proportioning of the metal in the various sections and the ribbing to give strength and long life. Truss rods are provided which bring all parts of the drum shell in compression, thus giving the greatest possible strength to withstand the severe service expected.

The motor equipment consists of two 2000-h. p., d-c. force-ventilated motors, direct-coupled to the drum shaft one on each side of the drum,—two motors instead of one in order to reduce the armature inertia and permit the use of commercial units.

The forced ventilation made it possible to reduce the motor capacity required by approximately 20 per cent, thereby reducing the armature inertia, as well as the cost of installation. Fan capacity for 35,000 cu. ft. of air per minute is provided, together with necessary air washing equipment. The use of the forced ventilated equipment made a net capitalized saving of approximately six per cent.

The motor generator-set converting the power to direct-current for the hoist as received consist of two 1650-kw., 500-volt generators at 575 rev. per min., driven by a 2200-h. p. wound-rotor type induction motor. A 90,000-lb. flywheel 12 ft. diameter provides the necessary energy for equalization of hoist peaks and a 50-kw. exciter furnished excitations for the generating and hoist motors. All units are mounted on a single built-up bed frame forming a four-unit, six-bearing motor generator set with flywheel.

The units are connected in the order, generator No. 1 to motor No. 1 to generator No. 2 to motor No. 2, thereby limiting the potential on any part of the equipment to 600 volts.

Slip-regulator control equipment automatically varies the speed of the set with the load and permits the flywheel to hold the input from the line below a predetermined valued and when hoisting on the given cycle will hold the power input to within five per cent of the mean.

The Ward-Leonard system of control is used and the generator field is varied by contactors controlled by a master controller and current limit relays which function during both acceleration and retardation.

To maintain the cycle necessary to secure the large output expected from this mine, high rates of acceleration and retardation are necessary. The calculated rates of acceleration of the down-going skip is 13⅛ ft. per sec. per sec. Tests on rates of accelerations place the maximum without a dangerous slackening of the rope between 15 and 19 ft. per sec. per sec. With current limit control of acceleration only, it would be possible to attain a rate of drum acceleration with light skip loads or empty skip which would be dangerous. To prevent this occurring, the field contactors are controlled in addition to the current limits by a traveling nut switch which regulates the rate of building-up the generator voltage, thereby giving approximately constant acceleration rates irrespective of the load.

While the control is equipped with current limit relays to limit the pump back load in retardation, there is a question whether these should be depended upon for retardation in emergencies when the maximum rate possible is desirable. The retardation time can be reduced as compared to retardation with current limit relays by the use of a Tirrill type relay which will hold the regenerative breaking current value to a constant which may be the maximum capacity of the generators, or at a safe retardation rate.

In either of these cases, however, if the main breaker goes out, control of the hoist must be maintained entirely by the brake. To take care of this emergency there is provided short-circuiting resistance controlled by contactors which will be immediately inserted across the terminals of the hoist motors with the opening of the main breaker. This resistance will give maximum

safe load for the motors and this load will be maintained as near constant as is possible by two additional sets of contactors which close successively when the motor armature voltage is reduced by the slowing down of motor or drum speed to a predetermined value.

Provision for prevention of overspeeding at any point in the shaft, for slowing down when approaching dumping point, for overtravel and for starting in wrong directions are provided in duplicate by traveling nut-limit switches, cam turnoff device and limit switches, with necessary back-out switches. Men are not handled by this hoist; therefore protection for man hoisting is not necessary.

In order to properly coordinate the operation of the bottom dump station with that of the hoist to secure maximum hoisting speed, signals are given and certain gates are operated by the hoist control devices.

To thoroughly explain this feature, a description of the method of handling coal at the dump station is necessary. The coal is dumped by means of an air-operated rotary dump directly into weigh pans. The weigh pans are manually operated and discharge the coal through a chute to the skips. At the end of this chute there is a safety gate which is open only when the skip is at the bottom. These gates are operated by a crank-shaft which revolves 180 deg. in opening and closing. To this crank shaft a motor is connected through a train of gears. On the second reduction gear shaft there is a magnetic clutch for each gate.

There are three lights in front of the operator on the dump floor to show respectively when the skip is in the shaft clear of the safety gate, when the safety gate is opening or closing, and when the skip is landed and the gate is in the proper position for loading.

When the descending skip has just passed the lip of the loading or safety gate, a contact on the traveling nut limit switch releases the gate brake and sets the clutch to rotate the gate to open. When the gate is open a cam switch on the crank shaft resets the brake and releases the clutch. This operation requires about $^4/_5$ of a second.

The dump operator opens the weigh pan with the signal that the safety gate is opening. The weigh pan, in opening, energizes a time relay after which an interval of eight seconds gives direct to the hoisting engineer the signal to hoist.

Similar operations control the closing of the safety gate. The operations are repeated for both skips so that the hoist works on a set schedule and relays reduced to practically nothing.

Man and Material Hoist

The handling of men and material with the coal-hoisting equipment consumes considerable time, and, in many incidences, seriously curtails productions. An auxiliary hoist, preferably at the air shaft where men and materials can be handled at all times without interfering with coal hoisting, is desirable at any mine and a real necessity where large capacities are involved.

When skips are used for hoisting, the cage equipment at the man and material shaft must be suitable for handling solid-end cars. To meet this requirement at the Thermal Mine of the Donk Brothers Coal & Coke Company, and the Orient No. 2 Mine of the Chicago, Wilmington & Franklin Coal Company, an overturning cage is used.

An auxiliary hoist may be used for the early development of the mine as well as for handling rock, coal, men and material throughout the life of the mine. Both mines mentioned were developed to a daily capacity of approximately 2000 tons before the main hoisting equipment was put into service.

The man and material hoisting equipment for the Thermal Mine consists of a pair of first-motion steam engines. The exhaust steam from these units is carried to the main exhaust system with the main hoist, permitting the utilization of its steam for the mixed-pressure turbine and feed-water heating as required.

At the Orient No. 2 Mine, where power is purchased, the main drive for the man and material hoist is a 450-h. p. induction motor geared to a 7-ft. by 10-ft. diameter cylindro-conical drum, giving a rope speed of approximately 900 ft. per min. To meet the requirements of the power company's peak limitations during their lighting peaks, an auxiliary induction motor of 200-h. p. capacity with an additional gear reduction, is provided.

In order to be able at all times to operate the hoist for handling of men and material, a twin steam-engine unit is also provided and geared to the drum to be used in case of power failure. The steam is provided from a small boiler plant which must always be under steam for heating water for wash house, drying sand and heating purposes.

The larger motor is connected permanently to the drum drive by a flexible coupling. The auxiliary motor and steam engine are arranged with an interlocked-clutch device, so that only one can be connected at the same time. The three drives are also electrically interlocked, making it impossible to move the hoist if more than one drive is connected.

Safety devices are provided for both coal and man hoisting for all three driving units. These safety devices are so interlocked that it is impossible to move the hoist unless the proper set for the drive to be used is connected. Signal devices are provided to show in the hoist house, at cage landing and at the bottom landing, to indicate whether the man-hoisting or coal-hoisting safety devices are connected.

Fan Drive

Normally, the mine-ventilating fan is electrically operated. However, there is provided, for emergencies' sake, such as motor trouble or failure of power, an

auxiliary steam engine which receives steam from the auxiliary steam plant mentioned above.

This steam engine has sufficient capacity to furnish the full air requirements of the mine and is direct-connected to the fan shaft by means of a jaw clutch. The engine is equipped with a heavy flywheel to eliminate any back lash.

Considerable saving in power consumption for mine ventilation can be made by reducing the fan speed at such times as the mine is not working. However, this method of operation, has not met with much favor in the middle west field, due to the effect of varying quantities of air on the roof and to the liability of gas accumulations. The general practise is to carry a constant volume at all times. This eliminates the necessity of using a variable speed motor, the most satisfactory type under these conditions being the squirrel-cage induction motor or a synchronous motor, provided the latter is of a type that will accelerate the fan to full speed or is provided with a satisfactory clutching device for connecting the fan after the motor is up to speed.

The fan at Orient No. 2 Mine was originally installed with a 50-h. p. squirrel-cage induction-type motor and using short-center belt drive with idler. This motor to be used during the early development period and later changed to a size adequate for supplying the mine requirements.

During the development of the mine, there is, of course, a steady increase in demand for air; this is provided for by changing the pulley on the driving motor and increasing the diameter as increase in fan speed is required. In the development of the mine, the increase in the demand for air can usually be cared for by four pulley changes.

This results in a lower first-cost installation as compared with the variable or adjustable speed type motor and better operating efficiencies.

Preparation Plant

In the process of preparation and loading at Orient No. 2, the coal is handled from the receiving hoppers to which coal is delivered by belt conveyor from the auxiliary shaft and skips in the main shaft, by two pan feeders delivering to two screens. From the screens the coal passes onto picking tables and thence to loading booms. The 1¼-in. screenings may be delivered by belt conveyor to the rescreening plant for further separation by shaking and vibrating screens. In addition to the preparation and loading equipment, there are a number of conveyers for conveying of coal, degradation and gob.

With exception of the main feeders and the main screens, the equipment is driven by constant speed induction motors. In normal operation of screens and feeders, there is usually required about 10 to 15 per cent variation in speed to take care of the different rates of feed required, and the different conditions of the coal which may require a faster or slower screen speed in order to secure the best separation.

Wound-rotor type motors do not prove entirely satisfactory, it being impossible to maintain a fixed partial motor speed with varying load conditions and for the most satisfactory results, motors having shunt characteristics must be used.

For this duty the a-c. motor commonly known as the brush-shifting type, should prove satisfactory, as it is not frequently necessary to vary the speed, hence the mechanical control of speed would not be objectionable.

There has always been considerable discussion regarding the proper housing for tipple motors and it is often argued that open type are satisfactory, since the coal dust does not materially injure a motor. There is no doubt but that the collection of coal dust even though it will not directly injure the installation, is detrimental to the motor, as it seriously impairs ventilation. This is apparently recognized even by one who advocates the open type motors, for it is a common sight on a tipple where open type motors are installed, to see small wooden enclosures built over the motors to protect them from dust.

Enclosed or totally enclosed ventilated type will give the best satisfaction in tipple operation. This policy was carried out to a certain extent at Orient No. 2 Mine, where motors which were subject to a great deal of dust were totally enclosed.

The controls of the motors are automatic and control-stations are located at two points in the tipple and two points on the rescreener, accessible to attendants normally overseeing the preparation or loading of the coal. The control-boards are fitted with signal lights and speed points at which they are operated.

Connecting conveyers, between the auxiliary tipple and the main tipple and between the main tipple and the rescreener, are provided with interlocking controls to prevent starting, or to stop units from the receiving end to prevent the possibility of delivering coal to equipment not in operation.

There is installed in the auxiliary tipple three motors aggregating 75 h. p., in the main tipple, seventeen motors aggregating 300 h. p., and in the rescreener twelve motors aggregating 290 h. p.

The motors all operate on 220-volt circuits, with power supplied through three banks of transformers. The motors on the auxiliary tipple are fed from the service transformers which supplies 220-volt power for the shop and other miscellaneous motors. For the main tipple and rescreener separator, 2200-volt lines are installed from the central switching station located in the auxiliary hoist house to two banks of transformers, one located in the main tipple and the second in the rescreener.

The motors are grouped as convenient on the distribution lines originating, at the transformer secondary busses and each line controlled by an oil circuit breaker. Control lines are provided from the low-voltage coil of these circuit breakers with cut-out

switches conveniently located so that the equipment can be shut down quickly in case of trouble.

The underground system of power distribution between the central switching station and the various buildings housing the surface equipment of a mining plant, is by far the most satisfactory. The original cost of such an installation is approximately 160 per cent of the cost of a well-designed overhead system. The saving in upkeep, longer life and elimination of power interruptions will more than effect this difference in first cost.

Very satisfactory results have been obtained with underground system using fiber conduit in concrete with lead-covered conductors.

The location of the central-switching station is dependent on local methods of operation. Wherever possible, location which would require extra operating labor should be avoided.

Converting Stations

For the operation of underground mining equipment consisting of locomotives, chain machines and mechanical loaders requiring d-c. power, converting stations must be supplied. These stations should be located at the various load centers at the face. The number of stations installed must be governed by the system of mine development, but should be as few as practical without necessitating excessive lengths of d-c. distributing stations. This can often be accomplished by combining two or more sections carrying the connecting cables along room-entries between the parallel cross or main entries. In mines of large capacity it is usually necessary to also provide a converting station near the shaft bottom to supply adequate power for the main-line haulage motors.

In general, the economical method of transmission of power from the central-mine switching station to the converting stations is by overhead lines carrying such voltage as is necessary to transmit the required load economically. High-voltage underground transmission should be avoided wherever possible as it is not only expensive, but introduces an element of danger.

The substations may be placed either above ground with the d-c. feeders entering the mine through bore holes, or the converting units may be placed below ground, thus necessitating the carrying of high-voltage feeders into the mine.

The substation on the surface has the advantage of—

Lower first-cost.

Keeping high voltage out of the mine.

Better ventilation.

Greater accessibility for repairs.

Better inspection facilities.

Can be kept in better condition.

The disadvantages are—

Makes it necessary to support heavier copper in bore holes.

Cannot be reached as quickly in case of trouble.

More liable to malicious damage.

Underground converter stations must be in fireproof rooms, preferably with concrete walls and floor and roof thoroughly secured by I-beams. Adequate provision must be made for ventilation, and, to secure the best results, an air-split should be provided with a continually moving current of air through the substation room.

The cost of an underground substation room and the installation of the equipment varies greatly because of the difference in the material underground conditions existing in different mines. In specific instances, this cost has been 200 per cent of the cost of the building and labor of installing on the surface, and it is safe to say that, with proper underground room, the cost of same will always exceed the surface installations.

Motor-generator sets have, in general, proven the most satisfactory for mine service. The driving unit should be of the synchronous type, wtih capacity for such power-factor correction as is necessary to bring the resultant power factor of the total mine load up to economical values.

Semi-automatic control,—that is, the equipment started by a push button and further operation automatic,—is desirable and will usually prove their worth in the elimination of station attendants for either surface or underground installations. These controls are now standard with manufacturers of electrical equipment and embody all the necessary safety features for protection of the units.

The necessity of providing load-limiting features can only be determined by a study of local condition. However, it is seldom that such protection has a material value in coal mining installations.

It is sometimes possible to eliminate the automatic features on the a-c. driving motor installed in underground substation. However, where the starting of the set is by manual control, there should be a man conveniently near to act as station attendant in connection with other duties.

Rotary converters are also used in mining installations, but before such an installation, a careful study should be made to determine their adaptability to local power and load conditions.

Based upon results that are being obtained with mercury arc rectifiers both in this country and abroad, it would appear that better results can be obtained by use on mining loads in place of either the rotary converter or motor generator set when cooling water is available.

The converting equipment for the Orient No. 2 Mine for a daily production of 6000 tons consists of three 300-kw. synchronous motor-generator sets, located in a concrete substation room approximately 1000 ft. from the main shaft.

When the producing entries have advanced a sufficient distance, substations will be installed near the face in at least two sections, leaving only sufficient

equipment in the present substation to take care of the main line haulage. These prospective stations will be moved from time to time as it is necessary, in order to maintain an efficient d-c. distribution system for the ever advancing working faces.

Mining Equipment

The choice of mechanical mining, mechanical loading and haulage equipment is largely determined by the natural mining conditions and mining system used. The advantages of various types of equipment for these operations from an electrical standpoint, are usually considered secondary to their ability to produce desired results in the mining, loading and transportation of coal.

The electrical features of the equipment furnished as standard by the manufacturers may often be altered to secure better economies under certain working conditions, but such changes must not be allowed to detract from the ability of the equipment to produce coal.

Discussion

Carl Lee: There are a few examples cited in this paper which might require amplification or qualification for an engineer not familiar with this particular industry. Estimates are made showing returns of 6 to 20 per cent on the investment made on labor and fuel-saving equipment. Yet the cost in some cases runs up to nearly half a million dollars. The ordinary coal company would hardly decide to add so much to its capital investment for a small return. The normal conditions of coal mining are so frequently interrupted by strikes, poor market and competition that the smaller the mine investment the nearer the operator can come to breaking even in the long run.

Without discussing the various points in detail, it might be said that this paper emphasizes the fact that in the mining industry, which has been so rapidly electrified, it is important that the electrical engineering problems be given due consideration. To do this certainly requires the services of an engineer who has had experience in that line. Haphazard decisions will likely mean a loss in one way or another, which might be avoided by the analysis and recommendations of a properly qualified engineer.

E. J. Gealy: The only correct basis of deciding whether a coal-mining company should purchase power or generate it is an economic one as presented by Mr. Adams. All phases of the question should be given due consideration.

If a coal company is not prepared to give its own power plant proper consideration and a fair opportunity to operate at best efficiencies, it is surely arguing itself into using purchased power. Often a coal company will do more to help reduce its power bill received from a utility company than it will to help its own power plant to operate economically. A power bill represented by cash to an outside company is always given more thought than costs to operate a mine power plant. Consequently a coal company will often do more to centralize its load, operate with a good load factor and good power factor when purchasing power than when generating its own energy.

At most coal mines, low-grade fuel is available for use in the power plant. This low-grade fuel if loaded, shipped and sold at a remote point often represents a loss. In such cases the coal company using this fuel in its own plant may rightly credit the plant with the amount of this loss rather than charge it for the fuel at its selling price.

Evidence of the fact that a coal-mine power plant can successfully compete with a public utility is found throughout the coal fields. Outstanding examples are the Consolidated Coal Company plant near Staunton, Illinois, which sells power to the town. Another large coal company in Central Pennsylvania has been selling power to a public utility company for years. Such examples indicate quite clearly that the advantages are not all conclusively upon one side or the other. Careful and thorough consideration must be given the question in all cases.

I also know of a company which generates most of its own power but ties in with the power company which carries the peak loads. Thus the load factor upon the coal company's plant is almost unbelievably good. The company has a large pumping load most of which is off in the day and on at night.

Specific examples show that it is quite possible to generate power at or near some coal mines. From the tipple of the No. 12 mine of the O'Gara Coal Company near Harrisburg, Illinois, one can see almost directly under his feet the coal company's power plant and also one of the largest public utility plants in Southern Illinois. Again, in Pennsylvania the Glen Alden Coal Company generates more power than the public utility company in its district.

However, there is another phase of this question to be considered. Many coal companies can effect much larger savings from capital invested in machinery used in the mining and preparation of coal than by owning their own power plant. Obviously a coal company should first consider placing capital where it will effect the largest savings or profits.

Turning to the use of motor-generator sets at coal mines, it will be interesting to note that records obtained early last year show that 73 per cent of the automatic power-converting substation equipments furnished by the large manufacturers was equipped with motor-generator sets.

Where considerable induction-motor load exists the synchronous motor-generator set lends itself admirably for power-factor correction. A large anthracite company is now using a 440-volt synchronous motor on one of its motor-generator sets and correcting power factor right where the poor power factor originates.

W. C. Adams: While an engineer's natural tendency is to put in equipment to secure the greatest efficiencies, sometimes, irrespective of the cost, there is a limit to the amount that should be expended to secure this better operating efficiency which is based on the earnings to be derived by such improvements. There is a difference of opinion of the yield required to warrant an expenditure. I believe that an investment should yield at least 10 per cent, and for conservative estimates, 15 per cent should be used.

There is a tendency and feeling among coal operators—and it is often right—that they should limit their investments in one operation to as low a figure as possible, with the thought that they can secure better returns on their money by other investments which may be operations in another field.

Often available capital is limited and it is necessary to sacrifice some efficiency to secure the essential features necessary for a successful operation.

With such a condition, an expenditure of $500,000 for a power plant might not be warranted, even though it would result in a yield of from 10 to 15 per cent. However, if such a power plant is dispensed with, it should be certain that a reliable source from which to purchase power can be secured.

Irrespective of efficiencies that may be obtained, the amount of capital expenditure warranted for a given operation must be based on all the facts covering possible earnings and return of investment. To secure the correct answer, a careful study and analysis of the existing conditions must be made and the decision must be an economic one.

Applications of Motors to Mine Locomotives

BY W. A. CLARK[1]

Associate, A. I. E. E.

Synopsis.—This paper discusses the rule of thumb method of applying motors to mine locomotives. It shows why the speed of a locomotive should be considered in selecting motor horse power. It indicates a rational method of selecting motors for locomotives for general application.

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IN successfully applying a motor to a mine locomotive, it is necessary to have the following information:

Weight of locomotive, gage, limiting height and width, maximum and average grade, minimum radius of curve and service to be performed. The weight required for a given service is determined by the weight required to haul, accelerate or brake the heaviest train on the steepest grade. The minimum radius of curve determines the maximum wheel base allowable. This, with the gage and the limiting height, determines the space available for the motor. The weight and speed of the locomotive determine the approximate horse power of the motor, and the all-day service determines the continuous capacity required of the motor to perform the service without overheating.

All manufacturers of mine locomotives have standardized on certain weights of locomotives and on definite motors for these weights varying with the gage and height limitation. The horse power of the motors for a given weight locomotive is approximately 10 h. p. per ton, but varies from this, depending upon the speed and other conditions. The basis of 10 h. p. per ton was established at the time all manufacturers built non-commutating pole motors, and all locomotives had rated speeds of six to seven mi. per hr. On this basis, the slipping point of the wheels did not require currents in the motors in excess of the commutating range of the motor. Speed should have been considered in selecting the motor horse power in addition to the weight.

If the hour rating current is not exceeded at the rated draw-bar pull of the motor, the motor will never be called upon to operate at currents beyond the commutating range and will have sufficient thermal capacity to stand the accelerating currents.

The horse power is readily determined from the formula:

$$\text{h. p.} = \frac{\text{Tractive effort} \times \text{mi. per hr.}}{375}$$

Tractive effort is equal to draw-bar pull plus locomotive friction, which may be assumed at one per cent or 20 lb. per ton. The rated draw-bar pull with steel-tired wheels is 25 per cent of the weight of the locomotive, or 500 lb. per ton. The tractive effort is the sum of these values, or 520 lb. per ton.

Substituting in the above formula it becomes:

$$\text{h. p.} = \frac{520\ T. \times \text{mi. per hr.}}{375}$$

Where T is the tons weight of the locomotive.

This gives the horse power at the wheel, at rated draw-bar pull and speed. However, mine motors are rated in horse power at the pinion, so that to find the motor horse power required, it is necessary to introduce into the formula the gear efficiency. With the standard arrangement of single reduction spur gear this is considered to be 95 per cent. The formula then becomes:

$$\text{h. p.} = \frac{520\ T \times \text{mi. per hr.}}{.95 \times 375}$$

$$\text{or} \quad \frac{\text{h. p.}}{T} = 1.46\ \text{mi. per hr.}$$

$$\text{or} \quad \frac{\dfrac{\text{h. p.}}{T}}{\text{mi. per hr.}} = 1.46$$

From this formula it is readily seen that if the horse power per ton per mile per hour is 1.5 or more, the hour rating current of the motors will not be exceeded at the rated draw-bar pull. The maximum starting draw-bar pull with steel-tired wheels is 30 to 33 per cent of the weight of the locomotive. The starting draw-bar pull will then be approximately 33/25 = 1.33 times the rated draw-bar pull. Using brakes to hold the wheels from slipping the maximum load on the motor will be the current corresponding to 40 per cent adhesion or 40/25 = 1.6 times the rated draw-bar pull. With motor outputs corresponding to 1.33 and 1.6 times the rated draw-bar pull, the corresponding current in the motors will be approximately 1.25 and 1.45 times the current at rated draw-bar pull. If the motor has a rating of 1.5 h. p. per ton per mile per hour, the maximum overload on the motor will be less than 50 per cent overload. A commutating pole motor will stand 100 per cent overload current for short periods without sparking or injurious heating. Therefore, a commutating pole motor with a horse-power rating of considerably less than 1.5 h. p. per ton per mile per hour will be large enough to satisfactorily handle the service provided it has sufficient continuous capacity.

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The ability of a motor to perform a given service without overheating depends on the continuous capacity and not on the hourly rating, although with similar designs, the continuous capacity is proportional to the hourly rating. The continuous rating of a mine motor is measured in amperes at reduced voltage in line with the standardization *Rules of the American Institute of Electrical Engineers.* The rating is in amperes so that it may be readily checked against the root-mean-square current as figured or measured for a given service. It is taken at reduced voltage because the average voltage impressed on a motor on a mine locomotive, during a definite time cycle of operation, will be approximately half the trolley voltage, or less, as the locomotive is standing with no voltage on the motors a large proportion of the time and is operating with resistance in series with the motors or with the motors in series during acceleration, at which time the voltage across the terminal of one of the motors is considerably less than the trolley voltage. This neglects the line-drop in the trolley wire from the substation to the locomotive.

The continuous rating will vary with the design and size of the motor. Since it depends principally upon the radiating surface, it will be a smaller percentage of the hour rating current on larger motors than on smaller since the surface per weight, or volume, is less on the larger motor. The continuous rating of an enclosed motor will vary from 30 per cent to 50 per cent of the hour rating current. If ventilating covers are supplied on the motor and the armature is equipped with a fan, the continuous rating will be increased in proportion to the efficiency of the ventilation. However, in a mine, a self-ventilated motor draws in a large amount of sand and coal dust. While the fan will keep the motor cleaner than an enclosed motor, the dirt in the ventilating air has a destructive affect on the surfaces over which it passes.

Another way to increase the continuous rating is the use of forced ventilation. A motor-driven blower is mounted on the locomotive and forces screened air through the main motors. By the use of forced ventilation, the continuous capacity is more than doubled. This method of increasing the continuous capacity has the disadvantage of requiring an additional piece of apparatus on the locomotive with additional space required for mounting the apparatus and the air conduits. On large locomotives this is not a serious disadvantage as it is easier to find place for the blower and air conduits than for the much larger motor required, if the motor were an enclosed motor without ventilation. The blower equipment is standard on the larger three-axle locomotives of several of the larger manufacturers of mine locomotives. It is also used on some smaller three-axle and two-axle locomotives, where mine clearances will not permit the use of an enclosed motor with sufficient continuous capacity to perform the service.

In applying motors to a locomotive for any given service, the first thing to determine is the weight of the locomotive required. As stated in the early part of this paper, this is determined by the weight required to haul, accelerate or brake the heaviest train on the steepest grade. Based on steel-tired wheels and level tangent tracks, the formula for determining the weight of locomotive is as follows:

$$W = \frac{L\,(R + A)}{0.30 \times 2000 - A}$$

Where W is the weight in tons of the locomotive required.

L is the weight in tons of trailing load $= N\,W$ where N is number of cars and W is weight in tons of car.

R is the frictional resistance in pounds per ton of the cars and varies from 15 lb. to 30 lb. depending on the weight of the car and type of bearings.

A is the acceleration resistance. This is 100 for one mi. per hr. per sec. acceleration and is usually taken as 10 or 20, corresponding to an acceleration of 0.1 or 0.2 mi. per hr. per sec.

30 per cent is the starting adhesion with steel-tired wheels.

2000 is a factor to give the adhesion in pounds per ton.

Where there are grades, the weight of locomotive required to haul the train up the grade is determined by the formula:

$$W = \frac{L\,(R + G)}{0.25 \times 2000 - G}$$

Where G is the grade resistance in pounds per ton or 20 lb. per per cent grade, and 25 per cent is the running adhesion of the locomotive.

The weight of locomotive necessary to start the train on this grade is determined by the formula:

$$W = \frac{L\,(R + G + A)}{0.30 \times 2000 - (G + A)}$$

A comparison of the last two formulas shows that a locomotive which will haul a train up the grade will have ample capacity for accelerating the train provided the grade is more than one and one-half to two per cent since the increase from 25 to 30 per cent adhesion on starting will more than make up for the addition of the accelerating resistance in the formula, since A is then relatively small in comparison with the sum of $R + G$.

Where the grade is in favor of the load, the formula becomes:

$$W = \frac{L\,(G - R)}{0.20 \times 2000 - G}$$

and to brake the train on the grade:

$$W = \frac{L\,(G + B - R)}{0.20 \times 2000 - (G + B)}$$

Where B is the braking effort in pounds per ton and equals 100 lb. per ton for a braking rate of one mi. per hr. per sec. or 10 lb. per ton for a braking rate of 0.1 mi. per hr. per sec. From the safety standpoint, the adhesion is taken as 20 per cent. It can, of course, be increased by the use of sand provided the grade is short.

If there are curves in the track of lengths equal to a train length, it is necessary to introduce in the above formulas the curve resistence, as for example,

$$W = \frac{L\,(R + G + C + A)}{0.30 \times 2000 - (G + C + A)}$$

where C is the curve resistance in pounds per ton. Curve resistance in pounds per ton for a train of mine cars may be determined by the formula:

$$\text{curve resistance} = \frac{WB \times 2000}{5 \times R}$$

Where $W\,B$ is the wheel base of the car in feet, and R is the radius of the curve in feet.

Having determined the required weight of the locomotive, it is necessary to determine whether the standard locomotive of the weight required has motors of sufficient capacity for the all-day service. If this is gathering service, it is rather difficult to figure the root-mean-square current of the motor, but the proportion of the time in which the locomotive is operating at high draw bar pull is very low, so that a motor applied in accordance with the formula of 1.5 h. p. per ton per mi. per hr. will have ample capacity for any gathering service, and a motor of smaller horse power than determined by this formula would usually have the required capacity.

In haulage service it is, however, comparatively easy to figure the root-mean-square from the motor curve, the profile and the loads. Where the hauls are not too long or the grades long and steep, the standard motor on the basis of 1.5 h. p. per ton per mi. per hr., or approximately 12 h. p. per ton, will usually have ample capacity for the service. But where the hauls are very long, so that the layover time at the end of the trip is very small in proportion to the total time, the root-mean-square current may figure higher than the continuous capacity of the standard motor. In this case, it is necessary to use a larger motor than standard for a given weight of locomotive, or apply forced ventilation.

In gathering service, the standard motors have ratings of approximately 10 h. p. per ton of the nominal weight of the locomotive, which is ample for locomotives with rated speeds up to seven mi. per hr. This speed is much higher than necessary for gathering service. Storage battery locomotives with rated speeds at 3½ mi. per hr. are able to gather almost as many cars per day as the higher speed trolley type gathering locomotives. Both speed and horse-power rating of the motors on gathering locomotives can be reduced considerably without reducing the daily output of the locomotive, but with a corresponding reduction in power consumption and peak loads. In gathering service the runs are short, and on most of the runs the load consists of one car followed by a run with the locomotive light. A locomotive which has its speed at rated draw-bar pull of 6½ to 7 mi. per hr. will have a balanced speed running light or with one car on the level, of 10 to 15 mi. per hr. This is a higher speed than can be reached in the short runs in the rooms, so it is necessary to operate with resistance in series with the motor a large percentage of the time. If the motors on gathering locomotives are designed for speeds of four to five mi. per hr. at rated draw-bar pull, the power required from the line will be 20 to 25 per cent less, the motor may have a correspondingly lower rating and the actual time of the trips of the light locomotive or of the locomotive hauling a light load will be approximately the same, since the motor will operate for a large proportion of the time with resistance cut out. On runs with heavy loads, the speed will be reduced, but if feeder capacity is limited, the reduced current draw of the lower speed locomotive will give a higher voltage at the locomotive which will increase the relative speed of the locomotive.

Discussion

C. Lee: The last paragraph of Mr. Clark's interesting paper brings out a point that has not been considered of much weight in the past, but which has recently been taken into account in some installations of gathering locomotives. It should be considered in all cases.

The duty cycle of a gathering locomotive and the physical condition of tracks, curves, etc., on which it operates certainly make it impractical to take advantage of the full-speed characteristics of a 7-mi. per hr. gathering locomotive. Locomotives built and rated at such speed have been sold to mine operators, because the operators have asked for a standard locomotive. The manufacturer offers a 7-mi. per hr. locomotive as a standard. In some cases the operator wants a locomotive that can be used interchangeably for gathering or hauling service. A locomotive built to meet such requirements is not the most practical.

Therefore, it seems that the manufacturers should build, rate and sell gathering locomotives rated at a speed of 3½ to 5 mi. per hour, as *standard* gathering locomotives.

E. J. Gealy: As an added bit of information necessary to apply a mine locomotive successfully to a given service, I might suggest that the size rail be considered. Most mining companies use smaller rails than advisable from a tractive point of view. To put a relatively heavy locomotive on a small rail is uncommon. Under such a condition the top of the rail is so narrow that a new set of locomotive wheels rides on a line contact for a considerable period before wearing down to the shape of the rail. Thus it is impossible for the locomotive to develop its full drawbar pull.

Another important detail about a mine locomotive is the clearance of the motor casings above the rails. At cross-overs and especially on uneven track, locomotive motors often drag and produce a heavy load. In service, wheels with false flanges and under-size wheels reduce the original clearance. The common practise of using undersized wheels from one type of locomotive on another type must be carried on with consideration. Excess loads may be placed on the motors by oversize wheels and the

clearance under the locomotive be greatly reduced when undersize wheels are used.

Ever since the first trolley locomotive was built the motor horse power per ton of locomotive weight has been increased. The ultimate limit for the ordinary haulage locomotive with the usual type controller, wheels and speeds, seems to have been reached. This ratio, about 12 h.p. per ton, appears to be nearly the limit because a locomotive so equipped will slip its wheels before it can be seriously overloaded. However, with the increased use of dynamic-braking controllers, which place the motors in service both when hauling and braking, still larger motor ratios per ton of locomotive weight may be required.

The ability of a slow-speed storage-battery locomotive to gather as much coal as a high-speed trolley locomotive, is significant. In gathering service, high speeds can rarely be attained, consequently much energy is used up in the control resistance to keep the locomotive running slowly. When we consider the reduced speed for which storage-battery locomotives are designed, we see a very good reason for their low h. p. per ton.

W. A. Clark: Mr. Lee feels that the manufacturers should sell as standard, gathering locomotives with rated speeds of 3½ to 5 mi. per hour. A motor which will fit in a definite space will have a horse-power rating varying almost directly with the speed of the motor. The horse power of the motor on low-speed locomotive would, therefore, be low. As shown in my paper the horse power of the motor need not be high but the general demand from mines is for large horse power irrespective of the speed. When the demand for slow-speed gathering locomotives becomes great enough, they will become standard.

Electric Lighting Equipment on Automobiles

BY J. H. HUNT[1]

Member, A. I. E. E.

Synopsis.—The convenience and flexibility of electric lighting is the principle reason for the development of the present type of automotive electrical equipment. The conflicting requirements of good illumination for the drivers and the elimination of glare for the driver of the on-coming car have lead to the adoption of light-directing devices according to specifications drawn up by the I. E. S. Lights conforming to these specifications give good results under ideal conditions but under bad road and car springing conditions cause a great deal of discomfort to the drivers of on-coming cars and considerable dissatisfaction with the results.

There is a great need of improved result. The possibility of obtaining improved result by the use of polarized light, by the means of light filters, by the use of more diffused light, is discussed very briefly.

THE convenience and flexibility of electric lighting is one of the principal reasons for the present universal use of complete electrical installations on automobiles. Electric lighting for automobiles was experimented with from very early days; the first installations provided parking lights from storage batteries, which were removed for recharging, as no charging generators were fitted. Headlights were not used, as the high-efficiency concentrated filament lamp was not available and the current requirements would have been prohibitive with the lamps available. Generator systems appeared with the development of the metal filament lamp, the early generators usually having complicated controls. However, intensive effort followed, and by 1910 generator-charging systems were applied as standard equipment on some makes of cars, to be followed by the complete starting and lighting systems a year later.

The metal filament of the six-volt lamp could be concentrated to such an extent that a very narrow beam could be obtained with a fairly short-focus (1¼ in.) parabolic reflector. This beam gave better penetration than the competing acetylene lamps, although the roadside illumination left much to be desired, which limited visibility when making turns, or when driving on winding roads. The high candle power in the center of the beam, combined with the existance of very little side illumination to aid the driver of oncoming vehicles, caused the public to protest because of glare, and laws were passed requiring dimming when meeting other vehicles.

The problem was studied by committees of the Illuminating Engineering Society and the Society of Automotive Engineers, with the final result of the adoption of the I. E. S. specifications for the distribution of the light from automobile headlamps. This distribution is shown in Fig. 1, which shows the permissible candle-power limits of the beam from a pair of lamps. This distribution curve is naturally a compromise between the needs of the man behind the lamp (who must be able to see a dark object 200 ft. ahead on the highway) and the needs of the driver facing the light, who should be able to see the roadway as well as the car in question, when illuminated by his own headlamps, and whose vision must not be impaired or dazzled for an appreciable interval after he passes the others lights. When the headlamp is placed according to the present S. A. E. recommended standard, 36 in. above the road, the light which is at the *B* point, the center of the highest permissible candle power, strikes the road 172 ft. ahead of the car. This is undesirably close, since it causes bright illumination immediately in front of the car, and hinders vision of more distant objects. The light at the *D* point is the light which strikes the eye of the on-coming driver after he has turned aside to pass. The 800 c. p., specified as a maximum, would be considered glaring by an appreciable percentage of individuals, but it does not leave persons of normal vision momentarily

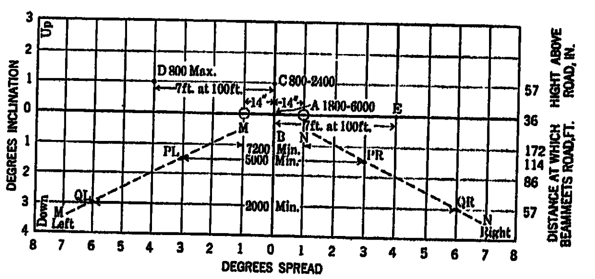

FIG. 1—S. A. E. SPECIFICATIONS FOR HEADLAMPS

All points on Curves *MM* and *NN* are six ft. from a line midway between and parallel to the axes of the lamps at the point where the beam meets the road.

blind. It must be remembered that a considerable amount of light, causing no objectionable glare when passing through a clear windshield, will make seeing quite impossible through a dirty windshield, or one covered with drops of water. Everyone could see fairly well with clear windshields, if, at the moment of passing a lighted vehicle, he concentrated his attention on the roadway about to be passed over, instead of glancing at the approaching lamps. Most of the states require that lights be left on full in passing, for investigation has shown that more accidents are caused by lack of conforming to the I. E. S. specifications than otherwise.

It must be admitted, however, that while lamps conforming completely to the I. E. S. specifications give excellent results on level, improved roads, combinations of conditions arise wherein the results are anything but

1. Head Electrical Division, General Motors Research Corp., Dayton, Ohio.

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satisfactory. On rolling roads, or even roads which cause the car frame to rock about a transverse horizontal axis (a thing which often occurs on paved roads with certain combinations of speed and car springs) the high c. p-beam is flashed into and out of the eye of an approaching driver. On narrow, high-crowned country roads, drivers head directly at each other until very near, and when they finally do swerve out, the rear end of the car tends to slide off the road, throwing the beam center of the light up and out to the left, directly into the eyes of any other approaching driver. As a result of this condition and the attendant dissatisfaction, Ohio and Michigan still require the dimming of headlights when within a specified distance of an approaching vehicle.

Today, gas-filled bulbs, giving 21 mean spherical c. p., are universally used for headlamps. These consume about 15 watts each. The 6-8-volt (rated) lamps give full c. p. at a terminal voltage of 6.5 volts. The 12-16-volt lamps give full c. p. at 14.6 volts. These voltages are the results of careful survey by the lamp manufacturers to determine the voltages actually existing on cars in service, and are determined by a weighted average, the computation being made on such a basis that the average life would be the designed life of 100 hours. Recently there has developed a very lively controversy between representatives of car manufacturers and lamp manufacturers as to whether the proper basis has been taken for rating lamps. For reasons that will be discussed in the section devoted to batteries, the voltages impressed on the lamps will vary over very wide limits, and since the designed life is very short and based on a voltage much lower than can and does exist on a considerable percentage of cars, unsatisfactorily short life occurs in enough cases to cause car manufacturers considerable annoyance. Some of the car manufacturers are experimenting with bulbs designed for longer life, and a reasonably satisfactory compromise will undoubtedly be worked out.

From the illumination standpoint, the effect of variation of voltage upon candle power, and, incidentally, upon glare, is clearly of great importance. Voltages at the lamp-terminals are found to vary from 5.9 volts to 8.3 volts, the lower voltage being due to undercharged batteries and poor wiring, and the high voltage to charging a very cold battery already pretty well charged. The law of candle power variation with voltage is the same for the automobile lamp as for the house lamp, and it is evident that the candle power at the *D* point of Fig. 1 can greatly exceed the specified minimum if the device has been so designed that 800 candle power is approached for a lamp at normal candle power.

Distribution, approaching that shown in the chart, was maintained for a considerable period exclusively by the use of a properly focused bulb in a simple parabolic reflector combined with a so-called lens, which, as a general proposition, is not a lens at all but simply a circular sheet of glass into which various combinations of prismatic sections have been incorporated. This results in rearranging the distribution of the light from the parabolic reflector, changing from a distribution giving a circular section perpendicular to the beam to one giving a rectangular section, or a very pronounced ellipse, with the shorter dimension vertical. The ideal device would take the beam from the parabolic reflector and, while leaving the full intensity in the horizontal and just below it, would divert the light above the horizontal to the sides and slightly below the horizontal, in conformity with the chart.

The pressure for reduced cost is just as effective in regard to lighting as with respect to any other details on a motor car. To meet this demand, some car manufacturers have recently adopted a headlamp in which the reflector itself has been modified from the parabolic form so as to give the desired lateral distribution of the light without increasing the depth of beam over that obtained by the plain parabolic reflector. No lens is used and it is obvious that there are many ways in which a mirror could be modified to accomplish this result.

One method which is used considerably at the present time is one having the mirror made up of small vertical sections, each of which is generated by a right line moving parallel to itself and following a parabola at its center point. This parabola has its focus at the lamp filament. Very satisfactory results are obtained by use of this principle and such reflectors have the additional advantage of being unaffected by the breakage of glass. High optical accuracy is not maintained in the manufacture of headlamp glassware, and apparently the commercial results of the use of the fluted reflector, in spite of theoretical difficulties in maintaining reflector shape, are as good as for the usual combinations of reflector and deflecting glassware.

The most frequent cause of complaint against the lighting equipment, so far as glare is concerned, is due, however, not to design but to faulty adjustment. The desire for low cost has led to the use of many devices which are not mechanically secure, with the result that lamp bulbs although once properly focused, can shake out of focus; and the driver carelessly continues to use them with this faulty adjustment. Trouble has also been caused by the lens which carries the light bending prisms turning around in the headlamp door, resulting in a light distribution entirely different from that intended. Another reason for the tendency of owners to leave lamps in faulty adjustment has been the difficulty of opening the lamp with some types of designs.

The Society of Automotive Engineers has recently adopted recommended practises calling for securing the lenses in place by a notch interlocking with a lug in the door itself, thus making turning impossible. They have also recommended a type of door more easily opened than the present type with the bayonet lock. The bulb manufacturers have made great progress in improving the accuracy of the lamp filament mounting

with respect to the base of the lamp, making an accuracy of plus or minus 3/64 in. With this degree of accuracy it is possible to design the reflector or the combination of reflector and lens so that with proper design of glassware there will be no necessity for provision for adjustment. Therefore, the distribution of the light would be within the permissible limits so long as the so-called precision lamps are used. To many familiar with the situation it seems that the commercial application of such a system will result in very real improvement for the whole situation. Replacing the bulb too often results in faulty adjustment.

Faulty tilting of the lamp is undoubtedly a cause of considerable trouble from glare. Inspection of the chart of Fig. 1 shows that if the minimum permissible candle power at the *B* point were raised to the *A* point, a light exceeding the maximum permissible would be obtained. This shift represents a shift of 1 deg. in tilt. The shorter wheel-base light cars on the market rock more than a degree, front and back, when the load is changed from one passenger to five passengers. If the headlamps have been adjusted to give a satisfactory driving light just inside the limits for one or two passengers, a glaring light inevitably results when the car is loaded. Of course, the lamps are supposed to be adjusted with the full load in the car, but when this is done the bright light of the center beam strikes the road so close to the driver that if he pays any attention whatsoever to lamp adjustment, he is almost certain to raise the angle of the beam.

Tilting headlamps, or rather a tilting of the beam, is a method of eliminating glare much more satisfactory than dimming. This can be accomplished by a mechanical control of the mirror from the dash, (a means which has been employed for several years by certain makers;) or by tilting the part of the mirror which reflects the greater part of the light flux directly back of the bulb. This tilting of the smaller part of the mirror can be accomplished by a combined spring and magnetic control without requiring a great amount of current. Recently a scheme involving the use of a double-filament bulb, having one filament at the focal point and one filament above, has been proposed. The beam is tilted down when switching from the filament at the focus to the upper filament, making a very simple electrical control. This latter method has not yet received universal approval by the state authorities but will probably ultimately find extensive use in one form or another.

Altogether we may say that the headlighting is a phase of the application of electricity to the motor car that is least satisfactory as regards relations between the motor car drivers and the public. It seems reasonable to believe that, as a result of the great demand for improvement, some method will be developed in the comparatively near future to provide real relief to the situation. However, the engineering complications are only a small part of the difficulties at the present time. We have forty-eight sovereign states, each with the right to set up their own inspection service and provide their own rules for inspection and their own regulations. While fortunately these regulating bodies are showing a great disposition to cooperate, it is obvious that they can cooperate only upon the basis of mutual consent. Such being the case, one or two gentlemen of rather fixed opinions, properly placed, can postpone the adoption of a device of merit for a considerable period, since the universal use of the motor car for touring makes it necessary that the equipment on a car sold in Texas shall be satisfactory to the authorities of California and Massachusetts, as well as all states between.

No discussion of the other lamps on the car will be undertaken here, except a brief reference to the so-called spotlight. This is a lamp small in size, but usually of high candle power at the center of the beam. 21-c. p. lamps are generally used with a reflector smaller than that in a headlamp. While it was originally intended to be a manually directed lamp used for picking up direction signs, in states requiring dimming it is usually used to light up the right edge of the road. However, being controllable, it is frequently abused and some states forbid its use entirely. Even when pointed down and to the right, the diffused light from the reflector is often so bright as to cause momentary confusion to the eye of a passing driver when the lamp is mounted high on the left windshield post as is usually the case. Therefore, its presence is somewhat objectionable. If mounted on the right windshield post, its use would be much less objectionable, especially if it were fixed. A committee of the National Conference on Street and Highway Safety has recently recommended that the mounting of spotlights on the left side of the car be forbidden.

Some possible methods of improvement might be mentioned as a matter of general interest, merely to point out possibilities, with no intention to pose as a prophet.

Dimming, tilting the headlight beam, or permanently limiting the height of the beam are all done so that an approaching driver will not be prevented from seeing objects illuminated by his own lamps, but any of these expedients on the part of the meeting drivers limits the vision of each. The most important area to be illuminated is that immediately about the approaching car, particularly the road to be passed over. Each driver might illuminate this space about his own car, and this could easily be done without glare to anyone, for very little forward-projected light would be required. This could be accomplished by switching from the regular headlamps to special lamps when cars are within 200 or 300 feet of each other. Sufficient illumination of the space about the car, especially at the left, would go far toward compensating for the blinding effect of the headlamps at full brilliancy. Since conditions of the road surface will greatly affect the results from the

illumination standpoint, it is doubtful if headlamps can be completely dispensed with by the adoption of such a scheme. An effort has been made in this direction by one car maker, but obviously the scheme cannot succeed unless universally applied, which would require legislation and impose somewhat of a hardship on owners of old cars, the electrical systems of which would not lend themselves easily to its application.

Illumination of the roadway by a permanent lighting system would also take care of the problem, and it is reasonable to expect that the main arteries of travel approaching our larger cities may be so lighted in the not-too-distant future. This is particularly desirable where they carry practically solid lines of traffic for several hours of the day, as is already often the case.

In the early days of the automobile, acetylene lamps were used without nearly so much complaint regarding glare as there is today. These acetylene lamps did not have so great an intensity in the central part of the bulb as that of the electric lamp, but they did give a large amount of side illumination immediately in front of the car, which was of considerable advantage to the driver of the oncoming vehicle. A return to this general type of illumination is being advocated today by several different interests, the claim being that the more uniform light distribution gives a much more satisfactory result, and that the lack of penetration in the center of the beam is more than compensated for by the fact that the driver's eyes need not to be adjusted to the effect of the very bright spot on the road directly in front of the car such as exists with the present type of lamp adjusted to the I. E. S. specifications. On the other hand the rather high intensity of illumination at the side of the car greatly aids the driver of the oncoming vehicle.

Two schemes of obtaining this general result have recently come to the author's attention. One of these, originated by W. D'A. Ryan, involves the use of a shallow, hyperbolic reflector, so mounted in a lamp with a bowl-shaped front glass that the filament itself projects beyond all opaque parts of the lamp and throws a light directly on the side of the road. The maximum candle power in the center of the beam is from 30 to 40 per cent of that obtained with the parabolic reflector, and with such a system, one can read the street numbers on houses being passed and still have the necessary vision of the road ahead. This side illumination is also of considerable assistance to oncoming drivers. Another system for obtaining much the same result involves the use of a large frosted bulb having the filament of greater candle power than is now used, say 36 or 40, mounted in a parabolic reflector. This lamp also gives a very satisfactory side illumination, but has much less candle power in the central beam. However, it has the advantage of simplicity and ease of application to cars already in service.

Everything indicates that such types of lighting will be given rather careful consideration in the near future, as there is a determined effort on the part of many influential people to have the whole question of road illumination reopened and the basis of the present approved design reconsidered. Preliminary tests indicate that very satisfactory road illumination can be obtained in this way, the penetration being quite sufficient for road speeds up to the usual 35 mi. per hr., and the side illumination gives an added feeling of security on winding roads, as compared to a narrow concentrated beam. However, a large amount of light rising from the lamps would cause trouble in fog sooner than a beam with a cut-off parallel to the road.

One reason for the adoption of the type of lighting just mentioned is the difficulty of maintaining satisfactory inspection, at reasonable cost, of lamps installed according to I. E. S. specifications. It is claimed that in states where inspection is maintained one can have his lamps adjusted at an official inspection station in one town and be stopped by the police in the next town, a new adjustment being required. Where inspection is not provided for, the results are naturally bad. Ohio required approximately I. E. S. results for a short time, all devices approved being subjected to test. There is every reason to believe that the preliminary inspection was carefully done, but as no inspection was maintained on service equipments, the glare situation became so bad, that the law requiring state approvals of devices was repealed and a dimming law enforced. It is claimed by the advocates of the lighting schemes involving the spreading light with the candle power of the central beam reduced in intensity, that inspection in service would become a very simple matter.

A system of some theoretical interest has been proposed, based on the idea of the use of polarized light. If the headlamps of a car emitted plane polarized light, polarized parallel to a plane inclined 45 deg. to the vertical with the lower end at the left, and a plate on the windshield having the property of polarizing by absorption, (as in the case with tourmaline), were arranged with plane of polarization at the same angle, it is obvious that a driver would see objects illuminated by his own lamps, and that the light coming directly from the lamps of a similarly equipped car directly facing the first, would be quite completely cut off with no glare. Goggles containing analyzers could be substituted for the polarizing sheet on the windshield. It would probably be commercially possible to polarize the light emitted by reflection without prohibitive cost or weight in the headlamps. High optical properties are required in the system through which the reflected light is returned, and no substances of the necessary properties for a polarizing windshield plate are yet known, while analyzing prisms for goggles are quite out of the question commercially. Therefore, it will probably be some time before such a system could be put in use. One cannot recommend the expenditure of a great amount of money in the development of the required materials, since such a system would require universal legislation

to put it into application, under which conditions the restrictions likely to be imposed would undoubtedly prevent any very great profit in its application. With the proper materials, however, it would seem that the head-lighting problem should be solved.

Glare can also be avoided by other schemes permitting the driver to see the light on the road as thrown by his own headlamps, but cutting out the light of the headlamps of oncoming cars. One method which might be used would be application of colored filters, the two-filter combination being so selected that the amount of light from an ordinary tungsten headlamp passing the filters of two colors, would be so small that the light which would pass through the filter on the headlamp of the one car and then through the filter on the windshield of the meeting car would be minimum and could, therefore, cause very little glare. Thus, if all drivers going east or north, used orange filters in their headlamps to give sharp cut-off, stopping all light of wave lengths shorter than 580 $m\ \mu$ carrying the same colored filters on their windshields, while all drivers in the reversed direction had blue-green filters, cutting off all wave lengths longer than 580 $m\ \mu$, it is obvious that there would not be much trouble from glare.

Roadside markers, informing drivers which filters to use, could be incorporated in the ordinary route markers, and simple means to change from one combination of filters to the other, controllable from the driver's seat, could be incorporated in the design. The dividing line for the colored filters should be chosen for wave lengths longer than that corresponding to maximum sensitivity of the light, since more energy would be available for the longer wave lengths than for the short.

However, compared to the polarized light scheme, and due to the loss of light of the most effective wave lengths, such a system would have the disadvantage of lower efficiency, since filters with a vertical cutoff line are not now or likely to be available, and the cutoff comes close to the point of maximum sensitivity. There would also be difficulty for a small percentage of persons having a very bad form of color blindness, and the affect of the colored light on the roadside might be objectionable to some people, although preliminary experiments indicate that the subjective effect is not as great as one might think. Changed color for danger signal lights would also become necessary although this would be only a minor part of the difficulty which would be caused by the fact that it will be necessary to have universal legislation.

An attempt has been recently made to sell such a system to motor car manufacturers. In this particular system, invented by Karl D. Chambers, a green filter is used for one combination and a magenta for the other, provision being made for the equivalent of two pairs of headlamps, one pair for each color and for means of switching from one colored headlamp to the other when switching filters. To take care of the rather large losses in light flux, due to the fact that only a small part of the visible light is used, 100-c. p-lamps are used in place of the usual 21-c. p. The light flux which passes through the filters on the headlamps is approximately equivalent to 21 c. p. of white light. Road tests have demonstrated that this system does eliminate glare, as when using it one can see the whole of the approaching automobile instead of simply the headlamps as at the present time and also see this automobile in its proper relation to the roadway. The greatly increased current consumption requiring a larger generator and a new current control make an undesirable increase in the cost. This would undoubtedly make it difficult to get the required legislation as the great majority of the cars on the road are of the type where it would be exceedingly difficult to fit generators of the capacity that would be needed.

The schemes which have been mentioned above will undoubtedly seem very wild to the average engineer. However, it appeared desirable to bring them to your attention in the hope that they might stimulate thought on this very important question and possibly cause the development of more practical plans than those discussed. A proper and universally satisfactory solution of the headlighting problem is one of the most important needs in the automobile industry. There is no disposition on the part of the author to seem to criticize unkindly the work of the I. E. S. in developing our present specifications. Under ideal conditions there would be very little criticism from anyone. Unfortunately, however, conditions are far from ideal and there is a great amount of adverse criticism. It, therefore, seems necessary to reconsider the whole question, as a joint committee is now doing. If it is found impossible to develop a more satisfactory method of meeting the problem that we now have, we shall at least have convinced more people that this is the fact.

Discussion

R. N. Falge: Mr. Hunt points out that with headlamp beams which meet the I. E. S.-S. A. E. specifications at the *B* point (175 ft. ahead of the car) the vision of distant objects is interfered with by high road brightness near the car. It is true that this criticism applies to some of the equipments which meet that specification, but only because they are poorly designed. The fault is not with the intensity directed to the *B* point but rather because of the excessive intensities directed to the lower angles and striking the road near the car. Such undesirable light distribution is not at all inherent in the specifications. The better equipments follow the *S. A. E. Recommended Practice* which provides for a relatively high intensity at the *B* point and a gradual reduction in intensity at the lower angles to provide satisfactory uniformity of road illumination for distances of several hundred feet ahead of the car.

Mr. Hunt says that very satisfactory road illumination, quite sufficient for speeds up to 35 mi. per hour, can be obtained from lamps provided with 36 to 40 candle power filaments and large frosted bulbs when used in parabolic reflectors with plain cover glasses. That there are conditions under which this would be true, I quite agree, but as a general statement covering the range of road, atmospheric, driving and car-voltage conditions over

which headlights must operate, there is ample evidence to disprove its adequacy. One might, to be sure, drive for a considerable time with these frosted lamps without encountering the more exacting conditions for vision and be quite oblivious of the potential hazards which the inadequate lighting entailed and of the extra eye strain and fatigue imposed. When, however, a car is equipped with two sets of headlamps, such that the driver can shift at will from the frosted-bulb distribution to the recommended beams, he very quickly arrives at an appreciation of the advantages offered by the latter. We have had much experience with such facilities and the various observers have all come to the conclusion that the light from the frosted bulbs is dangerously inadequate. There is furthermore, a large amount of data available as to the minimum illumination desired for safety under different road and driving conditions as determined by numerous observers in a car equipped so that they could vary the form of beam and the intensity at all angles while driving. These data indicate that devices are needed designed to direct to the road intensities of the order of those called for by the *S. A. E. Recommended Practice.* The determinations did not involve any unusual conditions as far as atmosphere or driving were concerned although they were made on roads with which the observer was not familiar. Memory and assurance of the absence of any special hazards could, therefore, not be counted upon to take the place of vision. The frosted bulb gives a maximum of only 3000 to 4000 candle power and this has most clearly been shown to be insufficient.

So much for the requirements of the man behind the lights. But what of the man approaching, who faces but 800 candle power with the conventional devices and 3000 to 4000 candle power with the frosted bulbs. Here again careful investigation and long experience has demonstrated that something of the order of 1000 candle power is the maximum value that one can face on the road without undue interference with vision. Even with that value there is glare; hence the advantage of the further relief afforded by the depressible beam, even on level roads. Without wishing in any way to discourage the careful study of all possibilities which may lead to better driving conditions at night, I feel it desirable to point out this tremendous gap between what has been found necessary on and above the road, and what is provided with the frosted bulb.

L. C. Porter: Mr. Hunt rather strongly advocates the use of the large frosted bulbs of 36 or 40 candle power, stating that this lamp gives very satisfactory side illumination. That is true; but I should like to call attention to the fact that the frosted bulb itself becomes the light source, or at least a partial light source, and the reason that it gives the side illumination is, because it is so far out of the focal point of the reflector. For the same reason the light is spread upward as well as to the sides, and it is spread upward at such a steep angle that lenses and other glare-reducing devices cannot control it. Headlamps using such bulbs, therefore, would be very glaring, and unquestionably exceed the glare limits set by the various state laws. A headlamp of this type reverts to the same general conclusion as the old Warner lenses, which were beautiful from the driver's point of view, as they illuminated all of the surrounding scenery,—even to the tops of the trees, but were abominable from the point of view of the approaching driver, and for that reason have been absolutely ruled off from the road.

W. D'A. Ryan (by letter): This paper is a direct and timely exposition of one of our greatest public menaces resulting in accidents and mortalities which are mounting at an alarming rate with each succeeding year. Unless something tangible is done to improve the situation the present automobile headlight laws and regulations must of necessity fall into disrepute. It must be admitted, however, that were it not for the I. E. S. specifications, which have been quite generally adopted, the situation would be much worse.

Some maintain that the majority of accidents are caused by glare, others claim that glare may be disagreeable but not particularly dangerous and that insufficient light on the road surface is the weak point. I am inclined to believe that if glare were eliminated we would find that we had sufficient illumination on the road surface at the present time when using the best units available. I am not opposed to increasing the illumination and believe it should be done if possible but this would be of little avail unless glare is eliminated. Just so long as we are allowed from 800 to 2400 candle power at the *C* point dangerous glare cannot be eliminated and we may as well realize this fact now. If the headlights are set for 800 to 2400 candle power at *C* point we will have glare but will comply with the range of 160 to 180 ft. as called for in the different states. If we cut the light down so that there is no glare at *C*, the range in most cases will be reduced to less than 100 ft. which is not sufficient for safe driving. This combination is brought about by the inherent defects in the design of a majority of the headlights in use to-day.

There is no rough and ready way of checking the I. E. S. specifications with any degree of accuracy in the existing testing stations. The rule now seems to be in a great many of them to tilt the headlights by bending the forks or otherwise until they consider that there is no glare, with the result that the maximum light strikes the road surface far short of the legal requirements for range. As Mr. Hunt points out, the lamp adjustment practised in one state does not of necessity comply with the adjustments in other states. I know of two cities within a comparatively short distance of each other which have entirely different ideas as to what is legal and what is not. You may have your lamps adjusted in one city and passed o. k. and be arrested in the other for improper adjustment. If the facts were known very few headlights, if any, in service would be found to comply strictly with the legal specifications on all points, and as matters now stand it is difficult to offer any useful constructive suggestions for a radical improvement before something tangible is available in the way of improved automobile headlights, other than to suggest federal control or other means of unifying the regulations in all of the states. Until this is accomplished the present absurd situation is bound to continue.

One serious objection to the use of frosted lamps, especially of high candle power, (36 or 40 as mentioned in the paper), is that the entire reflector becomes a disagreeable source of glare and as Mr. Hunt points out a great deal of this light would be reflected upwardly with sufficient intensity to give serious trouble in fog. This can be obviated with clear-bulb lamps by projecting the main beam so that the high candle power rays do not rise above the horizontal. I have found from experience that driving with such a light when you are looking over the main beam to the distant range and not through it, allows good visibility in the fog. This is further helped by direct light (not reflected) in the front and to the sides of the machine which, on account of the low candle power intensity, does not brilliantly illuminate the fog but is of sufficient intensity to make the sides of the road, edges and fences discernible.

After headlights permitting control of glare are placed on the market, high-candle power lamps to give greater road illumination may be approved, but until such time it will be difficult to convince the authorities having to do with automobile headlight regulations that a 21-candle power lamp, improperly focused, is a much greater source of glare than a 50-candle power lamp operated under proper conditions; so that for the present we are confined to the 21-candle power lamp, (which is standard for most of the states) and any improvements effected must be the result of better distribution, the utilization of stray light including that now producing glare and the elimination of losses in the housing and front door or lens. In view of the fact that there are few headlights today that are effectively utilizing more than one-third to two-thirds of the total lumens of the source, there is a fair opportunity to obtain considerably more useful light even under

the 21-candle power handicap and other limitations imposed by the diversified laws of the different states.

The requirements of a first-class headlight can be summed up as follows:

First. A non-glare unit having a range between 200 and 300 ft. on a level road. It should be non-focusing, capable of operating with lamps of any candle power without change of focal adjustment.

Second. The light distribution should be of fairly wide characteristic with reasonable depth and should be homogeneous with a gradually increasing intensity from a point near the machine to the most distant point and the reflected beam should not rise above the horizontal. There is always sufficient light from even a macadam road surface to take care of softening the cut-off above the horizontal at long range. The area of greatest intensity should not be concentrated in a small spot of high candle power but should have a reasonable lateral divergence. It is important to bear in mind that a very intense spot, particularly on a wet road surface introduces a new element of glare (reflected) which should be avoided.

Third. A reasonable amount of light should be projected at right angles to the plane of the main beam and even a few degrees to the rear so as to light up the gutters and make turns safe in difficult places and also make possible the reading of road directions on either side without the use of spotlights.

Fourth. Sufficient light should be thrown on the front of the machine, that is the radiator, forward wheels and bumper, so that they are clearly visible and if one light fails there should be no chance of mistaking an automobile for a motor-cycle. The cut-off of the beam should be such that there would be no upward high-candle power rays to scatter in the fog and reduce visibility.

Fifth. There should be a general dispersion of unreflected light to illuminate trees, telegraph poles and give general vista without glare so that distance can be judged at night as in daylight driving. If the non-glare feature of the unit is further improved by lighting up the front of the machine and the general surroundings, the intensity of the source becomes less brilliant by simultaneous contrast. Furthermore, the main beam should be of such a nature that it will become even more dead as the car is approached which in turn will improve the ability of the oncoming driver to see beyond the approaching car.

Sixth. The lights should be definitely focused for city and country driving so that there would be no necessity for dimming, tilting or other manual operations which in the majority of cases with the present increased automobile traffic is impracticable, unless operated at the low point practically all the time.

Now from a mechanical point of view:

First. The lamp should be adaptable to modification of designs to meet the aesthetic lines of the car and embody the elements of true art which at a glance suggest that the unit is primarily a functioning light source rather than a decoration.

Second. It must be sufficiently rigid in construction so that it cannot get out of adjustment.

Third. It should be dust and rainproof and a simple means of opening the door should be provided so that the replacement of lamp or cleaning of reflector can be done without the use of tools or unusual exertion.

Fourth. A simple means of adjustment of the beam should be provided which will not require bending of forks, difficult manipulations or technical knowledge; in fact, so simple that anyone can make the adjustment and there will be little excuse for failure to comply with state or police regulations.

Fifth. The headlights must be produced at a cost which will not make them prohibitive even for the low-priced cars.

I believe the above specifications are well within the range of possibility and can be made without great difficulty and at reasonable manufacturing cost.

J. H. Hunt: It seems to me that as far as the S. A. E. specifications are concerned, the greatest criticism is not against the unfavorable results that come under the special road conditions, but against the difficulty of enforcement. The regulations specify definite quantities of light flux at definite angles. The enforcing officers do not use instruments to make measurements on these requirements.

In line with what Mr. Falge has stated, it seems to me there is one defect in the specification. I wish to make the point that the maximum intensity strikes the road at 172 ft. If it strikes the road at this point, it is not available for greater distances. It would seem that an additional specification should be added to the S. A. E. specifications, limiting the light below the horizontal as well as above the horizontal.

Some comments have been made as to the speed limits of driving with cars with frosted bulbs. In the chart shown in the paper, the light along the horizontal for the frosted bulbs is about half the candle power available along the horizontal from lamps conforming to the S. A. E. specification. Now, the eye is not a physical instrument. It probably works in conformity to photochemical laws, and the relation between illumination and seeing is a logarithmic relation. As a result, the visibility of distant objects when using the frosted bulbs is very much greater than half the visibility when using lamps conforming to the specifications, particularly since there is not so much interference because of high illumination directly in front of the car. It is my personal opinion that it would be perfectly satisfactory to drive 35 mi. an hour with the frosted bulbs. This is based on considerable experience.

The only reason I am bringing these particular lamps to your attention is because they seem to offer a possibility which should be studied and they are absolutely fool-proof in their application. I will defy anybody to make an installation very much worse than the best installation can be with them.

With the windshield clean, it is perfectly possible for drivers of two cars, equipped with both of these lamps, to drive past each other on the road without dimming, and maintain 35 mi. an hour, without any more risk than is involved, I believe, in driving by with a tilted lamp. It would take quite a little investigation to find out whether they will be practicable under every conceivable condition.

With respect to driving in foggy weather, I have had a limited experience with the frosted bulbs in foggy weather, and I am convinced that in a light fog, I would be just as willing to use the frosted bulbs as the S. A. E. specification lamps. I have not had an opportunity to drive in a very dense fog and it is quite possible that they will not meet that situation. I had not expected, however, very much more trouble with them than that experienced in an ordinary fog.

Load-Building Possibilities of Industrial Heating

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Synopsis.—*The author gives a brief survey of the more recent achievements in industrial electric heating, and shows the increasing tendency of industries to adopt electric heating, particularly for high grade products. Specific installation for such processes as steel treating, copper and brass annealing, vitreous enamelling, glass annealing, baking japans, cores, bread, etc., on a large scale, are described and illustrated.*

It is shown that the quality of product has been improved by the use of electric heating with little if any increase in cost, and in many cases, at a lower "over all" cost.

The desirability of heating load for the central station is emphasized, due to its high power factor and load factor.

* * * * *

THE success attained during the past few years in the application of electric heat to manufacturing processes, indicates that the electric heating load will have a far-reaching affect on the future development and growth of central stations.

A general survey of some of the more important achievements in the industrial heating field will be attempted in the hope that such a survey will assist the various central stations of the country to analyze the load-building possibilities of industrial heating in their respective territories.

The use of heat in the preparation of raw materials, including the use of arc furnaces for steel melting and electrochemical processes, will not be discussed in this paper. Attention will be directed rather to the heat required in manufacturing operations subsequent to the preparation of raw materials.

An appreciation of the extent to which heat enters into such manufacturing operations can probably best be gained by considering the various steps in the manufacture of some well-known product. A familiar example, and one of which the manufacture probably best illustrates the extent to which electric heat has superseded other heat sources, is the electric motor. Heat must be supplied for annealing and enameling the punchings, annealing frame castings, and copper, for enameling and varnishing the wire, for japanning end shields, heating coil moulds, heating impregnating tanks and baking ovens, soldering, preheating rotors, melting aluminum used for rotor bars, and for heat-treating the dies and tools used in this line of manufacture. Electric heat is now being successfully applied to most of these processes.

An analysis of the heat and power required in the manufacturing of most products reveals the fact that the energy required for heating greatly exceeds that required for power. Whether the electric heating load will ever equal the power load, as has often been prophesied, will thus depend upon the extent to which electric heat will be able to supersede other heat sources. At present, only tendencies in this direction can be definitely stated. These tendencies can probably best be brought out by considering the achievements of electric heating in individual applications. Accordingly, a few installations in each of the following fields, where electric heat is now successfully competing with other heat sources, will be discussed:

I. Steel Treating
II. Copper and Brass Annealing
III. Vitreous Enameling
IV. Glass Annealing
V. Baking Ovens
VI. Impregnating Tanks

Steel Treating. Heat is used in steel treating chiefly for annealing, hardening, drawing and carburizing.

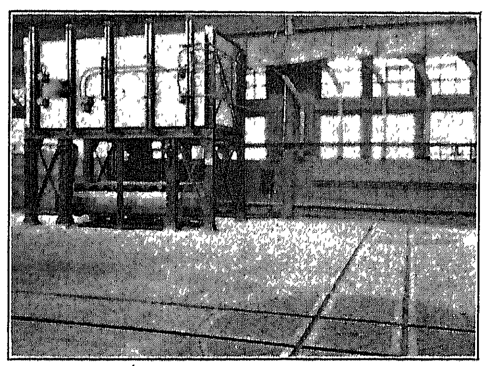

FIG. 1—ELECTRIC SHEET-STEEL-ANNEALING FURNACE

Annealing has for its main object, the softening of steel to increase machinability or ductility, the relieving of casting strain, or, in the case of electrical sheets, the reduction of eddy current and hysteresis losses.

Fig. 1 shows an electric-annealing furnace of the elevator type, for annealing sheet steel. The sheet steel is stacked directly on top of the furnace car without the use of the annealing boxes familiar in fuel-fired practise, and the car is charged into the furnace by a hydraulic elevator. This type of furnace is used in preference to the more common car bottom furnace with doors, since the air can be more effectively excluded and the furnace atmosphere controlled.

Since the heat in the fuel-fired furnace must be supplied by combustion within the furnace chamber, prod-

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ucts of combustion are present which have a strongly oxidizing effect on the punchings or other material being heated. Heavy steel or iron boxes which, in many cases, exceed the weight of the punchings themselves, are required to protect the punchings from this oxidizing atmosphere. Thus, the fuel-fired furnace labors under the disadvantage of heating practically twice the material heated in the electric furnace for a given output of punchings and, at the same time, is charged with a high annealing box depreciation. These two factors have made it possible to materially reduce annealing costs with the electric furnace.

This advantage, coupled with others of lower maintenance costs, more uniform product and better working conditions, indicate that the field for electric annealing of sheet steel is one capable of great future expansion. Fourteen electric furnaces of this type, having a total connected load of 2200 kw., are now being operated by one concern.

The widespread adoption during the past few years of electric-melting furnaces by steel foundries, is now being followed by the general adoption of electric furnaces for annealing. Since annealing can be carried on without attendance, it is usually done at night to secure the advantage of low off-peak rates. The advantages secured are a better and more uniform product, less scaling and better working conditions.

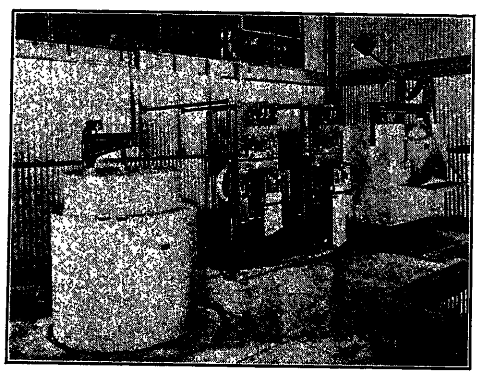

Fig. 2—Electric Tool-Hardening Furnaces

Steel is hardened by heating above its critical or transformation point and cooling quickly by immersion in some quenching medium, such as water or oil. This treatment greatly increases the hardness and tensile strength at the same time decreasing its ductility or toughness. The proper combination of hardness, tensile strength and toughness is secured by subsequent heating, known as drawing. Hardening and drawing are applied to tools and dies to give hardness and strength to the cutting edge, and to automobile parts, etc., to give maximum strength and toughness.

A typical tool-hardening room equipped with electric furnaces is shown in Fig. 2. Tool-room furnaces are usually of the box or pit type and require an energy input of from five to forty kw., depending on their size. These small electric furnaces have been found particularly useful in demonstrating to manufacturers the superiority of electric heating, and have in factories where they are used thus formed the basis for the more extensive use of industrial heating, for other processes.

Uniform heating of steel to the proper temperature throughout its mass is essential in order to secure maximum properties of hardness and strength. Even a slight deviation from these conditions will greatly reduce such properties, or may even cause the breakage of a tool or die in quenching. This clearly marks the hardening of dies and tools as the field of the electric furnace, with its accurate temperature control and uniformity of heating.

It is interesting to note that these advantages can be secured at average commercial rates for electricity without additional operating cost. To definitely establish this point accurate tests have been conducted in several plants. It might be well to add, however, that undue prominence is often attached to the cost of fuel or electricity for heating operations. A careful cost analysis, made of several steel parts, showed that the cost of electricity used for heat treating averaged only one-half of one per cent of the factory cost of the parts, while improvements in these parts equal to many per cent could, in most cases, be definitely credited to the electric furnaces.

The advent of the automobile, with its requirements for lightness and great strength, has given pronounced impetus to the heat treatment of steel. Some parts, such as ball bearings, are heated as many as six times during their manufacture. The same care and accuracy in heating are required as for tools and dies and, accordingly, many of the automobile manufacturers are turning to the electric furnace as a means of securing maximum results.

The operation of electric furnaces for this work over a period of three or four years has clearly demonstrated that not only can maximum results be obtained through their use but that these results can be secured at a lower "over-all" cost. In most cases the cost of electricity exceeds the cost of oil but this higher cost is more than offset by lower rejections, greater uniformity of hardness making possible the speeding up of subsequent machining operations, less cleaning, less labor, and more favorable working conditions which, in turn, results in lower labor turnover.

Continuous furnaces, in which the parts are pushed or carried through the furnace and quenched automatically, are used extensively for this work.

Fig. 3 shows a rotary-hearth furnace in common use for heating gears and small parts. A furnace of this type was installed by one concern four years ago. This concern now has seven such furnaces in operation with a total connected load of 1200 kw. Space permitting, numerous other examples could be given in the aggregate showing a very definite trend toward the complete

electrification of this important field with its enormous load-building possibilities.

Carburization of steel consists in increasing the carbon content of the surface of a soft steel part by heating it to high temperature in the presence of carbon. The steel parts are packed in a suitable container with carbonaceous material, heated in a furnace to the proper temperature and held at this temperature for a period

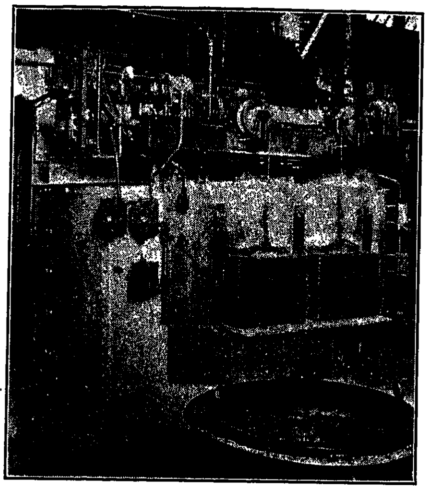

FIG. 3—ELECTRIC ROTARY-HEARTH FURNACE

of several hours. Numerous installations of electric furnaces have proved them ideal for carburizing and on the cost basis alone able to successfully compete with fuel-fired furnaces. This is due to the high efficiency of the electric furnace during the long "holding" period.

Longer life of carburizing boxes, less labor and more uniform results are advantages gained by the electric

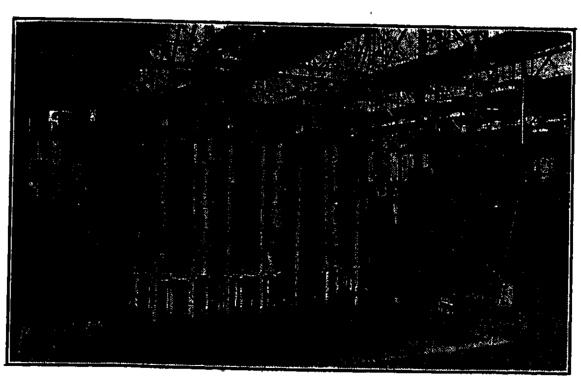

FIG. 4—ELECTRIC COPPER-ANNEALING FURNACE

carburizing furnace which promise to give great impetus to its future use.

Copper and Brass Annealing. The working of copper and brass increases its hardness and reduces its ductility. With continued working, the hardness of the metal reaches a point where any further reduction would cause it to break. It is then necessary to restore its softness and ductility by heating or annealing. In the manufacture of light gage sheet or wire it may be necessary to anneal several times. A final accurate annealing of some finished products is of vital importance. A good example is the copper used for electric motors and generators where great ductility is required to prevent crystallization and breakage due to vibration.

An electric furnace for bright annealing copper wire is shown in Fig. 4. In charging the furnace the platform, with its load of copper, is lowered by a hydraulic cylinder into a pit filled with water. The furnace is mounted on wheels and, by means of another hydraulic cylinder, is pushed into a position directly over the submerged charge of copper. Raising the platform to its original position then places the charge inside the furnace chamber. Two platforms are provided in order that one may be loaded or unloaded while the other is being heated.

Heating units are mounted on the walls of the furnace, radiating their heat directly to the charge.

FIG. 5—ELECTRIC VITREOUS ENAMELING FURNACE

Bright annealing is secured by keeping the furnace chamber filled with steam and thus precluding the air. Accurate temperature control and uniform heat distribution produce uniform results which can be absolutely duplicated in every heat.

In the fuel-fired furnace, steel muffles are required to exclude the products of combustion from the working chamber.

The chief advantages gained by electric furnaces of this type are lower costs and better and more uniform product. One company has five of these furnaces in operation with a total connected load of 900 kw.

For brass annealing, a large tunnel type of furnace is usually employed. One such furnace, with a connected load of 500 kw., has been in continuous operation for three years. The cost of electricity slightly exceeds the cost of oil but advantages, such as less scaling, greater uniformity of hardness, and lower rejections, have made the "over-all" cost in the electric furnace lower. Several such furnaces are now in operation.

Vitreous Enameling

In applying the vitreous enameled coating, familiar in lighting reflectors, kitchen ware, and sanitary ware, it is necessary to heat the metal to a high tempertuare to fuse the coating and render it adherent to the metal. The coating, consisting essentially of pulverized glass, is applied to the metal either as a thin paste or liquid by dipping, or, as a powder, by dusting it on the hot metal. The former, or wet process, is used chiefly on sheet metal parts such as kitchen ware, and the latter for cast iron parts such as sanitary ware.

Fig. 5 is a typical installation of the furnace used for enameling sheet metal. The chief requirements of the enameling furnace are: 1. High rate of production, since the labor of charging the furnace constitutes a large part of the enameling expense. 2. Pure furnace atmosphere to prevent contamination of the enamel. 3. Uniform heating to the correct temperature to maintain a high quality product. 4. Low maintenance cost. 5. A minimum of interruptions in production.

In order to meet the requirement of a pure atmosphere, it is necessary to either fire the fuel-fired furnace intermittently while the furnace is empty, thus reducing production, or to interpose a muffle between the combustion chamber and the working chamber. This also retards production and gives rise to frequency interruptions and high maintenance cost.

On the other hand, the electric furnace ideally fulfills all these requirements. At average rates for electricity, it suffers the disadvantage of a somewhat higher cost for heat. However, the large number of furnaces in operation have demonstrated conclusively, that in most cases other advantages of the electric furnace readily outweigh this higher heat cost and give a product of better quality at a lower "over-all" cost. One enameling company has, over a period of four years, increased its electric furnace installations to sixteen with a total connected load of 3500 kw.

Glass Annealing

Glass, as it comes from the moulds in which it is formed, is at a high temperature. If permitted to cool quickly in air, strains would be set up in it that would cause easy breakage. The formation of these strains can be prevented by cooling the glass slowly in a furnace or lehr. This lehr is usually of the continuous conveyer type, in which the temperature is graduated and the speed of the conveyer so set as to cool the glass to room temperature at the proper rate. There are certain critical temperatures in the cooling curve where great accuracy of temperature control is required to prevent the formation of strains.

Fig. 6 shows a typical electrically-heated glass lehr. It is heated by resistors mounted beneath the conveyer on the side walls and in the roof. These resistors are divided into five independent zones, each of which is arranged for automatic temperature control. The location of heating units gives a uniform temperature throughout the cross section of the lehr and the several automatically controlled zones make it possible to accurately govern the cooling of the glass along any desired curve.

This accuracy of control has made it possible to produce a glass of much greater strength, particularly noticeable in the reduction in breakage of electrically

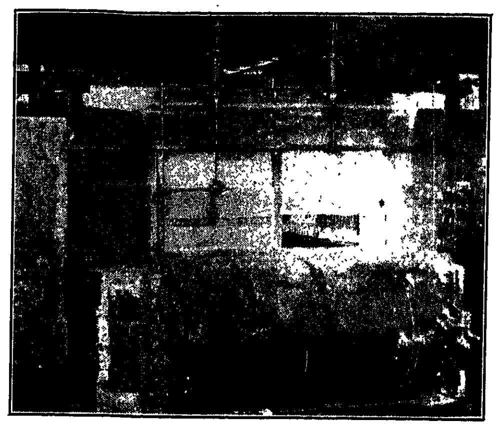

Fig. 6—Electric Glass Lehr

annealed glass in bottling machines. The cost of electricity for heating the lehr is, in most cases, higher than the cost of oil, but, as in many other instances referred to earlier in the paper, the cost of heat for annealing represents such a small portion of the cost of the finished product that even a slight improvement in quality will offset the additional cost. One company

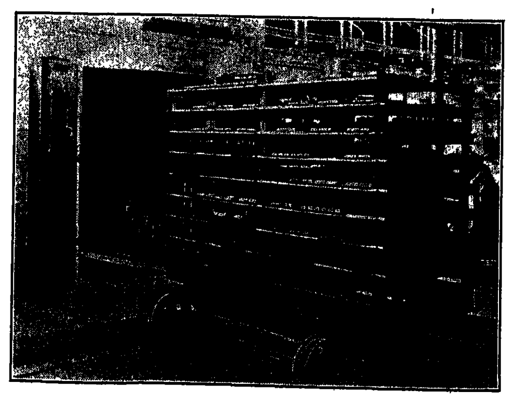

Fig. 7—Electric Core Oven

has installed eight lehrs during the past three years, with a total connected load of 1800 kw.

Baking Ovens

Baking ovens cover such a broad field that space will permit of only brief reference to a few of the more important applications, such as japanning, core baking, and bread baking.

The development of low temperature heaters for baking ovens preceded by several years the development

of suitable high temperature heaters for steel treating and other high temperature applications previously discussed. Accordingly, the low temperature applications have been more fully developed and still represent the greater part of the industrial heating load, exclusive of arc furnaces.

The extensive use of japanned metal for automobile parts, furniture, typewriters, cash registers, etc., make the field of the electric japanning oven from the standpoint of load building one of great importance. There are numerous installations throughout the country

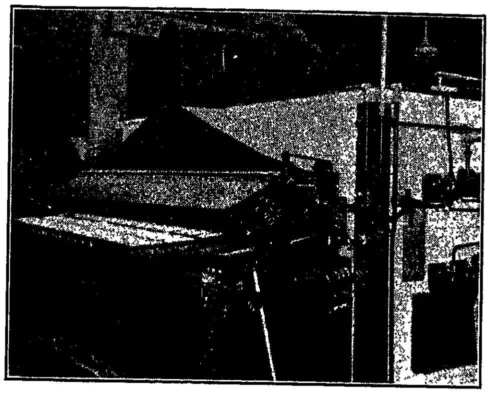

Fig. 8—Electric Bread-Baking Oven

ranging from small-box type ovens to great batteries of conveyor ovens. One company has a connected load of 12,000 kw. for japanning, built gradually up to this point over a period of ten years.

Because of the superior quality of electrically baked cores, electricity is becoming more generally used for core baking. Fig. 7 shows one of a battery of eighteen ovens used by one company. Another installation of 1000 kw. in core ovens is now in process of erection.

Bread baking by electricity has, for a long time, been successfully practised in homes, hotels and small bakeries. However, it is only recently, that the use of electricity has been considered for large bakeries. Fig. 8 shows a 450-kw. oven recently placed in successful operation. This oven is 9 ft. wide and 80 ft. long, and has an output of 4000 pounds of bread per hour. Accurate temperature control of the oven is secured by a large number of automatically controlled heating zones.

One 800-kw. oven and several smaller ovens are being used by one company for baking breakfast food. The cost of electric heat required for baking usually represents such a small part of the total cost of the product that even a slight improvement in quality will readily compensate for the additional cost of electricity for baking.

Impregnating Tanks

An interesting installation of electrically-heated vacuum impregnating tanks is shown in Fig. 9. Because of the fluctuations in steam pressure and the difficulty of securing high enough temperatures for all purposes, electric heating units were applied to these tanks. Contrary to expectations, the cost of heating electrically exceeded by only a few per cent the cost of heating with steam. Increased production as well as improved quality, made possible by uniform temperature distribution and automatic temperature control, actually reduced the cost per unit of product.

In addition to the large capacity installations discussed under these various headings, there are numerous other applications such as melting pots, soldering irons, glue pots, local heating units, etc., which, in the aggregate, represent a large load. These devices possess the advantage of low first cost and low operating cost and thus find a ready market.

The many advantages of electric heat over other heat sources thus established in numerous individual installations through a broad field of applications, indicate that the coming years will witness a great expansion of this central station load. It should not be overlooked that it represents a particularly attractive load from the standpoint of the central stations, for it is all at unity power and high load factor. Most of the furnaces

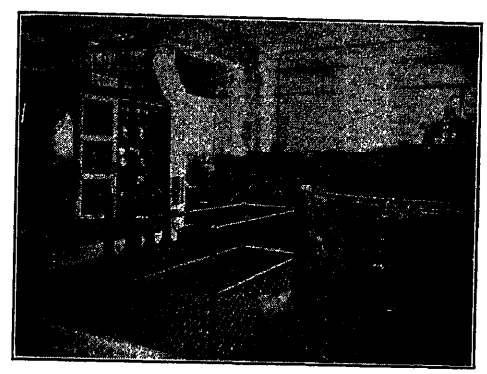

Fig. 9—Electric Impregnating Tanks

operate twenty-four hours a day but many can operate to advantage only during off-peak periods.

The rate of expansion of the heating load will depend, to a large extent, on the amount of effort spent in educating manufacturers to the advantages of electric heat. Effort was required to establish the electric light and electric motor as accepted standards in their respective fields; it now appears that similar effort on the part of electrical manufacturers and central stations will ultimately establish electricity as the accepted standard for industrial heating.

Discussion

H. N. Shaw: The maximum temperature of electric furnaces is fixed by the limitations of the heating elements available. Above 2000 deg. fahr. metallic elements are impractical due to their loss of mechanical strength, but non-metallic elements are now available which are practical for use in industrial furnaces operating at temperatures up to 2500 deg. fahr.

During the last few months, tests have been run on forging

furnaces operating at 2400 deg. fahr. in which a non-metallic element is used, and the results have proven that a forging furnace with automatic temperature control can be built so as to be as successful as the electric furnaces now used for steel treating.

This development opens up a new field for the sale of power in very large blocks to forging shops and steel mills. One large shop, for example, will require more than 20,000 kw. when all the oil furnaces are replaced by electric furnaces. Preliminary tests have shown that the electric furnace will cost less to operate and will require one less workman, due to the automatic temperature control. Besides this direct saving, the shop will be converted from a hot smoky place to a clean cool shop and thereby reduce the labor turnover.

The use of electric forging furnaces should bring about decided improvements on present-day forging practise, in a manner similar to that in which improvements in japanning practise were brought about, by the introduction of the electric japanning oven.

C. L. Ipsen: I was very much interested in Mr. Shaw's description of a new form of heating unit which he considers suitable for use in forging furnaces. A great variety of electric forging furnaces have been designed in the past which have clearly demonstrated the desirability of using electric heat for forging but none of these has proved successful on account of the high cost of maintenance.

The question has been raised concerning the power surges caused by electric furnaces. The various ovens and furnaces covered by my paper are all of the resistance type and operate at practically unity power factor. The load is of much the same nature as the incandescent-lighting load and consequently does not give rise to any power surges.

Concerning the desirability of this type of heating load to the central stations—it might be of interest to point out here that one large central-station company made a study of the relative returns from their heating load and from their lighting and power load. It was found that the lighting and power load returned a revenue of $26 per year for each kilowatt of demand and that the heating load returned $43 per year for each kilowatt of demand,—indicating very clearly the inherently high load factor of industrial heating equipment.

A High-Frequency Induction Furnace Plant
For the Manufacture of Special Alloys

BY P. H. BRACE*
Non-member

Synopsis.—*High-frequency induction furnaces have been used for some time for the laboratory preparation of special alloys on a relatively small scale, and, to a limited extent, on a commercial scale. High-frequency power has, in general, been secured from spark-gap oscillation generators. Recently the Westinghouse Electric & Manufacturing Company installed a plant having a nominal capacity of 20 tons per month, in which alloys are being produced in high-frequency induction furnaces supplied with power from a 100-kw., 5000-cycle inductor-type alternator. Zirconium silicate finds extensive use for furnace linings and thermal insulation. Alloys of great purity, which meet unusual and severe requirements, are being produced at a cost which compares favorably with that of oridnary commercial materials of the same nominal composition but having much inferior properties.*

THE plant described in the following paper was developed for the express purpose of manufacturing, on a commercial scale, metals and alloys of the same degree of purity, and having the properties of those heretofore available only as the result of costly small-scale laboratory production. The general plan of operation has been to melt the purest metals obtainable under conditions insuring the minimum of contamination, and very gratifying results have been obtained by the use of electrolytic metals and high-frequency induction furnaces of the type originated by Dr. E. F. Northrup.

The equipment of this plant falls into three main groups as follows:

1. Electrolytic iron refinery
2. High-frequency power plant
3. High-frequency furnace plant

The description which follows will be divided along these lines.

ELECTROLYTIC IRON REFINERY

The electrolytic refining of iron is no new thing, and the practise followed in this plant does not differ fundamentally from that found successful in the laboratory. The electrolyte is a distilled-water solution of the best grade of technical salts, as given by Table I.

Chemical control of the plant is maintained by periodical analysis of the electrolyte, to determine the acidity and concentration of iron as well as the proportions of the other components. The acidity is measured by electrometric titration in terms of the hydrogen-ion concentration.

In practise, it has been found that the concentration of the electrolyte and the relative proportions of the component salts may vary over a considerable range without causing serious difficulty. Satisfactory deposits are obtained with the iron concentration within limits of 45 and 55 grams per liter, and with the hydrogen-ion concentration between 1.5×10^{-6} and 0.7×10^{-6}.

The acidity is adjusted by addition of hydrochloric acid or ammonia, the latter being very conveniently added by injection into the circulating system from a tank of liquid ammonia.

The temperature of the electrolyte ranges between 25 deg. and 35 deg. cent. the only heat supplied being that due to the passage of the electrolyzing current.

In order to clarify the electrolyte and maintain uniformity of composition, it is circulated continuously through a nine-foot, five-tray Dorr thickener. The chief difficulties in the operation of the plant have been connected with the circulation and clarification of the electrolyte. These have been due to the fact that

TABLE I
Composition of Electrolyte

Salt	Quantity	Concentrations (group per liter) Fe	Cl	SO_4	NH_4
$FeCl_2 . 4\ H_2O$	75 grams per liter (4.7 lb. per cu. ft.)	21.1	26.8		
$FeSO_4 . 7\ H_2O$......	150 grams per liter (9.4 lb. per cu. ft.)	32.0		45.5	
$(NH_4)_2\ SO_4$.........	100 grams per liter (6.25 lb. per cu. ft.)			68.0	31.2
	Totals.....	53.1	26.8	113.5	31.2

aeration of the electrolyte from any cause results in oxidation of the ferrous salts and the precipitation of the iron as basic hydrates. As the removal of the iron proceeds, the acidity of the electrolyte rises, and shiny, brittle, deposits are produced, which peel from the cathode sheets and fall to the bottoms of the tanks. It has been necessary to arrange the pumps and piping and the inlets and outlets of the tanks so as to avoid turbulent flow and prevent entrainment of air. Clarification of the electrolyte is necessary to prevent contamination of the deposited iron by inclusions of sludge from the anodes.

The anodes are made from hot-rolled slabs of Armco iron two inches (5.1 cm.) thick by 28 inches (71.2 cm.) wide by 40 inches (102 cm.) long. They are supported in the tanks by two L-shaped lugs, arc-welded to the corners of the slabs at one end.

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Presented at the Spring Convention of the A. I. E. E., at St. Louis, Mo., April 13-17, 1925.

The cathodes are cut from ordinary soft steel sheets. They are clamped between brass bars which support them in the tanks. The general appearance of the anodes and cathodes is shown in Fig. 1. The anode at the right has been in service for some time and the uniformity of the corrosion is noteworthy. The cathode shown has received a deposit approximately ¼ inch (0.6 cm.) thick, and will be ready for stripping when the deposit is approximately ⅜ inch (0.9 cm.) thick.

Armco iron was chosen for anode material because it was but slightly more expensive than other suitable material and because its very small content of impurity would minimize the contamination of the electrolyte by foreign metals and sludge.

A large proportion of the insoluble impurities remains on the surfaces of the anodes as the iron is eaten away and forms a black, somewhat gelatinous, coating which interferes with the corrosive action of the electrolyte. If this sludge is allowed to accumulate, the bath will become impoverished in iron; therefore, the anodes are removed from the tanks about every

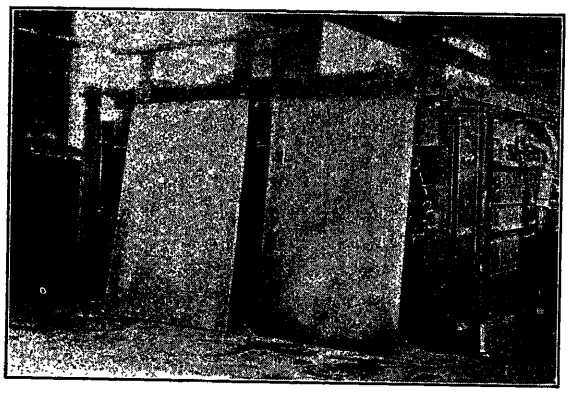

FIG. 1—ELECTROLYTIC IRON REFINERY CATHODE (LEFT) WITH ¼ IN. (0.6 CM.) DEPOSIT OF ELECTROLYTIC IRON; ANODE (RIGHT) AFTER APPROXIMATELY FOUR WEEKS' USE. NOTE UNIFORM CORROSION

third day and washed with a strong stream of water from a hose.

The cathode sheets are prepared for receiving deposits by cleaning to remove grease and rust and heating in a gas furnace to a dull red heat for a short time. A thin coating of black iron oxide is produced which prevents strong adhesion of the deposits and makes it an easy matter to strip the electrolytic iron from the sheets.

Carbon and sulphur are the most objectionable impurities. The former is removed very completely by electrolytic refining, as it remains in the tanks as part of the sludge. The sulphur in the original iron is not transferred, but some sulphur is introduced by the occlusion of electrolyte in the deposit. Analysis of a sample of good electrolytic iron by the "evolution" method will show a sulphur content of the order of 0.002 per cent or less. The same sample may show a sulphur content of 0.015 per cent when analyzed by the "oxidation" method. The latter method should always be used when analyzing electrolytic iron for sulphur because it determines the sulphur present as sulphates as well as that present as sulphide, while the evolution method shows only sulphide sulphur.

The deposits on the cathodes are allowed to build up to a thickness of approximately ⅜ inch (0.9 cm.) and are then washed for 48 hours in hot water agitated by steam jets. Two changes of water are used. By this means the total sulphur can be kept below 0.01 per cent and occasionally as low as 0.006 per cent. After washing, the deposits are stripped, broken by hand hammers to approximately two inch (5 cm.) size and stored ready for melting.

TABLE II.

Outline Specification for Electrolytic Iron Refinery

Total floor space	71 ft. (21.6 m.) x 32 ft. (9.7 m.)
Output	700 lb. (310 kg.) per 24 hr.
Current	2000 amperes maximum
Current density	10-12 amperes per sq. ft. (120-144 amperes per sq. m.)
Voltage	20 maximum
Power supply	50 kw., 2000 amperes, 20 volts separately excited, d-c. generator with field control, driven by 70-h.p., 2200-volts, 3-phase, 60-cycle induction motor.
Tanks	18 wooden tanks, connected in series, each containing 10 anodes and 9 cathodes. The tanks are 3 ft. (91 cm.) wide by 6 ft. (183 cm.) long and 4 ft. (122 cm.) deep inside.
Piping	Lead and stoneware.
Fitting, valves	Durion.
Pumps	Two Durion centrifugal pumps, rated at 100 gal. per min. (37.8 l. per min.) at 50 lb. per sq. in. (3.5 kg. per sq. cm.) driven by direct-connected 10-h. p., 220-volt, 3-phase induction motors.
Clarification	9 ft. (275 cm.) diameter, 5-tray Dorr thickener, operating with a flow of 50 gal. per min. (188 l. per min.).
Anodes	Armco iron, 2 inches (5.1 cm.) by 28 inches (69 cm.) by 40 inches (102 cm.)
Cathodes	Sheet steel, 1/16 in. (0.15 cm.) by 30 inches (76 cm.) by 40 inches (102 cm.) with coating of black oxide to prevent sticking of deposits.

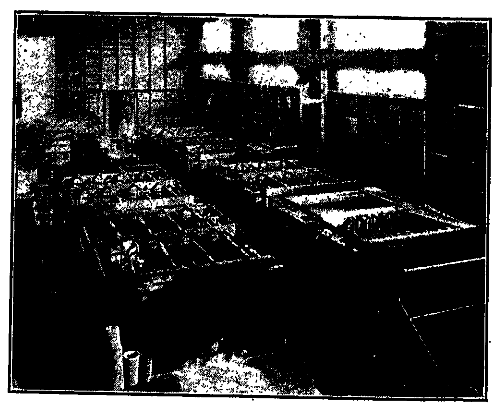

FIG. 2—GENERAL VIEW OF ELECTROLYTIC IRON REFINERY. DORR THICKENER INSTALLED BEYOND PARTITION AT FAR END. MIXING TANKS, LEFT-CENTER. WASHING TANKS, EXTREME LEFT

The principal data concerning this plant have been summarized in Table II, and Fig. 2 gives a general view of the installation.

High-Frequency Power Plant

High-frequency power for the induction furnaces is furnished by a motor-generator set designed and built especially for this work. This machine was designed by C. M. Laffoon of the Power Engineering Department, Westinghouse Electric and Manufacturing Company, and has been described by him in some detail*. The alternator is of the inductor type with a cylindrical rotor, and operates normally at 3750 rev. per min., giving a frequency of 5000 cycles per second at this speed. The machine delivers 400 amperes continuously at 250 to 300 volts, corresponding to an output of 100

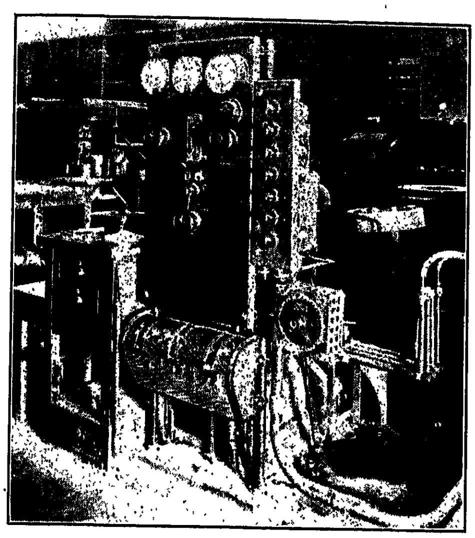

Fig. 3—Control Panel for High-Frequency Power Plant. Furnaces and Driving Motor of High-Frequency Alternator Seen in Background at Right.

to 120 kv-a. Under these conditions there have been no indications of excessive temperature rise in the windings, and the mechanical performance has been excellent under all conditions. The alternator is driven through a standard Westinghouse turbine reduction gear, giving a speed ratio from generator to motor of 4.12 to 1. The motor is a standard Westinghouse Type S K motor, rated at 200 h. p., 230 volts, 910 rev. per min. provided with field control to give a frequency range from 4500 to 6000 cycles per second.

The entire control of the high-frequency power supply is centered in the control panel shown in Fig. 3. This panel carries the following equipment.

1. Relay switch for operating the automatic motor-starting equipment.
2. Motor field rheostat for controlling motor speed.
3. Field switch for high-frequency generator.
4. Field rheostat for high-frequency generator.
5. Push-buttons controlling the solenoid-operated condenser switches.
6. Frequency meter calibrated in cycles per second and operating from a speed-indicating magneto direct-connected to the motor shaft.
7. Direct-current ammeter for measuring field current of high-frequency alternator.
8. Thermal ammeter for measuring high-frequency generator output.

Thus, one man has complete control of the starting and stopping of the high-frequency generator, of frequency, of condenser capacity, and of the voltage and current output of the alternator. For a given material and weight of charge, the furnace operations soon reduce to a routine matter of shoveling in the charge and the following of a definite current-time schedule.

The windings of the alternator are divided into twelve similar sections,—six on each end of the stator,—and these are operated in series-parallel connection, giving six groups, each having two coils in series. These six groups are connected in parallel to the busbars through 60-ampere, 500-volt fuses and the equalizing transformers. The purpose of the equalizing transformers is to ensure equal division of current among the paralleled

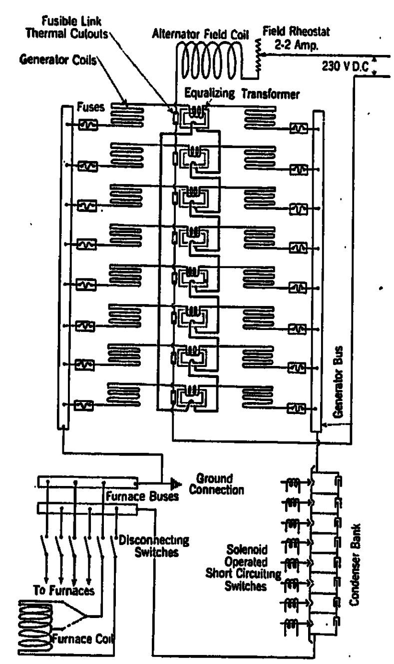

Fig. 4—Schematic Diagram of Circuits of High-Frequency Alternator, Condensers and Furnaces

sections of the windings. Previous experience with a small alternator of this same type showed that it was very difficult to get similar characteristics in all the coils, and that the relative characteristics of the coils would sometimes change appreciably with time, because of alterations produced in the dimensions or relative positions of the parts of the machine by temperature changes or other causes. As a result, it was impossible to form a grouping which would give an equal distribution of load under all conditions.

Fig. 4 is a schematic diagram of the generator, equal-

*High Frequency Alternators, by C. M. Laffoon: *Electric Journal*, Sept. 1924, Vol. XXI, No. 9, p. 416-420.

izing transformer, condenser and furnace circuits. From this it will be seen that the equalizing transformers consist of cores with a primary winding on each and that one of these primary windings is in series with each series pair of generator coils. All the cores are linked with a common secondary winding which is short-circuited on itself. Thus, the secondary current is always alike in all the transformers and the tendency is to maintain the equality of the primary currents; hence equal distribution of current among the various coil groups. If, for any reason, an open circuit develops in

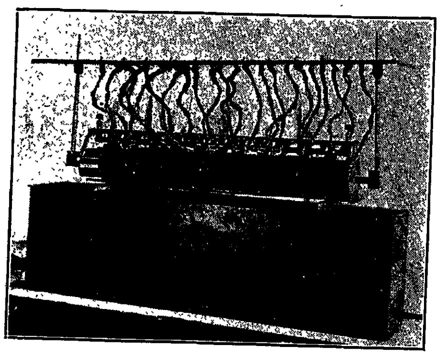

FIG. 5—SHOWING TRANSFORMER TANK, CORES SURROUNDED BY SECONDARY CIRCUIT, FUSIBLE-LINK THERMAL CUT-OUTS AND LEADS TO PRIMARY COILS

any coil group, the iron of the corresponding transformer core is immediately subjected to the full magnetizing force due to the secondary current impelled by the remaining transformers, and destructive temperatures are soon reached. The transformers are safeguarded against this contingency by placing a link of low-melting alloy in the oil immediately above each core. These links are connected in series with the field coil of the alternator. Hot oil rising from any overheated core melts the corresponding fusible link. This interrupts the field current and protects the transformer, while, at the same time it gives notice of trouble to the operator. This arrangement has worked satisfactorily on the few occasions when accidental overload has caused rupture of a fuse in series with one of the coils.

Fig. 5 shows the structural details of a group of equalizing transformers. The cases for the fusible links are seen above the cores. The common secondary consists of a copper tube, (to allow water cooling), which passes through the toroidal cores, and is provided with end-plates connected by flat copper bars to complete the circuit. The cores consist of stacks of 5 mil enameled four per cent silicon-steel ring punchings.

The alternator is connected in series with the furnace, inductor coil and the bank of condensers, as shown diagrammatically by Fig. 4. This condenser installation was designed by R. E. Marbury, Supply Engineering Department, Westinghouse Electric & Manufacturing Company and has been described elsewhere*. Westinghouse oil-insulated paper condensers, such as are regularly used for 2300-volt power-factor correction service, were used to build up this bank of condensers. The individual units have a capacity of 1.21 μf. and there are eight groups of them, each containing 20 units in parallel. Each group is provided with a remote controlled short-circuiting switch and adjustment of the condenser capacity is effected by manipulating these switches to vary the number of condensers in the circuit. Table III shows the great flexibility made possible by this arrangement.

TABLE III.
Capacity Range of Condenser Bank

No. of Groups in Circuit	Max. Total voltage	Capacitance microfarads	Max. Total Kv-a.
8	4650	3.2	2040
7	4050	3.46	1780
6	3500	4.02	1540
5	2900	4.85	1275
4	2320	6.05	1110
3	1740	8.07	765
2	1160	12.10	510
1	580	24.2	255

The condensers are mounted in grounded structural iron frames, from which they are insulated by porcelain insulators designed for a maximum stress of 10,000 volts. Much ingenuity was exercised in arranging the electrical circuits within the iron frame in such a way as to avoid

FIG. 6—GENERAL VIEW OF HIGH-FREQUENCY POWER PLANT. FROM LEFT TO RIGHT, AUTOMATIC CONTROLLER, HIGH-FREQUENCY MOTOR-GENERATOR SET, CONDENSER BANK, 225-LB. FURNACE, VACUUM PUMPS, CONTROL PANEL (FROM REAR), VACUUM FURNACE

heating of the iron work by the high-frequency magnetic field surrounding the busbars and no trouble has been experienced from this cause. The condenser frames and condenser switches are enclosed in a grounded, expanded, metal housing. Busbars are carried around the top of the condenser frame and disconnecting switches are placed at intervals, and the several furnaces are supplied with power through these switches

*The Application of Static Condensers to High Frequency Furnaces, by R. E. Marbury: *Electric Journal*, Sept. 1924, p. 421-422.

which are hand-operated by means of the usual wooden hook-stick.

This condenser installation is much more elaborate than would usually be necessary, and was made so purposely in order to provide flexibility, for, although the plant as a whole was designed as a productive manufacturing unit, the expectation was that it would also serve as an experiment station for the practical development of other applications for high frequency power, particularly in connection with heating problems. In passing, it may be said that this expectation has been fully realized.

Fig. 6 shows a general view of the motor-generator set, condenser bank, and a furnace being assembled for charging.

High-Frequency Furnaces

The high-frequency furnaces used in this plant are the same in principle as those which Dr. E. F. Northrup has so successfully developed for use with high frequency power derived from spark-gap oscillators. Slight modifications in design have resulted from the fact that the applied voltage is practically sinusoidal instead of being a series of impulses, as in the case of the spark-gap oscillator. From an electrical standpoint, a high-frequency furnace may be considered as a special case of the transformer. Physically it consists essentially of a water-cooled helix of copper tubing surrounding the material to be heated. The magnetic field produced by high-frequency current traversing the coil, or primary, induces voltage in the charge, or secondary, and the resulting currents cause heating. It will be obvious that if the resistivity of the charge is very high or very low the heating effect will be small, for, in the first case only negligible currents will flow, and in consequence, the product $I^2 R$ will be small, while in the second case, low resistance will result in a small $I^2 R$ product. In practise, satisfactory heating is obtained at 5000 cycles per second when the resistivity of the material to be heated lies between the approximate limits of 50 and 1000 microhms per centimeter cube and when the pitch of the coil and the current through it are such as to give approximately 500 ampere-turns per inch as a minimum. The efficiency of the furnace will increase as the rate of heating is increased because as the time required to reach a given temperature is reduced, the heat lost by conduction becomes a smaller proportion of the total amount supplied.

As in the case of the ordinary transformer, the closer the electromagnetic coupling between the primary and secondary, the higher the power factor and electrical efficiency. In the case of the furnace, however, close coupling requires close approach of inductor and charge, and a point is reached at which the gain in electrical efficiency due to improving coupling is counterbalanced by the loss in thermal efficiency resulting from decreasing thermal insulation. The optimum coupling is not a constant but depends upon the rate of energy supply, the temperature to be reached and the nature of the temperature cycle. The coupling coefficient will be denoted by ϕ and is defined as the fraction of the total magnetic flux produced by the coil which is linked with the charge. Roughly it is proportional to the ratio of the square of the diameter of the charge to the square of the diameter of the coil. For most high-temperature melting operations satisfactory performance is obtained when this ratio lies between the approximate limits of 0.5 and 0.7, and a usual value is 0.6. In a general way, the coupling giving the best results will approach the lower limit when it is desired to reach the maximum temperature with a given amount of power, and will approach the upper limit when a given temperature is to be quickly reached by generous application of energy.

The accurate calculation of the electrical characteristics of a given combination of inductor and charge is a matter of considerable difficulty, not only because of the purely mathematical problems of determining the effective resistances and reactances of the coil and the charge, but also because of our ignorance of the electrical and thermal properties of conductors and refractories at high temperatures. Northrup* has developed approximate formulas which are found to be sufficiently accurate for most engineering purposes when the frequencies are high enough to justify the assumptions:

1. That the effective ohmic resistance of the charge is equal to its reactance.

2. That the resistance of the inductor coil is negligible compared to its reactance.

Two of these formulas are particularly useful in designing furnaces to operate on given voltage and frequency, and are given below.

$$P = \frac{2\,\phi\,E^2}{2\,\pi^2 f L\,(\phi^4 + 2 - 2\,\phi^2)} \tag{1}$$

$$P = \frac{0.45\,\phi\,E\,I}{\sqrt{\phi^4 + 2 - 2\,\phi^2}} \tag{2}$$

Here, P is the power absorbed by the furnace: ϕ is the fraction of the total flux produced by the coil which is linked with the charge; E is the root-mean-square voltage across the coil; I is the current through it and L is its inductance.

In designing the furnaces for the plant here described, we have used somewhat different design methods, as outlined below.

1. Given: An ingot of a certain size and material to be melted.

2. Required: An estimate of the length, diameter, number of turns of the inductor, and the range of condenser capacity and kv-a. required for operation

*Electric Heating by Ironless Induction, by E. F. Northrup: *General Electric Review*, Nov. 1922, Vol. XXV, No. 11, p. 656-666.

*Principles of Inductive Heating by High Frequency Induction, by E. F. Northrup: *Transactions Amer. Electrochem. Society*, Vol. XXXV, 1919, p. 69.

with a power supply of which the voltage, current and frequency are given.

3. Experience was drawn upon to make a plausible estimate of the ampere turns per unit length of the inductor, and also to select a reasonable ratio for the diameters of inductor and charge.

4. The axial length of the inductor coil is determined by the length of the charge, and it has been our practise to make it somewhat greater than that of the charge, the excess in general not exceeding the diameter of the charge.

5. From (3) and (4) the dimensions of a trial coil are found. Its inductance is calculated, and from this and the given values of current and frequency, its reactive voltage and kv-a. are found.

6. The electrostatic capacity required to resonate at the assigned frequency with the inductance determined in (5) is calculated, using the formula,

$$C = \frac{1}{4\,\pi^2 f^2 L}$$

where C is the capacity in farads: f, the frequency in cycles per second: and L the inductance of the coil. This establishes the maximum voltage and kv-a. rating for the condenser and the minimum value for its electrostatic capacity.

7. The effective resistance of the inductor is calculated from its direct-current resistance in the light of test results. Test data are available which show that for frequencies between 5000 and 10,000 cycles per second the high-frequency resistance of the usual furnace coils will be from 5 to 25 times the direct-current resistance. For ½ inch (1.27 cm.) by ¾ inch (1.9 cm.) copper tubing flattened to 0.45 inch (1.14 cm.) and edge-wound with a pitch of two turns per inch (0.79 turns per cm.) the a-c. resistance is approximately 25 times the d-c. resistance for coils having proportions usually encountered in furnaces, the length-diameter ratio being of the order of two to three.

8. The resistance of the charge is calculated on the assumption that it is a thin cylindrical shell of which the length and outside diameter are those of the charge, and the thickness is given by Steinmetz's* formula for the depth of penetration of current in a conductor,

$$L_p = \frac{5030}{\sqrt{\lambda\,\mu\,f}}$$

where

L_p is the effective depth of penetration in centimeters:

λ the conductivity of the material in reciprocal ohms per cm.^{3}:

μ the magnetic permeability of the material and

f, the frequency in cycles per second.

9. The inductance of the charge is calculated by assuming that it is a very thin cylindrical shell whose mean length and mean diameter are those of the shell given by (8).

10. The mutual inductance of coil and charge are calculated by means of the approximate formula,

$$M = \frac{2\pi^2 A_2^2\, n_1\, n_2}{d \times 10^9} \quad \text{and}$$

$$d = X\sqrt{1 + \left(\frac{A_1}{X}\right)^2}$$

where

M is the mutual inductance in henries:

A_2 the means radius of effective current zone in charge in centimeters:

n_1, the total turns on coil;

n_2, the total turns on charge (= 1);

X, the half length of the coil, and A_1 the effective radius of the coil in centimeters.

11. The effective inductance of the coil when the charge is in place will be less than that of the coil alone, and is calculated from the formula given by Morecroft,*

$$L_1^1 = L_1 - \left(\frac{\omega M}{Z_2}\right)^2 L_2$$

where

L_1^1 is the effective inductance of coil with charge;

L_1, the inductance of the coil alone;

ω, the product $2\,\pi \times$ frequency in cycles per second;

M, the mutual inductance of coil and charge;

L_2, the inductance of the charge and

Z_2 the impedance ($\sqrt{R_2^2 + L_2^2}$) of the charge.

12. The effective resistance of the coil will be increased by the introduction of the charge because of energy absorption by the latter. The effective resistance of the coil when surrounding the charge is calculated by a formula given by Morecroft.†

$$R_1^1 = R_1 + \left(\frac{\omega M}{Z_2}\right)^2 R_2$$

where

R_1^1 is the effective resistance of the combination of coil and charge:

R_1, the high frequency resistance of the coil alone:

R_2, the resistance of the charge as calculated in (8).

13. The minimum values of the reactive kv-a. and voltage of the coil and hence of the condenser, and the maximum electrostatic capacity required, are determined by the effective inductance of coil and charge as calculated under (11) on the assumption that the condenser capacity will be adjusted to give resonance at the chosen frequency.

14. The effective resistance of the condenser is calculated from a knowledge of its power factor as

*Transient Electric Phenomena and Oscillations, by C. P. Steinmetz: 1909 Edition, p. 376.

*Principles of Radio Communication: Morecroft, p. 87.

† loc. cit.

determined by test, and the kv-a. corresponding to a given current.

15. The maximum power and current which will be taken by the furnace at the given generator voltage is then calculated by summing the resistances given by (12) and (14), and any circuit resistance which may enter, and applying Ohms' law, assuming operation at resonant frequency so that the inductive reactance of the furnace circuit is balanced by the capacitive reactance of the condenser, and hence that the apparent resistance between the terminals of the condenser-furnace circuit is equal to the true ohmic resistance.

16. The power input to the charge is found by subtracting the copper losses in the coil (determined from its high-frequency resistance and the current) from the total power as determined by (15). The maximum temperature obtainable can be found approximately by calculating the thermal resistance of the refractories separating charge and coil, and from this the temperature drop from charge to coil required to transfer the thermal equivalent of the power input to the charge.

17. Having thus arrived at a tentative design for the inductor coil and an approximate estimate of its performance, the final design can usually be obtained by a second approximation with sufficient accuracy.

In practise, it has been found that the method outlined above gives very good results so far as the determination of condenser capacity and coil voltage are concerned. The results for maximum temperature obtainable and the effective resistance of the coil with charge are less accurate. This is due to the approximations necessary in the calculation of the copper losses in the coil and the effective resistance and inductance of the charge (hence the effective resistance of the coil with charge) and to uncertainty as to the true values of the electrical and thermal properties of materials at high temperatures. Table IV gives a comparison of the results of calculation and test for a certain melting furnace.

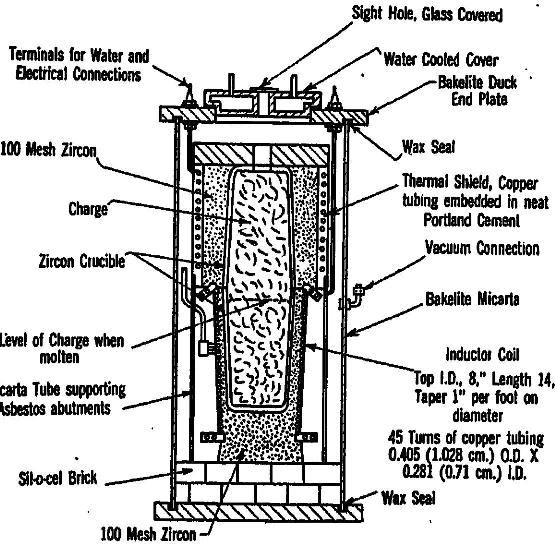

FIG. 7—50-LB. VACUUM FURNACE

The actual construction of two types of furnace now in use is shown by Figs. 7 and 8.

Fig. 7 is a vacuum furnace designed to melt 50-to 60 lb. charges of iron. The vacuum jacket enclosing the furnace proper consists of a micarta tube to which micarta duck-end plates are sealed by means of a special wax. The furnace proper consists of a slightly tapered helix of copper tubing supported on radial asbestos abutments carried within a second micarta tube. Water and electrical terminals are brought out from both ends and the middle of the coil. Above the coil is a water-cooled cylindrical shield formed by casting a shell of neat Portland cement around a helix of copper tubing. This shield supports the thermal insulation around the upper end of the furnace chamber and protects the micarta casing of the furnace against heat from the charge. The top end-plate is provided with an opening through which the furnace is filled. When the furnace is in operation, this opening is closed with a water-cooled cover provided with a glass observation window. A rubber gasket and stopcock grease form a vacuum-tight joint between the cover and the end plate.

TABLE IV.
Comparison of Calculation and Test Data for 50-lb. Vacuum Furnace

	Resonant Frequency (with full charge)	Furnace Current with full charge	Coil voltage (for 184 amperes)	Power Factor	Input kw.
Calculated	6200	300	735	6.5	23.8
Found	6100	184	740	10.7	14.5

The melting chamber is formed by placing a crucible within the coil and inverting a second one over it. A hole is cut through the bottom of the second crucible to allow for charging and for observation of the occurrences within. Zirconium silicate, ground to a uniform size of approximately 100 mesh, has been found a very satisfactory material for providing the necessary refractory thermal insulation around the furnace chamber. All the space between the crucibles and the coil and thermal shield is packed with this material, and the furnace is then ready for charging. The whole volume of the melting chamber is available for containing the ingredients of the charge because as melting proceeds in the bottom crucible fresh material settles down from the top.

With the furnace charged and the cover in place, evacuation is next in order. We have used No. 2 Trimount rotary oil pumps, exhausting into a system whose pressure is kept at approximately ½ inch (10-12 mm.) of mercury by a reciprocating vacuum pump. When the furnace is cold, the pressure within it can be reduced to a few hundredths of a millimeter of mercury, but it is usually impossible to get much below five millimeters of mercury when the furnace contains a molten 50-lb. charge of iron, for example, be-

cause of the evolution of gases from the charge and the hot refractories.

With the furnace exhausted and water flowing through the coils, it is ready for operation. It is connected to the busbars and the alternator is started and allowed to speed up with small field excitation. At first the current is imperceptible but it soon rises sharply to a maximum as the resonant frequency is passed. When the resonant frequency has been found, the alternator speed is reduced to give a slightly lower frequency and the field excitation adjusted to give the desired furnace current. Stable operating conditions are obtained by working slightly below resonance because tendency to increasing speed is counteracted by the increased load on the generator due to increases in both furnace current and power factor at the generator terminals which take place as resonance is approached.

TABLE V.
Log of High Frequency Vacuum Furnace Operation
Charge: 50 lb. 4 per cent Silicon-Iron Alloy

Time Mins.	Current	Voltage	Frequency cycles/second	Vacuum mm. Hg.	Remarks
0	0	0	....	0.5	Power on.
10	100	....	8000	....	Top half of coil.
20	150	500	8500	....	
30	175	600	9000	2.5	
50	80	500	10500	2.5	
65	140	750	→7000	30.	Changed to full coil.
75	180	1000	7400	20.	Melting evident.
90	192	1100	....	....	Melting complete.
130	210—180	Off scale	....	....	
150	200	Off scale	....	22.	
155	→100	....	....	....	
165	→ 50	....	....	15.	Freezing commences.
170	→ 0	....	....	....	Top solid, power off.
410				3.	Furnace opened.

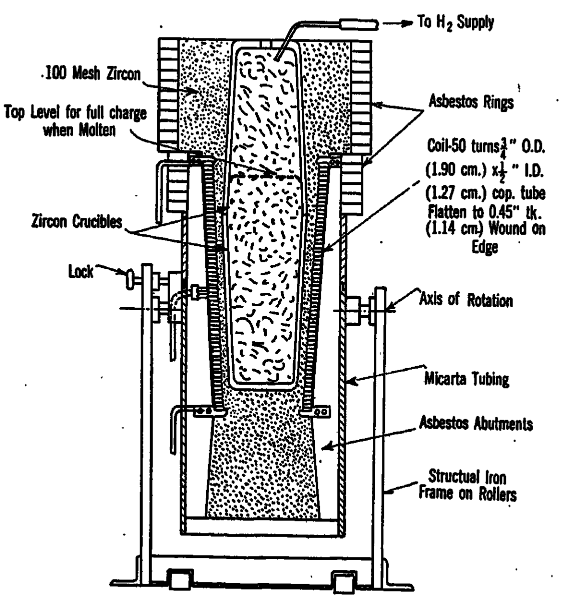

FIG. 8

After the melt has been brought to the proper condition, it is cooled by decreasing the power input and bringing the temperature of the charge down until it is close to the solidification point, and then cutting off the power, allowing it to freeze. The cooling schedule has an important bearing on the soundness of the ingot and is determined by experience. By using only the top half of the coil during cooling, the solidification of the top of the ingot can be retarded. This practise aids considerably in getting ingots which are free from shrinkage cavities.

The log of a typical run of one of these vacuum furnaces is given in Table V.

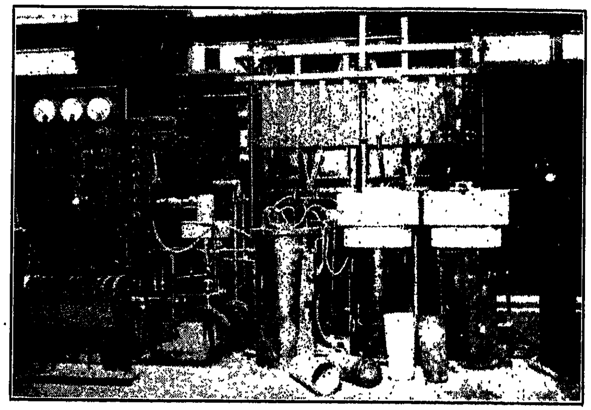

FIG. 9—HIGH-FREQUENCY FURNACES, 225-LB. (100 KG.) CAPACITY. HIGH-FREQUENCY MOTOR-GENERATOR SET AND CONDENSER BANK IN BACKGROUND

The variations in the frequency are due to the changes in the inductance of the furnace caused by the changes in the coupling coefficient and the resistance of the charge which occur as the latter is converted from a stack of irregularly shaped pieces of metal to a cylindrical molten mass.

When the ingot has cooled sufficiently, the vacuum is released, the zircon scooped out and the crucibles and contents lifted out with a pair of specially designed tongs. The taper of the furnace coil facilitates removal of the charge.

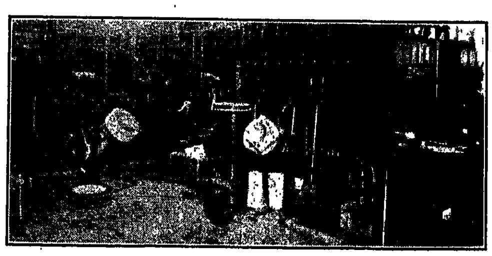

FIG. 10—HIGH-FREQUENCY FURNACES. FROM LEFT TO RIGHT; CONTROL PANEL, VACUUM PUMP, 50-LB. VACUUM FURNACE, TWO 225-LB. (100 KG.) FURNACES, HYDROGEN TANK. CRUCIBLES AND INGOTS IN THE FOREGROUND

The bottom crucible is used but once, but the top one usually lasts through five or six heats. The zircon is used repeatedly although there is some loss due to its

caking on the outside of the lower crucible, and being discarded.

Fig. 8 is a diagrammatic cross section of a furnace designed to melt 225-lb. charges of iron. The general appearance of some furnaces of this type is shown in Fig. 9. They are carried by structural iron frames mounted on rollers so that they may be easily moved about, and trunnions are provided so that they can be tilted for discharging. The melting chamber is formed by two crucibles, one inverted above the other as in the vacuum furnace. The hydrogen, or other protecting gas, is led in through a small side opening near the top end of the upper crucible. Zirconium silicate is packed between crucibles and coil and rings of asbestos lumber retain it around the upper crucible, above the end of the coil.

The necessity for using copper tubing of such large radial width may be questioned, but experience with some of the first furnaces built shows that the inductor coil must possess considerable mechanical strength if it is to resist the stresses caused by the expansion of the crucible. The zircon packing between the crucible and coil does not seem to possess a great deal of resilience, and water leaks in two coils were definitely traced to stretching due to the thermal expansion of the crucible and refractory packing.

The operating routine of these furnaces is not greatly different from that of the vacuum furnace just described. Melting is done at atmospheric pressure, usually under the protection of a suitable gas. Hydrogen is convenient and has been used in the majority of cases. The gas is supplied from a tank of compressed gas through a regulating valve. A typical log of a furnace run on a 225-lb. charge of iron-nickel alloy is shown by Table VI.

Fig. 10 gives a good idea of the general appearance of a portion of the melting floor as seen when looking toward the control panel and high-frequency motor-generator set. In the center is seen one of the vacuum furnaces described above, and between it and the control panel may be seen the vacuum piping and the oil pump. The mercury manometer and vacuum connection to the furnace appear on the right side of the furnace. The two right-hand furnaces are the 225-lb. type, shown diagrammatically in Fig. 8. The one next to the vacuum furnace was in operation when the photograph was taken. In the foreground are two ingots from the large furnaces and two of the crucibles used to make them.

In the early stages of our high-frequency furnace work the problem of refractory thermal insulation and of crucibles and furnace linings gave us much concern. Zirconium silicate is now used almost exclusively for thermal insulation, and the same material bonded with a small percentage of refractory clay is used to make furnace linings and crucibles. The process for making these crucibles was worked out by A. A. Frey, of the Research Department of the Westinghouse Electric & Manufacturing Company, and the crucibles are now being manufactured regularly at a cost which makes it economical to allow the ingots to solidify in the crucibles and use the crucibles but once. Thus, we avoid the pouring of ingots and the attendant complication and expense, eliminating the need for skilled personnel which would otherwise be required for this work.

Because of the unusual character of some of the processes and much of the equipment used in this plant, it might appear that it is nothing more than an overgrown laboratory, burdened with all the complications and expense usually associated with laboratory operations. A laboratory might be defined as a place where results are obtained without regard to cost or output, while the plant must produce the same results without regard to anything but cost and output.

When the plant is operating on a production schedule, there is one furnace connected to the power supply at all times, while others are being charged or emptied. The maximum output which has been required in any

TABLE VI.
Log of Run of 225-lb. High-Frequency Furnace Charge: 225-lb. Iron-Nickel Alloy

Time Mins.	Current in Furnace	Voltage		Kw. Generator Output	Kw. Motor Input	Frequency	Remarks
		Generator	Furnace				
0	280	252	1780	70.6	99	4650	
10	400	256	2330	102.5	146.5	4650	Start 80 lb. metal in furnace
20	400	232	2260	93.0	135.0	4800	
30	400	232	2120	93.0	115.0	5000	150 lb. of metal in furnace
40	400	210	2030	84.0	104.8	5200	
50	250	180	1250	45.0	60.0	....	Charging completed
60	250	87	1020	21.6	48.6	4700	Melting completed
70	205	71	800	14.5	42.0	4700	Cooling started
80	205	70	805	14.3	42.0	4700	
85	205	70	805	14.3	42.0	4700	Top frozen
90	0	0	0	0	0	....	Power off

TABLE VII.
Analysis of Production Costs based on Monthly Output of 30,000 lb. (13,500 kg.) (Costs in cents per pound)

	Electrolytic Iron	Iron-Nickel Alloy (ingots)
Labor	1.96	1.51
Incidental factory expense	10.36	6.81
Material	4.23	28.3
	.....	.....
Totals	16.55	36.62

one month was slightly more than 30,000 pounds, and this figure was reached without difficulty.

The operations of this plant have been subject to the same cost analysis and accounting procedure as other works departments of the Westinghouse Electric & Manufacturing Company, and the figures given by Table VII are believed to be trustworthy. The item "Incidental Factory Expense" covers capital charges, amortization, rental, supervision and all other overhead charges.

These figures are very satisfactory for they indicate that, by the extension of laboratory methods, it has been possible to produce a highly specialized material of great purity which meets very stringent specifications, at a cost not far different from the market price of ordinary commercial material having the same nominal composition but very much inferior properties.

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Discussion

H. N. Shaw: I should like to ask what the over-all efficiency of that high-frequency induction furnace really is? That is, the ratio of the B. t. u. put into the motor to the B. t. u. actually developed in the iron of the furnace. It seems to me that there would be other applications if that efficiency is at all high. Offhand, it would seem to me to be quite low.

P. H. Brace: In reply to Mr. Shaw's question as to the over-all thermal efficiency of the high-frequency furnace and generator installation, I should say that for an input of approximately 150 kw. to the motor we get slightly more than 100 kw. out of the generator. Twenty kw. are lost in the copper of the inductor coil of the furnace and five kw. in the condensers, thus leaving a balance of about 75 kw., which is converted into heat in the charge. This is not a particularly high thermal efficiency as compared with that obtained with large arc furnaces, but with the present type of furnace we can easily do things that are not practicable with other types. We have complete control of our furnace atmosphere and in our particular design of furnace we have avoided all the troubles due to the casting of ingots and done away with all of the heavy labor. The use of electrolytic metals has eliminated the fire refining steps required in ordinary steel-making processes and we have attained a product which we were unable to secure in any other way. I might emphasize the fact that this plant is more than a plaything; we are producing approximately 40,000 lb. per month of one particular material.

Synchronous Motor Drive for Rubber Mills

With Special Reference to Dynamic Braking Control for Safety Stopping

BY C. W. DRAKE[1]

Member, A. I. E. E.

Synopsis.—Since wound-rotor induction motors with clutch brakes have been very commonly used for this application, a review of the operating conditions and safety requirements is given, in order to show why synchronous motors without clutches may be used for the same application.

* * * * *

DURING 1920, the Rubber Subcommittee of the Industrial and Domestic Power Committee of the A. I. E. E. collected and prepared considerable data relative to the choice and selection of motors for mill-line drives (paper presented January 14, 1921 at the Akron-Cleveland meeting). It was mentioned in this report that synchronous motors had certain desirable characteristics for mill-line drives and that there were a few in operation which had given very satisfactory service. The real demand for synchronous motors, however, has come only since the interest in power-factor improvement has been given thoughtful consideration.

Mill-line drives may be divided into two groups; namely, the geared type, in which gear units with high or moderate speed motors are used, and the gearless type, in which the motors operate at the speed of the mill line or about 100 rev. per min. For the latter group, synchronous motors have been used principally on account of their lower cost and better performance as compared with induction motors, while those synchronous motors which have been used for gear drive were undoubtedly installed primarily for power factor purposes, since there is little advantage in efficiency or difference in cost as compared with the induction motor.

Rubber mill drives impose two severe conditions upon the motor equipment, first, a high starting torque when the mills are started with rubber in the rolls, and second, a quick stopping in case of accident or emergency. The risk and danger involved in the milling of rubber has been appreciated ever since the industry started, and these factors have always had an important bearing on the type of driving equipment used. In other words, the question of safety to employees has had, as it rightly should have had, preference over other factors in the layout of mill drives. A knowledge of these conditions explains why the wound-rotor induction motor, which has been most extensively used for mill drives, has in most cases been equipped with a clutch brake. From the standpoint of starting and running the clutch is unnecessary, as such motors have ample starting torque. However, since most of the stored energy of the system is in the rotor of the motor, it is not surprising to find that one of the first methods of obtaining quicker stopping was to disconnect the motor and then apply a brake on the remaining load. Although clutch brakes have been quite generally used, it has been found in many cases that it is the practise in starting to close the clutch first and then start the motor, thus using the clutch only as a safety feature and saving the wear which would be occasioned during starting to the clutch lining.

In view of the established or common practise employed in the installation of wound-rotor motors, it was to be expected that synchronous motors which had lower starting torque and, as a rule, more stored energy, would be equipped with similar clutch brakes, and that these would be used both during starting and stopping. Thus equipped, the synchronous motor should give service identical to that of the induction motor, since each may be designed for the same maximum torque, although the fact that the induction motor slows down considerably before reaching its maximum torque may sometimes give sufficient warning to prevent its pulling out and stalling. Many of the early installations of synchronous motors were of the gearless type, and since motors as slow as this have a low starting torque, the clutches were undoubtedly needed to obtain a starting torque comparable with that developed by the wound-rotor motors. Synchronous motors used with gear units have much higher starting torque but the question is just how much torque is required to start a mill line.

Under normal conditions the mills are started empty, since the rubber is always removed before shutting down at the end of each shift. The torque required to start an empty mill line is so low that it may be entirely neglected. If the mill line is shut down by accidentally pulling the safety switch or by loss of voltage, the torque required to start the mills will depend upon the condition of the rubber in the mills at that time, and that may take from 50 per cent to 150 per cent of full-load torque. If the motor is shut down due to pulling out or the opening of the overload relay, which is usually set at about 200 per cent load, it is evident that if the motor cannot carry the load at synchronous speed, it cannot start it from rest, and one must resort to other means in order to reduce the load. The most usual method is to reverse the motor, thus backing up the rubber in the mills. Depending on conditions, the rubber may

1. Westinghouse Elec. & Mfg. Co. East Pittsburgh, Pa.

Presented at the Spring Convention of the A. I. E. E., St. Louis, Mo., April 13-17, 1925.

either be removed if very stiff and cold, or, after again starting the motor in the normal direction, the rubber may be forced through the rolls, since by this method the motor gets a start before the rubber enters the wedge of the rolls. Experience has proven that with a motor of correct capacity for a given mill line the shut downs with loaded mills are not numerous, and consequently the question of starting torque should not unduly affect the rating or design of the motor. Synchronous motors designed to operate at 80 per cent power factor and at a speed of about 600 rev. per min., which is commonly used for geared drive, have a starting torque on full voltage varying from about 1.5 times full load torque for motors of 100 h. p. up to about 2.5 times full load torque for motors of 400 or 500 h. p. capacity. When started on the 80 per cent voltage tap of the starting transformer a torque varying from 1.0 to 1.5 times full-load torque will actually be obtained, and although this is not as high as could be obtained by a wound-rotor motor or by a clutch, it has been found sufficient to meet all requirements.

As long as a clutch was thought necessary to obtain sufficient starting torque, the simplest, and probably the cheapest, method of obtaining safety stopping was by means of a brake on the mill side of the clutch. The

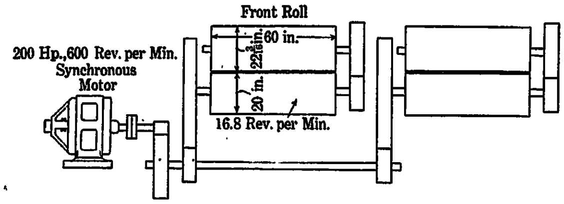


FIG. 1—SCHEMATIC ARRANGEMENT OF TWO 60-IN. MILLS WITH COUPLED-TYPE SYNCHRONOUS MOTOR

elimination of the clutch gave opportunity for other methods of braking, and electrical engineers naturally considered first the possibilities of electrical systems. Tests conducted a number of years ago on synchronous motors proved conclusively that very quick and uniform stopping could be accomplished by disconnecting the motor armature from the a-c. supply, and connecting it to a resistance of suitable value, while the field circuit of the motor is left energized at its normal full load value. Although the above facts and principles were known many years ago, it is only during the last two or three years that practical application has been made of them in the rubber industry, and to the best of the author's knowledge, no other industry has attempted a similar application.

Fig. 1 shows a characteristic layout of two 60 in. mills, which, in this case, are driven by a 200-h. p., 80 per cent power factor, synchronous motor, three-phase, 440-volt, 60-cycle, 600 rev. per min. This motor is connected to the gear unit by means of a flexible coupling and when delivering 200 b. h. p. will have a leading reactive component of 124 kv-a., while a wound-rotor induction motor of similar rating would have a lagging reactive component of 106 kv-a. Consequently the replacement of an induction motor by a synchronous motor in this case effects a reduction of 124 plus 106 or 230 reactive kv-a. in the total plant load.

The details of a control equipment for a synchronous motor with dynamic braking will vary considerably

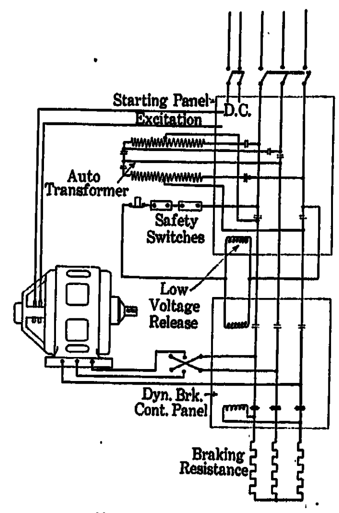


FIG. 2—SCHEMATIC DIAGRAM OF SYNCHRONOUS MOTOR CONTROL SHOWING STARTING PANEL, DYNAMIC BRAKING PANEL AND REVERSING SWITCH

with the size and voltage of the motor, also with the desires of the rubber company, but the fundamentals are the same in all cases. For instance, one company may desire the simplest type of manual starter while another desires a full automatic starter in order to install it in some remote location. Consequently the

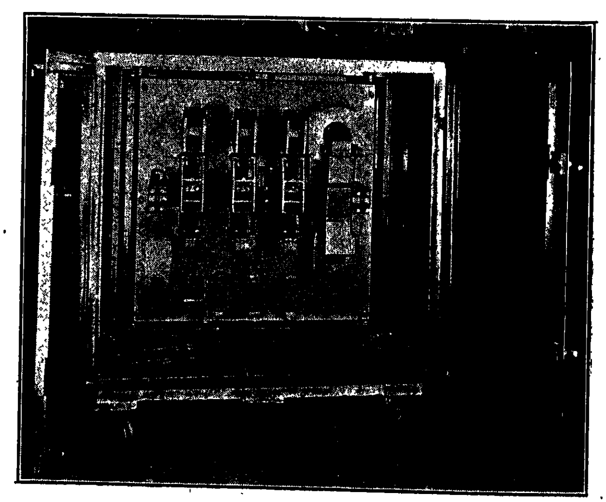
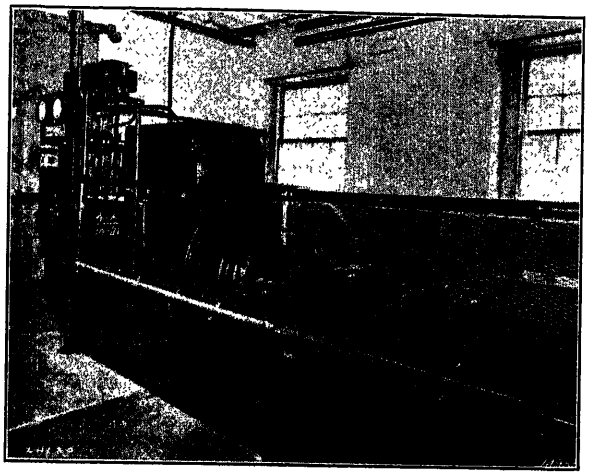
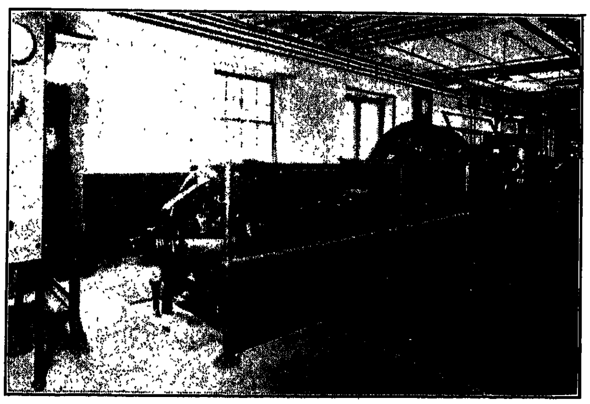


FIG. 3—GRAVITY-OPERATED DYNAMIC BRAKING CONTACTOR WITH REVERSING KNIFE SWITCH

dynamic braking contactor has been designed as a separate unit which may be used with any type of starting equipment. Fig. 2 is a schematic or simplified diagram of the control in which the motor starter may be either manual or automatic. The three-pole, double-throw, dynamic braking contactor is really the

heart of the system, for the stopping of the mills in case of emergency is entirely dependent on this and is independent of the rest of the control. As seen from the diagram, also from the photograph of this contactor in Fig. 3, the upper contacts open the circuit to the motor in series with those of the main controller thus introducing a double break and insuring opening the circuit even though the main circuit breaker or contactor does not open. The safety switches, of which there is at least one for each mill, are connected in series and upon the opening of any of these, the circuit is opened to the low-voltage release coil of the starter





Figs. 4 and 5—Installation of a 200-H.P. Synchronous Motor with Dynamic Braking Control Driving Two 60-in. Mills

and to the upper magnet coil of the dynamic braking contactor. Upon the opening of this circuit the upper contacts open and the lower contacts are instantly closed by the action of gravity aided by the spring pressure of the upper contacts. The closing of these lower contacts connects the motor armature to the dynamic braking resistance, and since the field circuit has not been opened, the voltage generated produces a current, the value of which may be regulated by the value of resistance used. The voltage generated during braking is also utilized to energize the lower magnet and hold the lower contacts firmly together until the motor stops.

The two-pole double-throw knife switch located below the three-pole contactor is used to reverse the direction of rotation of the motor when necessary and this switch is intended to be opened only when the motor is shut down. To provide against opening power current on this switch auxiliary contacts are provided which open the main circuit breaker or contactor of the starting equipment, also the dynamic braking contactor before the knife blades leave the jaws. Fig. 4 and 5 show installation views of a 200-h. p. synchronous motor mill line drive as shown diagrammatically in Fig. 1, together with the starting equipment. The starter is of the manual type panel-mounted consisting of a double-throw oil-immersed starting switch with separate mounting auto-transformers, the dynamic braking contactor, reversing knife switch and braking resistance are mounted in the cabinet at the rear of the starting panel.

To obtain the maximum possible safety on a rubber mill drive, the dynamic braking control should:

1. Operate by gravity and not depend upon closing a contactor electrically.

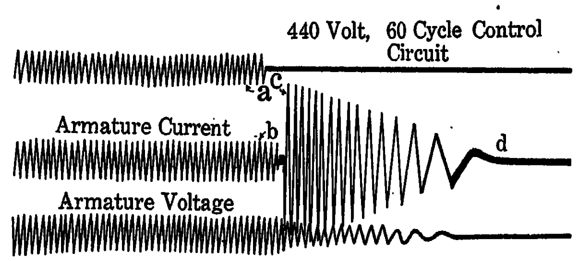


Fig. 6

2. Operate at high speed, in order to obtain as short a stop as possible. The special contactors developed for this work operate upon failure of the control circuit by any cause and, due to the method of construction used, a very high speed is obtained. Tests upon a 500-ampere contactor show a total time from the opening of the control circuit of the upper magnet coil to the closing of the lower contacts of about 1/20 of a second. High speed at this point is of great importance, for until the lower contacts are made, the rubber mills are traveling at full speed. The average speed of the front roll in the present case was about 100 ft. per min. or 20 in. per sec., so that the roll travel from the opening of the safety switch to the close of the dynamic braking curcuit of the motor would be about one inch.

A committee of the National Safety Council has been gathering data regarding the stopping distances of mill lines with a view of eventually preparing a safety code for rubber mills. No conclusions have as yet been reached but the Department of Labor of the State of New Jersey, in a tentative code, has decided upon 18 in. as a maximum travel for group-driven mills with roll diameters from 16 to 24 in. The travel is to be meas-

ured with the mills empty on the surface of the front roll by means of an electrical device which operates when the safety switch circuit is broken. This method of measuring the travel, although it gives the results desired by the safety engineers and the rubber engineers, does not allow the electrical engineer to analyze the entire operation. Consequently, oscillograph records have been taken using three elements and a characteristic film is reproduced in Fig. 6. The upper element indicates the current through the safety switch, the center element the current in the motor armature and the lower one the voltage at the motor terminals. At point *a* the safety switch opened, while the motor current was not interrupted until point *b* was reached. The small gap between *b* and *c* represents the time required by the contactor passing from the upper to the lower position. The distance from *c* to *d* indicates the time for the motor to stop after the braking current is completed, and by counting the number of cycles, it is possible to determine just how many revolutions the motor made. For instance, in this case there are 16 cycles, and since the motor has 12 poles or six cycles per revolution, the total number of revolutions is 16/6 or 2⅔ revolutions. On the mill line under consideration the ratio between the motor speed and the roll travel is approximately 600 rev. per min., to 100 ft. per min., or 10 revolutions equal 20 in. travel. Consequently 2⅔ revolutions represents about 5⅓ inch travel, to which it is necessary to add about one inch for travel during the operation of the controller, making a total travel of approximately 6⅓ in. The time required for the motor to stop can be adjusted to the maximum torque obtainable by adjusting the braking resistance. Probably the limiting condition in all cases will be the gears and mechanical parts of the mill drive, since the stresses in the motor when stopping in the minimum distance are not greater than those obtained when the motor is pulled out of step, due to overload. As a roll travel of about 10 in. with mills empty, is considered amply safe by motor engineers on mills of large diameter, there is little need of subjecting the equipment to unnecessary strain, and especially since every safety device should be operated and checked at least once a day to see that it is in satisfactory operating condition.

When stopping in a given distance, the dynamic braking will always be easier on the equipment than mechanical braking, since the torque developed in the motor is transmitted to the revolving field through the flexible medium of a magnetic field as compared with brake lining and mechanical friction in the latter case. Tests have also shown that the current values in the primary winding, when braking, are materially less than when starting under average conditions, so that as far as the motor is concerned, if it is of sufficient capacity to start and operate the mill line satisfactorily, those functions will be more severe than those encountered in stopping by dynamic braking.

Discussion

S. H. Mortensen: Mr. Drake's paper recognizes the importance of the application of dynamic braking to motors driving rubber mills and similar industrial installations where quick stops are necessary for safety reasons. The importance of this was brought to the speaker's attention in 1918, in connection with the design and operation of a 500-h. p., 450-rev. per min., self-starting synchronous motor, geared to a four-roll rubber-mill drive in the plant of the B. F. Goodrich Company in Akron, Ohio. This motor was installed for power-factor improvement and as it replaced a wound-rotor induction motor, it was connected to the mill by means of the magnetic clutch which formed part of the original motor drive, together with a solenoid-operated brake for stopping the mill in case of accident. As this installation was a novelty at that time, extensive tests were made on this motor. These proved not only its suitability for this type of drive but

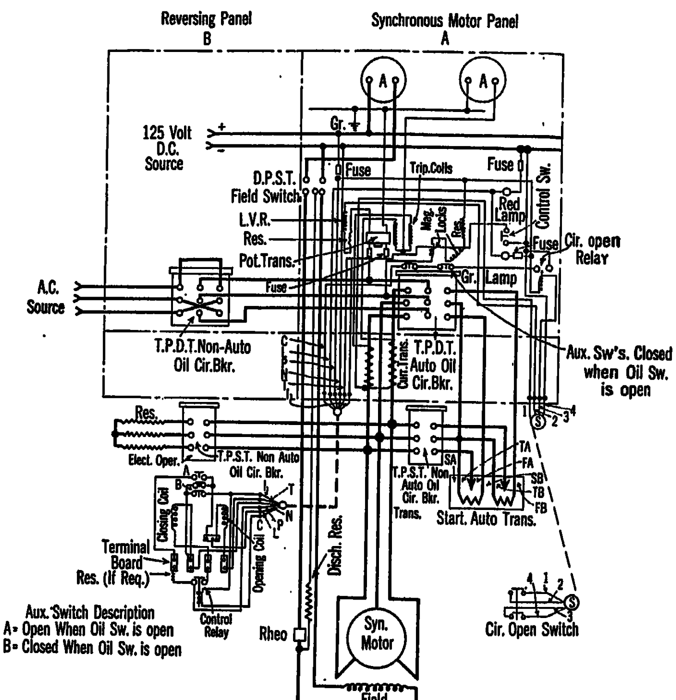


Fig. 1—Diagram of Connections as Seen from Rear of Panel. All Secondary Wire No. 12 R. I. F. P.

also that it did have ample starting torque to bring the mill up to speed with the clutch energized. Its pull-out was beyond any load that could be put upon it by overloading the mills with the toughest rubber available.

The first opportunity for applying dynamic braking in conjunction with a synchronous-motor rubber-mill installation occurred in 1920, when two 500-h. p., 450-rev. per min. synchronous motors were designed for direct gearing to two four-roll, rubber-grinding mills in the plant of the Fisk Rubber Company, Cudahy, Wis. In this installation the magnetic clutch was omitted from the mill drive, together with the mechanical brake, the motors being designed with a heavy squirrel-cage winding proportioned to develop a starting torque of 2.3 times full-load torque which enables it to start the fully loaded mills from rest and bring them up to synchronous speed. The motor was designed for dynamic braking. To stop the mills a safety switch located over the rolls is tripped and this automatically disconnects the fully excited motor from the power supply and short circuits its armature winding through a resistance, thereby producing the braking effect. The type of control used is shown in Fig. 1, herewith. This installation is to the best of the speaker's knowledge the first where the driving motor is geared

direct to the mill and dynamic braking is used for emergency stopping. After these motors had been in service for a considerable length of time they were subjected to a number of tests and the result of these tests together with a description of the general installation was written by the speaker and published in the August 4th, 1923, number of the *Electrical World*.

Fig. 2 is an oscillographic record of the voltage and currents obtained at the terminals of this motor during the period of dynamic braking. The upper records show that the time re-

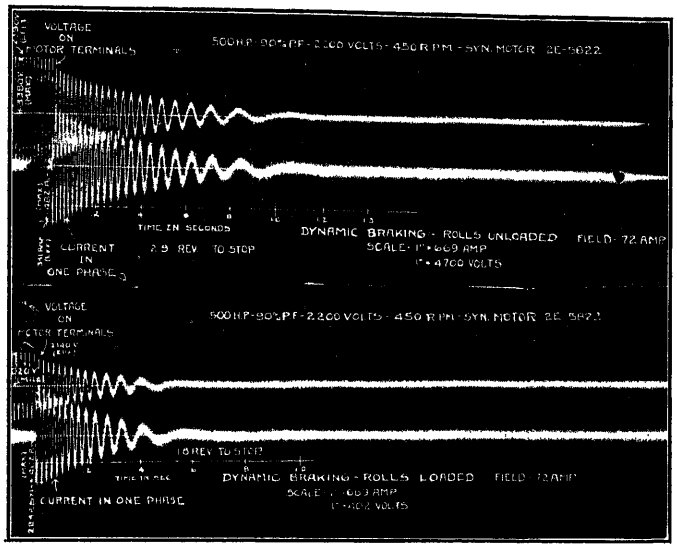


Fig. 2

quired from the instant the safety switch was operated to the time the empty mill came to a stop corresponds to 2.9 revolutions on the 450-rev. per min. motor. As the gear ratio on this mill is 1 to 20, the mill rolls actually came to a stop in 0.145 revolutions. The lower record of Fig. 2 shows the same condition as the upper part except that the mills were loaded when this test was made. Under these conditions the motor came to a stop in 1.8 revolutions. By making simple adjustments this stopping time could be further reduced.

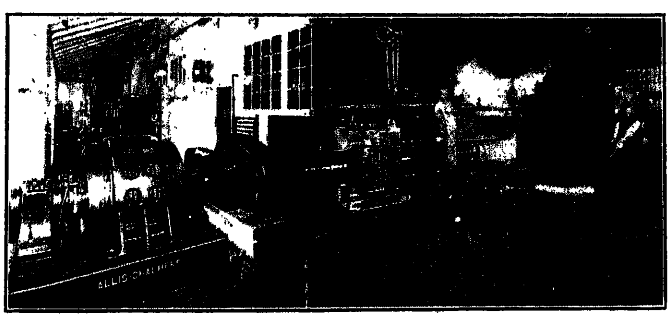


Fig. 3

Comparative tests were made on mills similar to the one described above, except that they were driven by induction motors through a magnetic clutch and stopped by disconnecting the magnetic clutch and setting the mechanical brake on the mill side of the drive. In all cases it was found that the dynamic braking was superior to the mechanical braking. It is consistent in value and acts more quickly than the mechanical brake and with less mechanical shock to the mill parts. The rapidity with which the rolls of a mill can be stopped by means of dynamic braking depends upon the stored energy of its rotating parts, the time elements of the switches, the characteristics of the motor and the amount of resistance connected in the armature circuits during braking. When the resistance is adjusted to give a maximum braking effect, the initial torque during dynamic braking is comparable to the pull-out torque of the motor with unchanged excitation. On the high-speed, gear-type synchronous motors the braking effect is as a rule ample to stop the mills in as short a time as the strain in the mechanical parts involved will permit, but where slow-speed direct-connected synchronous motors are involved the dynamic braking torque may not be sufficient to stop the mill in the desired time. In such installations the braking effect can be further increased by increasing the motor excitation simultaneously with its being disconnected from the line.

In connection with the installation described in Mr. Drake's paper, I should like to ask how this mill is stopped in case it becomes necessary to bring it to a sudden stop during the starting period, prior to the time when excitation is supplied to its fields.

In conclusion it may be of interest to the members of the American Institute of Electrical Engineers to know that dynamic braking has found successful application in connection with synchronous-motor-driven steel-rolling mills. An installation of this kind is shown in Fig. 3, which depicts a synchronous-motor-driven roughing mill. In this installation the dynamic-braking feature has proven its value as a safety measure and the rapid stops which it insures have, in several instances, either eliminated or minimized accidents which otherwise might have proven fatal to human life.

E. A. Hoener: The writer is connected with the Firestone Tire & Rubber Company; as engineer, also chairman of the Engineering Committee, Rubber Section, National Safety Council. We are collating data with reference to stopping distances on rubber mills and I am, therefore, very much interested in this discussion. In this connection I have come in contact with state commissions and have discussed with them the merits of the magnetic clutch brake and synchronous motors for stopping mill lines. One item which the synchronous-motor manufacturers have overlooked and which the state officials are giving not a little consideration is, what happens when the power goes off? We have had accidents due to power interruption on the part of the public utility or trouble in our own main power house causing power and lights to go off at the same time and mill men working on mills have been caught in the bite of the rolls. In cases of this kind dynamic braking does not do much good. We realize this is a very remote accident hazard, however state industrial boards think along these lines.

Another item to be considered is the changing over of existing equipment. I am interested in getting some of the rubber manufacturers in the United States to change their present equipment in such a way that it will become reasonably safe. It is hard to realize that there are rubber mills in existence which have only a motor direct-connected to the mill line without a brake or other safety device. This results in the line coasting to a stop after the power has been shut off. We also realize that when the manufacturer is making a new installation, he will seriously consider the installation of a synchronous drive on account of power-factor correction. However, this does not help out the manufacturer who desires to change over existing equipment in order to cut down the length of stop.

We have in Akron, particularly at Firestone, some very large mill lines direct-connected to 800-h. p. induction motors by means of magnetic clutch brakes. I shall grant that the stopping distances so far obtained on these lines have not been very good. The chief trouble has been in demagnetizing the clutch. In other words, the mechanical brake has to help overcome the magnetization that stays in the clutch. We, along with the manufacturers of equipment, have been working for some time to overcome this trouble.

A great deal has been done in correcting faulty equipment although there is a great deal still to be done. Mills, in my opinion, offer one of the greatest hazards in the rubber industry

because it is impossible to guard the bite of the rolls and still have them do the job for which they were installed.

Quentin Graham: The performance of a synchronous motor when used for dynamic braking, as described by Mr. Drake, can be predicted with reasonable accuracy in a fairly simple manner. During the braking period, the motor becomes a generator feeding a non-inductive load. The energy of rotation of the machine and its load is converted into heat which is dissipated mainly in the external braking resistor. Since the

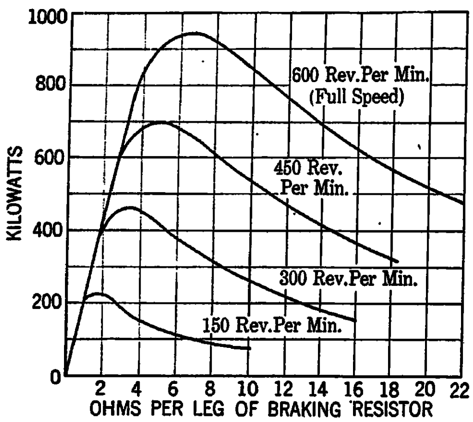


Fig. 4

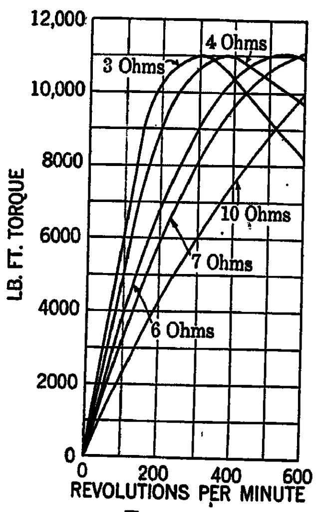


Fig. 5

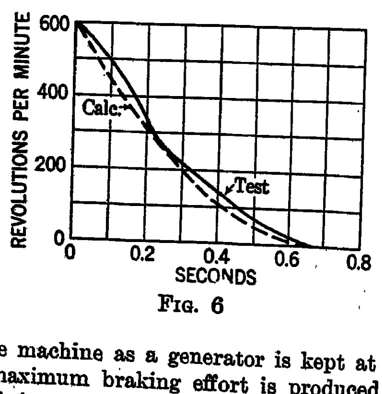


Fig. 6

output of the machine as a generator is kept at unity power factor, the maximum braking effort is produced for a given current. This is a decided contrast to the braking performance obtained with induction motors when reversed voltage is applied and the power factor and torque per ampere are quite low.

It is interesting to note the effect of changes in the ohmic value of the braking resistor. The curves in Fig. 4 show the variation in kilowatt output with changes in the braking resistance for a particular motor. Fig. 5 shows the corresponding values of braking torque determined from the kilowatt output curves. The losses within the motor, which also have a retarding effect, were neglected in plotting these curves since they are relatively unimportant. An inspection of the curves in Fig. 5 shows that at about 4 ohms per leg the braking torque would be most effective. Fig. 6 gives calculated and test curves showing the rate of retardation of the motor. For the case shown here the maximum braking torque was between two and a half and three times full-load torque.

For the retardation of heavy inertia loads which require much longer periods of time, there are two refinements of control which are used. First, the field current is increased to the maximum value so as to give the greatest possible output of the motor. Secondly, the braking resistance is varied as the speed decreases so as to obtain maximum output at all speeds. For most applications of dynamic braking, however, the retardation is too rapid to justify the use of these added complications.

P. C. Jones (communicated after adjournment): Mr. Drake's paper brings up a subject which should have been given more attention before, not only because it pertains to a distinct change in industrial drives but because the general use of such a method would vitally affect the entire central station industry by its decided affect on system power factor.

Thus, while there are any number of phases of the subject that could well be discussed, there are just two points I wish to bring up—both of them referring to the use of synchronous motors for dynamic braking.

It has always been recognized that braking equipment, particularly if used for emergency purposes, should be independent of any external source of energy. For this reason electrically operated brakes have been arranged so that the magnet, solenoid, or motor as the case may be, holds the brake released and a spring or weight sets the brake. Thus, even though the power should fail at the time the safety bar was pulled there would be no failure to stop. This principle is violated to some extent in the use of a synchronous motor as a dynamic brake. Where a direct-connected exciter is used the issue is met part way but not completely.

Of course, the chances of a power outage occurring at the time of braking are remote and generally will not be a sufficiently strong counter argument to have any appreciable weight in the ultimate conclusion, but they should always be considered if for no other purpose than to insure reliable excitation.

The second weakness I want to point out is inherent in all dynamic braking schemes. The stopping of a motor under a purely dynamic effect follows an exponential curve and is asymptotic to the axis of zero speed. Under pure dynamic effect, therefore, a motor will never stop. The expression for velocity is,

$$v = v_0 e^{-\frac{KT}{IZ}} \qquad \text{(a)}$$

where v is the speed of the motor at any moment, v_0 its initial speed, K a constant, I the moment of inertia of the system, and Z the impedance of the braking circuit.

In any actual case, however, in addition to the dynamic action there is a certain amount of line and motor friction and windage. Under this influence the velocity expression changes to the form below:

$$N v_0 = (N v_0 + 1) e^{-\frac{KT}{IZ}} - M \qquad \text{(b)}$$

If the expression (a) is integrated to obtain the distance traveled before stopping, the result will be infinity, but the expression (b) may be so integrated and yields a result in the form below:

$$s = (P I / Z) - Q \qquad \text{(c)}$$

P and Q are constants, s the distance traveled before stopping, and I and Z bearing the same significance as above. Actually both (b) and (c) are modified by armature reaction which cuts

down the initial torque and thus decreases the initial rate of deceleration, and, in the case of an a-c. generator, here being discussed, by the decreasing frequency which, by decreasing the reactance drop, has an opposite affect.

There is never any question as to the satisfactory stopping of a synchronous motor-driven mill line by dynamic braking under load conditions. Under unloaded conditions, however, where the friction load is low and the moment of inertia is high, and a very quick stop for safety purposes is desired, the conditions should be carefully checked.

Hans Weichsel (communicated after adjournment): It appears that the great number of applications to which the synchronous motor has proved to be useful, contradict the statement made by one gentleman that the synchronous motor cannot be considered as a machine of the future but rather one of the past.

It is interesting indeed to see from Mr. Drake's paper that the synchronous motor has been successfully used to replace induction motors with wound secondary, which, up to the introduction of the synchronous motor, had been used for this class of service on account of its superior starting characteristics. The operating results have shown that the synchronous motor can perform this work not only as well as an induction motor but has proved to be, in many respects, superior to it. There are two pronounced advantages obtained by the application of the synchronous motor, viz:

1. The power-factor correction
2. The possibility of applying dynamic braking

Referring to power-factor correction, the author properly calls attention to the fact that the obtainable correction is not equal to the leading component of the synchronous motor but equal to the sum of the lagging component of the replaced induction motor and the leading component of the replacing synchronous motor. This is a fact which quite often seems to have been overlooked in considering the field of usefulness of synchronous motors.

Referring more particularly to the contactor arrangement for obtaining the dynamic braking, the author states that the "voltage generated during braking is utilized to energize the lower magnet and hold the lower contacts together until the motor stops." No reason is given by the author why the voltage and not the current is used for this retaining action. I should think that in all possibility the decision was based on the following two points:

1. Mechanically, it is easier to arrange for a voltage coil than for a current coil.

2. Under the conditions existing during the braking period, the magnetism produced by a voltage coil is nearly constant for all speed conditions, while the magnetism produced by a current coil would decrease approximately proportionally with the decreasing speed.

It will be interesting to hear from Mr. Drake if these are the reasons which governed him in deciding for a voltage coil.

There are a few other questions which I should like to ask in this connection. A statement is made that, for maximum possible safety, the dynamic braking control should be "operated by gravity and not depend upon closing a contactor electrically." It would be interesting to have this statement explained more in detail. It appears to me that an electrically operated contactor is just as safe as a contactor operated by gravity as long as electric supply is in existence, but if the electric supply gives out, the dynamic braking effect fails no matter whether the contactor is operated by gravity or by electric means, because the possibility is very great that, with failure of the a-c. supply voltage, the d-c. supply voltage for exciting the synchronous motor also fails. Therefore, the braking effect in such an abnormal case would be zero no matter whether gravity or electric control is used for the contactor.

This reasoning assumes that the d-c. exciter is not directly connected to the synchronous motor, but is driven by an auxiliary unit. From the photographs and oscillograms given in the paper, it appears that this assumption is justified.

The very short time required for bringing these mills to a rest by aid of this dynamic braking equipment is extremely interesting, especially in connection with the fact that the current values in the primary winding when braking are materially less than when starting under average conditions. Naturally this is readily explained, because in starting, the motor has to overcome the inertia torque plus the torque necessary to move the rubber through the mills, while during the braking period, the generator torque plus the torque to move the rubber through the mill is equal to the inertia torque. It would be interesting to hear from Mr. Drake how often, under average working conditions, use is made of this dynamic braking equipment. It would seem that the number of braking periods during a day are so few and the time required for the actual braking so short that the braking conditions can be entirely neglected in selecting the frame size of the synchronous motor for a given horse power capacity.

E. W. Pilgrim (communicated after adjournment): It is my belief that dynamic braking of synchronous motors is by far the best method of emergency stopping for this type of motor. Tests we have made check with the data submitted in Mr. Drakes's paper and indicate that there are no severe strains imposed upon the motor or machine and I believe that this system which as far as I know is employed only in rubber mills, will become more or less the universal method in other industries.

Stopping by dynamic braking is always of the same value, whereas stopping by means of mechanical brakes is not, due to the fact that the brakes are not always in perfect adjustment. There is a definite resistance value for stopping the motor in the shortest time, and by varying this resistance, the time can be lengthened. We have found from experiments that increasing the resistance will lengthen the time and also decreasing the resistance over this definite value will also increase the length of time required for stopping.

I much prefer the automatic control as I believe that the mechanical control does not include all the features that should be included, and is not nearly so fool-proof. With hand control, if the operator throws the starting compensator into the starting position and the motor does not start, he is very apt to pull the compensator back to the "off" position which will cause severe arcing on the switch and perhaps its total destruction. The equipment should embrace a relay operated by the field current of the motor so that if the motor loses its field, it will stop. This would indicate to the operator that something was wrong and the trouble could be corrected. Otherwise, an emergency stop might be required when there is no field on the motor, in which case the dynamic braking would not be affective.

I do not think that it is absolutely necessary to operate the dynamic-braking contactors by gravity inasmuch as an electrically operated contactor functions fully as well and I can see no objections to this type of contactor, but the control should have other safety relays for stopping the motor in case of loss in field or loss in voltage. The chances that emergency stopping would be required at an instant when voltage fails, are very remote and I do not feel that it should be considered serious.

Dynamic braking of synchronous motors will, no doubt, be used very extensively now that experiments conducted by various electrical manufacturers have proven its reliability.

A. S. Rufsvold: The early synchronous motor dynamic braking installations described by Mr. Mortensen, are of considerable historical interest. Like any other electrical development, the scheme of control for dynamic braking has been considerably improved since it was first used.

In explaining the oscillograph records which he presented, Mr. Mortensen stated that the time required for the operation of the control immediately preceding the braking action was "very

short." The length of this time interval is of vital importance, because during the preliminary functioning of the control, the motor is traveling at full speed. In the apparatus described by Mr. Drake, this time interval is reduced to a minimum value of about 1/20 sec. by the use of a specially constructed contactor. This is an improvement over that type of control which depends upon the opening of one contactor and the closing of another by the use of relays, all of which takes up a considerable interval of time, and is less safe.

Mr. Mortensen has asked what provision has been made in the control described by Mr. Drake for dynamic braking during the starting period, before the excitation has been applied. On the latest type of control, this has been taken care of by arranging the control to automatically apply the excitation when a safety switch is opened during the starting period.

Mr. Hoener mentioned the possibility of the failure of power at the same instant a man is caught in the mill rolls. Although this is an exceedingly remote possibility, yet it is a point which is considered by safety committees. In the control described by Mr. Drake, since the dynamic braking contactor operates by gravity, failure of the a-c. power would not interfere with its operation. Of course, the braking action depends upon maintaining the field excitation, but should the main plant circuit breaker open, there is sufficient energy stored in the rotating parts of the motor-generator set supplying the direct current to maintain excitation during the brief dynamic braking operation. This brings up the question of what would happen in case a direct-connected exciter were used with the motor. On first thought it might be expected that no braking would be obtained, but tests have shown stopping distances quite comparable to those obtained by using separate excitation.

C. W. Drake: Mr. Hoener, Mr. Jones and Mr. Weichsel seem to be under the impression that no dynamic braking is obtained if the a-c. supply fails. This may be true with some types of control, but with the construction described, the failure of a-c. voltage automatically releases the upper contactors and the lower ones are closed by gravity. The d-c. field is maintained because any rotating d-c. machine, such as a motor-generator set or a synchronous converter, will, upon the failure of the a-c. supply, maintain its d-c. voltage for a sufficient length of time to give dynamic braking.

Mr. Jones, in his mathematical discussion, indicates that the synchronous motor losses and friction may materially affect the results of the various equations. He also states that for a quick stop with no load, such calculations should be carefully checked. Tests made on the stopping of synchronous motors in the factory with no load whatsoever indicate that there is no tendency for the motors to coast or drift at low speed. Fig. 6 in Mr. Graham's discussion shows a retardation curve under such conditions and it will be readily appreciated that the addition of a gear unit and mill line will add friction which will still further tend to eliminate drift.

Mr. Weichsel asks why a voltage, instead of a current coil, is used on the lower contactor for holding it closed during braking. The principal advantages of a voltage coil are as follows:

1. Easier to wind and connect
2. Fewer coil designs required to cover all applications
3. Gives more nearly uniform pull, since the impressed voltage and frequency decrease in proportion

Mr. Weichsel also questions the advantages of the gravity-operated contactor and quite definitely states there would be no braking in case of power failure. We have already explained why braking is obtained under these conditions and practise has proven it. Another advantage of the gravity-operated contactor is that any break in the control wiring immediately shuts down the mills, while if any circuit has to be made during the braking cycle, a failure in that wiring would remain unnoticed until it was required to operate.

Purchased Power as Applied to Plate Glass Manufacture

BY A. L. HARRINGTON[1]
Associate, A. I. E. E.

Synopsis:—The paper gives some general statistics on plate-glass manufacture, and estimates that the present total electrical requirements of the industry in the United States, are about 350,000,000 kw-hr. per annum.

About 60 per cent of this energy is purchased and is considered a very desirable load from the central station point of view, as the plants generally run almost continuously and shut down only a short time on Sundays, giving a very desirable load factor.

The paper briefly describes the manufacturing processes, and shows the different operations requiring the application of power. Specifically, it describes the 110-kv. station built for the Crystal City plant. This is designed along very simple lines to take the place of an isolated plant.

General Statistics, Plate Glass Manufacture

A SEARCH of the TRANSACTIONS of the Institute for quite a number of years back fails to show any papers presented on this subject, or, in fact, the application of electric power in any way to the manufacture of plate glass. In consequence of this, it might be well to present to the Society the general magnitude of the business as it affects central stations; then go into details.

It appears that the first attempt at plate-glass making in the United States dates back to about 1850. Nothing came of this and, in fact, various companies were formed and failed. Until 1883 no company seemed to be successful commercially. From that time on, plants have been built in small numbers until, at the present time, there are about sixteen separate factories operating.

In 1884 approximately one and one-half million square feet of glass were produced; now the American production is about one hundred million feet while approximately twenty-five million feet are imported.

Raw Materials Affecting Plant Location

Large amounts of very high grade silica sand, natural gas or coal and considerable quantities of water are required in the business, and most of the plants are located in a rough rectangle with Pittsburgh, Buffalo, Chicago and St. Louis as the four corners. In nearly all of this territory the supply of natural gas was plentiful and as it was particularly desirable as compared with producer gas which is now generally used, and as the melting end of the manufacture required about one-half of the total fuel used, a cheap and plentiful supply of natural gas largely dictated the location of the plant.

Previous Sources of Power

In the early plants most of the work was done by hand and most of the product was of relatively small dimensions. Gradually, however, individual high-speed engines were used for the grinding and polishing processes, to be later supplanted by Corliss engines, driving line shafts, from which principal units were driven through friction clutches. This later development required that the machines be driven at constant speed and about this time a development in the machinery itself made that feature possible. Line-shaft driving attained its maximum when 5000-h. p. engines were put in use.

Natural- and producer-gas engines were the next development, driving generators, many of which were 25-cycle, the principal units of the load being operated by means of belted motors. Overhead cranes and industrial locomotives were usually operated by 250 volts d-c., obtained by smaller gas engines or motor-generator sets.

Total Kilowatt-Hour Required

The present total requirements of the industry probably call for something like 350,000,000 kw-hr. per annum in the United States and an increasing percentage of this is being transferred to central station systems. At the present time, about 50 per cent of this is used in completely electrified plants and at least 60 per cent of this energy is purchased. In several of the larger plants, new power houses were built fifteen to twenty years ago and the normal depreciation of these plants has brought their life to an end within the last few years. So the question of changing over to central-station power has recently become a very important one.

Load Factors

Plants usually run continuously, shutting down only from eight to sixteen hours on Sundays during which time maybe 15 or 20 per cent of average power is required for operating machine shops, pump houses, cranes and locomotives. The annual shut-down, due to the necessity of making certain repairs about the furnaces, may decrease the demand on the power system to 10 or 15 per cent for two or three weeks, but in the event that a stock of semi-finished glass is available, the machinery part of the plant may be continued in operation, which makes a very desirable load from the central station man's point of view.

1. Pittsburgh Plate Glass Company, Pittsburgh, Pa.

Presented at the Spring Convention of the A. I. E. E., St. Louis, Mo., April 13-17, 1925.

Central Station Power Applied to Particular Plant

It so happens that at the present time one of the larger plants of the country, located within transmission distance of St. Louis,—namely, the Crystal City, Mo. plant of the Pittsburgh Plate Glass Company,—is being changed over to central station power and, as certain features of this change-over may be of interest to electrical engineers, this paper is prepared to make such information available.

In order to give a clearer understanding of the detailed application of power, we might describe briefly the process of plate glass manufacturing.

Detail of Manufacturing Process

The raw materials are chiefly a high grade silica sand, soda ash, limestone, together with a very small amount of sodium sulphate charcoal and arsenious oxide. The proper mixture of this raw material in a perfectly dry state is accomplished in a manner similar to that used with concrete. It is placed in fire-clay pots, which are, in turn, placed in the equivalent of an open-hearth furnace. The temperature is gradually raised until melting is complete. This is followed by a few hours of refining, after which the furnace is allowed to cool down so that the glass attains the consistency of "cold molasses." The pot is then removed from the furnace by an overhead crane and dumped on a water-cooled cast-iron table, somewhat larger than the largest plate desired to make. This is done by means of a special crane giving a motion to the pot very similar to a mason placing a trowelful of mortar on the brick.

A water-cooled cast-iron roller is then rolled over the hot glass, the thickness of the finished sheet being determined by flat strips of metal put under the ends of the roller. After the plate is cooled sufficiently to be capable of being pushed by its cold edge, it is forced into an annealing oven and is, by various mechanical methods, moved through a series of annealing ovens so that eventually it comes out at atmospheric temperature.

The "rough glass," approximately ½ in. thick, is roughly examined and cut in such a way as to remove defects, and stocked.

The next part of the cycle consists of grinding and polishing. The first step is to secure a sheet of rough glass to the approximately true top of a circular table which, when placed in the grinding or polishing machine, is in effect the same as a table of a boring mill. This attaching of the glass to the table is done by plaster of Paris so that but little time is consumed and yet the attachment is not so good but that it can later be quite easily separated.

Referring to illustration, Fig. 1, which is a vertical cross-section taken through the grinding or polishing machine, it will be seen that the table which is provided with four wheels to operate on a system of tracks around the works, is supported by and revolves with the circular casting at the top of the large shaft, which casting at all times remains secured to the shaft. It is evident that relative vertical motion of some part of the machine is necessary to place the table on the "spider" and this is usually accomplished in either one of two ways:

First, the tracks over the machine may be lowered by hydraulic jacks or equivalent, so that the table comes down on top of and rests on the spider, the rails going downward a sufficient amount to clear the wheel flanges: Second, the rails may be kept permanently in position and the shaft, together with the spider on the top, may be raised by a hydraulic jack until it lifts the table from the rails.

In the plant in question the last method is the one in use and the hydraulic jack at the bottom of the shaft,

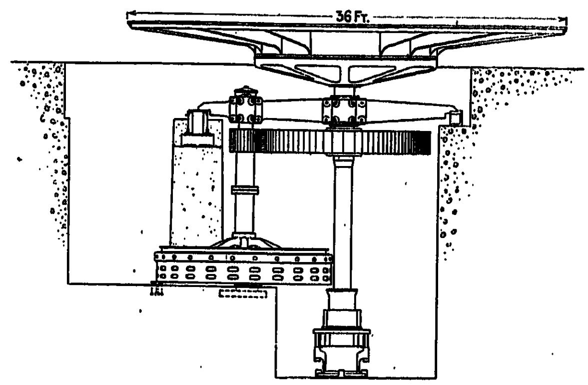


Fig. 1—General Cross-Section, Grinding and Polishing Machine

together with a safety lock or wedge, is introduced after the operation is completed, is shown at the bottom of the pit.

Grinding is accomplished with sand and water applied between the glass and iron blocks. These blocks are usually secured to disks, two or more of which are in contact with the top of the glass, with axes parallel but not in line with the main shaft. It is evident that rotation of the table produces a torque on these "runners," which revolve freely in their bearings, and pick up a speed intermediate to the table.

After coarse grades of sand have been applied, finer grades are usually followed by several grades of emery until the glass is as smooth as it is possible to make it. The table is then stopped, removed from the spider, transferred to another machine which is similar so far as the driving mechanism is concerned, but having in place of the cast iron blocks, pieces of felt saturated with iron oxide—"rouge"—and the glass is thus brought to a polish by the operation of the machine.

After one side of the glass is ground and polished, it is necessary to turn the plates over, bed them in plaster again, and repeat the cycle, after which the glass is removed from the table, washed with acid to remove the plaster, and, when dry, cut up into desired sizes and placed in stock.

In considering the application of electric power to the complete process of manufacture, we might first give consideration to the handling of the raw material. This is a proposition for handling raw material in which the usual methods are used and the only thing to be considered in applying a motor relates to the fact that the principal component, dry sand, is very highly abrasive. The other components, being alkaline, will attack certain parts of the equipment, particularly in the presence of moisture, and as a final item to look out for it is necessary to use every precaution to keep stray iron out of the batch material, which calls more or less for the use of magnetic separators,—usually pulleys.

The manufacture of the clay pots and other large amounts of fire-clay products is more or less similar to brick manufacture except that the clays are in a greater state of purity and must be kept so, and must be far more thoroughly mixed. The machinery consists of extra long pug mills in which the clay is mixed with water to the proper consistency, with rock crushers to crush up the raw clay, as well as fragments of previously burned clay, and chaser mills which grind the mixture of the different clays into powder.

Induction motors seem to be the proper driving medium and gear boxes, or similar devices taking up as little room as possible and closed against the entrance of dust, should be the connecting medium. As most of these machines either start up under load regularly or accidentally, they should be provided with slip-ring or double-wound rotor motors, 40- or 50-h. p. motors being about the limit as to size.

The entire melting or manufacture of the rough glass does not represent any features of interest to the electrical man, except in one or two small details. One of these is the "teeming crane" that pours the pots on the casting table. This teeming crane must move horizontally not more than twenty feet in two seconds, starting and stopping within this distance; and the motion must be combined with the pouring motion which occupies even less distance in this period. This power problem is usually met by applying a direct-current crane, or better still, mill motors and control that has practically no current or time limits.

The other item of interest is the considerable number of electric pyrometers required to follow the temperature of the glass throughout the various parts of the cycle.

The grinding and polishing part of the manufacturing process is the largest and by far the most important application of power as these machines consume upwards of 85 per cent of the entire power requirements. They also require the largest motors and are usually the only drives outside the ordinary daily applications.

Consideration of Main Drive

In the plant under consideration, 36-ft. tables are in use, a well-recognized maximum general size at many of the larger plants; and this size table requires upwards of 700 h. p. for the polishers, the speed of rotation being somewhere from 10 to 15 rev. per min., and the grinders require upwards of 500 h. p. at approximately double these speeds. We, therefore, have the problem of applying electric power to a vertical shaft running at very low speeds, and of course the first scheme would be a direct motor application. There are no examples of this kind of drive in use but it has commercial possibility.

The next solution is using a higher speed motor with a single gear reduction; if the machine in general is of the type in which the rails are lowered to permit the revolution of the table, then the high-speed shaft may be connected to the low-speed by either bevel gears, when the shaft would be horizontal, or, if the machine is of a type with fixed rails and the main shaft requires to be raised and lowered to permit the revolution of the table, then bevel gears are impossible and a high-speed vertical shaft with spur gears is necessary. With ordinary gear ratios, this will give a vertical shaft speed of somewhere from 50 to 150 revolutions and a vertical motor can be considered, as in the installation which we are describing.

The third scheme would be to apply bevel gears to the high-speed vertical shaft and on the horizontal shaft revolving at, say, 200 or more revolutions, the motor could be direct connected. Machines of this type are in successful operation.

Another scheme is to use a belted motor driving the last mentioned horizontal shaft, when, due to the motor being belted there are no particular limits as to the motor speed required. This last plan has been in use in this plant for some fifteen years and is in fact in quite general use in all plants.

In changing over this plant from 25 cycles produced locally to the 60-cycle central station power, (the 25-cycle system having been installed something over fifteen years ago) it was particularly necessary to consider the use of new motors rather than the use of the old motors with a frequency changer. This being the case, any type of motor could be selected; but due to mechanical troubles with belts, pulleys, bearings and bevel gears, which originally were designed for 300 and 500 h. p. but now with improved methods of manufacture, were loaded to peaks of 500 and 700 h. p., it was considered necessary to use a drive which would eliminate all bevel gears and belting. This decision reduced the methods of driving to the first two mentioned.

It is needless to point out that a motor revolving at 10 revolutions is a very large and expensive machine and nearly impossible at 60 cycles, although quite practical with the aid of a lower number of cycles and a frequency changer. Some of the saving due to a motor of this kind would be the reduction in size of the large pits required under the machines, formerly necessary to house the belted motors and various arrangements of gearings. Such a saving, however, could not be obtained except in a new plant designed particularly for motors of this kind and as it would also involve the

complete redesigning and rebuilding of the machine itself, it would mean a large loss in production; the machine being in use 144 hours per week. It was, therefore considered unfit for this case.

The second scheme of the vertical motor revolving at somewhere between 50 and 150 revolutions was found to be very satisfactory, if synchronous motors were used, both from an efficiency and a financial point of view.

Following is the problem which the motor manufacturers were called upon to meet:

Motors were to be three-phase, 60-cycle, 2200-volt, one rated 700 h. p. at 68 revolutions, and the other 500-h. p. at 106 revolutions; temperature rise being 50 deg. cent. and continuously loaded. Mechanically the motors were required to fit present pits and, specifically, not to exceed a certain diameter, approximately 15 ft. on account of their striking the main shaft or necessitating excessive cutting away of the concrete pillars supporting the upper part of the machinery. The stators were to overhang the pit without support. One bottom combined guide and thrust bearing was called for, this to be attached in the simplest possible manner to what remained of the concrete

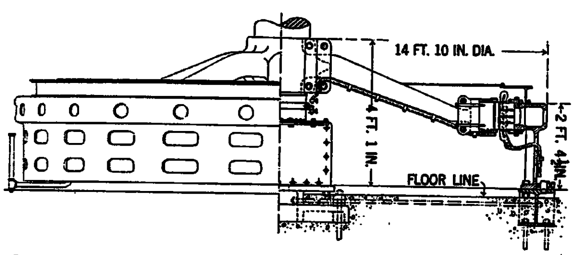


Fig. 2—Cross-Section, Synchronous Motor for Polishing Machine

after the block supporting the vertical shaft had been cut off to a level with the floor. The motors would be synchronous with 250-volt external excitation, self-starting and, when thrown directly across the line, were to develop sufficient torque to start. Due to the load at this plant being practically 95 per cent synchronous, unity power factor only was required.

Referring to the illustration, in Fig. 1, may be seen the manner in which the above specifications were carried out. The tooth reaction on the large motor is on the order of 45,000 pounds, which required the use of an 18-in. shaft to avoid excessive deflection causing unsatisfactory operation of the pinion and gear, or possible rubbing in the gap. The shaft is in two pieces, the idea being that pinion trouble, which has occurred in the past, could be quickly overcome by removing the top half of the shaft. The stator frame (Fig. 2) was made sufficiently deep to act as a girder in a vertical direction to take care of the overhang. The thrust bearing of the spring type (Fig. 3), was set upon a sole plate imbedded in the concrete floor, with arrangements to shift the bearing sideways and allow a lining-up of the gear teeth and similar arrangements provided on the stator frame to adjust the air-gap. The resultant side-pull at the bottom of the shaft due to reaction at the pinion, while not great because of the relative ratio of the lever arms, was, nevertheless, a considerable amount to be added to by variations in air-gap, and this side thrust was brought quite low on the bearing due to the fact that the thrust bearing is vertically rather thin.

The rotor spokes were made in umbrella type, bring-

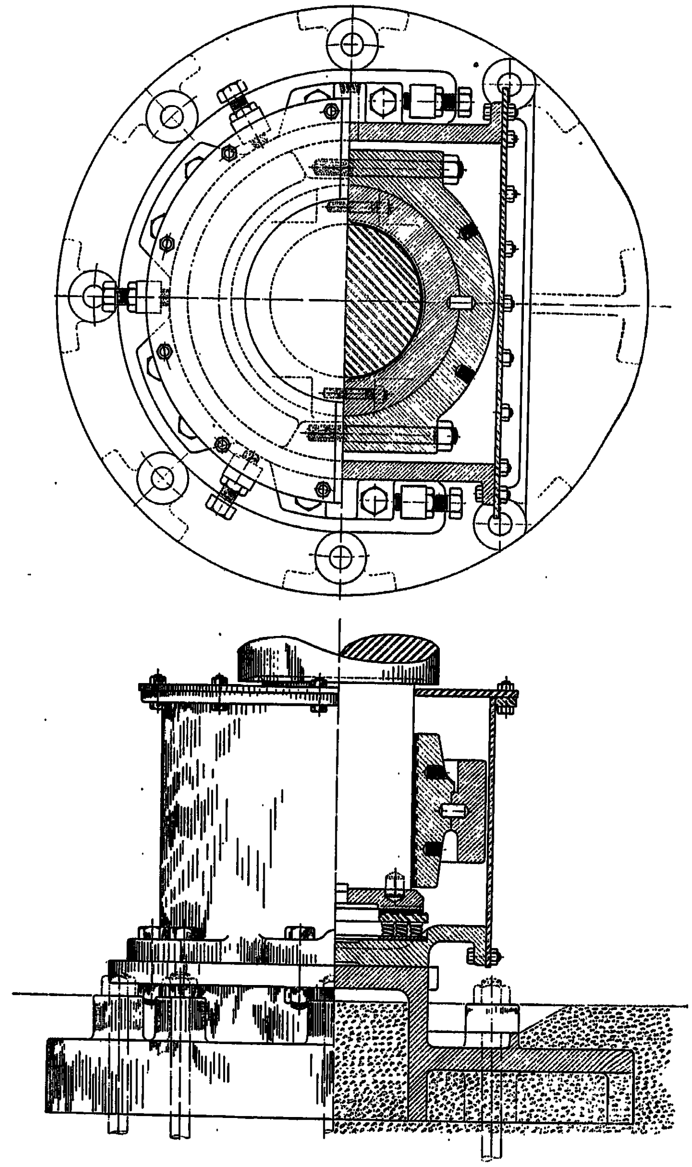


Fig. 3—Enlarged Views of Bearing of Motor Shown in Fig. 2

ing the center line of the active part of the motor down nearer the floor and quite close to the center of the lower bearing, the effect of this being that even considerable wear of the upper bearing would not cause any material change in the air-gap.

It will be seen from Fig. 3 that by taking the front plate off the bearing and raising the shaft slightly by means of jacks under the rotor rim, the bearing plates can if necessary, be very quickly removed and replaced.

These motors have been unusually successful in operation being put into regular continuous service within an hour or two after first being started. They show very high efficiency in spite of the low speed, the 700 h. p. being 92.3 per cent and 92.8 per cent for the 500 h. p. The motors are thrown directly across the line by means of a special contactor which is designed to have approximately 15,000-amperes rupturing capacity in the event of a short circuit; but this is sufficiently rapid in operation to permit of a jogging motion being given, so that the table may be slightly turned to bring the wheels above the track. The field is automatically closed by a definite time limit relay, set at something less than thirty seconds and put into operation by the throwing on of the stator current. The starting current, while not a limiting feature, is quite satisfactory, being less than the guaranteed 280 per cent of normal for the 700 h. p. and 250 per cent for the 500 h. p.

In conclusion, it might be stated that we are unable to locate any similar slow speed vertically geared synchronous motors in the United States, and that apparently those described herein are unique in several particulars.

High-Tension Substation

With reference to the central station power supply, the Union Electric Light and Power Company, of St. Louis, has built a double-circuit steel tower line, operating at approximately 115,000 volts from the Cahoka Power Plant to Crystal City, a transmission distance of 34 miles.

The power being purchased at 110,000 volts, the glass plant was required to build a complete receiving and transforming substation and other parts of the system required to receive this power from either or both of the transmission lines, the step-down transformer banks to be considered as a unit with the line. In other words, there were no cross connections.

The high-tension oil switches were required to have upwards of a million and a half-kv-a. rupturing capacity and their bushings, as well as the transformer bushings, were required to have at least leakage surface equivalent to that supplied on 132,000-volt equipment, because of the prevalence of coal smoke and similar dirty conditions of the atmosphere. The power factor of the load had to be above a certain amount, but this was more than met in that the bulk of the load consisted of synchronous apparatus.

From the consumers' viewpoint, in designing the high-tension substation the problem was presented of locating the station on a limited piece of ground adjacent to other high buildings and railroad tracks, compelling the transmission line to make a span of over 1600 feet across a number of the railroad yards of the company on which locomotive cranes were in constant use. This requirement in itself forced the use of a dead-ending tower approximately 135 ft. above the ground.

The peak load of the plant, amounting to 7500 kw. or more, together with some possible extensions, called for the consideration of 10,000 kv-a. banks. With these fundamental requirements in view, the substation shown in Fig. 4, was designed and built.

It will be seen that the two transmission lines are dead-ended on the upper platform of the tower and the

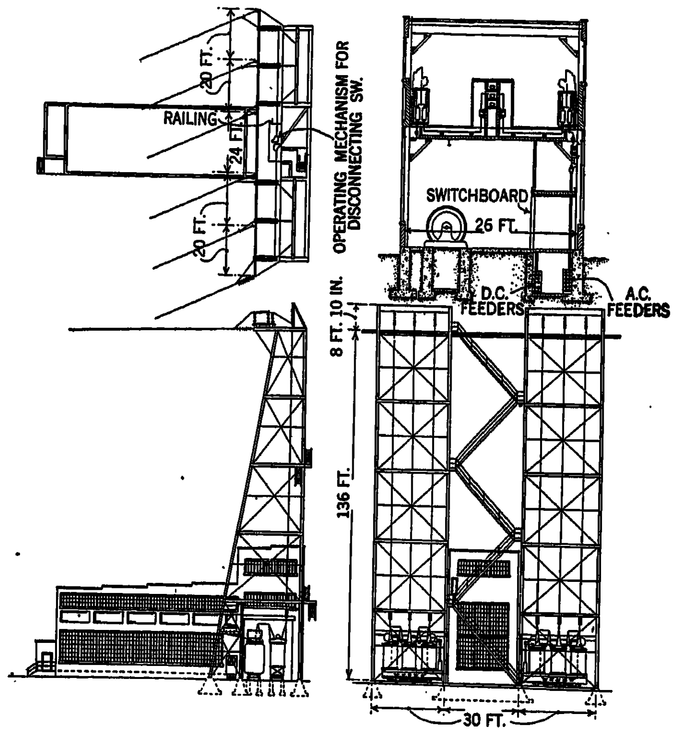


Fig. 3 A—High-Tension Transformer, with Four-Row Tubular Tank

wires are 20 feet apart. The circuit is then carried through motor-operated pole top switches having a single break six ft. long. The circuit continues down the inside of each of the outside sections of the tower, the wires being eight ft. apart, and having some five-ft. clearance to ground, supported at the upper end by strain disks, and at the lower end by a rigid insulator

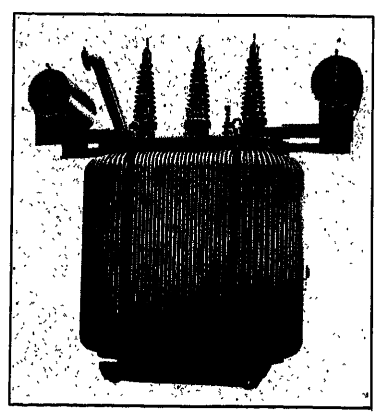


Fig. 4—View of 110-Kv., 20,000-Kw. Substation

hinged on the steelwork of the tower to swing in a vertical direction and clamped to the wire on the outside end. A weight of approximately a thousand pounds is attached to the head of the insulator, to put sufficient tension in the vertical wire to prevent side motion.

Running through the base of the tower are two standard gage tracks, on one of which are two 130,000-volt oil switches and on the other two 10,000-kw. three-phase, circular-coil core type transformers, provided with conservators and two-in. tubes for cooling. It will be noticed that the line side of the oil switch is within a few feet of the steadying insulator before mentioned, the relative arrangement being such that if the line should burn off at the upper end, the insulator and the weight are expected to swing downward and thus prevent damage to the oil-switch bushing. The oil switch and the transformer are as close together as possible so that no high-tension insulators are required between the two. The 2300-volt secondaries are connected to the leads going into the building by short lengths of flexible cable.

The central part of the tower, approximately 28 ft. square and 50 ft. high, constitutes the repair shed. Roll-up doors, 20 ft. wide and 23 ft. high, admit both the oil switch and the transformer to the building, where a 25-ton trolley of the standard type used on overhead cranes, operates on a fixed track at right angles to the transformer track. This crane is provided with a motor for the lift and hand motion for the cross travel. The lifting speed is approximately two feet per minute, which is very handy for transformer repair work.

The outstanding points of the high-tension part of the substation lie in the fact that only twelve high-tension insulators are in use outside of the apparatus bushings; 20,000 kw. with switching and repair facilities is located on approximately 2000 sq. ft. of ground; and the transformers, we understand, are the largest, or among the largest, provided with tubes, that have ever been shipped filled with oil.

The low-tension part of the substation is a two-story building on the second floor of which is the 2300-volt bus structure consisting of the usual concrete and brick arrangements, with disconnecting switches connected to the H-type oil switches, arranged along each side of the building by one-quarter by four bars run under the removable steel floor and above the four in. concrete ceiling. On the first floor, two 1200-kw. motor generators, with space for a third, are installed.

Along one entire side of the building is the remote control a-c. and the hand-operated d-c. switchboard of the usual design. A mezzanine floor located back of the panels and slightly above them, is provided to give access to the outgoing disconnecting switches and to cover part of the control conduit. There is also a partition wall running from the top of the switchboard panels to the ceiling which contains the balance of the control conduit and also protects the balance of the station in the event of a cable pothead or a current transformer blowing up. At the top of the panels is located a pull box, about 4 in. by 12 in. and extending the entire length.

In this connection it will be seen that considerable expense has been incurred to place barriers throughout the low-tension busbar system, as it was considered that short circuits of about 250,000 kv-a. might occur.

The outgoing a-c. feeders, in three-conductor leaded cable, go down the side of the building from above the mezzanine floor and, in fiber ducts encased in concrete, into the pit underneath the switchboard. The d-c. cables of the same general description enter the ducts on the other side of this pit immediately under the switchboard panels.

Substation Equipment

High-tension disconnecting switches:
- 2—L G 119—154 kv. on 135-kv. insulators, motor operated.

High-tension oil switches:
- 2—F H K O—72 C—135,000-volt, 400-amperes, solenoid operated.

High-tension transformers:
- 2—Type H. circular coil 10,000-kv-a.—110,000-2300-volt, three-phase, 60-cycles, air-cooled by 2 in. tubes, with conservators.

High-tension oil storage and treating system:
- 2—12,000 gal.
- 1—7 in. filter press.

High-tension repair shop:
- 1—25-ton crane.

Low-tension oil switches:
- 2—3000 amperes, 15,000 volt.
- 2—1200 " "
- 4— 800 " "
- 8— 500 " "
- Type H-206 and 203.
- 6—Motor-generator starting switches,—F. K. 132-A & B.

Bus insulators, 15,000 volt, heavy duty.

Motor generators:
- 2—1200-kw. 250-volt, with synchronous motors.

Switchboard:
- 12—A-c. remote control panels,
- 21—D-c., manual.

Above electric equipment by General Electric Company,

Substation peak, 10,000 kw. annual kw-hr., 50,000,000.

Plant Equipment:
- 8—700-h. p. synchronous motors, G. E. Co.
- 8—500-h. p. synchronous motors, G. E. Co.
- 1—500-h. p. synchronous motors for air compressors, G. E. Co.
  - Control for above.
  - 675-h. p. induction motors—for pumps—G. E. Co.
  - 950-h. p. motors various uses,
  - 500-motors, (approx) 250-volt, d-c. for all other uses.
  - 500-kw. total lighting transformer.
- Main distributing system—7 miles 3-conductor lead cable mostly underground.

Discussion

C. W. Fick (communicated after adjournment): Mr. Harrington's very interesting paper may raise the question of why this arrangement of vertical synchronous-motor drive is not universally adopted by the plate-glass manufacturers. There are some twenty plate-glass plants in this country with a total of 100 to 125 grinding and polishing machines. Of this number a majority must operate for a short time at slow speed at the beginning of each operating cycle, due to the method of lowering the grinder and polishing disks onto the glass. Thus, the synchronous motor is eliminated for these cases, unless a frequency changer and double bus arrangement is used.

A wound-rotor induction motor arranged as were the synchronous motors described by Mr. Harrington would have a power factor of between 65 and 68 per cent and would be more costly and require more control equipment.

However, there is no reason why the belts and ropes which are at present used on perhaps 75 per cent of these tables, cannot be eliminated by the use of horizontal slip-ring motors (as new motors are required) of 200 to 250 rev. per min. coupled to the bevel-gear shaft, with a saving of maintenance and power, and in some cases of first cost.

A. L. Harrington: The question has been asked why high-tension lightning arresters have been omitted from this substation. It was our privilege to be connected with a power system having some of the earliest outdoor substations. This system was in a locality subject to lightning any day of the year. Although the original main and substations were equipped with arresters, business increased so rapidly that it came about that transformers (in some cases) were on hand for new customers but no lightning arresters. A comparison was then available between protected and unprotected stations. Such failures as were experienced were never from line to ground but rather from turn to turn or coil to coil in the transformer. Much evidence was present to show that the dangerous disturbances were often of low potential but high frequency. In many cases results did not seem to justify the cost of lightning arresters.

With the greatly improved design of some modern transformers, we feel that they do not need the assistance of arresters. Recently these conclusions have been proven true in the Pittsburgh territory, but at a lower voltage than the substation under discussion. It is possible to add arresters to this job later if conditions of actual service seem to warrant them.

Sleet and Ice Troubles on Transmission Lines in New England

C. R. OLIVER[1]
Associate, A. I. E. E.

Synopsis.—This paper deals with the troubles one of the large power companies in New England has encountered from sleet and ice on transmission lines. The experience of this company should bring home to the transmission line designer the fact that in certain parts of the country sleet is a very serious problem and should be taken into account in designing all new transmission lines. It also shows the necessity for looking over the existing transmission systems with a view to providing some quick and easy method of thawing sleet from the lines already constructed.

* * * * *

UNTIL recent years, New England was not thought of as a section of country particularly susceptible to sleet; in fact, no serious trouble had been experienced by any of the communication or power companies before the storms of recent years, and outside of the usual one-half inch of ice and 8-lb. wind pressure, no particular attention was given by the designing engineer to the possibility of trouble from this source, either in the tower or in the conductor. Our recent experience has convinced us that there is no single tower line in New England built before 1922 that would stand up under the excessive loads that they may be called upon to carry. The only reason that any of them are standing today is that they have been fortunate enough to escape the sleet.

THE NEW ENGLAND COMPANY POWER SYSTEM

In describing the sleet troubles, the author will confine himself to those experienced by the New England Company Power System lines, with which he is most familiar.

This system, with its connections, extends from the New York State line across the State of Massachusetts to Boston, down into Rhode Island to Providence, and across to Fall River. In the northern section it extends half-way through Vermont, into New Hampshire and across the State of Connecticut to New London and Stafford Springs. Its main trunk lines are towers built in 1908 to 1914. Within recent years wooden pole lines have been constructed instead of tower lines. All of these lines operate at 66,000 volts. Last year a new double-circuit tower line was completed, 75 miles long across the center of Massachusetts, designed to operate at 110,000 volts. There are 240 miles of 66,000-volt, double-circuit tower line; 70 miles of 66,000-volt, double-circuit pole line; 275 miles of 66,000-volt, single-circuit, pole line; 75 miles of 110,000-volt, double-circuit, tower line, and approximately 75 miles of 22,000- and 13,000-volt feeder lines.

TROUBLE OF 1916

The first serious trouble on the system, due to sleet, came on December 22, 1916, on the tower line that connects the station near Hoosac Tunnel, Massachusetts, with the substation at North Adams, Massachusetts. This line was built in 1913, and consists entirely of 75-ft. standard square towers weighing approximately 5000 lb. each, with two circuits of No. 1 copper conductor, each circuit being arranged in a vertical plane, and with a ground wire at the peak of the towers. It is only 8.3 miles long, but in this length it crosses a section of country varying in elevation from 1000 to 2300 ft. above sea level. During the first winter of service on this line it was observed that quite frequently a slight sleet formation took place on the wires, and the engineers of the company thought that the No. 1 copper was too light to carry the sleet load. In the following summer, the No. 1 copper on five towers over the highest peak was replaced with ⅜-in. crucible steel conductors, and all of the towers over this peak were dead-ended.

This decision was very unwise, for two years later, in 1916, this section of the country was visited by a very heavy sleet storm, and ice of three to four inch diameter was sticking to the wires. All five of these towers on which the heavier conductors had been installed were completely wrecked. This failure occurred without a single break in the conductor, showing conclusively that had the towers been designed for the stronger conductor, no interruption would have occurred.

Only temporary repairs could be made, using low wooden structures, as the whole mountain side was frozen and covered with glare ice. Later, in the summer, the wooden structures were replaced with ten low towers, placing all wires in a horizontal plane. These towers were designed to withstand the sleet and wind loads for this section, and since their installation no trouble in this particular section has been experienced, although there have been some very heavy sleet storms on this line.

Fig. 1 shows the replacement towers which have been very satisfactory.

TROUBLE OF NOVEMBER, 1920

During Thanksgiving week of 1920 another sleet storm swept over this line, affecting a four-mile section varying in elevation from 1500 to 2300 ft. The sleet was

1. Assistant General Manager, New England Power Co., Worcester, Mass.

Presented at the Regional Meeting of the A. I. E. E., Swampscott, Mass., May 7, 1925.

again three and one-half in. to four in. thick, and although there was no tower failure, there were many span-wires broken; so many in fact, that later in the summer this whole section had to be resagged. The storm lasted five days and no headway could be made toward getting the line back into service, for as fast as breaks were repaired in one section trouble would develop in other spans. In desperation some 2300/220-volt transformers were borrowed from one of the manufacturers and, by a temporary connection, were so arranged that it was possible to secure approximately 6000 volts;

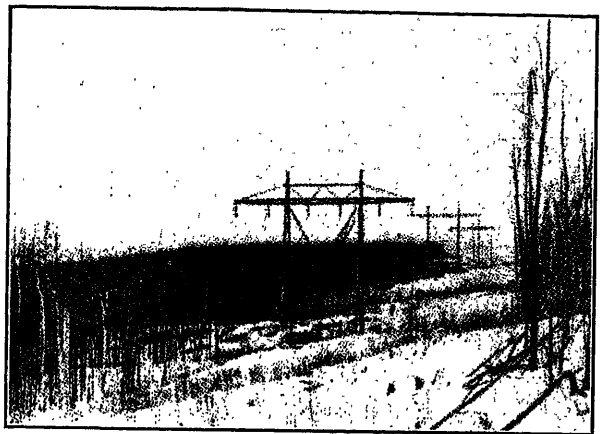


Fig. 1—A Section of the Line Showing Type of Replacement Towers

and with this pressure, using about 300 amperes, the sleet was cleared from the line by heat. During all this time the customers supplied from the North Adams substation were without power.

After the storm a careful examination of the line showed no signs of any of the towers being overstressed, nor were the wires stretched beyond their elastic limit. The only apparent damage was the broken conductors due to the overloading of the ice.

Fig. 2 shows the profile of the line from Hoosac Tunnel station to Adams substation, and the extent of ice trouble is indicated by the shaded part of the line.

Trouble in 1921

The most disastrous sleet storm on record in this section of the country started December 28, 1921. There was a steady, drizzling rain for three days with a temperature of around 32 deg. fahr. This froze as soon as it struck, thus accumulating ice three and four inches in diameter on wires, towers and trees. There was also a wind of about thirty miles an hour accompanying this storm, which appeared to have three distinct paths; one across the mountains between Shelbourne Falls and North Adams, one in the hills between Leverett and Ware, and one from Gardner east to the coast, the storm being heaviest within a 30-mi. radius of the city of Worcester, Massachusetts. Down the Connecticut Valley and in other sections of the system there was snow and very little damage to power and lines of communication. On the mountains around Hoosac Tunnel, where for years there had been so much trouble from sleet, the ice was just as heavy as in previous years but during this whole time service was maintained on two circuits 66,000-volt tower line without interruption. This was due entirely to the sleet-thawing method previously worked out. The telephone trunk-line over this mountain had, in early years, successfully withstood the sleet but was now completely wiped out. Lines affected in the company system were as follows:

1. Over 55 miles of double-circuit, 2/0 copper on 75-ft., square towers, weighing approximately 5000 lb. each, with the wires in a vertical plane and the ground wire at the peak: This line was built in 1913-1914 and extends from Shelbourne Falls to Millbury and Pawtucket. There were 69 tower failures, five towers with broken peaks and four with damaged crossarms. The tower failures consisted of broken masts, crippled legs, broken crossarms, damaged footings, both in tension and compression, and a number of towers was completely wrecked.

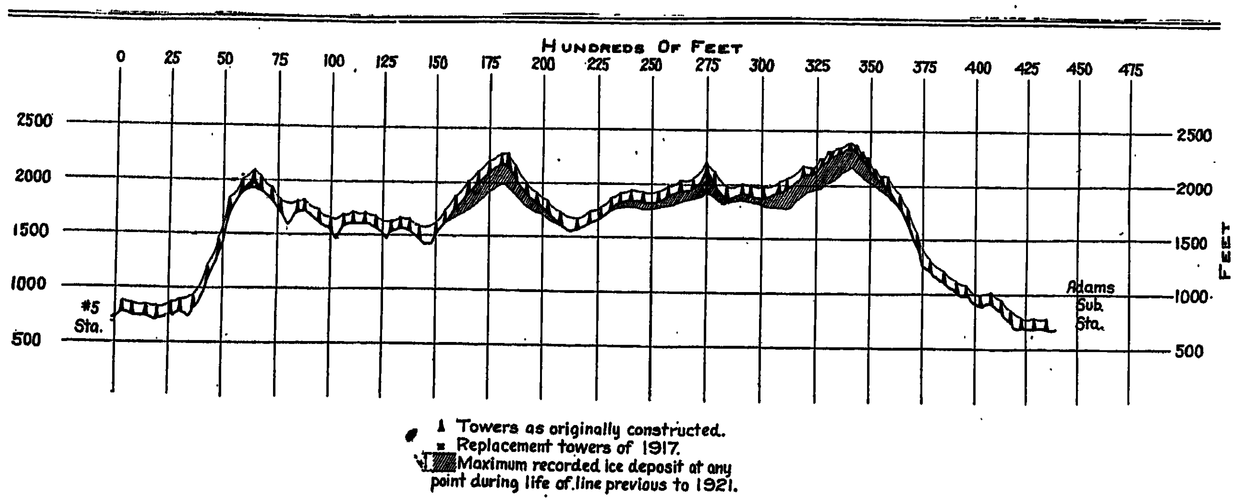


Fig. 2—Profile of Nos. 9-10 Lines from No. 5 Station to Adams Substation

Approximately 75 per cent of the damage was at

dead-end points, and in addition to the tower failures, there were many hundreds of broken spans on both power and ground wires.

Temporary repairs were made by building a 66,000-volt, single-circuit pole line along the edge of the right of way, and later, the good parts of the damaged towers were salvaged, the towers being replaced with new material purchased.

2. Over 40 miles of double-circuit, No. 2/0 and No. 2 copper line, with bayonet type towers, 50 ft. high, with pins, insulators and wires arranged in a triangular position: This line was constructed in 1908 and extends from Vernon, Vermont, to Worcester, Massachusetts, and all towers had been designed to break the conductor. Only three towers failed on this line, two being at angle points of over 45 deg., and the third tower was pulled down by the failure of the other two. Between Worcester and Gardner there were many

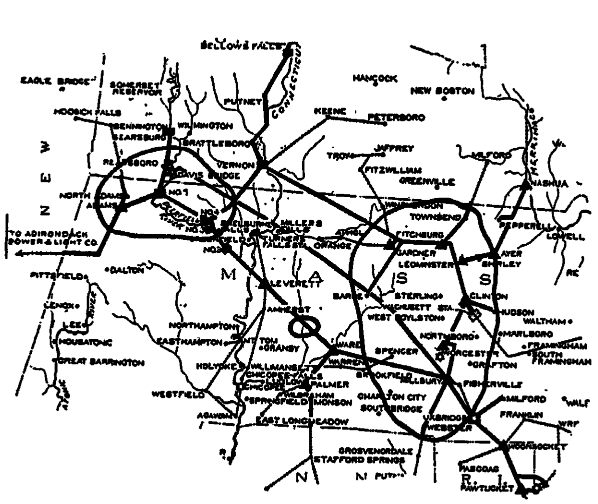


Fig. 3—Enclosures Show Sections Most Seriously Affected by Sleet

hundreds of broken spans, and damaged insulators and pins.

3. Over ten miles of "A" frame, double-circuit tower line with No. 2 copper in a vertical plane and ½-in., copper-clad ground wire at the peak of the tower: This line passes around the city of Worcester on a very crooked right of way, with many large angles. Among the "A" frames were mixed a few 75-ft., square towers at heavy angles and guyed to take the expected loading. Due to the excessive sleet loading, however, some of the angle towers gave way and 78 towers were completely wrecked. On the remaining towers a number of span wires were broken and the whole line was so completely wrecked that it was necessary to replace all of the damaged towers with two-pole structures.

4. Over 50 miles of wooden pole line with wishbone crossarms and suspension insulators: These poles were substantially guyed and double-pole construction used on all dead-end points. Four or five crossarms were broken and a number of poles was thrown out of the line, but there were no broken poles; there were also a few broken conductor spans of No. 2 copper, but it was a comparatively easy job to put these pole lines back in service.

5. In addition to the high-tension trouble, there were many miles of 13,000- and 22,000- volt feeder lines running along city streets carrying No. 2/0 copper and smaller. There were many broken span wires, but this was caused primarily by the trees, overloaded with ice, falling on the feeder lines and carrying them down. There was very little pole damage, and these lines were quickly put back into service as soon as the sleet was cleared.

Fig. 3 shows the area most affected by sleet during this storm.

Troubles of March, 1924

On the 11th of March, 1924, during a heavy snowstorm, the system began to experience trouble in the southern section, centering around Worcester and Providence. The wet snow packed around the insulators and stuck to the wires. By night it became so serious that Millbury, Mass., and Woonsocket and Warren, R. I., substations were completely isolated from the system. The storm centered in Rhode Island, and its effect on the balance of the system around Worcester was not so serious.

The trouble was caused by the very wet snow, sticking to the wires and towers, becoming solidified due to a drop in the temperature and this combination of snow load and a 60-mile wind completed the damage. In Rhode Island, snow accumulated to a diameter of four inches and actual measurements showed that it weighed 48 lb. per cubic foot. Around Worcester the snow was not so wet, and although the snowfall there was greater, the accumulation on the wires was much less. On lines north of Worcester there were very few broken spans on any of the circuits, and these circuits were all back in condition within a short while.

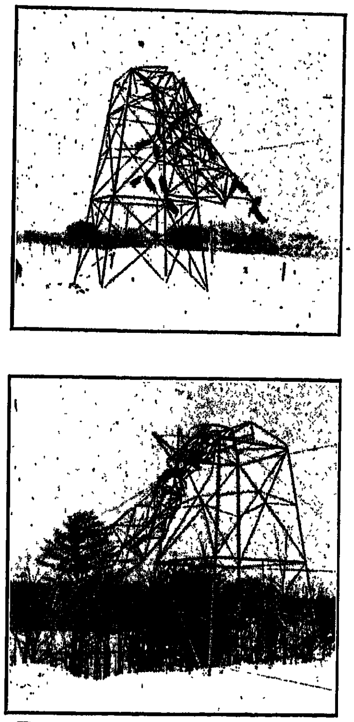
Between Worcester and Pawtucket, R. I., a distance of thirty miles, there were 36 broken span wires, 21 of them occuring in the last ten miles; but there was no structure damage. All of the towers in this section were standard 75-ft. square towers, weighing approximately 5000 lb., with the wires in a vertical plane.

The heaviest damage centered around Providence and on the transmission line to Fall River, (16.5 miles), 94 out of a total of 147 towers were damaged. This damage varied from broken masts to completely wrecked structures. The line consisted of 75-ft. standard square towers with the wires in the vertical plane and weighing about 3500 lb. Being lighter than the other 75-ft. towers on the system, the effect of the storm on these was much more disastrous. Temporary repairs were made by building a double-circuit, wooden-pole line along the edge of the right of way.

Figs. 4 and 5 show typical tower failures, illustrating how completely the towers were wrecked, and Fig. 6

is a photo of solidified snow on the wire, four inches in diameter.

An interesting thing developed during this storm; a 66,000-volt, double-pole, double circuit of No. 2/0 copper, wooden pole line with all wires in a horizontal plane runs around the north side of Pawtucket to East Providence. This pole line for quite a distance is on an adjacent right of way with a standard 75-ft. tower line. The ten miles of wooden pole line came through the storm with only three broken conductors and a few unhooked insulator strings, while the tower line had several towers completely wrecked and a great many broken span wires. One of these towers, due to the heavy load of ice, fell across the pole line and caused one of the above mentioned broken spans. The pole line had no broken or damaged structures and was put back in condition in one day's time.



Figs. 4-5—Tower Failures

Fig. 7 is a photograph of the 75-ft. tower which, in falling, put the wooden-pole line out of service.

Sleet Thawing

The first experience of sleet thawing on the company's system was during the storm of 1920. The lines were cleared of ice in western Massachusetts by using a group of 2300/220-volt transformers connected in series and paralleled, to secure from 6000 to 8000 volts at 300 amperes. This was only a temporary connection made up after six days of ineffective work trying to clear the lines, and much to the surprise of all it worked amazingly well. These were retained in the plant during the remainder of the winter. Plans were then made to provide the Hoosac Tunnel Station with permanent means of thawing the ice from the two No. 1 copper lines running over the mountain to North Adams, eight miles, and the two No. 1 circuits running over the mountains to Shelbourne Falls,—14 miles. Equipment in this station consisted of two 11,000-volt, 25-cycle generators, and two 2300-volt, 60-cycle generators; five 3000-kv-a., three-phase, 66,000- to 2300-volt transformers.

From many tests made in the field, it was evident

Fig. 6—Solidified-Snow-Covered Cable

that from 250 to 300 amperes at 6600 to 8000 volts would be required to clear in an hour the No. 1 copper of sleet. The engineers of the company were in favor of providing a separate auto-transformer for this service only, but on receiving the proposal from the manufacturers, the proposition was abandoned as too excessive in cost. Further study was made and it was finally discovered that by opening the delta on the low-tension side of one of the three-phase transformers and connecting the individual coils in series with the delta of a second three-phase transformer and

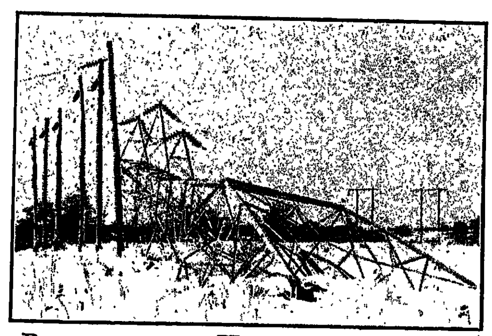


Fig. 7—Destruction of Wooden-Pole Line by Fall of 75-Ft. Tower

with the two high-tension sides connected in multiple, an extended delta could be secured, giving the desired voltage.

Fig. 8 shows the diagram of connection and the method of connecting the two deltas.

In order to make use of this combination for sleet thawing and still keep the transformers available for ordinary service during the other times of the year, all six secondary leads were brought out of one of the

three-phase transformers to disconnecting switches mounted on the ceiling overhead the unit. These switches were so arranged that the special combination for thawing could be made very quickly, and special cables were run from the extended delta up to the roof of the building, where a flexible connection of a hundred feet of three-phase cable was provided, by means of which connections could be made to any of the four lines on the roof to be thawed. The total apparatus tied up for sleet thawing consisted of one 60-cycle,

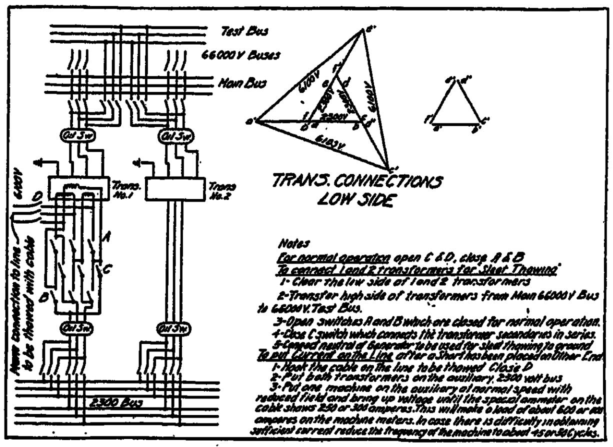


FIG. 8

2300-volt generator, and two three-phase transformers. This equipment, with a separate exciter, could be operated independent of the balance of the equipment in the station. The total cost of the sleet thawing outfit was, therefore, very moderate, and the only new equipment required was a few sets of 6600-volt, disconnecting switches and some cables. A test made under actual operating condition on the line to North Adams, (8.3 miles long and of No. 1 copper), showed the following result at 60 cycles.

Line Amperes	Generator Volts	Line Volts
200	1170	3150
250	1460	3930
300	1740	4760
390	2300	6150

A test on the line to Shelbourne Falls, (14.7 miles and of No. 1 copper), showed the following results:

Line Amperes	60 Cycle Generator Volts	Line Volts
200	2290	5600
250	2600	6970
	40 Cycle	
200	1670	4470
250	2090	5590
275	2300	6120
300	2500	6700

Within the last year, the third 25-cycle, 11,000-volt generator has been installed, and this will also be available for sleet thawing.

The success of the sleet-thawing arrangement depends entirely upon complete cooperation between the generating station at Hoosac Tunnel and the stations at the end of the line to be thawed. This cooperation normally requires telephone connection between the stations, but during sleet trouble, the telephone lines are usually the first ones to be put out of commission, so it was necessary to devise a means of working the sleet thawing device absolutely independent of telephone. Accordingly, a definite schedule was worked out for 24 hours, whereby sleet could be thawed on any or all of the four lines radiating from this station. This schedule is independent of dispatchers' orders or telephone, and it functions even when all telephone connections have been completely wiped out.

The sleet thawing is always started and carried on in accordance with this schedule regardless of the time of day and the number of lines to be thawed, so that the crews at both ends of the lines are thoroughly familiar with the routine of operation. A complete cycle of the schedule is given as follows:

Time	Shel. Falls Sta.	Hoosac Tunnel Station	North Adams Substa.
2:45 p.m.	Remove short from No. 7 Line	Start heat on No. 10 Line	..
3:00 p.m.	Close No. 7 Line when alive	Energize No. 7 Line at 66,000 volts	..
3:15 p.m.	Open and switch No. 8 Line	Open and switch No. 8 Line	..
3:30 p.m.	Attach short to No. 8 Line if dead	Stop heat on No. 10 Line	..
3:45 p.m.	..	Start heat on No. 8 Line	Remove short from No. 10 Line
4:00 p.m.	..	Energize No. 10 Line at 66,000 volts	Close No. 10 switch when alive
4:30 p.m.	..	Open and switch No. 9 Line	Open and switch No. 9 Line
4:45 p.m.	..	Stop heat on No. 8 Line	Attach short on No. 9 Line if dead
5:00 p.m.	Remove short from No. 8 Line	Start heat on No. 9 Line	..
5:15 p.m.	Close No. 8 Line switch when alive	Energize No. 8 Line at 66,000 volts	..
5:30 p.m.	Open and switch No. 7 Line	Open and switch No. 7 Line	..
5:45 p.m.	Attach short to No. 7 Line if dead	Stop heat on No. 9	..
6:00 p.m.	..	Start heat on No. 7 Line	Remove short from No. 9 Line
6:15 p.m.	..	Energize No. 9 Line at 66,000 volts	Close No. 9 Line switch when alive
6:45 p.m.	..	Open and switch No. 10 Line	Open and switch No. 10 Line
7:00 p.m.	..	Stop heat on No. 7 Line	Attach short on No. 10 Line if dead
7:15 p.m.	Remove short from No. 7 Line	Start heat on No. 10 Line	..
7:30 p.m.	Close No. 7 Line switch when alive	Energize No. 7 Line at 66,000 volts	..

The sleet thawing equipment was completed several weeks before the storm of 1921, but the crews at the station had not become proficient in its use, and when the sleet storm struck, it was not deemed wise to attempt to keep all four lines free from ice. All energy was devoted to keeping the lines over Florida Mountain

clear. As a result, these two lines came through the storm with a clean record and service was maintained to the customer during the whole time, in spite of the fact that all other communications and telephone lines in this part of the country were very seriously affected by the storm. This test thoroughly convinced the organization of the efficiency of the device, and since that time, sleet-thawing devices have been installed in several other points on the system.

Conclusions

The lessons learned during the past few years have been very expensive, but it is hoped that the future will benefit by so designing the lines of the system that they will come through a storm with a clean record. The following facts have been discovered:

1. That towers should be designed to stand regardless of the load on the conductors. A conductor failure is bad, but a structure failure is positively disastrous.

2. That earth footings fail very often in compression and this failure has wrecked many towers. There are also some failures of footings in tension.

3. That guying towers at angle points instead of using a heavy structure is very poor makeshift. The tower usually fails before the guy has a chance to do any work.

4. That "A" frame towers are not an ideal combination for sleet conditions even when all points are reinforced with heavy square towers installed every half mile in tangent and with a heavy ground wire overhead.

5. That vertical spacing of conductors will never prove satisfactory for sleet condition, not even with the middle crossarm extended.

6. That the so-called 75-ft. standard square towers as used in this section of the country for years will not withstand the sleet and wind load.

7. That wooden pole lines have suffered less damage and are easier to put back into service than light tower lines.

8. That in building important high-tension lines there is no half-way ground. To withstand the sleet it should be either a double-pole construction with steel crossarms or a steel tower with suitable factors of safety and with all wires in a horizontal plane.

9. That wherever possible it is advisable to install a thawing device if it can be done at a reasonable figure. This is a very desirable addition in New England, and will pay for itself many times on transmission lines thus protected from the sleet storm.

10. That over 60 per cent of trouble occurs at dead-end towers. These should be self supporting and not guyed, and as many angles as possible eliminated.

It is quite evident that ideas have had to be revised about transmission line design for this section of the country. During the past two years, the Company has constructed over 125 miles of 110,000-volt transmission lines and these lines have been designed with the thought that it was possible to build one that would withstand even the severe sleet storms of New England. Advantage has been taken of the lesson we have learned from the other sleet storms, and it is believed that the present transmission line will prove satisfactory.

Discussion

L. W. W. Morrow: The Pennsylvania Power & Light System, as well as other companies in central Pennsylvania, have encountered this sleet problem for several years, and have introduced a method of combating sleet which I think will prove of interest. It is somewhat along the same lines as the method used by the New England Power Company.

The method is based on the premise of catching sleet before it forms. In other words, we all know that it takes a large amount of heat to melt ice once formed. Down there most of their sleet occurs only in the high-altitude territory, so they have organized a system whereby, after studying all weather maps and close observation of approaching storms, all patrolmen are concentrated into the sleet areas to be on the job to start immediately if there is any chance whatsoever of sleet forming on any line.

If a report comes in that a sleet storm is approaching, they immediately organize to combat the sleet. This is done by isolating generators in stations sectionalizing lines as much as possible, improvising equipment that can short-circuit any line, and attempting to heat the lines before the sleet forms. It doesn't take much heat in a line to keep sleet from forming, but it takes an immense amount of heat to melt ice off the line once it is formed. So their method is sectionalization and crowding loads into lines, utilizing generators in stations, putting energy into the lines they are trying to warm to keep sleet from forming. It is a scheduled proposition, definitely arranged, with all men on the whole system alert to prevent sleet from damaging the system.

The result of their experience has been that in the last year or two they have had very little trouble and have been able to combat sleet very successfully.

J. Roubicek: I think it interesting to see how the sleet melting is effected in one of the recent additions to the New England Power Systems. I am referring to the Montauk plant in Fall River.

There is a 32,000-kw. generator operating on a bank of transformers temporarily connected to give 66,000 volts on the high-tension side. The generator cables are led to a small cable house in the outdoor switchyard, from which point bare copper connections are made to the transformer. A double-throw, disconnecting switch is located in the top of the cable house, by means of which it is possible to throw the generator output either on the transformer bank, or directly on the outgoing high-tension line, (hereby bridging switches, transformers and high-tension apparatus in the outdoor switchyard), and subject the high-tension lines to the passage of the heavy generator current. Lamps in the cable house indicate whether the transformer oil switches are closed or opened, so that the operator can make no mistakes in interrupting the transformer circuit while the breakers are in. The connection of the 14,000-volt sleet-melting cable to the high-tension line is made by solderless connectors which are disconnected when not in use.

H. S. Knowlton: The cost of modern transmission lines for high-voltage service is rapidly approaching a figure per mile

comparable with the cost of building steam railroads perhaps a generation ago, and it is certainly pretty nearly one-fourth of the cost of building steam railroads per mile through some of the canyons of the Far West. I think it would be very interesting if Mr. Oliver would say just a few words about what range of costs per mile have been encountered in his practise.

R. E. Argersinger: Some twelve years ago, before much trouble of sleet on transmission lines had been experienced in New England the company with which I am connected built a transmission line in Connecticut. This was a two-circuit line with No. 2 copper wires, each circuit in a vertical plane on one side of the tower.

Considerable sleet trouble was experienced, but a large portion of it was eliminated by extending the middle crossarm. However, it was later decided to attempt to melt the sleet from the wires and this was done successfully over a length of line approximating 20 mi., by short-circuiting one end of the circuit and applying voltage at the other end. A current of 175 amperes will melt sleet on these No. 2 wires with the air at about 32 deg. fahr. However, if there is a breeze and the temperature gets down to 20 deg. fahr., it will take nearly 300 amperes to obtain satisfactory results.

I did not clearly understand from Mr. Oliver the size of wire in the case to which he referred.

C. R. Oliver: It is our experience that 300 amperes will thaw the sleet if it is taken in time. The size of which I was speaking is No. 1.

R. E. Argersinger: If you start as soon as sleet begins to form, considerably less current is required to prevent further sleet formation than would be necessary to free the wire from a considerable thickness of sleet after definite formation.

I believe that in view of sleet troubles which have occurred in New England in recent years, more attention should be paid to the design of the line towers. In too many cases towers are purchased without sufficient attention to stresses allowed in the design and the compression formula employed; also to the proper factor of safety.

Last winter St. Louis experienced a very severe sleet storm and a great deal of damage was done throughout the district to overhead work. As reported to us the Keokuk-St. Louis steel tower line "stood like a rock,"—the result of careful structural analysis in the design and an ample factor of safety.

J. A. Johnson: Some years ago I had occasion to design a transmission line and carried out that old practise of going into the market and buying a standard product from a manufacturer. During the construction of the line, a heavy sleet storm occurred and several towers were wrecked before the line was ever put in service. The point I wish to bring out is that this sleet-thawing device ought to be the first thing provided when building a transmission line, and one should not wait until the line has been built and in service several months before beginning to think of sleet protection.

C. R. Oliver: It has been our experience that in the two or three hours between the time we stop thawing a line and get back to it, the sleet forms a little but not seriously. The whole trick that we have learned in sleet thawing is to get at it quickly. With very light sleet we have no trouble, but once the sleet gets two inches thick, you have real trouble on your hands, because your conductors sag down in the old vertical spacing and one span will unload before the other span does and a burned-off conductor results. We try to get to thawing the minute we find there is any sleet.

It has been true that there has not been as much thought put into the transmission-line tower design as we have put into our substation design or power house design.

We build a substation structure, and the structural engineer will calculate every member in that structure and all the stresses, and then add one hundred per cent in order to play safe. And by the time he gets through he adds another hundred per cent. We don't put that factor of safety into transmission lines; at least we haven't in the past. Very few men can actually calculate stresses in the mast of the tower, particularly in the old type of tower. A footing designed with delightful intention very often crumbles in compression, and if just one member crumbles in a particular footing a damaged tower results.

On the line that we finished just last year, we tried to put as much thought into the building of these individual towers as we did into the building of a structure of any kind. When the footings were excavated an engineer was there with a transit, and he stayed with the footing crew the whole time, and that footing was set just as carefully and just as much to grade as if we were setting the footing of a building.

Regarding the question of single-pole construction, this has been a standard on our system now for about six years; that is, up to angles of 15 deg. we use a so-called double-pole construction with a single crossarm, but over angles of 15 deg., we take an individual pole and dead-end of the wire with a pull-off construction for each phase.

Regarding the Moloney-type footing, we haven't used any of these. We know nothing about them except what some of our friends who have tested them have told us, but we hope to try some of them out in test if not in actual use.

Mr. Knowlton asked about the cost of transmission lines. Well, that depends much on the designing engineer, the type of country you are going through, the type of service you have to give. There are many indeterminate figures in the problem. We can give you what a 110,000-volt line costs, but it is dangerous to use it on any other section of the country; in fact, it is dangerous to use it in any way except as a guide.

We had that illustrated some time ago with one of our financial men. We were talking to some of the Southern fellows about building transmission lines, and had just put in an estimate of $15,000 a mile. He said, "We build that kind of lines for $8000 a mile," and we have had a very difficult time explaining to our people why we can't build them for $8000 a mile. But it is due to the character of the country, to the loadings we get, to the care we try to put into the lines, and it makes your cost go up. We have seen some recent estimates of 220,000 volt, running about $40,000 a mile for double circuit. On the 110,000, so-called flat construction with six wires in one plane, we run about $25,000 a mile, including the right of way and the fees.

Mr. Morrow spoke about sleet-thawing down in Pennsylvania. They have an ideal condition with their load at one point and power station at another part of the system, and can bunch their load therefore on one line. But we have a condition in New England quite different from that, that is, we have a 66,000-volt distribution system in effect, and there is no way that we can bunch the load over one line and keep the sleet off.

Transformer Tap Changing Under Load

BY H. C. ALBRECHT[1]

Associate, A. I. E. E.

Synopsis.—Attention is called to the need for voltage and power-factor regulating equipment on lines used for tying together large generating stations and for interconnection of systems. Such lines must be capable not only of transferring large amounts of energy in either direction, but must also be suitable for connecting generating sources operating at essentially equal voltages.

The important characteristics of the three principal methods for voltage regulation on interconnecting lines,—namely, synchronous condensers, tap changing under load, and induction regulators,—are presented, discussed and compared. The comparison includes such factors as first cost (including installation) reliability, ease of operation, losses, effect on system power factor and losses, maintenance, adaptability to reversible energy transfer and ability to give close voltage regulation.

The requirements of regulating equipment, particularly from the operator's standpoint, are discussed with the idea of bringing out the necessity of obtaining a high degree of reliability, ease of operation and flexibility to meet the various operating conditions.

The fields of application of various methods of regulation are discussed, showing the advantages of each for different requirements and pointing out that in some cases the best solution of the problem lies in the application of a combination of two different methods of regulation to secure the best over-all results.

Brief descriptions of three installations (two of tap changing under load, and one of induction regulators) for voltage regulation and power-factor control on lines of The Philadelphia Electric Company are included.

* * * * *

IN an effort to obtain satisfactory voltage regulation more economically for certain transmission requirements involving step-up transformation during the past two or three years, methods and equipment have been developed by which the voltage ratio of large power transformers can be changed while in service without interrupting the load. Changing taps by any such method is commonly referred to as "Tap Changing under Load."

The purpose of this paper is to indicate, so far as possible, the field in which transformers provided with tap-changing equipment might be economically and satisfactorily applied, comparing their characteristics, advantages and disadvantages with those of the other better-known types of regulating equipment, such as synchronous condensers and induction regulators. The development of equipment for tap changing and the various schemes by which this may be accomplished will be but briefly referred to, as this is to be the subject of other papers. As a result of experience in a recent installation of four 20,000-kv-a., 66-kv. transformers of this type on tie lines between two important generating stations of The Philadelphia Electric Company system, however, some suggestions will be made from the standpoint of the operating company relating to their design, construction and installation.

Where step-up transformation is involved, voltage regulation requirements have usually been those relating to one way transmission, but the tremendous growth in capacity of individual systems in the last decade, and the decided trend toward interconnection of systems, has brought with it another somewhat different problem of controlling voltage and transferring reactive kilovolt-amperes with reversible energy flow. The time is not long past when there were scarcely any ties between neighboring systems and but few of any considerable capacity between generating stations of the same system, since most companies could boast of but one large plant. The greatly increased demand for energy has resulted in the building of many large capacity generating stations, which, for economic operation, are tied together in any one system. In the past few years considerable progress has been made in the interconnection of adjacent systems to secure even greater economies. With the general spread of the "superpower" idea through the country and the consequent increase in the number of such ties, this newer problem of regulation, where energy transfer may be in either direction, is of growing importance.

Except for the induction regulator which has been applied only to a limited extent, until the recent development of tap-changing equipment; the synchronous condenser has been practically the only means of obtaining the voltage regulation to meet the requirements discussed in this paper. With tap-changing equipment available, however, the best solution of many problems may be a combination of this equipment with synchronous condensers, or, in certain cases, of tap-changing equipment alone where its range is sufficient. Of course, there are many cases in which bulk power is supplied to distant points, where it is practicable to obtain some or all of the required regulation by variation of voltage at the generating station, particularly when there is no local distribution from that point.

Application of the three above-mentioned types of equipment, for obtaining voltage regulation where step-up transformation is involved, may be commented on as follows:

SYNCHRONOUS CONDENSERS

This type of equipment has been extensively used for voltage regulation, particularly for one-way transmission, and has the following characteristics:

1. Engineer, Station Elec. Design & Const. Philadelphia Electric Co.

Presented at the Regional Meeting of the A. I. E. E., Swampscott, Mass., May 7, 1925.

a. Very important advantage of ability to carry reactive kilovolt-amperes, thus minimizing system losses and permitting best utilization of system capacity.

b. Ability to obtain smooth voltage regulation and, when necessary in cases where the charging kilovolt-amperes of the line is high, to prevent voltage rise at light loads.

c. Ability to maintain level-voltage transmission which may be of advantage where loads are tapped off along the line.

d. High cost of complete condenser installation, due not only to cost of the apparatus in the capacities required but also to the necessity of providing foundations and building.

e. Comparatively high operating and maintenance costs and losses; the latter, however, generally being more than balanced by savings in system losses.

f. Extremely high cost of installations to take care of reversible energy transfer, as duplicate equipment must be installed at each end of the line with possibly little appreciable additional saving in system capacity or losses to offset charges on the additional investment.

g. Units may be taken out of service purposely or accidentally without necessarily causing a serious disturbance.

Tap-Changing Transformers

This type of equipment of recent development, so far installed chiefly on tie lines between generating stations, is possessed of the following characteristics:

a. Very low cost, since the only amount involved is practically the difference between transformers provided with tap-changing equipment and those not so provided.

b. Adaptability to reversible energy transfer where line power factor is high or distances are relatively short.

c. Inability to improve system power factor and thus decrease losses and increase system capacity.

d. Inability to give smooth voltage regulation, changes being made in steps.

Induction Regulators

Induction regulators have been used, to a limited extent, on the low-tension side of transformers connected with high-voltage transmission lines and have the following characteristics:

a. Moderate cost.

b. Adaptability to reversible energy transfer.

c. Ability to give smooth voltage-regulation.

d. Inability to improve system power factor and thus decrease losses and increase system capacity.

e. Moderate losses.

f. Introduction of phase angle between line and bus voltage by polyphase regulators.

Comparisons

In considering these three types of regulating equipment, it will be seen that induction regulators and tap changers perform very much the same function—they vary the voltage, but have no beneficial effect on power factor, as in the case of synchronous condensers. Of these two equipments, tap changing is considerably cheaper, more efficient and occupies less space. For instance, a 7½ per cent buck-and-boost, polyphase, induction regulator, in series with the low-tension side of a 20,000-kv-a., 66-kv., three-phase transformer, would probably cost almost as much as the transformer itself, while a transformer of the same capacity provided with tap-changing equipment to give the same voltage range, would probably cost less than 150 per cent of the transformer without such equipment. In addition to this advantage in first cost, there would be a considerable annual saving by elimination of the regulator losses. There is also another advantage in favor of the tap-changing equipment; the regulator is almost as large as the transformer itself, requiring special foundations and more space than the tap-changer equipment. The regulator does give a smooth variation of voltage instead of in steps, but in many cases this is not of sufficient importance to overcome the other disadvantages. An application of tap-changing transformers of considerable interest is one in which the voltage change from one tap to the next is accomplished gradually through the use of a small induction regulator which may require an insulating transformer between it and the main transformer.

The polyphase regulator also introduces a phase angle between line voltage and bus voltage, which makes paralleling difficult. When voltage regulation is desired for existing installations, it may be quite difficult and expensive to safely apply tap changing and, in such cases, the induction regulator may work out very satisfactorily. It, therefore, seems that in new installations where step-up transformers are required, tap-changing equipment has decided advantages over regulators.

In a comparison of tap-changing equipment with synchronous condensers for voltage regulation, the two points of outstanding importance are, on one hand, the tremendous advantage of first cost of the tap-changing equipment over the condenser installation, and on the other hand, the ability of the condenser to supply reactive kv-a. in addition to voltage regulation, thus minimizing system losses and permitting best utilization of system capacity. It would seem, however, from an economic standpoint that, in many cases, the advantages of tap-changing equipment would outweigh those of condensers, if the tap changers would give the desired voltage range. As this is perhaps hardly possible, the best solution of many problems would very likely be a combination of tap-changing equipment with a smaller capacity of condensers than would otherwise be required. This compromise would permit of installation of condenser capacity sufficient to bring the power factor to the economical point and obtain the remaining regulation required with tap changing. Where the system power factor is already fairly high,

the desired voltage regulation can be obtained with tap-changing transformers of the maximum, practical voltage range, supplemented by condenser equipment if necessary. The combination of condensers and tap-changing transformers gives the advantages of both types of equipment and overcomes most of the disadvantages.

In many cases where the charging kv-a. of the line at no load is high, there may be no alternative other than the installation of condensers; but, even with this the case, the combination may work to advantage. It is again pointed out that where it is practical to vary the generating station voltage considerably, this, in combination with the use of condensers, may be a better solution than tap changing.

Development of Tap Changing

There has been a growing appreciation of the advantages of taps in transformer windings and a desire to increase their availability, and this even with recognition of the fact that taps do introduce some slight hazard and that the number should be kept down to the minimum advantageous for use. This first took the form of adding a terminal board with links whereby the ratio of transformation could be varied. However, this required the opening of a handhole over the terminal board or the lifting of the cover—an inconvenient procedure at any time and especially undesirable in bad weather. Thus, with the necessity for changing taps frequently to care for seasonal variations in load, this led to the development of ratio adjusters operated from without the transformer tank, with the unit out of service. As the necessity for making these changes became daily rather than seasonal, the next step was naturally towards means of changing taps without interrupting the load.

It may be of interest here to recall that many years ago schemes were developed and applications of small capacity were made in which taps were brought out from transformers to a dial-head and line connections were changed from one tap to another while the section of winding between taps was momentarily short-circuited through reactance.

There are several methods whereby tap changing under load may be accomplished; among them the use of (*a*) two parallel windings which can be cut out of circuit, one at a time, by oil circuit breakers or contactors in order to change taps; (*b*) one winding in which adjoining taps are momentarily tied together through oil circuit breakers or contactors during the change-over with or without "bridging" reactance or small induction regulator; (*c*) one winding in which taps are changed by taking each phase out of service in succession, the load being carried open-delta on the two remaining phases in the meantime.

In cases *a* and *b*, the windings from which the taps are brought out may be either those of the main transformer themselves or those of an auxiliary regulating transformer excited from the main bank. While the former is more compact and cheaper both in first cost and operating losses, it does not provide a means for keeping all the tap-changing equipment outside the main tank. Where regulation is desired on an existing transformer bank, this may be accomplished by the addition of such an auxiliary regulating transformer.

Where the voltage of the circuit is too high to permit the safe installation of tap-changing devices directly, it is possible to accomplish tap changing through insulating transformers, although, of course, this would be at increased cost.

The application of the general principle of tap changing under load presents possibilities for many ingenious arrangements to meet specific requirements, illustrated by the variety in the equipments already in service or on order. Undoubtedly there will be further developments involving new schemes or modification of existing ones.

While this paper treats mainly of tap changing applied to step-up or step-down transformers, it should be pointed out that tap changing can also be applied to series transformers where the line to be regulated includes no voltage transformation.

Philadelphia Electric Installations

It will be of interest to briefly describe an 80,000-kv-a. installation of tap-changing transformers at the Chester Generating Station of The Philadelphia Electric System. This station is fourteen miles from the Schuylkill Generating Station and is connected by two 66-kv. overhead circuits of No. 00 copper. Until recently two 18,750-kv-a. banks were installed at each end of the lines. Although these lines were intended to be for generating station ties only, it was not very long before two important, large customers for special reasons were tapped off not far from the center of the lines. Control of transfer of load and reactive kv-a. for the first two years was obtained by variation of the bus voltage of the Chester generating station, as the local distribution was not extensive and not particularly affected thereby. However, in 1920, this became less practicable and the need for some means of more effectively controlling the lines under various loading conditions was so great that after much consideration two 1750-kv-a., 13,600-volt induction regulators, to give 9 per cent buck-and-boost in a 18,750-kv-a. circuit, were purchased and installed at the Schuylkill end of the lines. These were and probably still are the largest regulators ever built. In 1924, however, it was decided to double the transformer capacity installed at the Chester Station end and extend a 66-kv. tap from the line to a distribution substation with requirement of 20,000 kv-a., the transformers from Chester being transferred to that point. The increased line loading brought a greater voltage drop and it was deemed best to take care of this by making the new transformers at Chester tap changing instead of purchasing additional

induction regulators or installing synchronous condensers. Five per cent buck-and-boost induction regulators were purchased for the tap off substation, as tap changers could be applied to the transformers available only with difficulty. Four 20,000-kv-a., 13.8/69-kv., delta-delta, three-phase, water-cooled transformers were placed in service the first of last November and arranged with six taps in the low-tension winding, to give up to 13 per cent rise at no load in the high-tension winding, or 78,000 volts. The low-tension windings are in two parallel circuits with taps in the middle of each winding, taps being changed on one winding, while the other is temporarily carrying the entire load. A single operation of a control switch carries to completion, the changing from one tap to another, in all three phases. Other papers give description of this equipment and its design problems in detail.

Since their installation these transformers have been operating very satisfactorily. Their control is simple and tap changes of 2.2 per cent cause no noticeable disturbances on the line. While the tap changers at Chester and the regulators at Schuylkill are non-automatic and under the control of the operators, the regulators at the substation are controlled from a contact-making voltmeter, thus insuring the maintenance of a constant voltage at that point irrespective of variations of the voltage of the 66-kv. line, due to direction and amount of energy transferred.

Construction work is well under way on the Richmond Station in Philadelphia, designed to ultimately house twelve turbo-generator units of at least 50,000-kw. capacity each, the first two of which are expected to be in operation by the end of 1925. It is anticipated that about one-half of the minimum ultimate capacity of 600,000 kw. will be distributed at 13.2 kv. and the remainder at 66 kv., and that half of the twelve generators will be connected to the 13.2 buses and the others to the 66-kv. busses through transformers connected directly to the generator leads. Provision is being made to tie the 13.2 and 66-kv. busses together through three 60,000-kv-a. transformer banks. As distribution within the next few years will be considerably greater at 13.2 kv. than at 66 kv., the first few generators will be connected to the 13.2-kv. busses. Two of the three 60,000-kv-a. tie transformer banks, each consisting of three 20,000-kv-a., single-phase, water-cooled units, connected delta-Y, have been ordered and will be placed in service the latter part of this year to provide for the demands for 66-kv. supply. In order to facilitate the ultimate reversible transfer of energy between the 13.2 and 66-kv. busses, these transformers have been ordered with tap-changing equipment of 10 per cent range in four 2½ per cent steps. An interesting feature of this installation will be that individual tap-changing equipments will be furnished with each single-phase transformer, no provision being made to have the taps on the three different phases changed simultaneously.

General Comments

In general, transformers, particularly in large capacities, have a record of high reliability. Tap changing necessarily adds complications to both the transformer itself, and the auxiliary equipment. It is extremely important that the design be developed in all its details so as to minimize any adverse affect upon the over-all reliability of the equipment. Operators are reluctant to add complications to important equipment, but in large capacity units, in which tap changing is of particular advantage, the extra expense to secure exceedingly substantial tap-changing equipment is a very small percentage of the total cost. A requirement of The Philadelphia Electric Company is that the tap-changing equipment inside the transformer tank be built so substantially as to require no attention under the expected duty other than that which would naturally be given on the rare occasions when it is necessary to untank the unit in case of winding failure.

It is true that some hazard is introduced by tap-changing equipment and some operators prefer schemes in which taps are brought outside the tank because of possibility of trouble in the tap-changing mechanism. This, however, brings about other disadvantages, such as the hazard in the additional exposure of important circuits and added first cost. Transformer windings are quite often somewhat special when built for tap-changing service and the design must be carefully worked out to obtain the best balance to meet all requirements; for instance, with schemes involving parallel windings, care must be taken to obtain the best compromise between circulating current and regulation when the load is carried for a short time on one winding during tap changing on the other.

The amount of tap-changing range that is practicable and the affect upon cost and characteristics are interesting subjects for discussion and have important bearing upon its application, particularly for reversible energy transfer. For instance, as previously indicated, the economic solution of many problems may involve a combination of synchronous condensers with tap-changing transformers in proportions determined largely by the maximum practicable range of the tap-changing equipment. It must be appreciated, however, that high ranges impress voltages higher than normal upon transformer, oil circuit breaker and other equipment, and the line itself.

It is of the highest importance that the design and construction of the tap-changing mechanism be such as to insure sturdiness and reliability; and there should be no tendency to skimp in this direction. Whether oil switches or contactors should be used for disconnecting devices, and the relative merits of different types of ratio adjusters, are matters to be carefully looked into for particular applications. Tap-changing equipment is well adapted to remote control and experience has indicated the value of but one operation of a control switch for a complete movement from one tap to

another. Of course, provision must be made for manual operation in case of motor trouble, or failure of control system. The complete movement, whether remotely or manually operated, should be made in the minimum length of time possible without any shock to the mechanism or sacrifice in durability. That portion of the tap-changing mechanism external to the transformer should be properly housed and specially designed to withstand moisture from condensation. The mechanism should be readily accessible for inspection and maintenance.

It is essential that the operator know the tap on which the bank is operating and that any change he desires has actually been completed. Therefore, a position indicator of some kind should be provided on the switchboard and also at the mechanism. It is often desirable to know the voltage on the high-tension side of transformer banks and costly high-voltage potential transformers may be avoided if the voltage is measured on the low-tension side (compensating for drop through the transformer). Since, in the case of tap changing, the actual ratio of transformation is varied, it is necessary to also provide some device such as a small auto-transformer the ratio of which is correspondingly varied by a dial-switch operated from the mechanism. Such a device has been operating satisfactorily in the installation at Chester Station, already described.

In providing the usual differential relay protection, this change in the ratio of transformation must be provided for either by the use of an auto-transformer the ratio of which is varied with that of the main unit or by designing the turns of the relay for the mid position of the tap changer and setting the relay sufficiently high to prevent its operation on through short circuits.

Any failure of the mechanism that may damage the transformer winding thermally should be guarded against, but should one occur, it should be indicated by an alarm. In the case of the double-winding transformers at Chester this is accomplished by relays connected differentially between the two windings which operate when either winding is carrying all the load or when the two windings are in parallel on different taps. These relays in turn operate a definite time limit relay which at the end of one minute gives an alarm.

Conclusion

The author believes that there is a real field for transformer tap changing under load and that this will continue to increase with the expansion of interconnection and superpower. Development will be stimulated as its possibilities are more widely recognized and appreciated.

Acknowledgment is made of the assistance of Messrs. R. A. Hentz, Raymond Bailey, B. E. Hagy and Jos. I. Tabakin, of The Philadelphia Electric Company, in the preparation of this paper.

Discussion

B. G. Jamieson: On the system of the Edison Company in Chicago, there has been built up within the last four years, a 33,000-volt nominal voltage, 60-cycle system of about 360,000 kilowatts. The transformers were generally equipped with the tap-changing system described, the earlier forms with the double winding, and in other cases, a system which involves the step switching scheme with a connected reactor giving the same effect that the last author described.

The operations have been about forty in number, and from the standpoint of the effect on the load, nothing more could have been desired. Changes were made promptly, and we found that with approximately two and a half per cent steps, we got all desired smoothness. It came to our notice, however, very early in the development of these schemes that the complication added to the transformers indicated the desirability of getting the maximum amount of this extra system outside the tank. We are not yet able to take a final position in the matter of requiring this development further than to call to the attention of engineers the many extraordinary things that happen in transformers and the undesirability of having minor troubles or difficulties augmented by the proximity or presence of supplementary internal devices of this character.

As to the schemes of regulation involving the regulator system, this refinement will, I believe, be necessary only in exceptional cases, but where we have used the double winding we are prepared to add the regulator when necessary.

In our committee work in the N. E. L. A., some discussion has arisen regarding the nomenclature, and I must say that there was not complete agreement on this score. The name tap changer and the name *ratio adjustor* were offered for consideration, but the choice seems to be still open. Generally speaking, one associates with the term tap changer a minor piece of equipment and with the term *ratio adjustor* a somewhat more comprehensive system. It would appear from what has been seen that perhaps the term *ratio adjustor* has a little more exact significance.

H. W. Smith: These papers point out the need for some scheme of regulating transformer voltages under load, and mention one solution. I wish to point out, however, that other schemes have been used, and in considering any given problem, these alternatives should be investigated. A tap-changing scheme has been used in which the taps have been brought out of the transformer and changed by oil circuit breakers using an auto-transformer in switching from tap to tap. This also provides a method by which additional voltage points can be obtained half-way between the taps so that, for instance, with five taps, nine voltage steps can be obtained. Four 15,000-kv-a., three-phase transformers using this scheme are in operation in Chicago. This scheme has also been used in connection with air-blast transformers supplying converters. The transformers having a primary voltage of 11,000 and standard contactors were used for changing the taps. This equipment has been in use in San Francisco for several years with satisfactory operation.

Another scheme which is particularly applicable to an interconnection between high-voltage systems, is that of using a combination of transformer with an induction regulator to bridge between the taps of the transformer. This equipment is termed a "step induction regulator." An interesting equipment involving this scheme and rated at 15,000 kv-a. has been used to interconnect the Tacoma Municipal System with the Seattle Municipal System. The Tacoma voltage is 50,000, three-phase, delta-connected, while the Seattle voltage is 57,000 volts, three-phase, star-connected. With the equipment, a voltage variation of 48,000 to 64,000 on the star side is provided. This installation has been described in an article "The Tacoma-Seattle Power Exchange Line," by R. E. Towne in the *Electric Journal* of June 1923. This same scheme has been applied to synchronous

converters to get a large range in voltage for reenergizing a d-c. Edison system when a shut-down occurs.

Another scheme which has been used is that of opening the delta on a bank of transformers, switching the taps on one transformer, then replacing it and doing the same with the remaining transformers. This has been applied to a bank of transformers supplying synchronous converters for obtaining low voltages to reenergize a dead Edison system.

There will be a large field for tap changing on transmission systems supplying scattered areas where the tap-changing equipment must be relatively inexpensive. An experimental installation is being supplied to one property which will use air-break switches with the ordinary type of tap changer. The operator will disconnect one transformer by means of air-break switches, operate the tap changer, close the disconnect switches again, and then carry on this same procedure in the case of the remaining two transformers. The scheme, of course, relies upon the operator to perform the functions in the correct rotation.

A. H. Kehoe: It is important from the operating standpoint that this equipment, which becomes a controlling element in our large capacity lines, should be specified and manufactured in a manner to make it reliable without constant inspection. On the other hand, all operators of such equipment should have regular inspections made until simplified designs have been developed which demonstrate in practise that such inspections are not required.

H. R. Wilson: I believe that we are indebted to C. R. Oliver for inaugurating the idea of changing transformer taps under load by using the scheme of parallel windings. About ten years ago, he installed at Pawtucket, two 18,625-kv-a. three-phase transformers operating in parallel. Each transformer was provided with *ratio adjustors*, and high- and low-tension oil circuit breakers. Tap changes were accomplished by opening the breakers of one bank, changing taps by means of the adjusters, and closing the breakers and the operation being repeated on the second bank.

As regards using disconnecting switches, The Consumers Power Company has had in operation for several years, a scheme whereby switches connected the taps in each leg of the delta so that the taps could be changed in one leg at a time; the transformer operating in open delta during the period the taps were being changed.

Universal Type Motors

BY L. C. PACKER[1]

Associate, A. I. E. E.

Synopsis.—*Heretofore the subject of universal motors has not received very much publicity but the field for these motors has been increasing so rapidly that a great deal more attention is now being given to them.*

The paper endeavors to bring out points of general interest rather than going into the fundamental design and other details.

The term "universal" as applied to this type of motor is defined, together with a description of the operating characteristics in general.

There are two types of universal motors; namely, compensated and non-compensated, both of which are described as to general design, construction and applications.

The question of ratings and limitations of the universal features o both types of motors brings out some interesting points.

Commutation and mechanical balance are questions demanding a great deal of attention due to the comparatively high speeds at which these motors usually operate.

Some of the applications best suited to the universal motor are described, showing the possibilities of this type of motor when properly designed, manufactured and applied.

* * * * *

THE term "universal" is applied to certain small series-wound motors, the performance of which is almost identical when operated on either alternating or direct current. (Alternating current is used in this paper to mean frequencies of 60 cycles and lower.) Universal motors have been in commercial use for fifteen or more years and are still most popular in the two applications in which they were first used in quantities; namely, portable drill and vacuum cleaner drive. However, with improvement in the motors and increase in the number of small motor-driven appliances, a number of new fields for the application of these motors are being found. If a manufacturer's product is sole over a large area, it is obviously to his advantagd to use a motor which will operate equally well on either alternating or direct current, and only the voltage of the circuit need be considered. This advantage has led some appliance manufacturers to modify their machines so that the operating characteristics of universal motors will be satisfactory for driving them.

All of the universal motors on the market are series wound and their performance characteristics are very much like those of the usual type, d-c. series motor. The no-load speed is quite high but seldom high enough to damage the motor, as is the case with larger d-c. series motors. When a load is placed on the motor, the speed decreases and continues to decrease as the load is increased. Although universal motors of several types of construction are manufactured, they all have the varying speed characteristic just mentioned.

Due to the difficulty in obtaining like performance on alternating and direct current from motors designed for operation at low speeds, most universal motors are designed for operation at speeds of 3500 rev. per min. and higher. Motors operating at load speed of 8000 rev. per min. to 10,000 rev. per min. are common. Practically all portable vacuum cleaner motors come within this range. Working speeds above 10,000 rev. per min. are not so common, due to there being few applications where such speeds are desirable and to the manufacturing difficulties in producing motors which will run at such speeds. Small stationary vacuum cleaners, truck-type vacuum cleaners and the larger size of portable tools have motors with operating speeds of 3500 rev. per min. to possibly 8000 rev. per min.

Ratings from zero h. p. to one h. p. are being manufactured. However, it is not possible to obtain the very low h. p. ratings at the lower speeds and get the same performance when operating over the whole range of frequencies from direct current to 60 cycles. In other words, the motors designed for very low horse powers and the lower speeds cease to be universal, for although they will operate on either alternating or direct current their speed for a given load varies over too wide a range with changes in the frequency of the supply circuit.

It is not possible to set a definite limit of difference in speed for a given load, inside of which a motor may be said to be universal and outside of which it would not be designated as universal when motors are operated on direct current or on various frequencies. That is, it cannot be said that when operating at rated load, the variation in speed must not be more than 10 per cent, 15 per cent or 20 per cent, if the supply circuit is changed from 60 cycles to any lower frequency or to direct current. In the usual universal motor applications, such speed variations will probably be found. However, in any application, the motor is considered as being "universal" if it will operate the apparatus satisfactorily when the power supply is varied over the entire range of commercial frequency with rated voltage applied in each case.

Vacuum cleaners and portable drills are two examples of very satisfactory applications for universal motors. In both of these applications, the variation in performance when operating on varying frequencies, can easily be kept within the required limits and it is desirable to have the speed vary with the load. If a vacuum cleaner is used under conditions which decrease the volume of air handled, the load on the motor is decreased and the speed increases. The increase in speed increases the vacuum, causing the cleaner to handle more air than it would if driven by a constant speed motor. Portable drills are almost ideal applications for universal

1. Westinghouse Electric & Mfg. Co., Springfield, Mass.

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motors. The cutting speed is automatically adjusted by the load, since the smaller the size of the drill being used the lighter the load and consequently the higher the speed, and vice versa, when larger drills are used.

The same conditions exist when universal motors are used in small pipe-threading machines and in certain other metal and woodworking machines.

The speed of a universal motor can be adjusted by connecting a resistance of proper value, in series with the motor. Advantage is taken of this characteristic in such applications as motor-driven sewing machines, where it is necessary to operate the motor over a wide range of speed. In such applications adjustable resistances are used and the speed is varied at will.

When considering the use of universal motors to drive any apparatus, the following characteristics of the motor should be considered:

Change in speed with change of load

Change in speed with change in frequency of power supply

Change in speed due to change in applied voltage

The last item has not before been referred to except indirectly in connection with the use of series resistance to adjust the speed. However, a larger percentage of all small motors is connected to lighting circuits and the voltage conditions are not always of the best. This condition must be kept in mind when determining the proper motor to use for any application regardless of type. In general, the speed of a universal motor varies directly as the voltage.

The starting torque of universal motors is usually much more than that required and in most applications does not have to be considered.

There are some motor applications where the motor runs all of the time and the load is connected and disconnected by means of a clutch. On such applications, if the operating characteristics of a universal motor are satisfactory while the load is applied but the no-load

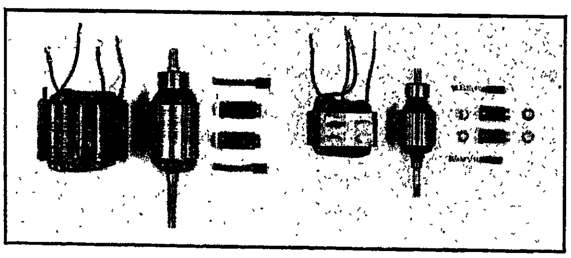

Fig. 1

speed is too high, it is sometimes possible to arrange to have a series resistance connected in the line during the time the clutch is in the position to release the load. The value of this resistance can be made such as to give the desired no-load speed.

Universal motors are applied with two kinds of construction—(1) concentrated-pole, non-compensated and (2) distributed-field, compensated. Most motors of low h-p. rating are of the concentrated-pole, non-compensated type, while those of the higher ratings are of the distributed-field, compensated type. The dividing line is somewhere near 1/4 h. p., but the type of motor to be used is determined by the severity of the service and the performance required. All of the motors have wound armatures of the same construction as the ordinary d-c. motor.

The concentrated-pole, non-compensated motor is exactly the same in construction as a d-c. motor except

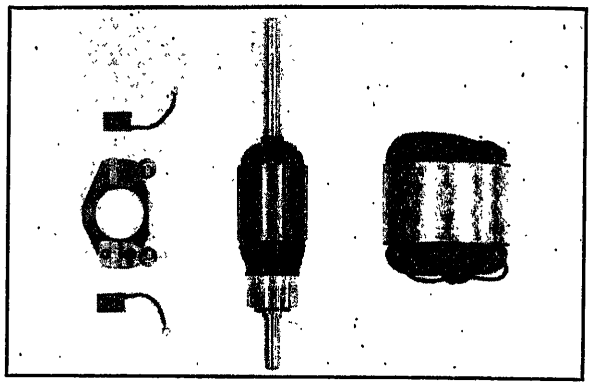

Fig. 2

that the complete magnetic path is made up of laminations. The laminated stator is made necessary because the magnetic field is alternating when the motor is operating on alternating current. The stator laminations are punched with the poles and the yoke in one piece. This makes a very simple construction and one which is very satisfactory to the appliance manufacturers who buy the motor parts as shown in Fig. 1, and assemble them in their apparatus.

The compensated type of motor has stator laminations of the same shape as those in an induction motor. These motors have stator windings of one of two different types. The parts of a compensated motor, as they are supplied to be built into the apparatus, are shown in Fig. 2.

The question might naturally arise as to why it is desirable or necessary to have the non-compensated and the compensated motors and why the ratings are usually limited to fractional horse powers and high speeds. The non-compensated motor is more simple and less expensive than the compensated motor and would be used exclusively over the entire range of ratings if its performance were as good as that of the compensated motor. However, as before stated, the non-compensated motor is used for the higher speeds and lower horse power ratings only. Figs. 3 and 4 show the speed-torque curves for a compensated motor and a non-compensated motor, respectively. It will be noted in Fig. 3 that although the rated speed is relatively low for a universal motor, the speed-torque curves for various frequencies lie very close together up to 50 per cent above the rated torque load. In Fig. 4, the performance of a much higher speed, non-compensated motor is shown. For most universal motor applica-

tions, the variation in speed at rated loads as shown on this curve is satisfactory. However, the speed curves separate rapidly above full load. If this motor had been designed for lower speed, the tendency of the speed-torque curves to separate would have been more pronounced. The chief cause of difficulty in keeping the speeds the same is the reactance voltage which exists when the motors are operated on alternating current. Most of this reactance voltage is produced in the field windings by the main working field. However, in the non-compensated motor some of it is produced in the armature winding by the field pro-

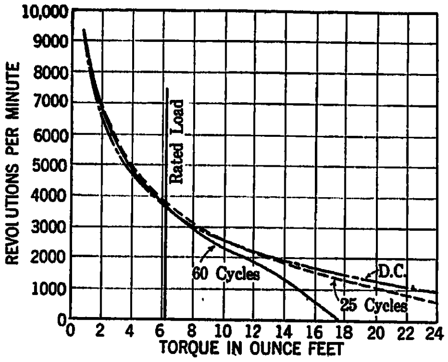

FIG. 3—UNIVERSAL MOTOR COMPENSATED ¼ H. P.—3400 REV. PER MIN.

duced by the armature ampere-turns. The true working voltage is obtained by subtracting the reactance voltage vectorially from the line voltage. If the reactance is high, the performance at a given load will be the same as though there were no reactance

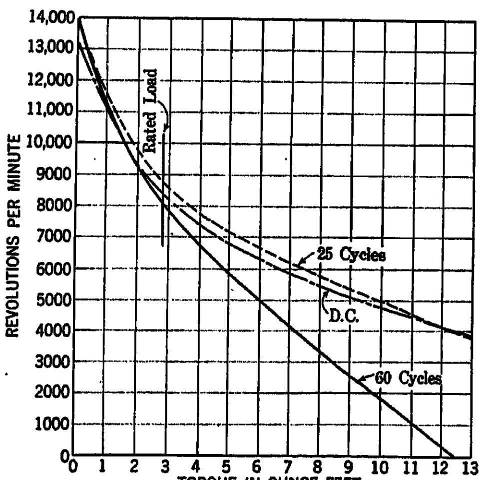

FIG. 4—UNIVERSAL MOTOR NON-COMPENSATED ¼ H. P. 800 REV. PER MIN.

voltage and the applied voltage had been reduced with consequent reduction in speed.

The reactance voltage varies with the frequency and is almost entirely responsible for the difference in performance on the different frequencies of alternating current. However, the difference in performance, when operating on alternating current and when on direct current, is caused not only by reactance voltage but also by the difference in the amount of flux produced by a given value of direct current and by the same value of alternating current. Due to the saturation of the iron in the magnetic path at the time of the maximum value of the alternating-current wave, the alternating current does not produce an effective value of flux equal to that produced by an equal value of direct current. While the reactance voltage reduces the speed, the reduced flux on alternating current increases the speed. The reactance voltage varies with the frequency, but the flux for a given value of alternating current remains constant, regardless of the frequency. When the motor is operated on low frequency the effect of reactance is almost negligible, while the reduction in flux is just as high as for the higher frequencies. The result is that on very low frequencies the speed is always higher than when the motor is operated on direct current. It is even possible to have conditions such that the speed on 60 cycles is higher than that on direct current. However, the latter condition can seldom be obtained in a motor design which is satisfactory in other respects.

It is now apparent that, to get like performance on alternating and direct current, the reactance voltage must be low and the difference between the flux on direct current and alternating current must be decreased wherever possible. The difference in magnetic flux produced by a given value of direct current and that produced by an equal value of alternating current, can be changed only slightly by changes in the material used or in the design of the parts. The reactance voltage can be reduced or increased within certain limits by changes in design. As mentioned before, the major part of the reactance voltage is produced by the field flux passing through the field winding. By keeping the product of the field turns times the flux to a low value, the reactance can be kept proportionally low. This results in what is usually termed a "weak field" motor. Now a weak field in an ordinary d-c. motor permits the field to be distorted by the ampere-turns in the armature, resulting in a tendency towards poor commutation and greater changes in speed with a given change in load. Identical conditions exist in a concentrated pole, non-compensated, universal motor. However, it is necessary to keep the reactance voltage low to prevent the speeds from becoming too low on the higher a-c. frequencies. Within a certain range of speed and h-p. ratings, the reactance voltage can be made low enough to keep the speed variation within reasonable limits and at the same time secure satisfactory performance from the motor. Much has been done in the matter of improving the commutation of weak field motors by improvement in the grades of carbons and by better motor design. When, in a desired rating, the horse power is too high or the speed too low to obtain satisfactory operation with a non-compensated motor a compensated motor is used.

In the compensated motor there is a series winding having a magnetizing force equal and opposite to the magnetizing force of the ampere-turns in the armature. This tends to eliminate field distortion and the resulting bad commutation. When the field distortion has been eliminated, a very weak field can be used with a resultant decrease in reactance voltage and the corresponding decrease in variation in speed for change in frequency. Due to mechanical reasons, it is impossible to distribute the compensated windings so as to exactly compensate for the ampere-turns in the armature. It is this lack of exact compensation which limits the horse power and speed to be obtained in the compensated type of universal motor.

Two types of stator windings are used in the compensated motor: In one type, two distinct field windings are employed. The compensating winding, which has just been referred to, is equal and opposite to the effective ampere-turns in the armature and is distributed to correspond to the armature winding. The main field winding is located 90 electrical degrees from the compensating winding and produces the working field. This winding usually occupies just two slots per pole. In the other type of field winding but one distributed winding is used. By shifting the brushes to the proper position, the armature ampere-turns are opposed by a certain portion of the field ampere-turns and the remaining portion of the field ampere-turns produce the working field.

When the two-winding type of stator winding is used, the brushes are located on the neutral with reference to the main field winding. This permits the direction of rotation to be reversed by reversing the main field. When assembling this type of motor, the brushes can easily be set in the proper position by applying alternating current to the compensating winding and operating the motor as a repulsion motor by short-circuiting the brushes. When the brushes are so set that the armature field is directly opposite the compensating field there will be no tendency in the armature to rotate. When the brushes are moved slightly from this position, the armature will turn in the direction in which the brushes have been moved. The brushes are in the correct position when the armature field is directly opposite the compensating field.

When designing a motor using the one-winding type of stator winding, if certain limiting proportions of field and armature turns are adhered to, it is possible to get exactly the same conditions as to compensation and main field as is obtained when using the two-winding type of stator winding. However, due to the restrictions in regard to relative strength of armature and field, it is difficult to get complete compensation for certain ratings. The fact that the brushes are located off of the neutral of the field winding makes it necessary, in order to get compensation, to shift the brushes when the direction is changed. This makes this type of motor unsatisfactory for reversing service.

The two most difficult problems in the manufacture of these high-speed motors, is that of obtaining good commutation and good mechanical balance. In addition to these, it is necessary to have all parts of the armature well made so that they will withstand the larger centrifugal force which is present when the motor is operating at the high speed to which it will be subjected.

Aside from the usual commutating conditions which are encountered in the ordinary d-c. motor, there is a transformer voltage generated in the short-circuited coil by the alternating main field flux. This transformer voltage is one of the chief causes of commutation trouble, and poor mechanical balance causes sparking at the commutator due to the inability of the brushes to make good contact with the commutator when the armature is vibrating at high frequency. In addition to this it will be realized that, in motors running at speeds around 10,000 rev. per min., the time permitted for the current to reverse in the coil while it is being commutated, is very short. In an ordinary vacuum-cleaner motor, this reversal of current in the coil being commutated must be accomplished in one-half of 1/1000 of a second. The fact that the coil is short-circuited for such a short time determines, to a certain extent, the amount of short-circuit current which exists in the coil.

When considering the mechanical balance, the unusual speed of these motors must be kept in mind. Referring again to the ordinary vacuum-cleaner motor, we find that the centrifugal force acting on an object at the surface of the armature when the motor is operating at normal speed, is approximately two thousand times the weight of the object. As an example, if a piece of No. 20 wire, 1½ in. long, were placed at the top of the armature slot, the centrifugal force acting on the wire would be equal to the total weight of the armature. With this in mind, it is quite obvious that the weights in the armature must be thoroughly balanced or there will be excessive vibration and excessive pressure on the bearings. The usual static balance of armatures improves any unbalance that may exist due to the uneven distribution of the windings or due to there being a slightly heavier coat of armature insulating varnish in one place than in another. However, if when balancing an armature statically the correction of weight is made at the end of the armature opposite that at which the unbalance exists, there is still a very bad condition of dynamic unbalance which will be destructive to bearings and brushes and cause the motor and driven appliance to be noisy. This condition can be eliminated only by very careful dynamic balancing.

The results which may be obtained by using extreme care in designing and manufacturing are shown by tests made on a particular vacuum-cleaner motor. This motor was so designed, electrically, as to get the best commutating conditions, and accurate dynamic balance was obtained. At the end of a 5000-hour endurance run, the indicated total brush life for one set of brushes was 9000 to 10,000 hours. As stated, this motor was very carefully built and such motors could not be produced commercially due to the increased manufacturing

cost. However, the tests on this motor did show the results of careful design and dynamic balance. There is no need in vacuum-cleaner service for a motor such as the one described. If a vacuum cleaner were used continuously two hours per week, which is probably the average, the total hours run per year would be approximately one hundred and the total life of one set of brushes would be 100 years. It is quite obvious that it is not necessary to go to the expense of building a motor of this quality when it is impossible for the driven appliance to remain in service such a great length of time. However, it must be remembered that the brush

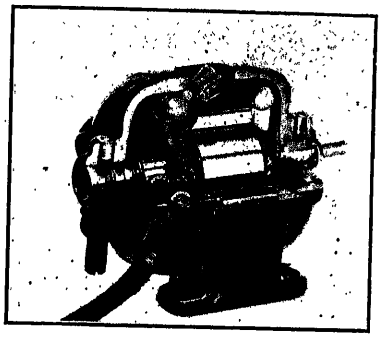

FIG. 5

life which has ordinarily been obtained in vacuum-cleaner motors has not been equal to the life of the vacuum cleaner. The vacuum cleaners being built today are of better quality than those of several years ago and are standing up better, with the result that better motors must be supplied. It is such tests as the one just mentioned which are pointing the way to better motors and are actually bringing about results in the commercial motors now being supplied for vacuum-cleaner service.

Vacuum cleaners and portable drills have already been mentioned as very satisfactory applications for universal motors. In addition to these, there are many other applications in which universal motors are being used. In many cases, the motor parts are supplied to the appliance manufacturer and built into his appliance. In other cases, complete motors, such as the one of which a part cross section view is shown in Fig. 5, are used to drive the appliances. Some of the devices to which these motors have been applied are advertising machines, portable, motion-picture machines, dish washers, hair clippers, sewing machines, drink-mixing machines, small ventilating fans, hair driers, pipe threaders, small grinders and small wood shapers and routers. This is a rather varied list of applications and the variety and number of applications are growing daily.

Discussion

W. L. Smith: I wish to emphasize the fact that on these little motors there is one thing that I believe is very important and that is that the greatest care should be given to perfection in insulation.

H. W. Hills: We can't blame the insulation for all trouble, for, because of the way these machines are used, the insulation becomes oily and hard and even the best insulation, after being subjected to such conditions, will eventually fail. The only thing to do is to provide a good ground on the machine itself. This can easily be done by incorporating a third wire for a ground wire along with the two supply wires. This ground wire can be attached to ground while the machine is in operation.

L. C. Packer: Recently the underwriters have specified insulated brushholder screws for commutator motors. That is a point that the appliance manufacturers have stressed a good bit, and the underwriters have specified certain design points that must be adhered to. Insulation of brush-holder screws is one very important item, especially in vacuum-cleaner and sewing-machine motors.

There is another point about the insulation. Most of the manufacturers of this type of motor test to ground at 1200 or 1400 volts, on a standard 110-volt motor. The motor itself is just as well insulated as higher types of motors. Most of the grounds are caused during the assembly of parts by the appliance manufacturers in grounding the connections against the frame: sometimes the insulation is pinched off the leads in assembly.

C. A. Adams: I should like to ask Mr. Packer how he can get substantially 100 per cent power factor.

L. C. Packer: In the compensated motor the field is usually fairly weak and its reactance voltage comparatively low; that is, from 10 to 20 volts on 25 cycles at rated load. The equivalent values on 60 cycles are 24 to 48 volts. The average motor, of course, will have about the average value between these figures. For illustration in an average design the reactance voltage due to the field is 15 volts on 25 cycles, and 36 volts on 60 cycles. Then suppose we work the motor over a range of load such that the value of reactance voltage would be from 10 to 20 on 25 cycles, and from 24 to 48 on 60 cycles. Since the reactance voltage varies directly with the flux, these values represent considerable change in load.

Since the true working voltage is the vectorial difference between the line and reactance voltages and since the power factor is equal to the true working volts divided by the line volts, it can be seen that it takes quite a material change in reactance voltage to show much change in power factor and it also takes a comparatively large amount of reactance voltage to drop the power factor an appreciable amount below unity.

As a rule, the power factor of the non-compensated type of motor is much lower than that of the compensated type. There is no compensating field; therefore, it is necessary to use as strong a main field as possible to keep down the distortion. This of course results in higher reactance voltage and lower power factor, which varies with different designs. There may be cases where the field may be comparatively weak, resulting in very good power factor, and again in cases of very low horse power rating at low speeds, the power factor may be as low as 70 per cent.

The reactance voltage in the armature is small compared with the reactance of the field of the non-compensated motor, and although larger in proportion to the main field in the compensated motor, in neither type has it much effect upon the ultimate result. For commutation reasons, however, it should be kept as low as possible.

In going through the complete design you would, of course, include all of the reactance voltages, even though some of them were negligible.

The air-gap in these motors is about 0.015 in. to 0.020 in. (single air-gap). The very small motors of cheaper design have from 0.020-in. to 0.025-in. and as much as 0.032-in. single air-gap, due to the fact that the appliances using them, in most cases, must be low in price, thus requiring cheap motors. The large air-gap permits of cheaper bearing design and lowers manufacturing costs.

Oil-Filled Terminals for High-Voltage Cables

BY EUGENE D. EBY[1]

Associate, A. I. E. E.

Synopsis.—Underground cables for transmission of power at 33,000 ft. and above have only recently come into use in America, or received much attention here. In connection with such cables suitable terminals are necessary and present an important problem in high-voltage design.

A marked tendency is noted toward the oil-filling of cable joints; terminal conditions make this procedure both logical and desirable.

Dielectric strength must first be specified, and should exceed that of the cable; flashover should occur without puncture; lightning voltages should be guarded against in the design. Proper d-c. tests are still undetermined for various combinations of solid and liquid dielectrics, and a rigid practise can not yet be established with assurance. At present, high-voltage cable lines are intended for a-c. operation, and safety factors should be determined for that kind of service. High-voltage d-c. operation may come into practise later, and research in d-c. testing should be pushed.

Standard ratings of terminals are proposed, corresponding to the accepted standard ratings for other high voltage apparatus. Consistency with other terminal insulation, such as apparatus bushings and line insulators, is desirable. Cable insulation may eventually experience similar standardization. The method of rating single-conductor and three-conductor cables should be harmonized, and both based on operating-line voltage.

Four typical designs of high-voltage cable terminals are described representing a carefully worked out and effective solution of the problem. These are (a) 37,000-volt three-conductor; (b) 50,000-volt single-conductor; (c) 73,000-volt, single-conductor; and (d). 110,000-volt single-conductor. Flashover tests and time tests, corresponding to breakdown and endurance tests on equivalent cables, are reported to illustrate the ability of the terminals to withstand factory and field tests on the cables, and to show the ample factors of safety under operating conditions. Results of an experimental installation of the 110,000-volt terminals demonstrate the safety of the design, predicted from calculations and confirmed by laboratory tests.

For temporary testing purposes these oil-filled terminals are most convenient and economical, and contribute to the uniformity and reliability of the results in cable testing, which are the factors of greatest importance.

Oil-Filled Terminals for High-Voltage Cables

WITH the introduction of lead-encased cables into the field of high-voltage power transmission, there has arisen the necessity of providing, at the ends of these cable lines, suitable terminals or end-bells, capable of maintaining safe connection between the cable and the apparatus or the overhead line with which it is to operate. Three-conductor, 33,000-volt cables have been used in this country for only a few years, and the number of such installations is still small. Operation of single-conductor cable lines at higher voltages is still more recent. Interest in high-voltage power transmission over cable lines has been rapidly increasing because of the large blocks of power which it is necessary to deliver through congested urban sections, and which cannot be handled on overhead lines. At present there is active interest in cable for 132,000-volt operation and a reasonable prospect of attaining this rating in the not-far-distant future. This rapid increase in cable voltage has led to an intensive effort in the development of the necessary joints and terminals for these higher voltage cables. It is the purpose of this paper to present some of the results of this work as related to the problem of terminals.

Tendency Toward Oil Filling

There is evident in the development of high-tension cable joints a definite tendency toward oil filling, *i. e.*, complete filling of the enclosing shell with a fluid oil under sufficient pressure from auxiliary reservoirs to eliminate all voids or pockets within the joint. The use of hard compounds is accompanied with certain well-known and serious disadvantages, among them being the shrinkage upon cooling which leaves unfilled cavities, separation from the surfaces of the insulating materials thus inviting local breakdown, and incomplete sealing of the joint against entrance of moisture through holes in the shell and the lead wipes. These disadvantages are effectively corrected by the use of a fluid filler under pressure.

The tendency toward the oil-filling of cable joints has already received expression by the substitution of softer compounds as fillers in place of the harder compounds commonly used with the lower voltage joints. Of the soft compounds petrolatum is the most common, while in one or two prominent cases a mixture of transil oil and petrolatum has been employed. Auxiliary pressure reservoirs of several types have been advocated and used to maintain complete filling of the joints. The use of these softer fillers has been fully justified by the results obtained, and these point the way to the oil-filled joint as a logical and promising solution of splicing in high-tension cable lines.

Terminal Conditions

From similar considerations the oil-filled terminal is the logical solution of the problem of insulating the ends of high-tension cables. In two prominent particulars the terminal offers a simpler problem than the joint, in that space limitations are largely removed, and the bared conductor does not have to be enclosed within grounded metal. On the other hand, the terminal presents difficulties not met with in joint design, since it combines the joint problem of insulating the cable end from the sheath by solid and liquid dielectrics, with the further problem of insulating it at the

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same time from the sheath through the air. There is also the greater temperature variation, and complete exposure to the elements. In addition to these physical differences, the location of a terminal at the junction point between underground cable and overhead line may at times impose far greater potential strains upon its insulation than most of the cable system is ever called upon to withstand. The cable terminal in general, therefore, is subject not only to those potential stresses originating or developing within the cable system, but also to those stresses occurring in the overhead system as well, including lightning. This latter source of over-voltage stress is usually far more destructive to solid insulations than to liquids. The use of oil in a cable terminal offers the best-known means of dealing with the lightning problem, at the same time contributing very effectively to the quality of the cable near the terminal by serving as an oil reservoir.

Dielectric Tests

The dielectric strength is the first feature of the design to be determined. Obviously the terminal must be able to withstand all tests applied to the cable after installation. This it might do but still be weaker than the cable. Properly, it should be *stronger* than the cable so that it will not fail at any voltage, either momentary or sustained, which the cable can withstand. By failure is meant internal breakdown. The internal strength should be greater than external flashover, the value of which should be consistent with the flashover of bushings on connected apparatus and insulators on connected lines. The desirability of flashover without puncture in the case of bushings and insulators is well established, and the same should be true of cable terminals. The terminal, therefore, should be able to withstand both the short breakdown tests and the longer time tests, it should flashover externally without internal failure at normal frequency, and it should be as nearly lightning proof as possible.

In the foregoing paragraph, alternating voltages have been in mind. Of course, where d-c. testing is employed the terminals must be able to withstand the applied d-c. voltages. Whether this means a greater strength than for a-c. testing will depend largely upon the kind of insulation and the ratio of d-c. to a-c. voltage. The proposed ratio of 2.4 is probably not far from correct for cable paper; for oil a smaller ratio exists, nearer 1.5; and for a combination of oil and paper an intermediate ratio would probably apply, expected to vary with the proportions of these materials. At present, data on the strength of various materials under high direct voltages are too meager to draw definite conclusions or establish a rigid practise. It should be remembered, however, that the terminals and cable are to operate with alternating current, and it is of first importance that they withstand a-c. potentials successfully. The difficulties of applying a-c. tests to long lines of high capacitance will encourage the use of d-c. testing equipment; and some cable lines installed at first for a-c. operation may even experience later on a conversion to d-c. operation. Research in d-c. testing must, therefore, be pushed vigorously, in order that only proper d-c. tests shall be employed. It is not unlikely that both kinds of tests will become a part of the designer's check upon his product.

Standard Ratings

In order that both manufacturer and consumer may benefit by standardization of parts and designs, standard ratings should be adopted and designs developed accordingly. It seems logical that the standard voltage ratings for high-tension apparatus should apply to cable terminals as well. This would harmonize terminal design with that of bushings, insulators, switches, metering transformers and lightning arresters. Standard ratings, as shown in the following tabulation, are already in general accepted use for these devices.

15,000	88,000
25,000	110,000
37,000	132,000
50,000	154,000
73,000	220,000

These ratings, when applied to cable terminals, should represent the practise for both *Y* and delta circuits, grounded and ungrounded, except as modified for other apparatus using terminal insulators. The cable should not be equipped with terminals of lower flashover voltage than that of the bushings of transformers, circuit breakers, and lightning arresters connected thereto.

Intermediate ratings of cable may be found necessary for economic reasons. This will not prevent the use of standard terminals, however, which is highly desirable so that their line-to-ground flashover strength shall not be inferior to other apparatus on the system. It is already generally recognized that the insulation of apparatus located at different points on a high-tension system should have a uniformly high value, even though some of the apparatus is located on a part of the system normally operating at lower voltage than another part. Cable insulation may eventually experience this same standardization.

This proposed standardization of voltage ratings of terminals for high-tension cables emphasizes the desirability of a uniform practise in applying voltage ratings to the cables themselves. Three-conductor cables are rated in terms of line-to-line voltage and their tests determined by this rating. In the case of single-conductor cables, however, it has been the practise to determine their test voltage on the basis of their working voltage between conductor and sheath. This has often resulted in the working voltage from conductor to sheath being used as an expression of the operating voltage, whereas in the case of all other apparatus the operating voltage is understood to be the voltage from line to line. It would appear rather inconsistent to rate single-conductor terminals in terms of the working voltage of

the cables, as this would give the terminal an entirely different rating from the other apparatus to which it will be connected. For example, a single conductor cable for a 50,000-volt system, although its working voltage is 28,900 volts, should have a terminal whose rating is 50,000 volts. Furthermore, if the cable and its terminals are to have their voltage ratings determined on the same basis, both would properly be assigned a rating of 50,000 volts.

Typical Designs

In light of the foregoing considerations, a careful study of the high-voltage cable terminal problem was made, which resulted in certain definite design features, including oil filling. To illustrate the principal features of these oil-filled terminals, four sizes will be described; (*a*) 37,000-volt, three-conductor, corresponding to the highest rated satisfactory three-conductor cable yet produced, viz., 33,000 volts; (*b*) 50,000-volt single-conductor, applicable to the lowest rated single cables above the present three-conductor range; (*c*) 73,000-volt, single-conductor, corresponding to the highest rated single cable in practical operation in this country; and (*d*) 110,000-volt, single-conductor, as now being used in the highest voltage experimental cable installation thus far undertaken. Terminals of this

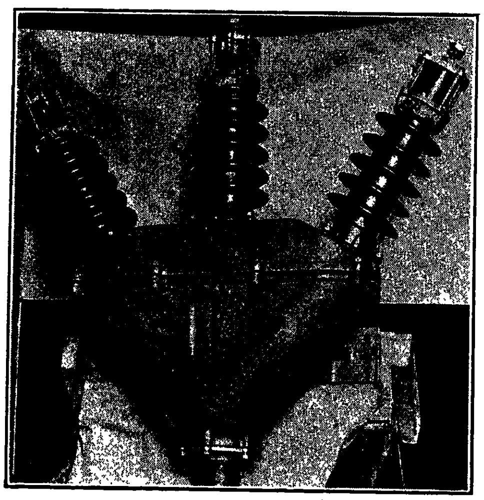

Fig. 1—37,000 Volt, Three-Conductor, Oil-Filled Terminal for Lead-Covered Cable

last rating were used also in the elaborate series of tests conducted by the Electrical Testing Laboratories of New York at the Pittsfield Works of the General Electric Company during the summer of 1924, when several manufacturers contributed samples of their best efforts toward 132,000-volt cable.

37,000-volt three-conductor terminal. This design is illustrated in Fig. 1. The three insulators are in the same plane, an arrangement which makes the tank very much larger than a triangular arrangement, and the spread of the insulators twice as great; but it lends itself readily to wall or pole mounting. Sufficient space in the tank has been allowed for transposition of conductors for phasing, without intermediate link connectors. No side opening in the tank is necessary, as the insulators and cover are removable. The insulators are of wet-process porcelain formed in one piece, with smoothly ground ends, against which bakelized cork gaskets are compressed by means of the metal clamping rings cemented around the ends of the porcelain.

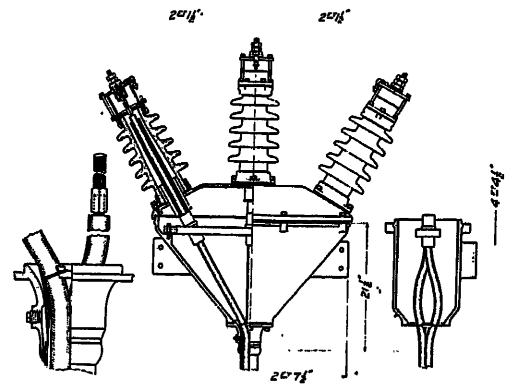

Fig. 2—Drawing Showing Construction of 37,000-Volt, Three-Conductor Oil-Filled Terminal for Lead-Covered Cable

The glass cylinder mounted above each insulator serves as a sight glass registering the level of the oil with which the terminal is filled. An auxiliary reservoir should be provided in connection with a tank of this size, to care for the expansion of the large volume of oil,—in this case about 22 gallons. The lower end of the iron tank is flanged and bolted to the brass-wiping sleeve. The joints here and from tank to cover, as well as at the ends of the porcelain insulators and glass gages, are made oil-tight with treated cork gaskets. Filling and draining is accomplished through a pipe connection to the wiping sleeve.

The internal construction is shown in Fig. 2. The cable sheath terminates just within the wiping sleeve. A thin copper band slipped under the end of the lead protects the paper from sharp edges and damage in soldering. The belt insulation is removed in steps, and the three conductors separated and spaced by a porcelain block. A reenforcement of insulating tape is built up around the outside of the three conductors to give predetermined shape and dimensions, so chosen as to secure a safe distribution of both radial and lateral stresses. Upon the lower portion of this reenforcement is wound an overlay of metal tape, soldered at the lower end to the cable sheath. The upper end of this metal overlay approaches close to the inner surface of the wiping sleeve, which then recedes from the reenforcement and the conductor insulation to meet the lower flange of the tank. The shape of these metal surfaces vitally influences the potential stresses radially and laterally, and largely controls the circumferential stress around the conductor insulation, which seems to be a large factor in so-called *crotch failures.*

Above the reenforcement, the conductors pass in gentle curves to porcelain tubes mounted in a treated wood support, and thence in straight lines to the terminal studs soldered to their bared upper ends. Fairly close fitting paper tubes preserve the concentric alinement of the conductors within the grounded metal shields in the cover. These shields present a smooth, uniform and definite surface to the electric field from the conductors, and also serve to improve the potential distribution on the external surface of the porcelain shells.

The terminal stud at the end of each conductor is locked in place by a double-threaded nut in the recessed depression of the top washer. A terminal cap engages with the stud and seals the top against moisture by an enclosed cork gasket. External connection is made through attachment to the threaded stud extending

Fig. 3—Flashover Test at 170,000 Volts from Cap-to-Cap on 37,000-Volts, Three-Conductor Oil-Filled Terminal for Lead-Covered Cable

from the top of the terminal cap. A drain cock in the top washer above each gage glass permits the escape of air during filling of the terminal with oil.

Samples of this terminal have been tested with three-phase potential to flashover at 170,000 volts, from cap to cap, as shown in Fig. 3. The flashover voltage from cap to case is about 150,000 volts. Time tests have been made at 76,000 volts for seven hours, followed by 100,000 volts for four hours. Subsequent examination failed to disclose any signs of deterioration.

While such terminals for three-conductor cables are perfectly practicable from the standpoint of design, they have some disadvantages in installation, cost, and maintenance, that encourage the use of single-conductor terminals on short single cables spliced to the three-conductor cable, a short distance from the end of the line. This practise is likely to displace the former for sound technical as well as physical reasons. With a length of single cable at the end of the line, extra insulation may be provided against the higher stresses in a connected overhead line, or, due to proximity to reflection points in connected apparatus. Single termi-

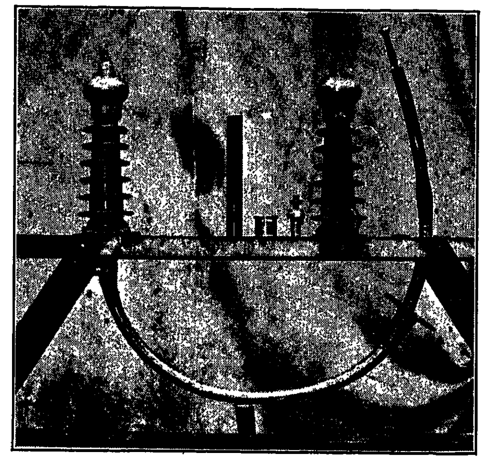

Fig. 4—50,000-Volt Single-Conductor Oil-Filled Cable Terminals with Test Piece of 20/32 In. Paper Insulated Lead-Covered Cable. Right-Hand Terminal Dismantled to Show Construction

nals which can be mounted directly underneath overhead lines of greater spacing than the three-conductor terminal would have, are much smaller and lighter in weight and hence easier to handle, and require much less oil for filling with consequently less expansion

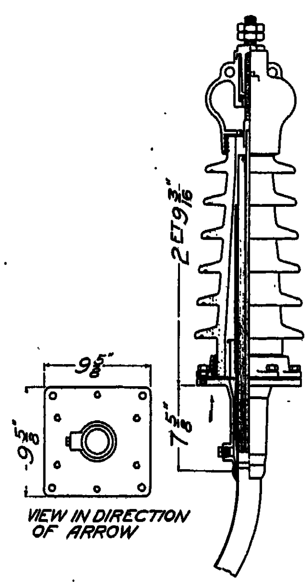

Fig. 5—Drawing Showing Construction of 50,000 Volt Single-Conductor Oil-Filled Terminal for Lead-Covered Cable

capacity in the oil reservoir. The glass gage at the top of the single type of terminal is sufficient for the expansion of the oil in the terminal itself. Single-conductor terminals for higher voltages are described in the following paragraphs. Designs for the lower voltages where

three-conductor cables are used, as at 22,000 volts and 33,000 volts, have been worked out along the same lines.

50,000-Volt Single Terminal

This design is illustrated in Fig. 4. In this case the wiping sleeve and support are in one piece, the latter taking the form of a square flange for bolting to horizontal brackets. The one-piece porcelain shell is cemented into the clamping ring at its lower end, and into the recessed cap at its upper end, with cork gaskets in the joints. The cap shown in the illustrations was designed for petrolatum-filled terminals; for oil-filled terminals a glass gage would be used to give a constantly visible indication of the oil level. The connection details at the top are similar to those described for the 37-kv. terminal.

The internal construction is shown in Fig. 5. The lead sheath terminates, as before, just within the wiping sleeve, and the copper band is inserted under the edge of the lead. A reenforcement of insulating tape is applied directly upon the cable insulation, and the lower end of this reenforcement is overlaid with a metal tape up to its greatest diameter. The reenforcement projects above the support flanges and within the grounded metal shield, with dimensions so chosen as to keep the radial and lateral stresses within safe values. A paper cylinder surrounding the conductor insulation, and spaced from it by means of narrow strips of press-board keeps the conductor straight and concentric within the ground shield, and divides the oil space into cylindrical ducts, thus increasing the insulating value of the oil, and directing its circulation.

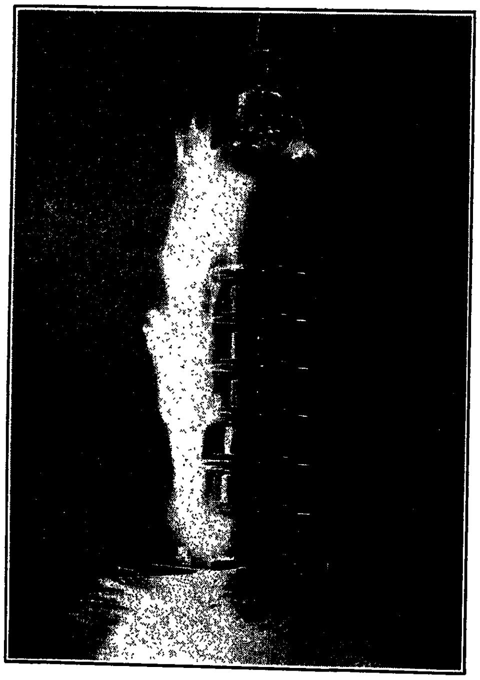

Fig. 6—Flashover Test at 190,000 Volts on a 50,000-Volt Single-Conductor Oil-Filled Terminal for Lead-Covered Cable

Samples of this terminal assembled with 500,000-cir. mils 20/32 in. paper cable[1] have received repeated flash-over tests at an average of 190,000 volts, as shown in Fig. 6, and have subsequently withstood, in consecutive order, the combination of all the several cable tests for 40,000-volt cable taken from the proposed Edison specifications, as follows:

114,000 volt for 5 min.	(breakdown test)
71,000 volt for 15 min.	(full reel test)
57,000 volt for 8 hours	(high voltage time test)

No disturbance of any kind developed during these tests, and no deterioration could be observed upon later examination.

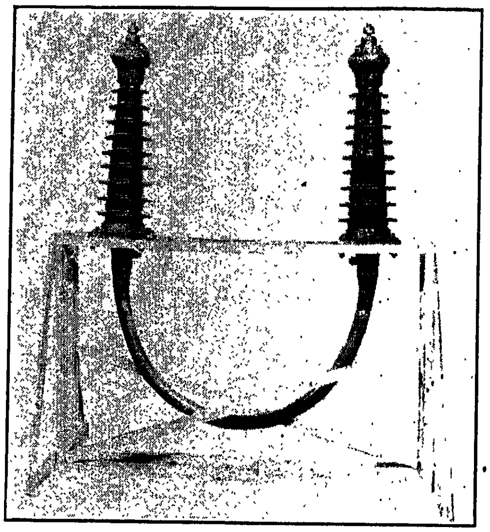

Fig. 7—Two 73,000-Volt Single-Conductor Oil-Filled Terminals for Lead-Covered Cable, Assembled with Test Piece of 66,000 Volt, 30/32-In. Paper Insulated Cable

73,000-Volt Single Terminal

In Fig. 7 there is shown the terminal developed for use with 66,000-volt cable. In external appearance and construction it closely resembles the 50,000-volt terminal just described. Internally, the only prominent difference is in the insulating of the metal ground shield by embedding its upper end in varnished cambric supported directly upon the paper cylinder. Perforations through the flange of the ground shield permit downward flow of the circulating oil in the duct outside of the cylinder. These details and the general construction are illustrated in Fig. 8.

A typical flashover test on this terminal at 290,000 volts is shown in Fig. 9. Time tests on sample terminals have been made at 200,000 volts for several hours. These terminals have also been used with great satisfaction in making time tests on samples of 30/32 in.

1. Cable of this size and insulation is being installed at Columbus, O., for operation at 40,000 volt, three-phase.

paper cables at 200,000 volts and in one instance, a test of 225,000 volts for 36 hours was made on such a cable with no terminal trouble.

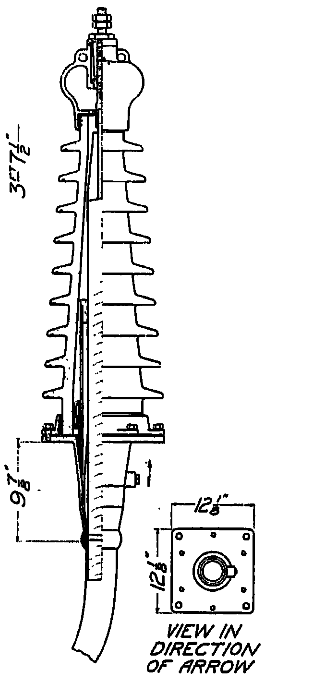

FIG. 8—DRAWING SHOWING CONSTRUCTION OF A 73,000-VOLT SINGLE-CONDUCTOR OIL-FILLED TERMINAL FOR LEAD-COVERED CABLE

110,000-VOLT SINGLE TERMINAL

The terminals shown in Figs. 10, 11 and 12 were developed first for testing of cables having 30/32

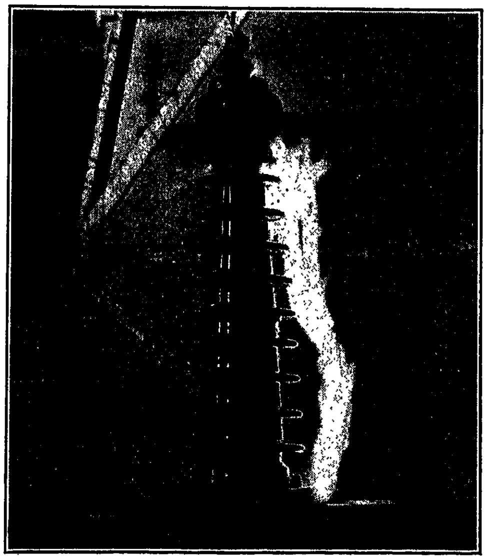

FIG. 9—FLASHOVER TEST AT 290,000 VOLTS ON A 73,000 VOLT SINGLE-CONDUCTOR OIL-FILLED TERMINAL FOR LEAD-COVERED CABLE

in. paper insulation, to determine how near an approach had been made to a safe and reliable 132,000-volt cable. For economy and convenience in assembly, the wiping sleeve is separate from the horn-shaped support casting, with a cork gasket between bolted flanges to form an oil-tight joint. A reenforcement of insulating tape encircles the cable insulation from the termination of the lead sheath to a

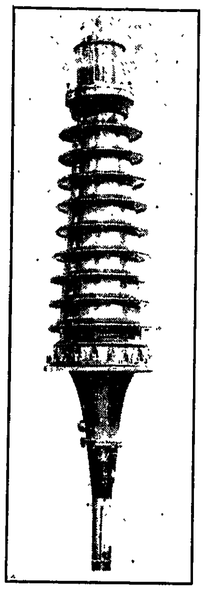

FIG. 10—110,000-VOLT SINGLE-CONDUCTOR OIL-FILLED TERMINAL FOR LEAD-COVERED CABLE

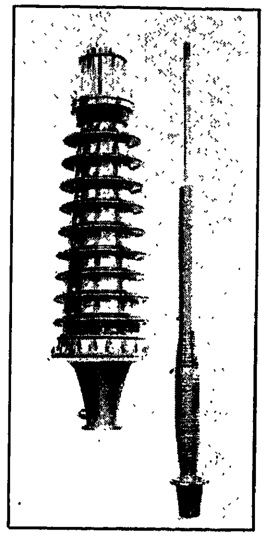

FIG. 11—110,000-VOLT SINGLE-CONDUCTOR OIL-FILLED TERMINAL FOR LEAD-COVERED CABLE, SHOWING TERMINAL REMOVED FROM CABLE, EXPOSING REENFORCEMENT OF CABLE INSULATION

point opposite the top of the ground shield. Radial and lateral stresses are controlled as before by this reenforcement, together with the configuration of the enclosing metal. A bare ground shield is possible by

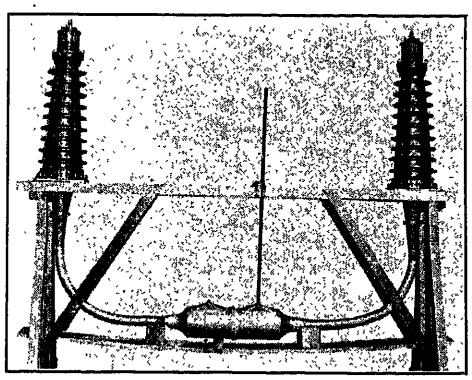

FIG. 12—TWO 110,000-VOLT SINGLE-CONDUCTOR OIL-FILLED TERMINALS FOR LEAD-COVERED CABLE, ASSEMBLED WITH TWO PIECES OF 30/32 IN. PAPER-INSULATED CABLE SPLICED WITH AN OIL-FILLED JOINT

reason of its diameter, and two paper cylinders break up the oil space into vertical ducts. In addition to the ground-shield, a terminal-shield is provided inside the upper end of the porcelain shell to improve the potential distribution along the outside surface of the porcelain.

The porcelain shell and its two clamping rings, and the glass oil-gage and the two adjacent castings are borrowed without change from the standard interchangeable oil-filled bushing for transformers, oil circuit breakers, and lightning arresters, and illustrate again the value, and convenience of standardized

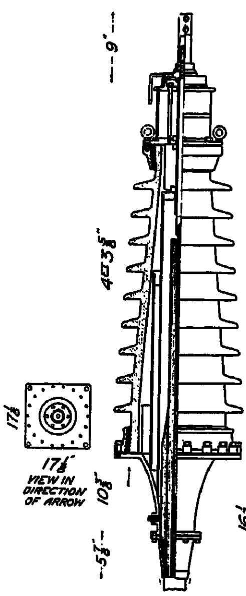

Fig. 13—Drawing Showing Construction of a 110,000-Volt Single-Conductor Oil-Filled Terminal for Lead-Covered Cable

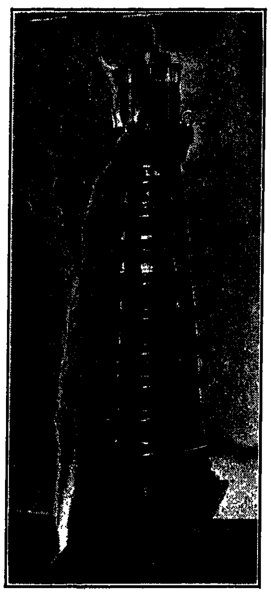

Fig. 14—Flashover Test at 400,000-Volt on a 110,000-Volt Single-Conductor Oil-Filled Terminal for Lead-Covered Cable

material. The details are shown in Fig. 13. The construction is such that, with the exception of the top connector and the wiping sleeve, this terminal may be assembled complete before installing over the prepared end of the cable. This is of great advantage in a terminal of this size and weight, which must be handled with a hoisting tackle of some kind. During installation the long stud, into which the end of the cable is soldered, is passed through the tube in the top of the terminal and secured at the proper elevation by a lock nut above the cover casting. The wiping sleeve is then raised into position, bolted to the support casting, and wiped to the lead sheath.

In the tests on the proposed 132-kv. cable, for which this terminal was first developed, it successfully withstood the breakdown tests up to its flashover voltage of 350,000 volts. These breakdown tests were made, as usual, by starting at some predetermined voltage, and increasing in 10 per cent steps at short intervals. Momentary flashover voltages of 400,000 volts, as shown on Fig. 14, have since been measured on these same terminals, when raising the voltage steadily up to the flashover point. Time tests of six hours at 275,000, and eight hours at 240,000 volts were obtained on some samples of cable, the terminals functioning with complete satisfaction. With sufficiently good cable, it is probable that time tests as high as 300,000 volts for eight hours could be made without trouble developing in these terminals.

It is of very real practical interest, also, that an experimental or trial installation of similar cable equipped with these terminals is receiving a service test on the 110,000-volt system of the Adirondack Power and Light Company, at Albany, N. Y. Fig. 15 shows the installation. This piece of cable is lying exposed on the ground with the ends extending into the terminals, which are mounted on a frame work several feet high. An auxiliary reservoir of oil maintains the oil in the terminals at the proper level, and supplies whatever absorption into the cable there may be. No current is carried by this cable, which fact, coupled with its exposure above ground, imposes a much more severe temperature variation than would be the case with a loaded cable buried in the ground. The test has been running now for four months (to Feb. 1, 1925), and the terminals have given no trouble whatever.

Terminals for Testing Purposes

It may not be inappropriate to say a word about the use of such terminals, as are here described, for temporary or testing purposes. While at low voltages it is usually sufficient to immerse the ends of the cable in

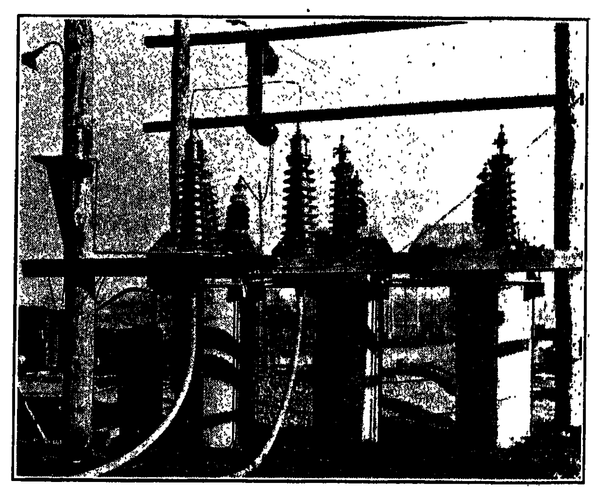

Fig. 15—Experimental 110,000 Volts Cable Undergoing Field Test at North Albany Station of Adirondack Power and Light Corporation, Equipped with 110,000-Volt Single-Conductor Oil-Filled Terminals

oil or compound, either in a tank or by means of cones of metal or paper surrounding the ends of the cable, yet at higher voltages, such as more than 200,000 volts, such temporary methods often become both inconvenient and unsatisfactory in results. The oil-filled type of porcelain terminal, as described above, lends itself most admirably to testing purposes,[2] as well as to permanent

2. See also paper on "Testing High-Tension Impregnated Paper-Insulated, Lead-Covered Cable" by Everett S. Lee, presented at the Midwinter Convention A. I. E. E., New York, Feb. 9-12, 1925.

service installations. The preparation of the cable ends for assembly with the terminal is a simple and rapid process, the terminal parts are easily and accurately adjusted to the cable, the oil filling can be done without delay, and the testing can proceed almost as soon as the filling is completed. With equal facility, the oil can be drained, the terminal removed, and a new piece of cable prepared for test with the same set of terminals. Not only does this method speed up the work, but it makes necessary only a small amount of equipment, avoids the use of large tanks of oil, and saves the loss of much temporary material wasted in some methods of testing. Of even far greater importance are the uniformity and reliability of results obtained, which factors are well nigh indispensible to the value of the test data on the cables themselves.

Discussion

A. O. Austin (by letter): There has apparently been a material improvement in potheads for high-voltage cable. There is no reason why a pothead cannot be made having any desired electrical characteristic, as the problem is not essentially different from making a high-voltage bushing.

The accompanying illustration shows a type of pothead which is quite similar to that shown in Mr. Eby's paper. This pothead was used for running tests on joints and cable for The Cleveland Electric Illuminating Company.

These potheads have an appreciable diameter at the center and form a good reservoir for oil which may be fed into the cable. With potheads of this size it was not possible to flash them over without breakdown of the cable.

In making the tests it was evident that if the cable was free from defects that high frequency had little or no effect. Since high frequency tends to magnify a defect, a much lower voltage may be used and I believe that in the end one of the most valuable cable tests will be that made at high frequency.

If the faulty sections can be eliminated it will be a comparatively easy matter to establish a high degree of reliability. In the potheads shown the stress is reduced to a very low value on the insulation surrounding the cable, as the cable projects up into the bushing. Hence, there is little or no danger of breakdown in the bushing.

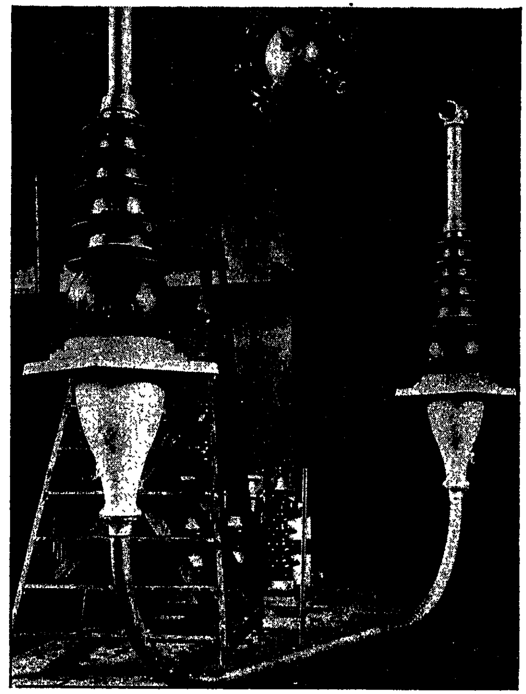

Fig. 1

The potheads were so designed that it is not necessary to use a wiped joint, as a lead sheath may be clamped to the lower end of the pothead bell using a soft gasket. With this arrangement an installation can be made in a few minutes and there is no danger of damaging the insulation from heat. Pin holes caused by the leaking of oil through the wiped joint are also avoidable.

Investigation of High-Tension Cable Joints

BY E. W. DAVIS[1] and G. J. CROWDES[1]
Member, A. I. E. E. Associate, A. I. E. E.

Synopsis.—With suitable cable, the successful operation of a high-tension cable system depends on the joints between consecutive lengths and this phase of high-voltage cable installation must receive considerable attention in the near future.

The essential properties of a good cable joint are:

1. Simplicity of design
2. High dielectric strength
3. Low dielectric loss

The dielectric strength of a joint, assuming proper care and method in assembly, depends upon the materials used. It does not necessarily follow that a joint with high dielectric strength will have low dielectric loss. Low dielectric loss is essential because of the high thermal resistance of the joint.

Data is given from experimental tests carried out on seven different types of joints, four with 25,000-volt cable, and three with 33,000-volt cable.

The joints were made up in the center of a ten foot piece of cable and dielectric loss tests made at various temperatures and voltages. Knowing the loss of the cable itself it was possible to determine the actual loss in the joint. In no case was this loss found to be lower than the loss of an equal length of cable.

The sample of cable with the joint in the center was then tested with high potential at room temperature.

THE demand for cables to operate at high voltages has resulted in much research work being done by cable manufacturers, engineering societies, and operating companies, the result of which has been a material improvement in cables for such service. Dielectric losses have been decreased, dielectric strength increased, and insulating materials developed that are not seriously affected by operating temperatures.

The constants of insulating materials are better known to-day than ever before, and cables can be designed for the normal operating voltages with greater assurance of success than would have been possible a few years ago.

There is, however, one phase of high-voltage cable installation that must receive considerable attention before satisfactory operation of high-tension cable systems can be assured. No matter how complete the design, how low the dielectric loss, or how high the dielectric strength of a cable may be, it is of little value if the joints between consecutive lengths are not equal in quality to the cable itself.

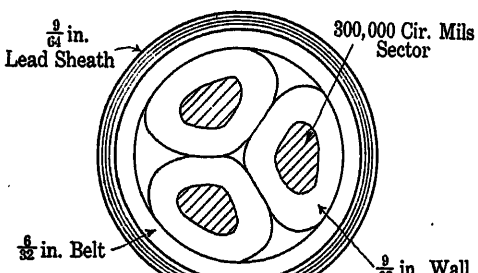

Fig. 1

Three-conductor, 300,000-cir. mil, sector conductor
9/32-in. wall paper on conductors
6/32-in. belt paper over three conductors
9/64-in. lead sheath

We believe that the essential properties of a good cable joint are as follows:

a. Simplicity of design
b. High dielectric strength
c. Low dielectric loss

In this paper, there will be discussions of the last two essential properties of a joint as just named, assuming that the *Simplicity of Design* needs little or no discussion.

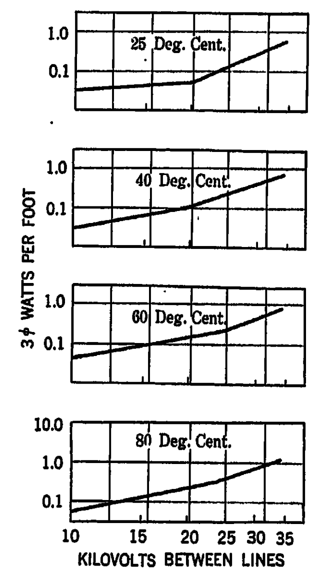

Fig. 2—Dielectric Loss—Three-Phase Watts per Foot vs. Kilovolts

Three-conductor, 300,000-cir. mil, sector, 25,000-volt cable

The dielectric strength and dielectric loss of a cable joint depend upon the insulating materials used, the method of application, and the exercising of extreme care in assembling. Low dielectric loss does not necessarily follow from the fact that a joint has high dielectric strength.

The insulating material on a joint is usually considerably thicker than on the cable itself, which results in a greater thermal resistivity for the joint. Even though the joint has a greater heat dissipating surface than the cable, it can be readily shown that temperature of the joint, with normal copper loss and dielectric loss, would

1. Simplex Wire & Cable Company, Boston, Mass.

Presented at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

be higher than that of the cable. Hence the necessity of making the dielectric loss of a joint as low as possible.

The following experimental work was carried out on seven different types of joints for cables to operate on 25,000 volts and 33,000 volts working pressure. The first four joints were made on a 25,000-volt cable, a cross section of which is shown in Fig. 1.

voltages as the temperature increases. Assuming that ionization is due to entrapped air, this phenomenon of the ionization point may be accounted for by the fact that the dielectric strength of air increases with increase of temperature and increase of pressure.

The first joint constructed was of varnished cambric insulation with dimensions as shown in Fig. 3.

The varnished cambric used was of the best grade,

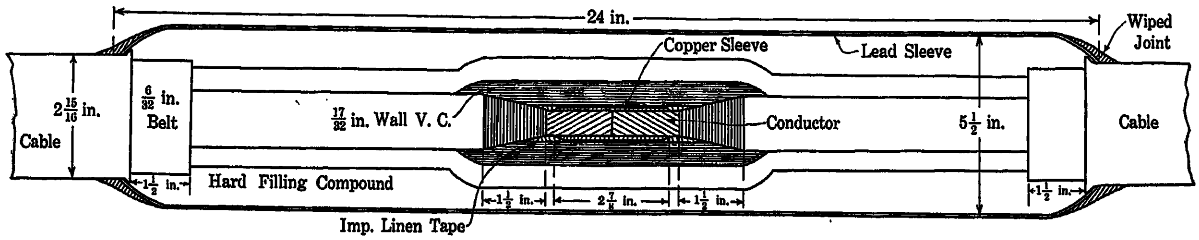

FIG. 3—CONSTRUCTION AND DIMENSIONS OF VARNISHED CAMBRIC JOINT
Three-conductor, 300,000-cir. mils. sector, 25,000-volt cable

A ten-foot length of the cable was placed in a rack, cut in two, and a joint constructed in the center of the length. The length, with the joint, was then put into a heat box and three-phase dielectric loss tests made at various temperatures and voltages. After the dielectric loss tests, the cable was removed from the heat box and dielectric strength tests made at room temperature.

From data of dielectric loss tests of the cable itself

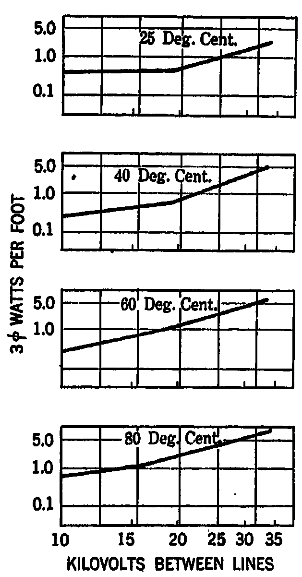

FIG. 4—DIELECTRIC LOSS—THREE-PHASE WATTS PER FOOT VS. KILOVOLTS
Varnished cambric joint—three-conductor, 300,000-cir. mils. sector, conductor 25,000-volt cable

and data of the dielectric loss tests of the cable with the joint, the dielectric loss of the joint only was calculated.

Fig. 2 shows the results of dielectric loss tests on the cable.

The loss curves plotted to logarithmic scale show a definite ionization point, and this point occurs at higher

bias cut, black cloth with an average dielectric strength of 1100 volts per mil. It may be noted in Fig. 3 that a soaked linen tape was applied over the copper sleeve. This tape was of the kind commonly used by cable splicers, boiled out in petrolatum grease. After wiping on the lead sleeve, the joint was filled with a standard high-grade filling compound, sealed, cooled, and made ready for test.

Fig. 4 shows the results of dielectric loss tests on the varnished cambric joint. The dielectric loss is expressed in watts per foot of joint for convenience in comparing the values with similar values for the cable.

It will be noted that while the ionization point of the cable occurred at from twenty to twenty-five thousand volts, (See Fig. 3), the ionization point of the joint occurred at the lower voltage of from fifteen to twenty thousand volts.

Table I gives the ratio of three-phase dielectric loss of the joint and the original cable at 25,000 volts working pressure.

TABLE I

Ratio of Three-Phase Dielectric Loss
Varnished Cambric Joint to that of Original Cable
Three-Conductor, 300,000-Cir. Mil Sector Conductor 25,000-Volt Cable

Temperature Deg. Cent.	Ratio—Three-Phase Dielectric Loss Joint to Cable
25	4/1
40	5/1
60	6.6/1
80	7/1

After the dielectric loss tests, the following dielectric strength test was made at room temperature. (Approximately 25 deg. cent.)

No. 1 Conductor vs. No. 2—No. 3 Conductor and Lead Sheath
75,000 volts—five minutes
100,000 volts—reached by 500-volt steps held 30 seconds at each step.
Return to 75,000 volts—30 minutes
No. 2 Conductor vs. No. 1—No. 3 Conductor and Lead Sheath
75,000 volts—30 minutes
No. 3 Conductor vs. No. 1—No. 2 Conductor and Lead Sheath
75,000 volts—180 minutes.

The dielectric loss of this joint is considerably lower than that of the varnished cambric joint, but still higher than that of the cable itself. The ionization point for this paper joint is higher than that of the varnished cambric joint—in fact, it occurs at practically the same voltage as for the cable.

The ratio of dielectric loss of the impregnated paper joint to that of the cable is shown in Table II.

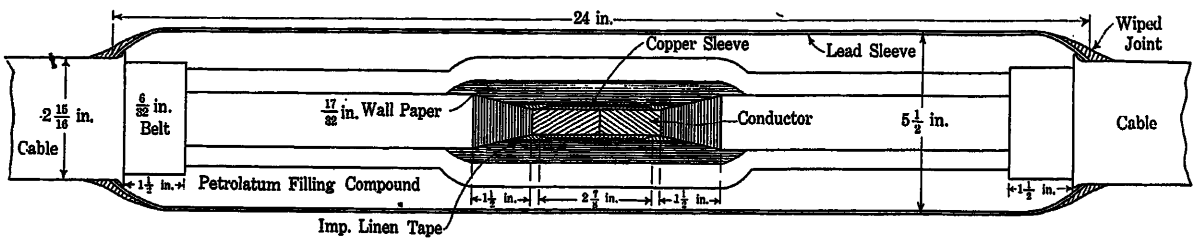

FIG. 5—CONSTRUCTION AND DIMENSIONS OF IMPREGNATED PAPER JOINT
Three-conductor, 300,000-cir. mils, sector, conductor 25,000-volt cable

The joint, after the high-voltage test, was cut open and examined, and there was no evidence of burning or charring of insulation.

The second joint tested was of practically the same dimensions as the first, with impregnated paper for insulation. The dimensions and construction were as shown in Fig. 5.

The impregnated paper used was a four mil tape,

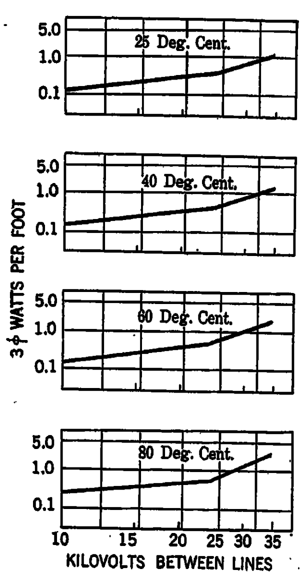

FIG. 6—DIELECTRIC LOSS—THREE-PHASE WATTS PER FOOT VS. KILOVOLTS
Impregnated-paper joint three-conductor, 300,000-cir. mil, sector, conductor 25,000-volt cable

kept immersed in cable compound in a sealed can until applied to the joint. The joint was filled with a paper cable compound. The results of dielectric loss tests on this joint are shown in Fig. 6.

TABLE II

Ratio of Three-Phase Dielectric Loss
Impregnated Paper Joint to that of the Original Cable
Three-Conductor, 300,000-Cir. Mil Sector Conductor-25,000-Volt Cable

Temperature Deg. Cent.	Ratio—Three-Phase Dielectric Loss Joint to Cable
25	2.4/1
40	2.3/1
60	2.1/1
80	2.1/1

After the dielectric loss, the following dielectric strength test was made at room temperature—Single-phase voltage.

No. 1 Conductor vs. Nos. 2-3 and Lead Sheath
75,000 volts—five minutes
100,000 volts by 5000-volt steps held 30 seconds at each step.
Return to 75,000 volts—30 minutes
No. 2 Conductor vs. Nos. 1-3 Conductor and Lead Sheath
75,000 volts—173 minutes when breakdown occurred.

Internal discharges were heard at the end of 160 minutes. These discharges grew gradually more frequent and severe until breakdown occurred.

Examination of the joint after breakdown showed the failure occurred $9\frac{3}{4}$ inches from the center of the joint. Severe burning was evident from the end of the copper sleeve along the conductor and up the insulation penciling. The paper on the joint showed burned spots at many places.

While this joint showed much less dielectric loss than the varnished cambric joint, its dielectric strength was considerably lower.

The third joint to be constructed and tested was a paper-insulated "step" joint with dimensions and construction as shown in Fig. 7.

In the "step" joint, the conductor insulation instead of being "penciled" or "scarfed" smoothly down to the conductor, was cut away in steps or terraces. In every other way the construction was similar to the previous paper joint.

The results of dielectric loss tests for this joint are shown in Fig. 8.

The dielectric losses in this joint are somewhat lower than those of the pencilled paper joint (see Fig. 6), but are still higher than those of the cable. Table III shows the ratio of the dielectric losses of the paper step joint to those of the cable.

An examination of the joint after failure showed extreme burning starting at the end of the first step and extending to the edge of the joint insulation.

The fourth and last joint for the 25,000-volt cable to be described was the so-called "Conducel" joint, a cross-section of which is shown in Fig. 9.

The conductor insulation was pencilled as in the second joint made, and the insulation over the copper sleeve built up even with the original conductor insulation. After the porcelain and conducel insulators were in place, the lead sleeve was wiped on and the joint filled with paper cable compound.

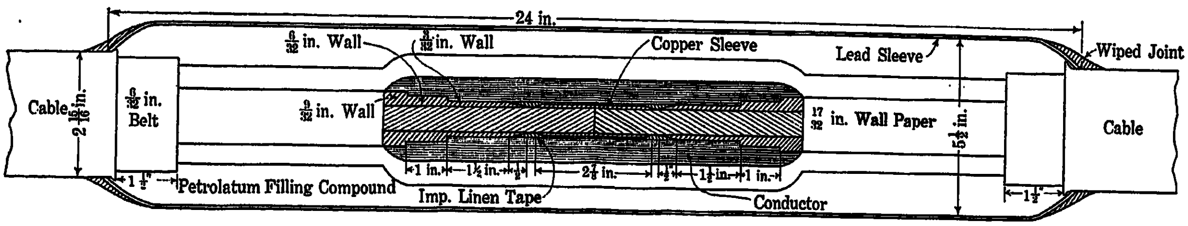

Fig. 7—Construction and Dimensions of Impregnated-Paper Step Joint
Three-conductor 300,000 cir. mil., sector conductor 25,000-volt cable

TABLE III

Ratio—Three-Phase Dielectric Loss
Impregnated-Paper Step Joint to that of the Original Cable
Three-Conductor, 300,000-Cir. Mil Sector Conductor 25,000-Volt Cable

Temperature Deg. Cent.	Ratio—Three-Phase Dielectric Loss Joint to Cable
40	1.7/1
60	1.7/1
80	1.7/1

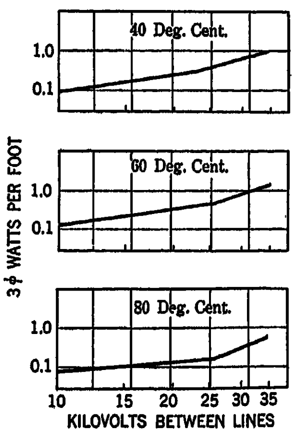

Fig. 8—Dielectric Loss—Three-Phase Watts per Foot vs. Kilovolts
Impregnated-paper "step" joint three-conductor—300,000-cir. mil, sector conductor 25,000-volt cable

After the dielectric loss test, the following dielectric strength test was made with single-phase voltage.

No. 1 Conductor vs. Nos. 2-3 and Lead Sheath
75,000 volts—five minutes
80,000 volts—30 seconds
85,000 volts—joint failure.

The results of dielectric loss tests on the Conducel joint are shown in Fig. 10.

Table IV gives the ratio of the dielectric loss in the Conducel joint to that of the original cable.

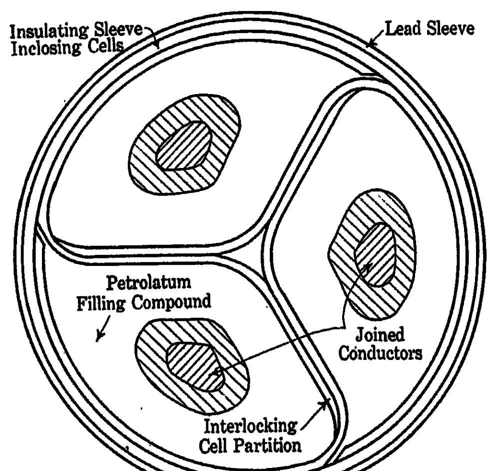

Fig. 9—Construction and Dimensions—Conducel Joint
Three-conductor 300,000-cir. mils, sector conductor 25,000-volt cable

TABLE IV

Ratio—Three-Phase Dielectric Loss
Conducel Joint to that of the Original Cable
Three-Conductor, 300,000-Cir. Mil Sector Conductor 25,000-Volt Cable

Temperature Deg. Cent.	Ratio—Three-Phase Dielectric Loss Joint to Cable
40	1.7/1
60	2.0/1
80	2.9/1

It should be noted that while the dielectric loss of this joint at low temperatures is practically the same as found for the other paper joints, it is very much greater at the high temperatures.

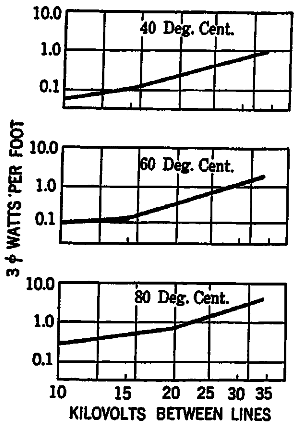

FIG. 10—DIELECTRIC LOSS—THREE-PHASE WATTS PER FOOT VS. KILOVOLTS

Conducel joint three-conductor 300,000-cir. mil sector conductor 25,000-volt cable

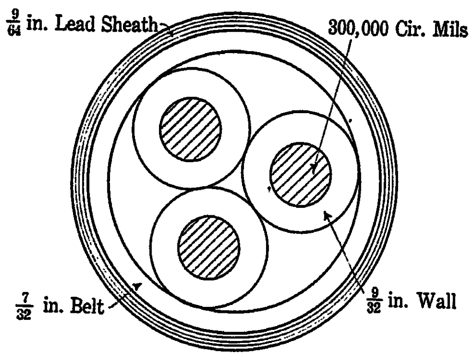

FIG. 11

Three-conductor 300,000-cir. mils, round conductor
10/32-in. wall paper on conductors
7/32-in. belt paper over three conductors
9/64-in. lead sheath, 33,000 volts

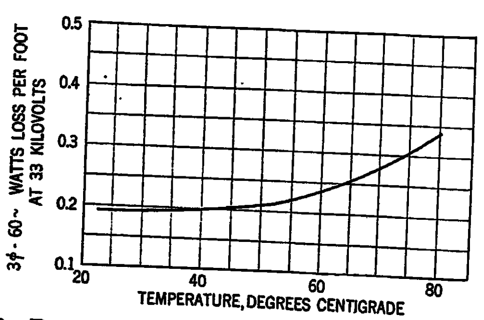

FIG. 12—DIELECTRIC LOSS—THREE-PHASE WATTS PER FOOT VS. TEMPERATURE

Three-conductor 330,000-cir. mils, round conductor 33,000-volt cable

The high potential tests on the Conducel joint showed very low dielectric strength.

No. 1 Conductor vs. Nos. 2-3 and Lead Sheath
75,000 volts—three minutes—failure between conductor.

At the end of two minutes, loud internal discharges were heard, which gradually increased in frequency and magnitude until failure occurred.

Examination after failure showed the conductor insulation badly burned and the conducel insulator punctured.

The remainder of the joints to be described in this paper were constructed for 33,000 volts.

The cable used was three conductors 300,000 round type, 10/32-in. wall paper by 7/32-in. belt paper, 9/64-in. lead sheath, designed and manufactured for 33,000 volts. A cross section of this cable is shown in Fig. 11.

Fig. 12 shows the results of three-phase dielectric loss tests on this cable.

The construction and dimensions of the first joint made in this cable was as shown in Fig. 13.

Results of dielectric loss tests at 33,000 volts at various temperatures are shown in Fig. 14.

Table V gives the ratio of dielectric loss in the joint to that of the original cable.

TABLE V

Ratio—Three-Phase Dielectric Loss
Impregnated paper joint to that of the original cable
Three-conductor, 300,000-cir. mil round conductor 33,000-volt cable

Temperature Deg. Cent.	Ratio—Three-Phase Dielectric Loss Joint to Cable
25	1.32/1
40	1.11/1
60	1.22/1
80	1.27/1

The dielectric loss of this joint is very nearly the same as that of the original cable.

The following dielectric strength test was made on the cable with joint after the above dielectric loss test had been completed. Three-phase, 60-cycle voltage used.

103,000 volts	................60 minutes	
110,000 "	................30 "	
120,000 "	................30 "	
130,000 "	................30 "	
140,000 "	................30 "	
150,000 "	................26 "	Failure.

The breakdown occurred inside the metal sleeve of the joint about one and one half inches from the end of the jacket. Excessive burning of the filling compound and the outside layers of tape on the conductors indicated that there was considerable stress between the edge of the jacket and the edge of the joint insulation. There were no signs of burning in the insulation over the copper sleeve nor along the penciling of the conductor insulation.

Throughout the high potential test after the first five minutes, internal discharges were clearly heard,

although they were rather intermittent and did not increase in frequency or magnitude during the test.

After careful examination of this first joint in the 33,000-volt cable, a second joint was designed which was substantially the same as the one just described except that the conductor insulation was reinforced from the cable crotch to the insulation over the copper sleeve. Construction and dimensions were as shown in Fig. 15.

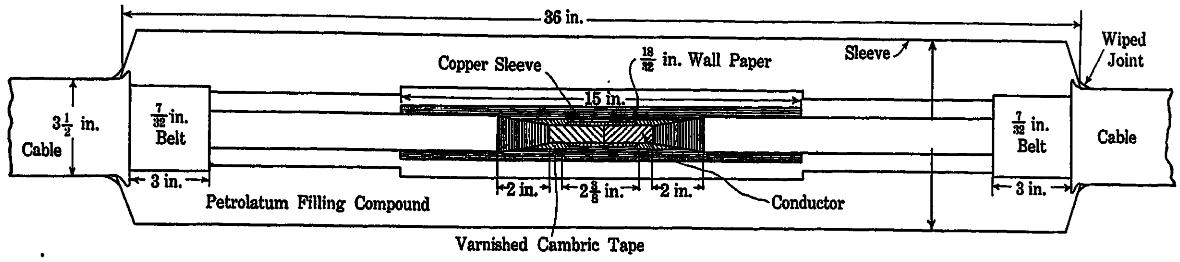

Fig. 13—Construction and Dimensions of Impregnated-Paper Joint

Three-conductor 300,000-cir. mil, round type, 33,000 volts

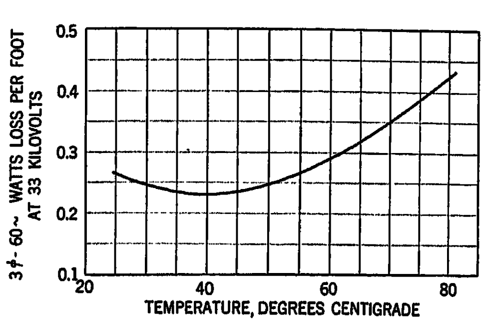

Fig. 14—Dielectric Loss—Three-Phase Watts per Foot vs. Temperature at 33,000 Volts

First impregnated-paper joint in three-conductor 300,000-round conductor 33,000-volt cable

The dielectric loss of this joint was very close to the same as that of the first joint in the 33,000-volt cable. (See Fig. 14.)

The following dielectric strength test was made on the second joint, three-phase, 60-cycle voltage.

103,000 volts	60 minutes
110,000 "	30 "
120,000 "	30 "
130,000 "	30 "
140,000 "	30 "
150,000 "	30 "
160,000 "	26 "

At the end of twenty-six minutes with 160,000 volts, flashover occurred and the test was discontinued.

Examination of the joint showed no signs of burning either in the joint insulation or the filling compound. No internal discharges were heard during the test nor was there any evidence of the joint temperature being higher than that of the cable.

The last 33,000-volt joint to be described in this paper was an "undercut curtain" joint, the construction and dimensions of which are shown in Fig. 16.

This type of joint was somewhat different from any o

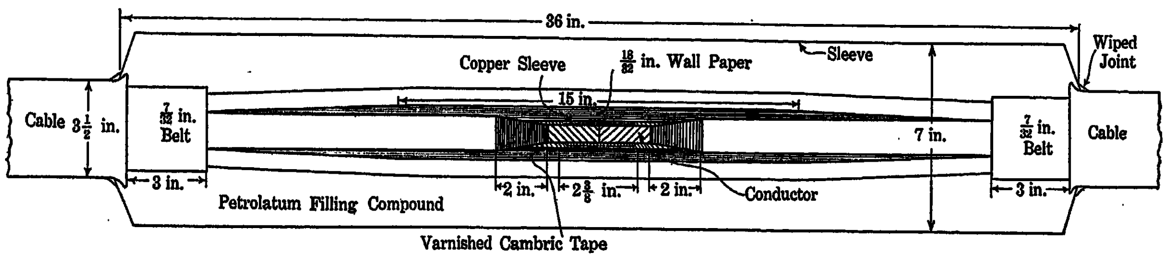

Fig. 15—Construction and Dimensions—Reinforced Paper Joint

Three-conductor 300,000, round conductor 33,000-volt cable

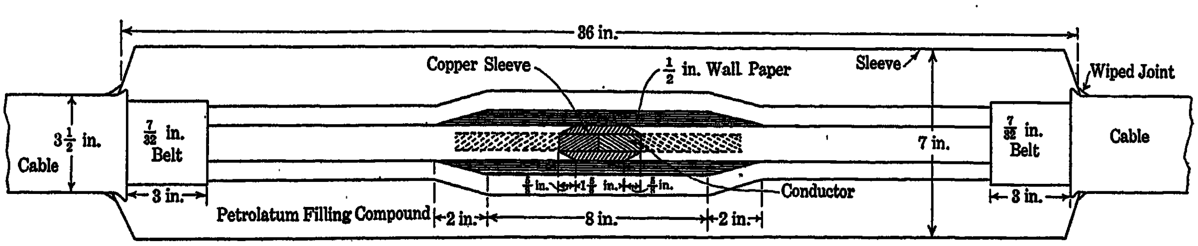

Fig. 16—Construction and Dimensions—"Undercut Curtain" Joint

Three-conductor 300,000 cir. mils, round conductor 33,000-volt cable

preceeding joints in both design and assembly. The conductor insulation was undercut by means of a special tool and copper sleeves of the same outside diameter as that of the insulated conductor, sweated on. The paper insulation, which was in the form of a roll of single sheet, was spiralled around the conductor and then "wound up" or tightened until it was as solid and firm as the conductor insulation itself.

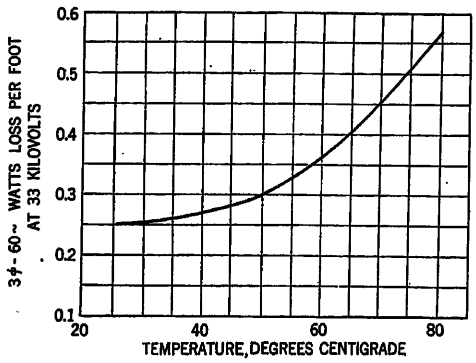

FIG. 17—DIELECTRIC LOSS—THREE-PHASE WATTS PER FOOT VS. TEMPERATURE AT 33,000 VOLTS

Undercut curtain joint on three-conductor 300,000 cir. mils, round conductor 33,000-volt cable

The dielectric loss at 33,000 volts at various temperatures are shown in Fig. 17.

Table VI gives the ratio of dielectric loss in this joint to that of the original cable.

TABLE VI

Ratio—Three-Phase Dielectric Loss
Undercut Curtain Joint to that of the Original Cable
Three-Conductor, 300,000-Cir. Mil, 33,000 Volt Cable

Temperature Deg. Cent.	Ratio—Three-Phase Dielectric Loss Joint to Cable
40	1.30/1
40	1.33/1
60	1.34/1
80	1.68/1

The following dielectric strength test was made on this joint with three-phase, 60-cycle voltage.

103,000 volts	60 minutes
110,000 "	30 "
120,000 "	30 "
130,000 "	30 "
140,000 "	30 "
150,000 "	10 "

At the end of ten minutes with 150,000 volts, failure occurred apparently in the joint. Examination of this joint showed burning at the edge of one of the paper tubes and much burned grease at one end of the joint.

We have not offered any theories as to the behavior of certain of the described joints nor have we drawn any definite conclusions as to the proper joint construction. This paper is merely a preliminary report of the work what has been completed to date. Conclusions will be drawn only after the work is completed.

The authors are greatly indebted to Mr. L. R. Hicks and Mr. C. A. Mayo of the C. H. Tenney Co., Boston—and Mr. Harry Tounge of McGonigle & Tounge, Boston—for materials and suggestions made during the experimental work.

Discussion

A. P. Thoms: I do not see why so much stress should be placed on the dielectric loss in a joint. E. S. Lee, in a paper before the A. I. E. E. Midwinter Convention[1], stated that the dielectric-strength test is the most important and the best test available at the present time; that indications are that the present standardized voltages should be increased and that increasing the application of the test two or three minutes is of doubtful value. The authors of this paper have used the standardized dielectric-strength test with the application increased a few minutes, and this test, according to Lee, is of doubtful value.

All the dielectric-strength tests were made at room temperature. They should be made at maximum and minimum temperatures experienced in normal operation of the cable, and should range down to at least 5 deg. cent.

Accelerated life tests should, in my opinion, be at from 2.5 to 5 times the rated voltage. A single dielectric-strength test will not determine whether a joint will be satisfactory in operation. We should have a test on a joint similar to that required on a cable. If the joint is not subjected to tests to which we subject a cable, such as those required by the N. E. L. A. specifications, we should at least standardize our tests so that we can compare them. I do not see why all the tests were not made with three-phase voltage.

After a joint fails on a test, it is very important that the joint be opened and examined in very minute detail. In looking over the paper, I could not determine the complete path of carbonization. This seems very essential, as it affords information whereby we may improve the weak points in the joints, either by changing the material or by changing dimensions.

The voltage at which sounds of distress in the joint are heard is also very important as this voltage denotes deterioration in the joint. This can be taken with a stethoscope or with a long paper tube held to the ear. Most of the large operating companies are testing their long transmission lines with a kenotron or d-c. test. If the proper ratio between the a-c. and d-c. for the cable is 2.4, it may be 1.7 for the joint, and it is very essential that we know beforehand that these joints are not going to break down when the line is subjected to the initial acceptance test.

Mario Puritz: In giving the results of the last 33,000-volt joint described, Mr. Davis speaks of an "undercut curtain" joint. The patented Pirelli, three-phase joint, with undercutting and tightened paper rolls, is exactly of that type and has been used in Europe for many years. A full description of this joint may be found in the *Electrical Review*, London, August 10, 1923, and a further description with the latest improvements will be published in the Report of the Underground Systems Committee, N. E. L. A., to be printed this year.

About 200 of these joints for 35,000 volts have lately been installed in Chicago and Boston and it may be interesting for you to know some of the results obtained in the tests made in the laboratories of companies of this country. The test joints were all of about the same dimensions as those described by Mr. Davis which were filled with soft compound and differed only in that the lead sleeve was of smaller diameter and the

1. Testing High-Tension Impregnated Paper-Insulated, Lead-Covered Cable, by E. S. Lee, A. I. E. E. JOURNAL, February, 1925, page 156.

reservoir on top. One joint was tested on three-phase, starting at 100 kv. and increasing 15 kv. every 30 sec. At 200 kv., after 5 sec., flashover occurred at the pothead. The voltage was lowered to 183 kv. and after 5 min., failure again occurred in the pothead. No sign of failure or carbonization was evident in the joint.

Four other joints of the same type were tested on single-phase, one conductor against the other two, with the lead at the middle point. The voltage was applied, starting at 120 kv. and increasing 10 kv. every 30 sec. One joint failed at 190 kv. after 21 sec. because of moisture in the compound. The three other joints withstood the test potential of 200 kv. (the maximum pressure available) without evident trouble, and all failures occurred *in the potheads* as follows: after 20 min. at 200 kv. for the first; after 20 min. 15 sec. for the second; and after 20 min. 5 sec. for the third. No signs of carbonization were found in the joints.

Tests of the three conductors in parallel against ground were also made on three other joints with failures of the joints at voltages corresponding to 225,000, 242,000 and 225,000 volts, delta pressure.

In a long-time test run in single-phase, 100-kv., one phase against the other two and the lead at the middle point, another joint was perfect after 18 hr. and 18 min. when the cable failed at a bend, due to excessive increase of temperature of the dielectric. Of course, after this test some leaf-shaped carbonizations were found on the conductors all along the cable and in the joint, and these carbonizations showed on about 15 of the outside layers of the conductors, especially due to the single-phase test, but the carbonizations in the joint were not so deep as carbonizations on the cable.

The cables on which the test joints were made were either three 350,000-cir. mil. or a three 300,000-cir. mil., 35,000-volt, round conductor, 8 × 4.5/32-in. insulation, 1/8 in. lead and 3-in. over-all diameter.

F. A. Brownell: The authors touch upon a vital point when they state that extreme care should be used in assembling the joint. We believe that is at least 50 per cent of the battle, and that the balance is in the right combination of materials entering into the joint.

From our experience with the right combination of low-dielectric-loss materials, we have found that the joint under stress is cooler than the cable. No actual temperature data are at hand, but in one case, after 13 hr. at 105 kv., the cable and joint were examined and while the cable was actually hot, the joint was at only slightly higher than room temperature.

If the theory of graded insulation is sound, then we are not following the correct principle by wrapping varnished cambric upon paper of a lower specific inductive capacity, as this will place a higher stress upon the penciled surface. By plotting voltage-gradient curves using a specific inductive capacity of 3.5 for paper and 4.5 for varnished cambric and taking a point 1/8 in. back on the penciled surface of the paper, we get a voltage gradient of 12 kv. per cm. more than if paper had been used for insulation.

We made up two single-conductor joints with stepped insulation and with the same thickness of insulation in each case; we wrapped one joint with varnished cambric and in the second paper was used.

Failure in each case was at 190 kv. and the surface of the stepped insulation was badly burned. Then we were at loss to know whether this was due to reverse grading or to voids, due to the fact that it is practically impossible to obtain a tightly wrapped joint with paper. The one advantage in the use of varnished cambric is that it makes a tighter wrapping.

It might be of interest to compare two joints we made, similar in design to joint No. 3 except that we used a special material for wrapping over the connector to a level of the factory insulation and then wrapped paper over this, giving the same thickness of insulation as in joint No. 3.

Tests made on these joints with single-phase voltage are as follows: 100 kv., 4 hr.; 120 kv., 2 hr.-45 min.; failure in crotch of joint.

On the second joint, we held 80 kv. for 4 hr. and raised the voltage 10 kv. each hour. After 13 min. at 130 kv., failure occurred in the end-bell. This was repaired and when voltage was rapidly applied, failure occurred at the same place at 160 kv. The ends were again cut back and failure occurred again in the end-bell at 166 kv.

While the second joint did not fail, it showed signs of high stressing. The surface of one conductor at the edge of the belt was charred and the petrolatum had changed in color with particles of carbon throughout. In neither joint were there signs of charring along the surface of the stepped insulation.

Our experience has been that we could get higher breakdowns with a stepped joint and we have found that the average cable splicer will make a better and more uniform stepped joint than penciled joint.

We have examined joints that have been in service for a number of years and found that where they had been penciled very uneven surfaces were left to tape over.

In the first 33-kv. joint, the failure and signs of distress near the edge of the belt showed plainly that this was due to the tangential stresses at this point. In the second joint the insulation was extended back to the crotch and as this joint did not fail, we are led to believe that this is due to the added insulation.

I cannot conceive of overcoming tangential stresses by adding more insulation, but would rather borrow Mr. Ely's idea (and this was done) of using an electrostatic shield to overcome the tangential stresses and smooth out the voltage gradient at the edge of the lead sheath. In fact, by using this and some slight refinements, we have developed a joint that is apparently stronger than the cable.

In the final joint, the use of the Cleveland idea of undercutting the insulation and extending the connector to the level of the factory insulation is no doubt good, but it is limited to round conductors.

From our observation of splicing cable in the field we could not conceive of using a paper roll on three-conductor cables, for it is seldom that the three conductors can be brought out on a plane and a tight wrapping cannot be obtained.

As the authors have drawn no conclusions from their experiments, we are tempted to advance the following for them:

That with varnished-cambric wrapping and the joint filled with a hard asphaltum compound, the dielectric strength on short-time breakdown is high, but on life tests at high potential, a well wrapped paper joint filled with petrolatum will give a higher breakdown. This is perhaps accounted for by lower dielectric loss.

That the difference of tests between the joints shown as Figs. 13 and 15 could be accounted for in workmanship.

To sum up, we have concluded from our experimental work that for a 33-kv. joint (*a*) the material entering into the construction of the joint should in characteristics approximate the material in the cable; (*b*) we cannot get a tightly wrapped joint with impregnated paper and we have substituted a material with the same characteristics but more pliable; (*c*) tangential stresses at the termination of the lead sheath should be taken into consideration and it is believed that they could be overcome by the use of static shields or by the proper slope of the lead sleeve; (*d*) to get a well-filled joint, it must be filled under pressure and (*e*) the education of the splicer is very essential.

The only apparent objection to the use of petrolatum in filling joints is due to migration of the oil from the joint and this we believe could be overcome to some extent by using a petrolatum with a higher viscosity than possessed by that used at present and still having approximately the same dielectric strength.

Regarding Joint 13, Mr. Crowder asked us if we didn't believe that, due to the fact that he has added more insulation, he has given a higher dielectric strength to that joint? I don't believe so. We made up joints of different thicknesses of insulation. In two cases I recall we made up joints with 40 per cent more insulation than used on the cable; in another case, 65 per cent more, and by some peculiar coincidence, the joints with the most insulation failed at a lower voltage. If we plot a curve between voltage gradient and distance from the conductor we would find that we would get what could be likened to a saturation curve, or a point out from the conductor at which there is a very small voltage gradient—in fact practically no stressing,—and from the little work I did I should imagine that they had just about reached that saturation point, or perhaps extended beyond it. In other words, they had more paper than they actually needed.

C. F. Hanson: In witnessing a dielectric-strength test on cable splices recently, I observed a phenomenon which, in my opinion, requires consideration.

The cable with the splice was bent into the form of a horse shoe and rested on wooden blocks supported by two I-beams laid across the top of a tank with the cable ends dipping into transformer oil in the tank. Between the sheath of the cable and the I-beam was an air-gap about 1 in. long.

The voltage was applied in steps and at each step the voltage was maintained constant for a given period of time. When the voltage reached a certain value a train of electrical discharges would occur intermittently across the air-gap. Coincident with these discharges, bubbles in the transformer oil would rise from the end of the cable. As the voltage was further increased, these intermittent discharges would generally cease for a while. Then later on, when the voltage had been increased still further, these discharges would again occur but with greater intensity. They would grow more violent until complete electrical breakdown occurred either in the cable or in the splice.

The phenomenon to which I alluded is the first train of discharges across the air-gap in conjunction with the bubbles rising in the transformer oil. The only logical explanation for this phenomenon which I can give, is as follows:

The discharges must have a high frequency in order that the impedance across the air-gap may be less than that of the return wire to the transformer. The thing which caused these high-frequency surges must have been a disturbance in the electric circuit. One source of disturbance is a surface discharge in the splice along the penciling of the original cable insulation. After a number of these surface discharges have occurred, the compound, which in this case was petrolatum, would have been heated locally and sufficiently to permit the compound to flow into the discharge path and temporarily oppose further discharges. The local heating would, of course, expand any air which might be present. The increased pressure could be relieved at the end of the cable, causing bubbles to rise. With further increase in voltage, the dielectric stress may become sufficient to expel the compound which had flowed in, when discharge would again occur. If the stress is sufficiently great to prevent the compound from flowing back into the path of discharge the splice will finally break down completely with a mass of carbonized compound. On the other hand the surges may break down the cable insulation before enough compound has become carbonized to form a complete short circuit in the splice.

A subsequent examination of one of the splices seems to substantiate the foregoing explanation. Apparently, this splice had not failed but the cable had. However, upon dissecting the splice, six layers of the original insulation and eight layers of the applied insulation were badly charred. Also a considerable amount of petrolatum was charred.

If the foregoing reasoning is correct then the breakdown voltage of a splice is that obtained when the first electrical disturbance occurs and not that obtained when a complete short circuit occurs. When the first disturbance occurs, the splice has become a menace in the electric circuit because it produces disturbances which are hazards to other electrical equipment in the circuit.

When reporting tests of splices it is frequently stated that the cable fails before the breakdown voltage of the splice is reached. If, however, the tests are interpreted as above suggested, it will probably be found that the splice has failed first.

C. F. Hood: The purchasers of high-tension cable have kept the manufacturers so busy in trying to meet their demand that the efforts which we have put forth in developing joints have followed somewhat behind the effort put into the development of cables.

In attempting to develop a joint which we feel can be recommended to the users of high-tension cables, we are now conducting a series of experiments similar to those outlined by Mr. Davis.

So far it appears that we can produce a wrapped joint which will be as good as the cable for which it is intended. We have made some experiments on wrapped joints in comparison with the so-called patented joints, and so far the wrapped joints have shown themselves to be considerably better than the type of joint built up with a barrier insulation.

Along these lines, I don't think that we can stress too strongly the necessity of having the jointers appreciate just what work they are trying to do. If they are not men who realize the importance of detail and following that very closely, as already pointed out, a good joint can be spoiled by a poor workman.

I think that a great deal can be gained by much closer cooperation between the manufacturers of the cable and the users.

A. H. Kehoe: The paper infers that due to the existing situation in joint design satisfactory operation of high-tension cable systems cannot be assured. I believe that satisfactory joints are now in operation on cable systems of all voltages for which we have been able to obtain cables. For ordinary joints we know that adequate factors of safety exist. For the high-voltage joints the factors of safety can only be estimated as we do not have good enough cable to test these properly.

The authors divide the essential properties of a good joint into three classes. We believe that simplicity of design is by far the most important element in comparing successful types of joints in service.

To further decrease the dielectric loss obtained with any practical joint today is not highly important, as cables in ducts run at a higher temperature than cables in manholes, so that the limiting condition is the heat of the cable in the duct.

Short-time applications of extremely high voltages may set up conditions not duplicated in actual service as certain types of dielectric have a high voltage-breakdown value for short-time applications, while other types do not. Results in practise at normal stress may thus give opposite results from such tests. In selecting test values it is important to ascertain whether the particular materials tested are stressed beyond their known breakdown values, rather than to make a large number of different joints some of which only demonstrate this breakdown limit which could have been positively predicted before the expense of testing was undertaken. The high-voltage accelerated-life test provides an important element of time saving in making tests but results are not directly comparative between varying types, as to normal voltage operation, particularly if high-voltage short-time values are used as is the case in these tests.

I believe that the essential element in cable joints and in cable installations is simplicity of design. From it, uniform results are most likely to be obtained. The greatest difficulty I have experienced in making joints is to be certain that all of them are tight, that is, that they are all waterproof. It is important in order to accommodate existing structures that a

short joint be used. This same condition causes many companies to use sector cable and it is more difficult to obtain a joint with proper factors of safety on sector cable than on round-conductor cable. Simplicity of design also makes it easy to eliminate voids. I believe that many of the joints tested failed because of voids in the filling compounds. Testing a few joints all of different types or even a few joints of one type does not give a definite indication of the factors of safety unless they all target very closely to the same value on the test curve.

I should like information from the authors as to how they handled the end-bells in testing. The paper infers that dielectric loss was obtained first on long cable lengths averaging so much per foot, and any increase was attributed to the joint. It is not clear how the dielectric loss of the end-bells was accounted for.

We have had several thousand joints of the fourth type tested by the authors, operating successfully at 28 kv. for over two years, and several other operating companies have had similar experience with this joint at similar voltage. The uniformly successful operation obtained with this type of joint for installation up to 33 kv. would not be predicted from the one joint tested.

In testing the first four types of joints, the dielectric strength to ground was tested, as three-phase potential should have been used to obtain the comparable results with the last three joints made from round-conductor cable. The initial voltage applied to the first four was equivalent to 130 kv. in so far as failure to ground was concerned. Certain of the joints tested have factors of safety to ground which are slightly lower than failure between phases, as this makes the best operating type of joint. The failure of No. 4 joint by puncture of the factory-formed insulator is the second case which has come to my attention; in the first one the workman did not understand the assembly of the material and the joint was poorly made with large voids existing in it. From the internal discharges reported on this test I judge that some similar phenomena occurred.

In discussing the last three types of joints tested it is well to emphasize the practical advantages in using a short-length joint where proper factors of safety can be obtained. It is evident, however, that the longer the joint the more reliable it will be in operation regardless of its type.

The method of accelerated-life testing is important in comparing results published by other authors. For instance, this is the first case which has come to my notice where thirty minutes per 10 kv. change has been used instead of increments of one hour for 10 kv. As there has been considerable doubt expressed in the past that even the one-hour increment gives proper comparative results for various types of joints, we believe that testing of this character should be longer and reduced voltage values used. Such a test method is more expensive but gives a better indication of the service results that are likely to be obtained.

The seventh joint of the so-called "curtain-roll" type is one with which we have had successful experience on a single-conductor 44-kv. installation. The joints are but 20 in. long, although the stresses are approximately the same as those tested by the authors on a 36-in. structure. This construction produces simple joints of uniform workmanship for round conductors, as long as they are straight, but it cannot be applied to sector conductors.

C. A. Adams: As yet we know practically nothing about the fundamental nature of dielectric phenomena. There are several hypotheses and much superficial misconception. As yet, we don't know even what are the significant variables with which we are concerned in this field. Moreover, much of the so-called "practical research work," which has been done bit by bit here and there without coordination is likely to be not only useless but in some cases actually misleading.

If manufacturers and users of insulated wires and cables and of other insulated apparatus would cooperate in a comprehensive research into the fundamental nature of dielectric phenomena at an expense to each which would be extremely modest as compared with the probable value of the results, there would be some hope of solving this most important problem. In such an undertaking, it would be necessary to enlist the assistance of the ablest physicists and chemists in the civilized world, but for work of this kind such assistance can be obtained at relatively small expense.

I have been endeavoring to start such a movement for the past seven years and a small start has been made by the organization of the Insulation Committee of the Engineering Division of the National Research Council, this Committee being also attached to the Research Committee of the A. I. E. E. But it is only recently that some of the users of cables are being forced to realize the need of such fundamental research.

L. A. Zima (by letter): The authors start out with an assumption of "Suitable Cable." This is a broad assumption to make for high-voltage cable, considering our limited operating experience to date. They further state that high-voltage cable jointing must receive considerable attention in the near future. Perhaps, from the manufacturer's standpoint, this is correct; but the cable user has given a great deal of attention to this problem ever since high-voltage cable has been manufactured, and has had reasonable success. This success is based on long practical experience following long tedious research work on experimental joints. Work along this line has been somewhat retarded waiting for the cable manufacturers to make suitable cable.

I rather question the three essential properties mentioned. Simplicity of design is certainly to be desired but it is not an essential property.

High dielectric strength is desirable but this does not necessarily mean very high instantaneous breakdown voltages or on short-life test. It is possible to make up a joint to give very high dielectric strength on instantaneous voltage application, but which will fail utterly to give satisfactory results in normal operation.

Dielectric loss, as low as that of the cable, is not an essential property of a cable joint. Due to the large radiating surface of the joint and the large volume of surrounding air in the manhole, the cable joint is at a much lower temperature than the cable inside the duct line.

The essential properties and requirements of a satisfactory cable joint are as follows:

a. Proper design, keeping the voltage gradient of all points in the joint within proper limits.

b. Materials of good quality, having high dielectric strength, with an specific induction capacity of relatively the same value as that of the cable. These should not vary in normal operation.

c. Workmanship of the highest order.

The voltage tests made on the various joints were of comparatively short duration, lower values and longer time would be more desirable and would more nearly approximate service conditions.

My attention is particularly directed to the electrical characteristics of Joint No. 4. Of the seven types of joints, the author finds this one to be the poorest of the lot as it has the lowest dielectric breakdown value. The reason for this, it seems to me, is not that of design but rather the general method of construction, especially of workmanship. The materials were not properly assembled and the filling probably faulty.

There are many of these joints in operation and they are giving satisfactory service. If properly assembled, using good material and with high-grade workmanship this joint will give a dielectric-breakdown test value, instantaneous and accelerative life, equal to that of the cable.

The Brooklyn Edison Company has had a large number of these joints in service over a year,—probably from 2500 to 3000,

—and the operating results have been satisfactory. Refinements on these joints have continued, slight changes made in the construction, strengthening the weakest points, properly training the splicer for this type joint, and maintaining close supervision to insure high-grade workmanship.

A number of joints constructed with these refinements are now under test. The result of the first eight joints is as follows:

Tests on Joints in Sector Cable. These were three-conductor 350,000-cir. mil, sector cables with $20 \times 10/64$-in. paper insulation, 9/64-in. lead. Tests were made with three-phase, 60-cycle voltage.

On all the joints except No. 1 the voltage was started at 70 kv. between phases and increased in 10-kv. steps remaining constant for one hour at each step. At 120 kv., voltage was held constant until breakdown occurred.

Joint No. 1 was subjected to 50 kv. for 405 hours before it was raised to 70 kv., after which the test was identical with those on the other joints.

Joint No.	Hours at 120 Kv.	Total Hours on Test
1	3.1	413.1
2	6.4	11.4
3	6.4	11.4
4	7.2	12.2
5	7.8	12.8
6	5.3	10.3
7	4.3	9.3
8	5.0	10.0

The failures occurred at 120,000 volts after a number of hours as shown. Examination showed the following:

Breakdown between conductors at sharp bend in conductor insulation near cable crotch.

Failure between conductors at porcelain spacer due to jamming and breaking conductor insulation.

Breakdown at edge of belt insulation due to sharp bend in conductor insulation.

A number of failures in cable crotch of the prepared cable ends.

These tests are a fair measure of the dielectric breakdown voltage values of the cable joint on a long-life test at comparatively high values.

As experience has demonstrated the value of long accelerated life tests to determine satisfactory cable, the same holds true for cable joints.

I would recommend that a standard method of long accelerative life tests be set up so that a direct comparison can be made on joints and cable of different types of construction as made by various manufacturers or operating companies These tests should then be supplemented by simi ar tests, using d-c. voltage instead of a-c

G. J. Crowdes: The work on joints, when completed, will comprise the construction and testing of about twenty joints. The following are results of tests on five more joints which may be of interest.

A duplicate of the varnished-cambric joint (see Fig. 3) was constructed with No. 00, $9 \times 6/32$ in. paper-insulated cable having round conductors. This joint had the same high loss characteristics as the previous varnished-cambric joint. The dielectric strength of this joint was quite high as shown by the following table:

3-Phase, 60-Cycle Kv.	Time (Min.)
92.5	60
110	30
120	30
130	30
140	28 to failure

The failure occurred between conductors about 4 in. from the edge of the belt. There was no evidence or charring or burning along the penciling.

Two joints were next built on 250,000-cir. mil. round-type cable with $9 \times 6/32$-in. of impregnated-paper insulation designed for 25 kv. working pressure. The construction of the joints was identical with the previous 25-kv. penciled paper joint (see Fig. 5) with two exceptions, (1) the length of penciling was increased to 2 in. and (2) each conductor was reinforced back to the crotch of the cable. One joint was filled with a standard hard filling compound and the other with petrolatum.

The loss in the joint with hard filling was considerably higher than the loss in the petrolatum-filled joint, at working voltage and a temperature of 80 deg. cent., the ratio being about 2 to 1. In dielectric strength there was practically no difference:

3-Phase, 60-Cycle Kv.	Hard filling Time (Min.)	Petrolatum filling Time (Min.)
92.5	60	60
110.0	30	30
120.0	30	30
130.0	30	30
140.0	30 failure	24 failure

The failure in both joints occurred at the edge of the belt insulation between conductors. There was no evidence of burning along the penciling.

The next two joints were constructed using 300,000-cir. mil. sector-type cable, with $9 \times 6/32$-in., impregnated-paper insulation. These joints were identical with the previous penciled paper joints described in the paper (see Fig. 3), with two exceptions, (1) the penciling was increased to 2 in. in length, and the built-up insulation was carried farther along the conductor for a distance of about 2 in. The conductors were not reinforced to the crotch. One joint contained hard filling compound and the other petrolatum.

Here again the joint with hard compound showed much higher loss characteristics. The slope of the loss curves with both voltage and temperature was considerably steeper in the hard-compound joint than in the petrolatum-filled one.

The dielectric strength of the hard-compound joint in this case was somewhat greater than the petrolatum joint as shown in the following table:

3-Phase, 60-Cycle Kv.	Hard Filling Time (Min.)	Petrolatum Filling Time (Min.)
92.5	60	60
100.0	30	12 failure
120.0	30 failure	

The failures on both of these joints were between conductors at the end of the tape-on insulation. Severe burning was evident along the penciling in each case.

There is one striking fact evident from the tests on the last four joints and that is the difference in dielectric strength between the sector-type cable joints and the joints on round cable. The joint construction is identical except that there is no reinforcing on the sector-cable joints. Yet, on the round-type cable joints it was possible to reach a voltage of 140 kv. and hold this stress for considerable time. On the sector-cable joints, 120 kv. was reached on one joint and 110 kv. on the other. The failure on the sector-type joints followed the penciling.

It does not seem possible that the reinforcing of the conductor to the crotch (which was the only difference between the two types of joints) could account for the great difference in dielectric strength.

E. W. Davis: We quite agree with those who have discussed our paper, that dielectric-strength tests should be standardized, but we are not ready to accept such tests as the sole criterion for

the value of an insulating material. Of all the electrical tests that we made on the various types of joint, we found that the dielectric-loss test was the most sensitive, and that it gave the most consistent and conclusive results. Experience has shown that a joint designed for normal dielectric strength and low dielectric loss will stand up better under severe operating conditions than one with high dielectric strength and high dielectric loss.

All the joints tested were opened and carefully examined and photographic records made of the paths of carbonization. A few of the joints were tested with radio apparatus installed in the ground circuit, and the tests stopped at the first signs of internal discharge. It was from a careful study of the paths of carbonization that the weak spots in the cable joints were found and the design changed so as to eliminate them as far as possible.

We have not discussed the question of workmanship in this paper. A poorly designed joint will not give satisfactory results, even though the workmanship is perfect. All of the joints tested were made by the same man so that the quality of workmanship was undoubtedly the same in all cases. In none of the joints that we opened and examined was it possible to say that the cause of the failure was due to poor workmanship.

Unfortunately, when the earlier work was done on the joints, we had only single-phase voltage available. Some of this work has been repeated with three-phase voltage with substantially the same results. Dielectric strength tests at high and low temperatures, show very little variation, especially if the results are comparative. The joints that show the higher dielectric strength at low temperatures, invariably have the higher dielectric strength at high temperatures.

We feel that the proper design of a joint to be put into a cable installation is well within the province of a cable manufacturer, but that the question of workmanship is a matter with which the operators should deal.

Predetermination of Self-Cooled Oil-Immersed Transformer Temperatures Before Conditions are Constant

BY W. H. COONEY[1]

Associate, A. I. E. E.

Synopsis.—*The temperature rises of dynamo-electric machines, in changing from one steady thermal state to another, follow an exponential law of the form $1 - \epsilon^{-\beta t}$, where $1/\beta$ is the thermal time constant, or the time required to attain 63. 2 per cent of the final change in temperature rise. It is shown here that while the winding temperature rise over room of self-cooled oil-immersed transformers follows this law only after a certain time has elapsed, quite accurate results may be obtained by calculating the time-temperature curves of the top oil temperature rise above room, and the winding temperature rise above top oil separately, since each follows the above exponential law quite closely, then adding them together to get the winding temperature rise above room at any time before conditions are constant. A procedure is explained in detail for calculating β, for either the top oil rise or the winding rise above top oil from the weights of materials the iron and copper losses and the winding and top oil constant temperature rises at any given load. Comparisons of temperatures by test and calculation are presented.*

* * * * *

Introduction

WHILE it is of interest to know the length of time necessary for a transformer to attain its final winding temperature rise at rated load, it may sometimes be desirable to know how long an overload may be carried, without exceeding a safe winding temperature, starting at room temperature or at any constant temperature rise less than that due to normal load.

For instance, it is required that railway transformers be tested at a load of 125 per cent or 150 per cent for two hours, starting at the temperature rise due to normal load, without exceeding a winding temperature rise over room of 60 deg. cent. This sort of test does not indicate how long the overload could be carried starting at some other initial steady state, such as room temperature.

To intelligently load the transformer, the operating engineer should have either the results of tests made under the various load conditions or the thermal time constants[2] for the different load and initial conditions for a given transformer. Of course, the most accurate results can be obtained from tests, but it is obvious that this method has its disadvantage in that it is not practical to make tests under all the various conditions under which the transformer might be required to perform. It is, therefore, highly desirable to be able to calculate the time constant, or constants, as the case may be.

In a recent paper[3] a method of calculating the time constant for machines where the parts generating heat (windings and iron of a generator, for example) are not thermally independent and the temperature rise is proportional to the loss, was outlined. It was shown in this paper that the winding temperature rise above room followed an exponential rise with time such that the ratio of temperature rise, at time, t, to final temperature rise $\left(\frac{\theta}{\theta_f}\right)$ could be expressed by $1 - \epsilon^{-\frac{t}{\tau}}$, where τ is a time constant which can be determined graphically from the tested time-temperature rise curve. It was also shown that the value $\left(1 - \frac{\theta}{\theta_f}\right)$

1. General Electric Co., Pittsfield, Mass.

2. The reason the value of the constant changes for different loads is because, as shown later, the temperature rise of self-cooled transformers for constant conditions is never proportional to the loss but to the loss raised to some definit power.

Presented at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

3. Kennelly: Thermal Time Constants of Dynamo-Electric Machines, Presented at Midwinter Convention, New York, Feb. 9-13, 1925.

which is equal to $\epsilon^{-\frac{t}{\tau}}$, could be plotted as a straight line against time on semi-log paper.

During the discussion[4] of the previously mentioned paper, it was pointed out that in an oil-immersed self-cooled transformer, the windings are separated from the core by an appreciable amount of insulation and oil, so that the windings may for all practical purposes be considered thermally independent of the rest of the transformer. Although the top oil temperature rise and the winding temperature rise above top oil may each be plotted in terms of $\left(1 - \frac{\theta}{\theta_f}\right)$ as a straight line on semi-log paper, the sum of the two, or winding temperature rise above room becomes a straight line only after the winding temperature rise above top oil has become constant. Moreover, the latter, when continued back as a straight line, does not pass through the origin.

The object of this discussion is to present a method, based on treating the winding temperature rise above top oil and the top oil temperature rise separately, whereby the winding temperature of a transformer can be calculated at any time after a change in load has occurred following a steady state. This method requires a knowledge of the weights of the materials in the transformer, the copper and iron losses, and the constant top oil and winding temperature rises above room for some given load such as that required in the acceptance tests.

It is not claimed that this formula is absolutely correct, since the first expression, Eq. No. 3, in its derivation, as pointed out later, is intended to cover not only winding temperature rise over top oil, but also the top oil temperature rise above room. However, it should be accurate enough for most practical purposes, as an inspection of Figs. 5 to 10 will show.

Final Temperature Rises

In order to calculate temperature rises before conditions are constant, it is, of course, necessary to know the final rise which would be obtained if the load were maintained until conditions become constant. Fortunately, the final rise for any assumed loss of most transformers can be estimated fairly accurately, especially if the rise for some given loss is known.

In the following exposition, the winding temperature rise above room temperature is separated into two parts: top oil temperature rise above room and winding rise above top oil. There are two reasons for this distinction:

In the first place, due to greater thermal capacity, top oil temperature rise, following a change in load, requires approximately as many hours to become constant as the winding rise over oil requires minutes. Secondly, the final values of the two rises do not vary at the same rate with change in load.

4. By V. M. Montsinger.

It has been found[5] that both the top oil rise of the usual self-cooled transformer and horizontal disk coil rise over top oil vary with loss approximately in accordance with the formula:

$$\theta = k\,P^{0.8} \qquad (1)$$

where θ is the temperature rise, k is a constant in which area of dissipation is one factor, and P is the loss. Temperature rises of vertical coils over top oil vary between the 0.9 and the first power of the loss, and can be expressed close enough for practical purposes by:

$$\theta = k\,P \qquad (2)$$

Formula (1) is shown for convenience on log-log paper in Fig. 1. A reference to this figure makes the distinction between top oil rise and coil rise over top oil more readily apparent. Assume that with a ratio of copper loss to core loss of 1:1 the final coil rise over top oil is 10 deg. cent. and the final top oil rise is 30 deg. cent. at some given load. If, due to an increase in load, the copper loss is doubled, the coil rise over top oil according to equation (1) for a relative loss of 2 becomes 1.75×10 deg. cent. = 17.5 deg. cent (and according to equation (2) becomes 20 deg. cent.) but the relative watts loss for top oil rise, since core loss remains constant, is $(2 + 1)/(1 + 1) = 1.5$ and the rise (Fig. 1) is 1.38×30 deg. cent. = 41.4 deg. cent.

By means of equations (1) and (2) the final temperature rise for any load can be found provided the rises are known for some definite load and the corrections for change in resistance from one temperature to another are made.

Increase of Temperature Rise with Time

It is a well-known fact that temperature rise varies with time according to the exponential curve as shown in Fig. 2. Immediately after the generation of loss in the body under consideration has begun, the temperature rise increases rapidly, since at first it is not sufficient to dissipate any loss, and, consequently, practically all the loss is stored. As the rise increases, the dissipation to the surrounding medium (oil or air, as the case may be) increases, until finally heat storage has been completed and all the generated loss is dissipated.

If the initial generated loss is W before the temperature rise has begun, the generated loss at time t and temperature rise θ for a constant current can be expressed as $W(1 + \alpha\,\theta)$ where α is the resistance coefficient of copper. The loss stored at time t is $C\frac{d\,\theta}{d\,t}$, where C is the thermal capacity of the body being heated. The loss dissipated can be expressed

5. Montsinger. Effect of Barometric Pressure on Temperature Rise of Self-Cooled Stationary Induction Apparatus. Proceedings A. I. E. E., Vol. 35, pp. 451-478, April, 1916.

oximately by $K\theta$. Multiplying the initial loss by stance correction is not strictly correct when using ormula for oil temperature rise, since core loss may decrease slightly with temperature. But $\alpha\theta$ is ly of the order of 15 or 20 per cent and copper loss, ially for overloads, is several times as large as the oss, so that no great error is caused. If α were plied by the ratio of copper loss to total loss, uld result not only in a complicated equation, but he necessity of a separate one, for oil top tempera-ise.

uating the loss generated to the loss stored and ated at time t and temperature rise θ the equation ned is

$$W(1+\alpha\theta) = C\frac{d\theta}{dt} + K\theta \tag{3}$$

integration of this equation, shown in the ndix, gives the formula:

$$\theta = \theta_f(1 - \epsilon^{-\beta t}) \tag{4}$$

θ_f is the final temperature rise caused by the loss initial value is W, ϵ is the base of Naperian thms and

$$\beta = \frac{W}{C\,\theta_f} \tag{5}$$

s constant may be more familiar in its reciprocal where it is known as the exponential thermal time nt, or the time necessary to attain 63.2 per cent l rise:

$$\tau_e = \frac{1}{\beta} = \frac{C\,\theta_f}{W},$$

the binary time constant[3]:

$$\tau_2 = 0.69315\,\frac{C\,\theta_f}{W}$$

is expressed in minutes and W in watts, for ng rise over top oil

$$C = 2.96\,\frac{A+a}{2a} \times \text{lb. of bare copper} \tag{6}$$

A and a are the insulated and bare cross-ns of conductor in the windings, respectively.

top oil temperature rise over room, when t is ssed in hours and W in watts,

$$C = \frac{3.5\,(\text{lb. Copper} + \text{lb. Core} + \tfrac{2}{3}\,\text{lb. Tank}) + 90\,(\text{Gallons of Oil})}{60} \tag{7}$$

e the calculation of C is of great importance in nethod, it may be of interest to show the reasons e above values of the constants.

c. g. s. units, the following values of specific heat ssumed constant for all temperatures, although actually is a small variation, which may be ted for the ranges of temperature considered.

Specific Heat Copper = 0.0935
Specific Heat Iron = 0.115
Specific Heat Oil = 0.47

The thermal capacity may be expressed in more practical units:

Copper = 2.96 watt minutes per pound per deg. cent.
Iron = 3.6 watt minutes per pound per deg. cent.
Oil = 105 watt minutes per U. S. gallon per deg. cent.

It is apparent that all the oil in a transformer is not heated to the same temperature as the top oil for the reason that the temperature at the bottom of the tank is cooler than at the top. The ratio of mean to maximum temperature in the gradients of self-cooled tanks ranges from about 75 per cent to 95 per cent, depending on the design. Assuming an average of these figures, or about 86 per cent as a representative figure, the constant for oil then becomes 0.86 × 105, or 90, as shown in formula (7).

A similar line of reasoning applies to the tank. The base of the tank, the bottom end of the wall, and, unless there is an oil conservator, the cover dissipate very little or no heat. For this reason, ⅔ of the tank weight is taken. Since the ratio of iron to copper in a transformer is about 5 or 6 to 1, the metal may be lumped together and multiplied by the factor of 3.5 instead of 2.96 and 3.6 for copper and iron respectively.

In considering the windings of a transformer, if the insulated cross-section of the wire is appreciably larger than the bare cross-section it should be taken into account. Fibrous insulation is usually so impregnated with insulating compounds or oil that its thermal capacity is practically that of the latter. Although oil has five times the thermal capacity of copper by weight, copper has about ten times the density of oil, so that the insulation may be considered to have ½ the thermal capacity of copper by volume as indicated in eq. (6).

When the weights of materials are not known, the values of C may be found by observing the time-temperature rise curves for both the top oil and the winding rise over oil when the transformer heats up, preferably starting at room temperature and 100 per cent load and excitation. The values of W and θ_f should be determined. β may be found by a graphical method[3], and C solved for in equation (5). This value of C will be constant regardless of the loads applied or the initial conditions.

Application of the Formula

In changing from one steady state to another, the problems encountered may be grouped into three classes:

a. The application of excitation and load to a transformer which is at room temperature.

b. Application of load after a constant oil rise over room temperature has been attained due to core loss.

c. Application of an increase in load after a constant

rise of windings and oil has been attained due to a smaller load and core loss.

Since the application of a formula becomes simpler, both in explanation and in use, when examples of it are shown, the following problems are worked out under each of the classifications just specified. The relative values of the characteristics given below are typical of a moderate sized power transformer.

Assume a transformer with the following characteristics:

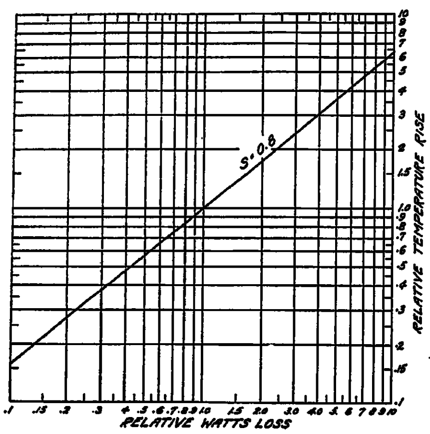

FIG. 1—REPRESENTATION OF $\theta = k\,P^{.8}$ (FORMULA 1) IN TERMS OF RELATIVE TEMPERATURE RISE AND RELATIVE WATTS

At 100 per cent load and excitation:
- 40 deg. cent. top oil temperature rise
- 15 deg. cent. winding temperature rise over top oil
- Copper loss, 9000 watts
- Core loss, 6000 watts

Weights:
- 900 gallons of oil
- 1000 lb. copper
- 4200 lb. core iron
- 4500 lb. tank

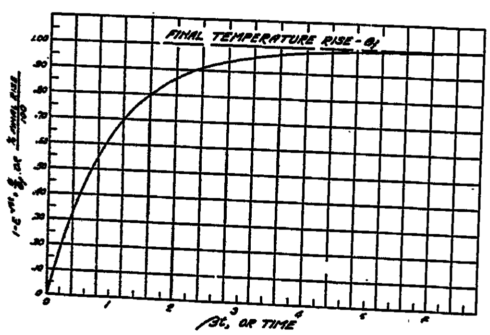

FIG. 2—VARIATION OF $1 - \epsilon^{-\beta t}$, WITH βt, OR INCREASE OF TEMPERATURE IN PER CENT OF FINAL RISE WITH TIME

(Neglect resistance corrections for simplicity.)

(a) 125 per cent load and excitation starting cold.

θ_f: For top oil rise, relative watts $= \dfrac{20080}{15000}$ $= 1.338$; from Fig. 1, oil rise $= 1.26 \times 40$ deg. $= 50.4$ deg. cent.

For coil rise over top oil, assume that the construction is such that formula (2) applies. Coil rise over oil $= 1.563 \times 15$ deg. $= 23.5$ deg. cent.

C: For oil rise

$$= \frac{3.5(1000 + 4200 + 3000) + 90 \times 900}{60}$$

$= 1830$

For coil rise over oil $= 2.96 \times 1000 = 2960$, (Neglecting insulation.)

W: For top oil rise $= 20{,}080$ watts.
For coil rise over top oil $= 14{,}080$ watts.

$$\beta: \text{ For oil rise} = \frac{20{,}080}{1830 \times 50.4} = 0.217$$

$$\text{For coil rise over oil} = \frac{14{,}080}{2960 \times 23.5}$$

$= 0.202$

When these values of β are multiplied by t in hours and minutes respectively, the corresponding values of $1 - \epsilon^{-\beta t}$ may be found from Fig. 2. The latter when substituted in formula (4) determines the value of θ at time t. The results are plotted in Figs. 3 and 4.

(b) Constant oil rise due to excitation, followed by 100 per cent load.

θ_f: For oil rise
Due to excitation, relative watts $= 0.4$, rise $= 19.2$ deg. cent.
Due to excitation and 100 per cent load, oil rise $= 40.0$ deg. cent.
Rise due to copper loss $= 20.8$ deg. cent.
For coil rise over oil at 100 per cent load $= 15$ deg. cent.

C: Same as in (a).

W: For oil rise $= 9000$ watts.
For coil rise over oil $= 9000$ watts.

$$\beta: \text{ For oil rise} = \frac{9000}{1830 \times 20.8} = 0.236$$

$$\text{For coil rise over oil} = \frac{9000}{2960 \times 15} = 0.202$$

The variation of θ, with time, following the same procedure as in (a), is plotted in Figs. 3 and 4.

(c) 125 per cent load following (b).

θ_f: For oil rise $= 50.4$ deg. $- 40$ deg. $= 10.4$ deg. cent.
For coil rise over oil $= 23.5$ deg. $- 15$ deg. $= 8.5$ deg. cent.

C: Same as in (a).

W: For oil rise $= 20{,}080 - 15{,}000 = 5080$ watts
For coil rise over oil $= 14{,}080 - 9000 = 5080$ watts

$$\beta: \text{ For oil rise} = \frac{5080}{1830 \times 10.4} = 0.267$$

$$\text{For coil rise over oil} = \frac{5080}{2960 \times 8.5} = 0.202$$

The results obtained from these values of β and θ_f are also plotted in Figs. 3 and 4.

Although corrections for resistance have been purposely omitted in the foregoing, the distinction should be made that θ_f is calculated for the loss at the final temperature whereas the W used in β is the loss at the initial temperature before the change.

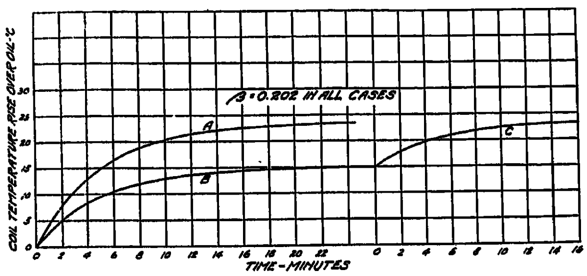

FIG. 3—WINDING TEMPERATURE RISE OVER TOP OIL, CALCULATED FOR THE TRANSFORMER OF ASSUMED CONSTANTS UNDER THE FOLLOWING CONDITIONS:

(A) 125 per cent load and excitation, starting at room temperature.
(B) 100 per cent load and excitation, starting at oil temperature rise due to core loss alone.
(C) 125 per cent load applied after conditions are constant for (B).
Calculations of β are shown in the text.
Note: 0 time for (C) corresponds to 18 hours in Fig. 4.

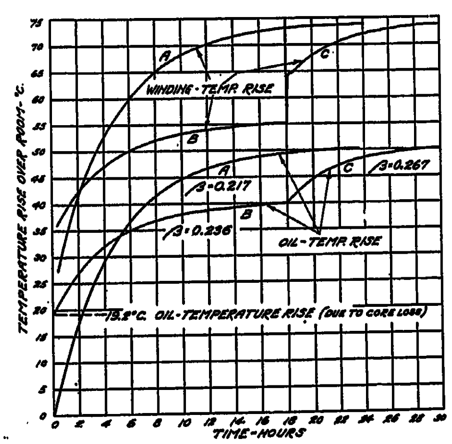

FIG. 4—OIL AND WINDING TEMPERATURE RISES ABOVE ROOM CALCULATED FOR THE TRANSFORMER OF ASSUMED CONSTANTS UNDER THE FOLLOWING CONDITIONS:

(A) 125 per cent load and excitation, starting at room temperature.
(B) 100 per cent load and excitation, starting at oil temperature rise due to core loss alone.
(C) 125 per cent load applied after conditions are constant for (B).
Winding temperature rises above room are obtained by adding the winding rises over oil (Fig. 3) to the oil rises. Calculations of β for the oil rises are shown in the text.

It will be noted that in all three cases the value of β for coil rise over top oil remains the same. This is due to the fact that θ_f is taken as proportional to W, which would not be the case if resistance corrections were considered or if a winding were assumed whose construction caused it to follow formula (1).

COMPARISON OF TESTED AND CALCULATED RISES

A comparison of tested coil temperature rise over top oil and that calculated by formula (4) is shown in Fig. 5. Insulated thermocouples were inserted in a horizontal disk coil stack, and various currents, in excess of normal to get higher readings to minimize any absolute errors, were put through the windings. A thermocouple was selected which was representative of the coil rise over oil by resistance. Readings were taken at intervals during the time required to attain final rise over oil. The calculated values are somewhat higher at first than test, but this may be due to the insulation, between the copper and the thermocouple, causing a temperature lag in the readings. The calculated values of β are shown in the figure.

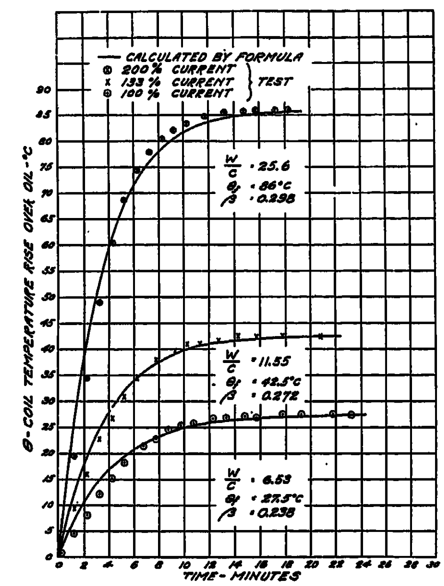

FIG. 5—VARIATION OF COIL TEMPERATURE RISE OVER OIL, WITH TIME, FOR VARIOUS CURRENTS, AS OBSERVED BY THERMOCOUPLE ON DISC COILS IN HORIZONTAL POSITION

An important point to be noted in connection with Fig. 5 is that, since θ_f does not increase as fast as W for this type of winding, β increases for the higher values of current. An inspection of the test values shows a confirmation of this method of calculating β, for the greater the current, the quicker is a given percentage of final rise attained.

To determine how closely tests would agree with the type of calculation shown in (c) of Fig. 4 for oil temperature rise, a 75-kv-a. transformer was given 100 per cent load and excitation starting at room temperature. Hourly readings were taken to determine the variation of top oil temperature rise over room with time until the readings were practically constant. Then 125 per cent load was applied, and the readings continued until conditions were again practically constant. The readings and calculation of β, and the factors on which it is based are shown in Fig. 6. The observations made in connection with Fig. 5 apply here

also. θ_f has not increased as rapidly as W (see formula 1) and consequently β has increased about 50 per cent in the overload. The general conclusion is that when the ratio of W to θ_f is not constant in all cases, β should be calculated for the particular load and initial condition under consideration.

For the purpose of checking up the form of calculation

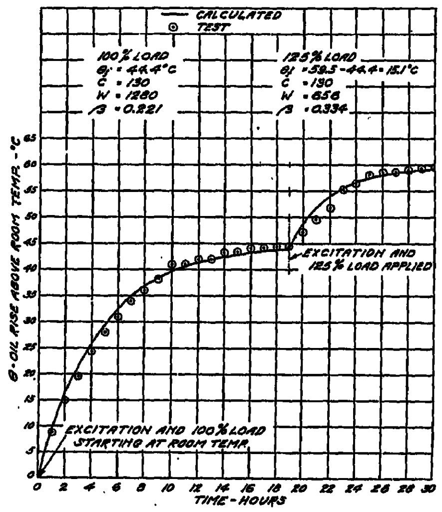

FIG. 6—VARIATION OF TOP OIL TEMPERATURE RISE, WITH TIME, IN A 75-KV-A. TRANSFORMER. 100 PER CENT LOAD AND EXCITATION APPLIED AT ROOM TEMPERATURE UNTIL RISE IS CONSTANT, FOLLOWED BY 125 PER CENT LOAD UNTIL CONDITIONS ARE AGAIN CONSTANT. (SIMILAR TO (C) IN FIG. 4)

shown in (b) of Fig. 4 for oil temperature rise, a 100-kv-a. transformer was put under excitation until the top oil temperature rise caused by the core loss was constant. Then normal load was applied and the top oil temperature rise observed hourly until practically final

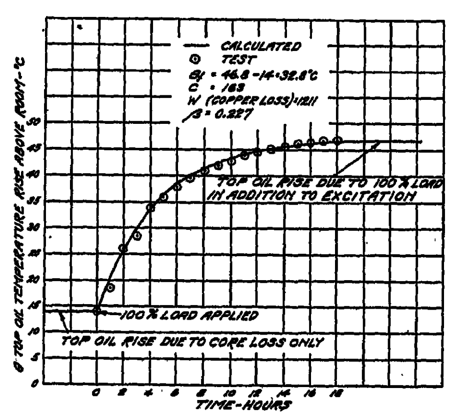

FIG. 7—VARIATION OF TOP OIL TEMPERATURE RISE, WITH TIME, IN A 100 KV-A. TRANSFORMER. NORMAL LOAD APPLIED AFTER CONSTANT TOP OIL TEMPERATURE RISE, DUE TO CORE LOSS ONLY, HAS BEEN ATTAINED. (SIMILAR TO (B) IN FIG. 4)

rise was attained. The observations obtained, and the calculation of β, are presented in Fig. 7. Hourly readings were also taken as the top oil was heating up due to core loss. They check up very closely with the calculated values, but since they are of minor importance they are omitted.

The 100-kv-a. transformer, whose performance is shown in Fig. 7, was also tested starting at room tem-

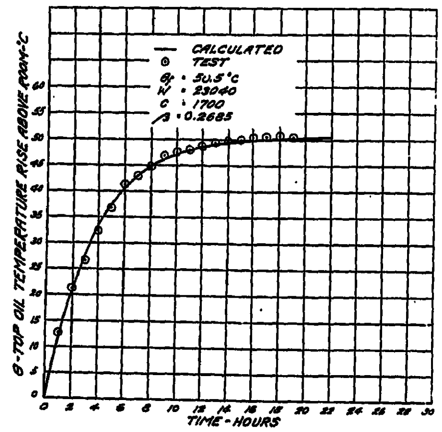

FIG. 8—VARIATION OF TOP OIL TEMPERATURE RISE, WTIH TIME, IN A 2000-KV-A. TRANSFORMER. NORMAL LOAD AND EXCITATION APPLIED STARTING AT ROOM TEMPERATURE. (SIMILAR TO (A) IN FIG. 4)

perature, with a good agreement of tested and calculated values, but due to the similarity of the test to the one starting with core loss temperature, it is not reproduced. Figs. 6, 8, 9, 10 show how well the formula checks tests starting at room temperature, regardless of the size of transformer, for 75, 2000, 5000 and 12,500 kv-a., respectively. These observations of top oil temperature were all taken hourly as the transformers

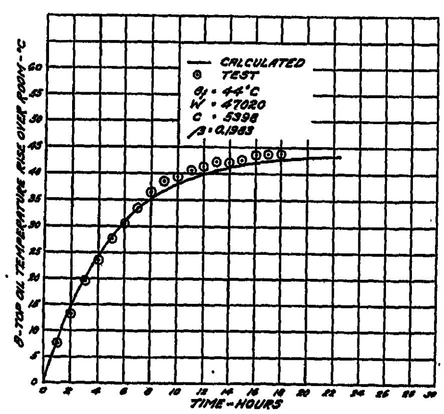

FIG. 9—VARIATION OF TOP OIL TEMPERATURE RISE, WITH TIME, IN A 5000-KV-A. TRANSFORMER. NORMAL LOAD AND EXCITATION APPLIED STARTING AT ROOM TEMPERATURE. (SIMILAR TO (A) IN FIG. 4)

were heating up, starting at room temperature, after 100 per cent load and excitation had been applied, conditions similar to those of (a) in Fig. 4. The calculated values of β are given in the respective figures in addition to the test data.

A general inspection of Figs. 5 to 10 shows that the calculated curve does not reach a maximum and flatten out quite as quickly as the test results. This is due to the fact that theoretically the exponential curve does

not become constant until infinity. After a certain per cent rise has been attained, extending the calculation several hours, in considering oil rise, may result in the addition of only one or two tenths of a degree. This is obviously absurd. Since commercial thermometers and thermocouples are seldom more accurate than ± 0.5 deg. cent., calculations may be shortened by carrying them only up to within 0.5 deg. cent. of final rise.

Restrictions in the Use of the Formula

While the proposed method should apply to the majority of self-cooled transformers, it would be misleading not to indicate some of the factors which may affect its applicability.

Considering first the question of coil rise over oil, it is apparent that the windings should be practically thermally independent of any surrounding medium except the oil, which carries away heat from the windings chiefly by convection. If, for instance, the coils

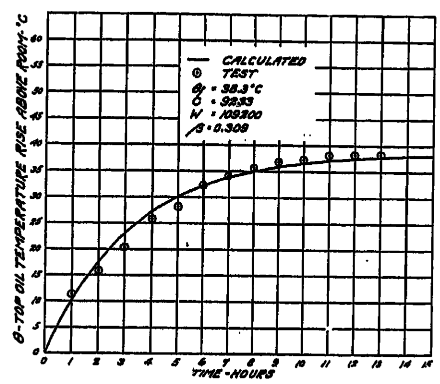

Fig. 10—Variation of Top Oil Temperature Rise, with Time, in a 12,500-Kv-a. Transformer. Normal Load and Excitation Applied Starting at Room Temperature. (Similar to (A) in Fig. 4)

are wound directly on the core, as in the case of many small transformers, there is a flow of heat into the core by conduction in addition to that given to the oil. The result is a very complicated thermal condition as far as calculation is concerned. Naturally, such a winding will not rise in temperature as fast as the proposed formula would indicate for a similar winding separated from the core by an insulating cylinder, for example, since, in such a case, the formula would indicate too high a rise. Tests would be necessary to show the true heating curve in order to get the safe maximum performance for any given design. It might be possible, by adding a portion of the core's thermal capacity to that of the copper, to get a good approximation but the amount added would be empirical, consequently it is impossible to give any general rule that would apply to all cases.

Before applying the formula to top oil temperature rise, it might be well to consider whether the transformer under consideration corresponds to the assumptions on which C in formula (7) is based. For instance, in small transformers of low voltage where it is not necessary to have large voltage striking distances the metal factor (weights of metal multiplied by 3.5) may be as large or larger than the oil factor (90 × gallons of oil). In that case, it may be advisable to give more attention to the former. The copper and iron may be multiplied by their respective factors of 2.96 and 3.6, more than ⅔ of the tank may be used, and the other metal in the transformer such as the core clamps, if appreciable, should be added in. Moreover, a core which runs at high density and consequently high mean temperature rise may have its weight multiplied by the ratio of mean core temperature rise above room to top oil temperature rise. If these factors are not taken into account the test points may be somewhat lower at first than the calculated. These additions should not be made unless it is desired to get maximum performance from the transformer. In any event, they should be made cautiously, for the formula as it stands will be only a little conservative for the cases mentioned above.

About the only factor which could possibly cause the tested top oil temperature rise to exceed the calculated would be a ratio of mean to top oil temperature rise of less than the 86 per cent used for the determination of C in Formula (7). This could conceivably happen through the use of a type of winding cooled in such a way, or so located, that all its heat is delivered to the oil near the top of the tank. Any doubts as to this point can be easily and quickly settled by placing thermometers in a vertical line from top to bottom of the outside of the tank in question and determining the ratio of the mean to the maximum temperatures read when the top oil temperature rise is near its normal value. If this ratio is less than 86 per cent, the factor of 90 should be decreased in like proportion.

Temperature Limits

Although the purpose of this paper is only to indicate a method of calculating temperature rises, it may not be amiss to indicate the temperature limits which should not be exceeded. According to the A. I. E. E. Rules, a maximum rated transformer should not exceed a 55 deg. cent. winding temperature rise by resistance over a 40 deg. cent. room at normal load. For railway transformers, a 2-hour overload resulting in a 60 deg. cent. rise is permitted. Therefore, this method of calculation should be used to show the length of time for which load may be maintained before the calculated winding temperature rise above room plus the room temperature, assumed for the transformer under consideration, exceeds a temperature of 95 deg. cent., by resistance, for maximum rated transformers, or 100 deg. cent. for railway transformers.

Acknowledgment

The author wishes to acknowledge the criticisms and suggestions of Mr. V. M. Montsinger, which have been of great assistance in the preparation of this paper.

In fact, the development of this particular method of applying the exponential law to the heating of transformers during a transient state was made by Mr. Montsinger several years ago, but it was never published. Experience has shown this method to be accurate enough for all practical purposes.

Appendix

Derivation of Formula (4)

$$W(1 + \alpha\, \theta) = C\frac{d\,\theta}{d\,t} + K\,\theta$$

$$C\frac{d\,\theta}{d\,t} + \theta\,(K - \alpha\, W) = W$$

$$C\frac{d\,\theta}{d\,t} + C'\,\theta = W, \text{ where } C' = K - \alpha\, W$$

$$\frac{d\,\theta}{d\,t} = \frac{C'}{C}\left(\frac{W}{C'} - \theta\right)$$

$$\int\frac{d\,\theta}{\theta - \dfrac{W}{C'}} = -\frac{C'}{C}\int d\,t$$

$$\log\left(\theta - \frac{W}{C'}\right) = -\frac{C'}{C}\,t + \log C''$$

$$\theta - \frac{W}{C'} = C''\,\epsilon^{-\frac{C'}{C}t}$$

$$\theta = \frac{W}{C'} + C''\,\epsilon^{-\frac{C'}{C}t}$$

When $t = 0$, $\theta = 0$, and

$$-\frac{W}{C'} = C''$$

Then $\theta = \dfrac{W}{C'}\left(1 - \epsilon^{-\frac{C'}{C}t}\right)$

Substituting for C',

$$\theta = \frac{W}{K - \alpha\, W}\left(1 - \epsilon^{-\frac{K - \alpha W}{C}t}\right)$$

When $t = \infty$, $\theta = \dfrac{W}{K - \alpha\, W} = \theta_f$

Letting $\beta = \dfrac{K - \alpha\, W}{C} = \dfrac{W}{C\,\theta_f}$,

$$\theta = \theta_f\,(1 - \epsilon^{-\beta t})$$

Accuracy of Alternating-Current Test Instruments

BY S. C. HOARE[1]

Associate, A. I. E. E.

Synopsis:—The paper deals mainly with the accuracy of instruments used under maintained conditions of load. Well designed voltmeters, ammeters, and wattmeters show very small errors due to self-heating and changes in ambient temperatures. Iron-vane ammeters are described which have small errors due to direct-current hysteresis and changes in wave-forms.

PRECISION a-c. instruments are today expected to meet demands for accuracy almost unthought of a few years ago. This applies particularly to acceptance and water-rate tests, which require maintained accuracy of instrument indication over long periods of time,—several hours. Compared with the d'Arsonval types, the a-c. instruments have relatively higher internal losses, and are more apt to be susceptible to error due to maintained conditions of load. It is the purpose of this paper to set forth some of the errors to be expected and show how well some modern instruments meet the exacting requirements.

Instruments for field service must incorporate sturdiness of construction with a torque sufficiently high to insure dependable behavior, provided the two features are not incompatible with good sensitivity and low losses. By torque, we refer to the absolute value and not necessarily the torque to weight ratio, though the latter must receive some consideration. It is not so much the torque to weight ratio, but torque itself which determines the life of accuracy in an instrument, other things being equal.

An instrument can be no more constant in indication than are its control springs, and we accordingly begin with springs and treat of the various factors tending to cause errors.

SPRINGS

A great deal of development work has insured that, with careful selection and treatment of the bronze, a control spring which is almost ideal can be made. This is fortunate inasmuch as other instrument refine-

1. Standardizing Laboratory, General Electric Company, West Lynn, Massachusetts.

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ments would be of little value with springs possessing appreciable "set" and "fatigue" characteristics.

Well made and properly mounted springs do not show "set" in instruments having the usual 90-deg. scale angle, and "fatigue" amounts to less than one minute of arc. This applies to the most severe conditions of stress, and means that the pointer should return to zero within one thickness of its thin knife-edge immediately after release from a four-hour 90-deg. deflection. With the ordinary four-inch pointer, the deviation from zero would not be greater than about 0.004 in., barely perceptible.

The temperature coefficients of both elasticity and resistivity, while not small in value, are, however, quite definite and can be compensated for in the layout of the various circuits o the instrument.

Temperature Errors

Present day demands are such that the precaution of removing instruments from the circuit after reading cannot always be observed. Instruments must, therefore, indicate, with good accuracy, continuously, over a period of hours instead of minutes. This requires among other things a consideration of the effects of temperatures due both to self-heating and to ambient changes.

We do not generally expect errors due to changes in mechanical dimensions, but some instruments do have a tendency for stickiness with an increase in temperature. This is due to the high expansivity of the shaft which makes it "freeze" in its jewel bearings. This especially applies to shafts of aluminum or its alloys. The defect could, of course, be corrected by setting the shaft loose in its jewels, but with a risk of "wobbly" and uncertain indication under normal temperature conditions. A better way is to use material of low expansivity.

The factors most apt to cause errors during sustained conditions of load has been that of inconstancy of the control springs. Though not always the case, many instruments today have springs of a quality which precludes further consideration of spring inconstancy in this paper.

Temperature effects, however, are inherent to all but the electrostatic types and offer a more serious problem. The elasticity of the control spring decreases about 0.04 per cent per deg. cent. rise in temperature at ordinary room temperatures, and by this amount, tends to make the instrument indicate high. The potential circuit, consisting of windings of copper or aluminum, in series with a "swamping" resistor, is increased in resistance with a rise in temperature. This makes for a lower instrument indication. The two effects tend to compensate each other to a degree, depending upon the proportion of windings and series resistor in the potential circuit.

At ordinary room temperatures the windings increase in resistance by about 0.4 per cent per deg. cent. rise, and the series resistor remains practically constant in value. If the circuit is adjusted so that the resistance of the windings is one-tenth that of the whole circuit, then its resistance will increase 0.04 per cent per deg. cent. rise. It might seem that this proportion would effect perfect compensation for effects of the control spring. However, due, to different rates of heat dissipation, it is generally necessary to use other circuit proportions than that of 1 to 10, depending upon the make and type of instrument.

In wattmeters, the resistance of the moving coil may be too low to effect any great degree of compensation. Thus, they may indicate high with increase of temperature. The comparatively high proportion of windings

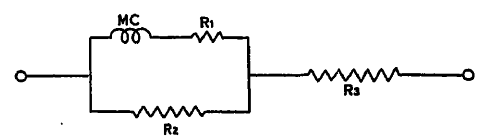

Fig. 1

resistance to total resistance,—unavoidable in voltmeters,—tends for overcompensation, and it is therefore found that voltmeters may indicate low with temperature rise.

Ammeters of the iron-vane type are inherently of a high-torque, which permits us to use windings of few turns and consequential low resistance. The self-heating errors are, therefore, small and we may expect errors due only to changes in ambient temperature.

The sign and magnitude of temperature errors in a-c. instruments may, therefore, vary with different conditions of load, instrument connections and room temperatures. The errors are largely eliminated by careful design and construction. Features receiving

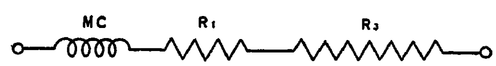

Fig. 2

special consideration are, (a) internal losses and overload capacities, (b) proportion of component parts of the various circuits, and (c) compensating circuits.

Accuracy of indication under sustained loads is not attained at the expense of other desirable features as, for example, high torque, low internal-phase angles, and low frequency and wave-form errors. Such compensating schemes as we find necessary to apply must vitiate none of these important features.

A common form of compensating scheme used for both wattmeters and voltmeters is that given in Fig. 1. Here $M\ C$ denotes the moving coil and R_1, R_2 and R_3 non-inductive, non-capacitive resistors. Effective compensation is secured with the proper values of both resistance and temperature coefficient of resistance of R_1 and R_2. The arrangement is not, however, critical of adjustment. If the initial error in the instrument is

positive, (as is often the case in wattmeters), then we may make R_1 of copper wire, and R_2 of wire possessing a negligible temperature coefficient of resistance. The effect of an increase in temperature then, is to reduce the current in the moving coil by the amount necessary for compensation. In voltmeters the circuit constants would be interchanged.

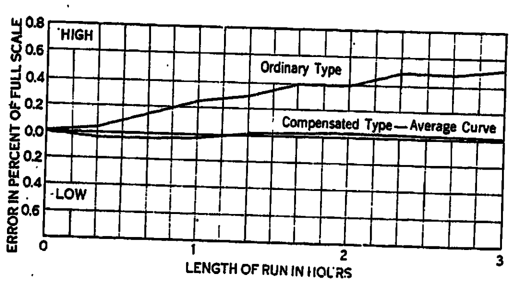

Fig. 3—Electrodynamic Wattmeters

Variation of instrument indication with time under maintained load conditions

Though effective in compensating for temperature, the circuit of Fig. 1 is almost certain to cause in wattmeters large and indefinite inductive errors. The shunting of the moving coil branch with the relatively low resistor R_2 may cause serious errors due to mutual inductance. It is not generally possible to secure good temperature characteristics with R_1 and R_2 of resistance values high enough to avoid complications due to phase angles.

This form of circuit should be reserved for voltmeters. Voltmeters are not sensitive to small differences of phase-angle between moving coil and field coil currents.

For wattmeters, the series method of compensation given in Fig. 2 is preferred. It is equally as effective as the shunted arrangement and does not disturb the internal phase-relations of the instrument. R_1 in this case is a small copper wire resistor placed near the spring.

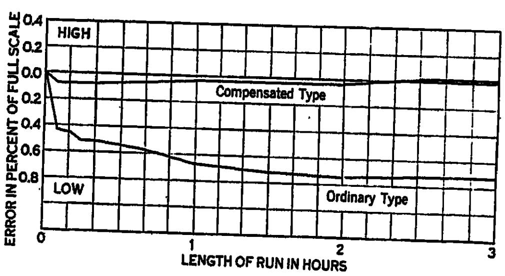

Fig. 4—Electrodynamic Voltmeters

Variation of instrument indication with time under different maintained load conditions

The curves of Figs. 3, 4 and 5 contrast the performance of some compensated instruments with those of the ordinary types. The curves for wattmeters show first, the initial dip due to increase in resistance of the moving coil, and second, as heating continues, the upward trend due to decrease in elasticity of the spring. Finally, as the compensation becomes more and more effective the curves gradually flatten out. The curves for voltmeters show much the same characteristics, though the effect of the spring is inconsiderable.

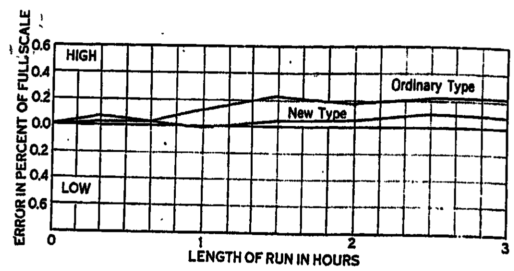

Fig. 5—Iron-Vane Ammeters

Variation of instrument indication with time under maintained load conditions

Figs. 6 and 7 are indicative of what may be expected in good instruments when used under widely

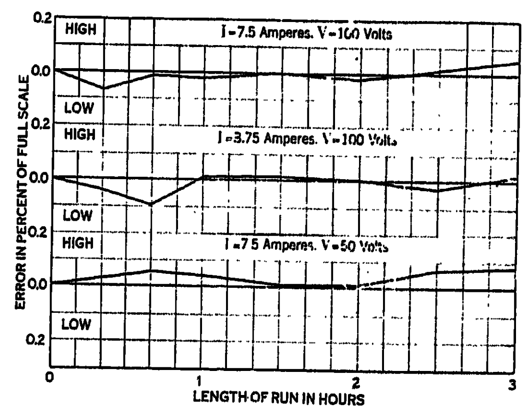

Fig. 6—Electrodynamic Wattmeter

Variation of instrument indication with time under different maintained load conditions

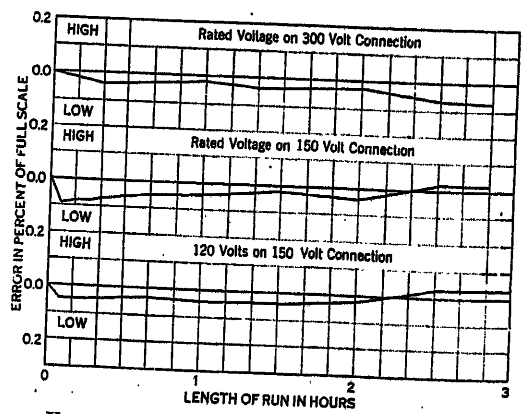

Fig. 7—Electrodynamic Voltmeter

Variation of instrument indication with time under different maintained load conditions

different conditions of loading. Errors observed with one deg. cent. rise in ambient temperature are given in Table I.

TABLE I

	Ordinary Type	Compensated Type
Wattmeters	+ 0.03%	+ 0.006%
Voltmeters	− 0.005%	+ 0.007%
Ammeters	+ 0.02%	+ 0.016%

Internal Phase Angles

These are due both to self inductance in the potential circuit and to mutual inductance in the windings. We are concerned ordinarily with the effects of phase-angles in wattmeters when used on circuits of low power factor, though it is also possible for voltmeters and ammeters to show errors due to phase-angles. Voltmeters and ammeters in which are incorporated certain compensating circuits can be quite sensitive to variations in both frequency and wave form due among other things to incorrect "time constants" of the circuits.

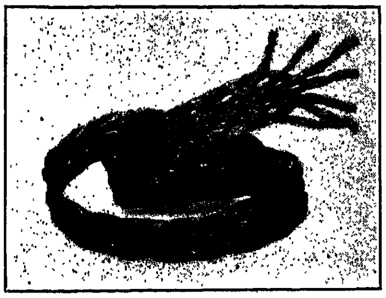

Fig. 8

Mutual inductance effects are negligible in well constructed instruments, and in fact the only possibility of errors from this cause lies either in short-circuit turns in the windings or in the use of low voltage connections in instruments designed for higher voltages, *i. e.*, the results of a 30-volt connection in the ordinary 150-volt instrument should be regarded with suspicion. Wattmeters compensated for temperature with the shunted arrangement of Fig. 1 are very apt to show errors due to mutual inductance.

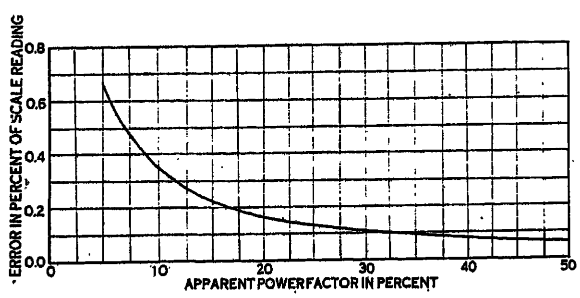

Fig. 9—Electrodynamic Wattmeter
Variation of calculated inductive correction with load power factor
Corrections to be added for leading current; subtracted for lagging current

Low phase-angles are, of course, very desirable but it is equally desirable that such as do occur be definite and amenable to calculation. Eddy currents induced in metal frame-work and certain temperature compensating circuits tend to give uncertainty both to sign and magnitude of the inductive errors. The multiple connection employed in the current winding to obtain higher ratings may add further complications, due to inequalities. Equality, which is necessary to avoid the presence of parasitic currents in the circuit, is more nearly approached with the multiple connection of two distinct conductors, than with two separate coils. Fig. 8 illustrates how equality is obtained in a high-capacity wattmeter current winding. Well designed series resistors are, for all practical purposes, both non-inductive and non-capacitive, though in some of the older forms capacitance predominated with the resulting complications.

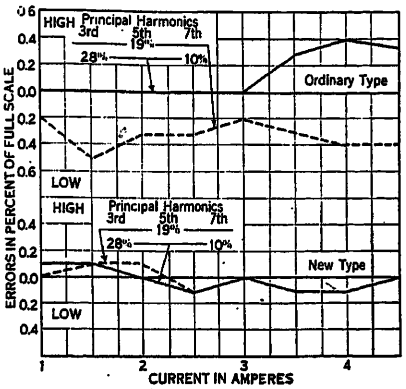

Fig. 10—Iron-Vane Ammeters
Variation in indication due to irregular wave form

A representative curve of inductive corrections for a wattmeter is given in Fig. 9. It is computed for 60 cycles.

Frequency and Wave Form

Except for induction errors, wattmeters are insensitive to variations in commercial frequency and wave form.

The relatively high self-inductance of voltmeters involves a careful consideration of their "time constants" if they are to be used interchangeably on direct- and alternating-current circuits. There should be no detectable variation in indication between mean reversed direct-current and an alternating-current of 60 cycles. In terms of 125 cycles, the variation ought to be no more than 0.05 per cent,—barely perceptible. Voltmeters of the electrodynamic type are unaffected by ordinary wave distortion, provided they are free of frequency errors.

The iron vane ammeter, though practically insensitive to variation in commercial frequency was, until recently, quite susceptible to changes in wave form. The use of new magnetic materials enables us to reduce errors from this cause from about 0.5 to 0.1 per cent. (Fig. 10.) These errors refer to wave forms having prominent third and fifth harmonics.

Hysteresis on Direct Current

Shielded wattmeters and voltmeters of the electrodynamic type show no differences in indication with reversal of direct-current if the shields are free of permanent magnetism. Extreme overloads may, however, give to the shields a definite magnetic polarity, and it is well to check the instruments occasionally for this condition.

Previously, the iron-vane ammeters were very susceptible to magnetic hysteresis, and the variation in indication with reversed direct-currents was great enough to prevent even a fair degree of accuracy. The magnitude of these errors depends upon the previous history of magnetization of the iron. Aside from the ordinary form of hysteresis, due to changing polarity and magnitude of the magnetic flux, the iron is also susceptible to changes in direction of the flux through it. We may call the latter "rotational hysteresis." Instruments having the irons mounted eccentric to the shaft exhibit effects of rotational hysteresis to a marked degree.

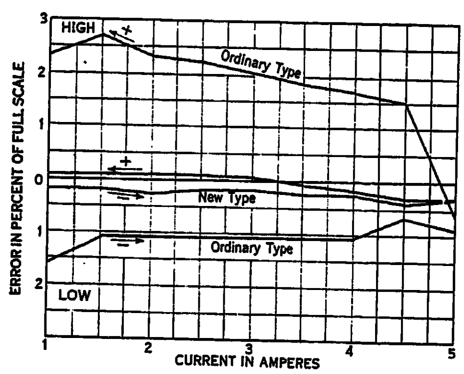

Fig. 11—Iron-Vane Ammeters

Variation in instrument indication with direct-current showing effect of magnetic hysteresis

New and specially prepared magnetic materials now enable us to construct iron-vane ammeters having little hysteresis error. Errors of two per cent so frequently observed in this type have been reduced to about 0.3 per cent (Fig. 11). For alternating-current work it would appear that the present-day iron-vane ammeter is a very dependable instrument. In view of the improved wave-form and hysteresis characteristics, this type is to be preferred over the low-torque and high-impedance type employing the shunted, moving-coil principle. The latter type is of course useful in the laboratory in making a-c. = d-c. comparisons. Laboratory standard ammeters are, therefore, of the shunted electrodynamic type.

Stray-Field Errors

High-grade a-c. instruments are sufficiently well shielded to prevent errors from stray fields under ordinary conditions. Possibility of error in instrument of high-current rating is remote if we use twisted conductors.

A field of 20 gauses "in-phase" is equivalent to about 100 times the horizontal component of the earth's magnetism, or the field 12 inches distant from a long straight conductor carrying 3000 amperes. In shielded instruments, this field should cause no error greater than about 0.4 per cent of full scale. With ordinary conditions of use, stray-field errors would than be undetectable. In general, we find that the higher-torque instruments are the least affected by stray fields, which is an argument for high torque.

To eliminate eddy currents and hysteresis, the shields are built of well-insulated laminations. Low power-factor errors in wattmeters have sometimes been traced to eddy currents in the shield. Improper methods of bolting the laminations together may be responsible for these errors.

The shunt compensating scheme of Fig. 1 in the paper is due to Swinburne (1887) and the series scheme of Fig. 2 was suggested by Dr. H. B. Brooks in his paper "The Accuracy of Commercial Electrical Measurements," (A. I. E. E. Transactions, Vol. XXXIX, part 1, p. 517). A complete and comprehensive treatment of the effects of self- and mutual-inductance is given in this paper of Dr. Brooks. Reference is also suggested to C. V. Drysdale's paper upon the subject of inductive errors and given in the *Electrician* (London), Vol. 46, pp. 774-8, 1901 and Vol. 76, pp. 523-5, 1916.

Discussion

C. G. Brown: As a-c. ammeters were previously made, the d-c. readings were so different according to the direction of flow of current that it was not feasible to calibrate on d-c. The metal in the moving system of the a-c. ammeter which Mr. Hoare has described has its direct and reversed d-c. readings so nearly alike that when necessary it can be checked on d-c.

B. W. St. Clair: One of the very important points of instrument construction mentioned in Mr. Hoare's paper that has a great deal to do with the dependability of instruments in service is that of instrument springs. The forces involved in indicating instruments are surprisingly small. The usual full-scale torque range is from 0.1 millimeter-gram to 10 millimeter-grams, which translated into more usual terms corresponds to 6×10^{-5} to 6×10^{-7} ft-lb. Despite the smallness of the forces involved, the spring does a remarkable piece of work, as in the better grade of test instruments the error of its indication is generally less than 0.1 of one small division, which is approximately one part in a thousand. Translated into other terms, the ordinary instrument spring on the better-grade instruments is dependable to about one minute of arc. Dependability of this high order is secured only by very careful choice of materials, unusual manufacturing facilities and by great care in the mounting of the spring in its final supports in the instrument.

I mention the springs as one of a number of very highly important members that contribute to the final dependability of the instrument. The magnets, the bearings, the constancy of shape of the windings, and many other factors are of equal importance.

The Measurement of Electrical Output of Large A-C. Turbo Generators During Water-Rate Tests

BY EVERETT S. LEE[1]

Associate, A. I. E. E.

Synopsis:—The water-rate of large a-c. turbo-generators is determined in place with the generator supplying power to the existing commercial load. Such procedure requires that the electrical output be accurately measured under conditions where the load is practically always varying slightly, and where it may be varying considerably. Portable test meters are frequently used for such measurement. The use of portable indicating wattmeters is absolutely feasible for such measurement even where the load variations are extreme, with the resultant accruing advantage that the superior operating characteristics of the portable indicating wattmeters, particularly as regards their permanency, are utilized.

Either the two-wattmeter or three-wattmeter method for measuring the power of a three-phase circuit may be employed.

Great care should be used in selecting instruments, particular reference being made to their past history. Comparisons against secondary standards should be made under conditions simulating those of the test.

Observations are made at frequent intervals depending upon the accuracy desired in the final result.

The accuracy of the final values of water-rate obtainable in practise is such that the per cent average deviation from the mean will be within ± 0.25 per cent. This means that practically all individual test results will fall in a belt 1 per cent wide, while the probably true value of water-rate will be located within a belt which will vary in width from 0.5 per cent to 0.25 per cent depending upon the conditions.

The results of a water-rate test on a highly variable load wherein the electrical output was measured with portable indicating wattmeters and the observations were obtained both with moving-picture cameras and by observers show the equality of the performance of the observers and the cameras under the existing conditions.

Introduction

THIS paper describes the methods used in measuring the electrical output in forty series of water-rate tests during the past five years on thirty-one machines located in twenty different power plants in this country. The machines tested ranged in rating from 10,000 kw. to 45,000 kw., and the test loads were the usual commercial loads supplied by the respective machines.

These tests were either guarantee acceptance tests or were for purposes of determining the improvement in water-rate produced by advance in machine design. Measurements of maximum possible accuracy were thus required and all means known to the art for attaining such results were available for use as far as they could be applied under the conditions imposed when testing on commercial loads.

Instrument Connections

The usual load being connected three-phase, the three-wattmeter method was used in practically all of the series of tests because of its symmetry. The connections are shown in Fig. 1. The generator neutral being available in practically all large turbo-generators, the connections for the three-wattmeter method are easily made, a set of instruments being connected into each phase. Values of phase voltage, current, and power are thus measured directly, from which measurements the phase power factor can be readily calculated. These values are required for making the necessary corrections to the readings of the wattmeters. Since the phase power unbalance is usually not more than two or three per cent, the three sets of instruments are operating under practically identical conditions which is an advantage both when comparing the instruments and applying corrections, and in operating with them. There is no disadvantage as regards measurement, however, if the phase unbalance is large, as the phase values are directly measured in each case. Since the phase power factor is the same as that at which the wattmeters are operating, and since the former is rarely

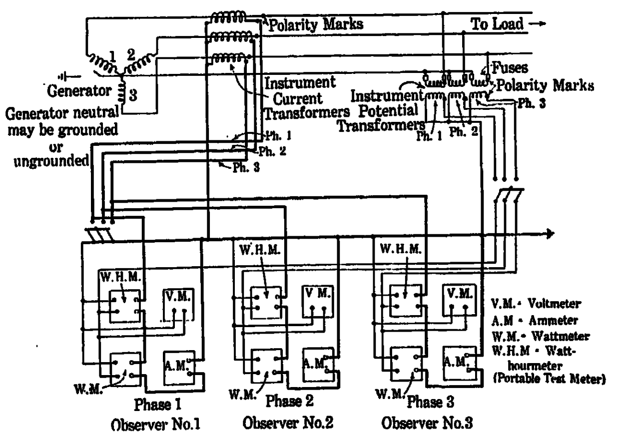

FIG. 1—DIAGRAM OF CONNECTIONS AND TABLE LAYOUT FOR MEASURING ELECTRICAL POWER IN A THREE-PHASE CIRCUIT USING THE THREE-WATTMETER METHOD

less than 0.8, the wattmeters are operating under their best condition which is near to unity power factor.

The two-wattmeter method was used in some series of tests. The connections are shown in Fig. 2. The two-wattmeter method requires less equipment and one less observer than the three-wattmeter method, but it is not symmetrical because the phase relation between the current and voltage supplied to the respective wattmeters differs from that of the load, and is different in

1. General Engineering Laboratory, General Electric Co., Schenectady, New York.

Presented at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925.

the respective wattmeters. The actual conditions as regards the current, voltage, and phase relation between them for the individual wattmeters can be determined from the well-known relations existing when measuring three-phase power by the two-wattmeter method. These values must be known when determining the conditions under which the wattmeters are to be compared, and when correcting the wattmeter readings later during test. Except at unity power-factor load the indications of the two wattmeters will be at different portions of the scales; this requires that consideration be given the wattmeters individually as to the suitability of scale range. When the load power factor is 0.8, current lagging, and load balanced, the low-

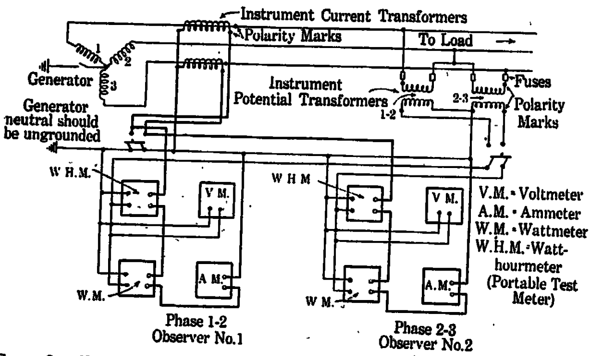

Fig. 2—Diagram of Connections and Table Layout for Measuring Electrical Power in a Three-Phase Circuit Using the Two-Wattmeter Method

reading wattmeter is operating at a power factor of 0.39. This is not at the best operating point of a wattmeter as regards power factor. If the generator neutral is grounded and a ground occurs at any other point on the system, the power may not be correctly measured. For this reason the generator neutral should be ungrounded during test.

Experience does not indicate that there is any real difference in the over-all accuracy of a water-rate determination whether the three-wattmeter method or the two-wattmeter method is used, provided that the existing conditions are determined and the proper corrections made for same in each method. Where a large number of series of tests are to be made under widely varying conditions, it is felt that these conditions can be determined and the necessary corrections applied more easily when using the three-wattmeter method. For this reason the latter method has been generally used, in spite of the increased equipment and additional observer required, and the uniformly satisfactory results obtained thereby have justified this selection.

Instruments

Portable indicating wattmeters have been chosen for measuring the electrical output because of their superior characteristics, particularly as regards their permanency. Portable test meters have usually been used in parallel with the indicating wattmeters as shown by the connections Figs. 1 and 2. This was done to study the operating characteristics of both indicating instruments and portable test meters.

For water-rate tests, where sustained accuracy is required, instruments exhibiting the most permanent calibration characteristics should be used. In order to select such instruments from any group available, they should be compared regularly with secondary standards and a careful record should be maintained of all such comparisons made. By means of such a record over a number of years the instruments of greatest permanency will be disclosed. Such a procedure is of greatest importance and should be regularly followed, especially with the wattmeters. Table I shows the results of successive comparisons made upon a portable indicating wattmeter thus selected from a group of similarly made instruments. The wattmeter was carried from laboratory to laboratory by messenger. The results show no difference practically greater than 0.1 of 1 small scale division. This is typical of the performance possible from selected instruments. From such a proof of permanency of calibration possible in high-grade portable indicating wattmeters, the surety for sustained high accuracy throughout a series of tests is evident.

Having selected the instruments, it is necessary that they be compared against secondary standards, under

TABLE I

SUCCESSIVE COMPARISONS OF PORTABLE INDICATING WATTMETER, RATED 5/10 AMPERES, 150/300 VOLTS, 500/1000/1000/2000 WATTS. COMPARISONS MADE ON 5-AMPERE 150-VOLT CIRCUIT

Standardizing Laboratory. Location. Date. Watts	G. E. Co. Schenectady N. Y. 9-13-20	G. E. Co. Pittsfield Mass. 9-16-20	G. E. Co. Schenectady N. Y. 9-18-20	G. E. Co. Erie Pa. 9-20-20	G. E. Co. Cleveland Ohio 9-22-20	G. E. Co. Ft. Wayne Ind. 9-25-20	G. E. Co. Schenectady N. Y. 10-8-20	G. E. Co West Lynn. Mass. 10-11-20	Bureau of Standards Washington D. C. 10-22-20	G. E. Co. Schenectady N. Y. 10-29-20
	Instrument Reads									
0	0	0	0	0	0	0	0	0	0	0
50	50.0	50.0+	50.0	no observation	50.5−	51.0	50.0	50.25	50.7	50.25
100	100.5	100.5	100.0+	100.0+	100.5−	100.5	100.0+	100.5−	100.8	100.5
200	200.0	200.5	200.0	200.0+	200.0−	199.5+	200.0	200.0	200.0	200.0−
300	300.0	300.0	300.0	300.0	300.0	300.5−	300.0	300.0	300.2	300.0
400	400.0	400.0	400.0	401.0	399.5	400.0	400.0−	399.5	399.9	400.0−
500	500.5	500.5	500.5+	500.0	500.5+	501.5+	500.5	501.0	501.2	500.5+

0.1 of 1 small scale division is 0.5 watts

conditions simulating test conditions. Ammeters and voltmeters should be compared at major scale points, with preferably the same kind of current and voltage to be measured during test. Wattmeters may be compared using direct voltage and current by using the average of direct and reversed readings. This assumes that previous measurements have been made on the instruments and standards employed to assure the correctness of such procedure. The value of the voltage maintained on the potential circuit of the wattmeter during comparison should be the test value. The interconnections of the potential and current coil maintained during the comparison should be likewise maintained during test. The value of the phase angle of the potential circuit of the wattmeter requires consideration, though in all high-grade wattmeters this is quite small, such as three to five minutes, and may usually be neglected without error in final result.

Attention should be directed to the small scale divisions of the wattmeters to see that these are properly located. A scale wherein the small scale divisions are improperly located should be replaced with a scale properly made. As mentioned above, when using the two-wattmeter method, one wattmeter may be indicating at the lower end of the scale; it is necessary, under these conditions, that the scale be open at the lower end.

In order to be assured that there is no heating error in the wattmeters due to continuous operation during the water-rate test, comparisons should be made at the beginning and end of a heat run on the wattmeters at normal voltage and maximum current to be measured. If a heating error is found, correction should be made for same by using the comparison obtained with the instrument hot and then allowing the instrument to become hot before use in test. However, the best high-grade portable wattmeters have practically a zero heating error, and such instruments should be used during all tests where maximum accuracy is required.

If watthour meters are to be used, they should be calibrated and adjusted in the laboratory so as to have their best operating characteristics in the range of test values. However, the accuracy curve employed for correction during test, should be obtained "in-place" using the test current and voltage. This should be done with portable indicating wattmeters during the water-rate test. The connections shown in Figs. 1 and 2 allow for such "in-place" checks to be made. Two-minute runs, with the observer reading the indicating wattmeter as rapidly as possible, which will give 40 to 60 observations per minute, are suggested as necessary and sufficient.

A frequency indicator, or other instruments as desired, may be included in the instrument test circuit.

Instrument current and potential transformers of proper rating and accuracy characteristics should be chosen. Values of ratio and phase angle for the condition of loading equivalent to the instruments, meters and leads used during test should be obtained by recognized means to cover the range of test values of current and voltage.

All instruments should be compared immediately before and immediately after the test series. Intermediate tests at two-week intervals may be advisable if the test series is long. Instrument transformers usually need to be calibrated only once during the test series, preferably immediately preceding it. Watthour meters must be calibrated "in-place," preferably immediately before and immediately after each individual test.

Observations

The instruments having been carefully selected and compared for the test conditions, they are connected into the generator line leads as shown either in Figs. 1 or 2. After checking over connections, and comparing the test-instrument readings with the available switchboard-instrument readings to see that there are no major errors, everything is ready to run a test as regards the measurement of electrical output.

Testing on commercial loads requires that means be provided for determining the readings of the indicating wattmeters over a wide range of load conditions. The load may be extremely variable, as in some railway systems, or quite steady as in a large system of combined power and lighting. The means employed has been to take a large number of observations during the test on the theory that if the number of observations is sufficiently large, the average value obtained therefrom will be accurate to within the desired limit, and experience has justified this procedure.

The uniformly satisfactory results of many tests indicate that a test of one-hour duration is ample. Table II shows results to substantiate this conclusion. If it is felt that the conditions at hand warrant tests of longer duration, several tests should be run at the longer duration and the results for each hour of the run be calculated. Such results will indicate whether the necessity for tests longer than one hour is justified. If such is the case, it is quite possible that some of the test conditions are not right and should be altered. In this connection it should be remembered that three one-hour tests at any load point run non-consecutively, are of greater value than three one-hour tests run consecutively.

The observers for the indicating instruments should be men who are capable of comprehending the nature of the work they are doing and who recognize that their efforts should be directed towards obtaining an honest answer as to the value of the water-rate of the machine under test. Their contribution to this effort is recording the readings of the indicating instruments as they honestly believe them to be. Experience obtained with high school graduates, college undergraduates and graduates, and construction men of mature years, indicates that any of these types of men are equally suitable as observers during a test series including some

dozen or two dozen tests and continuing for one or two weeks. The instructions given to all observers are as follows:

1. Place yourself so that you are comfortable and can see the instrument scale without hindrance.

2. Study the appearance of the instrument scale, pointer, and mirror, and have these fixed so clearly in mind that you can look at them and see them with the same familiarity as you would a book or newspaper.

3. Read the position of the pointer on the scale over its image in the mirror just as you see it when you look at it at the designated signal with both eyes open. Read freely and without constraint. Record the value observed.

4. At all other times rest the eyes by looking around as desired. Do not try to average readings during an interval.

5. Do not allow your readings to be influenced by any preconceived ideas as to what the value is probably going to be. Record the result just as it appears to you at the time.

Each observer is assigned to one group of instruments. Each observer reads the indicating wattmeter at a given signal, such as a bell. Unless the observers are widely separated, no preliminary signal is necessary. Instruments are read in regular order, the wattmeter first, followed by the ammeter and voltmeter. The two latter instruments need be read only at every other signal, or in some cases, every fifth signal. Watthour meters are read at the beginning and at the end of every test.

A relief observer will be necessary. The relief can be utilized also to read the values of field voltage and current, which as a rule need be read only at five-minutes intervals during a one-hour test. A supervisor is necessary, together with an assistant who acts as calculator. There are thus required four observers, one calculator, and a supervisor. The supervisor will have to help the calculator after the completion of a days run when the observers are released.

The supervisor and the observers will have to cooperate in order that the supervisor may help the observers to do their work and to detect undesirable tendencies in making observations. Rotating the observers is good practise. A good way to compare observers is, unknown to them, to connect all of the

TABLE II

WATER-RATE FOR EACH HOUR OF TESTS OF LONGER THAN 1-HOUR DURATION

Water-rate for each hour is expressed in per cent of the water-rate obtained from the test, which value is expressed as 100 per cent

Turbo-Generator Designation Number	Test Designation Number	Duration of Test	Water-Rate (Expressed on basis of 100% for Test Value)				Maximum Difference From Test Value Expressed In Per cent	Type of Load
			For Test	1st Hour	2nd Hour	3rd Hour		
1	1	2 Hrs.	100	100.0	99.8		0.2	Rapidly Fluctuating Railway Load
1	2	2 Hrs.	100	99.8	100.2		0.2	
1	3	2 Hrs.	100	100.9	99.2		0.9	
1	4	2 Hrs.	100	100.1	100.0		0.1	
2	1	3 Hrs.	100	100.3	100.0	99.8	0.3	Steady Lighting and power load
2	2	3 Hrs.	100	100.1	100.0	99.8	0.2	
2	3	2 Hrs.	100	100.0	100.0		0.0	
3	1	3 Hrs.	100	100.2	99.9	99.6	0.4	
3	2	3 Hrs.	100	100.2	99.9	99.9	0.2	Steady Lighting and Power Load
3	3	3 Hrs.	100	99.9	100.2	99.9	0.2	
3	4	3 Hrs.	100	99.9	100.2	99.9	0.2	
3	5	3 Hrs.	100	100.0	100.0		0.0	
3	6	2 Hrs.	100	100.0	100.0		0.0	
3	7	2 Hrs.	100	100.1	100.0		0.1	
3	8	2 Hrs.	100	100.0	99.9		0.1	

TABLE III

RESULTS OBTAINED BY CONNECTING ALL INSTRUMENTS IN SERIES UNKNOWN TO THE OBSERVERS

Turbo Generator Designation Number	Duration of Test in Minutes	Time Interval between Readings	Load Conditions	Observers A, B, C			Max. difference in per cent between results obtained by individual observers		
				Phase 1	Phase 2	Phase 3	Wattmeters	Ammeters	Voltmeters
4	60	1 min.	moderately variable	A	B	C	0.04	1.13*	0.45*
4	60	1 min.		C	A	B	0.25	0.72*	0.27*
4	60	1 min.		B	C	A	0.11	1.78*	0.27*
4	6	15 sec.	moderately variable	A	B	C	0.20	0.52	0.36
4	5	15 sec.		B	C	A	0.28	0.50	0.27
4	5	15 sec.		C	A	B	0.84	0.48	0.27
5	5	15 sec.	steady	A	B	C	0.33	0.41	0.31
6	2	2 sec.	steady lighting and power load	A	B	C	0.30		
6	2	2 sec.		A	B	C	0.06		
6	4	4 sec.		A	B	C	0.46		
6	10	20 sec.		A	B	C	0.40	1.00	0.30
6	10	20 sec.		A	B	C	0.40	0.85	0.25

*Ammeter and voltmeter read every two minutes

instruments in series on one set of current and potential transformers. Then run for a convenient time interval, preferably an hour, as during a test, and compare the results obtained by the individual observers. Results from such a procedure are shown in Table III.

The frequency of observation necessary has been determined from the following rule: Readings shall be taken at such intervals during a test that the average of all the observations does not differ from the average of all the alternate observations by more than an assigned value depending upon the accuracy desired. For most accurate results under practical conditions a value of 0. 1 per cent has been adopted. For most commercial loads this value has been attained on one-hour tests by taking readings every minute. Table IV shows some results in this connection.

TABLE IV

DIFFERENCE BETWEEN AVERAGE OF ALL THE OBSERVATIONS AND AVERAGE OF ALL THE ALTERNATE OBSERVATIONS

Turbo-Generator Designation Number	No. of tests	Duration Hours	Time Interval between Readings Seconds	Load Conditions	Percent Difference between all Observations and all the alternate Observations, for the number of tests indicated			Method of Measuring
					Average	Maximum	Minimum	
1	27	4 tests, 2 hrs. 23 tests, 1 hr.	10	Rapidly fluctuating Railway load	0.20	0.72	0.01	3-wattmeter
7	5	1	30	Very Steady	0.06	0.10	0.00	2-wattmeter
8	6	1	30	Very Steady	0.14	0.20	0.04	2-wattmeter
5	4	1	60	Steady Lighting	0.11	0.15	0.05	3-wattmeter
9	7	1	60	Steady Lighting	0.12	0.16	0.08	3-wattmeter
4	11	1	60	Variable unbalanced	0.44	1.10	0.01	3-wattmeter
6	11	1	60	Steady Lighting	0.20	0.62	0.02	3-wattmeter
3	3	1	60	Steady Lighting	0.18	0.39	0.07	3-wattmeter

CALCULATIONS

The average kilowatt output for any test is the sum of the corrected readings of the wattmeters. The corrected reading for each wattmeter for any test is equal to

$$W \times C.T. \times P.T. \times P_h$$

where: W is the average of all the corrected indications of the wattmeters, expressed in kilowatts. The correction used for each wattmeter is the average of all the comparisons which are made between it and the secondary standards.

$C.T.$ is the true ratio under test conditions of the current transformer to which the wattmeter is connected.

$P.T.$ is the true ratio under test conditions of the potential transformer to which the wattmeter is connected.

P_h is the combined correction factor for phase angle of the current and potential transformers and wattmeter.[2]

The kilowatt-hour output for any test is equal to the product of the average kilowatt output and the duration of the test in hours.

If, when comparing any wattmeter, the comparison at any point differs from the correction which has been used for previous corrections at this point by more than an assigned value (for most accurate work, taken to be 0.3 per cent of full scale value), an additional comparison or comparisons should be made immediately at this point until their results do not differ among themselves by more than 0.1 per cent of full scale value, and the average of these results will then be taken as the value of the point in question at the time of this comparison. In the event that the excess variation (0.3 per cent of full scale value) still remains, the new value found shall be taken as correct, and all tests in which this scale point may cause error should be investigated to determine the amount to which they have been affected. Such tests may have to be rejected. With the best, high-grade, portable indicating instruments now available, it has been found that the need for this procedure will be rare. However, a statement of such procedure is advisable for guidance if needed. Table V shows the results of successive comparisons of a wattmeter before, during, and after a test series including the suggested heat run. The test series was that described later in this paper (See Section entitled "Photographic Observation,") dated from Jan. 3, to Jan. 31, 1924, and the wattmeter was one of those read by observers. The performance of this wattmeter shows what has been found to be typical of high-grade portable indicating wattmeter performance during many test series.

ACCURACY OF FINAL RESULT

A value of accuracy of the final result of the electrical output alone may be arrived at from a knowledge of the precision of the wattmeters and instrument transformers used. However, a more satisfactory accuracy figure, is one which can be applied to the over-all measurement of the water-rate of the turbo-generator. This figure is the per cent averaged eviation from the mean, (defined below), and is applied to the results obtained from three or more tests at any given load point.[3] Enough tests can be run at any load point to

2. See "Revised Tables of Correction Factors for Phase Angle," C. T. Weller, *General Electric Review*, March 1925, and "Handbook for Electrical Engineers," *2nd Edition*, 1922, by H. Pender, Page 1929.

3. See "Discussion of the Precision of Measurements" by Silas W. Holman. Second Edition, 1904, chapter on "Direct Measurements."

TABLE V

SUCCESSIVE COMPARISONS OF PORTABLE INDICATING WATTMETER, RATED 5/10 AMPERES, 150 VOLTS, 500/1000 WATTS

Place	Schenectady	Interborough Rapid Transit Company				Schenectady	Maximum Change Small Scale Divisions
Compared by	Gen. Eng. Lab.	Electrical Testing Laboratories				Gen. Eng. Lab.	
Date	12-13-23	1-3-24	1-3*-24	1-17-24	1-31-24	2-11-24	
Scale		Instrument Readings					
0	0	0	0	0	0	0	0
10	9.96		10.0	9.9	10.0	9.92	0.10
15	14.96		15.0	15.0	15.0	14.88	0.12
20	19.94	19.9	20.0	19.9	19.9	19.92	0.10
25	24.94		25.0	24.9	24.9	24.98	0.10
30	29.90		29.9	29.9	29.9	29.92	0.02
35	34.96		35.0	35.0	35.0	35.00	0.04
40	39.94	40.0	40.0	40.0	40.0	39.98	0.06
45	44.96		45.0	45.0	45.0	44.98	0.04
50	49.94		50.0	50.0	50.0	49.98	0.06
55	54.95		55.0	55.0	55.0	55.00	0.05
60	59.94	60.0	60.0	59.9	60.0	60.00	0.10
65	64.94		65.0	65.0	65.0	64.94	0.06
70	70.00		70.0	70.0	70.0	70.00	0.00
75	74.98		75.0	75.0	75.0	75.00	0.02
80	80.00	80.0	80.1	80.1	80.1	79.98	0.12
100	99.90	100.0	100.1	100.0	100.0	99.96	0.20

*Comparison made after a four-hour heat run at 400 watts, 100 volts

bring the value of the per cent average deviation from the mean within any desired value. It has been found that a value within ± 0.25 per cent can be attained in water-rate tests wherein the measurements of electrical output are taken as are described herein and where equal care is given in making the steam-input measurements and corrections. Any individual test at a given load whose value of water rate differs from the mean value of all tests at that load by more than 0.75 per cent may be rejected. It is assumed in the above discussion that all determinate errors have been eliminated and that only indeterminate errors remain.

The per cent average deviation from the mean is given by the formula:

$$\text{Per cent } A.\,D. = \frac{(d_1 + d_2 + d_3 + d_n)\,100}{A\,N\sqrt{N}}$$

where A is the average of the values of water-rate obtained from

N tests

N is the number of tests

and $d_1 = W R_1 - A$

$d_2 = W R_2 - A$

$d_n = W R_n - A$, etc.

where $W R_1$, $W R_2$, $W R_n$ are the values of water-rate obtained on the 1st, 2nd and succeeding tests. The absolute values of d_1, d_2, and d_n, are used regardless of sign.

Table VI shows accuracy data from several water-rate tests on large a-c. turbo-generators. The values in the column headed "Per cent maximum difference between values of water-rate obtained from tests at each load," show that practically all individual test results fall within a belt one per cent wide. The values in the column headed "Per cent average deviation from the mean," show that the probably true value of water-

TABLE VI

PER CENT MAXIMUM DIFFERENCE BETWEEN VALUES OF WATER-RATE FOR SEVERAL TESTS AT A GIVEN LOAD, AND PER CENT AVERAGE DEVIATION FROM THE MEAN

Turbo-Generator Designation Number	Load Kilowatts	Number of tests at each load	Per cent Maximum difference between values of water rate obtained from tests at each load	Per cent average deviation from the mean	Type of Load
10	15,000	8	0.37	±0.09	Steady, lighting and power
	20,000	4	0.29	±0.09	
	25,000	8	0.77	±0.12	
	30,000	9	0.87	±0.08	
11	15,000	2	0.65	±0.32	Slightly variable
	20,000	2	0.19	±0.07	
	23,000	2	0.10	±0.04	
	28,000	5	0.90	±0.11	
12	23,000	2	0.49	±0.18	Slightly variable
	28,000	2	0.61	±0.20	
13	15,000	2	1.13	±0.40	Slightly variable
	20,000	2	0.00	±0.00	
	23,000	2	0.70	±0.25	
	28,000	2	0.71	±0.22	
14	10,000	3	0.66	±0.18	Steady, lighting and power
	15,000	2	0.90	±0.32	
	20,000	2	0.38	±0.13	
	25,000	4	0.00	±0.00	
	30,000	3	0.00	±0.00	
	35,000	2	0.00	±0.00	
15	10,000	3	0.00	±0.00	More variable
	15,000	4	0.18	±0.04	
	20,000	3	0.00	±0.00	
4	15,000	4	0.53	±0.11	More variable
	20,000	8	1.91	±0.18	
	25,000	4	0.09	±0.02	
	30,000	3	0.00	±0.00	
16	23,000	4	0.21	±0.05	Steady, lighting and power
	30,000	4	0.42	±0.15	

rate is located within a belt varying in width from 0.5 per cent to 0.25 per cent depending upon the conditions.

Photographic Observation

This paper would not be complete without a description of the method whereby Messrs. Kidder and Hall of the Interborough Rapid Transit Co. of New York measured the electrical output of a turbo-generator by means of portable indicating wattmeters, the observations of which were made with moving picture cameras. The use of a second set of portable indicating wattmeters in parallel with the test wattmeters, and read by observers, gave opportunity to determine the effectiveness of the observers.

The photographic measurement was made by mounting an indicating wattmeter in a vertical position together with a moving picture camera so focused that the scale of the instrument covered the entire width of the film. The wattmeter was provided with a special pear-shaped target pointer, and was properly balanced for operation with the shaft horizontal. Three such

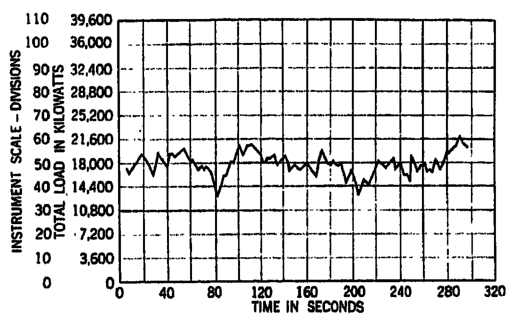

Fig. 3—Load Curve on One-Phase of Turbo-Generator Showing Fluctuations over Instrument Scale

Observations obtained photographically for five minutes of one-second intervals.

instruments and cameras were employed, one set for each phase. The shutter mechanisms of the cameras were operated in synchronism from a common motor-driven drive shaft. Number counters were so mounted on each instrument that the number appeared on the film, thus providing means for identifying all pictures. A watch was photographed on the film at the beginning and at the end of each test to obtain the rate of observation. Suitable illumination was produced from mercury vapor tubes. The entire installation was mounted in the generator room near the machine under test.

The shutter operating mechanism operated the shutters at one-second intervals for short-time duration. For long-time duration such as one-hour test periods, the fastest practicable rate was about one operation per seven seconds, which was the rate maintained during the water-rate tests.

Readings were obtained from the film after development by projecting the image on a screen through a projector and reading the value shown by the pointer. Readings were easily made to within 0.1 small scale division of the wattmeter scale, which is 0.5 watts.

The instruments were compared by setting up secondary standards at the instrument installation in the generator room. Holding scale values on the secondary standard, photographs were taken of the wattmeter indications. After developing the films, the readings were made from the projected image on the screen, and corrections thus obtained. These corrections were later used to correct the wattmeter readings during test. Comparisons were made of the instruments before and

TABLE VII

DIFFERENCE BETWEEN AVERAGE OF ALL OBSERVATIONS AND AVERAGE OF ALL ALTERNATE OBSERVATIONS

G. E. 30,000 Kw. Turbo-Generator, Interborough Rapid Transit Co., New York

Load Kilowatts	Difference between average of all observations and average of all alternate observations, expressed in per cent	
	Photographic Method	Visual Method
13,574	0.47	0.35
14,812	0.29	0.30
15,535	0.66	0.18
16,439	0.06	0.05
18,316	0.17	0.07
18,333	0.16	0.48
19,300	0.14	0.72
19,275	0.19	0.33
21,441	0.09	0.20
21,468	0.23	0.45
23,802	0.14	0.05
23,881	0.17	0.04
24,700	0.03	0.01
25,524	0.06	0.01
25,674	0.06	0.01
25,412	0.18	0.14
26,135	0.01	0.01
27,510	0.07	0.06
27,131	0.11	0.21
28,198	0.18	0.15
26,935	0.07	0.25
27,998	0.12	0.15
27,891	0.11	0.16
29,618	0.09	0.09
29,527	0.05	0.42
29,989	0.29	0.31
29,598	0.18	0.20
Maximum..........	0.66	0.72
Minimum...........	0.01	0.01
Average............	0.16	0.20

after the test series, and intermediate to these at two-week intervals. A preliminary heat run was also made, as previously indicated. The comparisons were all made by the Electrical Testing Laboratories, New York.

The wattmeters used in parallel with the test instruments were compared at the same time as the test instruments except that they were read by observers. Observations were made during test following the principles as described herein. Readings were made at 10-second intervals. Tests were of one-hour duration,

there being three tests each day for nine days during a period of four weeks. At no time did the observers complain of fatigue.

TABLE VIII
PER CENT AVERAGE DEVIATION FROM THE MEAN OF INDIVIDUAL TESTS FOR EACH LOAD POINT
G. E. 30,000 Kw. Turbo-Generator, Interborough Rapid Transit Co. New York

Load Kilowatts	Number of Tests	Per cent Average Deviation from Mean of Individual Tests: Photographic Method	Visual Method
14,000	2	±0.79	±0.78
16,000	2	±0.25	±0.21
18,000	2	±0.19	±0.02
20,000	2	±0.34	±0.22
22,000	2	±0.13	±0.09
24,000	3	±0.21	±0.24
26,000	4	±0.22	±0.27
28,000	6	±0.27	±0.28
30,000	4	±0.17	±0.21
Average (omitting 14,000 kw.).....		±0.22	±0.20
Average (including all loads).......		±0.28	±0.26

Fig. 3 shows a typical curve of the Interborough Rapid Transit Company's load. This is a railway load, and the continual variation was the reason for employing photographic means for obtaining accurate observations. This load is undoubtedly the most variable of the large commercial loads in this country, and results obtained thereon may be taken as representative of the attainment possible under the worst of conditions as regards variable load.

TABLE IX
DIFFERENCE BETWEEN WATER-RATE AS FINALLY ACCEPTED, AND WATER-RATE AS DETERMINED PHOTOGRAPHICALLY AND VISUALLY
G. E. 30,000 Kw. Turbo-Generator, Interborough Rapid Transit Co. New York

Load Kilowatts	Final Accepted Water Rate Expressed as 100 per cent	Water rate by photographic method, expressed as a percentage of final accepted water rate	Water rate by visual method expressed as a percentage of final accepted water rate
14,000	100	100	99.8
16,000	100	99.9	99.7
18,000	100	100.05	99.9
20,000	100	100.1	100.05
22,000	100	100.1	100.1
24,000	100	100.05	100.05
26,000	100	100.	99.95
28,000	100	100.15	99.95
30,000	100	100.1	100.

Differences in Per cent

	Between photographic and visual	Between photographic and finally accepted	Between visual and and finally accepted
Maximum.......	0.2	0.15	0.3
Minimum.......	0.0	0.0	0.0
Average..	0.1	0.07	0.1

The performances of the photographic method and the visual method are best shown by Tables VII, VIII and IX. These speak for themselves and show the truly remarkable results that may be obtained on a highly variable load with portable indicating wattmeters both as read with cameras and with observers. The equality of the results obtained by the two methods is evident. The art of electrical measurements is indebted to Messrs. Kidder and Hall for their persistent efforts in overcoming the difficulties in the photographic method, and in carrying it through to a successful conclusion.

Conclusion

The water-rate of large a-c. turbo-generators can be determined with such accuracy that the per cent average deviation from the mean, as defined, will be within ± 0.25 per cent. This means that practically all individual test results will fall in a belt one per cent wide, while the probably true value of water rate will be located within a belt varying in width from 0.5 per cent to 0.25 per cent, depending upon the conditions. Such results as these can be obtained on commercial loads, even when the load fluctuations are violent, by using portable indicating wattmeters read by observers following the methods herein described for measuring the electrical output, and with commensurate care in making the steam-input measurements and corrections.

The author wishes to acknowledge the helpful guidance of Mr. L. T. Robinson[4] in carrying on with the tests described in this paper, as well as the assistance received from Messrs. L. J. Cavannaugh and W. S. Vogel of the General Engineering Laboratory of the General Electric Company in conducting the electrical measurements in many of the tests. J. L. Roberts, of the Turbine Department of the General Electric Co., deserves great credit for his contribution to this work being in charge of the water-rate tests. The effective cooperation of the personnel in the Central Stations where tests were made was also most cordial and helpful, and their contribution to this work is hereby acknowledged.

Discussion

H. W. Oetinger: Reference is made to the checking of instrument transformers with equivalent secondary burdens. The usual procedure in determining this secondary burden is to calculate it from the published data of the instrument coil constants and the size and length of leads. Where test wiring is used exclusively, there can be no question regarding this procedure but where the test instruments are inserted in conjunction with station instruments and wiring, there is some question as to what is actually in the circuit. In one case it was found that coils of unknown and variable volt-ampere characteristics were left in circuit. In such cases it seems desirable to actually

4. See papers by Mr. L. T. Robinson as follows:

"Testing Steam Turbines and Steam Turbo-Generators," E. D. Dickinson and L. T. Robinson. Transactions A. I. E. E., 1910, Vol. XXIX, part II, page 1679.

"The Determination of Stray Losses from Input-Output Tests," L. T. Robinson. Transactions A. I. E. E., 1913, Vol. XXXI, part I, page 531.

measure the secondary burden and include all connections and instruments to be used during the test. A voltmeter, ammeter and wattmeter can be used in conjunction with a load box. The disconnection is made at the terminals of the instrument transformers and voltage applied to the leads at this point.

Under "Observations" mention is made of readings covering a wide range of load variations. For such load conditions it is important that the wattmeters used have practically no scale errors within the range of possible load swings. If the generator output is calculated strictly in accordance with the statements under "Calculations" this would not be necessary, but the usual procedure is to correct the average wattmeter reading only and not to average the corrected readings. The latter involves an immense amount of additional labor which can be eliminated by observing the above requirement.

With reference to "Photographic Observations," these have the distinct advantage of giving a permanent record of the instrument indications and any questionable data can be checked very readily by reading the film. There is, however, one point which should be recognized if speedy determination of the kilowatt output is required. By the visual method, final results of the test can be calculated within two hours. By the photographic method it is necessary to develop, dry and read the film which involves several times the delay required by the visual method.

J. A. Johnson: In the old days, when water was measured by means of weirs and piezometers we never had to worry much about the accuracy of our electrical measurements because the hydraulic measurements were so much worse. But within the last few years, there have been new methods perfected for measuring the water supplied to a turbine, so that we are now able to get hydraulic measurements so accurate that they have put the electrical engineer on his toes to produce electrical measurements of equal accuracy. We found that the discrepancies we were getting in our over-all results were apparently due to inaccuracies in the electrical measurements rather than the hydraulic. So I welcome Mr. Lee's paper showing that electrical measurements can be taken accurately by observers with indicating instruments, and that it is not necessary to use photographic methods, because it is always much easier to use apparatus which is available in any standard power company's laboratory than to have to develop special apparatus.

W. H. Pratt: Mr. Lee's paper shows what can be done by using instruments in a careful way, and it recalls to me work that I did about twenty-five years ago when we first had occasion to calibrate large watthour meters on very fluctuating railway loads. We found by averaging results on readings, simply taken as Mr. Lee describes, not attempting to average in the mind but putting down the readings as seen from moment to moment, that it was perfectly possible to get successive calibrations in agreement within a matter of a few tenths of a per cent, frequently within two-tenths of a per cent. So I think there is no doubt that this averaging of a moderate number of observations is an absolutely valid method.

In Mr. Lee's paper the use of a portable test meter is mentioned, and I think as a matter of record we should note that this is a use for which this meter was not originally intended. The accuracy that he seeks to obtain is higher than would be requisite when the meter is used as it is intended to be used. The meter in its proper field fully meets the conditions required.

F. V. Magalhaes: It is comforting, as well as interesting, to those handling tests involving the ordinary instruments and observations, to have the accepted methods of handling tests proved reliable by Mr. Lee's presentation of the use of the camera to check the readings of the instruments.

I should like to use Mr. Lee's paper as a vehicle for advancing a plea; and in advancing it, it is not intended as a criticism of this particular paper. The plea is for a careful consideration and statement of the necessities involved in any problem of measurement. These necessities or specifications, as well as the limitations of the instruments that may be selected for the test, should be clearly understood in the minds of those conducting the test.

The use of the rotating test meter for various purposes and its limitations have been quite actively under discussion during the past year or so and it is the use of this particular instrument that I wish to discuss. The uses which have been proposed or suggested for this instrument can be classified roughly into four necessities. The instrument was specifically designed to meet one of them but an effort is being made to use it for the other three. This effort has been attended with some lack of success and irritation on the part of those attempting it.

Firstly, there has been for many years a necessity for a convenient portable instrument for use in the field for making several hundred or several thousand tests a month of the house-type watthour meter installed on the lines of public utilities. For that purpose, experiments were started possibly twenty or twenty-five years ago, toward the development of a portable test meter or rotating test meter. The conditions to be met were primarily portability with a combination of as many ranges or scales within one instrument as it was possible to develop. The accuracies aimed at were, let us say, within 0.75 per cent.

Many are familiar with the successful use of the portable test meter in connection with the routine tests of service-type watthour meters in customers' premises.

The second problem is the one to which Mr. Lee's paper refers; namely, the measurement of energy during water-rate tests or acceptance tests on generators.

On tests of this character, the requirements for accuracy are more rigorous than for the field tests of commercial watthour meters and could probably be called $\pm$ 0.25 per cent rather than $\pm$0.75 per cent which was set up in case No. 1. Tests of this character would be made indoors in a station and it is at once apparent that the requirements for this test are different from the requirements which dictated the development of the rotating test meter for use in the field. There is no real necessity for using an instrument primarily designed for portability. Also the precision expected is much greater than that for which the portable instrument was originally designed.

The third problem that has arisen recently is the necessity for a reference standard to calibrate the watthour meters used for the interchange of power between large systems. These watthour meters are, as a general rule, very few in number and are always located within a power house. There would again be no necessity for the use of an instrument specially designed for portable use and an effort made to attain a precision of possibly $\pm$ 0.25 per cent.

A second variation of this same problem is the necessity for an instrument to calibrate the meters which sell energy directly to a small number of large customers, such as some of the utilities in the Niagara territory which have a total of possibly one or two hundred watthour meters, representing all of their customers as compared with other utilities having several thousand or several hundred thousand watthour meters. The instrument used to calibrate these relatively few watthour meters could properly be of different characteristics, possibly not quite so portable but providing a higher precision in the measurements. In this case a precision of possibly $\pm$ 0.20 per cent or 0.30 per cent might be required which again is much greater than the 0.75 per cent which the accepted type of rotating test meter now provides.

The fourth problem is the necessity for a reference standard of some description in the laboratories of public utilities or universities or public-service commissions as an instrument to certify the accuracy of the rotating test meters that are used in the field. Here again the conditions under which the instrument would be used are laboratory conditions so that there is no necessity for an instrument primarily designed for portable use

and the requirements for the precision of the measurements would be of the order of ± 0.20 per cent.

To sum it up, the present type of rotating test meter which is being used successfully by the utilities for making routine field tests of watthour meters very possibly will not meet the requirements of other forms of tests which I have just outlined. It is possible, however, that a different form of test meter may be developed which will meet all of these conditions satisfactorily.

To present an illustration from another field of measurement, I will refer to the subject of time measurements. The stop-watch was developed a great many years ago, primarily for the purpose of being carried around in a person's pocket to time races. Since this original development and use of the stop-watch, there have arisen the necessities of the electrical business with their incidental electrical and time measurements. An effort has been made to use this stop-watch in connection with more or less precise electrical measurements. The stop watch has continuously suffered from the comparison of the results which it can supply with the results obtained from good grade electrical instruments. If, precise measurements of time are required in the laboratory, it is quite obvious that some different form of measuring device should be used and this would undoubtedly be a device not primarily designed for portable use in a person's pocket.

The accuracy of the results which can be obtained from the stop-watch is probably adequate for timing a horse race or boat race, but no one should presume to use a stop-watch for the measurement of the time of flight of projectiles or the discharge of a condenser or other problems which have arisen since the stop-watch was originally designed. I am loath to class the rotating test meter with the stop-watch, but the comparison is only made to illustrate the difficulties which are being experienced with the use of the rotating test meter. These difficulties arise not with the instrument itself but from the use to which it has been applied.

C. G. Brown: There is one point to which I should like to call attention—probably a great many of Mr. Lee's readings are taken when the needle of the instrument is remaining fairly stationary. I think that if he were to have a large number of readings taken in every case when the needle is moving rapidly, (we shall say, up scale), he would find considerable difference in the observations from the different men. Some men would consistently read lower, while some would consistently read higher. It would also be interesting if, in an ordinary series of tests, those values that are taken when the needle is swinging rapidly up at the time of the observation could be marked, and then see how those results check up with the values obtained when the needle is swinging rapidly down.

B. W. St. Clair: Mr. Lee's suggestion that in making very important tests the previous history of the instruments involved should be investigated before putting undue dependence upon test results is a very good one. The demands made upon test equipment for constancy under unusual service conditions are very severe in the kind of tests referred to in Mr. Lee's paper. It is seldom in ordinary testing work that conditions as severe as these will be encountered. When consideration is given to the many factors that enter into instrument constancy it is really surprising that test equipment performs as well as it does under the adverse conditions often met with in turbine-room tests.

M. W. Leonard and E. J. Mommo (by letter): If any water-rate test is conducted throughout with the same care and attention to detail which Mr. Lee advocates for the electrical part of the work there should be no difficulty in obtaining a final accuracy of ± 0.25 per cent.

As the steam measurements are generally the greatest source of error, including not only these of initial pressure and superheat, but particularly that of weighed condensate, we should like to ask Mr. Lee if he considers the three-wattmeter method as most suitable for water-rate tests in general? Even where telltales are installed between double valves, and water lines are protected by blanks, there are still possibilities of error in calibrating weighing scales, measurement of gland leakage, condenser leakage, condensate from steam-jet pumps, etc.

The three-wattmeter method of measuring electrical energy is undoubtedly simpler than the two-wattmeter method so far as the application of the meter corrections is concerned. From a practical standpoint, however, we wish to point out that there are certain disadvantages to be encountered with either of these methods where indicating instruments are used. Under Mr. Lee's method three observers, one relief observer, and a computer are necessary. This means not only a multiplicity of instruments and high-test costs, but delay in computing and checking the increased amount of data. The eye strain on the observers, while not serious on a test of short duration, might well be a factor to be reckoned with on tests made every day for a week or so.

The polyphase-integrating-wattmeter method offers none of the above difficulties. Most of these instruments have a very constant calibration and involve very little difficulty in checking to within ± 0.1 per cent with a rotating master standard with 100 per cent power factor on each element. When equipped with a high-speed dial register ample precision can be obtained on tests of two-hour duration, readings of the dial being made at half-hour intervals. There is some question as to the advisability of running tests with a duration of less than two hours, not so much due to the precision of the integrating polyphase wattmeter but because of water levels, variations in condenser vacuum, and temperature changes in generator windings.

With the addition of a properly applied set of instrument-transformer calibrations it would appear that any further degree of refinement involving phase-angle corrections, etc., would be unnecessary with a single polyphase wattmeter because of the probably greater errors in the steam measurements. An over-all electrical accuracy of ± 0.25 per cent, including instrument transformers, may be reasonably expected even with the two elements of the polyphase meter checked at 100 per cent power factor. If greater accuracy is desired the wattmeter elements may be checked separately at 100 per cent and 50 per cent power factor, corresponding approximately to the electrical condition existing in the meter with generator loads at 85 per cent power factor, a value usually obtainable.

E. T. Brandon (by letter): I think that the refinements of measurement used would be justified only where a very large number of tests of first importance were to be made. Some of the corrections said to be accomplished are usually so very small that they are negligible no doubt in comparison with the error of observation of the meter reading itself. If the correction is made, I suppose it justifies itself by indicating that it has been accounted for, even if negligible.

The three-wattmeter method recommended has advantages, but would require temporary connection of potential transformers in some cases, and I think the advantage of having all wattmeters operating at high power factor is offset by the fact that three meters have to be read simultaneously, which is difficult. I think the trouble of having one wattmeter reading at low power factor, as is the case when the power factor of the load is much below 0.80 and the two-wattmeter method is used, could be overcome in most cases by an adjustment of power factor to unity before the test is started. We have been able to do this in our tests at all except light-load conditions for the generator under test.

No mention is made in the paper of the use of the polyphase wattmeter. This meter, in its portable form, usually has high accuracy and the additional advantage that it gives the total power on one meter, thus insuring the recording of a simultaneous reading. Adjustment of the load on the generator to approxi-

mately unity power factor would be necessary for the highest accuracy when the polyphase meter is used.

With reference to the electrical readings taken on the Gibson tests at Queenston, we have, in general, made corrections similar to those suggested and presumably used in the tests described in the paper. The Gibson test is different from the water-rate test in this respect, namely, that the power being delivered to the turbine is desired at the instant the gate starts to close. The quantity to be measured, therefore, is not the average power over a period of time, as in the case of the water-rate test. In correcting the meter reading obtained, we have taken account of the ratio and phase-angle errors of current and potential transformers, which have been previously calibrated, and the error in the meter itself. The wattmeter is always calibrated before and after test against our secondary standard, and is transported by messenger.

In noting that use was made of watthour meters, I might say that we have tried these, but were unable to get a very close check with the indicating wattmeter reading. This was due, no doubt, to the fact that the watthour meter would give the average kilowatts for the time immediately preceding the dropping of load, whereas the wattmeter gives the output at the instant of shut-down. The per cent error in the watthour meter is larger because the time during which it is reading is relatively short, not more than two minutes as a usual thing. In water-rate tests of an hour's duration, during which the load is held reasonably constant, this error would become very small.

I was interested to note the comparison between visual and photographic methods of obtaining the readings on the wattmeter. We have had this method in mind for some time, and expected that the extra cost of such a method would be justified by the increased accuracy obtainable. Apparently where the average of readings taken over a length of time is desired, competent visual observation can be depended on to give results as accurate as the results obtained with the camera. I still believe, however, that where an instantaneous reading is desired, greater accuracy would be obtained by photographing the position of the wattmeter needle.

E. S. Lee: With reference to using a polyphase integrating watthour meter as suggested by Messrs. Leonard and Mommo, the procedure, as described in the first part of the paper, regarding watthour meters should be followed. As regards the need for two-hour tests, the evidence submitted in the paper shows the sufficiency of tests of one-hour duration.

With particular reference to measurements of electrical power output from water-wheel units as mentioned by Mr. Brandon where the water flow is measured with the Gibson apparatus, conditions are quite different from turbine-generator testing in that the electrical power output is desired at a particular instant following a period of constant power output. The results of repeated tests will give the deviation. If the latter is too great, more refinements will have to be introduced into the measurement. Photographic observations would probably be of no advantage over the visual however, because of the steady conditions required preceding the observation which would allow the instruments to be read visually.

The use of a polyphase indicating wattmeter requires that the instrument be compared under the exact test conditions. The suggestion of adjusting the generator power factor to unity is not generally applicable as the generator must be tested at its rated power factor which is frequently less than unity.

The Engineer and Civilization

President's Address

BY FARLEY OSGOOD

The Institute

It seems fitting at this time to bring to your attention, very briefly, some of the outstanding results of the work of the membership of our Institute during the administrative year just closing.

I am sure most gratifying to all, is the increasing interest and activity in Institute affairs all over the country, the greater appreciation of the value of the Institute, professionally and practically, and finally the enjoyment of personal contact of scientists and engineers for the mutualization of thought toward the advancement of our art. These facts are clearly brought out in the statement of the number of meetings and attendance given in the Annual Report of your Board of Directors for the fiscal year ending April 30, 1925.

The work of our Standards Committee, which covers not only a revision of and additions to our Standards, but a rearrangement of their set-up, which will make for their much greater convenience for field use, is worthy of the greatest praise, as it required much untiring and unselfish effort on the part of a very large group of men determined to accomplish what has been done this year.

Our Meetings and Papers Committee has so coordinated its work with that of the Districts that regional conventions are now established on such a sound footing as to have made it desirable to abolish the National Spring Convention as such, which recommendation of the committee was concurred in by the Board of Directors at its meeting on May 15, 1925.

The revising of the constitution to bring it into line with the present scope of the Institute's activities, and the fast growing nationally spread membership has been accomplished, as was also reported at the Annual Meeting on May 15th in New York when the large affirmative vote by letter ballot was recorded.

The many Technical Committees have been even more active than usual, and through the Journal from time to time you have seen how well they have prepared the sessions of the conventions falling under their several responsibilities. Their following the electrical engineers into new fields of industry is particularly to be commended as it is bringing into our Institute many new groups, as members and authors, not previously interested in our possibilities. The detail of the constructive work of the committees will be recorded in the history of this convention, for it is now our practise to have their reports presented at this, the last and presumably the most widely representative convention of the administrative year, in order that their conclusions may be more far reaching in their orientation, and that through the discussion, views of members from the greatest number of sections of the Institute may be expressed.

The report of our auditors indicates a healthy financial condition, and that we have paid our way this year. The slight increase in income as provided by certain changes in our revised constitution, will be most welcome for the carrying forward in a broader way important activities as seen to be needed by the Board of Directors.

The excellent result from our Membership Committee is shown in the fact that on June 1, 1925 our list of members totalled 17,318—representing a very satisfactory continuous growth since the formation of the Institute.

The Reason

Now, the relation of these events concerning our activities and growth, which are not unlike those of our sister societies, should bring to our minds the thought that there must be a sound reason for it all.

There is a sound reason, and it is in the indisputable fact that the engineers have made the world what it is today; have brought to it industrial progress and economy; have given it the living comforts and, more than all, have shaped the very scheme of living of its inhabitants since civilization began.

History

The living in trees to escape the hazards of ground life, to the steel strengthened structure of many stories;

The dispatching of runners with messages, consuming days of time and covering moderate distances, to the instantaneous international transmission of intelligence and music by physical conductors and through the air;

The shaping of the business end of the club of the savage that the greatest destruction might be wrought on the head of an undesirable neighbor, to the sending through the air, accurately and speedily, tons of metal to a predetermined point;

The thought of the wheel bringing relief to transportation by hand or with manual carriers, with its evolution through all mechanical applications to the precision and delicacy of present-day timepieces;

The employment of steam as a mechanical agent to the point of mass energy for manufacture and transportation;

The development of highways, construction of bridges, furnishing of water supply, that humanity may dwell where it pleases;

The harnessing of electrical energy for nearly every purpose of industrial and domestic life;

The use of natural resources from the obtaining of

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

raw materials from the earth to the practical application of water streams and storage, which for centuries have been idle in their help to man;

The understanding of many of the mysteries of chemistry with their application to industry, health and comfort; . . . . all seem long steps in our notion of periods of advance, but all find their foundation in science and engineering. No one can say that but for these, and the very many related achievements, would the progress of civilization, the groupings of the peoples of this world, their interests, their methods and comforts of living, be such as we now behold.

Although centuries have passed while our present scheme of life has been perfected, it is only very recently that the rapid strides of applied science and engineering have been accomplished. To note the fact that as late as 1852 was founded America's first national engineering association, that of the Civil Engineers, is sufficient to indicate the rapidity of development of the engineering arts. The founding of the American Society of Civil Engineers in 1852 was followed by that of the American Institute of Mining Engineers in 1873. Then came the American Society of Mechanical Engineers in 1880, and finally our own American Institute of Electrical Engineers in 1884.

The very many and rapidly increasing special fields of engineering endeavor seem to have been the reasons for the birth of these now so-called "Four Founder Societies." As each new field opened up, and its immediate development was hastened by necessity, there was not the realization of its possible and essential coordination with the work of those in distinct lines already established. Now, however, that there is a breathing time sufficient to review the situation as a whole, coordination is being established, as can be recognized in the forming of the American Engineering Standards Committee, the American Engineering Council, the Society for Promotion of Engineering Education, wherein the needs of all engineering may be comprehensively discussed.

The electrical engineers whose art has such universal application in all branches of engineering for nearly every industry and every domestic service, have had much to do in the bringing to one center all the arts of engineering. If for technical reasons this move toward general coordination has been started, it is all the more important for the presentation of a unified national opinion of America's engineers in matters pertaining to our own welfare, and that of the nation where the knowledge and training of technical men may be useful.

That engineers have brought all this about, by no means signifies that their responsibilities have ceased or even lessened, for in fact they have become all the greater; as through scientific knowledge and engineering experience, must not only the material side of life but its relation to human existence, go forward.

Engineers by training are taught to think straight, to seek the plain truth without bias, to deal with facts only, and to marshall them toward the goal of practical and beneficial accomplishment. Who, better than they, can turn the knowledge and experience which has brought us to our present state, to its application in the less technical activities of life with which it has such a close relationship.

The advancement of science and engineering for its own sake is not enough, in its control of the forces and materials of nature, for the organizing and directing of men, and all that this means in its broadest sense, becomes the obligation of engineers in order that the human race may be fully benefited.

No finer example of full accomplishment of this ideal could be cited than the life of the renowned French chemist, Louis Pasteur, an almost fanatical devotee to his science, yet never without the parallel thought of the benefit of his results to all living people.

Here in America today we have our distinguished Secretary of Commerce, the Honorable Herbert Hoover, an engineer of proven accomplishment, a worker for humanity, beloved by those European nations so much helped by him during the World War; now devoting his power of discernment of fact to practical application for the industry and comfort of the people of our whole Nation.

Engineers up to now have been all too prone to become so engrossed in their own technique as to give little or no heed to the development of life about them, or to have any thought that they are a part of that life and should give to it of their ability and experience and judgment, as other men do, who too have vital interests at stake from which some time must be taken for the good of all.

President George Fillmore Swain in his address to the Civil Engineers as far back as 1913, advocated a more human engineering, while our past President, Arthur W. Beresford in his address to us in 1921, warned against the belief by engineers that they can run successfully all the jobs in the world; but surely between these two ideas is a middle road to follow where engineers can and should help in matters concerning which they possess useful knowledge. President John Lyle Harrington in his address to the Mechanical Engineers in 1923, speaks very definitely on the subject, and I commend for your careful reading and reflection these three addresses mentioned, which so clearly point the way we should go, that from engineers the world may get the fullest measure it so rightfully demands, and the position of the engineer may be recognized as it so surely deserves.

Of course, just being an engineer does not qualify a man as an executive, nor as a legislator, for such work requires broad human vision, balanced perspective, ability to sense relationships and effects, all of which faculties would seem to have been more generously bestowed upon lawyers and business men generally than upon men of technical training. Is this, however, the

fact? Have not the technical men these faculties, but have neglected to develop them by special education and experience so as to bring them sufficiently to a prominent place in their mental activities and desires?

Faculty for other than technical work must be present in the engineer if he is to be a worker in the broad fields of life, but having such aptitude, his very technical training should make him a more useful, more forceful worker than those non-technically trained.

If you reflect but for a moment, you will realize that many, in fact most of the problems of our people are basically engineering problems, if not strictly so technically as many really are, at least so generally, as they have to be solved by analysis, and the weighing in the balance what is found to be the logical and practical result if one plan is followed, with similar results of some other scheme under consideration. This method of analysis is peculiarly that of engineers who by training are well fitted to lend a hand.

In few places where the engineers could be helpful, do we see their names listed. To be sure here and there we find them, and how well these few perform their duty is a matter of record. The names of these men are mentioned with pride, but it is the exception rather than the general rule to find our members in such activities. In our principal body of national representation, our National Congress, how many names of engineers do we find? This body is composed largely of lawyers, over 65 per cent, and politicians whose business at home is not engineering. These are the men who pass on the problems of the country so many of which, as we have endeavored to show, are such as could be more logically determined by our technical men.

Truly, by now we have come to the time when our experience must be turned to broader fields of investigation than those confined to our technique; and how is this to be brought about?

A new state of mind in the engineer must be born. Those most advanced in our art must appreciate their debt of service to the needs of the people not having our training in the determination of facts; our educators must be brought to appreciate that the training of young men in the technique of their work, is but a part of the training for a suitable graduating degree. Our college students must be made to realize that their particular training incurs an obligation of its service for the benefit of all our people, wherever it is possible to bring it to bear.

In so many of our commissions and political bodies we find the make-up largely lawyers, with a few other non-technical men, whose work deals primarily with projects based on engineering, and as a consequence many engineering experts are called to bring in the scientific and technical facts that proper decisions may be reached.

Why, now that the legal and financial bases of most of the investigations by our public bodies have been so well established, should not the personnel consist largely of qualified engineers, and when legal or financial advice is needed, call in the experts from those fields? Would not this seem more logical? Has the training of our scientists and engineers been such as to make them unfit for this duty? If so, it is high time that intensive study should be undertaken to correct such a condition.

It would seem reasonable for a well balanced committee of the four founder engineering societies to be formed, selected from many branches of industry and the world work to cooperate with the Society for the Promotion of Engineering Education, in order to determine most broadly on a proper curriculum for our students, to bring to them a correct blending of technical work and training in human engineering; that our profession may perform its fair share of carrying on the welfare of our people.

Even if much which is now taught is abolished from our present curricula, and the fundamentals of the technique of our art made more thoroughly understood, with training in expression of their application, both orally and in forensic, so as to be clearly understood by non-technical men of the business world, a much broader viewpoint of the engineer, and a much wider appreciation of what he can do in the world's affairs will result.

No longer should we be looked upon as "glorified mechanics," when in our combined mentality and our essential training in the discernment and arrangement of facts, exists such a potentiality for helpfulness to all mankind. If we so continue we can have only ourselves to blame, and now that we have come so far, what a shame to rest in a feeling of complacent satisfaction.

Of course every engineer in the country cannot be placed in public office, either municipal, state or federal, but all engineers can lend aid in supporting those chosen for the more conspicuous places. Each has his own field of usefulness, by training or by choice, and his feeling of reward should be his satisfaction of service rendered.

The Master Mind of all creation holds the key of all knowledge; to the scientists is entrusted the unfolding of the fundamental laws of nature, to the engineers their practical application for the benefit of the human race in every possible way; the road is open, our duty is clearly defined, that our obligations will be fulfilled there is no doubt.

Present State of Transmission and Distribution Developments

By Committee on Power Transmission and Distribution[1]

THIS Committee submits the following report on the progress made during the past year in the field of Power Transmission and Distribution. In accordance with the plan proposed, a number of subjects are offered at the end of the report suitable for topical discussion at the Annual Convention.

In high-tension transmission, the year has been notable for the first normal operation of the two long-distance 220-kv. lines in California and the concrete proof of the entire feasibility of lines of this sort. The importance of this fact is difficult to exaggerate. Already several other 220-kv. lines are projected; one now going under construction. The physical construction of these lines will soon be pretty well standardized and no longer a matter of serious controversy.

While the evidence as to the feasibility of 220-kv. lines is clear and convincing, there remains much uncertainty as to the limitations of the capacity of such lines when of great length. Much quiet work has been done, since the forceful presentation at the Philadelphia A. I. E. E. meeting last year of the dependence of very long lines on the rigid maintenance of line voltage and the weakness of commercial synchronous condensers for this purpose. The prospect is bright for better operation than would have been expected. At present, the work is largely in the hands of the designers of the various sorts of apparatus involved in line-voltage support and regulation.

It may be well to mention that the explanation of the many mysterious insulator flashovers on high-voltage lines in central California, which have baffled investigations for some time, has been established as the presence of large birds roosting on the towers and guard rings. This difficulty has now been, or soon will be, practically eliminated.

An old problem has come into unusual prominence in certain places, viz., the more or less severe vibration of highly stressed conductors, due to wind action causing the formation of standing waves and endangering the cable strands. This subject is being worked upon.

Little progress has been made in the discussion of the various aspects of the subject of flux control in very high-tension insulator strings. The well-known guard ring seems to be holding its position.

Much field work has been done on corona losses. It is to be expected in the measurement of these losses on long lines, subject to varying voltage and climatic conditions along their length, and sometimes of varying conductor size and configuration, the results are somewhat conflicting among themselves and to some extent at variance with the accepted formulas. While there is a difference of opinion on the subject, no clear proof as yet exists that there is any material error in the established formulas when applied to ideal simple conditions. However, it is clear that the actual determination of this loss for the practical case of long 220-kv. lines is a very complex and difficult matter, since the conditions vary so much both along the line and from day to day.

With the increased study of important tie lines which may carry power in *either direction* and with the proposals for the transmission with the aid of synchronous condensers of amounts of power on high-voltage lines near their *maximum capacity*, the importance and the complexity of the designers' task is just now being realized and the all-important rôle of power factor recognized.

In the field of lead-covered cables, we have to record a most active year of great progress. With the marked advance in 66-kv. cable joints; the successful operation of a number of high-tension cables in 66-kv. circuits in actual commercial service; and with the growing belief that cable for 132-kv. circuits will be feasible and may soon be available, the whole aspect of the transfer of large amounts of power underground in congested districts is changed. The possibility of the use of these voltages leaves very little to be desired as far as capacity goes. However, the costs are extremely high and the technical details are not yet developed to any final stage.

There has been no marked development, but it is believed that a steady advance is being made in the quality of high-tension cables and, therefore, in the voltage at which cable can be used. In addition to the use of single-conductor cable the principal elements contributing to this advance are greater care in the selection of materials, in the fabrication of the cable particularly the more thorough elimination of air and impregnation with the insulating compound. In the higher-voltage cables there is a distinct tendency toward

1. Annual Report of Committee on Power Transmission and Distribution.

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Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 23, 1925.

a fluid impregnating compound. For very high-voltage cables it is proposed that this compound by means of reservoirs along the cable, be kept under a hydrostatic head, the conductor cable being hollow or of open structure to permit flow of compound. Much work has been done on all phases of this subject during the last year and many notable papers have been presented before the Institute.

Development in high-tension insulators, lightning arresters, protective relays and oil breakers proceeds steadily but no especially conspicuous advances are noted.

In spite of the number of years in which electric-power systems have been in general use and the vast numbers of distribution networks, there still exists a lively discussion as to the best type of low-voltage local distribution system—whether two-phase with neutral wire or three-phase with one modification or another.

This brief review should not close without mentioning the closer and closer operating cooperation between the well-known groups of large utilities, such as those in the southeastern States, where many necessary details are being worked out. Much more intimate mutual support is being realized, also the rapid interconnection of utilities by high-voltage lines and the establishment of large base-load plants in the great industrial district east of the Mississippi and west of the Alleghanies. This sort of interconnection is developing interesting and difficult problems in the metering of commercial power.

In accordance with the plan for making the Annual Convention a forum for the informal discussion of topics of current interest without the labor of the preparation of formal papers and without the consumption of the time necessary to present them, the following topics are presented for discussion.

A. What capacity in lead-covered cable can a present-day designer count upon in laying out connections for the transfer of power from a large base-load generating plant in or near a large city to the principal distributing substations? On what voltage, what size of conductor, what operating temperature, etc., would this be based?

B. How much load can be handled over a 100-kv. tie-line connecting two large independent systems and carrying equal amounts of power in both directions with dependence placed on tap-changing devices for the maintenance of stabilized voltage in both systems? Can such tap-changing devices be relied upon where the interchange between the systems is to be automatic in accordance with the variations of load and with the conditions of most economic operation in the two systems combined?

C. What is the exciting cause for mechanical standing waves in a high-tension, long-span line conductor? Is tight stringing necessarily a cause of vibration? What is the relation to conductor size, weight and elasticity? What sort of remedy is theoretically effective?

D. Where a large block of power is to be distributed over a considerable area, is it feasible or desirable to use single-circuit interconnected feeders carried by different routes to accomplish distribution and transmission simultaneously. In such a case, what are the limitations on tapping the single circuit feeders for local loads, the questions involved being relay protection, cost of installation, reliability of service, etc.

E. What can be done to reduce installation cost of high-voltage transmission lines? Can transmission towers be further standardized? Can more mechanical devices be used in hole digging and erection? Is the use of extra high-strength conductors and very tight stringing desirable?

F. What can be done to reduce to reasonable proportions the cost added to a transmission line construction by the necessity of considering the effect of heavy sleet?

Live Problems in Connection With Protection of Electrical Systems

By Committee on Protective Devices[1]

IN accordance with a practise established over a number of years, the work of the Committee on Protective Devices has been delegated to a number of subcommittees, the division being made with reference to the nature of the subject covered. Complete reports from each of these subcommittees are appended.

It will be observed that one new subcommittee is reporting in addition to those reporting last year. By a ruling of the Board of Directors, the subject of automatic substations was assigned to the Committee on Protective Devices. The work of this subcommittee has been done in cooperation with other committees, covering the apparatus used in automatic substations, and while the report deals particularly with protective devices, it will be found to cover rather completely the situation in automatic substations as found today.

It is felt that the scope of work assigned to the Committee on Protective Devices is very well covered by the

1. Annual Report of Committee on Protective Devices.

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R. A. Hentz,		A. H. Sweetnam.

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 23, 1925.

activities of the following subcommittees and the Chairman would not suggest any radical change in the organization as it now exists. It is heartily recommended, however, that the work begun by these various subcommittees be continued during the coming year and that the recommendations made in each case be followed.

Automatic Stations

Fundamental Difference Required in Design of Devices for Automatic Application from Those for Manual Application

One of the things which differentiate devices used in automatic stations from those used in manually-operated stations is the question of inspection. The designers of devices for manual stations tacitly assume that all of the devices will either be continuously under the eye of an attendant or will be inspected several times each day. This tacit assumption is borne out by the fact that the usual manual station is provided with instruments and switches. A human intelligence is required to read the instruments and thus operate the switches in accordance with the story told by the instruments. As a result, the designers of devices for manual stations have paid more attention to the performance of the devices in response to a specific circuit condition than to their continued successful performance with inspections at intervals exceeding a week.

Another item differentiating the design of devices for automatic stations from those in manual stations is the relative life to be expected from the two. Devices for automatic stations require in some instances a successful life of several million operations. The only devices in manual stations called upon for similar services are probably the automatic generator-voltage regulators and some of the control apparatus used for steel mill service. In each of these latter cases, life has been an important feature of the design and as much attention has been paid to this as to the operating characteristic of the devices.

It will be seen, therefore, that primarily the attitude of the designers of devices used in manual stations contemplates an attendant who will supply any deficiencies in the operation of the devices, while the design of devices for automatic stations requires that the devices function correctly or else make the station inoperative.

Minimum Safe Protection for Power Apparatus in Automatic Substations

From the standpoint of railway applications, the amount of protection that is afforded automatic substation equipment by the use of various devices, relays, etc., is a direct function of the type of installation with respect to its application, importance and whether full-automatic or partial-automatic.

This statement may be amplified by considering a typical case. It is the installation of automatic equipment on large urban properties where the successful and continued operation of the substations is of prime importance. Here it is customary to install all the various types of protection that will prevent, as far as possible, either an interruption to power service or damage to equipment.

On the other hand, automatic substations installed on some small interurban or suburban railways do not require the refinements of protection afforded more important substations. This is due, in some cases, to lack of capital for initial investment. In other cases the continuity of service is not the controlling factor and the protective features may be kept to a minimum. These substations quite frequently take the form of the so-called partial-automatic type where an attendant or a time clock is used to start and stop the station. Upon the occurrence of trouble these substations are generally arranged to cease operation and remain out of service until restarted by an attendant.

For standard size 600-volt converters, feeding into a metropolitan railway-distribution network, the following minimum protection is recommended:

A-c. overload (lock-out); d-c. overload; sustained overload; a-c. undervoltage; d-c. reverse current overheating of current limiting resistors; failure of field circuit; overspeed (lock-out); overheating of bearings (lock-out); failure to complete starting cycle (lock-out); flashover (lock-out).

D-c. Edison-system automatics are called upon for a high class of service and the protective features must be so chosen and applied that the service must be maintained. There is a tendency at present to so protect the station that the machines are tripped off at times when the condition of stress may be far below the limit of the machine. This has had a tendency to prejudice prospective purchasers of automatic apparatus by apparently complicating the switching equipment.

It has been the practise in some heavy manual systems to tie the converters or motor generators solidly to the bus without protection on the d-c. end. Within reasonable limits it is felt that this could be approached in the design of automatics on the same class of system.

For 250-volt motor-generator sets and converters the following minimum protection is recommended:

A-c. overload (lock-out); d-c. overload; sustained overload; overheating of machine; overheating of transformer; overheating of current limiting resistors; a-c. undervoltage; d-c reverse current; overspeed (lock-out); failure to complete starting cycle (lock-out); overheating of bearings (lock-out); flashover (lock-out).

Various devices for the protection of the service may be applied; in fact it is felt that this feature cannot be overdone. Several equipments have temperature protection which reduces the load on the machine upon the temperature rising within a few degrees of the point at

which the set would be taken off of the system. In this way the system voltage is held somewhat above the value which would obtain if the over-temperature device disconnected the machine from the system. In most cases the machine will cool while operating at partial load. As practically all machines are arranged to limit their output by means of regulating devices, the overheating is usually caused by inadequate ventilation or a failure of ventilating apparatus. If the ventilation is only partly retarded by the failure, the machine would probably operate indefinitely at partial load; the voltage, of course, being somewhat low but at least some power being delivered to the system. The liberal use of thermostats, operating alarms over supervisory circuits in the Dispatcher's Office, will protect the service to a large extent. An attendant may be dispatched to the station upon indication of even slight over-temperature and may be able to alleviate the trouble in time.

Automatic A-C. Distribution and Transformer Stations

The protection to the service should consist of reclosing features applied to the outgoing circuits. Power supply to the station should be assured by proper overload, balanced, or reverse-current protection of the parallel transmission lines supplying the station, or by automatic transfer devices if not desirable to have transmission circuits paralleled. Differential protection should be provided for each transformer so that a defective unit will clear from the system without interruption to service. The matter of apparatus protection may be reduced to that of transforming and regulating devices; their protection should be against overheating and groundings. All outgoing circuits should, of course, be provided with overload protection and in cases of automatically reclosed circuits, lock-out features should be provided.

Oil Circuit Breaker Operating Duty as Applied to Automatic Reclosing Circuits

It is suggested that an attempt be made to get the manufacturers of switchgear to furnish ratings with oil breakers supplied for this class of service that will enable the operating engineer to apply intelligently these breakers to his circuits which are to be reclosed after tripping on trouble. Referring to distribution circuits in the 2200- to 6600-volt class, it seems to be general practise to reclose these circuits three times before final lock-out. There seems to be a little difference of opinion here and there as to just how much time should elapse between reclosures, but as a general average we might say that the first reclosure is made in from two to five seconds after tripping; the second reclosure in about 30 seconds from zero, (initial tripping) and the third in anywhere from 60 to 180 seconds from zero. It is felt that if the manufacturers were to establish a tentative standard somewhere near the above values and rate the breakers on this basis, that the operating companies would either accept the standard or their own idea of the values would be close enough to the standard to make an intelligent application. Even if this tentative standard were based on three or four reclosures at five-second intervals it would be a decided advantage over what is now available.

Nomenclature for Types of Automatic Reclosing D-C. Feeders

The following is suggested:

A. Types of Feeders

1. Stub Feeder. A Stub Feeder is one which, at the time of reclosing, receives energy for the testing circuit from one source only.

2. Multiple Feeder. A Multiple Feeder is one which, at the time of reclosing, receives energy for the testing circuit from two or more sources.

3. Stub-Multiple Feeder. A Stub-Multiple Feeder is one which, at the time of reclosing, may receive energy for the testing circuit either from one, two or more sources.

B. Methods of Automatic Test Prior to Reclosing

1. Continuous Testing by Current. Continuous testing by current is a method which continuously furnishes to the feeder a limited current which operates a device or devices adjusted to function at or below some predetermined value of current flow into the feeder to effect reclosure of the circuit interrupter.

2. Intermittent Testing by Current. Intermittent testing by current is a method which intermittently furnishes to the feeder a limited current, operating a device or devices adjusted to function at or below some predetermined value of current flow into the feeder to effect reclosure of the circuit interrupter.

3. Testing by Voltage. Testing by voltage is a method which employs a voltage-actuated device connected between the source of energy and the feeder or between two sources of energy to effect reclosure of the circuit interrupter at a predetermined voltage condition.

Remote Supervision

The handling of various switching and receiving indications of operations from remote points has received much attention. The larger operating companies are discovering that, even with manual switching, something of this nature is necessary to expedite operations in times of trouble. As soon as a system grows to a size where it becomes necessary to establish a central dispatcher to direct switching, it is at once manifest that when trouble occurs, the switching necessary to bring the system to normal can be carried out only as fast as the dispatcher can obtain information. The use of the telephone for this involves the chance of human error and retards action as an operator cannot manipulate his switchgear while telephoning. The use of supervisory systems to provide the dis-

patcher immediately with an indication of changed condition shunts out many minutes of valuable time. In automatic stations, indications may be given of lamost anything required and it becomes only a matter of how far it is desirable to go in a given case. In Edison system automatics, remote supervision is particularly helpful as there are many conditions which can occur without warning and if allowed to continue may result in partial service interruption. For the dispatcher to feel that he has his system well in hand he should have at least the following indications from each of his Edison system automatics.

1. Continuous indication of d-c. amperes on each unit in the substation.
2. Positions of all oil circuit breakers.
3. Alarm upon failure of ventilating system.
4. Alarm when predetermined high temperature value is reached at the air discharge from any unit.
5. Alarm upon the operation of the lock-out relay on any unit as he may not otherwise know that the machine will not restart upon demand.

He should also have sufficient supervisory apparatus to enable him to shut down and "hold off" any or all units so that he may, if desired, restrict the output from a given station. In addition to the above it may be desirable, although not necessary, to have supervision over the closing and opening of the oil breakers on the supply feeders and junctions in the supply bus, etc.

Supervision of a-c. distribution automatics is, of course, less elaborate due to the simple nature of the equipment in such stations. There are comparatively few indications that the dispatcher really has any use for and the balance is rather a matter of convenience.

It is sometimes desirable for the dispatcher to know when the lock-out relay on any of the automatically reclosed circuits has operated, although it is not necessary for him to be able to release the lock-out as it is generally necessary to make repairs before the circuit is again closed. It is observed in some communities that sufficient trouble calls from customers pile up in two or three minutes to indicate definitely that the whole circuit is dead and therefore, before the dispatcher can inform the proper persons in the Trouble Department of his supervisory indication, they have already made their deductions.

It is sometimes desirable to give the dispatcher control over the breakers in the incoming supply lines but it is better engineering to arrange them to take care of themselves.

In special cases where water-cooled or air-blast equipment is used, it is almost necessary to give the dispatcher a warning of the failure of the auxiliary apparatus involved but in average cases such detail is not met with. A-c. distribution stations are usually in outlying sections and at a considerable distance from the center of operations. In this connection the matter of rental of telephone pair mileage becomes quite an item of cost.

In general, a supervisory system is a very desirable asset to a system but it should never be forgotten that the reliability of the whole scheme can be made no better than the wires used in the circuit—experience has indicated that it is impossible to maintain 100 per cent service on such small circuits. The station should be arranged, if possible, to take care of all conditions automatically without the aid of the remote supervision.

Load-Limiting Schemes in Event of Overload (D-C.)

Load Limiting by Series Resistance (Fig. 1). Load limiting by successive steps of series resistance in the main circuit is probably the oldest type of load limiting scheme used in automatic stations. It is applicable to synchronous converters and all types of generators. For this scheme the d-c. machine is connected directly to the negative. The positive is connected to the bus

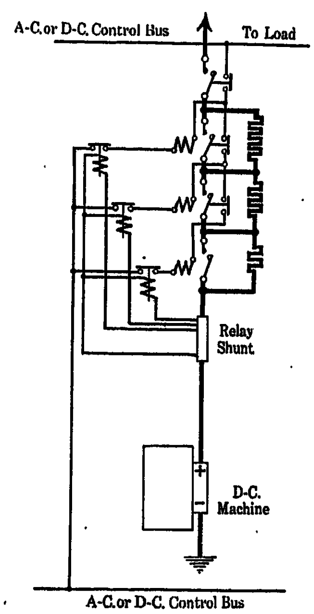

Fig. 1—Load Limiting by Steps of Series Resistance in the Main Circuit

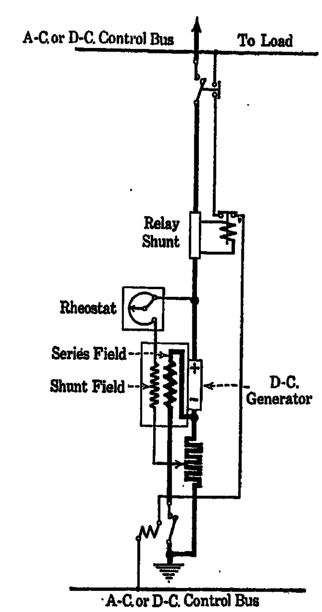

Fig. 2—Load Limiting by Accumulative Series, Field-Shunting Resistance

through a group of series resistors and a line contactor or circuit breaker. The usual design uses three steps of load limiting resistance with one shunting contactor or breaker for each step. The d-c. machine is connected to the bus with the shunting contactors open. Then the shunting contactors are closed successively; thus gradually making the bus and the machine pressures equal.

In case of overload, one or more steps of resistance are inserted into the line. If all the steps of resistance are inserted by means of the magnitude of the overload and if this overload persists for long enough time to overheat the resistors, then a thermal relay opens the line contactor and the machine is disconnected from the load until the load-limiting resistors cool. After

they have cooled, the machine is again connected to the bus through the resistors which are gradually shunted out, if the load permits.

Suitable interlocking is provided between the contactors to insure the correct sequence for closing and for opening.

Load Limiting by Accumulative Series Field Shunting Resistance (Fig. 2). Load limiting by accumulative series, field shunting resistance is used almost exclusively in connection with compound wound d-c. generators. The scheme is applicable with additional load-limiting resistors such as described above. It is permissable to eliminate such series resistors, however, if the characteristics of the load and the machine are within certain limits.

During normal operation the series field-shunting resistor has only a slight effect in reducing the amount of compounding. On overload a contactor is opened and this causes:

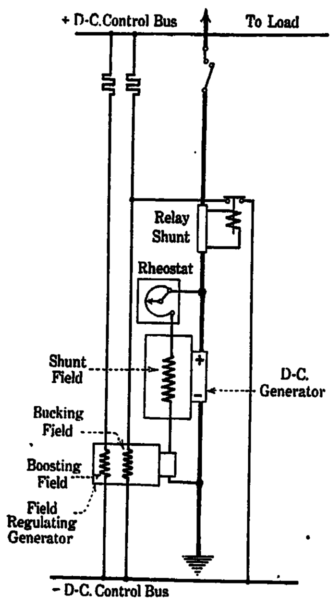

Fig. 3—Load Limiting by Counter E. M. F. Generator

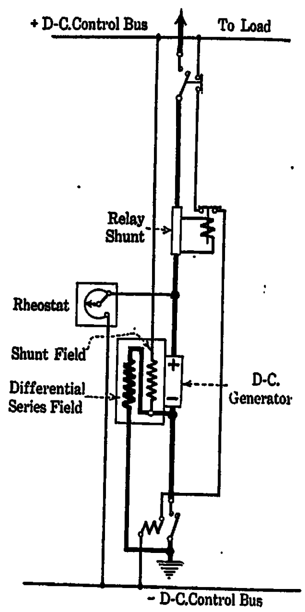

Fig. 4—Load Limiting by Differential Series Field

1. The generator to become a shunt machine.
2. The shunting resistance to be inserted in series with the machine.
3. The shunt field to be reduced due to voltage drop through the series field-shunting resistance.

On reduction in load to a predetermined value, a relay recloses the overload contactor as in scheme No. 1. Also, as in the previous scheme, interlocks are provided between the contactors to insure the correct sequence of opening and closing.

Load Limiting by Counter E. M. F. Generator (Fig. 3). Load limiting by counter e. m. f. generator is particularly applicable to shunt-wound generators. This scheme employs a small motor-generator set with the armature of the generator in series with the shunt field of the main generator. The small generator has two shunt-field windings. One is a comparatively weak boosting winding. The other is a stronger bucking winding. The boosting winding is used to supply a constant and practically separate excitation. The bucking winding is used to reduce the shunt field current of the main generator when it is inserted in the circuit by the opening of the contacts of a relay on overload.

The load is held at the setting of this relay by the regulating action of its contacts as long as the external circuit conditions require.

An excessive overload on the machine is prevented by control of the machine voltage up to a certain definite load beyond which the constant current relay in combination with the counter e. m. f. regulator makes the machine practically constant current.

Load Limiting by Differential Series Field (Fig. 4). Load limiting by differential series field is applicable to d-c. generators provided with a series field connected differentially. During normal operation, a contactor short circuits the differential series field. On overload, reverse current or short circuit, this contactor is opened by a suitable relay combination and thus inserts the differential series field into the circuit.

This method of load limiting is usually employed in conjunction with voltage and load regulating schemes, including a motor-operated shunt-field rheostat. Stability of operation on the differential series field characteristic is obtained through a constant amount of excitation from the d-c. control bus.

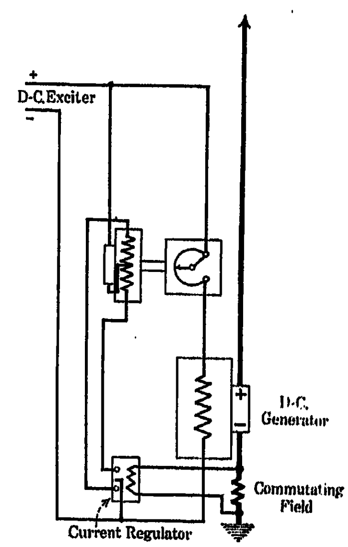

Fig. 5—Load Limiting by Current Regulator Controlling Shunt Field Rheostat

Load Limiting by Motor-Operated Field Rheostat (Fig. 5). Load limiting by motor-operated field rheostat is probably the simplest of all of the schemes employed. It is applicable particularly to d-c. generators and is suitable in general only for machines with separate field excitation, to insure stability at all voltages.

Essentially the scheme consists of a contact-making

ammeter or current regulator with a certain floating range. This device controls a motor-operated shunt-field rheostat which is operated to maintain definite voltage up to a certain load and then to reduce the voltage in order to back off from the load in case the load exceeds a certain given value.

Load Limiting by Step-Induction Regulator (Fig. 6). This scheme is applied to the transformer of a synchronous converter and affects the performance of the converter in the same manner as the well-known scheme of introducing an induction regulator between the a-c. supply and the rings of the converter, either ahead or behind the transformer. The old scheme, for various reasons, was limited to about 20 per cent range in voltage at the d-c. terminals of the converter, while this scheme is limited only by the ability of the converter to perform in a stable manner at low voltages. This limit is somewhere under 50 per cent of the rated voltage of the converter. The scheme consists essentially of an extended winding on the primary of the transformer with taps brought out at intervals. By means of a series of tap-changing switches coupled mechanically to an induction regulator, the steps of the winding are brought in or out of circuit. The various transitions are made at zero current in the tap-changing switch, the induction regulator acting as a booster to equalize the two points which are to be momentarily tied together during transition. The d-c. regulating range of the converter resembles a smooth curve. This scheme permits the use of a standard shunt-type converter, which is inherently a very stable machine.

Automatic Fire Extinguishment and Detection

Very little of a practical nature has been done along these lines except in power-plant work where the automatic liberation of inert gases into generator-cooling air has been more or less successfully carried out. There seems to be only two forms of extinguishing media which might be applied to automatic stations; inert gas and chemically-formed foam, the bubbles of which contain an inert gas. The objection to inert gas is the necessity for automatically closed dampers that will be tight enough to retain a sufficient amount of the liberated gas in the space affected. This method can best be applied to small stations using forced ventilation, as the whole station may be drenched with gas by closing the dampers at the intake and discharge openings. Stations provided with natural ventilation only are usually provided with a large number of openings both for intake and discharge and the problem of damper control would probably be too difficult. It is felt that this type of station, which is usually large in cubical capacity, can best be protected by the foam method, either in unit sections or by a system of sprinkler pipes. In this connection some experiments have been made in the application of a foam unit to a compartment or stall housing an induction regulator. The foam tank was suspended over the regulator and by means of a standard fire fuse, the chemicals were liberated into the tank where their mixing produces the foam. A section of the tank bottom was arranged to allow the foam as it was produced to drop down upon the body of the regulator. The effect produced was to cover the regulator with foam. In order to completely blanket the regulator it was necessary to close the open side of the stall with a screen which even of large mesh (½ in.) will cause the foam to be retained in the compartment. While this scheme indicated at least one good way of preventing a spread of fire from a regulator failure, it was felt that it could not always be applied in just this way. Fires generally originate in windings and except in cases of rotating equipment the devices possessing these windings may be installed in compartments or perhaps a group of such devices in an isolated chamber. This compartment or chamber may be protected by a foam unit or if the ventilating arrangement is just right by a gas unit. It was realized that wherever foam was used some method of heating the chemical tanks would be necessary to prevent freezing.

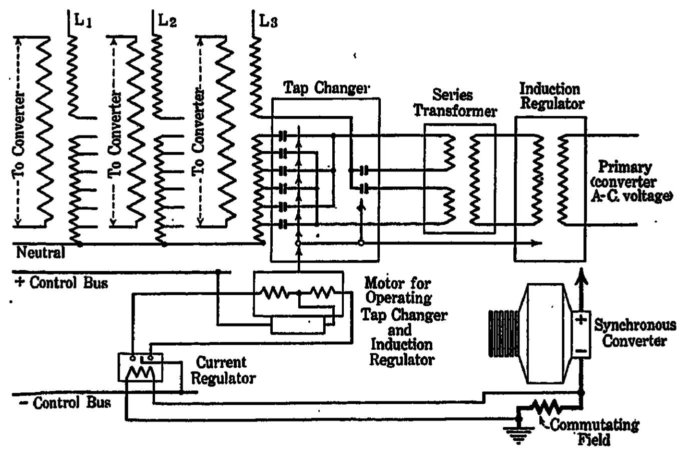

Fig. 6—Load Limiting by Step Induction Regulator in Transformer of Converter

A scheme was worked out for the protection of a compartment by gas where the nature of the device in the compartment was such as to necessitate one side of the stall being open for ventilation (this typical case being applied to an induction regulator). The use of the standard-roller, steel fire-curtain door was not thought advisable on account of its high cost and the fact that it was metal. A door of asbestos lumber, hinged at the top corners, was worked out. The door itself was hinged in the middle so that it could be hooked up to the ceiling in front of the compartment, jack-knife fashion. The operation of the fire fuse over the regulator released the gas supply and the door trigger at the same time, thus permitting the affected compartment to be flooded. It is necessary of course that an excess of gas be liberated in the stall as this type of door will have considerable leakage around it. It is felt that the use of the above door, even without liberating an

extinguishing agent, would be a step in the right direction and incidentally cheap enough to justify its use.

The matter of automatic fire detection has been given much thought. It is not very desirable to attempt to detect fire by temperature for more than one reason. First, the ordinary fire fuse used for building sprinkler systems has been known to liberate water when no fire has occurred at all. This sort of performance is, of course, unthinkable in automatic substations. Secondly, a fire such as we may experience first manifests itself in an odor followed by smoke and last by violent temperature rise. The most desirable thing to use to give the alarm would, of course, be the presence of a strong odor but as no automatic "smelling" devices have been produced, it is obviously impossible to consider this. Smoke, however, can be detected automatically and at present there is actually available a system, which by means of small suction pipes from various vital parts of a building, continually "samples" the air from these parts. This system is in use on shipboard and the indicating cabinet is located where some delegated officer can continually observe it. The smoke is made clearly visible by means of a properly directed light beam. It remains only to cause these light beams to act constantly on suitable devices to obtain automatic detection. This system in its present state of development is very elaborate and it is doubted if it can be justified in automatic stations at least for the present, but the manufacturers have indicated a desire to develop it further in the hope that it may be the answer to a good many of their problems.

Reactors

Since the most important feature of a current-limiting reactor is its reliability, the subcommittee has continued the collection of data on failures. The few reactors that have been called to its attention are mostly of the older design and the small number indicates that even in these, the weak points have been practically eliminated. Such failures which have occurred serve to keep before the manufacturers of reactors and engineers of power stations, the need for the greatest care in their manufacture and installation to the end that failures will be reduced to a minimum.

It is generally agreed that for medium and large size systems, reactors are a necessity not only because they reduce short-circuit currents and thereby lessen the duty on oil-circuit breakers and the electro-magnetic stresses on all equipment, but also maintain voltage on the rest of the system in the event of heavy short circuits in any one section of it. Were reactors omitted it is probable that an equal or greater cost would be involved in providing and installing oil circuit breakers of sufficient rupturing capacity to successfully open the very heavy short circuits that could flow, if indeed such oil circuit breakers could be built at the present time, and the system would still be without the stabilizing effect which reactors give.

Reactors being a protective device may be considered a form of insurance for which large premiums are paid in the form of losses and even more costly capital expenditures. Indicative of the extent of these capital expenditures is the fact that in one 200,000 kv-a. plant, the reactor installation (including compartments, building space, etc.), represents 15 per cent of the cost of the switchhouse. Another company, in order to add feeder reactors to one of its older plants, has had to erect a separate building for this purpose.

Thermal capacity continues to receive attention and the subject is being further investigated. There has been a distinct tendency towards increased copper cross-section. One operating company has increased the conductor size of its 3 per cent feeder reactors more than 100 per cent over that used about ten years ago, this increase being due to a desire for greater factor of safety, rather than the slightly greater duty imposed by the system's growth. The increased cost is but a small part of the total.

The smaller cross section of these older coils should receive the careful attention of all operating men. The increased use of reactors on outgoing feeders from substations offers a means of using these older coils advantageously, the newer coils being installed in the generating stations, where the short-circuit currents as well as the importance is usually less.

One or two operating companies have found it advantageous to order reactors with taps, so as to properly load parallel feeders of dissimilar characteristics (due to size or length, or both).

Recently attention has been focused on shielding the reactor leads so as to prevent damage from foreign magnetic material, such as bolts, nails, etc., from being accidentally dropped into the coils or drawn into them at times of short circuit. One American manufacturer has enclosed the coils in porcelain and in England a similar protection has been used. Another American manufacturer has designed coils with asbestos insulation around the copper conductors to avoid having the bare parts exposed to this danger. The committee expects to keep in close touch with this development.

The extension of the use of reactors is of interest. At first used in generating stations where their need is, of course, greatest, they are now being installed in substations on lines operating at generator voltages and also on the 2300- and 4000-volt distribution circuits. This latter application not only reduces the duty on the oil circuit breakers (a desirable step, especially where automatic circuit reclosing is employed), but also on the induction regulators, which on many systems would otherwise be subject to short circuits greater than those they are able to withstand.

At the last Annual Convention of the Institute, papers by Boyajian and Blake were presented on a new form of current-limiting reactor—the saturated core

type. The coils are wound on an iron core and the superimposition of a constant unidirectional flux (from coil-carrying direct current) gives a coil with low reactance on normal loads but high reactance at times of short circuit. The cost of such a coil will probably limit its use to special applications.

A paper on "Eight Years Experience with Protective Reactors" was presented before the Spring Convention (St. Louis) of the Institute by Lyman, Perry and Rossman.

In addition to points touched upon above, the Committee recommends that the succeeding Committee give consideration to two points which have recently been suggested, namely:

The question of the use of current-limiting reactors with static condensers, to—

1. Aid in smoothing out surges
2. Limit the fuse current
3. Increase the capacity of the condenser

The question as to whether the use of reactors actually increases the duty on oil circuit breakers.

Grounding of Systems

The principal activities of the subcommittee this year consisted in summarizing the information on grounding methods received in reply to an inquiry sent out in conjunction with the corresponding subcommittee of the National Electric Light Association, to thirty representative companies in the United States.

This subcommittee confined its study to the technical information gathered in this survey, paying particular attention to grounding methods used in a-c. substations of systems transmitting at higher than generator voltage. The prevailing practise of this class of systems is to dead ground the neutral, consequently in case of line to ground faults, large values of ground currents must be taken care of, which calls for a careful study of grounding methods.

The great increase in size of a-c. substations in recent years has forced attention to proper methods of grounding, because in large capacity substations ground fault currents attain such magnitude, that even when flowing through ground connections of only a fraction of an ohm will produce dangerous voltage gradients. In general, it may be stated that the problem of grounding is not so much that of obtaining individual grounds of low impedance, but rather one of obtaining a well distributed ground so as to approach an ideal equipotential area and thereby avoid dangerous potential gradients near ground electrodes when ground-fault currents flow. Another important requirement for a successful ground is the ability to dissipate, at times, large amounts of energy without a material change in the ground.

Grounding to Water Piping System. Water piping systems afford the best grounding systems obtainable and should be used wherever the necessary permission can be obtained. In fact, water systems have such comparatively low ground resistance that, where they are in proximity to other artificial grounds, a difference of potential will exist under fault conditions which will constitute a hazard to life unless the two are connected together. Unfortunately, water systems are available only in built up districts and generally can only be taken advantage of by indoor and moderate voltage substations.

Artificial Grounds. At high-tension substations in outlying districts, it is generally necessary to resort to artificial grounds such as plate or pipe grounds. It is interesting to note that the tendency is away from the use of plate grounds and toward the greater use of pipe grounds.

Although a single pipe ground has a higher resistance than a single-plate ground, a pipe ground of low resistance can be obtained by multiple grounding, that is, by connecting numerous pipes in parallel. In this way a ground of a given resistance can be obtained more economically with pipes than with plates. In addition, the multiple pipe ground will have the advantage of providing a well distributed ground which, as pointed out above, is a very important requirement. It may, therefore, be stated that the advantage of pipe grounds over plate grounds are that they:

a. Are more economical
b. Are more easily installed
c. Allow for convenient inspection and test
d. Provide a distributed ground over considerable area when used in multiple

The characteristics of ground pipes have been definitely determined in extensive tests conducted by the Bureau of Standards and others. Quantitative values on the properties of ground pipes may be summarized as follows:

a. The decrease in resistance with increased size of pipe is quite appreciable up to a pipe one inch in size, beyond which the curve becomes quite flat. From the standpoint of resistance there is, therefore, no economy in using pipe sizes larger than one inch.

b. Very little decrease in resistance is obtained by driving pipes to a greater depth than six feet. Pipes should, of course, be driven to a greater depth when the permanent moisture level is at a greater depth than this.

c. Ninety per cent of the resistance of a pipe ground falls within a radius of six to ten feet around the pipe. Pipes should, therefore, be spaced approximately six feet apart to keep one out of the dense current field of the other.

The effect of moisture and temperature on the resistivity of soils is surprisingly great below certain limits. Above a moisture content of 20 per cent there is very little variation in coil resistivity with variation in moisture content. Below a moisture content of 20 per cent, the resistivity rises very abruptly. With a moisture content of only 10 per cent, the resistivity

of red clay soil is 30 times as great as with a moisture content of 20 per cent.

The effect of temperature on resistivity of soil is not appreciable above 32 deg. fahr. Below the freezing point, however, the resistance increases very rapidly, being 50 times as great at a temperature of 5 deg. than at 32 deg.

In arid regions and in localities with sandy and rocky formations it is generally very difficult to obtain a good ground. In the West where such difficulties are frequently encountered, satisfactory grounds have been obtained by grounding to the steel casings of deep wells. Much benefit can also be obtained by treating the soil surrounding electrodes with chemicals such as ordinary salt, because, of the total resistance of a ground connection the most important part is contributed by the soil, the resistance of the electrode and contact resistance between electrode and soil under ordinary conditions being negligible. The electrical conductance of any soil is by means of the electrolytes formed by moisture combining with the soluble acids, alkalies, and salts, and where they are lacking, their artificial introduction will show excellent results. Such artificially treated soils require close attention and inspection as chemicals must be removed from time to time.

Potential Gradients. Because of the importance attached to the subject of potential gradients near ground electrodes, several tests were made to determine their values under normal operating conditions. These tests, which are reported in Appendixes A and B show that at least 50 per cent of the total voltage drop of a ground will fall within two feet of the ground electrode.

As reported in Appendix A, the potential gradient near the transmission tower when a line conductor becomes grounded to the tower is great enough to constitute a hazard to life. In this case the towers were near a highway and it was recommended that overhead ground wires be installed and carried back to the station ground, so as to improve grounding conditions on the line and eliminate potential gradient hazards.

In the test reported in Appendix B, heavy currents were used which, by heating and drying out the ground caused 80 per cent of the total voltage drop to fall within one foot of the ground electrode. This test emphasizes the importance of making ground connection capable of dissipating large amounts of energy without changing the character of the ground.

Value of Overhead Ground Wire. As pointed out in Appendix A, the overhead ground wire assists materially in reducing line and system ground resistances by connecting all tower and substation grounds in parallel. Several companies report improvement in relay action on grounded systems by connecting station grounds to the overhead ground wire on transmission lines. One company which formerly terminated overhead ground wire one or more spans away from substation reports that serious potential gradients were produced by returning ground currents when lines became grounded to towers, which hazard was eliminated by connecting the overhead ground wire to the substation grounds. It is evident, therefore, that in order to obtain the full benefit of overhead ground wires, they should be connected to the station grounds.

Petersen Earth Coil. Eleven months additional operating experience with the Alabama Power Company's Petersen Earth Coil is reported in a paper by J. M. Oliver and W. W. Eberhardt. The additional experience indicates that the high-voltage disturbances which were experienced when the coil was first placed in service have been entirely eliminated by making provisions to do all line switching, both hand and automatic, with the coil out of service—that is, with the system neutral solidly grounded.

From the experience to date, the application of Petersen Coils appear to be limited to comparatively low voltage lines (66,000 volts and less) of moderate length with a single source of power supply. On an interconnected network, where several sources of supply are maintained to all principle load centers, good service can be maintained without the use of flashover suppressing devices. In other words, the expense and complications of a Peterson Coil installation are justified only on radial feeder systems where it is desired to improve service to an important load center which is connected to the power source by only a single line. The Petersen Coil in such an application has the advantage over grounded neutral operation by reducing line outages due to flashovers, and over isolated neutral operation by eliminating arcing grounds.

Grounding at Generating Stations for Proper Functioning of Relays. In isolated phase installations, although the possibility of phase to phase short circuits are eliminated in case of two simultaneous faults to ground, the hazard of a fault to ground still remains.

Investigations have been made to determine the possibility, not of preventing ground fault currents, but of directing them into channels where they may be taken care of by connecting all metal parts, which are not normally at line potential but which are separated from parts at line potential by insulation, to a copper grounding system. The grounding busses of the three phases are joined together and taken through a current transformer to the station ground, the current transformer operating a relay when fault current flows to ground.

The application of this fault current relay scheme depends largely upon the arrangement of the grounding busses in which connection tests have been made to determine the proper arrangement and connection of the ground bus to various pieces of equipment. It is expected that further details upon this grounding scheme will be available at a later date and is one of the subjects which should be followed up by the grounding subcommittee next year.

Appendix A

Tests ot Determine Potential Gradients in the Vicinity of Transmission Line Towers

Object. To determine if the voltage drop along the surface of the ground in the vicinity of a 120,000-volt transmission-line tower would be dangerous to life at the instant when one line wire became grounded on the tower in question.

Method. One conductor of the line was grounded at

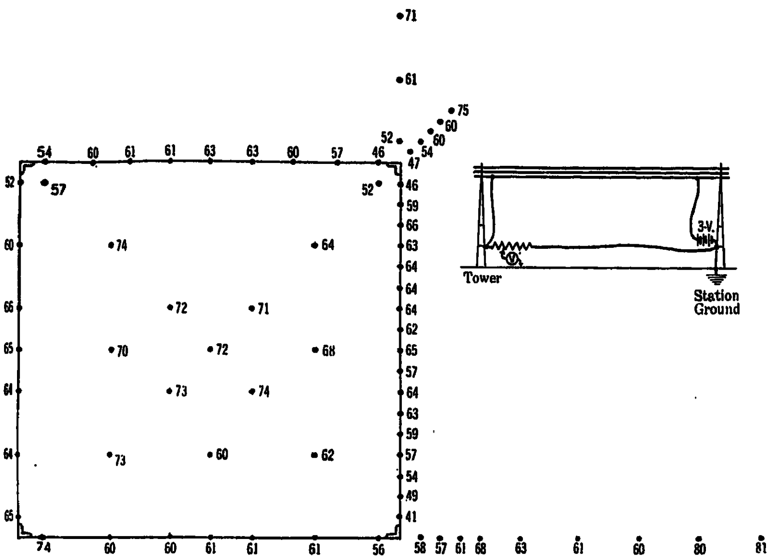

Fig. 7—Potential Gradient Test No. 1

Each number indicates the potential drop from the tower to that point in per cent of the total drop from tower to station ground
Ground resistance at tower = 0.467 ohms
Ground resistance at station = 0.147 ohms

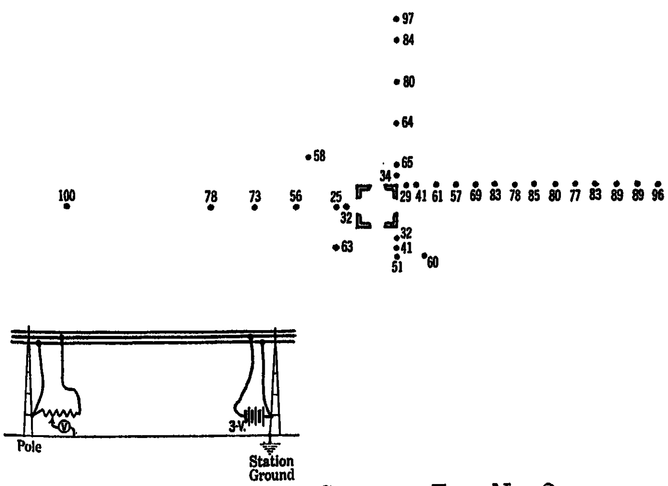

Fig. 8—Potential Gradient Test No. 2

Each number indicates the potential drop from the tower to that point in per cent of the total drop from tower to station ground
Ground resistance at tower = 3.40 ohms
Ground resistance at station = 0.147 ohms

the station, through a battery and the other end of the same conductor was grounded to the tower. A slide-wire rheostat was connected as a potentiometer, from the tower to the station ground, and the slide of the rheostat was at the same time connected through a millivoltmeter to a screw driver, as shown in diagrams 1, 2 and 3.

The screw driver was shoved into the ground at various points in the vicinity of the tower and the slide wire moved until the millivoltmeter read zero. A scale was placed on the rheostat dividing it into 100 parts, the position of the slide at zero reading of the millivoltmeter indicating directly the drop from screw driver to station ground as a per cent of total drop from the tower to the station ground.

Result of Tests:

Test Number	1.	2.	3.
Tower Resistance to Ground	0.467 Ohms	3.4 Ohms	2.6 Ohms
Potential Gredients	See Dia. 1.	See Dia. 2.	See Dia. 3.
Possible ground current in case a conductor becomes dead grounded to tower	5700 amperes	5700 amperes	2375 amperes
Computed voltage to ground	2650 volts	19,350 volts	6,175 volts
Voltage which person leaning against tower could be subjected to	1,590 volts	12,600 volts	4450 to 6175 volts
Voltage which person standing near tower would be subjected to from foot to foot	200 to 300 volts	3000 volts	1300 volts

Conclusion. These tests show that the potential near a tower is dangerously high at the instant a conductor becomes grounded to the tower.

In this particular case the towers were near a highway and the hazard was considered so great as to warrant the connection of the towers to the station ground with an overhead ground wire.

Appendix B

Ground Potential Gradient Test

The object of this test was to determine the potential gradient in the ground surface surrounding a driven ground pipe when a heavy current flow takes place to ground.

This test was made with the following equipment available:

1—6250-kv-a., 2300-volt, three-phase, 60-cycle turbine generator.

1—6300-kv-a., 22000/2300-volt transformer bank consisting of 3-2100 kv-a. single-phase units.

1—22000/110-volt potential transformer.

Switching equipment, instruments, insulator platform, etc.

Connections for the test were made as follows:

The test ground pipe was in a sandy soil which was the best location that could be found. Its measured resistance was 280 ohms.

The auxiliary ground, which was 300 feet distant, was in better soil and had a resistance of 53 ohms. By having an auxiliary ground of lower resistance than the test ground, assurance was made that no failure would occur in the circuit outside of the test ground.

The ground current was gradually increased from zero to a value which caused the test ground to steam. At this point the ground current was 30 amperes with an impressed voltage of 10,000 volts across the ground terminals. This value of current was considered the

maximum which could be maintained for any great length of time.

With the current held constant at 30 amperes, readings were taken of the voltage drop along the ground surface at one foot intervals in the direction of the current flow between the two grounds. The voltage readings were taken with a 22000/110 volt potential transformer and a 0-150 volt voltmeter; one

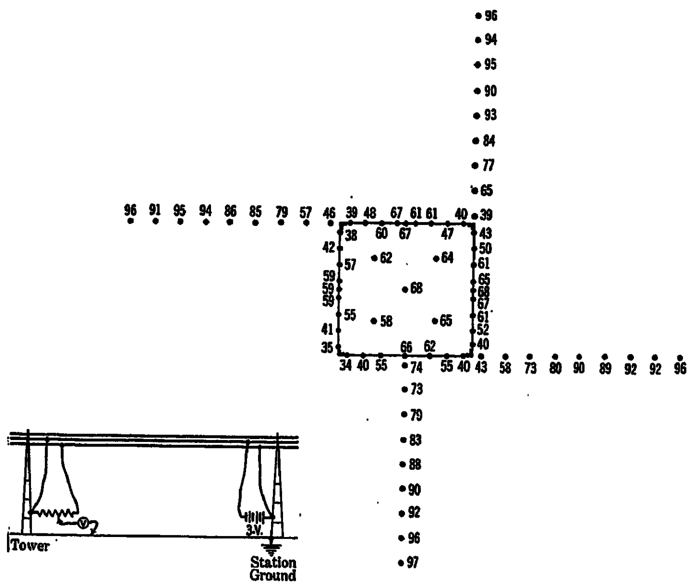

Fig. 9—Potential Gradient Test No. 3

Each number indicates the potential drop from the tower to that point in per cent of the total drop from tower to station ground
Ground resistance at tower = 2.6 ohms
Ground resistance at station = 0.147 ohms

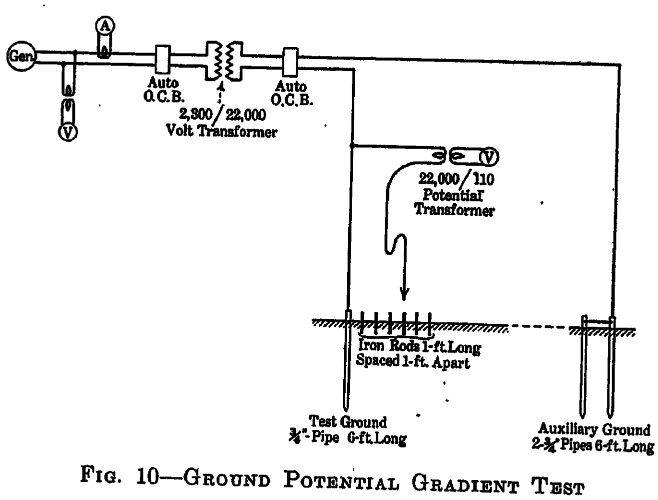

Fig. 10—Ground Potential Gradient Test

primary terminal of the potential transformer being connected directly to the test ground and the other terminal was successively touched to one foot iron stakes at various distances away from the test ground. This manipulation was done by an operator standing on an insulated platform and shifting the connections with a switch pole.

Following is a tabulation of the voltage readings taken:

Distance from Test Ground	Voltage Drop	Per cent of Total Voltage Drop at Test Ground
1 ft.	6900 volts	82.0%
2 "	7000 "	83.4%
3 "	7200 "	85.7%
4 "	7400 "	88.0%
5 "	7600 "	90.5%
6 "	7800 "	92.8%

These readings indicate that 82 per cent of the drop occurred within one foot and 92.8 per cent of the drop occurred within six feet of the test ground.

Another set of readings taken three minutes later averaged 700 to 800 volts higher than those shown above, indicating that the ground was baking out, which, was expected, since the ground was steaming in the vicinity of the ground pipe.

Tests made by the Bureau of Standards and reported in Technological Paper No. 108 showed that approximately 90 per cent of the total drop at a ground pipe occurs within a six-foot radius of the pipe. The Bureau of Standards tests, however, showed a much smaller drop within a one-foot radius of the ground pipe. The only explanation for this difference in results is that the resistivity of the soil around the pipe increased appreciably due to the baking out effect in this test. This emphasizes the importance of making grounds of sufficient current-carrying capacity to meet system requirements.

Lightning Arresters

The work of the Lightning Arrester Subcommittee this year has consisted almost entirely in an effort to list and evaluate the various electrical and mechanical features of lightning arresters.

In order to get an expression of opinion, the form given herewith was submitted as a suggestion in attempting the classification. Members of the subcommittee were requested to give their opinions as to the relative values of the listed features, proportioning each with respect to its importance in their own experience.

Electrical	Weighted Values
1. Spark-over Voltage..................................	
2. Dielectric Spark-Lag..................................	
3. Impedance—Normal Frequency....................	
4. " —200,000 Cycles..........................	
5. " —1,000,000 Cycles........................	
6. Discharge Capacity (amperes by time)..............	
7. Current Passed at Normal Voltage, Normal Frequency, with Gap Short-Circuited........................	
8. Time Required to Interrupt Dynamic Current........	
Mechanical	
9. Condition After Heavy Discharge..................	
10. Number of Heavy Discharges Arrester will take care of Without Repair..................................	
11. Attention Required in Service:	
(a) None..................................	
(b) Not More Than Once Per Season............	
(c) After Every Heavy Discharge................	
(d) Once a Day or Oftener......................	
12. Cost..................................	
13. Depreciation per year..................................	
Total..................................	100 per cent

This was sent out to members and was taken up for discussion at the midwinter meeting of the main committee. There was apparently quite a diversity of opinion among engineers as to the value and accuracy of such a classification. This diversity of opinion seemed to be, in a large measure, due to varying ideas as to the definition of the duty of a lightning arrester and of the various quantities involved.

The subcommittee was therefore, instructed to make an effort to state definitely the duties of a lightning arrester and to define clearly the quantities which make up the suggested classification. Following this, the subcommittee is to proceed with the classification along the lines suggested.

It is, therefore, possible to report progress only.

The measurement of some of the quantities necessary in making up the classification presents something of a problem. Particularly this is true of the measurement of dielectric spark-lag and the rate of discharge of lightning arresters. It is found that when discussing these quantities, there is a very great difference in the methods used by different experimenters. The different methods in use result in widely different readings of the quantities being sought. It seems that it is necessary to define very precisely, the size of the condenser to be used in measuring the discharge of an arrester; all the characteristics of the circuit to be used in measuring the voltage, and many other similar quantities.

It will undoubtedly be necessary to arrive at some standardization of these measurements before it will be possible to standardize or classify lightning arresters. In order to arrive at a definite method of determining these quantities, the Research Committee has been made acquainted with the problem, and it is hoped that through this cooperation some standard and practicable method of measurement may be obtained.

It is recommended that the succeeding subcommittee continue the work along similar lines.

Relays

Many difficulties have been encountered in the completion of the Relay Handbook. It is now in the hands of the printers and should be available for distribution in June.

This subcommittee is cooperating with the corresponding subcommittee of the Apparatus Committee, N. E. L. A., and plans within the next year to prepare the first supplement to the Relay Handbook. This supplement will be patterned after the Relay Handbook both as to form and size.

Although no new relay developments have been reported to date, several interesting installations have been received. One operating company has made an installation of protective equipment in a generating station, which is rather interesting in a number of respects, particularly in the matter of its completeness. A complete description of this installation appeared in the *Electric Journal* for April, 1924.

This same company has also made use of a variation of the split conductor scheme for the protection of a large frequency changer used for interconnecting 25- and 60-cycle systems. A description of this was given in a paper entitled "Use of Frequency Changers for Interconnection of Power Systems," by H. R. Woodrow, at the Spring Convention in St. Louis, April, 1925.

The problem of protecting generating station auxiliaries was investigated to some extent by this subcommittee, but the findings are not sufficiently definite to report more than progress. It is recommended that the succeeding subcommittee give this matter particular attention, and that the use of the so-called "Voltage Chaser" or automatic throwover switch be investigated. This latter gives some promise of considerable usefulness in assuring a continuous power supply to generation station auxiliaries, and it is felt that the subcommittee can do valuable service in coordinating experience and practise.

Oil Circuit Breakers

Progress in Standardization. Interrupting rating of oil circuit breakers was defined by the Protective Devices Committee last year. This definition has now received all of the necessary approvals and is before the Standards Committee for final adoption.

The proposed uniform procedure for testing oil-circuit breakers as prepared by this Committee last year has now been approved by the N. E. L. A. and is, therefore, in line to be used for future tests made by operating companies. Standardizing the methods of making system tests should make possible comparisons between tests on an equivalent basis so that maximum knowledge may be obtained from them.

The new edition of the A. I. E. E. Standards now being compiled will cover oil circuit breakers much more thoroughly than they are covered by the current edition. The most important new provisions are:

Conditions and Methods of Making Temperature Tests.

Tests of Dielectric Strength and Conditions and Methods of Making Same.

Protection from Voltage Surges.

Attention is called to the following definitions recently adopted by two prominent European technical societies covering certain essentials in connection with oil circuit breaker performance:

Working Voltage is the maximum potential occuring under any operating conditions at the point where the switch is installed in single and three-phase installations measured between the outer conductors. (Not including over-voltages).

Working Current is the effective value of the maximum current which flows through the switch continuously under any operating conditions (not including short circuit or over-current of short duration.)

Interrupting current is the effective value of the a-c. component which flows through the switch at the moment the contacts are opened during interruption.

Interrupting Voltage is the effective value of the potential which, at the interruption, occurs on the line that remains under voltage immediately upon the dying out of the arc of the interruption in all phases.

The present standard general definition of interrupting rating, paragraph 7090, A. I. E. E. Standards 1922, specifies the rating of current and voltage which the device will interrupt under "Prescribed Conditions." No attempt has been made to date to define prescribed conditions and much further study and experience will be necessary before anything could be done along this line. It appears, however, that sufficient knowledge and experience is available at least to begin studies to define the "Prescribed Conditions." In particular, the great difference in normal recovery voltage across circuit breaker contacts on systems with dead grounded neutrals, as compared with those whose neutral is free or grounded through resistance, makes it very desirable that the "Prescribed Conditions" be determined to the extent of covering this situation. Material economic advantage to many operating companies should result therefrom.

Standardization of interrupting ratings is in active progress at the present time by definite committees, which are now cooperating, to establish standard ratings, so as to reduce the number of circuit breaker designs which must be developed and the number of types of breakers required.

Recommendations. The following recommendations are offered:

a. Definite steps should be taken for a beginning, at least, toward formulating the "Prescribed Conditions" under which an oil circuit breaker is rated. First consideration should be given to the effect of conditions of operation of system neutral on interrupting rating of breaker.

b. Pending the working out of the above problem "Normal Voltage," in the present definition of interrupting duty should be defined so as to permit of no misunderstanding as to its relation to recovery voltage in any given case. Presumably for the present, at least, the breaker should be rated so that the recovery voltage is equal to normal voltage.

c. Definitions equivalent to those for Working Voltage, Working Current, Interrupting Voltage and Interrupting Current given above should be adopted by the A. I. E. E.

Latest Design and Practise in Power Plants

By Committee on Power Generation*

Introduction

PROGRESS in the art of steam station design and operation has been so rapid as to rather bewilder even the men who are giving their whole time and thought to this work. We have grown quite accustomed to seeing our dreams become actualities almost over night. The past year has witnessed the actual generation of power in an 80,000-kw. generating station at a coal rate of kw-hr. 20 per cent lower than any previous performance on a commercial scale. May we with confidence look forward to further gains of the same magnitude? May we expect that each new station built will establish a new record for operating performance? Why does each new steam generating station differ so radically from those already built? If we are to answer these questions, we must evaluate the present day tendencies, look backward a bit to see how far we have come, and attempt to look forward.

Important Technical Achievements of the Last Year

1. The operation of the two 40,000-kw. General Electric single-cylinder turbines, in the Philo Station of the Ohio Power Company: The steam is delivered to the turbine throttle at a pressure of 550 lb. per sq. in. gage, and a temperature of 725 deg. fahr., the steam being withdrawn from the turbine after being expanded down to a gage pressure of 155 lb., returned to the boiler room, reheated to a temperature of 725 deg. fahr., and then expanded through the remaining stages of the turbine to a pressure of approximately ½ lb. per sq. in., absolute at the turbine exhaust. This marks the consummation in this country of Ferranti's dream of a reheating cycle. The two units in this station have been in operation since October 14, 1924, and February 24, 1925, respectively. The water rate, corresponding to a load of 40,000-kw., steam conditions being as indicated above and no steam bled from the turbine for feed water heating, is reported to be approximately 8 lb. per kw-hr. In normal operation, steam is bled from two stages of the turbine for feed-water heating. The feed water then goes through economizers before

*Annual Report of Committee on Power Generation.

Vern E. Alden, Chairman,

H. A. Barre,	H. A. Kidder,	M. M. Samuels,
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Peter Junkersfeld,	John C. Parker,	

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 23, 1925.

being delivered to the boilers. The air for combustion passes through a preheater before going to the wind box of the chain grate stoker. The flue gases pass through the preheater on their way from the economizers to the stack. The heat consumption for this station has over a week's time been as low as 13,715 B. t. u. per kw-hr. of net station send-out, this performance corresponding to a load factor of 81 per cent for the week.

2. The initial operation of the Crawford Avenue Station of the Commonwealth Edison Company in Chicago: The turbines in this station are designed for substantially the same steam conditions as the turbines in the Philo Station. The three turbines in the Crawford Avenue Station are of the cross compound type and were built by three different manufacturers—

a. 50,000-kw. unit—C. A. Parsons
b. 60,000-kw. unit—General Electric Company
c. 50,000-kw. unit—Westinghouse

Two surface condensers with vertical tubes connect direct to the exhaust casing of the low pressure element of each of these turbines. Each of these three turbines was built on a new design and they present many interesting features. Due to the new design of low pressure exhaust nozzle and condenser, new features have been embodied in the turbine foundations, the building layout and the arrangement of auxiliaries. The Crawford Avenue Station has been in operation only a few months and as yet the single stage of reheating of the steam during its expansion has not been used.

3. The construction by the General Electric Company of a 3000-kw. turbine, running at 3600 rev. per min., for operation in the Weymouth Station of the Edison Electric Illuminating Company, with a steam pressure of 1200-lb. per sq. in. gage and a steam temperature of 700 deg. fahr. at the turbine throttle: This turbine will exhaust against a back pressure of approximately 350-lb. per sq. in. gage, its steam being first returned to the boiler room to be reheated to 700 deg. fahr. and then discharged into the main steam header of the station. Only one boiler for operation at 1200-lb. per sq. in. gage has been installed at the present time. This high pressure boiler and the turbine which serves as its reducing valve will soon be completely erected and ready for operation.

4. The operation in the Colfax Station of the Duquesne Light Company in Pittsburgh, of two 35,000-kw. Westinghouse turbines, with four-stage bleeding of steam to raise the feed water to a final temperature of approximately 350 deg. fahr. before it is returned to the boilers: This marks the extreme development of the regenerative cycle in turbine room operation.

5. The operation of four 30,600-sq. ft., double-ended Stirling boilers, with 22,464-sq. ft. economizers in the Lake Shore Station of the Cleveland Electric Illuminating Company: These boilers and economizers, equipped with pulverized fuel burners and furnaces—each of the latter having a volume of 29,150 cu. ft., have shown on test an efficiency of 92.9 per cent at 140 per cent of normal rating, and an efficiency of 89.8 per cent at 270 per cent of normal rating. These same boilers have operated at an average gross efficiency of as high as 90.4 per cent for a month's time. These results have been obtained with coal running as high as 11 per cent in ash and as low as 12,600 B. t. u. per lb. in heat value. The sulphur was as high as 3.5 per cent and the coal ash melted at approximately 2150 deg. fahr.

Operating efficiencies in connection with the 29,085 sq. ft. double-ended Stirling boilers in the Trenton Channel Station of the Detroit Edison Company, have been only slightly lower than the results quoted above for boilers installed in the Lake Shore Station in Cleveland. Each of these boilers in the Trenton Channel Station is equipped with two 9492 sq. ft. economizers. The coal is fired in pulverized form, and the furnace has a volume of 25,140 cu. ft. These operating results in the Lake Shore Station and the Trenton Channel Station, with the unburned fuel loss reduced to a small fraction of 1 per cent and the temperature of flue gases reduced to from 230 deg. fahr. to 250 deg. fahr., present an achievement undreamed of a few years ago.

6. The operation of the 18,010 sq. ft. Babcock & Wilcox cross-drum boilers in the Cahokia Station of the Union Electric Light & Power Company of St. Louis: These boilers are twenty tubes high in the main tube bank and are not equipped with economizers. The coal is burned in pulverized form. The furnace has a volume of 12,850 cu. ft. Burning a most inferior grade of southern Illinois coal, these boilers have shown on test an efficiency of 85.9 per cent at 148 per cent of normal rating and an efficiency of 82.1 per cent at 260 per cent of normal rating. The average gross boiler-room efficiency in the Cahokia Station has run as high as 81.2 per cent over a month's time. These results are remarkable for two reasons:

a. They were accomplished with very low grade coal.
b. They were accomplished without the use of economizers or air heaters.

7. The successful operation in the Chester Station of the Philadelphia Electric Company, of underfeed stokers, with air delivered to the stoker wind box preheated to a temperature 550 deg. fahr.: A 15,000 sq. ft. boiler, equipped with a 15-retort, 22-tuyere, Taylor stoker with clinker grinder has operated for extended periods at ratings in excess of 300 per cent of normal boiler rating, with air delivered to the stoker wind box at a temperature of approximately 550 deg. fahr. The fuel bed has been free from large clinkers and is as easy to maintain in good condition as in connection with other boilers not equipped with air preheaters. It appears that stoker maintenance will not be greatly increased by use of preheated air. It does appear, however, that furnace walls designed along conventional lines will not withstand the effects of the high furnace

temperatures which obtain in connection with the use of preheated air.

8. The further developments in use of water-cooled furnace side walls as exemplified in connection with the boilers installed in the Hell Gate Station and Sherman Creek Station, of the United Electric Light and Power Company in New York, and the boilers installed in the Zilwaukee Station, of the Consumers' Power Company in Michigan.

A novel wall has been in operation since November in connection with one of the stoker-fired boilers in the Lake Shore Station in Cleveland. By water cooling, the inner refractory lining having a thickness of less than 1 in. is maintained at a temperature of less than 2000 deg. fahr., but still at not too low a temperature to slow up combustion adjacent to the wall.

9. The successful use of Cottrell precipitators in the Trenton Channel Station, of the Detroit Edison Company, for the removal of fine ash from the flue gases before they pass out the stack: The ability of this equipment to remove 75 per cent of the solids in suspension in the flue gas has been demonstrated.

10. The successful operation of 70 in. Fuller air-separating type of pulverizing mill in the Cahokia Station, of the Union Electric Light and Power Company of St. Louis: This mill has pulverized 28 net tons of Illinois coal per hour, with a combined power consumption of 294 kw. for the motors driving the mill and the exhauster fan. Coal was pulverized to a fineness so that 65 per cent passed through a 200 mesh screen. This mill and its driving motor occupy a floor space of 13 by 21 ft.

11. The development by the Fuller-Lehigh Company of the "well type" furnace for burning coal in pulverized form: In an experimental furnace 8 ft. square by 8 ft. high, 32,700 lb. of coal per hour have been burned. Such observations as can be made, in connection with a furnace open at the top to atmosphere indicate that the coal is completely burned.

12. The successful operation in several stations of automatic combustion control equipment: Notable installations are those in the Lake Shore Station in Cleveland, Sherman Creek Station in New York, and Devon Station, of the Connecticut Power and Light Company, in Devon, Conn. Reference has already been made to the close agreement between average monthly operating efficiencies in the Lake Shore Station, and the efficiencies obtained under the most carefully made tests. It appears that equipment is now available which will enable elimination in large measure, of the losses incident to the lack of constant vigilance by the firemen.

13. The efficient operation of large single-pass condensers in the Waterside Station, of the New York Edison Company, and the Trenton Channel Station, of the Detroit Edison Company: In these installations less than 1 sq. ft. of surface is installed per kw. of turbine capacity. Ample provision has been made, however, for the penetration of steam into the tube bank. These installations point the way to possible reductions in the cost of condensing equipment without appreciable reductions in station economy.

14. The placing in operation of five 30,000-kw. and nine 20,000-kw. turbines of the new Westinghouse multi-exhaust Baümann type.

15. The construction by the General Electric Company of two 50,000-kw., 62,500-kv-a., 1800-rev. per min., tandem compound turbo-generators, for the new Richmond Station of the Philadelphia Electric Company, and two 60,000-kw., 60,000-kv-a., 1500 rev. per min., single cylinder turbo generators, for the Fourteenth Street Station of the New York Edison Company.

16. The use of three new methods for insuring the reliability of the power supply for the auxiliaries in large steam stations:

a. In ten new stations recently placed in operation or now in the course of construction, the generator which supplies power for the essential auxiliaries is directly coupled to the main generator and driven by the main turbine.

b. In the Trenton Channel Station, of the Detroit Edison Company, the power for the auxiliaries is obtained from separate turbo generators which operate as condensing machines and constitute a separate power plant within the larger power plant.

c. In connection with the Parsons turbine in the Crawford Avenue Station, Chicago, and the Richmond Station, Philadelphia, the power for the essential auxiliaries is to be obtained from a transformer which is tied direct to the leads of the main generator.

17. The extensive use of waste heat driers and steam driers for removing the moisture from coal before it is pulverized: This is best exemplified by the installations in the Cahokia Station, in St. Louis, and the Trenton Channel Station, in Detroit.

Important Developments of the Past Few Years

These things have come to pass within the last year. Looking back over a slightly longer period, we see the development of the steel-tube economizer for high pressure installations; the development of the inter-deck and radiant heat superheaters; the development of the air-cooled furnace for pulverized fuel, with a water screen for protecting the furnace floor; the development of the air preheater; the use of the closed system of ventilation with radiator type coolers for large turbo generators; the widespread use of deaerators and evaporators; the all but universal swing to the use of electric-driven auxiliaries; the newer developments in the design of boiler settings for chain grate stokers to insure a thorough mixing of the rich gas stream which rises from the front end of the stoker with the excess air which comes up through the rear end of the stoker, so as to enable a minimum of excess air to be used; and the development of the underfeed stoker for total furnace depths up to 19 ft. 8 in. All of these things were unknown as commercial achievements and most of them undreamed of five years ago.

The length of the strides which we have taken may be visualized by reference to the most interesting address

delivered by Mr. W. H. Patchell, at the time of his inauguration as president of the British Institution of Mechanical Engineers on February 21, 1924. As a result of a review of the situation in this country and abroad, Mr. Patchell presented the operating efficiencies for thirty-six stations in England, twenty-five stations in the United States, and one station—the Gennevilliers Station—in France. The heat consumption per kw-hr. for the most efficient station in England was given as 20,150 B. t. u. The heat consumption for the most efficient station in the United States was given as

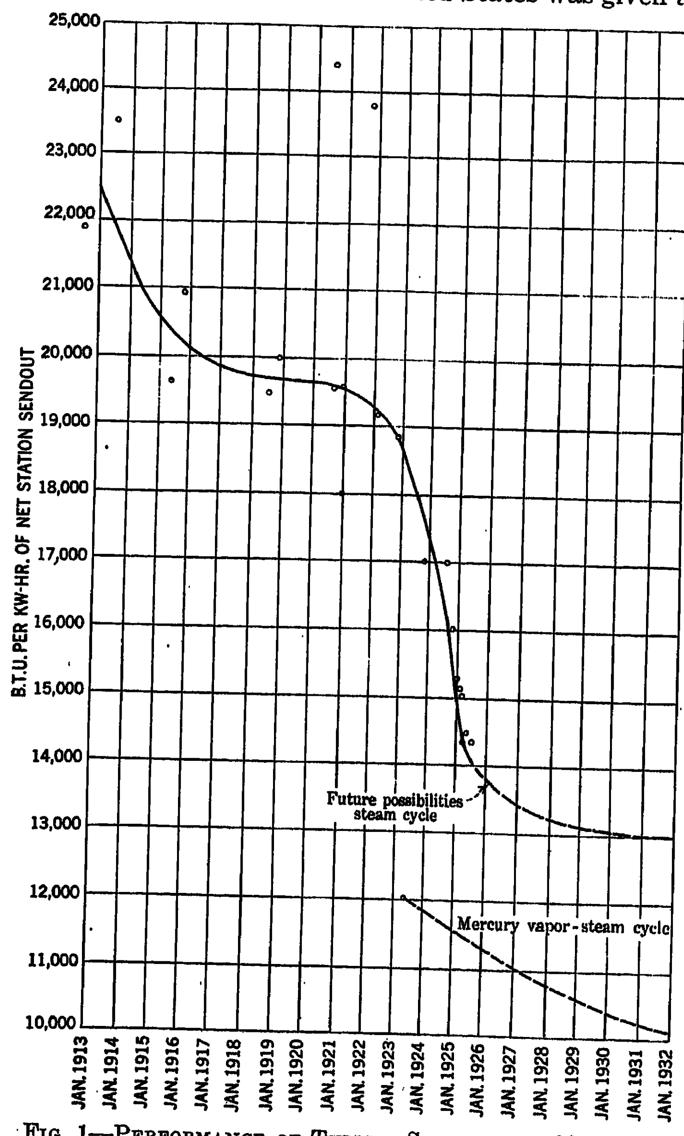

FIG. 1—PERFORMANCE OF TYPICAL STATIONS OF 60,000-KW. CAPACITY AND HIGHER PLOTTED AGAINST DATES OF INITIAL OPERATION OF STATIONS

18,030 B. t. u., and the heat consumption for the Gennevilliers Station in Paris was given as 22,240 B. t. u. We have referred earlier in this report to the remarkable performance of the Philo Station of the Ohio Power Company, which has been as low as 13,715 B. t. u. per kw-hr.

These same facts referred to in the preceding paragraph may be visualized by reference to the curves in Fig. 1, which show the trend of performance of typical stations plotted against the dates of initial operation of these stations. This curve is not the product of a disordered imagination, but represents the weighted average in connection with the performances plotted for twenty-two stations. At first glance, the curve appears to have a very peculiar shape. Further analysis indicates that there was a very definite reason for the slowing up of power-station development during the war period, and the extremely high prices of coal during the period from 1920 to 1922, inclusive, in no small measure account for the marked improvement in the performance of stations which have gone in service within the last six months.

The dotted extension of the curve shown on Fig. 1 is our estimate as to the future possibilities in the way of improved performance for a station designed to operate on the straight-steam cycle with a single stage of reheating. Obviously, this curve has to flatten out. We have also shown the over-all performance for a combined mercury vapor and steam station, which is indicated by the performance of the mercury-vapor turbine and boiler in the Dutch Point Station of the Hartford Electric Light Company, and we have shown by means of a dotted curve our estimate as to the future possibilities of the combined mercury vapor and steam cycle.

One might well gain the impression that the possibilities for further improvement in steam-station design have been almost exhausted. This is hardly the case. There are at least three major possibilities immediately ahead which will result in higher operating efficiencies for our steam generating stations:

a. The further development of commercial equipment for use in the application of the mercury vapor-steam cycle.

b. The development of superheaters, high-pressure steam piping, valves and turbines for operation in connection with steam temperatures of 800 deg. fahr. or higher.

c. The use of hydrogen or some equally suitable gas as the cooling medium in connection with closed ventilating systems for turbo generators, and the development of new generator designs which will take advantage of all the possibilities of this new cooling medium.

It is perfectly true, however, that while there has been a drop from approximately 18,000 B. t. u. per kw-hr. to 14,000 B. t. u. per kw-hr. in the last eighteen months, a further reduction from 14,000 to 10,000 B. t. u. per kw-hr. cannot be looked for unless use is made of the mercury vapor-steam cycle, with comparatively high pressure used in connection with the mercury vapor boiler and the most efficient possible layout in connection with the steam end of the station. As far as further reduction in the fuel cost in connection with large steam stations goes, work is being done on the law of diminishing returns. A point has already been reached where further gains are going to be very difficult of attainment.

The report thus far has dealt in large measure with the technical achievements and with the reductions in heat consumption in connection with our newer stations.

Our real function as power station engineers, however, is to deliver power on the station bus-bars at the lowest possible cost per kw-hr., and it is in the analysis of the elements which go to make up the cost per kw-hr. that we find the real answer as to the most profitable trend for future power station development.

The total cost of each kw-hr. delivered on the station bus-bars is made up of four major elements:

a. Operating labor and superintendence
b. Maintenance
c. Fuel cost
d. Fixed charges on the investment

The so-called operating cost which most of us talk

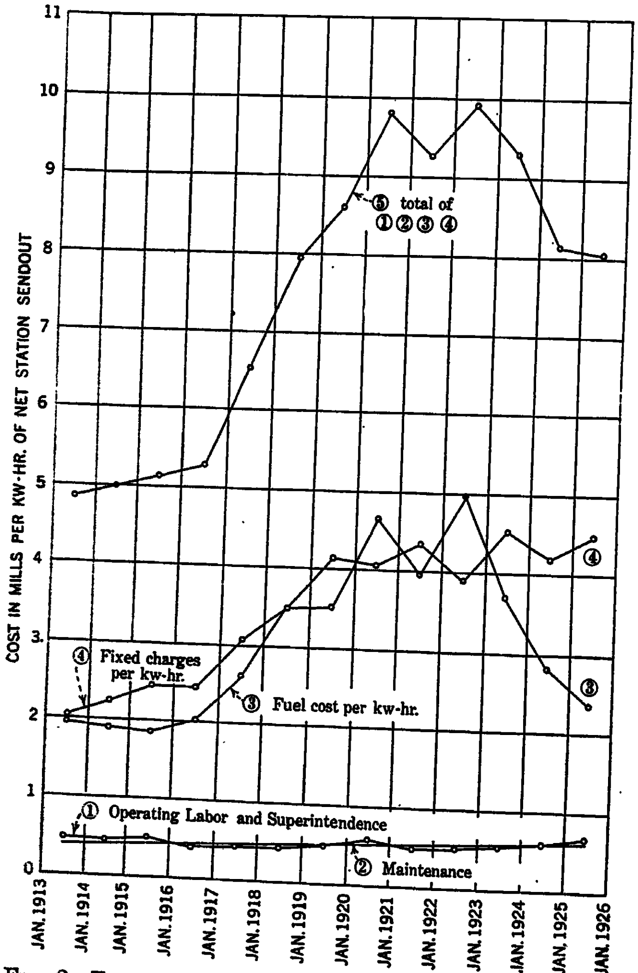

Fig. 2—Trend of Operating Costs and Fixed Charges for Typical Stations of 60,000-Kw. Capacity and Higher Plotted Against Date of Initial Operations of Stations

about when we discuss power costs, is made up of items a, b and c. Accounting systems commonly used do not spread fixed charges so as to allocate so many mils to each kw-hr. generated, and accordingly many of us lose sight of them. It is perfectly obvious, however, that the interest and the taxes paid in connection with an investment in power stations and the money which must be set aside each year to provide for renewals due to depreciation and obsolescence are just as tangible elements entering into the cost per kw-hr. as the money which must be paid for fuel to generate that same kw-hr.

In Fig. 2, is shown the trend of operating costs and fixed charges of typical large power stations plotted against the dates of initial operation for these stations. As in connection with Fig. 1, only power stations of 60,000-kw. capacity and higher are considered. An endeavor has been made to take into account all the

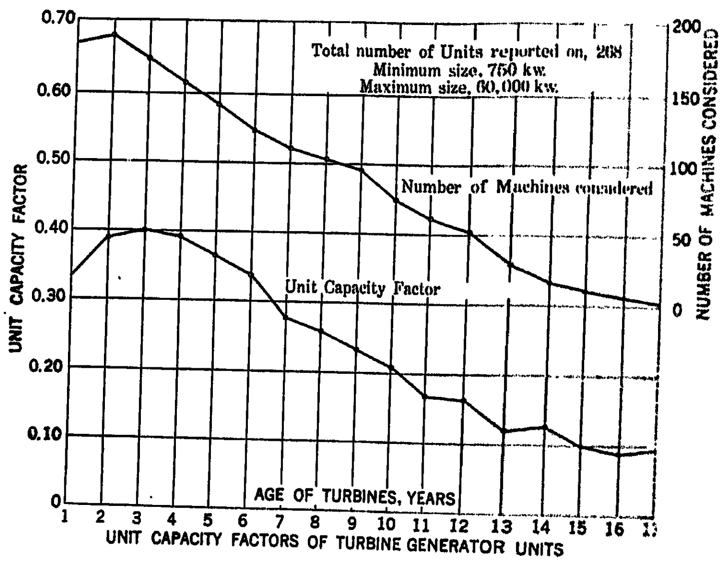

Fig. 3—Unit Capacity Factors of Turbine-Generator Units

different factors in the derivation of these curves. Some of these factors are as follows:

1. The variation in the cost of labor, both for operation, maintenance, and for construction work during the period from 1913 to 1925.
2. The variation in material costs and equipment costs.
3. The variation in coal prices (costs of coal delivered in the bunker for a power station in Baltimore has been taken as the basis).
4. The variation in the cost of new money in connection with financing which has been done during succeeding years.
5. The variation in taxes which have as their basis the investment in power stations.
6. The variation in the probable use factor of equipment placed in service during succeeding years. The average load factor of the system, the outages of equipment for inspection and repairs, and the amount of spare capacity considered necessary, are factors which go to make up the use factor in connection with the station.
7. The decrease in man hours of operating labor per kw-hr. incident to certain developments in power-station design.
8. The tendency towards the decrease in cost per kw. of installed capacity due to certain trends of power station design, and the conflicting tendency towards increased power station costs due to other trends of power station design.

The thing which stands out from an inspection of the curves in Fig. 2, is the relative trend of the fuel cost and the fixed charges during recent years. There has been a marked reduction in fuel cost due to the improvements in power station design and due to the declining price of coal. There is a definite upward tendency, however, in connection with the fixed charges per kw-hr., and as the matter stands at the present time the fixed charges

are 4.5 mils per kw-hr. as compared to a fuel cost of 2.4 mils per kw-hr. For those stations which are located in closer proximity to the coal mines and have the advantage of lower coal costs, the fuel costs will be still further reduced, assuming, of course, the same modern power station design for the best possible economy. The fixed charges, however, will remain at 4.5 mils.

The indications are that if we strive for lower fuel costs by the use of more efficient stations, the fixed charges per kw-hr. will rise still higher, and the increase in fixed charges per kw-hr. will more than offset the decrease in fuel cost per kw-hr.

The real job which the power station engineer has ahead of him is to decrease the fixed charges per kw-hr. and to reestablish the proper balance between fuel costs and fixed charges.

Certain executives and engineers will, no doubt, state that the fixed charges per kw-hr. in the newer stations which they are placing in operation, are very much lower than the curve indicates in Fig. 2, this being for the reason that these newer stations are carrying the base load for their system and are operating at an extremely high use-factor. Our answer is that this is a transient condition. Each new turbine or station operates on base load only for so long a time as it constitutes the most efficient turbine or the most efficient station generating power for that particular system. The curves given in Figs. 3 and 4 tell this story in a very striking manner. They show that the generating unit is heavily loaded during the second and third years of its life, but that from then on the loads carried during succeeding years gradually decline. These curves given in Figs. 3 and 4* constitute the actual operating history as regards kw-hr. generated during succeeding years by 268 turbines ranging in size from 750 kw. to 60,000 kw.

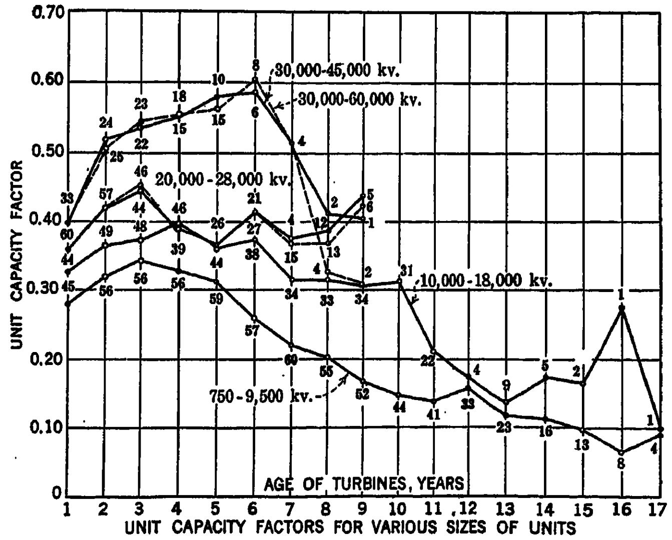

Fig. 4—Unit Capacity Factors for Various Sizes of Turbine-Generator Units

*The data given in Figs. 3 and 4 was compiled by the Turbine and Generator Committee, Association of Edison Illuminating Companies, and embodied in the report of that Committee for 1923.

It is perfectly obvious that fixed charges per kw-hr. must be determined by taking the total fixed charges on a piece of equipment or on a station during its life and spreading them over the total number of kw-hr. generated by that particular turbine or station during its useful life.

It is of interest to note from Fig. 3, that the average unit capacity factor during the 17-year period on the 268 turbines is approximately 25 per cent. The curve of fixed charges given as a part of Fig. 2 was based on the assumption that for generating stations placed in operation in 1913, the average unit capacity factor during their life would be 31 per cent, and for stations placed in operation in 1925, the average unit capacity factor would be 36 per cent. The evidence would tend to indicate, therefore, that if error has been made, it is on the side of showing the fixed charges per kw-hr. too low rather than as too high.

Tendencies in the design of steam generating stations may be classified under four heads:

1. Tendencies which improve the reliability of the power station, increase its cost, but do not appreciably affect the operating efficiency; for example:

a. The use of house turbines, auxiliary generators, and storage batteries for insuring the auxiliary power supply.

b. Isolated-phase layout and the use of reactors and other protective devices in the switchhouse.

c. The duplication of auxiliaries, and provision of excessive amounts of spare capacity in boilers and turbines.

2. Tendencies which decrease the coal consumption per kw-hr. and increase the cost of the power station; for example:

a. The use of excessively high steam pressures taken together with a single stage of steam reheating during its expansion.

b. The use of pulverized fuel-burning equipment.

c. The use of adjustable speed motors for driving auxiliaries where saving in power consumption at light loads is the consideration.

d. The use of air heaters or economizers usually falls in this classification.

e. The use of an excessively large amount of surface in the surface condensers for the main turbines.

3. Tendencies which decrease the coal consumption per kw-hr. and also result in a reduction in the cost of the power station and perhaps in the cost of operating labor; for example:

a. The use of electrically-driven auxiliaries.

b. The use of moderately high steam pressures without reheating.

c. The use of the highest steam temperatures which are possible with existing materials.

d. The use of large turbines and large boilers.

e. The use of three or four-stage bleeding for raising the temperature of feed water to within 75 deg. or 100 deg. of saturated steam temperature.

f. The use of large mills for pulverizing coal.

4. Tendencies which add to the cost of the station without improving either its reliability or appreciably decreasing its coal consumption; for example:

a. Insufficient care given to grouping of equipment and waste space in power station building.

b. Too many architectural frills.

For a particular set of operating conditions, some of the examples which have been cited above as falling in one classification may really classify themselves under an entirely new head. An inspection of the curve given in Fig. 2, however, presses home the conviction that every tendency which makes for an increase in the cost of steam generating stations, and correspondingly increases the fixed charges, must be viewed with suspicion. The burden of proof should be on the designing engineer to show why the particular feature should be embodied in the design. The same line of reasoning indicates that the designing engineer should give intensive study to those tendencies of power station design which hold forth promise of giving lower first costs and lower fixed charges, as well as lower fuel costs.

Now, turning to the field of hydroelectric plant design and operation, we find somewhat different conditions obtaining and no such revolutionary changes taking place as there are in connection with steam station design. There is one definite tendency in hydroelectric plant design very similar to that which has made itself evident in recent years in steam station design, namely, the trend toward the use of larger water-wheel turbines and larger water-wheel generators. In the main, the objective is not higher efficiency, but lower cost per unit of installed capacity. The tendency seems to be toward using as large water-wheels as the turbine manufacturers can build in single runner vertical turbines for the operating head that is to be developed. An effort is then made to match this water-wheel turbine with a generator which will permit delivering the full output of the water-wheel turbine to the station busbars. Water-wheel type generators of 65,000 kv-a. have been built. It is true, however, that these generators have been built for special plants, and that only in a few cases has a maximum rating of 35,000 kv-a. been exceeded. However, the size of generators is entirely economically dependent upon the individual case and its relation to the size and charging current of the transmission lines, if not limited by the maximum possible output from the water-wheel turbine.

In the future, it is going to be even more important than in the past to build water-power plants with a low first cost per unit of capacity. This must be done if the power from water-power plants is to compete with power from the larger and more efficient steam generating stations. The use of larger generating units is the one means that holds forth most promise for decreased station costs. Further, with the greatly increased total loads in connection with the big power systems, it becomes perfectly feasible to install generating units of large capacity. The tying of a number of plants into the transmission network of a large system makes single unit water power developments practical, and combined with remote or supervisory control, such developments permit a simple layout, resulting in low operating expense. This is particularly the case with some of the smaller power sites which are relatively close to existing developments. Simplicity of layout is very desirable, as it has a direct effect on reduction of costs, both capital and operating. The importance of this trend toward the use of larger generating units in connection with hydroelectric plants has made it seem worthwhile to prepare a symposium of the views of the engineers of the several different manufacturing companies on this particular subject. These statements are quoted herewith:

Statement by W. M. White. There are five principal factors which limit the maximum capacity of hydroelectric equipment. These reasons are given below:

a. Shipping facilities
b. Material size limits
c. Economical generator speeds
d. Manufacturing limits
e. Strength and life of parts

Shipping facilities are an important factor on many developments, the runner being the item most frequently affected. Runners for medium and low heads may be sectionalized, but this is expensive, and for high heads it is doubtful whether sectionalized runners can be made sufficiently strong. Other parts of the turbine can usually be sectionalized to accommodate shipping, but the runner is usually the limiting factor.

The sizes of available material is another important limiting factor. Shafts must be forged from steel ingots. At the present time, the largest ingots obtainable are about 80 in. in diameter, so that the shaft flange must come within a reasonable margin below this diameter. This factor seems to be one of the important points and limits size to about 100,000 h. p. at 200 ft. head, or 110,000 h. p. at 300 ft. head.

Economical generator speed is the most important factor for low head developments. The cost per kw. of generator equipment increases rapidly for speeds below 80 or 90 revolutions. Seventy rev. per min. is about the minimum for economical generator construction and this fact places the limit at about 12,500 h. p. at 20 ft. head and 30,000 h. p. at 40 ft. head.

Incidentally, the runners for large low head developments are usually larger than can be shipped in one piece, but propeller-type runners can readily be made in several sections so that shipping is not the most important factor for low heads.

Manufacturing limits are another important factor, that is, there is a maximum height and diameter of parts which can be handled with present shop tool equipment. Larger tools can be designed and constructed, but with the limited use for such equipment there is a question as to whether or not this is economical. At the present time, most of the manufacturers of hydroelectric equipment are able to handle diameters up to 28 or 30 ft. and heights of from 10 to 12 ft. Without exceeding these limitations, it is possible to build units for about 52,000 h. p. at 70 ft. head, 70,000 h. p. at 100 ft. head, and 90,000 h. p. at 150 ft. head.

Strength and life of parts is another important factor limiting the maximum capacity of hydroelectric units. For high head developments, the question of penstock material is sometimes a limiting feature and practically limits us to 100,000 h. p. capacity under 2000 ft. head and 60,000 h. p. capacity under 1000 ft. head. These turbines would be of the impulse type. For heads between 800 and 400, the question of strength and life of the runner is an important factor. Pitting occurs more frequently under these high head conditions and it is, therefore, advisable to make the diameters larger in order to decrease the relative velocities in an effort to decrease this pitting. Smaller runners of higher specific speeds could deliver the same h. p., but their life and strength is questionable and in all probability they would pit seriously in a short time. These factors seem to

indicate that 110,000 h. p. is the upper limit for heads of 400 to 800 ft., using the reaction type of runner.

These limitations are based not on a consideration of either hydraulic equipment or electrical equipment alone, but on a combination of the two as a hydroelectric unit. Undoubtedly, turbines of larger capacity could be constructed for heads below 100 ft. and generators of larger capacity for the high heads, but considering the combined hydroelectric unit, the values given represent the upper economical limit.

Statement by E. V. Gibbs. The curve shown on Fig. 5 indicates the maximum h. p. capacity for which it is possible to build water-wheel turbine units for heads ranging from 20 ft. to 600 ft. The specific speed, corresponding to the different heads, is also shown in this same figure—No. 5, and the formula for the specific

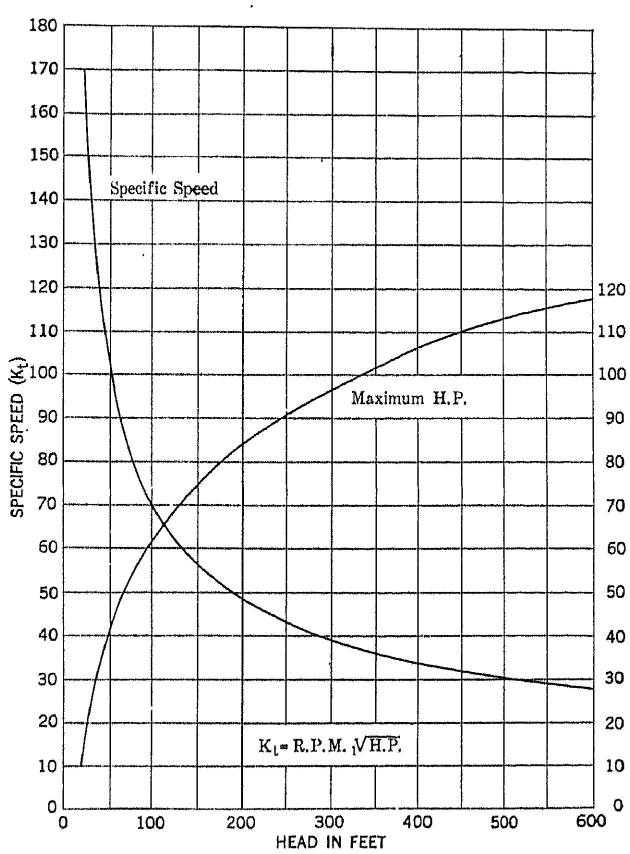

FIG. 5—CURVE SHOWING SPECIFIC SPEED AND MAXIMUM CAPACITY FOR WATER-WHEEL TURBINES DESIGNED FOR DIFFERENT OPERATING HEADS

speed at 1 ft. head is given. At the present time the maximum h. p. curve is considered the maximum output of a single runner turbine unit under the corresponding turbine head.

From any given head up to 600 ft., a specific speed from the curve can be found, thus leaving two unknown factors in the formula to be determined, viz., the speed and power. Either one of these unknown factors will have to be assumed under the given head, and this reduced to an equivalent at 1 ft. head by using the formula for speed which varies as the square root of the head.

The h. p. under the given head can be reduced to the equivalent at 1 ft. head by the formula that h. p. varies as the square root of the cube of the head. If it is desirable to obtain the speed of the maximum unit under the given head, take the h. p. from the curve for maximum h. p. shown on Fig. 5, and figure the same way to obtain the proper speed. While it is possible to build a turbine to operate under a 1000 ft. head, the field for pressure turbines under this high head needs further exploration before it would be wise to recommend them.

Statement by R. V. Terry. Relative to maximum h. p. of hydraulic turbines, the Newport News Shipbuilding and Dry Dock Company is prepared to build for different operating speeds and heads:

Since specific speed, $N_s = \dfrac{N\sqrt{P}}{H^{\frac{5}{4}}}$, the maximum power ($P$) for a given speed ($N$) and head ($H$) will depend upon the maximum specific speed selected for the particular head in question. The so-called Experience Curve, $N_s = \dfrac{5050}{H+32} + 19$ has been used in the past. This curve gives values of N_s too low for heads from 10 to 450 ft. and too high for 450 to 1000 ft. Specific speed should not be used as an absolute basis for the selection of the maximum head; other factors, including the draft head, should be given due consideration. However, for a number of years the formula, $\dfrac{632}{\sqrt{H}}$ has been used for the maximum specific speed to which it is possible to go, except under expecially favorable conditions as to low draft head. A few installations of the propeller type have been made with specific speed beyond values given by the latter formula, but they may be considered experimental installations. Therefore, in working up data for this report, a maximum specific speed for a given head equal to $\dfrac{632}{\sqrt{H}}$ was used.

Fig. 6 shows the maximum specific speed, maximum power and maximum speed for a given power that is proposed, using the head as a starting point. If the maximum power for a given speed and head is desired, this can be obtained from Fig. 6 or directly from the formula, $P_{max} = \dfrac{632^2\ H^{3/2}}{N^2}$. Fig. 7 shows the formula in graphical form. In using the formula, one must, of course, bear in mind the limitation of power for a given head due to physical dimensions and shipping facilities. This Company is prepared to build turbines as physically large as can be conveniently transported and erected.

A lower value of specific speed than that given above may be assumed for a given head. This simply means that for a given unit there is a certain choice of speeds. However, as stated previously, the power for a given head and speed will be higher, the higher the specific speed selected.

Statement by T. A. Worcester. With the larger units, the first cost, including building and operating cost, is less per kv-a. than with the smaller units. There is a physical limitation, however, to the size of units which can be built with present day materials and types of construction; and the kv-a. capacities vary with the speed of the units. These maximum size units are approximately as follows:

10,000	kv-a.	at	720	rev. per min.
20,000	"	"	600	"
30,000	"	"	514	"
55,000	"	"	400	"
80,000	"	"	300	"
110,000	"	"	200	"

This table might be carried to lower speeds and larger sizes. For instance, it is theoretically possible to build a 130,000 kw. machine at 100 rev. per min., but it is questionable if it is at all desirable to put so much capacity in a single unit.

Statement by F. C. Hanker. The 1924 water-power devel-

opments have been characterized by studies and investigations of a number of very large water-power developments in the north and northwest parts of the country. These developments have involved consideration of water-wheel generators approaching the limits of commercial design. The following gives an approximate tabulation of the economic design limitations of vertical water-wheel generators at the present time:

5,000 kv-a.	at 720	rev. per min.
12,000 "	" 600	"
40,000 "	" 514	"
45,000 "	" 400	"
70,000 "	" 300	"
100,000 "	" 200	"
75,000 "	" 100	"

This tabulation assumes that the more usual conditions of 60 cycles, 13,200 volts, 80 per cent overspeed, and normal flywheel effect apply and that standard commercial material is employed. At speeds below about 300 rev. per min., the physical dimensions of parts became a limiting factor. Low speed water-wheel generators are probably the largest pieces of material with which the electrical industry has to deal. When the dimensions become so great that special shop space and shop tools must be provided for the machine operation, the entire expense of this special equipment must be borne by such a relatively small number of machines that the cost becomes prohibitive. At the present time, the largest machine tools in this country will accommodate a frame bore (which is the same as the outside diameter of the armature punchings) of not more than 37 ft. This limiting dimension has been approached by the ratings given in the above table for speeds below 300 rev. per min. It appears quite probable that these limiting conditions will be reached by a few water-wheel generators in the near future. In general, machining limitations are reached before the transportation limitations become a serious factor since the generator can be separated into parts and a large amount of the assembling done in the power house location if necessary without greatly increasing the over-all expense.

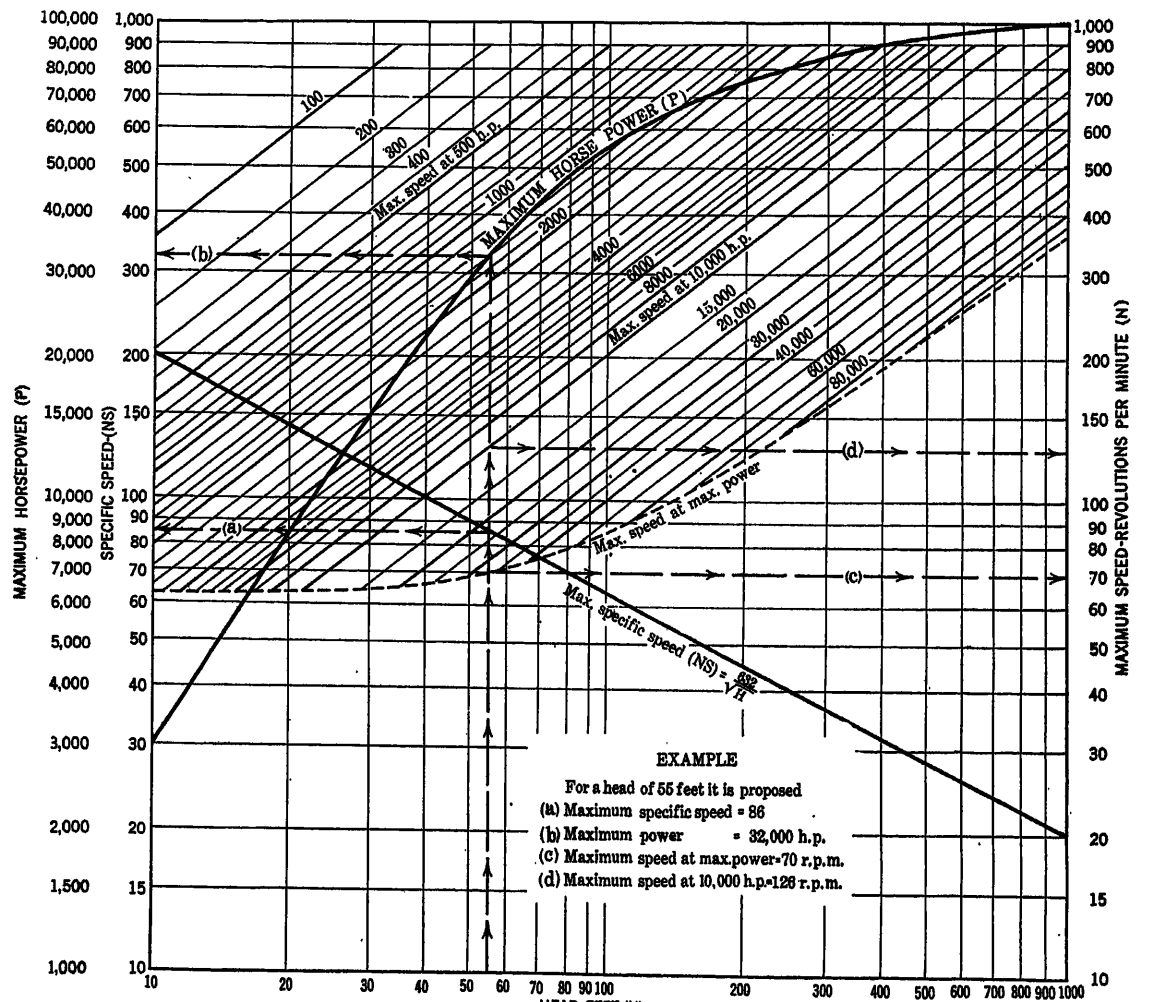

FIG. 6—CURVE SHOWING MAXIMUM SPECIFIC SPEED, MAXIMUM CAPACITY AND MAXIMUM OPERATING SPEED FOR WATER-WHEEL TURBINES DESIGNED FOR DIFFERENT OPERATING HEADS

TREND OF MODERN PRACTISE

The manufacturers seem to feel that automatic generating stations are growing in importance, but they, together with the majority of the operating men, feel that the complete automatic hydroelectric station is limited to the smaller sizes, there being two different reasons for this: In the first place, as the size of the installation increases, the complexity of the operating conditions determining the amount of power to be supplied from the station also increases, this being something that is controlled, *not* by the station itself, but rather by system conditions; also in important

installations, considerable judgment on the part of the load dispatcher is required for the best system operation, which cannot be supplied by any automatic station equipment. In the second place, there is no entirely satisfactory way of supplying the initial starting impulse for the control of the full automatic station. Voltage conditions which are satisfactory in railway stations will not do; drop in frequency is not satisfactory; so that, generally, the water level appears to be the most satisfactory means of control, and this very materially limits the scope of the automatic station.

mittee, who points out the advisability of having a unit or units in a given power plant controlled by a gate mechanism of another unit or units in the same station. In other words, the plan of operation consists in starting and stopping units automatically and from the gate motion of other units, so that the gate controls of all units are kept within the efficient range and probably within narrow limits where maximum efficiency is obtained, depending upon the number of units under such control.

While there has been a definite movement toward

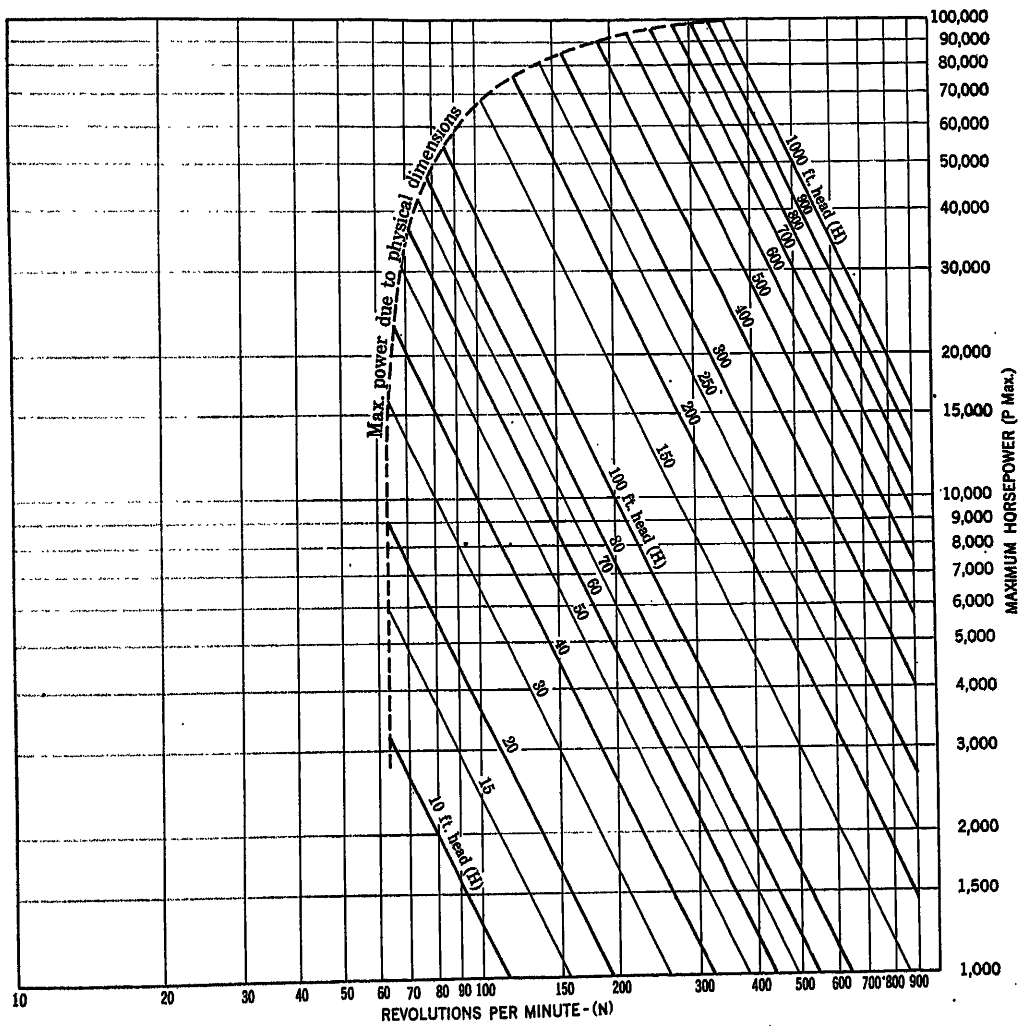

FIG. 7—CURVE SHOWING MAXIMUM CAPACITY FOR WATER-WHEELS DESIGNED FOR DIFFERENT OPERATING HEADS AND OPERATING SPEEDS

However, automatic stations with supervisory or remote control are quite different and developments along this line have been very rapid. Stations with three generating units and with capacities up to 10,000 kv-a. are now so controlled; several stations of 25,000 kv-a. with this type of control are under consideration. This type of control permits the operation of stations with fewer men and gives these men greater freedom to leave the operating floor to attend to duties at the head gates or in the switch yards.

A somewhat different phase of automatic operation has been proposed by one of the members of this Com-

outdoor substations and the location of all oil switches and transformers outdoors, progress in the matter of putting generating equipment outside has been very slow, the difficulty being, apparently, to provide suitable housing in case it is desired to dismantle a generator. However, several stations have been built with a very low superstructure with an outside gantry crane and with movable roofs over the individual generators, which can be rolled back so that the gantry can be used to dismantle any particular unit.

A noticeable tendency in connection with hydroelectric plants constructed in the west during the last

two or three years, has been the trend toward more permanent construction. Many of the earlier installations used open conduits. On all important developments now being made, these open conduits are being superseded by tunnels. Pipe lines that were formerly placed in trenches are now being placed entirely above the ground and carried in concrete saddles. The larger amounts of power that are being transmitted call for higher voltages, which in turn call for types of construction that are far more lasting and dependable than the wooden pole lines formerly used.

In the matter of station auxiliaries, there is considerable tendency to get away from direct-connected exciters on the main generator shaft, making the main generators easier to dismantle and reassemble. In the matter of excitation, many stations use a combination of a waterwheel and a-c. motor coupled to the same d-c. exciter, so that if one of the prime movers fails the other will continue to drive the exciter.

Several new European types of high-speed turbine runners are coming on the market in America. These appear to have considerable promise on account of their high efficiency.

There are instances this year of the use of babbitted bearings on vertical shaft turbines in place of lignum-vitae bearings to reduce maintenance expense.

Increased use of motor-driven, flyball governors is noticeable.

A subject which has received special study by both operators and manufacturers during the last year is the pitting of turbine runners, both as to the exact nature of the pitting, cavitation or corrosion itself, and as to the causes thereof. The three principal causes of pitting are believed to be excessive draft head, excessive specific speed and poor design. There is considerable difference of opinion as to the relative importance of these causes. The drive for high specific speeds in the last few years is blamed by some for the recent increase in pitting troubles.

A subject that has occasioned considerable discussion is the omission of governors from hydroelectric units, particularly when operating in large interconnected systems. The suggestion that governors be omitted in such cases except as safety shut-down devices, has met with considerable opposition among operating men. The proper design, characteristics and functioning of governors for hydroelectric units operating in parallel with steam electric units, are questions which are now receiving intensive study.

In the case of low head-water power developments on streams with widely fluctuating flow, there is an increasing tendency to secure the maximum possible output under existing limitations of property and flowage rights by the use of crest gates or movable dams of various types.

A considerable amount of experimental work has been done on draft tubes, but there does not seem to be any general agreement as to which type is the most satisfactory. Those which apparently indicate the highest efficiency are usually very much more expensive to build. It is in connection with the low head installations that the greatest care must be taken in the design of the draft tube. The great activity in draft tube design in connection with low head plants has been brought about by the increased specific speed of units wherein a greater percentage of energy is discharged from the runner. It seems unlikely that lower specific speeds will be adopted, but, on the contrary, more likely that an effort will be made to adopt higher ones, so that the problem of the draft tube will be with the Committee for sometime to come. It seems inevitable that some type of draft tube providing radially expanding passages must be adopted to best preserve the whirling energy from the high specific speed runners. It is important, however, that methods be developed whereby these draft tubes may be installed at moderate cost.

Conclusion

Much of the subject matter of this report falls more specifically within the scope of the American Society of Mechanical Engineers than of the Institute. We feel, however, that the members of the American Institute of Electrical Engineers should have called to their attention the trends in the art of power station design and operation. With this thought in mind, this report has been prepared, and deals not with details of design and operation, but with tendencies and trends. We had this same purpose in mind in scheduling one session of the Spring Convention at St. Louis for a symposium dealing with power station design. Certainly those of our members who are associated with companies generating large blocks of power, cannot afford to lose touch with the field of power station design and operation.

For those of our members who care to delve deeper into detailed discussions of the subjects briefly referred to in this report, we recommend a careful reading of the reports of the Prime Movers Committee, the Electric Apparatus Committee, and the Hydraulic Power Committee, of the National Electric Light Association. We also commend to your attention the wonderfully interesting series of papers presented at the World Power Conference in London. The papers presented before this Conference covered in an authoritative manner practically every problem of interest to the power station engineer.

A subcommittee of your Power Generation Committee, under the chairmanship of Mr. A. R. Smith, is working on a specific problem which we think is of very definite interest: There is a lack of common terminology and well-defined terms in the discussions which are heard, from time to time, dealing with power station operating costs and performance. It is felt it would be very much to the advantage of the industry that all should speak a common language. This Committee is

trying to set up a reasonable terminology and definition of terms which will have the best chance of acceptance by the engineers who have occasion to discuss the matters referred thereto. It is not the thought of the Power Generation Committee to attempt to impose any set of definitions on the engineers of other societies who also have very vital interest in discussions dealing with power station design and operation. It is our plan to call the importance of this matter to the attention of other committees working in this same field, and by cooperating with the members of these other committees, we hope to arrive at a set of definitions of terms which will be acceptable to all.

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This article deals with static pressure and flow control, the support of the heavy rotors, and the refinements in design such as governors, draft tubes and leakage prevention. It also gives efficiencies and test methods.

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Patchell, W. H. Address by the President. Address delivered before the I. M. E. at meeting in London, March 1924.

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An analysis of the future of power interpreted in the economies involved.

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Recent Developments, new adaptations of old instruments and their use in obtaining heat balance data.

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Woodworth, H. A. Central Station Heating Requirements. *Power Plant Engineering*, Vol. 28, No. 12, June 15, 1924, pp. 658-662.

Conservation of fuel due to double use of steam.

Developments in Electrical Machine Design

By Committee on Electrical Machinery[1]

THE past year has shown a marked advancement in the art of electrical machinery, not only in the successful starting and operation of machines larger than any ever built up to this time, but also in the purchase and partial completion of machines still greater in size. Accompanying the trend towards larger sizes of units, there has been much investigation along the lines of ventilation, insulation, mechanical strength of materials, losses in iron and copper, the effect of expansion and contraction on the insulation of long armature coils, balance of the moving parts and the dissipation of heat from flat surfaces.

New applications of electrical machinery have been made in various industrial lines and transportation. The design of an electric locomotive having d-c. motors and operating from a-c. feeders was adopted. Diesel-electric drives have been applied to locomotives in sizes heretofore not attempted for this type of drive.

TURBO-ALTERNATORS

During the past year a number of 50,000-kw., 62,500 kv-a. turbo-alternators were successfully put into operation. These machines operate at 1200 rev. per min. and represent the largest single shaft generators yet put into operation. The large size and weight of the various parts of these machines represent a difficult handling and transportation problem. The largest 3600-rev. per min. generator yet to be built was successfully placed in operation during the year, it being rated at 12,500 kv-a., 80 per cent power factor. Machines having 60,000-kw., 60,000-kv-a. ratings, and operating at 1500 rev. per min. are under construction. Quotations are being requested on 75,000-kv-a., 1800-rev. per min. machines, and 83,333-kv-a. cross-compound machines are being considered.

The increasing demand for turbo-alternators of larger and larger sizes has necessitated much development on the part of the manufacturers. This development has been along the lines of improvements in construction and improvements in materials for given purposes, such as retaining bands of greater strength to hold the field end turns in place, iron for stator cores having better magnetic properties, decrease in strand loss by use of twisted conductors and different arrangements of coils, and improvements in the mechanical strength of materials which make up the rotor to allow higher peripheral velocities.

The 62,500-kv-a., 0.8 power factor, 1200-rev. per min., self-ventilated units installed have rotors weighing approximately 200,000 lb., stators, 200,000 lb. and the copper in the machine, 41,250 lb. The stators are wound with approximately 40 miles of insulated wire. The successful operation of the rotors of these large machines has been largely due to many inventions made during the past few years in machines and machine tools.

Tests made recently show that the temperature of the bare copper in commercial machines insulated for 12,000 to 14,000 volts is approximately 15 deg. greater than the temperature determined by a detector placed between the top and bottom coils when the latter is from 55 to 60 deg. above the temperature of the ingoing air; also that the temperatures shown by resistance detectors and thermocouples are about the same, and that the temperatures at the half-way location in the slot are approximately two degrees lower than at the quarter-way.[2]

The problem of self-ventilation is becoming more complex with increasing size of machines, which causes one manufacturer to recommend the desirability of external blowers being used for ventilating future machines. In the past, operating engineers have objected to the use of external blowers on the ground that they are added auxiliaries and therefore an added source of difficulties which may lead to shut-downs. This manufacturer states that the use of closed-circuit ventilation with air coolers, now coming into use, offers added advantages in the use of external blowers and makes their consideration justifiable at this time. The chief advantage of using external blowers in connection with recirculating ventilation systems is that the blower can be placed before the air coolers and therefore the temperature rise in the air passing through the blower, due to rotational loss, can be absorbed by the air coolers and air supplied to the generator at a temperature some five or six degrees lower than possible with self-ventilating systems.

In most cases ventilation is obtained by a multiple radial system in which the air passes radially in and out through the stator core. During the past year there were published results of experiments on this type of ventilation showing that the air is distributed in accordance with a simple hyperbolic or trigometric sine law. Knowledge of the balance state of flow when several branches of air meet and divide in a tube, the intake and discharge, occurring normally to the walls of the

1. Annual Report of Committee on Electrical Machinery.

H. M. Hobart, Chairman

C. A. Adams,	G. Faccioli,	John C. Parker,
H. O. Albrecht,	W. J. Foster,	R. F. Schuchardt,
B. F. Bailey,	V. Karapetoff,	C. E. Skinner,
B. L. Barns,	A. E. Kennelly,	A. Still,
B. A. Behrend,	P. M. Lincoln,	A. H. Timmerman
A. C. Bunker,	A. M. MacCutcheon,	P. Torchio,
James Burke.	F. D. Newbury,	R. B. Williamson.
L. L. Elden,	J. M. Oliver,	

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 23, 1925.

2. JOURNAL of the A. I. E. E., October 1924, page 926.

tube, depends on the solution of a system of simultaneous transcendental equations. The total pressure required for a certain volume of air per unit time can be expressed by hyperbolic and trigometric cotangents of a certain argument which contains the geometrical dimensions of the air circuit.[3] Large models of turbo-alternators employing different types of ventilation were also set up and tested.

The use of closed ventilating systems has two distinct advantages, in that accumulation of dust on the windings is practically eliminated and fire can be easily extinguished by introducing an inert gas into the system. Several fire extinguishing systems have been installed during the past year. These consist of an arrangement for injecting carbon dioxide gas into the ventilating duct. The arrangement can be either manually operated or operated by the differential and balance relays used for protecting the machine against short-circuited turns or grounds. The results of exhaustive investigation along this line, published during the past year, show that the insertion of 25 per cent carbon dioxide gas, by volume, into the ventilating system will extinguish all flames.

Some of the inert gases have characteristics much better than air for use as the cooling mediums in high speed electrical machines with recirculating cooling systems. Many gases have been considered but the result of much study shows that hydrogen has the better properties if explosion dangers can be eliminated. The advantages which hydrogen offers for this use as compared with air are enumerated below:

Provided air can be excluded, hydrogen is a good fire extinguishing medium. The windage losses are greatly reduced due to the much lower specific gravity of hydrogen and its high thermal capacity. The temperature drop required to transmit heat through various parts of the machine is reduced. The temperature drop required to transmit heat from the hot surfaces of machine to the hydrogen and from the hydrogen to the coolers is less causing the hydrogen to return to the machine at a lower temperature.

Results of tests on generators filled with hydrogen show that it is possible to carry 30 per cent greater load on a machine with the same temperature rise when hydrogen is used as the cooling medium in place of air.

It would be comparatively simple to make a rotating machine hydrogen-tight if suitable shaft packing could be developed. Apparently shaft packing is at the present time, the largest obstacle hindering development of commercial hydrogen-filled machines. Nevertheless, this subject is worthy of further consideration and study.

Recent experiments show that there is considerable loss in the inactive magnetic parts and active magnetic parts of these machines. Miniature turbine-type generators were built for carrying out these investigations and the percentage losses were found to bear a similar relation to those of larger machines. Wood was substituted for various inactive parts and the magnitude of losses in these parts determined. Experiment showed that there is a high-frequency harmonic flux which causes considerable loss in the end structure and that these losses bear a relation to the magnetic loading of the field. These data together with the results of further study which is now being carried on, give rise to the hope that the efficiency of future machines will be still higher because the nature of losses in heads of machines will be definitely known.

Higher efficiency and lower temperature rises have been secured by grooving the surface of the rotor.

Synchronous Motors

A magnetic clutch has been applied to the synchronous motor allowing the motor to run at synchronous speed before the load is applied. By this means the motor can start heavy loads with a starting torque as high as the pull-out torque of the motor, and synchronous motors can therefore be used where good power factor characteristics are desired but where heavy starting duty is required.

The motor consists of a standard synchronous machine, having a rotor which is not keyed to the shaft, but free to move on a bearing carried by the spider. The field member of the clutch is bolted to the spider while its armature is fixed to the drive shaft. During the starting period, the rotor is brought up to synchronous speed and the drive shaft remains stationary. After the motor field is excited and line voltage applied to the stator winding, the magnetic clutch is excited, drawing its two halves together and bringing the load up to speed.

This motor has the advantage of being adaptable to full automatic, manual, or semi-automatic control. It is of sturdy construction, with desirable power factor characteristics, high load-starting torque and low starting current.

Another motor has been developed which accomplishes the above purposes in a different way. This motor is essentially a synchronous motor but differs mechanically from the ordinary type by having the stator mounted on auxiliary bearings. When the motor is started the rotor remains stationary and the stator comes up to synchronous speed. A friction brake is provided for bringing the stator to a standstill, thus causing the rotor to come up to synchronous speed.

By this method the relative speed between rotor and stator is the synchronous speed of the machine during the starting and running periods. In this motor, as in the one mentioned above, the starting torque available is the pull-out torque of the synchronous motor.

The other advantages are the same as enumerated for the motor above.

Induction motors have been arranged for operating at various speeds by being provided with armature windings which can be connected for various numbers

3. A. I. E. E. JOURNAL, March 1924, "The Multiple-Radial System of Cooling Large Turbo-Generators."

of poles, but until recently the synchronous motor has been inherently a single-speed machine when the frequency of power supply is constant. During the past year a synchronous motor has been developed, and a 5000-h. p. motor actually built, which has pole changing switches for both the stator and rotor windings.[4] This machine has pure synchronous motor characteristics for both speeds and requires no more attention than the ordinary synchronous motor. As the first cost of this motor is only slightly above that of the ordinary synchronous motor for the low speed, it is a practical machine which should broaden the field for synchronous-motor application.

The field poles are constructed so that they can be combined in groups of two having the same polarity, which gives the effect of one half the original number of poles and will therefore cause the machine to rotate at twice the speed when the stator winding is reconnected for the corresponding number of poles. The poles are shaped to compromise between the best shapes for each speed.

Frequency Converters

The year just past has seen not only the largest synchronous frequency changer yet built put into operation, but has also seen an induction-type frequency changer of the same capacity installed and successfully operated. The rating of these machines is 35,000 kw. A second induction frequency changer is now being built which will be rated at 40,000 kw. unity power factor for transfer of energy from the 60-cycle to the 25-cycle systems on which it is used and at 38,000 kw. unity power factor for transferring power from the 25-cycle to the 60-cycle system. This machine will be equipped with a phase-shifting device which will operate hydraulically.

The 35,000-kw. synchronous frequency changer rotates at 300 rev. per min. and has shown an efficiency of 96.1 per cent under operating conditions. This over-all efficiency represents an efficiency of each unit better than 98 per cent which is very high for rotating machinery. The machine is self-ventilated and has very good starting characteristics for a machine of its size. The induction frequency changer was put into operation with no difficulty and when used in connection with tap-changing switches in the air-blast transformer and reactors, showed a larger condenser capacity for power factor correction at reduced loads than could be obtained from a synchronous frequency converter of the same rating.

A new type of frequency converter was developed for tying together two systems, the frequency of each being susceptible to changes. This type is known as the Scherbius—controlled, load-regulating type of set. One machine is a synchronous generator and the other an induction motor having a Scherbius speed-regulating set. With this arrangement any desired amount of load can be delivered in either direction up to the limits for which the set is designed irrespective of the relative frequencies of the two systems. Two of these sets having ratings of 6000 kw. were installed during the past year. They form a tie between a 60-cycle and a 25-cycle system and are designed to operate with a total variation in frequency of nine per cent. If the 60-cycle frequency system is constant, the machine rotates at 720 rev. per min., the induction machine operating at above or below synchronism depending upon the frequency of the 25-cycle system.

4. Journal of the A. I. E. E., April 1925, page 339.

Transformers

The size of transformers installed and put under construction during the year represents a new high mark. Transformers were installed rated at 15,000 kv-a., 60 cycles, 7200-132,000 volts, three-phase, self-cooled, having a 12,470-volt tertiary winding. The efficiency of these transformers is better than 99 per cent and they have a regulation of 1.1 per cent. Since the largest self-cooled transformers constructed up to the year 1920 were of the order of the 10,000-kv-a., the above units represent a new level for three-phase transformers of this type.

Some 22,000-kv-a. single-phase, oil-immersed, water-cooled transformers, having 12,000-volt delta-connected primaries, and 39,500-volt secondaries, Y-connected to give 68,500 volts, were put into operation. These transformers do not have output ratings as large as other transformers now in service at 60 cycles, but, due to the fact that they operated on 25 cycles, the quantity of materials used in them was much greater than in any transformers of this type yet put into operation. Three-phase transformers, rated at 75,000 kv-a., have been built in Europe.

Improvements made in transformer design include a new type of ration adjuster switch by which the ratio can be altered while the transformer is carrying load. Potheads were developed for mounting on the side of transformers in order to provide a means for bringing underground cable into the transformer without exposing terminals or live parts. The pothead consists of two compartments, one in which the cable terminates and another in which disconnect links are provided. The lower compartment is filled with petrolatum and the other compartment with transformer oil. The conductors from the three-conductor cable are connected to lugs in the pothead which are arranged on the arc of a circle to make the reversing of conductors possible without splicing. A ground switch is now being designed to be located in the upper compartment and controlled remotely from the opposite side of the transformer. This switch will provide a convenient means for grounding the feeder connected to the transformer.

A new device has been developed for indicating the load on pole-type distribution transformers. This consists of two temperature detecting elements, one immersed in the oil and the other mounted in a case for

detecting ambient temperatures. These two temperature detectors are connected in series to a pointer which moves by the action of the temperature detector in the oil, which movement is modified by the action of the detector affected by the ambient temperature. When the temperature exceeds a safe value a target shows.

Several installations have been made of a transformer which is filled with nitrogen above the surface of the oil. The nitrogen used is automatically obtained from the atmosphere by passing the air breathed by the transformer through deoxidizing chemicals. The nitrogen, when formed, is conserved by a breathing regulator. The presence of nitrogen over the transformer-oil surface prevents the formation of sludges, (due to oil coming in contact with oxygen in the air), extinguishes fire, eliminates secondary explosions caused by mixture of the gases with air, and cushions primary explosive pressure from the sudden expansion of gases.

A radiator valve has been developed for use between the radiators and tank of large, self-cooling transformers, when it is necessary to remove the radiators for shipment. The valve consists of a tube projected into the radiator flange on the transformer tank and has a gasketed disk which is held against the end of the tube by means of a spring. It is operated by a handle. Rotating the handle 180 deg. in the proper direction opens or closes the valve. When the valve is open the handle may be removed to prevent unauthorized persons from operating it.

A new type of dehydrating breather for transformers has been developed, consisting of a screen basket suspended on a spring of large diameter. When the basket is filled with the proper amount of calcium chloride the spring is compressed and a pointer indicates the correct charge. As the calcium chloride absorbs moisture, the weight is increased and the spring is compressed until the pointer is opposite a refill designation. The calcium chloride is isolated from the humid atmosphere when the transformer is not breathing by means of check valves, and the device is designed so that the calcium chloride will not become saturated until after the refill designation has been reached by the pointer.

Synchronous Condensers

During the year, construction has been started on a 40,000-kv-a., 600-rev. per min. synchronous condenser representative of the largest machine of its type yet to be developed.

Spring Suspension for A-C. Generators

A new method has been developed for preventing the vibration of large a-c. generators from being transmitted from the stator to the supporting foundation. This arrangement consists of a spring mounted in the frame of the machine supporting the entire stator. The development of this device will aid in the successful operation of electric machinery in districts where vibration is objectionable, especially in the case of single-phase machines which have inherent vibration.

Water-Wheel Generators

The installation of the 65,000-kv-a., 12,000-volt, water-wheel generators, at Niagara Falls, has been carried on, and the first unit installed has already operated successfully for more than a year. These machines still represent the largest water-wheel generators built up to the present time. Among other large water-wheel generators about to be installed are a number of 32,500-kv-a., 12,000-volt machines, 23,160-kv-a., 13,200-volt machines, 18,750-kv-a., 6600-volt machines, and 19,500-kv-a., 6600-volt machines. The size of automatically-controlled, water-wheel generators has been advanced to 7300 kv-a.

Synchronous Converters

A number of synchronous converters have been developed for use in congested metropolitan districts. These are completely enclosed on the d-c. end and partially enclosed on the a-c. end. For cases in which systems of the same or different frequencies are tied together through the d-c. side of synchronous converters, a converter has been developed to incorporate special features for successfully withstanding current reversals feeding into a short circuit in either system.

Industrial Motors

New lines of single-phase, adjustable-speed, brush-shifting motors, having a speed range of two to one for constant-torque load have been developed. Single-phase, repulsion induction motors, having squirrel-cage rotor construction, without centrifugal devices, have been extended to include a reversing type capable of reversal from full speed in one direction to full speed in the opposite direction. Another new development in industrial motors is a new line of synchronous motors for driving ammonia and air compressors. These have double-bar, squirrel-cage windings, giving better control of current and torque under starting conditions and permitting the motors to be thrown on, full voltage. Induction motors of higher speeds with high-reactance secondary windings for full-voltage starting, totally enclosed motors with a special ventilation for operation in inflammable atmospheres and a new type of bearing which prevents air from getting into the bearing housing and oil from getting out, have been developed. Motors with enclosed collector rings have been put on the market for use in oil wells and similar service.

A New Self-Excited Synchronous Induction Motor[5]

The pure induction motor has characteristics which differ considerably from the pure synchronous motor. Each motor has advantages and disadvantages, the advantages of one being lacking in the other. A number of motors combining in part the advantages of both types of machines have been developed in the past and during the last year, there has been further development along this same line.

5. Journal of the A. I. E. E., August 1924.

A paper was recently published describing a new type of motor which has induction motor characteristics during the starting period and synchronous motor characteristics during the running period, the source of excitation when running as a synchronous motor being supplied from within the machine. The rotor of this motor carries a polyphase winding supplied with power through slip-rings and a commuted winding. The stator has an exciting winding displaced ninety electrical degrees from the axis of the brushes, and a neutralizing winding coaxial with the brush axis. Both these windings are connected to the brushes through separate variable resistances. A third variable resistance is inserted across the brushes to relieve the commuted winding under severe starting conditions.

When the polyphase winding is connected to the supply lines, a revolving flux is set up which revolves at synchronous speed when the rotor is at a standstill and is stationary in space when the rotor is revolving at synchronous speed. Since the revolving field cuts the two stator windings, displaced by ninety electrical degrees when the rotor is revolving at a speed less than synchronism, currents are set up in these windings which give a torque similar to that generated in an induction motor. These currents close over the brushes and the commuted windings and are regulated by the adjustable resistances which act in a way similar to the adjustable resistance inserted in the secondary of a slip-ring induction motor. As the rotor approaches synchronous speed additional torques are developed by the stator windings which pull the motor into step, at which time the magnetic field revolving with respect to the rotor becomes stationary in space and the brush voltage becomes unidirectional, thereby causing the magnetizations produced by the stator windings to be unidirectional.

Whenever the torque demand is greater than the maximum synchronous torque of the motor, the machine will drop out of step and become an induction motor in effect, giving the same asynchronous overload capacity as may be obtained from a corresponding slip-ring, induction motor. As soon as the torque demand is sufficiently reduced the machine will again fall into step. The motor is compounded to give power factor regulation by proper location of the brush axis with respect to the unidirectional magnetization of the secondary. All windings of the machine are utilized at all stages of starting and running.

A New Type of Single-Phase Motor[6]

Sometime ago a single-phase motor was developed which had series characteristics during the starting period and shunt characteristics during the running period, but as the motor contained a centrifugal device for decreasing the reactance of one of the windings, the motor was never adopted for production. However, this motor was a stepping-stone to a new type of motor having the same series and shunt characteristics and having no centrifugal device. The latter is termed the squirrel-cage repulsion motor and had been brought out during the past year.

This new motor consists of essentially a repulsion motor with a squirrel-cage winding inserted in slots which are located beneath the slots bearing the commuted winding. Since the induction motor winding is imbedded deeply in the rotor core, it has high reactance during the starting period, due to the relative high frequency of the induced current. However, when the speed of the rotor approaches synchronism, the frequency of the rotor currents is relatively low and the action of the squirrel-cage winding comes into play.

Since the reactance of the commuted winding is inherently low, the current flows mainly in this winding during the starting period, and torques are produced similar to those of a plain repulsion motor during the starting period. At full load the squirrel-cage and commutated windings deliver approximately equal amounts of energy.

Commutation is greatly aided by the action of the squirrel-cage winding due to the fact that it absorbs the energy which would otherwise cause sparking on account of induced electromotive force in the short circuited coil. To aid commutation further by absorbing the energy from the leakage flux which does not interlink with the squirrel-cage, a thin sheet of metal is inserted radially between the squirrel-cage winding and the commutated winding. This metal can be made of high resistance on account of the high frequency commutation and therefore does not materially interfere with the distribution of flux during the starting.

The motor has a very much better power factor, both at synchronism and below synchronism, than the plain repulsion motor. The commutation is practically perfect due to the action of the squirrel-cage winding in connection with the metal strips mentioned above. The efficiency is also considerably raised by the addition of the extra winding.

A New Alternating-Current General-Purpose Motor[7]

Another type of a-c. motor has been put on the market, which combines the starting characteristics of an induction motor with the good power factor characteristics of the synchronous motor, in a self-contained unit which needs no auxiliary machines for excitation. This motor will operate at unity or leading power factor and will carry very heavy temporary overload.

The motor consists of a rotor having a polyphase a-c. winding and a commutated winding, and a stator having a field winding and an auxiliary winding. The auxiliary winding is physically located at 90 deg. from the field winding and is closed upon itself. The field winding is connected to the commutator brushes. The power supply lines are connected to the polyphase winding on the rotor through slip rings.

6. Journal of the A. I. E. E., July 1924.

7. Journal of the A. I. E. E., April 1925.

The commutated winding does not appreciably affect the torque of the machine at standstill or low speed, but at higher speeds the commutator voltage enters into giving better synchronizing characteristics. When the machine is operating at synchronous speed a d-c. potential is present across the brushes, which serves as a direct-current source of supply for exciting the field. The machine, therefore, acts as an induction motor during the starting period and as a synchronous motor exciting itself during the running period. When the load reaches 150 to 200 per cent of full load, the machine drops out of step and continues to run as an induction motor. As soon as the load is again decreased to within these limits, the machine falls back into step and operates as a synchronous motor. The losses in this motor are comparable to those of an equivalent induction motor. However, the new motor has a tendency to be slightly less efficient at fractional loads and slightly more efficient at overload. The size of this motor is approximately the same as for the corresponding slip-ring induction motor.

Obviously there is a very large field for motors having the marked advantages enumerated above, not only for new installations where they can be made to operate at unity power factor, but also in older installations where a large number of induction motors have already been installed, in which case the lagging reactive kv-a. can be compensated by operating the new motor at leading power factor. However, the cost of these motors is considerably more than the corresponding induction motor, as may be expected.

A "Wool-Yarn" Oiling System for Small Motors

Many classes of service for small motors call for long operation of the motor without attention. Such service demands an oiling system for the motor bearings which is capable of supplying the bearings with the proper amount of oil for long periods without cleaning or the addition of oil.

A line of small motors has been put on the market which have a lubrication system called the "Wool-Yarn" System, consisting of a number of continuous strands of wool yarn placed over the shaft and projecting down into the oil in the well. In this system the oil is carried from the well to the shaft by capillary attraction instead of a revolving ring as is the usual practise. The compartment carrying the yarn and oil is practically dust proof, every precaution having been taken to prevent dust from entering around the shaft.

Even though this system is not applicable to motors larger than a few horsepower, it has marked advantages for small motors, in that the yarn acts as a filter for the oil and the oil in the well is not agitated which helps to confine it to the well. The oil capacity is increased by the amount held in the yarn and the motor will operate for long periods without re-oiling.

Surface Iron Losses with Reference to Laminated Materials[8]

Additional experimental work on surface iron losses with reference to laminated materials was carried on during the past year. The experiments were made on special test machines, the roters of which contained no windings other than exploring coils. The machines were rotated by direct-connected d-c. motors, which were calibrated in order that the losses in the test machines could be obtained from their inputs. The surface losses were obtained by separating the fundamental frequency losses from the total losses.

It was shown that the hysteresis and eddy-current components of the surface loss can be approximately separated graphically. Skin effect decreases the losses as the decrease in eddy current loss is considerably greater than the increase in hysteresis loss. In the case of salient poles the enamel on individual laminations decreases the surface losses only slightly, which would not justify the extra manufacturing expense in most cases, even though it may materially affect the relative hysteresis and eddy losses. It was stated that the hardening of lamination edges, due to punching, affects the hysteresis surface loss appreciably if the punchings are not annealed.

Repeated Thermal Expansions and Contractions Their Effect on Long Armature Coil Insulations[9]

During the past decade the length of armatures has been increased about five feet due to the increase in the capacity of machines. Since the coefficient of thermal expansion of the copper conductor in armature coils is about fifty per cent larger than that of the mica and paper insulation, there is a considerable difference in linear expansion of the two in armature coils. Recently much experimenting was done to determine the effect of this unequal expansion on long armature coils.

Experimental coils were placed in slots formed by stacking iron laminations in the same manner as they are placed in the slots of an alternator armature. Imitation vent ducts were situated at intervals in the iron so that the entire construction was comparable to a four-slot section of a large machine. The coils consisted of square brass tubes and were 118 in. in length. The insulation was standard for 13,200 volts. Thermocouples were placed at proper points for indicating temperature. The brass tubes were heated by passing current through them from a suitable transformer, and were cooled by passing them through air for certain experiments and water for other experiments where a greater change in temperature was desired.

Automatic equipment was used for heating and cooling the coils which consisted of time delay relays for putting on the current, starting the blower, etc. 7800 cycles of temperature changes of approximately 75 deg. cent. were given the coils, followed by 2512 cycles of

8. Journal of the A. I. E. E., August 1924.
9. Journal of the A. I. E. E., November 1924.

100 deg. cent. difference in temperature, 825 cycles of 130 deg. difference in temperature, and 400 cycles of 160 deg. temperature difference. The hottest temperature reached by the coils ranged betwen 150 and 180 for all tests.

After the coils were put through the above cycles, which were comparable to many years of service in a commercial machine, they were removed from the slots and inspected. The paper was somewhat darkened in color and was robbed of most of its mechanical strength. However, it was strong enough to retain its form about the conductor and to withstand the removal of the coils from the slot. The discoloration was greater at the imitation vent ducts than where the insulation was in contact with the stamping due to the fact that air could reach the paper more easily at those points. During the tests the coils were given a high potential test of 23,000 and 37,000 volts at intervals, and withstood these voltages without breakdown.

Tooth Pulsation in Rotating Machines[10]

Results of experiments have been published on the tooth pulsation in rotating machines, where both members are slotted. A method for checking the magnitude of flux pulsation was presented consisting of using metallic electrodes similar in shape to the machine teeth in connection with an electrolyte of mercury to represent air. Current is caused to flow through the teeth by applying voltage between the two members, with a magnitude proportional to the magnitude of flux that would flow under analogous magnetic conditions. The results obtained in this way are, in general, slightly lower than those shown by two methods of calculation; but the agreement is fairly good.

If the ratio of the mercury to electro resistance is small, corresponding to the effect of saturation in the iron, the effect of saturation may be experimentally determined. It was shown that for actual machines the effect of saturation on pulsation amplitude cannot be calculated by adding directly the air-gap and tooth reluctances, on account of the permeability of the iron not being constant.

Gaseous Ionization in Built-Up Insulation[11]

Experiments on the gaseous ionization in built-up insulation were conducted showing that the losses due to internal ionization caused a progressive deterioration of insulation, even though the absolute values of these losses in well constructed armature bars are small compared with dielectric losses of other types. The use of mica was shown to reduce the conductivity of the insulation and minimize the action of internal ionization. Even though the micafolium content can be somewhat reduced without seriously affecting the insulating properties over long periods, the use of high mica content appeared desirable due to variations in manufacturing process.

10. Journal of the A. I. E. E., July 1924.

11. Journal of the A. I. E. E., January 1924.

Effects of Time and Frequency on Insulation Tests of Transformers[12]

The use of the induced potential method for testing the insulation of transformers has increased during the past few years. In order to keep the exciting current within a reasonable range during these tests, it is necessary to use frequencies higher than normal. Since the frequency used affects the dielectric strength of most insulating materials, the fair length of time during which high voltage is applied in the case of higher frequencies than normal is somewhat less than that for normal frequencies.

During the past year the results of experiments made to determine the fair length of time for higher frequencies were published. It was shown that the rupture voltage of oil is the same for both sixty and 420 cycles. Due to the fact that the behavior of oil alone is very erratic no well defined relation can be made between time and dielectric strength. However, for the first few seconds, time decreases the strength quite rapidly, after which the effect decreases and is probably entirely absent after two or three minutes. The momentary strength ranges from 25 to 30 per cent higher than the one-minute strength.

The strength of solid insulation decreases with an increase in frequency. The effect of time on the strength of oil and solid insulation in series is approximately the same as for solid insulation alone until the oil distance exceeds the solid insulation thickness, after which it begins to be the same as for oil without barriers.

When solid insulation is under considerable stress the breakdown by creepage is not affected by time nearly so much as is the puncture voltage of solid insulation. Frequency does not materially affect the creepage failure of solid insulation which is under no stress. However, if the insulation is under considerable stress the voltage for failure decreases with increased frequency in about the same order as the puncture voltage of solid insulation. The effect of frequency on the puncture voltage of solid insulation and oil in series is approximately the same as for solid insulation, where the thickness of the solid insulation is greater than that of the oil. When the oil distance exceeds the thickness of the solid insulation the effect approaches that for oil without barriers.

Obtaining Steady High-Voltage Direct Current from Thermionic Rectifier without a Filter[13]

The ordinary polyphase high-voltage rectifier gives a practical constant direct-current potential, which has a ripple superimposed on each side of the mean of approximately five to seven per cent of the total d-c. voltage. This ripple may be ironed out to a considerable degree by the use of a filter consisting of condensers and reactors which are somewhat expensive, especially for high voltages.

12. Journal of the A. I. E. E., February 1924.

13. Journal of the A. I. E. E., November 1924.

During the past year work was done on the development of a special type of a-c. generator having the proper low voltage wave form for giving a more nearly smooth rectified direct potential and provisions were made for manually or automatically varying the wave form to maintain a steady potential for varying load demands.

The Application of the Saturated Core Reactor and Regulator[14]

The use of direct-current saturation in the iron cores of static a-c. apparatus in radio work has been in vogue for sometime, and of late is being used commercially in voltage regulators and current-limiting reactors. However, due to cost and inefficiency, the saturated-core voltage regulator and current-limiting reactor are at present confined to specific uses in which speed of operation is desired in the case of the voltage regulator and increasing reactance with alternating current is desired in the case of the current-limiting reactor.

During the past year, saturated iron-core current-limiting reactors were installed in a large central station between the essential auxiliary bus fed by a house generator and a miscellaneous auxiliary bus fed by a house transformer. These reactors are designed to carry 660 kv-a. under normal operating conditions, at a reactance drop of fifteen per cent. The short-circuit reactance is approximately 38.5 per cent, which will not allow more than 1710 kv-a. to pass through the reactors at normal voltage. Should the house generator be carrying full load and supplying 660 kv-a. to the miscellaneous auxiliary bus through the reactors, a disturbance on that bus could not overload the house generator more than 42 per cent due to the action of the reactor.

The Transverter

A machine has been developed in England for converting polyphase alternating current into direct current and vice versa. Polyphase a-c. voltages of values low enough to be generated in large commercial alternators can be transverted into continuous potentials on the order of 100,000 volts.

The a-c. supply lines are connected to a series of transformer banks which transform the original voltage to the desired value and, in addition, convert the fundamental number of phases into a large number of phases, 36 for example. The 36 phases are then connected in the proper order to a commutator which remains fixed in space, the brushes being the rotating member. The brushes are mounted on a shaft and rotated inside the commutator instead of being placed on the outside as is the usual practise. If only two brushes were used and there were only as many commutator segments as phases, the brushes would have to be revolved at the synchronous speed of a corresponding two-pole motor. However by using more brushes and segments, properly arranged, the speed can be made to correspond to that of a four- or six-pole motor.

The transverter recently built and displayed in England used ten commutators in series to give 20 d-c. amperes at 100,000 volts, and was designed to be supplied with 50-cycle, three-phase current at 6600 volts. The speed of the brushes is 1000 rev. per min.

This device can be used for a number of purposes, such as obtaining high-voltage direct current from low-voltage alternating currrent, low-voltage direct current from high-voltage alternating current, low-voltage direct current from high-voltage direct current (by adding another set of windings and commutators), and a given alternating frequency from another frequency. All these processes are reversible.

The Subcommittee wishes to express to Mr. J. A. Brooks its appreciation of the valuable assistance which he has rendered to it in the preparation of this Report.

Committee Organization

The organization of the Committee on Electrical Machinery comprises a number of Subcommittees including the following;

1. The review and preparation of technical papers in the field of electrical machinery, H.M. Hobart, Chairman.
2. The preparation of a résumé of the year's progress in the electrical machinery art for the presentation at the Annual Convention, J. C. Parker, Chairman.
3. Preparation of short memoranda of timely interest relating to electrical machinery, for publication from month to month in the Journal, E. H. Hubert, Chairman.
4. Subcommittee on Electrical Machinery Research, V. Karapetoff, Chairman.
5. Subcommittee on Electrical Machinery Standards, C. A. Adams, Chairman.

Provisions are made for further subcommittees which can be described as Regional Subcommittees. Each member of the Committee on Electrical Machinery is invited to examine the practicability of establishing in his vicinity a Regional Subcommittee to deal with subjects in which he is especially interested. The member includes in the Subcommittee a group of fellow-specialists and he is free to go outside of the Committee on Electrical Machinery for this purpose.

As an example of one of these Regional Subcommittee's may be mentioned that organized by Prof. B. F. Bailey. It deals with induction motors and generators and research subjects relating thereto. As Professor Bailey is located at the University of Michigan he has associated with himself in this Subcommittee Professor James F. Fairman, Mr. Norman S. Yost and Mr. L. N. Holland, all of whom are located at or near Ann Arbor.

The studies and discussion leading to the above organization of the Committee on Electrical Machinery took considerable time and the Subcommittees have not made as much progress this year as had been hoped. However, the Electrical Machinery Research Subcommittee has held meetings in which several matters have been profitably discussed.

14. Journal of the A. I. E. E., July 1924.

Precision Watthour Meters and High-Frequency Measurements

By Committee on Instruments and Measurements[1]

IN selecting subjects which would become the major study of the committee during the year, an effort was made to choose those which seemed to be lagging in progress because of unorganized attention, or to be only imperfectly coordinated and recorded because of their relatively recent development. Two subjects were chosen, one on each of these bases, and subcommittees (with chairmen, H. B. Brooks and C. M. Jansky respectively), were appointed to make the survey.

One of these subcommittees has been assigned the task of promoting the development of a device or devices for the measurement of energy directly in terms of watthours under the condition of excessively fluctuating power, for averaging this variable power with laboratory precision, and to produce results equivalent in accuracy to those obtained with laboratory equipment now used for the measurement of volts, amperes and watts. The committee is not concerned with the watthour meter as used by the million in the vending of electric energy; that is the concern of other agencies, and it may well be agreed that the watthour meter, in practically all respects, surpasses the measuring devices through which the public purchases other services and commodities. But this device, or modifications of it, thus far produced and used as a precision standard of comparison in laboratory and field testing, and as a means of averaging variable power over an extended time (as in water-rate determinations of large turbo-alternators, for example) leaves something to be desired.

The subcommittee charged with this problem realized that one of the outstanding sources of variation in accuracy of watthour meters was the variation in temperature of their component parts, arising from two principal causes; (1) changes in temperature of the ambient medium; (2) heating from the flow of current through the coils, and from iron losses. There was presented at the Midwinter Convention an important paper by Messrs. Kinnard and Faus, in which temperature errors in induction watthour meters were analyzed theoretically and experimentally and a device described for the compensation of the major of the two principal classes of temperature errors. The subcommittee considered it of little or no value to assemble and coordinate data on the temperature performance of the meters in current use, which are not equipped with any such device, but that its concern should be with meters which include this or some equivalent compensation. When such meters are regularly available, the subcommittee will consider the feasibility of having exhaustive tests made by suitable authorities.

Meanwhile, there remains a number of other disturbing effects to be considered, such as the influence of frequency, voltage, and wave-form variations, and of low power factor. These matters, and the methods by which improvement may be realized, will be included in the work of the subcommittee. The program thus briefly outlined will obviously involve a considerable amount of effort over a long time, and the subcommittee looks with confidence to the experts of the manufacturers, and others who specialize in this field, for full cooperation in the task.

The watthour meter, in different forms, has not been found entirely suitable for measuring the power output of large turbo-generators during water-rate tests and there is a tendency to prefer the wattmeter for this purpose, even in spite of the necessity of taking a very large number of readings during the progress of the test. The scope of the subcommittee's work includes a study of this method also, and by request, a valuable paper on the "Measurement of Electrical Output of Large Turbo-generators During Water-rate Tests" has been prepared by E. S. Lee; this paper was assigned to the program of the First District Regional Meeting at Swampscott in May. It was hoped that this paper would serve to bring out discussion which would be of value to the subcommittee in its further study of the problem.

The phenomenal development of radio communication naturally focussed the attention of the committee on the subject of electrical measuring instruments for use with frequencies in the audio and radio ranges. It seemed to be an opportune time to make a survey of the available instruments for these fields, their operating principles, limitations, and scope of applicability, and a second subcommittee was appointed to conduct the survey.

The most extensive use of high-frequency measurements is undoubtedly in the field of radio communication. However, frequencies ranging from 20,000 to 50,000 are much used in wire telephone and telegraph communication and in the so-called "wired wireless" or "carrier current" communication over power trans-

1. Annual Report of Committee on Instruments and Measurements.

A. E. Knowlton, Chairman
F. V. Magalhaes, Vice-Chairman

A. S. Albright,	E. D. Doyle,	L. T. Robinson,
O. J. Bliss,	R. C. Fryer,	Joseph Sachs,
Perry A. Borden,	C. M. Jansky,	Byron W. St. Clair,
H. B. Brooks,	P. M. Lincoln,	G. A. Sawin,
R. P. Brown,	W. M. McConahey,	Benjamin H. Smith,
J. R. Craighead,	Wm. J. Mowbray,	H. S. Vassar.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 23, 1925.

mission lines. Another possible application of high-frequency measurements is the determination of the periodicity of transients and the measurements of higher harmonics on power lines and in electrical machinery.

Low values of inductance and capacitance are of great importance where high frequencies are involved. The power engineer has heretofore considered them negligible. With a better knowledge of high-frequency measurements, quantities that have been neglected may assume greater importance even in power engineering. These low values of inductance and capacitance are usually measured by some resonance method; that is, by "tuning" the circuit with the unknown inductance or capacitance to the frequency of another circuit with known inductance and capacitance and then calculating the unknown quantity in terms of the known values.

There are two common methods of measuring resistance at high or radio frequencies; the reactance variation method, and the resistance variation method. The application of the reactance variation method requires a knowledge of the frequency used. It is, therefore, more useful in measuring logarithmic decrement and sharpness of resonance. Bridge methods are but little used for the measurement of resistance at radio frequencies but a somewhat similar method, which utilizes a differential transformer, is coming into use. There is also the method employing a voltmeter designed for accuracy at radio frequencies.

Great need has been evidenced for the development of more accurate measurement of resistance, inductance, and capacitance at high frequencies, but there is greater need for the more general utilization of instruments of adequate accuracy for measuring the frequencies of the electromagnetic waves emitted by radio-frequency generators used in broadcasting and other stations. These frequencies are usually measured by means of an instrument known as a frequency or wave meter. The extensive use of certain radio-frequency bands requires that radio stations be assigned frequencies which are, in some cases, only 10,000 cycles apart. If there is to be no interference between stations, each must maintain its assigned frequency within a small fraction of one per cent; there is a call for the development and use of frequency meters having the degree of accuracy required for this purpose.

This subcommittee is also continuing its studies.

The Activities in Research

By Committee on Research[1]

Part I. General

Activity in the field of electrical research, as noted in last year's report, has continued unabated during the past year. The range of problems has been much the same, and while no striking research of outstanding importance may be mentioned, noteworthy progress has been made in many directions.

In the field of high-voltage transmission and power distribution there is an increasing tendency to investigate the performance of such systems through experimentation with models, miniature systems and equivalent net works. With these should also be included several important analytical studies of the transients occurring in such systems. Several accounts have appeared of experiences with 220-kv. systems based on special methods of observation and measurement, and these have brought forward facts and conditions sufficiently new to warrant their mention in this report. Important new data are available as to lightning disturbances and methods of protection; a new type of suspension insulator of great promise has appeared and further studies have been made of the properties of protective reactors. New data as to the law of loss due to corona have been brought forward and definite statements made as to the value of corona as a stabilizing, if not a protective measure for high-voltage systems.

In the wide field of magnetism much new material has been produced. Investigations have extended from studies of the core loss in induction machinery, the losses in laminated surfaces next to air-gaps, the influence of slot openings on wave form, and other similar questions in machinery, to the further study of the properties of magnetic materials in relation to their constituent substances. Noteworthy in this class is the substance permalloy and its adaptation to the submarine cable; a thing of great promise, but so far, upon this relatively little definite information is available. An important accomplishment in this field is the completion of a complete bibliography of the literature by the Core Loss Committee of the National Research Council. The bibliography embraces all branches of the field of core losses and is admirably classified and indexed. It is hoped that some one of our large research organizations will find it possible to publish this bibliography so that it may become generally available.

Among other studies in the general field of electrical engineering there may be mentioned as either completed or under way the following variety of problems: The

1. Annual Report of Committee on Research.

John B. Whitehead, Chairman

Edward Bennett,	C. I. Hall,	E. W. Rice, Jr.,
V. Bush,	V. Karapetoff,	D. W. Roper,
E. H. Colpitts,	A. E. Kennelly,	C. H. Sharp,
E. E. F. Creighton,	M. G. Lloyd,	C. E. Skinner,
W. A. Del Mar,	F. W. Peek, Jr.,	Harold B. Smith,
B. Gherardi,	Harold Pender,	R. W. Sorensen.

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 22-26, 1925.

influence of impurities in storage-battery electrolytes; many studies of rectifying devices, including further methods of obtaining high, continuous voltage; the construction of an absolute electrometer for very high voltages; a redetermination of the unit of resistance, (these two latter at the National Bureau of Standards); the determination of the temperature errors in induction wattmeters, and other important developments in methods and instruments of measurement.

In the field of electrical communication special mention should be made of the increasing expansion of research work carried on by the Bell Telephone Laboratories. The range of problem handled in these laboratories is extremely wide and very noteworthy contributions are appearing at regular intervals. These are not confined to the immediate problems of conversion between speech and electric circuits and vice versa, but reach out into the field of pure physics on the one hand, and to studies of the performance characteristics of all types of electric circuit on the other. Of special note is our increasing knowledge, obtained through these laboratories, of the performance of all types of circuits under a very wide range of frequency, and the development of filters, relays, and other forms of auxiliary equipment for circuit control for special purposes.

Extension of range and improvement of methods, in radio, telegraphy and telephony is of almost daily occurrence. It is a highly specialized field, but one in which, many members of the Institute are nevertheless participating. Among the great number of improvements in transmitting and receiving equipment, there stand out as of recent, more conspicuous achievements, the control of static in transatlantic service; the increasing use of the shorter wave lengths; the control of "fading"; and the further perfection of generating equipment.

In the field of pure physics, principal attention appears still to be devoted to questions of molecular and atomic structure and the nature of the ultimate constitution of matter. Progress has been very rapid and the results obtained are of greatest importance as well as of absorbing interest. Up to this time, however, these studies do not appear to touch, in any close manner, the laws underlying the various arts in electrical engineering. It is an interesting fact that while much certain knowledge has been gained, as to the structure of the atom and the electrons therein, nevertheless it has not been found possible to adapt this knowledge in any certain way to explanations of the great fundamental phenomena of electric conduction, magnetism, and dielectric induction. For this reason, as well as because of the vast variety and quantity of material, only this passing mention is made of this great field of the highest type of scientific research.

PART II. ELECTRICAL INSULATION

This report has reserved for separate comment the subject of research in the field of electrical insulation. The year has seen a notable continuance of the general interest in this important problem. Numerous papers have been published giving new technical data, thus slowly increasing our knowledge. Among the subjects treated may be mentioned distribution of flux density in cables, the relation between breakdown voltage and times of application and rest, experimental data on the breakdown of cable insulation under standard tests with special reference to the duration of application of the test voltage, the influence of temperature on impregnated paper insulation, and ionization in impregnated paper insulation. Discussion of these papers has indicated the desirability of certain changes in present tests standard for high-voltage cables.

Particular mention should be made of the work being done by the committee of the National Electric Light Association on Cable Insulation Research. This committee has formulated a well considered plan and has raised sufficient money to pay for "whole-time" research assistants, the work to be carried on in the electrical engineering laboratories of Harvard University, Johns Hopkins University and Massachusetts Institute of Technology. Problems of attack are those bearing on the life of high-voltage cable insulation and the causes of its failure, the work is certain to produce results of value.

As indicated in last year's report, the Committee has had before it as its principal object the complete review and digest of the literature of electrical insulation. This work, formulated first by the Engineering Division of the National Research Council, in its Committee on Electrical Insulation, is being carried on largely by members of the Committee on Research of the American Institute of Electrical Engineers. The particular value of this work will be found in the summaries and conclusions to be prepared by the chairman of the several committees. The Committee hopes to include in these summaries statements as to the present problems under the respective headings, with suggestions for the most profitable lines of experimental attack. Steady progress has been made, although it can scarcely be said to be rapid; the character of the undertaking is such as to require considerable time and sustained effort. Surveys in the field of insulation can be made only by experienced and competent men, and under our present plan, we are relying entirely on the voluntary efforts of a comparatively small number. All of these are busy men with whom the work of the Committee must, of necessity, take a subordinate place.

The present state of the work is approximately as follows: The Subcommittee on Dielectric Absorption, J. B. Whitehead, Chairman, has practically completed its review of literature; Subcommittee on Phase Difference, Delafield DuBois, Chairman, has completed about eighty-five per cent of the literature; on Electric Strength, W. A. Del Mar, Chairman, has completed all the literature on solids, and its report is well advanced towards completion. The review of literature

on liquids is well advanced. The Subcommittee on Flashover Voltage, F. W. Peek, Jr., Chairman, has covered the field of the literature in English, and is making progress in foreign literature. The Subcommittee on Theories, J. B. Whitehead, Chairman, has practically completed the review of the literature. The Subcommittees on Dielectric Constant and on Resistivity have not been able to make any considerable progress.

The reviews of the literature referred to above consist of a separate report in standardized form for each paper reviewed. The results of this work therefore will constitute not only a valuable bibliography of the separate divisions of insulation, but also a most valuable concentration in one place of the important results of each worker, and, therefore, a combined picture of the entire problem which should prove of very great value. As the publication of this large mass of data will be a matter of some expense, the Committee hopes to make the more important results of its work available to all through the preparation of reviews and summaries by the respective chairmen.

PART III. THE ORGANIZATION OF RESEARCH

A general review of the work in electrical engineering research during the past year reveals two striking facts: First, the great amount of experimental research under way and the wide range of problem; and second, the lack of coordination among various workers in the same or allied fields. Obviously, a function of the Committee on Research should be the bringing about of such coordination if in any way possible. The difficulty here is in obtaining information as to the work undertaken in widely separated localities. Often the Committee's first knowledge of a piece of work is the appearance of the paper presenting the results. This state of affairs must continue so long as the original conception of the problem arises in the interest of some individual, in a university laboratory, or in the special needs of some manufacturing process.

The foregoing condition has been recognized for some time and the desirability for organization and coordination of electrical research has been frequently emphasized; in fact, the National Research Council in its Engineering Division, the Engineering Foundation, the National Bureau of Standards, and, to a less extent, the national engineering societies, all conceive it their definite function not only to stimulate but to coordinate research. The Committee on Research of the Institute acts as an advisory committee on questions of electrical engineering to the Engineering Division of the National Research Council, the body which initiated the work on electrical insulation. In addition to the Committee on Electrical Insulation, it has formed a number of other important committees, not only in the electrical, but also in other fields of engineering. If the purposes of these committees are carried out, they will result in authoritative statements of the problems in the various fields, will serve as important guides for future work, will avoid duplication of effort and, naturally, will result in a much more rapid extension of our knowledge in the respective fields.

The picture so presented is an inspiring one, but it has one serious and fundamental defect. On closer examination it will be found that the function of the comprehensive review of the field of any problem is relegated to volunteer workers. The men who are capable of making these studies and reviews, and of laying out subsequent programs, occupy important positions the duties of which must necessarily require most of their time; committee work of necessity takes a subordinate place. Money has been appropriated for particular research problems, but none for greater expert reviews of the entire field. This is unfortunate, the more so since it should not require any considerable annual expenditure to avoid it. The Committee on Research has spent nearly two years in accumulating data on insulation outlined above. This could have perhaps been done in one-fourth of the time by a single well trained man devoting his entire time to it.

The concrete suggestion, therefore, is that large organizations, the chief function of which is to promote experimental research, could, with profit, maintain as a part of their organizations a few competent men trained in science, engineering and in the methods of research, with their principal duty the presentation of the work of the past, problems of the present, and plans for concerted experimental attack for the future. The volunteer research workers would do the rest.

Developments in Applying Electricity to Industrial Uses

By Committee on General Power Application[1]

THE annual report of the Committee on General Power Applications has been divided into three parts, namely—an account of the Committee's activities in the past year and two appendices which are believed to contain data of interest and value to the membership.

Soon after its appointment, this Committee offered its services to all Institute Sections, in the preparation of papers or in locating speakers on any subject within its scope. In answer to an inquiry from the British Consulate General, the Committee conducted inquiry into the use of magnetic hoists in shipyards and at docks, with especial reference to their effect on the ship's compass, due to its possible derangement or the magnetization of the ship's frames.

A session of the St. Louis Convention was assigned to this Committee and five papers were presented as follows:

Load-Building Possibilities of Industrial Heating, C. L. Ipsen;

A High-Frequency, Induction Furnace Plant for the Manufacture of Special Alloys, P. H. Brace;

Electrically-Heated Lead, Solder and Babbitt Pots, J. C. Woodson;

Synchronous-Motor Drive for Rubber Mills, C.W. Drake;

Use of Purchased Power in Glass Manufacture, A. L. Harrington.

The Committee has given special thought to the manner in which the wide variety of subjects under its jurisdiction may be most suitably presented to the membership. The preparation and presentation of papers alone is not sufficient. The space available in the JOURNAL and the Convention sessions which may be devoted to General Power Applications, without unduly curtailing other subjects of equal or greater importance, leave the Committee embarrassed by a too plentiful supply of new and valuable material.

As a consequence, the Committee submits as Appendix A of this report, a brief summary of progress covering important developments in General Power Applications which have come to its attention during the past year. In Appendix B is submitted a bibliography of articles which have appeared in the technical press, on subjects related to the Committee's activities. It is believed that this bibliography will be found valuable by those who wish to investigate the subjects which have been included.

The Chairman wishes at this time to record his sincere appreciation of the efforts of D. H. Braymer, member of the Committee, and also the courteous and efficient help rendered by other members of the McGraw-Hill organization. Without their assistance, the appendices would have lost much of their value.

The Committee submits the following recommendations to its successor:

a. The annual preparation of a similar summary of progress and a similar bibliography, if the reception of these by the membership indicates that they are useful and desirable.

b. Suggestion of the subject of "Group Control of Motors, Sectionalized Drive" as one which should be developed in a group of papers.

c. The recommendation of subjects on "Power Factor and Load Factor, Their Control in Industrial Applications" for special development.

The Chairman gratefully acknowledges the interest shown by the members of the Committee and the Institute Staff in their cooperation in carrying on the work of the past year.

Appendix A

A condensed summary of general progress in power applications as recorded in engineering publications during the past year.

TRENDS IN INDUSTRIAL MOTOR APPLICATIONS

Through the cooperation of engineering departments of manufacturers, consulting engineers and electrical engineers in industrial plants, progress is being steadily made along the line of more efficient power drives and control equipment and drive arrangements better suited to the changing operating conditions or tending toward lower maintenance with improved operation from the standpoint of production in manufacturing operations.

Outstanding points in this connection are, improved motor construction; modifications in motor applications to speed up production; new types of control; improved resistors; new forms of limit stops and safety features in control applications; installation of automatic equipment in steel mill substations; extended use of electric heating, and the like.

Steel Mills. The most noteworthy feature of the electrification of steel mills has been the large number of installations of new equipment that were started during

1. Annual Report of the Committee on General Power Application.

A. E. Waller, Chairman
A. M. MacCutcheon, Vice-Chairman

P. H. Adams,	W. B. Hall,	T. D. Montgomery,
D. H. Braymer,	H. D. James,	H. W. Rogers,
H. E. Bussey,	A. C. Lanier,	A. F. Rolf,
R. F. Chamberlain,	W. S. Maddocks,	W. H. Timbie,
C. W. Drake,		W. C. Yates.

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 24, 1925.

a year of comparatively dull times for the steel industry. This fact is a most striking indication of the economy and reliability of electric operation.

The Westinghouse Electric & Manufacturing Company reports that five reversing equipments have been sold, including two large reversing blooming mills, two roughing structural mills and one finishing structural mill. This company further states that one of the greatest advances in the art was made this year when the company placed in operation a 500-h. p., single-unit, reversing motor, receiving its power from two 2100-kw. generators operating in parallel. It has heretofore been considered necessary to drive one motor from a single generator, and if two motors were used two generators were required. By this new arrangement it is said to be possible to design both motors and generators to best meet their operating conditions, rather than to have the design determined by the number of units of which the equipment is composed.

A 40-in. and a 44-in. blooming mill are each being equipped, by this company with 7000-h. p., single-unit motors. These are said to be the largest single-unit, d-c. motors ever built. A 500-h. p., d-c. motor is being installed by this company to operate at 155 to 500 rev. per min., at any voltage from 220 to 250. This application is for a tire mill and constitutes a speed range of 3.7 to 1, which is exceptionally high for motors of this capacity.

The Allis-Chalmers Manufacturing Company of Milwaukee, Wis., reports that it has installed and put in successful operation three mill-type synchronous motors on main roll drives of steel mills. The motors are coupled to the mills without the use of a clutch.

By the addition of 45,350 h. p. (normal continuous rating) of main roll motors during the year, the total of General Electric main roll drives has been brought to over 700,000 h. p. This company reports that an installation of particular interest is a 14-in. continuous merchant mill at the plant of the Jones & Laughlin Steel Corporation. The nine stands of this mill are driven by seven, adjustable-speed, d-c. motors. This mill will roll a very large tonnage of diversified products and the employment of individual drives with a very wide speed range is said to enable it to do the work of two or three less flexible mills.

The 26-in. rail mill at the Sparrows Point Plant of the Bethlehem Steel Company was changed over from steam to electric drive by the installation of two 3000-h. p., 500-rev. per min., 660-volt, constant-speed, induction motors made by the General Electric Company.

The Reliance Electric & Engineering Company reports that one of its most important application developments is an individual wire block driven by an adjustable-speed, d-c. motor with special automatic control equipment.

Textile Mills. An application of the brush-shifting motor to the operation of textile finishing machinery, where the operation of several motors in tandem may be required, is reported by the General Electric Company. In each case the speed of the main motor is controlled by push buttons through the operation of a pilot motor, which actuates the brush-shifting mechanism, while the speed of the other motor is controlled either mechanically or electrically by the motion of a compensating gate or floating roll between sections of the machinery.

Rubber Mills. The Westinghouse Electric & Manufacturing Company reports that for the past two years, its mill drive using synchronous motors without clutches or brakes has proved highly satisfactory. This is said to be due to the use of dynamic braking as a means of making safety stops, which has proved thoroughly reliable and effective. During the past year further improvements and developments in the control have been undertaken with a view to obtaining even greater reliability and quicker stopping. This company states that the development of a three-pole, double-throw, gravity-operated contactor makes it possible to use dynamic braking with either manual or automatic starters, the braking feature being entirely independent of any external control circuit or relay operation.

The elimination of the use of low-voltage and other relays in the braking circuit, together with the use of gravity operation, is said to give a reliability exactly comparable with d-c. drive for calenders, which has been in use for so many years.

Lumber and Woodworking Mills. A variable-voltage log carriage, consisting of a d-c. motor and motor-generator set, has recently been installed on the Pacific Coast by the Westinghouse Electric & Manufacturing Co. The motor is rated at 35 h. p., and has a speed range of from 10 to 80 rev. per min. Power is supplied by a 30-kw., 320-volt motor-generator set.

The General Electric Company reports that small, drawn-shell type motors of unusual speed were provided for the operation of woodworking machinery. A typical application of this character consists of five small motors with operating speeds of 25,200 rev. per min., which are utilized by a single woodworking machine, the current for the small high-speed motors being supplied by means of a frequency changer. The motors operate on three-phase, 240-volt, 420-cycle circuits.

Paper Mills. A new rotary contactor regulator for sectional paper-machine drive is reported by the Westinghouse Electric & Manufacturing Company. This regulator is an improvement over past apparatus, in that great simplicity and a more flexible action is obtained.

By the same company there has been placed in operation a paper winder drive with automatic regenerative tension control for sheets, including automatic regenerative and dynamic braking for the winder and unwinder. This is said to permit much faster operation of the winder and much better control of the finished roll, as it produces a uniform tightness throughout. Thus is avoided the expense and annoyance of frequent replacement of friction brake linings on the unwinding roll.

Also by the Westinghouse Electric & Manufacturing Company an automatic speed regulation of the vibrating regulator type for single motor paper machine drive has been developed and placed in operation. The use of this device is said to make possible a degree of regulation not previously obtainable through inherent speed regulation characteristics of the motor and generator. This is a decided advance in the art and contributes in a marked degree to precision control, which is becoming of greater and greater importance in this industry.

An installation of a sectional paper-machine drive, for a 175-in., 1000-ft. per min., Kraft paper machine, has been placed in operation by this company. This is the largest Kraft paper machine built to date.

An installation of five automatic grinder load regulators for the control of as many 1800-h. p. synchronous motors driving pulp grinders has been placed in operation by this company. The control of the load on these machines is said to relieve the governors of the power plant of an enormous amount of work and to greatly improve the frequency and voltage regulation of the entire system.

Industrial Haulage. Induction motors designed to give a high starting torque with a relatively low current and a high efficiency when operating under load were applied for the first time to large systems of conveyers by the General Electric Company.

The first haulage electrification of an open-pit iron mine in America was effected by the adoption of three 60-ton, double-truck, 500-volt mine locomotives made by the General Electric Company at a mine of the M. A. Hanna Mining Company at Duluth, Minn. These locomotives are provided with auxiliary cable reels, and current supply is effected by means of an overhead trolley.

Machine Shops. With the improvement in cutting tools and quality of rails, has come a demand for more driving power for frog and switch planers. During the past year, the Reliance Electric & Engineering Company has developed and applied motors in sizes up to 100 h. p., 250-1000 rev. per min. to such planer drives. This company further states that reversing the motor drive with automatic control of such work means the elimination of belt troubles, ample power, accurate speed adjustments for a cutting speed range of 1 to 2, and a separate speed adjustment for return strokes from 50 to 100 ft. per min. Quick return strokes reduce the non-productive operating time to one-third that of belt drive.

A complete line of variable-voltage, reversing planer equipments has been developed by the Westinghouse Electric & Manufacturing Company. Each unit makes use of a separate motor-generator set; consequently, it is possible to install a variable-voltage equipment on a planer at any point in a plant having alternating-current service. Other outstanding features that this drive is said to have are a greater speed range, smoother acceleration and braking, and very simple control, combined with a power saving in service where reversals are frequent.

Elevators. The General Electric Company reports that radical improvements were made in the operating characteristics of its high-speed elevator equipment. One of the principal difficulties in the past has been that the speed, on any one speed position of the controller, is decidedly variable with the load, being slowest when the motor is hoisting the maximum load and fastest when the motor is being overhauled by the load. The improved system consists of a motor and motor-generator controlled on the Ward Leonard principle but with the novel feature of an auxiliary series generator by which a considerable degree of compounding can be successfully applied to the main generator to an extent entirely impracticable to apply in the ordinary manner.

A new gearless traction elevator motor for passenger service has been developed by the Allis-Chalmers Manufacturing Company. It is of the shunt-wound type designed for operation at slow speeds suitable for giving normal car speeds, using standard diameter sheaves. The system of control used is the Ward-Leonard requiring separate motor-generator set and variable-voltage control.

Elevator equipment and the necessary control has been developed to such a point of perfection that speeds of 1000 ft. per min. are now entirely possible, is reported by the Westinghouse Electric and Manufacturing Company. In fact, there seems to be no reason why speeds cannot be increased up to the limit prescribed by the speed at which humans can be transported, in vertical position, without discomfort.

Oil Wells. The Westinghouse Electric & Manufacturing Company reports that field tests of the Hild differential drive for oil wells have been made. Two holes, 3700 ft. deep, are said to have been completed successfully.

This drive is designed primarily for rotary drilling. Its function is the automatic regulation of the downward bit feed according to the resistance of the formation encountered. Essentially, it consists of a differential gear unit, two slip-ring motors and a rotary drawworks. One of the motors—the drill motor—drives the rotary table which revolves the drill pipe and at the same time drives one-half of the differential. The other—the regulating motor—drives the other half of the differential. The two motors operate the differential in opposite directions, the operating motor in a direction tending to raise the drill pipe and the drilling motor in a direction tending to lower it. By adjusting the speed of the drilling motor slightly higher than that of the regulating motor, a slow downward feed of the drill pipe is produced. If the resistance to the drill is increased due to change in formation, the increased load slows down the drill motor and a slower feed results. Conversely, a lighter load means a more rapid feeding of the bit. High resistance encountered in drilling through hard rock causes the bit to rise until it clears itself.

Automatic Controllers. Because of the very nature of industry and the years of study of application and performance of motor devices and control, there is always a certain amount of development and refinement in controller design and construction that is accomplished through the increased knowledge of the requirements of industrial power drives. The year 1924 was no exception with regard to progress and developments.

One of the outstanding developments is the inductive, time-limit controller devised by The Cutler-Hammer Manufacturing Company of Milwaukee, Wis. In the design of this time-limit starter the inductive principle is utilized to obtain the accelerating time, a transformer being used in place of relays, interlocks, dashpots or other moving parts to control the time of acceleration. The manufacturer states that, through the medium of the transformer, a holding-out current of transient nature is obtained in successive accelerating switches. Transfer of connections takes place automatically with the cutting out of successive steps of resistance without disconnection of the coil circuits. It is said that in this new development, the acceleration period is very uniform under ordinary conditions of load variations, the time being somewhat increased on heavy loads; thus the machine driven, whether reversing table, screw-down or other auxiliary machine, is always brought up to speed in the same period to insure the productive synchronism and plant efficiency desired.

Another development in time-limit acceleration is reported by the General Electric Company in the case of a 300-h. p., a-c. motor on the hoist and a 75-h. p., a-c. motor on the trolley of a coal tower at Clairton, Pa. Current-limit acceleration was previously used on this application and the change to time-limit acceleration was secured by using a suitable number of d-c. contactors and allowing them to close in sequence, each one being interlocked through the preceding contactor. D-c. contactors were used to secure a slow time of closing, so that fewer contact elements were required than would have been necessary with alternating-current contactors. No time element relays were used.

Two forms of resistor-type magnetic starters for a-c. motors have been produced by the General Electric Company; one for squirrel-cage induction motors, and one for slip-ring motors. Mechanically, these starters are very much alike, the difference being chiefly in their connections. They both employ a new type of time-element relay for the accelerating period, which can be adjusted to about six seconds. The relay consists of an armature that is drawn across the face of an a-c. magnet by a spring which is distended when the line contactor closes. The magnetism resulting from the alternating current intermittently attracts and releases the armature as it slides by the pole face, thus giving the desired time adjustment.

A great advance has been made in the control of blooming mill auxiliaries, by the installation of two entirely separate control circuits, either of which can be used at will, is the report received from the Westinghouse Electric & Manufacturing Company. Each master switch is plugged into a receptacle, and in case of trouble can quickly be replaced by a spare. A complete spare control panel is included which can be quickly transferred to any motor.

Similar information comes from the Rowan Controller Company of Baltimore, Md., in citing the case of a switchboard containing the necessary control equipment for the auxiliaries for a large blooming mill in the Cleveland steel district. A unique feature of this installation is that all the controllers for the front and rear tables and the screw-down are exactly alike. An additional spare control panel is provided so that by means of throwover switches this spare panel can be rapidly connected in place of any of the other similar controllers.

A new type master has been developed by the Cutler-Hammer Manufacturing Company for use particularly with Cutler-Hammer magnetic-type controllers. This master is a compactly enclosed structure with a contact cylinder mounted on a square shaft. Standard non-stubbing contact fingers such as are used in Cutler-Hammer drum controllers, are mounted on the stationary support, the leads to which they are connected also remaining stationary. Pyroplax arc barriers are placed between the fingers and the contact segments which are automatically lubricated by means of an oil wick which holds sufficient oil for about six months use.

A new automatic reversing planer controller has been developed by this company. The scheme of control employed plugs the planer motor in stopping and reversing the planer, and the company reports that this results in the fastest method of operation. Two shunt-field rheostats are provided in the control—one is for regulating the cutting speed and the other is for adjusting the return stroke speed.

For several years past it has been evident that the trend of safety regulations is toward the enclosure of all types of motor starters and speed controllers. During the year, the General Electric Company reports that work along these lines was practically completed, and the remaining open-type starters and speed controllers were provided with either self-contained enclosing cases with external handle or were redesigned to accommodate enclosing cases when required.

Monitor Thermaload starters, manufactured by the Monitor Controller Company, Baltimore, Md., are now being built with standard Monitor side-arm contactors instead of the special contactor previously employed. Hairpin-shaped thermal elements are also being supplied, instead of the coiled elements previously used.

The thermal relay of the Monitor Thermaload starter operates on the thermal expansion principle. The heat produced by the thermal element under overload causes a liquid confined in a tubular receptacle to expand and to elongate two expansion units. These expansion units in turn operate an arm containing two contacts which control the operating circuit of the

contactor. The liquid used is carbon tetrachloride, a non-corrosive and non-freezing liquid which is used extensively for fire extinguishing purposes. This starter is intended for starting small induction motors, both single-phase and polyphase, and is said to protect the motor against light overloads dangerously prolonged, yet permits the motor to carry heavy overloads momentarily.

A new steel enclosed, dust-proof drum controller is being marketed by the C. H. McCullough Engineering Company, of Pittsburgh, Pa. This company reports that the diameter of the drum of this controller is exceptionally large, giving a greater wearing surface and thereby increasing the arcing distance between points. This company also points out that due to its dustproof qualities the controller is especially adapted to foundaries, steel works, cement plants and similar dusty locations.

The Condit Electrical Manufacturing Company has announced that during 1924 it has arranged its type N-3 oil motor starters for push-button control, thus making them especially adaptable to industrial service. The type N-4 starter which was developed in 1923 is now equipped with thermal cutouts.

The Allen-Bradley Company of Milwaukee, Wis., has within the last year or so added refinements to its lines of across-the-line starters, automatic starters and semi-automatic starters, according to reports received from them. It also informs us that these lines have been standardized so that they can be made on a large quantity basis at a very reasonable price and yet keep the same standard of quality for which these starters have been noted.

Heretofore, fractional horse-power motors have, to a very large extent, utilized standard, wiring device switches, which were primarily designed for lighting circuits. A new design of single-pole and double-pole enclosed tumbler switches, assembled in boxes for conduit wiring for throwing small motors on the line, now gives these motors a class of control equipment comparable with that provided for large motors, reports the General Electric Company.

This company also informs us that a new thermal overload relay was designed to follow the heating and cooling curve of the average induction motor, and that this is particularly adapted to service where it is important for the motor to carry heavy, short-time overloads intermittently, without being tripped out by the overload device. It is said that this device will permit the motor to do any kind of work that does not run it above a safe operating temperature.

The company states that it has developed a definite time relay which was utilized for the first time in an automatic compensator, and is now used to provide the accelerating period. It consists essentially of a motor-driven train of gears, magnetic clutch and switching mechanism to provide for either opening or closing the contacts at the end of the time for which the device is set, being adjustable from a few seconds up to several minutes, and adaptable to many applications where a definite time adjustment is needed.

The Cutler-Hammer Manufacturing Company states that a complete line of control equipment for various sizes of Fynn-Weichsel, alternating-current motors (made by the Wagner Electric Corporation, St. Louis, Mo.) was brought out in 1924. For the smaller sizes, the face-plate type with the operating handle arranged for manipulating from the exterior of the enclosing case is used. A drum type starter is employed with the larger motors incorporating in its design non-stubbing fingers, a square steel shaft for carrying the contact segments, and blowout shields. In all of these starters the design takes care of simultaneously cutting out resistance in each of the two secondary windings found in this type of motor.

For the automatic control of motor-driven pumps, air compressors and the like, the General Electric Company has developed a new pressure governor for use in connection with automatic starters. This governor is of the Bourdon tube type, and can be used on any liquid or gas system which will not corrode the Bourdon tube. The equipment includes an "impulse" magnetically operated relay of a quick throwover type breaking its own operating circuit as soon as it functions.

Three, across-the-line, automatic starters have been placed on the market by the Sundh Electric Company of Newark, N. J., one having undervoltage release or undervoltage protection, the second in addition, having, inverse-time-limit overload thermal relay protection and the third being provided with inverse-time-limit overload relay protection. All these starters are manufactured in either the open or enclosed type, two-pole or three-pole, for motors up to and including 10 h. p., 220-440-550 volts.

Heretofore this company's principal activities have been more or less confined to furnishing control for large office buildings and pump concerns. This company now plans to enter the industrial market more intensively with the present line of equipment rounded out with the additions that have just been described.

Compensators. Two new automatic compensators have appeared on the market during the past year. One of these is a high-voltage compensator made by The Electric Controller & Manufacturing Company of Cleveland, Ohio. It is built for voltages of 2500 and below, is push-button operated and entirely automatic. With the exception of the overload panel, which is mounted on the top of the tank, the compensator is entirely submerged in oil and the tank is so designed that the compensator is said to be dustproof, weatherproof, vaporproof and fireproof. It may be installed either indoors or outdoors.

The power supply for the push-button circuit comes from an independent low-voltage circuit which is taken from a separate transformer so that there is no danger of the operator ever coming into contact with the high-voltage circuit. The compensator is so

designed that continuous torque is applied to the motor from the time the push-button is pressed until the motor has been brought up to speed.

A new automatic starting compensator for squirrel-cage induction motors, has been placed on the market by the General Electric Company. This motor starter is for remote control of constant-speed, two- or three-phase, squirrel-cage motors up to 600 volts and general applications driving lineshafting, pumps, compressors, blowers, conveyors and the like. With it, such equipment may from a distance be started or stopped by means of one or more small hand-operated push buttons or snap-switches located within convenient reach of the operator or automatically operated by a pressure governor, float switch, thermostat or similar arrangement.

Resistors. A ribbon-type resistor, wound on edge, has recently been developed by the Monitor Controller Company, Baltimore, Md., and is intended for service where cast-iron grids would otherwise be employed. It consists of a high-resistance alloy ribbon, wound on edge in helical form and mounted on a steel-reinforced, porcelain support which passes through the entire length of the unit, supporting and separating every convolution at two diametrically opposite points. This method of construction relieves the resistor ribbon from mechanical strain and permits of thorough ventilation. The maker states that the ribbon will operate at any temperature up to red heat without sagging or in any way injuring the resistor as a whole.

A system of terminals and taps enables a unit to be connected into a circuit, and to be interconnected with other units. Two simple forms of clamps provide all these facilities. These clamps may be placed at any desired point along the resistor and changed at will. This permits of accurate adjustment of the resistance steps. The maker reports that a saving of weight and space is obtained by the use of these resistors as compared to cast-iron resistors. These resistors are shown in an accompanying illustration.

The *EMB* grid resistor, manufactured abroad, is being introduced by the C. H. McCullough Engineering Company, of Pittsburgh, Pa. This resistor is made of drawn material and in one continuous length for an amount equal to one frame, or bank. The maker states that it is unbreakable, rustless, scaleless, of uniform section, jointless in the frame and is covered by a five-year guarantee. He also states that the weight of same is less than that of cast-iron grids of equal rating.

Safety Switches. A new, small safety switch has been placed on the market by The Trumbull Electric Manufacturing Company, of Plainville, Conn. This switch is now made in the 100-ampere, 250-volt size and will shortly be made in the 440- and 550-volt size. The maker reports that the switch is characterized by its small size, yet the parts are accessible since the switch may easily be removed from the box. The switch is constructed on the double-break principle, the blades being carried on a rotor made of molded material.

A new line of quick-make, quick-break, enclosed safety switches has recently been put on the market by the Westinghouse Electric & Manufacturing Company. This line was designed to meet the demand for a simplified enclosed switch without the full safety features.

This switch, known as the WK-60 switch, is unique in that the quick operating mechanism has been condensed into a few simplified parts, located inside the operating handle. The maker states that this feature alone is a marked advance in the design of safety switches, since it lends greater reliability, lesser maintenance cost, and at the same time, increases the ease of plant operation with regard to inspection testing and the making of repairs.

A new "Lumenized" finish has been added to the line of Bull Dog safety switches made by the Mutual Electric & Machine Company of Detroit, Mich. The company states that this finish involves the depositing of aluminum flakes, like tiny fish scales, upon the basic metal under high temperature. It is said that the result is giving the switch the following qualities: making it luminous in the dark; rust, acids, and alkalies resisting; easily grounded; and the last word in cleanliness.

A new type of cross-bar construction has been incorporated in safety switches made by the Square D Company of Detroit, Mich. A steel bar is heavily insulated with molded composition tubes, possessing not only superior mechanical strength but greater dielectric strength as well. This construction has an extremely low absorption of moisture, which is said to prevent warping and the consequent distortion of blade alinement. Wide fiber washers have been provided to prevent accumulation or adherence of dust, breaking up any continuous path of dust to ground. The insulating tubes have an offset at the ends so that dust cannot work in under the washers, thus causing leakage.

During the past year the Super-Safety Electric Company has developed a new design of safety switches of the larger sizes. Instead of the contacts being on the base of the cabinet, as is usual, they are placed on sides of the cabinet, thus leaving the base entirely clear. This constitutes the principal difference between the new switch and the more usual types of safety switches.

The maker reports that this type of construction results in the following advantages; greatly increased wiring space within the switch with, at the same time, a reduction in overall dimensions; double break—a break on each side of the fuse; greater flashover distance between polarities; and perfect and even contact between all contact members by reason of their tandem arrangement and their self-alining construction. This switch is shown in one of the accompanying illustrations.

Limit Switches. A new safety limit stop has been developed by The Morgan Engineering Company, Alliance, Ohio. This device consists of a cast-iron box in which are mounted two double-throw, single-pole switches. These two switches, when actuated by contact with the hook block, cause the limit switch to function. The maker states that this limit switch

differs from others in that a resistance is thrown in series with the armature as the limit switch starts to function, thereby slowing down the armature and greatly reducing the amount of current broken by the contactors. Complete stopping of the hoist motor is accomplished by an usual arrangement of dynamic braking. The maker also points out that another feature of this limit switch is that the lowering circuits are established immediately upon the reversal of the controller through the action of a solenoid; there is no waiting to lower through resistance. Other features cited by the manufacturer are: absence of external banks of resistance, for the limit switch is self contained; positive action, since an upward movement of the weight of even a fraction of an inch will actuate the limit stop; foolproof, having only one adjustment; the use of extra large copper to carbon contactors; having only one swinging weight, guided by one of the hoisting ropes; and sequence of operation of the contactors obtained by a single system of levers, with no cams, springs or counterweights.

The Cutler-Hammer Manufacturing Company of Milwaukee, Wis., reports a new development in safety limit stops which is said to be a compact, rugged unit with a number of features of great importance in the functioning of a safety device. The working parts are liberally designed and arranged to move freely, this ease of operation being still further insured by the provision of ball bearings.

Miscellaneous. A new quick-acting, electric, solenoid brake has been developed by the Whiting Corporation of Harvey, Ill. This brake was especially designed for application to crane service.

The maker points out that the brake arms are so pivoted that the shoes release equally at all points; leaving no chance for shoes to drag at the lower ends. This is said to be a great advantage in applying the brake as the shoes bear equally at all points, resulting in quick braking action as well as uniform wear.

Nichols-Lintern Company of Cleveland, Ohio, reports improvements in its electromagnetic sander. An electrically-heated chamber has been placed around the sand box, thus keeping the sand warm and dry, thereby greatly increasing the efficiency of the sander. These sanders are intended to replace dangerous and wasteful hand sanding methods and are said to save crane and runway maintenance.

The use of static condensers for power-factor correction in industrial plants has increased considerably. The Westinghouse Electric & Manufacturing Company reports that it has developed a new low-voltage unit which may be connected at the motor or place where the power is used, thereby reducing the line losses in the feeder circuits. This new line of low-voltage condensers is made for 220-, 440-, and 550-volt, 60-cycle service.

The National Electric Condenser Company of New Haven, Conn., is also putting out a line of low-voltage condensers for direct connection at the motor terminals.

Automatic Substations. The automatic station has become firmly established as an economic, operating necessity in the electrical industry. With its inherent advantages established beyond dispute a marked advance in the application of this type of control was made during 1924.

There were no radical changes in the design of automatic switching equipments. The tendency during 1924 was to produce unit equipment which might be easily handled and installed, standardizing wherever possible. The General Electric Company reports the progress of standardization to such an extent that complete automatic stations for mining and industrial service are now being stocked.

To meet the requirements for this class of service, the company states that one type of standardized design is so arranged that a single machine automatic control equipment may be used in either single- or multiple-unit stations with any number of reclosing feeders, thus giving maximum flexibility, since it allows of arranging converting units in any manner desired with a possibility of later re-arranging if called for by a change in load conditions. This may then be accomplished without modifications of the control.

Theater Dimmers. A combination stage dimmer and switchboard termed "Controlite" has recently been developed by the Ward Leonard Electric Company. A quick make and quick break switch opens the circuit after the lights are dimmed. This simplifies the control of the lights by reducing the number of movements which the operator has to make. It has previously been considered good practise to dim the lights with one lever and reach elsewhere on the switchboard to open the switch.

Space on the stage is limited and the control handles have frequently been placed eight and nine feet from the floor. *Controlite* by its simplification makes it possible to place all operating handles within easy reach of the operator.

Motor Driven Dimmers. There has been an increasing demand by the trade for motor-driven remote controlled dimmers to control the lights in the auditoriums of moving picture theaters, churches and masonic halls. The Ward Leonard Electric Company has developed several very efficient types of motor drive for their dimmers.

Control Equipment

A new line of motor and generator field rheostats has been completed by the Ward Leonard Electric Company. The smaller sizes are of the Vitrohm (vitreous enameled) type and the larger capacities are of the "Ribohm" (stamped metal grid type). Due to new developments in the art of enameled rheostat construction, the Vitrohm plates are made of stamped steel, instead of being of the former cast-iron construction. Many advantages are claimed as a result of this change.

This company has also recently placed a dead-front

type sectional battery charging panel on the market. It is especially adapted for charging electric vehicle batteries. The special features are compactness, ease of operation, and the total enclosure of all live parts. A unique design of enclosing cover renders all operating parts readily accessible when necessary.

The Ward Leonard Electric Company has also produced a line of totally enclosed contactor type motor starters which are unique in that the resistors form part of the enclosing case but are arranged so that all heat radiation takes place outside of the enclosure.

Appendix B

Motor Applications

Engineering and Mining Journal Press has published the following article on subjects listed:

Industrial Haulage

Determining Proper Mine Locomotive for Specific Service, Graham Birght, Nov. 18, 1922.

Scope—Factors to be considered in making selection of equipment of both storage battery and trolley types.

Coal Age

Specially Designed Motors for Larry Cars, Atwell, C. A., Feb. 14, 1924, Vol. 25, p. 237.

Motors for Mine Fans, B. W. Chadbourne, May 15, 1924, Vol. 25 p. 722.

Electrical Mine Machinery, T. F. McCarthy, Feb. 28, 1924, Vol. 25, p. 307.

Automatic Mine Electrical Equipment, E. J. Gealy, Jan. 17, 1924, Vol. 25, p. 77.

Air Lift Pumping, E. J. Gealy, Dec. 18, 24, Vol. 26, p. 868.

Locomotive Control, H. H. Johnson, Sept. 11, 1924, Vol. 26, p. 367.

Operation of Mine Locomotives, F. L. Stone, Nov. 13, 1924, Vol. 26, p. 681.

Fan Motor Applications, F. L. Stone, Nov. 6, 1924, Vol. 26, p. 644.

Battery Locomotive Haulage, J. B. Hicks, Jan. 8, 1925, Vol. 27, p. 39.

Savings made by Electricity, E. J. Gealy, Jan. 15, 1925, Vol. 27, p. 77.

Aerial Tram for Rock Dump, F. H. Kneeland, Feb. 12, 1925, Vol. 27, p. 249.

Coal Cutting and Loading Machines, N. D. Levin, April 2, 1925, Vol. 27, p. 499.

Huge Mine Hoist, Delbert Kay, April 9, 1925, Vol. 27, p. 535.

Flexible Coal Conveyor, N. D. Levin, May 7, 1925, Vol. 27, p. 675.

American Machinist

1-9. Cranes

Direct or Alternating Current for Overhead Traveling Cranes, William L. Laing, Vol. 60, pp. 133-134.

Advantages and disadvantages of each type—Selecting a crane motor with characteristics to fit the work, speed, braking, overload capacity, service and other characteristics.

Solenoid Load Brakes for Crane Hoist Motors, H. H. Vernon, Vol. 61, pp. 23-24.

Benefit of Synchronous Motors by L. J. Moore, San Joaquin Light & Power Corp., Application 5-2, Oct. 4, 1924, Vol. 84, No. 14, p. 733,

Discusses induction motors—recommends that they be installed to operate at full load—start without compensators.

Push Self-Starting Motors by J. C. Hobart, President, Triumph Electric Co., Application 2-2, March 14, 1925, Vol. 85, No. 11, p. 554.

Solenoid load brake saves weight and space, electrical connections, regenerative braking by the motor.

Electrical World

Analysis of benefits of use of synchronous motors.

Transactions A. I. E. E.

Another New Self-Excited Synchronous Induction Motor by Val. A. Fynn, 1925, p. 64.

A New Self-Excited Synchronous Induction Motor, p. 660.

Brown-Boveri Mitteilungen

Motors with Forced-Air Cooling by P. Suter, Jan. 1925.

Journal of the Institution of Electrical Engineers, (England)

Predetermination of the Performance of Induction Motors by D. B. Hoseason, March, 1925.

Electric Journal

Lap Windings with Unequal Coil Groups by C. R. Riker and A. M. Dudley, January 1925.

Effect of Unbalanced Voltages on the Operation of Induction Motors by O. C. Schoenfeld, January, 1925.

Electritchestwo,

Method of Starting Asynchronous Motors without a Rheostat by Prof. K. I. Shenfer, No. 2, 1924.

Transactions A. I. E. E.

Complete Synchronous-Motor Characteristics by John F. H. Douglas, Eric D. Engeset and Robert H. Jones, Vol. 43, 1925, p. 982.

Electritchestwo

Problem of Collectorless Direct-Current Generators by G. A. Akomovperetz, No. 8, 1924.

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Squirrel-Cage Induction-Motor Core Losses by T. Spooner, Vol. 43, 1925, p. 262.

Electritchestwo (Russia)

Single-Phase Collector Motor with Independent Excitation by A. V. Lebederv, No. 3, 1924.

Electrician (England)

Steel and Electrical Engineering by Sir Robert Hadfield, December 5, 1924.

Journal of A. I. E. E.

Electric Power Application in Pacific Northwest Fir Mills by J. L. Wright, December, 1924.

Electric Railway Journal

Boston "L" Woodworking Shop Uses Individual Drive, Dec. 27, 1924.

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Electrician (England)

Skip Hoists by George Frederick Zimmer, December 26, and January 2, 1925.

Iron and Steel Engineer

Main-Roll Drives in the United States and Canada, Jan. 1925

Electrician, (England)

Spinning and Weaving Mills by H. Fuhrken, Feb. 27, 1925.

Electric Drive in the Cotton Industry by F. B. Holt, Feb. 27, 1925.

Elektrotechnische Zeitschrift

Development of Electric Rolling-Mill Drive by F. Rohde, Feb. 12 and 19, 1925.

Blast Furnace and Steel Plant

Electric Motor Drives for Centrifugal Compressors by Gordon Fox, Jan. 1925.

Operation of Electric Elevator Machines—Traction Type.

Revue Generale de l'Electricite, Nov. 29, 1924.

Industrial Electric Heating Installations in Publication, No. 24-80 of the N. E. L. A.

Electrical World

Electrifying Ten-Inch Merchant Mill by M. J. Wohlgemuth and M. H. Morgan, Jr., Westinghouse Electric & Mfg. Co., Application 1-1, Vol. 84, No. 10, p. 465, Sept. 6, 1924.

Details of power design and installation.

A Completely Electrified Foundry by E. J. Apperly, G. E. Co.,

Los Angeles, Application 1-1, Vol. 84, No. 11, p. 515, Sept. 13, 1924.

Description of installation in foundry of Alloy Steel and Metals Co., Los Angeles.

Asynchronous Unity-Power-Factor Motor by Val A. Fynn, St. Louis, Mo., Application 5-2, April 18, 1925, Vol. 85, No. 16, p. 816,

Description and discussion of Fynn-Weichsel motor.

Dual-Frequency Supercalender Drive by W. H. Scott, Application 1-5, April 4, 1925, Vol. 85, No. 14, p. 705.

Electricity in a Modern Lumber Project by Frank Innes, Sessions Engineering Co., Chicago, Application 1-4, Jan. 17, 1925, Vol. 85, No. 3, p. 137.

General Plan. Describes original installation generally.

Electricity in a Modern Lumber Project—Part II, Application 1-4, 5-4, Jan. 24, 1925, Vol. 85, No. 4, p. 199.

Electricity in a Modern Lumber Project—Part III, Application 1-4, 5-4, Vol. 85, No. 4, p. 199,

Primary Distribution and Substations.

Features of Electric Truck Motors by C. A. Atwell, Westinghouse Electric & Mfg. Co., Application 1-6, Feb. 28, 1925, Vol. 85, No. 9, p. 453.

Discussion of speed, efficiency, torque characteristics of motors used.

Transformer Maintenance Schedule by Editor, Application 4-4, Jan. 12, 1924, Vol. 83, No. 2, p. 87.

Discusses maintenance routine and supervision of transformers.

Electricity in the Oil Fields by W. G. Taylor, Industrial Eng. Dept., General Electric Co., Application 1-8, Feb. 2, 1924, Vol. 83, No. 5, p. 229.

Type of motors, costs, Power consumption.

Selecting Motors for Cotton Mills by W. S. Maddocks, Electrical Engineer, Lockwood, Greene & Co., Boston, Application 1-2, Feb. 9, 1924, Vol. 83, No. 6, p. 273.

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Bonusing Use of Synchronous Motors by Herbert, Application 5-2, Vol. 83, No. 6, p. 285.

Control Applications

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Care and Adjustment of Controllers, H. D. James, April 17, 1924, Vol. 25, p. 580, Class 2.

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Avoiding Electrical Accidents, J. F. MacWilliams, Oct. 2, 1924, Vol. 26, p. 468, Class 4.

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Automatic Reclosing Circuit Breakers, Raymond Howey, July 31, 1924, Vol. 26, p. 152, Class 1.

Protecting Mine Equipment, J. F. MacWilliams, March 19, 1925, Vol. 27, p. 431.

Iron and Steel Electrical Engineer

Direct-Current Hoist and Trolley Control for Ore Bridges by N. L. Mortensen, December, 1924.

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1. Automatic Controllers
2. Control Application

Electrical Equipment for Machine Tools, John W. Harper, Vol. 61, pp. 261-264.

Devices giving protection from overload—Dynamic braking—Push-button and jogging control—Alternating and direct-current motors and their uses—Types of Winding.

Principal Development in Shop Equipment, L. C. Morrow, Vol. 62, p. 89.

Resumé of first six months of 1924, marked by tendency toward greater use of motor-drive and push-button control.

Machine Tools with Built-In Electrical Equipment, John W. Harper, Vol. 62, pp. 780-782.

Changes of motor drive in machine tools, Individual drive replaces group drive, Increased use of machines having motors built in, Auxiliary motors used on large tools.

Electrical Equipment for Machine Tools, John W. Harper, Vol. 61, pp. 261-264.

Conclusion—Devices giving protection from overload, Dynamic braking, Push button and jogging control, Alternating and direct-current motors and their uses, Types of winding.

Swing Frame, Portable and Hand Grinding Machines, Special Correspondent, Vol. 61, pp. 645-648.

Early swing frames, Greater freedom of movement needed, More power and better wheels required, The electric motor and the solid wheel aid in development.

Principal Developments in Shop Equipment, L. C. Morrow, Vol. 61, p. 89.

Resumé of first six months of 1924, marked by tendency toward greater use of motor drive and push-button control.

Developments in Shop Equipment, L. C. Morrow, Vol. 62, p. 81.

Resumé of last six months of 1924, marked by trend toward the use of the motor drive and the adaption of ball bearings.

1-10. Machine Tools

Electrical Equipment for Machine Tools, John W. Harper, Vol. 61, pp. 215-217.

General features—Characteristics of various types of motor controllers, Duty each type is best fitted to perform, Undervoltage protection, Magnetic or push button control.

Electrical World

Automatically Controlled Asbestos Mill by A. A. St. Aubin, Canadian Comstock Ltd., Application 2-1, March 28, 1925, Vol. 85, No. 13, p. 655.

Description of plant and system of control from one control room.

Power

Machine Tools with Built-In Electrical Equipment, John W. Harper, Vol. 62, pp. 261-264.

Changes of motor drive in machine tools, Individual drive replaces group drive, Increased use of machines having motors built in, Auxiliary motors used on large tools.

2. Control Applications

Operation and Care of Drum-Type Manual Controllers for Motors by H. D. James, March 4, 1924.

Operation and Care of Cam Type Manual Controllers for Electric Motors by H. D. James, March 25, 1924.

Limitations of Manual Controllers for Electric Motors by H. D. James, April 29, 1924.

Control of Auxiliary Drive Motors, A. L. Harvey, May 13, 1924.

Engineering and Mining Journal-Press

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A discussion of the characteristics and design of the various types in use.

Electrical World

Dual-frequency Supercalender Drive Increases Production 25 per cent, Application 1-5, 3-a, Nov. 15, 1924, Vol. 84, No. 20, p. 1048.

Description of Installation at Kalamazoo Paper Co.

Mechanical End of Motor Drive

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3. Mechanical End of Motor Drive
   - 3-1. Lineshafting
     - 3-1-b. Bearings

Advanced Practise in Anti-Friction Bearings, P. W. Liebermann, Vol. 60, pp. 333-334.

Data on two important conveyor installations, Application to electric motors, Desirability of making comparative tests in laboratory.

Bearing Pressures and Friction, Louis Illmer and Leonard N. Linsley, Vol. 61, pp. 959-962.

Abstract of two papers presented at A. S. M. E. annual meeting, Effects of quality and quantity of oil, Friction coefficient, Ultimate load capacities and breakdown pressures.

Principal Developments in Shop Equipment, L. C. Morrow, Vol. 62, p. 81.

Resumé of last six months of 1924, marked by trend toward the use of motor drive and the adoption of ball bearings.

Installation and Maintenance of Ball Bearings, H. N. Parsons, Vol. 62, pp. 731-732.

Care in handling, Instructions covering application, Removing and cleaning bearings, Lubricants; the kinds to use and the kinds to be avoided and the reasons therefore.

3-2-b. Leather Belts

American Machinist

Slip, Friction and Stretch Tests for Leather Belting, L. C. Morrow, Vol. 60, pp. 469-471.

How the slip test is performed, Measuring stretch, Determination of coefficient of friction, Example of an overload belt running without tension on the slack side.

Belting Standardization at the Du Pont Plants, William Staniar, Vol. 60, pp. 977-978.

Installations made and tested under severe service conditions to compare various makes and grades, Purchases made on basis of record, Description of system.

3-3. Gearing

3-3-a. Speed Reducers

Annular Gearing, Its Design and Application, P. H. Bryant, Vol. 60, pp. 125-128.

Principle and method of operation, action of the gear teeth, points of design of annular gear sets, construction employed to give uniform speed. Applicable to drives requiring large reduction ratios.

Laboratory Tests of Non-Metallic Gears, R. H. Moyer, Vol. 60, pp. 505-507.

Work done to determine strength and endurance, Evolution of the testing equipment, Tests for vibration and the effects of oil Reproducing service conditions.

Worm Gear Reductions for Heavy Service, H. Edsil Barr, Vol. 60, pp. 931-932.

Description of reducer in restricted position from 100-h. p. motor to 60-in. rubber mill.

Spur Gear Speed Reducers. What Are They? Why and Where Are They Used? Warren G. Jones, Vol. 61, pp. 689-690.

Range of reduction ratios, Straight line and offset drives, Origin of the inclosed reducer, Advantage in space economies, Comparative cost and upkeep.

Wear of Reduction Gear Teeth Measured from Plaster Casts, H. B. Chapman, Vol. 62, pp. 51-52

How an accurate cast of helix or tooth can be made, Method serves to keep complete record of reduction gear wear.

3-3-b. Chains

The Application of Silent-Chain Drive to Machine Tools, Charles W. Weiss, Vol. 61, pp. 873-876.

Development and characteristics of the bushed silent chain, Advantages of silent chain drive, Applications of silent chains to machine tools, Principles of automatic chain tightener.

An Improved Design of Chain Sprocket Teeth, Charles W. Weiss, Vol. 61, pp. 993-996.

New tooth form results in considerably increased chain life, Principles applicable to cut teeth as well as cast teeth, Pitch line clearance and pressure angle.

Manufacturing Silent Chain Parts, C. J. Priebe, Vol. 61, pp. 145-148.

Where silent chain is used, Applications for power transmission, Wide adaptability, Heavy, high-speed and large reduction installations, Use on machine tools.

3-4. Direct Drive 3-4-a. Couplings

Safety Couplings for Machines Drives, F. Osgood Hickling, Vol. 61, pp. 921-923.

Devices to prevent damage to machines when drives or feeds are overloaded and which take the place of the braking pin in light machines.

3-4-b. Clutches

Formulas and Data for Friction Clutches, George W. Drake, Vol. 61, pp. 955-957.

Formulas and data for cone, split-ring and disk friction clutches, Practical examples of the formulas.

Coal Age

Anti-Friction Bearings Lower Transportation Costs, F. H. Kneeland, April 24, 1924, Vol. 25, p. 603, Class 1-b.

Twin-Rope Skip Hoist, H. F. Hebley, A. T. Allen, June 26, 1924, Vol. 25, p. 935, Class 2-c.

Prolonging Life of Wire Rope, L W. Bevan, June 12, 1924, Vol. 25, p. 872, Class 2-c.

Power

3. Mechanical End of Motor Drives

Relative Merits of Different Types of Motor Bearings by F. E. Boyd, April 22, 1924.

Substation

Coal Age

Flashing of Substation Generator, E. B. Cameron, June 12, 1924, Vol. 25, p. 887, Class 5-c.

Automatic Substation Equipment, Chester Litchlenberg, June 19, 1924, Vol. 25, p. 912, Class 6.

Protecting Converters & Motor-Generator Sets, N. S. Taylor, April 10. 1924, Vol. 25, p. 545, Class 6.

Fitting Switchboards to Appliances, W. L. Newmeyer, August 7, 1924, Vol. 26, p. 184, Class 6.

Preventing Transformers from Burning up, D. E. Kenworthy, July 10. 1924, Vol. 26, p. 57, Class 4.

Power

Transformers Connected for Operation on a Two-Phase Circuit by B. A. Briggs, Jan. 15, 1924.

Some High Points of Modern Transformers by Arthur Palme, March 11, 1924.

Transformers Connected for Operation on a Three-Phase Circuit by B. A. Briggs, April 22, 1924.

Grounding Transformer Secondaries by J. B. Gibbs, August 26, 1924.

Special Applications of Standard Transformers by J. B. Gibbs, November 25, 1924.

High-Power Mercury-Arc Rectifiers by F. C. Bailey, January 22, 1924.

Scherbius Controlled Induction-Synchronous Frequency Converter by R. E. Greene, February 17, 1925.

Electrical World

Fields for Power Sales Development by E. W. Lloyd, Commonwealth Edison, General, Sept. 13, 1924, Vol. 84, No. 11, p. 518.

Description of various power applications of electricity possible.

Electric Power Distribution Systems in Industrial Plants

Coal Age

Mine Power Plant Generates Cheaply, C. L. Moorman, Feb. 7, 1924, Vol. 25, p. 210, Class 4.

Wasted Power, A. J. Hoskins, April 3, 1924, Vol. 25, p. 498, Class 4.

Mine Generated Power, A. J. Hoskins, May 1, 1924, Vol. 25, p. 631.

Generate Power Safely, H. F. Hebley, and A. Allen, July 3, 1924, Vol. 26, p. 3, Class 6.

Bonds for Tracks, J. B. Austin, July 3, 1924, Vol. 26, p. 330, Class 6.

Electric Mine Equipment Guarded Against Delays, E. J. Gealy, July 24, 1924, Vol. 26, p. 117, Class 6.

Mining Co. Generates more Power Than Public Utility, E. J. Gealy, July 17, 1924, Vol. 26, p. 82, Class 6.

Dangers of Ground Currents, E. E. Jones, Oct. 23, 1924, Vol. 26, p. 574, Class 6.

Central Metering Point Saves Power Costs, A. MacDonald, December 4, 1924, Vol. 26, p. 793, Class 3.

Choosing the Proper Power Contract, W. H. Russel, Sept. 25, 1924, Vol. 26, p. 437, Class 3.

Reducing Energy Consumption of Mine Apparatus, W. H. Russel, Oct. 9, 1924, Vol. 26, p. 499, Class 4.

Mines can Generate Power Economically, C. H. Matthews, Jan. 1, 1925, Vol. 27, p. 11, Class 4.

Power Factor Correction at Mines, W. B. Synder, Feb. 19, 1925, Vol. 27, p. 281, Class 2.

Dangers of Stray Currents, C. S. Hurter, Feb. 19, 1925, Vol. 27, p. 284, Class 4.

Static Condensers Cut one-third off Power Bill, E. J. Gealy, May 14, 1925, Vol. 27, p. 709, Class 2.

Power

Making Electrical-Machinery Tests in Industrial Plants by J. Elmer Housley, Dec. 2, 1924.

Testing Direct-Current Motors by J. Elmer Housley, Jan. 20, 1925.

Testing Alternating-Current Motors by J. Elmer Housley, Feb. 24, 1925.

Selection, Use and Care of Recording Electrical Motors by J. Elmer Housley, May 5, 1925.

How to Calculate Three-Phase Power Distributing Circuits by J. E. Berger, June 24, 1924.

Application of Static Condensers to Power-Factor Correction by R. E. Marbury, April 28, 1925.

Electrical World

Power-Factor Correction in Practise—I, by L. W. W. Morrow, Application 5-2, Nov. 22, 1924, Vol. 84, No. 21, p. 1096.

Analysis of practical accomplishments in industrial power factor correction.

Practical Ways to Correct Power Factor by H. B. Dwight, Application 5-2, Dec. 6, 1924, Vol. 84, No. 23, p. 1199.

Discussion of synchronous motors and other correcting methods.

Economics of Power-Factor Correction by Raymond Schulze, Duquesne Light Co., Application 5-2, Dec. 15, 1924, Vol. 84, No. 24, p. 1247.

Analysis of power-factor correction methods.

Economic Aspects of Power-Factor Correction by A. S. Knight, Boston, Application 5-2, March 28, 1925, Vol. 85, No. 13, p. 662.

Discussion of power rate problem.

Secondary Power Distribution, Application 2-1, Jan. 24, 1925.

Description of motor applications and control systems.

Benefits of Power-Factor Correction by A. M. Perry and H. C. Anderson, Application 5-2, Feb. 14, 1925, Vol 85, No. 7, p. 34.5

Description of experience with correction methods in various industrial plants.

How central-stations should encourage high power-factor in industrial plants, Bristol Dwight, Feb. 9, 1924.

Selection of Corrective Equipment by M. A. Hyde, Westinghouse Elec. & Mfg. Co., Application 5-2, March 8, 1924, Vol. 83, No. 10, p. 472.

Comparison between Static Condensers and Synchronous Condensers.

Power Costs in Cotton Mills by Editor, Application 1-2, March 29, 1924, Vol. 83, No. 13, p. 621.

Survey of Conditions in Twenty Mills.

Modern Hotel Electrification by Karr Parker, Manager, McCarthy Bros. & Ford, Application 1-7, August 30, 1924, Vol. 84, No. 9, p. 405.

Discusses electrical application for elevators, ventilation, lighting with wiring diagrams.

Electricity's Progress in the Iron and Steel Industry

By Committee on Applications to Iron and Steel Production[1]

AT the first meeting of this Committee held in the Schenley Hotel, Pittsburgh, Pennsylvania, September 17, 1924, every member being present, it was agreed that the Annual Report for 1924-25 should be a topical consideration of progress during the current year, and the field was duly apportioned to the several members of the Committee.

One of the papers secured by the Committee and deserving of special mention in this Report is that, prepared and presented by Mr. K. A. Pauly before the Annual Convention at Pasadena, California, September 1924, entitled "Contributions of Electricity to the Steel Industry."

1. Annual Report of Committee on Applications to Iron and Steel Production.

F. B. Crosby, Chairman
E. Gordon Fox, E. S. Jefferies, G. E. Stoltz,
Eugene Friedlaender, D. M. Petty, J. D. Wright.
A. G. Pierce,

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 24, 1925.

It was also agreed that each member of this Committee should actively interest himself in arranging for one or more joint meetings of the Local Sections of the A. I. E. E. and the Association of Iron and Steel Electrical Engineers (A. I. & S. E. E.).

Although many members of this latter organization are also members of the A. I. E. E., your Committee was of the opinion that closer cooperation and unity of interests might be brought about by the formation on the part of the Association of Iron and Steel Electrical Engineers of a committee empowered to act directly with your Committee in all matters relating to the mutual interests of the two organizations.

This matter was brought to the attention of your Board of Directors, who, on September 26, 1924—

VOTED: That the Board of Directors recognizes the importance of the work being carried on by the Association of Iron and Steel Electrical Engineers in the industry with which it is associated, and appreciates the desirability of closer cooperation between the Institute and the A. I. & S. E. E., and requests its

Committee on Applications to Iron and Steel Production to consider and report to the Board methods of cooperation between the two organizations and means of bringing it about.

This action on the part of the Directors was duly transmitted by this Committee to the Board of Directors of the A. I. & S. E. E., who, through an incomplete appreciation of the objects in view, at first rejected this suggestion. Further consideration, however, has resulted in the consent of the Directors of the A. I. & S. E. E. to give the proposition more thorough study before taking final action.

Every twelve month period shows the steady and truly remarkable, if not always spectacular, increase in the application of electricity in the manufacture of steel. The current year of 1924-25 has been no exception. Generally speaking, however, the growth has been along lines already well defined and presents little that is fundamentally new or novel.

I. Generating Units

The Steam turbine continues to hold first place as a prime mover in the steel plant. That there has been a substantial increase in the generating capacity of steel-mill power plants is evident from the fact that one manufacturer reports the sale during this past year of fourteen units ranging from 750 to 15,000 kw. and aggregating 111,000 kw.

It is an open question, however, as to whether the greater economies which are being sought throughout the industry may not, together with improved design of engines and gas cleaning equipment, bring the slow speed gas engine back into favor. The economic value of blast furnace gas is increasing, due to its successful use in various metallurgical processes, soaking pits, heating furnaces, etc. This increasing value will automatically necessitate its use at the highest possible efficiency and as a given quantity of gas can be converted into more kw-hrs. of electrical energy through a gas engine than when burned under a boiler, the return of the gas engine, in spite of its present high first cost and maintenance charges, is a possibility.

Very satisfactory progress is being made in the development and use of Diesel Engine units up to 5000-kw. capacity.

II. Distribution

No marked improvements in distribution have been reported, but the use of purchased power has rapidly increased as the size and reliability of commercial power systems has increased. Largely for this reason, 60-cycle current now predominates.

Automatically controlled railway and power substations have been in successful operation for several years, and during the past year there have been installed several such equipments in steel plants. One large steel plant has changed over all of its manual stations, involving five motor-generator sets and a large number of a-c. and d-c. feeders, to full automatic control. At another plant a full automatic substation has been installed to control two 1000-kw. motor-generator sets and all a-c. and d-c. feeders. A third plant has installed three equipments for the operation of motor-generator sets and feeders. This increase in the use of automatic substation control has been due primarily to the extreme reliability of operation, together with the operating saving that can be shown, as compared with manual control, to take place where power is used, thereby reducing the line losses in the feeder circuits.

III. Main-Roll Drives

To an even greater extent than usual, this past year has been marked by a replacement of existing engine drives by modern electrical equipment. The sturdy induction motor still meets, in some one of its several forms, most requirements for main roll drives, except for reversing mill duty where, of course, nothing is likely to replace the d-c. machine with generator field control.

The growing demand for high tonnage mills with great flexibility in range of product is, however, requiring more and more adjustable speed, d-c. motors and this in turn is giving more importance to the relative merits of motor-generators, rotaries and mercury-arc rectifiers as a means of transforming from alternating to direct current. For certain types of mills the synchronous motor is being considered very favorably, although no installations of importance have as yet been made.

The following tabulation includes only main-roll motors on a continuous rated basis in units above 300 h. p. as reported by the three principal electrical manufacturers in this country up to June 1, 1925.

	1923	1924	1925
60 cycle	452840	478390	543440
25 cycle	475825	490225	538450
Direct current	299670	324860	430610
Totals	1228335	1293475	1512500

It is interesting to note that out of 95,225 h. p. reported by one company as sold during the year units (13) totaling 23,750 h. p. or approximately 25 per cent of the total represents motors which have been purchased to replace existing steam engine drives.

For the operation of a continuous skelp mill there has been purchased a 7500-h. p. induction motor with Kraemer speed regulating set to adjust the speed of this motor from 250 to 134 rev. per min. This is the largest motor with Kraemer equipment that has ever been built.

Another manufacturer reports the construction of a 5000-h. p., 75/150-rev. per min. motor for reversing service built with a single armature and supplied at 700 volts from two generators in parallel forming a part of a three-unit set.

This is a distinct departure from the usual practise of having several armatures connected in series where

units of this size are involved. This motor replaces an engine on a 48 in. universal plate mill.

Other notable units reported by this manufacturer are:

2—3500 h. p.—50/120 r. p. m.—700 volt—reversing units
2—5000 h. p.—25/150 r. p. m.—700 volt—reversing units
1—5000 h. p.—50/120 r. p. m.—700 volt—reversing unit
4—7000 h. p.—50/120 r. p. m.—700 volt—reversing units
1—8000 h. p.—40/80 r. p. m.—700 volt—reversing unit

Physically, the 7000 h. p. motors are the largest single armature reversing motors yet built and the 8000 h. p. will be, when completed, approximately 50 per cent larger. This latter motor will drive a 54 in. blooming mill which is likewise the largest mill of the kind in this country.

Another notable installation is reported by the same manufacturer, namely, a tandem hot-strip mill driven by one 1500 h. p. induction motor and six individual adjustable speed d-c. motors with an aggregate rating of 10700 h. p. for the d-c. machines which are of the compensated type.

A strong and sturdy yet easily operated foot-master switch has been developed for the control of reversing mill motors. The use of a foot-operated master switch requires one less operator in the mill pulpit than with a hand-operated controller and some mills have also found that production is increased as the concentration of control to one man permits closer coordination of the various operations.

Two speed regulating equipments of the frequency converter type are being built for use with existing 600 h. p. motors.

Truck-type switching equipment is being used to a greatly increased extent in many mills. Less time is required for assembly in the field, greater safety is secured as live parts are better protected and shutdowns are fewer as a spare truck may be kept available to quickly replace a damaged one.

IV. Automatic Control

An interesting tendency in auxiliary mill-motor control is toward the use of time limit acceleration instead of current limit which has long been the accepted standard. In one system the time interval between the closing of successive accelerating contactors is secured by an ingenious application of the well-known principle that when a constant d-c. potential is applied to a circuit containing inductance, an appreciable time elapses before the current reaches its maximum value.

Another control system recently placed on the market secures the time element of acceleration by the delay in building up or down of a magnetic field; when the relay coil is short-circuited, a definite time is required for the flux to decrease to a point permitting the release of the relay armature. This armature is forced out by a spring of adjustable tension which, together with other features, gives a timing range of 0.2 to 2.0 seconds for the relay. Once adjusted, this timing remains constant under all operating conditions.

In one case where this control was substituted for a series-contactor control, the current peaks were reduced from nearly 800 amperes to a maximum of approximately 350 amperes. This reduction in current peaks is due to the fact that, with the series-contactor control, it is necessary to set the contactors to close at a relatively high value of current in order to take care of maximum load conditions which are encountered when a mill is first started. This adjustment usually is not changed after the mill has been limbered up, and as a result the motor and control are forced to handle all loads with the same effort as required for the maximum load. With a definite time control, the motor is forced to exert its maximum torque only when the load conditions demand it.

V. Yard Electrification

The search for the best all around system of yard transportation continues slowly because of the excessive investment charges. Installations now in operation for two years or more seem to have demonstrated conclusively that a third-rail system can be operated successfully in a steel-plant yard without excessive maintenance charges or danger to employees. The first cost is high—approximately one dollar ($1.00) per foot of trackage electrified.

The development of the Diesel Electric Locomotive for this purpose is proceeding very satisfactorily. When production permits reasonable first costs, its very attractive operating characteristics will undoubtedly bring it into general use in steel plants.

VI. Electric Heating

With the expiration of patents covering certain alloys of great value for use in resistor units, the development of annealing and heat-treating furnaces has received marked impetus. Furnaces of large capacity with temperatures of 1800-2000 deg. fahr. are being installed in considerable numbers.

Two electric furnaces for the continuous hardening, quenching and tempering of carbon-steel wire have been recently installed in wire mills. This process results in a higher quality of wire than when treated in a gas-fired furnace. A bright unoxidized surface on the wire is produced. These are the first electric furnaces to successfully do this work and have proven economically desirable as well as producing a more satisfactory product.

VII. Arc Welding

The use of arc welding as a means of repair of worn and broken parts in steel mills has shown a great increase within the past year. The savings effected by arc welding of a single worn or broken part in a number of cases more than paid for the first cost of the welding equipment. A number of steel plants are using the automatic arc-welding process for the building up of worn shafts and similar operations.

VIII. ELECTRIC FURNACES

Conditions in the development of "arc furnaces" for melting steel and iron have become fairly well standardzied, both as to installed kw. per ton, voltage between electrodes, etc. Continued investigation of the two-voltage system of operation, a high voltage for melting followed by a low voltage for refining, indicates that there is no deleterious effect on the product while there is a decided saving in the power and electrode consumption, together with the time required to make a heat.

In this field, there continues a steady, if slow, development of low frequency "induction furnaces" for making high grade steels and alloys, but the field of high frequency induction furnaces has witnessed one of the most interesting developments of recent years.

For many years it was thought that the high frequency induction furnace could not operate successfully in melting metals at frequencies lower than 5000 cycles. However, it has recently been demonstrated that frequencies approximating 500 cycles will melt metals on a commercial basis with very good efficiency. For instance, common brass has been melted starting with a hot crucible with power consumption of approximately 225 kw-hr. per short ton—and other metals in proportion.

The equipment is simple, consisting of a motor generator set taking power from standard power circuits and supplying approximately 500-kw. single-phase to the furnace. The furnace consists of a standard clay graphite crucible, as developed for steel melting, with heat insulating casing around which is placed an edge-wound copper strip coil. To the terminals of this coil is applied 900 or 1800 volts at 500 cycles, single phase; a capacitor unit being connected across the coil in order to bring the power factor up to approximately 100 per cent.

In view of this recent development, no definite data are available, but it is confidently expected that this condition will be changed in time for the next report.

Advances in Use of Electricity in Mines

By Committee on Applications to Mining Work[1]

THE work of this committee is, of necessity, limited very largely to the securing of papers from representative mining engineers.

During the year several such papers have been presented, including one by W. C. Adams of the Allen & Garcia Co., on "Coal Mine Electrification;" one by Shelton and Stoetzel on "Electric Shovels;" one by W. C. Clark of the Westinghouse Co. on "Application of Motors to Mine Locomotives;" and one by your Chairman, entitled "Electricity in Mines."

The subject of mining does not seem to particularly interest many A. I. E. E. members, and it was very difficult to draw out a good discussion on any of these papers. The reason for this, the author believes, is more or less obvious and is due, at least in part, to the fact that the mining profession in general feels that the A. I. E. E. as a body is not particularly interested in its problems. The Chairman is convinced that the only way to stimulate interest at a mining meeting and possibly break down this impression, is by holding the mining session in conjunction with some local mining society. If this is done, it seems certain that the members of the mining society will turn out and will be glad to cooperate and participate in the discussion of such points of interest as may arise.

There seems to be very little constructive work for the committee on mine applications. This field has been very well covered by other organizations; thus it would seem that the only work left for this committee is to keep constant watch on the development and assist, whenever possible, in the improvement of mine electrification.

During the year there has been no outstanding and novel development in the application of electricity to mining projects, but, on the other hand, there has been a steady and healthy growth of this application. Larger projects have been undertaken and put through to a successful finish than ever before. One notable instance is the five-mile belt conveyor underground at the Colonial Mine of the H. C. Frick Coke Co. Here we have coal transmitted on a belt conveyer from the mine face to the docks, a distance of approximately five miles, on twenty sections of conveyer at a speed of 450 to 500 ft. per minute. This conveyer has a potential capacity of some 10,000 tons per day. It has been in operation for some considerable time and can be stamped as entirely successful.

Another notable installation is that of the largest coal mine hoist in the world; namely, the 4000-h. p. Ward-Leonard coal mine hoist which operates in the Orient No. 2 Mine of the Chicago, Wilmington & Franklin Coal Co.

The Old Ben Coal Co. has purchased three 2200-h. p. coal mine hoists, two of which are in operation, the third to be installed shortly.

From a business standpoint, the bituminous mining industry finds itself today in a very unfortunate condition.

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1. Annual Report of Committee on Applications to Mining Work.

F. L. Stone, Chairman,
R. T. Andrea, C. N. Beebe, M. C. Benedict, Graham Bright, H. W. Eales, L. C. Ilsley, G. M. Kennedy, R. L. Kingsland, A. B. Kiser, John A. Malady, D. C. McKeehan, W. F. Schwedes, W. A. Thomas, C. D. Woodward.

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y. June 24, 1925.

The majority of coal operators feel that cheaper production of their product will help matters to a considerable extent. The author believes the actual savings that result from complete electrification are not fully appreciated by all operators. Reliable records show savings as high as 25 cents per ton, resulting from change-over from steam to electricity in the same mine. Further, the stand-by losses when mines are electrified are very greatly reduced, so that shut-downs and idle periods are not such serious matters from a cost standpoint with electrified mines as they are with steam-driven mines.

The electrification of coal-loading machinery is receiving a great deal of attention from the manufacturing engineers as well as from the coal mine operators. There is no other practical method of drive for these machines, and, like the majority of mine applications, improper motors have been applied on early machines with the usual result.

There is still a great deal of work to be done of an educational nature so far as the mine operator is concerned. The motors required for his work should be, almost invariably, of a special design, laid out to meet mining conditions. Standard industrial motors are very rarely applicable to this class of work, and their application usually ends in delays and dissatisfaction.

In conclusion, it is suggested that something might be accomplished by the formation of a joint committee on Application of Apparatus to Mines; this committee being made up of the chairmen of the various committees at work on this subject in other societies such as the American Institute of Mining and Metallurgical Engineers, the American Mining Congress and the United States Bureau of Mines; these gentlemen meeting with the chairman of the committee on "Application of Electricity to Mines" of the American Institute of Electrical Engineers; this latter body possibly acting as sponsor for such a committee. Such a committee could review the work already done along this line and suggest changes or give its approval.

The most outstanding work along this line has been done by the committee on Underground Transmission and the committee on Power Equipment of the American Mining Congress. These committees, working jointly, have prepared a set of rules and suggestions for the installation and care of electrical apparatus in and around mines. These rules and suggestions are, at the present time, before the American Engineering Standards Committee for its approval. A committee made up of the personnel suggested above might go over these rules and suggestions in a constructive manner, and, if their approval is received, would undoubtedly help the mining industry to some extent at least.

Rules and Personnel Problems of the Marine Field

By Committee on Applications to Marine Work[1]

It is believed that the report of the Committee on Application to Marine Work for this year may be said to be the most comprehensive of any similar reports for the past few years.

The main proposition on hand for this year was the revision of the existing Marine Rules. These rules were issued about five years ago, and at that time were more or less of a tentative draft, and had become somewhat outlawed by changes in the art. Due to the fact that the Sectional Committee of the American Engineering Standards Committee did not seem to be functioning with sufficient rapidity to insure a recognized standard within the near future, the Marine Committee voted last Fall to re-edit and publish a revised set of Marine Rules, and for the past year, practically all of the time of the Committee and Subcommittees has been devoted to this line.

In the above connection, exceptional credit must be given to last year's Chairman and the Chairmen of the other Subcommittees for their untiring efforts and cooperation to bring about the desired results.

Among a few of the details which have been under consideration, and in some cases accomplished, may be mentioned the following:

a. A member of our Committee has been appointed to represent the A. I. E. E. on the Marine Standards Committee.

b. The 1923 proceedings of the National Fire & Protective Association adopted the Marine Rules in so far as they applied to insurance.

c. The British Consul requested the Committee's advice in regard to using magnetic cranes in shipyards.

d. A member of this Committee addressed the Boston Section in March, on the subject of the Mer-

1. Annual Report of the Committee on Applications to Marine Work.

L. C. Brooks, Chairman
H. Franklin Harvey, Jr., Vice-Chairman
J. S. Jones, Secretary

R. A. Beekman,	H. L. Hibbard,	Arthur Parker,
J. F. Clinton,	William F. James,	G. A. Pierce,
M. W. Day,	J. S. Jones,	H. M. Southgate,
C. S. Gillette,	M. A. Libbey,	W. E. Thau,
Wm. Hetherington, Jr.,	W. F. Meschenmoser,	A. E. Waller.
	I. H. Osborne,	

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 24, 1925.

chant Marine, especially covering the point of responsibility of electrical operators, and the failure of the U. S. Steamboat Inspection Service to recognize its responsibilities.

e. In the report of the meeting of November 1924, was included an item of Publicity, giving a bibliograph of recent data which had been published in various magazines, as applying to Marine Electrical Industry.

It is believed that an extension of this service by the Institute would be a desirable undertaking, (possibly in cooperation with Mechanical Engineers) in connection with the Engineering Index. We understand that this matter is now under consideration, and the Committee most heartily endorses the proposition.

The most important work of the Marine Committee of this year is contained in the conference held on Thursday morning at the St. Louis Convention, at which session there were three articles presented: one covering the History of Electrical Application to Marine Work up to date; another, the Application of Electrical Propulsion; and a third, covering Merchant Installations and Electrical Operators, with special reference to the licensing of electrical engineers.

It is believed that a survey of these three articles may be considered as a very definite progressive report of the work of this Committee for the year, and suggestion is made that all interested might peruse these three articles to advantage.

In connection with the suggested work of next year, the recommendation would be, if possible, to develop an inspection to include electrical devices for marine use. Also, possibly, to extend the scope of the Committee to include certain branches of manufacturing as are not already included within the scope of our Marine Rules, such as Instruments, etc. Also, there will no doubt develop certain phases of the present rules that will need further revision, and the development of rules as suitably applying to electric propulsion.

Progress in Diverse Lines of Electrochemistry and Electrometallurgy

By Committee on Electrochemistry and Electrometallurgy*

TWO papers on electrochemical subjects have been presented at meetings of the Institute by members of this Committee. The first of these entitled "Electrometallurgical Applications" by J. L. McK. Yardley was presented at the Pacific Coast Convention in October 1924. This paper dealt with developments in the utilization of electrical power based upon fundamental principles of electrochemistry and the relation of the electrical engineer to the chemist and metallurgist. The second paper, by G. W. Vinal and G. N. Schramm, was entitled "Storage Battery Electrolytes." This was presented at the A. I. E. E. Midwinter Convention, February 1925. Measurements to determine the effect of a wide variety of impurities in the electrolyte were summarized and a proposed specification for using sulphuric acid in storage batteries presented for discussion.

The Committee has suggested to the Standards Committee the desirability of revising and extending the section on storage batteries in the Standards of the Institute. The section on the method of rating storage batteries is not entirely clear at the present time. Serious difficulties in the industry have recently arisen in the matter of rating the various sizes and kinds of storage batteries for use on automobiles and for radio purposes. However, the point to be considered, does not relate to the rating of particular types of batteries, but rather to the general principles for rating batteries and to clearer indication of how time and current ratings may be used. Revision of the definition of the word "charge" has been suggested as well as the possible addition of specifications for the purity of the solutions. The Standards Committee has acted favorably on these suggestions and the appointment of a working committee to include a wide range of those interested in the subject of storage batteries has been authorized. Mr. Vinal has been chosen chairman of this working committee.

A request for information on the physiological effects of electric currents, addressed to the Institute, has been referred to this Committee. In response to this inquiry a short bibliography on the subject has been prepared. This bibliography does not contain references to the therapeutical or strictly medical aspects of the subject nor to electrocardiographs. Since information on this subject does not seem to be readily available elsewhere and as it may be of interest to others, the bibliography is appended hereto.

The field of electrochemistry and electrometallurgy is so diverse, including as it does such contrasting subjects as potential measurements and electric furnaces; electro-plating and storage batteries; electrolytic rectifiers and the production of materials, that it is difficult to define its boundaries or to estimate the relative importance of achievements in the various lines of

*Annual Report of Committee on Electrochemistry and Electrometallurgy.

George W. Vinal, Chairman

A. M. Hamann,	W. A. Moore,	John B. Whitehead,
Carl Hering,	W. E. Moore,	C. D. Woodward,
E. T. Moore,	J. A. Seede,	J. L. McK. Yardley.

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 24, 1925.

activity. Fortunately for the electrical engineer, interested in electrochemistry and electrometallurgy, there are available several up-to-date lists of titles of publications covering the entire field. These are found in both the monthly *Bulletin* of the American Electrochemical Society and the monthly issues of *Mining and Metallurgy*. These lists are generally more up-to-date than the abstract journals.

It is inevitable that no two summaries of the progress of the art in any particular field can be alike because of the limitations of individual knowledge and the trend of personal opinions. The following resumé, for which the Chairman of the Committee is largely responsible, aims to present very briefly significant points about the recent developments within the field of electrochemistry and electrometallurgy.

The rapid growth of the automobile industry and the still more recent development of radio have resulted in a great expansion of the storage-battery industry. The vexed question of how to rate storage batteries of certain types is still being discussed. Radio batteries are marketed as having a certain number of ampere-hours capacity, but no standard method of rating them has been adopted. One of the most notable, but less conspicuous achievements, has been the production of thin plate batteries for airplane service. Plates of 0.050 inch which would have been thought impossible not long ago are now in use. Contrasting the output of these batteries with the familiar automobile battery it is found that the capacity per pound has been increased from 50 to 75 per cent in spite of the handicap imposed in making them nonspillable. The life of the plates is from 50 to 100 cycles which together with the cost, limits their use to certain kinds of service. Our knowledge of the low temperature characteristics of these batteries has been extended.

Primary batteries especially dry cells have experienced notable developments chiefly as a result of the interest in radio. The production of dry cells which runs into the hundreds of millions per year has been accompanied by a general improvement in the quality. The establishment of Standards of Performance and the Specification of Tests have been instrumental in making this advance. The quality of well established brands has been improved, and by comparing records of performance of 1918 with 1920 and 1924, it is found that there are more manufacturers today whose cells comply with the government requirements than in 1920 when the requirements were not as severe. The American Engineering Standards Committee has asked the Bureau of Standards to act as sponsor for a representative sectional committee to undertake the standardization of dry cells and batteries. This committee is now being formed.

Further development of electric furnaces involves the important question of improving the refractories. A symposium on the subject of refractories was held by the American Electrochemical Society at its Spring Meeting in Philadelphia last year. During the slump after the War, there was some question as to whether electric melting of steel and brass would continue to increase at its former rate when conditions were again normal. The rate of increase now indicates that the "saturation" point is still far from being reached. Experiments have been reported that show a saving for the electric melting of brass over coke-fired pit furnaces. Improvements in automatic electrode control devices have been made. In the low-temperature furnace work and ovens for enameling, the adverse effect has been felt of certain recently developed auto finishes.

High frequency induction furnaces are finding increased use for laboratory and experimental purposes and also for small scale production. The larger sizes are now operated on high frequency generators which have replaced the oscillatory spark gap. The smaller units may be operated by electron tubes, obviating the risk of escaping mercury vapor. It is expected that applications to large scale uses will ultimately be made.

Ring-type horizontal induction furnaces of six tons capacity and 800 kw. are now in use.

From the standpoint of the electrical engineer the fertilizer industry represents an outlet for electrical energy that is dependent upon the demand for more concentrated fertilizers. Phosphoric acid is now made in the electric furnace. The development of processes for the fixation of nitrogen has indicated a trend toward decreased power requirements per unit of nitrogen. The arc process which was first developed involved a large power consumption. The cyanamid process, next in order of time, required only one fourth as much and more recently has come the synthetic ammonia process requiring only one sixteenth the power (without electrolytic hydrogen) of the arc process. But in this last case it is evident that the electrical power requirements will depend very largely upon how successfully the electrolytic production of hydrogen can compete with more strictly chemical methods. Electrolytic hydrogen is produced directly in a high state of purity which makes it well suited to the purpose. At the September meeting of the American Electrochemical Society to be held in Chattanooga there will be a symposium on the relation of the electrochemical industry to the production of fertilizers. This is to include papers on phosphoric acid, the production of hydrogen and the fixation of nitrogen.

Production of the lighter metals by electrolysis of fused salts has been stimulated by the demand for light-weight materials of construction and by the radio industry. Aluminum and magnesium are perhaps the most important. Developments in the technique of the electrolytic production of aluminum have been made during the past year. The ability to produce aluminum of very high purity will doubtless have an effect on its uses within the field of the electrical engineer. Experiments on beryllium and calcium have also been mentioned. A symposium on fused electro-

lytes was held at the Niagara Falls meeting of the Electrochemical Society, April 23-25, 1925.

Improvements in the electrolytic production of other metals have been recorded and electrolytic tin has been added to the list of commercial products.

Three methods for the production of electrolytic iron were described at the World Power Conference. During the discussion it was stated that electrolytic iron was an assured commercial possibility, but that it would not become a serious competitor of ordinary iron for some time to come. About 28 per cent of the caustic soda produced in this country is made by the electrolytic process. New outlets for chlorine products are being sought by the newly established Chlorine Institute, Inc. which has been created by a group of manufacturers.

Electrolytic rectifiers have become a subject of renewed interest as a result of the phenomenal growth of the radio industry. Improved forms of both the aluminum and tantalum rectifiers have recently appeared. These are designed for the charging of small batteries for radio receiving sets and the latter has also been found useful by railroads for charging signal batteries. Our knowledge of output and efficiency of these rectifiers in relation to the impressed voltage and the battery voltage has been clarified and the operating characteristics materially improved.

A promising method for the study of the instantaneous values of electrode potentials has been developed by the use of a resistance coupled vacuum-tube amplifier in combination with the oscillograph. By means of this amplifier it is possible to obtain sufficient power to operate the oscillograph without polarization of the cell, and on the other hand with current flowing through the cell it is possible to distinguish between the electrode potential and the $I R$ drop.

Scientific methods are being extended in the field of electroplating. Results of research and much practical experience are gradually being welded into a comprehensive theory of electrodeposition. The structure of electrodeposited metals has been studied and new methods for the regulation of plating baths and hydrogen-ion control have been introduced. Electrolytically deposited coverings for the prevention of corrosion have recently been the object of much study. The use of zinc for this purpose is increasing. Nickel does not afford complete protection at the present time, but the technique of nickel plating is being improved. Experiments have also been made on chromium and cadium. Other preventives of corrosion include alternate layers of copper and nickel and the plating of alloy deposits such as brass, bronze and mercury-zinc.

Engineers are taking cognizance of the importance of the corrosion problem. The American Society of Testing Materials held a symposium on this subject in June 1924 and the American Electrochemical Society did likewise in October of the same year. More recently the American Chemical Society has taken the matter up and it is proposed to establish a corrosion institute. Whatever may be the outcome of conflicting opinions on the fundamental causes of corrosion, a theory for its prevention, involving both chemical and electrical agencies, will probably be ultimately agreed upon.

Use of the earth current meter during the past year has been gradually extended and there appears to be a general recognition that it affords a more accurate means for determining the rate of stray current corrosion than has been available heretofore. Its use is limited by the time and expense involved in making the excavations necessary for its use.

The recently published transactions of the first World Power Conference, London 1924 (Volume IV) contain papers on electrochemistry and electrometallurgy as follows:

The Austrian Electrochemical Industry, Paweck.

Small Waterpowers and Electrothermal and Electrochemical Loads, Bodex.

Nitrogen Fixation, Halvorsen.

A New Resistance Furnace with Reaction Zone, Holmgren.

Electrochemical Industry in Sweden, Palmaer.

Power in Electrochemical and Electrothermal Industries, Fitzgerald.

Electrical Engineering as a Leading Factor in the Development of Modern Steelworks, Geyer.

Electrolytic Iron, Hutchins.

Electrometallurgy in Italy, Giolitti.

Power in Electrometallurgy in the U. S., Mathewson.

The electrical engineer who is interested in electrochemistry as an outlet for power will find statistics of interest in several of these papers.

Appendix

PHYSIOLOGICAL EFFECTS OF ELECTRIC CURRENT

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2. Resistance and Conductivity

Electrical Resistance of Human Body, Hooper, *Elec. Rev.*, 38, p. 821, 1901.

Electrical Resistance of Human Body Tissues, Bordier, *Arch. d'elect. med.*, 11, p. 544, 1903.

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The Human Body as an Electrical Conductor, Gildemeister, *Elekt. Zeit.*, 40, p. 463, 1919.

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Resistance of Human Body, C. M. Dowse and C. E. Iredell, *Arch. Radiol. & Electrother.*, 25, p. 33, 1920.

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Experiences sur l'electrocution au laboratoire central electricite par la commission charges d'elaborer le texte de l'instruction sur les premiers soins a donner aux victimes des accidents electriques. Zacon, *Bull. de la Soc. Internat. des Elect.* Ser. III, Vol. 1, p. 437, 1911.

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A Year's Progress in Lighting

By Committee on Production and Application of Light[1]

THE past year resembles the preceding one, in that the notable advances were those of better and more intensive application of available equipment and methods rather than of fundamental discoveries.

The really outstanding event of the year was a nation-wide campaign of education on home lighting. This activity was conducted by the entire electric lighting industry during the fall of 1924 and represented the most extensive cooperative movement ever undertaken by the electrical industry. Because of its noncommercial character it received widespread endorsement from school authorities, and culminated in an essay contest in which about one million high school pupils competed for valuable prizes.

It is safe to say that, as a result of this campaign, the American public has a better understanding of lighting, especially in the home, and is approaching such problems more intelligently.

1. Annual Report of Committee on Production and Application of Light.

G. H. Stickney, Chairman

W. T. Blackwell,	F. F. Fowle,	F. H. Murphy,
J. M. Bryant,	G. C. Hall,	Charles F. Scott,
W. T. Dempsey,	H. H. Higbie,	B. E. Shackelford,
H. W. Eales,	A. S. McAllister,	W. M. Skiff,
F. M. Feiker,	P. S. Millar,	C. J. Stahl.

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, June 24, 1925.

In general, lighting has enjoyed a very healthful advance both in extent and quality of application. It has been a year of prosperity to the industry and of enhanced service to the people.

As pointed out in previous reports, the best numerical figures available to indicate the growth of lighting application are in the number of incandescent lamps consumed.

The large incandescent lamps represent the lighting of factories, stores, homes, streets, trains and similar places on circuits of electric service companies, electric and steam railways and other power plants. Of these large lamps, 263 million were sold in 1924, an increase of 7½ per cent over 1923.

The other major class is the miniature lamps of which about two-thirds are used on motor vehicles, one-sixth for flashlights and other small battery lamps, and one-sixth for Christmas trees and similar decorative purposes. Of this class about 188 million lamps were sold, or an increase of 8 per cent over 1923.

These figures are quite conservative, since they indicate only the number of lamps and are not weighted according to the size of lamps. Particularly in the large lamp group there is a tendency toward the use of higher power lamps. Had the measure been in terms of wattage capacity, the increase would presumably have been greater, or if in lumen capacity, still greater.

Such figures will soon be available and are desirable for certain kinds of comparison. The simple, numerical figure is, however, suitable for the present purpose.

Illuminants. Present-day illuminants are still far below the scientific ideals of efficiency, and considerable experimental investigation is underway. While favorable indications have been observed, no important practical improvements have been reported.

Frequent minor improvements have been made in incandescent lamps, but these have been mostly in the nature of refinements that do not receive prominent recognition by the users. That the improved manufacturing appliances mentioned in previous reports have brought results, is evidenced by the several reductions in lamp prices. It is also reported that the average quality of the lamps has improved during the year.

A general exposition of the advance of incandescent lamp quality since Mr. Edison's invention was given by J. W. Howell, in his address[2] (on receiving the Edison Medal) at the 1925 Midwinter Convention.

In last year's report mention was made of the tendency toward the use of the ring-shaped filament, that is, a helically coiled filament formed into an open ring. More recently there has been a tendency to warp the ring so as to give a higher horizontal component of light.

With the application of incandescent lamps to new uses, there is a constant tendency to multiply the number of special types, bases, and other features, which interfere with interchangeability, increase lamp costs, and often introduce confusion in the selection of lamps, as well as delay in securing them. Lamp Engineers are constantly studying means of simplifying and standardizing. The great advantage of standardization to light users is particularly evident to those familiar with conditions abroad where less progress has been made toward eliminating unnecessary variations.

Considerable experimental work has been done on diffusing finishes for lamp bulbs, with the view of effectively meeting the various demands of lighting practise. Some of these promise real improvement in the near future.

During the past year or two there has been an increase in accuracy of incandescent lamp manufacture, particularly in the focus types where small tolerance of light center length is important.

The automobile rear lamp has been raised from two to three candlepower and improved in efficiency. Experiments have been made with automobile headlight lamps, having two equal power filaments, to produce a suitable dipping of beam by switching from one filament to the other. Much experimental work has been carried on for the Government and others to adapt the incandescent lamp to the various requirements of night flying, and considerable improvement has been made during the year.

2. See A. I. E. E. Journal, March 1925, p. 310.

Candle Power Standard. A remarkable research extending over several years and reported in 1924, culminated in a proposal of a primary standard of candle power. All previous primary standards have been so subject to variation for one cause or another, as to be too inaccurate for standardizing purposes. In 1910, the United States Bureau of Standards cooperated with the English and French National Laboratories in establishing the international unit of candle power recorded in the construction of incandescent lamps. No other method compared with this for accuracy. The desirability of an absolute standard, which can be reproduced from specification and which will avoid any possibility of drift, is obvious. While it is too early to say absolutely that this has been accomplished, the indications are that it has. The standard device consists of a cylindrical platinum fuse with a longitudinal slit. This is supported by conductors at both ends and heated by an electric current. A certain section of the interior as viewed through the slit has been found to follow the "black body" law, and readings are taken with increasing temperature up to the point where the platinum melts, blowing the fuse. In other words, the method measures the light emitted by a black body at the melting point of platinum, a condition which has been regarded as most likely to give a suitable primary standard.

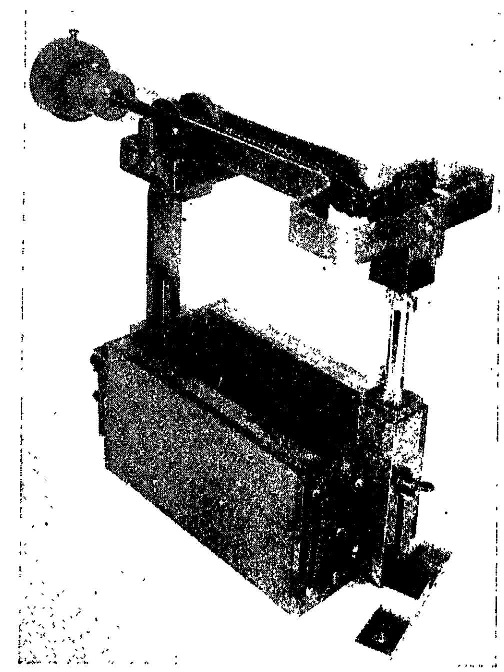

Fig. 1—New Primary Standard of Candle Power Proposed by Dr. H. E. Ives

Device for producing black body radiation at melting point of platinum showing slitted cylinder of platinum and support. (See text.)

Lighting Practise. No radically new devices have come out. The general tendencies recorded in last

year's report have continued. In commercial lighting, the shallow enclosing globe has seemed the most popular, although there has been a growing use of certain forms of indirect and semi-indirect units, especially in the highest grades of installations where accurate vision is important.

In the interest of eyesight conservation it is gratifying to note the continuing spread of appreciation of the need of good lighting in schools. This will no doubt be stimulated by the new American Engineering Standard Code of School Lighting. There is still a serious economic resistance, and it is still true that when daylight fails, the student is less adequately cared for than the office worker. Nevertheless the year has witnessed a large increase in the number of reasonably adequate installations.

Store window lighting has been a field of increased activity. The data referred to in last year's report have apparently been effective in convincing merchants of the sales value of light. Colored light and spot lights are being extensively used and are making the merchandise more interesting to look at. In the business sections, the practise of providing strong illumination in the daytime for the purpose of eliminating external reflections or rendering the displays more effective, has grown rapidly.

The advance in levels of window lighting is indicated by the fact that equipment manufacturers have found it desirable to develop and market reflectors for 300- and 500-watt lamps, whereas a year ago the reflector for 150-watt lamps was the largest in common use.

In industrial lighting the steel dome, because of its economy, coupled with moderate diffusion, is still the most common equipment, but in many processes a combination glass and steel unit, and the various forms of opal and prismatic globe equipments, are being preferred because of their greater diffusion. Illumination levels are still being raised. After a survey of the situation the electric lighting industry has concluded that the time is ripe for increased activity in the industrial field, and an extensive campaign is projected for the fall of 1925.

Home lighting is more essentially an artistic problem than an engineering problem. However, certain engineering and utilitarian features are deserving of more attention, and it is important that artistic considerations include the lighting effect as well as the design of the equipment. The trend of practise is toward better diffusion and more illumination, avoiding what has been aptly termed "glare and gloom."

The use of portable lamps is spreading rapidly, with an increasing use of those which direct considerable light to the ceiling, for redirection. Portable equipment has certain features of flexibility which permit the exercise of personal taste. It is also becoming quite common for housewives to make their own shades. While some of these are not particularly effective, they permit a rather free expression of personal taste and improvements may be expected through a better understanding of the possibilities. The best practise for general rooms usually requires a combination of fixed and portable equipment, and it is desirable that wiring should be planned to accommodate both types.

The kitchen lighting movement which was reported as very active last year has been less conspicuous, in contrast to the educational movement of 1925. Nevertheless it is probable that the number of improved installations in kitchens has continued to increase.

Electric Light Wiring. One of the problems confronting the engineers who are endeavoring to improve lighting practise is the tendency to stint in the wiring of buildings and fail to provide outlets in locations necessary to produce suitable illumination. In order to call attention to this question and encourage better provision, a so-called "Red Seal" campaign has been instituted under the direction of the Society for Electrical Development and installations complying with an accepted standard are indicated by a red seal.

FIG. 2—SIGN LIGHTING, SHOWING ONE OF THE WORLD'S LARGEST SIGNS ON BROADWAY, NEW YORK

The upper sign employs nearly 20,000 incandescent lamps.

The progress in the application of the elexit or disconnecting support for lighting units has been somewhat disappointing, in view of the inherent advantages of such a scheme. They are being found exceedingly useful in luminaire display rooms, and it is probable that such use will stimulate their application elsewhere.

Outdoor Lighting. Larger and better things are being done in sign lighting and the use of higher power lamps which was formerly confined to a few important centers, is becoming common in smaller cities and less central locations. To what an extent illuminated advertising is contributing to the illumination of metropolitan business districts can only be appreciated by visiting those sections at the very late hours when it has ceased to function. By contrast, the regular street lighting

seems surprisingly faint. One large sign installed in 1924 contained 19,000 lamp sockets.

Colored floodlighting, especially in connection with flashing effects, is finding considerable application, both for advertising signs and for buildings.

The trend is still toward the ornamental units and poles. Twin units have been used to some extent in white-way lighting, although these are usually less economical than the single higher power unit.

Recent statistics on the sale of series incandescent lamps indicate that over 60 per cent are of 1000 lumens or less. In the opinion of engineers who have made general analysis of street lighting costs this percentage is too high and better economic conditions would exist if larger lamps were being used. This is based on the principle that the cost of service increases much less rapidly than the volume of light. The tendency is in the right direction, but greater progress is desirable.

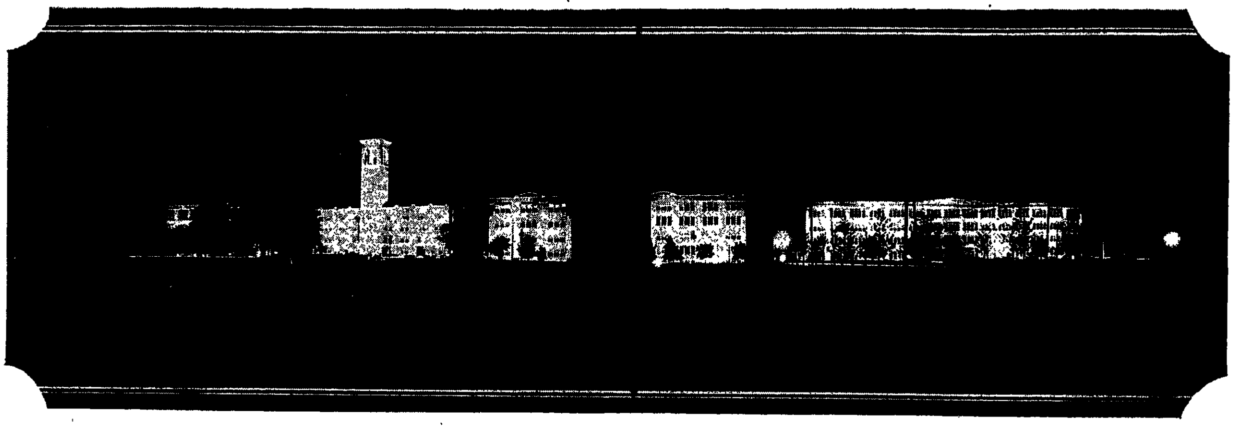

Fig. 3—Floodlighting for Publicity Purposes

A large playing card factory at Cincinnati has one of the largest of the recent installations. Eight 500-watt floodlights illumine the tower, twenty-four 1000-watt floodlights light the facades facing the camera, and eight more are projected on surfaces facing in another direction and therefore not shown.

Street Lighting. Street lighting has been quite active during the year. It is estimated that the new and replacing installations of 1924 required about fifteen per cent more equipment than those of 1923, which in itself had shown quite an advance. The vast majority of these installations utilize the gas-filled tungsten lamp.

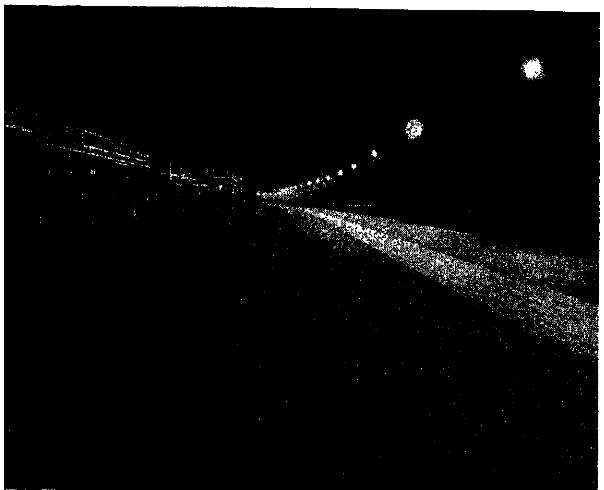

Fig. 4—The Boardwalk

Atlantic City has, in some sections, to be illuminated from one edge only. Specially devised ornamental units direct the larger portion of the light across the walk and still provide a lower illumination on the beach without shadows on the globes. Width of walk 60 ft., spacing of posts 70 ft., size of lamps 750-watt. Photo taken late at night after sign and show window lights were extinguished.

Each unit is arranged so that light from a 200-watt lamp symmetrically directed can be switched on.

There have been minor improvements in transforming and other accessory equipment, as well as a few improved designs of luminaires.

In residence and other secondary streets, the asymmetrical types of distribution seem to have proved their worth. Some discussion is still going on as to the merit of various characteristics of asymmetrical distribution, and it is probable that the next few years experience will bring out a better general understanding as to the types and degrees suited for different street lighting problems.

Asymmetrical lighting of highways, utilizing equipment designed to deliver the maximum illumination on the road surface is meeting with considerable success, and promises to provide the best solution for the problem of handling heavy night traffic on important roads.

Extensive experimental studies preliminary to establishing improved street lighting have been underway in a number of cities, notably Indianapolis, Indiana; Columbus, Ohio and St. Louis, Missouri. It is reported that the investigation in Indianapolis has led to a decision to install a new unified system through that city.

During the year, a cooperative study of urban street lighting was made by a committee appointed jointly by the New York State Conference of Mayors and other City Officials and the Empire State Gas and Electric Association. This committee has made a

constructive recommendation regarding practises to be followed and standards of lighting to be adopted for various classes of streets.

A general committee of the National Electric Light Association has been studying the problems of street lighting and is preparing a report which should be of considerable value.

Still other street lighting activities are underway, and studies by simultaneous comparison in demonstrations are being made. With the existing need and the widespread study of the various problems, it is probable that the next few years will witness a rapid advance in this field.

Traffic Signal Lighting. Closely associated with street lighting is the traffic signal lighting. This falls in two main classes.

The flashing traffic beacons which merely warn of an intersection, are placed at moderately congested points of cities, towns and interurban roads. Electric signals of this sort are being installed in considerable number.

The second group is the traffic control signals for controling the movement of traffic in congested districts. It has been found expedient to synchronize the various signals of each district, and to extend the system some distance beyond the points of traffic congestion. Not only does this sort of a system reduce accidents by speeding traffic, but it reduces the congestion, not to mention the convenience resulting to the motorist. A considerable number of such installations have been made during the past year and many others are projected.

FIG. 5—HIGHWAY LIGHTING—PLEASANTVILLE BOULEVARD, ATLANTIC CITY, ILLUMINATED WITH 2500 LUMEN (250 C. P.) LAMPS IN UNIT DESIGNED TO CONCENTRATE LIGHT ON ROADWAY

Units about 25 ft. high, spaced one for each 225 ft. of road, staggered. Central motor way, glossy surface, about 28 ft. wide, wagon way on each side about 15 ft. wide.

A notable example is that on Broadway, New York City, from Rector Street to 86th Street, a distance of six miles. This system contains a number of new and interesting features. It is arranged so as to avoid obstructing the roadway and requires only six police officers for its operations, where an earlier form would have required at least twenty-six. Moreover, these officers are on the street level and so quickly available for emergencies. A parallel system is about to be installed in Seventh Avenue.

Other Practises. Lighting practises in the operation of railroads are extending rapidly. While floodlighting of railway classification yards is not new, recent investigations have brought about a considerable extension of such lighting and a more definite crystallization of the practise. The Association of Railway Electrical Engineers is undertaking the preparation of a comprehensive manual of railroad lighting practises.

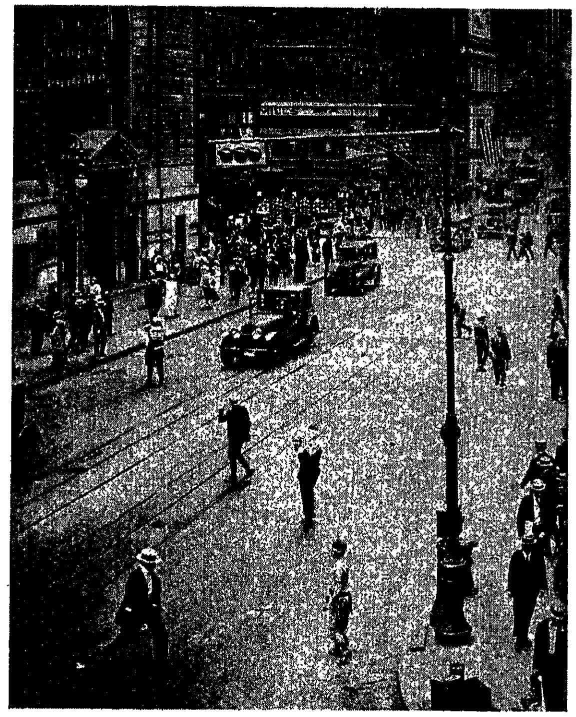

FIG. 6—TRAFFIC SIGNAL LIGHTING

New installation on Broadway, New York, showing arrangement for locating signals conspicuously over line of travel without obstructing traffic.

Electric lighting is contributing much to the facility and safety of surgical operations. Much better lighting is being provided in some of the leading operating rooms, and various small lamps are playing an important part in lighting internal organs. Although not new during the year, there appears to have been but little mention in engineering circles of an instrument for entering the stomach or lungs through the mouth. By the light of the smallest lamp made, it is possible to examine the walls of these organs, remove foreign matter, such as tacks or pins, and perform other operations in the saving of life.

In the newer applications of artificial light, such as night flying, plant growth control, the use of polarized light to study internal strains of structural forms, considerable progress has been made.

Education. Educational work has been carried on extensively. Besides a number of classes which have been operated by various organizations, notably the incandescent lamp manufacturers, an extensive course to train illuminating engineers for electric service companies was operated jointly by the Illuminating Engineering Society and the National Electric Light Association. This group of students came together in Chicago, visited South Bend, Detroit, Cleveland, Washington, New York and vicinity, Boston and Lynn. Lectures were given in Cleveland, New York and Harrison and representative illuminating engineering departments were visited and studied.

Another large lighting demonstration has been opened since the beginning of 1925 and in a number of European cities, demonstrations have been patterned after the American practise. The European activity in this field has no doubt been considerably stimulated by American engineers who have been abroad during the year, and several of whom presented important papers on lighting practise at the Geneva meeting of the International Commission on Illumination.

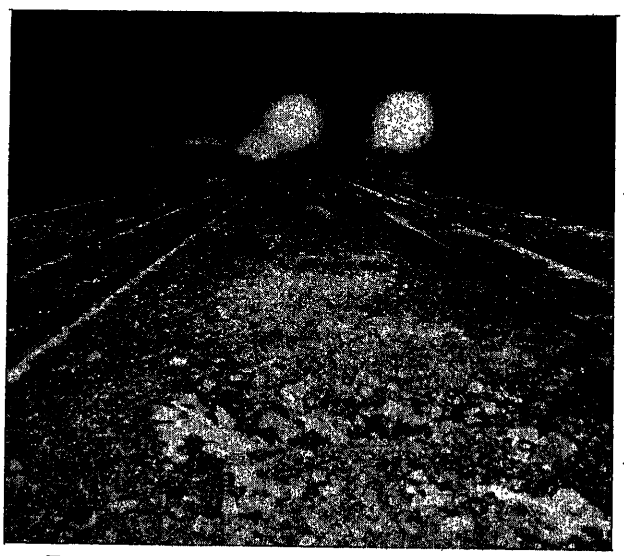

Fig. 7—Railway Classification Yard Lighting
An installation partly completed at Selkirk, N. Y. which employs nearly 100 floodlights, 1000 watts each, projecting light along the road in both directions.

At this meeting, the commission which has in the past centered its attention on standards, and highly technical phases, decided to place more emphasis on practise and applications. The 1927 meeting is scheduled to be held in the United States.

During 1924, the State of Washington adopted an industrial lighting code, making the tenth state to take such action. These ten states represent a population of about 42,000,000.

The revised School Lighting Code, initiated by the Illuminating Engineering Society, has become an American Engineering Standard under the joint sponsorship of the American Institute of Architects and the Illuminating Engineering Society.

Industrial Lighting Tests. It has been generally assumed that good illumination was warranted economically through its influence in speeding production. Data supporting this view has been accumulating for several years. The National Research Council is undertaking an extensive series of tests, which when completed should provide authoritative figures over a range of representative industries. The Council is undertaking to study the welfare aspect as well as the economic. Because of the extent of the problem as well as the thoroughness of the methods, the results are not expected to be available for at least another year.

Committee Activity

The Committee on Production and Application of Light is composed for the most part of members who are widely separated geographically.

This is suitable for the general work of the committee but precludes full attendance at meetings. As a result, after the original organization meeting each year, practically all business has been conducted by correspondence.

The organization meeting for the current year was held at A. I. E. E. headquarters, October 9, 1924, three members being in attendance. Because of this small attendance, the Chairman submitted the minutes to the entire committee for comment before considering the action final. No criticism was received.

The committee organization of last year was retained; Dr. B. E. Shackelford being detailed to solicit papers for conventions, Mr. W. M. Skiff being retained in charge of securing Illumination Items and fillers for the A. I. E. E. Journal and Mr. G. H. Stickney was asked to supervise the preparation of the Annual Report. It was decided to continue previous policies and plans.

Later in the year the question of representation on the Standards Committee came up and was handled by correspondence. Since the Chairman was already attending the meetings of that committee in another capacity, it was considered expedient that he represent this committee.

Illumination Items. Because of the relation of the Institute membership to the lighting art, this has seemed to provide the best method of presenting a larger part of the lighting material to the organization. Throughout the year, the Editor of the Journal has been kept supplied with items in advance of his requirements in ample quantity to fill all of the space which he deemed it expedient to devote to this subject.

Convention Papers. Considering the state of the art and the interests of the membership, the committee has not considered it expedient to place many papers on the convention programs. Two papers have been arranged for presentation at national conventions during the year, viz., "Street Lighting—A Municipal Problem," by Rich D. Whitney—Pacific Coast Convention, Pasadena, October, 1924, "Automotive Headlighting," by J. H. Hunt,—St. Louis Convention, April, 1925.

Preliminary arrangements are underway for securing several papers for future conventions. A paper had been arranged for the regional meeting at Cleveland, but has been held in abeyance since the withdrawal of this meeting.

Cooperation with Branches. No convenient means seems to be yet available for cooperating with Branches. The Chairman has been informally experimenting, not in the name of the Institute, with one university in the hope of arriving at a plan. A course of lectures was arranged and is now being carried out. At the termination of this course, it may be possible to draw some conclusion as to its effect. A subcommittee under the chairmanship of Professor H. H. Higbie has been studying the problem, and it is hoped that a report will be received in time to be of assistance to next year's committee.

Lighting Publicity. A system is in vogue whereby members of the committee reviewing original Illumination Items indicate other publications likely to be interested in the material, whereupon the Editor of the JOURNAL undertakes to furnish proof of article to the designated periodicals with release dates.

In conclusion, the Chairman wishes to acknowledge the cordiality which has been extended to him by the entire committee, and especially the active cooperation of those members who have assumed the specific tasks already mentioned.

Recent Advances in the Communication Art

By Committee on Communication[1]

THE name of this committee has been changed from "Telegraphy and Telephony Committee" to "Committee on Communication." This was done in accordance with a recommendation of the special committee who reviewed the technical activities of the Institute. The Communication Committee feels that this is a desirable change in name.

In this report, under appropriate headings, are briefly summarized the advances which have been made, or which have come into prominence in the communication art during the past year. Thirteen papers have been presented to the Institute under the auspices of this Committee. These papers are in general a record of advances in the art, and are mentioned under the appropriate headings in the report.

Telegraphy. The development of long telephone cables of 19 and 16-gage, capable of operating for distances of 1000 miles and more, has led to the development of telegraph systems suitable for operating through these cables. These telegraph systems, developed by the engineers of the Bell Telephone System, were described in three papers presented at the Midwinter Convention. The titles of these papers were "Metallic Polar Duplex Telegraph System for Long Small-Gage Cables," "Voice Frequency Carrier Telegraph Systems for Cables" and "Polarized Telegraph Relays."

The first paper describes a type of telegraph circuit, designed to operate over cable telephone wires at frequencies below the telephone range, and without interfering with the telephone circuits using these same wires. In this system it was necessary to reduce the telegraph operating currents to values comparable with those used in the telephone circuits in order to prevent interference with the telephone circuits by the telegraph circuits. One form of interference which has thus been avoided is called the "flutter effect" and is caused by the telegraph currents affecting the magnetic characteristics of the loading coils in the telephone circuit, so as to cause a rapid fluctuation of received current. This telegraph system is on a balanced two-wire basis in order to diminish cross-fire between telegraph currents and the effect of any currents induced in the cable.

The second paper describes a carrier type of telegraph system which makes use of a regular four-wire long distance cable circuit. No telephoning is carried on over the circuit when employed for the telegraph system, but the frequencies which are ordinarily involved in a telephone conversation are used to transmit ten telegraph messages in each direction. A separate frequency is employed for each message spaced in the frequency range from about 400 to 2000 cycles. No change is required in the telephone circuit except at its terminals.

These telegraph systems required the development of very sensitive and reliable relays which are described in the third paper noted.

Submarine Telegraphy. The Western Union Telegraph Company has put into service its new permalloy loaded submarine telegraph cable between New York and the Azores Islands. This type of cable, developed by the Western Electric Company, was described in last year's report. It represents the most radical change in the submarine cable art since its earliest days.

1. Annual Report of Committee on Communication.

O. B. Blackwell, Chairman

F. L. Baer,	Sergius P. Grace,	F. A. Raymond,
H. P. Charlesworth,	P. J. Howe,	Chester W. Rice,
L. W. Chubb,	F. H. Kroger,	J. K. Roosevelt,
C. E. Davies,	N. M. Lash,	Edgar Russel,
H. W. Drake,	Ray H. Manson,	H. A. Shepard,
R. D. Evans,	G. W. McRae,	E. B. Tuttle,
L. F. Fuller,	R. D. Parker,	F. A. Wolff,
D. H. Gage,	H. S. Phelps,	C. A. Wright,

Presented at the A. I. E. E. Annual Convention, Saratoga Springs, N. Y., June 24, 1925.

The finished cable has fully met the expectations stated in the last report, and has so well established the success of this type of cable that the Western Union has ordered a second loaded cable to be laid between New York and Penzance, England, by way of Newfoundland. A paper describing the development of this type of cable will be presented at this convention.

During the year 1924, the Western Union Telegraph Company developed a printing telegraph system for use on ocean cables. It has been in commercial operation on a transatlantic cable circuit between London and New York. In this system, signals sent from a transmitter at London are translated and printed in Roman characters at New York without manual handling at any of the intermediate repeater stations.

The initial circuit consists of an underground pair between London and Penzance, England, 312 miles; ocean cable, Penzance to Valentia, Ireland, 307 nautical miles; ocean cable, Valentia to Heart's Content, Newfoundland, 1874 nautical miles; Heart's Content to North Sydney, Nova Scotia, 335 nautical miles; overhead land-line North Sydney to New York, 1110 miles; or a total of approximately 3900 miles. Regenerative automatic repeaters which regenerate the signaling impulses as to strength, shape and time are used at Penzance, Valentia, Heart's Content and North Sydney and universal duplex repeaters are used at St. John (New Brunswick) and Boston.

The code used is of the five-unit type, the same as is used in the Western Union Multiplex system. Tape printers are used at New York for translating the signals. The printer prints in Roman characters upon a gummed strip of paper ⅜ in. wide and this strip is gummed to regular cable message receiving forms for delivery.

Telephone Signaling. The use of 16 to 20 cycles alternating current for telephone signaling was developed in the very early days of the art, and is still employed, not only for ringing subscribers, but also for signaling on many toll lines.

The introduction of composite telegraph circuits brought about the introduction of 135 cycles for signaling, in order that the signaling should be at a frequency which was not used in the telegraph signals. Considerable improvements have been made recently in 135-cycle signaling, and it is being employed on very long circuits requiring many repeaters.

More recently voice-frequency signaling systems have been developed particularly for operation over very long repeater circuits. By thus using a frequency in the voice range, any circuit over which speech can be effectively transmitted also effectively transmits signaling currents of this frequency. Since the signaling is carried out at intervals when there is no talking on the circuits, there is no interference with the speech currents. It is evidently necessary, however, that some means be provided to prevent the speech currents from operating the relays employed with this system. This is taken care of by interrupting the signaling current approximately 20 times per second, thus producing a form of current which is not produced by the voice. A relay system arranged to operate with such current will not be operated, therefore, by any voice current.

An improved signaling system for taking care of the signaling and control arrangements necessary for handling calls over the wires of a toll circuit group, is being applied in the toll plant. By the use of this system the signaling and control arrangements for as many as 30 circuits are handled over a single pair of wires, thus freeing a large number of channels for telegraph purposes.

Machine Switching. During the past year, continued progress has been made in perfecting improvements in machine-switching equipments. These improvements are the result of intensive development work and have been made with a view of facilitating manufacture, installation and operation of central offices and private branch exchanges of this type. Progress has also been made in the development of new maintenance methods and tools, and in stabilizing the maintenance methods employed. This is of importance due to the large amount of apparatus involved in automatic operation, and the dependence that must be placed on it when connections are completed by machine switching.

The theory of probability plays a very important part in the design of switching systems, and in the determination of the amount of facilities required to meet various traffic conditions. A paper on "The Theory of Probability and Some Applications to Engineering Problems" was presented at the Midwinter Convention. Only a very few papers on this important subject have ever been presented to the Institute.

On December 31, 1924, there were 993,000 stations operating on a machine switching basis in the Bell System as compared with 567,800 switching stations at the end of 1923. In New York City there were 185,300 machine switching stations in service on December 31, 1924, operated from a total of 21 central offices.

The application of machine switching to the toll plant is being studied with a view to determining where it can be employed to advantage. An installation is being made of this system on a commercial basis and if found successful it will be extended.

Coincident with the development work on machine-switching equipments, considerable progress has been made in manual switching, particularly with respect to trunking methods. These improvements have been effected largely by the increased use of automatic methods in connection with the manual handling of calls and have resulted in reducing the manual labor necessary in completing telephone connections.

Telephone Distributing Frame Wire. Improvements

in means for flame-proofing wires, developed in the Bell System's Laboratories, have resulted in the production of rubber-insulated wire showing resistance to burning fully equivalent to that obtained in flame-proofed enamel and textile insulated wires. This wire is particularly adapted for use in the distributing frames of central offices. The rubber-insulated wire, so flame-proofed, possesses advantages in electrical characteristics and cost over alternative forms.

Telephone Transmission. An extensive investigation has been carried out by the Bell Telephone engineers with regard to the characteristics of speech and of hearing. A considerable number of papers have been published on this work. This analysis of the characteristics of speech has made it possible to determine accurately the effect of different frequency ranges on the intelligibility, loudness and naturalness of the spoken word. It is interesting to note that a large proportion of the energy of the voice is in the low range of frequencies of a few hundred cycles, while the higher frequencies up to several thousand cycles are of importance in providing clear articulation. As a by-product of these researches, important scientific results are being obtained regarding the nature of hearing, and methods of aiding the deaf and the dumb.

A paper entitled "Telephone Transmission Maintenance Practises," read at the Pacific Coast Convention, described the practise employed by the telephone companies in this country to guarantee their circuits maintaining a condition for giving the best transmission results. These practises are of the greatest importance in maintaining both toll and local circuits.

A paper read at the San Francisco Convention, entitled "Guided and Radiated Energy in Wire Transmission," gives a good discussion of transmission over wire circuits, and the conditions under which part of the energy is radiated from such circuits. This is a matter of fundamental technical importance since so much of our electrical art depends on the fact that electromagnetic waves may be guided by conducting wires.

Telephone Circuits have become so long, both geographically and electrically that echoes may be set up in them. The effects produced are very similar to those with sound waves. Whenever a voice wave meets an electrical irregularity in a circuit, some part of the wave is reflected. If the time required for the speech waves to travel to the irregularity and for the reflected waves to return to the speaker or listener is sufficiently great, the effect becomes an echo. A discussion of such echo effects and arrangements devised for avoiding them where they become of importance was described in a paper before the St. Louis Convention entitled "Echo Suppressors for Long Telephone Circuits." It is in telephone circuits of such length as to require a number of repeaters, particularly in long cable circuits that the effects may become sufficiently serious to justify the echo suppressors which are described. With these suppressors, the speech currents operate relays which block the echoes without disturbing the main transmission.

Outside Plant Practises. Tests of full size poles of the various timbers used in telephone work have recently been made by the Bell System engineers, in order to redetermine their moduli of rupture. These tests are probably the most comprehensive that have ever been made on full size specimens, and it is hoped that a paper can be obtained during the coming year describing the tests and giving the results obtained.

Drop wire having the two insulated conductors placed parallel under a common braid has been standardized for use in the Bell System. This is the type of wire used generally for connecting subscribers' stations with the nearest cable terminals. The new construction, in place of the twisted pair construction formerly employed, results in reduced accumulation of ice loads, better resistance to abrasion, and in simplified insulating supports. It costs less because of reduction in material required and simplification of the manufacturing operation. Extensive trials have shown that the long twists which occur in the removal of the wire from the coils, in view of the conditions under which it is used, are sufficient to prevent cross-talk between adjacent circuits.

Radio. International radio telegraph circuits have continued to grow during the year 1924. The Radio Corporation of America has inaugurated a direct service between the United States and Sweden; and between the United States and Argentine—this being the first direct radio link between New York and South America. This Company has also established a station at Belfast, Maine, for reception of messages from Europe. The signals received are automatically relayed by radio to the Riverhead receiving station. Because of reduced static, and also the somewhat shorter distance, the signals received at Maine and thus relayed to Riverhead are generally considerably better than those received directly at Riverhead.

Considerable progress has been made during the year in the substitution in the marine field of tube transmitters generating sustained oscillations for spark transmitters. This results in increased range of communication and decreased interference, both in the marine field itself, and to broadcast listeners. The Radio Corporation has completed its program for installing tube transmitters in all of its shore stations, and has inaugurated a program for converting the ship spark transmitters in which it is interested into tube transmitters.

Marked interest is being taken by shipping companies in the use of radio direction finders aboard ships. With the increased number of new ship installations, the Lighthouse Division of the Bureau of Commerce and Labor are increasing the number of radio beacons which will be equipped with tube transmitters. Ship installations of this kind are not only of importance in

determining the position of a ship, but also enable it to determine quickly the direction of a ship in distress.

An interesting development in ship-to-ship and ship-to-shore telephony are the radio sets built by the Western Electric Company for the United States Coast Guard. These sets were designed to give voice transmission up to 50 miles, and telegraph transmission up to 100 miles. They use one-wave length only, which will lie between 100 and 200 meters. The transmitter is of the coupled oscillator type, of 50 watts output, and the receiver is of the double detection (so-called super-heterodyne) type.

A brief statement was given in last year's report of the developments in the use of short waves of 100 meters and less, and the surprising results which had been obtained with them under some conditions. A large amount of further information has been obtained as to the characteristics of these waves, and attempts have been made to explain their action from the theoretical standpoint by Larmor (*Phil. Mag.* December 1924) and by Nichols and Schelling (*Bell System Technical Journal*), April 1925). While much more remains to be done before it will be possible to predict definitely the action of such waves, they are already being put into important practical use for telegraph purposes. Four of the long transatlantic circuits are now being supplemented by short-wave systems. These systems are still largely experimental, but it is understood they give promise of having considerable value. It has not yet developed, however, whether these short waves will displace the present use of long waves, or will rather enlarge the possibilities of radio.

The interest in these short waves led to a conference in England during July last, attended by representatives of the large radio telegraph companies in Europe and America. This conference discussed ways and means of intensively studying the short wave field. The use of short waves involves interesting and important possibilities of using directive systems at both the transmitting and receiving stations.

Under the auspices of the American Engineering Standards Committee, a Sectional Committee on Radio has been organized for the purpose of formulating standards in the radio field, particularly with respect to nomenclature and methods of rating and testing apparatus. The sponsor bodies are the American Institute of Electrical Engineers and the Institute of Radio Engineers.

Last year's report mentioned the fact that weekly tests of telephone transmission from America to Europe were being carried out, and that a committee appointed by the British Post Office had recommended the installation of a 200-kw. telephone transmitter at its new Rugby station for transmission to America. These weekly tests have been continued throughout the past year and the British Post Office is proceeding with the installation of the transmitter as noted.

The production in considerable quantity of high-frequency alternating currents for radio transmission involves some very important technical problems. A paper on "Frequency Multiplication," read at the St. Louis Convention, discusses one of the means of producing such high frequency currents.

Radio Broadcasting. Radio telephone broadcasting continues with unabated public interest. From the transmitting standpoint, there has been a steady increase in the number of better grade broadcasting stations. The problem of providing a sufficient number of frequency bands in which to accommodate them has become a serious one to the authorities of the Department of Commerce, no satisfactory solution having as yet been found. This matter, as well as other aspects of the broadcasting situation, was the subject of a National Radio Conference in October last—the third to have been called by Secretary Hoover.

The developments which have been made in the art of connecting together a considerable number of broadcasting stations by long distance wire telephone circuits is strikingly indicated by the arrangements which are now made for broadcasting national events. For example, a speech by President Coolidge on November 3, was broadcast simultaneously by 28 radio broadcasting stations scattered over the entire country from the eastern to the western seaboard.

Considerable development has been made in receiving sets. Regenerative equipment in which self oscillation is prevented only by proper manipulation by the operator, is coming into disfavor and being supplanted by sets employing multi-stage radio frequency amplification, or sets of the so-called "super-heterodyne" type. Sets of these types do not interfere with each other and give good speech and music quality together with high selectivity.

The loud speakers associated with radio sets have in general presented more difficult problems in attaining high quality reproduction than have the sets themselves. Last year's report mentioned two papers which had been presented during that year to the Institute on this matter. With one of these papers was demonstrated the so-called cone-type of loud speaker developed by the Bell System engineers. This loud speaker has since been put on the market. It properly reproduces the low frequencies whose lack has been perhaps the most unfortunate characteristic of broadcast reception from a musical standpoint. At the same time it permits a better reproduction of the higher frequencies.

A paper entitled "A New Hornless Type of Loud Speaker," read at the St. Louis Convention, describes the development work on loud speakers carried out by the General Electric engineers and in particular a form of hornless loud speaker which they have developed. It is expected that this development will form the basis of a high-grade loud speaker to be put on the market.

It is very important with high-grade loud speakers that the voice frequency amplifiers associated with them, as well as the radio set, be free from distortion. A paper entitled "The Design of Distortionless Power Amplifiers," read at the Midwinter Convention, discusses the problems which arise in the design of such amplifiers and the means for overcoming them.

Inductive Relations of Power and Communication Circuits. In response to a demand for a body representative of all wire-using utilities to carefully consider the technical problems of inductive coordination, the American Committee on Inductive Coordination was organized. This committee is made up of representatives of the American Railway Association, the American Electric Railway Association, the National Electric Light Association, the Western Union Telegraph Company, the Postal Telegraph-Cable Company and the Bell System. This committee has perfected an organization, agreed upon a program, appointed subcommittees and has issued its first report.

The Joint General Committee of the N. E. L. A. and the Bell System has continued its cooperative work throughout the year. The comprehensive investigation by the Development and Research Subcommittee has been divided into eleven projects, and each project put in charge of a small Project Committee. The program includes both theoretical and experimental work in the laboratory and in the field. Subjects under investigation include; coupling coefficients between power and telephone circuits; effectiveness of coordinated transpositions; effects of unbalances in telephone circuits and means for locating and clearing them; the cumulative effects of successive exposures; protection of telephone circuits from acoustic and electric shocks; residual voltages and currents in power circuits and methods for their control, including effects of isolating and grounding the neutral by different means; noise in telephone circuits, its effects, frequency composition, methods of measurement, and survey of its magnitude and distribution; wave shapes of voltage and current in power circuits and the development of means for improvement and rating; inductive effects under joint use of poles or other conditions of close exposure; the effects of changes in the power level and sensitivity of telephone circuits; selective devices for suppressing undesirable frequencies in either or both systems; characteristics of telephone receivers and transmitters; special devices for application to either or both systems; and interference with carrier frequency channels. It will require a period of several years to complete these investigations, but it is planned that important results will be made generally available from time to time.

Fig. 1

The paper on "Telephone Circuit Unbalances—Determination of Magnitude and Location," read at the Pacific Coast Convention, covers a matter of large

importance in considerations of inductive interference.

Electrical Protection. The National Fire Protection Association, through its Committee on Signaling Systems, has undertaken a complete revision of the Regulations for Municipal Fire Alarm Systems, and for Protective Signaling Systems. The Electrical Committee of the Association is engaged in a triennial revision of the National Electric Code, including the Regulation for Wiring of Signaling Systems and Radio.

Communication in Railroad Operation. Communication plays a very important part in railway work.

FIG. 2

FIG. 3

A paper on "Communication in Railroad Operation" read at the St. Louis Convention discusses this matter from the railroad standpoint.

Solar Eclipse. The eclipse of the sun in January was of special interest to communication engineers, in that communication facilities played a very important part in the scientific observations which were made, because of the effect of the eclipse on radio transmission.

At the request of the American Astronomical Society, the telephone companies arranged circuits connecting observing parties at Buffalo, Ithaca, Poughkeepsie, East Hampton and New York, N. Y., and Middletown and New Haven, Connecticut. Telegraph signals, set by observers at five points at the instants when the eclipse became total to them, were recorded on a chronograph in conjunction with seconds beat by a very accurate electric clock.

Careful observations were made by a number of the communication companies, and by others interested in radio, as to the effect of the eclipse on radio transmission. Definite correlation between the passage of the shadow of the eclipse and radio transmission appears to have been well established in some cases.

Electrical Transmission of Pictures. There has been a large amount of activity during the year in the electrical transmission of pictures.

The report for last year gave a brief description of a system developed by the Bell Telephone System engineers for transmitting pictures over telephone lines.

Further development has so far perfected this system that the transmitted pictures present an appearance differing very little from that of the original photograph. Fig. 1 is a reproduction of a picture so transmitted, and shows the great amount of detail obtained. The picture was transmitted directly from an ordinary positive film, and about seven minutes was taken in transmission. Pictures of the inauguration of President Coolidge on March 4 were transmitted from Washington, D. C. and received simultaneously at New York, Chicago and San Francisco, and were published in the afternoon editions of the newspapers of that same day. Commercial picture service with this system has been inaugurated between the three cities of New York, Chicago and San Francisco. The system is described in a paper appearing in the *Bell Systems Technical Journal* for April, 1925.

The lines of the Western Union Telegraph Company have been used commercially by a number of newspapers for the transmission of photographs by a method devised by M. Ferree and J. Wissmar. In this system, the elements constituting the picture are sent over the line wire and handled in exactly the same manner as the signal impulses used in the transmission of ordinary telegraph messages. At the sending end a stylus moves in spirals over a specially prepared cylindrical plate. Current impulses flow through an electric circuit, of which the stylus and plate form a portion, whenever the stylus rests upon a light portion of the plate and operate a telegraph pole changer. The current reversals caused by the pole changer operation are transmitted to the receiving station or stations through the line wires and such telegraph repeating apparatus as is necessary. At the receiving station a relay opens and closes an electric circuit, in which is included a stylus which travels spirally over a chemically prepared paper. By the action of these electrical impulses upon the treated paper, marks are made so that a picture is formed corresponding to that on the sending plate. The system described has been successfully used on a circuit lay-out arranged for duplex operation, and made up of about 5300 miles of line wire connecting to fifteen cities. Fig. 2 is a reproduction of a picture transmitted in this way. About one hour is required for transmission.

The Radio Corporation of America announced during the year transmission of pictures by radio from London to New York. Fig. 3 is a reproduction of a picture so transmitted. The method employed is, in brief, as follows: At the transmitting end, the picture to be sent is wrapped around a glass cylinder which is caused to revolve. As the drum revolves, a small concentrated electric light on the inside sends a beam through successive portions of the picture to a sensitive photoelectric cell. The current variations resulting are highly amplified, and are caused to charge a condenser in such a manner that the rate of charge is determined by the amount of light piercing the transmitting film. When a given voltage is reached on the condenser, the circuit trips and gives a pulse which works a relay to start a signal. When the amount of light increases beyond a certain point, the current not only is able to charge the condenser more quickly, but is so arranged that it will hold the relay for a greater length of time. This gives rise to a peculiar characteristic of widely separated dots for one end of the light scale, of close dots for the middle ground and long-drawn-out dashes for the other end of the photographic scale.

At the receiving end a paper sheet is fastened to a drum, which rotates in synchronism with the drum at the sending end. The radio signals, highly amplified, actuate a small fountain pen suspended just above the paper, which thus records on the paper the dots and dashes received by the radio, and thus forms the picture.

The rugged character of the dot and dash signals, such as has made possible long distance communication by radio telegraphy, carry these pictures through what would otherwise be considerable interference. The question of detail is a matter of the amount of time which can be devoted to the transmission. A portrait with the amount of detail shown can be handled in about twenty minutes.

A system devised by Edouard Belin, who has long been interested in picture transmission, has been tried out during the year between St. Louis and New York, and pictures so transmitted have been published in newspapers in both of these cities.

A number of other investigators have been carrying on experiments in picture transmission, among these being C. F. Jenkins, Austin G. Cooley, M. L. D. McFarlane and H. G. Bartholomew.

Education in Communication Engineering. There has been a steady growth of interest in communication courses in the teaching of electrical engineering, although the growth is perhaps less rapid than in the years immediately following the war. The tendency seems to be toward required rather than elective courses in electrical communication, probably due to the growing realization that such courses introduce the student to certain important fundamental conceptions and ideas which he would not get from his other courses. In August, 1924, an educational conference was held in New York City, in which a group of college professors met with officials of the Bell Telephone System. The discussion of various phases of electrical communication and the methods of teaching them were of much value to all those who attended the conference.

A considerable number of educational institutions are undertaking the broadcasting of courses of lectures, music, athletic events, and other matters of interest. This promises to become an important function of some of our universities.

Power Possibilities at Muscle Shoals, Alabama

BY SAMUEL S. WYER[1]

Synopsis.—*This study was made in February and March, 1925. The author had access to the public records of the War Department but his responsibility for all statements is complete.*

Factual conveyance has been more important than euphony or euphemism. The object of the report is to present the basic facts pertaining to the whole Muscle Shoals situation in a way that the layman can make his own evaluation of,

1. What Muscle Shoals is

2. What has, and can be done at Muscle Shoals

and from these facts deduce as to what, in the public interest, on the basis of the greatest good to the greatest number, ought to be done with it.

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I. Fundamental Features

1. Public's Understanding of Term "Muscle Shoals." In the public's mind, the term "Muscle Shoals" is a definite spot and not the stretch of river described in Section 3. As used by the layman, Muscle Shoals means the water-power development now nearing completion at the Wilson Dam—named for President Wilson—and the term is so used in this study.

While the Wilson Dam is merely a small part of the proposed Tennessee River development, it is the only part now under construction and this is why the public's attention has been focused on it, rather than on the project as a whole.

2. Public's Interest in Muscle Shoals. At present Muscle Shoals is the most talked of power project in the world. No other water power has ever received so much oratorical attention, or has had expended on it so much printers' ink. The very name "Muscle Shoals" has become a symbol for power.

In the forum, in the press and in the conversation of the public, Muscle Shoals has frequently been the dominating topic of the day. From its inception as a war measure, public interest in it has grown until at times the preponderant public opinion has been that the military and economic strength of the nation was dependent upon Muscle Shoals.

3. Location of Muscle Shoals. The general geographical location of Muscle Shoals in northwestern

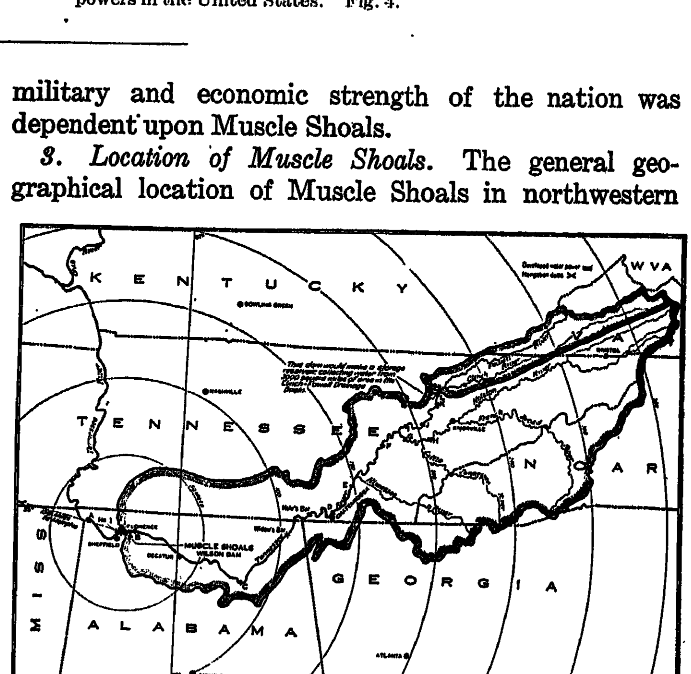

FIG. 1—MAP OF MUSCLE SHOALS DRAINAGE BASIN OF TENNESSEE RIVER

Possible future dam sites for navigation and power

Riverton........	A	Sale Creek.......	E	Senator.........	I
Number 3.......	B	White Creek.....	F	Melton Hill.....	J
Guntersville.....	C	Marble Bluff.....	G	Clinton..........	K
Sherman........	D	Coulter Shoals...	H	Cove Creek......	L

Alabama is shown by Fig. 1, with local details given on page 4.

1. Consulting Engineer, Columbus, Ohio.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

For a distance of about 37 miles, between the railroad bridge at Florence and Brown's Island near Decatur, Alabama, the bed of the Tennessee River is really a series of shoals, one right after the other, with a fall of 140 feet in that distance.

The shoals near Florence are known as "Muscle Shoals," as the muscle shell fish—earlier and now alternate spelling "Mussel"—were found in abundance in the bed of the shoals. The present geographical name "Muscle" has been derived from this fact. The Muscle Shoals section includes the stretch of river from Muscle Shoals up to Brown's Island.

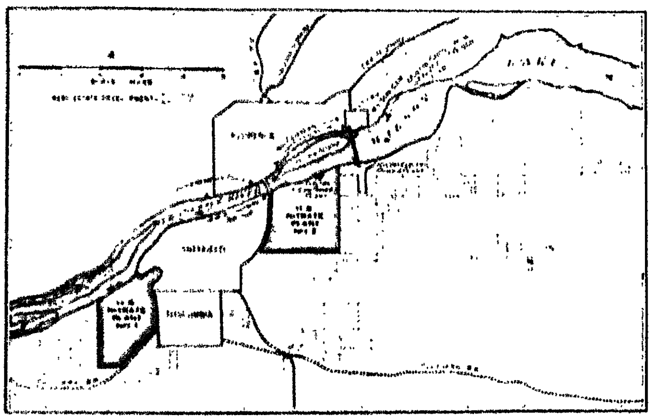

FIG. 2—MUSCLE SHOALS DISTRICT, ALABAMA, MARCH 1925

4. Current Terms Applied to Muscle Shoals. The following are commonplace expressions for and representing current ideas of Muscle Shoals:

"There is no place in the world where greater advantages are to be found for the harnessing of water power for the uses of industry, or where there is greater power awaiting development, within reach of the seaboard by water and rail transportation, than at Muscle Shoals."[1]

"If I were greedy for power over my fellow men, I should rather control Muscle Shoals than to be continuously elected President of the United States."[2]

"The completed Muscle Shoals is worth more than all the gold currency in the world."[3]

"The destiny of the American people for centuries to come lies at Muscle Shoals."[4]

"America's Gibraltar—Muscle Shoals. In peace, prosperity for the farmer; in war, preparedness for the Nation."

"Muscle Shoals, waiting through the ages for man to tame their roaring waters and harness their mighty power."

"The Niagara of the South."

"Standards of living at stake in Muscle Shoals decision."

5. Characteristics of Tennessee River Drainage Basin. The area of the drainage basin supplying the Wilson Dam at Muscle Shoals is 30,800 sq. mi. Much of this is hilly or mountainous so that the rain falls on sufficient of a slope to produce a rapid run off from the surface of ground to the stream. Daily stream-flow readings for the Tennessee River, made at the railroad bridge at Florence, about 2½ mi. below the Wilson Dam and recorded by the U. S. Geological Survey, are available since 1871[5].

The average rainfall for the entire drainage basin is 51 in., of which 13 in. occurs in the winter, 15 in. in the spring, 14 in. in the summer, and 9 in. in the autumn.

6. Why Variable Stream Flow is Natural. Rainfall is the initial source of the water in all rivers. Rainfall is not continuous but periodic and varies largely with different seasons. Therefore, unless there is natural or artificial storage to equalize the volume, the flow of a river will vary approximately with the rainfall, Variable flow is obviously the natural condition of a river.

The Tennessee has neither natural or artificial storage at present, and its variable flow characteristics are, therefore, typical, varying merely in degree with other rivers.

7. Characteristics of the Tennessee River. The banks of the Tennessee River are well cut, clearly defined and there has been little change, or tendency to change, in its course from year to year. The bottom of the river does not present any serious difficulties for dam foundations.

"The Tennessee is subject to sudden and frequent fluctuations in discharge. Its headwaters drain a mountainous region noted for heavy and prolonged rains and generally high annual precipitation. The high-water season is in the late winter and early spring and is caused by copious rainfalls, while the melting of snow, as a contributive factor, is only of minor significance.

"The maximum rates of flood flow in the Tennessee are not so high as might ordinarily be expected of a

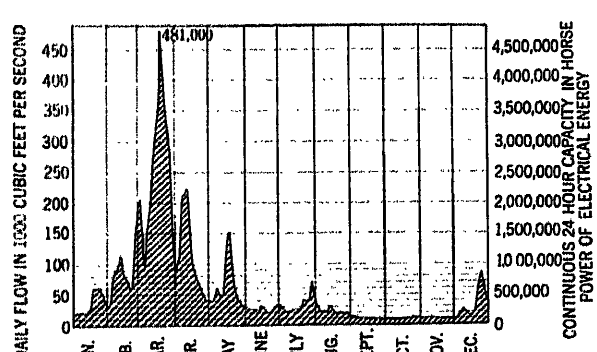

FIG. 3—VARIATION IN DAILY FLOW AND HORSE POWER OF TENNESSEE RIVER AT MUSCLE SHOALS

On the basis of 80 per cent. efficiency of the hydroelectric generators, 10 h. p. of electrical energy will be generated for each cubic foot per second flow at the Wilson Dam at Muscle Shoals. The scale at the right of figure shows h. p. capacity. (Left). Daily stream flow readings of Tennessee River at Florence, Ala., near the Wilson Dam, have been recorded by U. S. Geological Survey for 53 years. The data shown are for 1898.

river basin subject to the high rates of storm rainfall that are common there. This is attributable to three factors:

a. The peculiar configuration of the drainage basin which, near the middle, is constructed to a width of less than 40 mi.

b. The general westerly flow of the river in a direction contrary to the movement of storms.

1. Senate Document No. 83. 59th Congress, First Session.
2. Newton D. Baker.
3. Thomas A. Edison.
4. The New York Times credits this to Henry Ford.

5. Recorded in the U. S. Geological Survey Water Supply Papers Nos. 353, 383, 403, 433, 453, 473, 503.

c. The arrangement of the tributaries, which does not favor rapid collection and concentration of run off.

"On account of the mild winters, ice troubles, so common at hydroelectric plants in the northern United States and Canada, are rare and become a negligible factor in the operation of such installations in this region."[6]

8. Variation in Daily Flow of Tennessee River. This is shown in the yearly hydrograph chart, Fig. 2. In the extreme year shown—1898—the daily flow in "1000 cu. ft. per second" varied from 481 to 9, or 53 to 1.

In 1923, the daily flow varied in "1000 cu. ft. per second" from 215 to 8, or 27 to 1.

The maximum discharge of the Tennessee River recorded at Florence was 499 "1000 cu. ft. per second" and occurred on March 19, 1897.

It is important to bear in mind that these flow data are not deductions from rainfall statistics, but are actual measurements, and show the high and low flow conditions that must be coped with in water-power developments on this river.

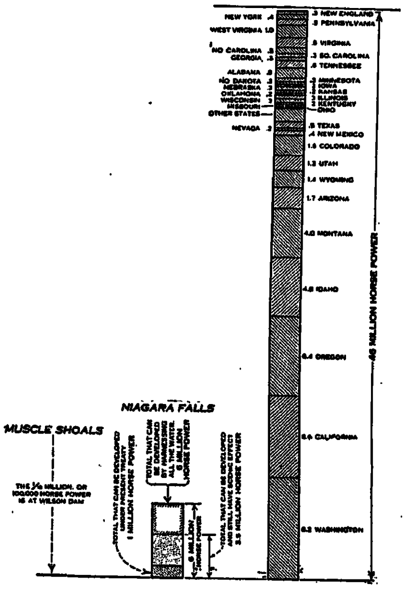

Fig. 4—Muscle Shoals Compared with Niagara Falls and Undeveloped Water Power in United States in Million H. P.

The diagram at the right, the geographic distribution of potential undeveloped water power in United States, is based on data from the U. S. Geological Survey. This total is without artificial or stream flow control and is larger than it would be feasible to develop. Probably not over 25,000,000 h. p. will be worth developing for many years. The amount of power that may be alloted to the United States in any future joint development of the Niagara and St. Lawrence Rivers is not included.

The two comparisons above are for 24-hr. capacity without artificial or stream flow control; because of scenic preservation and ice flushing conditions, not more than three and one-half million h. p. can be depended upon at Niagara Falls.

The horse power that may be developed for various conditions of river flow at the Wilson Dam at Muscle Shoals, is shown at the right on page 8.

9. Water Storage Limitations. At the Wilson Dam at Muscle Shoals, the area of the pool known as Wilson Lake is 14,500 acres, or in round numbers, 23 sq. mi. This pool gives enough ponding to meet daily fluctuations in loads but affords practically no storage.

10. Meaning of "Primary Power" and "Secondary Power." Primary power is the power that can be developed continuously for 24 hours of the day and for every day in the year. Secondary power is power that can be developed for merely a part of the time, and which may be for certain seasons of the year or for certain hours of the day only. The thing in which the ultimate consumer is interested is not primary power per se or secondary power per se, but continuous power; that is, continuity of service; and secondary power can be made continuous power only by having some other source of power to make up for the deficiency. This usually means a steam plant.

11. Primary Power of Wilson Dam at Muscle Shoals. In general, the primary power that may be depended upon at the Wilson Dam at Muscle Shoals is 100,000 h. p., continuous output. However, there will be occasional days when the primary power will drop to about 87,000 h. p. For the larger turbine capacity needed to utilize this primary power see §23.

12. Secondary Power of Wilson Dam at Muscle Shoals. In addition to the 100,000 h. p. of primary power that can be developed at the Wilson Dam, the following quantities of additional secondary power can be developed for the respective per cents of total time shown below for the two conditions of river flow shown:

Additional secondary horse power	Approximate per cent of total time available[7] Low year daily discharge basis	Mean monthy discharge basis
50,000	56 per cent	85 per cent
100,000	44 per cent	72 per cent
150,000	32 per cent	63 per cent
200,000	24 per cent	55 per cent
300,000	15 per cent	42 per cent

13. Muscle Shoals Compared with Niagara Falls.

	Muscle Shoals Wilson Dam	Niagara Falls[8]
Area drainage basin, square miles	30,800	263,400
Storage	none	1/3 area of drainage basin is water storage.
Head—that is, fall—that can be developed	95 ft.	300 ft.
Ice problem	none	Flushing the ice and maintaining scenic effect reduces the capacity from 6,000,000 h. p. to 3,500,000 h. p.
24 hour continuous capacity in horse power	100,000	3,500,000

Thus the continuous capacity that can be obtained from Muscle Shoals is 1/35 of the capacity of Niagara Falls, or it would take 35 Muscle Shoals to equal one Niagara Falls.

In using Niagara Falls as a comparative yard stick,

6. House Document No. 319, 67th Congress, Second Session.

7. House Document No. 1262, Plates 107 and 108, 64th Congress, First Session.

8. For further discussion see Smithsonian Institution Paper "Niagara Falls: Its Power Possibilities and Preservation."

it is important to bear in mind that the Niagara River is jointly owned by the United States and Canada and that the treaty limits the total development at present to 1,000,000 h. p. and there can be no further development until the treaty is amended. Furthermore, if the treaty is amended and the development is pushed to the limit—on the basis of still maintaining an adequate scenic effect—of 3,500,000 h. p., one-half of this, or 1,700,000 h. p., would belong to the United States.

14. Muscle Shoals Compared with Undeveloped Water Power in United States. While the potential undeveloped water power in the United States is, in round numbers, 46,000,000 h. p.—as shown on page 10 probably not over 25,000,000 h. p., of this 24-hour continuous capacity without artificial storage, will be worth developing for many years. Therefore, the Wilson Dam at Muscle Shoals compares with this probable undeveloped water power, available for the near future, as follows:

Total undeveloped water power in the United States worth developing	25,000,000 h. p.
Wilson Dam at Muscle Shoals	100,000 h. p.

That is, the Wilson Dam at Muscle Shoals represents 1/250 of the undeveloped water-power capacity of the United States worth developing; or it would take 250 Wilson Dams to equal the undeveloped water power now worth developing in the United States.

15. Muscle Shoals Compared with Stationary Primary Power in United States.

Using the term primary power—as defined in Section 10—with the knowledge that the aggregate horsepower capacity of all stationary steam, oil and gas engines, and steam and water turbines is in excess of 50,000,000 h. p. at the present time it is estimated that the strictly primary stationary horse power capacity in the United States is at least	40,000,000 h. p.
The primary power at the Wilson Dam at Muscle Shoals is	100,000 h. p.

That is, it would take 400 Muscle Shoals to furnish the primary stationary power that is now in use in the United States.

16. Water Power Possibilities of the Tennessee River Compared. The Wilson Dam is not the whole Muscle Shoals project, but merely a part of the proposed project; and this is but a small part of a much larger proposed development which Congress is now having studied; viz., that of the power-navigation-industrial possibilities of the entire Tennessee Basin.

The water-power possibilities of the Tennessee River compare as follows:

	Primary horse power
Undeveloped water power in the United States worth developing	25,000,000
United States share of total possible development at Niagara Falls	1,700,000
Probable total development on Tennessee River	1,000,000
Wilson Dam at Muscle Shoals	100,000

This shows that the proposed development on the Tennessee River is 1/25 of the probable power development in the United States and about 60 per cent of the United States' share of the total power that can be developed at Niagara Falls; and, further, that the present Wilson Dam development represents but one-tenth of the probable total Tennessee River development.

17. Water-Power Limitations.

1. Water power is not free. In the consideration of water power, many well meaning persons think only of the free gift of nature and entirely overlook the part that man's labor and money must necessarily play in the realization of the bounties of this gift. There seems to be an impression among some people that, since water runs down hill, water power can be developed at very little cost and with very little risk. Such an impression is, in most cases, very far from the truth. Water power is worth just what can be gotten out of it in competition with power manufactured from fuel.
2. While nature has made the water, it is no more natural or free than the nature-made coal. Both must be brought under control to be of service to man. "Much has been said about harnessing water falls. However, there has been little appreciation that someone must furnish the money for securing, maintaining and ultimately replacing the harness and hiring labor for the continuous direction of the harnessed energy."
3. Coal is relatively a small part of total power cost.
4. The cost of water-power plants per horse power of plant capacity is usually more than for steam plants, but the operating cost of a water-power plant should always be less than for a steam plant.
5. The capacity is dependent on stream fluctuation. With storage reservoirs, the maximum operating capacity is merely the average stream flow; without reservoirs, the maximum capacity that can be depended upon is merely the minimum flow. The flow of the stream may be injuriously affected by the operations of man or the agencies of nature and the available flow may be much less than anticipated.
6. While auxiliary plants can be installed to furnish power during the low water period,the increased cost would ordinarily not make this attractive in single plants.
7. Actual delivering capacity is usually much lower than the published figures.
8. Water-power development in the United States has been strewn with wrecks, due to failure to appreciate the clearly defined economic limitations under which water-power developments can take place and failure to cope with the nature-made limitations in variation in stream flow.

18. Proximity of Coal to Muscle Shoals. Extensive coal deposits are located in central Kentucky, central Tennessee and northwestern Alabama. These are, of course, advantageous from the viewpoint of furnishing fuel for any industries that would use Muscle Shoals power. These coal deposits, however, because of their proximity, make steam-power generation relatively easy and are, therefore, a determining factor in appraising Muscle Shoals power, since, in the long run, Muscle Shoals power is worth no more than power that can be generated from a steam plant.

19. Confusion of Power and Fertilizer. Much of the misunderstanding regarding Muscle Shoals arises from the confusion of the power project with nitrate production for either munitions or fertilizer. Power production must stand on its own feet and should be self-

sustaining; fertilizer production should also stand on its own feet and be self sustaining.

Fertilizer manufacture would require power. But there is no mysterious advantage whatsoever in Muscle Shoals power; power from any other source would function just as effectively. The crux is one of economic cost and not of mere geographical position.

II.—What has been Done to Date

20. History. "In 1824, J. C. Calhoun, Secretary of War, asserted that a canal around Muscle Shoals was of great national importance." The first investigation was authorized by Congress in 1828 and this was followed by others.

The first study "undertaken by the United States with a view to the possible development of the extensive potential water powers in this section of the river" was made in 1907. The whole project was finally brought to a head in the report of the Chief of Engineers, June 4, 1916,[9] describing the then-proposed dam construction, which later was named after President Wilson and officially designated the "Wilson Dam."

What is now known as the Wilson Dam was started November 9, 1918—two days before the armistice—but the work was suspended from April 15, 1921, to October 1, 1922, and the present indications are that the work will now be completed about January 1, 1926.

21. Wilson Dam. The Wilson Dam is located at the foot of Muscle Shoals, 2½ miles east of Florence, Alabama, as shown in Fig. 2.

The dam is a huge concrete structure with navigation locks at the north end, a power plant at the south end and a roadway on top. The total length of the structure is 4500 feet, and contains 1,350,000 cu. yds. of masonry, which is the largest volume of masonry in any dam in the world. The dam is 96 ft. high, 105 ft. thick at the base and rests on a hard blue limestone.

22. How the Work has been Handled. The entire project has been ably and efficiently handled by the War Department, under the general direction of the Chief of Engineers. All of the work has been done on a day labor basis and practically no contract work, except minor subcontracts, has been used on the project.

23. Hydroelectric Power Plant. This is located at the southern end of the dam, and is in fact, an integral part of the main dam construction.

The power-house building is 1250 ft. long, 160 ft. wide and 134 ft. high, built of monolithic and reinforced concrete.

Four complete hydroelectric generators, each of 30,000 h. p., are now being installed, also four at 35,000 h. p. each. The room is large enough so that 10 additional units, each of 35,000 h. p. can be installed later.

9. Published as House Document No. 1262, 64th Congress, First Session.

The total installed capacity (above auxiliary equipment used in the plant itself) contracted for now is:

	Horse power
4 units, at 30,000 h. p. each	120,000
4 " " 35,000 " "	140,000
Present capacity	260,000
Additional units proposed:	
10 units, at 35,000 h.p. each	350,000
Proposed future capacity	610,000

However, not all of the 260,000 h. p. of installed capacity is available for continuous use for the reasons given in the next paragraph.

While the stream-flow conditions will permit of a continuous output of but 100,000 h. p., the demands for electric energy are not uniformly distributed over the 24 hours and the maximum hour would ordinarily be 50 per cent more than the average hour, so that 150,000 h. p. of turbine capacity would be required to meet the probable daily peak of the primary horsepower serving capacity. One unit should be in reserve and ready to at all times take care of any breakdown in operating machines.

The additional equipment which is for secondary power can be used in general only, as arrangements are made for getting some other source of power to supplement the low stream flow periods.

24. Navigation Locks. Two navigation locks, each having a lift of 46-ft. 6-in., are a part of the north end of the Wilson Dam. The depth available in each lock is 7½ ft.

25. Highway Bridge. The extreme top of the dam structure will be used as a public highway bridge and arrangements will be made so that the Lee and Jackson Highways, now going into Florence, will connect with this bridge.

26. No. 1 Dam for Navigation. This is located near the railroad bridge at Florence, as shown on the map, Fig. 2. It is about 15 feet high and by means of a lock with a 10-ft. lift, will connect to the Wilson Dam locks by a canal about 2½ miles long on the north side of the Tennessee River. This dam will be used for navigation only.

The cost of this is entirely separate and in addition to the figures given for the Wilson Dam in the next section, it is estimated at $1,600,000.

27. Estimated Cost of the Wilson Dam. The total amount of money spent directly on the Wilson Dam up to March 1, 1925, was $38,340,822. The total cost of completing the dam and installing four 30,000-h. p. units and four 35,000-h. p. units, with all auxiliary equipment—see Sections 29 and 30—is estimated at $45,800,000.[10]

28. Wilson Dam Built with Borrowed Money. The

10. The estimated cost of the additional ten 35,000-h. p. units, or a total of 350,000 h. p., is $5,323,000.

United States Government, in order to get money to build the dam, sold bonds to a large number of private individuals. In this way, money that was earned and saved by private individuals was borrowed by the United States Government on a rental or interest basis and mobilized; that is, these individual savings were put to work collectively in building this project. In this way, the government hired capital just as it hired labor, and the rental of the money is just as much a part of the cost of the dam as the hire of the labor. Furthermore, in order to ultimately return the money borrowed from the individual owners, provision should be made within the period of the useful life of the property so that enough money will be set aside out of the income from the project to ultimately pay back the money to private owners from whom it was borrowed.

29. Omitted Cost Items. The estimate in paragraph 27 has not included all items of cost that must ultimately come out of the public treasury. In addition to the appropriations for building the dam, there is:

a. The money used for paying for the material and labor has been borrowed for a number of years and the government has been paying interest on this amount. The total interest during construction, which must come out of the public treasury, up to the completion of the dam, January 1, 1926, is, in round numbers..................	$4,000,000
b. Preliminary engineering service in the survey upon which the construction work was based, about......................	150,000
c. Services of the Army Engineers, who have directed the work, paid out of the public treasury and therefore chargeable to this work..........................	150,000
	$4,300,000

30. Cost of Wilson Dam to the Public. Taking the estimated cost of $45,800,000, plus the omitted cost items in the preceding section, $4,300,000, we get a total cost to the public of the Wilson Dam of $50,110,000

In the more than six years since the construction work on this dam was started, the United States Government has purchased construction equipment which is a part of the above dam cost. If the No. 3 Dam is built at once, a part of this equipment can be used for the additional work. If the No. 3 Dam is not built, then this equipment may be sold or transferred to another job and the Wilson Dam cost should be credited with whatever can be realized from such sale or transfer. It has been impossible to reconcile the widely divergent views as to the credit that could be allowed for this equipment. For the purposes of comparison, an allowance of $1,000,000 is made as a credit to the Wilson Dam cost, thus making the net cost in round numbers to the public on the Wilson Dam, $49,000,000.

31. Wilson Dam Cost Compared with Steam Plant Cost.

Taking the total cost given in the preceding section for the Wilson Dam as $49,000,000, and with the frank recognition that this gives but 100,000 h. p. of primary power, we at once get an investment cost per horse power of plant capacity of............	$490.00
The per horse power cost per actual primary horse power output of a high grade steam plant, with $^1/_5$ of the capacity in reserve at all times to guard against breakdown to insure continuity of service, would be........	100.00

The additional secondary power that can be generated at Muscle Shoals will, of course, be of service but it can be of service only as it is combined with other plants, which will keep the combined capital investment above a single steam plant.

32. Navigation's Share of Wilson Dam Cost. The water-power development of the Wilson Dam at Muscle Shoals is not sufficiently attractive to subsidize navigation. The comparative cost figures given in the preceding section indicate the financial handicap for this water power project, so far as investment is concerned.

It has been suggested that 25 per cent of the total cost could be charged to navigation, but it must be remembered that this would be merely a bookkeeping transfer and would, in no way, change the blunt fact that the public must ultimately foot the entire bill, for, under our present navigation policy, the contribution would come out of the public treasury and not from the people who benefit directly by the navigation advantages.

33. Real Estate Speculation. There naturally has been an abnormal real estate boom in the Muscle Shoals district. Detailed records of 109 subdivisions totalling 48,381 lots and aggregating 6750 acres or 10.7 sq. mi. area, were examined. This is merely part of the development and some of the allotments are indicated on Fig. 2.

Alluring but deceptive advertising matter has been persistently and widely circulated; lots have been sold to widely scattered buyers. Many of these allotments are in corn fields or pasture land, and practically all are without improvements other than a few graded dirt streets and cheaply constructed sidewalks. Twelve miles distant from the Wilson Dam is not an uncommon location for these additions that are advertised as being right at Muscle Shoals.

34. Origin of Public Misconceptions about Muscle Shoals. We have a wide gap between current popular beliefs regarding Muscle Shoals and the fundamental facts because:

1. The narcotic and hypnotic effect of superlative terms, like those cited in Section 4, used repeatedly, has produced a distorted perspective.
2. As a nation, we have been more interested in debating than in fact-finding, fact-recording, and fact-facing. There has been all too little appreciation that while slogans may change beliefs and mental attitudes, they never can alter facts. Imagination plays an important role in our national life. To many, symbols are the same as realities and the symbol rather than the reality aspect has been stressed at Muscle Shoals.

3. The current newspaper expression of "white coal" for water power is creating an erroneous idea as to the value of water power and the public has jumped to the unwarranted conclusion that when you have a water power, you are getting something for nothing. Deflation of many of our water power ideas is desirable.

4. There is nothing magical about water power and it is worth just whatever can be obtained from it in competition with coal,—and no more.

5. Even though the Muscle Shoals dam is the largest concrete monolith and the largest dam in the world, mere size of structure—that is, volume of masonry—does not make power. Water power requires water fall and continuity of flow and has been much over-estimated at Muscle Shoals.

6. Optimism run wild in real estate development has not only disseminated many erroneous statements but has secured the financial interest of a large number of individuals scattered all over the United States in the unwarranted real estate developments which have tended to still further mislead the public as to the possibilities that might be realized at Muscle Shoals.

III—What can be Done at Muscle Shoals

35. Possible Future Dams. The map, Fig 1 shows the locations of possibly twelve future dams that may be built in the Tennessee River drainage basin. "Examinations made of the rocks which will be used show that excellent foundations can be obtained throughout this region."[11] The aggregate primary horse power that can be developed at the twelve future dam sites shown, is 751,000. In addition, the Cove Creek Dam would make a storage reservoir from which some regulation of stream flow can be secured and this would increase the primary horse power capacity at the present Wilson Dam and Hale's Bar Dam.

The estimated cost of these twelve dams is $125,000,000. The land that would have to be condemned aggregates 109,000 acres; and the above estimated cost has allowed $30,000,000 for land acquisition. Obviously, any tendency on the part of the present land owners to hold up prices could easily so increase the cost as to make future projects from this aspect alone undesirable from a business viewpoint.

36. Tennessee River Power Possibilities. The 100,000 primary h. p. capacity of the present Wilson Dam at Muscle Shoals plus the 751,000 primary horse power of the proposed 12 dams, referred to in the preceding section, plus the increase in primary horse power capacity that regulated stream flow would produce in the present Wilson and Hale's Bar Dams, would give a primary horse power capacity of the Tennessee River, in round numbers, of 1,000,000 horse power.

37. Navigation Aspects. Building the additional dams would give satisfactory navigation conditions for this part of the river, however, there still would be a stretch between the Riverton Dam going north to the mouth of the Tennessee River at Paducah that needs improvement.

38. Muscle Shoals a Mere Incident in the Tennessee River. From the preceding, especially in paragraph 35, it is obvious that Muscle Shoals is a mere incident in the Tennessee River development program and it must be evaluated from this point of view.

39. United States Government's Position at Completion of Wilson Dam. Toward the end of this year, when the Wilson Dam will be completed, the United States Government will be in the position of having about $49,000,000 invested in a water power project that can be depended upon for 100,000 primary horse power, but will be without transmission lines or a market. That is, the Muscle Shoals power project will be "all dressed up and no place to go."

The secondary power at Muscle Shoals can be made available only as it can be combined by transmission lines with other power sources.

40. United States Government Steam Plant. An excellent steam plant was built at Nitrate Plant No. 2 in 1918. It contains a 60,000-kw. steam turbine in three elements, of 80,000 h. p. capacity.[12] It is in good physical shape and could be used for supplementing the secondary power of the Wilson Dam.

41. Common Sense on Fertilizer Situation.[13] Fertilizer will be made by fixing the nitrogen of the air or any other process whenever it is good business to do so; that is, when the process can be carried on at a fair profit. No sane man would go into an enterprise unless it could be carried on at a profit. In other words, the same motive that prompts the farmer to expend labor and capital in raising a crop (which is merely getting a living wage or return for labor and capital) will insure the manufacturer of fertilizer. It is no more wicked for a fertilizer manufacturer to make a fair profit than for a farmer to make a fair profit. Without profit there is no incentive; without incentive, there can be no progress.

The fixation of nitrogen is a long way from fertilizer. To compare the cost of segregated nitrogen at Muscle Shoals before its proper combination with other ingredients to make usable fertilizer, is not unlike comparing the cost of raw wool with the cost after it becomes a suit.

42. U. S. Nitrate Plant No. 1. This was an experimental synthetic ammonia plant. It never operated, and, as it now stands, is not of interest for commercial production. It probably would be cheaper to construct a new plant than to redesign and rebuild Plant No. 1 to permit of its competing with plants based upon the latest developments. This represents an investment of about $12,000,000.

43. U. S. Nitrate Plant No. 2. This is constructed to operate on the cyanamid process, which won its place because it requires less than one-fourth the power necessary for the earlier, but now out of date, arc process and because the raw materials—coal, limestone,

11. Wilbur A. Nelson, State Geologist, Nashville, Tennessee.

12. One horse power is equal to 0.746 kw., or one kilowatt is equal to 1.34 h. p.

13. For further discussion see Part 3 "The Air-Nitrogen Processes"—U. S. Department of Commerce, Trade Information Bulletin No. 240.

and nitrogen—are obtainable in many places. The synthetic ammonia process is rapidly rendering the cyanamid process commercially obsolete as a method for fixing atmospheric nitrogen for fertilizer purposes. It uses less than one-third of the power required for the cyanamid process and this power need not be electric energy. Progress in the synthetic ammonia process makes the cost of pure hydrogen, and not power, the controlling economic factor. This plant, including the Waco Quarry, represents an investment of about $67,000,000.

44. Future Market for Muscle Shoals Power. Electric power has completely revolutionized industrial living and social conditions in parts of the South. In Alabama, east Tennessee, Georgia, and the Carolinas, hydro power is largely employed in various industrial and textile activities. Western Tennessee and practically all of Mississippi have already been touched and are waiting for this development program.

The absence of income or inheritance taxes in Alabama and other taxation inducements, make this state an exceptionally favorable one for the investment of capital for carrying on industrial activities.

Abundant reserves of non-migratory, reliable, English speaking, intelligent, white labor, accustomed to simple living and living close to food supplies with small fuel needs for house heating and lack of labor concentration with attendant housing problems, give the South an advantageous mill labor situation for converting its raw cotton into the finished product right in the South, providing dependable and continuous electric power service is available.

The shortened transportation of the raw cotton, by finishing the product in the South, is, of course, also advantageous.

The southern negro, living here in his natural and normal habitat, employed extensively for common labor, because of his tractability, is rarely susceptible to the disturbing influence common where the foreign element dominates the labor field, and is a distinct labor asset for the heavier tasks.

The South is growing rapidly and there has been a marked exodus of the cotton industry from the New England states to the South. The number of wage earners, the annual million pounds of cotton manufactured, number of active spindles, and primary horse power employed in cotton industry is increasing much faster in the southern states than in the north.

The per cent of cotton produced, that is, converted into the manufactured product is for:

Mississippi..................	5 per cent
Louisiana....................	13 per cent
Tennessee....................	31 per cent
Alabama......................	44 per cent
Georgia......................	111 per cent
North Carolina..............	141 per cent
South Carolina..............	174 per cent

That is, the last three states are fabricating more cotton than they are producing and are, therefore, importing from other producing states, due primarily to the advantageous labor and electric power conditions.

Within the limits of the Tennessee River Basin and where power will be required for development, at least 50 mineral substances of economic value are found.

"Special emphasis should be placed on the large phosphate deposits of middle Tennessee; the extensive coal fields of northern Alabama and eastern Tennessee; the fine ball clays of western Tennessee, which are used in making electrical insulators and other high-grade articles; the great marble copper mines at Ducktown; the unlimited deposits of limestone of the purest quality, suitable for making high grade lime and other similar uses; to the iron ores of Alabama and Tennessee and the bauxite deposits of eastern Tennessee and northern Georgia."[14]

Securing lower labor and lower production costs is, of course, of vital interest to the public at large. There has been all too little appreciation of the fact that every man's wage is a part of some other man's cost of living.

"It therefore appears that a broad and well-founded judgment would dictate that the Muscle Shoals development should be interconnected for exchange of power with the existing power systems of the Southern States, and that this interconnection and exchange should be arranged for without delay, so that future construction, both at Muscle Shoals and elsewhere, can be directed for the production of plants which will supplement each other for economy of construction and operation."[15]

In conclusion, within easy transmitting distance from Muscle Shoals, there is an adequate, existing, electric-power market that can promptly absorb all of the electric power that can be developed. This market can be economically reached and effectively served only as Muscle Shoals is made part of an integrated, interconnected transmission system, so that advantageously located steam plants and storage reservoirs can be used for supplementing the otherwise practically valueless secondary power at Muscle Shoals.

14. Wilbur A. Nelson, State Geologist, Nashville, Tennessee.

15. War Department Document No. 1039, Office Chief Engineer, "The Power Situation During the War." p. 267.

Hydroelectric Development of the Saguenay River

The Duke-Price Power Company, Ltd., at Isle Maligne, Quebec, Canada

BY W. S. LEE[1]

Fellow, A. I. E. E.

Synopsis.—*The building of the Isle Maligne Station in a remote section with extreme difficulties due to weather and floods, called for very careful planning and coordination of the various parts of the work. The steps in the construction of this plant are shown perhaps at rather extreme length, but this is done largely to bring out the value and importance of designing with the idea of cooperating and coordinating with the contractor or the construction forces. The design and layout of the construction plant were given as much attention on this work as the design of the powerhouse. The writer maintains and endeavors to impress upon designing and construction engineers of any large project, this very important cooperative feature. Had this not been done at the Isle Maligne Station, the great amount of work could never have been executed in the same length of time.*

THE largest single installation in water-power development ever undertaken is at Isle Maligne on the Saguenay River, Province of Quebec, Canada, where a total power-house installation of 540,000 h. p. is being made, of which 360,000 h. p. is completely, and 180,000 h. p. partially installed.

The Saguenay River is one of the great tributaries of the St. Lawrence River and flows out of Lake St. John by two channels, the Grand Discharge and the Little Discharge. These two channels unite at a point nine miles below the Lake outlet, and thirteen miles further downstream the Saguenay River reaches tidewater.

Lake St. John is located about 100 miles north of the City of Quebec. Its normal water level is 310 ft. above mean sea-level and the drainage area of the Lake is estimated to be 30,000 sq. mi. Since 1913, the Quebec Development Company, Ltd., has been maintaining a gaging station on Lake St. John and the zero point of the gage corresponds to the mean low-water level in January. By a grant of the Government, the Power Company is permitted to utilize the storage capacity of Lake St. John between ordinary low water and ordinary high water, which is estimated to be 187.1 billion cu. ft. in a depth of water of 17.5 ft., based on the survey of the Quebec Streams Commission. The superficial areas of Lake St. John for various gage readings are as follows:

TABLE GIVING AREAS OF LAKE ST. JOHN FOR DIFFERENT ELEVATIONS OF ITS WATER-SURFACE

Gage Reading Ft.	Area of Lake Sq. Mi.
0	328
5	380
10	394
15	403
17.5	408

The daily discharges of Lake St. John, observed during the year 1922 before the water power development was commenced, are shown on curve, Fig. 1. It will be noticed that the curve is pretty regular, due to the natural storage effect of Lake St. John. The bulk of the water from Lake St. John passes through the Grand Discharge and in winter time the Little Discharge is practically dry.

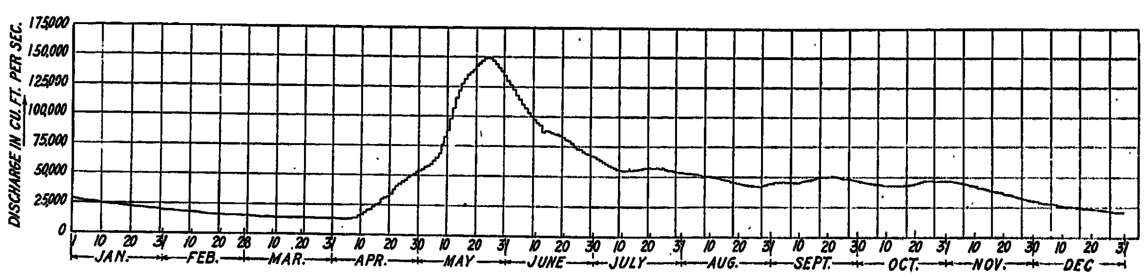

FIG. 1—DAILY DISCHARGE CURVE OF SAGUENAY RIVER 1922

Isle Maligne, the last of the many islands in the Grand Discharge is about 1¼ miles long and divides the stream-bed into two rocky gorges. Its foot is two miles above the junction of the Grand Discharge with the Little Discharge and at the first-named point the mean low-water level of the river is 105 ft. below the ordinary low-water level of Lake St. John and 122.5 ft. below the level of the fully impounded Lake.

1. Southern Power Co., Charlotte, N. C.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

After exhaustive studies, it was decided to locate the power house in the left channel of the Grand Discharge at the downstream end of Isle Maligne and to close up the right channel at the upstream end of the Isle Maligne by a spillway, the difference in normal water elevations between these two points being 45 ft. At this spillway site, called No. 4 on the general map, Fig. 2, the river is deeply cut in rock and it was necessary to provide additional spillways on Isle Maligne, which is barely covered with soil above its granitic formation. Finally an earth dam was required in the ravine near Spillway No. 4, on Alma Island between the Grand Discharge and

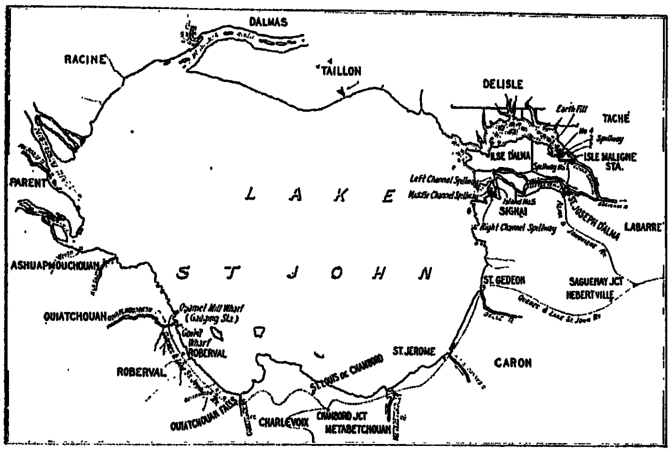

FIG. 2—GENERAL PLAN OF DEVELOPMENT

the Little Discharge, so as to extend Lake St. John to the intake structure of the power house. As previously mentioned, the Little Discharge is practically dry in winter time and its head consists of three small, rocky channels, calling for a comparatively small quantity of masonry for the confinement at this point of the impounded Lake St. John.

It will be noticed that this layout permits the drawing of the water into the turbines directly from the natural storage of Lake St. John, from an average depth of about 20 ft. under its surface, and no trouble is anticipated by ice which is from two to three feet thick on Lake St. John in the winter time. Furthermore, the ice will melt directly and the Lake is usually drawn down at the end of the winter season, thus making the water available for use through the turbines at a very opportune time.

RAILWAY TO POWER SITE

Authority to begin with the construction of this undertaking was given in December 1922 and the schedule called for the completion of the entire development by January 1926. At that time, Isle Maligne, where the principal amount of the construction was to be done, was practically inaccessible and severe winters of long duration were to be faced. The nearest railway station was Hebertville, on the Canadian National Railway Line, about eleven miles south of the power site, and the nearest settlement was St. Joseph d'Alma on the Little Discharge, which was connected with Hebertville by a fairly good highway, but with heavy grades, extending to, and crossing, the Grand Discharge on a covered wooden bridge one mile below Isle Maligne. Beside getting in the employes and doing some sledge haulage of materials and supplies for the preliminary work of the first winter, the highway was not considered of much use. A railway provided a short haul, with reasonable grades, unbroken shipments, load capacity, volume of traffic and means of handling the snow, none of which were available by any practicable road. Moreover, a railway was an integral parcel of a complete power development, giving a shipping outlet to any further industrial development, such as the planned paper mills at St. Joseph d'Alma, and, extended by service tracks to the spillways and other parts of the work, it completed a positive construction function. Therefore construction for an eleven-mile railway line from Hebertville Station to Isle Maligne, as shown on the map, Fig. 2, was begun immediately. This line crosses the Bedard River, a low- and high-water channel of the Little Discharge, above St. Joseph d'Alma and the right channel of the Grand Discharge at the foot of Isle Maligne. The track is ballasted and laid with 70-lb. rail and all bridges are steel on concrete piers, the longest being that to Isle Maligne. As shown on Fig. 3, this latter bridge consists of two 90-ft. long and one 220-ft. long truss spans for double track service. Because the rapid and deep current in the river was practically doubled in volume by the diversion of flow from the left channel, by a cofferdam at the head of

FIG. 3—VIEW OF BRIDGE ACROSS RIGHT CHANNEL OF GRAND DISCHARGE AT FOOT OF ISLE MALIGNE, LOOKING DOWNSTREAM

Isle Maligne, (described elsewhere in this article), and did not permit the use of falsework, the large span was erected by the cantilever method from the 90-ft. approach span. The railroad was completed to the terminal, at the Grand Discharge opposite Isle Maligne, the middle of August 1923, and transportation, across the bridge to Isle Maligne, was started the end of October 1923.

CONSTRUCTION CAMP

The next important step was to provide means of housing and feeding sixteen hundred workmen and to furnish homes for the company's employes and their

families. The plateau above the railroad terminal on Alma Island opposite Isle Maligne and overlooking the construction operations in the south, was the most favorable location for a construction camp, inasmuch as it was close to the works and provided natural drainage. It is called Isle Maligne and has been made an official postal station. Fig. 4 is a general plan of the camp, and Fig. 5 is a partial view showing the type of buildings

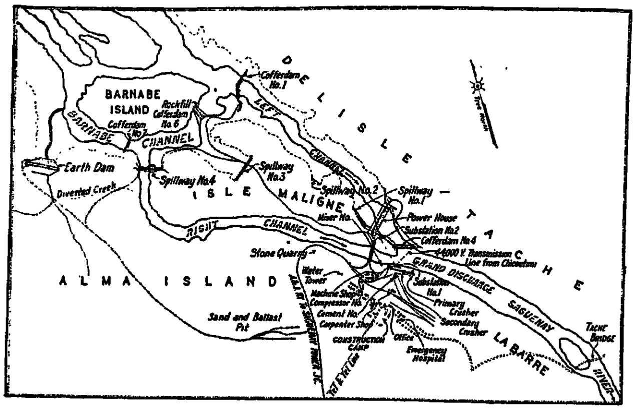

FIG. 4

constructed. All buildings have sewerage and water supply, are electrically lighted and furnished with stoves.

Particular attention has been given to the water supply. Water from the river is pumped to a 20,000-gal. tank on a hill above the camp, and thence distributed over the camp and works. The distribution was rather expensive because of the cold climate. All

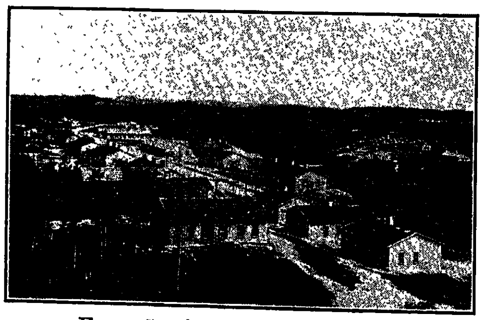

FIG. 5—CONSTRUCTION CAMP

pipes had to be placed at least six feet under ground to prevent freezing and much of the trenching was in rock. Chlorination was also employed. While the large volume of river flow made the danger from pollution rather remote, it was felt necessary to guard against typhoid as fully as possible and a chlorination plant was installed.

The camp was made self-contained, since the community is isolated, particularly in wintertime; a store, bank, post office and hospital were provided in addition to the usual dining halls, also bunk houses and cottages and one of the common rooms provide means for a school. There is also a resident physician.

A chart showing the number of laborers employed each day during the year 1924 is given in Fig. 6.

Construction Plant

Having the location and type of permanent structures fixed, and the volume of work to be done determined, progress schedules were prepared, giving the time allowed for the completion of each structure, keeping always in mind the fact that climate and stream flow set the time when certain construction operations could be begun and must be finished.

As the illustrations indicate, the main construction items are concrete masonry and excavation, largely in

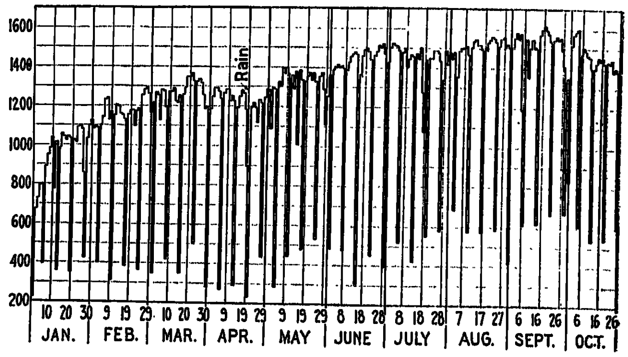

FIG. 6

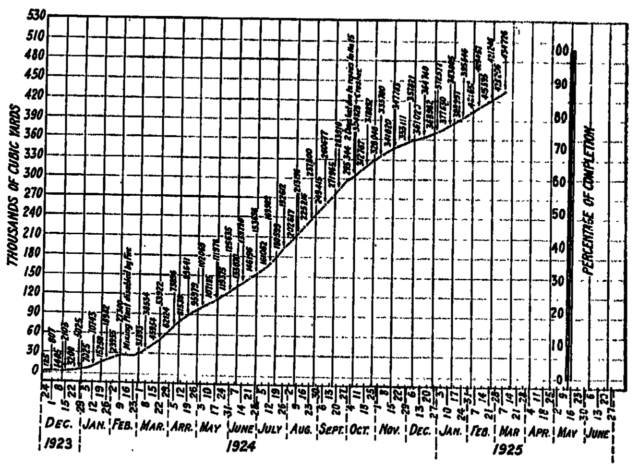

FIG. 7

rock, besides the steelwork for the turbines and powerhouse.

The estimated masonry quantities in the Grand Discharge are as follows:

	Cu. Yds.
Powerhouse Bulkhead & Substructure	288,000
Powerhouse Bulkhead Extension	40,500
Splashwall & Retaining Walls	10,000
Spillway No. 1	51,500
Spillway No. 2	3,000
Spillway No. 3	29,000
Spillway No. 4	80,000
Total	502,000

The estimated quantity of masonry in the Little Discharge is 20,000 cu. yd.

A weekly progress chart, showing the amounts of masonry placed in all the structures combined, to end of February 1925, is shown on Fig. 7.

The estimated quantities of excavation are as follows:

Tail-race	165,000 cu. yd.
Masonry Foundations	155,000 " "
Total	320,000 " "

Connecting with the railroad terminal on Alma Island, a track system was laid out to all structures and con-

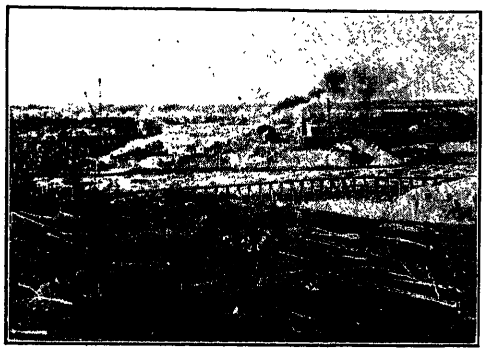

Fig. 8—Crusher and Mixer Plant

way No. 1 are 1000 ft. away, Spillway No. 2, 2500 ft., Spillway No. 3, 4000 ft. and Spillway No. 4, which is approached at both ends, is two miles away, on Isle Maligne and 2.7 miles on Alma Island. In the Little Discharge for the construction of the isolated spillways containing a comparatively small quantity of masonry, an individual mixer plant was provided.

As shown in Fig. 9, with a typical elevation of the crusher and mixer plant, the equipment consists of the following:

A. Crusher Plant:

One 66 by 84 in. manganese steel fitted jaw-crusher, driven by one 400-h. p. motor, 360 rev. per min.

One No. 15 manganese steel fitted gyratory crusher, driven by one 200-h. p. motor, 860 rev. per min.

Four No. 6 manganese steel fitted gyratory-crushers, driven by two 150-h. p. motors, 860 rev. per min.

Two 60 in. by 12 ft. 0 in. long scalping screens driven by one 50-h. p., 860-rev. per min., motor.

Two 72 in. by 24 ft. 0 in. long finishing screens driven by one 75-h. p., 860-rev. per min. motor, each.

One 42 in. belt conveyor, 174 ft. long, driven by one 75-h. p., 860-rev. per min. motor.

One 42 in. belt conveyor, 274 ft. long, driven by one 100-h. p., 860-rev. per min. motor.

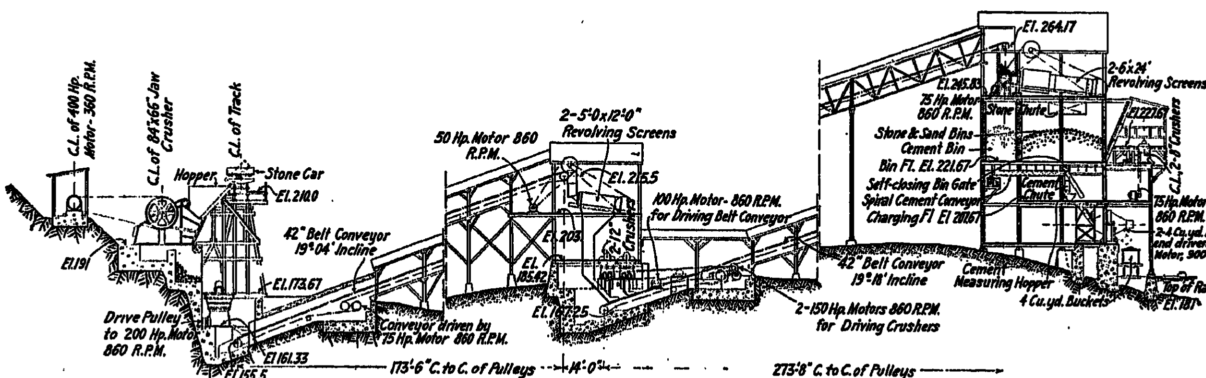

Fig. 9

struction plant units, practically eliminating job haulage by any other means. The maximum grade is three per cent, allowing for compensation in the curves and 70-lb. rails were used for the 15 miles of service tracks.

Since the bulk of masonry was to be placed near the foot of Isle Maligne, the crusher and mixer plant was placed on the Alma shore near the double track bridge across the right channel of the Grand Discharge. There, the topographical conditions were favorable for the layout of a gravity crushing plant; a large sand pit and quarry, supplementing the stone supply from the tail-race excavation, were also close by along the railroad to Hebertville, and cement could be quickly brought in. A panoramic view of the crusher and mixer plant is shown on Fig. 8 and its location and the layout of the service track system is indicated on the map, Fig. 4. Considering the mixing plant as the center, it will be noticed that the power house and spill-

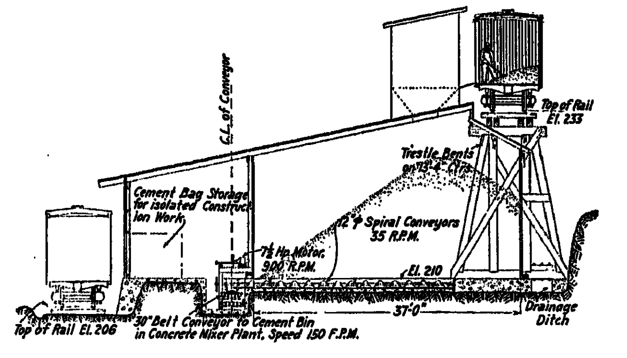

Fig. 10

One set transmission machinery for driving the four No. 6 crushers with two motors.

One set transmission machinery for driving the 203-foot long belt conveyor.

One set transmission machinery for driving the 293-foot long belt conveyor.

One set transmission machinery for driving scalping screens.

One set transmission machinery for driving finishing screens.

All motors are of the slip-ring type, 40-deg. rating, 2200-volt, 60-cycle, three-phase, each equipped with drum starter, with resistance for two-minute starting duty. Each motor (with the exception of the 400-h. p. motor which has a rope sheave for driving the jaw-crusher), is provided with sliding base rails and pulley. All these motors have also dust proof bearings and covers for the slip rings.

The scalping screens are perforated with two-in. round holes, and the finishing screens have dust jackets perforated with ½-in. diameter holes.

The maximum capacity of the crusher plant is 400 tons of two-inch ring-stone per hour.

B. Mixer Plant:

Two stone bins, of 510 cu. yd. capacity each.

One sand bin, of 540 cu. yd. capacity.

Two 4-yd. Smith tilting mixers on steel skids, with gated batch hoppers on steel frames, each driven by a 75 h. p., 2200-volt, 900-rev. per min. induction motor.

One 30-in. wide cement belt conveyer driven by one 10-h. p., 220-volt, 900-rev. per. min. induction motor.

Two 100-h. p. boilers for heating mixing water.

One 90-h. p. boiler for heating sand.

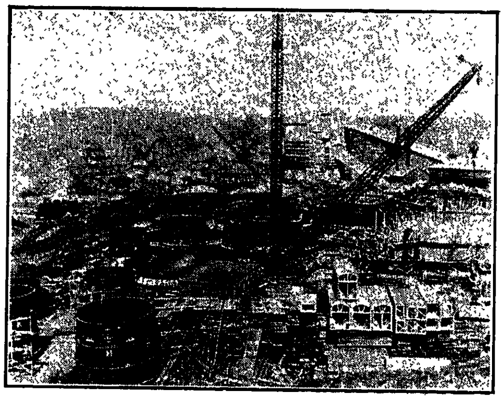

Fig. 11—Shop Manufacture of Draft Tube Forms

The cement house, a typical cross-section of which is shown on Fig. 10, is laid out for the handling of cement in bulk, with a storage capacity corresponding to 75,000 bags. A screw conveyer, running in a groove at the bottom of each cement bin, feeds the belt conveyer to the mixer plant. Only a small quantity of cement is shipped and stored in bags, available for isolated work.

The maximum output of the concrete mixer plant was, on September 26, 1924, amounting to 676 batches of 4 cu. yds. concrete.

Excavating Plant

For the earlier operations in connection with the cofferdam construction at the head of Isle Maligne and the service track construction, an 8 by 8 in. gasoline engine-driven portable compressor of 210 cu. ft. per min. displacement and a stationary steam-driven compressor of 340 cu. ft. per min. displacement supplied from one 100-h. p. boiler, were used.

The main air-compressor plant is located on Alma Island, 300 ft. south of the bridge to Isle Maligne. It consists of two 350-h. p. 2200-volt, 200-rev. per min., three-phase, 60-cycle synchronous motors, direct-connected to cross-compound compressors of the clearance-control type. Each compressor has cylinders of 25- and 15¾-in. diameter, with 18-in. stroke, and a capacity of 2033 cu. ft. of free air per minute.

The rock-drill equipment includes high speed drills on tripods and jack hammers. The drill steels are sharpened by three "Leyner" sharpeners after the drills are heated in oil furnaces.

A separate compressor plant, of 350 cu. ft. per min. free air capacity, is provided for riveting. In the plate-steel work of each scroll case for the twelve turbines, there are contained 12,000 rivets, and a uniform pressure is required which cannot be continuously maintained in the service pipes of the main compressor plant.

For bulk excavation and loading of the blasted rock in the tail-race, steam shovels are employed. Three shovels are of the railway type and two smaller size shovels are fitted with caterpillar traction.

For soft excavation, wherever practicable, a hydraulic monitor is used, consisting of a four-stage, 520-ft.-head centrifugal pump, direct-connected to a 400- h. p. induction motor, having an eight-in. delivery pipe and 2½-in. nozzle.

Work Shop and Saw Mill

The machine shop, near the compressor plant, is fitted with planers, drills, lathe, shears, bolt and pipe machines and forges. It is used for maintenance of rolling stock, general repair work and the manufacture of smaller articles; it is, however, large enough for the construction of flat cars, frogs and switches, of which a number were built.

The carpenter shop, located further away from the congested track system near the crusher and mixer plant, is equipped with band, swing and rip saws, planing and jointing machines. It is principally engaged in making the multitude of forms required for the construction of the concrete masonry structures. A view of a completely assembled draft-tube form for one of the twelve turbines is shown in Fig. 11.

A saw mill is in operation during the winter months when the snow permits of logs being hauled to the site from local sources. However, all structural timber is brought in by rail and is mostly British Columbia fir. About 10,000,000 ft. b. m. timber were used on the job to the end of 1924.

Electric Power Supply

With the exception of steam shovels, locomotive cranes and railway locomotives, all construction and shop equipment is motor operated and the aggregate motor equipment is 5000 h. p.

Price Bros. and Co., Ltd. which are connected with the Saguenay Power development, have a hydroelectric plant at Chicoutimi from which they furnish the electric power. Early in 1923 a 40-mile, 44000-volt, three-

phase, transmission line was built from the Chicoutimi plant to Isle Maligne and at the works, the current is stepped down to 2300 volts for major motors and to 440 volts and 220 volts, respectively, for the minor motors and lighting circuits distributed about the camp and works, as illustrated by Fig. 4. A study of this map and Tables I and II, will furnish the essential details of power distribution and equipment.

Altogether, about fifty odd motors are required and all but two or three are of the a-c. induction type. The largest motors are 400 h. p., for the jaw-crusher and sluicing pump, 350 h. p. for the air compressors and 200-h. p. for the No. 15 crusher. The derrick motors are all 100 h. p. The advantages of electric power, for the construction plant of the Isle Maligne development, are a continuous supply from a central source, ease of distribution with the smallest loss and reduction of attendance to machines.

TABLE I
LIST OF MOTORS FOR CONSTRUCTION PLANT

Application	Type	H. P.	Volts	R.P.M.	Number
Jaw-Crusher	Ind.	400	2200	360	1
Sluicing Pump	"	400	2200	1800	1
No. 15 Crusher	"	200	2200	860	1
Conveyor No. 1	"	75	2200	860	1
No. 6 Crushers	"	150	2200	860	2
Conveyor No. 2	"	100	2200	860	1
Scalping Screens	"	50	2200	860	1
Concrete Mixers	"	75	2200	900	2
Cement Elevator	"	10	220	900	1
Screw Conveyors	"	7.5	220	900	6
Compressors	Syn.	350	2200	200	2
Exciter for Compr.	D-c.		125	900	1
Water Supply Pump	Ind.	25	440	1745	1
Carpenter Shop	"	5	220	1800	1
Carpenter Shop	"	50	2200	900	1
Carpenter Shop	"	30	440	900	1
Machine Shop	"	15	220	1150	1
Sand Derrick	"	100	440	900	1
Stone Derrick	"	100	440	900	1
Carpenter Derrick	"	100	440	900	1
Cement Unloaders	"	5	440	720	3
Sand Pump	"	30	440	1720	1
Traveler Cranes	"	100	440	900	10
Construction Derricks	"	100	440	900	9
Well Machines	"	10	440	1720	2
Log-Handling Hoist	"	100	440	900	1
Pumps	"	40	440	1200	2
12 in. Pump	"	100	440	1800	1

TABLE II
LIST OF ELECTRIC TRANSFORMERS FOR CONSTRUCTION PLANT

Location	Capacity Kv-a.	Primary Voltage	Secondary Voltage	Number
Substation No. 1	500	44000	2300–460	3
Substation No. 1	75	2200	220–440	2
Cement Shed	10	1100–2200	110–220	3
Derrick for Jaw-Crusher	75	2200	220–440	1
Derrick for Carpenter Shop	75	2200	220–440	1
Jaw-Crusher Motor House	7.5	2200	110–220	1
Machine Shop	37.5	2200	110–220	3
Substation No. 1	5	2200	110–220	1
Carpenter Shop	5	2200	110–220	2
Water Tank	10	2200	220–440	3
Foreman's Camp	10	2200	110–220	1
Camp 4	5	2200	110–220	1
Camp 6	7.5	2200	110–220	1
Camp 6	5	2200	110–220	1
Camp Staff	10	2200	110–220	1
Rumfeldt's House	5	2200	110–220	1
Drill Sharpening Shop	5	440	110–220	1
Substation No. 2	500	44000	2300–460	3
Lights, Isle Maligne	10	460	110–220	3
Substation No. 3	200	2300	460–230	3
Spare Transformer	500	44000	2300–460	1

Equipment for Delivering Concrete

For the delivery of the concrete from the mixer plant, full-batch buckets, consisting of four cu. yd. bottom-dumping buckets, are used; also two-cu. yd. buckets for structures of smaller volume.

The concrete is handled on flat cars of 80,000- to 100,000-lb. capacity, which take four four-cu. yd. or six two-cu. yd. buckets. Certain work such as portions of the power-house substructure were built by depositing the concrete by means of chutes. For this purpose, special four-cu. yd. wooden boxes provided with steel gates were built and four of them placed on a flat car, as shown in Fig. 12. By suitable operation of the concrete train, a box is brought opposite a chute and the gate then opened for discharging the concrete directly through the chute to its final place.

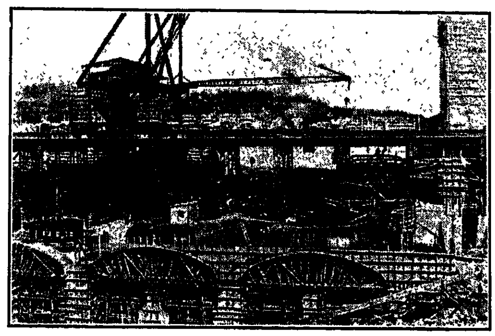

Fig. 12—Train of Four-Yard Buckets under Traveler Crane in Process of Depositing Concrete to Chutes

Hoisting Equipment

With the exception of two small wooden stiff-leg derricks, all derricks are built of steel and their capacity is in proportion to the power required for handling the four-cu. yd. bottom-dump concrete buckets. There are distributed over the various parts of the works nine steel stiff-leg derricks and six steel guy derricks. Of special interest are the five traveler cranes built for the construction of the power-house bulkhead and substructure and for subsequent use in connection with the construction of Spillway No. 4. As shown on the view, Fig. 13, each traveler crane consists of two stiff-leg derricks with individual hoisting engines mounted on top of a portal of steel construction. This portal has a length of 36 feet, a span of 43.5 ft. between centers of runways and a clearance of 19 feet above the three railway tracks, arranged within the traveler span permitting trains and locomotive cranes to pass under it. These two derricks handle a load of 10 tons per boom at a radius of 80 ft. and of 20 tons per boom at a radius of 40 ft.

The twenty-five hoisting engines for the various derricks are of the double-drum type for ¾-in. and ⅝-in. diameter rope and have interchangeable parts. Each

hoist is fitted with a 100-h. p. motor at 900 rev. per min. with suitable drum controller and resistance for three-phase, 60-cycle, 440-volt current. Based on 75 per cent of the motor horse power, the rating of the hoist is 9000 lb. at a speed of 275 ft. per min. on a single line.

The two steam locomotive cranes used on the works have a lifting capacity of 25 tons at 15-ft. radius and of 5 tons at 50-ft. radius.

Rolling Stock

As stated before, the railroad from Hebertville to Alma Island opposite Isle Maligne is 11 miles long and in addition there are 15 miles of service tracks in use on the works, all of standard gage.

The sixteen construction locomotives in operation on these tracks are 33- and 40-ton saddle tanks with 14,400-lb. and 17,400-lb. traction power, respectively. For hauling the concrete and other construction materials, twenty-two flat cars of 80,000- to 100,000-lb. capacity are used. For excavation and earth-fill work, there are available thirty-six 20-cu. yd. air-dump cars and twenty 6-cu. yd. hand-dump cars.

Designing Features of Structures

Preliminary examinations made at the sites finally selected for the power-house bulkhead and substructure, for Spillways No. 1 to No. 4, inclusive in the Grand Discharge, and for the spillways in the Little Discharge, indicated that excellent rock foundations, consisting of hard granite, were available.

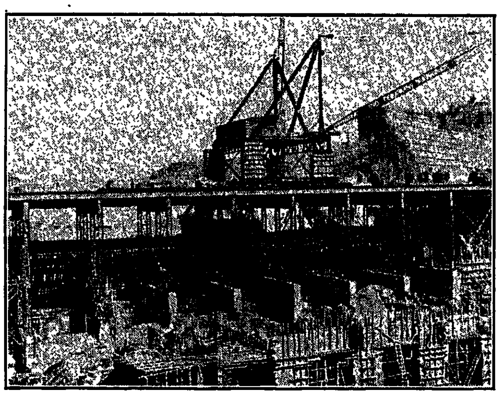

Fig. 13—View of Traveler Crane used in Construction of Concrete Work for Powerhouse

Spillways

Since 1913, daily stream-flow records of the Grand Discharge and of the Little Discharge of the Saguenay River (as well as the daily stages of Lake St. John), were taken by the Power Company.

By Government grant the ordinary high-water level of Lake St. John was fixed at el. 257.5 which is 17.5 ft. above the zero point of the gage at Roberval on Lake St. John, and, to have the full benefit of the available storage capacity, it was decided to install movable weirs or regulating gates on top of the concrete masonry spillways wherever it could be economically done. As shown in Figs. 14 and 16, the regulating gates are built of steel and are of the Stoney type, with suspended roller trains. All the gates are 40 ft. wide in the clear between piers and their top is at el. 257.5 when in closed position. The number and height of gates in each spillway and the corresponding discharge capacities with all the gates raised are given in Table III.

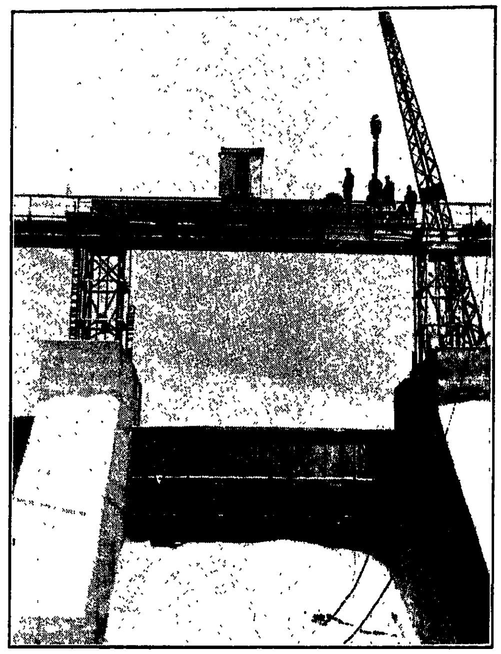

Fig. 14—Flood Gate Installed in Spillway No. 3

TABLE III
DISCHARGE CAPACITIES OF 40-FT. WIDE FLOODGATE OPENINGS IN SPILLWAYS OF GRAND DISCHARGE AND LITTLE DISCHARGE CONSIDERING WATER ELEVATION 257.5

Number or Name of Spillway	Number of Gates	Height of Gates	Discharge Capacity Cu. Ft. per Sec. Per Spillway	Total
		Grand Discharge		
1	6	17 ft. 6 in.	63,400	
2	0	0 in.	0	
3	12	17 ft. 6 in.	126,800	
4	11	27 ft. 6 in.	226,100	416,300
		Little Discharge		
Right Channel	3	17 ft. 6 in.	31,700	
Middle "	0	0 in.	0	
Left "	2	17 ft. 6 in.	21,100	52,800
		Total Discharge of	Lake St. John.	469,100

Extending for a distance of about three-quarters of a mile from the outlet of Lake St. John, the Narrows of the Grand Discharge will be enlarged, thus increasing the available water areas both below the low-water level, when the present Lake discharge is not sufficient for operating all the turbines, and below the high-water level, to eliminate the gorging condition at this point.

It is not deemed practical to make any excavations in the turbulent waters of the Narrows of the Grand Discharge before the spillways are completed and the water backed up against the rapids and falls in the river. According to the construction program, this excavation work will be only partially done before the Isle Maligne Station begins operation.

Serious consideration was given to the possible effect of ice pressure. As known, the tendency of the ice is to expand at rising temperatures and where this expansion cannot freely act, it will exert pressure against the structures, causing obstruction. The maximum thickness of ice on Lake St. John is from two to three feet; however, its layers are formed under all kinds of weather

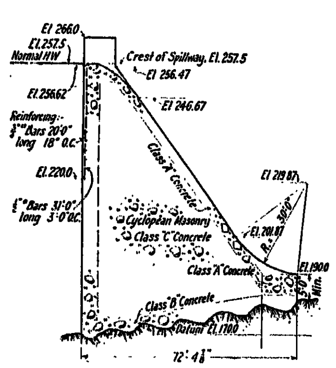

FIG. 15

conditions, which explains the fact that the unit crushing strength of the ice decreases with the increase in thickness of the ice. Furthermore, the heat transmitted to the ice, due to the rise in temperature of the atmosphere, gradually loses its intensity in the bottom layers, so that only the top layers of ice will expand and exert pressure if obstructed.

For the structures which are likely to be subjected to ice pressure from Lake St. John, an allowance of 20,000 lb. per lin. ft. of structure is made in the design. Referring to Fig. 15, showing a typical cross-section of Spillway No. 1 with masonry crest at el. 257.5, it will be noticed that steel reinforcement is provided on the upstream face to take care of the ice pressure.

A typical cross-section of Spillway No. 4, showing the arrangement of the regulating gate is given in Fig. 16. The masonry in the spillway structures is mass concrete with large stones ranging up to five tons, embedded in same to secure a good bond between the successive layers of concrete.

No silt pressure against the up-stream face of the spillways is considered, since the water carries no silt in suspension. An allowance for possible uplift at the base of the spillway is made, assuming that full water pressure, acting at the heel, decreases to zero at the toe in a parabolic function.

With the exception of spillway No. 2 and the spillway in the middle channel of the Little Discharge, neither of which have regulating gates, each spillway is provided with a crane runway, supported by steel towers, anchored to the 10-ft. thick piers between the gates. On top of each runway, there is a traveling hoist of the screw-type, designed for raising and lowering each gate at a speed of two ft. per min. under the most adverse conditions. For handling stop logs, for which vertical grooves are provided at the up-stream end of the piers, the traveling hoist is equipped with an auxiliary hoist, having a capacity of 10 tons at each end; it can be operated by the main hoisting motor and is designed for a vertical lifting speed of 10 ft. per min.

All five traveling hoists have interchangeable parts and the motors of the marine type, 550-volt, three-phase, 60-cycle are fully enclosed. The main hoist motors have a capacity of 30 h. p., each, which is based on the power required for handling the 27-ft. 6-in. gates; the motors for the traverse have a capacity of 10 h. p., each. Each screw stem of 6-in. outside diameter, is operated by a manganese-bronze operating nut, to which the power is transmitted by steel gears. The cast-iron case enclosing each hoisting mechanism contains a nest of cushion springs sufficiently strong to create gradually the necessary overload in the hoisting motor and to trip the overload-release switch in case the gate should meet an obstruction while being lowered; this device will prevent the screw stems from buckling. Four rail grippers are provided for each hoist, holding

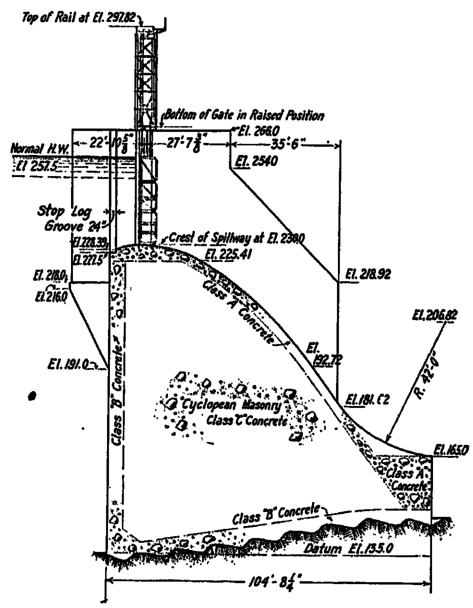

FIG. 16

the hoist to the rails while a gate is in operation. The gate to be operated is connected by steel pins to the end of the two hoisting-screw stems. On top of the piers there are stops, consisting of heavy cast-iron brackets with heavy forged steel pawls, which are strong enough to support the weight of the heaviest gate when it is in raised position and disconnected from the screw stems of the traveling hoist.

Power-House

Considering the 720-ft. over-all length of the power-house and the 163.5-ft. width of the power-house bulkhead and substructure at the base, it was deemed necessary to provide for expansion and contraction of the structure due to temperature changes. Referring to Fig. 17, which is a typical cross-section of the power-house, an expansion joint is shown between the power-house substructure and the forebay structure, beginning at el. 137.0 or 1.4 feet below the mean tail-water level and ending at the generator floor level. This joint, provided along the entire building, separates the massy bulkhead structure from the cut-up substructure containing the turbines. The plate-steel intake pipes embedded in the bulkhead masonry are connected to the respective turbine spiral casings by

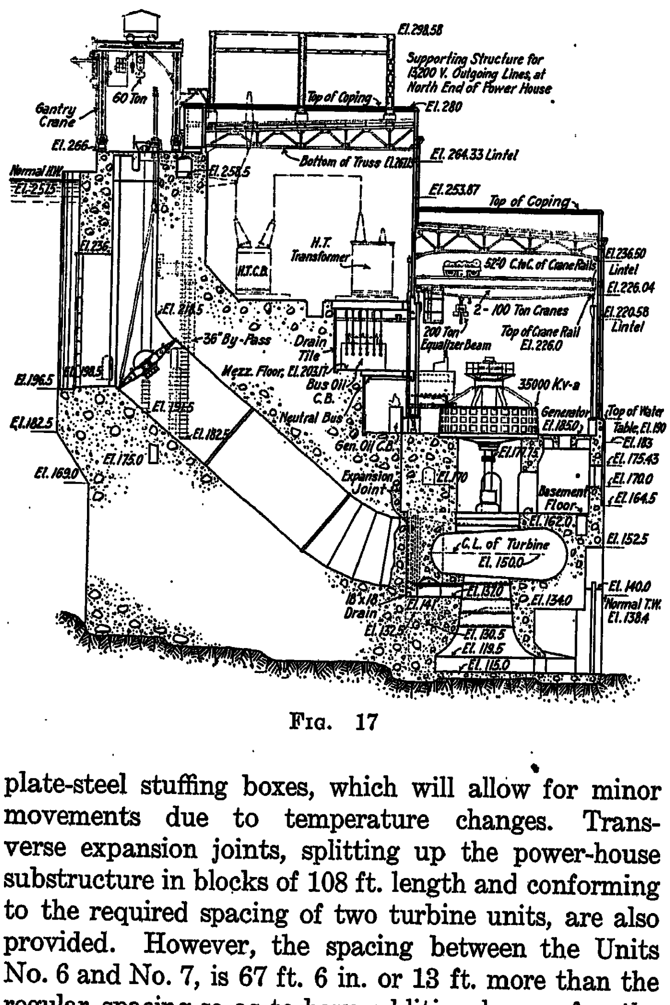

Fig. 17

plate-steel stuffing boxes, which will allow for minor movements due to temperature changes. Transverse expansion joints, splitting up the power-house substructure in blocks of 108 ft. length and conforming to the required spacing of two turbine units, are also provided. However, the spacing between the Units No. 6 and No. 7, is 67 ft. 6 in. or 13 ft. more than the regular spacing so as to have additional room for the accommodation of the electric bench-board.

The rack structure at the up-stream face of the power-house bulkhead is built of reinforced concrete and structural steel. The top of the rack bars is at el. 236.0 or 4 ft. below the ordinary low-water level of Lake St. John. The spacing of the 6 by 3/8-in. rack bars is 4 1/2 in., center to center.

The intake to each of the twelve turbine flumes consists of two 16 by 22 ft. openings formed in the concrete masonry. Each opening contains a rectangular gate of the butterfly-valve type, having in the center a cast-steel girder of 2 ft.-8 in. depth and of 2 ft.-6 in. width, with bronze-bushed trunnions of 18 in. outside diameter and 24 in. length at each end. The wings of cast iron are firmly attached to the girder by machine bolts and shrink rods. A built-up cast-iron wall frame securely anchored to the masonry is provided with machine-finished surfaces at its lower half at which point the surfaces of the gate and wall frame come in direct contact. The upper half of the wall frame has machine finished bevelled faces conforming to similar surfaces of the gate. The slot between the upper half of the gate and the wall frame is sealed by a set of brass flaps attached to the upstream side of the wall frame. Each gate is operated by an individual, completely enclosed hoist located on top of the power-house bulkhead as outlined on Fig. 17. The design of the gate is such that the connecting rod, consisting of a 9 in. extra heavy steel pipe, is in tension under all operating conditions. To prevent buckling of the connecting rod when the gate is reaching its lowest position the upper pin connection of the rod is machine-slotted for the pin in the cross-head to allow a 4 in. overtravel on the downward stroke. There is also a nest of heavy coil springs in the gear housing of the hoist which will act as a cushion for the operating stand in the range of over-travel on the downward stroke and gradually create the necessary overload, causing a tripping of the overload release switch. An allowance of 24 in. for overtravel on the upward stroke is made by proportioning the length of the screw stem and cross-head guide. A gate-position indicator is provided on top of the hoist, also a limit switch arrangement of moisture and spray-proof construction.

Direct-current, 230-volt, electric power is supplied for the operation of the hoists. Each headgate motor is completely enclosed, and has a starting torque of 275 ft.-lb. and a running torque of 140 ft.-lb., based on six minutes' service with a temperature rise not exceeding 55 deg. cent. Under the most severe operating conditions, the gate may be completely closed or opened in five minutes. A weather-proof push-button control station is provided for each hoisting mechanism on top of the bulkhead, and a non-weather proof control station on the power-house benchboard for distant closing of each gate. The emergency push-button control on the switchboard is for use in accidents only as the intake gates can be readily closed when the generator is operating under full load.

Each of the twelve turbine flumes formed in the concrete masonry of the power-house bulkhead gradually converged into a 22-ft. diameter opening which is the beginning of the plate-steel intake pipe forming an extension of the turbine spiral casing embedded in the power-house substructure.

Due to the fact that the turbine draft tubes are of the White hydro-cone design, calling for large openings in the lower part of the power-house substructure, considerable steel reinforcement was required in this portion of the structure to take care of the superimposed loading.

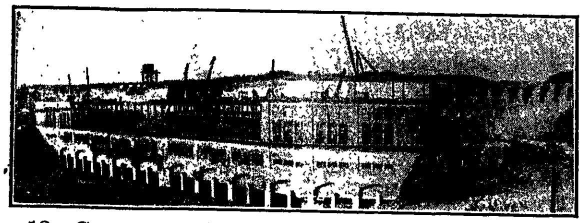

FIG. 18—GENERAL VIEW OF INCOMPLETE POWERHOUSE, LOOKING UPSTREAM

The power-house superstructure, shown in Fig. 18, is built of hollow tile and face brick of buff color which encase the structural steel frame supporting the roof

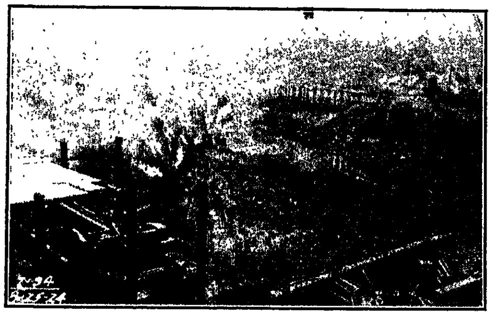

FIG. 19—EXCAVATION WORK IN TAILRACE

trusses and the steel runways for the two 100-ton capacity electric cranes in the generator room. The hollow tile face in the interior of the power-house is covered with plaster and finished with a coat of paint.

the tail-race is estimated to be 50,400 cu. ft. per sec; for eight units it is 33,600 cu. ft. per sec. Measured at the bottom the width of the tail-race immediately below the downstream face of the powerhouse is 657.5 ft; the depth below normal watersurface is 23 ft. The total length of the tail-race to the foot of Isle Maligne where it joins the Grand Discharge is 800 feet, of which the downstream portion of 240-ft. length has a bottom width of 300 ft. The maximum velocity of water in the tail-race is estimated to be 8.1 ft. per sec. with all 12 turbines operating at full load.

Practically all of the tail-race excavation consisted of hard rock, as may be seen on the illustration, Fig. 19.

EARTH DAM

The only structure to withstand water pressure, which is not built of masonry, is the earth dam across the ravine near Spillway No. 4 on Alma Island. Preliminary examinations and test borings made at the site disclosed the fact that below the 10 ft. thick overburden of muskeg there is a layer of impervious clay of about 15 ft. thickness before rock formation is struck. It was, therefore, decided to wash out the layer of muskeg by means of a hydraulic monitor and to build a semihydraulic fill with a clay core in the center and slopes as indicated on the cross-section, Fig. 20.

The earth dam has a length of 650 ft. measured on top. The upstream slope is covered with riprap for protection against the action of waves and ice.

STATION INSTALLATION

The power-house design calls for the installation of 12 vertical-shaft, single-runner, Francis-type turbines, having plate-steel spiral casings and operating at a speed of 112.5 rev. per min. Eight turbine units are now completely installed and all the parts which go into the concrete for the four additional units are in place.

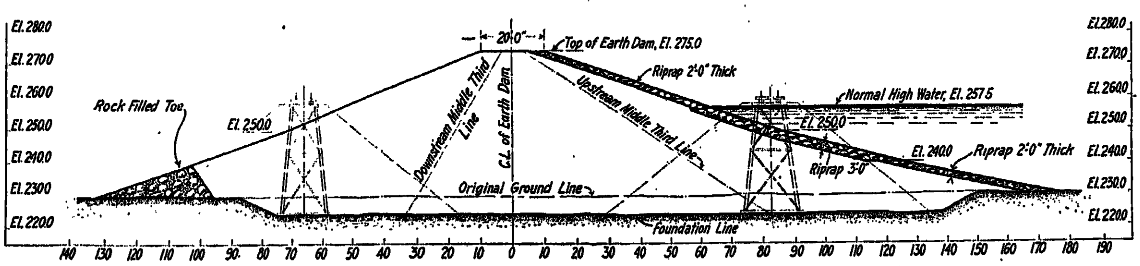

FIG. 20

The generator floor and the 5-ft. high inside face of the concrete water table are covered with glazed tile. The large window sashes are built of steel and are securely attached to the steel columns supporting the roof trusses. The roof consists of reinforced gypsum slabs resting on steel purlins and the top of the slabs has a finish of tar and gravel.

TAIL-RACE

When all twelve turbines are operating at full load and under 110 ft. effective head, the total discharge into

The range in head on these turbines is between 100 ft. and 120 ft. and within these limits the rating of each unit is as follows:

Head in feet	100	105	110	115	120
Horse power at best efficiency	35,000	37,500	40,000	42,500	45,000
Maximum horse power	40,000	42,500	45,000	47,500	50,000

An individual governor system operating under an oil pressure up to 220 lb. per sq. in. is provided for each turbine, and the piping is so arranged that the governors

may be operated separately or as an interconnected system in units of four, each. The governor fly-balls are directly connected to the turbine shaft below the rotor. The governor stands, gear-type oil pumps, pressure tanks and receiving tanks are located in the spacious power-house basement or, better expressed, in the turbine room. An emergency-stop device is installed for each turbine, enabling the operator at the benchboard to shut the turbine gates and stop any machine without the assistance of an operator in the turbine room.

The closing time for the governor for the full stroke of the servo-motors is three seconds.

The eight generators directly connected to the turbines now completely installed have a continuous full load rating of 30,000-kv-a. or 24,000-kw. at 80 per cent powerfactor, 13,200-volt, three-phase, 60-cycles, 112.5-rev. per min. They are guaranteed for a temperature rise, not to exceed 55 deg. cent.; the required exciter capacity is 185 kw. Each generator will carry 35,000 kv-a., continuously at normal voltage and 80 per cent power factor, with a temperature rise, not exceeding

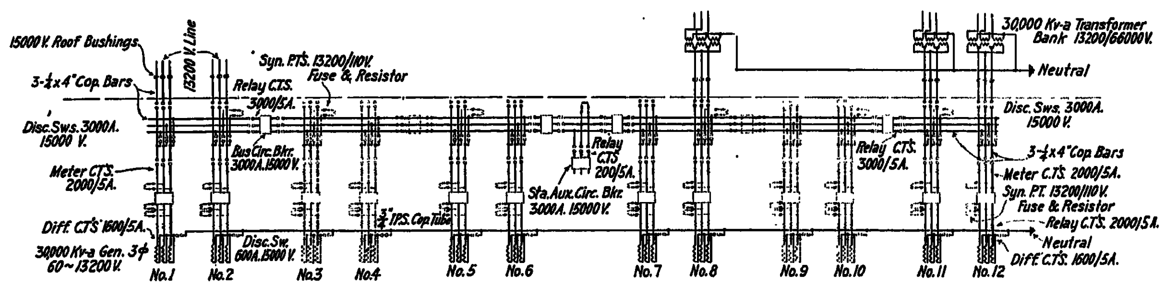

FIG. 21

70 deg. cent. by thermometer, and with a required exciter capacity of 210 kw. It will also carry 24,000 kw. continuously at 15,180 volt and 80 per cent power factor. The guarantees are based on the cooling air having a temperature of 40 deg. cent. or lower. Each generator is equipped with an upper and lower guide bearing. The thrust bearing on top of the generator is of the Kingsbury type and its housing is provided with water cooling coils.

The individual exciters placed above the thrust bearings have a full load rating of 185 kw. at 250 volts and 112.5 rev. per min. and are guaranteed for a temperature rise not to exceed 40 deg. cent. Followed by 225 kw. load for 24 hours, the temperature rise is not to exceed 50 deg. cent.

The oiling system of each generator unit is self contained and automatic in operation. The thrust bearing is immersed in an oil reservoir containing water-cooling coils located on top of the upper bearing bracket. A gear pump, mounted in a smaller oil reservoir located below the lower bearing bracket, is driven by a gear on the generator shaft and forces the oil to the upper thrust bearing reservoir. Proper pipes also conduct the necessary oil to the two guide bearings and the overflow oil from each bearing is carried back to the lower reservoir for recirculation.

Large brakes are provided for four of the eight arms of the lower generator bearing bracket. If operated by air pressure, the brakes will stop the generator with the turbine gates closed and if operated by oil pressure from the governor system, the brakes may be used as jacks to lift the rotor. Cast-iron stools are provided for mounting on the bracket arms to support the rotor when lifted.

The generator armature is star-connected and the six leads are brought out from the upstream side of the generator. By means of a 15,000-volt disconnecting switch, the neutral tie is connected to the neutral bus consisting of a ¾ in. copper tube suspended by porcelain insulators to the mezzanine floor along the upstream side of the power-house and it is then grounded outside of the station.

A diagram showing the connection of the main generator leads with the low-tension bus system is shown on Fig. 21. It will be noticed that the connections from the generators No. 3, 4, 9 and 10, which are left blank, are indicated by dotted lines. The generator leads and the transfer busses are built up of three ¼ by 4 in. copper bars and the spacing between conductors is 18 in. Normally each generator feeds its own transmission line and the duty of the low-tension bus system is to transfer power from another generator in case of emergency and to provide the means for delivering power to the transformers for the station auxiliaries.

The triple-pole, single-throw generator and bus oil circuit breakers are identical in construction and of the same capacity. The rating of each circuit breaker is 3000 amperes at 15,000 volts and the guaranteed interrupting capacity is 65,000 r. m. s. amperes at 13,200 volts. The cell structure for the breaker is of reinforced concrete and each pole is mounted on a casting provided with wheels to enable rapid roll-out for inspection. An electrically-operated motor mechanism is provided for each breaker and arrangement is made for emergency manual operation. Direct-current power at 230 volts, for the operation of the mechanism, is furnished from a 25-kw. motor-generator set.

The benchboard is located on an extension of the mezzanine floor into the generator room between units No. 6 and 7. It is of the continuous closed type and

has a length of 40 ft. The auxiliary vertical board containing recording ammeters, volt-meters, watthour meters and relays is 16 ft. 4 in. long and 7 ft. 6 in. high.

The control circuits are for 230 volts and the lighting circuits for 115 volts. They are placed in 1½ in. diameter pipe conduits embedded in the generator and mezzanine floors and the estimated aggregate length of conduits in the power-house, including a number of flexible conduits, is 108,500 ft.

For station use, there are installed two banks of transformers which are located in the center of the high-tension room and each bank consists of three 200-kv-a. single-phase transformers, stepping down the

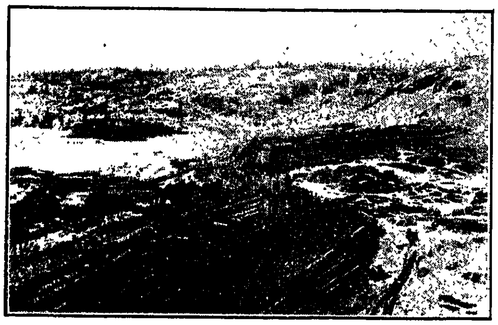

Fig. 22—Cofferdam No. 1 under Construction

voltage from 13,200 to 550 volts. The two 100-ton capacity power-house cranes, the 100-ton capacity crane for lifting and transferring apparatus from the generator room to the high-tension room, the 60-ton capacity gantry crane on top of the power-house bulkhead and the sixteen headgate hoist motors now installed, are using 230-volt direct current. The power for operating these motors is furnished by a 200-kw. motor-generator set. The three air compressors supplying compressed air for the governor system and the two air compressors for general station use are driven by alternating-current motors at 220 volts; power is furnished through three 25-kv-a. single-phase 550- to 220-volt step-down transformers. For station lighting the current is stepped down in three 25-kv-a., 550/110-volt single-phase transformers.

All these motors, including the 550-volt induction motors driving two 1200-gal. per min., 65-ft. head, centrifugal pumps furnishing cooling water for the high-tension power transformers, the 25-kw. motor generator set and all station lighting circuits, are supplied by the first bank of transformers. At present, the other bank of transformers is delivering power at 550 volts to one 150-h. p. induction motor, driving a centrifugal pump of 1200-gal. per min. capacity against a head of 400 ft., forming part of the general water supply system for the entire Isle Maligne development; provision is made for the installation of two additional pumps of the same capacity for future use.

Method of Construction

With the exception of some isolated bridge piers which were built by contract, the entire construction work for the Isle Maligne development was done by force account. The work was prosecuted uninterruptedly and while the winters were long and severe, with temperatures reaching 45 deg. below zero fahr., the snow on the ground permitting haulage on sledges; the formation of ice on the river where the current was not too swift made it easier to build cofferdams. Beginning with November, the Saguenay River gradually falls until extreme low-water is reached around the end of March; advantage of this season was taken to build the main cofferdams and temporary rock fill dams. The procedure was to build up the cribs on the ice, then to cut out the surface to be occupied by each crib and to drop and sink the crib into position. The extreme high-water stage of the Saguenay River is reached around the end of May, flood conditions lasting until the end of June. It was, therefore, necessary to build high cofferdams so as to have no delay in the construction work during the late spring and to have the full benefit of the early summer season with long hours of daylight.

Power-House Construction

The power-house construction and the tail-race excavation involved the largest volume of work to be done. The first step taken was to immediately construct the cofferdams across the left channel of the Grand Discharge at both the head and foot of Isle Maligne, so as to be able to unwater the power-house foundations and the tail-race area. The upstream cofferdam, marked No. 1 on Fig. 4, was commenced in January 1923 after a construction force had been organized. With the exception of a 200-ft. long section

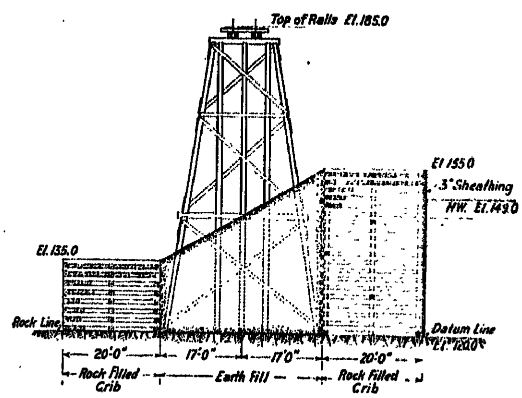

Fig. 23

across a highwater channel on the Isle Maligne shore, which was closed in the following July as the water subsided, it was completed in two months. This cofferdam is of the usual crib construction, rock filled, with two layers of 1½ in. sheathing on the up-stream face; however, its length is, 1143 ft. and its maximum height 42 ft. Fig. 22 is a view showing the cofferdam under construction:

The cofferdam at the foot of Isle Maligne, marked No. 4 on Fig. 4, was built to shut off the backwater of the Grand Discharge from the tail-race excavation and to give a footing for a timber trestle, forming part of the

construction railroad system for the transportation of excavated rock from the tail-race to the crusher plant. The length of cofferdam No. 4 is 627 ft. and a typical cross-section of same is shown on Fig. 23.

With the power-house foundations unwatered, a steel trestle of 715 ft. 6 in. length, carrying three lines of standard track and a runway of 43 ft. 6 in. gage to accommodate four traveler cranes was built. This trestle, a view of which is shown in Fig. 24, covered the entire length of the power-house and the top of rail for the three tracks was at el. 188.0, which is 50 ft. above

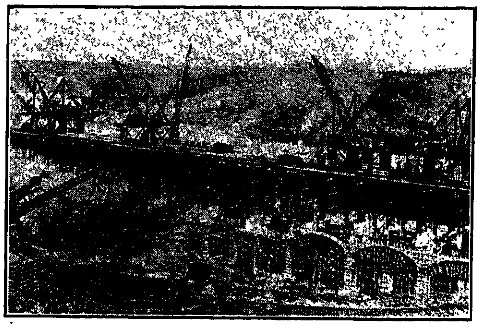

Fig. 24—Construction Trestle for Powerhouse

the normal tail-water level; it was completed during the end of December 1923, two months later traffic was started over the bridge connecting with the railroad system on Alma Island.

The steps followed in the construction of the power-house bulkhead and substructure, the relative location

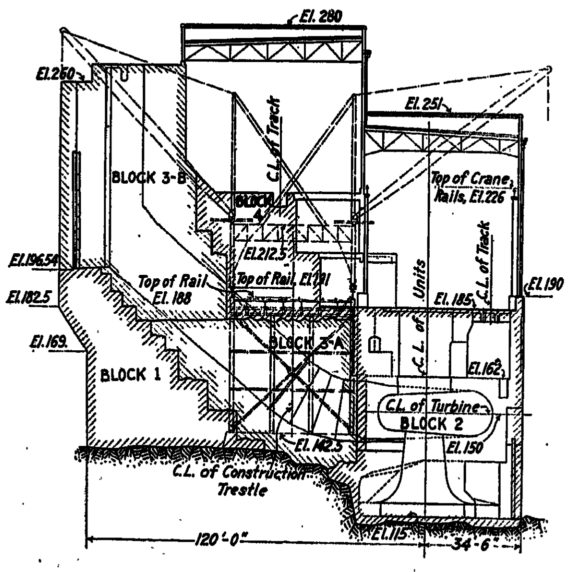

Fig. 25

of the steel trestle and an outline of the traveler crane are indicated on the section, Fig. 25. It will be noticed that the masonry is subdivided into five blocks. According to the schedule, Block No. 1 and the power-house bulkhead extension to el. 197.5, containing altogether 89,000 cu. yd., were built first so as to eliminate any possible danger from extreme high-water which cofferdam No. 1 could not safely hold back. This work was completed the middle of April 1924, six weeks before the river reached its highest stage, records being taken by the Quebec Development Company. At that time the peak of the high-water was within one ft. of over-

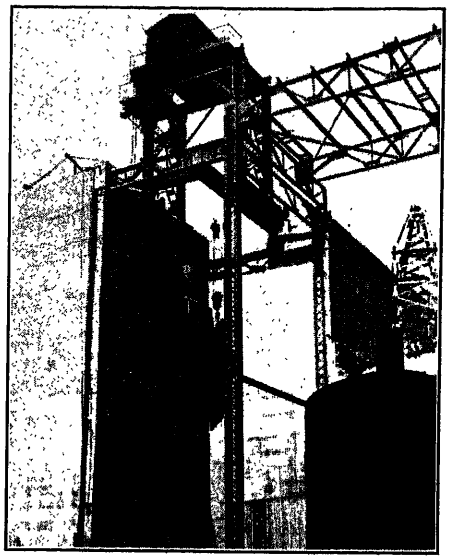

Fig. 26—Gantry Crane Lifting Headgate Weighing 54 Tons

topping the cofferdam. Block No. 2, forming the power-house substructure proper and containing 60,711 cu. yd. was completed end of November 1924; Block No. 3-b of 61,474 cu. yd. end of December 1924. In Block No. 3-a containing 53,961 cu. yd., the 28 steel bents of the construction trestle were embedded and this

Fig. 27—Uncompleted Spillway No. 1 and Its Relative Location to Right Channel of Grand Discharge

work was completed middle of October 1924. Block No. 4 of 39,527 cu. yd., the last portion of power-house masonry, was completed in March 1925.

The power-house design called for a gantry crane running on top of the bulkhead and capable of lifting any of the 24 headgates provided for the penstock intakes. This crane has a capacity of 60 tons when the main hoist is operated and of five tons with the auxiliary hoist running; in addition two bracket hoists of five tons capacity, each, are provided for lowering and raising stop logs in front of the rack structure. Immediately after completion of Block 3-b, the gantry crane was

erected, being used for unloading and lifting each headgate on top of the bulkhead and transferring and lowering same to its respective position. Each 16-ft. by 22-ft. headgate weighs 108,000 lb. and railroad clearances permitted the gates to be shipped assembled in special flat cars. A view showing a headgate being lifted by the gantry crane above the special railroad car is shown on Fig. 26.

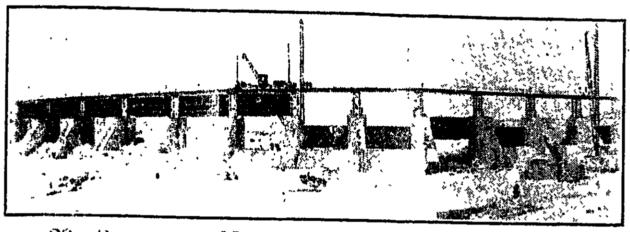

Fig. 28—Spillway No. 3 with Flood-Gates in Place

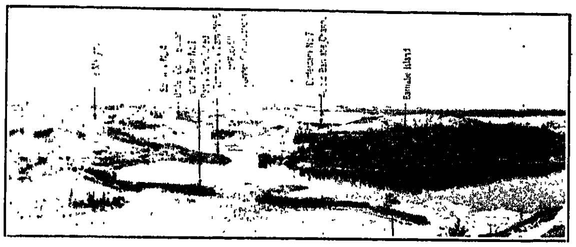

Fig. 29—Incomplete Temporary Dam No.6 of Rock-Fill Construction Between Barnabe Island and Isle Maligne, Above Spillway Site No. 4

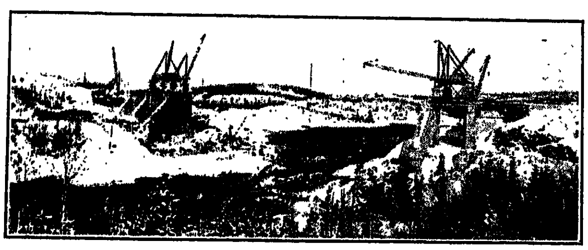

Fig. 30—Spillway No. 4 under Construction, Showing Traveler Cranes above Deep River Section

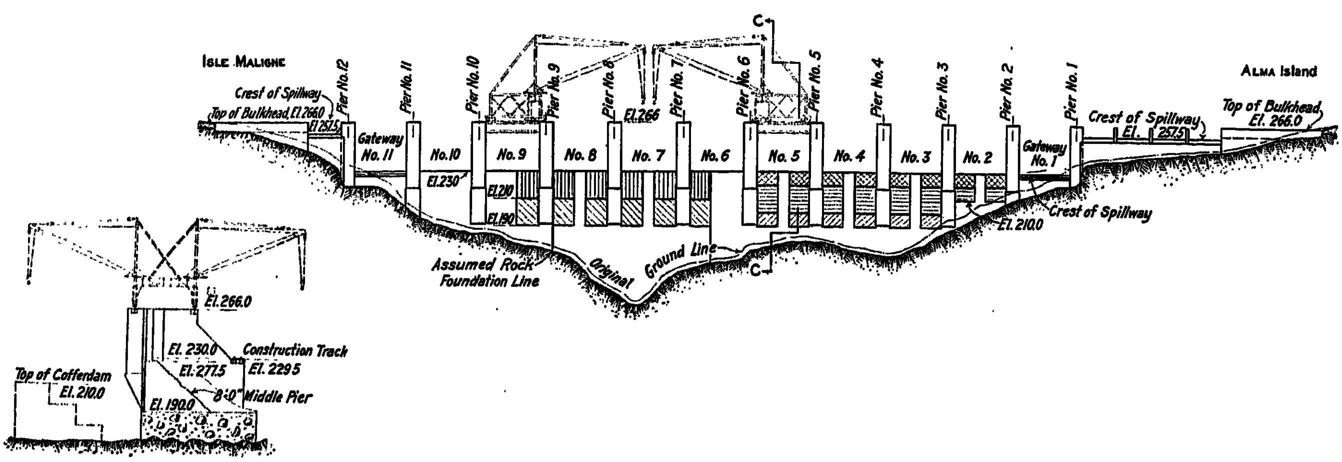

Fig. 31

Spillway Construction

The major part of Spillway No. 1, which is located on Isle Maligne and forms an extension of the power-house bulkhead at right angles to same, was built simultaneously with the power-house masonry. According to the program the right channel of the Grand Discharge at the head of Isle Maligne was closed up (as described later), on February 15, 1925; and the cofferdam No. 1 blown out so as to return the riverflow to the Left Channel and to discharge water through two of the turbines operating temporarily under a head of 70 ft. The gaps left in the masonry of Spillway No. 1 provided additional discharge capacity so that the normal flow of the Grand Discharge could be controlled during the critical construction period when the deep foundations for Spillway No. 4 were being unwatered and the masonry built up. Fig. 27 shows a view of the uncompleted Spillway No. 1 and its relative location to the Right Channel of the Grand Discharge.

Spillways No. 2 and 3 are located in natural depressions of Isle Maligne above the present highwater line, but below the final raised water level, and no special difficulties were encountered in the construction of these structures. Spillway No. 3 was completed in January 1925 and a view of same is shown on Fig. 28.

The final control of the Saguenay River and the raising of the water to the level of the Lake St. John was accomplished by the construction of Spillway No. 4 and this work called for the most careful planning, so as to complete the structure at the scheduled date. Due to the great depth and the swift current it was not found practical to build a cofferdam across the right channel of the Grand Discharge immediately above the site of Spillway No. 4 to hold back the entire stream. It was, therefore, decided to go further upstream and use the channel between Barnabe Island and Isle Maligne, where much more favorable conditions for the location of a temporary dam were encountered. This layout called, however, for the location of another cofferdam of 475 ft. length across the so-called Barnabe Channel between Barnabe and Alma Islands; but this channel was practically dry during the construction period and offered no difficulties. The temporary dam between Barnabe Island and Isle Maligne is marked No. 6 on the

general plan, Fig. 4, and it is of rock fill construction as shown on Fig. 29. This arrangement made it possible to divert the river flow into the left channel of the Grand Discharge after the cofferdam No. 1 at the head of Isle Maligne was blown out, and to discharge the water through the turbines operating under a reduced

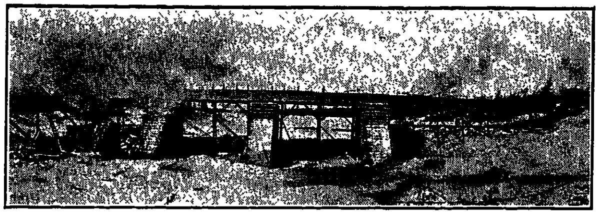

FIG. 32—SPILLWAY ACROSS RIGHT CHANNEL OF LITTLE DISCHARGE UNDER CONSTRUCTION

head of 70 ft. and through the temporary gaps left in Spillway No. 1, which accomplished the unwatering of the deep portion of the riverbed at the site of Spillway No. 4 for the construction of the masonry.

During the time that these two temporary dams were being built, concrete masonry was laid from both ends of the spillway site and steel runways placed on top of each of the completed masonry sections for the accommodation of two traveler cranes which were previously used in the construction of the power-house masonry. As shown in the illustration, Fig. 30, these two traveler cranes cover the entire deep river-bed area, which was unwatered. The method of building up the masonry for Spillway No. 4 and making the final closure is indicated on the profile, Fig. 31.

The spillways in the three channels of the Little Discharge at the outlet of Lake St. John were constructed during the winter season of 1924-1925 when only the small flow through the right channel was interfering with the work. A view showing the spillway across the right channel of the Little Discharge under construction is shown on Fig 32.

CONSTRUCTION OF EARTH DAM

The construction of the earth dam across the ravine near Spillway No. 4 on Alma Island was commenced in May, 1924. The foundation was cleared of all vegetable matter and the muskeg over-burden in the low part of the ravine removed by use of a hydraulic Monitor.

Suitable borrow pits for both the gravel dikes forming the outer thirds and the clay core of the fill were found within a short haul of the dam. The fill material was excavated with a 2 yd. steam shovel and loaded into 6 yd. dump cars made up in trains and transported by

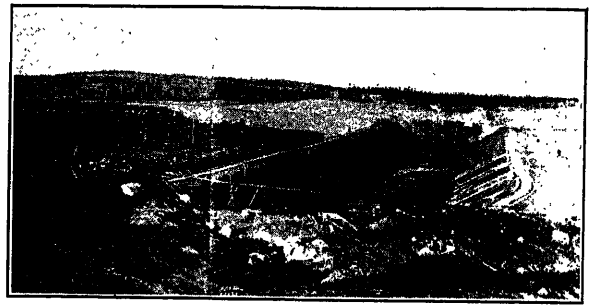

FIG. 33—EARTH DAM ACROSS RAVINE NEAR SPILLWAY NO. 4, UNDER CONSTRUCTION

locomotives to the upstream and downstream construction trestles as indicated on the typical section through the earth-fill, Fig. 20. The material was then dumped into wooden chutes placed at intervals and washed down into the pool between the two dikes by means of a hose pipe with the nozzle attached to a push car moved along the train track, as shown on the view, Fig. 33.

Work on the fill progressed until middle of December 1924, at which time the dam was 82 per cent complete and during freezing weather, occurring previous to this date, the areas of the fill on which no work was done were covered with tarpaulins and a series of steam pipes run underneath the cover.

Two-Phase, Five-Wire Distribution

Its Engineering and Economic Elements

BY P. H. CHASE[1]

Member, A. I. E. E.

***Synopsis:**—In view of the present trend toward three-phase secondary distribution involving, in some cases, a change from a two-phase system, an analysis of the engineering and economic elements of a two-phase system may be of value. The two-phase five-wire secondary system is examined in the light of fundamental requirements, such as service continuity, safety, standard voltages, flexibility, low cost, etc., and compared particularly with the three-phase, four-wire star system. Many of the advantages of the two-phase, five-wire system result from the diametrical connection of the two phases, from the inherent balance thereby obtained, and from the greater power carried per wire. An important advantage of the two-phase, five-wire system lies in the fact that single-phase, two-wire and three-wire loads and two-phase loads can be supplied at standard voltages from combination lighting and power secondaries and that new loads can be flexibly supplied through all stages of load growth. There are marked advantages from a construction and operating point of view in having ordinarily only two transformers in banks which supply two-phase secondaries from either two-phase or three-phase primaries. The two-phase, five-wire system has certain advantages as to metering and a comparison of the first cost and the annual cost of two-phase and three-phase motor installations, with wiring, shows small differences. The inherent cost differential between two- and three-phase secondaries with several types of primary systems is shown to be of such a small magnitude that the cost of change over from one type of system to another may over shadow the theoretical savings. Accordingly, with a relatively small inherent cost differential between the existent system and one having certain more or less proved advantages and disadvantages, the central station engineer must produce extremely strong arguments leading out of his local situation in order to justify a change from the existing system.*

* * * * *

ENGINEERING AND ECONOMIC ELEMENTS OF TWO-PHASE, FIVE-WIRE DISTRIBUTION

SINCE the inception of alternating current distribution, initially single-phase, there have been developed various schemes of polyphase distribution, the oldest of these being two-phase. For more than the past decade the change-over of distribution systems from two-phase to three-phase has been considerable. Usually the justification for the change-over of any distribution system lies in the particular combination of conditions which appertain to the territory and system in question. It may be unsafe to derive conclusions for any particular locality from those reached in another locality or from generalized or purely theoretical grounds.

The subject of the best type of distribution system has received intensive study by many electric companies and, in view of the present general trend to three-phase, it is felt that many of the features of a recent analysis of a system which still has two-phase distribution for its secondary and 2300-volt primary systems, will be of particular interest at this time.

For reasons which will be stated later, this paper gives consideration to secondary distribution suitable for the supply from the same mains of both light and power, with only such references to primary distribution as appear necessary for completeness. Distribution transformers, however, are treated as a part of the secondary system.

FUNDAMENTAL REQUIREMENTS

As a basis for further discussion, the fundamental requirements which must be met by any secondary distribution system, in order to meet the requirements of the customer with safety and economy, are outlined.

CUSTOMERS' AND UTILIZATION REQUIREMENTS

1. *Service Continuity.* A high degree of service continuity is required for all classes of service. This is afforded by various feeder, main and service combinations with their requisite protective equipment to afford the desired service insurance.

2. *Safety.* For a secondary distribution system, the requirement for safety is met according to present accepted ideas by a service voltage of approximately 115 from any wire to ground. Existing wiring standards and appliances allow this voltage to be handled safely for a wide diversity of applications.

3. *Standard Voltage.* The voltage for lamps has been standardized at 115 volts, with 120 volts as an allowable higher figure. The voltage for motors for combined light and power systems has been standardized at 110 and 220. Any system of distribution should conform to these basic voltage standards, established through years of experience and investigation.

4. *Voltage Regulation and Balance.* The inherent voltage regulation and balance of the secondary distribution system must be such that, with the usual voltage regulating equipment at the substation, the variations of service voltage, on all phases, and under all load conditions, will be kept within a value which will not impair illumination from lamps nor service from motors and appliances.

5. *Customers' Equipment and Wiring.* The type of distribution system should be such that the customers' equipment may be of standard, readily secured types and the wiring simple, efficient, and inexpensive.

1. Asst. Engr. Trans. and Distr., Philadelphia Electric Co., Philadelphia, Pa.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

Distribution System Requirements

1. Simplicity and Standardization. Separate or combination light and power service should be readily available from a single set of mains in order to take the maximum advantage of diversity between power and lighting loads. The distribution system should make use of standard equipment.

2. Load Balance. . The secondary distribution system should be readily adapted for service both to single-phase and to polyphase loads without material load unbalance between the phases.

3. Voltage Balance. The voltages of the different phases should remain balanced with respect to each other and to ground.

4. Adaptability to Physical Conditions. The distribution system should be equally adaptable to aerial or underground construction, to urban, suburban, and rural conditions, and to residential, commercial, and manufacturing districts.

5. Growth Adaptability. The secondary distribution system should be adaptable to all stages of load growth and load density, by means of additions and reinforcement, and with the minimum amount of reconstruction work at any stage. When the initial single-phase lighting load in residential areas eventually develops demands for polyphase service, the distribution system should be sufficiently flexible to render such service without extensive changes.

6. Investment. There should be the minimum investment in mains, services, transformers, meters, etc. The annual charges on the investment are usually the largest portion of the total annual cost of the secondary distribution system.

7. Power Losses. The secondary distribution system should have low power losses. This requirement is usually met when the requirements for voltage regulation are properly met.

8. Operating and Maintenance Costs. These costs should be low. They usually will be a fairly constant percentage of the investment.

Combined Light and Power Secondaries

The treatment of the subject throughout is based on the requirement of a system suitable for the supply of power and lighting, either single-phase or polyphase, from the same mains. A study of the economic advantages of combination mains, under assumed average conditions, points toward savings of the order of twenty per cent in the total annual cost (fixed charges and losses) of transformers, mains and services, as compared with separate transformers, mains and services for lighting and power. It is recognized that there are and will be many instances, particularly on aerial systems, where the use of separate lighting and power mains is necessary to prevent fluctuating power loads or the frequent starting of motors from impairing lighting service, and that on many systems the mileage of separate lighting and power mains greatly outweighs the mileage of combined mains.

The Two-Phase, Five-Wire Secondary System

The two-phase, five-wire secondary system is shown schematically in Fig. 1. The system is strictly a four-phase, five-wire *diametrical* system, with 115 volts to neutral. This *diametrical* connection is responsible for many advantages of the two-phase system, owing to the relative independence of the phases. If there is any virtue in the *number* of phases, surely such a four-phase system should compare favorably with the three-phase!

The two-phase, five-wire system has the following outstanding features:

1. Standard 115 volts for lamps and appliances.
2. Standard 115 or 230 volts for motors.
3. Voltage to ground balanced.
4. Load balanced.
5. Two transformers per bank.
6. Transformers or standard ratio and voltage.

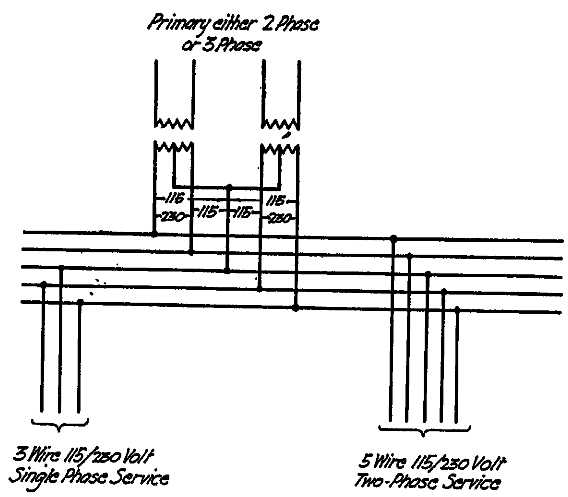

Fig. 1—Schematic Diagram of Connections for Two-Phase, Five-Wire Secondary Mains

Power per Wire

Considering the secondary distribution problem from a fundamental point of view, it will be realized that the condition which largely influences the economic advantages of one system over another is the *amount of power carried per wire.* Table I shows the amount of power per wire for various systems, all with 115-volt lamp voltage (E), assuming the same current (I), per

TABLE I
POWER PER WIRE

System	No. of Wires	Power Carried	Power Carried Per Wire
5-wire 2 ϕ 115-230 Volt...	5	4 $E\,I.$	4/5 $E\,I.$ = 0.80 $E\,I.$
4-wire 3 ϕ 115-199 Volt...	4	3 $E\,I.$	3/4 $E\,I.$ = 0.75 $E\,I.$
3-wire 1 ϕ 115-230 Volt...	3	2 $E\,I.$	2/3 $E\,I.$ = 0.667 $E\,I.$
2-wire 1 ϕ 115 Volt.......	2	1 $E\,I.$	1/2 $E\,I.$ = 0.50 $E\,I.$*

*Voltage regulation requirements will often operate to reduce this value as a comparative measure of power per wire.

wire and the same size wire. This comparison is based on equal voltage drop and therefore represents equivalent conditions for the distribution systems set forth.

The two-phase, five-wire system carries 6.6 per cent greater power per wire because, with 20 per cent more copper, it carries 33 ⅓ per cent more power than the three-phase, four-wire system having the same lamp voltage.

Inherent Advantages

Consider the inherent advantages of the two-phase, five-wire system. It has a fifth or neutral wire common to two *independent* 115-230-volt phases. A current in the neutral is caused only by a load unbalance on one side of a phase and this is ironed out in the transformers.

The neutral of the three-phase, four-wire star system, and also the transformers, will carry the resultant current due to any line—neutral load unbalance, and thus transfer the unbalance to the primary mains.

In a secondary distribution system, which ordinarily supplied both single-phase and polyphase loads of various types, there is inevitably unbalance between line and neutral loads and the effect of this upon the voltage balance of the system is inherently greater with the three-phase, four-wire star system than with the two-phase, five-wire system.

In the aerial plant, standard cross arms and racks can be utilized practically as efficiently by the two-phase, five-wire system as by the three-phase, four-wire system. Four wires utilize practically as much space as five wires and the four wires will have to be larger for the same amount of power carried under the same conditions.

In the underground plant the two-phase, five-wire system can make use of single conductor, two-conductor, three-conductor, or four-conductor cable as local duct, heating, loading, and service conditions warrant. Under many conditions, where it is economically preferable to install mains on two sides of a street, the two-phase, five-wire system affords a very flexible method of rendering service to loads largely single-phase on each side of such a street while still maintaining balanced load on the mains and phases.

With heavy load densities, the two-phase, five-wire system may readily be accommodated with one phase in each of two ducts. There is a consequent improvement in the transmission of heat from the cables and a decrease of the hazard to service from one phase in the event of a fault on the other phase. The three-phase, four-wire star system requires a voltage unbalancing arrangement of the cables if two ducts are used, unless all three phase conductors are in each duct.

Although local conditions will materially affect the handling of any underground system, it is felt that the two-phase, five-wire system has at least every advantage that the three-phase, four-wire has in best utilization of pole and duct space.

Comparison with Three-Phase, Four-Wire Star System

On the other hand, the three-phase, four-wire secondary system forces the distribution engineer into a choice of one of the following dilemmas:

1. A decrease in the motor service voltage to 199 volts nominal, with 115-volt lamp voltage. This may require the development of a new line of motors, or the de-rating or under-loading of the present line of 220-volt motors.
2. An increase in the lamp voltage to 133, thus bringing the motor service voltage to 230. This requires the introduction of a new line of lamps and appliances and a new line of distribution transformers, or the marked over-excitation of the present standard ratio transformers to give 133 volts, secondary. Also changes to present standards for substation equipment might be necessary.
3. A compromise raising the lamp voltage to around 125, and decreasing the motor voltage to around 216. This presents complications with lighting, appliance, and motor loads, as the compromise voltages are often high for lamps and appliances and materially, if not seriously, below the 230-volt standard for motor service.
4. The use of auto-transformers for stepping up the main voltage of 115-199 to 230 volts for motors. Under conditions where the lighting and power loads on a secondary main system are approximately equal, it would then be necessary to transform one-half of the load from 115-199 volts up to 133-230 volts by means of auto-transformers. Assuming an average connected load per power customer of 12.5 kv-a. in power, a fair average figure for an underground city area, it is estimated that the necessary auto-transformers (excluding installation costs) would cost more than twenty-five per cent of the entire investment in secondary mains, conduits, and manholes.

The service which on the two-phase, five-wire system would be three-wire, 115-230 volts, single-phase, on the three-phase, four-wire system must be either (a) two-wire, 115-volt, single-phase, or (b) three-wire, 115-199 volt, open "Y", or (c) three-phase, four-wire. Each of these solutions results in higher losses, poorer voltage regulation, unbalanced voltage, higher investment, singly or in combination.

The two-phase, five-wire system requires none of these compromises or sacrifices.

In this comparison, consideration has not been given to the various other three-phase secondary systems which do not have some of the disadvantages of the three-phase "Y"—connected system. They have other combinations of disadvantages which are summarized in "Alternating-Current, Low-Voltage Networks," Serial Report of the N. E. L. A., Publication No. 25-1.

Transformer Installations

For the purpose of this paper, the transformer installation for supplying the secondary mains from the

primary distribution system is considered as a part of the secondary system. In considering the transformer installation, it is fully as important that attention be directed to the construction and cost elements of the supporting structure for the transformers if on a pole, or to the manhole or vault if the transformers are subway type.

For the two-phase, five-wire, 115-230 volt secondary distribution system, the connections will be as shown in Figs. 2A and 2B for a two-phase primary and in Fig. 2C for a three-phase primary. In all of these combinations, for a given size bank, only two transformers, or a multiple of two, are required. For the three-phase, four-wire system, three transformers, or a multiple of three, are required. The use of polyphase transformer units is practicable for either two- or three-phase, but certain service and operating advantages are generally felt to be lost as compared with standard single-phase units.

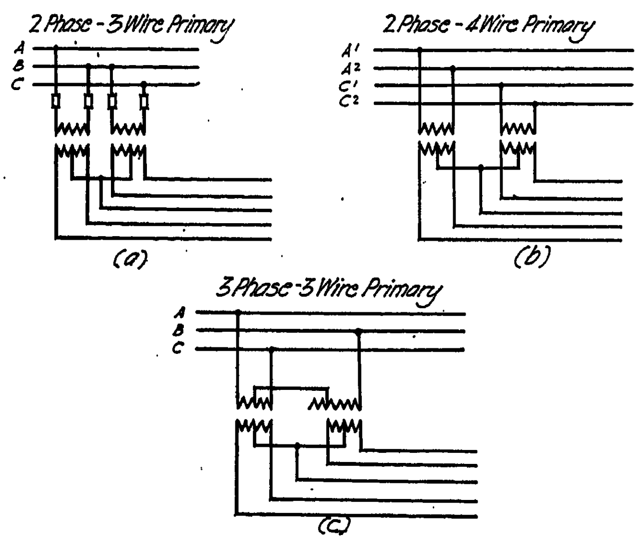

Fig. 2—Transformer Connections for Supplying Two-Phase, Five-Wire Secondary Mains from Two-Phase or Three-Phase Primary Circuits

There are marked advantages of having ordinarily two transformers per bank, with the two-phase system. On the aerial system, two units can be hung or arranged much more readily on a pole, with minimum decrease of working and climbing space for linemen. The mechanical advantages of a balanced transformer arrangement are apparent from Fig. 3. In the underground system, the use of two transformers is generally more economical of manhole space than the use of three transformers.

With two-phase primaries, standard transformers may be used in connection with the five-wire, two-phase secondary system. In the case of three-phase primaries, standard transformers with Scott taps are readily available at a slight increase in cost per unit.

The Scott connection and the combination of three-phase primaries with two-phase secondaries has sometimes been referred to as a hybrid scheme. Let us reserve judgment on this matter and base our conclusions upon common-sense engineering which must recognize construction, operating, and cost facts in addition to purely theoretical considerations.

In the modern Scott-connected transformer bank, there is a voltage unbalance of the order of one and one-half per cent, and a phase displacement of the order of one and one-half degree at eighty per cent power factor and much less at higher power factors. It can be readily shown that the per cent voltage unbalance caused

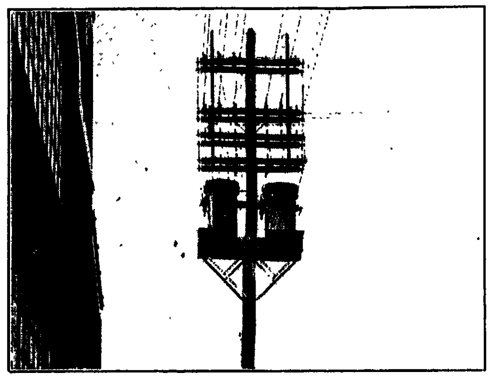

Fig. 3—Typical Construction—Two-Phase Transformer Bank

by 100 amperes per wire in approximately 500 feet of three-phase, 115-199 volt, four-wire No. 00 secondaries on a cross arm with standard 14½-inch spacing, is about the same as in a Scott-connected transformer bank having approximately five per cent impedance. There is no such inherent unbalance in two-phase, five-wire secondaries, when properly grouped, as the phases are separate and diametrically connected. The existence of current in the neutral will affect the voltage balance

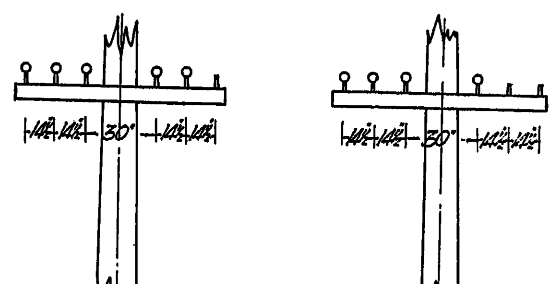

Fig. 4—Typical Arrangements of Secondary Mains on Cross Arms

of the three-phase system more seriously than that of the two-phase system.

See Fig. 4 for typical arrangements of wires on a cross arm.

For an economic statement of the cost differentials between aerial transformer banks of the standard type connected (1) two-phase to two-phase, (2) three-phase to two-phase, and (3) three-phase to three-phase, refer to Table II, which shows the comparative invest-

TABLE II
COMPARATIVE COST OF AERIAL TRANSFORMER INSTALLATIONS

Distribution System	Two-Phase—Two-Phase 2300-115/230 Volt	Three-Phase—Two-Phase 2300/3984-115/230 Volt	Three-Phase—Three-Phase 2300/3984-115/199 Volt
Number of Transformers per Bank	2	2	3
Voltage Rating of Transformers	2300–115/230	2300/4600–115/230.*	2300–115/230
150-Kv-a. transformer bank			
Total Investment	$1,096.00	$1,249.00	$1,353.00
Investment per kv-a	7.31	8.33	9.02
Total annual cost per kv-a.†	1.30	1.46	1.54
75-Kv-a. transformer bank			
Total Investment	678.00	783.00	774.00
Investment per kv-a	9.04	10.43	10.32
Total annual cost per kv-a.†	1.55	1.81	1.77
15-Kv-a. transformer bank			
Total Investment	286.00	339.00	337.00
Investment per kv-a	19.07	22.60	22.47
Total annual cost per kv-a.†	3.10	3.74	3.62

*Transformers are the type rated at 2300/4600-115/230 volts with taps for 3984 and 3444 volts and will give full rated output when Scott-connected.
†Total annual cost includes fixed charges on investment and cost of losses evaluated on increment cost basis.

ments and annual cost (fixed charges and cost of losses) estimated on a comparative basis.

It will be noted that the costs for the two-phase to two-phase banks are distinctly less than for the other connections, and that the three-phase to two-phase and the three-phase to three-phase costs run very close.

The three-phase to two-phase transformer costs are based on a relatively costly unit with taps suitable for either a 4000-or 4600-volt primary, while the three-phase to three-phase transformer costs are based on the standard 2300-volt unit. The three-phase to two-phase scheme therefore would allow a primary voltage of 4600 with consequent appreciable primary feeder savings as compared with 4000 volts.

It is therefore apparent that the two-phase, five-wire secondary system may be readily supplied economically from either two-phase or three-phase primaries.

Growth Adaptability

The two-phase, five-wire system, a combination of two single-phase, three-wire systems, possesses a high degree of flexibility and adaptability to all stages of load growth in a distribution system. Generally a district initially is supplied from single-phase, three-wire, 115-230-volt mains. As the district develops demands for polyphase supply to motors, this stage of development is readily met, especially with a two-phase primary system, by extensions of existing adjacent three-wire, single-phase secondaries supplied from different primary phases, thus affording a five-wire, two-phase service with the minimum additional investment in transformer installations and secondary mains, as diagrammatically shown in Fig. 5. If the primaries are three-phase, and as the amount of motor load grows, the transformers will be erected in banks of two, and part of the secondary mains will become five-wire. A three-wire service from mains originally three-wire, when the mains are increased to five-wire, requires no change in the motor or to the customer's wiring. There is, accordingly, a large degree of growth flexibility with the two-phase system. This is also a result of the inherent high capacity per wire, of particular value with continuing growth of load where theoretical considerations often are completely outweighed by the necessity of reasonably providing for development.

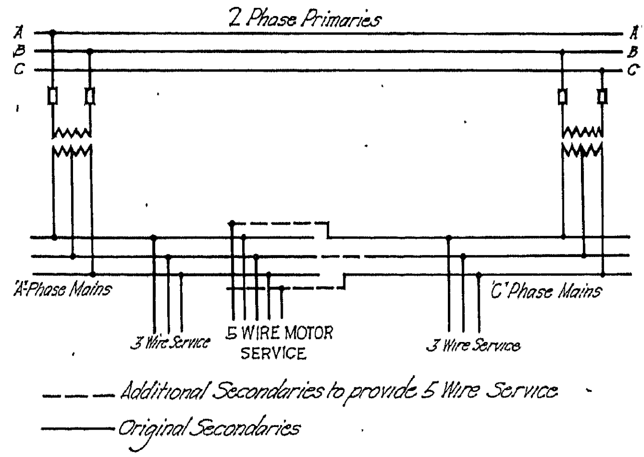

Fig. 5—Typical Method of Development for Two-Phase, Five-Wire System

With the three-phase, four-wire system, it is apparent that if a district is permitted to originally develop on a single-phase, three-wire basis, when it becomes necessary to provide for polyphase loads, the erection of two additional transformers will be necessary and also the conversion of the three-wire, single-phase secondary mains to four-wire, three-phase, unless such expedients as open "Y", three-wire mains are resorted to.

In such a transition it will be necessary to care for every three-wire, 115/230-volt, single-phase customers' service in one of the following ways:

1. Change to four-wire, three-phase. This requires running an additional wire, replacement of the single-phase meter by a polyphase meter, and changing the customers' wiring for a four-wire service.

2. Change to two-wire, 115-volt, single-phase. This may require larger service wires, and a change to a two-wire meter. In many situations, as with electric range loads, such a change would greatly increase the voltage

drop or necessitate a separate service to maintain the proper voltage on lamps.

3. Change to three-wire, three-phase 115/199-volt open-"Y." This will require a second meter or replacement of three-wire meters by polyphase metering. The service voltage will be unbalanced by the current in the neutral and the losses will be much higher in the service wires and the main neutral.

If the mains are made four-wire initially, there is the initial extra investment in transformers and mains, and the unavoidable choice must be made between the above three types of services, each with its disadvantages as compared with the three-wire, 115/230-volt service. The importance of this situation is apparent from the relatively large proportion of services which are single-phase on most systems.

Although many situations of the above nature are being solved by the use of separate mains for light and power, the substantial savings in combined mains, it is felt, compel consideration of any secondary system from the point of view of its suitability to the ultimate supply of practically all types of small and medium size loads from the same mains. To this end stricter requirements in motor starting currents and improved methods of distribution should be considered.

Relation of Primary Distribution

Although this paper deals particularly with secondary distribution, there are a number of related points in connection with primary distribution which should be borne in mind.

Primary distribution is affected by a large number of variable elements, including:

1. The size and spacing of substations.
2. Length and capacity of feeders.
3. Density and character of load, both primary and secondary.
4. Type of system, radial, parallel, loop, network feed, aerial, underground, etc.
5. Configuration of streets.
6. Tree conditions.
7. Governmental restrictions relating to voltage, aerial construction, tree trimming, etc.
8. Contractual restrictions such as those relating to voltage, type of construction, etc., in joint pole use.

Again, as in the case of secondary distribution, the influence of local conditions upon the type of system is very great, and it is often difficult to make comparisons between systems in different localities.

It is desired in connection with primary distribution to call attention to one factor which in recent years has become of increasing importance in this involved problem. Pressure of increasing-load demands by the public and the desire for greater economies by central station engineers in the use of copper and the saving of losses, have of late greatly accelerated the increase from 2300 to 4000 volts, to 6600 and 11,000 volts, and to 13,200 and 22,000 volts. The economies have resulted, not particularly from any change in the number of phases as such, but from *the higher voltage*. This voltage increase has progressed in a way undreamed of twenty, or even ten, years ago.

Improvements in insulators, protective devices, and all materials used in the distribution plant have resulted in gratifying operating experience and have practically wiped out the older, natural distrust of voltages higher than 4000 volts. The great economic savings of these high voltages therefore have been taken advantage of, in recent years, by many companies.

The situation in some instances is that of a superposition of a primary distribution system with voltages such as 11,000 and 13,200, over the present 2300-or 4000-volt primaries. There is either direct transformation to the secondary mains or utilization of the present 2300- or 4000-volt primaries, merely as an intermediate short-haul facility.

Thus, this tendency in primary distribution, where prompted by local conditions, appears to be toward what may be called super-distribution circuits, often at generated voltage, with consequent lower substation costs from the omission of transformers and with lower losses, instead of toward a moderate change of voltage such as from 2300 to 4000 volts.

The strength of this idea lies in the probability that a moderate increase in the primary voltage, so often resulting in marked theoretical economies in primary copper investment and losses, may require expensive reconstruction of substations, distribution plant, and, often more serious, costly change-over of primary customers' installations. The additional investment, power losses, operating and maintenance costs of the duplicate plant during a long transition period require very careful consideration in order to avoid a long postponement of actual net savings from the change. Further, during the change-over, the load conditions may have so changed that the primary system has become inadequate to meet the new conditions and another increase in voltage may be required. A glance backward over the history of power distribution should constitute a warning as to the possible futility of taking too early what may later stand out as only a make-shift economic step. These statements apply with equal force to secondary distribution.

Metering, Motors, and Customers' Installations

In the two-phase, five-wire system, single-phase service loops generally are two-wire for loads up to a prescribed figure and three-wire for loads in excess. For the former range of loads, which are representative of small residential consumers, the metering is identical, whether from a two-phase or a three-phase system. For the latter range of loads, the metering costs involve a comparison between a three-wire, single-phase meter for the two-phase system and polyphase metering or raising the limit for two-wire services for the three-phase system. The cost of a three-wire, single-phase

meter installation is slightly lower than the cost of a two-wire, single-phase meter installation of the same kilovolt-ampere capacity and is very much less than any type of polyphase metering now in use.

For metering polyphase power load, there is practically no difference in the meter costs for two-phase or three-phase.

A study of metering costs for a typical system including all types of metering indicates that the total investment in metering equipment would be about fifteen per cent greater for the three-phase system than for the two-phase system.

The cost of two-phase motors of the usual voltage and speed ratings is identical with that of three-phase motors. In some cases there is a slight additional cost for the starting compensators for two-phase motors. The two-phase motor at rated full load has an efficiency slightly less than the three-phase motor, for the usual voltage and speed ratings, which is of the order of one per cent for sizes up to about 25 h. p. and less for larger sizes. This difference is due partly to the slightly less efficient coil design for two-phase and partly to the fact that most manufacturers design parts which are interchangeable for two-phase and three-phase motors but which are not always quite the most efficient design for two-phase motors. The torque characteristics of the two-phase motor are as good as, if not better than, the three-phase motor, according to manufacturer's rating sheets. The effect of reduced voltage under some of the three-phase, four-wire schemes is to materially decrease the pull-out torque of the standard 220-volt motor.

The two-phase, five-wire system uses standard motors of standard 220-volt rating while the three-phase, four-wire system with 115/199-volt mains requires the development of a new line of 199-volt motors or the material de-rating of the present line of motors,(unless auto-transformers are resorted to), thus resulting in a distinctly unfavorable situation.

It has been claimed that the two-phase, five-wire system, which for motor service requires four wires as against three wires for the three-phase system, requires more expensive wiring. Comparative estimates prepared for motor sizes from 10 to 50 h. p., both three-phase, three-wire, and two-phase, four-wire, show differences of the order of one per cent, in some cases in favor of the two-phase and in some cases in favor of the three-phase.

These figures are based on 220-volt motors for both two-phase and three-phase. If the cost of the necessary auto-transformers were included with the three-phase motor or allowance made for the additional cost of a 199-volt motor with its wiring, to be supplied from 115/199-volt mains, the comparison would be favorable to the two-phase system.

A typical example will serve to illustrate the small magnitude of the difference of the losses in a two-phase motor with its wiring, and a three-phase motor with its wiring. Assuming a 10-h. p. motor first three-phase, 199-volt, and second, two-phase 220-volt, the difference in the value of losses for a period of operation of 2000 hours per year at an average of 75 per cent full load, at two cents per kw-hr., amounts to less than $2.75 per year in favor of the three-phase motor. This is less than one per cent of the value of the power input to the motor and is of such a small order of magnitude as compared with the usual variables of installation costs, over-sized motors, operating hours, length of wiring, etc., that it should not be considered important in the choice between a three-phase and a two-phase system.

Thus, if the three-phase motor is supplied through an auto-transformer from 115/199-volt mains or the motor is a 199-volt, the carrying charges on the extra investment in motor and wiring, or in auto-transformers, and the additional losses would throw the saving in favor of two-phase.

With a three-phase motor service voltage of 199 volts, the voltage drop in the wiring in many cases will result in a materially lower voltage at the motor terminals. Using the same per cent in this case as is nominally allowed between a 230-volt service and a 220-volt motor, the rated motor voltage should be 191 volts.

For lighting loads there are difficulties with the three-phase, four-wire system which do not exist with the two-phase, five-wire system. The main feeds to panel boards must be either four-wire, 115/199-volt, with a more expensive four-wire panel board, or they must be three-wire, open "Y", with the losses in the wiring approximately 50 per cent higher than with three-wire, single-phase feeds. There will also be a voltage unbalance due to the assymetrical phase relations of the current in the wires, its magnitude depending upon the power factor, and an increased voltage drop. Single-phase, three-wire feeds from a two-phase, five-wire system do not have these disadvantages.

General Economic Features of Two-Phase, Five-Wire Distribution

It has been seen from the foregoing discussion that the character of service from the two-phase, five-wire secondary system, so far as it meets the various distribution, engineering, and customers' requirements, has no serious disadvantages as compared with the three-phase system, and in many respects has outstanding advantages.

Therefore, under such conditions, it will be of interest to compare the overall economies of the two systems, in order to determine the probable magnitude of the cost differential and the importance of this factor as compared with costs of change-over and other factors not readily measured in dollars.

For this measurement of their relative inherent economic standing, an analysis of several combinations of underground primary and secondary distribution systems with two- and three-phase secondaries, was made

TABLE III

TOTAL ANNUAL COST OF NETWORK SYSTEMS

Excluding Operation and Maintenance

No.	Primary	Secondary	Substation	Feeders	Transformers	Mains	Total
		Range of Primary Voltages: 2300–4600 Volts					
1	2-ph. 3-wire 2300/3252-volt	4-ph. 5-wire 115/230-volt	$216,050	$92,900	$118,200	$108,760	$535,910
2	3-ph. 4-wire 2300/3984-volt	3-ph. 4-wire 133/230-volt	211,700	67,120	122,800	97,680	499,300
3	3-ph. 4-wire 2300/3984-volt	3-ph. 4-wire 115/199-volt	211,700	67,120	122,800	101,820	503,440
4	3-ph. 3-wire 300/3984-volt	4-ph. 5-wire 115/230-volt	211,700	67,120	125,300	108,760	512,880
5	4-ph. 5-wire 2300/4600-volt	4-ph. 5-wire 115/230-volt	218,900	93,580	119,600	108,760	540,840
		13,200-Volt Primary					
1	3-ph. 3-wire 13,200-volt	3-ph. 4-wire 133/230-volt	122,900	44,100	147,900	97,680	412,580
2	3-ph. 3-wire 13,200-volt	3-ph. 4-wire 115/199-volt	122,900	44,100	147,900	101,820	416,720
3	3-ph. 3-wire 13,200-volt	4-ph. 5-wire 115/230-volt	122,900	44,100	153,800	108,760	420,560

on a comparative basis, as applied to a definite load and area. Each type of system was given the full benefit of its most economical design. Tables III and IV summarize the results of this investigation. Table III shows the total annual cost of an underground network plant, including in the total annual cost the fixed charges on the investment and the value of the energy losses in the various parts. Table IV shows the investment in the various parts of the plant for the systems considered.

The cost of the mains for the different types of secondary systems does not vary widely, being of the order of magnitude of 10 per cent for the total annual main cost and for the investment in mains. For the transformers and mains together, the cost variation is even smaller in either the 2300-to 4600-volt primary range or with 13,200-volt primary. It will also be noted that the investment in mains is only about 20 per cent of the total plant investment in substations, feeders, transformers, and secondary mains. The tables also indicate that the part of the problem demanding further engineering attention is that pertaining to the primary, where higher voltages than 4600 show marked possible economies.

These analyses, although on a comparable basis, cannot evaluate in dollars many of the advantages of the

TABLE IV

INVESTMENT FOR NETWORK SYSTEMS

No.	Primary	Secondary	Substation	Feeders	Transformers	Mains	Total
		Range of Primary Voltages: 2300-4600 Volts					
1	2-ph. 3-wire 2300/3252-volt	4-ph. 5-wire 115/230-volt	$1,695,000	$792,000	$773,000	$938,300	$4,198,300
2	3-ph. 4-wire 2300/3984-volt	3-ph. 4-wire 133/230-volt	1,665,900	584,000	790,000	851,700	3,891,600
3	3-ph. 4-wire 2300/3984-volt	3-ph. 4-wire 115/199-volt	1,665,900	584,000	790,000	880,700	3,920,600
4	3-ph. 3-wire 2300/3984-volt	4-ph. 5-wire 115/230-volt	1,665,900	584,000	830,000	938,300	4,018,200
5	4-ph. 5-wire 2300/4600-volt	4-ph. 5-wire 115/230-volt	1,728,600	816,000	783,000	938,300	4,265,900
		13,200-Volt Primary					
1	3-ph. 3-wire 13,200-volt	3-ph. 4-wire 133/230-volt	1,023,000	402,000	954,200	851,700	3,230,900
2	3-ph. 3-wire 13,200-volt	3-ph. 4-wire 115/199-volt	1,023,000	402,000	954,200	880,700	3,259,900
3	3-ph. 3-wire 13,200-volt	4-ph. 5-wire 115/230-volt	1,023,000	402,000	1,019,000	938,300	3,382,300

two-phase, five-wire secondary system, which it is felt fairly match or outweigh the advantages of the three-phase, four-wire system for a densely loaded underground district.

Other figures which have been prepared lead to the same conclusions for the aerial system.

In the change-over of a d-c. network system to alternating current, it may be possible in the choice of the best a-c. system to consider the particular district as a separate problem, because the d-c. equipment will require replacement in any event. However, existing a-c. areas, both aerial and underground, which eventually will form part of the network area, and the need for standardization on one universal type of system will have a necessarily large, if not a predominating, influence on the decision.

Conclusion

With an existing a-c. system of any type, now rendering adequate service satisfactory to the consumers, and economical and adaptable to growth, the cost of change-over to any other system becomes a very important factor. In addition to those costs of a fairly determinable nature, there are others less susceptible of accurate prediction, such as the extra cost of operating two types of systems during the change-over and the effect of a longer or shorter change-over period.

Accordingly, with a relatively small inherent cost differential between the existing system and one having certain more or less proved advantages and some known disadvantages, the central station engineer must have extremely strong arguments leading out of his local situation in order to justfy a change from the existing system.

In conclusion, I desire to express my appreciation of the assistance and suggestions rendered by a number of my associates.

Discussion

A. H. Kehoe: If one first reads the conclusions of this paper (which are carefully drawn), he is apt to take a more generous view of the statements which appear earlier in the text. For instance, the emphasis on cost of making system changes is well placed.

Concerning the subject of a combined light and power system, we believe it can be demonstrated that starting *de novo*, the advantages and disadvantages of two- and three-phase will be so nearly balanced that any difference in cost is well within the accuracy of the original assumptions. In practical applications, however, there are three factors which should always receive consideration:

First, the sources of power and transmission, if these exist, are three-phase, and the country in general is, and will doubtless remain, on a three-phase basis for general polyphase utilization. This necessitates the use of special polyphase devices where two-phase, five-wire distribution is adopted.

Second, the three-phase, four-wire system requires an odd voltage according to present standards, for either one or both of the single-phase or polyphase loads.

In addition to the above the adaptation of existing equipment for combined light and power distribution should be accomplished with minimum cost.

In the two locations where two-phase, five-wire light and power distribution systems are contemplated at the present time, there is little doubt that due to the existence of a two-phase system it is much cheaper to retain the two-phase than it is to change over to a three-phase, four-wire supply. These two locations are the exceptions however, and the same elements of cost have the opposite effect on the typical systems of the country which, of course, are three-phase. The systems adopting two-phase, five-wire will perpetuate non-standard polyphase equipment while the three-phase, four-wire systems will force certain rating compromises but should be able to utilize standard equipment, both existing and new. Just what this later effect will be, depends upon what voltages are selected. The author has everywhere in his paper ignored 120/208-volt, three-phase, four-wire which seems to me to affect seriously some of his deductions. This voltage appears to be the only compromise which can be adopted that will allow all existing standard equipment to be utilized successfully.

I have noted such a large number of exceptions in the detailed text of the paper that it would be impossible to comment upon all of them. I shall therefore deal with but four or five matters which seem to be of most importance.

Carrying the matter of "power per wire" to its absurd conclusion, the system proposed by the author is but 80 per cent efficient. However, we all must appreciate the tremendous advantage of the simple three-wire, d-c. system. It is the obvious necessity of generating and transmitting polyphase as well as having some polyphase utilization, that makes it necessary to even consider the complications of going to four-wire or five-wire systems to obtain balanced loading on a single system. It may be well, however, to bring out the fact that neutral wires have to be sufficient to carry the unbalance which depends upon the utilization equipment and not the type of system.

Under "inherent advantages" on the third page, the question of accommodating phases in separate ducts introduces the element of polyphase motors acting as phase converters while running single-phase. This condition at times of secondary short circuits is one of the problems with which there has not been sufficient operating experience to obtain a positive solution. It seems certain, however, that separate polyphase secondary cables in separate ducts will not improve the hazard which already exists in the matter.

On the fourth page, mention is made of polyphase transformer units. I wish to emphasize this, as in the writer's opinion we have scarcely begun the economic use of polyphase units that will come automatically as soon as the light and power systems are established in a reasonable number of places.

With reference to the Scott connection as a hybrid scheme, some of us who do not agree with the author believe we are basing our conclusion upon common-sense engineering. Regarding the unbalance due to Scott transformation, it should be realized that this occurs at the source of the supply while the voltage unbalance, set up in the case of secondaries, occurs at the end points of the line. It is doubtful whether any quantity of secondary light and power mains would be installed in the country with spacing as indicated on the fourth page. Our telephone friends might be interested.

In discussing primary voltages higher than 4000 volts, the author speaks of the older natural distrust as if the distribution conditions of the last ten years were not justified. We believe that it has just been demonstrated by a different design of system that it is possible to distribute at the generator voltage and any distrust which has existed in the past was a very proper one for the systems then in use.

R. A. Paine: In the first part of the paper a discussion is given of the fundamental requirements which any distribution

system must meet. The field has been covered in a very complete manner but there appear to be several points on which there is disagreement as to just how nearly certain types of distribution systems meet all of the requirements. A comparison has been made in the paper principally between two types of secondary distribution systems, *i. e.*, (1) two-phase, five-wire system, and (2) three-phase, four-wire system.

Throughout the paper the voltage of the three-phase, four-wire system has been stressed as being 115/199 volts. This apparently has been done for a purpose. Since the two-phase, five-wire system considered was 115/230 volts, the voltage to ground for the three-phase system for sake of convenience was also selected as 115 volts. We believe the comparison should be made either on the basis of a two-phase system voltage of 120/240 with a corresponding three-phase voltage of 120/208, or on the basis of a two-phase system voltage of 115/230 and a three-phase system voltage of 120/208.

The comparison should be made in accordance with one of the above stated methods because in all cases where a distribution voltage of 115/230 exists, it is entirely possible by proper notification to customers, after authority has been received from the public service commission, to raise the voltage to 120/240. If a particular company does not desire to raise its voltage to the above standard, the benefits to be derived from so doing, such as decreased losses on account of the higher voltage, etc., should not be withheld in making a comparative economic study of the two systems.

Accepting this as fact,—namely, that with a three-phase, four-wire system the distribution voltage would be 120/208 volts if operation similar to present practise is followed out,—it should be possible to maintain practically this voltage at the customer's service. Hence, if the motor wiring is at all suitable, voltage at the motor should be well above 200 volts, three-phase. When standard 220-volt motors are used, and assuming the motor terminal voltage to be 205 volts, the efficiency at full load is not materially less than when the motor is operated at 220 volts. The efficiencies at fractional loads will be somewhat higher when the motor is operated below 220 volts. The pull-out torque will be reduced to roughly 86.6 per cent of the pull-out torque at 220 volts. Assuming the pull-out torque value to be 250 per cent of normal full-load torque, and that the motor is operated at rated voltage, the new figure will be approximately 217 per cent which should prove ample.

There should be no trouble in operating three-phase motors at 120/208 volts, since the heating of the motors should not be materially increased and because the majority of motors operate at somewhat less than their rating. Further advantage may also be taken due to the fact that the motors are rated on a 40-deg., ambient temperature basis and the usual ambient temperatures encountered are somewhat lower than this figure.

A study of the performance curves of present-day motors indicates that motors rated at 220 volts are apparently designed for a voltage between 200 and 210. Unless the design of motors is changed, cases where 220-volt motors will not operate entirely satisfactorily at 205 volts will be very rare.

The "Outstanding Features" of a two-phase, five-wire distribution system which have been outlined in the paper are, with two exceptions, equally applicable to a three-phase, four-wire system. Of these exceptions, one (namely, use of 208 volts three-phase as the motor voltage for the three-phase system as against 230 volts for motor voltage of the two-phase system) is not a serious disadvantage, if it can be called a disadvantage at all. This is on account of the reasons given above. The second exception (namely, the use of three transformers per bank for the three-phase system instead of two transformers per bank for the two-phase system) is, of course, somewhat of a handicap in the case of the three-phase system. Usually, however, there is considerable flexibility possible in the manner in which loads are cared for in the three-phase system so that this disadvantage can probably be compensated for.

Voltage unbalances which take place or are inherent in a three-phase, four-wire system are in all cases extremely small and practically no difficulty is encountered from an operating standpoint in satisfactorily compensating for them.

This suggestion is made in the paper that a common set of secondary mains be used for supplying both power and lighting loads. This practise is quite commonly followed by several companies at the present time. From the economical point of view, it is very desirable and works out very nicely in practise except in cases where extremely severe conditions are imposed upon the circuits by some types of power load which necessitate separate sets of mains for the power and lighting loads. Stress has been placed upon the saving in the capacity of mains due to diversity between lighting loads and power loads. Any saving due to diversity of load in this part of the system will be exceedingly small as a rule. While the value of diversity is exceedingly great it is felt that the actual diversity existing in the part of the system being discussed in this paper is generally greatly over-estimated. The character of the service which must be rendered has a far greater effect upon the design of the secondary mains than does diversity. Any advantage which might exist in certain special cases may be realized equally well for either the two-phase or three-phase distribution system.

An advantage is claimed for the two-phase, five-wire system, particularly in underground districts, in that multiple-conductor cable can be more readily utilized than with the three-phase system. While this might be an advantage in some cases it would be obtained at the expense of lower quality of service due to the fact that any trouble in the cable would affect a larger number of customers than if single conductor cable were used. The three-phase, four-wire system has a very distinct advantage over the two-phase, five-wire system in underground districts in that for the same power to be transmitted, four single-conductor cables of somewhat larger size can be installed instead of five single-conductor cables with considerable economy in investment and duct space occupied.

In the case where secondary circuits are extended into new territory, a three-wire, open-Y, 120/208-volt installation may initially be made. This arrangement is the equivalent of installing a three-wire, 115/230-volt, secondary line in the two-phase system. If later, polyphase service is required, it is necessary only to run an additional wire or cable, while with the two-phase system to render polyphase service two additional wires must be installed.

Referring to Table No. 4, it is seen that the total annual cost of the three-phase, four-wire primary, three-phase, four-wire secondary system is approximately 7 per cent less than the two-phase systems shown as Nos. 1 and 2 in the same table. This amounts to upwards of $30,000 annually. Capitalizing this saving at 12½ per cent the above figures correspond to an investment saving of about a quarter of a million dollars.

With regard to all of the points at issue, it may be confidently stated that the three-phase, four-wire distribution system with voltages 120/208 is quite able to hold its own with the two-phase, five-wire system with voltages 115/230 and has the decided advantage of using recognized standard apparatus, including three-phase motors. It is very likely that future developments and refinements will benefit the three-phase system to a greater extent than they would benefit the two-phase system since developments are usually made for the benefit of the majority.

In discussing a common system of a-c. secondary mains for both lights and motors, the author suggests stricter requirements in motor-starting currents. If lower starting currents than now exist are a requisite for any particular type of distribution system, all factors entering into the problem should be carefully evalu-

ated as, in general, the larger the permissible starting current, the lower the cost of the motors. In Brooklyn the use of a combined lighting and power secondary is practically universal and has been for a great many years. We have made our motor-starting current requirements very broad and have found that we have encouraged the use of electricity as a power source, by lowering the customer's initial investment. Any motor installation conforming with the 1923 N. E. L. A. Motor Rules is accepted. In addition, we shall accept motor installations having starting currents in excess of these rules, provided no objectional voltage fluctuations are experienced by other customers connected to the same secondary, as the customer having the large starting current. We believe that all forward looking engineers should plan and design their distribution systems so that any motor installation meeting the requirements of the 1923 N. E. L. A. Rules is an acceptable installation. When distribution systems are brought up to this standard we may be able to lower still further the initial costs of motor installations by broadening the present motor rules.

H. R. Woodrow: Mr. Chase's paper attacks the problem from the standpoint of the existing system and for a new layout he is in agreement that the two-phase system is more costly than the three-phase.

In attacking a remodeling problem for a rapidly growing system, I should like first to see how we should develop the system without limitations of existing equipment and then determine how the new layout can be worked into the existing plant or the existing plant worked into the new system, for in a 10- to 12-year period the existing equipment is only one-third of new additions required.

The statement is made in the paper, "If there is any virtue in the number of phases surely it should compare favorably with the three-phase." It would seem to me that this should represent an indirect function in place of a direct one, as the best system, if it gives the same economy, would be a single-wire system.

Referring to the second page, the number of wires per customer would, in many cases, more nearly represent the cost than the power carried per wire, as in the majority of cases the limitation is determined by the minimum size of wire which is practical from the mechanical strength standpoint. The four-phase, five-wire system has 33⅓ per cent more current-carrying wires per service than the three-phase, four-wire system.

Referring to the tabulation under "Feeders" and the comparison of the three-phase, 2300/3900-volt system with four-phase, the four-phase distribution is 40 per cent more expensive than the three-phase. In the "Mains" the annual cost of the four-phase, five-wire system is represented as 7 per cent greater than the three-phase, four-wire system and the transformers under the 2300-volt heading show an increase in cost of the four-phase, five-wire system as 4 per cent. In other words, the tabulation shows an advantage of from 5 to 10 per cent in annual cost for the three-phase, four-wire system in comparison to the four-phase, five-wire, which is in agreement with the general studies we have made.

The only conclusion to which I can come from the study of the new system is that the central station is required to spend from 4 to 10 per cent more for the four-phase, five-wire system than the three-phase, four-wire without increased economy, and that the customer is required to spend more money to take the service and have increased losses in his system. This condition would naturally produce a rapidly increasing demand for three-phase equipment with a reducing demand for two-phase, and therefore the complications of the two-phase system would become more and more involved each year.

Although these factors favoring the three-phase, four-wire system may not in some cases justify the expense of changing over an existing plant, I do think these factors should be very carefully weighed before definitely perpetuating an inferior system.

P. H. Adams: I think Mr. Woodrow overlooked the fact that Mr. Chase's five-wire circuit is made up of two two-phase, three-wire circuits, using a common neutral. In his comparison of costs, he forgot the fact that each customer is connected as a two-phase, three-wire customer.

I agree with Mr. Woodrow in his criticism of the five-wire, two-phase system as a continuation of something that is inherently bad, and that we should tackle the problem of changing from two-phase, 2400 volts to a higher voltage by considering the existing system as something that will be comparatively small as contrasted with the system five or ten years hence.

We had the same problem in New Jersey, and while our system is a four-wire, two-phase instead of a three-wire, two-phase, as the one with which Mr. Chase has dealt, when we made our change we chose the 4150-volt, three-phase, four-wire system. I think we were right in making this decision, as our system is growing rapidly and in five years we expect to have a system at least four or five times as large as that which existed when we started the change.

L. T. Robinson: I think there is another side to this question of distribution systems that should not be overlooked; that is, that systems and convenience in the arrangement of systems is not everything; you should do as much as you can to make it possible to employ standard apparatus and have conditions under which the apparatus must be used as uniform as possible.

The question of lamp voltages, transformer ratios, the torque and heating of motors, their starting current and efficiency are all involved. While the idea that standard apparatus may be used on both the four-wire, three-phase, and five-wire, quarter-phase systems runs through the presentation and discussion, you must recognize the fact that to the difficulties always present due to the regulation of systems for voltage and frequency, there will be added the conditions of having to meet variable base voltages.

Possibly the apparatus can be successfully made to cover the base-voltage range required as well as the variations found in practise, but it will be more difficult for the designers, and I feel, to some extent,—I can't say off-hand to what extent—it is going to make the apparatus more bulky and more expensive.

H. Richter: Throughout the paper there are references indicating that the author intended his arguments to apply particularly to areas that now have two-phase distribution, where a combined light and power secondary system is contemplated and it is necessary to solve the involved problem of choosing the type of system best suited to those particular areas. I wish to emphasize the importance of confining the meaning of the paper to such two-phase systems and of not considering it to apply to systems that are now three-phase or to entirely new distribution layouts which may be started in the future.

In my estimation the paper refers almost exclusively to underground and overhead secondary network systems and not to purely radial systems. The idea of extensively using the combined light and power secondary system has been entertained seriously only since the advent of what might be called the latest type of underground a-c. low-voltage network; that is, where a number of primary feeders supply in common an interconnected secondary network. Isolated cases where the combined scheme has been tried out on typical radial systems are known, but the majority of companies have kept away from it because, unless an excessive expenditure is made, there is likely to be winking of lights when motors are started and burnout of polyphase motors due to insufficient voltage. It should therefore be recognized that the paper applies only to network systems. The inclusion of overhead secondary systems in the analysis is sound, I believe, for the surprisingly rapid spread of the network idea indicates that in the future, ten to twenty years from now, practically all

of the underground systems of the cities will not only be networked, but a good portion of the overhead systems will also have networks, possibly in a simpler form and using simpler apparatus.

The comparison has been confirmed to the 115/199-volt three-phase system. I should like to forestall consideration of the 110/190-volt and 125/216-volt, three-phase systems in the discussion. Satisfactory operation of polyphase apparatus on 190 volts would entail so great an expenditure, both in taking care of existing installations and development of new apparatus, that 110/190 volts is practically out of the running. 125/216 volts is also more or less out of the picture, partly because 125 volts is not even a recognized exception as a lamp standard, and partly due to the excessive burden that would be thrown on the industry as a whole by the necessity of developing new lines for the many types of apparatus now standardized at 115 volts or considered satisfactory for operation on 120 volts. The comparison of two-phase with three-phase should therefore be confined to 115/199, or possibly 120/208 volts for the three-phase systems.

Wherever low-voltage networks are being installed or planned, the companies are figuring on employing three-phase for the primaries, either at potentials in the class below 5000 volts or at some such higher voltage as 13,200. Even the two companies that have indicated a leaning towards two-phase secondary distribution have expressed the intention of employing 13,200-volt, three-phase feeders for future growth. Consequently, we must keep in mind, as of five or ten years from now, only the comparison of the Scott connection with its attendant disadvantages, for transformer banks serving two-phase as against the straight step-down for three-phase distribution.

While it is true that in the past there has been a general tendency to avoid such high primary voltages as 13,200 for miscellaneous distribution in cities, it is also a fact that this is no longer the case. The change in sentiment has been brought about almost entirely by the ability to employ a simple system of distribution in which all primary protective and sectionalizing devices are eliminated, and the only protective apparatus on the distribution system is a low-voltage device of proved performance.

These remarks have been made in an endeavor to clarify the discussion and in that way, if possible, simplify the comparison of the various network propositions. As Mr. Robinson has said, this is quite necessary for it is extremely important from the standpoint of the manufacturers, customers and, indirectly, the operating companies, that the latter get together without further delay and standardize as far as possible on such a combined light and power scheme as can be used to the greatest advantage on the majority of systems introducing the least possible extra expense in the manufacture and stocking of different types of apparatus.

Even though the including of items for operation and maintenance gaged over a period of ten years would probably have increased the difference in favor of the three-phase secondary system, Tables III and IV do not show a large economic advantage for three-phase. Further, it is apparent that despite a weighing of all evidence of engineering and operating nature a balance shows in favor of three-phase, and also, this balance is not large. It is therefore admissible that if the standpoint of the operating company only is considered and the analysis is confined to the usual period of ten years to come, the net advantage of three-phase over two-phase may not be great enough to outweigh the obvious benefits of having a uniform system of secondary distribution throughout a city. In connection with that which follows it should be borne in mind that the greater part of these benefits accrue from the attitude of the public in general.

Likewise, where a large number of two-phase motors now exist in an area that it is proposed to network, it is evident that the expense of changing them over to three-phase, or in all cases providing auto-transformers for a three-phase to two-phase transformation, could probably not be counterbalanced even in ten years by the small economic advantage of the three-phase.

However, it is conceivable that in this particular problem, an unusually broad point of view should be applied. The paper touches on the conditions that are tending to make two-phase obsolete. Exact data in this regard are difficult to obtain. There are but two large distribution systems and a few smaller ones still two-phase. The ratio of present investment in two-phase systems to that in all systems in the country may be gaged approximately by the fact that in 1923 only 6 per cent of all the polyphase motors sold by one prominent manufacturer were two-phase. It does not require a stretch of the imagination to foresee that within the next ten or twenty years this percentage is likely to decrease to even half its present value as the tendency for smaller companies operating two-phase to change to three-phase continues and as new systems in rapidly growing parts of the country start up, using the present standard of three-phase. With that reduced percentage of two-phase business it is natural to expect that prices and deliveries of two-phase apparatus will be adversely affected. Thereupon, customers in two-phase areas will set up a demand for three-phase service and this will have to be complied with by the operating companies. At the current rate of load increase, in ten years there will be about four times as many motors on the two-phase systems as at present and in twenty years about sixteen times as many. The expense of change-over to three-phase at that time will be correspondingly greater than at present. Even though measures may be taken to postpone the change-over, these will of necessity have only a temporary effect. The greater the delay the worse will be the situation when the change to three-phase finally takes place.

Coming now to systems where three-phase already exists, enough has been said to prove that it is almost out of the question to expect them to go to two-phase, five-wire secondaries. To review a few of the reasons, there is the great expense of changing three-phase motors to two-phase, replacing three-phase transformer banks by Scott-connected banks, and pulling in a fifth secondary service as well as sometimes in many places riser wire. Where ducts and service pipes are too small for the fifth wire, reconstruction would be necessary.

Finally, there are the companies that will install networks in what might be termed virgin territory. The paper shows certain definite though small advantages for three-phase, four-wire over two-phase, five-wire, such as in annual charges and first cost. In the majority of cities the cost of operation and maintenance of the two-phase system would be greater, due to the added maintenance of the fifth wire and five-wire protective equipment. There are also the extra losses due to the Scott transformation to two-phase. Three-phase motors and utilization devices are standard, while two-phase apparatus is becoming obsolete. In some makes of motors there is quite an appreciable difference in performance in favor of three-phase, assuming operation at rated voltage. Even though the advantages for each of these factors may not individually be great, when added together they point unmistakably to the wisdom of making networks in all new distribution layouts three-phase.

P. H. Chase: In some of the discussions on my paper there has been a tendency to confuse what is common practise with that which is good, and what is not common practise with that which is bad. Though two-phase, secondary distribution is not so common as three-phase, proper use of the fundamental distribution requirements as yard sticks certainly cannot lead to the opinion expressed by two of the critics that the two-phase system is bad or inferior.

It is interesting to note that advocates of "Three-Phase" are by no means unanimous in their choice of which is best of the various types of three-phase systems. One type requires what one supporter diplomatically calls "certain rating compromises", but these may result in widespread changes to equipment, to

standard lamp voltages, or to motor ratings. Another type does not afford load balance or balanced voltages to ground. Expensive expedients, such as auto-transformers and the like, are offered to overcome these drawbacks in order to adhere to three-phase and continue to render standard voltages to customers.

Most of the operating companies will continue to have the greatest part of their *secondary* load neither two-phase nor three-phase, but two-wire and three-wire, single-phase. In addition, the number of customers supplied with single-phase as compared with those supplied with polyphase service will be greater than the proportion of the actual single-phase and polyphase loads. Any system which penalizes single-phase customers is laboring under a handicap.

The flexibility and adaptability of the two-phase, five-wire system in handling a combination of two-wire single-phase, three-wire single-phase, and two-phase loads, with superior regulation and load and voltage balance, has not been successfully challenged. These valuable advantages have been demonstrated by long experience but are difficult of expression in dollars.

The small percentage of two-phase motors does not tell the story. From early days there has been a general tendency to operate motors on separate secondaries (and often separate primaries) from lighting. Consequently, the motor voltage and number of phases could be determined by considerations relating to and affecting the motor alone. Lighting was two-wire and three-wire, single-phase, because of the utilization devices, simpler metering, and lower service costs, which conditions still obtain. The trend of generation and of transmission toward three-phase naturally directed motor development and primary and secondary polyphase distribution along three-phase lines. Accordingly, three-phase motors predominate but lighting has remained single-phase.

Of recent years, the advent of a-c. secondary networks and the advantages of combined light and power secondaries have brought to the front a requirement which was originally unimportant,—that is, the flexible supply from the same secondary polyphase mains of all types of load, at established standard voltages. The three-phase system cannot meet this without expensive changes or doubtful compromises and such expedients as three-wire, open-Y, or four-wire services and polyphase metering for residence lighting. The two-phase, five-wire system now meets this need without change for both radial and network systems.

Standard two-phase motors are neither obsolete nor penalized in price, as reference to motor price-books will show. The adoption of a *new* line of 199-volt, three-phase motors would be a greater deviation from the principle of standardization than the retention of the already established line of two-phase motors.

Three-phase primary distribution can readily be utilized for supplying two-phase secondaries by the Scott connection of transformers which has been proved by years of operation and does not involve a deviation from standard voltages. The Scott connection deserves no more attention or criticism than many of the common modifications to "standard" transformers which are asked for and furnished almost as a matter of course.

We must keep this matter of standardization in mind. Standardization refers particularly to voltage, frequency, capacity, speeds, and types of equipment. On the two-phase, secondary, five-wire system, apparatus, equipment and devices are used that are standard as to voltage, frequency, capacity, and kind.

From the standpoint of the operating man, the matter reduces down to whether he is giving adequate service at reasonable cost, by adhering to his existing secondary system, and whether—although there may be a slight theoretical differential in cost, if he were starting new—he is justified in paying the high cost of change-over from the existing system. It is universally admitted that the cost of change-over is extremely high because of the cost of change-over of customers' installations. If the theoretical saving, over a long period of years—say ten—does not pay for the cost of change-over, how can a change be justified?

I referred particularly to 115 volts as the lamp voltage. The same relative situation applies to both two-phase and three-phase taking 120 volts instead of 115 volts. Even with 120/208 volts, three-phase, I believe most operating engineers would feel apprehensive about giving 208 volts to motor customers where they are used to receiving 225 to 235 volts for 220-volt motors.

Regarding "power per wire," this comparison indicates the efficiency in using the capacity of the installed wires including the neutral, which as a matter of practise is comparable in size to the other conductors. One critic states that the two-phase, five-wire system is only 80 per cent efficient. This is true as regards the utilization of the installed copper and on the same basis the three-phase, four-wire system is only 75 per cent efficient.

As to the matter of voltage unbalance on three-phase secondary mains, with a wire spacing of 14½ in., I might say the unbalance is not much decreased even with a spacing of 4½ in. such as on a bracket. That can be easily checked by calculation. Whether voltage unbalance is caused at the source or along the mains makes no difference in the effect.

As I think the paper brought out, I favor the higher primary voltages. However, it is true that there has been in the past some distrust, at any rate on the part of the public and some engineers, of the higher primary voltages for distribution.

Comparing systems on the basis of starting off new would be interesting, but hardly justifiable. But the words of the old adage still hold true:—"No matter where you are going, you must start from where you are." That is the point from which my paper started.

In the matter of relative costs of distribution systems and the relative savings in the light of growth, Tables No. III and IV were based, for the particular area under investigation, on a doubling of the present dense load. Whether doubling or quadrupling of load is assumed, there must be taken into account one factor which seems very important—the increasing loads supplied to customers at primary voltage. As larger buildings are erected and the load grows, it does not necessarily follow, particularly in a congested area, that delivery of all the increased load will be at the same low voltage. Often delivery will be at primary voltage, with the customer providing his own transformers and deciding on the voltage and number of phases on his premises. The primary system will probably supply the medium and large customers and thus absorb a large percentage of the increased load, and the secondary system will supply the smaller customers.

Reference has been made by two speakers to the two-phase system as being "inferior" or "inherently bad." Is a system inferior or inherently bad unless it is either extravagant or inadequate with regard to service? The two-phase, five-wire, secondary system is not a "proposed" system, but has been an actual reality for years. It has given adequate service since the inception of polyphase systems and still is giving economical service. It delivers standard voltages, and this cannot be said of some of the present and proposed three-phase systems. It employs standard apparatus. It affords maximum voltage and load balance in the system. It is flexible in growth and adequately serves the customers' conveniences.

Therefore, my conclusions still are:—The two-phase, five-wire secondary system is adequate and economical, and has a recognized place in distribution practise not inferior to any other system. The two-phase, five-wire system as well as any other adequate and economical system where once established should be continued unless another system presents greatly superior engineering and economic advantages.

The Oil Circuit Breaker Situation from an Operator's Viewpoint

BY E. C. STONE[1]

Member, A. I. E. E.

Synopsis.—This paper is an outline of the oil circuit breaker situation from the operator's standpoint, particularly with reference to interrupting duty, as it appears today to the author. It is not an original study, but rather an assembly of previously existing information, and is arranged by topics with a view to bringing out a large amount of discussion, in the hope that it may result in further clarifying the very complex problem of interrupting electric currents.

The topics taken up are as follows:

I Factors determining interrupting capacity, namely, intensity and duration of the arc.

II-III Essential features of breaker design and their functions.

IV Factors affecting interrupting duty, with especial emphasis on effect on same of system and fault grounding conditions.

V Relations between interrupting ratings and costs.

VI Status of interrupting ratings with reference to maximum nature of such ratings and facts upon which they are based; relative ratings on different operating duties and desirable modifications in method of rating.

VII Applications, particularly possibilities for improved practise in future, and necessity of adequate maintenance on all breakers in service.

At the 1918 Midwinter Convention of the American Institute of Electrical Engineers, a paper on "The Rating and Selection of Oil Circuit Breakers" was read by Messrs. Hewlett, Mahoney, and Burnham. In this paper the manufacturers presented their interpretation of the A. I. E. E. standards on oil circuit breakers as they existed at that time and a discussion of the features involved in the selection of oil circuit breakers for use on the various power systems. Since that time, much additional experience has been gained in the design and operation of oil circuit breakers and a definition of interrupting duty has been added to the standardization rules of the A. I. E. E.

In that which follows, no attempt at original research into the problem of oil circuit breaker interrupting duty has been made. Rather, the purpose is to present briefly a bird's-eye view of the circuit breaker situation from an operator's standpoint, as it appears to the author today. It is hoped that the outline which follows, of some of the more prominent phases of the subject, will call forth discussions which will bring out much important information.

The rating of an oil circuit breaker includes normal voltage, normal current, normal frequency, maximum momentary current which the breaker can withstand, and interrupting capacity. Of these items, all except the last are perfectly simple and not subject to argument. Hence, interrupting capacity is the only item of rating which is involved in the following discussion.

I—Factors Determining Interrupting Capacity of Oil Circuit Breakers

The interrupting duty imposed upon an oil circuit breaker when it opens a circuit depends on:

a. Intensity of arc between contacts, which is a function of magnitude of current interrupted.

b. Duration of arc, which is a function of the voltage tending to maintain the arc; that is, the "recovery voltage" which appears across the terminals of the switch at the moment the arc is interrupted.

The effect of the arc is to release an amount of energy in the tank which is determined by the intensity and duration of the arc, as stated above. This energy appears as heat which breaks down a portion of the oil in which the contacts are submerged. Gases are emitted, which develop high pressures, and carbon is deposited, causing loss of insulation strength of the oil. Also, the contacts are usually more or less burned. If, due either to too great intensity of arc (excessive current), or too great duration of arc (excessive voltage), or both, excessive amount of energy is released, the breaker will be unable to withstand the pressure developed and will give way, accompanied by more or less severe explosion, due to which the oil may ignite. The maximum amount of energy which the breaker safely can take care of marks the limit of its interrupting capacity.

After a breaker has opened the circuit, the gas pressure disappears in a comparatively short time, as the gas escapes through the vent, but the burning of contacts, carbonization of oil, and depositing of carbon on insulated surfaces inside the tank are accumulative. That is, every time a circuit breaker opens a circuit through which current is flowing, the arc produced causes burning of contacts, oil carbonization, and carbon deposit which effect a definite reduction in the interrupting capacity of the breaker. After a sufficient number of circuit openings, interrupting capacity of the breaker is exhausted and can only be restored by repairing contacts, cleaning inside insulating surfaces, and replacing old oil with new.

The loss of interrupting capacity of a breaker, as it operates, may be likened to loss of capacity in a storage battery as it discharges. After a certain duty has been performed, the breaker, like the storage battery, becomes exhausted. As the duty is intensified, the life

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of the breaker is shortened, just as the life of the battery is shortened on heavy load duty. Again, just as the battery may be recharged after its life has been run, so a breaker can be brought back to its original rating by a little simple and inexpensive maintenance.

II—Essential Features of Oil Circuit Breaker Design which Determine Interrupting Capacity

The essential features of design which determine interrupting capacity are:

a. For interruption of arc—
- Contact break distance
- Speed of contact travel
- Contact pressure
- Magnetic blow-out effect

b. For absorption of energy of arc (gas pressure)—
- Volume of oil
- Oil head above contacts
- Air space above oil
- Venting

c. Mechanical Strength—Great mechanical strength is required throughout to withstand the strains due both to pressures developed by release of gas and to electromagnetic effects of the heavy currents handled.

d. Thermal capacity—All current-carrying parts must have sufficient thermal capacity to carry the maximum currents while they last.

Operating tendency is to require increased thermal capacity to take care of the longer duration of short-circuit current due to the higher relay settings used for obtaining selectivity of circuit-breaker operation and for taking advantage of the current decrement curve of the synchronous equipment.

e. Oil quality—

III—Design Features on which Difference of Opinion Exists

Opinion is divided as to the value of the following design features:

High Speed contacts

Explosion chamber

Multiple contacts in series

Resistance introduced into breaker circuit to reduce energy released by arc in breakers

Opinion is also divided as to the best means of absorbing the gas pressure and the different designs take care of this feature by various methods.

IV—Factors Determining the Interrupting Duty to which an Oil Circuit Breaker is to be Subjected

Since interrupting capacity depends on current interrupted and recovery voltage, the interrupting duty in any case varies with these two factors:

1. Factors affecting current to be interrupted—The maximum current which a circuit breaker may be required to interrupt is obviously a dead short circuit at its terminal.

The current at any point of a system, under short-circuit conditions, is affected by the following factors:[2]

a. The total kv-a. reactance, and transient characteristics of the synchronous machines connected to the system.

b. Number, reactance, resistance, capacitance, and arrangement of all circuits over which power can be supplied to the point of short circuit.

c. The kv-a. arrangement, resistance, reactance, and capacitance of all reactors and transformers, through which power can be supplied to the point of short circuit.

d. Contact resistance at the short circuit.

e. The nature of the short circuit, whether single-phase or multiphase.

f. The kv-a. and power factor of the load being carried at the time of short circuit.

g. The point of the pressure wave at which the short circuit was established.

h. The use of automatic voltage regulators.

i. Conditions as to grounding of system neutral and grounding of short circuit.

j. Elapsed time from first cycle of short circuit to interruption of arc on breaker contacts.

2. Factors affecting recovery voltage—Very little consideration, so far, has been given to these factors although they are fully as important as those affecting current. The most obvious are:

a. Phase angle between interrupted current and recovery voltage.

b. Conditions as to grounding of system neutral and grounding of short circuit.

c. The kv-a. and power factor of the load existing at the time of short circuit.

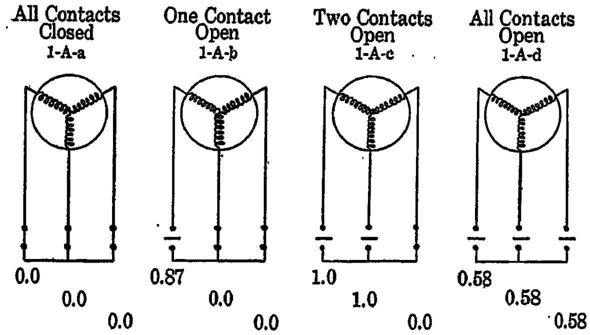
d. Arrangement and characteristics of circuits and apparatus.

The effect of item (a) has been discussed in a previous paper[3]. Under ordinary operating conditions item (c) should not vary sufficiently to affect radically the interrupting duty of the circuit breakers. The effect of Item (d) is to cause a high frequency transient voltage to be superimposed on the normal frequency recovery voltage, as a result of the discharge of stored energy in the system. Comparatively little data are available on the magnitudes of these transient voltages under varying system conditions—in most cases, however, it does not seem probable that they will increase the interrupting duties on breakers beyond the factors of safety intended to be in the breaker ratings.

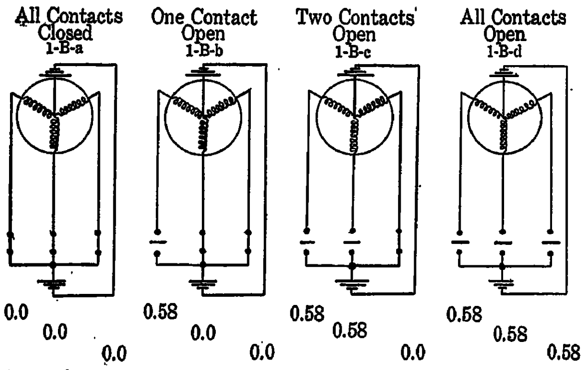
Conditions as to grounding of system neutral and grounding of short circuit, item (b), however, have very marked effect on the recovery voltage and therefore

2. Items (a) to (h) listed herein are taken from the paper "Rating and Selection of Oil Circuit Breakers" by Hewlett, Mahoney, and Burnham, A. I. E. E., 1918.

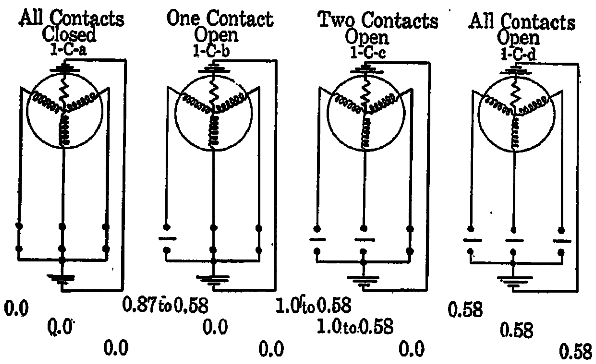
3. "The Rating and Selection of Oil Circuit Breakers" by Hewlett, Mahoney, and Burnham, A. I. E. E., 1918.



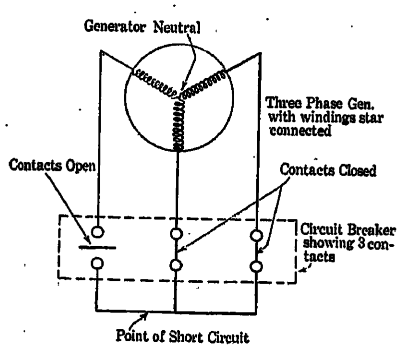
1A. Generator Neutral Grounded or Ungrounded Fault Ungrounded



1B. Generator Neutral Solidly Grounded Fault Solidly Grounded



1C. Generator Neutral Resistance Grounded Fault Grounded



1D. Explanation of Diagrams 1A to Ac

Fig. 1—Relative Recovery Voltages for Various System Conditions

In Figs. 1A to 1C the numbers directly under the breaker contacts give the normal voltages, expressed in decimals of normal system line voltage, which will exist across the respective contacts under the conditions shown. Under usual operating conditions, these values may be assumed as the limiting normal frequency recovery voltages on which interrupting rating should be based.

The parts of the circuit in which short-circuit current flows are shown by heavy lines.

on the interrupting duties imposed on the breakers. The relative values of recovery voltage under the various possible grounding conditions are shown in Figs. 1a to 1c.

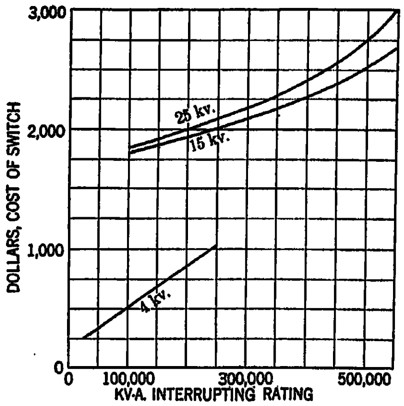


Fig. 2—Cost vs. Interrupting Rating for Oil Circuit Breakers. 25 Kv. and Below

V—Relation of Rated Voltage and Interrupting Capacity to Cost of Oil Circuit Breakers

For the study of problems in economics of system design a series of curves showing comparative costs

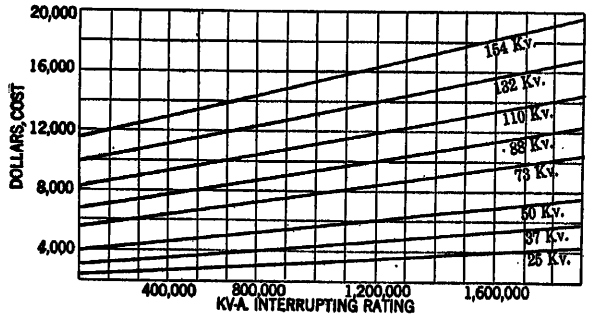


Fig. 3—Cost vs. Interrupting Rating for Oil Circuit Breakers, 25-Kv. and Above

of oil circuit breakers has been compiled. These curves are based on present day prices and are believed to present a fairly accurate picture of the relation of

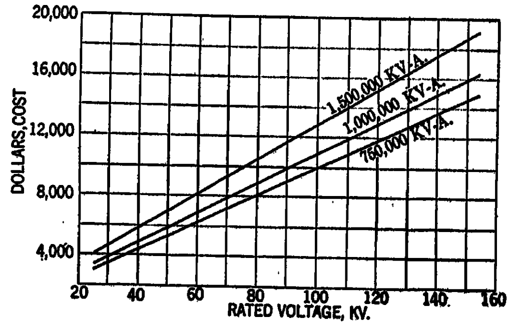


Fig. 4—Cost vs. Voltage Rating for Oil Circuit Breakers Interrupting Ratings 750,000-Kv-a. to 1,500,000-Kv-a.

costs to interrupting ratings as a whole, although in some individual cases the price of a given breaker may be considerably out of line. These curves are shown in Figs. 2, 3, and 4.

On low-voltage breakers interrupting capacity is the chief factor in determining price, while voltage has less effect. Thus, a 15-kv. breaker of 300,000-kv-a. interrupting capacity costs approximately 16 per cent more than a 15-kv. breaker of 130,000-kv-a. capacity, while a 25-kv. breaker of 500,000-kv-a. capacity costs approximately only 9 per cent more than a 15-kv-a. breaker of the same capacity. (Fig. 2.)

On high-voltage breakers, however, voltage is the chief factor in determining the price, while interrupting capacity has a lesser effect. Thus, a 37-kv. breaker of 1,500,000-kv-a. capacity costs approximately only 28 per cent more than a 37-kv. breaker of 750,000-kv. capacity, but a 73-kv. breaker of 750,000-kv-a. capacity costs approximately 60 per cent more than a 37-kv. breaker of the same capacity. (Fig. 2.)

VI—Status of Interrupting Ratings

Interrupting ratings of oil circuit breakers are admittedly maximum ratings so that breakers cannot be expected to function properly even slightly beyond their ratings. It is understood, however, that these ratings are based on interrupting an ungrounded short-circuit, under which condition the normal frequency recovery voltage is a maximum. (Fig. 1A.)

Interrupting ratings are based on a constantly increasing fund of knowledge resulting from tests in factory and field and from operating experience. Special attention is called to the exhaustive factory tests made by one manufacturing company. Some of the interrupting duties under which breakers have been tested in the field are shown in Table 1.

TABLE I
FIELD TESTS ON OIL CIRCUIT BREAKERS OF HIGH INTERRUPTING CAPACITY

Normal Line Voltage	Limiting Recovery Voltage	Maximum kv-a. three phase Interrupted	Amperes per Pole	Grounding Characteristic: System Neutral	Grounding Characteristic: Short Circuit
	(See Note)				
132 kv.	77 kv.	725,000	3150	Dead Grounded	Grounded
110 kv.	63 kv.	542,000	2850	" "	Grounded
44 kv.	44 kv.	280,000	3660	" "	Ungrounded
44 kv.	25 kv.	246,000	3220	" "	Grounded
24 kv.	14 kv.	580,000	14,000	" "	Grounded
23 kv.	23 kv.	450,000	11,400	Grounded through Resistance	Grounded
13 kv.	7.5 kv.	545,000	23,700	Dead Grounded	Grounded

Note: Limiting recovery voltage, as used in above table, is defined as the maximum or limiting normal frequency recovery voltage which can appear across the contacts of one switch pole after the arc is interrupted, under the grounding conditions existing in the test. For explanation of method of obtaining these values see Figs. 1A to 1D.

On the actual tests the recovery voltage did not in any case reach the full limiting value as given above, even taking into account such higher frequency harmonics as were developed.

So far as the writer has been able to ascertain, no field tests have been made where more than 725,000-kv-a. has been interrupted. Most of these field tests, as shown in the table, have been made on grounded short-circuits in systems with dead grounded neutral. In such cases the recovery voltage is approximately only 58 per cent of that on a system with neutral ungrounded or grounded through a high resistance. The interrupting duty under such tests was therefore only about 58 per cent of that which would have been imposed if the tests had been made on systems with high resistance instead of dead grounded neutral. In general, also, tests were made under conditions where the transient voltages resulting from the interruptions of the short circuit have been very small, but on the other hand, metallic short-circuits were used which would give more severe conditions than probably exist ordinarily in service.

Oil circuit breakers, with their designs based on the results of tests within the range of their interrupting rating, may be expected to perform satisfactorily up to these ratings, where they are installed under system conditions similar to those under which the field tests were made, particularly in regard to conditions as to grounding of system neutral and grounding of short circuits which the breaker must interrupt.

For ratings beyond the range of tests, it must be clearly borne in mind that very little direct data have been obtained and that designs are based on data deduced from tests made on lower interrupting duties. It remains to be seen how accurate the conclusions thus drawn will prove to be.

In oil circuit breakers for high voltages, the tank size is determined by the voltage rather than by interrupting duty. It is quite possible that the interrupting capacity of high-voltage breakers may prove to be larger than expected for this reason.

All published interrupting ratings are now based on the standard operating duty approved last year by the Protective Devices Committee and printed in the A. I. E. E. Journal for October, 1924.

Referring to the standard definition of interrupting rating on which the standard operating duty is based, it is probable that the term "Normal Voltage" will require further definition in view of the wide differences in the recovery voltage which may prevail with the same normal system voltage. Meanwhile it is assumed as stated above that this term is interpreted to mean that the switch will be able to perform its full operating duty under the conditions which will give a recovery voltage up to the magnitude which results when the short circuit is ungrounded. It is not clear, at the present time, whether this is always the case.

Further attention must also be given to the relative interrupting ratings of the same circuit breaker under different operating duties.

The following relative ratings have been proposed by the Power Club:

a. One-unit operating duty...... 100 per cent to 125 per cent
Rating varies between limits given with design of breaker

b. Two-unit operating duty, two-minute intervals (*standard duty*)........................... 100 per cent

c. Four-unit operating duty, two-minute intervals	70 per cent
d. Four-unit operating duty, one-half minute intervals	60 per cent
e. Four-unit operating duty, no time intervals	25 per cent
f. 300-unit operating duty, 15-minute intervals	30 per cent
g. Four-unit operating duty, successive intervals of 0, 30, 75 seconds	30 per cent
h. Four-unit operating duty, successive intervals of 15, 30, 75 seconds	40 per cent
i. Three-unit operating duty, one-minute intervals	70 per cent

It is recommended that operating duties (d), (e), (g), and (h) be confined to oil circuit breakers having interrupting ratings on standard operating duty not over 250,000 kv-a. and voltage ratings not over 37 kv.

Taking the above figures as a basis, it is probable that a breaker that will perform satisfactorily under operating duty (a) or (b), one or two interruptions, will also perform satisfactorily on any of the other operating duties at the percentage rating given. On the other hand, it does not seem at all certain that a breaker which will satisfactorily interrupt the lower values of kv-a. on multiple interrupting duty will also be able to interrupt the higher kv-a. of the one or two interruption operating duties on the percentage relation shown in the table.

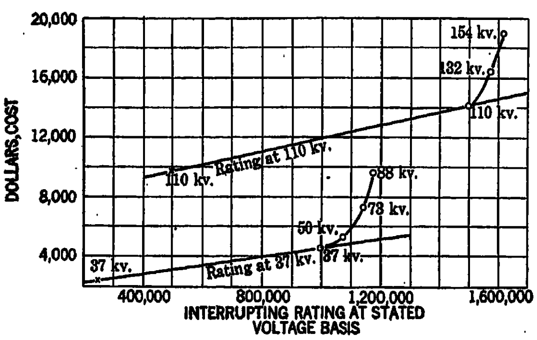


FIG. 5—COMPARATIVE COSTS OF INTERRUPTING RATINGS IN OIL CIRCUIT BREAKERS OPERATED AT VOLTAGES BELOW NORMAL

In Fig. 5 are shown the relative costs of obtaining interrupting capacity when using breakers at voltages below their normal rating. Thus, while a 37-kv. breaker of 1,150,000-kv-a. interrupting rating will cost $5000, a 73-kv-a. breaker, giving the same interrupting rating on 37-kv. operation, will cost $7300, an increase of 46 per cent. The effect of this relation of ratings is to make very much more expensive the use of breakers of higher-voltage ratings than the normal operating voltage of the system in which they are connected, although such practise becomes vitally necessary on some systems. Certain operating data which have come to the attention of the writer lead him to believe that in many cases the permissible increase in interrupting ratings at reduced voltage operation is too conservative and it is earnestly suggested that the manufacturers give careful consideration to revision of their present standards in this respect.

Tests are still urgently needed to prove performance at greater interrupting capacities and it is hoped that the operating companies will continue the practise of testing oil circuit breakers at progressively greater interrupting duties as the available short-circuit capacity on their systems increases. To assist in carrying out these tests and to provide for getting the greatest benefit from results obtained, a proposed uniform procedure for testing the interrupting rating of oil circuit breakers was approved by the Protective Devices Committee at its meeting last spring. This procedure now has the approval of the Electrical Apparatus Committee of the N. E. L. A. and is recommended as a basis of procedure for all future oil circuit breaker tests on power systems.

VII—APPLICATION OF OIL CIRCUIT BREAKERS

In considering the application of an oil circuit breaker to a specific situation, the maximum interrupting duty must be determined by calculation of maximum current to be interrupted and maximum recovery voltage. The interrupting rating of the breaker specified will depend upon the maximum interrupting duty as thus determined and the particular operating duty which will be demanded of the breaker in service. At the present time conservative practise requires that calculations be made on a basis that will give a maximum interrupting duty equal to or somewhat higher than that which can actually be imposed on the breaker.

Calculation of short-circuit current is thoroughly understood and can be made with almost any desired degree of accuracy.[4] Calculation of recovery voltage is not so well understood. The important factor to be taken account of in determining recovery voltage is the condition as to grounding of system neutral. (Refer to Figs. 1A to 1C). Up to the present time most of the heavy-duty oil circuit breaker experience has been derived from systems with dead, or nearly dead, grounded neutrals, on which the interrupting duty for a given current is of the order of only 58 per cent of that on a system with ungrounded or high-resistance grounded neutral. Hence on systems of the later type, which are becoming more numerous all the time, it becomes of prime importance to give careful consideration to the more severe conditions.

Up to the present time comparatively little attention has been given to refinement in the application of oil circuit breakers. As the interrupting ratings of breakers become more accurate it should be possible to fit breakers more closely to their individual duties. Many factors under operating conditions tend to make the current which must be interrupted on most faults of much less magnitude than the maximum current of a

4. "Application of Decrement Factors in Short Circuit Studies" by W. R. Woodward, *Electrical Journal*, May 1924.

dead short circuit at the breaker terminals. Some of the more obvious of these factors are:

Resistance in fault.

Faults resulting in grounds or three-phase shorts, whereas duty may be calculated on the more severe single-phase line to line short.

Neglect of system resistance in calculating fault currents.

In networks, the interruption of the fault in two or more steps.[5]

When these factors are known and circuit breaker ratings are also accurately determined, it may be found that the maximum conditions assumed in the selection of the breaker at the present time occur so seldom that they may be treated as a special case. This special treatment might involve allowing the breaker to function above its rating in these few instances, or by prescribing a less severe operating duty under especially heavy short circuits, or by installing breakers in group arrangement in such a manner that one breaker of sufficient interrupting capacity to meet maximum duty would be in series with a number of smaller breakers designed for average conditions and would only open

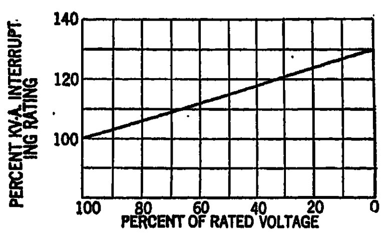


FIG. 6—INCREASE OF INTERRUPTING RATING OF AN OIL CIRCUIT BREAKER WHEN OPERATED AT A VOLTAGE BELOW ITS NORMAL VOLTAGE RATING

when the rating of the small breakers was exceeded by the short circuit. All of these methods are in use to a limited extent at the present time, but their effectiveness is uncertain, because of lack of exact knowledge as to conditions. Data on actual values of fault currents as experienced in every day operation are much needed, and it is believed that if such data could be obtained and analyzed, material savings in overall circuit-breaker investment might be effected without undue hazard to service.

A number of companies have found it necessary to use breakers designed for normal voltage higher than those of the systems on which they are to be used, in order to obtain adequate factors of safety against insulation failure. On the basis of ratings now standard, this factor of safety is obtained only at marked increase in cost. (Fig. 6.)

The standard definition of operating duty of a circuit breaker contemplates very clearly that after the rated interrupting duty of the breaker is performed, the breaker is no longer good for its rating until suitable maintenance has been performed. The reasonableness of this condition is found in the explanation in Section I above of what happens within the breakers when a current is interrupted. It is of the utmost importance that this limitation be recognized. An adequate system of inspection and maintenance must be set up if satisfactory service is to be obtained. In this connection it is obvious that the cost of maintenance will be less where the rated interrupting capacity of the breaker is in excess of the maximum duty it is called upon to perform. In circuit-breaker installations, therefore, some consideration should be given to striking a balance between first cost and maintenance cost. Due consideration of both of these factors will sometimes call for a larger breaker than would otherwise be specified.

5. See discussions on papers on Baltimore tests, especially that by A. F. Bang, A. I. E. E., 1922.

Discussion

W. S. Edsall: We feel that there is not a great deal in this paper that can be discussed by the manufacturers, because it is a paper primarily presenting the operators' viewpoint.

This showing of the various conditions of the grounding of the system and generator is going to help the general situation. The effect of grounding upon recovery voltage and the effect of recovery voltage upon the duty of an oil circuit breaker have not been given the attention the subject merits by either the manufacturer or the operator. The manufacturer has known that very severe duties are imposed upon the breaker under high recovery-voltage conditions. The operators in many cases have not known that the systems contained certain combinations of reactance and capacitance which would, under certain conditions, give high recovery voltages.

We feel it important that the recommendation of the Protective Devices Committee regarding definitions for normal voltage, recovery voltage, normal current, etc., should be followed out. The publication of such definitions will tend to call attention to the conditions that exist, and will lead to more accurate application of breakers to the system.

Some European engineering associations have already made definitions that would distinguish between normal voltage and recovery voltage.

O. K. Marti: Mr. Stone brings out in his paper that it would be highly essential, in the future designs of breakers, to have more data regarding four distinguished designs (see his paper under III) since the opinion is greatly divided as to their value. I shall refer below to two designs which European engineers, at present, believe to be the right steps in the right direction, namely, to the application of multiple contacts in series and to the introduction of a resistance into the breaker circuit.

Very little information has been actually published regarding the operation of breakers embodying the above designs. To my knowledge, there are only two references on the foregoing information, the first in a report to the Swiss Commission by Dr. B. Bauer[1], and the second in an article by Mr. G. Bruehlmann[2] in the *Brown Boveri Review*. Both articles are based upon elaborate tests which led to facts not suspected at the time of starting the investigations.

In Fig. 1 herewith is shown an arrangement whereby a protective resistance may be introduced by means of multiple contacts in series, and its effect realized from Fig. 2. An a-c. arc is extinguished at the moment the current passes through zero if the recovery voltage does not increase high enough so that an arc can be struck anew. Thus it may be noted from the oscillograms taken at Section I of Fig. 1 and shown in Fig. 2 that the increase of the voltage immediately after the arc in Section I extinguished, in case of introducing a protective resistance (see curve denoted by 4) was much more favorable than when no

1. *Bulletin*, Ass. Suisse des Electriciens, 1915.
2. *Brown Boveri Review*, March, 1923, page 43.

resistance was introduced (see dotted curve denoted by 7). The advantage of the new design lies in the slow increase of the voltage due to the fact that it retards its recovery speed and thus gives the produced arc gases time to cool. Furthermore, there may be noted the very favorable influence of the recovery voltage immediately before the arc extinguishes, having then the value of the ignition voltage. Tests with breakers having various sizes of protective resistances, in addition to many unexpected facts, gave the following results:

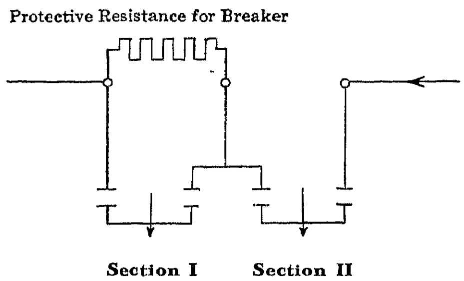


FIG. 1—DIAGRAM OF CONNECTION OF A BREAKER WITH PROTECTIVE RESISTANCE

The best interrupting conditions in any breaker circuit are obtained when a protective resistance eight or ten times the short-circuit impedance is introduced during the short circuit.

The duration of the arc decreased considerably, while the breaker duty, volume of arc gases, and the pressure of the oil decreased accordingly. It was actually found in several tests that the released energy in the arc was less than 1/10th.

Other tests which were conducted by the Swiss Federal Railroads on single-pole breakers rated 15,000 volts, 350 amperes, revealed the fact that, in a single tank of less than 5.5 cu. ft.

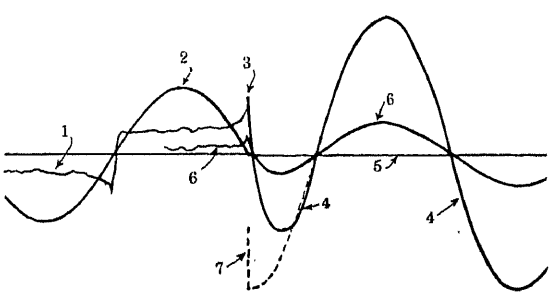


FIG. 2—VOLTAGE AND CURRENT WHEN A CIRCUIT BREAKER HAVING PROTECTIVE RESISTANCE IS TRIPPED BY A SHORT-CIRCUIT AS SHOWN IN SECTION 1

1. Arc Recovery Voltage
2. Arc Current
3. Arc Extinction, Section 1
4. Voltage Across Electrodes and Resistance
5. Zero Line for Current in Arc
6. Current in Resistance
7. Voltage Curve Without Resistance

volume over 100,000 kv-a. interrupting capacity could easily be handled by introducing in the breaker circuit a protective resistance with multiple-break arrangement. The protective resistance in the above breaker is an integral of the breaker and is inserted in the tank. The result of this test was the adoption of such breakers as standard designs by two state railways in Europe.

The addition of a resistance to a breaker or to the system of a power station should not be considered as a step in the wrong direction in breaker design, since similar steps are being made at the present time along this line in protecting electric equipment, as, for instance, by the addition of reactors, a limitation of the short-circuit current is obtained.

A schematic arrangement of a multiple-contact breaker is shown in Fig. 3. The purpose of this design is mainly for the reduction of the produced arc gases and the deposit of carbon, the latter causing loss of insulating strength of the oil during interruption. By dividing up the arcing distance, less energy is released, and since the gas is produced at several places throughout the oil volume, the latter is therefore much more quickly cooled. Moreover the arc can be much more easily controlled, and, therefore, several dielectric problems overcome without a complicated design. Realizing that 100,000 volts require an arcing distance of over 70 in., the foregoing fact would be greatly appreciated in the design of high-voltage breakers.

It might also be of interest to know that there have been investigations by a European company concerning a new contact device using two series coils so arranged as to form a compact piece of apparatus, which produces a powerful action with a minimum of space. The field produced by these coils closes the breaker against the forces which are especially severe at a short circuit just as easily as at normal load. The forces tending to part the contacts are due to the electrodynamic action and the gases produced between the contacts resulting from the current and the arc. The magnitude of such forces and their influence

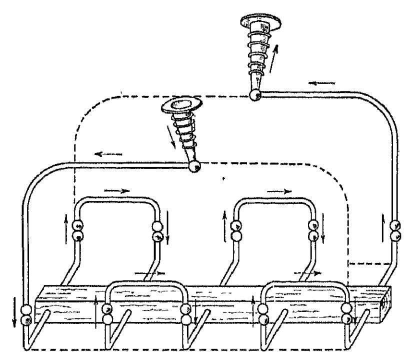


FIG. 3—ARRANGEMENT OF BREAKS AND PATH OF CURRENT IN CASE OF TEN-SERIES BREAKS

upon the operation of the breaker and its control mechanism may be better realized after considering the following figures; by breaking a current of about 25,000 amperes and assuming that its maximum value including the asymmetrical component is about 50,000 amperes, the foregoing forces would be approximately 400 lb. which have to be given due consideration in the design of breakers.

The factors determining the duty of a breaker are tabulated under Table IV of Mr. Stone's paper. It follows that the duty depends upon a great number of factors which require a very tedious and rather drawn-out procedure when given due consideration by selecting a breaker. It may be of interest, therefore, to know that a table for ratings of circuit breakers has been published by the Swiss Commission of Oil Circuit Breakers[3] which may be considered as a first step to simplify the foregoing mentioned procedure. This table is shown as Table I herewith and has already been adopted in various parts of Europe.

A. H. Kehoe: Many of the large capacity circuit breakers require extrapolation of published test data to establish their rating. This applies particularly to the restoring voltage after final rupture takes place. Future tests of circuit breakers should be with restoring voltage at least equal to the system voltage on which the circuit breaker is to be used, and for some situations double normal voltage should be used on the test circuit in order to obtain correct performance data of the breaker.

In several places the paper mentions contact resistance at the short-circuit as being one factor to be considered in selecting

3. *Bulletin No.* 2 Ass. Suisse des Electriciens, 1925.

the size of breaker required. While it may be possible to arrange a circuit so that faults will be in the form of arcs, yet short-length arcs have voltage drops in the order of 100 to 200 volts for currents from 1000 to 15,000 amperes so that on high-voltage systems contact resistance is negligible. This fact is of importance in considering the destructive forces at a fault, particularly on high-voltage cable systems. Instead of in-

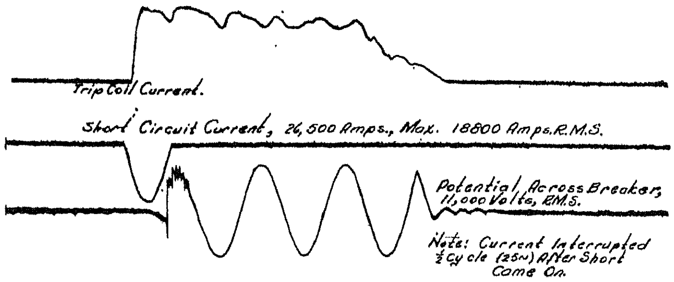


FIG. 4—HIGH-SPEED, OIL CIRCUIT BREAKER TEST, ALL POSSIBLE LOAD

creasing with the voltage, the destructive effect is quite the reverse, as high-voltage lines generally have a considerably lower value of short-circuit current than do the circuits now commonly used.

J. B. MacNeill: On the second page of his paper Mr. Stone gives certain features of design which affect rupturing capacity, and states that there exist differences of opinion regarding the most effective principle for a particular range of circuit breaker sizes and capacity is limited in its use to such sizes.

It is generally known that the magnetic blowout effect of the current passing through the loop formed by the contacts and

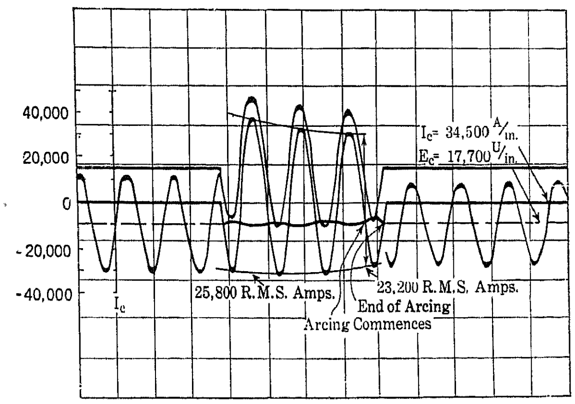


FIG. 5—OSCILLOGRAM OF OIL CIRCUIT BREAKER TEST OPENING APPROXIMATELY 25,000 R. M. S. AMPERES AT 13,200 VOLTS, 25 CYCLES

terminals of an ordinary moderate-voltage breaker has a tremendous effect in accelerating the arc formed between the contacts. The length of the arc thus formed may be pretty much independent of the mechanical speed of the breaker al-

TABLE I

DUTY IMPOSED ON ONE BREAKER POLE ON TWO- AND SINGLE-PHASE SHORT CIRCUITS COMPARED WITH DUTY ON THREE-PHASE SHORTS

		Short Circuit via transmission lines and transformers	Short Circuit at generator terminals (without appreciable line impedance)	
Quantity	Nature of Short Circuit	Initial Short equals sustained Short	Initial Short	Sustained Short
	Three-phase	100%	100%	100%
Current interrupted..........	Two-phase	$\frac{\sqrt{3}}{2} \times 100 = 87\%$	abt. 100%	abt. 115-150%
	Single-phase	100%	abt. 115%	abt. 150-200%
	Three-phase	100%	100%	100%
Voltage.....................	Two-phase	$\frac{1}{\sqrt{3}} \times 100 = 58\%$	$\frac{1}{\sqrt{3}} \times 100 = 58\%$	abt. 65- 85%
	Single-phase	$\frac{100}{1.5} = 67\%$	$\frac{100}{1.5} = 67\%$	abt. 85-115%
	Three-phase	100%	100%	100%
Interrupting capacity........	Two-phase	50%	$\frac{1}{\sqrt{3}} \times 100 = 58\%$	abt. 75-125%
	Single-phase	$\frac{100}{1.5} = 67\%$	c.a. $\frac{115}{1.5}$ 77%	abt. 125-230%

The table is based on the assumption that the impedance of the generators in each phase is smaller by about 15% on two-phase and single-phase shorts when compared with three-phase shorts, due to the magnetic interlinkage of the phases.

In the last column the first figure refers to low-speed generators, the second figure to turbo generators.

merits of some of these features. The general use of certain of these features over a large range of voltage classes and interrupting capacities necessarily results in the application in places where a given principle may not be used to its full advantage as well as in places where the principle may be particularly useful. For instance, the magnetic blowout effect referred to by Mr. Stone is of negligible importance on high-voltage breakers where the current to be interrupted is only a few thousand amperes. On the other hand the use of high-speed breaks on low-voltage breakers where the interrupting current is large, would be entirely superfluous. The best results are obtained when the though sluggish breaker action is not to be advocated. As an illustration of what can be accomplished with magnetic blowout effects, Fig. 4 herewith shows an oscillogram of a special circuit breaker opening a current of 18,000 r. m. s. amperes on a single-phase grounded circuit at 11,000 volts. The total time of operation of the circuit breaker including the tripping time, movement of the contacts, and blowing out of the arc, is only one-half cycle on 25 cycles. Fig. 5 shows an oscillogram of a standard circuit breaker of the dead-tank variety opening a current of approximately 20,000 amperes and in which the total period of arcing is approximately one-half cycle on 25 cycles.

inasmuch as the time of arcing cannot be reduced below one-half cycle provided the arc starts at the beginning of the voltage wave, it can readily be seen that the addition of high-speed breaks or other features of design in an attempt to reduce arcing further would simply be adding complication to the breaker structure.

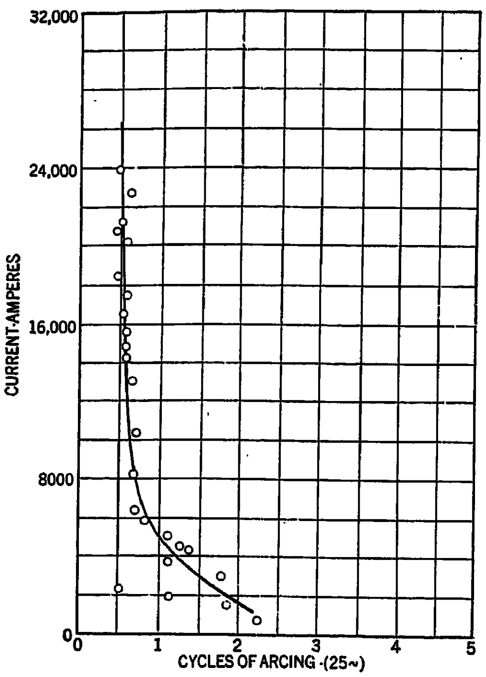


Fig. 6—Duration of Arcing as Obtained from Average of More than 200 Oil Circuit Breaker Test Oscillograms

The increased efficiency of the magnetic blowout with increasing currents is shown in Fig. 6 in which ordinates are r. m. s. currents and abscissas are cycles of arcing on a 13,200-volt, 25-cycle circuit. It is evident that the curve becomes asymptotic at one-half cycle. This curve represents the data from a large number of tests on several sizes of breakers and

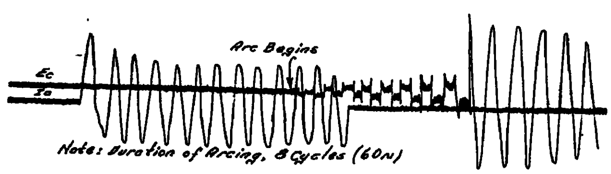


Fig. 7—Oscillogram of Oil Circuit Breaker Test, Opening Approximately 3000 Amperes at 44 Kv., 60 Cycles

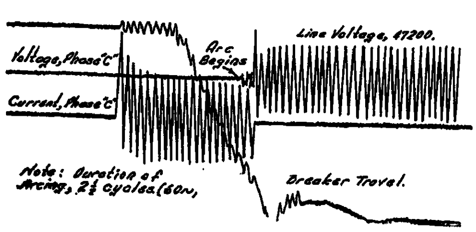


Fig. 8—Circuit Breaker Test

various oil conditions and illustrates the dependability and regularity of operation under these conditions of breakers using this principle.

On the other hand, for high-voltage breakers in which the magnetic blowout effect is inherently small, some device must be resorted to to prevent the building up between the contacts of a pillar of arc gas which may result in an unnecessary long period of arcing. For this purpose high-speed breaks are particularly useful. Anyone who has had the experience of opening an ordinary knife switch will realize the difference that high-speed contacts make on arc duration and the amount of burning on the contact parts. Fig. 7 shows the operation of an ordinary speed breaker at 44,000 volts on opening approximately 3000 amperes at 60 cycles. The arc extends over several cycles which, of course, represents a certain amount of burning of contact parts and oil and the generation of gas pressure in the breaker structure. In Fig. 8 is shown an oscillogram of a breaker with high-speed contacts opening approximately 4500 amperes at 44,000 volts, 60 cycles and, therefore, the amount of arcing can be compared directly to that in Fig. 7. The total duration of arcing with the high-speed contact is 2½ cycles on a 60-cycle circuit and consequently the amount of burning on the breaker contacts, the pressure generated in the tank, and the amount of oil burned up are reduced to a minimum.

Fig. 9 shows curves of the relative speed of the arcing members of a breaker with ordinary breaks and of the same breaker

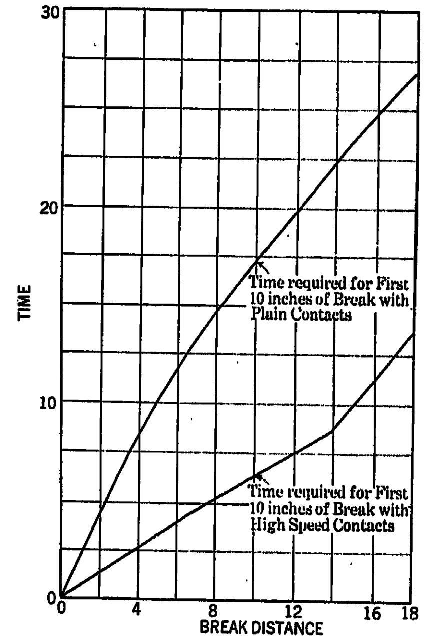


Fig. 9—Relative Speeds of Plain and High-Speed Contacts

equipped with high-speed contacts. These data were taken with oscillographs but represent mechanical action of the parts only. The time for a given break with high-speed contacts is approximately one-third that required for the same displacement with ordinary break and this ratio holds closely over a large range of voltage classes. Reference to Figs. 7 and 8 indicates that the ratio of arcing period for the two constructions favors the high speed contacts more than does the comparison of mechanical speeds.

M. I. Pupin: In listening to the discussion on circuit breakers, I observed that there was one fundamental principle which I think is never taken account of in the design of circuit breakers. We always take note of the principle of conservation of energy. The energy,—that is, the electrical energy,—must be transformed into some other form of energy. That, of course, is correct. But when you do that, you go only half-way. There is another principle in the science of electricity which must be taken into account, and that is the principle of conservation of momentum. You must provide something which will take up the electromagnetic momentum accompanying, say, 50,000 or 100,000 amperes, and this so far as I can see, is not so easy to do.

Arnold Roth: Perhaps I may take up the question of the voltage. As I am acquainted with the work of the European

committee mentioned in Mr. Stone's report, I might tell you how we came to the conclusion to introduce the "recovery voltage" in the duty cycle and in the duty imposed on a circuit breaker.

For the time lag of our relays, we use one second and two seconds, and there are generating stations using four and five seconds. As generally known, there is a very big difference for generating stations between the value of the initial rush of short-circuit current and the sustained value. This difference is due to the decreasing of the field. But there is a second effect of this weakening of the field, that is, the decreasing of the induced voltage. It takes place in quite the same relation as the decrease of current and I wonder that more attention has not been given to this fact. As a consequence of this effect, if you have a ratio of 4 to 1 between momentary and sustained short circuit, you might have almost the same relation between the induced voltage at the beginning and at the end of the short circuit. The induced voltage during short circuit is identical with the recovery voltage after rupturing the circuit. That is to say, in a 15-kv. network, you would have at the end of a short circuit a voltage of only 3700. You have a relation between the sustained and the momentary short-circuit kv-a. imposed on your breaker, not of 1 to 4, as we used to calculate, but of 1 to 16. That is to say, if you cut the short circuit in four, or to 0.1 sec. for this special case, you would have a difference from 1 to 16 that might be in one case 500,000 kv-a. for momentary tripping and in the other 30,000 kv-a. with time delay. You see a big difference, and it is absolutely necessary to introduce this relation in order to have a clear comparison between different tests and different kinds of breakers.

I would not uphold our European practise to delay tripping as long as we do. I think we shall have to build breakers able to break the initial short circuit. But even for "momentary" breaking, if you analyze the short-circuit tests made, you will find it very difficult to have the whole voltage of your station as recovery voltage in the moment you are breaking your short circuit. That is to say, take a 100,000-volt network, and you will always find that if, in the moment of the establishment of the short circuit, the voltage has been 100,000 volts, you will have to break only about 85,000 volts if you don't employ special apparatus to trip the breaker before the short circuit occurs, as is the practise in artificial tests.

One of the reasons of European operating companies for introducing long relay setting might be of some interest. These companies installed breakers for the calculated short-circuit capacity when they built the generating stations; afterwards the short-circuit capacity grew and the breakers were no longer sufficient to break the short circuits. Then the operators gave a very long time-setting to the relays in order to reduce the duty. They had really had some reserve for their breakers in the time setting of the relays.

I shall not say that it is good practise, but it is a practise with which we have to calculate in Europe, and to make possible the calculation, we recently introduced the definition of the recovery voltage into our regulation. I would not dare to say that this manner of calculation has become general practise, but we hope it may.

I should like to ask you some questions. I have had opportunity to see many of the leading engineers of this country and have been astonished to find how different are their opinions on the actual situation of the short-circuit-breaker question. I have been in companies where they told me it was quite all right and that they had no difficulties at all. Then there were other networks where it was quite the opposite. Now if you calculate in a general way the short-circuit capacity of the different networks and see the general conditions you would think they are the same. Of course, there must be some reason for this different behaviour. I think it would be a very interesting subject to investigate.

Another question is that of secondary explosions. I have heard many times that one of the most important reasons for the blowing up of breakers is the secondary explosion. We used to make some tests on secondary explosions by exploding gas mixtures, and we found that it takes a relatively big spark to cause an explosion. If we only had on insulators those small sparks which we have as "static" or something like that, it would be quite impossible to ignite the gas. I understand that after a breaker has operated, there is a mixture of explosive gas in the tank of this breaker, but I can't understand how you get the ignition. I should be very much interested if it could be explained.

Perhaps it would be interesting to do some work on the voltage drop in the arcs of short circuits. We made a test with two parallel iron tubes as electrodes and a copper wire acting as fuse with some 90,000-kv-a., short-circuit capacity behind 8000 volts induced voltage. We found about the same values of drop in the arc you had here of 2 to 3 per cent, but this was true only for the first ten or twenty cycles, and afterwards the length of the arc grew and the voltage drop became quite considerable compared to the induced voltage of the circuit, so an influence of the arc resistance on the value of the short-circuit current and consequently on the duty imposed on the breaker is possible if the tripping of the breaker occurs in the moment the arc has a great length. The influence was so great that in some cases the arc was extinguished without operation of the breaker. It took the form of a horseshoe growing bigger and bigger, up to a diameter of 15 to 20 ft. and then extinguishing.

O. R. Schurig (by letter): The following discussion refers particularly to the subject of secondary gas explosions. According to Dr. Roth's discussion, data obtained in Europe appear to show that secondary explosions are less likely to occur than American experience has indicated. It is, therefore, the object of this discussion to present some data obtained in this country on the subject of secondary gas explosions.

Secondary explosions in oil-circuit breakers take place when properly proportioned mixtures of air and gases of decomposition from the oil are ignited. Hence the necessary requirements for secondary explosions are:

1. A combustible gas (or vapor),
2. The presence of oxygen (as, for instance, in air),
3. The proportions of gas and air must be within the explosions limits, and
4. A source of high temperature, such as a flame, an arc, or high-temperature gases.

Tests have shown that the conditions for secondary explosions are likely to be met in oil-circuit breakers unless preventives are applied.

It is well known that the *gases evolved during circuit* interruption in oil are combustible and capable of explosion when suitably mixed with air. The analysis of the gases discharged by an arc between brass electrodes under oil (a commonly used mineral switch oil) showed a hydrogen content of the order of 60 per cent by volume at room temperature. Other explosive gases such as methane and ethylene are also present.

An investigation conducted at Schenectady in 1920 and 1921 to determine the *proportions of gas and air required for explosive mixtures* showed that mixtures containing 5 per cent gas or more up to 50 per cent are explosive, while mixtures not within this range of proportions are not explosive.[4]

The *maximum pressures of explosion* were eight times as high as the initial pressures, all pressures being measured in absolute

4. For the present purpose, the term explosive is defined as capable of rapid combustion in a manner creating a material pressure rise. The maximum velocity of flame propagation was found to be of the order of 30 ft. per sec. as indicated by the experimentally observed time interval between the beginning of the ignition arc and the occurrence of the maximum pressure.

units, in tests made with initial pressures ranging from atmospheric to 40 lb. per sq. in. absolute (*i. e.*, from 0 to 25 lb. per sq. in. gage). In other words, the maximum pressures of explosion for mixtures of circuit-breaker gases and air are, roughly, 100 lb. per sq. in. gage for a mixture initially at atmospheric pressure, and 300 lb. per sq. in. gage for a mixture initially at 25 lb. per sq. in. gage pressure. Mixtures containing, roughly, 20 per cent oil-circuit-breaker gases and 80 per cent air (by volume) gave the highest pressures above indicated.

In regard to the *ignition of the gases*, investigations have shown that high-pressure secondary explosions are brought about not only by an igniting arc, (either the main circuit breaker arc or some other arc reaching the explosive gas mixture), but also by hot gases rising out of the oil and mixing with the air in the air space, *i. e.*, in the absence of an igniting arc. Thus secondary gas explosions are within common possibility unless suitable preventives are applied (to effect cooling of the rising gases, proper confinement of the arc, or elimination of sparks or arcs in the air space).

In addition to special investigations on this subject, the occurrence of secondary gas explosions from each of the causes stated above has been observed on numerous occasions during oil circuit breaker tests when the breakers were over-loaded beyond their rupturing capacity or when means for preventing secondary explosions were omitted.

E. C. Stone: I shall refer the question of secondary explosion to the circuit breaker designers.

With reference to the circuit breaker situation in transmission net works, present operating conditions probably depend on whether the circuit breakers at the present time are subjected to less or more than the duties which they can stand. In some cases over-capacity breakers may have been originally installed, or the growth may have been slow, while in other cases the breakers, when installed, may have had little factor of safety and systems may have grown very materially since.

I was very much interested in Doctor Roth's statement as to how the consideration of recovery voltage came about in Europe. In this country, two seconds is about the highest relay setting used. This limit has probably been set by the possible physical damage to equipment and disturbances to motor service. It seems to me that if we allowed five seconds duration of short circuit on our systems as they are operated in this country, we would get into all kinds of trouble, both with our executives and with our customers.

The recovery-voltage proposition came to my attention through a comparison of tests made on circuit breakers of the order of 500,000 kv-a. interrupting capacity on systems with dead-grounded neutral with those tested on systems where the neutral was grounded through fairly high resistance. A study of the oscillograms from the two systems brought out very clearly the difference in the recovery voltage under the two different conditions as to neutral grounding.

It seems to me very important, at the present time, that we should give very serious consideration to "recovery voltage." The fact that along the "decrement" curve the recovery voltage decreases as well as the current is another reason why recovery voltage should be given more serious consideration.

In this discussion the question of rebuilding existing systems has come up. In such a matter economics plays a very important part. We cannot afford to spend a very large amount of money for a benefit from which dividends will not be derived for a good many years in the future. On the contrary, we must see an almost immediate dividend on all money that is to be spent. The "fixed charges" saved in a step-by-step development will often be many times more than the increased cost of such step development.

There was an expression of difference of opinion between Mr. Kehoe and Doctor Roth with reference to the voltage at the arc in a fault. I am inclined to think that both were right. Mr. Kehoe referred particularly to underground systems where the arc is confined within close limits, while Doctor Roth referred to overhead systems where there is ample opportunity, particularly with a slight wind, for the arc to spread. Our experience on the system with which I am connected indicates quite clearly that the duty on oil circuit breakers on overhead systems is less than the duty on underground systems, where the theoretical maximum short-circuit current is the same in both cases. Perhaps the behavior of the arc as the breaker opens is one of the factors. Short circuits may occur on pole lines, as well as in cables, by failure of two insulators simultaneously, but in this case, if a wooden crossarm is used, there is considerable resistance in the short circuit.

One of our men recently reported a failure from lightning on a 66,000-volt line which was rather interesting. He saw the flash start at the insulator on a tower, spread to an arc between wires, which were about 7 ft. apart, and travel down the span for probably 15 ft. or more, after which the arc broke. It was, however, immediately reestablished at the insulator and the same action occurred again. This was repeated several times until the opening of circuit breakers cleared the line. If the circuit breaker had opened at the moment the arc was broken its duty would have been, of course, light. This illustrates the possibilities that are to be found on overhead systems where an unstable arc is involved.

The Quadrant Electrometer

BY W. B. KOUWENHOVEN*

Member, A. I. E. E.

Synopsis.—*The quadrant electrometer has long been recognized as a valuable instrument for measuring power in a-c. circuits, especially at low values of the power factor. Nevertheless, it has been used but little. This has been caused to some extent by the fact that Maxwell's treatment does not hold for all electrometers, and by the difficulty of determining its constant with alternating current.*

In this paper, the author has discussed the theory of the instrument from the viewpoint of power measurements and has reduced its general equation to a simple form. He has shown how its constants may be determined with continuous current, and that these constants also apply in a-c. power measurement. He has checked the theoretical calculations by experiments. Data of the determination of the constants of a given instrument with continuous current are given in the paper, as well as examples of the use of the instrument in the measurement of power in a number of alternating circuits having different characteristics.

THE quadrant electrometer has had long recognition as an instrument possessing excellent characteristics for the purpose of measurement. Nevertheless, it has been used but little. To a great extent, this has been due to the fact that its theory is not available in any one place, but one must search in foreign publications[1] to find it. With this instrument, it is possible to measure small alternating voltages and currents; also small amounts of power at very low values of power factor.

I. Theoretical

The instrument depends upon electrostatic repulsions and attractions for its torque. It consists of a needle,

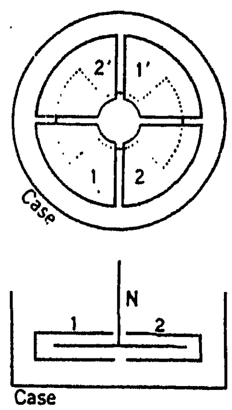


Fig. 1—Quadrant Electrometer

or disk, cut in the shape of a figure eight and suspended by means of a conducting fiber in a cylindrical metal box, cut into four equal quadrants. The needle and the quadrants are all mounted inside of a metal case, which protects them from external fields.

The case is fitted with a window through which the motion of the needle may be observed. The needle and the quadrants are insulated from each other and the case, and the latter is provided with leveling screws.

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1. For list of references see end of paper.

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The theory of this instrument was developed by Maxwell[2] as follows:

Let N = Needle potential
Let P_1 = Potential of quadrant pair No. 1
Let P_2 = Potential of quadrant pair No. 2
Let Q_0 = Charge on needle
Let Q_1 = Charge on quadrant pair No. 1
Let Q_2 = Charge on quadrant pair No. 2
Let Q_c = Charge on case

The case is grounded in practise and its potential is therefore zero.

The charges will be

$$\left.\begin{aligned} Q_0 &= K_{00}\,N + K_{01}\,P_1 + K_{02}\,P_2 \\ Q_1 &= K_{10}\,N + K_{11}\,P_1 + K_{12}\,P_2 \\ Q_2 &= K_{20}\,N + K_{21}\,P_1 + K_{22}\,P_2 \\ Q_c &= K_{c0}\,N + K_{c1}\,P_1 + K_{c2}\,P_2 \end{aligned}\right\} \quad (1)$$

Here, K_{00}, K_{01}, K_{02}, etc., are coefficients of induction of the conductors for electrostatic charges.

Since the needle and the quadrant pairs are completely enclosed by the case, the algebraic sum of the charges must equal zero and therefore,

$$Q_0 + Q_1 + Q_2 + Q_c = 0 \quad (2)$$

From equations (1) and (2), it follows that

$$(K_{00}+K_{10}+K_{20}+K_{c0})\,N + (K_{01}+K_{11}+K_{21}+K_{c1})\,P_1 + (K_{02}+K_{12}+K_{22}+K_{c2})\,P_2 = 0$$

and, since this relation must hold for all values of the potentials N, P_1 and P_2, we have:

$$K_{00} + K_{10} + K_{20} + K_{c0} = 0 \quad (3)$$

$$K_{01} + K_{11} + K_{21} + K_{c1} = 0 \quad (4)$$

$$K_{02} + K_{12} + K_{22} + K_{c2} = 0 \quad (5)$$

Now, assume that the needle is connected to quadrant pair No. 1, and at the same time grounded on the case; and that a positive potential P_2 is placed on quadrant pair No. 2. Then the needle will swing from quadrant pair No. 1 into quadrant pair No. 2. As the needle moves, some of the coefficients of electrostatic induction will change while others will remain constant.

If we let K' designate the new values of these coefficients we will have, for example, instead of (4),

$$K_{01}' + K_{11}' + K_{21}' + K_{c1}' = 0 \quad (6)$$

Now, $K_{21} = K_{21}'$ and $K_{c1} = K_{c1}'$ because there is no change in the relative positions between the two quadrant pairs or between the case and quadrants No. 1.

Therefore

$$K_{01}' + K_{11}' = K_{01} + K_{11}$$

As the needle swings from quadrant pair No. 1 into quadrant pair No. 2, K_{01} must decrease, and Maxwell assumed that

$$K_{01}' = K_{01} - X\,\theta \qquad (7)$$

where X is a constant and θ the angle through which the needle swings.

It follows from (7) that:

$$K_{11}' = K_{11} + X\,\theta \qquad (8)$$

In like manner, Maxwell showed that

$$K_{02}' = K_{02} + X\,\theta$$
$$K_{22}' = K_{22} - X\,\theta$$

and

$$K_{c0} = K_{c0}'$$
$$K_{c2} = K_{c2}'$$
$$K_{00} = K_{00}'$$

It is evident that the increase in the value of one coefficient of electrostatic induction, as the needle swings, must be balanced by an equal decrea sein the corresponding coefficient if equations (3), (4) and (5) are to hold.

The energy that the system possesses after the deflection, θ, is given by the relation:

$$W' = \tfrac{1}{2} K_{00}' N_0^2 + \tfrac{1}{2} K_{11}' P_1^2 + \tfrac{1}{2} K_{22}' P_2^2 + K_{01}' N P_1 + K_{02}' N P_2 + K_{12}' P_1 P_2 \qquad (9)$$

The change in energy produced by the deflection is given by the first differential of (9) with respect to the deflection, and keeping in mind the values of the coefficients of electrostatic induction we find:

$$\frac{dW'}{d\theta} = \tfrac{1}{2} X P_1^2 - \tfrac{1}{2} X P_2^2 - X N P_1 + X N P_2 \qquad (10)$$

The change in energy caused by the deflection is equal to the work done in twisting the suspension. If one assumes that k is the constant of the suspension per unit angle of deflection, one has,

$$k\,\theta = X(P_1 - P_2)\left(N - \frac{P_1 + P_2}{2}\right)$$

This is usually written

$$D\,\theta = (P_1 - P_2)\left(N - \frac{P_1 + P_2}{2}\right) \qquad (11)$$

This is Maxwell's equation for the quadrant electrometer and according to him, D is a constant.

The deflection of the electrometer depends upon the forces of attraction and repulsion between the charges on the needle and quadrants and the counter torque produced by twisting the suspension. Orlich and Schultz[3] showed that the D in Maxwell's equation is not a constant except for very small deflections and for some special electrometers. In most electrometers, D varies with the needle potential and the potential difference between the quadrants, and when the electrometer is used as a deflection instrument, this variation must be taken into account.

There are at least two reasons for the variation in D. The first is that the directing force changes as the needle deflects and, under these circumstances,

$$K_{01}' \neq K_{01} - X\,\theta$$

but the function is more complicated and is better expressed by

$$K_{01}' = K_{01} \pm x\,\theta \pm y\,\theta^2$$

The same is true of the electrostatic coefficients of induction, K_{02}', K_{11}', K_{22}' and K_{00}'. That the coefficient for the needle, K_{00}, should vary as the needle deflects is to be expected when you consider that it is practically impossible to get the needle exactly central.

The second reason for the variation in D is caused by the fact that the quadrant electrometer is very sensitive to small differences of potential when used in quadrant or power connection. Contact differences of potential exist between the needle, the case and the quadrants. If we let

p_0 = contact potential difference between needle and case

p_1 = contact potential difference between quadrant No. 1 and case

p_2 = contact potential difference between quadrant No. 2 and case

Then

$$N = V_0 + p_0$$
$$P_1 = V_1 + p_1$$
$$P_2 = V_2 + p_2$$

where V_0, V_1 and V_2 equal the outside potentials applied to the needle, quadrant pair No. 1 and quadrant pair No. 2, respectively.

We shall rewrite the equation (9) for the energy in the system and substitute the new values of N, P_1 and P_2, at the same time substituting

$$A_0 = \tfrac{1}{2} K_{00}' \qquad B_0 = K_{12}'$$
$$A_1 = \tfrac{1}{2} K_{11}' \qquad B_1 = K_{01}'$$
$$A_2 = \tfrac{1}{2} K_{22}' \qquad B_2 = K_{02}'$$

$$W' = A_0 (V_0 + p_0)^2 + A_1 (V_1 + p_1)^2 + A_2 (V_2 + p_2)^2 + B_0 (V_1 + p_1)(V_2 + p_2) + B_1 (V_0 + p_0)(V_1 + p_1) + B_2 (V_0 + p_0)(V_2 + p_2) \qquad (12)$$

Expanding the equation (12) for the energy in the system and grouping terms, we have

$$W' = A_0 V_0^2 + A_1 V_1^2 + A_2 V_2^2 + B_0 V_1 V_2 + B_0 V_0 V_1 + B_2 V_0 V_2 + B_1 V_0 V_1 + B_2 V_0 V_2 + (2 p_0 A_0 + p_1 B_1 + p_2 B_2) V_0 + (2 p_1 A_1 + p_2 B_0 + p_0 B_1) V_1 + (2 p_0 A_2 + p_1 B_0 + p_0 B_2) V_2 + A_0 p_0^2 + A_1 p_1^2 + A_2 p_2^2 + B_0 p_1 p_2 + B_1 p_0 p_1 + B_2 p_0 p_2 \qquad (13)$$

Let

$$C_0 = 2 p_0 A_0 + p_1 B_1 + p_2 B_2$$

Let

$$C_1 = 2 p_1 A_1 + p_2 B_0 + p_0 B_1$$

Let

$$C_2 = 2 p_2 A_2 + p_1 B_0 + p_0 B_2$$

Let

$$H = A_0 p_0^2 + A_1 p_1^2 + A_2 p_2^2 + B_0 p_1 p_2 + B_1 p_0 p_1 + B_2 p_0 p_2$$

Substitute these values in equation (**13**) and rewrite

$$W' = A_0 V_0^2 + A_1 V_1^2 + A_2 V_2^2 + B_0 V_1 V_2 + B_1 V_0 V_1 + B_2 V_0 V_2 + C_0 V_0 + C_1 V_1 + C_2 V_2 + H \quad (14)$$

The first six terms of equation (**14**) correspond to those in equation (**12**) and they are independent of the contact potentials. The C_0, C_1 and C_2 terms depend upon both the applied and contact potentials and terms containing contact potentials only are covered by the H term.

The new torque equation, whose derivation will be found in Part III of this paper, is derived from equation (**14**) and is given below:

$$\theta\,[1 + R\,(V_0 - V_1)\,(V_0 - V_2) + S\,(V_1^2 - V_2^2)] = a_0 V_0^2 + a_1 V_1^2 + a_2 V_2^2 + b_1 V_0 V_1 + b_2 V_0 V_2 + c_0 V_0 + c_1 V_1 + c_2 V_2 \quad (18)$$

The D of Maxwell's equation (**11**) equals the terms embraced in the brackets of equation (**18**) and it is no longer a constant, but varies with the potentials applied to the needle and quadrants. The right-hand side of equation (**18**) contains four terms corresponding to the four terms of Maxwell's equation (**11**) and in addition to these, it contains an $a_0 V_0^2$ term and three terms, $c_0 V_0$, $c_1 V_1$ and $c_2 V_2$, which depend upon the contact potentials and are not included in equation (**11**.) Equation (**18**) is the general equation of the quadrant electrometer. This equation will be further simplified in the experimental work that follows.

A study of equation (**18**) shows that there are two controling forces present in the quadrant electrometer. These are the mechanical control produced by the torsion of the suspension and the electrostatic control caused mainly by lack of symmetry in the instrument. The presence of the electrostatic control is indicated by the terms embraced in the brackets of equation (**18**). The algebraic sum of these two controling forces opposes the deflecting torque set up by the applied potentials. By proper adjustment and construction the electrostatic control may be made to add to the restoring torque of the suspension, to vanish entirely, or to neutralize the suspension torque. This last condition is used in the Crompton electrometer[1] to increase its sensitivity. In power measurements it is usual to reduce the effect of electrostatic control to as low a value as possible.

Setting Up of the Electrometer

The setting up of the electrometer may be divided into four parts:

1: *Leveling*. Remove the case and sighting through the quadrant slits, level in one direction. Turn through ninety degrees and level again. Continue turning and leveling until the instrument is perfectly level in all directions.

2. *Setting horizontal position of needle.* Bring the needle in position so that it is bisected by the quadrant slits and adjust its height until as nearly as can be determined by the eye, it is midway between the quadrants. Replace the case and make the final adjustment for the horizontal position. To accomplish this, connect both quadrants to the case and to ground and apply a high continuous positive potential to the needle. Note the deflection. Then reverse the continuous potential, making the needle negative, and note the deflection. Adjust the horizontal position of the needle with respect to the quadrant slits until the two deflections are equal. The needle is then accurately centered horizontally.

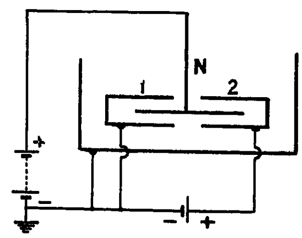

Fig. 2—Connections for Adjusting the Height of the Electrometer Needle

3. *Adjust for height.* Apply a high continuous potential to the needle and a small continuous potential between the quadrants as shown in Fig. 2. Adjust the height of the needle until the deflection is a minimum. The needle is then midway between the quadrant faces.

4. *Adjust so that the mechanical and electrical zeros*

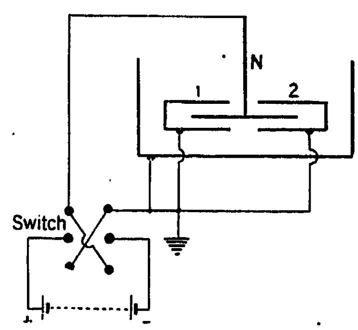

Fig. 3—Connections for Bringing the Mechanical and Electrical Zeros into Coincidence

coincide. Connect both quadrants to the case and to ground and apply a high continuous or alternating potential to the needle. Tilt the instrument by means of the leveling screws until the zero position remains the same with the voltage either on or off the needle. When this adjustment has been properly made, the position of maximum electrostatic capacity coincides with the natural zero of the instrument, and the mechanical and electrical zeros coincide. This adjustment is very important and should be checked when ever the instrument is used.

The connections used for bringing the mechanical and electrical zeros into coincidence is shown in Fig. 3. When this adjustment has been made, the $a_0\,V_0^2$ term of equation (**18**) is eliminated. This may be seen from a study of the values of the terms of the right hand side of equation (**18**)

Value of terms of equation (18)

Position of Switch	$a_0\,V_0^2$	$+a_1\,V_1^2$	$+a_2\,V_2^2$	$+b_1\,V_0\,V_1$	$+b_2\,V_0\,V_2$	$+c_0\,V_0$	$+c_1\,V_1$	$+c_2\,V_2$	Deflection
Up	+	0	0	0	0	+	0	0	$=\alpha$
Down	+	0	0	0	0	−	0	0	$=\beta$

When this adjustment has been properly made, $\alpha = \beta = 0$. Then if we take the sum of the two deflections we shall have

$$\alpha + \beta = 2\,a_0\,V_0^2 = 0$$

Therefore

$$a_0 = 0$$

It does not follow, however, that the $c_0\,V_0$ term also equals zero when the mechanical and electrical zeros coincide. The constant, c_0, includes contact potentials, and when the quadrants are not connected and grounded, as in Fig. 3, these may become of importance. The constant, a_0, however, does not depend upon the contact potentials, and it may safely be taken as zero when the electrometer is adjusted.

Neglecting the $a_0\,V_0^2$ term, equation (**18**) reduces to

$$\theta\,[1 + R\,(V_0 - V_1)\,(V_0 - V_2) + S\,(V_1 - V_2)^2] = a_1\,V_1^2 + a_2\,V_2^2 + b_1\,V_0\,V_1 + b_2\,V_0\,V_2 + c_0\,V_0 + c_1\,V_1 + c_2\,V_2 \qquad (19)$$

DETERMINATION OF THE CONSTANTS

In determining the constants, the quadrant connection is employed. In this connection, the needle is

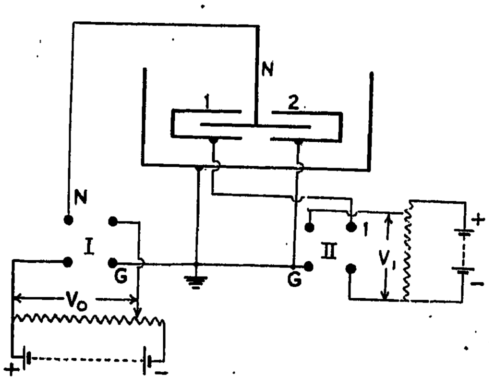

FIG. 4—QUADRANT CONNECTION FOR DETERMINING THE ELECTROMETER CONSTANTS

at a high potential, one quadrant pair is connected to the case and grounded and the other is at a low potential. The constants may be determined by using alternating current[4] and measuring the power consumed in a load of known characteristics, but they are most readily determined with continuous current. The connections are shown in Fig. 4.

In the diagram of connections, V_2 is zero, V_1 is a relatively low potential and is very small compared to V_0, the needle voltage. Under these conditions, it is possible to neglect certain of the terms which form the left hand side of equation (**19**) and we may write

$$\theta\,[1+R\,V_0^2] = a_1\,V_1^2 + a_2\,V_2^2 + b_1\,V_0\,V_1 + b_2\,V_0\,V_2 + c_0\,V_0 + c_1\,V_1 + c_2\,V_2 \qquad (20)$$

In Fig. 4 there are two reversing switches or commutators marked with the Roman numerals, I and II respectively. There are four possible positions of the two commutators and the terms forming the right hand side of equation (**20**) have different signs depending upon the positions of the commutators. We obtain a deflection for each of the four positions of the commutators. The deflections and the signs or values of the corresponding terms are given below:

Position of Commutator I	II	Value of Terms $a_1 V_1^2$	$+a_2 V_2^2$	$+b_1 V_0 V_1$	$+b_2 V_0 V_2$	$+c_0 V_0$	$+c_1 V_1$	$+c_2 V_2$	$=[1+R V_0^2]$	Deflection θ
‖	‖	+	0	−	0	+	−	0	=["]	α
=	‖	+	0	+	0	−	−	0	=["]	β
=	=	+	0	−	0	−	+	0	=["]	γ
‖	=	+	0	+	0	+	+	0	=["]	δ

The constants are determined from the above deflections as follows:

$$[1 + R\,V_0^2]\,(\alpha - \beta + \gamma - \delta) = -4\,b_1\,V_0\,V_1 \qquad (21)$$

$$[1 + R\,V_0^2]\,(\alpha - \beta - \gamma + \delta) = +4\,c_0\,V_0 \qquad (22)$$

$$[1 + R\,V_0^2]\,(\alpha + \beta - \gamma - \delta) = -4\,c_1\,V_1 \qquad (23)$$

$$[1 + R\,V_0^2]\,(\alpha + \beta + \gamma + \delta) = +4\,a_1\,V_1^2 \qquad (24)$$

If we take readings of the deflections for two or more values of the voltages V_0 and V_1 we shall obtain sufficient data to solve the above equations for the constants. For example, let α', β', γ' and δ' equal the deflections obtained when V_0' and V_1' are the voltages applied to the needle and quadrant 1 respectively. Then from equation (**21**) we obtain

$$\frac{[1+R\,V_0'^2]}{b_1} = -\frac{4\,V_0'\,V_1'}{\alpha' - \beta' + \gamma' - \delta'} = X_1 \qquad (25)$$

For applied voltages V_0'' and V_1'' we get the deflections $\alpha'', \beta'', \gamma''$ and δ'' for the four positions of the commutations respectively, and equation (**26**) follows:

$$\frac{[1+R\,V_0''^2]}{b_1} = -\frac{4\,V_0'\,V_1''}{\alpha'' - \beta'' + \gamma'' - \delta''} = X_2 \qquad (26)$$

Here X_1 and X_2 are the numerical values of the right hand sides of equations (**25**) and (**26**) respectively.

Solving these two simultaneous equations for b_1 we find that,

$$b_1 = \frac{V_0'^2 - V_0''^2}{X_2\,V_0'^2 - X_1\,V_0''^2} \qquad (27)$$

and substituting the value of b_1 in equation (25) we obtain,

$$R = \frac{b_1 X_1 - 1}{V_0^{\prime 2}} \tag{28}$$

In a similar manner, we can obtain the other constants from equations (22), (23) and (24).

It is evident from the symmetrical construction of the electrometer that

$$\begin{aligned} a_2 &= -a_1 \\ b_1 &= -b_2 \\ c_1 &= -c_2 \end{aligned}$$

We may also prove that:

$$\begin{aligned} a_2 &= \tfrac{1}{2} b_1 \\ a_1 &= -\tfrac{1}{2} b_1 \end{aligned} \tag{33}$$

The mathematical proof of equation (33) is given in the mathematical part of this paper and the confirming experimental data will be found in the experimental part.

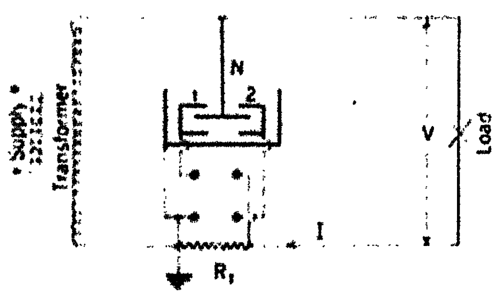

FIG. 5 CONNECTIONS FOR MEASURING POWER WITH FULL VOLTAGE ON NEEDLE

If we substitute these values of the constants in equation (19) it becomes,

$$[1 + R(V_0 - V_1)(V_0 - V_2) + S(V_1^2 - V_2^2)]\,\theta$$

$$= -\frac{b_1}{2} V_1^2 + \frac{b_1}{2} V_2^2 + b_1 V_0 V_1 - b_1 V_0 V_2 + c_0 V_0$$

$$+ c_1 V_1 + c_2 V_2 \tag{34}$$

If the quadrant electrometer is to be used in measuring power in a-c. circuits, it is only necessary to know the constants R and b_1. The other constants may either be reduced to zero or made negligible.

POWER MEASUREMENT

The quadrant connection is the one employed in measuring power in a-c. circuits. The quadrant electrometer may be connected so that there is either full voltage on the needle or only a fraction of the full voltage. A diagram of the connections for full voltage on the needle is shown in Fig. 5. The effective voltage across the load is V and the effective value of the load current is I. R_1 is a non-inductive capacity free resistance. Only one commutator is used.

We shall assume that the instantaneous value of the voltage and current are designated by v and i respectively and that Φ is the phase angle between them. Then

$$\begin{aligned} v &= \sqrt{2}\, V \sin \omega t \\ i &= \sqrt{2}\, I \sin (\omega t \pm \Phi) \end{aligned}$$

When a quadrant electrometer is supplied with alternating current, the needle potential V_0, and the quadrant potentials, V_1 and V_2, are the instantaneous values of the applied voltages. The electrometer reads the integral of these voltages over a complete period T.

For example:

$$a_1 V_1^2 = \frac{1}{T} \int_0^T 2 I^2 R_1^2 \sin^2 (\omega t \pm \Phi) = I^2 R_1^2$$

$$c_0 V_0 = \frac{1}{T} \int_0^T \sqrt{2}\, V_0 \sin \omega t = 0$$

It follows, from the above, that the c_0, c_1 and c_2 terms will all equal zero when the electrometer is used to measure power in a-c. circuits.

On alternating power measurements, V_1 and V_2, the quadrant potentials, are small compared to V_0, the needle potential and we may therefore neglect the S term, and as the c_0, c_1 and c_2 terms are equal to zero, we can write equation (19)

$$[1 + R V_0^2]\,\theta = -\frac{b_1}{2} V_1^2 + \frac{b_1}{2} V_2^2$$

$$+ b_1 V_0 V_1 - b_1 V_0 V_2 \tag{35}$$

We shall apply this equation to the measurement of the power consumed by the load of Fig. 5, bearing in mind the fact that V_0 is the instantaneous value of the potential from the needle to ground, and that V_1 and V_2 are the values of the instantaneous potentials from quadrant pairs No. 1 and No. 2 to ground respectively.

In Fig. 5, we see that,

$$\begin{aligned} V_0 &= v + i R_1 \\ V_2 &= i R_1 \text{ or } 0 \\ V_1 &= 0 \text{ or } i R_1 \end{aligned}$$

The values of V_1 and V_2 depend upon the position of the commutator.

The integral of the $b_1 V_0 V_1$ term over a complete period is as follows:

$$\frac{b_1}{T} \int_0^T V_0 V_1 = \frac{b_1}{T} \int_0^T (v + i R_1)\,(i R_1)$$

$$= b_1 (R_1 I V \cos \Phi + I^2 R_1^2)$$

There are two positions of the commutator and the value and signs of terms of equation (35) are given below. The circuit conditions and the applied voltage must remain constant while the deflections are measured.

VALUE OF TERMS

Position of Commutator	$-\frac{b_1}{2} V_1^2$	$+\frac{b_1}{2} V_2^2$	$+b_1 V_0 V_1$	$-b_1 V_0 V_2$	Deflection
‖	0	$+\frac{b_1}{2} I^2 R_1^2$	0	$-b_1(R_1 I V \cos \Phi + I^2 R_1^2)$	$= [1 + R V_0^2]\ \alpha$
=	$-\frac{b_1}{2} I^2 R_1^2$	0	$+b_1(R_1 I V \cos \Phi + I^2 R_1^2)$		$= [1 + R V_0^2]\ \beta$

Taking the algebraic sum of these deflections, we get

$$[1 + R V_0^2] (\beta - \alpha)$$
$$= - b_1 I^2 R_1^2 + 2 b_1 (R_1 I V \cos \Phi + I^2 R_1^2) \quad (36)$$

The needle voltage, V_0, is equal to the vector sum of the load voltage, V, plus the drop across the non-inductive resistance R_1. Since the drop is very small compared to the load voltage we may assume that V_0 on the left hand side of equation (36) is equal to V.

Rewriting equation (36) and combining like terms we obtain

$$\frac{[1 + R V^2] (\beta - \alpha)}{2 b_1 R_1} = I V \cos \Phi + \frac{I^2 R_1}{2} \quad (37)$$

It is evident from equation (37) that the quadrant electrometer, with full voltage on the needle, measures

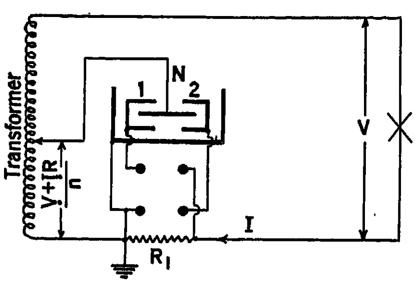

Fig. 6—Measurement of Power with Less than Full Line Voltage on Needle

the power consumed by the load, plus one-half the power lost in the resistance, R_1.

The power consumed by the load is

$$I V \cos \Phi = \frac{[1 + R V^2]}{2 b_1} \cdot \frac{1}{R_1} (\beta - \alpha) - \frac{I^2 R_1}{2} \quad (38)$$

Equation (37) holds for all values of power factor, both leading and lagging. The only constants involved are b_1 and R which may be determined as already outlined.

If R has a negligible value or if readings are taken on a number of loads at a constant voltage V, then the expression

$$\frac{1 + R V^2}{2 b_1} = K$$

where K is a constant.

The deflections α and β for the two positions of the commutators are right and left deflections respectively and the resulting deflection $(\beta - \alpha)$ is usually written equal to θ. Equation (37) may then be written as follows:

$$\frac{K \theta}{R_1} = I V \cos \Phi + \frac{I^2 R_1}{2} \quad (39)$$

If the quadrant electrometer is used to measure power with less than full voltage on the needle, the diagram of connections are given in Fig. 6 is applicable. Here V, I and R_1 have the same meanings as in Fig. 5. In

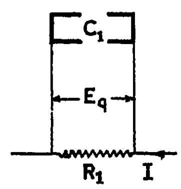

Fig. 7—Condenser C_1 Formed by the Electrometer Quadrants

this case the needle voltage equals the vector sum of the load voltage, V, plus the $I R_1$ drop divided by n, where n is the factor by which the voltage across the

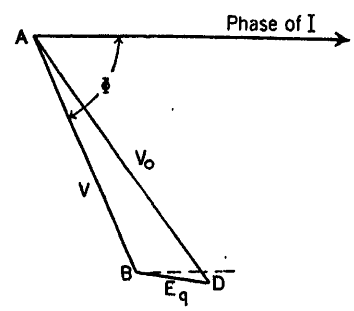

Fig. 8—Vector Diagram of the Circuit Shown in Fig. 5 for a Condenser Load

needle must be multiplied to give the total voltage of the circuit.

The instantaneous value of the voltage across the needle is

$$\frac{v + i R_1}{n}$$

There are two positions of the commutator and the value and signs of the terms of equation (35) for reduced voltage on the needle are given as follows:

VALUE OF TERMS

Position of Commutator	$-\frac{b_1}{2} V_1^2$	$+\frac{b_1}{2} V_2^2$	$+b_1 V_0 V_1$	$-b_1 V_0 V_2$	Deflection
||	0	$+\frac{b_1}{2} I^2 R_1^2$	0	$-b_1\left(\frac{R_1 I V \cos\Phi + I^2 R_1^2}{n}\right)$	$= [I + R V_0^2]\ \alpha$
=	$-\frac{B_1}{2} I^2 R_1^2$	0	$+b_1\left(\frac{R_1 I V \cos\Phi + I^2 R_1^2}{n}\right)$	0	$= [\ + R V_0^2]\ \beta$

Taking the algebraic sum of these two deflections, we obtain,

$$[1 + R V_0^2]\ \theta$$

$$= -b_1 I^2 R_1^2 + 2 b_1 \frac{R_1 I V \cos\Phi}{n} + 2 b_1 \frac{I^2 R_1^2}{n} \tag{40}$$

$$[1 + R V_0^2]\ \theta$$

$$= \frac{2 b_1}{n} R_1 \left(I V \cos\Phi + \frac{2 - n}{2} I^2 R_1 \right) \tag{41}$$

as stated above, the needle voltage is equal to

$$V_0 = \frac{\dot{V} + \dot{I} R_1}{n}$$

In practise, $\dot{I} R_1$ is very small compared with V. It is usually less than one per cent of the load voltage and for practical purposes we may write

$$V_0 = \frac{V}{n}$$

We obtain, when we make this substitution in equation (41),

$$\frac{\left[1 + R\left(\frac{V}{n}\right)^2\right]}{2 b_1} \cdot \frac{n}{R_1} \cdot \theta$$

$$= I V \cos\Phi + \frac{2 - n}{2} I^2 R_1 \tag{42}$$

If n equals one, equation (42) reduces to equation (37), which is the power equation for full voltage on the needle of the quadrant electrometer. A further study of equation (42) shows that if n is equal to two, the last term of the equation vanishes; if n is greater, the last term becomes negative. This is true at all values of power factor. These deductions are verified by experiments reported in the experimental part of the paper.

In special cases, equation (42) may be written

$$\frac{K n \theta}{R_1} = I V \cos\Phi + \frac{2 - n}{2} I^2 R_1 \tag{43}$$

Equation (43), however, holds only for the conditions where R is negligible or where readings are taken with a constant voltage applied to the needle.

Equation (35) gives the simplified form of the quadrant electrometer power relations. In this equation θ equals the algebraic sum of the two deflections α and β for the two positions of the commutator. The circuit conditions and the applied voltage are assumed to remain constant while these readings are being taken. Under these conditions we noted that

$$V_1 = I R_1 \text{ or } 0$$

and

$$V_2 = 0 \text{ or } I R_1$$

Therefore, if we determine the signs and the values of the terms of equation (35) for the two commutator positions and take their algebraic sum, we may write

$$[1 + R V_0^2]\ \theta = 2 b_1 V_0 V_1 - b_1 V_1^2 \tag{44}$$

This equation is a simplified form of equation (35). If we substitute in equation (44), the values of the needle voltage to ground, V_0, and the drop across the resistances, V_1, expressed in the form of their vector relations, we obtain exactly the same result as was found by the method used in determining equations (37) and (41). An example of this method is given in the paragraphs devoted to the question of errors.

Errors

The sources of error in the measurement of power with the quadrant electrometer have already been ably discussed in an Institute paper[5]. Therefore only a single case, illustrating the applications of equation (44), will be considered in this paper.

If we study the diagrams of connections in Fig. 5 and 6, we see that the electrostatic capacity between the quadrants pairs, No. 1 and No. 2, and their leads is in parallel with the non-inductive resistance R_1. The drop across R_1 and this condenser in parallel with it will not be in phase with I as assumed. Let C_1 equal the capacity of this condenser in farads. Then if E_q equals the voltage across the quadrants we find that

$$E_q = \frac{I R_1 (1 - j \omega C_1 R_1)}{1 + \omega^2 C_1^2 R_1^2} \tag{45}$$

The vector diagram of the electrometer, with full voltage across the needle and a capacity load in the circuit, is shown in Fig. 8. In Fig. 8 $A B = V$, the voltage across the load; $B D = E q$, the voltage drop across R_1 and C_1 in parallel, and $A D = V_0$, the voltage across the needle. $A D$ is also the vector sum of V and $E q$.

With I as the reference vector, the vector equations are as follows:

$$\dot{V} = V \cos\Phi - j \sin\Phi$$

$$\dot{V}_1 = \frac{I R_1}{1 + \omega^2 C_1^2 R_1^2} - j \frac{I \omega C_1 R_1^2}{1 + \omega^2 C_1^2 R_1^2}$$

$$V_0 = V\cos\Phi - j\,V\sin\Phi + \frac{I\,R_1}{1+\omega^2 C_1^2 R_1^2} - j\,\frac{I\,\omega\,C_1 R_1^2}{1+\omega^2 C_1^2 R_1^2}$$

Substitute the vector relations of V_0 and V_1 in equation (44) and take the power product and simplify

$$\frac{(1+R\,V^2)\,(1+\omega^2 C_1^2 R_1^2)}{2\,b_1 R_1}\,\theta = I\,V\cos\Phi + \tfrac{1}{2}\,\frac{I^2 R_1}{(1+\omega^2 C_1^2 R_1^2)} + I\,(\omega\,C_1 R_1)\,V\sin\Phi + \tfrac{1}{2}\,\frac{I^2 R_1\,(\omega\,C_1 R_1)^2}{1+\omega^2 C_1^2 R_1^2} \quad (46)$$

The capacity C_1 is, however, very small and at commercial frequencies $\omega^2\delta C_1^2 R_1^2$ is negligible. Therefore, we may write

$$\frac{(1+R\,V^2)}{2\,b_1 R_1}\,\theta = I\,V\cos\Phi + \frac{I^2 R_1}{2} + I\,(\omega\,C_1 R_1)\,V\sin\Phi \quad (47)$$

Equation (47) is the same as that derived by Messrs. Simons and Brown[5] using a different method.

Other sources of error are the charging current flowing from the needle to the high quadrant and through the shunt resistance R_1, the phase shift of the needle voltage introduced by the use of a potential divider, the phase shift due to the resistance of the needle suspension, and any energy loss that may be present in the electrometer itself.

II—Experimental

Determination of the Constants b_1 and R. The constants, b_1 and R, are best determined with continuous potentials, although they may be determined with alternating potentials.

The diagram of connections is given in Fig. 4. The voltage applied to the needle could be varied through a range of from about 50 to 600 volts. It was measured with a Weston laboratory standard voltmeter. The voltage applied to the quadrants was varied from 2 to 15 volts and was measured with a Siemens and Halske instrument.

The quadrant electrometer was set up, leveled, and carefully adjusted, with about 600 volts on its needle, until its electrical and mechanical zeros coincided. Then readings of the deflections for the different positions of the two commutators were taken for various values of V_0 and V_1. Four readings were taken for each set of voltages, which were maintained constant while the deflections were noted. In taking a set of readings it is best to commutate in a regular manner and read at regular intervals after commutating.

The electrometer used was one constructed in the electrical laboratories of the Johns Hopkins University. The quadrants were of aluminum. The inside diameter of the cylindrical box from which they were cut was 15.24 cm. and its height was 3.81 cm. The needle was also of aluminum, and the length of its phosphor bronze suspension was approximately 20 cm. The quadrant pairs and the needle were insulated by special bakelite supports.

The readings upon which the calculations of the constant b_1 and R are based are given in Table I.

TABLE I

V_0 Volts	V_1 Volts	Position of Commutator No. I	No. II	cm.	cm. $\theta = \alpha-\beta+\gamma-\delta$	$\frac{4V_0V_1}{\theta}$
100	5	‖	‖	$-3.6 = \alpha$		
		=	‖	$+3.35 = \beta$		
		=	=	$-3.6 = \gamma$		
		‖	=	$+3.35 = \delta$	−13.9	+143.9
100	10	‖	‖	−7.35		
		=	‖	+6.52		
		=	=	−7.32		
		‖	=	+6.55	−27.74	+144.1
100	15	‖	‖	−11.3		
		=	‖	+9.65		
		=	=	−11.3		
		‖	=	+9.65	−41.9	+143.2
270	5	‖	‖	−9.6		
		=	‖	+9.3		
		=	=	−9.6		
		‖	=	+9.3	−37.8	+142.5
270	10	‖	‖	−19.6		
		=	‖	+18.35		
		=	=	−19.6		
		‖	=	+18.35	−75.9	+143.5
500	2	‖	‖	−7.12		
		=	‖	+7.55		
		=	=	−7.1		
		‖	=	+7.5	−29.27	+136.6
500	5	‖	‖	−18.3		
		=	‖	+18.15		
		=	=	−18.25		
		‖	=	+18.15	−72.85	+137.4

The first three sets of readings were taken with V_0 equal to 100 volts and they give an average X_1 of 143.73. The next two sets of readings are for V_0 equal to 270 volts, and X_2 equals 143.0. The last two sets of readings were at 500 volts and X_3 average equals 137.0.

Calculating b_1 from these results in accordance with equation (27) we get an average b_1 of

$$b_1 = +\,0.00695$$

calculating R from equation (28) gives

$$R = -\,1.95 \times 10^{-7}$$

Determination of the Constant a_1. In order to check equation (24)

$$[1 + R\,V_0^2]\,(\alpha+\beta+\gamma+\delta) = +\,4a_1\,V_1^2 \quad (24)$$

an attempt was made to determine a_1 by this connec-

tion. Since a_1 depends upon the value of the quadrant voltage, V_1 was made equal to V_0 in this test. The results are given in Table II.

TABLE II

Position of Commutator No. I	Position of Commutator No. II	V_0 Volts	V_1 Volts	cm.	$\theta = \alpha + \beta + \gamma + \delta$ cm.
‖	‖	25	25	$-6.6 = \alpha$	
=	‖			$+2.35 = \beta$	
=	=			$-6.6 = \gamma$	
‖	=			$+2.35 = \delta$	-8.5
‖	‖	40	40	-17.25	
=	‖			$+5.65$	
=	=			-17.3	
‖	=			$+5.65$	-23.25

The calculation of a_1 from the results in Table II and in accordance to equation (24) gave

$$a_1 = +0.00351$$

According to the theory a_1 has half the value and a sign opposite to that of b_1 and it is clear that this test gives a fair check of the theoretical calculation. This method is, however, not suited for the determination of a_1, because in order to get readings of sufficient difference to make it possible to calculate a_1 with any degree of accuracy we must make the quadrant voltage large compared to the needle voltage. This is not in accordance with the theory of the quadrant connection which presupposes a high needle voltage and a low quadrant voltage.

A better way to prove that a_1 has half the value of b_1 is to use the two idiostatic connections A and B of the electrometer which are shown in Fig. 12.

The readings for connections A and B are given in Table III for an applied voltage of 44.5 volts in each case.

TABLE III

Position of Commutators III	Position of Commutators II	Connection A Deflections cm.	Connection α Deflections cm.
‖	‖	+6.6	+6.6
=	‖	+6.6	+6.6
=	=	−6.1	−6.1
‖	=	−6.1	−6.1

Exactly the same deflections were found for both connections A and B and they were in the same directions. Therefore equation (33), whose proof is given in the mathematical part of the paper is true.

$$a_2 = \tfrac{1}{2} b_1 \qquad (33)$$
$$a_1 = -\tfrac{1}{2} b_1$$

Proof of the Power Equation with Full Voltage on Needle. Equation (37) states that the deflection of a quadrant electrometer is proportional to the power consumed in the load plus one-half the power loss in the shunt resistance R_1 at any value of the power factor.

In order to check this relation the quadrant electrometer was connected up as in Fig. 9, and three loads were used in turn. The first load was a non-inductive resistance R_0. The second load consisted of an inductance, L, and the third, of a mica condenser, C.

The test at unity power-factor load will be considered first. In this test, the secondary voltage of the trans-

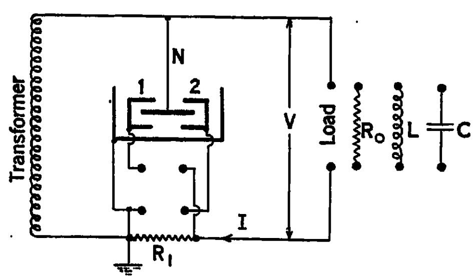

FIG. 9—QUADRANT ELECTROMETER WITH FULL VOLTAGE ON NEEDLE AND DIFFERENT LOADS

former was maintained constant at 100 volts and 60 cycles. Readings of the deflections were taken with three values of R_1, namely, 100, 2000, and 7000 ohms respectively. The value of the load resistance, R_0, was adjusted for each value of R_1 so that the power loss in R_0 was maintained constant at 0.32 watts.

The constant of the electrometer with 100 volts applied to the needle is

$$K = \frac{1 + R V^2}{2 b_1} = \frac{(1 - 1.95 \times 10^{-7} \times 100^2)}{2 \times 0.00695} = 71.8$$

The results of the unity power factor test are given in Table IV.

TABLE IV

R_0 Ohms	R_1 Ohms	I Milli-Ampere	$I^2 R_0$ Load Watts	$\frac{I^2 R_1}{2}$ Watts	Electrometer θ cm.	Electrometer $\frac{K\theta}{R_1}$ Watts	$I^2 R_0 + \frac{I^2 R_1}{2}$ Watts calculated
29250	1000	3.31	0.32	0.006	4.5	0.324	0.326
27125	2000	3.44	0.32	0.013	9.45	0.339	0.333
13650	7000	4.83	0.32	0.082	39.6	0.406	0.402

In Table IV, the current, I, is calculated from the known constants of the circuit. The watts consumed in the load, I^2R_0, and one-half the loss in the shunt resistance R_1 are also calculated, as are the values given in the last column.

A comparison of the watts registered by the electrometer for the different values of R_1 and the watts calculated, as given in the last column of Table IV, shows a fairly close agreement. A comparison of the last two columns of Table IV shows conclusively that at unity power factor equation (37) holds and the instruments read the watts lost in the load plus one-half the loss in the resistance shunting the quadrants.

The inductive load, lagging power factor test followed the resistance test. The inductance used as the load consisted of an air cored copper coil with a resistance of

1650 ohms and a coefficient of self-induction of 7.73 Henrys. At a frequency of 60 cycles the reactance was 2915 ohms. Readings were taken at two values of R_1 and the applied voltage was adjusted so as to maintain the current constant and thus keep the load at a uniform amount.

The voltages used were 55 and 46.5 volts respectively, and at this value of the voltage the constant of the electrometer = 72.

The results are given in Table V. In this test the power factor was 56.6 per cent lagging.

TABLE V

Voltage	R_1 Ohms	Circuit Impedance Ohms	I Milli-amp.	Load Watts	$\frac{I^2 R_1}{2}$ Watts	Electrometer θ cm.	Electrometer $\frac{K\theta}{R_1}$ Watts	Load $+\frac{I^2 R_1}{2}$ Watts Calculated
46.5	1000	4664	1.178	0.23	0.069	4.1	0.296	0.299
55.0	2000	3940	1.178	0.23	0.138	10.2	0.367	0.368

A comparison of the watts as measured by the quadrant electrometer with the calculated watts shows a good agreement. A study of the last two columns of Table V shows conclusively that, at a lagging power factor of 56.6 per cent, the electrometer reads the power consumed by the load plus one-half the power lost in the shunt resistance.

The capacity load test was made with a standard mica condenser whose capacity was 1/10 of a microfarad. The frequency used was 60 cycles. Readings were taken at three values of the shunt resistance R_1 and the applied voltage was varied between the limits of 396 and 400 volts so as to maintain the current through the circuit and the load constant. At 400 volts the constant of the electrometer is 69.7. The results are given in Table VI.

TABLE VI

Voltage	R_1 Ohms	Circuit Impedance Ohms	I Milli-amp.	Load Watts	$\frac{I^2 R_1}{2}$ Watts	Electrometer θ cm.	Electrometer $\frac{K\theta}{R_1}$ Watts	Load $+\frac{I^2 R_1}{2}$ Watts
396	1000	26520	14.9	0.027	0.111	1.95	0.136	0.138
398	2000	26570	14.9	0.027	0.222	7.15	0.249	0.249
400	4000	26800	14.9	0.027	0.444	27.10	0.472	0.471

The watts consumed by the load are determined by subtracting from the watts measured by the electrometer one half of the loss in the shunt resistance R_1. Another method of determining the watts load is to plot the watts measured by the electrometer as ordinates against the values of the shunt resistance R_1 as abcissas. These points should lie on a straight line, as shown in Fig. 10. If this line is extended to the ordinate axis it will cut that axis at a value equal to watts lost in the load. From Fig. 10 we see that the load loss is equal to 0.027 watts. The power factor of the condenser load used in this test is 0.47 per cent.

A comparison of the results given in the last two columns of Table VI show clearly that the quadrant electrometer with full voltage on the needle reads the loss in the load plus one-half the loss in the shunt resistance R_1. The curve plotted from the results in Fig. 10 also confirms this conclusion.

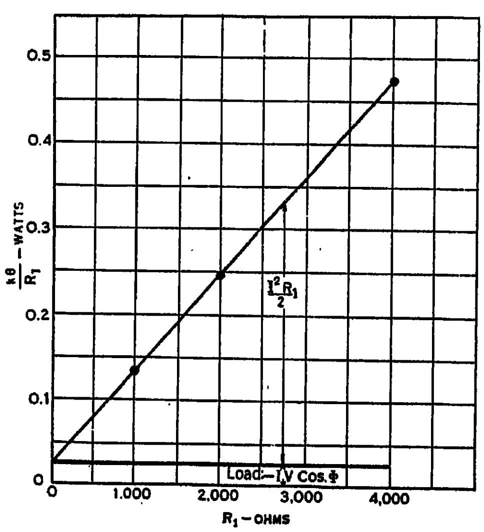

Fig. 10—Curve Showing the Relation Between Watts Measured by the Electrometer and the Shunt Resistance R_1 with Full Voltage on the Needle. Plotted from the Results of Table VI

The results of these three tests prove conclusively the correctness of equation (37), namely, that the quadrant electrometer with full voltage on the needle reads the power consumed by the load plus one-half the loss in the shunt resistance, irrespective of the value of the power factor.

Proof of the Power Equation with Reduced Voltage on the Needle. Equation (42) applies to this type of measurement.

$$\frac{\left[1 + R\left(\frac{V}{n}\right)^2\right]}{2\, b_1} \cdot \frac{n}{R_1} \cdot \theta = I\, V \cos \Phi + \frac{2-n}{2} I^2 R_1 \tag{42}$$

The sign of the last term, or correction term, depends upon the value of n. If n is equal to two, the term vanishes. If n is greater than two, the term becomes negative. In order to check this relation, readings were taken at unity power factor load for n equal to three different values. The connections used are those given in Fig. 6.

The first test was run with n equal to two and the voltage of the transformer was 160 volts at 60 cycles. The voltage applied to the electrometer needle was one-half of the transformer voltage and equaled 80 volts.

The constant of the electrometer for this needle voltage was 71.9.

Readings were taken at three values of the shunt resistance, and the load which consisted of a non-inductive resistance was maintained constant. The results are given in Table VII.

TABLE VII

R_0 Ohms	R_1 Ohms	I Milli-amp.	$I^2 R_0$ Load Watts	Electrometer θ cm.	$\frac{n\,K\,\theta}{R_1}$ Watts
29250	1000	5.29	0.825	5.75	0.826
27125	2000	5.49	0.825	11.5	0.826
20000	5000	6.4	0.82	28.7	0.825

It is apparent from the results given in Table VII that when n equals two, the quadrant electrometer reads directly the power consumed in the load, and the correction term vanishes.

The second unity power-factor test was made with one-third of the transformer voltage applied to the needle, that is, n equaled three. The transformer voltage in this test was 240 volts at 60 cycles and this gave a needle voltage of 80, the same as used in the run with n equal to two. The electrometer constant equaled 71.9 in this test also.

For the value of n equal to three, the correction of equation (42) becomes negative and equals

$$-\frac{I^2 R_1}{2}$$

The results of this test are given in Table VIII. Readings were taken for three different values of shunt resistance R_1.

TABLE VIII

R_0 Ohms	R_1 Ohms	I Milli-amp.	$I^2 R_0$ Load Watts	$\frac{I^2 R_1}{2}$ Watts	Electrometer θ cm.	$\frac{n\,K\,\theta}{R_1}$ Watts	$I^2 R_0 - \frac{I^2 R_1}{2}$ Watts Calculated
29250	1000	7.93	1.84	0.032	8.45	1.82	1.81
27125	2000	8.24	1.84	0.068	16.2	1.75	1.77
13650	7000	11.62	1.84	0.473	44.2	1.36	1.37

It is evident from the results given in Table VIII that when n is equal to three the correction term becomes negative. The check between the calculated watts and the watts as measured by the electrometer is good.

The third unity power-factor test was made with n equal to four. Under this condition equation (42) becomes

$$\frac{\left[1 + R\left(\frac{V}{n}\right)^2\right]}{2\,b_1}\,\frac{n}{R_1}\,\theta = I\,V\cos\Phi - I^2 R_1$$

Readings were taken for three conditions. In the first, the loss in the shunt resistance was less than the load. In the second, the loss in the load and the loss in the shunt resistance were equal, and in the third, the loss in the shunt resistance was greater than the loss in the load. This last condition should give a negative deflection of the electrometer.

The voltage of the transformer was 400 volts at 60 cycles. This gives, with n equal to four, a needle voltage of 50 volts and an electrometer constant of 72. The results are given in Table IX, which is divided into two parts. Part I gives the calculated watts and Part 2 gives the electrometer readings.

TABLE IX

Part I

R_0 Ohms	R_1 Ohms	I Milli-Amp.	$I^2 R_0$ Load Watts	$I^2 R_1$ Watts	$I^2 R_0 - I^2 R_1$ Watts Calculated
15000	10000	8.	0.96	0.64	+0.32
10000	10000	10.	1.0	1.0	0
10000	15000	8.	0.64	0.96	−0.32

Part II

R_0 Ohms	R_1 Ohms	Electrometer: Position of Commutator	Readings cm.	Deflection cm.	$\frac{n\,K\,\theta}{R_1}$ Watts
15000	10000	‖ =	+6.2 −5.3	+11.5	+0.33
10000	10000	‖ =	−0.7 −0.7	0	0
10000	15000	‖ =	−7.15 +9.1	−16.25	−0.312

A study of Table IX shows that the electrometer reading reversed and became negative when the loss in the shunt resistance was greater than the load loss as predicted. A comparison of Parts I and II shows clearly that the experimental results check the theory.

The experimental results of Tables VII, VIII, and IX definitely prove that the sign of the correction term of equation (42) depends upon the value of n and not upon the value of the power factor.

In order to prove that equation (42) holds at low power factor, a set of results is given in Table X that were taken on another electrometer which was convenient. The load consisted of a high-voltage condenser of about 0.02 microfarads capacity. The voltage applied was 7500 volts at 60 cycles. The value of n was six and the electrometer constant was 83.5. The current was measured by another electrometer which was used as a voltmeter.

For n equal to six, the correction term becomes $-2\,I^2 R_1$.

TABLE X

R_1 Ohms	I Milli-ampere	Load Watts	$2\,I^2 R_1$ Watts	Electrometer θ cm.	$\frac{n\,K\,\theta}{R_1}$ Watts	Load $-2\,I^2 R_1$ Watts Calculated
2000	4.7	0.264	0.085	+0.7	+0.175	+0.179
4000	4.7	0.264	0.177	+0.7	+0.0875	+0.087
5000	4.7	0.264	0.221	+0.4	+0.040	+0.043
8000	4.7	0.264	0.354	−1.45	−0.09	−0.09
10000	4.7	0.264	0.442	−3.55	−0.178	−0.178

The watts measured by the electrometer are plotted against R_1 in Fig. 11. The readings lie on a straight line and the intercept of this line on the Y axis gives a loss in the condenser under test of 0.264 watts. A comparison of the last two columns of Table X shows an excellent agreement between the watts as measured by the electrometer and the calculated watts. The power factor of the load used in this test was 0.75 per cent. The results in Table X also show that the electrometer deflection changed to negative when the loss $2\,I_2\,R_1$ became greater than the load.

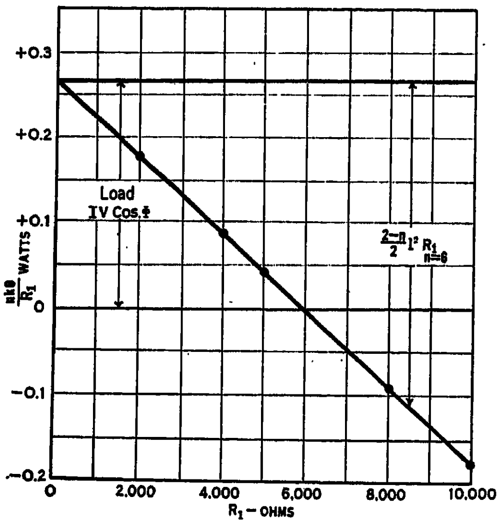

Fig. 11—Curve Showing the Relation Between Watts Measured by the Electrometer and the Shunt Resistance R_1 with $n = 6$. Plotted from the Results of Table X

The results of this test prove that equation (**42**) holds at low power factor.

The experimental results prove conclusively that equations (**37**) and (**42**) are derived correctly and that they are applicable at all values of power factor.

III. Mathematical

Derivation of the torque equation:

We assume that

$$A_0 = a_0'' + a_0\,\theta + a_0'\,\theta^2$$
$$A_1 = a_1'' + a_1\,\theta + a_1'\,\theta^2$$
$$A_2 = a_2'' + a_2\,\theta + a_2'\,\theta^2$$
$$B_1 = b_1'' + b_1\,\theta + b_1'\,\theta^2$$
$$B_2 = b_2'' + b_2\,\theta + b_2'\,\theta^2$$
$$C_0 = c_0'' + c_0\,\theta + c_0'\,\theta^2$$
$$C_1 = c_1'' + c_1\,\theta + c_1'\,\theta^2$$
$$C_2 = c_2'' + c_2\,\theta + c_2''\,\theta^2$$

If we study the electrometer, we see that B_0, the coefficient of electrostatic induction between quadrant pairs 1 and 2 will not change with the deflection because the quadrants are fixed in position and the motion of the needle does not intercept lines of force passing between them. The terms covered by H are also independent of the deflection if the instrument is properly set up as explained in the body of the paper.

In order to determine the torque, differentiate equation (**14**) with respect to the deflection, and we get

$$\frac{d\,w'}{d\theta} = a_0\,V_0^2 + a_1\,V_1^2 + a_2\,V_2^2 + b_1\,V_0\,V_1 + b_2\,V_0\,V_2 + c_0\,V_0 + c_1\,V_1 + c_2\,V_2 + \theta\,(2\,a_0'\,V_0^2 + 2\,a_1'\,V_1^2 + 2\,a_2'\,V_2^2 + 2\,b_1'\,V_0\,V_1 + 2\,b_2'\,V_0\,V_2 + 2\,c_0'\,V_0 + 2\,c_1'\,V_1 + 2\,c_2'\,V_2) \qquad (\mathbf{15})$$

Substituting the value of $\frac{d\,w'}{d\,\theta} = C\,\theta$; we find

$$\theta\,(c - 2\,a_0'\,V_0^2 - 2\,a_1'\,V_1^2 - 2\,a_2'\,V_2^2 - 2\,b_1'\,V_0\,V_1 - 2\,b_2'V_0\,V_2 - 2\,c_0'\,V_0 - 2\,c_1'\,V_1 - 2\,c_2'\,V_2) = a_0\,V_0^2 + a_1\,V_1^2 + a_2\,V_2^2 + b_1\,V_0\,V_1 + b_2\,V_0\,V_2 + c_0\,V_0 + c_1\,V_1 + c_2\,V_2 \qquad (\mathbf{16})$$

A study of the symmetry shows that $a_1' = -\,a_2'$ and that $b_1' = -\,b_2'$. We also see that the c_0', c_1' and c_2' terms, which include the small contact potentials, depend upon only the first power of the applied voltages and therefore they may be neglected.

Rewriting equation (**16**) we get

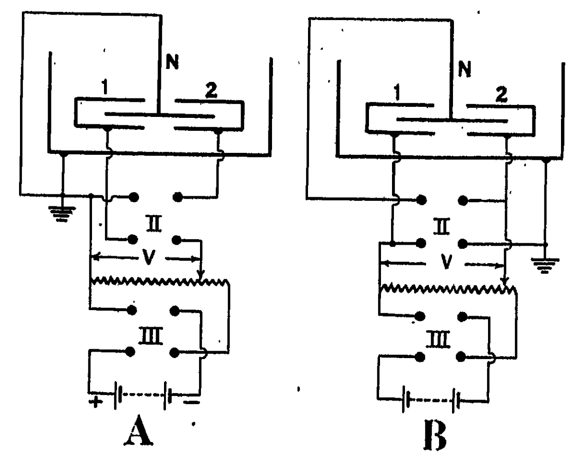

Fig. 12—Idiostatic Connections

$$\theta\,[c - \{2\,a_0'\,V_0^2 + 2\,a_1'\,(V_1^2 - V_2^2) + 2\,b_1'\,V_0\,(V_1 - V_2)\}] = a_0\,V_0^2 + a_1\,V_1^2 + a_2\,V_2^2 + b_1\,V_0\,V_1 + b_2\,V_0\,V_2 + c_0\,V_0 + c_1\,V_1 + c_2\,V_2. \qquad (\mathbf{17})$$

Experimental results show that the torque equation (**17**) may be written in a more useful form; namely

$$\theta\,[1 + R\,(V_0 - V_1)\,(V_0 - V_2) + S\,(V_1^2 - V_2^2)] = a_0\,V_0 + a_1\,V_1^2 + a_2\,V_2 + b_1\,V_0\,V_1 + b_2\,V_0\,V_2 + c_0\,V_0 + c_1\,V_1 + c_2\,V_2 \qquad (\mathbf{18})$$

Proof that $a_2 - a_1 = b_1$:

In order to prove this relation the connections shown in Fig. 12 are used. These connections are known as the idiostatic connections and are also used where the electrometer is used as a voltmeter.

There are two possible arrangements of the connections which are shown in Figs. 12 A and B. In connection A the needle and either quadrant, 1 or 2, are grounded and in connection B the needle is not grounded but quadrants 1 or 2 are grounded depending upon the position of the commutators.

For connection A, V_0, the needle potential is zero and equation (**19**) reduces to

$$\theta \{1 + R\, V_1\, V_2 + S\,(V_1 - V_2)^2\} = a_1\, V_1^2 + a_2\, V_2^2 + c_1\, V_1 + c_2\, V_2 \qquad (29)$$

because all of the terms containing V_0 will be zero.

We get a deflection for each of the four positions of the commutators and the signs of the terms of equation (**29**) are given below

Position of Commutator III	II	Values of the Terms $a_1 V_1^2$	$+ a_2 V_2^2$	$+ c_1 V_1$	$+ c_2 V_2$	Deflection
||	||	0	+	0	−	= { } α
=	||	0	+	0	+	= { } β
=	=	+	0	+	0	= { } γ
||	=	+	0	−	0	= { } δ

Taking the algebraic sum of these deflections as follows, we get

$$\{1 + R\, V_1\, V_2 + S\,(V_1 - V_2)^2\} = M$$

$$\{M\}\,(\alpha + \beta - \gamma - \delta) = 2\,(a_2 - a_1)\, V^2 \qquad (30)$$

In connection B, Fig. 12, equation (**19**) holds and contains all of its terms. Here we see that, depending upon the positions of the commutators,

$$V_0 = V_1 = V \text{ or } V_0 = V_2 = V$$

$$V_1 = V \text{ or zero and } V_2 = V \text{ or zero}$$

The deflections for each of the four commutator positions and the signs of the terms are as follows:

Position of Commutator III	II	Values of the Terms $a_1 V_1^2$	$+ a_2 V_2^2$	$+ b_1 V_0 V_1$	$+ b_2 V_0 V_2$	$+ c_0 V_0$	$+ c_1 V_1$	$+ c_2 V_2$	Deflection
||	||	+	0	+	0	+	+	0	= [M] α
=	||	+	0	+	0	−	−	0	= [M] β
=	=	0	+	0	+	+	0	+	= [M] γ
||	=	0	+	0	+	−	0	−	= [M] δ

Taking the algebraic sum of these deflections as follows, we obtain

$$[M]\,(\alpha + \beta - \gamma - \delta) = 2\,(a_1 - a_2)\, V^2 + 2\,(b_1 - b_2)\, V^2 \qquad (31)$$

If the same voltage is used in connections A and B of Fig. 12, then both will give equal deflections in the same directions. That the deflections will be in the same directions may be seen from a study of the connections. Consider connection A with both commutators vertical; then the needle and quadrant pair No. 1 will be grounded and therefore at same potential and quadrant pair No. 2 will be negative and the needle will swing into quadrant pair No. 2. In connection B, with both commutators vertical, the needle and quadrant pair No. 1 are again connected together, and therefore at the same potential which in this case is positive. Quadrant pair No. 2 will be grounded, and the needle will again swing into quadrant pair No. 2. Since the same voltage is used in both A and B connections, the deflections will be equal.

Experiments show that the two connections give equal deflections and also that they are in the same direction. Therefore it follows that the right hand sides of equations (**30**) and (**31**) must be equal, and we have

$$2\,(a_2 - a_1)\, V^2 = 2\,(a_1 - a_2)\, V^2 + 2\,(b_1 - b_2)\, V^2 \qquad (32)$$

Solving, we find that

$$2\,(a_2 - a_1) = b_1 - b_2.$$

or

$$a_2 - a_1 = \tfrac{1}{2}\,(b_1 - b_2)$$

Owing to the symmetrical construction of the electrometer

$$b_1 = -\, b_2 \text{ and } a_1 = -\, a_2$$

Therefore

$$a_2 = \tfrac{1}{2}\, b_1 \qquad (33)$$
$$a_1 = -\, \tfrac{1}{2}\, b_1$$

Conclusion

In order to use the quadrant electrometer for the measurement of power, it is necessary only to determine two constants, namely b_1 and R. This may be done with continuous current. In some instruments, R has a negligible or zero value.

The quadrant electrometer may be used in measuring a-c. power with either full or a fraction of the full voltage applied to its needle. In the case where full voltage is used on the needle, equation (**37**) holds.

$$\frac{[1 + R\, V^2]}{2\, b_1} \times \frac{\theta}{R_1} = I\, V \cos \Phi + \frac{I^2\, R_1}{2} \qquad (37)$$

In cases where a fractional part of the full voltage is applied to the needle, equation (**42**) holds

$$\frac{\left[1 + R\left(\frac{V}{n}\right)^2\right]}{2\, b_1} \times \frac{n\, \theta}{R_1} = I\, V \cos \Phi + \frac{2 - n}{2}\, I^2\, R_1 \qquad (42)$$

These equations hold for all values of the power factor both lagging and leading.

The general power equation of the quadrant electrometer for a-c. circuits is given by equation (**44**).

$$\frac{[1 + R\, \dot{V}_0^2]}{2\, b_1}\, \theta = \dot{V}_0\, \dot{V}_1 - \frac{\dot{V}_1^2}{2} \qquad (44)$$

Here $\dot{V}_0$ is the voltage applied to the needle and $\dot{V}_1$ is the voltage applied to the quadrant pairs. In using this equation both are to be expressed in their vector form. By means of equation (**44**) it is a simple matter to determine what the instrument will read under any conditions in an a-c. circuit.

The experimental data presented amply proves the theory given in this paper.

The author wishes to thank Mr. W. W. Hill and

Mr. G. A. Irland for their assistance in the experimental work.

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Discussion

J. B. Whitehead: Those of us who have had occasion to measure small values of dielectric loss with the quadrant electrometer instrument, know of the apparent facility and convenience of that instrument. I say "apparent" because we also know that we soon run into many entirely unexpected and apparently inexplicable phenomena in connection with the instrument. In the literature we find a great deal on the instrument and think we will be able to clear up our difficulties. As we go on, however, we find that the points of view of the several people who have written on the quadrant electrometer often differ. The several authors introduce different constants and we are left with a large number of constants, whose relative importance we are unable to determine easily.

I think that Dr. Kouwenhoven has rendered a very important service, in that he has considered the quadrant electrometer from its simplest stage as represented by Maxwell's equations, in which we find every coefficient of induction which is required by a system involving three elements. Some of the constants will change with the deflection of the instrument. The service he has rendered is to show that used as a power instrument, many of these constants disappear, either through mutual neutralization or because they happen to be so small as to be negligible. He has thus brought the instrument to a point where it is shown to be reliable through the determination of only two constants, and these two constants can be determined by d-c. measurement.

D. M. Simons: Anyone who has had much practical experience with the quadrant electrometer has found that the so-called constant of the instrument is usually by no means what its name implies; we usually found that the constant would have different values at different voltages, and until certain adjustments were made, it would vary with the magnitude of deflection. It gave one rather a lack of confidence in the fundamentals of the theory of the instrument, because Maxwell's equation was based on the constancy of the proportion between the deflection and the total load. There have been some indications in print that the relationship did not hold good, such as Orlich's article in the *Zeitschrift fur Instrumentenkunde* about ten years ago and the article by Cantutti in *l'Elettricista* in 1923, and the matter has been in the minds of a great many people, without any definite solution. I think, therefore, that Dr. Kouwenhoven is all the more to be congratulated on having solved the question so thoroughly in a quantitative manner.

Personally, I find it rather difficult to mention the quadrant electrometer without discussing the zero method, and the subject is not entirely inappropriate, because one of the main advantages of the zero method is that no knowledge of the constant is required. We described a zero method last year before the Institute,[1] which I believe is very practical, the deflection being brought back to zero merely by the insertion of a resistance in the needle circuit, the value of power factor of the load being calculated in terms of the resistance required. Using any zero method with the electrometer, there are certain great advantages; a straight-line constant is not necessary, and thus there is much greater freedom in the design of the instrument. The setting is practically independent of voltage fluctuations, and thus the measurements may be made much more accurately and quickly than with a deflecting instrument where the needle will swing with every fluctuation of voltage. The reading with the zero method is proportional to power factor, while in the deflection method it is proportional to watts, and thus the zero method is more suitable for the measurement of ionization.

S. L. Gokhale: 1. To begin with, I suggest that the title of the paper be changed to "Electrostatic Wattmeter;" in the past, all the valuable literature on this subject has been indexed under the head "quadrant electrometer," and it is generally overlooked by those who are searching for information on methods for measurement of power.

2. Tables IV, V, and VI constitute a very convincing demonstration of the reliability of this type of wattmeter, for loads of low power factor, but all the cases treated by the author are loads of no distortion. The particular case I am interested in, is the case of core loss in iron for flux density near saturation, involving not only a very low power factor but also a serious distortion of the current wave, with a possibility of distortion of the voltage wave also. It seems to me that the instrument would prove very satisfactory for this purpose also, but I would prefer to have the fact demonstrated, if possible.

3. The question of instrument error, that is, error characteristic of the instrument, irrespective of the nature of the load, is also quite important. On this point, I suggest, by way of experiment, the measurement of core loss in an air-core transformer (*i. e.*, mutual inductor); as there is no real loss to be measured, any apparent loss would obviously be the instrument error. Another experiment directed towards the same end would be the measurement of core loss in iron-core transformers at low flux density by the electrostatic wattmeter, and also by the electro-dynamometer wattmeter.

4. As to the mathematical development of the final working formula (omitting the correction for errors), it may be noted that the author has given two equations, Nos. (42) and (43). Equation (42) is theoretically the final form in which the variation in the coefficient of torque due to variation in the voltage of the needle has been taken into account. Equation (43) is a simplified practical final form of equation (42), assuming the coefficient of torque to be constant and independent of voltage. In the several experimental demonstrations given by the author, this coefficient is either really constant, the working voltage having been maintained constant for each series of tests, or practically constant and regarded as such by the author himself. In other words the final working formula is equation (43), not (42). In view of this fact, it may be of interest to know that equation (43) can be derived directly from the well-known fundamental equation, that is, from equation (11) of Prof. Kouwenhoven, without the intermediate thirty equations which needlessly discourage an average reader. I do not intend to suggest that equation (42) together with the mathematical development which leads to it, is all unnecessary. It is easy to believe that occasions will arise where the nature of the work in hand calls for a

1. The Quadrant Electrometer for the Measurement of Dielectric Loss, by D. M. Simons and W. S. Brown, A. I. E. E. JOURNAL, December 1924, page 1147.

variation of voltage, which would make equation (42) indispensable. In view of this possibility, I might suggest that Prof. Kouwenhoven extend his experiments to the case of varying voltage, other variables being maintained constant for the time. It would be interesting to have it demonstrated that the coefficient of torque follows the law formulated in equation (42).

W. B. Kouwenhoven: The quadrant electrometer is an electrostatic instrument and measures root-mean-square values exactly. Its readings are independent of wave form and frequency. The capacity-load measurements referred to in Table VI were made with energy supplied from a generator that does not give a pure sine wave. Under these conditions the current wave was badly distorted; nevertheless, the results checked closely. I am sure that the instrument could be successfully used in measuring core loss in iron samples.

As stated in the paper, equation (43) is only a special case of equation (42). Equation (43) may be directly derived from Maxwell's equation (11).

In using an instrument it is always desirable to have a straight-line constant, but this is not possible in all electrometers for the reasons mentioned in the paper. In some electrometers the constant varies considerably with the needle voltage. In the electrometer referred to in the paper the constant varied as follows:

Needle Voltage	55 volts;	Constant	72	Table	V
" "	100 "	"	71.8	"	IV
" "	400 "	"	69.7	"	VI

This electrometer was used later with voltages as high as 1800 volts on the needle. At this voltage its constant was found to be 33.8. This clearly shows the effect of the variation in voltage upon the constant of the instrument.

A New Method and Means for Measuring Dielectric Absorption

BY RALPH E. MARBURY*

Associate, A. I. E. E.

Synopsis.—*The progress of practically all electrical apparatus is to a large degree dependent on progress in insulation. The successful selection and use of insulation, particularly insulation operated with maximum economy as in the case of static condensers, depends on our progress in the understanding of the factors on which insulation quality depends.*

It has long been known that for a given working condition, reliability depends on dielectric losses, particularly the losses commonly referred to as hysteresis losses.

The hysteresis loss is definitely related to the phenomenon of absorption or residual charge, especially when the residual voltage quickly builds up.

In order to further analyze dielectric losses, and study the progress of treatment of insulation, a device has been developed for measuring dielectric absorption.

This article describes the "Dielectric Lag Meter" and gives some test data secured by it.

* * * * *

I. Introduction

The importance of insulation to the progress of the electrical art is well recognized. This is especially so in apparatus where insulation is operated with maximum economy, as in the case of condensers and cables. Possibly the most important of all insulation from the standpoint of relation to the cost and reliability of the apparatus is that used in condensers, since in condensers the insulation is the vital element, the active part, and fundamental to the device itself. More than that, the quantity of insulation varies inversely as the square of the stress at which the material is worked. The working stress must at all times be well within limits which have been established as reliable; nevertheless as we acquire knowledge of the fundamentals of insulation, we shall be able to fabricate it more economically and operate with less, but more definite, factors of safety.

Insulation may be divided into two general classes; viz., (1) that occurring in a natural state, (2) that which is created by combination of other materials. Under the first class we have mica and similar substances, while under the second class we have fibrous spacer materials impregnated with oils, waxes, or similar substances. The practical disadvantage of the first class is that the quality is not under our control. We can only make suitable tests to select the material of satisfactory quality. Insulation of the second class, while of a complicated nature, is subject to closer control since practically all the materials making up the structure may be controlled to a greater or lesser degree. It is only necessary to exercise the proper control when a means is available by which the quality of the various materials can be studied separately, combined, and during the process of combination.

The advantages of oil-impregnated paper insulation are now fully recognized and this type of insulation has been used with success for many years. This form of insulation can be used in large quantities due to its inherent uniformity. A closer control, however, during impregnation, will result in still greater gain, since a

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large mass of insulation may be treated until every unit of its volume is of satisfactory quality. The principal advantages of oil-treated paper insulation are the high quality of oil as an impregnating medium, the circulating or heat distribution qualities, and the thoroughness with which it impregnates the material.

It is extremely important that practically all the moisture be removed from the paper before the introduction of oil, and, in fact, the characteristics of the dielectric consisting of high grade paper and oil depend almost entirely on the completeness of moisture removal before impregnation.

Many methods have been developed whereby the finished condenser, or cable, may be tested for dielectric losses. Such tests are extremely important, especially when one is concerned with the efficiency or temperature rise. On the other hand, these tests give only the result of imperfections and not the nature of the losses. In other words, the losses in the condenser may be caused by losses in the metal plates, conduction losses, or the so-called hysteresis losses in the dielectric itself.

It is desirable not only to analyze these losses but to make the tests on the condenser during treatment.

In the case of 60 cycles, practically all of the loss is in the insulation itself, rather than in the metal plates or due to conduction through the dielectric. It is generally agreed that the losses in the insulation are directly related to the phenomenon of absorption. A measure of absorption during treatment should serve as a very effective quality control.

A device for conveniently measuring absorption has been worked out and it is the purpose of this paper to describe this device in detail.

II. Dielectric Absorption

When a condenser having a perfect dielectric is connected to a source of electromotive force, it immediately takes up a quantity of electricity, Q_0, which is proportional to the voltage

$$Q_0 = C_0 E$$

where C_0 is the geometrical capacity, or the capacity based on the physical dimensions of the insulation and specific inductive capacity.

However, in the case of an imperfect dielectric, the above does not complete the process. The quantity Q_0 increases with time until a final value, Q_∞, which exceeds the value Q_0 by a certain fraction K, is reached.

$$Q_\infty = (1 + K)\, Q_0 E$$

This characteristic of insulation has been given the name of dielectric absorption and the key to an understanding of insulation losses lies in the thorough understanding of this phenomenon.

As stated above, the initial quantity of electricity may be represented by Q_0. This is also the quantity that will be dissipated on a quick short circuit or discharge through a low resistance. Thus, when a condenser is charged, discharged, and then left open-circuited, a voltage will build up with time across its terminals. The rate at which this voltage builds up, and the magnitude of the voltage for a given condenser, depends on the time of charging and applied voltage. With different condensers, the shape of this residual curve, especially when different applied voltages are considered, depends on the quality of the insulation.

Our work has indicated that these curves, if fully understood and analyzed, will provide a measure of at least the major factors on which dielectric quality depends.

The most desirable method for measuring absorption direct is to charge the condenser for an arbitrary, but short, length of time, then discharge it through a very low resistance to eliminate the quantity Q_0, and measure the residual voltage as it builds up after discharge.

The subject of dielectric absorption has occupied the minds of many investigators over a long period. There are some who account for it entirely on the basis of heterogeneity, while others consider it of a more complicated nature.

F. W. Grover* has reviewed a number of these theories and analyzed them quantitatively.

K. W. Wagner† has made a careful study of the Maxwell‡ theory of absorption and has applied it to a more complicated dielectric structure.

Difficulty of Measuring Absorption

Many investigators have measured absorption by charging the condenser, discharging it, and immediately connecting across its terminals an electrostatic voltmeter. This is not wholly satisfactory, since the static voltmeter has a large time lag and fails to indicate the values of voltage during the most important part of the residual-voltage curve. Our tests of this kind showed that the part of the residual curve occurring during the first ten seconds is not secured. Further investigations showed first, that the most important part of the residual voltage curve occurs within the first second, and secondly, that after a few seconds the true shape of the residual curve is distorted by leakage current due to the over-all insulation conductivity.

It is desirable, therefore, to make the absorption measurements within one second after discharge, and a sufficient number of measurements should be taken during this period of time to establish with reasonable accuracy the shape of the curves.

The Dielectric Lag or Absorption Meter

The dielectric lag meter measures the instantaneous values of voltage across the condenser by direct comparison with known voltages.

*F. W. Grover, Bulletin of the Bureau of Standards No. 4, December 15, 1911.

†K. W. Wagner, Theory of Dielectric After Effect in the Light of Maxwell's Notions.

Archiv fur Elektrotechnik, Vol. 2, No. 9, 1914.

‡Clerk Maxwell, *Electricity and Magnetism*, Vol. 1, page 452.

The apparatus consists of an arm switch rotating at a constant and known speed, a potentiometer for varying the value of the known voltage, a galvanometer circuit, and a means for selecting the instant of time at which the measurement is to be made.

The diagram in Fig. 1 shows the rotating arm switch with twelve stationary contacts, the galvanometer and potentiometer circuits, and, in addition, a variable resistance in parallel to the condensers. The variable resistance is not used in making residual voltage curves, but is provided for making discharge curves which are described later.

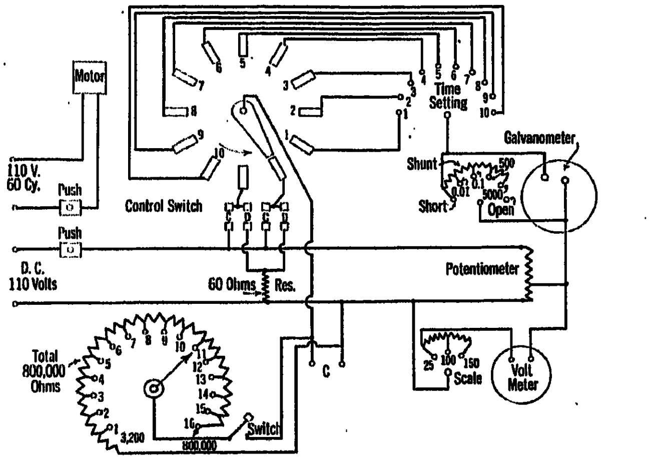

FIG. 1—DIAGRAM OF CONNECTIONS OF DIELECTRIC LAG METER

Three different types of test which may be made by means of this instrument have been found to be of particular value:

A. *Residual voltage against time,*
B. *Residual voltage against applied voltage,*
C. *Discharge curves with condenser and resistance.*

The methods of making these tests are discussed separately and in detail to illustrate the operation of the instrument.

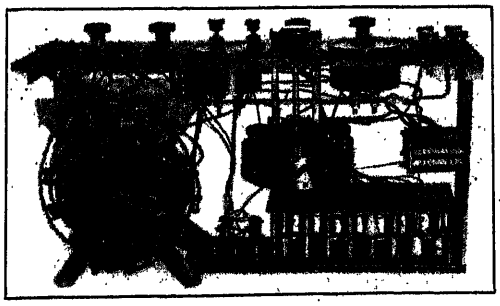

FIG. 2—INTERNAL VIEW OF DIELECTRIC LAG METER

A. Residual Voltage Against Time. The condenser to be tested is connected at terminals marked *C* in Fig. 1. The shunt resistance is disconnected for this test. The two terminals marked *D. C.* are connected to a source of direct current. The other two terminals are connected to the 110 volts, 60 cycles, for operating the synchronous motor driving the arm switch.

Two segments are used for charging and discharging the condenser, while the other ten segments are used for measuring the residual voltage at various values of time after discharge.

The charge and discharge segments are connected to the line and discharge resistance, marked 60 ohms on Fig. 1, by means of the switches *C, D, C, D,* so that when the arm is rotating counter-clockwise the first segment will charge the condenser to the applied voltage and the second will discharge it. The width of the charging segments and the space between it and the discharge segment are such as to fix the charging time at 0.111 seconds. The discharge resistance is made just high enough to prevent damage to the segments on the discharge and not high enough to fail to discharge the initial quantity Q_0.

In making the tests, the arm makesc ontact first with the charging segment, then with the discharge segment, and then makes a complete revolution and charges again and so on at a constant rate. Each time the arm leaves the discharge segment and as it makes its revolution a residual voltage develops each time taking on the same values for the periods of time corresponding to the segments 1, 2, 3 to 10, consecutively.

While this is going on, the time-setting switch, shown

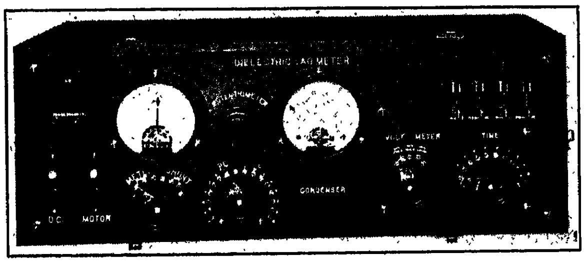

FIG. 3—FRONT VIEW OF DIELECTRIC LAG METER

on diagram Fig. 1, is set on contact one and the potentiometer voltage adjusted until the potential of segment No. 1 is the same as that of the rotating brush at the particular instant of time that it comes in contact with segment No. 1. This condition is obtained by adjusting the potentiometer until the galvanometer fails to reflect in either direction, when the arm makes contact with this segment. When the balance is obtained, the voltage setting of the potentiometer is equal to the condenser voltage at the instant of time corresponding to the position of segment No. 1, which is located 0.111 seconds after discharge. This is repeated for segments 2, 3, 4, etc., thus giving 10 points on the residual curve, 0.111 seconds apart.

B. Residual Voltage Against Applied Voltage. In this case the time-setting switch is left in some one position and the residual voltage measured for different applied voltages by balancing with the potentiometer as in *a.*

C. Discharge Curves with Condenser and Resistance.

Discharge curves are less important than the residual voltage curves, since more factors are involved. The departure of the curve on a condenser from the theoretical equations

$$E = E_0 \times e^{-\frac{T}{rc}}$$

is quite noticeable with the slightest amount of absorption. This is especially true with discharge curves of very short duration and the lag meter is especially suited for making these discharge curves.

In making discharge curves, the charge and discharge segments are changed so that the condenser is first discharged and then charged to line voltage. This is accomplished by means of switches C, D, as will be seen by referring to Fig. 1. In this case the condenser voltage is that of the line voltage when the arm starts its revolution and drops during the course of the revolution, due to the discharge resistance which is shunted across it. The variable discharge resistance should be set at a value depending upon the capacity of the condenser, so that the voltage at segment 10 will be of the order of one volt, thus giving 10 points on the discharge curve. Steeper discharge curves may be made, if desired, where the voltage has already reached practically zero value at segment two or four or even one. The values of voltage for segments from one to ten consecutively are determined as before.

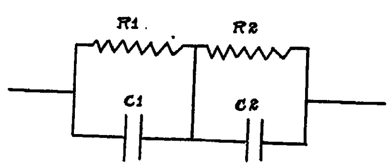

FIG. 4—MODEL CONDENSER TO ILLUSTRATE MAXWELL'S THEORY OF ABSORPTION

The details of the device, especially the arm switch, are shown in Fig. 2. The front with all control knobs is shown in Fig. 3.

DIELECTRIC ABSORPTION MEASUREMENTS WITH "LAG METER"

Three types of test are described below to illustrate the performance of the device:

1. Dielectric absorption tests on a model condenser made to illustrate Maxwell's theory of absorption.
2. Dielectric absorption tests on a paper condenser during removal of moisture to show effect of moisture removal on absorption.
3. Tests on miscellaneous power condensers.

1. MODEL CONDENSER

Clerk Maxwell showed in his famous Treatise on Electricity and Magnetism that a dielectric having a variation in the product of R and C at varous points would exhibit the phenomenon of absorption. That is, the initial voltage distribution would follow the values of C and a later or final distribution would follow the values of R. If the distribution according to values of R does not agree with the distribution for the values of C correspondingly, a shift will take place in the stored charges. This charge represented by the shift from the normal condition will not come out instantaneously when the condenser is discharged, but will come out with time after discharge, resulting in a residual voltage.

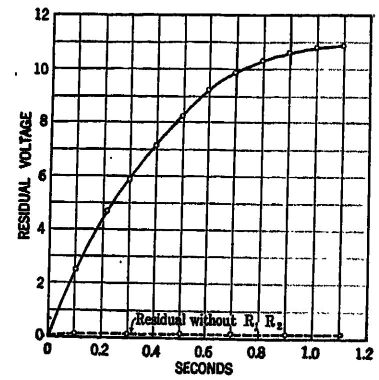

FIG. 5—RESIDUAL VOLTAGE AGAINST TIME FOR MODEL CONDENSER SHOWN IN FIG. 4

A model condenser was constructed to illustrate this, and measurements of the residual voltage were made with the lag meter. This model consisted of two condensers C_1 and C_2 shunted by a resistance R_1 and R_2 as shown in Fig. 4. No attempt was made to make the values representative of an actual condenser. The two condensers used to make this test had extremely low residual voltages, so that for all practical purposes the residual values obtained were due only to the fact that $R_1 C_1$ was not equal to $R_2 C_2$. Tests were also made with values of R_1 and R_2, so that the two products $R\,C$ were equal and no residual voltage was obtained in excess of that which would exist without the resistances.

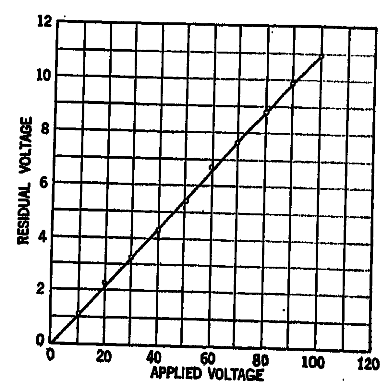

FIG. 6—RESIDUAL VOLTAGE AGAINST APPLIED VOLTAGE FOR MODEL CONDENSER SHOWN IN FIG. 4. TIME AFTER DISCHARGE EQUALS 1.11 SECONDS

The residual voltage against time for the model as shown in Fig. 4 is given in Fig. 5.

The residual voltage against applied voltage for T after discharge = 1.11 seconds, is given in Fig. 6.

This shows that the residual voltage is proportional to the applied voltage for such a model as would be expected from Maxwell's equations.

Fig. 7 shows discharge curves made, using the above model and with resistances of 31,000, 50,000, 135,000,

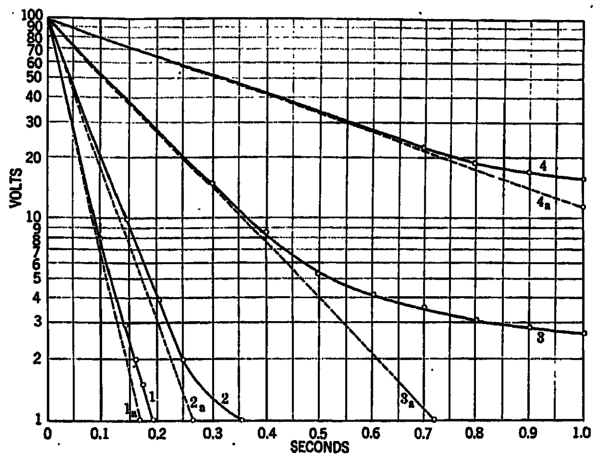

Fig. 7—Shows Discharge Curves Made Using Model Fig. 4

and 400,000 ohms. The model follows qualitatively Maxwell's theory of absorption.

2. Dielectric Absorption Tests on Condensers During Treatment to Remove Moisture

A stacked paper condenser having a capacity of approximately 3 microfarads was dried in an oven with vacuum in order to remove moisture, in accordance with

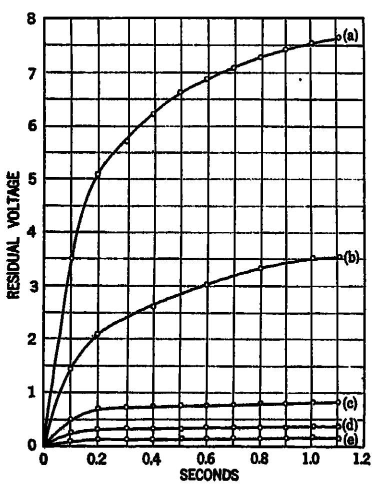

Fig. 8—Residual Voltage Against Time on a Condenser During Moisture Removal

the usual process. Five residual voltage curves were taken, one just after heat treatment before vacuum was applied and the other four during heat treatment with vacuum. In each case the condenser was allowed to cool to 22 deg. cent. before making the test. The five curves in Fig. 8 are identified as follows:

a. Heated the day before 6 hours at 60 deg. cent.

b. Heated the day before 6 hours at 100 deg. cent. and 27 in. vacuum.

c. Heated the day before 6 hours at 100 deg. cent. and 28 in. vacuum.

d. Heated the day before 6 hours at 100 deg. cent. and 28 in. vacuum.

e. Heated 16 additional hours at 100 deg. cent. and 28 in. vacuum.

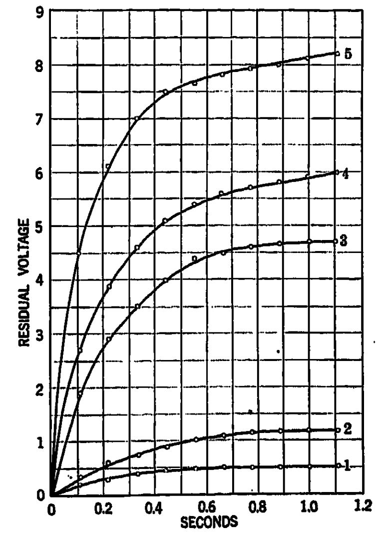

Fig. 9—Residual Voltage Curves on Typical Power Condensers

As the moisture is removed from the dry paper condenser, the residual voltage decreases to a value which is practically immeasurable, as shown by the above curves. This is due to the fact that the dry paper has very little absorption in itself.

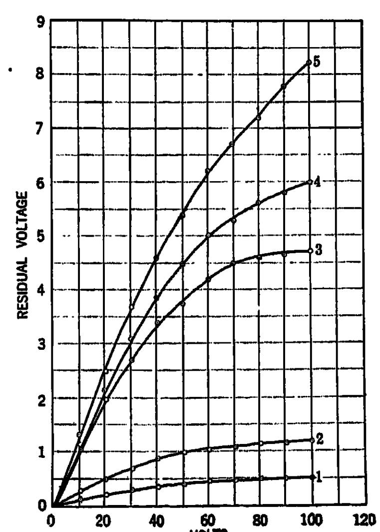

Fig. 10—Residual Voltage Curves Against Applied Voltage for Typical Power Condensers

3. Tests on Miscellaneous Power Condensers

In order to illustrate the values of residual voltage obtained with average commercial power condensers, curves are given in Fig. 9 for five different condensers, each of the same style but varying somewhat in quality. These curves show that for a very high

quality condenser, having losses of about 0.2 per cent as designated by Curve No. 2, the residual voltage is very low, while for a condenser having power losses such as 0.5 per cent the residual voltage is very much higher, as shown in Fig. 8, Curve No. 5. It must be borne in mind, however, that a direct comparison cannot be made between losses and residual voltage unless all factors are taken into consideration, and in fact, the exact relation existing has not been definitely established. These tests are made on 5-kv-a., 2300-volt condenser units.

Fig. 10 gives the five residual voltage curves against the applied voltage for the same condensers as covered by Fig. 9. Corresponding numbers are given to the two sets of curves for comparison.

It is of greatest importance to note that in Fig. 10 the residual voltage curves against applied voltage have a saturating characteristic, that is, at some voltage for each condenser the residual voltage reaches a maximum value or approaches a maximum value as a limit, instead of being at all times proportional. It is of even more interest to note that the higher the quality of the condenser, the smaller the angle of rise becomes.

Conclusions

All tests to date indicate that the residual voltage against applied voltage and residual voltage against time are of greatest value.

The saturating characteristics of residual voltage against applied voltage appear to be the most important of all, in that it follows so closely the over-all quality of the condenser and it is our opinion that a thorough understanding of this curve will result in a better understanding of the factors on which insulation quality depends. This saturating characteristic is not accounted for by previous work or theories, so far as we have been able to learn.

The immediate value of the absorption test is in determining when the moisture has been removed from the paper during treatment, so that in no case will the oil be applied until the moisture has been removed.

A measure of absorption may show the progress of the oil in reaching the microscopic crevices of the fiber.

After the factors affecting the absorption are understood, absorption tests may be of value in periodically testing cables in service, to show stratified deterioration and serve as a means of anticipating cable failures.

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Discussion

J. B. Whitehead: We have not had sufficient recognition of the importance of dielectric absorption as a factor in the question of dielectric loss. Modern theory of dielectric behavior, such as it is, is directed more and more toward the explanation of dielectric loss in terms of dielectric absorption. That is not to say, however, that we are getting to understand the fundamental character of this loss any better, because there is no more obscure phenomenon than that of dielectric absorption.

We have Maxwell's theory of absorption, but Maxwell's theory has not been confirmed by quantitative measurements, and, indeed, there are many indications that it must be modified

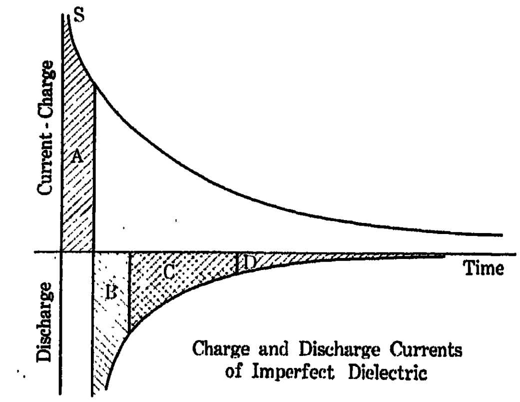

Fig. 1

in some way. However, the actual phenomenon itself is relatively simple; that is to say, we can make certain measurements on dielectrics which will give us certain curves, and if we express these curves in terms of mathematical functions it is possible to substitute these functions in our simple a-c. power equations and we get expressions for dielectric loss, phase difference, and specific conductive capacity, which go far toward explaining their variations with such quantities as voltage, frequency, and to somewhat less extent, the temperature. So it is particularly important that we should have a paper on this subject.

I find, however, in considering Mr. Marbury's method that it is subject to one very serious limitation. One of the best ways of representing the phenomenon of absorption is to plot a curve as in Fig. 1 between the charging current of the condenser containing the dielectric and the time; that is to say, if we take a condenser which has been lying idle a long time, and suddenly apply a continuous voltage, measuring the current, we find a curve approximately asymtotic to the axis of time, but which, in most cases, reaches a final steady value. In a few very perfect dielectrics only does the curve come down to the horizontal axis. This curve can extend over a very long period of time, days or even months.

For example, in 60-cycle circuits, we have complete reversal of voltage in a very short interval of time and consequently any influence of this absorption curve must pertain to a portion of the curve which is extremely near its starting point, *S*. The great trouble that has been found in linking up the phenomenon of absorption with losses as we observe them has been the dif-

ficulty of determining the shape of this curve for extremely short intervals of time.

In the cycle represented by Mr. Marbury's instrument, he charges the condenser for 1/10 sec., the quantity of charge being represented by the area A of the figure. He then discharges it for about the same interval of time (1/10 sec.), the quantity discharged being shown by area B. Now, there is a law not very generally spoken of, connected with this phenomenon of absorption. This is called the "Principle of Superposition." It was first noted by Sir John Hopkinson in some of the earliest and best work that has ever been done on dielectrics, and it was confirmed beyond question by J. Curie. It states that if you start one of these absorption curves, and then make any change, whatever, in voltage, the succeeding behavior of the dielectric will be as though you superimposed upon the initial curve the curve represented by the change of voltage when acting alone. In the figure, this means that when dielectric is short-circuited it behaves just as though we had applied a negative value of the voltage equal to the charging voltage. The discharge curve is exactly similar to the charging curve, but refers to the charging curve instead of the horizontal axis. In other words, it says the preceding state of the dielectric persists with the change in voltage applied to the initial curve, and not to the horizontal axis. So, also, the discharge curve which Mr. Marbury gets is subject to the same law.

But what does he do? He has discharge for only a period of 1/10 sec. and then allows the residual voltage to rise for a period of 9/10 sec. but then he stops! His voltage residual due to the first cycle is represented by the progressive integration of the area C; that is, all the residual due to areas B and D are not included.

If the discharge interval stops before the dielectric is completely discharged, represented by the full area under the discharge curve, he has not measured his residual voltage accurately. The observations he makes appertain to the particular cycle represented by his instrument, because his next charging cycle rises to a lower value and the successive discharge curve will also be a little less; but something more will be added to the foregoing residual. He is measuring the sum of a succession of these intervals, each one being less than the preceding one. So he is measuring something that is certainly due to absorption, but it is peculiar to the particular cycle he is using; namely, 1/10-sec. charge and discharge and a 9/10-sec. residual. The 1/10-sec. interval is of no great interest because it is not short enough for information as to 60 cycles. Hopkinson's investigations showed that a condenser, so far as the initial static charge is concerned, will be discharged in an interval in the order of a 1/17000 sec., the residual curve then starting. So in Mr. Marbury's instrument the discharge interval allows a large portion of the absorption to escape in the short circuit.

I am sure that Mr. Marbury's instrument will be of great value in testing the relative absorption properties of dielectrics, particularly those in which most of the discharge will take place within the interval of the instrument itself, but I think it ought to be clearly understood that it does not measure the true absorption curve and that what it measures is peculiar to this particular instrument.

Arnold Roth: It is interesting to note the different opinions on the importance of the losses in cables. We have worked on this question in Europe and our cable manufacturers also used to give much importance to this factor.

I should like to point out the difference in the importance of these losses in two kinds of materials, namely, (a) cable with relatively moderate outside temperature, (about 40 deg. cent.), and (b) materials which I should like to call "high-loss" materials, like bakelite, shellac, paper, etc., in oil of high temperature (70 to 80 deg. cent.). The importance of the losses for those two kinds of materials from the practical point of view is quite different.

To make it clear, I should like to speak of two kinds of breakdowns. On one side we have what we call the "real electrical" breakdown. It is known to all of us. It occurs in one-minute tests, one-second tests, or 1/10-second tests. Its physical details are not explained. On the other side there is another kind of breakdown which I should call "heat breakdown" or "loss breakdown." It was explained by Steinmetz, Wagner, Dreyfuss, and others, and is based only on the specific losses and the heat conductivity. You will remember that in insulating materials there are losses, and the losses increase very fast as the temperature increases. If you have some kinds of material and apply a voltage, it will create losses and the losses are transformed into heat. The heat makes the temperature rise, and you may reach a balanced state or an unbalanced state, depending entirely upon the voltage impressed.

This effect might be illustrated by the curves in Fig. 2 herewith, representing losses plotted against time. Curve E_1 refers to a voltage E_1; Curve E_2 to a somewhat higher voltage E_2. The object measured might be a bushing of the paper type (with or without condenser layers). In the first case, an equilibrium is reached after some hours; in the second case breakdown occurs.

It is possible to predict the voltage at which this breakdown occurs before having made any tests, provided you know the specific losses of the material and the heat conductivity. An accuracy of 10 to 15 per cent is possible. I say that in order to show that this phenomenon is not simply an assumption but that it is a calculable fact.

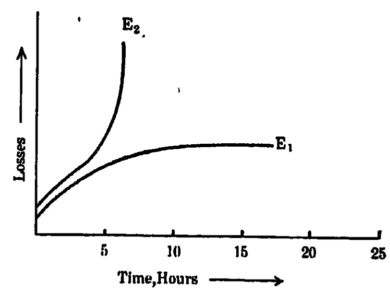

FIG. 2

Now, to get back to the point from which I started: I repeat that there are two kinds of breakdown voltages which may exist, namely, the heat breakdown voltage, due only to the losses, and the electrical breakdown voltage which, so far as known today, has no direct relation to the losses. In your cables, you have now very low losses and if you calculate the heat breakdown voltage for cables, you will see that you will come to about 180,000 volts in sustained service. You will see that today this heat breakdown voltage is of no importance whatsoever in connection with cables.

I do not wish to be misunderstood. Although the direct effect of the losses does not enter into consideration, that does not mean that loss measurements are of no importance. Loss measurements do give very interesting indications about moisture, regularity of manufacturing processes, and so on.

The heat breakdown voltage is quite another thing in bakelite. You are using it in hot oil in transformers; it may be at 60 to 90 deg. cent. In addition there are the specific losses of the material itself, so that you get there a limiting voltage of 60 kv. It might vary from 40 to 70 kv. You may be astonished because I apply such high voltages to bakelite. In speaking of those breakdown voltages, I am speaking of the case where you have only a unidirectional flow of heat. In a bushing you may have a dissipation of heat in the direction of the axis also, and the tension will be higher than that.

You see now that the electrical breakdown voltage for the cable is below this heat breakdown voltage and so the heat breakdown voltage is not of interest for cables. But it is quite a

different thing for bakelite. For bakelite the electrical breakdown voltage is above 60 kv., so the heat breakdown voltage is interesting. I know very many cases of commercial design where the heat breakdown is the governing factor.

Delafield DuBois: Mr. Marbury's paper, and Fig. 8 in particular, add to the data that we already have linking absorption, dielectric phase difference, and dielectric loss with moisture. Now the behavior of moisture in a dielectric under electrical stress, and particularly in a fibrous dielectric, is a most complex phenomenon. The moisture is strongly held by surface tension but is acted on by electrical forces tending to form it into conducting paths. Complicating this is the fact that the passage of current through these paths tends to disrupt them by heat generated in the paths.

If we had a dielectric containing a high resistance path embedded in it and partially bridging it, we would have the equivalent of the model condenser shown in Fig. 4 of the paper, and such a model would give a residual voltage curve as given in Fig. 6. But if this conducting path were a path of moisture it would not remain a fixed path of constant resistance, but would be constantly changing in length and resistance and interconnections with other moisture paths under the application and removal of voltage, and we should no longer expect a curve as Fig. 6. There are so many factors affecting this change that it is difficult to give them all proper weight in drawing conclusions. But, referring now to Fig. 10, it would seem that above a certain voltage the moisture paths, if we may consider them such, tend to increase in their effective resistance, acting to improve the dielectric. It is as if the dielectric became dryer with the increase of voltage.

It is obvious that for a higher voltage the conducting paths carry more current, and it does not seem unlikely that it is this current, dispersing the moisture paths, that is responsible for the shape of the curves of Fig. 10.

W. F. Davidson: Mr. Marbury's paper calls our attention to a very important phase of insulation behavior and one which gives promise of telling much about the fundamental behavior of electrical insulating material. The paper also presents a very interesting shop method for determining certain aspects of dielectric absorption, but I think it would be a mistake to classify the method as truly scientific.

Towards the end of the paper the author calls attention to an apparent saturation of the dielectric. In an effort to explain this we find a disconcerting lack of detailed information as to the apparatus used and the test procedure. For instance, the diagram of connection indicates a 110-volt d-c. supply for charging the condenser and we are without information as to the means used for varying the voltage in the individual tests. If this is done by means of a series resistance, certain results would be expected, while if it is done by varying the voltage of a battery with low internal resistance, the results would have a somewhat different characteristic. Furthermore we have no exact data as to the duration of the contact, either for the purpose of charging or for the purpose of discharging the condenser. An effort to determine this time by scaling from the drawing indicates that the contact has a duration in the order of 0.03 seconds, which can hardly be considered as a quick short circuit. Neither can the 60-ohm resistance be placed in the discharge circuit, be called low resistance, although it is quite permissible to use this sort of method for shop work.

In the early part of this paper the author referred to the "inherent uniformity" of impregnated paper, but I fear that his statements are somewhat misleading and a little too optimistic. Those of us who have had experience with high-voltage cable with impregnated-paper insulation fully realize that this material does not have an inherent uniformity; if it did have, many of the difficulties of the cable manufacturer would be things of the past. I must also question the thoroughness with which oil impregnates the material, for numerous observations have indicated that the thoroughness of impregnation depends in a very large measure on the type of fibre from which the paper is made. Probably Mr. Marbury's statement is quite correct for the type of paper used in manufacturing condensers, but it is hardly correct for some sorts of paper which have been suggested for use in high-voltage cable.

In the last paragraph of his paper the author makes the prediction that a study of absorption may afford a means of predicting cable failure in operating systems. A method of cable testing based upon this idea was described before the Institute in 1923 by Messrs. Phelps and Tanzer[2] and had been further developed by several operating companies. In a discussion of a paper on testing cable by Mr. Lee[3] presented at the Midwinter Convention of this Institute several aspects of the problem were discussed. Special high-speed, curve-drawing instruments for recording the data were described and some of the results presented.

Due to the large amounts of energy involved, the "discharge and recovery system" such as used for small condensers is very difficult to handle on long cable. Better results seem to be obtained by observing the characteristic during the period of charge. In addition to the advantage just mentioned for the charging system as distinguished from the discharge and recovery systems, there is the point that the readings are somewhat less influenced by the previous history of the cable. This is of great importance since the absorption of many of our cables is of very large magnitude and unless long times are allowed to elapse between successive tests the readings on any one test may be largely influenced by the preceding test value.

Probably the value of Mr. Marbury's paper could be very considerably increased if he could include some data taken with a electrostatic oscillograph showing the voltage across the condenser terminals during a complete cycle of the test, that is during the charge, the standing, the discharge, and the recovery periods. Such data would be very helpful in explaining the apparent saturation of the dielectric previously referred to. It would also give a better idea as to the behavior of the contacts and the effectiveness of the discharge circuit in removing basic quantity of electricity on the basic charge.

W. B. Kouwenhoven: The device developed by Mr. Marbury possesses many valuable features. It is similar in certain respects to the apparatus developed by the Bureau of Standards for charging and discharging condensers by the method of mixtures.[4] All who have used this apparatus know that different results will be obtained when the time of the charge, mix and discharge cycle is varied in any manner.

Mr. Marbury in his paper mentions the operation of his device at only one speed. It would be valuable to know what results would be obtained with some other cycle of charge and discharge and perhaps it would be possible to find some speed which will give results that would indicate more definitely the relative values of different types of insulating materials.

W. A. Del Mar (communicated after adjournment): A century and a half ago, Franklin made a Leyden Jar with removable coatings and found that the charge adhered to the glass and not to the coatings. Until three years ago this experiment was regarded as proof that the electric charge was held by the dielectric. In 1922, however, Addenbrooke upset this theory by repeating Franklin's experiment but taking special precautions to keep the surfaces of the glass absolutely dry, when he found that the charge adhered completely to the metal coatings. (*Phil. Mag.* 1922, Vol. 3, pp. 489 to 493). In Franklin's experiments, the charge which appeared to be in the glass was really bound to the moisture films on the surface of the glass.

2. A New Method for the Testing of A-C. High-Voltage Paper-Insulated Cables. A. I. E. E. JOURNAL, Vol. XLII, March 1923, p. 247.
3. Abridged A. I. E. E. JOURNAL, Vol. XLIV, February 1925, p. 156.
4. Curtis, H. L. *Bull. Bur. Stds.*, Vol. 6, p. 441, 1911.

There is moisture within most insulation and it can hold charges when the coatings are discharged. They would dissipate slowly through the high resistance of the dielectric, giving rise to residual-charge effects. It is therefore not necessary to assume the movement of moisture to explain dielectric absorption. The moisture merely acts as secondary or internal electrodes.

Mr. DuBois explains the effect of moisture by assuming that it collects in threads, stretching between electrodes, absorption being due both to the mechanical energy required to build the threads and to their influence in promoting the Clerk-Maxwell effect by shunting parts of the dielectric. If this were correct, the presence of moisture in sufficient quantities to produce distinct absorption effects should materially lower the breakdown voltage due to the short circuiting of parts of the insulation by these moisture filaments. This, however, is not the case, as tests with manila-rope paper impregnated with petrolatum show a distinctly higher dielectric strength when the paper is not dried prior to impregnation than when it is dried.

Research work is now needed to determine definitely whether the Clerk-Maxwell effect holds quantitatively in the absence of moisture and whether the added effect of moisture can be explained quantitatively by internal charges. If not, it will be time to look into the more complex theories that have been suggested.

E. S. Lee (by letter): The methods of measurement of residual voltage are of long-time standing. The particular feature ascribed to Mr. Marbury's instrument is that readings are obtained at intervals of 0.1 sec. up to 1.1 sec. after the condenser has been discharged. The means adopted for doing this excludes the usual caution that the condenser must be entirely discharged so that no residual charge remains for succeeding voltage applications. For this reason it would appear that results obtained by the instrument described by Mr. Marbury would be a function of the instrument constants.

Although not specifically stated, it would appear that the curves in Figs. 8 and 9 are obtained on comparable condensers. On the basis of this assumption it is interesting to note that the residual-voltage characteristics of an untreated-paper condenser during the drying (Fig. 8) are practically identical with the residual-voltage characteristics of treated-paper condensers having values of power factor from 0.2 per cent to 0.5 per cent (Fig. 9). From a standpoint of voltage rating and effectiveness of operation, the untreated-paper condensers would in no wise compare with the treated-paper condensers. The similarity of residual-voltage characteristics for such dissimilar insulations indicates the limitedness of the residual-voltage curves as a criterion for the effectiveness of insulation.

While claims are made by Mr. Marbury that all tests to date indicate that the residual-voltage curves are of the greatest value, it is interesting to note that while nothing new is pointed out by the author resulting from these curves, there is a correlation made between these curves and the values of power factor of the condensers measured. It would appear, therefore, that the correlation between the residual-voltage curves and the life of the insulation is not different from what we now know as between power factor and life.

R. E. Marbury: There are of course many causes of insulation losses, such as conduction losses, losses in the metal plates, and the so called hysteresis loss. The magnitude of the various losses depends a great deal on operating conditions; for example, on extremely high frequency the $I^2 R$ losses in the metal plates might be of great importance. On commercial frequencies such as 60 cycles the hysteresis type of loss is of greater importance, since the other losses can be reduced to neglible figures very easily. There is every indication that at least the most important cause of this hysteresis effect is absorption.

The long-transient residual curves have been observed for many years. It is quite common to receive a severe shock from a condenser after repeated attempts to discharge it. Maxwell's theory of absorption explained these long transients very well, and many investigators have plotted the residual voltage against time over long periods such as minutes, hours or even days after discharge. It can be easily shown mathematically that a condenser will develop these residual voltages if the products $R\ C$ for the various layers of insulation are not constant.

It has been recognized for a long time that the residual phenomenon of this type could not account for 60-cycle losses, due to the short duration of charge on 60 cycles, and that if 60-cycle loss was to be explained on this basis the same form of phenomenon would have to occur at a high rate of speed. The latter would require some very low products $R\ C$ as compared with the values capable of producing the common type of residual curves. In view of this, other more complicated theories of absorption were conceived, most of them admitting an actual lag in polarization. It is quite possible that there is a lag of this type but on 60 cycles it must be so slight as to be beyond measurement, and relatively unimportant from the point of view of losses.

While the model condenser may be only a rough approximation of a real condenser it may be used to a good advantage in discussing the phenomenon. Maxwell's theory is very useful. A model condenser such as shown in Fig. 4 of the paper will give a residual curve as shown in Fig. 5. If the values R_1 and R_2 are increased, the time required for the residual to reach its maximum value increases. The time required for the model to reach its steady state on charging also increases. The values R_1 and R_2 may be varied over unlimited range but the nature of the residual curves will be always the same. If we use an actual condenser, which in reality is composed of many such models in parallel, and charge it for a long time, we will obtain a residual curve which may require hours to build up to its maximum value. If we repeat this, using shorter and shorter charging time, we will find that as the time of charging is reduced the residual reaches its maximum value sooner. As long as we can continue reducing the charging time and obtain the same form of residual curves we are apparently correct in assuming that the cause of the residual is the same. Since the condenser is composed of many values $R_1\ C_1$, or $R_2\ C_2$, and since it is easily shown that the smaller these products the more rapid is the residual transient, it naturally follows that as the charging time is reduced the lower products $R\ C$ participate the most in producing the residual curves. The same may be said for long charging intervals until the charging time is long enough for the steady state to be reached for the largest products $R\ C$. If the charging time is augmented beyond this point the residual curve will not be affected.

The dielectric lag meter was developed to measure the residual curves with very short charging periods. The work that has been done has shown that the residual curves thus obtained are of the same type as with longer charging, thus proving that there are in a dielectric products $R\ C$ which are low enough to explain 60-cycle losses, and that the only difference between these residual curves and those heretofore obtained is that they build up very quickly, as is to be expected.

The principle of Maxwell's theory need not be modified. However, certain secondary phenomena do exist. The resistivity does not remain constant with voltage apparently due to movement of moisture under the influence of the field. Furthermore, the lines of force are not always perpendicular to the various materials, or to the metal electrodes, but become refracted in passing through media having different specific inductive capacities.

Dr. Whitehead spoke of the inability of the lag meter to record the entire or true residual curve. If Dr. Whitehead considers the true residual curve as that which would be obtained if the condenser remained on charge until a steady state is reached, then the lag meter will not measure it. Such a residual would require hours to develop, and the charging time would be of the same order. As stated above, the special object in view in the design of this device was to measure the residual that may be

secured with very short charging periods, or where the residual obtained is a function of low products R C.

In the paper the charging period mentioned in several illustrations is 1/10 sec. It was not intended that this figure be considered as important, as the charging time may be varied over a wide range during any investigation. The duration of discharge may also be varied depending on the data desired.

Dr. Whitehead mentions the effect of previous charge on residual curves, or in other words the "past history" of the dielectric. It is a fact, as can be readily shown, that if a dielectric is subjected to a routine charge and discharge a cyclic condition is quickly reached where the residual curves will repeat themselves very accurately for each charge and discharge, and the effect of previous charge disappears. With the lag meter this cyclic condition is reached long before the first reading can be taken, and in any case the galvanometer could not be balanced until this condition existed.

A graphic check on the results that may be obtained in so far as accuracy is concerned may be shown as follows:

Take the case of a model condenser such as shown in Fig. 4, but having the following values:

C_1 = 1.89 microfarads, C_2 = 3.11 microfarads,
R_1 = 796,000 ohms, R_2 = 4000 ohms.

It would be desirable to use for C_1 and C_2 perfect condensers, that is condensers having no residual of their own; otherwise the formula would not hold perfectly. The condensers used were not perfect but had a residual which was small compared to the residual caused by the model.

If we charge this condenser for a time T at a voltage V_o, the voltage being applied instantaneously, and then discharge it through an infinitely small resistance, the connection being left for an infinitely short time, we find mathematically that the residual voltage V at any time t after discharge follows the formula

$$V = V_o \frac{R_1 C_1 - R_2 C_2}{(C_1 + C_2)(R_1 + R_2)} \left[1 - e^{-\frac{T}{(C_1 \times C_2)} \times \frac{R_1 + R_2}{R_1 R_2}} \right] \left[e^{-\frac{t}{(R_1 c_1)}} - e^{-\frac{t}{R_2 c_2}} \right]$$

Calculating the residual on the above basis and using 100 volts for V_o and 0.07 sec. for T we obtain the dotted curve shown herewith in Fig. 3.

The solid curve in Fig. 3 was made on the same model using the lag meter but with a discharge resistance of 10 ohms left on for 0.0003 sec. If the two condensers C_1 and C_2 had been ideal condensers the curves would have practically checked.

Mr. DuBois states that the moisture films are elongated by the field sufficiently to bridge a part of the dielectric. He states that the current probably disrupts these paths and causes the unproportionality of residual to applied voltage.

It might be added that while this phenomenon may exist under certain conditions there is a movement of moisture that takes place very quickly. Probably the moisture movement that causes the shape of curve Fig. 10 in the paper is of a different nature from that Mr. DuBois has in mind. To illustrate this Fig. 4 is given herewith. This shows five residual curves made on the same condenser but with different charging times from 0.0014 sec. to 0.07 sec. When the charging time in this particular case exceeded 0.0035 sec. the curve took on a saturating shape. This became quite marked at a time of charging of 0.0065 sec., and the curve became almost horizontal with a charging time of 0.07 sec. This shows that a certain time is required for the moisture movement to take place. It is even more interesting to note that Curves A, B, C, D, E, may be made in any order and the same curves obtained, or any one curve may be made with increasing or decreasing voltage. This shows the wonderful accuracy of movement of the moisture, and suggests a movement within the limit of the surface-tension restoring force, rather than an extensive elongation and volatization as suggested by Mr. DuBois. If the condition exists as described by Mr. DuBois it would probably not repeat itself so accurately.

In reply to Mr. Davidson, residual-voltage curves as shown in Fig. 10 of the paper are made as follows. As far as balancing is concerned the lag meter is operated in the same way as when taking a residual curve as it builds up with time. The time-setting switch is left on one point and the residual thus obtained for this given time after discharge is plotted against applied

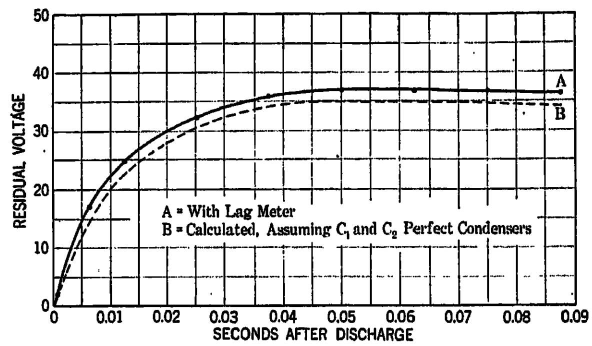

FIG. 3

voltage. Another way of course would be to run complete residual curves as in Fig. 9 for different applied voltages, then take the residual values corresponding to any desired time and replot against applied voltage. The same curve would be obtained of course in either case. The applied voltage was varied by means of a three-point rheostat having such characteristics as not to affect the results. If there were an effect of this kind the straight line as in Fig. 6 could not be obtained, nor would the straight lines as in Fig. 4 herewith be possible.

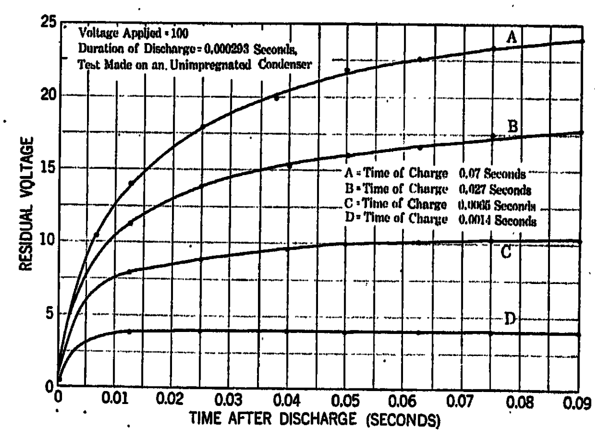

FIG. 4

Mr. Davidson mentioned the difficulties experienced in attempting to test cables by the charge-and-recovery scheme, owing to the large amount of energy handled. With the lag meter it is possible to read residual voltages as low as 0.1 volt with accuracy. There is no need, therefore, to use more than 100 volts. The lag meter works very satisfactorily on condensers having capacities as high as 10 microfarads. By slight adjustments it works equally as well on 100 microfarads.

Mr. Davidson asked how one could be sure that the discharge resistance was of the proper value, or if the discharge time was correct. This paper was written to describe the principle of measurement rather than the actual details and physical nature of absorption. It is possible that we may later have some

thoughts to offer on the subject of absorption in the light of data being obtained. Shortly after the contact drum described in the paper was made, a new drum was made which made possible

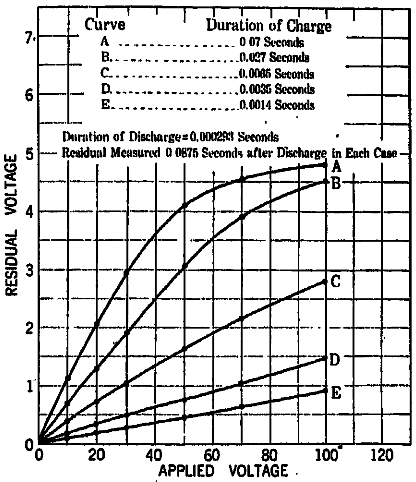

Fig. 5

a wide range of variation of time of charge, discharge, and the values of time following discharge were made smaller. This new drum was arranged so that the first point on the residual curve was only about 0.00035 sec. after discharge. This made it possible to observe at once if the condenser was discharged to practically zero.

In reply to Mr. Kouwenhoven regarding the speed, the speed may be changed at will by changing the worm-gear ratio on the driving mechanism. The speed which has been found of greatest value is such as to permit a charging time of 0.0025 to 0.07 sec.

Mr. Delmar refers to some of the more complicated theories of absorption, many of which assume an actual lag in polarization. Maxwell's theory will hold wherever the product $R\,C$ is not the same throughout the entire mass of material. We have only to show that in a dielectric there are products $R\,C$ low enough to cause or explain 60-cycle losses. The lag meter has proven the existence of low products $R\,C$ by the fact that with it, it is possible to record residual curves which build up to their maximum value in short spaces of time. Fig. 5 herewith gives a few such curves which show definitely the existence of such a condition.

Mr. Lee has called attention to the comparison between curves Fig. 8 and Fig. 9, stating that the residual is as high on the impregnated condenser as before. The statement that a condenser having power losses of the order of 0.5 per cent has higher residuals as shown by curve No. 5 Fig. 8 is incorrect. It should read Fig. 9. No comparison can be drawn directly between Figs. 8 and 9, as far as values of residual are concerned. Curves 1, 2, 3, 4 and 5 of Fig. 9 bear a relation to 60-cycle losses.

Separate Leakage Reactance of Transformer Windings

BY O. G. C. DAHL[1]
Associate, A. I. E. E.

Synopsis.—The paper discusses a method for determining the separate leakage reactances of transformer windings, originally suggested in 1921 by W. V. Lyon.[2] The method is applicable only to three-phase banks of three identical transformers, and makes use of the third harmonic electromotive force and current which are introduced into the windings by the inherent magnetizing characteristics of the iron. Attempts made to determine the separate leakage reactances of a single transformer did not meet with success.

The method may be used both with "two-winding" and "multi-winding" transformers. However, it is particularly convenient when the transformers have more than two windings.

Laboratory tests have been made on small experimental, two-winding and three-winding transformers, and field tests on a bank of three-winding power transformers. The results in each case warrant the conclusion that the separate leakage reactances may be obtained with sufficient accuracy for engineering purposes.

Introduction

The separate leakage reactances of transformer windings cannot be calculated with accuracy. The standard formulas found in textbooks on principles and design of transformers are all based on broad assumptions in regard to the distribution of the leakage flux, and may easily give results which are in error to a considerable extent. Furthermore, it seems to be doubtful whether more rigorous and reliable formulas are capable of being developed.

So far, no method has been available for experimental determination of the individual leakage reactances. It is easy to obtain the equivalent leakage reactance by a short-circuit test, but when it comes to assigning proper fractions of this reactance to the separate windings, difficulties are encountered. Usually the equivalent reactance is split equally between the two windings. This procedure, however, is frequently far from correct although often the only one which can be resorted to.

The purpose of this paper is to present a method, originally suggested in 1921 by W. V. Lyon,[2] by which the separate leakage reactances of transformer windings may be determined experimentally. This work was done as an introduction to a general study of transformer harmonics undertaken in the Electrical Research

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Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

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The theory of the method is briefly discussed and data and results as obtained from tests on three small experimental transformers are given.

The method has also been applied with success to a bank of three 2100-kv-a., 110,000/22,000/2300-volt transformers.[4]

Conception of Leakage Reactances

The mutual flux in an iron-core transformer is usually considered to be exclusively confined to the core. This assumption is not entirely rigorous. Evi-

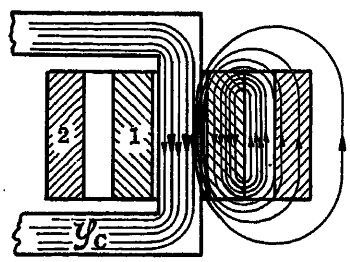

Fig. 1—Flux Distribution (Very Approximate) When the Inner Coil (1) Carries Current

dently part of the flux which the current in any one coil sets up in the air will produce linkages with the other coils and hence be a mutual flux in the true sense of the word.

All fluxes which have their entire path in air are proportional to the current producing them. This is true also for fluxes which only partly exist in the air, since the reluctance of the iron path is insignificant as compared to the reluctance of the air path.

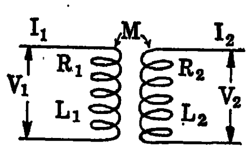

Fig. 2

Fig. 1 roughly illustrates the fluxes when the inner coil (1) carries current. As seen, a small part of the flux set up in the air will link with turns of coil (2) and produce a voltage in this coil in addition to the voltage produced by the flux in the core. This additional voltage will be in time quadrature with the current in coil (1).

Fig. 2 represents a two-winding transformer with both windings carrying current. Considering voltages and currents of a *single frequency* only the vector equations for the voltage drops in the two windings may be written:

$$V_1 = R_1 I_1 + j\,\omega L_1 I_1 + j\,\omega M_{12} I_2 \qquad (1)$$

$$V_2 = R_2 I_2 + j\,\omega L_2 I_2 + j\,\omega M_{21} I_1 \qquad (2)$$

Since the permeability of the iron is a function of the instantaneous flux density and hence of the instantaneous current, the self and mutual inductances will be some function of the current. On the contrary, the inductances which are due to fluxes in the air will be constant.

The flux in the core itself contributes the part, M_c, of the mutual inductance. Introducing this quantity, the two equations above may be rewritten in the following form:

$$V_1 = R_1 I_1 + j\,\omega\,(L_1 - M_c)\,I_1 + j\,\omega\,(M_{12} - M_c)\,I_2 + j\,\omega\,M_c\,(I_1 + I_2) \qquad (3)$$

$$V_2 = R_2 I_2 + j\,\omega\,(L_2 - M_c)\,I_2 + j\,\omega\,(M_{21} - M_c)\,I_1 + j\,\omega\,M_c\,(I_1 + I_2) \qquad (4)$$

In equation (3), $L_1 - M_c$ is a *constant* inductance due to all the flux in the air set up by the current in coil (1). It is the "self leakage inductance" of this coil. $M_{12} - M_c$ is the constant "mutual leakage inductance" of coil (1). The term $j\,\omega\,M_c\,(I_1 + I_2)$ evidently represents the voltage induced in coil (1) by the flux which exclusively exists in the core. The corresponding quantities in equation (4) may be similarly interpreted with reference to coil (2).

Introducing

$$X_{11} = \omega\,(L_1 - M_c) \qquad (5)$$

$$X_{22} = \omega\,(L_2 - M_c) \qquad (6)$$

$$X_{12} = X_{21} = \omega\,(M_{12} - M_c) = \omega\,(M_{21} - M_c) \qquad (7)$$

$$E_{1c} = E_{2c} = j\,\omega\,M_c\,(I_1 + I_2) \qquad (8)$$

equations (3) and (4) reduce to

$$V_1 = E_{1c} + (R_1 + j\,\omega\,X_{11})\,I_1 + j\,\omega\,X_{12}\,I_2 \qquad (9)$$

$$V_2 = E_{2c} + (R_2 + j\,\omega\,X_{22})\,I_2 + j\,\omega\,X_{21}\,I_1 \qquad (10)$$

Neglecting the exciting current, the primary and secondary currents are equal and opposite and the equivalent impedance drop $V_1 - V_2$ becomes

$$\begin{aligned} V_1 - V_2 &= [R_1 + j(X_{11} - X_{12})]\,I_1 - [R_2 + j\,(X_{22} - X_{21})]\,I_2 \\ &= (R_1 + j\,X_1)\,I_1 - (R_2 + j\,X_2)\,I_2 \\ &= [\,R_1 + R_2 + j\,(X_1 + X_2)\,]\,I_1 \\ &= (R_e + j\,X_e)\,I_1 \end{aligned} \qquad (11)$$

The reactances, X_1 and X_2, are the true leakage reactances of windings (1) and (2), respectively, with respect to the other winding. The relative aspect of the leakage reactances should be carefully noted. The leakage reactance of a winding is not a quantity which is dependent upon and characteristic of that winding alone; it must, of necessity, be defined with respect to some other winding. This fact becomes particularly important in multi-winding transformers. Thus in a transformer having three windings designated Nos. 1, 2 and 3, the leakage reactance of winding No. 1 with respect to winding No. 2 will be in general different from the leakage reactance of the same winding with respect to winding No. 3.

3. Some results of these researches have been incorporated in the author's report, "Transformer Harmonics", published in the report of the Inductive Coordination Committee of the National Electric Light Association in June 1923.

4. See the paper "Transformer Harmonics and Their Distribution."

In the following the true leakage reactance will be termed the leakage reactance and the mutual leakage reactance will be called the mutual reactance.

The relative magnitude of the leakage and the mutual reactance of a winding depends upon the spacing and arrangement of the coils. If the spacing is large the mutual reactance is small and in some cases may even become negligible.

The standard short-circuit test gives the equivalent leakage impedance of any two windings of a transformer. Equation (**11**) expresses short-circuit conditions when V_2 is zero. Fig. 3 shows the vector diagram of a short-circuited transformer, the exciting current being neglected and the primary vectors being rotated

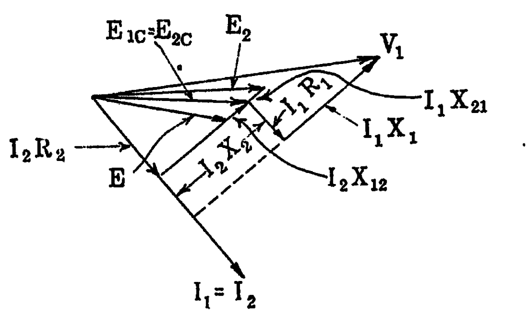

FIG. 3—VECTOR DIAGRAM OF A SHORT-CIRCUITED TRANSFORMER, SHOWING RESISTANCE, LEAKAGE REACTANCE AND MUTUAL REACTANCE DROPS

through 180 deg. Both leakage reactance drops and mutual reactance drops are indicated on the diagram.

EXPERIMENTAL DETERMINATION OF SEPARATE LEAKAGE REACTANCES

a. General. The experimental method by which the separate leakage reactance of the windings of a transformer may be determined makes use of the third harmonic component which inherently exists in the magnetizing current of a transformer when a sinusoidal voltage is impressed. The method is applicable only when a three-phase bank of three identical transformers is available. Attempts made to determine the leakage reactance of a single transformer did not meet with success.

The principle of the method is as follows: If sinusoidal voltages are impressed on a Y-Δ-connected bank of transformers, the third harmonic component of the magnetizing current will be confined to the delta where it appears as a circulating current. If the transformers are perfectly balanced and there is no external load on the secondary, no current other than the third harmonic and its multiples can exist in the delta. Usually the ninth and fifteenth harmonics, etc., are negligible and need not be considered. The third harmonic electromotive force induced per phase of the delta is just balanced by the triple frequency impedance drop due to the circulatory third harmonic current. The problem is then to measure with precision the proper third harmonic electromotive force and current, which, by simple division, will give the desired triple frequency leakage impedance.

b. Two-winding Transformers. The bank is connected Y-Δ and balanced sinusoidal voltages impressed. The third harmonic current in the delta and the third harmonic electromotive force per phase on the primary side are recorded. The latter is most practically obtained by connecting a Y-connected resistor bank between the lines and measuring the voltage between the resistor and transformer neutrals.

If the generator is Y-connected and its phase voltage is free from a third harmonic (and multiples), the bank of resistors may be omitted and the voltage measured between generator and transformer neutrals. No commercial Y-connected generator, however, is entirely without a third harmonic component in its voltage to neutral, so this method is scarcely of practical interest.

As a rule, it will be necessary to take oscillographic records and take out the third harmonics by analysis. While the current in the delta is sensibly third harmonic, a fundamental and also other harmonics are unavoidable between the two neutrals if even the slightest unbalance in the impressed voltages, the resistors or the transformers themselves is present.

The diagram of connections and the third harmonic vector diagram are given in Fig. 4 and Fig. 5, respectively. One to one ratio of transformation is assumed.

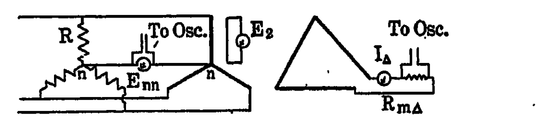

FIG. 4—DIAGRAM OF CONNECTIONS FOR LEAKAGE IMPEDANCE TEST ON TWO-WINDING TRANSFORMER

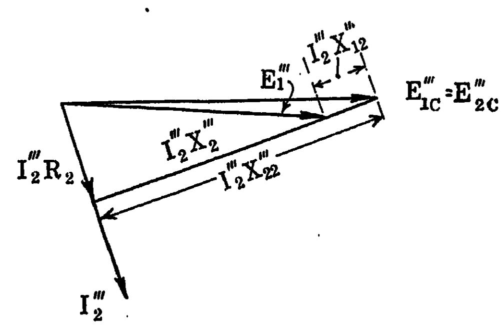

FIG. 5—VECTOR DIAGRAM OF THIRD HARMONIC QUANTITIES INVOLVED IN THE "TWO-WINDING METHOD"

If the transformers have another ratio, the quantities in the various equations given below should all be referred to the same side.

If R is the resistance of the resistors per phase, r the resistance of the voltmeter, and E_{nn}''' the third harmonic voltage between the neutrals, then

$$E_1''' = E_{nn}''' \left(1 + \frac{R}{3\,r}\right) \qquad (12)$$

This voltage is the vector sum of two components. One component (E_{c1}''') is induced in each primary winding by the triple frequency flux (ϕ_c''') in the core.

The second component ($-jX_{12}'''I_2'''$) is induced by the part of the third harmonic flux in the air which produces linkages with the primary winding. Hence from equation (**10**), considering voltage rises,

$$E_1''' = E_{1c}''' - jX_{12}'''I_2''' = E_{2c}''' - jX_{12}'''I_2''' \quad (13)$$

The triple frequency leakage impedance and reactance of the secondary windings are now found by

$$Z_2''' = \frac{E_1'''}{I_2'''} \quad (14)$$

$$X_2''' = \sqrt{(Z_2''')^2 - (R_2)^2} \quad (15)$$

This X_2''' is a *true triple frequency leakage reactance.* By repeating the measurements with the original primary winding as secondary and vice versa, the leakage reactance of the other winding may be found in a similar manner. Dividing the triple frequency reactances by three, the fundamental reactances are found.

Having determined the individual leakage reactances, the equivalent impedance of the transformer becomes (1 to 1 ratio assumed):

$$Z_e' = \sqrt{(R_1 + R_2)^2 + (X_1' + X_2')^2} \quad (16)$$

This value should, if correct, check the short-circuit impedance of the transformer to at least engineering accuracy.

c. Three-winding Transformers. When the transformers have more than two windings, a more convenient method may be used which eliminates the necessity of establishing the artificial neutral on the primary side. As before, the primaries are Y-connected, while the other two windings are Δ-connected. One delta is closed and the circulating third harmonic current in it recorded. The other delta is not closed; the third harmonic voltage appearing across the open corner of this delta may therefore be measured and will for balanced conditions be equal to three times the third harmonic electromotive force per phase. Recording these two quantities makes it possible to determine the leakage reactance of the closed delta winding with respect to the open delta winding. In many cases oscillographic records are unnecessary when the transformers are well balanced, as both the voltage and the current will be sensibly third harmonic.

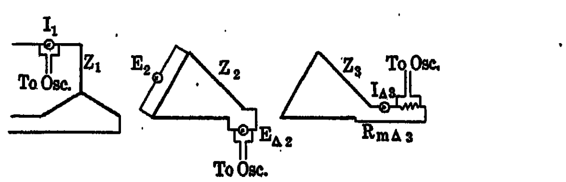

FIG. 6—DIAGRAM OF CONNECTIONS FOR LEAKAGE IMPEDANCE TEST ON THREE-WINDING TRANSFORMER

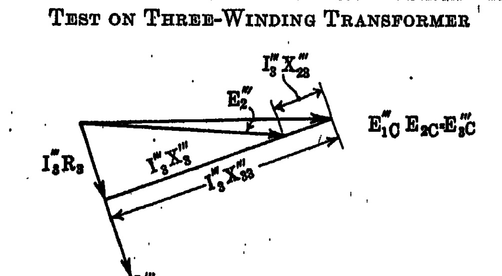

FIG. 7—VECTOR DIAGRAM OF THIRD HARMONIC QUANTITIES INVOLVED IN THE "THREE-WINDING METHOD"

The connections are shown in Fig. 6 and the third harmonic vector diagram in Fig. 7.

Solution of the vector diagram exactly as in the preceding case gives the leakage reactance of winding No. 3 with respect to winding No. 2. By repeating the measurements with changed connections, the other individual reactances are obtained.

It is beyond the scope of this paper to discuss how the separate leakage impedances in a three-winding transformer may be combined for the purpose of calculating load division, etc., between the various windings when they all carry currents.

EXPERIMENTAL WORK

a. Apparatus. Fig. 8 shows one of the test transformers used for the experimental work in the Electrical Research Laboratories of the Massachusetts Institute of Technology. Three transformers of this type were used, the approximate rating of each being two kw.

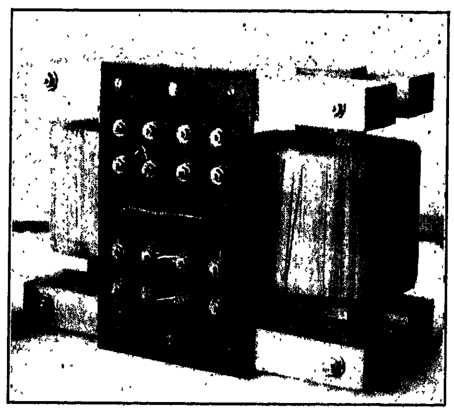

FIG. 8—CORE-TYPE EXPERIMENTAL TRANSFORMER USED IN THE TESTS AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY

The cores, built up of silicon steel laminations with lap joints, are firmly held together by wooden frames with through-going brass bolts. The dimensions of each lamination are 8 in. by 1½ in. by 0.014 in. The gross thickness of the cores is 2 in., giving a gross cross-sectional area of 3 sq. in. (19.36 sq. cm.). Assuming 95 per cent lamination factor, the net area becomes 2.85 sq. in. (18.4 sq. cm.).

There are four coils on each leg, numbered 1, 2, 3, and 4, in the order of their proximity to the core. Each coil consists of 100 turns, double cotton covered copper wire, wound in two layers with the thickness of the insulation only between layers. Coils No. 1 and No. 2 are wound with No. 12 B. & S. wire and are separated by the thickness of the insulation only. Coil No. 3, also of No. 12 wire, is spaced ¾ in. from coil No. 2, beind supported by square wooden spacers. Coil No. 3 has wound with it a search coil of No. 24 wire, which is designated as coil No. 4.

Both ends of each coil are brought out to binding posts on a fibre terminal board. During most of the tests the corresponding coils on the two legs were connected in series, forming windings of 200 turns.

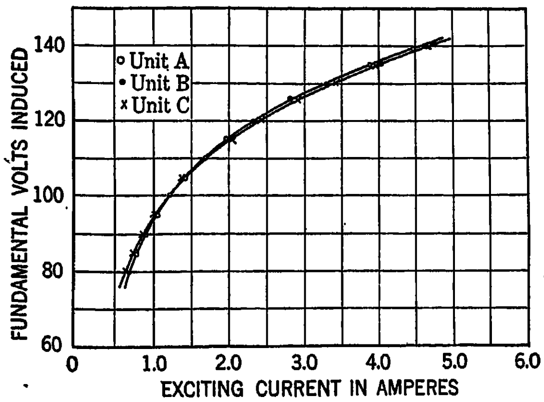

FIG. 9—SINGLE-PHASE EXCITING CURRENT OF THE EXPERIMENTAL TRANSFORMERS. THE CURVES SHOW THAT THE TRANSFORMERS WERE WELL BALANCED ELECTRICALLY

The transformers were well balanced electrically. This is apparent from Fig. 9, which shows curves of the exciting currents of the three units as obtained from single-phase tests with a sinusoidal voltage impressed.

As source of power a three-phase, 60-cycle, 5-kw., 230-115-volt sine-wave generator was used. This generator gave a very staisfactory wave shape at all balanced loads.

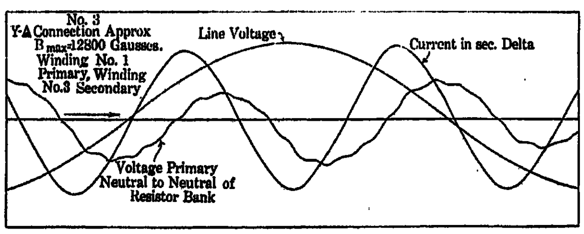

FIG. 10—OSCILLOGRAM FROM LEAKAGE IMPEDANCE TEST ON TWO-WINDING TRANSFORMER. CIRCUIT CONNECTIONS SHOWN IN FIG. 4.

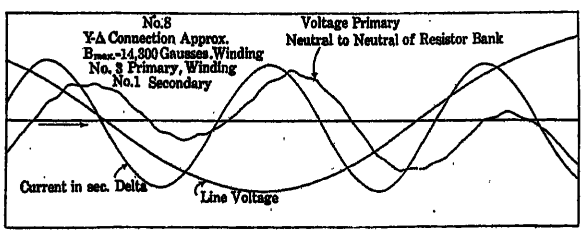

FIG. 11—OSCILLOGRAM FROM LEAKAGE IMPEDANCE TEST ON TWO-WINDING TRANSFORMER. CIRCUIT CONNECTIONS SHOWN IN FIG. 4

Vacuum thermocouples were used for the measurements whenever it was necessary to record voltages without drawing appreciable current, and also in order to measure currents in circuits where commercial ammeters would cause serious disturbance in existing conditions. Either shunt or series resistance was used in the heater circuit in order to adapt the thermocouple to any desired range. A microammeter was used as indicator in the circuit of the thermo-element.

b. Test Results.[5] Figs. 10 and 11 show two of a series of oscillograms when the "two-winding" test was applied to the experimental transformers. Fig. 10 was taken while the winding designated No. 1 was used as primary, winding No. 3 being secondary. In Fig. 11, winding No. 3 was primary, No. 1 secondary. The spacing between these two windings is large enough so as to make the mutual effect of the air fluxes small.

The necessity of using oscillographic records when this connection with artificial neutral on the primary side is used is best illustrated by the curves (Fig. 12) which give the triple frequency leakage impedance of coil No. 1 and coil No. 3 as computed directly from meter readings. According to these curves the leakage

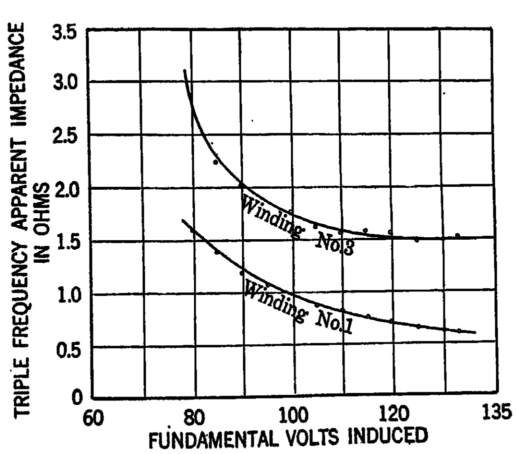

FIG. 12—THE CURVES SHOW THE "APPARENT" LEAKAGE IMPEDANCE AS OBTAINED FROM METER READINGS ALONE USING THE CONNECTIONS SHOWN IN FIG. 4. THE "APPARENT" VARIATION WITH SATURATION IS MAINLY DUE TO A PRONOUNCED FUNDAMENTAL VOLTAGE OF VARYING MAGNITUDE IN ADDITION TO THE THIRD HARMONIC BETWEEN THE TRANSFORMER AND RESISTOR NEUTRALS. HENCE IT IS IN GENERAL NECESSARY TO TAKE OSCILLOGRAPHIC RECORDS WHEN ARTIFICIAL PRIMARY NEUTRAL IS USED.

impedances apparently vary with the saturation, which in reality is not the case. The apparent variation is mainly due to a pronounced fundamental voltage of varying magnitude in addition to the third harmonic between the transformer and resistor neutrals (see Figs. 10 and 11).

Tables I and II give the data and results from this test performed at four values of flux density. The oscillograms were analyzed for their third harmonic components, the current waves by the 5-ordinate, and the voltage by the 11-ordinate schedule method. It will be noticed, however, that the current, without appreciable error, might have been taken directly from the meter readings since it appears to be practically

5. Part of the given experimental data were recorded by W. J. Miller, formerly of the Massachusetts Institute of Technology.

TABLE I

Connections	Oscillogram No.	Approx. flux density (Gausses)	Volts induced in winding No. 2			Amperes in delta I_Δ	Volts between neutrals E_{nn}	Ohms Resistance in metering circuit $R_{n\Delta}$
			Unit A	Unit B	Unit C			
Windings No. 1 primary Y; No. 3 secondary Δ.	1	9200	90.0	90.0	89.0	0.339	0.645	1.34
	2	11200	110.0	110.0	109.0	0.670	1.120	1.25
	3	12800	125.0	125.0	124.3	1.270	1.980	1.14
	4	14300	140.0	140.0	139.3	2.470	3.885	0.89
Windings No. 3 primary Y; No. 1 secondary Δ	5	9200	90.0	90.0	89.0	0.330	0.972	1.45
	6	11200	110.0	110.0	109.3	0.710	1.240	1.25
	7	12800	125.0	125.0	124.0	1.330	1.620	1.25
	8	14300	140.0	140.0	139.0	2.670	2.300	1.00

TABLE II

Connections	Oscillogram No.	Third harmonic e. m. f. per phase E'''	Third harmonic current in delta I_Δ'''	Total third harmonic impedance per phase of delta Z_t'''	Resistance of delta winding per phase R	Total resistance per phase of delta R_t	Third harmonic reactance per phase of delta X'''	Average third harmonic reactance X'''
Windings No. 1 primary Y; No. 3 secondary Δ.	1	0.521	0.339	1.537	0.466	0.913	1.235	
	2	1.023	0.670	1.530	0.466	0.883	1.249	
	3	1.922	1.270	1.513	0.466	0.846	1.254	
	4	3.855	2.470	1.560	0.466	0.763	1.360	1.275
Windings No. 3 primary Y; No. 1 secondary Δ.	5	0.282	0.329	0.858	0.284	0.767	0.384	
	6	0.598	0.710	0.842	0.284	0.701	0.466	
	7	1.085	1.330	0.816	0.284	0.701	0.417	
	8	1.970	2.660	0.741	0.284	0.617	0.410	0.414

third harmonic. Inspection shows that the maximum difference between any single value of the third harmonic leakage reactance and the average is about 10 per cent.

The leakage reactance of winding No. 3 was also found by the "three-winding method." Fig. 13 shows

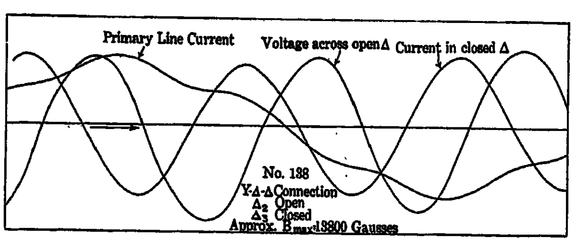

FIG. 13—OSCILLOGRAM FROM LEAKAGE IMPEDANCE ON THREE-WINDING TRANSFORMER. CIRCUIT CONNECTIONS SHOWN IN FIG. 6.

one of the oscillograms, while Table III gives the data and results from this test, which also was performed at four densities. It will be noted that the test gives the leakage reactance of winding No. 3 with respect to winding No. 2. Windings No. 1 and No. 2, however, were so close together that the leakage between them was entirely negligible; hence the leakage reactance of winding No. 3, with respect to winding No. 2, coincides with the leakage reactance of winding No. 3 with respect to winding No. 1.

As seen from the oscillogram, the current in delta No. 3 is practically a pure third harmonic, while the voltage across the open corner of delta No. 2 is very nearly third harmonic. It appears from Table III that the discrepancy between results obtained from wave analyses and those obtained directly from meter readings is only about 3½ per cent.

It will also be noted that this method gives much more consistent results than the Y-Δ method with artificial neutral point on the primary side. The four values of the leakage impedance are practically coinciding.

The value of the triple frequency leakage reactance of winding No. 3 from this test, is 1.336 ohms as compared with 1.275 ohms from the other test. The difference between the two is about 4½ per cent.

Since more consistent results are obtained in this test, the larger value of X_3''' is assumed to be the better one. Using then

$$X_1''' = 0.414 \text{ ohm}$$
$$X_3''' = 1.336 \text{ ohms}$$

the fundamental reactances become

$$X_1' = 0.138 \text{ ohm}$$
$$X_3' = 0.445 \text{ ohm}$$

The average short-circuited impedance of windings No. 1 and No. 3 obtained from a series of short-circuit tests on all transformers is 1.023 ohms, and the ohmic resistances, measured directly after the short-circuit tests, are 0.308 ohm and 0.513 ohm for windings No. 1 and No. 3 respectively. It will be noticed that these resistances are slightly larger than those used in the tables. This, of course, is due to the fact that the transformers became heated during the short-circuit tests, while the current during the other tests was entirely too small to cause any appreciable temperature rise.

Using the individual reactances in connection with the hot resistances gives for the equivalent impedance:

$$Z_e' = \sqrt{(0.308+0.513)^2 + (0.138 + 0.445)^2} = 1.007 \text{ ohm}$$

TABLE III

Connections	Oscillogram Number	Approx. flux density Gausses	Volts induced in winding No. 2 E_2	Volts across open delta $E_{\Delta 2}$	Amperes in closed delta $I_{\Delta 3}$	Ohms impedance $\frac{E_{\Delta 2}}{3\, I_{\Delta 3}}$	Ohms resistance in metering circuit $R_{m \Delta 3}$	Third harmonic volts across open delta $E_{\Delta 2}'''$	Third harmonic amperes in closed delta $I_{\Delta 3}'''$	Ohms impedance $\frac{E_{\Delta 2}'''}{3\, I_{\Delta 3}'''}$
Winding No. 1 primary Y, No. 2 open Δ, No. 3 closed Δ.	137	13100	128.0	6.10	1.310	1.552	0.66	5.88	1.308	1.500
	138	13800	135.0	8.32	1.785	1.554	0.66	8.02	1.781	1.501
	139	12300	121.0	4.60	0.987	1.553	0.66	4.43	0.984	1.502
	140	11300	111.0	3.22	0.691	1.554	0.66	3.10	0.689	1.500

Average $\frac{E_{\Delta_2}}{3\, I_{\Delta_3}} = 1.553$ ohms Average $\frac{E_{\Delta_2}'''}{3\, I_{\Delta_3}'''} = 1.501$ ohms Discrepancy = 3.46 per cent.

Average $X_3''' = \sqrt{(1.501)^2 - (0.466 + 0.66)^2} = 1.336$ ohms

The discrepancy between the two equivalent impedances is

$$\frac{1.023 - 1.007}{1.023}\, 100 = 1.56 \text{ per cent}$$

Summary

The paper has presented a method for experimental determination of separate leakage reactances of transformer windings. It is applicable only to three-phase banks of identical transformers and is based on simultaneous measurements of a third harmonic current and the third harmonic electromotive force producing it.

The method can be used both with two-winding and multi-winding transformers. It is particularly convenient, however, when the transformers have more than two windings since the necessity of establishing an artificial neutral point on the primary side is eliminated. Furthermore, oscillographic records are less important in this case due to the fact that the quantities measured are very nearly of triple frequency when the transformers are well balanced. Meter readings alone will therefore often be sufficient.

The experimental data from the tests in the Electrical Research Laboratories of the Massachusetts Institute of Technology, which have been reproduced, show that the obtained results are consistent and accurate enough for engineering purposes. This was also substantiated by the field tests previously referred to on the bank of three 2100-kv-a., 110,000/22,000/2300-volt transformers.

List of Symbols

V = Terminal voltage
E = Induced voltage
E_c = Voltage induced by flux in the core
E_Δ = Voltage across the corner of an open delta winding
I = Current
I_Δ = Circulatory current in a closed delta winding
ϕ_c = Flux exclusively confined to the core
$R_{m\Delta}$ = resistance in the corner of a closed delta winding

Subscripts attached to the above symbols refer the quantities to the particular winding designated by the subscript.

E_{nn} = Voltage between resistor and transformer neutrals
R = Resistance of resistors per phase
r = Resistance of voltmeter
R_1, R_2 = Resistance of windings No. 1 and No. 2
L_1, L_2 = Self-inductance of windings No. 1 and No. 2
$M_{12} = M_{21}$ = Mutual inductance between windings No. 1 and No. 2
M_c = Mutual inductance due to flux in the core
X_{11}, X_{22} = Self-reactance of windings No. 1 and No. 2
$X_{12} = X_{21}$ = Mutual reactance of windings No. 1 and No. 2
X_1, X_2 = Leakage reactance of windings No. 1 and No. 2 with respect to some specified winding
R_e = Equivalent resistance of two windings
X_e = Equivalent reactance of two windings
$\omega = 2\pi f$ = Angular velocity

Several of the above quantities are different for the various harmonics. Primes attached to these quantities indicate the order of the harmonic.

Discussion

For discussion of this paper see page 810.

Transformer Harmonics and Their Distribution

BY O. G. C. DAHL*

Associate, A. I. E. E.

Synopsis.—This paper discusses briefly the distribution of harmonics in single-phase transformers and in three-phase banks of single-phase transformers. Two-winding transformers or three-winding transformers, where the harmonic current exists in two of the windings only, are considered.

Formulas for the distribution of harmonic currents between primary and secondary circuits are given.

Data and results from tests on small experimental transformers in the Research Laboratories of the Electrical Engineering Department, Massachusetts Institute of Technology, and also from field tests on a bank of power transformers and a transmission line, have been reproduced.

Both in laboratory and field tests calculated and measured values check to engineering accuracy.

INTRODUCTION

DURING the last few years an investigation of the general subject of transformer harmonics has been undertaken by the Research Division of the Electrical Engineering Department, Massachusetts Institute of Technology. In a concurrent paper, the writer, who has been intimately associated with the various phases of this research, has discussed a method for experimental determination of the separate leakage reactances of transformer windings.

The purpose of this paper is to give a brief discussion of the problem of transformer harmonics and their distribution between primary and secondary circuits. Some data from laboratory tests on small experimental transformers and also some data and results from tests on a bank of three 2100-kv-a., 110,000/22,000/2300-volt transformers are presented.

SINGLE-PHASE TRANSFORMERS

It is a well-known fact that when a sinuoidal voltage is impressed upon a single-phase transformer the exciting current that it takes will be non-sinusoidal. The number and magnitude of the harmonics which the exciting current contains depend upon the characteristics of the iron and the maximum density at which it is operated.

The third harmonic is generally by far the most prominent, while usually an appreciable fifth also is present. Higher odd harmonics are also easily traced, but their magnitude is, in general, very small. These higher harmonics are, therefore, of minor importance as far as the operation of the transformer is concerned. At normal saturation the fundamental is about 90 per cent, the third harmonic about 45 per cent, and the fifth harmonic about 15 per cent of the equivalent sinusoidal exciting current. The percentage harmonics increase with the flux density as long as this is not forced up to abnormal values. At such abnormal saturations the percentage harmonics may be expected to decrease.

In the discussion of distribution of harmonics which follows, the third harmonic alone is mentioned and the equations have been established for this harmonic. It should be noted, however, that in single-phase connections all harmonics follow the same laws and what is said in regard to the third harmonic will hold for any harmonic.

Consider the third harmonic components in the transformer circuit shown in Fig. 1. In general, third harmonic currents will flow in the secondary as well as in the primary circuit.

The voltage of the generator is assumed to be strictly sinusoidal. Since third harmonic currents exist they must be produced by triple frequency electromotive forces. Being sinusoidal, the impressed voltage cannot directly give rise to third harmonic currents. Hence the triple frequency electromotive forces are generated by a triple frequency flux in the iron core. Assuming unity ratio of transformation, or referring all quantities to the same side, the third harmonic electromotive

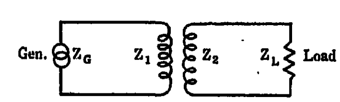

FIG. 1—LOADED SINGLE-PHASE TRANSFORMER

forces induced in the two windings by this flux are equal in magnitude and phase. The following equations inter-relate the third harmonic quantities:

$$V_1''' = I_1''' Z_G''' = E_{1C}''' - I_1''' Z_{11}''' - j X_{12}''' I_2''' \quad (1)$$

$$V_2''' = I_2''' Z_L''' = E_{2C}''' - I_2''' Z_{22}''' - j X_{21}''' I_1''' \quad (2)$$

$$Z_{11}''' = R_1 + j (X_1''' + X_{12}''') \quad (3)$$

$$Z_{22}''' = R_2 + j (X_2''' + X_{21}''') \quad (4)$$

The symbols used in these equations have the following meaning:

$E_{1C}''' = E_{2C}'''$ = third harmonic electromotive forces induced in the windings by the flux in the core

$V_1''', \ V_2'''$ = third harmonic terminal voltages

$I_1''', \ I_2'''$ = third harmonic currents

$Z_{11}''', \ Z_{22}'''$ = third harmonic self-impedances

$Z_1''', \ Z_2'''$ = third harmonic leakage impedances

X_{12}''', X_{21}''' = third harmonic mutual reactances

Z_G''' = third harmonic impedance of generator and primary leads

Z_L''' = third harmonic impedance of load and secondary leads

*Massachusetts Institute of Technology.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

It is apparent that it is impossible to get entirely rid of a small third harmonic component in the terminal voltages of a transformer, even if the generator voltage is a pure sine wave. The complete elimination of the third harmonic voltage components would require zero triple frequency impedance of the circuits (or one of the circuits) where the third harmonic current flows, a condition which of course never can be fulfilled.

In single-phase or polyphase connections, however, where no appreciable impedance is offered to the flow of the third harmonic current, the triple frequency voltages will usually be so small as to be entirely negligible in comparison with the fundamental component. The resultant voltages, therefore, will be sensibly sinusoidal.

Not only the magnitude, but also the phase of the impedance which the third harmonic current must overcome, has an important effect on the magnitude of the third harmonic voltages and currents. Without going into the question in further detail it may be said that a lagging third harmonic current in general reduces the triple frequency flux in the core, while a leading current tends to amplify it. The author intends to discuss the mechanism of this interaction in a future paper.

Transposing terms, equations (1) and (2) may be written:

$$E_{1C}''' = I_1''' (Z_{11}''' + Z_G''') + j\, I_2''' X_{12}''' \qquad (5)$$

$$E_{2C}''' = I_2''' (Z_{22}''' + Z_L''') + j\, I_1''' X_{21}''' \qquad (6)$$

By equating these expressions the distribution of the third harmonic current between the primary and secondary is obtained. The ratio of the currents is:

$$\frac{I_1'''}{I_2'''} = \frac{Z_{22}''' + Z_L''' - j\, X_{12}'''}{Z_{11}''' + Z_G''' - j\, X_{21}'''} = \frac{Z_2''' + Z_L'''}{Z_1''' + Z_G'''} \qquad (7)$$

Equation (7) shows that the distribution of third harmonic current between the two windings depends upon the leakage impedance of the windings and upon the external impedances. The distribution is inversely proportional to the ratio of the total impedances of the two circuits.

Usually the load impedance will be very much greater than the other impedances involved (the two leakage impedances and the generator impedance) when referred to the secondary side. Hence, in general, the third harmonic current in the primary will be many times greater than in the secondary when both currents are referred to the same side. If they are not referred to the same side, the actual third harmonic current in the primary may of course be the smaller in the case of a step-down transformer of large ratio.

It may be said, then, that the distribution of third harmonic current between the two windings of a transformer is largely regulated by the magnitude and character of the triple frequency impedance of the load. The minimum fundamental impedance of the load is determined by the rating of the transformer. The load, however, may easily be made large to the fundamental and still extremely small to the third harmonic by a suitable combination of inductance and capacitance in series. Let

$$Z_2''' + Z_L''' = R_2 + j\, X_2 + R_L + j\, (X_L''' - X_C) \qquad (8)$$

If then

$$X_2''' + X_L''' - X_C''' = 0 \qquad (9)$$

the secondary is tuned to series resonance for the third harmonic current. If at the same time the load resistance is zero, the ratio of the secondary third harmonic current to the primary third harmonic current is a maximum, the distribution being given by

$$\frac{I_1'''}{I_2'''} = \frac{R_2}{Z_1''' + Z_G'''} \qquad (10)$$

It should not be inferred from this statement that the secondary "leakage resonance" condition gives rise to the maximum amount of third harmonic current. Much larger currents as well as voltages may be obtained at other capacitive loads and at a second resonance condition. Operation in this region is also at certain points accompanied by peculiar instability phenomena, the discussion of which is beyond the scope of this paper.*

There has been considerable discussion of the causes for the harmonics in the magnetizing current and the voltage of a transformer. While opinions on this question have differed a good deal in earlier years, most engineers now agree they are caused both by the varying permeability of the iron and by hysteresis.†

The relation of the harmonics to the power losses in the core, however, has not, as far as the writer is aware, been settled to everybody's satisfaction.‡ The writer's conception of this question is outlined in the following.

If the voltage of the generator is strictly sinusoidal, then none of the harmonic currents can produce power in conjunction with this voltage. Hence, power is input to the core at fundamental frequency only.

The harmonic currents, however, will necessarily give rise to a copper loss in the circuits where they exist. The power corresponding to this copper loss, plus the losses in the core caused by the non-fundamental fluxes, is then evidently conveyed to the core as power of fundamental frequency. In the core, it is converted to power of other frequencies, a part of which is given out to the circuits where the currents of the higher frequencies flow.

The transformer core is in this respect nothing but a frequency converter, and the harmonics add to what

*These instability effects have been and are still being investigated in the Electrical Research Laboratories of the Massachusetts Institute of Technology.

†Excellently discussed by J. J. Frank, Bibliography 20.

‡The reading of some of the papers and the discussions given in the bibliography will show this.

may be called the "apparent core loss," or to the fundamental input to the core, while in reality a part of this power is expended as copper loss by harmonic currents.

Conceive a hypothetical transformer having core loss but requiring no harmonics in the magnetizing current for impressed sinusoidal voltage. The vector diagram on open circuit is given in Fig. 2, and the exciting current taken is I_n'. The hypothetical core is now exchanged for a regular iron core requiring harmonics in the magnetizing current. Fig. 2 may now be used as a vector diagram of fundamental quantities only. Neglecting the slight change in the fundamental leakage impedance drop of the primary, the only effect of the sudden introduction of harmonics in the current would be to increase the excitation conductance of the transformer. It would now draw a fundamental current I_n and the apparent increase in core loss is:

$$E_1\,(I_c - I_c')\text{ watts} \qquad (11)$$

This power is, as already pointed out, not utilized as power of fundamental frequency but is converted to power of higher frequencies, part of which is absorbed by the core (and this part may be very small if the non-fundamental fluxes are small), while the rest is

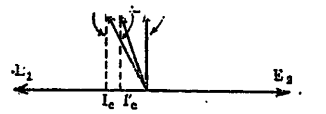

FIG. 2—VECTOR DIAGRAM SHOWING INCREASE IN "APPARENT CORE LOSS"

dissipated as copper loss in the circuits carrying the harmonic currents.

THREE-PHASE CONNECTIONS OF SINGLE-PHASE TRANSFORMERS

When single-phase transformers are connected for three-phase operation, the method of connection constitutes a means by which the distribution of the third harmonics and multiples may be partly controlled independent of the external circuits. This depends upon the well-known fact that the triple frequency voltages and currents in a balanced system are in phase in the three phases; in other words they appear as residuals.

The third harmonic voltage, therefore, cannot appear between lines while it may be found as a component of the voltage to neutral. The third harmonic current can appear on the lines only in a Y-connection with neutral. In a Δ-connection it will circulate in the closed delta but cannot escape from this and enter the lines.

The other harmonics which are not multiples of three and hence are phase-displaced 120 deg. in the three phases, cannot be controlled by transformer connections independent of the external circuits.

In the following discussion of the distribution of the harmonics strictly sinusoidal impressed voltages and balanced transformers are assumed. The ratio of transformation of the transformers is assumed to be unity.

Since the same laws do not govern the distribution of the two classes of harmonics, *i. e.*, the third harmonic and its multiples and those which are not multiples of three, equations for the fifth as well as for the third harmonics have been established. The generator and the balanced load are both assumed Y-connected for all transformer connections. The equations given, however, are easily modified to hold when either generator or load, or both, are delta-connected. "No load" in all cases means that the secondary lines are opened at the transformer terminals.

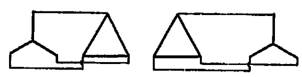

FIG. 3

It should be noted that voltages and currents of triple frequency are given *per transformer*. In a Y-connection this is equivalent to *voltage to neutral* and *line current*. Fifth harmonic voltages and currents are to neutral and per line, respectively, for any connection used.

This significance of the symbols used in the equations below should be kept in mind, as otherwise the equations are easily open to incorrect interpretation.

A. Δ–Δ CONNECTION

With this connection (Fig. 3), the third harmonic currents cannot appear on the lines, but will exist as circulating currents in the two deltas. Conditions are the same whether the transformer bank is loaded or not.

The induced third harmonic electromotive forces are given by

$$\begin{aligned} E_{1c}''' = E_{2c}''' &= I_1'''\,Z_{11}''' + j\,I_2'''\,X_{12}''' \\ &= I_2'''\,Z_{22}''' + j\,I_1'''\,X_{21}''' \end{aligned} \qquad (12)$$

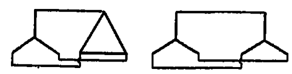

FIG. 4

and the division of third harmonic current between primary and secondary is given by

$$\frac{I_1'''}{I_2'''} = \frac{Z_2'''}{Z_1'''} \qquad (13)$$

The fifth harmonic currents will appear on the lines. When the transformers are loaded the following equations hold:

$$\begin{aligned} E_{1c}^{\mathrm{v}} = E_{2c}^{\mathrm{v}} &= I_1^{\mathrm{v}}\left(\frac{Z_{11}^{\mathrm{v}}}{3} + Z_G^{\mathrm{v}}\right) + j\,I_2^{\mathrm{v}}\,\frac{X_{12}^{\mathrm{v}}}{3} \\ &= I_2^{\mathrm{v}}\left(\frac{Z_{22}^{\mathrm{v}}}{3} + Z_L^{\mathrm{v}}\right) + j\,I_1^{\mathrm{v}}\,\frac{X_{21}^{\mathrm{v}}}{3} \end{aligned} \qquad (14)$$

$$\frac{I_1^{\mathrm{v}}}{I_2^{\mathrm{v}}} = \frac{Z_2^{\mathrm{v}} + 3\,Z_L^{\mathrm{v}}}{Z_1^{\mathrm{v}} + 3\,Z_G^{\mathrm{v}}} \qquad (15)$$

When the load is disconnected (at the secondary terminals of the bank) the fifth harmonic current ceases to flow in the secondary windings, while a fifth harmonic voltage V_2^{v} still appears on the secondary side. The following relations hold:

$$E_{1c}^{\mathrm{v}} = E_{2c}^{\mathrm{v}} = I_1^{\mathrm{v}}\left(\frac{Z_{11}^{\mathrm{v}}}{3} + Z_G^{\mathrm{v}}\right) \tag{16}$$

$$V_2^{\mathrm{v}} = E_{2c}^{\mathrm{v}} - j\, I_1^{\mathrm{v}} \frac{X_{21}^{\mathrm{v}}}{3} = I_1^{\mathrm{v}}\left(\frac{Z_1^{\mathrm{v}}}{3} + Z_G^{\mathrm{v}}\right) \tag{17}$$

B. Δ-Y CONNECTION

Isolated neutrals. With this connection (Fig. 4), the

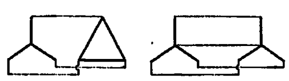

FIG. 5

third harmonic current will be confined to the primary delta both when the bank is loaded and when it is on open circuit. The following equations will hold:

$$E_{1c}''' = E_{2c}''' = I_1''' Z_{11}''' \tag{18}$$

$$V_2''' = E_{2c}''' - j\, I_1''' X_{21}''' = I_1''' Z_1''' \tag{19}$$

The equations for the fifth harmonic when the bank is loaded are

$$E_{1c}^{\mathrm{v}} = \frac{E_{2c}^{\mathrm{v}}}{\sqrt{3}} = I_1^{\mathrm{v}}\left(\frac{Z_{11}^{\mathrm{v}}}{3} + Z_G^{\mathrm{v}}\right) + j\,\frac{I_2^{\mathrm{v}}}{\sqrt{3}}\, X_{12}^{\mathrm{v}}$$

$$= \frac{I_2^{\mathrm{v}}}{\sqrt{3}}\left(Z_{22}^{\mathrm{v}} + Z_L^{\mathrm{v}}\right) + j\, I_1^{\mathrm{v}} \frac{X_{21}^{\mathrm{v}}}{3} \tag{20}$$

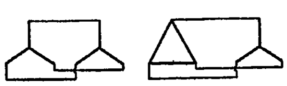

FIG. 6

$$\frac{I_1^{\mathrm{v}}}{I_2^{\mathrm{v}}} = \frac{\sqrt{3}\,(Z_2^{\mathrm{v}} + Z_L^{\mathrm{v}})}{Z_1^{\mathrm{v}} + 3\, Z_G^{\mathrm{v}}} \tag{21}$$

Disconnecting the load gives

$$E_{1c}^{\mathrm{v}} = \frac{E_{2c}^{\mathrm{v}}}{\sqrt{3}} = I_1^{\mathrm{v}}\left(\frac{Z_{11}^{\mathrm{v}}}{3} + Z_G^{\mathrm{v}}\right) \tag{22}$$

$$V_2^{\mathrm{v}} = E_{2c}^{\mathrm{v}} - j\,\frac{I_1^{\mathrm{v}}}{\sqrt{3}}\, X_{21}^{\mathrm{v}} = \sqrt{3}\, I_1^{\mathrm{v}}\left(\frac{Z_1^{\mathrm{v}}}{3} + Z_G^{\mathrm{v}}\right) \tag{23}$$

Interconnected neutrals. When the bank is loaded (Fig. 5) the following equations hold for the third harmonic:

$$E_{1c}''' = E_{2c}''' = I_1''' Z_{11}''' + j\, I_2''' X_{12}'''$$

$$= I_2\,(Z_{22}''' + Z_L''' + 3\, Z_{2n}''') + j\, I_1''' X_{21}''' \tag{24}$$

$$\frac{I_1'''}{I_2'''} = \frac{Z_2''' + Z_L''' + 3\, Z_{2n}'''}{Z_1'''} \tag{25}$$

When the load is removed equations (18) and (19) express the triple frequency relations.

The fifth harmonic will follow the same laws as in the case with isolated neutrals and is hence determined by equations (20), (21), (22) and (23).

C. Y-Δ CONNECTION.

Isolated neutrals. With this connection (Fig. 6), the third harmonic current will be exclusively confined to

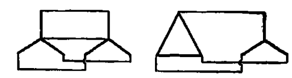

FIG. 7

the secondary delta both for load and no-load conditions. The following equations hold:

$$E_{c1}''' = E_{c2}''' = I_2''' Z_{22}''' \tag{26}$$

$$V_1''' = E_{c1}''' - j\, I_2''' X_{12}''' = I_2''' Z_2''' \tag{27}$$

When the bank is loaded the fifth harmonic relations are

$$E_{1c}^{\mathrm{v}} = \sqrt{3}\, E_{2c}^{\mathrm{v}} = I_1^{\mathrm{v}}\,(Z_{11}^{\mathrm{v}} + Z_G^{\mathrm{v}}) + j\,\frac{I_2^{\mathrm{v}}}{\sqrt{3}}\, X_{12}^{\mathrm{v}}$$

$$= \sqrt{3}\, I_2^{\mathrm{v}}\left(\frac{Z_{22}^{\mathrm{v}}}{3} + Z_L^{\mathrm{v}}\right) + j\, I_1^{\mathrm{v}} X_{21}^{\mathrm{v}} \tag{28}$$

$$\frac{I_1^{\mathrm{v}}}{I_2^{\mathrm{v}}} = \frac{Z_2^{\mathrm{v}} + 3\, Z_L^{\mathrm{v}}}{\sqrt{3}\,(Z_1^{\mathrm{v}} + Z_G^{\mathrm{v}})} \tag{29}$$

Removing the load gives

$$E_{1c}^{\mathrm{v}} = \sqrt{3}\, E_{2c}^{\mathrm{v}} = I_1^{\mathrm{v}}\,(Z_{11}^{\mathrm{v}} + Z_G^{\mathrm{v}}) \tag{30}$$

$$V_2^{\mathrm{v}} = E_{2c}^{\mathrm{v}} - j\,\frac{I_1^{\mathrm{v}}}{\sqrt{3}}\, X_{21}^{\mathrm{v}} = \frac{I_1^{\mathrm{v}}}{\sqrt{3}}\,(Z_1^{\mathrm{v}} + Z_G^{\mathrm{v}}) \tag{31}$$

Interconnected neutrals. With this connection (Fig. 7), the third harmonic current will flow in the primary lines and the secondary delta both at no-load and when the bank is loaded. The following equations hold:

$$E_{1c}''' = E_{2c}''' = I_1'''(Z_{11}''' + Z_G''' + 3\, Z_{1n}''')$$

$$+ j\, I_2''' X_{12}''' = I_2''' Z_{22}''' + j\, I_1''' X_{21}''' \tag{32}$$

$$\frac{I_1'''}{I_2'''} = \frac{Z_2'''}{Z_1''' + Z_G''' + 3\, Z_{1n}'''} \tag{33}$$

The neutral connection does not affect the fifth

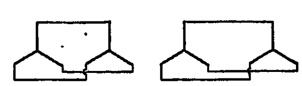

FIG. 8

harmonics; they still follow equations (28), (29), (30) and (31).

D. Y-Y CONNECTION

All neutrals Isolated. With this connection (Fig. 8), no third harmonic current can flow on either side under

any condition. A third harmonic voltage, however, which may be very large, exists on both sides.

$$V_1''' = V_2''' = E_{1C}''' = E_{2C}''' \quad (34)$$

When the bank is loaded the equations for the fifth harmonic are

$$E_{1C}^{\mathrm{v}} = E_{2C}^{\mathrm{v}} = I_1^{\mathrm{v}}(Z_{11}^{\mathrm{v}} + Z_G^{\mathrm{v}}) + j\,I_2^{\mathrm{v}} X_{12}^{\mathrm{v}}$$
$$= I_2^{\mathrm{v}}(Z_{22}^{\mathrm{v}} + Z_L^{\mathrm{v}}) + j\,I_1^{\mathrm{v}} X_{21}^{\mathrm{v}} \quad (35)$$

$$\frac{I_1^{\mathrm{v}}}{I_2^{\mathrm{v}}} = \frac{Z_2^{\mathrm{v}} + Z_L^{\mathrm{v}}}{Z_1^{\mathrm{v}} + Z_G^{\mathrm{v}}} \quad (36)$$

Disconnecting the load gives

$$E_{1C}^{\mathrm{v}} = E_{2C}^{\mathrm{v}} = I_1^{\mathrm{v}}(Z_{11}^{\mathrm{v}} + Z_G^{\mathrm{v}}) \quad (37)$$

$$V_2^{\mathrm{v}} = E_{2C}^{\mathrm{v}} - j\,I_1^{\mathrm{v}} X_2^{\mathrm{v}} = I_1^{\mathrm{v}}(Z_1^{\mathrm{v}} + Z_G^{\mathrm{v}}) \quad (38)$$

Primary and generator neutrals interconnected. With this connection (Fig. 9), the third harmonic current

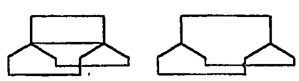

FIG. 9

will flow on the primary side both at no-load and when load is put on the bank. The equations for the third harmonics are

$$E_{1C}''' = E_{2C}''' = I_1'''(Z_{11}''' + Z_G''' + 3\,Z_{1n}''') \quad (39)$$

$$V_2''' = E_{2C}''' - j\,I_1''' X_{21}'''$$
$$= I_1'''(Z_1''' + Z_G''' + 3\,Z_{1n}''') \quad (40)$$

The conditions as far as the fifth harmonics are concerned are identical with those existing when all

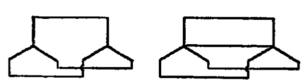

FIG. 10

neutrals are isolated as expressed by equations (35), (36), (37), and (38).

Secondary and load neutrals interconnected. When the bank is loaded, (Fig. 10), the third harmonic current will flow on the secondary side.

$$E_{1C}''' = E_{2C}''' = I_2'''(Z_{22}''' + Z_L''' + 3\,Z_{2n}''') \quad (41)$$

$$V_1''' = E_{1C}''' - j\,I_2''' X_{12}'''$$
$$= I_2'''(Z_2''' + Z_L''' + 3\,Z_{2n}''') \quad (42)$$

At no-load there is no circuit in which the third harmonic current can flow. Equation (34) holds for the voltages.

The fifth harmonics behave as in the other Y–Y connections. Equations (35), (36), (37) and (38).

Neutrals interconnected on both sides. With this connection, (Fig. 11), a third harmonic current will flow on both sides when the bank is loaded.

$$E_{1C}''' = E_{2C}''' = I_1'''(Z_{11}''' + Z_G''' + 3\,Z_{1n}''') + j\,I_2''' X_{12}'''$$
$$= I_2'''(Z_{22}''' + Z_L''' + 3\,Z_{2n}''') + j\,I_1''' X_{21}''' \quad (43)$$

$$\frac{I_1'''}{I_2'''} = \frac{Z_2''' + Z_L''' + 3\,Z_{2n}'''}{Z_1''' + Z_G''' + 3\,Z_{1n}'''} \quad (44)$$

When the load is removed, Equations (39) and (40) will evidently hold. The fifth harmonics follow Equations (35), (36), (37) and (38).

E. Y–Y CONNECTION WITH TERTIARY DELTA

Neutrals isolated. With this connection, (Fig. 12), the third harmonic current will always be confined to

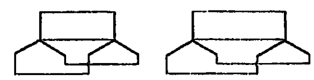

FIG. 11

the tertiary delta independent of whether the bank is loaded or not. No harmonic, other than the third and its multiples, can flow in the tertiary winding.

Designating the tertiary winding as winding No. 3 and assuming unity ratio of transformation also for this winding, the following equations may be written:

$$E_{1C}''' = E_{2C}''' = E_{3C}''' = I_3''' Z_{33}''' \quad (45)$$

$$V_1''' = E_{1C}''' - j\,I_3''' X_{13}''' = I_3''' Z_{3A}''' \quad (46)$$

$$V_2''' = E_{2C}''' - j\,X_3''' X_{23}''' = I_3''' Z_{3B}''' \quad (47)$$

In equations (46) and (47) Z_{3A}''' and Z_{3B}''' represent the third harmonic leakage impedance with respect to windings No. 1 and No. 2 respectively.

The fifth harmonics behave as in the other Y–Y connections. Equations (35), (36), (37) and (38).

LABORATORY TESTS

The data which are reproduced below were obtained in the Electrical Research Laboratories of the Massachusetts Institute of Technology by tests on three small core type transformers.*

A. SINGLE-PHASE EXCITING CURRENT

It is important in three-phase tests that the transformers are as nearly balanced as possible. The experimental transformers were well balanced electrically. This is apparent from Fig. 13, which shows curves of the exciting currents of the three units as obtained from single-phase tests with a sinusoidal voltage impressed.

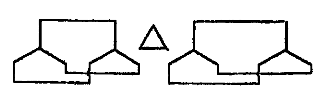

FIG. 12

Fig. 14 shows one of a series of oscillograms of the single-phase exciting current. The magnitudes of the fundamental, third harmonic and fifth harmonic components as found from analysis of this series of oscillograms are given by the curves in Fig. 15.

The curves are plotted against fundamental flux density and it will be noted that the percentage harmonics increases with the density.

*The transformers and other equipment are briefly described in the paper, "Separate Leakage Reactance of Transformer Windings."

As a typical example, the results of the analysis of the exciting-current curve in the oscillograms Fig. 14 are plotted in Fig. 16. The relative magnitudes and phase relations are correctly reproduced. The phase relation of each harmonic current, for instance the third, and the corresponding third harmonic electromotive force evidently depend on the triple frequency impedance of the circuits where it flows as established by equations (5) and (6).

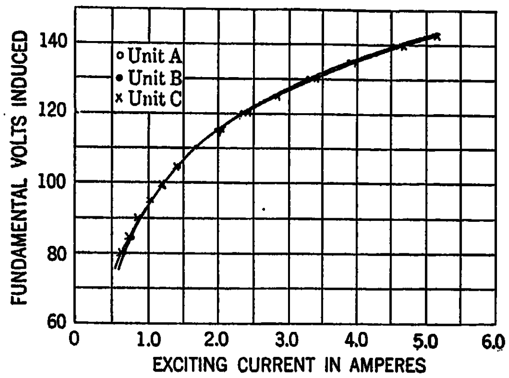

FIG. 13—EXCITING CURRENTS OF EXPERIMENTAL TRANSFORMERS SINGLE PHASE

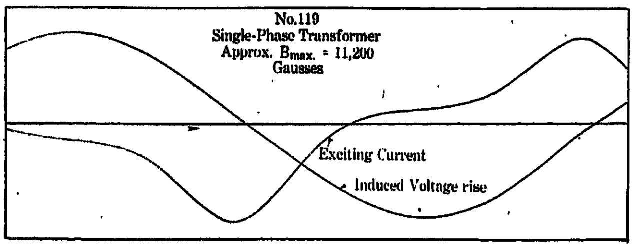

FIG. 14—OSCILLOGRAM OF SINGLE-PHASE EXCITING CURRENT

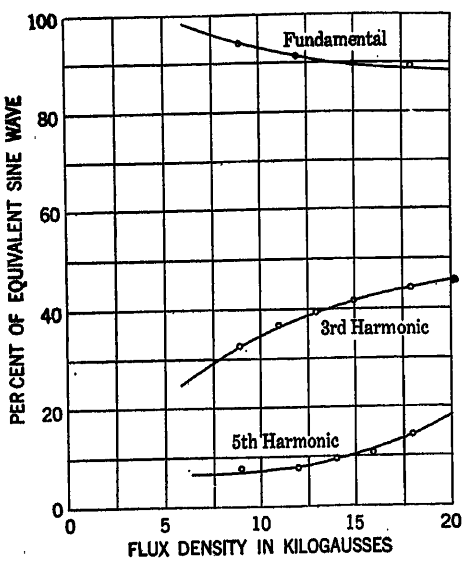

FIG. 15—COMPONENTS OF MAGNETIZING CURRENT OF EXPERIMENTAL TARNSFORMERS. SINGLE PHASE

These curves of the single-phase magnetizing current of the transformers have been included in order to give a picture of the magnetic characteristics of the transformers. The single-phase data are also interesting when compared with some of the data from three-phase tests.

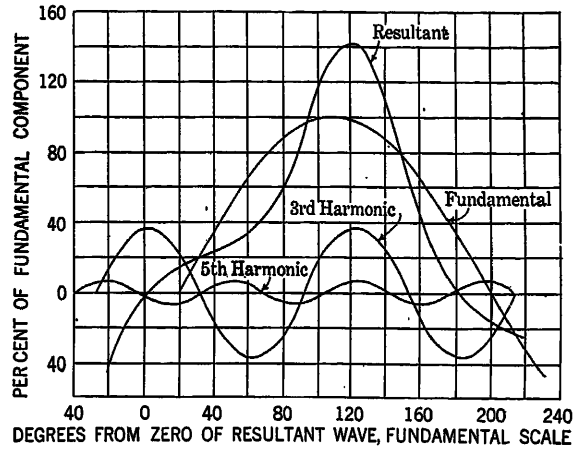

FIG. 16—COMPONENTS OF EXCITING CURRENT FROM OSCILLOGRAM FIG. 14

B. DISTRIBUTION OF THIRD HARMONICS IN THREE-PHASE BANKS

In order to investigate the distribution of third harmonic current between two circuits, a series of tests was performed with circuits of various nature provided for this current. It will be remembered from the preceding discussion that, when any harmonic electromotive force is introduced exclusively by the transformer itself, the corresponding currents set up should divide between any two circuits inversely as the ratio of their total impedances at the frequency considered.

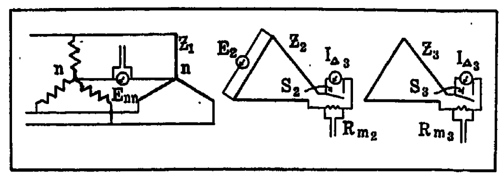

FIG. 17—DISTRIBUTION OF THIRD HARMONIC CURRENT. CIRCUIT DIAGRAM FOR Y-Δ-Δ TESTS. NO EXTERNAL LOAD

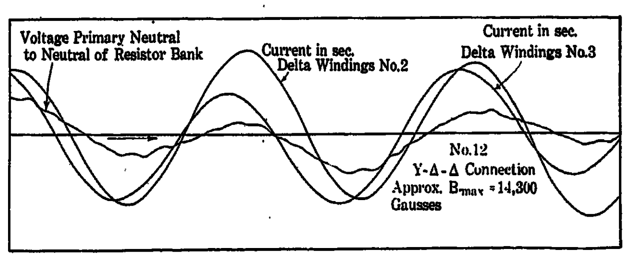

FIG. 18—DISTRIBUTION OF THIRD HARMONIC CURRENT. OSCILLOGRAM FROM *Y*-Δ-Δ TESTS. CIRCUIT CONNECTIONS SHOWN IN FIG. 17

Y–Δ–Δ *Connection.* This test was performed in order to check the theory of third harmonic current division between two closed deltas. The connections used are shown diagrammatically in Fig. 17.

Windings No. 1 were used as primaries and were Y-connected; windings No. 2 and No. 3 were used as secondaries and Δ-connected. The currents in the two deltas were recorded at five values of the flux density.

Fig. 18 shows one of the oscillograms taken during these tests. The voltage between the resistor and transformer neutrals (not necessary for this particular test) and the delta currents was recorded. The triple frequency current was separated out by analysis.

Tables I and II give the data and results from these tests. Since there was only one vacuum thermocouple ammeter suitable for measuring the current in the two deltas, it was necessary first to put the meter in delta No. 2 while the switch S_3 was closed and, after reading its deflection, to remove it and put it in delta No. 3 in order to read the current here with S_2 closed and S_3 open.

It is not possible to apply a rational correction to this current because of the change in the third harmonic electromotive force which accompanies the change in current. This change in electromotive force cannot be estimated with precision from the data at hand. The ratio of the currents would therefore be expected to be slightly too large.

Inspection of the results given in the last three columns of Table II shows this to be the case. The discrepancies, however, are not large, and bearing the above consideration in mind, the check between the measured and computed current ratios seems satisfactory.

Δ-Y connection with capacity load. These tests were made as an attempt to simulate the conditions when a Δ-Y connected transformer bank is connected to a cable or long transmission line of considerable capacity. The circuit diagram is shown in Fig. 19.

The capacities used for the tests recorded in oscillogram No. 121 and No. 123 were not quite balanced. This fact accounts for the large fundamental current in the secondary neutral. The capacities used in the other tests, however, were practically perfectly balanced and analyses show that with these capacities the third

TABLE I*
DISTRIBUTION OF THIRD HARMONIC CURRENT
Y-Δ-Δ TESTS—NO EXTERNAL LOAD

Transformer connections	Oscillogram Number	Frequency cycles per second f	Approximate flux density (Gausses)	Volts induced in winding No. 2, Unit A	Unit B	Unit C	Amperes in delta No. 2 $I_{\Delta 2}$	Amperes in delta No. 3 $I_{\Delta 3}$	Ohms resistance in metering circuits, Delta No. 2 R_{m2}	Delta No. 3 R_{m3}
Windings No. 1 primary Y, No. 2 and No. 3 secondary Δ's.	9	60	9200	90.6	90.0	89.5	0.275	0.20	1.08	1.61
	10	60	11200	110.5	110.0	109.5	0.520	0.30	1.10	2.69
	11	60	12800	125.5	125.0	124.5	0.900	0.52	1.13	1.30
	12	60	14300	140.8	140.0	139.5	1.780	0.93	1.14	1.07
	13	60	9200	90.5	90.0	89.3	0.212	0.17	4.10	5.20

Triple frequency leakage impedance of winding No. 1 = $0.284 + j\,0.414$ ohms per phase
" " " " " " No. 2 = $0.466 + j\,1.336$ " " "

*The experimental data in the Y-Δ-Δ tests were recorded by W. J. Miller, formerly of the Massachusetts Institute of Technology.

TABLE II
DISTRIBUTION OF THIRD HARMONIC CURRENT
Y-Δ-Δ TESTS—NO EXTERNAL LOAD

Oscillogram Number	Third harmonic amperes, In delta No. 2 $I_{\Delta 2}'''$	In delta No. 3 $I_{\Delta 3}'''$	Triple frequency impedance, Delta No. 2 $Z_{\Delta 2}'''$	Delta No. 3 $Z_{\Delta 3}'''$	Impedance ratio $\frac{Z_{\Delta 3}'''}{Z_{\Delta 2}'''}$	Current ratio $\frac{I_{\Delta 2}'''}{I_{\Delta 3}'''}$	Per cent discrepancy referred to impedance ratio
9	0.211	0.108	2.55	5.20	2.040	1.955	4.17
10	0.494	0.239	2.57	5.94	2.310	2.065	10.60
11	0.870	0.460	2.39	4.86	2.035	1.890	7.12
12	1.757	0.891	2.40	4.73	1.971	1.970	0.05
13	0.192	0.142	5.40	7.97	1.476	1.352	8.40

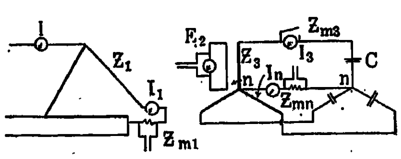

FIG. 19—DISTRIBUTION OF THIRD HARMONIC CURRENT. CIRCUIT DIAGRAM FOR Δ-Y TESTS. CAPACITY LOAD

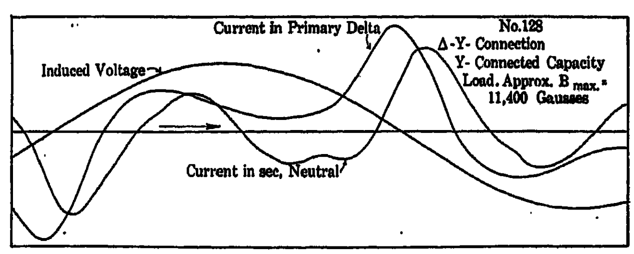

FIG. 20—DISTRIBUTION OF THIRD HARMONIC CURRENT. OSCILLOGRAM FROM Δ-Y TESTS. CIRCUIT CONNECTIONS SHOWN IN FIG. 19

harmonic current in the secondary neutral is by far predominant.

Fig. 20 is a sample oscillogram from this series and shows induced voltage (E_2), current in the primary delta, and current in the secondary neutral. The third harmonic current in the latter is assumed to be the algebraic sum of the third harmonic currents in each phase and equal to three times the third harmonic current per phase, two conditions which are true when perfect balance exists.

Tables III and IV give the data and results. The details of the computations are not reproduced. In obtaining the theoretical current division, the average the theoretical and measured quantities is unsatisfactory. However, a more thorough analysis of the conditions actually existing in the network during these tests permits and substantiates a much more favorable interpretation of the results.

Unfortunately, a commercial ammeter was used in the primary delta, giving a third harmonic impedance of the metering circuit in one branch of the delta of $1.14 + j\,0.665$ ohms, which is appreciable as compared with the impedance of the rest of the delta. Due to the unbalance thus introduced, the sum of the third harmonic electromotive forces will not be exactly balanced by the third harmonic impedance drop around

TABLE III
DISTRIBUTION OF THIRD HARMONIC CURRENT
Δ-Y TESTS—CAPACITY LOAD

Transformer connections	Oscillogram Number	Frequency cycles per second f	Volts induced in winding No. 2	Primary amperes		Secondary amperes		Load capacity in microfarads C
				in lines I	in delta I_1	in lines I_3	in neutral I_n	
Windings No. 1 primary Y, windings No. 3 secondary Y, with Y-connected capacity load. neutrals interconnected.	121	60	110.0	2.38	1.57	0.622	0.59*	15.0
	123	60	110.0	2.64	1.815	0.366	0.37*	9.0
	127	60	111.5	1.87	1.10	1.17	0.0657	25.85
	128	60	111.5	1.56	1.09	1.15	0.0640	25.85
	129	60	112.0	1.625	1.15	1.15	0.0685	25.85
	130	60	111.0	1.55	1.08	1.14	0.0662	25.85
	131	60	111.0	1.54	1.07	1.14	0.0660	25.85
	132	60	111.8	2.24	1.40	0.63	0.0464	15.30

Switch S_3, (Fig. 19), was closed while all other meters were read, and when oscillograms were taken, except for oscillograms No. 121 and No. 123.
*Appreciable fundamental component introduced by unbalance of condensers.

TABLE IV
DISTRIBUTION OF THIRD HARMONIC CURRENT
Δ-Y TESTS—CAPACITY LOAD

Oscillogram Number	Third harmonic amperes		Triple frequency impedance per phase		Current ratio	Impedance ratio	Per cent discrepancy referred to impedance ratio
	in primary I_1'''	in secondary neutral I_{3n}'''	primary $Z_{pri.}'''$	secondary $Z_{sec.}'''$	$\frac{3\,I_1'''}{I_{3n}'''}$	$\frac{Z_{sec.}'''}{Z_{pri}'''}$	
121	0.810	0.0470	$0.925\,\underline{/44.^{\circ}1}$	$56.3\,\overline{\backslash 87.^{\circ}9}$	51.7	60.9	15.1
123	0.830	0.0242	"	$95.7\,\overline{\backslash 88.^{\circ}8}$	103.0	103.5	0.48
127	0.705	0.0593	"	$34.37\,\overline{\backslash 72.^{\circ}9}$	35.7	37.1	3.77
128	0.698	0.0577	"	"	36.3	37.1	2.16
129	0.730	0.0619	"	"	35.4	37.1	4.58
130	0.680	0.0589	"	"	34.7	37.1	6.47
131	0.674	0.0587	"	"	34.4	37.1	7.27
132	0.702	0.0418	"	$57.3\,\overline{\backslash 79.^{\circ}8}$	50.4	62.0	18.7

third harmonic impedance per phase on each side was calculated. The average primary impedance per phase is given as the impedance of one primary winding plus one third of the impedance of the metering circuit in the primary delta, *i. e.*, ammeter in series with shunt and vibrator in parallel.

In comparing the results from these tests, (Table IV), it will be noted that the observed current ratios are consistently less than the computed ratios by amounts varying from 0.48 per cent to 18.7 per cent. Since, in general, experimental errors should be divided equally between plus and minus, one may at a first glance be inclined to conclude that the check between the delta, with the result that a third harmonic current will appear on two of the lines and in two of the phases of the Y-connected generator. No third harmonic current will appear on the third line.

The third harmonic current in the two phases of the primary delta which do not contain the ammeter is equal and in phase. On the contrary, the third harmonic current in the phase where the ammeter is inserted, is neither equal to, nor in phase with, the two other currents. Since on the secondary side the third harmonic current per phase is considered equal to one third of the third harmonic current in the neutral, it is necessary for correct comparison of the third

harmonic currents on the two sides also to use the *average third harmonic current* in the primary delta, and the corresponding average triple frequency impedance per phase.

It is, therefore, evident that computing current ratios as done in Table IV, based on values of the primary third harmonic current which are smaller than the average values, discrepancies in the direction indicated might have been anticipated. Hence it seems safe to conclude that the results check to engineering accuracy.

Y–Δ connection with resonant load. Fig. 21 shows an oscillogram from a series of tests performed with the

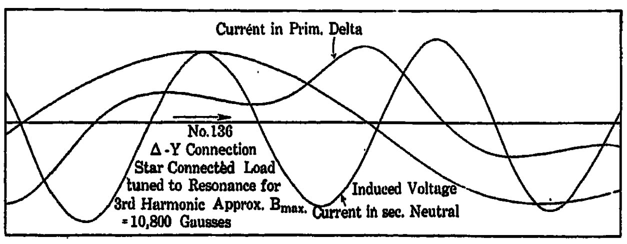

FIG. 21—DISTRIBUTION OF THIRD HARMONIC CURRENT. CIRCUIT DIAGRAM FOR Δ-*Y* TESTS. INDUCTANCE AND CAPACITY LOAD. SECONDARY CIRCUIT TUNED TO TRIPLE FREQUENCY RESONANCE

connections shown in Fig. 22, using a combined inductive and capacitive load so adjusted that the total secondary circuit was tuned to series resonance for the third harmonic.

The oscillogram shows that in this case the current in the secondary neutral, as might be expected, is practically all third harmonic, while in the test with

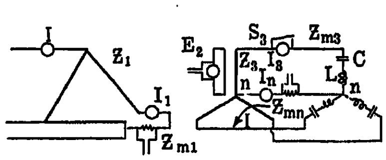

FIG. 22—OSCILLOGRAM FROM Δ-*Y* TESTS. CIRCUIT DIAGRAM SHOWN IN FIG. 21

pure capacity load this current also contained a considerable amount of higher harmonics. With the resonant circuit these are all damped out by the inductance.

The complete data from these tests will not be reproduced here as they cannot be used to verify the third harmonic current distribution. This is due to the fact that iron-core reactors (with air gaps) were used for producing resonance in the secondary circuit. Having iron cores, the reactors themselves would tend to introduce harmonics. The effect of the reactors in this respect cannot be quantitatively analyzed with sufficient precision to render the results of value for checking the theoretical distribution of third harmonic current.

FIELD TESTS

a. General. In July 1922, the Research Division of the Electrical Engineering Department, Massachusetts Institute of Technology, made a series of tests on the Gadsden-Lindale Tie Line belonging to the Alabama Power Company and the Georgia Railway and Power Company. By courtesy of these companies this line was placed at the disposal of the Institute for testing purposes. F. S. Dellenbaugh, Jr.,* was in charge of the work.

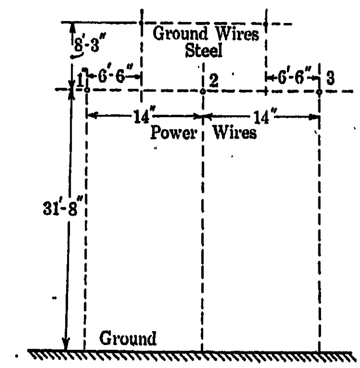

FIG. 23—CONFIGURATION OF TRANSMISSION LINE USED IN FIELD TESTS

The tests comprised mainly investigation of d-c. transients, harmonics due to transformers and determination of the separate leakage reactances of a bank

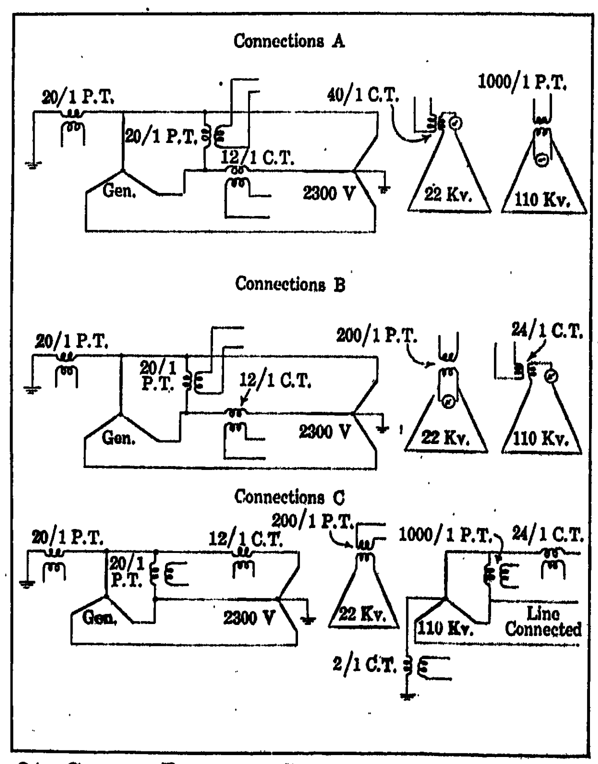

FIG. 24—CIRCUIT DIAGRAMS SHOWING CONNECTIONS USED IN FIELD TESTS

of power transformers at Gadsden. At the same time engineers of the American Telephone & Telegraph Co. made bridge measurements of the impedance to ground

*Massachusetts Institute of Technology.

†See the paper, "Separate Leakage Reactance of Transformer Windings."

of the line at frequencies from about 180 to 2400 cycles per second. Several of the tests may, therefore, be used to check some of the theory of transformer harmonics was well as the method of measurement developed in the laboratory.

Fig. 23 shows the configuration of the line from Gadsden to the State border (Alabama-Georgia). The configuration of the line from the State border to Lindale was essentially the same and will not be reproduced here. The lines were not transposed. Detailed data of the lines have no particular interest for this paper.

The transformer bank at Gadsden consisted of three 2100-kv-a. transformers made by the General Electric Company. These had three windings each and were normally connected 110,000 - 22,000 - 2300 volts in Δ - Δ - Δ.

Three series of tests are of particular value for checking the theory of transformer harmonics. Diagrams of connections used in these are shown in Fig. 24. In connections *A* and *B* will be recognized the "three-winding method"† for the determination of leakage reactances. The diagram marked "Connections *C*" shows the bank connected *Y*-open Δ-*Y* with the line connected to the 110-kv. side, the neutral being

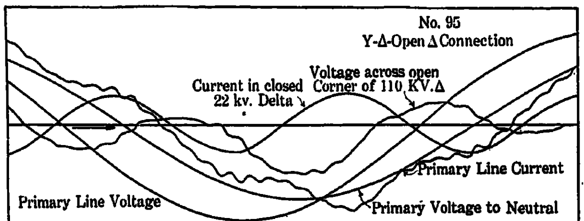

Fig. 25—Oscillogram from Leakage Impedance Tests. Circuit Connections Shown in Fig. 24-A

grounded. The distant end of the line was free. This test was also performed with the 22-kv. delta closed. As the circulating current in this delta, was not recorded, however, this test cannot be used for checking purposes.

Referring to the circuit diagrams in Fig. 24, it will be seen that primary line voltage, line current, and voltage to ground were recorded for all three connections. With connections *A* and *B*, the voltages across the open corner or the circulating currents in the deltas were read. With connection *C*, the voltage across the open corner of the 22-kv. delta and line voltage, line current and potential to ground on the *Y*-connected, 110-kv. side were recorded. All these data, however, will not be reproduced here; only those strictly required will be given, corrected for instrument transformer ratios, instrument errors, etc.

b. Separate Leakage Reactances. Fig. 25 shows one of the oscillograms taken during the tests with connections *A*. It will be seen that while the circulating current in the closed 22-kv. delta is practically a pure third harmonic, the voltage across the open corner of the 110-kv. delta contains a pronounced fundamental. This is due to unbalance of the transformers. Hence in this case it would have been impossible to rely upon meter readings alone; oscillographic records had to be used throughout the series of tests.

Table V gives the values of the voltages across the open corner of the deltas and the circulating currents necessary for determining the leakage reactances. The actual readings, the per cent third harmonic as obtained from analyses of the oscillograms, the third harmonic voltages, and currents referred to the high-tension (110,000-volt) side, and the computed leakage impedances are given.

Table VI gives the ohmic and 60-cycle effective resistances of the three windings. The separate effective resistances have been computed from the equivalent resistances as obtained from short-circuit tests, by assuming that the ratio of the separate effective resistances is equal to the ratio of the separate ohmic resistances. This assumption evidently involves a small approximation.

The 180-cycle effective resistances will be considered to be the same as at 60 cycles. The error which may result from this assumption is negligibly small, since the resistances of the windings are insignificant as compared to the reactances. Hence the third harmonic leakage impedances in Table V may be reduced to exactly 60 cycles by multiplying by the ratio of 60 and the average frequency recorded. Furthermore, the third harmonic and the fundamental (60-cycle) reactances may be found.

Table VI gives the desired values. It should be carefully noted that the leakage reactance of winding No. 2 is with respect to winding No. 3, and that of winding No. 3 with respect to winding No. 2. The leakage reactance of these windings with respect to winding No. 1 will in all probability be different from the leakage reactances given. This, however, is entirely dependent upon the arrangement and the spacing of the coils in the transformers.

c. Third Harmonic Current Distribution with Line Connected to Transformer Bank. Data from two tests performed with "Connections *C*", Fig. 24, will be checked. The third harmonic current in the neutral on the line side will be computed from the voltage across the open corner of the 22-kv. delta, the separate leakage impedance of the 110-kv. windings, and the measured impedance to ground of the line.

In the first test (oscillogram No. 106) the part of the line from Gadsden to the State border only was used; in the second test (oscillogram No. 113), however, the entire line Gadsden-Lindale was connected to the transformers. In both cases the distant end of the line was open.

Fig. 26 shows one of the oscillograms (No. 106). While the neutral current appears to be nearly third harmonic, the main part of the voltage across the open corner of the 22-kv. is delta fundamental. This is partly due to unbalance of the transformers themselves and

partly due to unbalance of the line admittances. Table VII gives the recorded data. From a set of

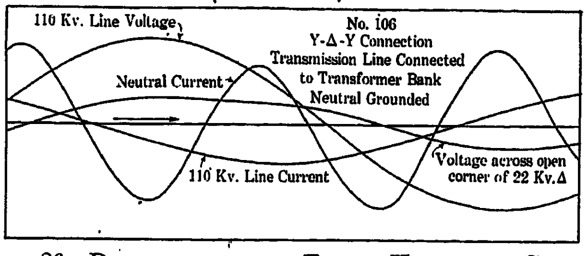

FIG. 26—DISTRIBUTION OF THIRD HARMONIC CURRENT. OSCILLOGRAM FROM FIELD TESTS. CIRCUIT CONNECTIONS SHOWN IN FIG. 25-C.

curves (not reproduced here) of line impedance to ground plotted versus frequency, the necessary line impedances can be found. With the distant end open and the three conductors in parallel, the following values at the triple frequency corresponding to the fundamental frequency recorded in Table VII are obtained:

Part of line	f cycles	Z ohms
Gadsden—State Border........	176.7	$18 - j\,775$
Gadsden—Lindale.............	176.1	$4 - j\,540$

The leakage impedance per phase of the 110-kv. windings is

$$Z_3''' = 26.3 + j\,649 \text{ ohms}$$

The total triple-frequency impedances of the high-

TABLE V
LEAKAGE IMPEDANCE TESTS

Transformer connections	Oscillogram Number	f cycles	V_Δ Reading volts	V_Δ Per cent third harmonic	I_Δ Reading amperes	I_Δ Per cent third harmonic	Referred to 110-kv. side V_Δ''' volts	Referred to 110-kv. side I_Δ''' amperes	Referred to 110-kv. side Z''' ohms	Average Z''' ohms
No. 1 primary Y, No. 2 open Δ, No. 3 closed Δ	92	59.4	158.5	61.8	0.68	99.0	98.0	0.1347	243	
	93	59.4	148.5	62.3	0.68	99.4	98.6	0.1352	243	
	93A	59.8	159.3	..	0.69	..	99.3	0.1370	242	
	95	59.8	306.4	66.5	1.20	100	204.0	0.2400	283	253
No. 1 primary Y, No. 2 closed Δ, No. 3 open Δ	98	60.0	..	91.0	0.184	92.7	..	0.1706	..	
	98A	60.3	67.2	..	0.179	..	306.0	0.1660	615	
	100	59.7	101.6	..	0.244	95.3	462.5	0.2325	663	639

No. 1 = 2300 volt winding
No. 2 = 22,000 volt winding
No. 3 = 110,000 volt winding

V_Δ = voltage across corner of open delta
I_Δ = current in closed delta
$Z''' = \dfrac{V_\Delta'''}{3\, I_\Delta'''}$ = leakage impedance per phase of closed delta winding with respect to open delta winding

TABLE VI
SEPARATE WINDING CONSTANTS

Winding	Actual ohmic resistance ohms	Referred to 110 kv. side: Ohmic resistance ohms	60-cycle effective resistance ohms	Leakage reactance in ohms, With respect to No. 2: 60 cycles	With respect to No. 2: 180 cycles	With respect to No. 3: 60 cycles	With respect to No. 3: 180 cycles
No. 1-110,000 volt	0.00892	20.40	22.4	..	..	..	..
No. 2-22,000 "	0.699	17.48	19.7	..	..	84.7	254
No. 3- 2300 "	23.64	23.64	26.3	213	639	..	..

TABLE VII
DISTRIBUTION OF THIRD HARMONIC CURRENT WITH LINE CONNECTED

Oscillogram Number	H. T. line voltage kv.	f cycles	$V_{\Delta 2}$ Reading volts	$V_{\Delta 2}$ Per cent third harmonic	I_{n3} Reading amperes	I_{n3} Per cent third harmonic	$V_{\Delta 2}'''$ volts	I_{n3}''' amperes
106	130.5	58.9	1114	12.35	0.404	97.8	137.8	0.395
113	129.9	58.7	970	7.6	0.368	90.7	73.7	0.334

tension circuit to be used in connection with the third harmonic currents in the neutral are:

Gadsen—State border:

$$Z''' = 18 - j\,775 + \frac{26.3}{3} + j\,\frac{639 \times 176.7}{3 \times 180}$$

$$= 26.8 - j\,556 = 566 \diagdown 87.°3 \text{ ohms}$$

Gadsden—Lindale:

$$Z''' = 4 - j\,540 + \frac{26.3}{3} + j\,\frac{639 \times 176.1}{3 \times 180}$$

$$= 12.8 - j\,332 = 332 \diagdown 87.°8 \text{ ohms}$$

The voltage to be used in connection with these impedances is the third harmonic voltage per phase in the 22-kv. winding referred to the 110-kv. side. The third harmonic voltage per phase is one-third of the third harmonic voltage across the open corner of the 22-kv. delta.

The above procedure is in accordance with equation (42) holding for a Y–Y connected bank with secondary neutral. This is evidently correct since the Y–Y connection and the open Δ–Y connection are essentially the same as far as the effect on the third harmonics is concerned.

In Table VIII, the third harmonic currents in the neutral are calculated and compared with the measured values. The check obtained must be considered very satisfactory, particularly when consideration is given to the fact that the tests were performed under difficult circumstances.

TABLE VIII
DISTRIBUTION OF THIRD HARMONIC CURRENT WITH LINE CONNECTED

All quantities referred to 110-kv. side

Oscillogram Number	f cycles	E_3''' volts	Z''' ohms	Calculated I_{n3}''' amperes	Measured I_{n3}''' amperes	Discrepancy per cent
106	58.9	229.7	566	0.405	0.395	2.53
113	58.7	122.9	332	0.370	0.334	10.8

In the tests previously referred to, where the 22-kv. delta was closed, the triple frequency component of the neutral current on the line side was 0.568 amperes when the line to the State border was used, and 1.125 amperes when the entire line to Lindale was connected to the bank. The frequency was 59 cycles.

It is possible to calculate the third harmonic current which should flow in the closed delta, but since this current unfortunately was not measured, no check can be obtained.

In accordance with equation (25) the distribution of third harmonic currents is given by

$$I_2''' = I_n'''\,\frac{Z'''}{Z_2'''}$$

Using this equation the third harmonic currents in the delta winding referred to the high-tension side become:

Line Gadsden—State border:

$$I_2''' = 0.568\,\frac{26.8 - j\,566 \times \dfrac{177}{176.7}}{19.7 + j\,254 \times \dfrac{177}{180}}$$

$$= 0.568\,\frac{567}{250} = 1.29 \text{ amperes.}$$

Line Gadsden—Lindale:

$$I_2''' = 1.125\,\frac{12.8 - j\,332 \times \dfrac{177}{176.7}}{19.7 + j\,254 \times \dfrac{177}{180}}$$

$$= 1.125\,\frac{333}{250} = 1.50 \text{ amperes.}$$

Applying the ratio of transformation, the actual third harmonic circulating current in the 22-kv. delta would be 6.45 amperes and 7.50 amperes respectively for the two cases considered.

SUMMARY

The paper has discussed the distribution of harmonics in single-phase transformers and in three-phase banks of single-phase transformers. Two-winding transformers, or three-winding transformers where the harmonic current exists in two of the windings only, have been considered. The impressed voltages have in every case been assumed strictly sinusoidal so that all harmonics originate in the transformers themselves. If the impressed voltages are non-sinusoidal the problem becomes much more involved and may be considered beyond the scope of this paper.

Formulas for the distribution of harmonics between primary and secondary circuits have been given for a series of connections. These formulas, slightly modified, may also be applied to two secondary circuits, and in general formulas of the same type will give the distribution between *any two circuits* provided no other circuit carries any of the harmonic currents whose distribution is desired.

The harmonic currents distribute themselves between two circuits in inverse proportion to the ratio of the total impedances of the two circuits at the particular frequency considered. The part that the transformer or each transformer in a bank contributes to these impedances is the true leakage impedance of each of the two windings involved with respect to the other winding.

The distribution of the third harmonic current between two circuits has been examined by laboratory tests on small experimental transformers. Y–Δ–Δ and Δ–Y connections were used. The Δ–Y bank was loaded by a Y-connected capacity load and also by a Y-connected load consisting of inductance and capacity. The inductance and capacity in the latter

case were adjusted so as to produce triple frequency resonance in the secondary circuit. The neutral point of the Y-connected load and the neutral point of the transformer bank were in every case interconnected.

The measured and calculated distribution of third harmonic current in the first two tests check to engineering accuracy. The results from the third test could not be used for checking purposes since the reactors employed as a part of the load had iron cores and, therefore, had a tendency to introduce harmonics themselves.

The paper has further given some data and results from a series of field tests on a bank of three 2100-kv-a., 110,000-22,000-2300-volt transformers and a short transmission line. The tests of particular interest in connection with the present subject consisted of determining the separate leakage impedances of two of the windings of the transformers and of measuring the third harmonic current in the line-side neutral when the line was connected to the bank. The transformer connections during the latter tests were Y-open Δ-Y with secondary neutral point grounded.

The third harmonic current in the neutral has been computated from the measured value of the third harmonic voltage across the open corner of the delta winding in connection with the proper combination of transformer leakage impedance and line impedance. The check between the computed and measured third harmonic currents must be considered satisfactory.

List of Symbols

V = Terminal voltage
V_Δ = Voltage across the corner of an open delta winding
E = Induced voltage
E_C = Voltage induced by flux in the core
I = Current
I_Δ = Circulating current in a closed delta winding
I_n = Current in neutral
R_m = Resistance in the corner of a closed delta winding containing meters, etc.
Z_Δ = Total impedance of a closed delta circuit
Z_n = Impedance of neutral

Subscripts attached to the above symbols refer the quantities to the particular winding designated by the subscript.

f = Frequency
C = Capacity
Z_{pri} = Impedance of a primary circuit
Z_{sec} = Impedance of a secondary circuit
Z_G = Generator impedance
Z_L = Load impedance
Z_{11}, Z_{22}, Z_{33} = Self impedance of windings No. 1, No. 2, and No. 3
Z_1, Z_2, Z_3 = Leakage impedance of windings No. 1, No. 2, and No. 3 with respect to some specified winding
R_1, R_2, R_3 = Resistance of windings No. 1, No. 2, and No. 3
X_{11}, X_{22}, X_{33} = Self reactance of windings No. 1, No. 2, and No. 3
$X_{12} = X_{21}$ = Mutual reactance of winding No. 1 and No. 2
$X_{13} = X_{31}$ = Mutual reactance of winding No. 1 and No. 3
$X_{23} = X_{32}$ = Mutual reactance of winding No. 2 and No. 3
X_1, X_2, X_3 = Leakage reactance of windings No. 1, No. 2, and No. 3 with respect to some specified winding
X_L = Load reactance, inductive reactance
X_C = Capacitive reactance

Several of the above quantities are different for the various harmonics. Primes attached to these quantities indicate the order of the harmonic.

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Discussion

For discussion of this paper see page 810.

Resolution of Transformer Reactance

Into Primary and Secondary Reactances

A. BOYAJIAN[1]

***Synopsis.**—The stand is taken that the resolution of the leakage reactance of a pair of windings into the individual reactances of the two windings is indeterminate unless referred to a third winding and that therefore it varies with the object in view when making the resolution. If the object is the influence of exciting current on performance, the problem is converted into a three-winding transformer problem by conceiving of the exciting current as produced by a (fictitious) load in a (fictitious) third winding. It is pointed out that the resolution of leakage reactance into individual reactances is possible only in the case of three windings, and that therefore in a transformer with three real windings and a fictitious exciting-current-load winding, constituting the equivalent of a four-winding transformer, the simple resolution fails, each winding having a different individual leakage reactance when associated with one pair of the remaining windings than when associated with another pair. Furthermore, the resolution made from the standpoint of real load currents will be different from that made from the standpoint of exciting current. Formulas are given for such resolutions, and experimental methods are described. The problem is also considered from the standpoint of flux distribution and linkages, and the limitations of some common views are pointed out.*

INTRODUCTION

THE old puzzle of the division of the leakage reactance of a transformer into primary and secondary reactances is usually discussed with the tacit assumption that such a division exists in nature,—definite, inherent and absolute,—and that our problem is merely to ascertain its value. But the resolution of the leakage reacttance of two windings into the reactances of the individual windings is no more determinate than the resolution of the distance between two points, A and B, into two parts, one as A's distance, and the other as B's distance. For this reason, it is frequently reported by

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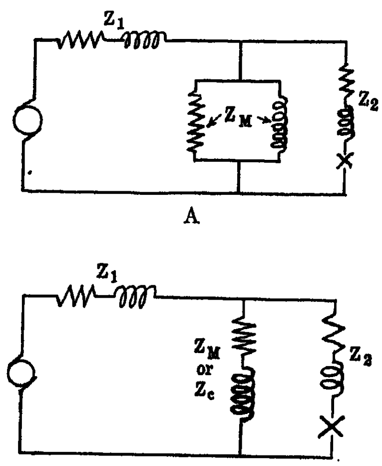
investigators that the experimental separation of the primary and secondary reactances has failed.

However, if the resolution is desired, not for pure philosophic interest but for a definite concrete object in view, then this object furnishes us the point of view relative to which the resolution is to be made, and the problem becomes determinate. In every instance



A

B

FIG. 1

where an investigator reports that he has found a means of experimentally segregating the primary and secondary reactances, it will be found that he has referred the resolution to some arbitrary point of reference, probably very useful and valid for his purpose, but arbitrary just the same.

The resolution of leakage reactance into primary and secondary reactances is sometimes necessary for certain practical purposes, as shall be illustrated below, but it will be found in every instance that the resolution which is valid for one purpose is not valid for other purposes, and that, therefore, any such resolution has to be relative and conditional.

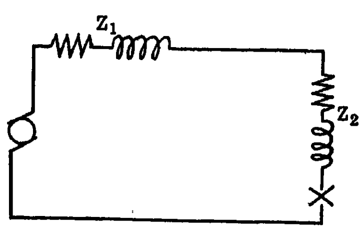


FIG. 2

THE EQUIVALENT CIRCUIT

The classical treatment of a transformer is by means of its equivalent circuit (Fig. 1A or 1B). If the exciting current is ignored, the equivalent circuit becomes like that shown in Fig. 2. If the exciting current is ignored, and the transformer is assumed to have both a secondary and a tertiary winding, then the equivalent network becomes like that shown in Fig. 3. Comparing the equivalent network of a two-winding transformer, which draws an exciting current, (Fig. 1A or 1B, especially the latter), with the equivalent network of a three-winding transformer which draws no exciting current, (Fig. 3), the essential identity of the two becomes evident. That is, the standard equivalent circuit of a transformer (Fig. 1) treats the exciting current as though it were a (fictitious) load-current drawn from a (fictitious) third winding. From this standpoint, the resolution of leakage reactance into separate primary and secondary reactances becomes a three-winding transformer problem, the general solution of which has been accomplished within the last few years[2] and which we may briefly review here.

GENERAL THEORY OF THREE-WINDING TRANSFORMER

In considering the effect of the impedance of a three-winding transformer on its performance, such as regulation, short-circuit current, division of load, etc., we may resolve the leakage impedances between pairs of windings into impedances of the individual windings, the only condition to be satisfied beings, that, symbolically,

$$Z_1 + Z_2 = Z_{12} \quad (1)$$

$$Z_2 + Z_3 = Z_{23} \quad (2)$$

$$Z_1 + Z_3 = Z_{13} \quad (3)$$

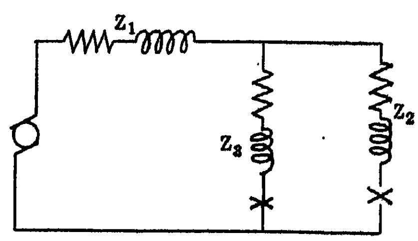


FIG. 3

It may be evident from these equations that the resolution of Z_{12} into Z_1 and Z_2 is made dependent on winding No. 3 by involving Z_{13} and Z_{23}, so that if winding No. 3 is altered the resolution of the leakage reactance between windings No. 1 and No. 2 into Z_1 and Z_2 is also altered. Certainly, any such resolution that is made variable with changes in a third winding cannot be considered absolute or inherent but must be considered purely relative. Generalizing, we may say that a winding may be said to have a definite individual leakage reactance only with reference to two other windings. Referred to less than two other windings, the individual leakage reactance of a winding is indeterminate, any resolution being as good as any other, and all of them being equally needless. Referred to more than two other windings, the reactance characteristics of a winding cannot be completely specified by a single value.

2. Transformers for Interconnecting High Voltage Transmission Systems by J. F. Peters and M. E. Skinner. JOURNAL A. I. E. E., June 1921, Vol. XL, page 483.

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Application to Two-Winding Transformers

In a two-winding transformer the resolution of the leakage reactance into primary and secondary reactances having a significance only in relation to the exciting current, and the exciting current being conceivable as occasioned by a load in a fictitious third winding, the desired resolution, in the light of the theory of three-winding transformers, may be defined as follows. Let C represent the fictitious third winding which simulates the characteristics resulting from the core. The leakage reactances between this fictitious winding C and the real windings No. 1 and No. 2,

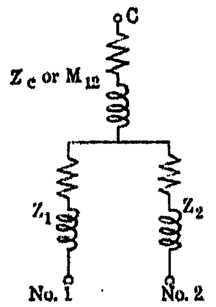

Fig. 4

respectively, may be designated as X_{1c} and X_{2c}. The individual leakage reactances X_1 and X_2, resolved with respect to the exciting current, will be given by

$$X_1 = (X_{1c} + X_{12} - X_{2c})/2 \qquad (7)$$

$$X_2 = (X_{2c} + X_{12} - X_{1c})/2 \qquad (8)$$

$$X_c = (X_{1c} + X_{2c} - X_{12})/2 \qquad (9)$$

The equivalent network of this resolution, as that of a three-winding transformer, is shown in Fig. 4, which is another form of Fig. 1B.

It may be noted that the core exercises a controlling influence on the resolution when made with reference to exciting current, (even though its effect on the total leakage reactance is ordinarily small), but it exercises no direct influence on the resolution when made with reference to load currents as in a three-winding transformer.

If it is desired to consider the element of exciting current in a three-winding transformer, the problem becomes one of four-windings, the exciting current being represented as a load in a fourth (fictitious) winding, as outlined above. In this case no definite, consistent reactance values can be assigned to the windings to indicate their performance rigorously from the standpoint of exciting-current. Furthermore, even if this could be done, the reactance values so assigned would be inconsistent with the values required to represent their load characteristics properly.

Individual Leakage Reactances in Terms of Self and Mutual Reactances

Considering the equivalent network of a two-winding transformer with its fictitious third circuit, C, (Figs. 1B and 4), it will be evident that the impedance, Z_c, is in series with and common to both of the circuits No. 1 and No. 2, and may, therefore, be recognized as the mutual reactance M_{12} between the two circuits. Z_{1c} will be recognized as the open-circuit, self inductive reactance of circuit No. 1, which we may designate as Z_1', Z_{2c} as that of No. 2, which we may designate as Z_2'. Evidently,

$$Z_1 = Z_{1c} - Z_c \qquad (10a)$$

$$= Z_1' - M_{12} \qquad (10b)$$

$$Z_2 = Z_{2c} - Z_c \qquad (11a)$$

$$= Z_2' - M_{12} \qquad (11b)$$

$$Z_{12} = Z_1 + Z_2 \qquad (12a)$$

$$= Z_1' + Z_2' - 2\,M_{12} \qquad (12b)$$

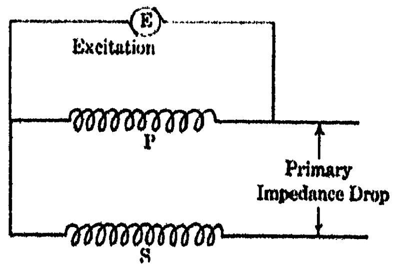

Fig. 5

Equation (12b) will be recognized as classcial. Equations (10b) and (11b) have been used in the past[3] to define the individual reactances Z_1 and Z_2, but no recognition has been made of their relativity with respect to the exciting current so far as the present writer is aware. These equations (10b) and (11b) if derived directly, based on the reasoning that the total

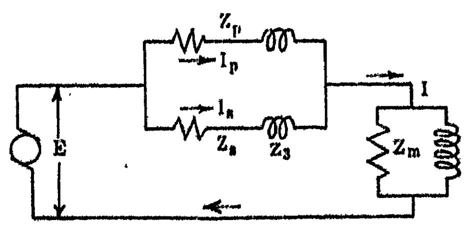

Fig. 6

voltage induced in the winding is the difference between the self-inductive voltage drop and the mutual voltage induced from the other winding, have the appearance of finality. However, when they are derived, as a special case of a three-winding transformer, even though the derivation is roundabout, it is believed that the relativity of the resolution is thereby made clear. So far as utility for practical computation is concerned, neither the set of equations (7), (8), and (9), nor the set (10b), (11b), and (12b) is of a great deal of value. Their primary use is to define the quantities to which

3. See, for instance, Rogowski, Dispersion in Transformers, *ETZ*, Vol. XXXI, pp. 1033-1036, 1069-1071; also, Mitteilungen uber Forschungsarbeiten, Heft. No. 71.

they refer and thus suggest and guide practical methods of calculation.

Experimental Resolution

Consideration of the equivalent circuit of a two-winding transformer, (Figs. 1 and 4), suggests a variety of test methods for the determination of X_1 and X_2. They are very simple and convenient for transformers with 1:1 ratio. Those with other ratios will require either current or potential transformers of the same ratio as their own and the results will naturally be complicated on account of errors of ratio and phase angle introduced by these auxiliary transformers.

1. *Regulation due to Exciting Current.* The primary impedance drop, due to exciting current, can be measured directly and very exactly in a 1:1 ratio transformer by using the connection shown in Fig. 5. Knowing the current and the impedance drop, the value of the impedance follows by Ohm's law. If the transformer ratio is not unity, a potential transformer of the same ratio would be necessary for the test.

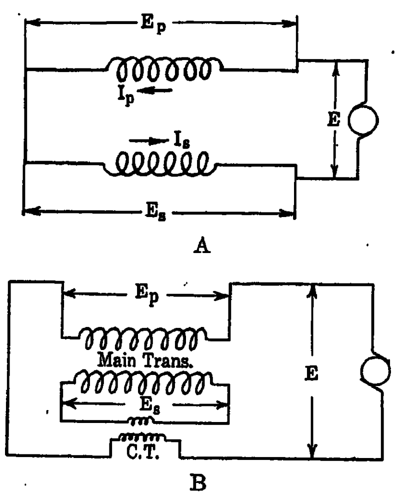

Fig. 7

2. *Division of Exciting Current.* If the two windings (1:1 ratio) are excited in parallel, they must divide the exciting current inversely as their respective leakage impedances. The equivalent circuit diagram of this test condition is shown in Fig. 6. When the ratio is not unity, the windings may be paralleled through a suitable current transformer.

3. *Series Impedance Test.* With the two windings connected in series opposition, (Fig. 7A), the voltage-drop across each winding can be measured directly, and knowing the current, the individual leakage impedances follow by Ohm's law. With ratios other than unity, the two windings may be connected in series through a suitable current transformer (Fig. 7B), although the results are likely to be disturbed by the errors of ratio and phase angle of the current transformer.

4. *Parallel Impedance Test.* If the two windings are excited in parallel-opposition, (Fig. 8), so that the core cannot be magnetized as a whole, the impressed voltage must be balanced by the leakage impedance of the two windings, and the division of current between them will be inversely as their respective leakage impedances.

Absolutely consistent results need not be expected from all these tests. In fact, the results will vary somewhat with varying values of current and voltage even when using the same connection. The reason for this is that changes in the permeability of the core at various densities affect differently the reluctances of the external and internal parts of the magnetic circuit, neither the

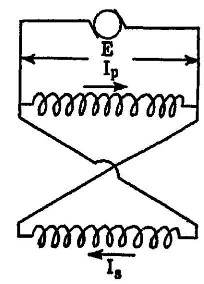

Fig. 8

iron magnetic circuit nor the air magnetic circuit (which are in parallel) having uniform section or length. This fact was forced to the attention of the writer in 1921 while investigating the division of third harmonic current between transmission lines and internal deltas. The method of test and some of the results bearing on the present discussion were as follows:

5. *Third Harmonic Tests.* The division of exciting current, described under test method No. 2, would, of course, apply to the higher harmonics of the exciting current as well as to its fundamental, at least to a first approximation. Hence, if two parallel paths are offered to the flow of the third harmonic exciting current, as for instance by two delta-connected wind-

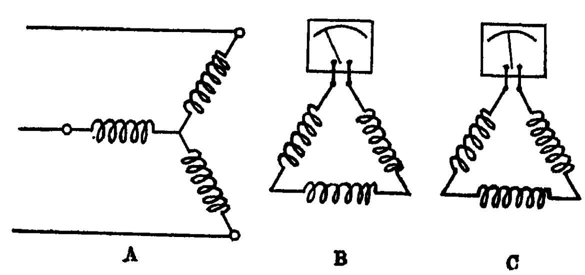

Fig. 9

ings, (Fig. 9), the division of the third harmonic exciting ampere-turns between them must be inversely as their respective leakage impedances. Hence, having obtained the division of current by test, the division of the leakage impedance between the two windings follows as indicated.

The foregoing method was tested on a three-phase bank of two-legged core-type transformers having a number of concentric windings on both of the legs. One set of windings was connected in Y for excitation, and two sets were connected in delta, (Fig. 9). The

two delta windings had the same current and voltage rating and had low impedance between them, their 60-cycle leakage reactance being about three times their resistance. Both of the delta windings were on one and the same leg of the core. The *Y* primary windings were chosen on the same leg as the delta windings in one set of tests (marked Case I. below), and on opposite leg in another set of tests (marked Case II. below), to observe the effects of low and high reactances between the primary winding and the two secondary delta windings, with results as follows:

	Nominal flux density in core	Third harmonic circulating current in each unit in per cent of the total third harmonic current Delta *B*	Delta *C*
Case I.	38 Kilolines/sq. in.	51 per cent	49 per cent
	112 " " "	44 per cent	56 per cent
Low Reactance			
Case II.	43 " " "	48 per cent	52 per cent
	113 " " "	53 per cent	47 per cent
High Reactance			

It is of interest to note how the division of current (and reactance) changes with a change in core density, and also how the direction of change with high reactance primary is opposite from that with low reactance primary. It may be remembered from earlier discussion that this is really a four-winding problem,—three real and one fictitious windings, and that the treatment of this as a three-winding problem, that is as two real (delta) windings and one fictitious winding representing the core, is justifiable only at the lower densities, where the primary exciting current and the corresponding flux in air are small, as confirmed by the fact that Cases I and II given above approach each other at the lower density.

The Viewpoint of Flux Distribution and Linkages

Discussion of reactances by transformer engineers is frequently expressed in terms of flux linkages. This is the designer's point of view, and has its uses in the determination of losses. As this involves a more detailed analysis of the transformer, one might be led to consider it more basic and accurate. However, as ordinarily handled, it has serious limitations. For instance, a common statement by transformer specialists is that most of the reactance of a transformer (say 80 per cent to 90 per cent), is primary reactance and that only a small portion (say 10 per cent to 20 per cent), is secondary reactance. Such a view, although conceding the relativity of individual reactances by assigning the reactance primarily to the excited winding, fails by not distinguishing between main and leakage fluxes and their composite. The basis commonly cited for the assignment of the total or major part of the reactance to the excited winding is briefly as follows: Ignoring the small resistance drops of the windings for simplicity, the net flux-linkages of each winding must be proportional to its terminal voltage. But on the impedance test the secondary terminal voltage is zero, and hence its net flux-linkages must be zero. This condition, however, does not require that no portion of the secondary should link any flux. Since windings must have some thickness, and leakage flux within their thickness, partial flux-linkages of the short-circuited secondary cannot be avoided, the magnitude and sign of these partial linkages being such as to add up to zero. In making such an analysis it is found that the major part of the flux links exclusively the excited winding, a small part links both the excited winding and the short-circuited secondary, and another small part (equivalent and opposite to the preceding) links exclusively the short-circuited secondary. Basing the resolution of reactance on such a resolution of fluxes, if net flux-linkages alone are considered all reactance becomes primary and none secondary; and if the partial exclusive flux-linkages of the secondary are featured, a small fraction of the total reactance becomes secondary. However, this plausible reasoning

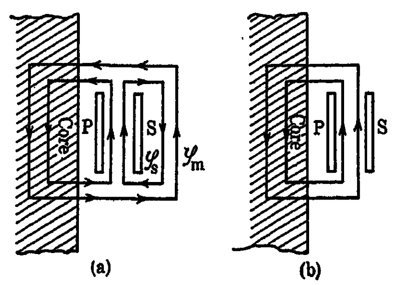

Fig. 10

fails by confusing the total short-circuit flux with the leakage flux. The flux distribution and linkages hinted above are true only for the resultant of the main and leakage fluxes, and therefore the distribution or linkages of the latter cannot be considered a direct measure of the distribution of the leakage reactance. It would be erroneous to assume that there is no main flux under the impedance test condition, even ignoring the resistance of the windings. Main and leakage components of the flux may cancel in the short-circuited winding, and add up in the excited winding, as illustrated in Figs. 10a and 10b; but the fact that the linkages of the *resultant* flux with the secondary is zero could not be taken as a proof that the leakage reactance of that winding is zero, as this can be proven only by segregating the leakage flux proper and determining its linkages. The two component fluxes, under the impedance test condition, are so intimately merged into each other that their segregation, if attempted by measurements made on the composite flux, is most discouraging, as has been the experience of many an

investigator[4]. The practical solution of the problem would rest in making measurements under conditions in which only one kind of flux exists at a time: (a) leakage flux unmixed with main flux in one case, (b) main flux unmixed with leakage flux in the other. Condition (a) is realized by connecting primary and secondary windings in series-opposition (one to one ratio) in which case no exciting current and hence no main flux can exist. Condition (b) is realized by open-circuiting the secondary, so that no load current, and hence no leakage flux, can exist, and impressing on the primary a voltage equal to the secondary impedance drop measured in the preceding case. One small limitation must be borne in mind that due to the differing permeability of the core at different densities, the flux distribution and linkages for the resultant flux will not follow exactly the calculated values based on linear combinations of the two components.

CONCLUSIONS

(1) The idea of individual leakage reactance for circuits inductively related to each other is a very convenient conception for certain purposes; however, the resolution is not inherent and absolute, but is purely relative, and is workable only when the problem is theoretically resolvable into three circuits. With less than three circuits, the resolution is indeterminate, and with more than three circuits, it is not possible.

(2) In a two-winding transformer, such a resolution can have a significance only with respect to the exciting current. The resolution is then defined (in terms of three-winding theory) by conceiving the exciting current as caused by a load in a fictitious third winding.

(3) As a practical definition, the individual reactances of a two-winding transformer may be defined as the apparent individual reactances of the two windings in series-opposition connection, on the basis of 1: 1 ratio. Other methods of test are also described in the text, but perfect agreement among the various methods cannot be expected on account of the varying permeability of the core at the different densities involved. The different densities will influence the division of reactance very sensibly, even though they may not affect the total reactance appreciably.

(4) In a three-winding transformer, it is not rigorously possible to assign a definite individual leakage reactance to each winding with reference to exciting current, but it is possible to do so with reference to their load currents.

(5) Flux distribution and linkages under the short-circuit test condition apply to the resultant of a main flux and a leakage flux and therefore may not be taken as a direct measure of the division of leakage reactance between primary and secondary windings.

4. See for instance, K. B. McEachron, "Magnetic Flux Distribution in Transformers," JOURNAL A. I. E. E., 1922, pp. 281-287.

It may be noted that in the simple case of a perfectly symmetrical magnetic circuit (ignoring the resistance and thickness of the secondary), the main flux ϕ_m and the leakage flux ϕ_s (shown in Fig. 10A for the impedance test condition) wipe out each other bodily where they fall together, as in the return leakage field of the secondary, and the resultant flux in this region becomes zero, as shown in Fig. 10B. In actual transformers, the distribution of main and leakage fluxes in the return field are neither alike nor uniform, and, therefore, although the two will still neutralize one another's total linkages, they will not wipe out each other's fluxes at every point in the return path of the secondary leakage field, and some of the secondary leakage flux will retain its identity. It is this partial flux which is sometimes erroneously taken as the measure of secondary leakage reactance.

Discussion

SEPARATE LEAKAGE REACTANCE OF TRANSFORMER WINDINGS
(DAHL)

TRANSFORMER HARMONICS AND THEIR DISTRIBUTION
(DAHL)

RESOLUTION OF TRANSFORMER REACTANCES INTO PRIMARY AND SECONDARY REACTANCES
(BOYAJIAN)

SARATOGA SPRINGS, N. Y., JUNE 25, 1925

J. F. Peters: In Mr. Dahl's paper, as far as I can see, the assumption is made that the triple-frequency component of magnetizing current follows Ohms' law, that is, there is inherently in the transformer a triple-frequency voltage and the triple-frequency component of current that will flow is that voltage divided by a triple-frequency impedance. If this is the case, then by decreasing the triple-frequency impedance to a small value, the corresponding current could be made quite large. Obviously this cannot be the case because when the triple-frequency current reaches a certain value, which is approximately 40 to 45 per cent of the fundamental-frequency current, the voltage wave takes on a true sine shape in which case the triple-frequency voltage disappears. It may not be possible to decrease this impedance to a very small value within the transformer, but if it is a true impedance, it can be counteracted to any desired degree externally. Also, if no triple-frequency current is permitted to flow, there will be a large triple-frequency voltage appear across each of the phases. In a transformer of commercial proportions and flux density, this voltage would amount to approximately 75 per cent of the fundamental-frequency voltage, which, in the transformer analyzed by Dahl, would amount to 100 volts triple-frequency. Then the triple-frequency current that should flow in any winding should be that voltage divided by this triple frequency impedance. He finds in winding one a triple-frequency impedance of approximately one-half ohm. This should give a triple-frequency current in the order of 200 amperes. Actual measurements show approximately two amperes.

Since this triple-frequency impedance is not a true impedance, that is, its X value is not a consonant, there are a number of operations used by Mr. Dahl that may be questionable. For instance, in his paper on *Transformer Harmonics, etc.*, on the seventh page, first column, paragraph 3, in measuring the triple-frequency current in two delta windings, when measuring the current in one winding, the corresponding resistance in the other winding was short-circuited in order to use the same measuring instrument in both, and vice versa. This resistance in the measuring instrument is quite large. It is of the order of the total impedance of the winding. That means,

in shifting from one winding to another, the triple-frequency current distribution between the windings has been changed, and since the triple-frequency impedance obtained applies only to the particular flux density and triple-frequency current under which the test was made, there is a chance for considerable error being introduced.

There is another point in connection with the paper that makes me feel that perhaps something not permissible was done. Mr. Dahl determined the triple-frequency impedance of his transformer by two methods: In the two-winding methods, he determined X_3 between coils 1 and 3 and found the value to be 1.275. By means of the three-winding method, he determined X_3 between windings 2 and 3 and found it to be 1.336. The distance between coils 2 and 3 is three-quarters of an inch. The distance between coils 1 and 3 is approximately 0.95 in. That being the case, the reactance X_3 measured between coils 1 and 3 should be approximately 25 per cent higher than measured between coils 2 and 3. Actually, he finds it to be approximately 96 per cent, so that there is a considerable discrepancy between X_3 measured by the two methods of the order of 25 or 30 per cent. That could hardly be considered within engineering accuracy.

The methods used by Mr. Boyajian are so simple and direct that I feel the chances of error are very remote and you will note in his one set of tests that he gives the distribution of triple-frequency current between two delta windings, that distribution changes with flux density which also shows the fictitious triple-frequency impedance is not a true impedance but changes with conditions.

V. Karapetoff: Consider a two-winding transformer, with or without an iron core, and disregard saturation and core loss. Such a transformer can be first replaced by one of one-to-one ratio of turns, and then by an equivalent diagram shown in Fig. 1 of Mr. Boyajian's paper (with a pure reactance in the exciting branch). In this diagram, the values of the leakage reactances Z_1 and Z_2 are perfectly definite, namely $Z_1 = Z_1{}^1 - X_{12}$ and $Z_2 = Z_2{}^1 - X_{12}$, where $Z_1{}^1$ and $Z_2{}^1$ are the total impedances of the windings (leakage plus mutual flux) and X_{12} is the mutual reactance. Therefore, rather than convey an impression that eakage reactances are fictions valid for one purpose and not valid for other purposes, would it not be better to say that in a two-winding transformer these reactances have a definite meaning in the equivalent diagram and must be interpreted in the actual transformer with reference to this equivalent diagram?

In a transformer with more than two secondaries, the indefiniteness of the division of the leakage fluxes among the individual windings lies in the very nature of the phenomenon. Consider for example a "split" single-phase line, consisting of two lines on adjacent cross-arms. The inductance of such a line depends upon the fractions of the total load carried by the two loops, and is different for different divisions of the load.

R. G. McCurdy: It seems to me that Mr. Boyajian is in error in stating that the difficulties which have arisen in dividing leakage impedances between windings are due to any lack of definiteness of the problem. The generally acceptable definitions of the individual leakage reactances are given by equations (**10b**) and (**11b**) of his paper. In two-winding transformers, when the exciting current is ignored, a single leakage impedance equal to the sum of the two individual leakage impedances may be used in computing regulation under load, and with three-winding transformers ignoring the exciting current leads to the set of equations (1) (2) and (3) of his paper which express individual impedances in terms of the total leakage impedances between pairs of windings. I think it leads to confusion, however, to call such individual impedances of three-winding transformers "individual leakage impedances."

When it is desired to determine division of exciting currents between different windings the above simplifying assumptions may not be made. Knowledge of the individual leakage reactances is then necessary. With two-winding transformers the simple equivalent network of Fig. 1 of Mr. Boyajian's paper may be used. This same network will also apply when considering the division of harmonic exciting currents in three-winding transformers when one of the windings is connected so that current of that frequency is suppressed. When current may flow in all three windings a more complicated network must be set up involving in general seven impedance elements.

While it is true that fairly simple arrangements may be used for determining the individual impedance of unity-ratio transformers, the application of such methods to transformers of other ratios presents difficulties. Take for example the circuit of Fig. 7B of Mr. Boyajian's paper. An equivalent circuit is shown in the appended Fig. 1. It will be evident by inspection that if the ratio of the secondary leakage impedance to the exciting impedance is the same for both the transformer under test and the current transformer, the secondary voltage will be zero and the primary voltage will be the same as that obtained on a

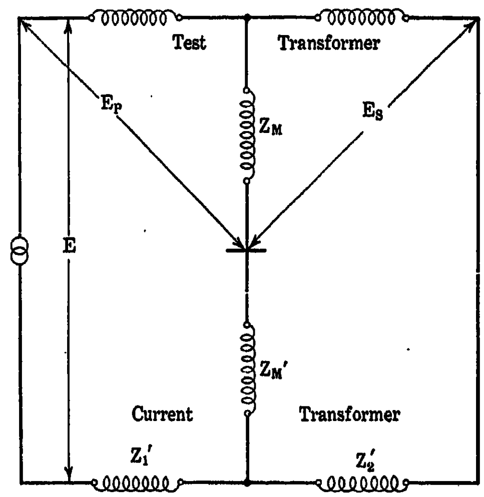

FIG. 1—CIRCUIT EQUIVALENT TO FIG. 7 OF BOYAJIAN PAPER WITH CURRENT TRANSFORMER

short-circuit test. To determine the individual leakage impedances accurately by such a scheme, the secondary leakage impedance of the current transformer would need to be very low and the exciting impedance very high as compared to the corresponding quantities of the transformer under test. Similar difficulties arise in the use of potential transformers in determining the division of exciting currents between the windings. Mr. Boyajian erroneously refers to the use of current transformers for this purpose.

For these reasons such methods for determining the individual leakage impedances do not seem to have much practical importance. Methods based upon the division of triple-harmonic exciting currents as discussed by Mr. Dahl appear to be more useful.

I note in comparing the test results of Mr. Boyajian and Mr. Dahl a considerable difference in the effect of magnetic density on the individual leakage impedances. I wonder if they have been equally careful to eliminate the impedances of measuring instruments and the effects of fundamental and harmonics other than the third. If I read Mr. Dahl's paper correctly he has largely eliminated errors due to these sources.

J. F. Peters: In view of what Mr. McCurdy stated I would like to amplify a point that I intended to make in the first discussion. I feel that the method that Mr. Dahl has used is of considerable value in many applications but it must be carried on with great care. The point I wanted to bring out was that the impedance obtained by this method applies only to that particular flux

density and value of triple-frequency current, that it is not a true impedance. Care should be taken to retain the currents in their normal values. It is very unfortunate in the tests of the two deltas that Mr. Dahl made that it was necessary to short circuit the resistance in one delta while measuring current in the second. This is more nearly a constant-current proposition than it is a constant-potential proposition. That is, if you were to insert in the corner of the delta sufficient impedance to reduce the current to one-half of the value, the residual triple-frequency voltage would be more than double. It will not follow that law at all and I feel that there may be some considerable errors in the actual values obtained by means of changing the distribution of current.

L. P. Ferris: Without entering into the technical controversy that appears between the papers by Messrs. Dahl and Boyajian, I wish to point out one example of the practical importance of a solution of this problem of the division of impedances between transformer windings. The example to which I have reference arises in connection with the inductive relations between power circuits and neighboring telephone circuits. When the neutrals of transformers are grounded it is important to know how the triple-harmonic exciting currents divide between the several windings of the transformer and particularly how these currents divide between closed deltas and the windings which are connected to the lines with grounded neutrals. This is a practical problem of great interest in addition to the problems of the effect of these impedances on regulation and on the division of load between windings.

It will be appreciated that this distribution of the triple-harmonic currents is a perfectly definite thing which is determined in practise as soon as the transformer is connected to the line and is controlled by the characteristics of the transformer and of the line. The transformer impedances involved may be called by the term "leakage impedance" or by some other term. However, there is this very definite distribution which takes place and it is necessary to carry our analysis to the point where this distribution may be predetermined. Thus it may be possible to predetermine the effect of a given change in connections or the effect of the addition of a new bank of transformers in a complicated network.

This problem was encountered by Mr. McCurdy, Mr. Cone and myself about ten years ago in connection with our work in California with the Joint Committee on Inductive Interference. There are discussions of it in several of the technical reports of that committee which have been published by the California Railroad Commission.

The interest which is now being manifested in this subject by the colleges and several manufacturing concerns is very gratifying and I hope that it will continue until a common understanding is reached. This problem is one which is now engaging the attention of the Joint Subcommittee on Development and Research of the National Electric Light Association and the Bell System and I am sure that this committee will welcome any contributions to a solution by engineers either from the manufacturers or from the colleges.

C. T. Weller: I would like to discuss the subject of transformer reactance with particular reference to instrument transformers.

The use of instrument transformers has been referred to by Mr. Boyajian, so I will first comment on that. He states, under "Experimental Resolution," that when instrument transformers are used, the results will be complicated on account of the errors of ratio and phase angle introduced thereby. The ratio and phase-angle errors of properly designed instrument transformers are very small and can be very accurately determined if the condition of use can be reproduced in the calibration. If I understand the diagrams correctly, the condition described in Paragraph 1 can be accurately reproduced, but this is not the case with respect to Paragraphs 2 and 3, because it is entirely possible that the excitation of the instrument transformer may be wholly or partly supplied from the secondary due to differences in the characteristics of the transformers. I believe that the reference in Paragraph 2 should be to a potential rather than to a current transformer. In this connection, I would like to emphasize the point, which has already been made, that the detectors used in these tests may considerably complicate the results.

Instrument transformers are usually two-winding transformers with the primary surrounding the secondary. We have determined the approximate individual leakage reactance of the windings of a few of our standard transformers by calculation from the ratio and phase-angle and exciting-current results. The results are somewhat indeterminate due to the very small errors of the transformers. I might add here that in transformers of this grade, the effect of third and higher harmonics is negligible.

For potential transformers of moderate voltages, the primary leakage reactance may be taken to be about one-half of the total reactance obtained from the short-circuit impedance test. The range of phase angle, for the limiting cases of first assuming that the primary has zero leakage reactance, and second, that all the leakage reactance is concentrated in the primary, may not exceed five minutes. This indicates that the phase-angle error is very small. The leakage reactance appears to be constant under all normal load conditions. The total impedance and reactance appear to be the same at rated voltage as at the low voltage obtained in the impedance test. This makes it possible to determine in advance rather than to obtain by trial the power factor of the output which will give the maximum ratio error.

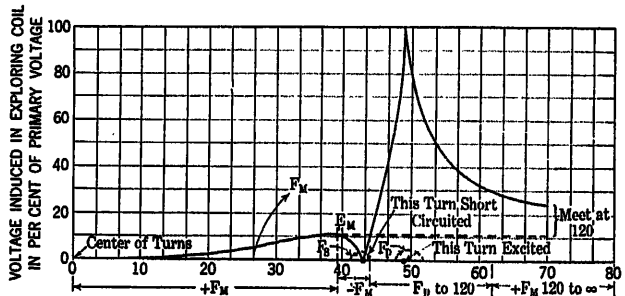

Fig. 2

For current transformers of the ring-core type, with evenly distributed windings, the secondary leakage reactance may be taken to be zero. For current transformers of moderate voltage ratings and of low ratios of the square-core type, the leakage reactance may be taken to be approximately one-quarter of the total leakage reactance obtained from the short-circuit impedance test. This value applies over the range of rated current. For moderate overloads, the value may reach over one-half of the total leakage reactance. At heavy overloads, the saturation of the core greatly complicates the results.

To sum up, the approximate division of the leakage reactance of the windings of instrument transformers has been given for a few cases. These results were obtained by calculation from the ratio and phase-angle and exciting-current results and were referred to the results of the short-circuit impedance tests. This method is applicable to any two-winding transformers.

K. K. Palueff: In regard to the Resolution of Reactances, it is perhaps of interest to see the results of theoretical analysis of an elementary transformer. They are two concentric circles in air (one of which serves as primary and the other as secondary). These results can best be presented in graphic form.

Fig. 2 herewith gives voltage induced in exploring coils of various diameters placed concentrically with the circles and in the same plane. This voltage is expressed in per cent of total voltage induced in primary (which in this case is the outer circle).

Fig. 3 gives voltage distribution in case the inside circle is primary.

In both cases the secondary circle is short circuited and is assumed to be of zero resistance. These examples are calculated for circles of 43 and 49 centimeter radius and round conductors of one centimeter diameter.

From the curves the direction and density of the flux in the plane of the circles can be determined.

In Fig. 2 we find that beginning from the center the voltage increases to a certain point (E_m) inside of the region enclosed by the secondary circle, and then gradually diminishes reaching zero in the region occupied by the shorter circle. This means that in the region encircled by the secondary there are two fluxes ($+ F_m$, $- F_m$) equal and opposite and with boundary line coinciding with the point of maximum voltage of this region, (E_m).

As we proceed the short-circuited circle toward the excited outer circle voltage is steadily increasing. At a certain point it reaches the value of maximum of the region encircled by a secondary (E_m). This point therefore lies on the outer boundary of the secondary flux ($- F_m$).

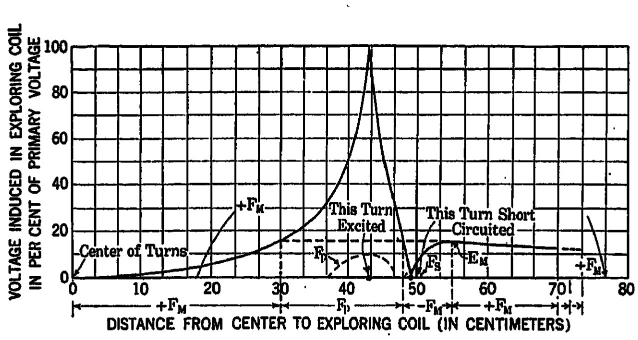

FIG. 3

After passing the primary turns voltage begins to diminish and at certain points reaches again the value of maximum of the region enclosed by the secondary, thus marking the outer boundary of flux linking with the primary circle alone (F_p) and of the flux linking with both turns ($+ F_m$).

Fig. 3 gives voltage distribution in case the inner circle is excited and the outer short-circuited.

W. V. Lyon: The method of analyzing the performance of a transformer that Mr. Dahl outlines has some interesting possibilities when applied to three-circuit transformers. It seems to me that it makes clearer some points that Mr. Boyajian brings up in his recent paper on the theory of such transformers.[1]

When three circuits are magnetically coupled the instantaneous terminal potential of one of them may be equated to the sum of four components.

$$v_1 = r_1 i_1 + L_1 \frac{d i_1}{d t} + M_{12} \frac{d i_2}{d t} + M_{13} \frac{d i_3}{d t} \qquad (1)$$

The difficulty met in handling this equation is due to the fact that with an iron magnetic circuit the inductances, both self and mutual, are variables, being functions of the currents. As far as the solution of a great number of problems goes, however, this difficulty can be removed entirely by the following device. While the principle of the device is old, the way of presenting it is not so common, especially in the case of transformers with three windings. The magnetic flux distributions which account for the self and mutual inductances may be divided into two components. By far the greater portion of the flux is confined to the iron core, but a smaller part exists wholly or partially in air. This smaller part, however, determines almost entirely the operating characteristics of the transformer. This thought is interesting.

1. Theory of Three-Circuit Transformer, A. Boyajian. A. I. E. E. JOURNAL, April 1924, p. 345.

At this point we will assume that the flux that is wholly within the iron core depends only upon the value of the ampere-turns producing it, and is entirely unaffected by the position of these ampere-turns with respect to the core. While this is not precisely true the error must be very small. That is to say, this flux that is wholly within the iron is the same whether produced by a given number of ampere-turns in circuit one, two, or three. Let M be the value of inductance assigned to each circuit on account of this flux. This inductance is variable. The remainder of the flux due to any of the circuits is wholly or partly in air and for this reason is essentially proportional to the current in the circuit. In all that follows we will assume that each of the three circuits has the same number of turns. The method of treatment when the number of turns is different is well known. Thus if we subtract from the self and mutual inductances given in equation (1) the inductance M the result in each case will be an inductance of *constant* magnitude, being due to flux that exists wholly or partly in air. Equation (1) becomes

$$V_1 = r_1 i_1 + (L_1 - M) \frac{d i_1}{d t} + (M_{12} - M) \frac{d i_2}{d t}$$

$$+ (M_{13} - M) \frac{d i_3}{d t} + M \frac{d}{d t} (i_1 + i_2 + i_3)$$

The last component is the electromotive force produced in each of the windings by the action of the flux that exists wholly in the iron. Hereafter we will represent it by the letter e, or E, if the equation is written in the vector form. Following the nomenclature that Dahl uses we will write

$$(L_1 - M) \frac{d}{d t} = j x_{11}$$

$$(M_{12} - M) \frac{d}{d t} = j x_{12}$$

and

$$(M_{13} = M) \frac{d}{d t} = j x_{13}$$

Therefore we will now write equation (1) thus:

$$\overline{V}_1 = (r_1 + j x_{11}) I_1 + j x_{12} I_{12} + j x_{13} I_3 + E \qquad (2)$$

Likewise

$$\overline{V}_2 = (r_2 + j x_{22}) I_2 + j x_{12} I_1 + j x_{23} I_3 + E \qquad (3)$$

and

$$V_2 = (r_3 + j x_{33}) I_3 + j x_{13} I_1 + j x_{23} I_2 + E \qquad (4)$$

x_{11} is the self reactance of circuit (1) due to flux that is wholly or partly in air. It is thus a constant quantity. Similarly x_{12} is the mutual reactance between circuits (1) and (2) due to flux that is wholly or partly in air. It is likewise a constant quantity. The meaning of the other reactances x_{22}, x_{33}, x_{23} and x_{13} is at once evident. In order to proceed with the problem effectively and without too much difficulty it is necessary at this point to assume that the sum of the currents in the three circuits, that is the net magnetizing current, is insignificant in comparison with the individual currents themselves. This assumption limits the scope of the solution, but with about full-load current in the circuit it is nearly true. At any rate it is a customary assumption and need cause no concern. Thus we will write equation (5) as

$$I_1 + I_2 + I_3 = 0 \qquad (5)$$

Equations 2, 3, 4, and 5 are fundamental in determining the operation of three-circuit transformers.

The voltage drop between the terminals of circuits (1) and (2) *i. e.* ($V_1 - V_2$) is, if it is noted that $I_3 = - I_1 - I_2$

$V_1 - V_2 = [r_1 + j\ [(x_{11} - x_{12} + x_{23} - x_{13})]$

$$I_1 - [r_2 + j\,(x_{22} - x_{12} - x_{23} + x_{13})]I_2 \qquad (6)$$

Similarly

$$V_2 - V_3 = [r_2 + j\,(x_{22} - x_{23} - x_{12} + x_{13})]\,I_2 - [r_3 + j\,(x_{33} - x_{23} + x_{12} - x_{13})]\,I_3 \qquad (7)$$

Also

$$V_3 - V_1 = [r_3 + j\,(x_{33} - x_{13} + x_{12} - x_{23})]I_3 - [r_1 + j\,(x_{11} - x_{13} + x_{23} - x_{12})]\,I_1 \qquad (8)$$

It will be noted that these last three equations may be written as follows:

$$V_1 - V_2 = Z_1 I_1 - Z_2 I_2$$
$$V_2 - V_3 = Z_2 I_2 - Z_3 I_3$$
$$V_3 - V_1 = Z_3 I_3 - Z_1 I_1$$

The impedances Z_1, Z_2 and Z_3 in these equations are the same as Z_A, Z_B and Z_C that Boyajian uses. In the same way that a two-circuit transformer may be represented by a single coil as in Fig. 4, so a three-circuit transformer may be represented by three coils as in Fig. 5.

Equations (6), (7) and (8) throw some interesting light on the composition of the impedances that are associated with the three equivalent coils shown in Fig. 5; *e. g.*, $x_{11} - x_{12}$ is the leakage reactance between circuits (1) and (2) while $x_{11} - x_{13}$

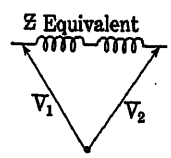

Fig. 4

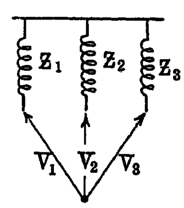

Fig. 5

is the leakage reactance between circuits (1) and (3). The equivalent reactance of circuit (1) is the leakage reactance of (1) with respect to (2) plus the differential effect of (3) upon (1) and (2). If the third circuit were symmetrically located with respect to the other two circuits its mutual effect upon each would be the same; x_{23} would equal x_{13} and the equivalent reactance of (1) would be its leakage with respect to (2) alone. Or, if the third circuit did not carry any current the terms x_{23} and x_{13} would not enter into the equivalent reactance of the first circuit. This also appears if in equation (6) I_2 is equal to $-I_1$, *i. e.* $I_3 = 0$. It will be seen that the drop between the first two circuits is then

$$V_1 - V_2 = [r_1 + j\,(x_{11} - x_{12}) + r_2 + j\,(x_{22} - x_{12})]\,I_1$$

Now it is evident that it is only when $x_{11} = x_{22}$ that the leakage reactance of (1) with respect to (2) is equal to the leakage reactance of (2) with respect to (1). Again, if it is possible to arrange the circuits so that the mutual reactances between circuits (1) and (2) and between (1) and (3) are relatively large in comparison with the mutual reactance between (2) and (3) it may be that the equivalent reactance of circuit (1) will be negative. Boyajian mentions this.

If one or two of the three circuits delivers power to a load to which constants R and X may be assigned, the terminal potential of the circuit may be replaced by $-I\,(R + j\,X)$; *e. g.*, in the case of a load on circuit (3) equation (4) may be written:

$$I_3\,(R + j\,X + r_3 + j\,x_{33})\,I_3 + j\,x_{13}\,I_1 + j\,x_{23}\,I_2 + E = 0$$

In this way it becomes a simple matter to determine the power currents in each of the windings of a three-circuit transformer.

Another even more interesting point is the operation of three-circuit transformers on three-phase circuits. In this case the third circuits of each transformer are usually connected in delta and spoken of as the tertiary delta. One of the principal reasons for this is to provide a sure path for the third harmonic exciting current if the other windings are both connected in Y. The tertiary Δ is also used to supply power, often at a relatively low voltage. The general method of analysis is as follows: Write the voltage equations similar to equations (2), (3) and (4) for each winding of each transformer, nine equations in all. Add the voltages across the three windings numbered (1) and divide by 3. By definition this is the zero-sequence voltage in the first winding[2]. This gives

$$V_{10} = (r_1 + j\,x_{11})\,I_{10} + j\,x_{12}\,I_{20} + j\,x_{13}\,I_{30} + E_0 \qquad (9)$$

This assumes that the three transformers are in all essential respects identical. The added subscript 0 signifies that the quantity is the zero-sequence component. Likewise, we may write

$$V_{20} = (r_2 + j\,x_{22})\,I_{20} + j\,x_{12}\,I_{10} + j\,x_{23}\,I_{30} + E_0 \qquad (10)$$

$$V_{30} = (r_3 + j\,x_{33})\,I_{30} + j\,x_{13}\,I_{10} + j\,x_{23}\,I_{20} + E_0 \qquad (11)$$

If the third windings form the tertiary delta, V_{30} is zero. This gives a useful relation for E_0 which may be substituted in the other equations (9) and (10). Making this substitution gives

$$V_{10} = (r_1 + j\,x_{11} - j\,x_{13})\;I_{10} + j\,(x_{12} - x_{23})\,I_{20} - (r_3 + j\,x_{33} - j\,x_{13})\,I_{30} \qquad (12)$$

and

$$V_{20} = (r_2 + j\,x_{22} - j\,x_{23})\,I_{20} + j\,(x_{12} - x_{13})\,I_{10} - (r_3 + j\,x_{33} - j\,x_{23})\,I_{30} \qquad (13)$$

We also have the general relation that

$$I_{10} + I_{20} + I_{30} = 0 \qquad (5)$$

Let us consider a few of the cases that may commonly arise.

1. Primaries and secondaries in Δ.

In this case $V_{10} = 0$ and $V_{20} = 0$, and the only values of the zero-sequence currents that will satisfy these conditions are zero. That is, there can be no zero-sequence load current in any of the windings under any condition even if it should exist in some unbalanced Δ-connected load, as it might. Or, it may be said that any current that may exist in either of the other deltas can only affect the current in the tertiary delta to a minor degree. That is, additional current in either delta will probably produce a small change in the terminal potentials of the tertiary windings and so alter the currents they may be delivering. Other than this the tertiary current is not affected. Indeed, as is well known, it is only when there is the possibility that the primary or secondary may carry zero-sequence currents that there can be produced any important modification of the tertiary currents, except, of course, as the load directly connected to the tertiary windings is changed.

2. Primaries in Δ, secondaries in Y with neutral grounded and with a current to ground of $3\,I_G$.

In this case the zero-sequence current in the secondary is I_G and from equation (5)

$$I_{10} + I_{30} = -I_G$$

Since the primaries are in Δ, $V_{10} = 0$ and we thus have

$$(r_1 + j\,x_{11} - j\,x_{13})\,I_{10} + j\,(x_{12} - x_{23})\,I_G - (r_3 + j\,x_{33} - j\,x_{13})\,I_{30} = 0$$

Eliminating I_{10} from three equations gives

$$I_{30} = -I_G \frac{Z_1}{Z_1 + Z_3}$$

For the meaning of Z_1 and Z_3 see equations (6,) (7), and (8) and the three following ones. That is, the total current to ground divides between the primary and tertiary as it would

2. Methods of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks. C. L. Fortescue, A. I. E. E., Transactions, 1918, Vol. XXXVII, p. 1027.

between two impedances Z_1 and Z_3 connected in parallel. There are also the positive- and negative-sequence currents. But they are equal and opposite in the primary and secondary just as if the tertiary were not present. The zero-sequence component in the secondary voltages is from (13)

$$V_{20} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1 + Z_3} I_{20}$$

For the meaning of these impedances see equations (6), (7) and (8) and the three following ones.

Another interesting point about this connection is that the third-harmonic component of the exciting current divides between the primary and tertiary but not in the same proportion as do the load currents. Since in this case the secondary can carry no third-harmonic exciting current,[3] it is without effect on its division between the primary and tertiary, and the impedances that are involved are the true leakages between these windings. The ratio of the third-harmonic components is:

$$\frac{I_3}{I_1} = \frac{r_1 + j\,3\,(x_{11} - x_{13})}{r_3 + j\,3\,(x_{33} - x_{13})}$$

In the first case, however, in which all of the windings are in Δ, the third-harmonic components of the exciting current will divide between all three windings inversely as their third-harmonic impedances. That is, the third-harmonic components divide as:

$$\frac{I_1}{r_1 + j\,3\,(x_{11} - x_{13} + x_{23} - x_{12})} = \frac{I_2}{r_2 + j\,3\,(x_{22} - x_{23} - x_{12} + x_{13})} = \frac{I_3}{r_3 + j\,3\,(x_{33} - x_{13} - x_{23} + x_{12})}$$

3. Primaries in Y, secondaries in Y with neutral grounded.

In this case the sum of the primary currents must be zero and I_{10} is thus equal to zero. From this I_{30} equals $-I_{20}$ which is one-third of the current to ground. Again there are also positive- and negative-sequence currents of the same magnitude which are equal and opposite in the primaries and secondaries but which do not exist in the tertiaries. In this case grounding the generator neutral has no effect on the current distribution since the generator is otherwise insulated from the secondaries. The displacement of the primary neutral, viz. V_{10}, is from equation (12)

$$V_{10} = I_{20} Z_3$$

The displacement of the secondary neutral is

$$V_{20} = I_{20} (Z_2 + Z_3)$$

4. Primaries in Y with neutral grounded; secondaries in Y with neutral grounded; generator neutral also grounded.

If there is no impedance between the generator and the primaries and the generator is balanced, the zero-sequence primary voltage is zero. That is $V_{10} = 0$. Thus we have: (see equation (12))

$$(r_1 + j\,x_{11} - j\,x_{13})\,I_{10} + j\,(x_{12} - x_{23})\,I_{20} - (r_3 + j\,x_{33} - j\,x_{13})\,I_{30} = 0$$

also

$$I_{10} + I_{20} + I_{30} = 0$$

But $I_{20} = I_G$ where $3\,I_G$ is the current to ground on the secondary side. Solving these relationships gives, as in case 2,

$$I_{30} = -I_G \frac{Z_1}{Z_1 + Z_3}$$

The displacement of the secondary neutral or the zero-sequence component of the secondary voltages is the same as in case 2 if there is no impedance between the generator and the primaries.

If there are equal line impedances of Z between the generator and the primaries and an impedance of Z_n between their neutrals the current in the tertiary is

$$I_{30} = -I_G \frac{Z_1 + Z + Z_n}{Z_1 + Z + Z_n + Z_3}$$

The general effect of this is to increase the tertiary current. If there is any unsymmetry in the lines between the generator and the primaries the solution is somewhat more difficult involving as it does the positive- and negative-sequence currents. For in this case a positive sequence load current will give zero-sequence line drop due to the negative-sequence component of the line impedance. Grounding the neutral of a Y-connected system has of itself no effect in determining whether or not there is a zero-sequence component in the Y voltages, for it is possible even with the neutral grounded that the sum of the Y-voltages should not be zero.

There are several interesting cases involving auto-transformers,[4] of which we will consider three. The fundamental equations are now in slightly different form. Refer to Fig. 6 herewith. All of the resistances and reactances will be given on the basis of a transformer which has the same number of turns in windings (1), (2) and (3). The voltages in the individual windings are given in equations (2), (3) and (4). The primary potential, V_{1n}, is however V_1 plus V_2 and is

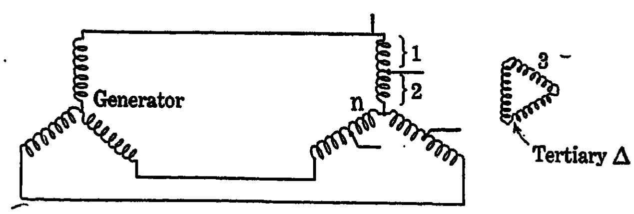

FIG. 6

$$V_{1n} = I_1 (r_1 + j\,x_{11} + j\,x_{12}) + I_2 (r_2 + j\,x_{22} + j\,x_{12}) + I_3 (j\,x_{13} + j\,x_{23}) + 2\,E$$

Eliminating E as before gives

$$V_{n0} = I_{10}(r_1 + j\,x_{11} + j\,x_{12} - j\,2\,x_{13}) + I_{20}(r_2 + j\,x_{22} + j\,x_{12} - j\,2\,x_{23}) - I_{30}\,(2\,r_3 + j\,2\,x_{33} - j\,x_{13} - j\,x_{23})$$

Substituting the relation that $I_{20} = -I_{10} - I_{30}$ we have

$$V_{n0} = I_{10}\,(Z_1 - Z_2) - I_{30}\,(Z_2 + 2\,Z_3)$$

If both the generator and transformers are grounded and there is no impedance between the generator and the transformers and the generator voltages are balanced, V_{n0}, equals zero, and

$$I_{30} = I_{10} \frac{(Z_1 - Z_2)}{Z_2 + 2\,Z_3}$$

Thus if the tertiary is symmetrically located with respect to both of the other windings of the auto-transformer and they themselves are symmetrically located with respect to the iron core the tertiary will carry no current since in this case Z_1 and Z_2 are equal. If, however, the generator is unbalanced or there is impedance between it and the transformer, the tertiary will carry current even though the windings are symmetrically located as described.

In case the generator is not grounded the zero-sequence current I_{10} is zero and I_{30} equals $-I_{20}$, that is, it is one-third the current flowing into the neutral of the transformers. The displacement of the transformer neutral in this case is

$$V_{n0} = I_{20}\,(Z_2 + 2\,Z_3)$$

If the generator is grounded but the neutral of the transformers

3. The secondary terminals are assumed to be free.

4. Performance of Auto-Transformers with Tertiary Windings under Short-Circuit Conditions. J. Mini, Jr., J. L. Moore, R. Wilkins. A.I. E.E. TRANSACTIONS 1923, page 1060.

is not grounded, the zero-sequence current in the tertiary must be equal and opposite to the zero-sequence current in the primary since $I_{20} = 0$. That is, it is one-third of the current to ground either at the generator or load. The displacement of the transformer neutral is now

$$V_{n0} = I_{10} (Z_1 + 2 Z_3)$$

The performance of the four-circuit and, in general, of the n-circuit transformer can be calculated by the same methods. When the transformer has more than three circuits little, if anything, is gained by attempting to represent it by an equivalent net work.

Using the same nomenclature as before we may write:

$$V_1 = I_1 (r_1 + j x_{11}) + j x_{12} I_2 + j x_{13} I_3 + j x_{14} I_4 + E \tag{14}$$

$$V_2 = I_2 (r_2 + j x_{22}) + j x_{12} I_1 + j x_{23} I_3 + j x_{24} I_4 + E \tag{15}$$

$$V_3 = I_3 (r_3 + j x_{33}) + j x_{13} I_1 + j x_{23} I_2 + j x_{34} I_4 + E \tag{16}$$

$$V_4 = I_4 (r_4 + j x_{44}) + j x_{14} I_2 + j x_{24} I_2 + j x_{34} I_3 + E \tag{17}$$

Also

$$I_1 + I_2 + I_3 + I_4 = 0 \tag{18}$$

These five fundamental equations may be handled by a variety of ways. Since there is some advantage in having them in symmetrical form we will use the following method of elimination. First eliminate E by taking successive differences. Then eliminate I_4 from the first difference, I_1 from the second difference, and I_2 from the third difference. These three equations together with equation (18) are sufficient to determine the currents in any case. If it is of any advantage, the loads on any of the windings may be replaced by their equivalent impedances, in which case the terminal voltages of these circuits will of course be taken as zero. We thus have:

$$V_1 - V_2 - I_1 [r_1 + j (x_{11} - x_{12} - x_{14} + x_{24})]$$
$$- I_2 [r_2 + j (x_{22} - x_{12} + x_{14} - x_{24})] + j (x_{13} - x_{23} + x_{24} - x_{14}) I_3 \tag{19}$$

$$V_2 - V_3 = I_2 [r_2 + j(x_{22} - x_{23} - x_{12} + x_{13})]$$
$$- I_3 [r_3 + j (x_{33} - x_{23} + x_{12} - x_{13})] + j(x_{13} - x_{34} + x_{24} - x_{12}) I_4 \tag{20}$$

$$V_3 - V_4 = I_3 [r_3 + j(x_{33} - x_{34} - x_{23} + x_{24})]$$
$$- I_4 [r_4 + j (x_{44} - x_{34} + x_{23} - x_{24})] + j (x_{24} - x_{14} + x_{13} - x_{23}) I_1 \tag{21}$$

and the relation that

$$I_1 + I_2 + I_3 + I_4 = 0 \tag{18}$$

An examination of these equations shows some interesting facts in regard to the impedances. In the first equation $r_1 + j (x_1 - x_{12} - x_{14} + x_{24})$ is the impedance that would be assigned to the first winding if the first, second and fourth windings were considered as a three-circuit transformer. We will represent this by Z_{124}. The impedance $r_2 + j (x_{22} - x_{12} - x_{14} - x_{24})$ is that which would be assigned to the second winding if the first, second and fourth were considered as a three-circuit transformer. We will represent this by Z_{214}. The first subscript shows to which winding the impedance is attached. The second and third subscripts indicate which of the other windings are grouped with the first to form a three-circuit transformer. The order of the second and third subscripts is unimportant; that is, there is no difference between Z_{124} and Z_{142}. It will also be noticed that the coefficient of I_3 in the equation (19) is $Z_{213} - Z_{214}$, that the coefficient of I_4 in equation (20) is $Z_{342} - Z_{312}$, and that the coefficient of I_1 in equation (21) is $Z_{413} - Z_{423}$. Thus the equations of differences may be written as follows:

$$V_1 - V_2 = I_1 Z_{124} - I_2 Z_{214} + I_3 (Z_{213} - Z_{214}) \tag{22}$$

$$V_2 - V_3 = I_2 Z_{231} - I_3 Z_{321} + I_4 (Z_{342} - Z_{312}) \tag{23}$$

$$V_3 - V_4 = I_3 Z_{342} - I_4 Z_{432} + I_1 (Z_{413} - Z_{423}) \tag{24}$$

There are some other interesting relations between these impedances.

For example,

$$Z_{124} - Z_{123} = - Z_{214} + Z_{213}$$
$$Z_{134} - Z_{132} = - Z_{314} + Z_{312}$$
$$Z_{421} - Z_{423} = - Z_{241} + Z_{243}$$

The values of these impedances can be determined by measurement if voltage is applied to one winding and *one* of the other windings is short-circuited. If, for example, Z_{12} is the leakage impedance of the first and second windings when neither of the others carry current, then it follows[3] that:

$$Z_{123} = \frac{Z_{12} + Z_{13} - Z_{23}}{2}$$

O. G. C. Dahl: I first want to discuss the paper by Mr. Boyajian. Mr. Boyajian states that the separate leakage reactance of one winding of a transformer is not a definite quantity. It seems to me, however, that before we make any attempt to determine whether leakage reactance is an explicit quantity or not, we ought to have an accepted definition of what we mean by the term leakage reactance. Leakage reactance of one winding with respect to another ought, in my opinion, to be the reactance caused by the part of the flux set up fully or partly in air by a current in the first winding which does not produce any linkages whatever with the second winding. I believe that on the basis of this definition, the separate leakage reactance of a winding is a perfectly definite quantity and does not in any manner depend upon the connections or the load conditions.

Of course, this reactance is not the reactance which should be assigned to a winding in a three- or four-circuit transformer when more than two of the windings carry current. The reactance of a winding in this case is a leakage reactance in the sense that it is caused by fluxes which do not exist exclusively in the core. These fluxes, however, are not entirely leakage fluxes according to the definition given above. They are partly mutual fluxes, and give rise to mutual reactances ("mutual leakage reactances") which enter as a part of the total or effective reactances. This idea, I think, is very clearly brought out in the discussion by Prof. Lyon.

Mr. Boyajian makes use of the classical representation of the two-circuit transformer and states that the splitting of the reactance into primary and secondary has no meaning unless the circuit representing excitation is considered. It should be noted that such a three-circuit network is not an *exact* representation of a two-circuit transformer, although it represents the transformer very closely.

Let us assume then that Fig. 3 in Mr. Boyajian's paper represents a two-circuit transformer. According to the classical theory the voltage across the excitation impedance (Z_3) should equal the voltage induced in the windings by the flux which exclusively exists in the core (usually called the mutual flux). If this is assumed and the network is analyzed on the three-circuit basis, it can be shown that the presence of the fictitious excitation circuit does not affect the distribution of reactance between primary and secondary. Considering the currents positive when flowing toward the common point, the equations for a three-circuit transformer are:

$$V_1 = (r_1 + j x_{11}) I_1 + j x_{12} I_2 + j x_{13} I_3 + E \tag{1}$$

$$V_2 = (r_2 + j x_{22}) I_2 + j x_{12} I_1 + j x_{23} I_3 + E \tag{2}$$

$$V_3 = (r_3 + j x_{33}) I_3 + j x_{13} I_1 + j x_{23} I_2 + E \tag{3}$$

$$I_1 + I_2 + I_3 = 0 \tag{4}$$

The reader is referred to Prof. Lyon's discussion for a complete explanation of the symbols. Equation (4) signifies that the exciting current is neglected in the three-circuit transformer, while for the two-circuit transformer it simply indicates that the sum of the primary and secondary currents equals the current required for excitation. When these equations apply to the two-circuit transformer, evidently $V_3 = 0$. Introducing this in equation (3), the following expressions may be obtained from the equations above:

$$V_1 = Z_1 I_1 - Z_3 I_3 = [r_1 + j (x_{11} - x_{13} + x_{23} - x_{12})] I_1$$
$$- [r_3 + j (x_{33} - x_{13} + x_{12} - x_{23})] I_3 \tag{5}$$

$$V_2 = Z_2 I_2 - Z_3 I_3 = [r_2 + j (x_{22} - x_{23} + x_{13} - x_{12})] I_2$$

$$- [r_3 + j (x_{33} - x_{23} + x_{12} - x_{13})] I_3 \quad (6)$$

which may be written

$$V_1 = Z_3 (I_1 + I_2) + Z_1 I_1 \quad (7)$$

$$V_2 = Z_3 (I_1 + I_2) + Z_2 I_2 \quad (8)$$

The term $Z_3 (I_1 + I_2)$ represents the voltage across the excitation circuit and shall be equal to the voltage (Ec) induced in the windings of the *two-circuit* transformer by the flux exclusively existing in the core. This voltage (Ec) is not the same as the voltage (E) induced in the windings of the three-circuit transformer, and the two should not be confused.

Equations (7) and (8) may now be written

$$V_1 = E\mathrm{c} + Z_1 I_1 = E\mathrm{c} + [R_1 + j (x_{11} - x_{12} + x_{23} - x_{13})] I_1 \quad (9)$$

$$V_2 = E\mathrm{c} + Z_2 I_2 = E\mathrm{c} + [R_2 + j (x_{22} - x_{12} + x_{13} - x_{23})] I_2 \quad (10)$$

These equations must be equivalent to equations (9) and (10) in my paper on *Separate Leakage Reactance of Transformer Windings*, namely,

$$V_1 = E_1\mathrm{c} + (R_1 + j x_{11}) I_1 + j x_{12} I_2 \quad (11)$$

$$V_2 = E_2\mathrm{c} + (R_2 + j x_{22}) I_2 + j x_{21} I_1 \quad (12)$$

From equations (9), (10), (11) and (12) we obtain

$$(x_{23} - x_{13}) (I_1 + I_2) = 0 \quad (13)$$

Since $I_1 + I_2$ is different from zero, being equal to the exciting current I_3, $x_{23} - x_{13}$ must be equal to zero. In other words we have $x_{13} = x_{23}$, which means that the mutual reactance ("mutual leakage reactance") between the fictitious excitation winding and the primary is the same as between the fictitious winding and the secondary, a result which appears entirely logical. By introducing this equality in the composite or effective reactances of the primary and secondary windings, we find that these reduce to the true leakage reactances given by my original definition. Hence, even in a two-circuit transformer the distribution of leakage reactance between primary and secondary is definite and is independent of whether or not a third circuit representing excitation is considered.

It is not improbable that the same idea might be extended to the three-circuit transformer which, when excitation is considered, represents a four-circuit problem. Also in this case it might be found that the presence of an excitation circuit does not affect the reactances to be assigned to the three actual windings. Of course, the reactances would still be *composite* reactances, the same as would be obtained for the three-circuit transformer when excitation is neglected.

Mr. Boyajian described four tests for experimental separation of the leakage reactances in a single, two-winding transformer. Theoretically these tests all appear valid, but no doubt when applied in practise the first two will give rise to considerable inaccuracies. This is mainly due to the presence of harmonics. It will, presumably, be preferable to use the fundamental component of the voltages and currents in question. Since these fundamental quantities in each case are associated with higher harmonics of considerable magnitude, it will be necessary to take oscillograms and to separate out the fundamental by analysis. Since, as mentioned, the order of magnitude of the harmonics, particularly the third, being large as compared to the fundamental components, this separation is difficult to perform with sufficient accuracy.

These tests were tried in our Research Laboratory. However, they did not yield good results at all for the reason mentioned above, although they were all performed on 1 : 1 ratio transformers. It seems to me, therefore, that the only way in which sufficient accuracy may be obtained is by making use of such connections as will separate out currents and voltages of one frequency. This is accomplished in tests No. 3 and No. 4 where the voltages and currents theoretically should be of impressed frequency. These tests, therefore, seem entirely rational when applied to transformers of unity ratio of transformation. When used with other ratios of transformation in connection with instrument transformers, the results may easily be subject to errors, as also pointed out by Mr. Boyajian. These tests will also fail to take into account the effect of saturation on the leakage reactance, if any. Personally, I think this effect is very small.

The three-phase, third-harmonic test seems to me to be the one to which the fewest objections may be raised. It determines the leakage reactances at normal (or any desired) saturation, and instrument transformers are not likely to affect the results seriously since their secondaries are connected directly to indicating meters. All doubt in regard to the calibration of the instrument transformers is thus eliminated. Of course, due to unbalance and other causes, it may be impracticable to obtain an entirely pure wave; but in any event the quantities of triple frequency which it is desired to measure will be entirely predominant and hence correct determination is highly facilitated even though oscillogram analysis may be necessary.

Mr. Boyajian's tests (1) and (2) make provisions for taking the effect of the exciting circuit into account, while his tests (3) and (4) eliminate any effect of the exciting current, because in each case the connections are such as to suppress the flux exclusively existing in the core. Since Mr. Boyajian is of the opinion that the exciting circuit affects the distribution of leakage reactance between primary and secondary, the two former tests should be expected to give results which differ from those obtained by the two latter tests. Mr. Boyajian does not mention this point at all, which to the writer seems quite important if it is assumed that the exciting circuit affects the distribution. Since, as shown, this is not the case, it would not make any difference in practise which test was employed as far as this particular point is concerned.

In connection with test No. 5 in which the Y-Δ-Δ connection is used, Mr. Boyajian presents a table giving the distribution of third-harmonic circulating current between the two closed deltas. His figures show that the distribution of this current, although to a comparatively small extent, depends upon the flux density in the core. Experiments performed in our Research Laboratory show less dependency on flux density. The table also indicates that the distribution of third-harmonic current between the two deltas to a certain extent depends on the reactance between the Y-connected winding and the two delta windings. There does not seem to be any reason why this reactance should enter into the problem at all. I am inclined to attribute the change in distribution to unbalance of the transformers rather than to any other cause. Even if the transformers are very well balanced and the ratio of transformation is uniform, it is practically impossible to avoid a small amount of fundamental in addition to the third harmonic in the closed delta circuits. According to Mr. Boyajian's sketch, Fig. 9, the third-harmonic currents were measured by inserting meters only in the closed windings. It is hardly safe to assume that the currents measured were of purely triple frequency unless this was checked by oscillograph.

Next, I want to say a few words in regard to Mr. Peters' discussion. Mr. Peters accuses me of saying that when the triple-frequency voltage, which appears when the third-harmonic current is fully suppressed, is divided by the triple-frequency leakage impedance, the triple-frequency current is obtained. I am surprised indeed that any of my statements can be interpreted in such a manner. I fully agree with Mr. Peters that the relation between the triple-frequency "open-circuit voltage" and the triple-frequency current does not follow Ohm's law. The relation, however, between the triple-frequency voltage, which is induced in the winding by the triple-frequency flux actually existing in the core, and the triple-frequency current, does follow Ohm's law.

It is well known that the third-harmonic voltage depends upon the third-harmonic current which flows. I briefly mentioned this in my paper on *Transformer Harmonics and Their Distribution* but did not discuss the mechanism of the interaction in detail. I stated, however, that it was my intention to do so in a

future paper. Since this particular question, however, is brought up, I would like to say that the effect of the third-harmonic current on the third-harmonic flux and corresponding voltage may very illustratively be compared to the effect of the armature reaction in a synchronous alternator. In the latter, the excitation voltage will exist between the terminals if the machine carries no load. As soon as load is applied to the machine, however, the voltage which is induced in the winding is changed due to the effect of armature reaction, and the excitation voltage becomes entirely fictitious. If the actually induced voltage is divided by the sum of the leakage impedance and the external impedance of the machine, we obtain the current which flows in the circuit. If the synchronous impedance corresponding to the particular flux density is known, then the same current may be obtained by dividing the fictitious excitation voltage by the sum of the synchronous impedance and the external impedance. What I have done for the transformer is equivalent to the former operation. If it were possible to ascertain the value of the transformer impedance which would correspond to the synchronous impedance in the rotating machine, then the triple-frequency current could be obtained by dividing the open-circuit third-harmonic voltage by the sum of this impedance and the external impedance, if any. Of course, the internal impedance to be used in this case would depend upon and vary with the flux density, and hence with the triple-frequency current. There is a possibility that an impedance of this sort may be determined on an empirical basis, and I have looked into that question to quite an extent. However, the work has not as yet proceeded far enough, and I am not ready at this time to make a definite statement.

I agree with Mr. Peters that it was unfortunate that the ammeter had to be shifted from one of the closed deltas to the other in the Y-Δ-Δ test for determination of third-harmonic current division. I called attention to this fact in the paper. It should be noted, however, that while the meter was changed the oscillograph shunts and vibrators, which constituted the main part of the resistance in the corners of the two deltas, were never short-circuited, but always left in the circuits even when the ammeter was removed. The resistance of the shunted vacuum thermocouple ammeter was 0.3 ohm when oscillograms Nos. 9, 10 and 13 were recorded, and 0.06 ohm when Nos. 11 and 12 were taken. Hence, the resistance of the measuring instrument was in all cases far from being of the order of magnitude of the total impedance of the winding, as Mr. Peters states. In the most unfavorable cases (oscillograms Nos. 10 and 11) the effect of a change in resistance of 0.3 ohm on the magnitude of the total impedance of the low-impedance delta winding is about 10 per cent.

I cannot give any definite reason for the discrepancy between the values of leakage impedance of winding No. 3 as determined by the two-winding and three-winding methods. As Mr. Peters points out, the one found by the latter ought to have been the smaller, since it was taken with respect to winding No. 2 instead of with respect to No. 3. I do not think, however, that the difference would have to be as large as 25 per cent, even though the distance between the coils is reduced by approximately this amount. Such a direct relation between spacing and leakage reactance would exist only with a very ideal distribution of leakage flux of uniform density, which does not obtain in an actual transformer.

The discrepancy may be due to the effect of unbalance or to inaccuracies in determining the reactances by the two-winding method, or both. In the two-winding tests the third-harmonic voltages which had to be measured, particularly at the lower densities, were very small and this fact may have affected the precision of the measurements.

I fully agree in the remarks made by Mr. McCurdy. The matter of whether a leakage reactance is definite depends upon definition, a point which I have tried to amplify in my discussion of Mr. Boyajian's paper. Of course, as Mr. McCurdy also states, the leakage impedance is not the impedance which should be assigned to a winding in all cases. It depends upon connections used and the number of circuits carrying current.

Prof. Lyon's discussion of applications to three- and four-circuit transformers is very interesting and illuminating. It brings out very clearly the difference between the leakage reactance of one winding with respect to some other winding, and the reactance which has to be assigned to the same winding in case three or more windings carry current of the same frequency.

Aram Boyajian: Professors Karapetoff, Lyon, and Dahl and Mr. Peters seem to be in agreement with the speaker on the main points of the paper. Thus Prof. Karapetoff says, "In a transformer with more than two secondaries, the indefiniteness of the division of the leakage fluxes among the individual windings lies in the very nature of the phenomenon." Prof. Lyon offers a number of equations because, he says, "it makes clearer some points that Mr. Boyajian brings up in his recent paper on the theory of such transformers." Prof. Dahl is no less explicit when he says (following his equation **11**), "The relative aspect of the leakage reactances should be carefully noted. The leakage reactance of a winding is not a quantity which is dependent upon and characteristic of that winding alone; it must, of necessity, be defined with respect to some other winding. This fact becomes particularly important in multi-winding transformers. Thus, in a transformer having three windings designated Nos. 1, 2 and 3, the leakage reactance of winding No. 1 with respect to winding No. 2 will be in general different from the leakage reactance of the same winding with respect to winding No. 3. . . "

Mr. Ferris' attitude is, I think, entirely reasonable. Without generalizing and dogmatizing beyond his immediate problem, he states that in dealing with third-harmonic troubles it would be very desirable to formulate some sort of individual leakage reactances which could be used in the computation of residuals and third-harmonic line currents. This probably can be done to some tolerable extent, subject of course to variation with changing degree of saturation. More investigation would be desirable in this direction.

Mr. McCurdy claims that the speaker "is in error in stating that the difficulties which have arisen in dividing leakage impedances between windings are due to any lack of definiteness of the problem." What I have claimed was that (a) a single resolution of universal application is impossible, not merely difficult, and (b), that resolutions for particular applications are easy, describing half-a-dozen methods for exciting-current applications. Mr. McCurdy's interest in the subject being confined exclusively to a very definite particular problem, *viz.*, third-harmonic telephone interference, *his* difficulty was not due to the indefiniteness of the general problem, even though the general problem is indefinite. His difficulty apparently lay in the belief that the third-harmonic method is the only possible method of resolution for exciting-current applications.

Ordinarily, we speak of two kinds of transformer reactance: (1) open-circuit or magnetizing reactance, (2) short-circuit or leakage reactance. Thus, leakage reactance is the reaction of the transformer to the load currents. Still, Mr. McCurdy wants to restrict the use of the term "individual leakage reactances" only to magnetizing-current applications by saying that "it leads to confusion to call such individual impedances of three-winding transformers 'individual leakage impedances'." If the impedance offered by a transformer winding to its load current may not be called its leakage impedance, I fail to see what else may be called leakage impedance. I have justified the use of the term "leakage impedance" in connection with exciting-current applications, only by considering the excitation kv-a. as a load in a fictitious auxiliary winding.

Relative to the third-harmonic tests mentioned by the author, Mr. McCurdy asks if the effect of meter impedances and extraneous harmonics were eliminated. So far as extraneous harmonics

are concerned, they were previously found to be absent by oscillographic records, and in these particular tests no oscillograms were taken. So far as meter impedances are concerned no corrections or eliminations were necessary. The test consisted in this: we had two independent windings A and B dividing between them the third-harmonic excitation of the transformer. We noted that with a certain voltage impressed on the transformer, A took more than half of the third-harmonic exciting current. Leaving all meter and connections absolutely unchanged, but increasing the voltage impressed on the transformer considerably, we observed that this time B took more than half of the third-harmonic current. This showed, regardless of meter impedances, that the division of the third-harmonic impedance between the two windings had changed in changing the impressed voltage of the transformer. The phenomenon is explainable qualitatively by a consideration of the return magnetic circuits of the two windings.

Mr. McCurdy admits that the various test methods which I have described are entirely feasible with one-to-one ratio transformers, but he overestimates the difficulties which instrument transformers would introduce, the difficulties which I have already mentioned in my paper. As against his opinion that "these methods do not have much practical importance" and that the third-harmonic method is the only way, we have Mr. Peters' opinion that "These methods are so simple and direct that chances of error are very remote." We also have Mr. Weller's statement that test methods No. 1 can be accurately reproduced, and that he has resolved experimentally the reactances of a number of potential transformers without the aid of third harmonics. Mr. Weller's results are the more significant in view of the fact that the resolution of the leakage reactances of potential transformers is much more difficult than that of power transformers.

I wish to emphasize here the inherent weakness in giving the third-harmonic method a privileged position in this resolution.

In the first place, we must recognize that third harmonics are not a necessary inherent and unavoidable accompaniment of transformer action such as resistance or reactance or magnetizing current, but they are purely incidental and even avoidable if one wishes to pay the price. Leakage reactance, on the other hand, is an inherent accompaniment of transformer action, and is primarily formulated for, and effective to, the normal-frequency currents regardless of the presence or absence of third-harmonic phenomena. Does it appear reasonable to imply that the normal-frequency performance characteristics of the leakage reactances of the various windings cannot be determined except through the aid of the non-essential incidental third harmonics? Let us assume an air-core transformer; it certainly will have a large magnetizing current and a large leakage reactance, but no third harmonics. Does the resolution of leakage reactance become indeterminate now on account of the absence of third harmonics? Or assume an iron-core transformer: are we to think that, when it is excited at such high densities as to generate considerable third harmonics, the windings show definite individual reactances to fundamental-frequency currents; and that, when the transformer is so underexcited as to yield inappreciable third harmonics, the windings will not show definite individual reactances? Or again we ask, is the effect of the division of leakage reactance upon the normal-frequency characteristics so insignificant that it cannot be detected or measured by normal-frequency phenomena? If so, the resolution would become largely a third-harmonic phenomenon indifferent to fundamental frequency, a conclusion which the third-harmonic champions would hardly admit. We should, I believe, admit that if the resolution does make a sensible difference to fundamental-frequency characteristics, then that difference ought to give us a line on the desired resolution.

Mr. Weller mentions the interesting case of ring-wound current transformers. These constitute the simplest case for theoretical analysis. In them, all leakage reactance belongs to the outer winding, subject to a small correction due to the thickness of the inner winding.

Mr. Weller and Mr. McCurdy are right in stating that the passing reference at the end of the paragraph numbered 2 in the text should be applied to a potential transformer, not to a current transformer.

Prof. Dahl, commenting on the classical equivalent-circuit diagram of a two-winding transformer as used in my paper, says that it is not an *exact* representation, although he is willing to tolerate it as an approximation. One would be led to infer from such a comment that Prof. Dahl's equations do not imply such a diagram and that the leakage reactances which he has formulated, and tried to determine, are not the leakage reactances shown in this diagram. As a matter of fact, this is exactly the diagram implied by the classical equations which Prof. Dahl has used, and the leakage impedances shown therein are the very impedances which he is trying to determine. The impedance links of the equivalent circuit are somewhat variable depending on the saturation of the core exactly as the self and mutual reactances used in his equations are variable with density.

This case is probably illustrative of other seeming differences between the two papers.

Prof. Dahl offers a definition of leakage reactance in the belief that by it individual reactance will become a single-valued quantity incapable of more than one interpretation. Thus, the leakage reactance of a winding is defined as the "reactance caused . . . by a current in the first winding which does not produce any leakages whatever with the second winding." Referring to the clause "caused by a current," we ask, caused by which current? Any current and every current? Or is it some particular kind of current such as exciting current or load current? If it is the latter, we then ask, which load current, the one occasioned by the load in winding X or that due to the load in winding Y? The burden of my paper was to show that the leakage reactance which a winding offers to an exciting current is different from that which it offers to a load current, similar to the fact that the total reactance which a transformer offers to exciting current is different from the total reactance which it offers to load currents, and that if the load can be applied at more than one point, the leakage reactance (for instance of the primary) is different for each different location of the load and of the secondary with respect to which the primary leakage is being considered. Thus, even the definition which Prof. Dahl offers does not necessarily make the general resolution a single-valued operation.

Prof. Dahl, like Mr. McCurdy, seems to wish to restrict the individual leakage reactances to currents considered as exciting currents. Such restriction, however, fails to reckon with the historical fact that the term "leakage reactance" is something that has always been applied exclusively to the reactance of a machine for its load current, not only in the case of transformers but also in the case of induction motors and synchronous machines. What linguistic or historical justification can there be now in trying to restrict the leakage reactances of windings to exciting current? In my treatment, the exciting current comes in as a particular kind of load, a load in an auxiliary fictitious winding, and thus takes its place alongside of other loads, coordinate with them and entitled to no unique privileges. The problem then becomes that of a three-winding or multi-winding transformer.

I am fully aware that, in contrast to the foregoing system of considering all currents as load currents, it is possible to construct an alternative system in which all currents including load currents are considered as magnetizing currents, and are thus given the same mathematical treatment as the no-load magnetizing current. The initial steps in this latter method are quite elementary, and, therefore, the first impulse of nearly everybody on first approaching the subject of three-winding transformers is to try to attack it by beginning with the familiar equations,

$$E_1 = I_1 X_1 + I_2 X_{12} + \ldots$$
$$E_2 = I_2 X_2 + I_1 X_{12} + \ldots$$

where the voltages are total voltages and the currents are the total currents in the corresponding windings, and X_1, X_2, etc., X_{12}, X_{13}, etc., are the total (magnetizing) self and mutual reactances respectively. This latter method, however, leads to great complexity and an unnecessarily burdensome system of equations, whereas, the consideration of the load currents as load currents and the reactances as leakage reactances leads to wonderful simplification of equations, even though at first it may be a little difficult to grasp the point of view. It is certain that any system dealing with multi-winding transformers and starting with total currents and magnetizing self and mutual reactances must at some stage convert into load currents and leakage reactances, to be of any practical use. All this can be accomplished directly and with very little labor and mathematics once the comprehensive physical point of view is appreciated. The work done by Mr. Peters and the writer on the theory of three-winding transformers, as well as references in American and European technical literature that have come to my attention, seem to confirm this conclusion. Judging from very old treatises on transformers, the pioneers in transformer theory used to calculate load characteristics by the aid of self inductances and mutual inductances. Later it was discovered that the idea of leakage or short-circuit reactance introduces a wonderful simplification and precision into calculations by eliminating the need of any reference to the awkward magnetizing self and mutual inductances. It appears to me, therefore, that in dealing with load-current problems, to use magnetizing self- and mutual-reactance conceptions instead of leakage-reactance conceptions, is somewhat like using the instantaneous values of currents instead of their effective values. The question here is not which is correct (because both are correct), but which is more comprehensive? Which is more practical? Which utilizes to a greater extent the accumulated technical experience of the profession? Wouldn't we handicap ourselves unnecessarily if in every problem we should start with Maxwell's fundamental equations?

The conclusion which Prof. Dahl draws from the 13 equations in his discussion is that the reactances of the three branches of the equivalent circuit are definite quantities. This conclusion hardly needs a proof, but only the caution that this resolution refers to the exciting current because the various impedances used in the equations are the magnetizing impedances. Prof. Dahl further wishes to conclude that the resolution is independent of whether or not a fictitious excitation winding is assumed. However, whether or not a fictitious excitation-load circuit representing the kv-a. loss in the iron is expressly assumed, one is implied. For instance, if Z_1 is given as the self-inductive impedance of winding No. 1, it represents the leakage impedance between winding No. 1 and the excitation-load winding which we have postulated as hidden in the core to represent the excitation loss (kv-a., not only kw.) in the iron. Although I have called this a fictitious circuit, it is not altogether fictitious, because the core does constitute many little circuits and these may be represented by a single equivalent winding. The position and shape of this equivalent winding may not be arbitrarily assumed. It has to be such as to agree with the characteristics of each particular core.

Referring to the test methods described in my paper, Prof. Dahl states that some of them were tried out with one-to-one ratio transformers and found unsatisfactory due to the influence of the harmonics of the exciting current. It is very unfortunate that Prof. Dahl does not given any data at all, neither does he state just why the tests were considered unsatisfactory. It is possible that these tests were made after the publication of these papers and the data have not as yet been put into suitable shape for presentation. It is also possible that these tests at fundamental frequency did not check the previous third-harmonic tests and were therefore considered unsatisfactory. Now, so far as the effect of harmonics of exciting current on these tests is concerned, I think we can decide the matter rather definitely on theoretical grounds as follows:

The resolution of the normal-frequency leakage reactance by the aid of the third-harmonic test is based on the assumption that the same resolution holds at triple-frequency as at fundamental-frequency or at any other frequency. Now, if this assumption is true, that is, if all harmonics require the same resolution so that the ratio of the individual leakage reactances of the two windings is the same for all of the harmonics, then the division of the various harmonics of current between two windings ought to be the same whether one harmonic is tested or the other or whether their composite is tested, assuming that the meter impedances are not influencing the circuit conditions to any serious extent. Harmonics of exciting current could disturb the results only when the resolution of leakage reactance is different for different harmonics, and this I do not think Prof. Dahl would concede, because it would undermine the applicability of third-harmonic test results to fundamental-frequency phenomena. Furthermore, the resolution may be tested at various flux densities in the core, starting with very low densities to which harmonics are practically absent, and plotting a curve of resolution against flux density. This may then be compared instructively with the resolutions obtained by a single harmonic at the corresponding densities.

It is to be hoped that Prof. Dahl will investigate these various test methods thoroughly and that he will present his data to the Institute in another paper soon.

The curves presented by Mr. Palueff are interesting and have evidently been prepared with much care. It must be noted that he is dealing with the composite or resultant of two fluxes (produced by the magnetizing and load currents) and, therefore, the various voltages measured by the exploring coil are not directly indicative of individual reactances. Of course, so far as the individual leakage reactances (with respect to exciting current) of a transformer consisting of two circles in space are concerned, they can be very easily solved in accordance with equations (10) and (11) of my paper without having to plot any curve. It is unfortunate tht Mr. Palueff did not segregate the two components of fluxes and voltages.

Professor Karapetoff as an educator is rightfully somewhat concerned over the sense of uncertainty and indefiniteness that people may get if told of the relativity of the resolution of leakage reactance. It was attempted to guard against this opposite extreme by tabulating at the end of my paper a number of definite conclusions indicating specific applications and interpretations.

A New Two-Phase to Six-Phase Transformer

Connection of One Hundred Per cent Apparatus Economy

BY A. BOYAJIAN[1]

Member, A. I. E. E.

Synopsis.—A new transformer connection of 100 per cent apparatus economy is described for transformation from two-phase to six-phase and vice versa. Its merits from various standpoints are compared with those of the Scott and the Woodbridge connections.

Introduction

THIS paper describes a new two-phase to six-phase transformer connection which has an "apparatus economy" of one hundred per cent.

By apparatus economy is meant the ratio of the single-phase transformer kv-a. rating of the device to its kv-a. rating as a phase transformer. This ratio is sometimes called "internal power factor". This latter term, however, is not so good, because of the fact that, although at unity power factor loads the value of the internal power factor is identically the same as the apparatus economy, yet the internal power factor varies with the load power factor, while the apparatus economy is independent of the load power factor.

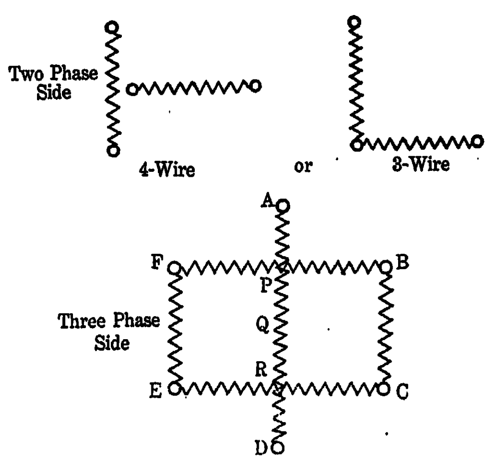

Fig. 1—Two-Phase to Diametric Three-Phase or Six-Phase Transformers

Three-phase side:

Voltages: *AD—BE—CF*—100 per cent

AP—PQ—QR—RD—25 per cent

BC—PR—FE—50 per cent

BP—PF—CR—RE—43.3 per cent

Currents in Lines—100 per cent

Current in *AP—PQ—QR—RD*—100 per cent

" " *BC—FE*—50 per cent

" " *BP—PF—CR—RE*—86.6 per cent

Description of the Connection

Fig. 1 shows the connection diagrammatically.

The core and flux are two-phase, so that two single-phase cores or one two-phase core would be necessary and sufficient.

Considering the windings, two windings are needed on the two-phase side, one for each phase. They may be interconnected (as *T*, *L*, or diametrically), or they may be entirely independent. This freedom is advantageous in being adaptable to any two-phase system.

On the six-phase side, we have a rectangle, *B C E F*, crossed by the line *A D*. The voltages of the various parts are shown in the illustration.

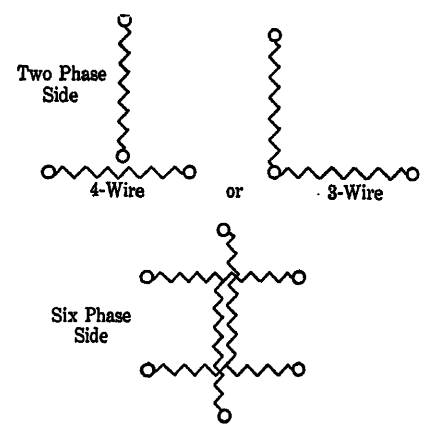

Fig. 2—Double-T or Double-Scott Connection

It may be noted that the six-phase side may be also considered a diametric three-phase system, since any diametric three-phase device may be operated from it.

New Connection Compared with the Double-T Connection

The common connection for two-phase to six-phase transformation is the so-called double-T or double-Scott connection shown in Fig. 2. While structurally there is a great deal of resemblance between this and the new connection, what little difference there is accounts for the difference in the apparatus economy of the two connections. The maximum apparatus economy which the Scott connection is capable of is 96.6 per cent (using non-interchangeable units and properly interlaced three-phase main windings) as against the 100 per cent apparatus economy of the new connection. Besides this difference in economy, the new connection is free from the trouble of the interlacing requirement for the two halves of the main winding in the Scott connection. This advantage is more appreciated if it is considered that six-phase circuits are usually of low voltage and high current, and proper interlacing of the two halves of the main is not a simple matter.

1. Of the General Electric Company, Pittsfield, Mass.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, N. Y., June 22-27, 1925.

It will be observed that the Scott connection can be converted into the new connection by redistributing the two teaser windings around the two main windings in accordance with Fig. 1.

A disadvantage of the new connection in comparison with the Scott connection is that the former is not adaptable for the use of interchangeable units. If a spare is needed, it must be a complete polyphase unit, while in Scott connection a single-phase spare may be sufficient. In this interchangeable scheme, however, the apparatus economy of the Scott connection is at best only 93.8 per cent.

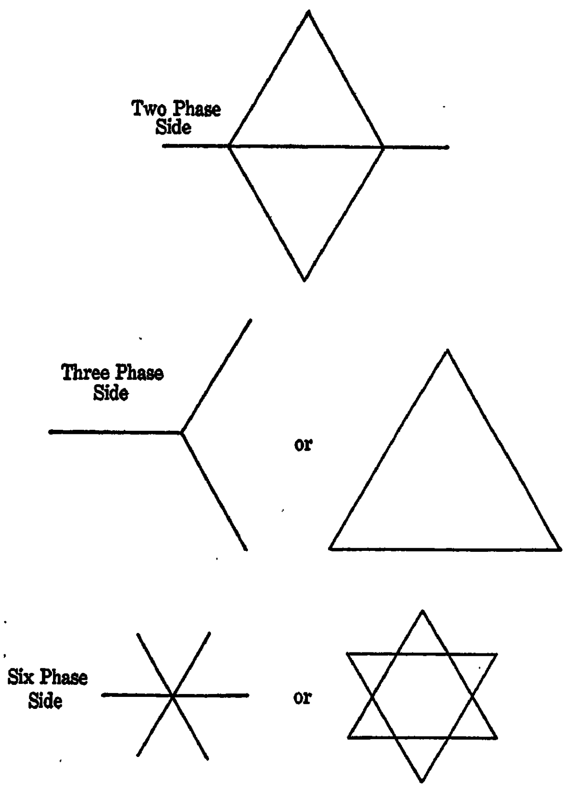

FIG. 3—THE WOODBRIDGE CONNECTION

NEW CONNECTION COMPARED WITH THE WOODBRIDGE CONNECTION

The Woodbridge connection is a 100 per cent economy connection shown in Fig. 3. Originally this connection was intended for two-phase to three-phase transformation, but its applicability to two-phase to six-phase transformation is obvious. The Woodbridge connection, although of high economy, is seldom used on account of three limitations: (1) being a 4-wire two-phase connection, it is not adaptable to 3-wire two-phase circuits; (2) the two-phase side is unsuited for taps; and (3) it is as complicated for two-phase to three-phase service as for two-phase to six-phase service, while in the matter of simplicity, adaptability and convenience for two-phase three-phase service the Scott connection is far ahead of any of the other many connections so far invented. The new connection here described is not adaptable at all to three-wire, three-phase service.

For six-phase service, the Scott, the Woodbridge and the new connections are of the same order of complexity in the matter of windings, but the latter two do not need the particular interlacing required by the Scott connection. Since taps are or would be placed on the two-phase side, in this service the Woodbridge connection is at a very serious disadvantage.

A point of great theoretical interest in this comparison is that both of the two 100 per cent economy connections are "diametric" in their derived phases, but free and flexible in their original. The Woodbridge connection uses three-phase flux, and is flexible in its three-phase side, its original side, being capable of either diametric or mesh connection. However, on its derived side,—viz., the two-phase side,—the connection is entirely limited to four-wire diametric connection. The new connection here described utilizes two-phase flux, and this being its original phase, it is capable of any two-phase connection. However, on its derived side, the three-phase side, it has to be diametric, that is, six-wire three-phase, and is therefore not adaptable to ordinary three-wire three-phase circuits.

Discussion

Vladimir Karapetoff: A general solution of two-phase to six-phase transformation is shown in Fig. 1 presented with this discussion. $O\,a$, $O\,b$, $O\,c$, $O\,d$, $O\,f$, $O\,g$, are six "star" voltages desired to be had on the six-phase side. As a simplest case,

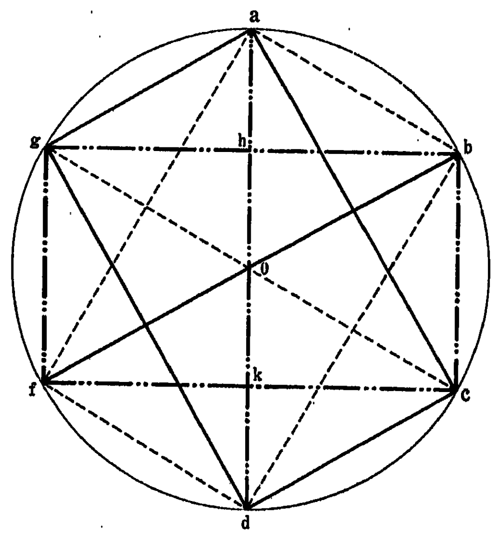

FIG. 1

imagine a six-phase synchronous motor adjusted to run at unity power factor and used as a load on a two-phase line. Then the foregoing six voltages may be considered, as those across the individual armature phases (star connected) of the motor. The problem of phase transformation consists in connecting the vertices of the hexagon by lines of two mutually perpendicular directions. Starting with point a, there are only three possible independent beginnings, viz., $a\,b$, $a\,c$, and $a\,d$. Beginning with $a\,b$, we have to draw $a\,b$, $g\,c$, $f\,d$, for phase I, and $a\,f$, $b\,d$, for phase II. This will give the figure drawn by dotted lines and

identical with that shown in Boyajian's Fig. 1.[1] Beginning with $a\ c$ gives again a Φ figure shown in full lines. Beginning with $a\ d$ gives an identical Φ figure drawn in "dash and dot." Thus, the Φ transformation is not one of several possible transformations, but the only possible *perfect* transformation.

By a perfect transformation I understand one in which only the vertices of the hexagon are used. With the double-T transformation, auxiliary points, h and k are necessary, and this causes reactive currents, increased kv-a. rating of the transformers, necessity for interlacing, etc. It will be seen from my Fig. 1 that no more lines interconnecting the vertices in a "perfect" way can be drawn; therefore, all possible solutions are included in this figure.

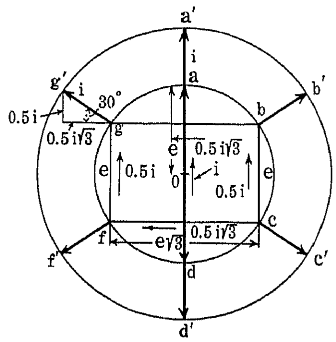

Fig. 2

For example, the six-pointed star shown in Boyajian's Fig. 3 may be readily seen in my Fig. 1. It may be of interest to point out that a sketch identical with Fig. 1 was included in the first edition of my "Experimental Electrical Engineering" published in 1909, and the general method of obtaining perfect phase transformations explained. However, I did not follow the matter any farther and did not realize that I had the Φ figure until I saw it in the paper under discussion.

The reason for which a perfect transformation gives 100 per kv-a. efficiency may be seen in Fig. 2. The six line currents flowing into the load are shown by the vectors $a\ a'$, $b\ b'$, etc. The six star voltages are in phase with these currents. Only two voltages, $O\ a$ and $O\ d$, are shown. Phase I of the transformer furnishes the currents $a\ a'$ and $d\ d'$ directly, and the kv-a. delivered is $2\,ie$. The current $g\ g'$ is furnished by both phases. Phase I supplies 0.51 and phase II supplies $0.5\ 1\ \sqrt{3}$. Thus the phase I furnishes altogether

$$2\,i\,e + 2\,(0.5\,i)\,e = 3\,i\,e.$$

Phase II furnishes

$$2 \times (0.51\sqrt{3}) \times (e\sqrt{3}) = 3\,i\,e.$$

In other words, both phases furnish equal kv-a., and all the currents in the transformer windings are in phase with the respective voltages. This means 100 per cent apparatus efficiency.

1. I suggest calling this kind of transformer connection the "Φ connection," because of some resemblance of the diagram to this Greek letter.

It will be seen that the Φ connection involves two "stub ends," a and d, and four "junction points," b, c, f, g, where two transformer windings are joined together. For a "perfect" transformation, the stub ends should be used only where the induced voltages in both systems are in phase with each other. At all other points, junctions should be used in order to produce a resultant current out of two components, each of which is in phase with its induced voltage for that particular transformer case.

It is shown in Fig. 3 herewith the usual T connection does not satisfy this condition, and for this reason its kv-a. efficiency is less than 100 per cent. Stub ends, a and b, are used in the phases in which the induced voltages differ by 30 deg. As a result, the three-phase currents $a\ a'$ and $b\ b'$ are out of phase with the induced voltages when the two-phase side is loaded at 100 per cent power factor. This causes unbalanced magnetomotive forces, increased magnetic leakage, and necessity for interlacing.

Aram Boyajian: Professor Karapetoff has proven for us that the transformer connection here described is the only possible two-phase-six-phase connection of 100 per cent apparatus economy utilizing two-phase fluxes and voltages. If his argument is true, as it appears to be, he saves us from futile effort at further invention along this line.

The connection here described originated in an effort to devise a two-phase-three-phase connection that would be free from the complications and limitations of the Woodbridge connection. Since the Woodbridge connection was based on three-phase flux and voltages, the solution of the problem was sought in the use of

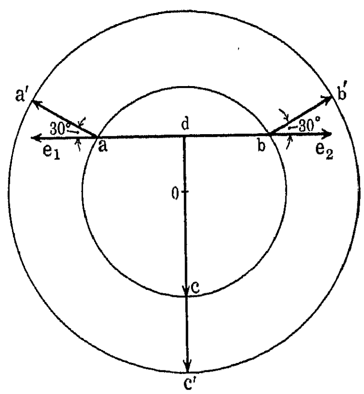

Fig. 3

two-phase flux and voltages. However, the three-phase system so derived turned out to be a diametric (that is, six-wire) three-phase system, unadaptable to three-wire three-phase systems, and was equivalent to a six-phase system. As stated in the paper, it appears to the author that this is inherent in the nature of things, that is, the derived system has to be diametric, such as four-wire two-phase, or six-wire three-phase. Maybe Professor Karapetoff can definitely prove this to us some time.

Losses in Iron Under the Action of Superposed Alternating- and Direct-Current Excitations

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Synopsis:—*The paper presents the results of wattmeter measurements of iron loss in a small experimental reactor designed for a-c. and d-c. excitations. All the resistances were measured by the d-c. voltmeter-ammeter method, and the $I^2 R$ losses were subtracted from the total wattmeter readings. A-c. core losses are plotted against d-c. excitation for various a-c. flux densities. These curves were checked qualitatively by means of hysteresis loops taken with a special bilateral oscillograph.*

All results show a comparatively small change in core loss as d-c. excitation is added, the core loss even showing a decrease when the a-c. saturation is high.

The core loss proper is distinguished from the double frequency circulating current copper loss, and means are given for decreasing this $I^2 R$ loss.

* * * * *

Introduction

In thesis work at Massachusetts Institute of Technology in 1924, the authors investigated the losses in iron under the action of superposed alternating- and direct-current excitations, and the results of this investigation are given herewith. This paper was prepared while the authors were connected with that Institute.

The question of superposed excitation is one of wide interest in electrical engineering as it occurs in one form or another in many electrical machines. Perhaps the best known example is the d-c. saturated regulator and reactor. Some rectifier circuits have transformers that are partially saturated with direct current, and the question of iron loss may be very important in connection with the proposed high-voltage, d-c. transmission. Practically all rotating machinery is subject to magnetization at two or more frequencies at the same time, especially in the teeth and pole tips. Telephone and telegraph circuits also carry both direct and alternating currents in many cases.

Previous investigators in this field have found, in the main, a very decided increase in a-c. iron loss with an increase in d-c. bias, sometimes amounting to fifteen or twenty times the usual core loss. Only a few reports show any decrease in the iron loss. In most of those which show excessive losses, the circulating double frequency current which appears in the d-c. circuit or in some other winding on the core when direct-current excitation is added, has either been neglected or there have been other possible errors in the method of measuring the core losses.

They all agree, however, that the classical formula of Steinmetz expressing hysteresis loss does not hold at high-flux densities. Ball and others have attempted to fit the Steinmetz equation to the case of superposed fluxes by changing the value of the exponent and using some value of B which is a function of both the a-c. and d-c. flux densities. Although the authors cannot, at the present time, propose an exact formula, they believe that the hysteresis loss actually begins to decrease at very high d-c. flux densities, and finally becomes zero when the total change in flux caused by the a-c. component takes place above the saturation point of the iron.

The eddy-current component of core loss depends upon the a-c. flux change and not upon the d-c. component, and if the a-c. flux wave remains sinusoidal, the eddy-current losses should not change with an increase of d-c. excitation. As a matter of fact, the wave forms do become somewhat distorted, and the eddy-current loss probably goes up somewhat when direct current is added.

The authors found that total core losses remain practically constant for a given a-c. flux density, regardless of the amount of d-c. flux in the iron, though some change did occur at high and low a-c. flux densities. The losses increased when d-c. excitation was added if the a-c. saturation was low, and decreased if high. The loss curves were taken from wattmeter readings and were checked by dynamic hysteresis loops taken with a bilateral oscillograph. A study of the accuracy of this oscillograph was also made and they believe the loops shown are substantially correct.

Method of Test

The method consisted of measuring the total input to an iron-core reactor by a low power factor wattmeter and subtracting the $I^2 R$ losses in the windings, leaving the difference as the core loss of the reactor. Two circuits were used. One had the direct current and alternating current in series in the same coil, forcing all of the $I^2 R$ loss into this one coil where it could be easily measured; the other had three coils, two carrying direct current and the third alternating current alone. The latter had the advantage of better wave form but still brought the second harmonic current into one winding where it could be measured.

The series circuit is shown in Fig. 1. A storage battery supplied the direct current and a series resist-

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ance regulated its value. Dynamometer ammeters were used to measure the current. A transformer kept the direct current out of the alternator windings. All voltage and frequency regulations were made with the field controls of the 5-kw. motor-generator set. The core under investigation weighed 20 lb. It was built up of 25-gage (0.022-in.) U. S. electric sheet with lap-joints. The outside dimensions of the core were 9 by 9 by 2 in. The cross-section of the middle leg was 2 by 2 in. while that of the rest of the core was 1 by 2 in. This gave 4 sq. in. for the a-c. flux path and 2 sq. in. for the d-c. flux when the three-coil circuit was used.

In taking readings, it was found necessary to measure the resistance at every point because of the effect of temperature changes while taking data. This was done

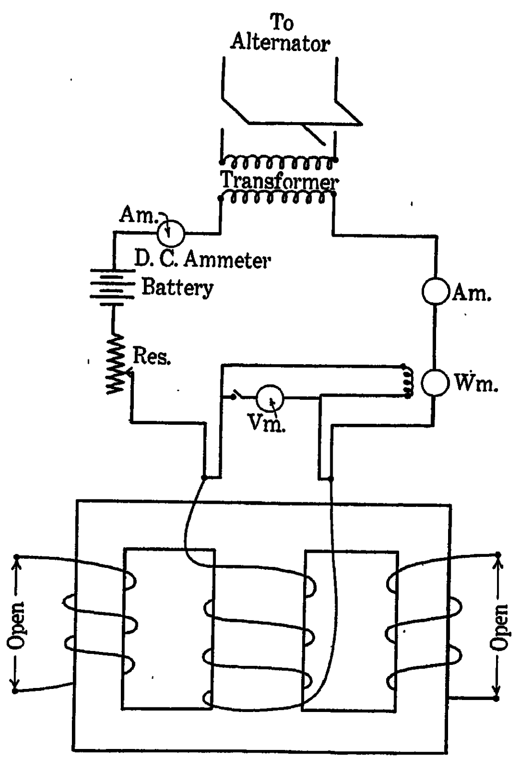

Fig. 1—Series Connection Showing Positions of Instruments

by the voltmeter-ammeter method using double-pole, double-throw switches to put the direct current alone in the windings. Carefully calibrated d-c. instruments were used, as the resistance had to be known with great accuracy in order to make the "subtraction method" at all reliable. With this arrangement, however, no difficulty was found in checking readings at different temperatures.

Fig. 2 shows a set of curves of core loss at various flux densities taken with the series circuit. The densities marked on the curves were figured from the applied voltage, and sine waves were assumed. If corrected for wave form and $I\,R$ drop, the values would be slightly lower than those indicated.

Since these curves did not agree at all with some similar curves taken at Massachusetts Institute of Technology in 1923, or with the results of other investigators, an attempt was made to discover the discrepancies. The circuit used in the M. I. T. report of 1923 is shown in Fig. 3. On considering this circuit, it was found that an alternating current of double frequency must flow in the direct current circuit, even though the

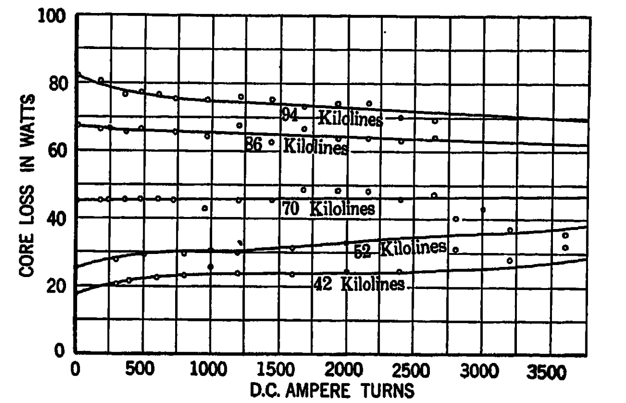

Fig. 2—Core Loss Curves for Various A-C. Flux Densities Plotted Against D-C. Ampere-Turns. Alternating and Direct Currents in Series in the Same Winding.

d-c. coils were balanced with respect to the fundamental when no current was flowing in them. If the direction of the winding is such that the a-c. voltages in the d-c. coils buck, it will be found that the a-c. flux will be *with* the d-c. flux in one core and *against* it in the other. This means that there will be a greater change of flux in one

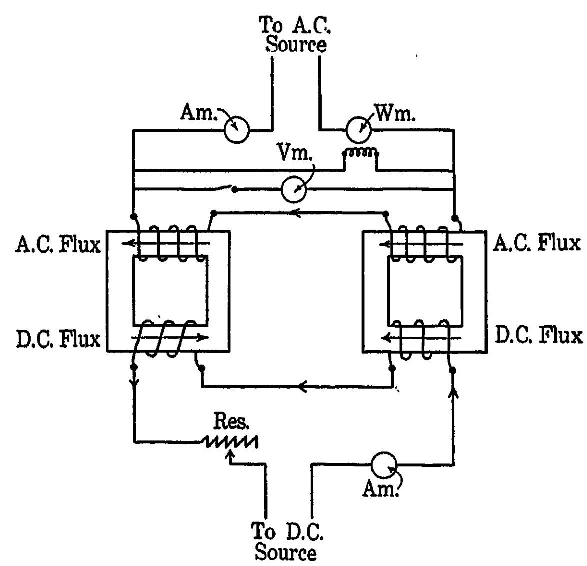

Fig. 3—Connection used in taking Curves of Fig.

core than in the other, due to the shape of the magnetization curve of iron, and hence more voltage will be induced in one coil than in the other at a given instant. This will produce a circulating current of double frequency. In fact, this is almost the identical circuit used at the Sayville Radio Station as a frequency doubler, and was first developed by Count von Arco.

When this circuit was duplicated with the identical reactors used in the work of 1923 but with an ammeter in the direct-current side that would read the square root of the sum of the squares of the r. m .s. alternating current and the direct current, and when the $I^2 R$ loss due to the circulating current was subtracted, the core loss checked that found by the simpler series circuit. At the a-c. flux density used it decreased slightly as direct current was added.

There was one peculiarity that is shown by Fig. 4. The upper line is the curve of "core loss" given in the

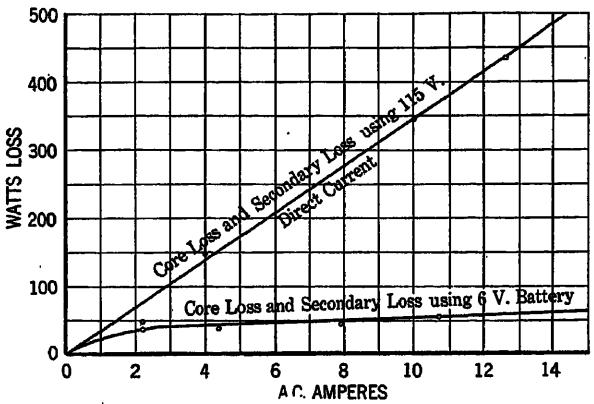

Fig. 4—Curves Showing Effect of Resistance in D-C. Circuit on Loss Due to Double Frequency Circulating Current

previous report, which really included the circulating current loss in the d-c. circuit; the lower curve is the *same thing* as read by our wattmeter. The true core loss lies under the lower curve. An explanation of the large difference is found in the different constants of the d-c. circuits. The upper curve was taken using direct current from the d-c. mains at 115 volts—a large resistance was necessary to cut it down to a value required for the windings on the reactors. In the lower curve, direct current was supplied from a storage battery of only a few cells, and a very small resistance was needed to regulate the current. In this case the second harmonic path was practically a short circuit, while in the first case it had a large resistance load in it.

It was discovered that as the resistance of the d-c. circuit was varied and the direct current kept constant by varying the voltage at its source, the circulating double frequency current remained substantially constant and approximately equal to $I_{dc}/\sqrt{2}$. This led to the suggestion that a short-circuited winding be provided in parallel with the d-c. winding in which this current could flow. This was tried and the total losses were still further decreased. The loss due to this current can be made about the same value as the iron loss without using an excessive amount of copper in this short-circuited coil. Since the r. m. s. value of the circulating current is $I_{dc}/\sqrt{2}$, if no parallel short-circuited coil is provided for the circulating current, the loss due to it will be equal to one-half the direct-current $I^2 R$ loss in the entire d-c. circuit, including the source of supply. It should be noted that any short-circuited winding should be placed next to the core in order to be most effective, as its leakage reactance will be less than if placed outside the d-c. coil.

The three-coil circuit mentioned here is shown in Fig. 5. This scheme has the disadvantage of having no d-c. flux in the middle leg of the core and consequently this flux is not so effective in changing the reactance as it might be, but the wave-form of the current is much improved by this form of core. Some core-loss curves were taken with this circuit and found to be substantially the same as those of Fig. 2, except for small differences due to the unsaturated portion of iron in the middle leg, and the increased reluctance of the direct current path. This was about three times as high as when the direct current was in series with the

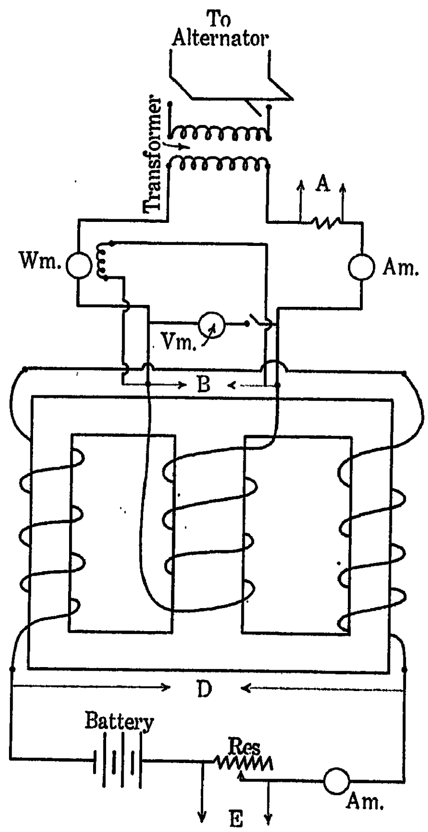

Fig. 5—Three-Coil Circuit Which Gives Better Wave Form Than the Series Circuit of Fig. 1 But Same Iron Loss

alternating current, as the area of the path was less and the length greater.

Families of curves taken at different frequencies are very similar to those shown in Fig. 2 except that they move up as the frequency is increased, much as they would if no direct current were present. The middle or straight curve comes at a higher a-c. flux density as the frequency goes up, although its actual position, the authors believe, would depend upon the relative amounts of hysteresis and eddy-current loss in the given core.

Oscillograms

Several oscillograms of the current and voltage in both the series and three-coil circuits were taken when the d-c. ampere-turns were of different values. The single-coil connection of Fig. 1 will be considered first. The addition of direct current caused the voltage wave to become quite peaked. This was due in part to the effects of the direct current on the supply transformer, as it had to circulate through the secondary of this transformer as well as through the reactor being tested. The volt-ampere load on the transformer was about half its rating when the largest value of direct current was used. The $I R$ drop across the regulating resistance also distorted this wave. The combined effects resulted in a flat flux wave, which in itself, would tend to cause less iron loss. That this was not the entire cause of decreased iron losses, however, will be apparent from the results of the other circuit.

The current wave with no direct current is the usual magnetizing current of iron with a high percentage of the third harmonic and other higher harmonics. When direct current is superposed, the lower half of the wave is cut off, more or less completely, depending upon the direct current value. Fig. 6 shows the voltage and current as well as the direct current alone before the alternator switch was closed. This peculiar current wave is due to the various harmonics which it contains. About 30 per cent each of a third and second harmonic, together with several higher ones, will account for this shape.

Fig. 6—Current and Voltage of the Series Circuit of Fig. 1 When Direct Current was Flowing. The D-C. Value is Shown by the Upper Horizontal Line, which Was Taken Before Closing the A-C. Switch

To make sure that there was not also some valve action in the storage battery that prevented the current wave from reversing at the a-c. frequency, a battery was put in series with an alternator giving the same r. m. s. voltage as the battery, and an oscillogram taken of the current and voltage waves. The current remained sinusoidal but was displaced above the zero axis. This displaced current changed sign without any indication of a valve action, although the lower peaks of the wave just barely extended across the zero line.

Fig. 7 shows the current and applied voltage of the three-coil circuit of Fig. 5 when the a-c. flux density was about 94 kilolines, and 4000 d-c. ampere-turns were in the outer legs. There is no distortion of the voltage wave as when the direct current was in series. There is also a decided improvement in the wave form of the current, because the iron was saturated by the direct current and was operated upon the straight-line portion of its magnetization curve. Since there are two possible paths through the iron for the a-c. flux in this arrangement, the current wave remains symmetrical. The third harmonic still remaining is due partly to the unsaturated portion of iron in the middle leg, and partly to the incomplete d-c. saturation of the outside legs.

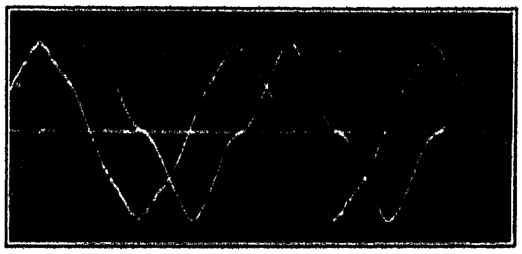

Fig. 7—Current and Voltage of the Three-Coil Circuit of Fig. 5 when 4000 D-C. Ampere-Turns Were in the Outer Legs of the Core

Fig. 8 shows the current and voltage in the d-c. circuit, and the a-c. voltage applied to the middle leg. The circulating current is of just double frequency and is practically free from other harmonics.

These oscillograms are typical of the thirty or more that were taken under different conditions, and show what wave forms may be expected from a saturated core reactor.

Fig. 8—Exciting Voltage (with Tooth Ripple) and Double-Frequency Circulating Current and Voltage in the D-C. Circuit. The Upper Horizontal Line is the Direct Current before the A-C. Switch was Closed, and the Double-Frequency Current is Symmetrical about this Current as an Axis

Hysteresis Loops

Dynamic hysteresis loops were taken with a two-dimensional oscillograph constructed at M. I. T. by T. W. Kenyon, and they check the wattmeter readings of core loss, at least qualitatively. There is a description of such an oscillograph by E. L. Bowles in the Journal of the A. I. E. E. for August, 1923. The instrument consists of two ordinary vibrators mounted

with their axes at right angles and the light beam is reflected by both mirrors, one giving it motion in one plane and the other deflecting it in a plane at 90 deg. to the first. One vibrator carries a shunted portion of the magnetizing current, and the other, a current which is proportional to the flux in the core. This current is obtained by inserting a very large inductance in series with an exploring coil on the core, and if the ratio of $\frac{R}{L}$ in this circuit is very small, the current is very nearly proportional to flux. If this ratio is not small, negative loops will appear at the tips of the hysteresis loop; but this effect was negligible in the circuit as used, for the ratio could be increased to four times its normal value without causing any suggestion of negative tips.

Fig. 9 shows a normal loop taken at about 94 kilolines, a-c. flux density, and one taken at the same a-c. impressed voltage, but with 8 amperes direct current flowing in the same winding. Both loops are taken to the same scale, and the area of the distorted loop does

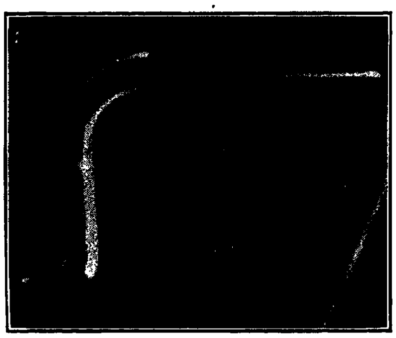

Fig. 9—Normal and Displaced Hysteresis Loops Taken with Bilateral Oscillograph. Their Area Includes Both Hysteresis and Eddy-Current Losses

not differ greatly from that of the normal one. The loop containing d-c. flux shows practically no area in the portion representing high flux density, and therefore practically no loss on this part of the cycle. The addition of direct current causes a shift to the right, of the current vibrator, but there is no effect on the flux vibrator by the steady d-c. flux in the core.

Numerous other loops were taken at various values of a-c. and d-c. flux, and an extensive study was made of the errors in the bilateral oscillograph. It is felt that the loops shown above are free from any but very small errors, and that they very nearly represent the iron loss (both hysteresis and eddy current) by their area.

However, as a further check on the accuracy of these loops, ordinary oscillograms were taken of the exciting current and the voltage induced in an exploring coil on the core. Fig. 10 shows a trace of this negative. The flux wave was plotted by taking the area under the voltage wave, and the hysteresis loop was constructed from this flux wave and the current wave. The constructed loop checks the distorted loop of Fig. 9 very well indeed, if allowance is made for the difference in scales. The applied d-c. voltage and direct-current values were the same in both cases.

Fig. 11 shows two loops taken with the three-coil circuit. The flux vibrator was connected to an exploring coil of five turns wound outside of the main coil on the middle leg. The current vibrator carried a part

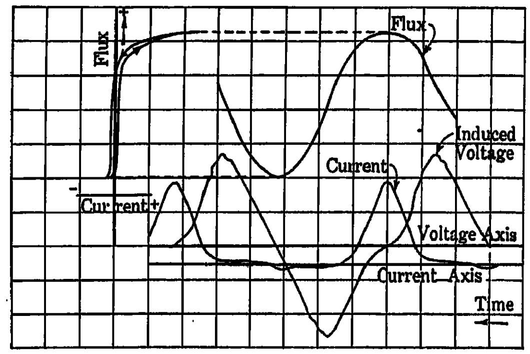

Fig. 10—Displaced Hysteresis Loop Constructed From Actual Current and Voltage Waves. Note that this Loop is Substantially the Same as the One taken with the Bilateral Oscillograph

of the exciting current shunted off at (a) in Fig. 5. The currents were so high that the errors due to leakage and $I\,R$ drop were very pronounced. The area of the loop is actually negative, and is therefore valueless in indicating losses. It does show the change in permeability, though, by the decrease in its slope when the direct current was added. The vertical loop is a normal one, and has very little error, since the exciting current was quite low when no direct current was present. The

Fig. 11—Normal and Displaced Hysteresis Loops of Three-Coil Circuit of Fig. 5, Showing Change in Impedance by a Decrease in Slope when Direct Current is Added. Area of Distorted Loop is not Representative of Iron Loss Because of Errors

distorted loop shows the effect of leakage and $I\,R$ drop at high values of d-c. flux by a decrease in vertical height. The exploring coil did not link as much of the flux when the core was saturated, and the flux was less by the $I\,R$ drop, since the applied voltage was held constant. It should be noted that the average slope of the whole loop from the horizontal axis is a measure of

the a-c. permeability of the iron, or impedance of the reactor.

Impedance Curves

The design which gives the greatest change in impedance for the least number of d-c. ampere-turns is the best, other things being equal. Fig. 12 shows the change in impedance of the reactor as direct current is added. All of these curves are at 60 cycles. In each case the impedance, when there is no direct current, is taken as 100 per cent and the percentage of this impedance plotted for different d-c. ampere-turns. Curve No. 1 is at a low a-c. flux density (42 kilolines) and has the direct current in series (only the center leg of the reactor being used). The initial decrease in impedance

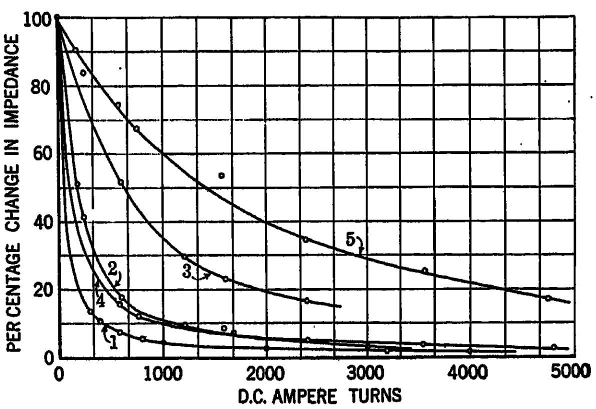

Fig. 12—Percentage Change in Impedance of Reactor with an Increase of D-C. Excitation for Various Connections and Different Values of A-C. Excitation

is seen to be quite large for small values of direct current. No. 2 is the same curve for 70 kilolines, and No. 3 for 94 kilolines. The direct current is most effective at low a-c. flux densities. Curve No. 4 is also at 42 kilolines, but has direct current in the outside legs only. It lies above Curve No. 1 because the middle leg is not affected by the d-c. flux, and the reluctance of the d-c. path is higher than for the series connection. No. 5 is at 90 kilolines and corresponds to No. 3 of the series circuit.

Conclusions

The results of this work indicate that the *iron losses* of a d-c. excited reactor are not excessive. For high a-c. saturations, such as would be used in shunt reactors for voltage regulation on a transmission line, the iron losses proper may even be decreased. However, no scheme has yet been discovered for eliminating the necessary evil of the double frequency circulating current, and its copper loss must really be charged up to the iron. This loss, however, may be made as small as desired by using sufficient copper, since for a given core the double frequency current is a function of the direct current excitation, and by its circulation, tends to cut down the voltage that produces it. Its limiting peak value is the value of direct current if it all flows in the d-c. circuit.

In conclusion, the authors wish to express their appreciation to Dr. V. Bush for his inspiration and assistance during the course of the work and to Messrs. G. Faccioli, A. Boyajian, and O. R. Schurig, for their helpful criticisms and suggestions.

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Discussion

J. D. Ball: About ten years ago I was very much interested in investigating the amount and nature of magnetic losses in iron when subjected to a superimposed a-c. and d-c. excitation. We spent three years on this investigation and collected what we could find of the data available at that time. All the results showed quite conclusively that for a given flux change due to alternating current, there was a definite increased loss in the hysteresis if d-c. flux was superimposed upon it, and the greater the superimposed d-c. excitation, the greater the loss. The same conclusion was verified by experiments made at the United States Bureau of Standards, at the Pittsfield Laboratory of the General Electric Company, and also by tests made in the Standardizing Laboratory of the General Electric Company at Schenectady. The results from the various publications studied led us to the same conclusions.

In the present paper the writers state that an attempt was made to account for the additional losses due to a superimposed d-c. excitation by a formula in which the Steinmetz equation was used with changed exponents. This is not accurate. In our mathematical equation the Steinmetz formula was left exactly as it was given by him but it was extended by adding a supplementary expression which would indicate the increased

losses. It is readily seen that to be mathematically correct this additional expression must be added and cannot be multiplied; otherwise the entire expression would become irrational when the d-c. excitation is zero.

There was no certainty expressed at the time that the expression, as supplemented by ourselves, was necessarily correct, but this equation did agree with the results of various experimenters and was satisfied by the data which we obtained.

Our investigation was made for two reasons, first, to assist in the design of machines where those conditions applied, such as in the inductor generator, and in the iron losses in rotor teeth, and second, in the hope that it would stimulate inquiry and result in the subject being studied by other investigators.

The present paper disputes the results obtained up to this time. It is not essential to defend our position taken some time ago; the only thing in which we are interested is in finding the actual facts. The present results are unquestionably correct for the tests made, but I should question whether the authors had the same conditions in their circuits which they thought they had.

I am more than glad additional inquiry has been made on superimposed losses. I do not think it is fair to take the results of this paper as the results of anything except the particular circuits in which they were applied. Since the tests, as the authors are aware, have been made in accordance with what has not been considered good practise for a number of years, I feel they will agree with me on this.

Turning to the paper, the first thing is the question of taking the instrument losses from a primary coil. This was the way it was done for a good many years, and up to fifteen years ago, it was considered the best way possible. The instrument losses and additional current consumed by the instruments are nicely taken care of and it is a simple matter to take out those instrument losses which can be mathematically calculated and you always get something, although not always what is aimed at.

It has been shown by experimenters in magnetic materials that it is necessary to use a potential or secondary coil for a voltage coil to measure the flux, and also to use the potential coil, either the same one or another potential coil, to excite the voltage winding of the wattmeter. I think it was Prof. Epstein who first pointed this out and the fact has been thoroughly established.

Some elaborate tests were made at the Bureau of Standards in Washington and it was pointed out by Dr. Lloyd, and afterwards by Dr. Burroughs, that it was absolutely necessary to use a potential coil to get consistent results in any magnetic testing or investigation which they attempted. That is why I was surprised in glancing over this paper to find a secondary potential coil was not used. I feel that if definite conclusions are drawn, it would be very well to check these results, using the approved method of employing a potential coil.

A second point I wish to make refers to the nature of the magnetic circuit used. Possibly I misread this; but taking Fig. 3, if this represents the same connection as shown in Fig. 1, in which there is a d-c. excitation on the center leg of a three-legged core, and another excitation, a-c. or d-c., on the outer legs, I should take exception to that method of procedure. That isn't exactly superimposing alternating current on direct current. In a three-legged core, flux does quite a number of things and it doesn't all go through the same paths.

Another point: in superimposing alternating current and direct current, I know from experience that you get wild things unless it is done in one definite manner, that is, to have the alternating current and the direct current with one winding right on top of the other, so that any leakage or trouble of any kind would have to be more or less similar.

Another point is that the results obtained from any sample except a ring sample didn't seem to be very good. True, with a ring sample the flux density is greater at the inner circumference than at the outer circumference, but corrections have been established by the Bureau of Standards. We found the only reliable method was to use a large ring in which the mean diameter of the ring was comparatively large in comparison with the width of the sheet, or the difference between the outer and inner radii of the test specimen. Even then certain corrections should be applied as pointed out by the Bureau of Standards and others.

There are at least two definite ways of superimposing alternating current and direct current: One is to subject the test specimen to alternating current and to superimpose on it a direct current in another winding immediately under or over it; another way is the step-by-step method, measuring the losses by the ballistic galvanometer. I think it would be well to check the present tests by the step-by-step method. First, a definite flux change should be assumed which would represent the iron when subjected to the a-c. excitation. The hysteresis loop should then be obtained by the well-known step-by-step method. This is the normal hysteresis loop. To obtain the effect of superimposing d-c. excitation, another loop which I have termed the "unsymmetrical loop" should be taken. This can be done by going up on the saturation curve to some point higher than the maximum of the first loop, then dropping down on the normal hysteresis loop from this second point to a point where the flux change from this new maximum is in the same amount as the total flux change in case of the first loop. Then from this new bottom point, return step-by-step to this new maximum. Rather unique figures are obtained by this method, some of which were published in 1915[1]. We invariably found that with the same flux change the area was always greater when the maximum point from which the loop started was raised. This is the same situation as when you have an a-c. excitation and superimpose upon that excitation a d-c. excitation represented by the mean density between maximum and minimum of the various loops. A study of these figures, the characteristics of any hysteresis loop, and the characteristics of any magnetization curve will show that invariably the area of the unsymmetrical loop increases with increasing mean density, the flux change remaining constant.

A. C. Lanier: The oscillographic record, Fig. 5 in this paper, seems to show a reduction in the density range as well as a change in shape of the dynamic hysteresis loop due to combined a-c.-d-c. excitation as compared with the normal loop. Both effects should increase progressively with the increase in d-c. excitation for large a-c. excitations, and both should cause a diminution in the hysteretic component of the iron loss. The effect upon the eddy-current component is less clear. With small a-c. excitations there should be no appreciable reduction in density range except for very large d-c. excitations. The area of the displaced hysteresis loop for a given density range has also been shown to increase with increasing average density. Therefore, the curves of Fig. 2 appear reasonable.

In connection with the study of surface iron losses, the speaker has noticed that sometimes, with high average gap densities, the measured losses are less than the expected values. The magnetic structure used when these results were observed was a homopolar structure consisting of a slotted cylindrical member rotating within a smooth cylindrical member. The excitation produced radial average flux distribution with a superposed tooth ripple.

At high densities the amplitude of flux pulsation increased less with a given rise in gap density than it did at low densities. The loss increase was correspondingly lower. The seeming discrepancy is traceable to the effect of high tooth saturation which causes a departure from the straight-line relationship between the tooth-ripple density and the average gap density.

Considering these results it seems probable that if the average gap densities had been carried high enough, the flux ripple and

1. The Unsymmetrical Hysteresis Loop, by J. D. Ball, A. I. E. E. TRANSACTIONS, 1915, Vol. 34, page 2693.

the losses due to it might have shown actual decreases with further increase in average gap density.

O. R. Schurig: Mr. Ball called attention to the possibility of errors resulting from the method of iron-loss measurements employed by the authors, if proper corrections are not made.

It is true that the power given by direct primary measurement, that is without the use of a separate potential winding on the iron core, includes losses other than iron losses. If, however, the extra losses are evaluated and corrected correct iron losses are obtained. I believe the paper gives evidence to show that the proper corrections were applied and that reliable results were secured.

It must be pointed out here that the method of iron-loss measurement employing a separate potential-circuit winding on the iron core—the method advocated by Mr. Ball—also involves possibilities of grave errors. The potential-coil method is particularly subject to error where there is leakage flux between the primary (exciting) winding and the potential (exploring) winding to which the wattmeter potential circuit is connected—a well-known fact. Thus, the reliability of any test results on iron losses depends on the thoroughness with which the extra losses have been corrected, unless the circuits have been specially designed to make the excess losses negligible.

It so happens that within the last year the potential-exploring-coil method has been applied at Schenectady to iron-loss measurements with superposed d-c. and a-c. excitations, *i. e.*, a series of tests similar to those presented in the Charlton and Jackson paper, but utilizing, for measurement purposes, a separate potential winding on the iron core under test. The results of these more recent tests, to be published on a future occasion, show, as do those of the paper, a slightly decreasing iron loss with increasing d-c. excitation when high a-c. excitations at constant impressed a-c. voltage were employed.

K. K. Palueff: I should like to mention that if a core as used by the authors is long, a certain portion of the total flux produced by windings placed on the outer legs may not reach the center leg but take an air path for return. The same thing would be true for the flux created by the winding on the inner leg. This "stray" flux will increase as the flux density increases. Thus, the superposition of two fluxes—one created by the winding on the outer legs and another by the winding on the inner leg—may not be as complete as anticipated.

W. R. Weeks (by letter): In the measurement of iron losses under normal conditions, what is known as the Epstein circuit is used to eliminate the $I^2 R$ losses of the exciting winding and wattmeter current coils from the readings. If it is possible perfectly to interlink the excitation coil and the exploring coil, then the readings obtained will be an accurate measure of the losses in the iron. The errors due to circuit-resistance losses will be automatically eliminated.

However, errors may be introduced if there are other circuits which produce flux that links the exploring coil but does not, at the same time, link the exciting coil. Such leakage flux between the two windings will cause errors that will be positive or negative depending on the phase relation of the leakage flux.

There are three possible conditions that may be encountered. (1) If there is no leakage between the exploring and excitation coils, the readings will be correct. (2) If there is leakage between the exploring and excitation coils, and there is no third excitation coil, the readings of the wattmeter will be low. (3) If there is leakage between the exploring and exciting coils and there is a third coil (such as the d-c. winding) which sends flux through the leakage path, the readings of the wattmeter may be high.

All of the above leakage factors are quite difficult to determine for any magnetic circuit such as that used by the authors of the paper, and I believe that using the older method which introduced additional losses, losses that could be accurately determined, was much preferable to trying to use the Epstein circuit.

The Epstein circuit would undoubtedly be better if the exciting coil were the outside strands of a cable of which the central strand was used as the exploring or potential coil.

J. E. Jackson: Mr. Ball has pointed out the fact that the paper was inaccurate as to the exact methods that had been applied in correcting the Steinmetz formula. I am sorry that the mistake was made, but it was intended mainly to indicate that an attempt of some kind *had* been made to adapt existing formulas to the case of superposed excitations.

In answer to the question of why the Epstein method was not used, it was simply because it was felt that with the facilities available, the "subtraction method" was much more reliable. Mr. Weeks has pointed out some of the difficulties encountered in the Epstein method, and, that for the very reason that no one knew what the different fluxes would do when they were superposed, it was impossible to calculate the errors to be expected. If all the leakage factors were accurately known, the calculations would still be more involved than the ones used, and there would be that much more chance of error.

Whether the two circuits used were identical in their behavior or not is still an open question, but the data showed that they were at least substantially alike. Our wattmeter method, when used in the series circuit, was felt to be correct, and the fact that the curves were practically the same when taken with the three-coil circuit tends to prove that the flux paths were very nearly identical in the two cases.

Mr. Ball stated that a study of magnetic characteristics would show that invariably the area of the unsymmetrical loop increases with increasing mean density. I do not believe that this is necessarily true if the analysis is carried far enough. Our hysteresis loops taken with the bilateral oscillograph show clearly that the area is less at the top of a distorted loop, and a consideration of the magnetic theories of Ewing, Poisson, and others shows that this must be the case. Hysteresis is generally regarded as some sort of molecular friction, and when d-c. excitation is added to the core the molecules are lined up and held so tightly that they cannot turn over completely with the reversing a-c. flux, but only vibrate slightly. If the d-c. excitation is strong enough to hold them tightly clamped, the hysteresis loss must disappear completely. The eddy-current loss should not change at all unless the wave form changes, so the net result would be a decrease in iron losses with d-c. saturation. As a matter of fact, the wave forms do become slightly distorted, and the eddy losses tend to go up to some extent.

Our work may not be accurate in the highest degree, but we feel that it is not grossly in error, and that certainly the iron losses do not increase ten or twenty times when d-c. excitation is added, although they may change as much as 100 per cent one way or the other.

Study of Time Lag of the Needle Gap

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Synopsis.—*The study of high-voltage, steep-wave-front transients is difficult from the experimental standpoint because of the very short times involved. Due to the improvement which the cathode-ray oscillograph has enjoyed in recent years, a device is now available by the use of which transients occurring in times as short as one-millionth of a second or less may be photographed. In the paper, the authors used an oscillograph developed by Dufour in France, with which a brief study was made of the time lag of needle gaps and of a needle to a plane.*

A description of the oscillograph is given including a discussion of the method of operation. The photographic film is placed inside the tube so that the electrons impinge directly on the film. The wave is drawn out along a time axis by the combined action of a sweeping motion and a perpendicular oscillating motion imparted to the electron stream by the action of proper electromagnetic fields.

Tests were made with a wave which was nearly perpendicular, reaching its maximum in about one microsecond. Such a wave was obtained by the discharge of a condenser through a suitable circuit. An oscillogram which shows the wave front used is given, and attention is directed to the 20,800-kilocycle oscillation which appears superimposed on the wave front.

The results of tests in which this wave front was applied to gaps are given and it is shown that with any given gap setting and sparking voltage that the time lags vary through wide limits. It is also shown that, for the same voltage, increased gap settings mean increased lag. The per cent overvoltage required to keep the lag to two microseconds or less decreases as the gap spacing increases.

The Study of Transients

ONE of the most difficult and perhaps also one of the most fascinating problems which the electrical engineer of today is called upon to study is that of the transient phenomena occurring in electrical circuits. Failure of apparatus, caused by the puncture or flash-over of insulation due to overvoltages the duration of which may be of the order of a few microseconds, has made desirable the use of lightning arresters which limit the voltage to safe values. Since in practise many of the steep front traveling waves are the result of the sudden releasing of a bound electrostatic charge, lightning arrester laboratories have used the discharge of a suitable condenser to simulate the actual line condition. For this purpose, and for the study of the action of insulation and gaps, impulse generators have been built which may be charged to values as high as 2,000,000 volts.

The calculation of such transients is not troublesome so long as the voltage is low and the wave front not too steep. However, with the higher voltages and very steep wave fronts, assumptions are often made, the correctness of which is at least open to question. The sphere gaps which are used are assumed to be instantaneous, and the resistance of the gap after arc-over is assumed to be negligible. The inductance and capacitance of the connecting leads is often neglected, while the influence of the test piece itself on the wave shape is frequently unknown. The circuit usually comprises loops which anyone familiar with radio knows are likely to mean coupling that may give rise to erroneous results.

The limitations and some, at least, of the possible sources of error involved in the use of the impulse generator have been recognized by lightning-arrester engineers for some time. Three years ago the authors of this paper began to search for means for recording, on a photographic film, transient phenomena the frequency of which might be a million cycles per second or more. As a result of this search of the literature, the device described in this paper was found.

Oscillographs

The frequency limit of the ordinary Duddell oscillograph is in the neighborhood of 15,000 cycles. At this frequency the errors due to mechanical inertia are large and it is impossible to do more than count the cycles; the wave form cannot be distinguished.

A satisfactory oscillograph for the delineation of the volt or current-time characteristic of a short-time transient must satisfy the following conditions:

1. The device must have no appreciable inertia and must be capable of operating at a frequency of at least one million cycles per second.
2. The device must use little energy in its operation, so that its use will not appreciably disturb the original circuit.
3. The device should be capable of registering both voltage and current simultaneously.
4. The apparatus must be so arranged that a single impulse will be sufficient for a satisfactory photographic impression.
5. The oscillograph should be as simple as possible and have sufficient accuracy so that the results may be used with confidence.

The first point can be satisfied only by some device using the flow of electrons. When thinking of the available devices making use of such a flow, one naturally turns to the Braun tube which has been used for many years as an oscillographic device. As originally developed, the Braun tube consisted of a cathode and an anode in an exhausted tube, together with a fluorescent screen. Unidirectional voltage from a static machine causes a flow of electrons from the cathode to the anode, some of which pass through a small hole in the anode and are deflected by magnetic or electrostatic fields produced by the phenomena being studied. The rays then pass on to the fluorescent screen where a graph is

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traced the coordinates of which are determined by the deflecting fields. If the phenomenon repeats itself the graph appears as a stationary pattern, and may be recorded photographically using an exposure of several seconds.

The Braun tube has negligible inertia, and very little energy is required to cause the deflection of the cathode beam. Its speed is the speed of the electron which may be varied between quite wide limits especially if using a heated cathode as in the Western Electric tube[1]. The upper limit of velocity is perhaps one-half that of light.

The fourth condition mentioned, that of recording a single impulse, may be satisfied by placing the photographic film inside the tube in such a way that the electron stream impinges directly on the film. This has been done by several investigators with marked success. Some additional photographic effect can probably be obtained by the use of the heated cathode, but to get high electron velocities a very good vacuum is required which makes the operating technique more difficult.

Many references have been found in the literature concerning the Braun tube and its uses. Most of this material has been the result of investigations done in foreign countries. All of the early work was done with fluorescent screens which limited the oscillograph to use with phenomena which could be duplicated, since the time of exposure was very long compared to the duration of most transient phenomena.

S. R. Milnor[2] in 1912 reported a volt-ampere characteristic of an arc at a frequency of two million cycles. This work was done with a cold cathode and a fluorescent screen, but he was able to make the phenomenon repeat itself exactly.

The first reference to the use of the photographic film inside the tube appears in an article by Alexander Dufour[3] in *Comptes Rendus* in 1914. He stated that he had obtained on a film a space of 1 millimeter corresponding to an interval of time of one three-millionth of a second. Dufour succeeded in doing this by the use of a method of registration which is described more in detail later in this paper.

In 1919, Sir J. J. Thompson[4] in a lecture before the Royal Institution stated that, when using a hot cathode with the film inside the tube, an exposure of one-hundred thousandth of a second would be sufficient to excite the photographic plate. This reference also mentions the use of an auxiliary deflecting alternating current which produces the time axis and which could also be used for timing the phenomenon.

In a valuable contribution appearing in the *L'Onde Electrique* in 1922 Dufour[5] describes a cathode ray tube which he used successfully with transients up to a fundamental frequency of 750,000 cycles with harmonics showing plainly at a frequency of 3,000,000 cycles. To produce a time axis, Dufour used the rise of current in an inductive circuit to draw out continuous oscillations from an arc generator. He used a cold cathode with the photographic plate inside the tube, the tube being connected to a continuously operating air pump. This apparatus was used in studying the antenna currents in the wireless system on the Eiffel Tower.

In 1921, Dr. D. A. Keys[6], in an article entitled *A Piezoelectric Method of Measuring Explosion Pressure,* used a cathode oscillograph as the indicating device for the determination of instantaneous pressures in explosions under water. In this work Dr. Keys used a heated cathode with the photographic plate inside the tube.

In 1922 Johnson[1] described a tube developed by the Western Electric Company which makes use of the hot cathode. This tube has a much higher sensitivity than the cold cathode tube used by Dufour. With any given tube the sensitivity depends on the speed of the electrons in the cathode stream and on the deflecting force. If high sensitivity is required, the use of the heated cathode will give very good results as it is possible to apply low cathode voltages and secure slow moving electrons. Since the cold cathode depends on positive ion bombardment for the liberation of electrons, quite high voltages are necessary, which means high velocity electrons.

The name of Professor Harris Ryan[7] has been associated with the cathode ray oscillograph for many years, he had a paper on this subject before the institute as early as 1903.

Important work has been done by several investigators among whom may be mentioned Wood[8], Minton[9], and Chaffee[10], but enough has been mentioned to indicate the development of the cathode ray oscillograph in recent years.

THE DUFOUR OSCILLOGRAPH

For the study of high-speed transients then, it appears that no better device is available than the oscillograph developed by Dufour. In the description of this oscillograph which follows, the tube itself is identical with that described by Dufour[5]. The switching apparatus furnished by Dufour has not been used in this investigation*.

Referring to Fig. 1 the oscillograph consists essentially of glass tubes *a* and *b*, fitted by means of a ground joint into the bronze chamber *c*. The upper glass tube *a* carries the cathode and anode. The tube *b* has one pair of deflecting plates for electrostatic deflection of the electron stream. For magnetic deflection two sets of coils, 1–1 and 2–2, Fig. 2, perpendicular to each other, are placed external to the tube and located slightly below the deflecting plates. The coils are arranged so that they may be rotated about the axis of the tube, thus allowing adjustment of the angle between the axes of the deflecting fields.

To operate the oscillograph expeditiously, easy means

1. See list of references at end of paper.

*For a description of this part of the apparatus reference should be had to the work of Dufour.

must be provided for changing films quickly. It is also necessary that a fluorescent screen be arranged so that it can be removed when making an exposure. How this is done in the Dufour oscillograph may be seen by

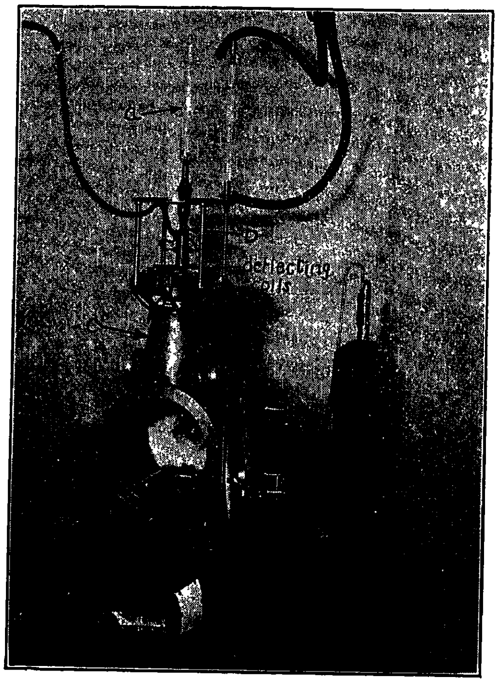

Fig. 1—Dufour Oscillograph

referring again to Fig. 1. The drum, which in the illustration appears in the foreground, is provided with a film magazine which allows six films to be taken in

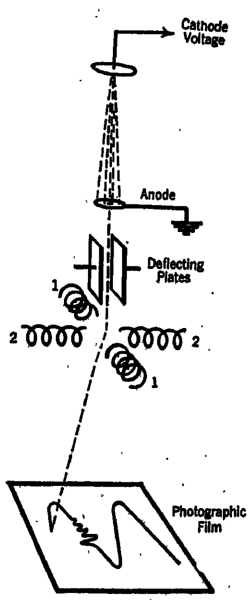

Fig. 2—Diagramatic Representation of Oscillograph Operation

succession. When viewing the phenomenon, a fluorescent screen is turned up into position covering the opening into the interior of the drum so that the films are not exposed when using the screen. After placing the films in the drum, it is placed inside the bronze chamber and locked in position. The opening is closed by a door having a very carefully constructed joint so that the tube may be made air tight. Three cocks, turning in ground joints placed in the door, serve to operate the mechanism for changing films and moving the fluorescent screen. Two glass windows, one on either side of the bronze chamber, permit of easy view of the fluorescent screen.

Operation of the Oscillograph

For slow speed work, a moving drum to take the place of the magazine drum may be used. This drum is driven by means of an external motor and magnetic clutch. A simple calculation shows that such a drum

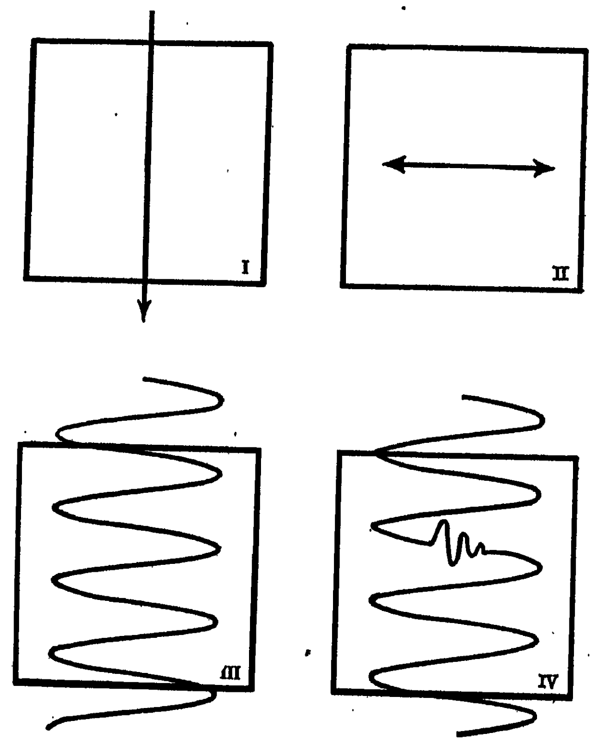

Fig. 3—Method of Registering Transient Phenomena
I. Sweeping III. Sweeping and Oscillator
II. Oscillator IV. Transient superimposed on III

cannot be rotated at a sufficiently high velocity to draw out the oscillations so that they may be studied. To draw out a one-million-cycle wave in a manner similar to that used with the ordinary Duddell oscillograph, so that two millimeters are allowed per cycle, would require a film velocity of 2000 meters (6650 feet) per second.

This problem has been solved by Dufour in a very satisfactory manner. Rather than move the film, the electron stream is subjected to the action of auxiliary fields which draw out the wave without the limitations of a mechanical system, as shown in Fig. 2. The method by which this is accomplished may be understood by reference to Fig. 3. In I is shown the effect of passing a transient current through coils 1-1. With the proper circuit arrangements the beam is held

off the film at the top until ready for the photograph to be taken, when a transient takes place which sweeps the beam across the film holding it off the film at the bottom. This transient current will be referred to as the sweeping current.

A source of high frequency (a vacuum tube oscillator) is connected to coils 2-2, which are mechanically spaced 90 deg. from coils 1-1. With coils 2-2 energized, the oscillator traces a straight line on the film, the amplitude usually being adjusted to utilize the entire width of the film. When coils 1-1 and 2-2 are operated together the oscillator waves are drawn out as seen in III. If the oscillator frequency is 50,000 cycles and the effective width of the film 100 m. m. (3.9 in.), then the average distance corresponding to one microsecond would be 10 m. m. (0.39 in.). This means that if a million-cycle wave were impressed on the deflecting

FIG. 4—IMPULSE GENERATOR

plates so that the beam was deflected thereby in the same direction as by the sweeping current, it would be drawn out sufficiently so that the wave form could be determined and IV shows the effect of the combination of the three fields. The oscillator wave is the zero line for the transient being studied, and it is a time axis whose unit of measure is varying according to the sine law. The speed of sweeping is always a compromise between drawing out the oscillator wave and the difficulty of getting the unknown phenomena timed so as to appear on the film.• With much slower speed phenomena, the sweeping field may be placed 90 deg. from that of the unknown transient, so that the time axis becomes a straight line across the film. This axis may be conveniently calibrated by superimposing a known high frequency using the oscillator coils.

Thus it is possible to get volt-time or ampere-time curves with a time axis which can be calibrated with considerable accuracy. Volt-ampere characteristics may be taken by applying to the cathode stream fields proportional to the voltage and current and spaced 90 deg. apart.

The Cathode Stream

The best registration on the film is obtained when conditions are such that a fine pencil of rays strikes the film only when required. Not only is it desirable to hold the rays off from the film before and after the transient, but the operation of the tube is much improved if the cathode voltage is applied for just sufficient time to allow the proper registration of the unknown transient.

For a given voltage impressed on the cathode the sharpness of the trace on the film will depend on the degree of exhaustion of the tube. For the work described in this paper the authors have used a Langmuir condensation pump, backed up by a two-stage oil pump. A McLeod gage was used to give an indication of the condition of the vacuum but the appearance of the spot on the viewing screen and the character of the discharge from the cathode itself, as viewed with the eye, gives a more accurate determination of the proper operating condition. A small tray of phosphorus pentoxide was kept inside the vacuum chamber which serves to keep the vapor pressure low. The entire tube is always kept in the evacuated condition except when actually changing films. This procedure aids in keeping the gas evolved by the walls of the tube down to a sufficiently low value so that the tube can be worked easily and quickly.

The necessary cathode potential may be obtained by the use of either a high-voltage direct current, or a few degrees of the crest of an alternating potential. The latter method may only be used with phenomena which are fast compared to the change of potential during its registration. This method was mentioned by Dufour as being particularily adapted to the study of very short time transients, and as this method is very convenient it was adopted for use in this study of gap characteristics.

Timing the Transient

The spot made on the photographic film by the electron stream may travel as fast as 80 km. (50 mi.) a second across the film; and since it is not feasible to get a developed registration length of more than 10 or 12 meters (32.8 to 39.2 ft.) on the film, the transient must be initiated during the very short interval of time in which the spot is sweeping across the film.

A rotary switching device has been built which makes the necessary contacts so that voltage is applied to the cathode, the sweeping started, and the unknown phenomenon so timed as to appear on the film. The oscillator is connected before voltage is applied to the cathode, and remains connected until after the exposure has been made. The arrangements are such that only

the pushing of a button is required to set in operation a mechanism which makes all connections automatically.

Time Lag of Needle Gaps

It is known that a needle gap shows considerable lag when subjected to steep wave front impulses. The brief study which is presented herewith measures definitely the lags encountered under the given conditions. The results are not complete, but do give for the first time, as far as the authors are aware, a direct measurement of lags as short as a few microseconds. The methods used here are being applied to the study of the problems encountered in lightning arrester practise and will yield results of great importance.

The time lag of a gap may be taken as the time elapsing until breakdown occurs during which the applied potential exceeds the low frequency spark potential. For a voltage only slightly in excess of the low frequency spark potential the time lag may be long, while with steep wave fronts of high voltage it will be extremely short. The lag with any given gap is determined not only by the voltage at the time of spark-over but also by the shape of the wave front used.

The purpose of these tests is to find the effect of successive increments of overvoltage on the time lag. To avoid the complication of a sloping wave front, it was thought best to use a wave as nearly rectangular as could be obtained. It is impossible to produce a perfect rectangular wave but if the time required for the voltage to reach a constant value is small in comparison with the time taken by the gap under test to spark over, the error will be negligible.

Test Arrangements

An impulse generator, which was built for use in connection with the testing of lightning arresters, was used as a source of voltage. This generator, which may be operated up to 100 kv., consists of two hundred glass plates with tinfoil coatings. The plates are divided into four groups connected in series, each group consisting of 50 plates in parallel, giving a capacity of 0.13 microfarads. The photograph (Fig. 4) shows the compact arrangement of the plates and the kenotron equipment which is used to charge the condenser. A connection diagram is given in Fig. 5 and shows the limiting sphere gap which determines the voltage at which discharge will take place. The water tube resistance, R_1, allows the sphere gap to charge up properly, while R_2 is used to control the wave front as will be shown later. The dividing condensers, C_1 and C_2, were used for reducing the voltage to the proper value for the deflecting plates on the oscillograph. The use of a resistance potentiometer was considered, but if a value low enough to record the wave form accurately was used, it would discharge the condenser too rapidly, so that the voltage applied to the needle gap would not be sustained.

The oil-immersed dividing condensers are shown in Fig. 6, together with the needle gap being tested. These condensers consist of two fixed plates spaced 0.8 mm. (1/32 in.) with a movable third plate having a

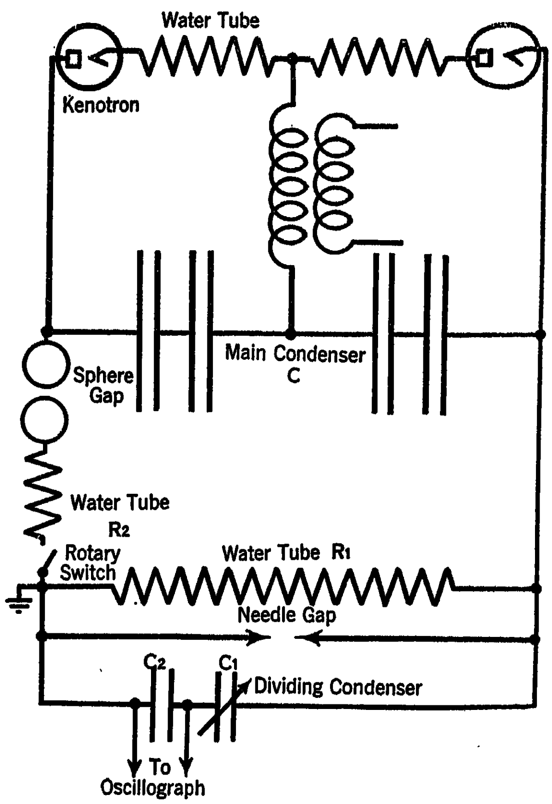

Fig. 5—Connection Diagram

micrometer adjustment. The capacity of C_1 at the setting used on the tests was about 20 micro-micro-farads. Variable stray capacities to ground and inductive effects between the condenser and the oscillograph

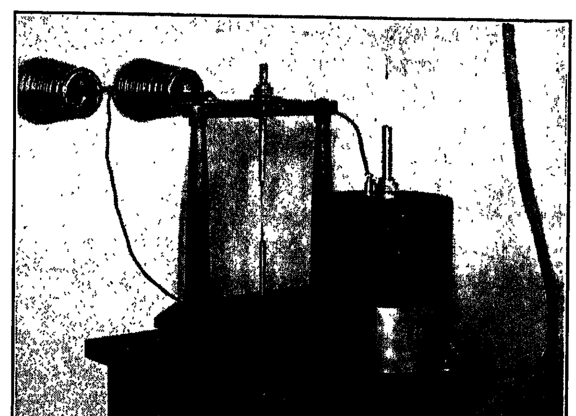

Fig. 6—Dividing Condensers and Test Needle-Gap

were eliminated by using a ground shield around the dividing condenser and by the use of a concentric cable connection to the oscillograph. The grounded sheath of this cable was made of braided bronze wire, the inner conductor being a fine manganin wire having a resist-

ance of about 100 ohms. This resistance was used to damp out oscillations originating in the dividing condensers and lines.

Voltage calibrations were obtained from capacity measurements, and more directly by taking oscillograms when holding a known 60-cycle voltage on the dividing condensers.

The general arrangement of the different elements used in taking a cathode ray oscillogram is shown in Fig. 7. The vacuum tube oscillator may be seen just in front of the oscillograph. The vacuum pumps are not visible, being located under the table directly beneath the oscillograph. The rotary switch is housed in the square box shown at the right. Between the rotary switch and the oscillograph are located the dividing condensers and the test needle gap. The entrance bushings back of the test gap lead to the 200-plate condenser, which is located close to the other side of the wall. When setting up this circuit, care was taken to make all connections as short and direct as possible.

Fig. 7—Laboratory Set-up for the Study of Impulses

Wave Form

Fundamentally, the circuit shown in Fig. 5 represents the discharge of one capacity into another with small series inductance and considerable series resistance. The circuit is of course complicated by the use of series gaps, wires leading to oscillograph and etc. With such a circuit, the series resistance R_2 will increase the time required to charge the capacity of the dividing condenser and connections.

The effect of changing R_2 may be seen by referring to Fig. 8, which shows the wave fronts with three different values of resistance. The method of registration used is the same as that described in connection with Fig. 3 and consists in applying an upward sweeping motion, combined with the horizontal motion of the oscillator. Superimposed is the discharge of the condenser which is initiated by the action of the rotating switch.

On the oscillograms given in Fig. 8 will be found two sets of oscillations, the first being damped out rather quickly. This oscillation, which has a frequency of

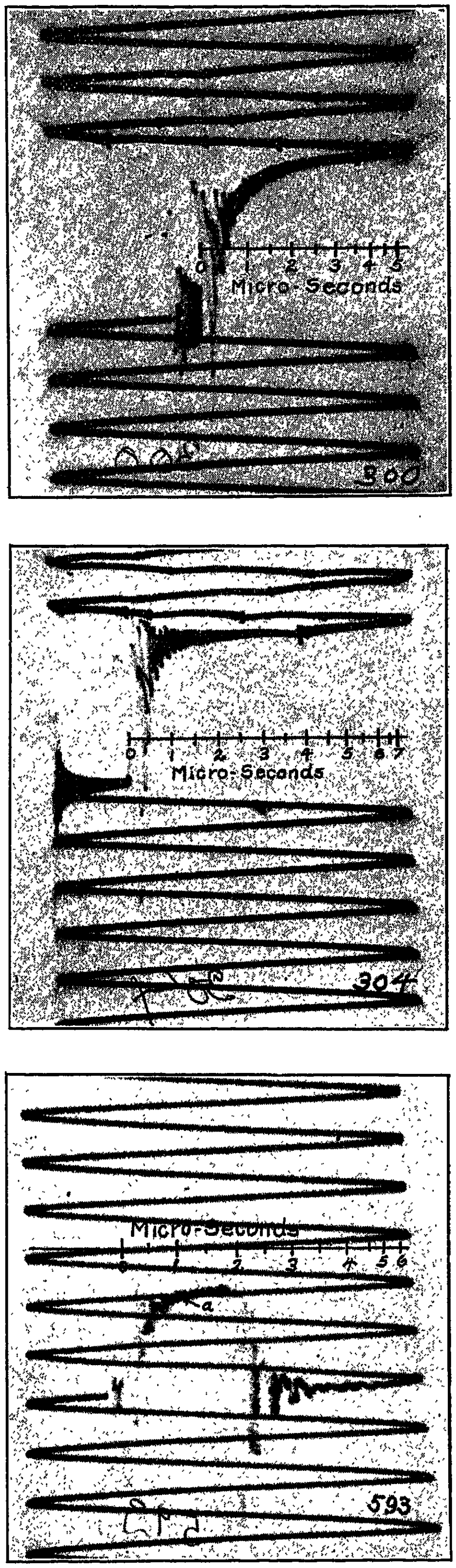

Fig. 8—Oscillograms Showing the Wave Front Used on Needle Gap Tests

Film 300—Oscillator frequency 42.6 kc showing wave form with 1100 ohms series resistance. Voltage reaches constant value in 2.5 microseconds.

Film 304—Oscillator frequency 42.6 kc. Wave form with 570 ohms series resistance. Voltage reaches constant value in 0.4 microseconds.

Film 593—Oscillator frequency 50 kc. Wave form with 700 ohms series resistance. Voltage reaches constant value in 1.0 microseconds. Needle gap sparks after 0.83 microsecond. Gap setting 65mm. voltage 75 kv.

approximately 20,000 kilocycles, occurs when the rotating switch sparks and is followed by another when the limiting gap sparks. There is a certain variable time interval between the sparking of these two gaps. However, for the purpose of this paper the wave to be studied is that found when the limiting sphere gap sparks. On each of these films time is measured along an oscillator wave beginning at the time when the main condenser discharge begins.

In making these tests the aim was to obtain a steep wave front but at the same time to prevent the voltage from over shooting. Film 300 (Fig. 8) shows the main transient rising to its maximum value in about 2.5 micro-seconds. Superimposed on this wave front are oscillations which are made up of a combination of several frequencies. Film 300, which was taken with 1100 ohms in series, shows that none of the oscillations has a voltage exceeding the maximum value of the main transient.

The series resistance was reduced to 570 ohms and film 304 taken. This film shows that the main transient rises to its maximum in about 0.4 microsecond. This resistance is too small, however, as some of the crest values of the superimposed oscillation exceed the final voltage.

A value of 700 ohms was chosen as being the best compromise between the steepness of wave front and the condition of over-shooting. Film 593 was taken, using this series resistance, and it was found that a time of one microsecond was required for the voltage of the main transient to reach its full value (marked a on the film). This film is interesting as it shows the sparking of a needle gap 0.8 microsecond after full voltage had been applied.

Results of Tests

The time lags under most test conditions used exceeded two microseconds, which made the use of the oscillator undesirable except for timing purposes; therefore, nearly all results were taken with the sweeping only, as this allowed several exposures on one film. With six films and five tests per film, it was possible to get 30 tests before releasing the vacuum and changing the magazine drum. The use of the sweeping also gives a uniform time scale for the measurement of the lag.

The results of nine representative tests are given in Fig. 9. As this type of oscillogram is probably new to most of the readers of this paper, a brief explanation is given. The different tests are numbered in the order in which they were made. In the first test, for instance, which is at the bottom of the film, (No. 543), the cathode spot comes on the film from the left, being swept across the film at a uniform rate corresponding to 4.5 microseconds per mm. About 190 microseconds later the voltage is applied by the operation of the rotating switch. The cathode beam is deflected upward and traces a horizontal line, parallel with the zero axis, until after 140 microseconds the needle gap under test breaks down and the cathode spot falls to zero and so continues, passing off the film at the right. Although the wave front in this film appears perpendicular, it is really as shown in Fig. 8, fi m 593.

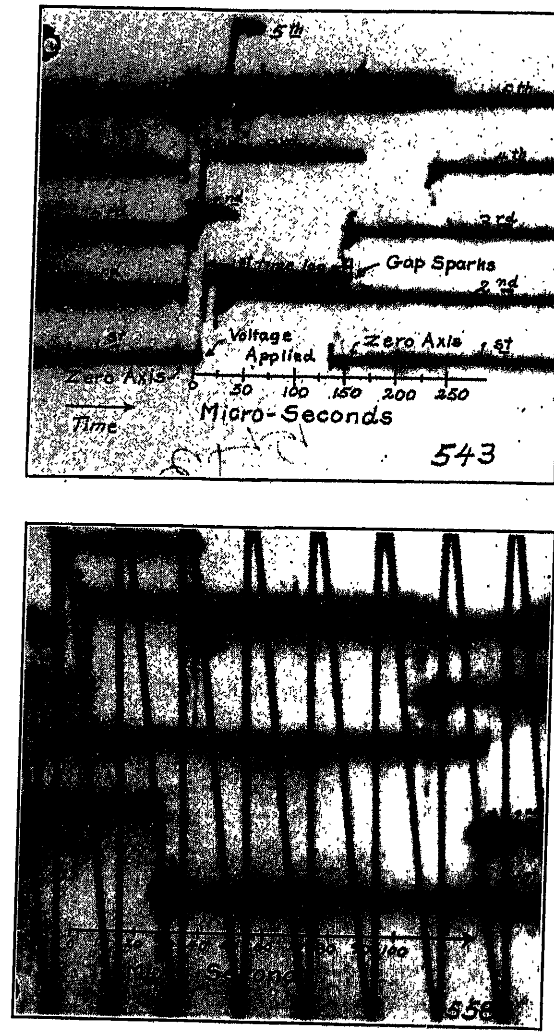

Fig. 9—Oscillograms Showing the Time Lag of Needle Gap
Film 543—Five tests on needl gap at 15 mm 22 kv.
Film 558—Four tests on needle gap at 60 mm spacing 58 kv. 50 K C Timing wave.

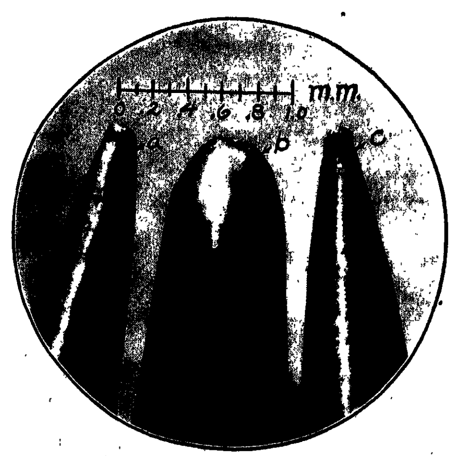

Fig. 10—Condition of Needles after Thirty Discharges
a. New point. b. Dulled point.
c. Extra sharp point.

Four needle-gap breakdowns are given in film 558,

Fig. 9, the voltage being 58 kv. with a needle gap spacing of 60 mm. This film shows a 50-kilocycle timing wave which fixes the time calibration. Fig. 9, Film 543, shows the result of tests on a 15 mm. needle gap with 22 kv. applied. These oscillograms show very nicely the steepness of the wave front compared with the time lags, and also how well the cathode ray oscillograph is adapted to the study of short time phenomena.

Results obtained from a series of oscillograms for three different voltages and with different needle-gap spacings are given in Tables I to III. Tests were also made with a needle to a plane and between needles having different degrees of sharpness. A photomicrograph is given in Fig. 10 showing the different needle points magnified to 30 diameters. This picture shows the condition of the needles after each had been given thirty discharges.

An analysis of the results given in these tables brings out certain relations which are briefly discussed.

For each voltage used the gap setting, corresponding to infinite lag, will be slightly above the 60-cycle setting for that voltage. As the voltage applied to the gap decreased about five per cent in 4000 microseconds due to leakage of the condenser, no attempt was made in these tests to get extremely long lags. The per cent overvoltage above the 60-cycle spark potential necessary to obtain lags of one microsecond or less was found to decrease with increased spacing.

With spacings of 10, 40 and 65 millimeters the per cent overvoltages are 75, 40 and 29 respectively. It is, of course, to be expected that the greater the per cent of over voltage the shorter the time lag. The results show that this is true, in general, although wide variations in time lag occur with every setting and at all voltages.

TEST DATA
TABLE I
LAGS GIVEN IN SAME ORDER AS TESTS WERE MADE
VOLTAGE 22,000

Spacing and Gap	Atmospheric Conditions	Time* Calibration microseconds per m. m.	Time Lags in Microseconds	Max. Min. Av.	Per cent Sparking† 1st range	2nd range	3rd range	4th range
18.5 m. m. Needle Gap	Bar. 29.03 in. Temp. 21.5 deg. cent	18.5	354 710 354 222 205 112 205 615 298 850 93 700 205 205 55 240 280 850 75 75 370 93 175	Max. 850 Min. 55 Av. 323	48	30	0	22
15 m. m. Needle Gap	Bar. 29.03 in. Temp. 22 deg. cent for 1st 31 tests. Rel. humidity 22 per cent Bar. 29.3 in. Temp. 18 deg. cent. for next 10 tests. Rel. humidity 55 per cent Bar. 28.5 in. Temp. 21 deg. cent. for last tests.	4.5	140 10 25 210 13 120 27 18 50 34 120 100 73 18 18 18 145 160 170 272 236 244 226 226 280 122 258 240 218 140 140 23 172 191 145 23 204 18 272 23 245 145 45 136 222 182 182 163 172	Max. 280 Min. 13 Av. 132	32	18	26	24
15 m. m. Needle Gap Extra Sharp Points	Rel. humidity 28 per cent. Bar. 28.5 in. Temp. 18 deg. cent. for 1st 10 tests. Rel. humidity 55 per cent. Bar. 28.5 in. Temp. 21 deg. cent. for last tests.	4.5	181 90 14 136 41 75 115 81 213 107 100 127 226 204 236 23 27 272 23 250 264 82 240 130 226 182 190 226 168 190	Max. 272 Min. 14 Av. 147	17	33	20	30
15 m. m. Needle Gap Dulled Points	Rel. humidity 55 per cent. Bar. 28.5 in. Temp. 18 deg. cent. for 1st 10 tests. Rel. humidity 55 per cent Bar. 28.5 in. Temp. 21 deg. cent. for last tests.	4.5	86 14 90 158 14 208 100 82 45 59 177 18 9 13 13 23 23 32 9 14 172 19 218 18 172 226 18 13 132 100	Max. 226 Min. 9 Av. 75	53	20	7	20
12 m. m. Needle Gap	Bar. 29.05 in. Temp. 22 deg. cent. for first 20 tests. Rel. humidity 32 per cent Bar. 28.5 in. Temp. 22 deg. cent. for remaining tests.	1.75	4 10 7 5 19 4 7 42 10 7 3.5 42 7 7 31 21 2 4 9 4 5 16 4 7 9 5 27 62 35 43 5 4 14 9 60 43 9 9 46 32 5 35 4 43 26 21 16 50 64 26	Max. 64 Min. 2 Av. 16.1	57	10	16	8

*Time calibrations are given to show that the accuracy of time measurement is comparable at different lags.

†In many cases the lags occurred in definite zones and to show this in a comparative way the maximum lag was divided into four equal times the percentage of lags occurring in each of these ranges being given.

TEST DATA
TABLE IA
LAGS GIVEN IN SAME ORDER AS TESTS WERE MADE
VOLTAGE 22,000
*Needle to plane—Needle Negative

Spacing and Gap	Atmospheric Conditions	Time Calibration microsec. per m. m.	Time Lags in Microseconds	Max. min. Av.	Per cent Sparking			
					1st range	2nd range	3rd range	4th range
12 m. m. Gap †New Needles	Rel. humidity 55 per cent Bar. 29.3 in. Temp. 20 deg. cent. for 1st 25 tests. Rel. humidity 26 per cent Bar. 28.6 in. Temp. 20 deg. cent. for last 5 tests.	4.5	110— 9— 4—155— 9 4—100— 9— 9— 9 4— 9—140— 14— 9 Put in new needle 100—110—104— 86— 91 86—104—110—100—114 Put in new needle 32— 41—109— 66—133	Max. 155 Min. 4 Av. 62	41	7	41	11
†Extra Sharp Points	Rel. humidity 26 per cent. Bar. 28.6 in. Temp. 20 deg. cent.	4.5	126— 64— 32— 9—114 68—104— 86— 82— 32	Max. 126 Min. 32 Av. 74	20	10	40	30
†Dulled Points	Rel. humidity 26 per cent Bar. 28.6 in. Temp. 20 deg. cent.	4.5	14— 60—380— 23— 18 380— 9— 9— 36— 9	Max. 380 Min. 9 Av. 94	80	0	0	20

*Plane was a disk 2¼ in. in diameter.
†See Fig. 10 for degree of sharpness of points.

An examination of the tabulated results discloses the existance of time-lag zones, which indicates that breakdown is more likely to occur within these zones than outside. The existence of these zones is doubtful in some cases, while in others it seems well defined, as for instance at 75 kv. with a 95-mm. spacing (Table III).

In general, the tests show that dull needles give shorter time lags than sharp needles, although more tests should be made to be certain of the relationship.

Comparing the point-plane tests (Point negative, Table IA) with the needle points having the same spacing, the data show that the lags are of the same order of magnitude although the maximum lag with the point-plane is considerably greater than the corresponding value for the points. Tests made with the point positive (Table IB) show that the lag is less than two microseconds while with the point negative with the same spacing and voltage, an average lag of 62 microseconds was obtained. When the point was negative with a spacing of 13 mm. sparking occurred with approximately 50 per cent of the voltage applications. With the point positive a similiar condition was obtained with a spacing of 19 mm. These results give some conception of the effect of polarity on the lag of a point-plane gap.

Measurements of atmospheric conditions were made as a matter of record. Although no correlation with the variation in time-lag could be found, these factors

TEST DATA
TABLE IB
LAGS GIVEN IN SAME ORDER AS TESTS WERE MADE
VOLTAGE 22,000
Needle to Plane—Needle Positive

Spacing and Gap	Atmospheric Conditions	Time Calibration microsecond per m. m.	Time Lags in Microseconds	Max. Min. Av.	Per cent Sparking			
					1st range	2nd range	3rd range	4th range
18 m. m. gap	Humidity 57 per cent Bar. 29.3 in. Temp. 25 deg. cent.	18.5	465—465—130—205—335 700— 37—240—410—410 150—260—220—700	Max. 700 Min. 37 Av. 338	22	35	28	15
16 m. m. Gap New point	Humidity 26 per cent Bar. 28.6 in. Temp. 20 deg. cent.	4.5	61—210—116— 66—188 77— 33— 61—232— 66	Max. 232 Min. 33 Av. 111	11	56	0	33
Extra Sharp Point			221—116—183— 28— 44 80— 28—110—188	Max. 221 Min. 28 Av. 108	45	11	11	33
Dulled Point			100—200—300—222—338 360—210—188—288	Max. 360 Min. 100 Av. 236	0	10	45	45
14 m. m. gap	Humidity 57 per cent Bar. 29.3 in. Temp 25 deg. cent.	4.5	95— 9— 11— 13— 9 11— 13— 5— 9— 23 13— 18— 13— 18— 11	Max. 95 Min. 5 Av. 18.1	94	0	0	6
					If 95 point is included			
				(Av. 12.6 omitting 95 point)	7	43	29	21

TEST DATA
TABLE II
LAGS GIVEN IN SAME ORDER AS TESTS WERE MADE
VOLTAGE 58,000

Spacing and Gap	Atmospheric Conditions	Time Calibration microsecond per m. m.	Time Lags is Microseconds	Max. Min. Av.	Per cent Sparking 1st range	2nd range	3rd range	4th range
60 m. m. Needle Gap	Relative humidity 48 per cent Bar. 28.5 in. Temp. 21 deg. cent.	2.8	22—14—251—251—182 33—215—212—296—22 22—48—31—140—28 48—25—112—28—134 36—42—45—34—14 87—210—196—87	Max. 296 Min. 14 Av. 78	52	21	17	10
50 m. m. Needle Gap	Relative humidity 35 per cent Bar. 28.5 in. Temp. 20 deg. cent.	4.5	13—18—68—59—9 136—91—68—9—68 91—9—9—9—9 9—100—68—9—82 5—77—9—9—68 100—68	Max. 136 Min. 9 Av. 47	48	4	44	4
45 m. m. Needle Gap	Relative humidity 15 per cent Bar. 29.1 in. Temp. 21 deg. cent.	2.8	6—25—6—3—3 6—6—6—6—112 6—42—112—6—3 3—3—3—3—3 3—3—3—63—3 6—39—3—6—3	Max. 112 Min. 3 Av. 16.5	83	7	3	7

undoubtedly have a great effect. To determine the effect of these factors much greater variations are necessary than those which occur naturally.

Conclusions

An oscillograph is now available, as represented by that made by Dufour, by the use of which single transients may be photographed, without being limited by the inertia of a mechanical system. By its use, wave forms are shown in the paper having measureable oscillations up to 20,000 kilocycles. The authors have worked with an oscillator frequency of 250 kilocycles which allows the registration of a frequency of 100,000 kilocycles. As the frequency increases, the problem of the characteristics of the circuit used become increasingly important and great difficulty is experienced in keeping the oscillograph circuits free from disturbances emanating from the main impulse circuit. The cathode ray oscillograph, as used here, becomes a tool of the greatest value in the study of transient phenomena.

The lag tests, with constant voltage on the needle gaps, show that the lags vary between wide limits, the average lag increasing with increased gap settings. The limits could probably be narrowed considerably by the use of careful control of air and electrode conditions. The per cent overvoltage, required to keep the lag to two microseconds or less, decreases as the gap spacing increases. The lag is shown to depend on the condition of the needle, the dull needle tending to have the shorter lags.

The authors are continuing the use of the oscillograph, intending to apply it to the study of transients on

TEST DATA
TABLE III
LAGS GIVEN IN SAME ORDER AS TESTS WERE MADE
VOLTAGE 75,000

Spacing and Gap	Atmospheric Conditions	Time Calibration microsecond per m. m.	Time Lags in Microseconds	Max. Min. Av.	Per cent Sparking 1st range	2nd range	3rd range	4th range
95 m. m. Needle Gap 70 per cent sparking at this setting	Relative humidity 14 per cent Bar. 29.1 in. Temp. 22 deg. cent.	8.5	120—21—88—42—25 491—406—34—340—466 339—440—400—460—400	Max. 491 Min. 21 Av. 260	40	0	14	46
80 m. m. Needle Gap	Relative humidity 14 per cent Bar. 29.1 in. Temp. 22 deg. cent.	5	35—45—280—240—250 35—40—30—235—30 35—30—35—35—335 235—40—15—210—230 300—200—250—45—30 35—35—40—35—210	Max. 335 Min. 15 Av. 120	58	0	28	14
72 m. m. Needle Gap	Relative humidity 26 per cent Bar. 28.8 in. Temp. 19 deg. cent.	2.4	38—38—43—48—26 43—29—38—21—48 29—45—29—29—29 29—2—17—55—60 31—53—41—31—31	Max. 60 Min. 2 Av. 35	4	48	28	20

transmission lines due to lightning and other causes. The breakdown of insulation and the operation of lightning arresters is also being investigated. Acknowledgment is gladly given to the work of Alexander Dufour, who constructed the oscillograph used by the authors in the work described in the paper.

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Discussion

Swampscott, Mass., May 9, 1925

C. H. Dagnall: I should like to ask Mr. McEachron what method he used to have the specified amount of gas—inert gas—inside of a tube to concentrate a stream of electrons into a point. It has been found that when a definite amount of inert gas is present in the cathode-ray tube, due to ionization of this gas, the electrons flowing are concentrated into a very fine stream, instead of spreading out due to repulsion between the electrons and the stream itself. I should like to know what method Mr. McEachron used to produce this definite amount of gas.

H. B. Smith: I think this paper of Mr. McEachron's deserves discussion, and, in my own case, appreciative discussion, because in 1914 we attempted at Worcester to carry out a little of the fundamental research on dielectrics which Professor Adams pointed out the other day as important, as a basis for cable design as well as other things. We wanted to see what could be determined with regard to the analysis of dielectric phenomena occurring when approaching the point of breakdown of the dielectric. We wanted to get polar oscillograms of half waves of current and voltage on a sample of dielectric just previous to the point of breakdown, in order that we might analyze the phenomena going on in the dielectric under that condition.

Now, our first attempt, beginning about 1914, although recognizing the inherent difficulties of any instrument involving inertia, was with the application of the Einthoven galvanometer, which minimizes inertia of parts for the current wave. An electrostatic instrument with a very fine metal filament deflected in an electrostatic field was used to get the voltage wave simultaneously, with a suitable photographic optical system, shutter, etc., so that we might obtain a half wave of both current and e. m. f. on a polar diagram of sixty cycles.

That was not the high speed with which Mr. McEachron has worked, but still it is fairly high, and we obtained very good polar diagrams under those conditions. But the diagrams were of such a character that the more we studied them, the more we were afraid of the inertia effect.

So the next step naturally taken up was the application of the cathode ray, and in that case, we made use of the Western Electric Company's development of the cathode-ray tube, and ran into just the difficulty mentioned by Mr. McEachron with regard to getting a satisfactory photographic impression at that speed. We couldn't do it. We had just about reached that point when we learned of the Dufour instrument and attempted to make use of that. This was around 1917, and the war conditions made it impossible to continue the work.

However, experience with this instrument shows that we can secure such records as are needed for half periods on samples of dielectric, under the unstable conditions approaching breakdown. It offers opportunity for this in a most promising fashion.

W. L. Smith: I was wondering how far it is possible to carry this speed before one runs into the difficulty of the change of inertia of the electron with its speed, limiting the speed of the phenomenon to be measured.

Saratoga Springs, N. Y., June 26, 1925.

E. E. F. Creighton: In the early years of lightning arresters and protective devices development, I spent many hours with Dr. Steinmetz speculating on what might be taking place in the earlier parts of our artificial lightning discharges. We could estimate, in a way, what extremely high frequencies might be brought into existence—superposed on our lightning frequencies which were, as a matter of fact, carried as high as 5,000,000 cycles per second—but there was no assurance that these superposed higher frequencies actually existed to any appreciable degree. The mathematical calculations of the resistance of the "skin effect," for example, indicated that such high frequencies would be damped out immediately. These puzzles of more than twenty years' standing have been carried in our minds. Therefore, to me, personally, it is a most peculiar pleasure to look on these oscillograms of records of millions of cycles per second and see the complete solution of our speculative problem.

I wish to make one correction as to the authors' credit given to Mr. Duddell in the matter of the early work on the oscillographs. There are three men who stand preeminent in the development of the oscillograph—Blondell in France, Duddell in England, and our good friend, Louis Robinson, in America.

André Blondell is, without doubt, the father of the oscillograph. In 1898 I passed through London on my way to Paris and spent a day with Mr. Duddell. He was just getting his oscillographic work under way at that time. Upon arrival in Paris I found that M. Blondell had had his oscillographs in use for a long time. He had both the magnetic-needle type and the bifilar type which is in such universal use today. The practical oscillographic apparatus which all of us know and find so convenient to use is entirely the work, one might say, of Mr. Robinson.

L. R. Golladay: I was interested in the author's investigation of the impulse circuit resistance to give most nearly a vertical wave front. It seems to me that a moderate amount of overshoot with a steeper wave front might not be objectionable. If the frequency of oscillation were high enough, the effects on the time lag of the alternate half-waves would, at least approximately, cancel out.

The authors conclude that the percentage of over-voltage required to keep the lag at two microseconds or less, decreases as the gap spacing increases. This conclusion must be based on data not included in the paper. It is to be inferred from Pedersen's work that the percentage of over-voltage to keep the time lag constant with increasing gap length would increase.

I have referred above to the work of P. O. Pedersen who has published two important papers on spark lag in the *Annalen der Physik*. His method makes use of the velocity of spreading of Litchtenberg's figures, and of the velocity of propagation in conductors, to measure time lags of the order of 10^{-8} seconds. The present paper indicates that very careful work is required to obtain consistent results in measuring times as short as those measured by Pedersen. A few of the latter's conclusions are as follows:

(a) For constant gap length, the time lag decreases as the excess voltage increases. A similar conclusion is reached in the present paper.

(b) For constant surge voltage, the time lag decreases to zero as the gap length is shortened.

(c) Effects of electrode shape.

1. For equal gap lengths, needles and points are faster than sphere-gaps.

2. For equal time lags, a needle gap is about one-third longer than a sphere-gap.

3. With a gap of 3 mm., and about 100 per cent excess voltage, the time lag of a 10-mm., diameter sphere-gap was found to be 10.5×10^{-8} sec., and of a needle gap, 5.6×10^{-8} sec., at the same voltage.

4. The time lag of a gap is determined almost entirely by the shape of the anode, and the cathode has practically no effect. This accounts for the results obtained with a gap between a point and a plane in the present paper.

(d) A minimum time lag is obtained with clean electrodes, which may be multiplied by five or more by contamination of the electrodes surface. The effect of a spark is to increase the time lag for subsequent sparks due to corrosion of the electrode surface.

(e) Time lag depends more upon the ratio of impressed voltage to breakdown voltage, than upon gap length, although there is a slow increase with increasing gap length.

The above results were obtained with relatively short gap lengths—usually of a few millimeters. It is interesting to note that Pedersen found that the time lag is more affected by the electrode surface than by the electrode shape.

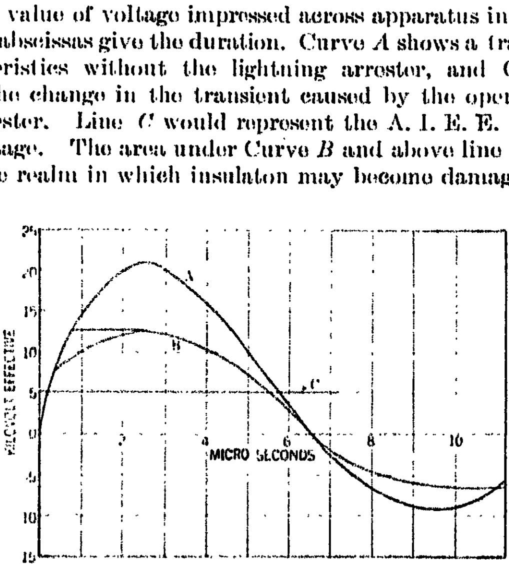
In the course of lightning-arrester development work, the writer had occasion to develop a means for measuring time lag of insulator flashover, breakdown of spark-gap, and discharge of lightning arresters. Satisfactory answers to these problems were obtained with a "timing" circuit consisting of a condenser and resistance connected in series and shunted across the apparatus to be timed. The peak voltage of the condenser is measured by a small sphere-gap in the case of long time lags, or by a klydonograph in the case of short time lags. With the assumption that the voltage impressed on the timing circuit has a rectangular wave shape, the time is given by the formula $t = R\,C \log_e \dfrac{E}{E - e}$, where R and C are the constants of the timing circuit, E the surge voltage, and e the condenser voltage. With this method we have obtained a figure of 2.7 microseconds for the time lag of a 25-kv., pin-type insulator with 150 kv. impressed. The time lag of a 25-cm., sphere-gap set at 8.5 mm., was 0.24 microseconds with 36 kv. While the sphere-gap is fast, it is not instantaneous. It has a measurable time lag.

As a matter of interest some tests were made under one of the conditions of the present paper, a 12-mm. needle gap at 22 kv. A 0.1-microfarad condenser supplied the surge through a 675-ohm resistor. The results obtained were: minimum time lag, 0.83; average 3.3; and maximum 13.8 microseconds. These results are of the same order but somewhat lower than those of the paper. I believe that the discrepancy is due to differences in tests conditions. The results with the needle gap were quite erratic compared to those obtained with sphere-gaps. The major inconsistencies are probably due to microscopic differences in the needles. The smaller differences are encountered when testing with sphere-gaps and are probably due to minor changes in test conditions.

E. E. Burger: One of the most important uses of the cathode-ray oscillograph at the present time is to study the operation of lightning arresters. Heretofore, we have had to rely chiefly upon sphere-gaps for measurements but since only maximum values could be measured and the time element was not considered, questions of uncertainty always accompanied the measurements. By applying this oscillograph to lightning-arrester test circuits a new field of information has been opened for study.

In the study of lightning arresters, there are two important points that we wish to consider from the protection standpoint. First, how much above the insulation test voltage do the arresters allow the voltage of the transient to go, and second, what is the duration of this over voltage? Most insulation has a test voltage of approximately two times normal for one minute. Studies of the mechanism of insulation breakdown seem to indicate that insulation failure depends upon an overvoltage-time characteristic.

The accompanying curves show two cathode-ray oscillograms transcribed into rectangular coordinates. The ordinates represent the value of voltage impressed across apparatus insulation, and the abscissas give the duration. Curve A shows a transient's characteristics without the lightning arrester, and Curve B shows the change in the transient caused by the operation of the arrester. Line C would represent the A. I. E. E. value of test voltage. The area under Curve B and above line C represents the realm in which insulaton may become damaged.



Oscillograms Showing Rise of Voltage (A) Without and (B) With Lightning Arrester.

Now the question is: what kinds and shapes of transients shall we apply to arresters? In the laboratory it is possible to produce transients of almost any wave front, voltage or duration, but we do not know how these transients compare with what we actually get in practise. This is one of the most important points we should know, and possibly through the use of this oscillograph or other instruments, we may be able to determine some of these factors.

L. T. Robinson: In connection with this instrument, it is interesting to recall that as early as 1903 a device making use of the cathode-ray tube for obtaining photographic records of electrical wave forms was developed and described by our past-president, Harris J. Ryan.[2]

K. B. McEachron: No direct determination of the correct amount of residual gas is made. The condition of the vacuum for proper focusing is determined by observing the appearance of the cathode stream. To be able to judge properly the size of the spot in this manner requires some experience on the part of the operator. The gage shows a pressure of from 2 to 6 microns, but the gage indication is not depended upon for determining the proper vacuum condition.

The question of limitation of the oscillograph raised by Professor W. L. Smith is an interesting question. The electron speed corresponding to the voltage used is about one-third that of light. At this speed the limitation does not depend on the velocity of the electrons but rather on means of drawing out the wave along a time axis.

We have used oscillator frequencies as high as 1,000,000 cycles, detecting frequencies on the unknown transient of more than 100,000,000 cycles. Still higher frequencies may be studied but the sweeping rate becomes so high that it is difficult to time the phenomena properly. For most work it is doubtful if much is to be gained by going to frequencies higher than this.

Concerning the effect of spacing on the percentage of overvoltage required to keep the lag to less than two microseconds, mentioned by Mr. Golladay, the detailed data were not included in the paper, but the data for three different spacings were given

1. The Cathode-Ray, Alternating-Wave Indicator, by Harris J. Ryan, A. I. E. E. Transactions, Vol. XXII, 1903, page 539.

in the text near the end of the paper. The results in the table are for lags of two microseconds or longer.

The results obtained by Pedersen are of interest, but as Mr. Golladay states, they were obtained on relatively short-gap lengths with which electrode conditions are of greater importance than with larger gaps.

The time lags on the insulation and sphere-gap as given by Mr. Golladay are of note, but the results will depend considerably on the wave shape applied. Considerable error may be introduced when using this method by assuming a perpendicular wave form for calculation purposes.

Mr. Golladay used a much smaller sphere-gap setting than is standard for 25 cm. spheres which is probably the reason for the rather long time lag given. Peek[3] has shown that considerable lag may be expected with gaps much smaller than $0.3\sqrt{R}$, where R is the radius of the sphere in centimeters.

Oscillographic Solution of Electromechanical Systems

BY C. A. NICKLE[1]

Associate, A. I. E. E.

Synopsis.—*A simple and practical method has been developed for investigating certain important classes of dynamic systems, as represented, for instance, by a power system comprising synchronous or induction generating units, with prime movers, connected through transmission lines to receiving apparatus of the same character. The behavior of individual units, of course, can also be investigated,—such as the important case of determining the current pulsation of a synchronous motor driving a reciprocating compressor, or purely mechanical systems involving moving masses and resilient members, such as beams, bus bars, etc., under the influence of suddenly applied load. In the present paper, the method is described and its application to a few of the possible cases is illustrated.*

The method is to have an "equivalent electrical circuit" solve the problem, and the oscillograph plot the solution. The idea is based on the fact that if the differential equation for an electric circuit is identical with that for the dynamic mechanical system in question, then the corresponding electrical quantities can be taken to represent quantitatively the actual mechanical quantities. The equivalent circuit can be easily set up, and oscillographic records of the voltages and currents constitute the plotted solutions.

The chief value of the scheme lies not only in one's being able to easily solve a given complicated problem, but also in the facility with which the effect of change in design factors may be determined.

*The method is capable of considerable extension. Its application is limited only by the extent to which circuit elements of the proper characteristics can be found. The treatment given here considers only those cases which involve the circuit elements L, C, and R, and in which these are constant.**

MATHEMATICAL analysis of mechanical and electro-mechanical dynamic systems, such as, for instance, a modern power system as a whole, or individual power units, becomes complicated very rapidly as the number of degrees of freedom is extended. When the limit of practical mathematical solution is reached, it is possible to resort to graphical methods which may be effectively employed within a limited field. Such methods were carried to unusual limits in a recent investigation by Booth and Bush[2] which demonstrated the efficacy, and also indicated the limitations, of those methods. Many practical power systems, nevertheless, extend far beyond such limits, yet the importance of their solution is certainly no less. On the contrary, it is perhaps greater.

A simple method has been developed for investigating systems of this general character. It involves the use of the oscillograph in connection with electrical circuits in which the electrical quantities represent the mechanical quantities of the actual system. In any given problem the procedure is to sketch out the equivalent circuit according to definite principles, merely by an inspection of the diagram of the actual system. The equivalent circuit, described later, usually comprises batteries, inductances, capacities, and resistances of appropriate values. Oscillographic records of the currents and voltages in the various circuit branches, following some disturbance, constitute the *plotted solutions* of the differential equations of the system—plotted against time to a definitely known scale of torque, speed, displacement, power, etc. The method thus brings within practical reach the solution of extremely complicated problems relating not only to power systems but also to *any* system for which an equivalent circuit can be set up.

HISTORICAL

The idea of analogous systems has been employed for many years as an aid in visualization, explanation, and mathematical analysis. The same differential equation may represent many different natural phenomena; which means, of course, in each case, that the quantities expressed vary with respect to each other in precisely the same manner. Thus, analogies follow, and electrical engineers have been quick to utilize them in the analysis of many problems. Arnold[3] has extended such analogies to the use of equivalent electrical circuits in the mathematical analysis of problems. While this usually affords a clearer conception of the problem, and may sometimes facilitate the handling of mathematical expressions, nevertheless, the expressions still remain to be handled.

In the present paper, the idea is carried a step further. The mathematical processes are eliminated altogether by having the electrical circuit solve the problem, and

1. Of the General Electric Co., Schenectady, N. Y.

2. Power System Transients, JOURNAL, A. I. E. E., March, 1925, p. 229.

Presented at the Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925, and at the Annual Convention, Saratoga, N. Y., June 22-26, 1925.

3. *Die Wechselstromtechnic*, Vol. IV, page 379.

the oscillograph plot it. So, instead of tedious equations perhaps hopeless of solution even with such aid in mathematical treatment as the analogous circuits may give, one has available in the present oscillographic method, practical and simple facilities for analysis.

PREMISES

The fundamental basis for the method is that the differential equations for the equivalent circuit are of exactly the same form as those for the actual system. Since the equations are of the same form, the electrical quantities which occur in the equations for the equivalent circuit may be considered to represent the physical quantities which occur in the equations for the actual system. As will be shown later, if we choose inductance to represent mass, and electrical charge to represent displacement or length, time being common to both systems, it will be found that the differential equations are of exactly the same form, and, furthermore, that the various phenomena occurring in the physical system are represented by quantities in the equivalent circuit which are convenient to measure, that is, voltage and current.

Assuming for the present that these representations are the most advantageous, Table I can at once be constructed.

TABLE I

Mechanical System				Electrical System	
mass	M			$\mathfrak{L}$	inductance
length	L			Q	charge
time	T			t	time
Derived quantities		Dimensions	Dimensions		
force	F	$M L T^{-2}$	$\mathfrak{L} Q t^{-2}$	E	voltage
velocity	V	$L T^{-1}$	$Q t^{-1}$	I	current
damping constant	K_d	$\frac{F}{V}$	$\frac{E}{I}$	R	resistance
resilience constant	K_r	$\frac{L}{F}$	$\frac{Q}{E}$	C	capacity

In general, however, it is not desirable to use Table I for quantitative representation, since this would often require very large and impracticable electrical units. In order that the electrical units shall be of reasonable magnitude, we can let one unit of mass be represented by a units of inductance, one unit of time in the physical system by b units of time in the electrical system, and one unit of length by c units of charge. Then, by arbitrary choice of the conversion factors a, b, and c, the order of magnitude of the quantities in the electrical circuit can be made any desired value. Introducing the conversion factors, Table II may be constructed.

In the differential equations for systems in rotation, the moment of inertia corresponds to mass in systems in translation, angle corresponds to length, and time is the same for both systems. Hence the table for systems in rotation may be written at once, as shown in Table III.

TABLE II

Mechanical System		Conversion Factors[4]	Electrical System	
mass	M	a	$\mathfrak{L}$	inductance
length	L	c	Q	charge
time	T	b	t	time
Derived quantities				
force	F	$\frac{a\,c}{b^2}$	E	voltage
velocity	V	c/b	I	current
damping constant	K_d	a/b	R	resistance
resilience constant	K_r	b^2/a	C	capacity
power	$F\,V$	$\frac{a\,c^2}{b^3}$	$E\,i$	power

TABLE III

Rotational System		Conversion Factor	Electrical System	
moment of inertia	I	a	$\mathfrak{L}$	inductance
angle	θ	c	Q	charge
time	T	b	t	time
Derived quantities				
torque	$\mathfrak{J}$	$\frac{a\,c}{b^2}$	E	voltage
angular velocity	ω	c/b	i	current
damping constant	$\mathfrak{J}_d$	a/b	R	resistance
resilience constant	$\mathfrak{J}_r$	b^2/a	C	capacity
power	$\mathfrak{J}\,\omega$	$\frac{a\,c^2}{b^3}$	$E\,i$	power

In the present treatment, only physical systems, in translation or rotation, in which existing forces are proportional to $\frac{d^2 x}{d t^2}$, $\frac{d x}{d t}$, and x, will be considered, where x represents length or angle; in other words, only physical systems which may be expressed by linear differential equations will be taken up.

A composite system, of which the component parts are expressible by linear differential equation, is itself expressible by a linear equation of higher order and may be represented by a composite electrical circuit.

Obviously the method is not limited to problems in mechanics alone, but may be used as a means of solution of any problem which may be expressed by linear differential equations. However, the method is not necessarily limited to linear systems only. If electrical circuits can be devised which have characteristics varying in exactly the same manner as the characteristics of the system to be solved, then the method is

4. (Mechanical quantity) × (conversion factor) = (electrical quantity). Thus, $C\,L$ feet = Q Coulombs, $V\,c/b$ feet per sec. = amperes, etc.

applicable. In general, this would require resistances, capacities, and voltages which should be variable in accordance with certain prescribed laws. This might, in many cases, be entirely feasible, as is demonstrated in the representation of governor action described in Appendix A.

Equivalent Circuits

Fig. 1 illustrates some elementary physical systems and their equivalent electrical circuit.

A brief inspection of the familiar differential equations for the various cases shows that they are of exactly the same form and, hence, their solutions must be of the same form.

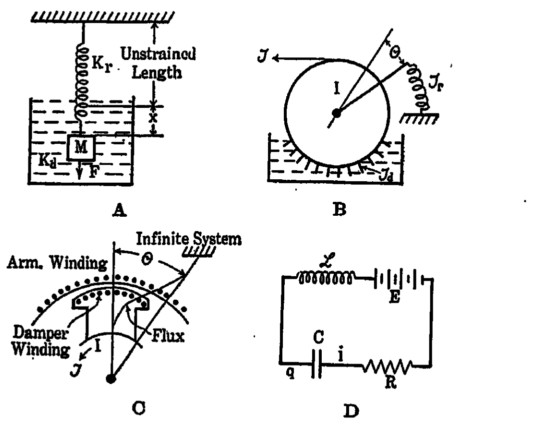


FIG. 1—SYSTEMS REPRESENTED BY THE SAME DIFFERENTIAL EQUATION

(A)

$$M\frac{d^2x}{dt^2} + K_d\frac{dx}{dt} + \frac{1}{K_r}x = F$$

$$M\frac{dV}{dt} + K_d V + \frac{1}{K_r}\int V\,dt = F$$

(B)

$$I\frac{d^2\theta}{dt^2} + \mathfrak{J}_d\frac{d\theta}{dt} + \frac{1}{\mathfrak{J}_r}\theta = \mathfrak{J}$$

$$I\frac{d\omega}{dt} + \mathfrak{J}_d\omega + \frac{1}{\mathfrak{J}_r}\int\omega\,dt = \mathfrak{J}$$

(C)

$$I\frac{d^2\theta}{dt^2} + \mathfrak{J}_d\frac{d\theta}{dt} + \frac{1}{\mathfrak{J}_r}\theta = \mathfrak{J}$$

$$I\frac{d\omega}{dt} + \mathfrak{J}_d\omega + \frac{1}{\mathfrak{J}_r}\int\omega\,dt = \mathfrak{J}$$

(D)

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,t\,d = E$$

Case A represents a mass and spring with one end of the spring attached to an immovable support, and having the mass constrained to move in a bath which is assumed to give a damping force proportional to the velocity of the mass M. Comparison of the equations for Case A and Case D shows at once that mass in the physical system is represented by the inductance in the electrical circuit, and length, by charge, time being the same for both systems. From Table I, force should be represented by voltage, the damping constant by resistance, the resilience constant by capacity, and velocity by current. These relations all exist respectively in the two sets of equations, and therefore, Case D is a true representation of Case A. A sudden application of force on the mass M is given by a sudden application of voltage. The voltage across the inductance then gives the inertial force of the mass; the voltage across the condenser C is a measure of its charge, and thus gives both the displacement of the spring and the displacement force; the voltage across the resistance R gives the damping force of the liquid on the mass; and the current in the circuit gives both the velocity of the mass and the rate of change of displacement between the two ends of the spring, these velocities being the same since the support is immovable.

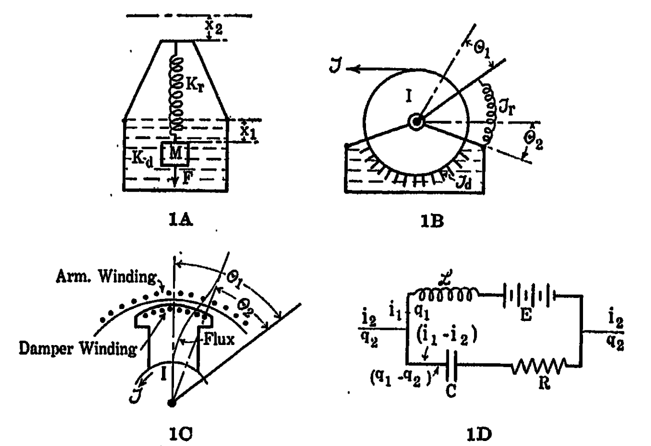


FIG. 2—SYSTEMS REPRESENTED BY THE SAME DIFFERENTIAL EQUATION

(A)

$$M\frac{d^2x_1}{dt^2} + K_d\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + \frac{1}{K_r}\left(x_1 - x_2\right) = F$$

$$M\frac{dV_1}{dt} + K_d\left(V_1 - V_2\right) + \frac{1}{K_r}\int\left(x_1 - x_2\right)dt = F.$$

(B)

$$I\frac{d\theta_1}{dt^2} + \mathfrak{J}_d\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) + \frac{1}{\mathfrak{J}_r}\left(\theta_1 - \theta_2\right) = \mathfrak{J}$$

$$I\frac{d\omega_1}{dt} + \mathfrak{J}_d\left(\omega_1 - \omega_2\right) + \frac{1}{\mathfrak{J}_r}\int\left(\omega_1 - \omega_2\right)dt = \mathfrak{J}$$

(C)

$$I\frac{d^2\theta_1}{dt_2} + \mathfrak{J}_d\left(\frac{d\theta_1}{dt} - \frac{d\theta_2}{dt}\right) + \frac{1}{\mathfrak{J}_r}\left(\theta_1 - \theta_2\right) = \mathfrak{J}$$

$$I\frac{d\omega_1}{dt} + \mathfrak{J}_d\left(\omega_1 - \omega_2\right) + \frac{1}{\mathfrak{J}_r}\int\left(\omega_1 - \omega_2\right)dt = \mathfrak{J}$$

(D)

$$L\frac{d^2q_1}{dt^2} + R\left(\frac{dq_1}{dt} - \frac{dq_2}{dt}\right) + \frac{1}{C}\left(q_1 - q_2\right) = E$$

$$L\frac{di_1}{dt} + R\left(i_1 - i_2\right) + \frac{1}{C}\int\left(i_1 - i_2\right)dt = E$$

Similar reasoning applies to Case B, except that the electrical quantities are interpreted in terms of rotation instead of translation.

Case C represents a synchronous machine connected to an infinite system. In this case the voltage across the condenser gives the synchronous torque of the machine due to the displacement of the rotor from the

terminal voltage; the charge on the condenser gives the angular displacement of the rotor from the phase of the terminal voltage; the voltage across the resistance gives the damping torque due to the amortisseur winding, and the current gives the velocity of the rotor with respect to the terminal voltage, that is, the departure of the rotor from synchronous speed. It should be noted that the assumption of direct proportionality between synchronous torque and displacement angle has been made. Although this is not rigorously true, nevertheless for small angles the assumption is, of course, justifiable.

If the capacity in the circuit is made infinite, the synchronous torque becomes zero, leaving only the damping torque. *The circuit is then evidently the representation of an induction motor.*

In the more general case, the supports are not immovable, but also have a velocity and displacement which are functions of time. Fig. 2 illustrates this condition.

Inspection of the respective differential equations shows that they are of exactly the same form as those for the equivalent circuit. Hence, for Case A and Case D the voltage across the inductance gives the inertial force of the mass; the current in the inductance gives the absolute velocity of the mass; the voltage across the condenser, the displacement force of the spring; the charge on the condenser, the displacement

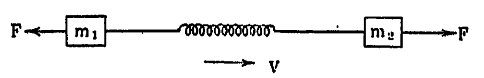


Fig. 3

between the two ends of the spring; the voltage across the resistance, the damping force acting on the mass; the current in the condenser, the rate of change of displacement of the two ends of the spring; and the current in the circuit terminals, the speed of the end of the spring.

The reasoning for Case B is the same except that the electrical quantities are interpreted in terms of rotation instead of translation.

For Case C, the velocity of the rotor is represented by the current in the inductance; the velocity of the terminal voltage is represented by the current in the circuit terminals; the displacement of the rotor from the bus is given by the charge on the condenser; the synchronous torque, by the voltage on the condenser; the damping torque, by the voltage across the resistance; and the relative velocity of the rotor with respect to the terminal voltage, by the current in the condenser.

The characteristics of a transmission line are such that the power transmitted over the line, and the torque corresponding to it, are proportional approximately to the sine of the angle of displacement between the terminal voltages. Rigorous representation of such a characteristic would require a condenser the capacity of which is a sine function of the charge. It has been found, however, that for displacements of the magnitude customarily encountered in practise, the torque may be taken as proportional to the angular displacement[5].

For such an assumption, the transmission line will evidently be represented in the equivalent circuit by a condenser, just as in the case of the spring.

In certain cases, such as power systems, the variation in speed of the component parts may be of relatively small magnitude compared with the total speeds. Since motion is purely relative, any constant value of speed may be taken as a reference. The actual choice of this reference will be largely determined by the nature of the problem under consideration. If every part of the system is initially moving at the same speed, the logical reference point is this initial speed. For such a case, the relative initial speed of each component part with respect to this reference is, of course, zero, and since current in the equivalent circuit represents speed in the actual system, the initial currents in every part of the equivalent circuit must be zero.

Now, for constant speed in the actual system, Σ torques acting on the rotor of each unit must be zero. These conditions impose on the equivalent circuit that

(a) all initial currents must be constant and equal to zero, and

(b) Σ voltages acting on the individual inductances must be zero.

As an illustration, let Fig. 3 represent a mechanical system in equilibrium, the entire system traveling at an absolute velocity V. To an observer also moving at the velocity V, the system will, of course, appear stationary; that is, with respect to the velocity V as a reference, the relative velocity of the entire system is zero. Now, if one of the forces is suddenly changed to a new value, the observer at velocity V will note certain velocities in the various parts of the system as functions of time, while to a stationary observer every velocity will be different by V. Evidently, therefore, the speed of the observer will have no effect upon the other phenomena such as forces, relative displacements in the system, etc.

For systems initially in motion, it is thus possible to obtain an equivalent circuit in which the currents all differ from the actual equivalent currents by a constant, without changing the performance of the circuit.

Use of this principle is made in developing a representation of certain prime mover torque-speed characteristics.

Fig. 4 shows a typical torque-speed characteristic for a waterwheel with full load gate opening held constant. If the waterwheel is initially operating at point C, then for considerable variations in speed, the characteristic is essentially the straight line A tangent to the

5. The relation between power and angular displacements between the voltages at the ends of a transmission line is shown in the paper by Bush and Booth, on Power System Transients, Journal, A. I. E. E., March, 1925, Vol. XLIV, No. 3, page 233.

actual characteristic at C. The equation of line A is

$$\mathfrak{T} = \mathfrak{T}_0 - Ks \qquad (1)$$

where

$\mathfrak{T}$ = torque at any speed s

$\mathfrak{T}_0$ = intercept of line A with the Y axis

and

K = the numerical value of the slope of the line A.

In accordance with the foregoing, since it is desirable in the interest of accuracy to choose some speed other

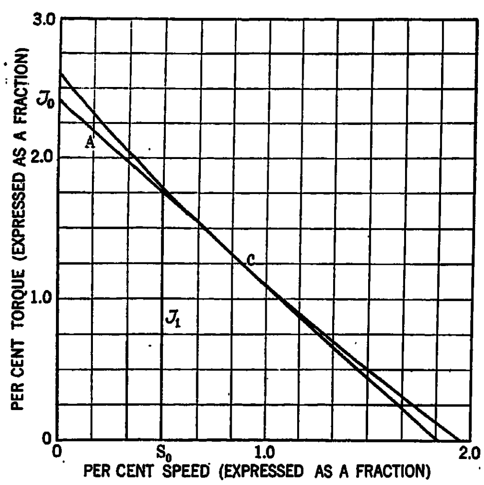


FIG. 4—SPEED-TORQUE CHARACTERISTIC

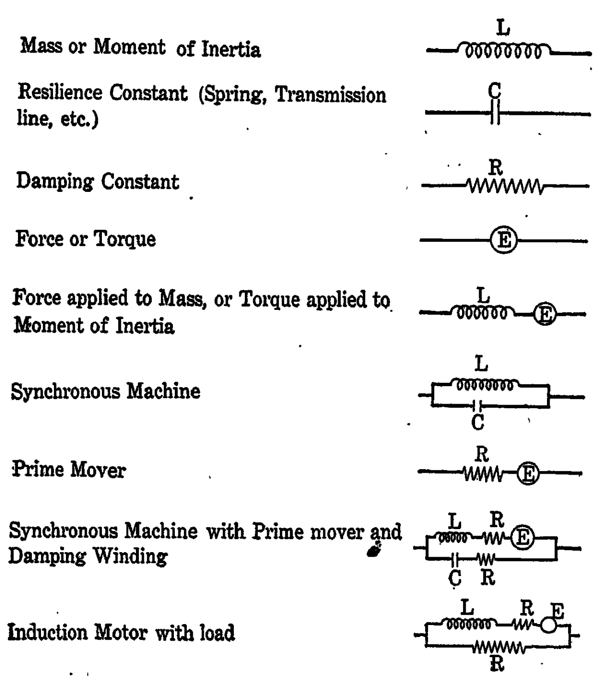


FIG. 5—EQUIVALENT CIRCUITS

than zero as the reference speed, let s_0 be the reference speed with respect to which other speeds are to be measured. Then, since the slope of the line is of course not changed by the choice of reference, the general equation is

$$\mathfrak{T}' = \mathfrak{T}_1 - K(s - s_0) \qquad (2)$$

where $\mathfrak{T}'$ = the torque at any relative speed $(s - s_0)$

and $\mathfrak{T}_1$ = the torque when the relative speed is zero.

That is,

$\mathfrak{T}_1$ = the ordinate of the line A at the reference speed s_0.

In the equivalent circuit, torque is represented by voltage and speed by current. Hence, corresponding to Eq. 2, the prime mover torque at any relative speed will be represented in the equivalent circuit by

$$e = E - Ri \qquad (3)$$

The quantity K, or the slope of the torque speed characteristic, is thus represented in the equivalent circuit by a resistance.

For other gate openings, the slope of the line A and its intercept with the Y axis will have other values. Under actual conditions, the governor will change the gate opening as a certain function of time thus necessitating values of E and R in equation (3) which are also functions of time. An approximate method of obtaining the proper functions for these quantities is illustrated in the representation of the governor given in Appendix A. However, under transient conditions it requires from two to four seconds for the governor to function; hence, the phenomena may be investigated, at least approximately, during the first second or two under the assumption of constant gate opening.

In all equivalent circuits, the inductances, which are used to represent inertia, possess a certain amount of inherent resistance. This resistance, however, can always be considered as a part of the resistance required to represent the torque-speed characteristic. Oscillograms of voltage across the terminals of the various inductances to obtain inertial forces will include the small resistance drop due to the inherent resistance of the reactor. To obtain the true inductive voltage it is, of course, only necessary to subtract the resistance voltage at each instant from the total voltage.

The equivalent circuits for the individual elements of the system, such as generators, motors, transmission line, etc., have been established, and are shown in Fig. 5. It is now necessary to show how these must be connected in a combined, equivalent circuit to represent the system as a whole.

Referring to Fig. 6, let any number of synchronous and induction machines A, B, C, etc., including generators, motors and synchronous condensers, be connected to the bus m m. The conditions which must be satisfied are:

(1) Σ power flowing *to* point o must equal zero.

(2) The speed ω of all branches must be the same at the point o, being the synchronous speed of the bus.

(3) Therefore, Σ torques, corresponding to the various values of power, and to that speed, must be zero.

Thus, by (1),

$$\Sigma\, p = o = p_a + p_b + p_c + p_d + \ldots .$$

By (2)

$$\omega_a = \omega_b = \omega_c = \omega_d \ldots\ldots\ldots = \omega_0$$

By (3)

$$\Sigma\, T = O$$

Fig. 6B shows the equivalent circuits for the various

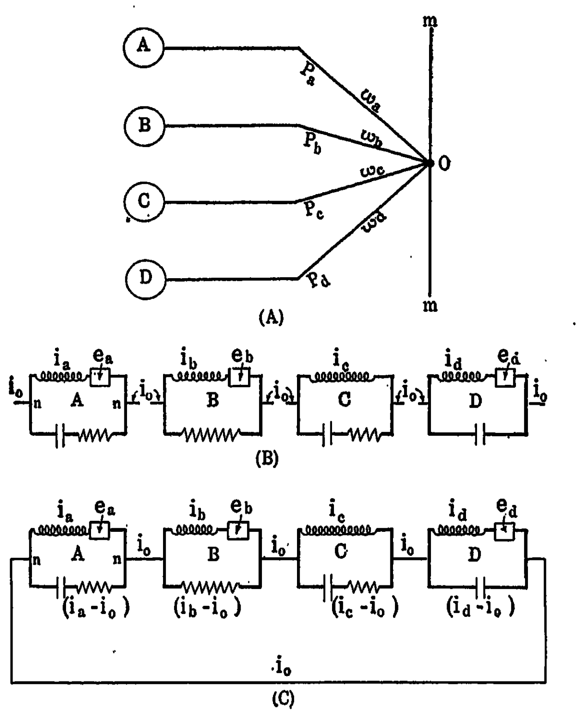


FIG. 6—SYNCHRONOUS AND INDUCTION MACHINES CONNECTED TO THE SAME BUS

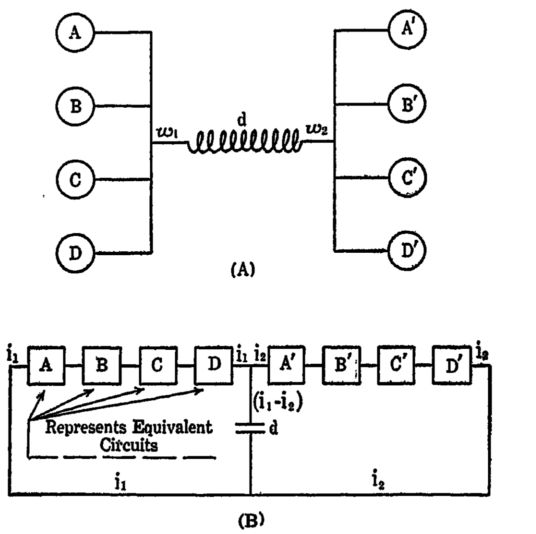


FIG. 7—EQUIVALENT CIRCUIT FOR TWO BUSSES CONNECTED BY A TRANSMISSION LINE

units A, B, C, etc. The speed of the terminal voltage—that is, the voltage at O, Fig. 6A—is represented by the current i_0. Since this must be the same for all units, the circuits must be connected in series; and since Σ torques must equal zero—the torque evidently being the voltage[6] between the junction points $n\ n$—it follows that Σ voltages must equal zero. Hence by Kirchoff's Law, the circuit must be closed, as in Fig. 6C. Therefore, *for any number of machines connected to a bus, as in Fig. 1A, the equivalent circuits of the various units should be connected in series*, as in Fig. 6C.

Thus, if the synchronous generator A were suddenly dropped from the system (by short-circuiting the points $n\ n$), the variation in the speed of the bus voltage, the instantaneous values of phase displacement, torque, speed, slip, etc. of the various machines can be read at once from the currents and voltages of the various branches. This is shown in more detail in the *Numerical Illustrations.*

Next, consider two such busses connected by a transmission line, as in Fig. 7A. The difference between the speeds ω_1 and ω_2 of the two-bus voltages is, of course, the rate at which the angular displacement between the two voltages is changing. Thus, in Fig. 7B, if i_1 and i_2 represent respectively those two speeds, then $i_1 - i_2$ would be the current to the condenser, which represents the spring-like characteristic of the line. That is, the charge on the condenser d represents the phase displacement between the ends of the line, and the current $(i_1 - i_2)$ is the rate at which it is changing. The connection must, therefore, be as in Fig. 7B. The rectangles A, B, C, and D in Fig. 7B represent the equivalent circuits as shown in Fig. 6B.

Consider next a system as shown in Fig. 8A. At the junction point or bus at G, the speed for all branches is the same, namely ω_2. There must therefore be a main series circuit, which connects the various individual circuits, and in which a current i_2 flows. Yet each bus will likewise have its own series circuit as in Fig. 6. The transmission lines a, b, and c will each be represented by a condenser which takes a current equal to the difference between i_2 and the current representing the speed of the particular bus. Thus, the component circuits are shown separately in Fig. 8B. The connections are completed by inspection, as in Fig. 8C. Thus the series circuit for each bus is shunted by a condenser representing the line which connects that bus to the common point at G, and the groups are then all connected in series.

Following the same reasoning, the equivalent circuit for the complicated system in Fig. 9A can be constructed. Here, each circle marked ω represents a bus with any number of units on it, such as in Fig. 6A. From the individual circuits shown in Fig. 9B as

6. The voltage $n\ n$ represents the electro-magnetic torque, and is obviously the impressed torque minus that consumed in acceleration:

$$T = T_m - I\,\frac{d\,\omega}{d\,t}; \text{ or, in the electrical circuit}$$

$$e_{nn} = e_a - L\,\frac{d\,i_a}{d\,t}.$$

sketched from inspection of Fig. 9A, the final connections are made as in Fig. 9C.

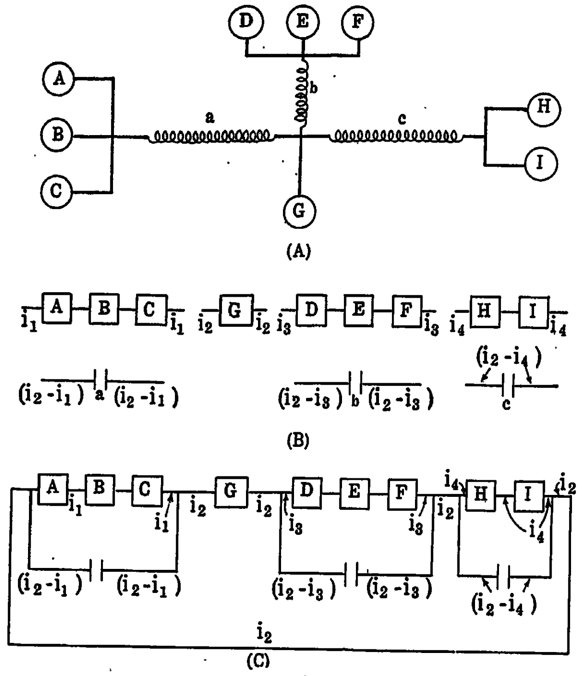


FIG. 8—EQUIVALENT CIRCUIT FOR A FOUR-BRANCH SYSTEM

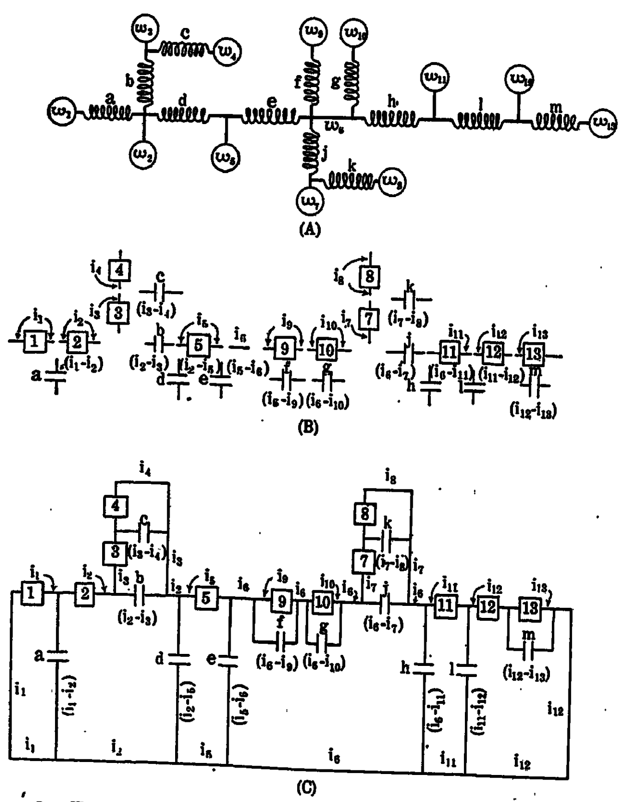


FIG. 9—EQUIVALENT CIRCUIT FOR AN EXTENDED SYSTEM

Likewise, the equivalent circuit is constructed for the network shown in Fig. 10.

Thus the guiding principles for setting up the equivalent circuits are:

1. For any number of machines connected in parallel to a bus, the equivalent circuits should be connected in series, as in Fig. 6C.

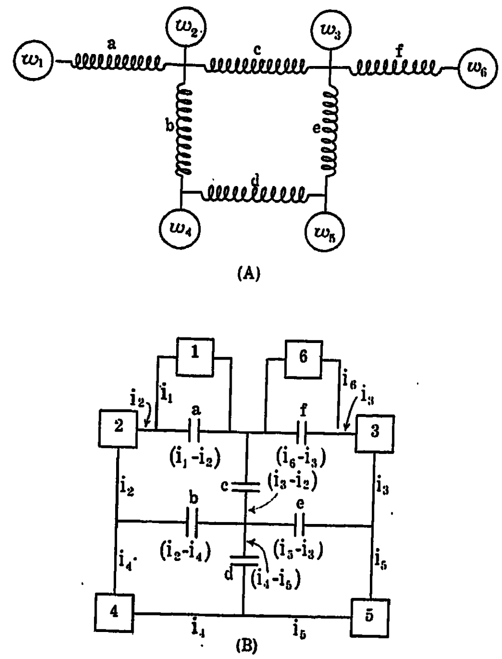
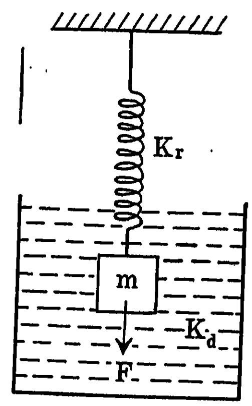
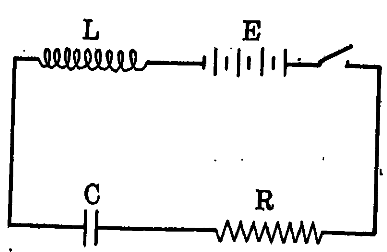


FIG. 10—EQUIVALENT CIRCUIT FOR A CLOSED SYSTEM



m = 0.1475 (4.75 lb.)
K_r = 0.111 ft. per lb.
K_d = 0.116 lb. per ft. per sec.
F = 4.53 lb.

a = 1.11
b = 0.0407
c = 0.0396



L = 0.164 henrys
C = 165 μf
R = 3.18 ohms
E = 120 volts

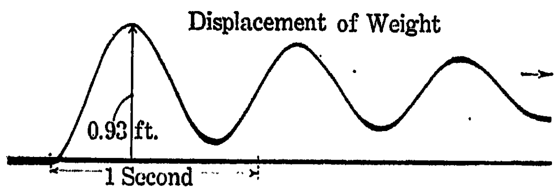
11A

2. If this bus is connected to another through a transmission line, or other inductance (the line or inductance thus being a series connection), the condenser representing the line should be connected in shunt as in Fig. 7B.

3. Condensers representing transmission lines must be always connected in such a way that the condenser current will be the difference between the two currents, representing respectively the speeds of the two buses which the line connects.

Numerical Illustrations

The differential equations, shown in connection with Figs. 1 and 2, are set up in terms of actual mechanical speeds and displacements. However, on account of the fact that the actual speeds of the various units comprising the system are different on account of the different numbers of poles, it has been found more convenient, in



11B

Fig 11—Case 1.

(A) Equivalent Circuit

(B) Oscillographic Solution

handling actual numerical examples, to reduce all quantities to the basis of a two-pole machine. This has been done in numerical examples here considered. The conversion factors are given in Table IV, Appendix B.

Case 1. Fig. 11A shows a mechanical system, consisting of a mass and spring suspended from a rigid support, having the mass m constrained to move in a damping bath. The constants of the mechanical system and the available values of L, C, and E for use in the equivalent electrical circuit are given.

The conversion factors are then obtained as follows:

$$a = \frac{\text{inductance}}{\text{mass}} = \frac{0.164}{0.1475} = 1.11$$

$$b^2/a = \frac{\text{capacity}}{\text{resilience constant}} = \frac{165 \times 10^{-6}}{0.111}$$

$$= 0.00149$$

From which

$$b = 0.0407$$

$$\text{also,} \quad \frac{a\,c}{b^2} = \frac{\text{voltage}}{\text{face}} = \frac{120}{4.53} = 26.5$$

Then,

$$c = 26.5\, b^2/a = 0.0396$$

Of the three constants, L, C, and R, if two of them are chosen arbitrarily, the third is thereby fixed. Since resistance values are easily obtained, it is convenient to take L and C as are available, and let the resistance be what is thus required.

The conversion factor for the damping constant is

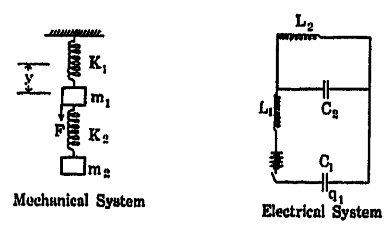
$$\frac{a}{b} = 27.3$$

Therefore,

$$R = \frac{a}{b}\, K_d = 3.18 \text{ ohms.}$$

The oscillogram, Fig. 11B, was taken of the voltage across the condenser, C, for a sudden application of voltage, E, thus giving a measure of the displacement force of the spring. The actual displacement force is obtained by dividing the voltage by its conversion factor, $\frac{a\,c}{b^2}$. The values of time for the actual system are obtained by dividing the time on the oscillogram by the factor b. Therefore, the oscillogram gives the plotted curve of either the actual displacement or the displacement force as functions of time, by merely giving it the proper scales.

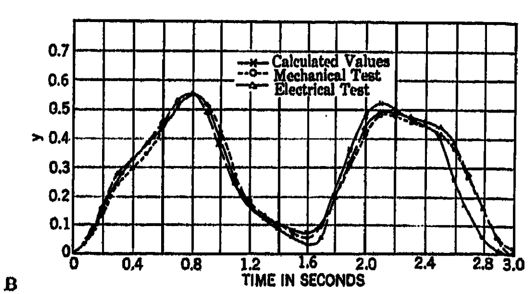
Case 2. Fig. 12A shows a composite mechanical system with its equivalent circuit. The constants for the two systems are



A

$y = .286 - .239 \cos 4.25t - .047 \cos 11t$

$q_1 = k(.286 - .239 \cos -4.25t - .047 \cos 11t)$



B

Fig. 12—Case 2

(A) Equivalent circuit

(B) Comparison of mathematical and oscillographic solutions with actual test.

Mechanical	Electrical
$m_1 = 0.218$	$L_1 = 0.246$ henrys
$m_2 = 0.1475$	$L_2 = 0.167$ henrys

Mechanical	Electrical
$k_{r1} = 0.131$ ft-lb.	$C_1 = 54.9$ microfarad
$k_{r2} = 0.111$ ft-lb.	$C_2 = 46.5$ microfarad
$F = 2.18$ lb.	$E = 264$ volts

The mass, m_1, was raised to a point 0.286 feet above its normal equilibrium position and then suddenly released. The displacement as a function of time was then calculated mathematically, and also solved by means of the equivalent circuit with the aid of an os-

cillograph. Actual test was also made on the mechanical system. The results of the three methods plotted in Fig. 12B show the remarkable agreement between the different methods.

Case 3. Fig. 13A shows a power system and its equivalent circuit. The generating apparatus consists of a shunt motor driven synchronous generator which delivers power over a transmission line to a synchronous motor. The synchronous motor is direct-connected to a (direct-current) shunt generator which furnishes power to a resistance load. The load was suddently thrown on the shunt generator. The consequent variation of power over the line was calculated mathematically, measured by means of the equivalent circuit, and obtained from actual test.

The constants used in the analysis are:

Power System	Equivalent circuit
$I_1 = I_2 = 2.78$	$L_1 = L_2 = 0.328$ henrys
$\mathfrak{J}_{r1} = \mathfrak{J}_{r2} = 0.00157$ rad/lb. ft.	$C_1 = C_2 = 27.45$ mfd.
$\mathfrak{J}_{d1} = \mathfrak{J}_{d2} = 6.3$ lb. ft/rad/sec.	$R_1{}^1 = R_2{}^1 = 16.4$ ohms
$\mathfrak{J}_r{}^3 = 0.00193$ rad/lb. ft.	$C_3 = 33.6$ mfd.

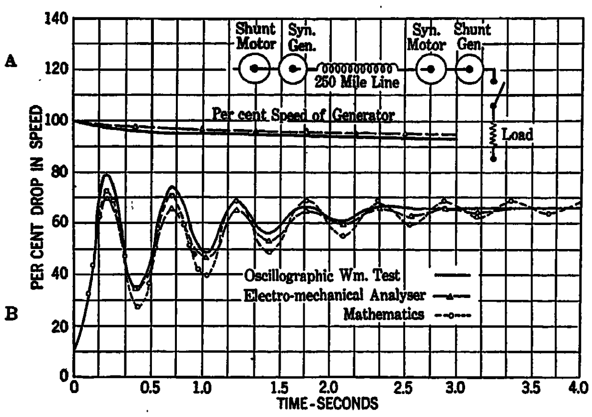


FIG. 13—CASE 3

(A) Equivalent circuit
(B) Comparison of mathematical and oscillographic solutions with actual tests.

Shunt motor torque-speed characteristic,

$K_1 = 3.3$ lb. ft/rad/sec. $R_1 = 8.6$ ohms

Shunt generator torque-speed characteristic,

$K_2 = 0.335$ lb. ft/rad/sec. $R_2 = 0.875$ ohms

Applied torque

$\mathfrak{J}_2 = 127$ lb. ft. $E_2 = 130$ volts

I = moment of inertia

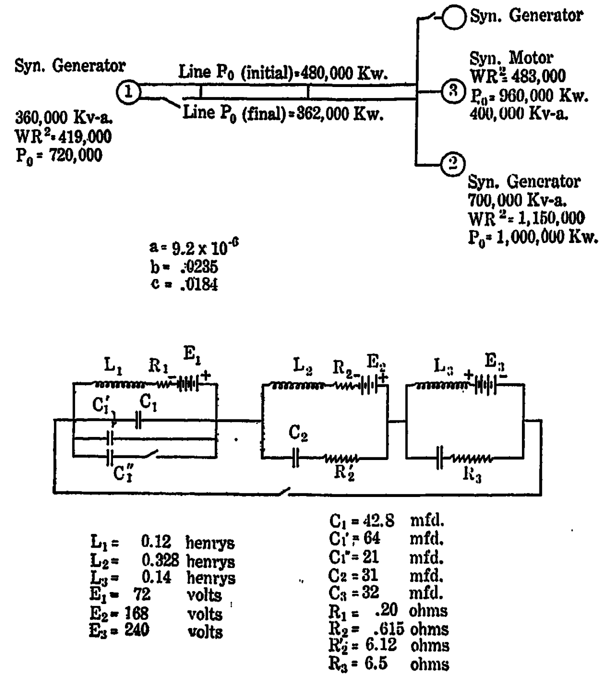
$\mathfrak{J}_r$ = resilience constant

$\mathfrak{J}_d$ = damping constant

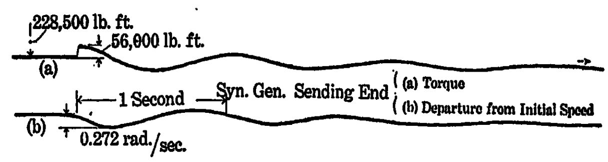
Subscripts, 1, 2, and 3 refer to sending apparatus, receiving apparatus, and line, respectively.

Case 4. Fig. 14A shows a composite power system and its equivalent circuit. The system is comprised of a waterwheel driven generator (1) delivering power over a two circuit transmission line to a bus, to which may be connected a turbo-driven generator (2), a synchronous motor (3), and a synchronous generator (4).

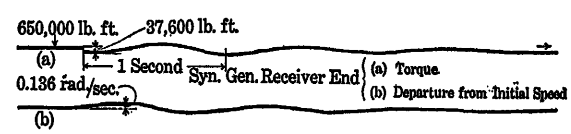
Tests were made by means of the equivalent circuit to



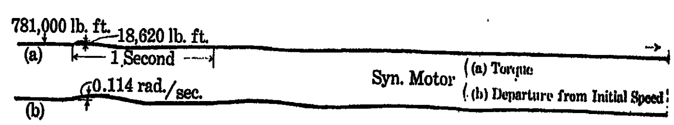
14A



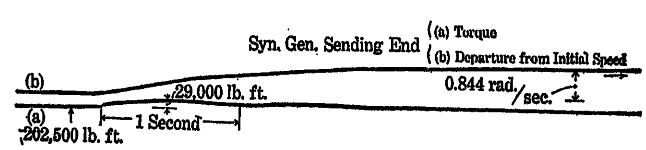
14B



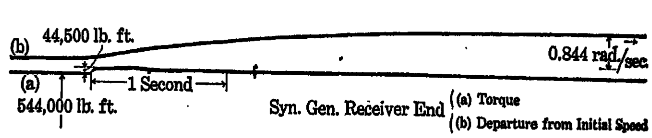
14C



14D



14E



14F

FIG. 14—CASE 4

(A) Equivalent circuit
(B), (C), (D), (E) and (F), Oscillographic solutions

determine the nature and magnitude of the disturbances in various parts of the system when (a) a section of the line is opened, and (b) when synchronous generator

(4) is dropped from the system. Figs. 14B, 14C, and 14D show the effect in the various units of dropping a section of line. In the equivalent circuit representing this condition, switch (2) is permanently closed and switch (1) is initially open, but is closed to produce the disturbance due to dropping a section. For this condition, the synchronous motor (3) was assumed to be carrying 400,000-kw. load, generator (1), 120,000 kw; and generator (2), 280,000-kw. Generator (4) is not connected.

Initially the system is running in equilibrium with a resultant distribution of power and displacement angles throughout the system. When the section of line is dropped, the power-angle characteristics for the system are instantly changed. For the first moment, however, the rotors, due to the inertia, remain in their original relative positions. Therefore, due to the change in power-angle characteristics of the system, there is an instantaneous redistribution and change of power and torque for the various units. Oscillograms (a), Figs. 14B, 14C, and 14D show this instantaneous redistribution and subsequent variations of torque with time for units (1), (2), and (3) respectively.

The final distribution and magnitude of power and torque is the same as the initial distribution and magnitude, since these are determined only by the governor characteristics of the prime movers and the applied load, which remains constant. However, the angular positions of the rotors with respect to each other, are different in the final condition from the initial condition since the power-angle characteristics are different. In changing from the initial to the final positions, the rotors go through certain cyclic changes in speed as shown in oscillograms (b), Figs. 14B, 14C, and 14D.

The effect of dropping generator (4) was next investigated. For this condition switch (1), in the equivalent circuit, remains open and switch (2), across which the voltage normally represents the torque of generator (4), is closed, thus eliminating the torque of generator (4) from the system. In this case the synchronous motor (3) is assumed to be carrying a 400,000-kw. load; generator (1), 100,000-kw; generator (2), 280,000-kw; and generator (4), 20,000 kw.

The consequent variations of torque in units (1) and (2) are shown in oscillograms (a), Figs. 14E and 14F, respectively. The final speed of the system is not the same as the initial speed, and the variation of speed in units (1) and (2) are shown in oscillograms (b), Figs. 14E and 14F, respectively.

Case 5. Fig. 15A shows a system consisting of an induction-motor and a shunt-motor-driven synchronous generator connected to the same bus. Fig. 15B shows variation of induction motor slip and synchronous generator torque consequent to suddenly applying full load to the induction motor. The constants shown in Fig. 15A were used in this test. The curves shown in Fig. 15B indicate that the disturbance was slightly oscillatory. To make this more pronounced, the oscillograms in Fig. 15C were taken for an induction motor which would carry full load at ½ of one per cent slip. For this condition, R_2, in the equivalent circuit, was changed to 280 ohms. Oscillogram (b), Fig. 15C, shows the speed of the synchronous generator rotor with respect to its terminal voltage and oscillogram (a) shows the torque of the synchronous generator. In this case the disturbance was decidedly oscillatory, thus showing that the combination of an induction motor with a synchronous machine is not necessarily logarithmic.

ACKNOWLEDGMENTS

The author gratefully acknowledges the valuable suggestions of Mr. R. H. Park, Professor V. Karapetoff, and Dr. A. R. Stevenson, Jr., also the assistance of Messrs. K. C. Mobarry and L. M. Best, in making tests and preparing the data.

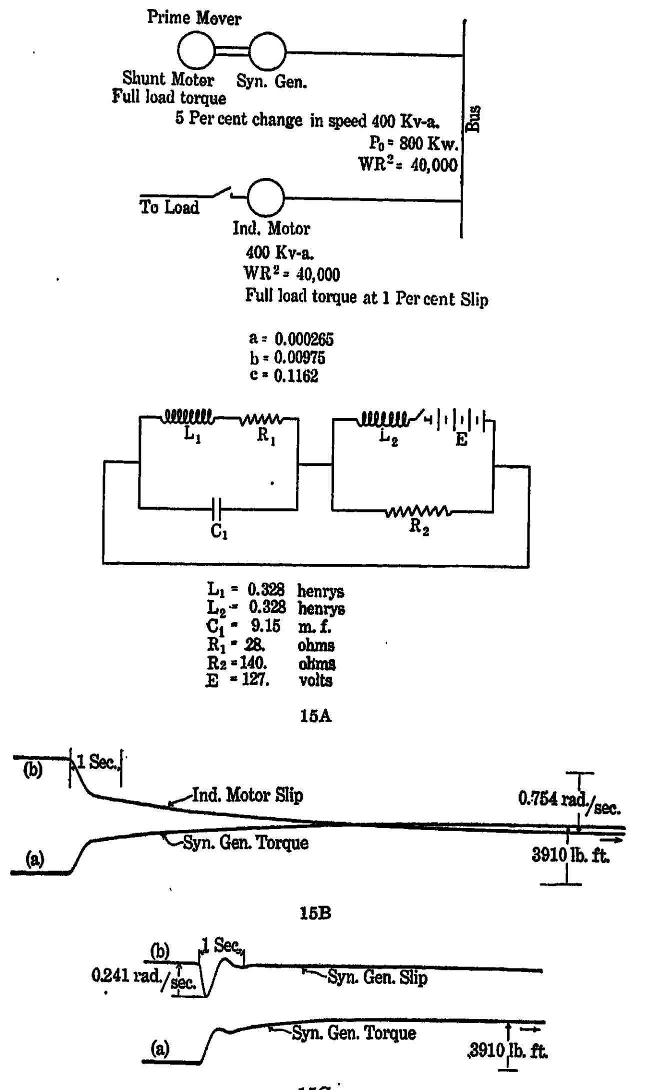


FIG. 15—CASE 5
(A) Equivalent circuit
(B) and (C), Oscillographic solutions

Appendix A

(Suggested by Professor V. Karapetoff)

CENTRIFUGAL GOVERNORS

Due to the inertia and sluggishness of the centrifugal governors controlling the prime movers—especially the water-wheels—additional periodic forces may be called into play in the hunting machines, which sometimes aggravate the conditions. For small changes in position, a governor may be assumed to add an input of steam or water proportional to the departure of its actual position from that corresponding to the steady load. Thus, calling this departure x, Arnold's[7] equation for a synchronous machine with an applied torque becomes

$$(J/p)\,\Omega_m\,(d\omega/dt) + W_s(\theta - \theta_m) + W_d(\omega - \omega_k) = G\,x \quad (1)$$

where G is a coefficient of proportionality equal to additional power per unit length of governor displacement[8]. Since the moving part of the governor has some inertia and is subjected both to a restoring force and to friction, its equation of motion is of the form

$$L_g(d^2x/d\,t^2) + r_g(d\,x/d\,t) + s_g x = r_0\,\omega \qquad (2)$$

In this expression, L_g is the mass of the moving part,

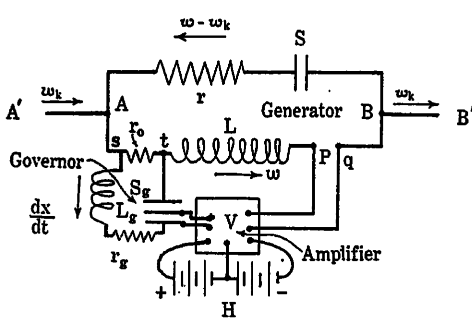


FIG. 16—EQUIVALENT CIRCUIT FOR CENTRIFUGAL GOVERNOR

r_g the friction factor, and S_g the restoring force per unit displacement. The right-hand side represents the external force, that is, the action of the generating unit, and may be assumed to be proportional to the deviation ω of its instantaneous angular velocity from the mean angular velocity ω_m; r_0 is a coefficient of proportionality. Eq. (2) is identical with Arnold's eq. (289) on p. 410, except that his x represents the total travel of the governor body from the position of zero speed, so that the departure from the mean operating position is denoted by $x - x_0$. The same applies to his value of ω.

Eqs. (1) and (2) are simultaneous differential equations of a generating unit which is hunting and the governor of which is also slowly oscillating. Both equations are of the general type

$$L\,\frac{d\,i_0}{d\,t} + S\,q + r\,i = E$$

and can be represented by two coupled, equivalent, electric circuits, Fig. 16. This equation is the familiar expression for Kirchoff's second law in a network around a closed loop containing an inductance L, an elastance S[9], a resistance r, and an external applied e. m. f. E. It must be kept in mind, of course, that in eq. (2), x corresponds to the electric charge q, and that, therefore, the current in the equivalent circuit of the governor is equal to $(d\,x/d\,t)$. The energy transformations in the governor itself are small as compared to those in the alternator, and the governor acts merely as a relay—controlling energy input into the prime mover. Therefore, in the equivalent electric circuit the storage battery H is shown, the energy of which may be added at will between the points P and q, which correspond to the disturbing e. m. f. e_a in Fig. 6C. An amplifier, V, is connected with its input side in the governor circuit and the output side in the alternator circuit.

In eq. (1), the term $G\,x$ on the right-hand side represents the voltage between the points P and q, proportional to the instantaneous displacement of electricity, x, in the governor circuit. For this reason, the input leads to the amplifier are shown connected across part of the condenser S_g. The elastance of this part is so chosen, that the voltage across it, when properly amplified, will give the required voltage, G, on the output side.

In eq. (2), the right-hand side represents the drop of voltage due to the current ω through a resistance r_0. This resistance is shown connected between the points s and t of the inductance branch of the main circuit. Since the currents and the voltages in the governor circuit are much smaller than those in the main generator circuit, the resistance r_0 is of the nature of an ammeter shunt. This means that the voltage drop across $s\,t$ is small in comparison with that across $A\,B$, and the current $d\,x/d\,t$ is small in comparison with ω.

Since the main circuit in Fig. 16 is practically identical with that shown for the individual unit in Fig. 6C, the other connections as shown in Fig. 6C still hold true when some or all of the machines in parallel are controlled by centrifugal governors.

When the action of the gate, controlled by a centrifugal governor, causes a surging motion of the whole column of intake water, the inertia of this column and the elastance of the surge tank may have to be taken into consideration. This can be done by including in the circuit of the battery H, Fig. 16, some inductance, resistance, and elastance, connected somewhat as in the equivalent circuit of the generator itself.

Appendix B

CONVERSION FACTORS FOR REDUCING QUANTITIES TO THE BASIS OF A TWO-POLE MACHINE

Since power systems, in general, are comprised of multipolar machines having unequal numbers of poles, the relation between mechanical and electrical speeds

7. *Die Wechselstromtechnik*, Vol. 4, Second Edition, page, 381.
8. *Die Wechselstromtechnik*, Vol. 4, Second Edition pp. 415 and 418.
9. An elastance is a reciprocal of a capacitance; see V. Karapetoff, "The Electric Circuit," page 148.

for the various units is not uniform. For the sake of uniformity and convenience in numerical examples, it is advantageous to substitute for the respective units an equivalent two-pole machine. The synchronous electrical speed of the system then becomes identical with the synchronous mechanical speeds of the various units.

If p is the number of poles of any given unit, the speed of the equivalent two-pole machine must be $p/2$ times as great since the the electrical speed or frequency must remain the same.

Also since torque is inversely proportional to speed for a given power, the torque of the equivalent two-pole machine will always be $2/p$ times the torque of the actual machine.

Since the speed for the two-pole unit is $p/2$ times as great as the actual mechanical speed, it is evident that the acceleration for the equivalent machine will also be $p/2$ times as great as the actual mechanical acceleration. Now in general, the torque consumed in acceleration is $T = I\,a$.

Or, $$I = \frac{T}{a}$$

where I is the moment of inertia and a is the acceleration produced by the torque T. The conversion factor for I is then evidently the ratio of the factor for torque to the factor for acceleration, or $4/p^2$.

The mechanical angle traveled through in a given time will be proportional to the speed or $p/2$ times as great in the equivalent two-pole machine as for the actual unit.

Let P_0 be the power required to produce a displacement of one electrical radian in the synchronous machine. Since the conversion factors are to reduce all quantities to the basis of a two-pole machine, and P_0 is already expressed in these terms, the factor for it is unity.

Following the foregoing reasoning, the factor for converting the actual resilience constant

$$\mathfrak{J}_r = \frac{\text{Mechanical displacement}}{\text{torque required to produce it}}$$

to a two-pole basis, is $p^2/4$.

Likewise the factor for converting the damping constant $\mathfrak{J}_d$, is $4/p^2$.

The conversion factors are tabulated in Table IV.

TABLE IV

Quantity for Actual Machine		×	Conversion Factor =	Quantity for Equivalent Two-Pole Machine
Mechanical torque	$\mathfrak{J}$		$2/p$	
Mechanical angle	θ		$p/2$	
Moment of inertia	I		$4/p^2$	
Mechanical resilience constant	$\mathfrak{J}_r$		$p^2/4$	
Mechanical damping constant	$\mathfrak{J}_d$		$4/p^2$	
Mechanical angular velocity	ω_m		$p/2$	
Kw. per elec. radian	P_0		1	

Discussion

R. E. Doherty: Anyone who has wrestled with differential equations in attempting to solve those of higher order must certainly be impressed with the possibility of having an electric circuit solve them and an oscillograph plot the solution. This is precisely what Mr. Nickle's scheme does.

The extent to which it has been developed in this paper is, in my opinion, only a beginning, touching as it does only those cases in which the phenomena can be represented by circuit elements which are constant. However, the scheme is not limited to such cases. The method is perfectly general and applicable to any problems for which circuit elements of proper characteristics are available.

Regarding its present possible application to problems involving constant circuit elements, it should not be inferred from the paper that it is applicable only to problems of power transmission; rather, it applies to many other problems involving mechanical oscillation, for instance, the flywheel problem of synchronous machines connected to reciprocating apparatus, short-circuit forces in busbars, etc. Such problems are beautifully solved by this method.

I wish to call attention, also, to its educational value in drawing out the parallel between various physical phenomena. In this, it is of great value and I commend it to the attention of educators.

R. D. Evans: In regard to the application of Mr. Nickle's work to the solution of the problems which are facing the industry at the present time—such as the stability problem—there are a number of points which I would like to discuss. The method as described in the paper is of limited application in the study of stability, for the reason that the fundamental assumptions involve power relations proportional to the angle of phase difference between e. m. f.'s instead of a trigonometric function of the angle. Stability is of importance in the vicinity of the limit, and for this condition the assumption that the power is directly proportional to the angle is entirely too crude for practical application.

The method described by Mr. Nickle has application in the determination of the hunting condition rather than for the stability condition where pull-out may occur. For the condition of hunting, the power changes in almost direct proportion to the angle, and the approximation that the power is directly proportional to the angle is sufficiently accurate for the purpose.

It is possible that Mr. Nickle's methods may be extended, not to solve the stability problem as a whole, but to solve a particular part. For example, it may be possible that his method may be applied to the receiving network of a transmission system in order to find a simple equivalent. Such a possibility seems likely because the power limits of the individual parts of the receiving network are relatively high in comparison with the power limits of the transmission network. Consequently the phase difference is small and the approximation assumed in this paper is sufficiently good for the purpose Having found a simple equivalent of a receiving network, the problem of determining the stability would be much simplified and could be handled analytically from this point.

We have been working along the same general lines, but in a somewhat different manner. Instead of converting the electromechanical transients to an electrical transient, we have sought to convert the electro-mechanical transients to a transient of a mechanical system. One of my associates, Mr. Griscom, has investigated a mechanical system which avoids the limitation in Mr. Nickle's paper, due to the power being proportional to the angle of phase difference. Mr. Griscom was able to represent the trigometric function accurately. In the original form of the method, there were limitations due to the inability to represent losses with absolute accuracy.

These comments are made merely to point out the limitations in the method so far developed and described, and with the

hope of stimulating further work along these lines, for the electromechanical problem which Mr. Nickle has studied is one which in general cannot be solved analytically, but must involve either a solution of the type which Mr. Nickle has presented or involve a method of simplifying networks, so that the system, as a whole, will be sufficiently simple to handle analytically.

C. A. Nickle: The differential equations for any electrical circuit are linear if the values of inductance, capacitance, and resistance remain constant. Evidently, such electrical circuits offer a rigorous solution for only those systems which may themselves be represented by linear equations.

The equations for a transmission system are not linear and hence the method cannot be used for values of power near the power limit. However, for values of power well below this limit, such a system may be represented for all practical purposes by linear differential equations and the equivalent-circuit method may be used. In this way the effect of various types of disturbances, such as dropping a generator, opening a line section, adding load, etc., may be studied. The relative severity of different types of disturbances, the effect of various physical constants on the behavior of the system, etc., may be investigated.

It should be realized, however, that the investigation of power-transmission problems is only one of the many possible applications of this method of solution. For instance, the sustained oscillation of synchronous machines connected to reciprocating apparatus may be easily and accurately determined by this method.

As already emphasized, the use of constant values of inductance, capacitance, and resistance limits the method to systems represented by linear differential equations, but the use of circuit elements which vary in accordance with certain prescribed laws is quite within the range of possibility, thus making the method applicable to systems which are not linear.

The Klydonograph and Its Application to Surge Investigation

J. H. COX[1] and J. W. LEGG[1]

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—In the past few years the need of a device for recording voltage surges on transmission lines has been felt more and more. Realizing this need, J. F. Peters, in the fall of nineteen twenty-three, developed the klydonograph which utilizes the Litchtenberg figure to record the characteristics of transient voltages. The principle of the instrument and practical connections to a line are discussed. The results obtained in the field from four investigations are given.

Parts II and IV describe the first experimental model of the klydonograph which uses a stationary glass photographic plate in removable plateholders with a moving electrode, and the commercial type of klydonograph which uses a day-light loading roll film of sufficient length to last seven days. This latter model has three electrodes for connection to a three-phase line.

* * * * *

I. Principle and Characteristics

SINCE the beginning of high voltage transmission the question of the nature of transient voltages on transmission systems has been a troublesome problem. Many troubles have been attributed to surges, but without positive evidence of their existence. However, there is considerable evidence that transient overvoltages of short duration do exist on transmission lines and information regarding them is very desirable. There has been developed a considerable amount of theory regarding these transients but very little verification of this theory by test has been produced, due to the absence of a satisfactory means of measurement. Heretofore, the spark gap has been the principal means of measurement. It was found that the spark gap had considerable time lag when the voltage in excess of its flashover value was small. Other objections to the spark gap for the measurement of transients are well known. Various other devices that have been tried had practically the same objections; that is, they had time lag, were not graphic, indicated only one value, introduced hazards to the line or were excessively expensive.

Principle. The klydonograph[2] was the successful culmination of the effort of J. F. Peters and his assistant, W. L. Teague, to develop a satisfactory surge recorder, one that would give a continuous graphic record of detailed information regarding magnitude, time of day, polarity, steepness of wave front, direction of travel, and whether or not the surge was oscillatory. In producing this instrument a phenomenon was utilized that has been known for a century and a half. In 1777, Dr. G. C. Lichtenberg first observed that when a condenser was discharged onto a terminal in contact with a plate of insulating material such as ebonite between it and a grounded metallic plate, and particular kinds of powder, such as flour of sulphur, were sprinkled on the insulating plate, the powder would arrange itself about the position of the terminal in a distinctive and consistent manner. The powder could be applied either before of after the discharge. Figures thus formed are called Lichtenberg figures.

Since the original observations, there has followed a long series of investigations on these figures. In 1888, J. Brown and E. Trouvelot discovered that these figures could be produced by replacing the insulating plate with a photographic plate, the emulsion being in contact with the terminal. The plate, of course, had to be kept in a dark box. The glass plate acted as the insulating material and the emulsion, when developed, replaced the dust. It was found that figures thus produced were more definite and clean cut than those produced by the use of dust. The most recent and most complete exposition of the Lichtenberg figures was that by P. O. Pederson of Copenhagen, Denmark, in a paper for "Det Kgl. Danske Videnskabernes Selskab, 1919." This paper carefully analyzes the characteristics of the figures and gives a number of curves of these characteristics. However, the curves are small, only indicate the general shape of the variations, and are not suitable for calibration. The method used by Pederson was that developed by S. Mikola in 1917.

Considerable investigation has been made, but with little success, to determine the nature of the phenomena that causes these figures. No attempt was made in the development of the klydonograph to explain the phenomena, but the work was confined to a study of the characteristics of the figures for their use in gaining information regarding surges. As far as was known, Lichtenberg figures had been used only in an academic way and never for the measurement of transients as such.

The essential elements of the klydonograph are shown in Fig. 1. The form in which these were assembled for the laboratory work is shown in Fig. 2. This laboratory instrument had six terminals and took

1. Both of the Westinghouse Electric & Mfg. Co.

2. The word "Klydonograph" was suggested by Dr. Roscoe M. Ihrig of the Carnegie Institute of Technology. It is derived from two Greek words "Kludon" and "Graphos." Kludon means "billow" or "wave," and a related adjective means "surging" or "dashing." Graphos means a writing. Thus the Klydonograph means an instrument for recording surges.

The Klydonograph, J. F. Peters, *Electrical World*, April 19, 1924.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

a 5 in. by 7 in. plate. It is not suitable for commercial work, as it is not graphic. For this work, it was found that any plate of high photographic speed was satisfactory. No work was done using dust figures.

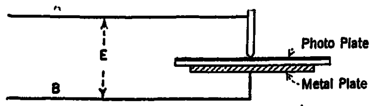


FIG. 1—ELEMENTS OF THE KLYDONOGRAPH

Characteristics. When a voltage above the critical voltage is impressed between A and B of Fig. 1, and the plate developed, a figure will be found surrounding the

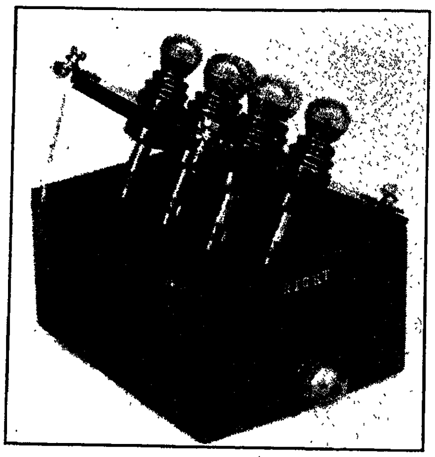


FIG. 2—LABORATORY KLYDONOGRAPH

spot where the terminal has been in contact. The critical voltage is approximately 2.0 kv., below which, at atmospheric pressure, no figure is produced. The

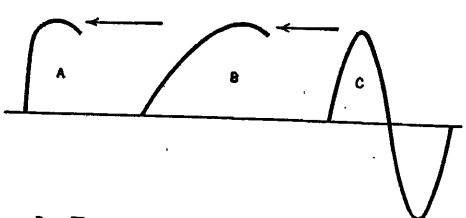


FIG. 3—TYPICAL WAVES: (a) ABRUPT, (b) SLOPING, (c) ALTERNATING

figures consist of branches, or lines, emanating uniformly from the center. They maintain this form up to about 18 kv., when, in addition to the uniform branches, main trunks extend out from the center and in turn act as emission points for other branches. These trunks do not form in a consistent manner as do the branches. If the voltage is further increased a point will be reached where a visible spark will occur and the entire plate will become exposed. If the voltage impressed is unidirectional there will be a striking difference between the figures produced by positive and negative potentials. In the case of an oscillating voltage the two will be superimposed. By the figures one may also distinguish between an abrupt-front surge, such as indicated in Fig. 3A, and a tapered-front surge, as indicated in Fig. 3B. Figs. 4 and 5 show a positive and a negative surge, respectively, having a front as abrupt as could be produced by ordinary means in the laboratory. Figs. 6 and 7 show a positive and a nega-

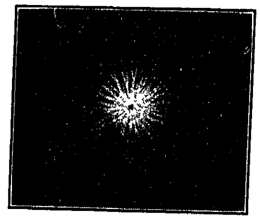


FIG. 4—POSITIVE SURGE, ABRUPT FRONT

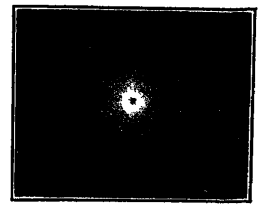


FIG. 5—NEGATIVE SURGE, ABRUPT FRONT

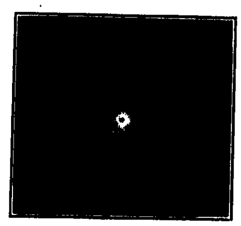


FIG. 6—POSITIVE SURGE, FIVE-MICROSECOND FRONT

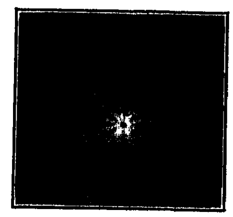


FIG. 7—NEGATIVE SURGE, FIVE-MICROSECOND FRONT

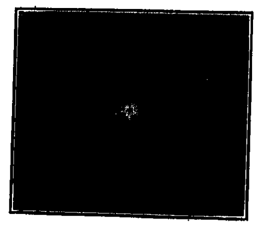


FIG. 8—POSITIVE SURGE, 200-MICROSECOND FRONT

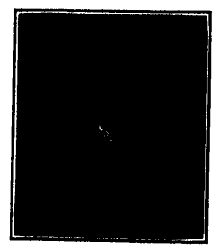


FIG. 9—NEGATIVE SURGE, 200-MICROSECOND FRONT

FIG. 10—POSITIVE SURGE, ABOVE RANGE OF INSTRUMENT

FIG. 11—NEGATIVE SURGE, ABOVE RANGE OF INSTRUMENT

tive surge, respectively, having a front of five microseconds; that is, it required five-millionths of a second to rise from zero to its maximum value. This corresponds to a traveling wave having a front of one mile on a transmission line. Figs. 8 and 9 show similar surges having fronts of 200 microseconds or 40 miles. Distinct differences in the figures formed by these three lengths of wave front can be noticed. Figs. 10 and 11

show a positive and a negative surge that were above the range of the instrument. Fig. 12 shows a figure produced by an alternating potential such as in Fig. 3c.

The paper by P. O. Pederson goes into considerable detail regarding the make-up of the figures and their variations due to differences in plate thickness and pressure of surroundinggas. In this development, these

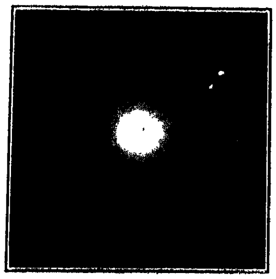


FIG. 12—ALTERNATING VOLTAGE

phases were investigated only to the extent of determining that the figures were practically constant within the range of atmospheric pressure in air and of thick-

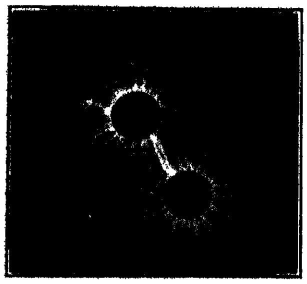


FIG. 13—POSITIVE FIGURES, 30 SECONDS BETWEEN SURGES

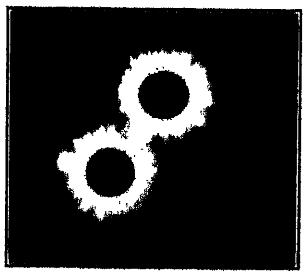


FIG. 14—NEGATIVE FIGURES, 30 SECONDS BETWEEN SURGES

nesses of commercial photographic plates. Further special tests were made and it was found that, so far as could be detected, the figures were identical whether

FIG. 15—POSITIVE FIGURES, TERMINAL MOVED BETWEEN SURGES

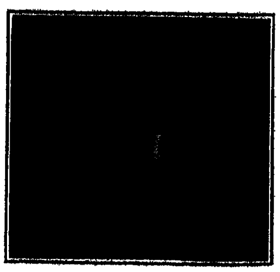


FIG. 16—NEGATIVE FIGURES, TERMINAL MOVED BETWEEN SURGES

produced in fairly dry or in saturated air, and whether the plates were developed immediately or after a lapse of several days. According to Dr. Pederson, in order to form a figure the voltage must be impulsive so as to produce an intense field on the surface of the plate. It was found that a 25-cycle wave would produce a figure of the same size as a wave of short front. However, if a d-c. potential was gradually applied, the figure produced was peculiar in shape and did not confrom to the calibration curve in size.

Tests were made to determine whether or not one application affected the plate to succeeding applications. The rays or branches of a single figure never cross. It was suggested that if one surge were recorded and a comparatively short time later another surge appeared, the figures might not overlap and the second surge would make no impression. Tests were made

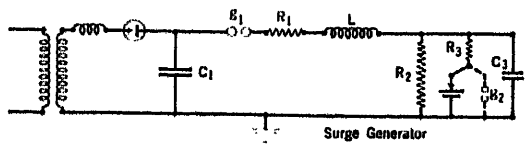


FIG. 17—NETWORK USED IN LABORATORY

which proved that one impression has no effect on succeeding records. If two surges are impressed simultaneously on adjacent terminals, the figures will not overlap. If a surge is applied on the position of a previous surge it superimposes its figure exactly as if there had been no previous figure and the rays of the two figures cross each other promiscuously. In this manner a succession of figures merely increases the density and with a large number the spot becomes black. This is what happens in an a-c. figure. Figs. 13 and 14 show positive and negative surges, respectively, where one surge was impressed and a few seconds later a surge was impressed on the adjacent terminal. Figs. 15 and 16 show similar surges where there was a similar interval and each terminal was in contact with the plate only when the surge was im-

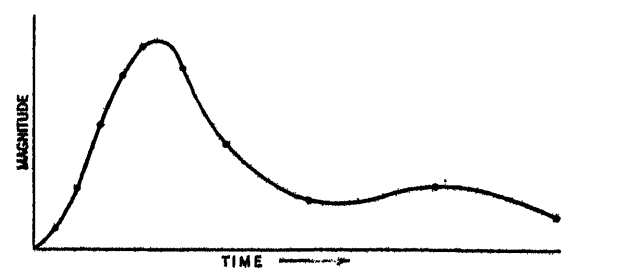


FIG. 18—TYPICAL SURGE PRODUCED BY NETWORK OF FIG. 17

pressed. Further, the size of each succeeding figure was determined only by the applied voltage. This was checked by applying surges of equal values a various number of times, one to six, on the six terminals of the instrument. The figures were of the same size and only differed in intensity.

Calibration. Fig. 17 shows the network used in the laboratory as a surge generator in the development of the klydonograph. Fig. 18 shows the shape of a typical surge as calculated from the constants of the network. By varying the constants of the network a surge of any desired wave shape could be produced. It was found

to be no easy matter to obtain surges of the desired characteristics. The greatest trouble was encountered with the reflections of the initial traveling wave. Before the condenser, c_3, at the end of the network, was added, when the sphere-gap sparked over a wave would proceed out through the series inductance and when it hit the klydonograph it would reflect with increased voltage. The klydonograph itself forms a small condenser so an oscillation would be set up in the circuit. The klydonograph would record these oscillations. Further, due to the reflection the klydonograph would record a higher voltage than the setting of the spark-gap. By placing the condenser c_3 in parallel with the klydonograph the initial wave would be absorbed. This would be reflected but in this case since the surges were tapered, it did no harm and would join with the principal part of the surge in bringing the voltage up to the proper value. However, it was found necessary to place resistance R_3 in series with the klydonograph to critically damp the loop consisting of leads, klydonograph and c_3 to prevent small reflections at the plate. This did not disturb results as a non-inductive resistance was used and the voltage drop was negligible since the current was very small. It was interesting to notice that a sphere-gap would not indicate these reflections but the klydonograph would. To obtain the desired setting the klydonograph was removed and gap g_2 inserted and set at the desired voltage. Gap g_1 was then varied until g_2 would just spark over at the breakdown of g_1. Gap g_2 was then replaced by the klydonograph, and leaving g_1 as set, a series of figures were made.

The diameter of the figure is a measure of the magnitude of the surge. Positive and negative surges have quite different calibrations, a positive figure being considerably larger for the same voltage. Since the positive is the larger, the a-c. calibration is the same as the positive. All the work for the calibration curves was done with five-microsecond and 200-microsecond surges. Tests were made which indicated that for a surge with a front as long as 5 microseconds the time lag of the measuring sphere-gap was negligible. In addition to this, the characteristics of the surge generator used gave a surge with a comparatively flat top. It was found that the calibration was the same for unidirectional surges of both these wave fronts of each polarity as well as for a 60-cycle and a 25-cycle a-c. wave. It was realized that even with no coil inserted, the inductance of the leads and the time lag of a sphere-gap would give a slight taper to the wave. The figures shown as abrupt-front surges were as abrupt as could be made with ordinary methods. They were made with no inductance between the spark-gap and klydonograph except that of a short lead. It takes a certain time for a sphere-gap to break down and become highly conductive, so it was felt that surges thus produced could not be called absolutely abrupt nor be depended upon to be of the same degree of steepness. They were, therefore, not used in calibration work.

It was found in calibrating that the radius of a figure made at a setting of the generator would vary as much as 20 per cent from the average reading of the same value but most of them were within 10 per cent of the average. This would not seem to be very accurate at first glance, but it is maintained that the variation was impressed in the voltage due to inaccurate setting of the spark-gap and inaccuracies in the method used, and not in the figure made by the klydonograph at a given voltage. The network would be set for a given voltage and condition of surge. The voltage was tested in the position of the klydonograph by a sphere-gap. A series of pictures would then be taken and the network changed to some other value. The curves were made up from the results of a great many settings for a given value. These settings were made on different days over months and it is impossible to duplicate such a set-up with exactness. The basis for attributing the error to set-up rather than klydonograph is the fact that for a single setting the variation between the figures was exceedingly small. To test this quality further, surges were thrown on six leads connected to the six terminals of the test instrument, simultaneously, and in practically every case there were no variations that could be measured. In the few cases where there were measurable variations, these variations were very slight.

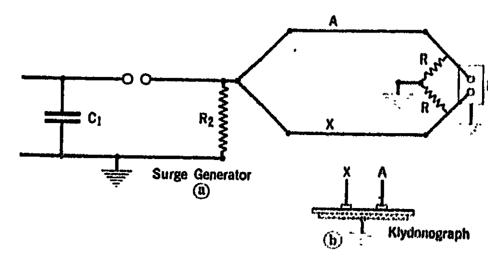


FIG. 19—ARRANGEMENT USED TO MEASURE SHORT-TIME INTERVALS
(a) SURGE GENERATOR (b) TWO-TERMINAL KLYDONOGRAPH

Rapidity. Since surges may be of extremely short duration it was highly important that the klydonograph be a rapid instrument. Many tests were made to determine the speed of formation of the figures. The diagram of the set-up used to obtain these data is shown in Fig. 19. This merely consisted of a circuit which discharged a condenser through a sphere-gap into two open-wire lines which were shunted to ground by a high resistance. The lines A and X were connected at the far end to a special klydonograph having two ⅜-in. terminals in contact with the plate at a short distance apart. The time lag of the sphere-gap used to discharge the condenser played no part as the data obtained was of conditions after the gap had broken down. When the sphere-gap broke down a surge was impressed on the two lines simultaneously, and would proceed as a traveling wave at approxi-

mately the speed of light. If the lines were of equal length, the wave would arrive at the ends at the same time. Figs. 20 and 21 show positive and negative figures, respectively, made with equal length lines. The division line between the two figures was distinct and its action positive in that its position was consistent with repeated trials. Tests were then made having one line longer than the other. Figs. 22, 23, and 24 show surges impressed on unequal lines. Fig. 22 is for a positive surge with $A = 30$ ft. and $X = 50$ ft. From the rate of propagation of a surge on an aerial line this difference of 20 ft. represents a time interval of 2 by 10^{-8}, or twenty billionths of a second. It is to be noticed that the surge from the shorter line took possession of more than one-half the space between the figures. Fig. 23 is for a positive surge with $A = 50$ ft. and $X = 140$ ft. This difference of 90 ft. corresponded to a difference in time of the arrival of the wave at the instrument of 9 by 10^{-8} seconds or ninety billionths of a second. It is noted here that A is practically fully developed when the wave arrived at X, so that the figure became nearly complete in ninety billionths of a second. Tests were made varying the difference in

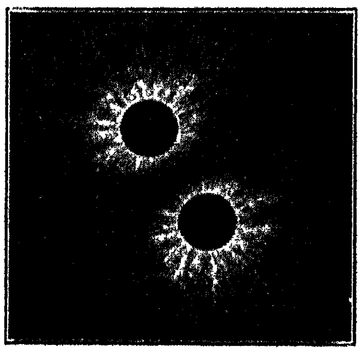


FIG. 20—POSITIVE FIGURES IMPRESSED OVER EQUAL LINES $A = X$

line lengths and the speed of the figures was found to be consistent. Fig. 24 is for a negative surge with $A = 30$ ft., $X = 60$ ft. The negative figures were found to be the slower by about five times. This is evident from a comparison of Figs. 22 and 24. However, either is amply rapid for the purpose of recording practical surges.

It was interesting to note the evidence of the tapering of the wave front as it traveled down the line even in such short distances as 100 ft. Fig. 20 shows the two figures made with lines of equal length to be identical in form. On the other hand, Fig. 23 shows the figure produced at the end of the longer line as indicating, by its form, a longer wave-front than the other.

Since it takes time for a spark-gap to spark over and become highly conductive, the surges were somewhat tapered, and if the lines were short the reflection of the foot of the wave would return from the shorter line and disturb results. In fact this would sometimes cause the figure to form on the more distant terminal first. Therefore, the lines had to be long enough so that the complete wave would arrive before the reflected wave of the shorter line disturbed the principal wave. When a wave strikes the open end of a line, it theoretically doubles. The voltage was found to increase at the end of the line but did not double in value except where the line was of considerable length. Lines could not be made long enough so that the entire wave front could

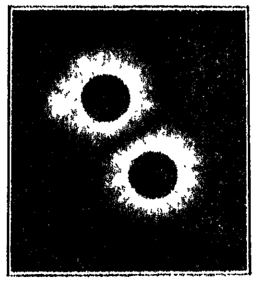


FIG. 21—NEGATIVE FIGURES IMPRESSED OVER EQUAL LINES $A = X$

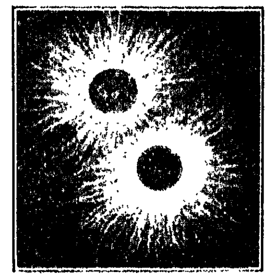


FIG. 22—POSITIVE FIGURES IMPRESSED OVER UNEQUAL LINES $A = 30$ $X = 50$

start before the reflected foot of the wave would return and complicate the wave. In order to cut down this reflection as much as possible, a resistance equal to the surge impedance of the line was placed in parallel with the klydonograph. This was found to absorb the wave with practically no reflection and the true surge would be recorded with no increase in voltage. Before this grounding impedance was added, it was interesting to note that a sphere-gap at the end of the line, while it indicated a rise in voltage would not indicate the full

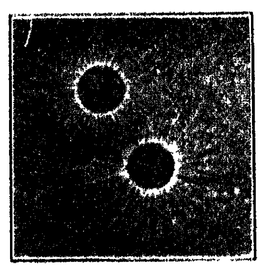


FIG. 23—POSITIVE FIGURES IMPRESSED OVER UNEQUAL LINES $A = 50$ $X = 140$

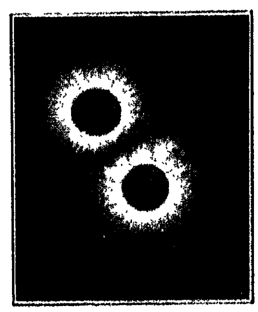


FIG. 24—NEGATIVE FIGURES IMPRESSED OVER UNEQUAL LINES $A = 30$ $X = 60$

value of the reflection as recorded on the klydonograph. In the case of a long line, the klydonograph indicated practically double voltage.

Extensive tests were made by Pederson to measure the speed of formation of the figures, that is, the rate at which the figures grow from the center outwards. This he found to vary somewhat with the distance from the center and with the voltage, but to remain of

the same order of magnitude. He gives the following values:

Speed at 8 kv. positive — 4 by 10^7 cm. per sec.
" " 8 kv. negative— 1 by 10^7 cm. per sec.
" " 15 kv. positive—6.5 by 10^7 cm. per sec.
" " 15 kv. negative—2.8 by 10^7 cm. per sec.

Special Applications. The Lichtenberg figure promises, in addition to its use in surge investigation on transmission systems, to be a valuable aid in special tests both in the laboratory and field. It can be used in the measurement of exceedingly small time intervals

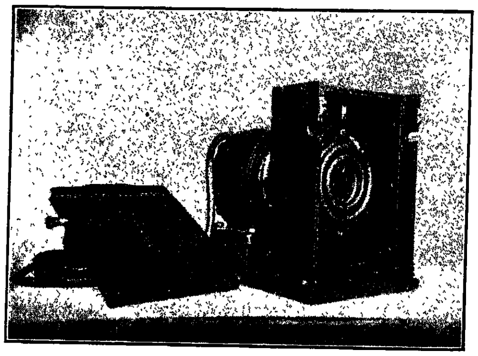


FIG. 25—SYNCHRONOUS KLYDONOGRAPH

wherever a capacity of the order of 10^{-11} farads is negligible and the voltage is above 2.5 kv. Some work has been done by P. O. Pederson on the time lag of a spark-gap. There has not been time, since the development of the klydonograph, for much special investigation, using it as an instrument. Fig. 25 shows a

FIG. 26—FIGURE PRODUCED BY SYNCHRONOUS KLYDONOGRAPH

synchronous klydonograph which was developed for the purpose of separating the positive and negative figures of an a-c. wave. Fig. 26 shows a typical picture produced by a 60-cycle wave.

Electrostatic Potentiometer. To be applicable to transmission systems in general the instrument must have a wide range of voltage. The range of the instrument itself is from 2.5 kv. to 18 kv. Since the instrument is an exceedingly low current device, it can be connected by means of an electrostatic potentiometer to lines of practically any voltage without introducing an insulation hazard. An electromagnetic multiplier is, of course, out of the question, since there must be no time lag. Fig. 27 shows a satisfactory method of connecting the klydonograph to a line. The lower ring in this set-up will maintain at all times the same

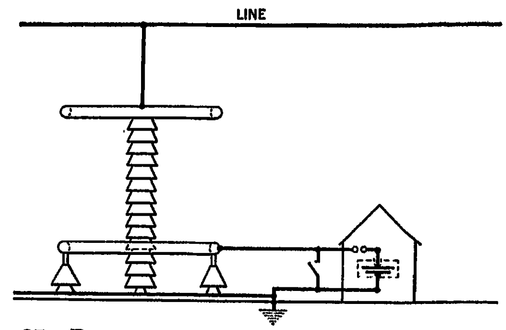


FIG. 27—DIAGRAM OF ELECTROSTATIC POTENTIOMETER

proportion of the potential on the upper ring. A klydonograph connected in this manner will record figures giving the magnitude and polarity of the surges.

A gap in series with the instrument may be included as shown, or left out as desired. If included there will be no record except in case of a surge the voltage of which is in the same ratio to normal as the setting of the gap is to the normal voltage on the lower ring. If no gap is used, there will be a uniform band on the plate corresponding in width to the diameter of the positive figure at the normal voltage on the lower ring. A surge figure will then be superimposed on this band. The width of the band will also indicate wide variation in line voltage. Whether or not the instrument is operated with a series-gap, the potentiometer must be calibrated with the gap closed. The reason for this is that the capacity of the instrument, while small, cannot be ignored in comparison with that of a practical low potential ring. The voltage on the lower ring after the gap breaks down is not the same as before since there is a division of charge.

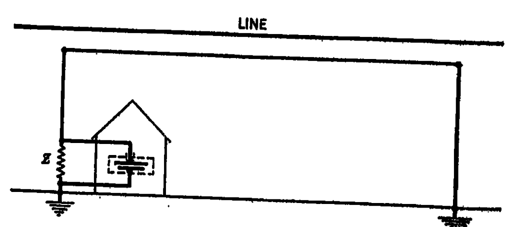


FIG. 28—DIAGRAM ON ANTENNA

Antenna. While some idea of the steepness of wave front can be gained from the appearance of the figures obtained on the potentiometer instruments, this is extremely approximate. Fig. 28 shows a means of connection that will give the steepness of wave front more accurately. This consists of running a wire parallel to the transmission-line conductors as close as is consistent with the insulation requirements,

grounding one end solidly and the other end through a high impedance. The klydonograph is connected across this impedance and if the impedance is high compared to the impedance of the antenna loop itself, the klydonograph will measure any voltage above 2.5 kv. which may be induced in the loop. This antenna loop will have a certain mutual inductance in relation to the various conductors of the transmission line. At a given distance the length is varied to give the desired amount. The voltage induced in the antenna loop will be proportional to $d\,i/d\,t$ on the transmission line; that is, it is proportional to the steepness of the current wave. Under traveling wave conditions the current and voltage waves are absolutely inseparable. The current is always equal to the voltage divided by the surge impedance of the line and the wave shape is the same. Therefore, the steepness of the voltage wave front $d\,V/d\,t$ can be calculated by the relation

$$d\,V/d\,t = \frac{Z_0}{M}\,e \quad \text{where}$$

e = Voltage induced in antenna loop
M = Mutual inductance between loop and line conductor
Z_0 = Surge impedance of line conductor calculated with respect to the ground

By a comparison of the polarity of simultaneous readings on the klydonograph connected to the line through the electrostatic potentiometers and that connected on the antenna, the direction of travel can be determined. That is, a surge of a particular polarity will induce a potential of one polarity in the antenna if traveling in one direction, and of the opposite polarity if traveling in the other direction. The direction is readily determined from an examination of the connections. Thus, magnitude and polarity are obtained directly from the potentiometer klydonographs, steepness of wave front from the antenna klydonograph, and the direction of travel from a comparison of the polarity of the two.

II. Preliminary Field Model, Plate Type

A *time component* must be added to the camera already described before the same is effective in obtaining data on disturbances in transmission systems. To obtain a time component, the electrode must move relative to the plate or photographic film. Either the plate may move and the electrode remain at rest, or the electrode may move while the plate remains at rest. Each of these two cases has its advantages and its disadvantages. If the plate is the moving element, the electrode capacity to ground may be kept a minimum, but the case must be large enough to clear the carriage of the moving plate. The apparatus as a whole is cumbersome and is not readily loaded without taking a large part of it into a dark room.

Stationary Plate. The first field model, Fig. 29, was made with a stationary plate in a removable holder, enclosed within the same light-tight cabinet with the clock-driven electrode. The holder was of the single-plate type having a stiff metal bottom. Above this metal bottom was a sheet of soft felt, cut away at the center so as to press a very thin disk of hard-sheet-metal against the under surface of the active part of the photographic plate. The metal disk was grounded through the plate holder bottom, and did not extend to the edges of the photographic plate. Thus, a uniform capacity was assured, and sufficient insulation between the electrode and ground. A number of these removable plate-holders were provided for each recorder.

Clock Driven Electrode. A one-day clock was used in most recorders, driving the electrode in a 7-in. circle, one revolution in 24 hours. A few instruments were equipped with 7-day clocks making one revolution in 168 hours. The electrode was mounted vertically near the edge of a light disk of insulating material, having a wire-collector ring on its periphery. The lead-in terminal, passing through an insulating bushing

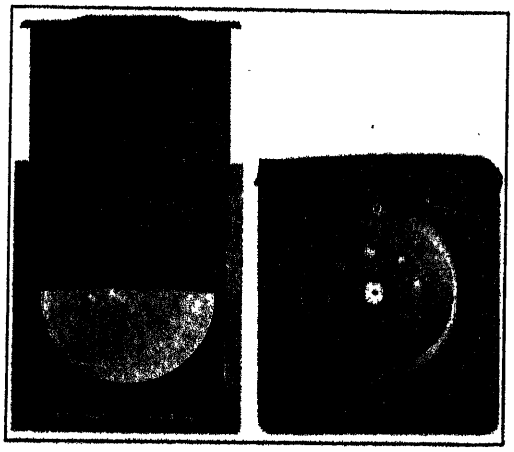


Fig. 29—Plate-Type Klydonograph, Showing Rotating Electrode and Hour Circle

in the case, had a brush on the bottom end which bore on the collector and thus carried the potential to be measured to the electrode proper. This electrode was free to move up and down and pressed on the sensitive side of the photographic plate with its own weight only.

Time Locating Device. In order that the time at which each surge occurred might be known by a glance at the finished photographic plate, a photographic template was prepared with opaque background and transparent dial and figures. All klydonograph plates were exposed to light with this template over them, so that when the plate was developed a circular scale, marked in 24 hours with "Midnight" at top and "Noon" at bottom, appeared. An hour circle on the electrode disk was also graduated into 24 hours. If this was set correctly before starting the test, by slipping the friction clutch on the clock shaft until the figures on the disk corresponded to true time, the klydonograph figures would appear after development

in their proper time relation with the scale on the plate.

The instrument was inverted to set the electrode disk through a large hole in bottom of the case, and before turning the instrument right side up the plate holder was inserted. The removal of the rubber slide from the plate holder permitted the electrode to drop to the sensitive surface of the photographic plate, and thus be in condition to take a klydonogram. After a 24-hour test, the instrument was again inverted, the rubber slide replaced, and the plate holder removed.

III. Field Experience

Instrument. Since the film-type instrument has only now been made available, all the field work has been done using the moving electrode, plate-type of klydonograph. With the electrode speed of this instrument, the time of a record could not be distinguished to a greater accuracy than 15 min. No practical attachment could be made with this type of instrument to speed up the electrode. Besides, this would have required changing of plates more frequently than once a day which would have unduly increased the expense of material and labor. When an operation is performed on a line requiring several switches to be thrown at various points, these are often thrown at intervals of a minute or so. Where a figure was recorded at the time of such a performance it was impossible to assign any particular operation as the cause. In order to obtain more concrete information as to the causes of surges, arrangements were made to have switches thrown at 15 min. intervals, when an operation was required, for certain periods during tests. During such periods much more satisfactory information was obtained as to the causes and actions of the surges recorded.

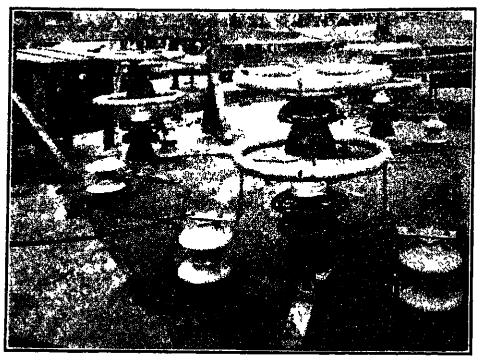


Fig. 30—30-Kv. Electrostatic Potentiometer

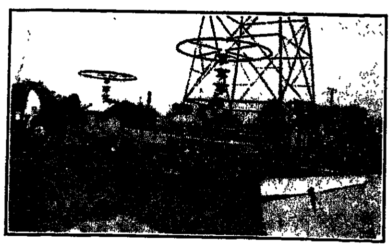


Fig. 31—140-Kv. Electrostatic Potentiometer

Potentiometers. Fig. 30 shows the potentiometer rings which were used on the 26,400-volt system. It was found that a 30-in. ring of 2-in. iron pipe, mounted as shown, was satisfactory on this class of voltage. The high potential element was mounted about 30 in. above the ground plate. The lower ring was placed at such a height that it had a potential of 3-kv. crest with normal voltage on the upper ring. This proved to be about 15 in. above the ground plate. For the higher values of voltage, the high potential element was raised and the ratio of distances changed to fulfill both the insulation and calibration requirements. Thus, to keep the proportions reasonable and a more uniform field, the sizes of the rings were increased with the voltage. Six- to eight-feet rings were used on the higher voltage lines. Fig. 31 shows the set-up used on the 140-kv. line and Fig. 32 shows the set-up used on the 220-kv. line. The height of the upper ring was between 8 and 12 ft. from the ground plate and the lower ring about 15 in., as before, to give 3-kv. crest with normal voltage on the upper ring.

Fig. 32—220-Kv. Electrostatic Potentiometer

In order to make the position of the ground beneath the rings constant, the rings were mounted on a metallic sheet consisting of either sheet iron or close-woven mesh. This ground plate was made large enough to extend about a foot beyond the outside circumference of the rings and solidly grounded.

An alternate form of potentiometer, shown in Fig. 33, was found feasible on the 220-kv. test and was used at three of the stations. The station busses on this system consisted of 4-in. iron pipes mounted parallel to, and 12 ft. from, the ground. A 10-ft. length of 3-in. iron pipe was mounted on insulators parallel to the bus at the proper distance above the ground plate, which was a 10- by 9-ft. plate of sheet iron lying on the ground and tied solidly to the station ground. This form of potentiometer has certain disadvantages which will be pointed out later.

The potentiometers were calibrated by measuring the

voltage on the lower ring with a 3/8-in. sphere-gap with normal voltage on the upper ring. When no series-gap is used, this calibration can be roughly checked by the width of the normal voltage line on the plate. With the potentiometer set to give 3-kv. crest with normal line voltage, the upper limit of the instrument, or 18-kv. crest, permits the measurement of a surge of six times normal voltage and the recording of a surge eight times normal before the instrument will spark over.

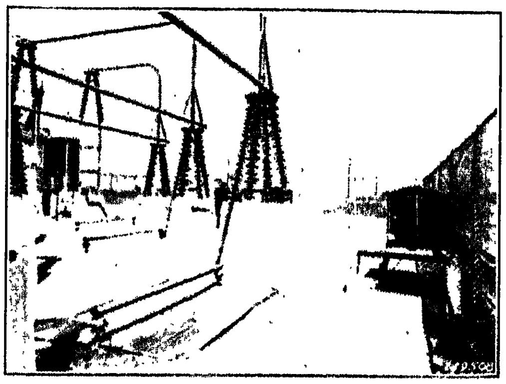


FIG. 33—220-KV. BUS TYPE POTENTIOMETER

At first some difficulty was experienced as a result of calibrating the potentiometers with the series-gap open. It was found that when a surge just high enough to break down the gap occurred, the figure on the plate indicated a somewhat lower voltage than the gap setting. The instrument itself had approximately 25 by 10^{-12}-farads capacity. By voltage tests with and without a series-gap the capacity of the lower potentiometer element to ground, in the case of the bus type, was about 125 by 10^{-12}-farads. Thus, there was a reduction of voltage on the klydonograph bus of about

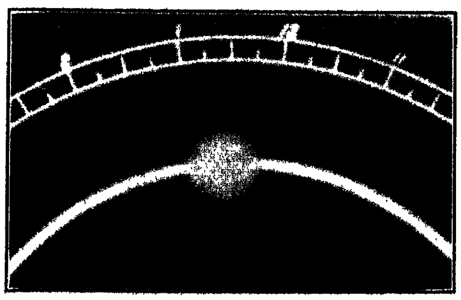


FIG. 34—SECTION OF PLATE WITHOUT SERIES GAP. OSCILLATORY SURGE

20 per cent upon breakdown of the gap. This error was eliminated by doing all calibration of the potentiometer with the instrument in place and the series-gap closed. It was also found that due to the capacities of the various elements in series it took a voltage 15 per cent above the setting of the gap to spark it over. This merely altered the lower limit at which the instrument began to record and could be taken care of by changing the gap setting. On two of the tests the series-gap was used and on the other two it was not. When no gap is used the smaller negative records are obscured by the band of normal positive figures. However, these are not important since a negative surge large enough to cause concern would show itself above this band. To operate without a series-gap gives a continuous check on the calibration of the potentiometers, a check on wide variations of the line voltage, and indicates a static potential on the line where this occurs. Fig. 34 is a section of a plate taken with the klydonograph operating without a series-gap. The surge shown indicates an oscillating surge. Fig. 35 is a section of a plate where a series-gap was used. The surge shown indicates a positive surge.

It was found that on a polyphase set-up the potentiometers must be set at some distances apart to prevent mutual effects between them. This presents no particular difficulty when rings are used as these can generally be placed at intervals along the line. In the case of the bus-type potentiometer, the separation of the station busses being fixed, this effect cannot be avoided. This makes the latter type a less accurate

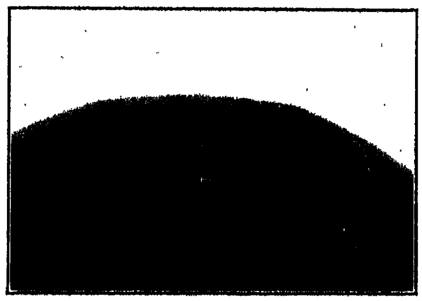


FIG. 35—SECTION OF PLATE WITH SERIES GAP. POSITIVE SURGE

set-up. It was found that in a particular case, where the height above ground was 12 ft. and the separation was 12 ft., one bus would affect the pipe of an adjacent bus to an extent of about 30 per cent. It was assumed in general that, except in the case of a lightning surge, a surge was unlikely to occur on two phases simultaneously and that, therefore, the calibration was correct when made with the adjacent busses dead.

In situations where the potentiometers were out in the weather, it was found to be imperative for all metallic parts to be painted. When this was neglected, the rust, running down on the supporting insulators, would create leakage and drop the potential on the lower ring to very low values.

Antenna. It was found that where the transmission line had a ground wire, this was generally in sufficiently close proximity to the line to act as an antenna. By insulating the wire on one tower, extending it down and grounding it through a suitable resistance, the antenna loop was complete. This could be made as long as desired by insulating the wire from as many

towers adjacent to the end tower as necessary. Fig. 36 shows an antenna set-up effected in this way. On lines where no ground wire was used, it was a simple matter to mount a wire on the transmission line poles or towers.

The antenna loop had a definite mutual inductance with each conductor and could be calculated with fair accuracy. At a fixed separation this can be varied by changing the length of the loop. At a given value of mutual inductance, the range of the klydonograph permits only a given range of steepness of wave fronts to be recorded. Such a value of mutual inductance was chosen that the steepest of the wave fronts encountered would not cause a flashover of the plate. A surge with a front too tapered to come within the range of the klydonograph with such a set-up was probably not abrupt enough to be serious.

The value of the mutual inductance with the various conductors was found to vary about 25 per cent in practical cases due to their different positions. In the case of an antenna record with a simultaneous potentiometer record on a particular phase, the mutual inductance with that phase was used in calculating dV/dt. In the case of a record caused by a steep-front surge, too small in magnitude to be recorded by the potentiometers, an average value of mutual inductance was used. In the case of simultaneous waves on the three conductors, the effects add algebraically. It was assumed that, in general, surges on three conductors caused by a switching operation would not be simultaneous when considering the propagation speed of electric waves. When the evidence indicated a lightning surge, the sum of the three mutual inductances was used.

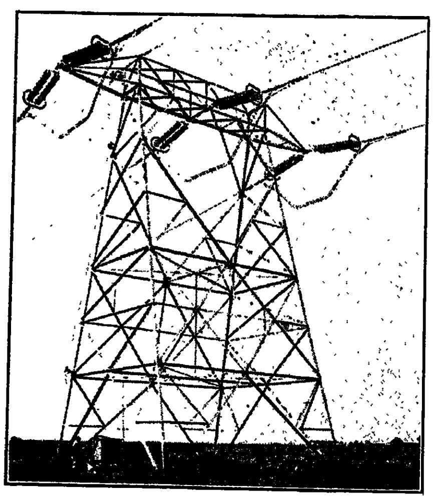


FIG. 36—ANTENNA CONNECTION ON 220-KV. LINE

Tests and Results. While up to the present writing there has not been time for as much field experience as could be desired, rather extensive tests have been made on four systems representing a wide range of conditions of electric power transmission.

a. These tests were made on a 26,400-volt system consisting of connected cable and open wire. Grounding was varied between solid and 150-ohm resistance. One instrument was installed on one phase at a substation having cables extending in each direction and another instrument on the same phase on the open-wire line, 18 miles from the junction of cable to open-wire. The tests were continued 20 days.

During this 20-day investigation 18 surges were recorded. The largest number in one day was three. All were obtained on the instrument on the open-wire line. A possible reason for this was the fact that the potentiometer was not painted for the greater part of the tests on the instrument recording no surges, with the resultant lowering of the calibration. Of these surges, seven were alternating, seven were positive, and four negative. As to magnitude of voltage, two were twice normal, six were 1.5 times normal, and ten were 1.3 times normal or less. As to weather conditions, eight occurred in clear weather, seven during rain, two in cloudy weather, and one during an electric storm. Both surges twice normal came during rain and wind. As to attendant causes, the two surges twice normal were not caused by a switch operation but switches were tripped by the disturbances. The one surge during an electric storm was 1.3 times normal. Of the total, nine were caused by switching and one by lightning. No cause could be assigned to the remainder, of which three came in clear weather and five during rain.

b. These tests were made on a 27,400-volt cable system. Only one instrument was installed at a pot head of a station with cables extending in both directions. The system was solidly grounded at one point approximately 10 mi. from the klydonograph. The duration of the test was 104 days.

During this investigation, 24 surges were recorded. One was 1.5 times normal, one was 1.4 times normal, and 22 were 1.3 times normal or less. Nine occurred in rain and 15 on clear days. The probable cause of the surge 1.5 times normal was a switch operation within three miles of the instrument. The 1.4 times normal surge occurred during switching operations.

c. These tests were made on a 240-mile, 140-kv. open-wire line with a free neutral. Three instruments each on potentiometers were installed at the receiving end, 65.5 mi. and 193 mi. from the receiving end and at the sending end. One antenna instrument was installed at the receiving end. The duration of these tests was 65 continuous days.

During this investigation, 124 surges were recorded on the potentiometer instruments. Of these, 22 were at the sending end, eight at the second station, 27 at the third station, and 67 at the receiving end. Fifty-four of these were 1.4 times normal or above and were distributed according to Table No. I.

TABLE NO. I

Surge Magnitude times normal	Sending end	Second Station	Third Station	Receiving end
3.5 to 4.2				2
2.5 to 3.5	1		2	6
2.0 to 2.5	5	3	1	7
1.4 to 2.0	5	3	4	15

In only two cases were surges recorded at more than one station simultaneously. In both of these cases, the surge was oscillatory and the maximum values of the surges were 4.2 and 2.8 times normal, respectively. Of these 54 separate figures, 15 were alternating caused by three surges; that is, some surges occurred on more than one phase and extended to more than one station; 21 were positive, caused by 13 surges, and 18 were negative caused by 18 surges.

Lightning caused nine a-c. figures in two surges, maximum values 3.3 and 2.8 times normal. Only one of these caused figures at more than one station. Lightning caused nine positive figures, all at the receiving end, in four surges, with maximum values of 4.2, 2.5, 1.5 and 1.3 times normal. No negative figures were caused by lightning. The other surge that was recorded at more than one station, causing seven figures in all, was caused by a telephone wire falling into and grounding one phase at about the middle of the line. The maximum value of the surge was 4.2 times normal at the receiving end.

Of the 54 figures, 1.4 times normal or above, switching caused nine. None of these were over two times normal. To about 80 per cent of the 70 figures below 1.4 times normal, no cause could be assigned. The others occurred at times of switch operations.

Except for a few mild lightning storms, the weather was in general fair.

The steepnesses of a total of 106 surges were recorded on the antenna instrument at the receiving end. Of these, 16 were simultaneous with records of magnitude on the potentiometer instruments. The maximum was due to one of the oscillating surges caused by lightning and was 28 by 10^6-kv. per sec. or indicated a frequency of 16,000 cycles. The next largest was due to an arcing ground and indicated 24 by 10^6-kv. per sec. or 9000 cycles. The steepest front surge caused by switching had a slope of 14 by 10^6-kv. per sec. and was unidirectional. Of the six lightning surges on the potentiometer, there were four to which there were corresponding antenna records. Of the 90 antenna figures with no simultaneous figures on the potentiometers, 20 could be connected with switching operations. There was no apparent cause for the remainder.

The varying relationship between the widths of the positive and negative portions of the normal voltage band indicated that this system would pick up and maintain for several hours a static potential of one or the other polarity.

This investigation has not been completed.

d. These tests were made on a 270-mi., 220-kv., open-wire line with neutral solidly grounded at four points along its length. Klydonographs were installed as follows: three on potentiometers at the sending end, three on potentiometers and one on an antenna at stations 105 mi. from the sending end, 140 mi. from the sending end; and at the receiving end. Tests were made on 120 days, covering 4½ months.

The results of this investigation are being discussed in a contemporary paper by R. J. C. Wood, of the Southern California Edison Company. They will therefore not be discussed in detail here. There was an unusual amount of system switching performed during the investigation and a resulting large number of surges were recorded. However, none of these were of alarming magnitude. The maximum surge recorded was 3.2 times normal voltage to ground. The next highest was 2.7 times normal. During the tests, there were two flashovers of line insulators, neither of which caused a higher voltage than 1.9 times normal, and this might have been caused by a switch operation within a few minutes of the flashover. The weather was fair during the investigation with no lightning.

Oscillations. There have been certain theories advanced regarding the existence of high-voltage, high-frequency oscillations on transmission systems. The above tests while not broad enough to be conclusive on all systems have indicated nothing to substantiate these theories except in the case of an arcing ground. Further, arcing grounds on a grounded-neutral high-tension line produced no high voltage oscillations, and in the case of the free neutral system, the frequency of oscillation did not exceed 16,000 cycles.

IV. Commercial Type of Roll-Film Recorder

A roll-film type of single-electrode klydonograph was under consideration before the construction of the glass-plate type. However, in order to cause as little delay as possible in obtaining a few field instruments, the rotating-electrode plate-type was chosen because of the certainty of its operation with standard clock and standard dry plates.

As soon as the plate-type of recorder proved that the klydonograph was very valuable in obtaining data on surges on transmission systems, the complete design and construction of the roll-film type was undertaken.

Some of the desirable features to be incorporated in such a recorder are as follows:

1. Daylight loading and unloading
2. Independent of power for operation, hence clock-driven
3. At least three electrodes desired
4. A long record possible, hence roll film to be used
5. Time markings correct without resorting to cog-wheel drive and holes in edges of film
6. Capable of standing considerable over-voltage
7. Reasonably constant in its calibration
8. Simple and reliable in operation

9. Truly portable; reasonably small and light
10. Rugged and not too expensive in construction

In order to harmonize these desirable features, a design radically different from that of any existing graphic instrument is required for this klydonograph. In such instruments. the torque required to reroll the chart is constantly increasing from beginning to end, and hence must be supplied by a separate clock spring or motor. The same type of reroll was chosen for this klydonograph as was designed for the long-film attachment for the three-element portable oscillograph. This incorporated the daylight-loading feature and constant torque for driving film and reroll.

Daylight Loading Film. A 6½-in. width of roll-film was chosen as a desirable width for a good three-electrode chart. This film was obtained on a metal, flanged spool having extra deep holes in the ends to permit the introduction of steel pins to act as shafts. These special films are put up in cartons marked for the klydonograph. A film length of 8 ft. was chosen to give a one-week record with a film velocity of ½-in. per hr. Fig. 37 shows this film in place in the klydonograph. The system consists of a main clock-driven drum over which the film passes on its way under the electrodes. The outer shell of the drum is micarta insulation. Beneath this is a thin metal sheet moulded into the micarta tubing. The metal sheet is grounded, through the metal ends of the drum, to the frame. It is practically impossible to make a thin film lay flat on a glass plate and touch all parts of the surface. However, it is much easier to make a film roll over a micarta cylinder and hug the cylinder so as to have no air spaces between the film and the cylinder under the electrodes of the klydonograph. In this design, we have a repetition of the essential features of the original klydonograph. The film used in this klydonograph has black paper on each end but none under the film itself. This permits of daylight-loading and daylight-unloading without introducing an undesirable member between the film and the internally grounded drum.

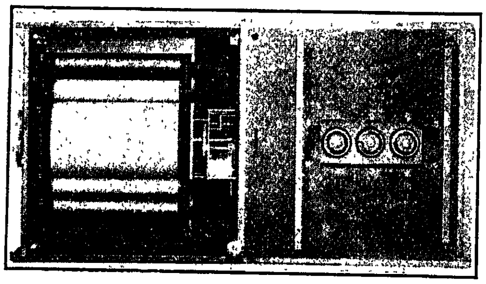


FIG. 37—THE KLYDONOGRAPH, COVER REMOVED, SHOWING FILM IN PLACE

Reroll Scheme. The general construction of the rerolling system can be seen in Fig. 37. On one side of the main drum is the unexposed film; on the other side is the exposed film. Beyond each of these is a roller with pulleys attached. Two helical-spring belts hold the rollers against the film cartridges, and, in turn, the cartridges against the main drum. If the under side of each belt is tight and the upper side relatively slack, the film will be drawn tight over the top half of the main drum. Now, as the main drum is turned by hand, or by the clock, the unrolling action from one spool will cause a rolling-up action on the other spool by means of the rollers and the two belts. The driving pulleys are made larger than the driven pulleys so that the under side of each belt will remain tight and the upper side relatively slack. Thus the film is urged to roll up faster than it is unrolled from the first spool. The rollers and film spools are held in alinement by shaft extension, free to slide in longitudinal grooves in the metal frame.

No slipping of either film or rollers takes place, for the ratio of the pulley diameters is so chosen as to keep the twisting torque as high as possible and yet well below the point where the rollers would slip on the film. In other words, the sum of the four tensions in the belts, multiplied by the coefficient of friction between the film and the rollers, is always greater than the sum of the differences between the tensions in the tight sides and the slack sides. The belts have no tendency to continue to accumulate tension on one side. If each driving pulley is a certain per cent larger in diameter than the driven pulley, then the slack side of the belt will have that same percentage greater number of turns than the tight side. The belt stretches as it leaves the driven pulley and contracts again as it leaves the driver pulley, and so on, indefinitely. Thus, there is no slipping of any member and no tendency to break the helical-spring belt.

It is easily seen that the peripheral speed of the main-drum surface, the film surface, the driving-roller surface, and the driven-roller surface is the same. Also the torque required to drive the drum, and thence the reroll, etc., is constant. As the unexposed film-roll decreases in diameter, the exposed roll increases in diameter, and the film shafts and the roller shafts shift along in the horizontal groove. The center distance between the pulleys remains practically constant with this arrangement and hence the tension in the belts does not change. All of these novel features go to make a simple but reliable reroll system which may be driven by a single clock.

Time Marker. Since the photographic chart is friction driven, it is necessary to devise a time-marking system which will show no error, due to any slight creepage of the film as it passes on its way over the main drum. The clock is geared to the drum so as to drive the film at a uniform speed of approximately one foot per day. The drum actually makes just one revolution in 24 hours, yet the film-travel, in seven days, may vary as much as an inch. If the time markings had a

3. JOURNAL of the A. I. E. E., Vol. XLII, p. 109—Legg: Portable Oscillograph.

proportionate error this would represent two hours. By means of a stationary speck of *luminous paint,* just under one edge of the drum cylinder, and twenty-four equally distributed holes through the outer edge of the cylinder, a dot is exposed on the edge of the photographic film every hour, and a dash every sixth hour. Each hole passes over the speck of luminous paint just as that hole lines up with the three electrodes. Hence, there can be no error in time marking, even if the film should vary several inches in its travel during one week, provided the 8-day clock keeps good time.

An *Hour Circle* is provided on the end of the main-drum shaft on the outside of the klydonograph. This has 24 graduations, each of which lines up with an index at the same time as the corresponding hole in the edge of the cylinder lines up with the electrodes. If the clock loses or gains a few minutes during the week, this can be checked by the reading on the hour circle and later allowed for in estimating the true time of occurrence of surges which may appear on the developed film.

The four dashes, per 24 hours, are all alike so that any one of them may be 6 o'clock or 12 o'clock, day or

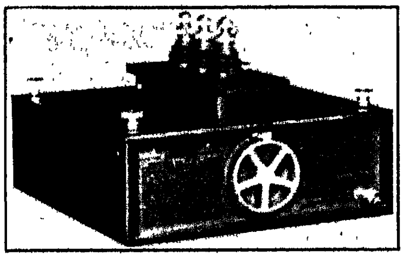


FIG. 38—FILM-TYPE KLYDONOGRAPH—GENERAL VIEW

night. This is done to save film when initially setting the instrument. *A daylight intensity* record appears on the opposite edge of the film from the dots and dashes. Daylight enters the case through a horizontal hole in the upper part of the klydonograph and is reflected downward through a vertical hole in the cover to a point on the film in line with the electrodes, thus giving the record of day and night, by reference to which it is easy to determine which time-dash is 6 a. m., which 12 m., which 6 p. m., and which midnight. A 45 deg. whitened surface, on the inside end of a screw, at the conjunction of the horizontal and vertical light holes, may be twisted so as to regulate the amount of light reaching the edge of the film. At any setting, daytime can be told from night, but by adjusting the amount of reflection according to the locality and position of the klydonograph, it is possible to get a record of the variation in intensity of daylight as caused by changing cloud conditions. This last feature is not essential but is often very helpful in checking up the causes of surges which appear on the film.

Complete Instrument. The complete roll-film type of klydonograph is shown in Fig. 38. The parts which appear on the outside are: the three electrode-terminals; their lead-in bushings; the daylight-intensity adjustor, on one side of the electrode chamber; the cover thumb-nuts; the hour circle, on the instruction panel; and the ground-binding-post. The outfit is but 12 by 12 by $8\frac{3}{8}$ in. over all, the case proper being but half that height. This three-electrode seven-day

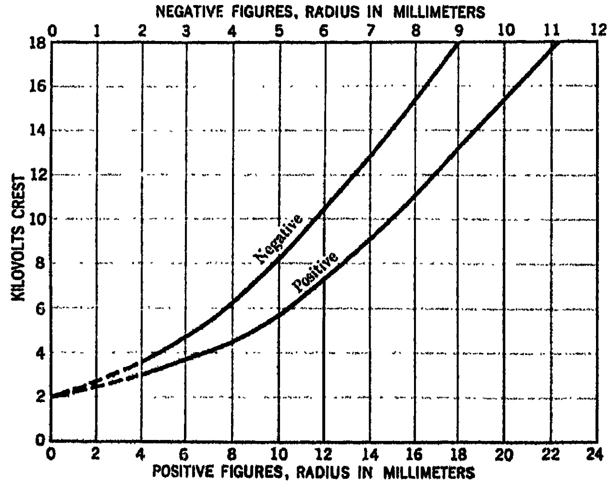


FIG. 39—CALIBRATION OF KLYDONOGRAPH—FILM-TYPE, EXPERIMENTAL MODEL

recorder is no larger than the original single-electrode 24-hour plate-model. Its weight is much less than many single-element graphic meters. It is so portable that it may be taken to any desired location on a transmission system, and in fact, in some cases located up in a transmission tower. For accurate work, special electro-static potentiometers, previously described in

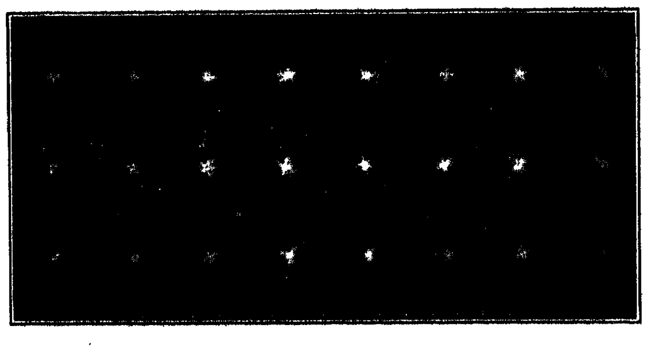


FIG. 40—SECTION OF TYPICAL FILM. ARTIFICIAL SURGES PRODUCED IN LABORATORY

this article, must be used. However, some information can often be obtained by tapping in between the first and second insulators, when of the multiple unit type, so as to lead a fraction of the voltage of each of the lines, of a three-phase system, to an electrode of the klydonograph.

Condensed instructions for loading and operating the instrument are moulded into the micarta panel each

side of the hour-circle. The hour-circle and the ground binding-post are in a recess, so that there is no protrusion beyond the 12 by 12 cases. The top of the movable part of each electrode is protected by a cap-nut over the electrode binding-post. With this arrangement, no delicate mechanism of any kind is exposed, and the instrument may be carried or shipped without danger of breakage.

Introduction and Removal of Film. In loading the klydonograph the whole top is removed, with the electrodes attached. The 8-ft. roll-film, with steel pins in spool ends, is inserted between the main drum and the roller having the larger pulleys. The opaque-paper end is brought up over the main drum and inserted in the slot in the empty spool between the other roller and the drum. This spool is turned until the opaque paper is tight over the upper half of the drum and the second roller is twisted until the lower halves of both spring belts are tight and the upper halves relatively slack. The cover is then replaced, with the electrodes bearing on the opaque paper on top of the drum. The hour circle is then drawn out, thus disengaging the drum from the clock pinion, and is turned until the joint between the opaque paper and the sensitized film is detected by the upward movement of the electrodes (easily seen, or felt, when the cap-nuts are removed). The hour circle is turned in until the graduations indicate the proper hour and fraction thereof, taking the nearest long mark to represent the nearest quarter of the day (6 or 12 o'clock, day or night). At this proper setting the hour circle is released so that the gear on the other end of the drum shaft engages with the clock pinion, thus driving the film ½ in. per hr., for seven days. Meanwhile the daylight-intensity indications, the luminous-paint time markings, and any surge phenomena have made their impressions on the photographic film. After the seven days are up, the hour circle is pulled out and turned onward so as to roll up the last of the film and opaque paper on the second spool. The cover is then lifted and this rerolled film-cartridge removed, without resort to a dark room. Later the films may be developed by any photographer or by the operator in a suitable dark room.

Calibration and Tests. The calibration of this roll-film-type of klydonograph proved to be very nearly the same as that for the plate type, the main difference occurring at the lowest perceptible voltages where the effect of the dielectric constant, and thickness, is most apparent. Fig. 39 shows the calibration chart of radius of figures (both positive and negative) to crest volts applied. Fig. 40 shows a short length of film with artificial surges, timing dots and dashes, and daylight-intensity record. The surge figures are described in detail in earlier parts of this article. Although the normal continuous potential on the electrodes of this instrument will be adjusted at 2500 volts, approximately, tests were made with a-c. potentials as high as 25,000 volts without damaging the instrument or puncturing the film. The calibration is not satisfactory beyond 18,000 volts, but this is 700 per cent above normal, and hence gives a very great range to the instrument. Since there is no power backing the surges as picked off an electrostatic potentiometer, and since the instrument stood higher voltages, backed with power, than can be met in practise, it is undoubtedly safe for the service intended.

Discussion

D. W. Roper: One of the first things that is necessary in order to make improvements in power cables is to know what causes the failure. A paper on dielectric losses in relation to cable failures[1] presented before the Niagara Convention outlined one cause of failure and the method of determining when such failures occur. A study of the failures on the cable system in Chicago, however, indicated that only 40 per cent of the failures of the transmission cables could be accounted for on this basis, and we had to look elsewhere for the balance.

Then it was noted that there were occasional simultaneous failures of switches in generating stations or substations and cables. There was a continuous discussion as to which was the cause and which the effect, and the cause was difficult to determine until the office cat wandered into the switch-house and came in contact with a line reactor. That gave us a very definite cause and when a cable failure occurred at the same time we knew the station trouble was the cause and the cable failure the effect.

That was verified later at another station where a rat got across the insulation of a switch connected to the generator bus and on that occasion two cable failures resulted.

The interesting part is that we know definitely that from these transient voltages cable failure is caused, and the next thing to do is to get some idea of the nature and voltage of these transients. For a number of years we have been securing records by means of needle gaps, but it was hard to tell just what they meant. When a transient sparks across a needle gap, you know that the voltage of the transient is greater than the voltage required to spark across a needle gap, but you don't know how much greater; it may be 10 per cent greater or 100 per cent greater. But in the klydonograph we get an instrument which will tell us that particular quantity.

In the Dufour oscillograph we get an instrument which will give us more intimate information regarding the nature of these transients, their shape and their voltage. The interesting thing about these transients and their frequency is that the theoretical men tell us that high-frequency transients do not travel, and that they cannot travel on underground systems. Then we have a disturbance like this cat and the reactor performance at one place and the cable failure occurs a few miles away. They say that the transient must be due to the effect of the discharge of the magnetic energy in the transformer, but it sometimes occurs a mile or two away from a transformer, and in such a way that the only path from the switch that fails to the cable that fails is through two line reactors which are said to stamp out transient voltages of that frequency.

With the advent of these new tools, we may call them, for attacking the problem of the troubles which occur on our cables, we should be able very shortly, with the continued assistance of the companies in whose laboratories these tools have been developed, to determine first the cause of the failures of our cables and afterwards the cure.

K. B. McEachron: We have been making a few preliminary tests, using the klydonograph and we believe it will be a very useful tool. There is just one point that I wish to bring up

1. Dielectric Losses and Stresses in Relation to Cable Failures, by D. W. Roper, A. I. E. E. TRANSACTIONS, Vol. XLI, 1922, p 547

regarding the calibration for different wave fronts in connection with the work that we have done recently.

Near the beginning of the paper it is stated that a 25-cycle wave would produce a figure of the same size as a wave of short front. We have briefly investigated the effect of using single half cycles of a 60-cycle wave so as to separate the positive and negative figures. This was done by a special synchronous switch, recording the wave form by an oscillograph. To give only one case as an example, we found that a 60-cycle wave of 15 kv. maximum gave a negative figure with a radius of about 1 mm. while a steep-wave front impulse, about 0.01 microsecond, of the same voltage, gave about 10 mm. While there may have been reflections in the impulse circuit tending to increase the voltage, we could hardly expect more than two or three times the voltage due to reflections so that it seems probable that the difference in wave front is responsible for at least a part of the change in the size of the figures.

Based on such data, I want to question the statement that the size of figure is independent of the wave front. We believe that the same size of figure may represent considerably different voltages if the wave fronts vary through a wide range.

B. E. Hagy (communicated after adjournment): It may be of interest to describe briefly the results of some recent tests on the system of The Philadelphia Electric Company in which two of the klydonograph instruments were used.

The tests were made on one line of a double-circuit, 66-kv., ungrounded, open-wire tie line between two generating stations which are about 14 mi. apart. Both lines on the transmission towers are insulated for 110 kv. and have overhead ground wires. The oil switch at the end of the line farthest away from the point where the actual switching was done remained open throughout the tests.

The purpose of the tests was to determine the transient disturbances set up on this line by normal switching operations on the system. Attempts to propagate surges on the line were made by charging electrolytic lightning arresters, by shifting load from line to line, by picking up and dropping an unloaded but fully energized line, and by changing the ratio of some tap-changing transformers at one end of the line, etc. This testing was carried on over a period of two days and several readings of value were obtained.

On one of the two days, a polyphase klydonograph instrument was located at each end of the line, the purpose being to measure the surge voltages at both the sending end and at the distant open end which were expected to be of different magnitude due to attenuation, damping, corona, or other effects. On the second day, both klydonograph instruments were located at the same end of the line but on opposite sides of the line choke coil for the purpose of determining the voltage difference caused by the inductance of the coil. There were considerable differences in simultaneous readings at the two ends of the line and across the choke coil.

The highest value of surge voltage reached during the two days of miscellaneous testing was 4.3 times normal or 240 kv. crest to ground while there were several readings of over three times normal. The reading of 4.3 times normal as well as the majority of the other high readings were recorded when the unloaded but energized line was disconnected from the source by opening the high-voltage line switch.

An interesting point of information indicating desirable operating procedure is noted in comparing the results of switching on the high- and low-voltage sides of the step-up transformers. These results show that while almost no surges are produced when the switching is done on the low-voltage side, surge voltages of more than four times normal are produced when the switching is done on the line side of the transformer.

The changing of transformer ratio under load, the shifting of load at an intermediate substation, the energizing of the line, and the charging of lightning arresters produced only nominal surges, which is somewhat surprising particularly with respect to the operation of charging the electrolytic lightning arresters which was expected to cause surges of fair magnitude. In some cases, the voltages built up by reflection at the distant end of the line as well as the voltage on the line side of the choke coil were simultaneously greater than the voltages on the same phase at the source end and sometimes less. Only a few of these surges were oscillatory in character and these were rapidly damped.

These tests are, of course, of different significance from those described in the paper where the usual application of the instrument was to have it connected more or less permanently to the system over a fairly long period to record normal system surges due to atmospheric conditions as well as switching, while the purpose in the tests described was to reproduce several normal switching operations in a short period and in good weather for the purpose of determining the probable magnitude of switching surges in ordinary operation. Even under the less severe conditions, it will be noted that the voltages recorded are relatively greater than any previously reported. It may be that on a good many systems routine switching operations will produce surges of surprisingly high value.

The latest film-type polyphase instruments were used in these tests and the results secured and the experience with them show the device is convenient to use and gives ample voltage records the meaning of which is easily interpreted.

J. H. Cox: Mr. McEachron raised two questions; one was calibration of the klydonograph when the figures begin to slide, and the other was the accuracy of calibration for various frequencies or steepness of wave fronts. As brought out in the paper, when the figures begin to slide they do not behave in a consistent manner and the voltage is approaching the limit of the instrument. However, when these slides do occur there are nearly always some rays that enanate directly from the center and the length of these conforms to the calibration curve.

In the development of the klydonograph little work was done using wave fronts beyond two limits of length, that is, shorter than five microseconds and longer than that given by a 25-cycle wave. Most of the work was confined between five and 200 microseconds. A great many records were taken with surges as abrupt as could be produced by ordinary means and our work leads us to believe that these conform to the calibration curve of the more tapered surges. They were not used in obtaining these curves for the reason that the voltages produced in such a circuit are indefinite. When a spark-gap is discharged into a simple circuit, as done by Mr. McEachron, you cannot say with any degree of certainty what the voltage is at any part of the circuit. It required a great amount of work to set up a circuit that did not give reflections and oscillations. Successive records taken without changing a set-up did not vary as much as 1 mm. Further, when a single surge was impressed on six terminals tied together, no variation could be measured.

The klydonograph was developed for the purpose of recording transients on transmission systems. It has been gratifying to find that the surges present on practical lines have a wave front between one and 200 microseconds. In this range at least the klydonograph is entirely satisfactory. It must be remembered that no greater accuracy than 15 per cent is possible in such an instrument. This is ample for practical use.

When the application is as slow as a 60-cycle or a 25-cycle wave the conductivity of the film surface has an influence. A slight conductivity in the case of a slow application will allow some charge to leak off the electrode and thus tend to lessen the intensity of the field at the emulsion surface which is necessary to produce a figure. We have found commercial films to vary somewhat in this respect, but not enough to cause concern when dealing with surges. The performance of the instrument at commercial frequencies is not particularly important since there are more convenient methods of measuring such potentials.

Over-Voltage on Transmission Systems Due to Dropping of Load

E. J. BURNHAM[1]
Associate, A. I. E. E.

Synopsis:—*When a waterwheel-driven generator, or a hydroelectric station, loses all or a large part of its load, the voltage rises more rapidly and to a greater extent than has been generally realized.*

The purpose of this paper is to show the manner in which the voltage will rise under different conditions, and a method of calculating the voltage rise of a waterwheel-driven generator when load is lost.

The results of a large number of tests are given showing the rise in voltage on an 8500-kv-a., 6600-volt waterwheel generator when load was tripped under several different conditions at three different places, namely, at the generator, on the high-voltage side of a transformer bank, and at a substation forty miles away.

As it is not considered good practise to subject a transmission system to high over-voltages, consideration is given to methods of checking the voltage rise by use of relays as soon as possible after load has been dropped.

MUCH has been written about instantaneous rises of voltage due to switching surges, and transients due to short circuits, but very little has been said regarding the rise of voltage due to dropping of load on an a-c. system.

Great interest has been shown in various sections of the country during the past year in the voltage obtained when a waterwheel-driven generator, or a hydroelectric station, loses all or a large part of its load, since voltages obtained in this manner are much higher and develop more rapidly than would ordinarily be expected.

When load is lost on an a-c. generator, or group of a-c. generators, the manner in which the voltage rises and the time interval taken for such a rise varies according to the following factors:

1. Voltage regulation and design of the generator.
2. Type and speed regulation of prime mover connected to generator.
3. Generator excitation obtained from direct-connected exciter, exciter motor-generator set, or d-c. bus.
4. Amount and power factor of load dropped.
5. Voltage regulators used or not used.
6. Place in circuit where load is tripped.
7. Connections including lines, transformers, etc., between generator and load.
8. Over-voltage or over-speed devices used or not used.

With the above points in mind, calculations were made to determine the maximum rise in voltage on ordinary waterwheel-driven generators when load was lost under different conditions. The results of these calculations are shown in Table I. Details regarding method of making calculations are given later.

In order to check the calculations and to obtain further information, a series of tests[2] were made, in which load was dropped in different ways and under many different conditions on an 8500-kv-a., three-phase, 60-cycle, 164-rev. per min., 6600-volt, 80-per cent power-factor waterwheel generator, connected to a large 60-cycle system, through an 18,000-kv-a. bank of transformers and a 40-mile, 66,000-volt transmission line.

TABLE NO. I
VOLTAGE RISE IN PER CENT OF NORMAL VOLTAGE

Type of Prime Mover	Over-Speed	With Voltage Regulator		Without Voltage Regulator	
		With direct-connected Exciter	Without direct-connected Exciter	With direct-connected Exciter	Without direct-connected Exciter
Water-turbine....	25–35%	50–80%	40–65%	90–100%	60–85%

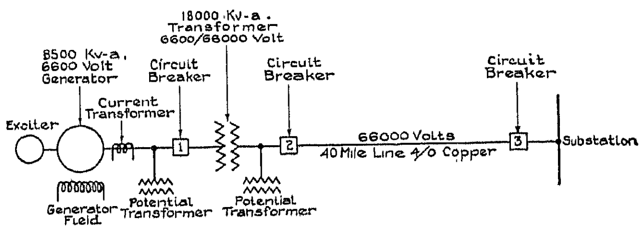


FIG. 1—ARRANGEMENT OF EQUIPMENT FOR TEST

Connections and arrangement of equipment between the generator and substation are shown by Fig. 1. By use of circuit breakers 1, 2, and 3, load could be tripped at any one of three different places. These circuit breakers will be referred to as generator, transformer-high-voltage, and substation circuit breakers, respectively.

The generator had a direct-connected exciter, but in order to also make tests with the generator separately excited, a temporary connection was made to a small, waterwheel-driven exciter. However, the limited capacity of this exciter would permit the generator when separately excited to carry only 4000-kw. load at 98 per cent power factor.

A vibrating type, 60-cycle voltage regulator was in the station, so plans were made to use that in some of the tests.

Connections were made to an oscillograph so that

1. Central Station Engineering Dept., General Electric Company, Schenectady, New York.

2. Tests were taken at the Spier Falls Station of the Adirondack Power & Light Corporation.

Presented at Regional Meeting of Dist. No. 1, Swampscott, Mass., May 7-9, 1925, and at the Annual Convention of the A. I. E. E., Saratoga Springs, N. Y., June 22-26, 1925.

TABLE II

Test	Oscillogram	Load dropped-kw.	Power Factor per cent	Direct-connected Exciter	Regulator	Over-voltage Relay	Over-Frequency Relay	Breaker tripped by Hand	Breaker tripped by Relay	Maximum Voltage in per cent of normal
1	97300	5900	83	Yes	No	No	No	1	–	195
2	97304	6100	83	Yes	Yes	No	No	1	–	151
3	97325	4000	98	No	No	No	No	1	–	128
4	97321	6500	83	Yes	No	No	No	2	–	174
5	97313	6400	90	Yes	Yes	No	No	2	–	157
6	97312	6400	90	Yes	No	No	No	2	–	185
7	97317	6700	83	Yes	No	No	No	3	–	218
8	97323	6100	87	Yes	Yes	No	No	3	–	168
9	97322	6500	83	Yes	Yes	Yes	No	2	1	113
10	97324	6150	83	Yes	Yes	No	Yes	2	1	126
11	97318	6700	83	Yes	No	Yes	No	3	2	183
12	97319	6700	83	Yes	No	Yes	No	3	1	201
13	97310	3500	70	Yes	No	Yes	Yes	2	1	138
14	97320	6700	83	Yes	Yes	No	No	3	–	206
15	97309	3500	66	Yes	No	Yes	No	2	1	138

any three of the following waves could be recorded at the same time:

1. 60-cycle or 40-cycle timing wave.
2. Generator field or exciter voltage.
3. Generator current.
4. Generator voltage.
5. High-tension voltage at a point between transformer and transformer high-voltage circuit breaker.

Table II gives details regarding tests in which load was dropped in different ways and under different conditions.

It will be noted that 6700 kw. was the largest load dropped. The generator could not carry a greater load than this because of the water conditions at the time the tests were made.

Commercial load was used for these tests; therefore, because of the system operating conditions, it was not always possible to drop the load most desirable for a particular test. However, by giving consideration to the amount of load dropped, proper comparisons may always be made between the different tests.

It will also be noted that after a load was dropped, in some cases the speed rise reached 152 per cent of normal, which is a much greater speed increase than generally obtained on waterwheels under similar conditions. This large speed increase is accounted for by the fact that the waterwheel governor had not received final adjustment, but this does not make the tests less valuable, since the speed characteristics for each test can be determined by making a comparison between the timing wave and the generator-voltage wave. Furthermore, the large speed regulation of the waterwheel was offset, to some extent, by the load dropped, which was less than normal; therefore, the results are not far from what might ordinarily be expected, as will be shown later.

Load Dropped by Tripping Generator Breaker Direct-Connected Exciter Used

Fig. 2 shows the results of two tests, a voltage regulator being used in Case 2, but not in Case 1. Curves 1 *A* and 2 *A* show the rise in speed that took place as soon as load was dropped. Curves 1 *B* and 2 *B* show the corresponding rise in generator voltage.

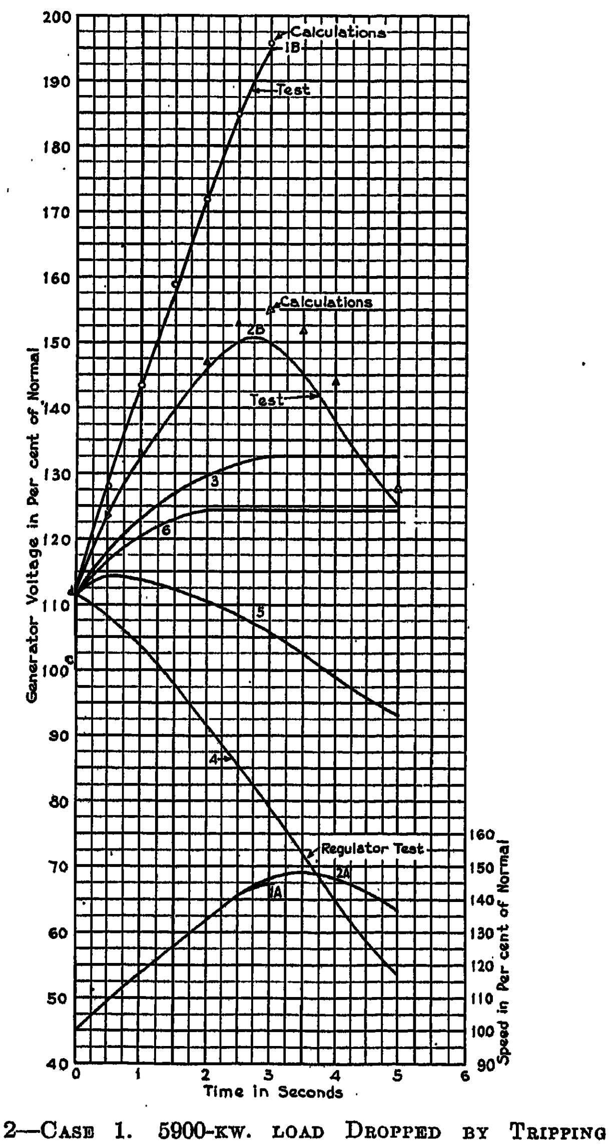


FIG. 2—CASE 1. 5900-KW. LOAD DROPPED BY TRIPPING GENERATOR BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR.

CASE 2. 6100-KW. LOAD DROPPED BY TRIPPING GENERATOR BREAKER. DIRECT-CONNECTED EXCITER. REGULATOR IN OPERATION.

The curves illustrate that as soon as the generator breaker was tripped, the generator voltage rose instantly to approximately 112 per cent of normal, and then increased in accordance with Curve 1 *B* or 2 *B*, 1 *B* being without voltage regulator and 2 *B* being with voltage regulator. The effectiveness of the voltage regulator is plainly seen when a comparison is made between Curves 1 *B* and 2 *B*. The maximum over-speed is greater in Case 2 than in Case 1, as shown by speed Curves 2 *A* and 1 *A*, due to the fact that a slightly larger amount of load was dropped in Case 2 than in Case 1.

Referring to Case 1, Fig. 2, the solid line 1 *B* represents test values as recorded on an oscillogram, while the small circles were plotted from calculations. In Case 2, solid line 2 *B* represents test values, while the small triangles are taken from calculations. The method by which the calculations were made will be given later.

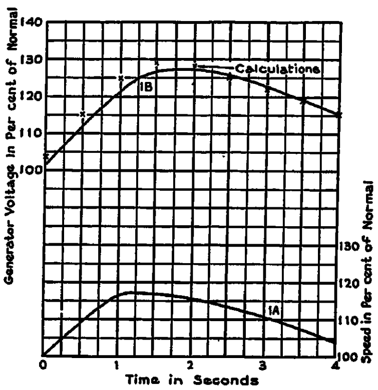


FIG. 3—4000-KW. LOAD DROPPED BY TRIPPING GENERATOR BREAKER. GENERATOR SEPARATELY EXCITED. NO REGULATOR.

LOAD DROPPED BY TRIPPING GENERATOR BREAKER GENERATOR SEPARATELY EXCITED

Results of a test in which the generator was separately excited are shown in Fig. 3. As only 4000-kw. load was dropped, the speed did not rise as high as it did in tests previously described.

After the load was dropped, the voltage increased in accordance with Curve 1 *B*. The small crosses represent points calculated.

LOAD DROPPED BY TRIPPING HIGH-VOLTAGE TRANSFORMER BREAKER DIRECT-CONNECTED EXCITER USED

Fig. 4 shows results of two tests, one taken with and one without voltage regulator. In each case, load was dropped by opening the high-voltage transformer breaker. Here, as before, the voltage regulator was very effective in limiting the rise of voltage.

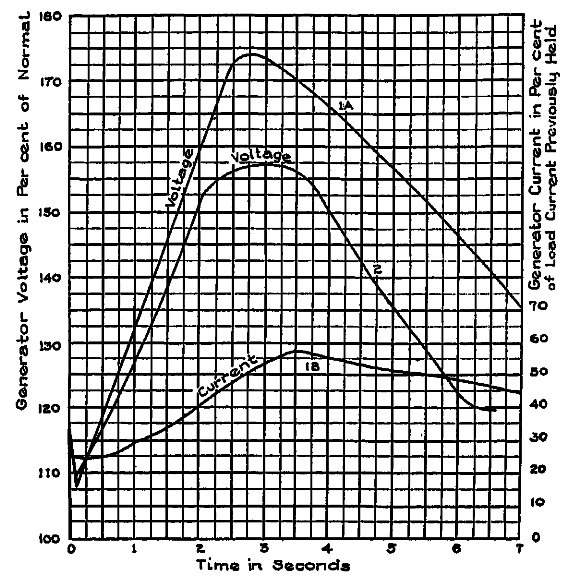
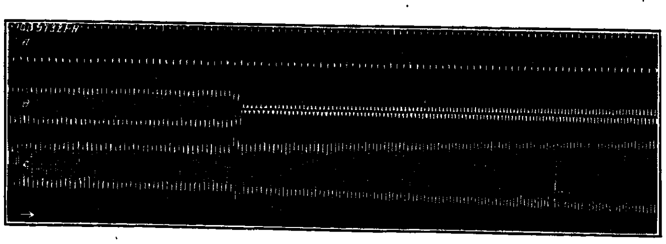


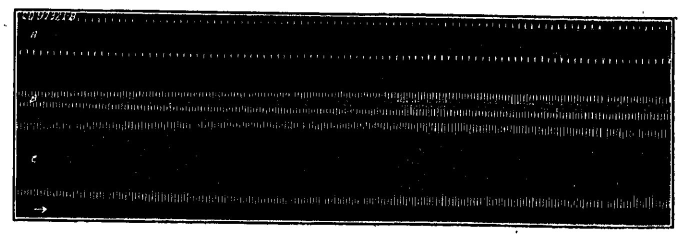
FIG. 4—CASE 1. 6500-KW. LOAD DROPPED BY TRIPPING TRANSFORMER HIGH-VOLTAGE BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR.

CASE 2. 6400-KW. LOAD DROPPED BY TRIPPING TRANSFORMER HIGH-VOLTAGE BREAKER. DIRECT-CONNECTED EXCITER. REGULATOR IN OPERATION.

Allowing for the different amounts of load dropped, it is seen that when the load was tripped by the transformer high-voltage circuit breaker, the voltage did not rise as high as when the load was tripped by the generator circuit breaker. This difference was due to the exciting current taken by the transformer after the high-voltage circuit breaker had been opened, which was load current for the generator. Curve 1 *B*, Fig. 4, shows how the transformer exciting current, or in other words, the generator-load current, increased as the voltage became greater and greater after load had been dropped.

Figs. 5A–B show part of the oscillogram from





FIGS. 5A-B—6500-KW. LOAD DROPPED BY TRIPPING TRANSFORMER HIGH-VOLTAGE BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR.

Curve *A*—40-cycle timing wave Curve *B*—Generator current
Curve *C*—Generator voltage

which Curves 1 A and 1 B, Fig. 4, were taken. The oscillogram shows how the transformer exciting current and the generator voltage increased in value after load was dropped, the middle wave being transformer exciting current and the lower wave generator voltage.

Fig. 6 shows part of an oscillogram similar to that of Fig. 5A, with the exception that a slightly different load was dropped. The middle wave indicates exciter voltage instead of generator current. This middle wave gives a good indication of how the exciter voltage increased in value as the speed increased, after the load was dropped.

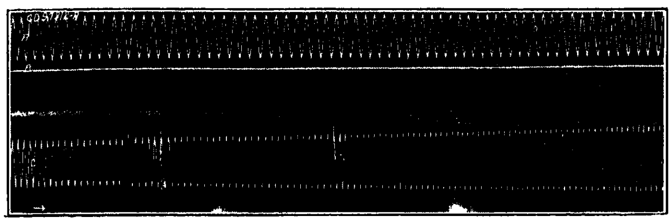


FIG. 6—6400-KW. LOAD DROPPED BY TRIPPING TRANSFORMER HIGH-VOLTAGE BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR.

Curve A—40-cycle timing wave
Curve B—Exciter voltage
Curve C—Generator voltage

LOAD DROPPED BY TRIPPING SUBSTATION BREAKER DIRECT-CONNECTED EXCITER USED

Fig. 7 shows the results of two tests, one taken with, and one taken without, voltage regulator, in which load was dropped by opening substation circuit breaker. The voltage regulator was again very effective in limiting the voltage rise.

Curve 1 of Fig. 7 shows a 118 per cent voltage rise, which is a greater rise than obtained in any of the tests. This is only natural when consideration is given to the fact that the charging current of the 40-mile line more than offset the exciting current of the transformer, the effect of which was to boost the excitation of the generator.

Curve 2 B, Fig. 7, shows the generator current which is a resultant of transformer exciting current and line-charging current. In this instance, the current curve has two peak values, the dip in the middle occurring at the time the voltage reached maximum.

Figs. 8A–B show parts of the oscillogram from which Curves 2A and 2B of Fig. 7 were taken. The middle wave of the oscillogram shows clearly the dip in the generator currents mentioned above, and also the many harmonics in the current wave. The lower wave of the oscillogram represents the generator voltage and shows the switching surge at the time the high-voltage transformer circuit breaker was tripped, and just how the voltage increased after the load was tripped.

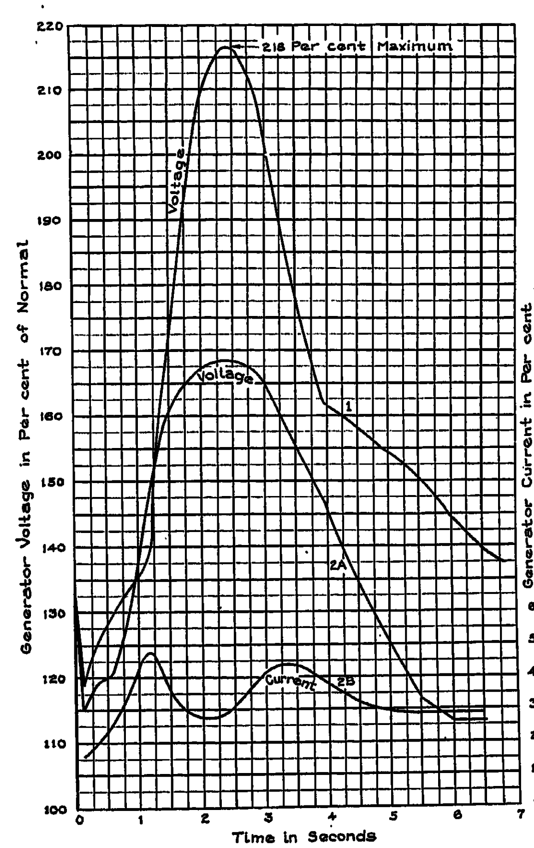
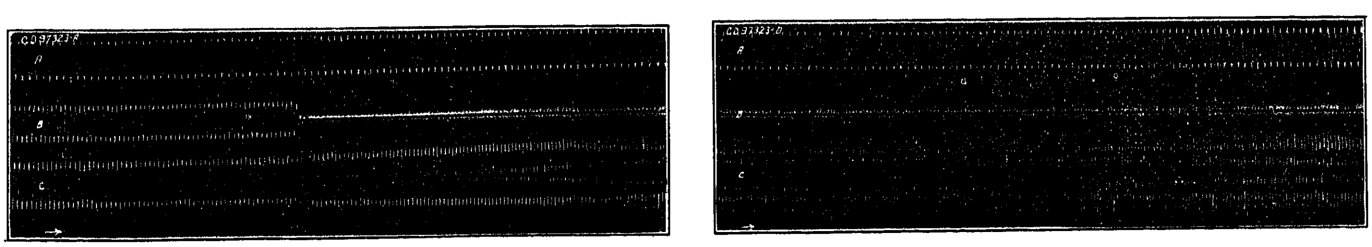


FIG. 7—CASE 1. 6700-KW. LOAD DROPPED BY TRIPPING SUBSTATION BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR.

CASE 2. 6100-KW. LOAD DROPPED BY TRIPPING SUBSTATION BREAKER. DIRECT-CONNECTED EXCITER. REGULATOR IN OPERATION.



FIGS. 8A-B—6100-KW. LOAD DROPPED BY TRIPPING SUBSTATION BREAKER. DIRECT-CONNECTED EXCITER. REGULATOR IN OPERATION.

Curve A—40-cycle timing wave Curve B—Generator current
Curve C—Generator voltage

Voltage Reduction

The tests already described show the rapid and extensive rise of voltage that occurs when load is lost on a waterwheel-driven generator. Believing it desirable to check the voltage rise as soon as possible after load is

TABLE NO. III
REDUCTION IN GENERATOR OPEN-CIRCUIT VOLTAGE

Test	Oscillo-gram	Previous Voltage in % of Normal	Method of Reducing Voltage	Time in Seconds for Voltage to decrease 50%
16	97302	143	Insert Resistance in Exciter Field Circuit	4.75
17	97306	149	Open Exciter Field Switch	3.75
18	97305	52	Trip Generator Field Breaker	0.90
19	97316	100	Trip Generator Field Breaker	1.00

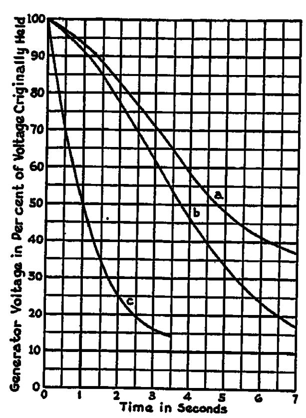


Fig. 9—Generator Open-Circuit Voltage Decreased: Case *a*: by Inserting Resistance in Exciter Field Circuit.
Case *b*: by Opening Exciter Field Switch.
Case *c*: by Tripping Generator Field Circuit Breaker.

dropped, a series of tests was made to determine the speed with which the generator open-circuit voltage could be reduced by three different methods. A tabulation covering these tests is given in Table III. Fig. 9 shows the results of three of these tests, each

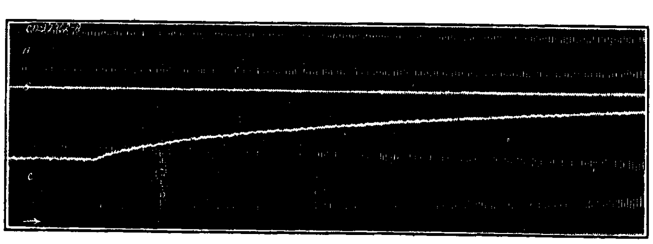


Fig. 10—Generator Open-Circuit Voltage Decreased by Inserting Resistance in Exciter Field Circuit
Curve *A*—60-cycle timing wave
Curve *B*—Exciter voltage
Curve *C*—Generator voltage

method being represented. Curve *a* shows the rate of voltage decrease when a block of resistance was placed in the exciter field circuit, this being accomplished by holding the voltage regulator contacts open. Curve *b* shows the rate of voltage decrease when the exciter field switch was opened, and Curve *c* the rate of voltage decrease when the generator field circuit breaker was tripped. Field discharge resistors were, of course, used in the last two cases.

Fig. 9 shows the advantage of tripping the generator

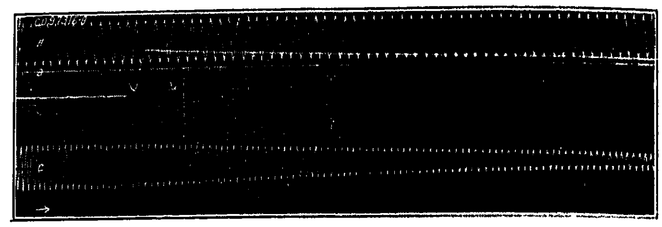


Fig. 11—Generator Open-Circuit Voltage Decreased by Tripping Generator Field Circuit Breaker
Curve *A*—40-cycle timing wave
Curve *B*—Generator field voltage
Curve *C*—Generator voltage

field when it is desirable to reduce the generator voltage at a very fast rate.

Fig. 10 shows the oscillogram from which Curve *a* of Fig. 9 was taken. The middle wave of oscillogram

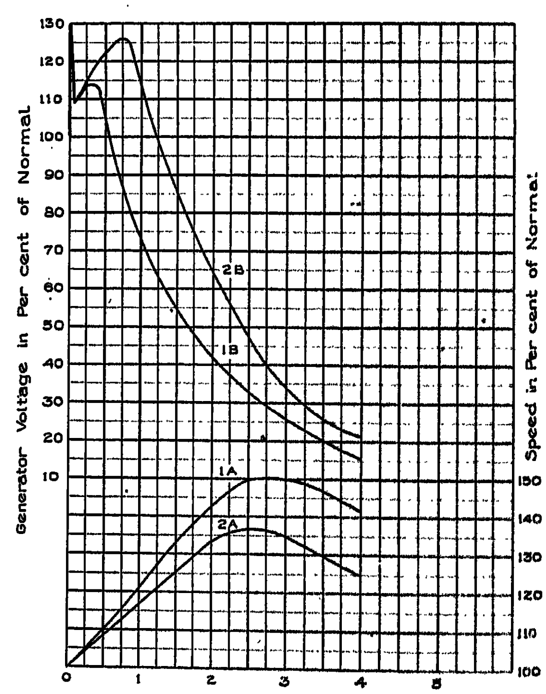


Fig. 12—Case 1. 6500-kw. Load Dropped by Tripping Transformer High-Voltage Breaker. Direct-Connected Exciter. Regulator in Operation. Over-Voltage Relay Tripped Generator Breaker.
Case 2. 6150-kw. Load Dropped by Tripping Transformer High-Voltage Breaker. Direct-Connected Exciter. Regulator in Operation. Over-Frequency Relay Tripped Generator Breaker.

shows the way the exciter voltage decreased when a block of resistance was placed in the exciter field circuit.

Fig. 11 shows the oscillogram from which Curve *c* of Fig. 9 was taken, the generator field circuit breaker

being tripped at a time when the generator was operating at normal voltage, no load. The middle wave of oscillogram shows how the generator field voltage decreased after the field circuit breaker was opened. It will be noted that the generator field excitation was reversed in polarity at the time the field circuit breaker was tripped.

Fig. 12 shows the effect of tripping the generator field circuit breaker, as well as the generator main circuit breaker, after load had been dropped by opening high-voltage transformer circuit breaker by hand.

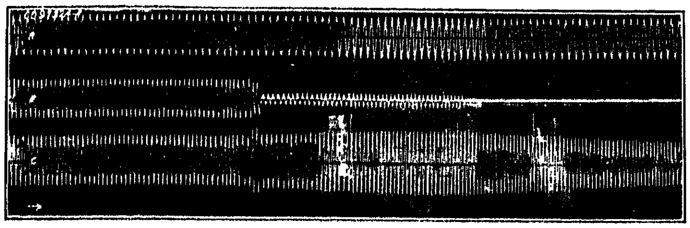


FIG. 13—6150-KW. LOAD DROPPED BY TRIPPING TRANSFORMER HIGH-VOLTAGE BREAKER. DIRECT-CONNECTED EXCITER. REGULATOR IN OPERATION. OVER-FREQUENCY RELAY TRIPPED GENERATOR BREAKER.

Curve *A*—40-cycle timing wave
Curve *B*—Generator current
Curve *C*—Generator voltage

Curves 1 *A* and 2 *A* show the rise in speed and Curves 1 *B* and 2 *B* show the corresponding voltage changes, when load was dropped. The difference in the two speed curves is due to the fact that more load was dropped in Case 1 than in Case 2.

Voltage Curves 1 *B* and 2 *B* are also different. In Case 1, the generator and field circuit breakers were tripped by an over-voltage relay set at 115.5 per cent of normal voltage. In Case 2, the generator and field circuit breakers were tripped by an over-frequency relay set at 63 cycles or 105 per cent of normal frequency. In each case, auxiliary contactors were used with the relays so arranged that the trip circuits of the generators and field circuit breakers would be energized at the same time.

OVER-VOLTAGE AND OVER-FREQUENCY RELAYS

The curves in Fig. 12 give information regarding the operation of the relays, as well as information regarding the voltage and speed changes. In Case 1, the generator and field circuit breakers were tripped sooner than in Case 2. This was due to the fact that the over-voltage relay in Case 1 was operated by the initial voltage impulse which reached approximately 130 per cent of normal, while the over-voltage relay in Case 2 did not operate until approximately a quarter of a second later when the speed reached 105 per cent of normal.

Fig. 13 shows part of the oscillogram from which Curves 2 *A* and 2 *B* of Fig. 12 were taken. The middle wave on the oscillogram represents the generator current and indicates clearly the time when the high-voltage transformer circuit breaker opened and also the time when the generator circuit breaker opened. By use of the 40-cycle timing wave, it is found that the generator circuit breaker opened approximately 0.73

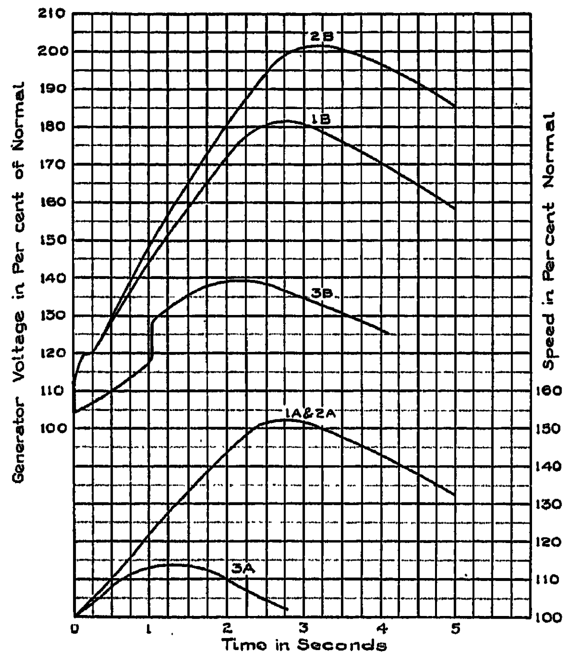


FIG. 14—CASE 1. 6700-KW. LOAD DROPPED BY TRIPPING SUBSTATION BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR. OVER-VOLTAGE RELAY TRIPPED TRANSFORMER HIGH-VOLTAGE BREAKER.

CASE 2. 6700-KW. LOAD DROPPED BY TRIPPING SUBSTATION BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR. OVER-VOLTAGE RELAY TRIPPED GENERATOR BREAKER.

CASE 3. 3500-KW. LOAD DROPPED BY TRIPPING TRANSFORMER HIGH-VOLTAGE BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR. COMBINATION OVER-VOLTAGE AND OVER-FREQUENCY RELAY TRIPPED GENERATOR BREAKER.

seconds after the high-voltage transformer circuit breaker was tripped.

Three different tests are represented in Fig. 14. An over-voltage relay was used in Case 1 and Case 2, but

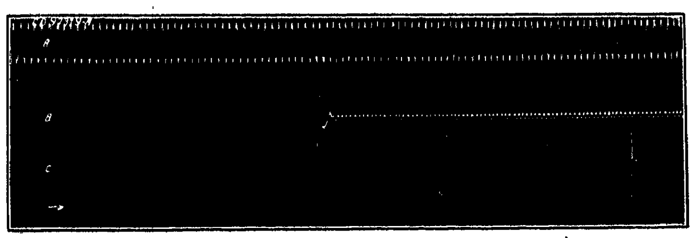


FIG. 15—6700-KW. LOAD DROPPED BY TRIPPING SUBSTATION BREAKER. DIRECT-CONNECTED EXCITER. NO REGULATOR. OVER-VOLTAGE RELAY TRIPPED GENERATOR BREAKER.

Curve *A*—40-cycle timing wave
Curve *B*—High-tension voltage at transformer terminals
Curve *C*—Generator voltage

in Case 3 a combination relay of both over-voltage and over-frequency was used. In the latter case, the over-voltage element was set at 115 per cent normal voltage, and the over-frequency element was set at 105 per cent

normal frequency. The contacts of both elements were placed in series, so the tripping circuit of the generator breaker would not be energized until each set of contacts had been closed. An analysis of the curves shows that the contacts of the over-voltage relay closed approximately 0.5 seconds later than the closing of the over-frequency relay contacts.

In Case 1, the relay tripped the high-voltage transformer circuit breaker, while in Case 2, the relay tripped the generator circuit breaker. In Case 1, the transformer exciting current furnished load for the generator; therefore, the voltage did not rise as high as it did in Case 2, where the generator had no load after the generator breaker was tripped.

Fig. 15 shows part of the oscillogram from which Curve 2 *B* of Fig. 14 is taken. The middle wave of the oscillogram represents the voltage on the high-voltage side of the transformer. The small voltage indicated after the generator breaker was opened was induced from a 110,000-volt, 60-cycle line, which parallelled the line being used for these tests.

Generator Circuit Breaker and Relay Tests

In order to check the operation of the relays, two tests were made with the use of the oscillograph, one to determine the operating time of the generator circuit breaker and the other to determine the operating time of the over-voltage relay. The results of the tests are shown in Table IV.

TABLE IV
OPERATING TIME
GENERATOR CIRCUIT BREAKER AND OVER-VOLTAGE RELAY

Test	Oscillogram	Equipment	Operating Time Seconds
20	97314	Generator Breaker and Relay	0.21
21	97315	Generator Breaker	0.11

General Considerations and Calculations

Voltage Regulation. Due to regulation, the voltage of an a-c. generator carrying full load at normal speed and voltage will generally rise between 25 and 35 per cent above normal when load is lost, providing the speed and excitation remain constant.

Part of this rise in voltage is instantaneous and part takes place over an interval of time. Consider the 8500-kv-a., 6600-volt a-c. generator used in the tests just described, and assume that it is carrying a load of 5900 kw., 0.83 per cent power factor, as in test No. 1.

When carrying this load, the calculated internal or generated voltage is equal to 111 per cent of the terminal voltage. The internal voltage differs from the terminal voltage by the amount of impedance drop in the stator winding. If the load just mentioned is dropped, the terminal voltage will instantly rise to a value equal to the internal voltage because the flux to produce this voltage is already present in the stator and because the impedance drop in the stator winding disappears as soon as load is dropped. Referring to Fig. 2, this instantaneous voltage rise is shown by line *c d*, the calculated value being 111 per cent and the test value being 112 per cent of normal voltage.

Using calculations and assuming, further, that the speed and excitation remain constant, the voltage will increase from 111 per cent voltage to the open-circuit voltage. This rise of voltage is shown by Curve 6, Fig. 2, and takes place over an interval of time, because after load is tripped and armature reaction disappears, time is consumed in building the stator flux up to a value corresponding to the field m. m. f.

For an average generator on which full load is lost, this time may be assumed to be two seconds, but varies to some extent, according to the size and design of the machine in question.

Prime Mover. As the governors of waterwheels and steam turbines are speed devices, a change in load is always accompanied by a change in speed. Given sufficient time, the new output of energy will be produced at a new steady speed. (Only by readjustment of the governor can the new output be produced at the previous speed.) When the decrease of load is great and sudden, the speed will rise higher than the ultimate speed, this over-shooting being due to the inability of the governor system to follow quickly enough the speed of the turbine rotor.

The overspeed of water turbines when full load is lost varies according to the horse power of the turbine, the time taken to close the guide vanes, the $W R^2$ of combined generator and turbine unit, the rev. per min., and the length of penstocks. However, these different factors combine in such a way that for an ordinary design of water turbine, the overspeed is usually between 25 and 35 per cent when load is dropped.

Fig. 16 shows some typical speed-time curves indicating the rise in speed on a large water turbine when different amounts of load were dropped. In this particular case, the speed rose to 131.5 per cent of normal, when full load was dropped.

In the tests referred to in the first part of this article, the speed regulation was greater than customary for the reason already given, namely, the waterwheel governor had not received final adjustment.

The maximum speed attained by a steam turbine when load is suddenly removed is much less than that attained by a water turbine under similar conditions.

Tests taken on a 30,000-kw. steam turbine show that when full load was dropped the governor action was such that the maximum speed attained was only four per cent above normal. This maximum speed was reached in one second. Upon dropping a 75 per cent load on the same turbine a maximum speed rise of 2.75 per cent normal was attained in 0.9 seconds. Even if the emergency governor of a steam turbine is called upon to operate, the speed will generally not rise above 110 per cent of normal.

In view of the foregoing, the danger from the over-

voltage on steam stations is not as great as on hydroelectric stations.

Generator Excitation. Field excitation for a-c. generators is generally obtained by one of the following methods:

1. D-c. bus.
2. Direct-connected exciter.
3. Exciter motor-generator set.

The maximum over-voltage obtained when load is dropped depends, to some extent, on which method of excitation is used, as a direct-connected exciter will overspeed with the generator when load is dropped. The same is true of an exciter motor-generator set if the motor of the set receives power from a circuit connected to the a-c. generator which loses load.

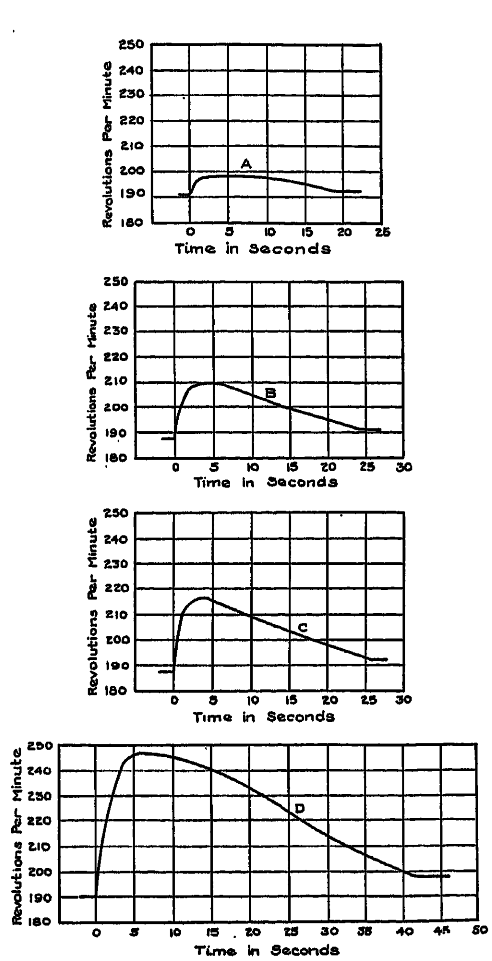


FIG. 16—LOAD REJECTION ON A 44,000-KW. WATER TURBINE
Case *A*— 39.8 per cent load dropped
Case *B*— 58.0 per cent load dropped
Case *C*— 67.0 per cent load dropped
Case *D*—100.0 per cent load dropped

An increase in the speed of the exciter causes an increase in the exciter voltage, which, in turn, causes an increase in the a-c. generator field current and, therefore, a further rise in the a-c. generator voltage.

For exciters generally used, the armature voltage increases approximately as the square of the speed, as the speed is increased above normal. The a-c. generator field current increases in direct proportion to the increase in exciter voltage providing the resistance in the field circuit remains the same, but the a-c. generator voltage will not increase in direct proportion to the increase in field current due to saturation of the iron in the generator magnetic circuit. However, if a waterwheel-driven generator, having a direct-connected exciter, drops load, the field current will rise to such an extent that the open-circuit voltage will usually reach the upper part of the saturation curve, which is nearly flat.

Referring again to Fig. 2, the voltage rise shown by Curve 6 assumes no increase in speed when load is dropped, and assumes the generator to be separately excited. If the speed is increased and field excitation held constant, the voltage will actually rise above this Curve 6 in direct proportion to the increase in speed. This is true because the open-circuit voltage of an a-c. generator is always proportional to the generator speed, providing the excitation remains constant.

Now assume that the 8500-kv-a. generator has a direct-connected exciter and that it is connected to a waterwheel having a speed characteristic as shown by Curve 1 *A*, Fig. 2. Curve 3, Fig. 2, shows how the generator voltage increases to 132.5 per cent of normal assuming an overspeed on the exciter, but no over-speed on the generator. (This is a physical impossibility, but is suggested as a step toward determining the actual voltage). Instantaneous increase in voltage *c d* is the same as before. For the voltage rise indicated by Curve 3, a time lag of three seconds has been assumed, which takes into consideration the lag of flux change in both the exciter and the a-c. generator.

When the load is dropped, the generator, as well as the exciter, overspeeds; therefore, the calculation of the actual generator voltage curve for this condition involves increasing the value of the voltage points on Curve 3, in accordance with the increase in speed, as shown on speed Curve 1 *A*. The resulting calculated values of actual generator voltage are indicated by the circle points falling on or near Curve 1 *B*.

Voltage Regulators. In case a generator voltage regulator is used in connection with a generator that loses load, the regulator will, of course, try to keep the voltage from rising. As soon as load is lost, the regulator contacts at once open and a block of resistance is inserted in the exciter field circuit. If a large amount of load is dropped on a waterwheel-driven generator, the generator voltage rises in spite of the fact that a block of resistance has been placed in the exciter field circuit, principally because flux change in the exciter and the generator is relatively slower than the speed change.

Curve *A* of Fig. 9 represents a voltage regulator test, as already mentioned. This curve is reproduced in Curve 4 of Fig. 2 for use in making calculations. For convenience, the starting point of the curve is raised to 111 per cent generator voltage and all other points are raised in the same proportion.

It was previously stated that Curve 3 of Fig. 2 shows how the generator voltage increases, assuming an over-speed on the exciter, but no over-speed on the generator. By combining Curve 3 and Curve 4, a resultant curve is obtained, as shown by Curve 5, which represents approximately the change in voltage when a load of 6100 kw., 0.83 power factor is dropped, a direct-connected exciter and a voltage regulator being used.

Curve 3 is used to take care of a 6100-kw. drop of load, as well as a 5900-kw. drop of load, because, if two curves were plotted, one would fall practically on the other. Now, increasing the voltage points on Curve 5, in accordance with the increase of speed, as shown by Curve 2 *A*, final calculated results are obtained as shown on the small triangles. These calculated points fall reasonably close to the test Curve 2 *B*.

Amount and Power Factor of Load Dropped. The per cent voltage regulation of a generator and the per cent speed regulation of a turbine being greater for full load than for part load, it is obvious that the loss of full load on a generator will cause a greater rise in voltage than dropping part load.

As the power factor of a load being furnished by a generator is decreased below unity (assuming constant kw. output), the field excitation, the kv-a. output, and the internal generated voltage increase. Therefore, the lower the power factor of the load dropped, the greater will be the rise in voltage.

Conclusions

It is not considered good practise to subject a transmission system to high over-voltages, especially if they are of long duration, because such over-voltage might cause damage to the generators, motors, transformers, lightning arresters, lines, insulators, and other equipment connected to the system.

Each piece of apparatus is designed for a particular maximum voltage, should be tested in accordance with A. I. E. E. Standardization Rules, and should operate continuously and successfully at that voltage under normal operating conditions. If the voltage is increased above normal, apparatus or equipment may or may not fail, according to the following factors:

1. Maximum over-voltage obtained.
2. Duration of over-voltage, taking into consideration the speed with which the voltage is increased or decreased.
3. Factor of safety in insulation and design of apparatus or equipment.

In other words, the time element of over-voltage must be taken into consideration as well as the maximum overvoltage. For example, a piece of apparatus may withstand an impulse voltage of double normal or more, when the duration of the impulse is a fraction of a cycle, but may fail if subjected to a voltage of much less value when applied continuously for several seconds at normal frequency.

In general, it is desirable to design any piece of apparatus so that it has as high a factor of safety against failure as is consistent with an economical design. The proper balance between factor of safety and economy of design depends largely on the principle underlying the design. In some kinds of apparatus, such as transformers, a high factor of safety is possible; in others, as lightning arresters, the same high degree of safety with over-voltage is incompatible with the proper function of the device.

An attempt to capitalize the factor of safety by operating a piece of apparatus at voltages above normal should never be made. Furthermore, it should not be assumed that the factor of safety will take care of any over-voltage that may exist momentarily on a system.

Although the apparatus treated thus may not fail immediately, the insulation or other parts may be weakened, so that by repeating or continuing the conditions of over-voltage, eventual failure is invited.

Total loss of load, and therefore dangerous over-voltage, may occur on any hydroelectric station having only one or two outgoing lines. If all, or nearly all, of the power is fed out from the station over one line, then in case of trouble it is quite possible that load on this line will be lost, particularly if the line is adequately protected by relays. If a station has two outgoing lines, both line breakers may occasionally trip automatically; also, if two outgoing lines are used, one line may be under repair, in which case all of the power would be transmitted over the second line. The situation would then be the same as though only one line existed.

When the load is fed out over three or more lines, there is much less likelihood of complete loss of load; therefore, the same precautions are not as essential as in the case of stations having only one or two circuits.

The contact of the over-speed and the over-voltage relays may be connected either in multiple or in series. If they are connected in multiple, the over-voltage relay should be given a sufficiently high setting to prevent it operating in case of ordinary switching surges. If the contacts are connected in series, the over-voltage, as well as the over-speed relay, may be given a rather low setting.

Discussion

H. W. Smith: The paper by Mr. Burnham is valuable in that it gives us actual tests on the over-voltages due to over-speeding of waterwheel generators. This problem has also been encountered by the Niagara Falls Power Company who have had lightning arresters fail due to the rise in voltage on dropping of load. Tests have shown that with load suddenly dropped, the generator voltage has increased from 12,000 to 21,000 volts. There was an instantaneous increase up to about 30 per cent above normal voltage. An over-voltage relay has been used to correct this difficulty, and the primary relay of the standard induction feeder regulator has been used as the voltage relay. This relay operates in about ten cycles of a 25-cycle system, and has been set to operate at 30 per cent above normal voltage.

At the Mitchell Dam plant of the Alabama Power Company,

the combination of over-voltage and over-frequency relays is used.

W. F. Dawson: There seems to be a difference in practise in respect to waterwheel governing. We in this country do not, so far as I know, provide by-passes to the penstock. Therefore it is necessary to have the governors operate very slowly. Otherwise the inertia of the water column would destroy the penstock. I do not know how far they have carried the practise, but I happened to be in England about eighteen years ago when a prominent London firm was asking for tenders for d-c. generators to be connected with water-wheels at the Lockleven plant of the British Aluminum Company. Following our American practise, I designed the proposed d-c. generators for about 50 per cent overspeed. It is the custom of consulting engineers in this country to require that generators and water-wheels be perfectly safe from mechanical overspeed stresses, at the anticipated runaway speed of the water-wheels. I told the consulting engineer that we had made such provision in the machines upon which we were tendering, and he was surprised.

He said, "We do not have to allow for that. Our waterwheel builders provide by-passes about the penstock, so that the governors can operate with sufficient rapidity to prevent overspeeding."

That is the only case I have ever had brought to my attention suggesting that by-passes were provided. If any waterwheel designers are here, we should have information from them, because it is certainly a tremendous handicap to the designers of generators for waterwheel plants, not to speak of the bad effect on over-voltage, to have to provide for from 50 to 75 per cent over-speed.

I realize, of course, that by-passes mean increased capital account, but the high over-speeding that has to be provided for also means big increase in the cost, not to speak of the disadvantage due to high over-voltage.

H. C. Don Carlos: I should like to correct the impression which might be gained from the last remarks, which intimate that pressure-regulating valves are not commonly used in this country. I think that a large majority of the high-head plants in this country are designed to use relief valves which are operated from the governor mechanically to give a by-pass in the case of heavy load rejections, which would tend to produce a heavy pressure rise in the penstock.

There is another factor which I believe has not been mentioned in the paper or the discussion, which should not be overlooked in a consideration of the over-speed of hydroelectric units, that is, the characteristic of the Francis-type turbine itself, which protects it against an over-speed of more than 65 to 70 per cent. Without any governors at all, most water wheels of this type will not attain an over-speed of more than 65 to 70 per cent on account of the choking of the water in the wheel.

E. J. Burnham: It has been suggested that resistance might be placed in the exciter field, and also in the generator field, in order to check the rise of voltage on a generator that has lost load.

Curve *a*, Fig. 9, of the paper shows the way generator voltage decreases when resistance is placed in the exciter field circuit. As the resistance in the field circuit is increased, Curve *a* approaches Curve *b* as the limit.

The results of inserting resistance in the generator field circuit at the time of dropping load would be represented by a curve lying between Curve *b* and Curve *c*, of Fig. 9. The method of inserting resistance in the generator field circuit could easily be adopted in cases where face-plate regulators are used.

Regarding the time of reaching maximum speed, after full load has been dropped, the time of three seconds is not uncommon on waterwheels of ordinary design, as shown by Curve *D*, of Typical Water Wheel Characteristics, Fig. 16.

The water-turbine characteristic curves shown by Fig. 16 apply, in a general way, to water turbines of different sizes, and used under different conditions, because, taking into consideration the different factors, such as size of turbine, length of penstock, and $W R^2$ of revolving parts, the results will be approximately the same in any case.

In accordance with Mr. Don Carlos' remarks, by-passes are commonly used in this country for stations having high heads.

The Loaded Submarine Telegraph Cable

BY OLIVER E. BUCKLEY[1]
Member, A. I. E. E.

Synopsis.—*With an increase of traffic carrying capacity of 300 per cent over that of corresponding cables of the previous art, the New York-Azores permalloy-loaded cable marks a revolution in submarine cable practise. This cable represents the first practical application of inductive loading to transoceanic cables. The copper conductor of the cable is surrounded by a thin layer of the new magnetic material, permalloy, which serves to increase its inductance and consequently its ability to transmit a rapid succession of telegraph signals.*

This paper explains the part played by loading in the operation of a cable of the new type and discusses some of the problems which were involved in the development leading up to the first commercial installation. Particular attention is given to those features of the transmission problem wherein a practical cable differs from the ideal cable of previous theoretical discussions.

Brief mention is made of means of operating loaded cables and the possible trend of future development.

* * * * *

THE announcement on September 24, 1924, that an operating speed of over 1500 letters per minute had been obtained with the new 2300-mile New York-Azores permalloy-loaded cable of the Western Union Telegraph Company brought to the attention of the public a development which promises to revolutionize the art of submarine cable telegraphy. This announcement was based on the result of the first test of the operation of the new cable. A few weeks later, with an improved adjustment of the terminal apparatus, a speed of over 1900 letters per minute was obtained. Since this speed represents about four times the traffic capacity of an ordinary cable of the same size and length, it is clear that the permalloy-loaded cable marks a new era in transoceanic communication.

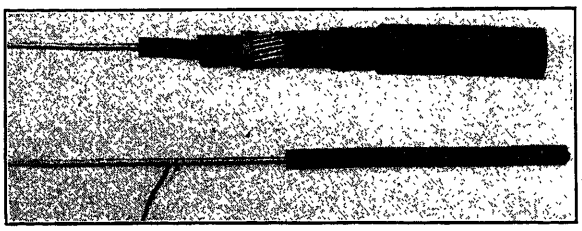


FIG. 1—PERMALLOY-LOADED CABLE
Above, section of deep sea type showing construction.
Below, section of core showing permalloy tape partly unwound.

The New York-Azores cable represents the first practical attempt to secure increased speed of a long submarine telegraph cable by inductive loading and it is the large distributed inductance of this cable which is principally responsible for its remarkable performance. This inductance is secured by surrounding the conductor of the cable with a thin layer of permalloy. Fig. 1 shows the construction of the deep sea section of the cable. In appearance it differs from the ordinary type of cable principally in having a permalloy tape, 0.006 in. thick and 0.125 in. wide, wrapped in a close helix around the stranded copper conductor.

Permalloy, which has been described by Arnold and Elmen[2], is an alloy consisting principally of nickel and iron, characterized by very high permeability at low magnetizing forces. The relative proportion of nickel and iron in permalloy may be varied through a wide range, or additional elements, as for example, chromium, may be added to secure high resistivity or other desirable properties. On account of its extremely high initial permeability, a thin layer of permalloy wrapped around the copper conductor of a cable greatly increases its inductance even for the smallest currents.

In the case of the New York-Azores cable the permalloy tape is composed of approximately 78½ per cent nickel and 21½ per cent iron and gives the cable an inductance of about 54 millihenries per nautical mile. An approximate value of the initial permeability of the permalloy in that cable may be obtained by assuming the helical tape replaced by a continuous cylinder of magnetic material of the same thickness. This material would have to have a permeability of about 2300[3] to give the observed inductance. A better appreciation of the extraordinary properties of the new loading material may be obtained by comparing this permeability with that which has previously been obtained with iron as the loading material. The Key West-Havana telephone cables are loaded with 0.008-in. diameter soft iron wire. The permeability of this wire, which was the best which could be obtained commercially when that cable was made, is only about 115, or approximately one-twentieth of that of the permalloy tape of the New York-Azores cable.

The proposal to use permalloy loading to increase the

1. Bell Telephone Laboratories, Inc.

Presented at the Annual Convention of the A. I. E. E., Saratoga Springs, June 22-26, 1925.

2. *Journ. Franklin Inst.*, Vol. 195, pp. 621-632, May 1923.

3. The true initial permeability is slightly higher. To compute it, account must be taken of the fact that, contrary to what has been sometimes assumed, the magnetic lines of induction in the tape do not form closed loops around the wire but tend to follow the tape in a helical path. The pitch of the helical path of the lines of induction is slightly less than that of the permalloy tape with the result that a line of induction takes a number of turns around the conductor, then crosses an airgap between two adjacent turns of tape and continues along the tape to a point where it again slips back across an airgap. O. E. Buckley, British Patent No. 206,104, March 27, 1924; also K. W. Wagner, *E. N. T.*, Vol. I, No. 5, p. 157, 1924.

speed of long telegraph cables was one outcome of an investigation undertaken by the author, soon after the war, to determine whether some of the new methods and materials developed primarily for telephony might not find important application to submarine telegraphy. In the subsequent development of the permalloy loaded cable a large number of new problems, both theoretical and practical, had to be solved before the manufacture of a cable for a commercial project could be undertaken with reasonable assurance of success. The problems encountered were of three principal kinds. First was that of the transmission of signals over a cable having the characteristics of the trial conductors made in the laboratory. Although the theory of transmission over a loaded cable had been previously treated by others, the problem considered had been that of an ideal loaded cable with simple assumptions as to its electrical constants and without regard to the practical limitations of a real cable. The second class of problems had to do with the practical aspects of design, manufacture, and installation. In this connection an extensive series of experiments was conducted to determine the

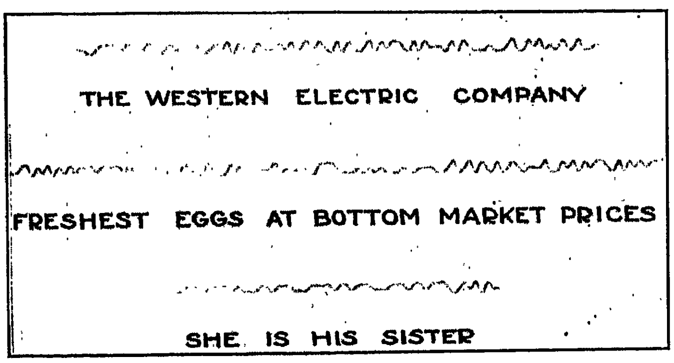


Fig. 2—Test Message—Western Union New York-Azores Permalloy Loaded Cable

Sent from Horta (Azores) and received at New York November 14, 1924.
Speed—1920 letters per minute.
Recorded with special high speed siphon recorder.

means required to secure, at the ocean bottom, the characteristics of the laboratory samples on which the transmission studies were based. Among the numerous problems which arose in this connection were those concerned with protecting the copper conductor from any possible damage in the heat-treating operation which was necessary to secure the desired magnetic characteristics, and those concerned with protecting the strain-sensitive permalloy tape from being damaged by submerging the cable to a great depth. The third class of problems had to do with terminal apparatus and methods of operation. The prospective speed of the new cable was quite beyond the capabilities of standard cable equipment and, accordingly, new apparatus and operating methods suited to the loaded cable had to be worked out. In particular it was necessary to develop and construct instruments which could be used to demonstrate that the speed which had been predicted could actually be secured. The success of the investigations along all three lines is attested by the results which were obtained with the New York-Azores cable. Fig. 2 shows a section of cable recorder slip, the easily legible message of which was sent from Horta, Fayal, and received at New York at a speed of 1920 letters per minute.

It is principally with regard to the first of these classes of problems, that of the transmission of signals, that the following discussion is concerned. No attempt will be made here to discuss the details of design and development of the physical structure of the cable, nor will there be given a detailed description of the operating results or how they were obtained; these subjects must be reserved for later publication. It is desired in what follows to explain how inductive loading improves the operation of a submarine cable and to point out some of the problems concerned with the transmission of signals which had to be considered in engineering the first long loaded cable.

In order to understand the part played by loading in the transmission of signals, it is desirable first to review briefly the status of the cable art prior to the introduction of loading, and to consider the factors then limiting cable speed and the possible means of overcoming them. A cable of the ordinary type, without loading, is essentially, so far as its electrical properties are concerned, a resistance with a capacity to earth distributed along its length. Although it does have some inductance, this is too small to affect transmission at ordinary speeds of operation except on cables with extremely heavy conductors. The operating speed of a non-loaded cable is approximately inversely proportional to the product of the total resistance by the total capacity; that is,

$$S = \frac{k}{C R l^2}$$

where C is capacity and R resistance per unit length, and l is the length of the cable. The coefficient, k, is generally referred to as the speed constant. It is, of course, not a constant since it depends on such factors as terminal interference and method of operation, but is a convenient basis for comparing the efficiency of operation of cables of different electrical dimensions. As the technique of operating cables has improved, the accepted value of k has increased, its value, at any time, being dependent on the factor then limiting the maximum speed obtainable. This factor has, at times, been the sensitiveness of the receiving apparatus, at other times, the distortion of signals, and in recent years, interference. During a great part of the history of submarine cable telegraphy, distortion was considered the factor which limited the speed of operation of long cables and on this account most of the previous discussions of submarine cable transmission have been concerned principally with distortion and means for correcting it. As terminal apparatus was gradually

improved, means of correcting distortion were developed which practically eliminated distortion as an important factor in the operation of long cables. With distortion thus eliminated, the speed was found to be limited principally by the sensitiveness of the receiving apparatus. This limit was, however, in turn eliminated by the development of signal magnifiers. During recent years, in which numerous cable signal magnifiers have been available and methods of correcting distortion have been understood, the only factor limiting cable speed has been the mutilation of the feeble received signals by interference. Most cables are operated duplex, and in these, the speed is usually limited by interference between the outgoing and incoming signals. In cables operated simplex, and also in cables operated duplex where terminal conditions are unfavorable, speed is limited by extraneous interference which may be from natural or man-made sources and which varies greatly in different locations. The strength of the received current must, in either case, be great enough to make the signals legible through the superposed interference current. Owing to the

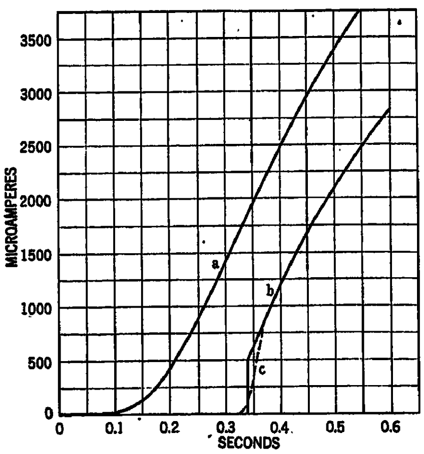


Fig. 3—Arrival Curves

a. Non-loaded cable
b. Ideal loaded cable
c. Real loaded cable (approximate)

rapidity with which the received signal amplitude is decreased as the speed of sending is increased, the limiting speed is quite sharply defined by the interference to which the cable is subject.

With the speed of operation thus limited, there were two ways in which the limiting speed could be increased; the interference could be reduced, or the strength of signals made greater. No great reduction in interference due to lack of perfect duplex balance could be expected, as balancing networks had already been greatly refined. Extraneous interference in certain cases could be reduced by the use of long, properly terminated sea-earths. The signal strength could be increased either by increasing the sending voltage or by decreasing the attenuation of the cable. Nothing at all is gained, however, by increasing the voltage in duplex operation where lack of perfect duplex balance limits the speed, and in simplex operation any gain from raising the voltage is obtained at the cost of increased risk to the cable, the sending voltage being usually limited to about 50 volts by considerations of safety. The attenuation of the cable could be reduced and the strength of the signal increased by use of a larger copper conductor or by using thicker or better insulating material. None of these possible improvements, however, seemed to offer a prospect of very radical advance in the art.

In telephony, both on land and submarine lines, an advantage had been obtained by adding inductance[4] in either of two ways, by coils inserted in series with the line or by wrapping the conductor with a layer of iron. The insertion of coils in a long deep-sea cable was practically prohibited by difficulties of installation and maintenance. Accordingly, only the second method of adding inductance, commonly known as continuous loading, could be considered for a transoceanic telegraph cable and it is primarily with regard to continuous loading that the following discussion is concerned.

Most of the proposals to load telegraph cables have had the object of reducing, or eliminating, distortion, and accordingly most of the mathematical treatments of loading have been from that point of view. The reduction of distortion is, however, not the only benefit to be obtained from loading and, in fact, may not always be secured in the high-speed operation of a loaded cable. The principal benefit of loading from the practical standpoint is to decrease the attenuation of the signals so that for a given frequency more current will be received or so that the minimum permissible current may be received with a greater speed of signaling. From the mathematical standpoint, there are two ways of treating the problem of the loaded

4. The idea of improving the transmission of signals over a line by adding distributed inductance to it originated with Oliver Heaviside in 1887, (*Electrician*, Vol. XIX, p. 79, and *Electromagnetic Theory*, Vol. 1, p. 441, 1893), who was the first to call attention to the part played by inductance in the transmission of current impulses over the cable. He suggested, as a means for obtaining increased inductance, the use of iron as a part of the conductor or of iron dust embedded in the gutta percha insulation. He also proposed inserting inductance coils at intervals in a long line. Other types of coil loading were proposed by S. P. Thompson (British Patent 22,304—1891, and U. S. Patents 571,706 and 571,707—1896), and by C. J. Reed (U. S. Patents 510,612 and 510,613—1893). M. I. Pupin (A. I. E. E. Trans., Vol. XVI, p. 93, 1899, and Vol. XVII, p. 445, 1900) was the first to formulate the criterion on the basis of which coil loaded telephone cables could be designed. Continuous loading, by means of a longitudinally discontinuous layer of iron covering the conductor, was proposed by J. S. Stone in 1897 (U. S. Patent 578,275). Breisig (*E. T. Z.*, Nov. 30, 1899) suggested the use of an open helix of iron wire wound around the conductor and Krarup (*E. T. Z.*, April 17, 1902) proposed using a closed spiral so that the adjacent turns were in contact. J. H. Cuntz (U. S. Patent 977,713 filed March 29, 1901) proposed another form of continuous loading. Recent general discussion of loaded telegraph cable problems has been given by Malcolm (Theory of Submarine Telegraph and Telephone Cable, London, 1917), and by K. W. Wagner, (E. N. T., Oct. 1924).

cable: first, with regard to the transmission of a transient impulse, and second, with regard to setting up steady alternating currents of definite frequency. In the ultimate analysis the solution of either problem can be got from the other. However, for practical purposes they are two distinct means of attack. The choice of the one to be used depends on the object to be secured. If one is concerned primarily with the effect of the cable on the wave shape of the signal transmitted over it, it is fairly obvious that the transient treatment has advantages. If, however, one is concerned only with the strength of the received signal, as is the case if there is assurance that the signal shape can, in any event, be corrected by terminal networks, then the steady state treatment is sufficient and much more convenient to apply. In the case of the real loaded cable the complete transient solution is extremely complex and the steady state treatment relatively simple. The solution of the transient problem of an ideal loaded cable is, however, very valuable to give a physical picture of how inductive loading aids the high speed transmission of signals.

The transient solution of the problem of an ideal heavily loaded cable has been worked out by Malcolm[5] and more rigorously by Carson,[6] who have determined the curve showing the change of current with time at one end of the cable if a steady e. m. f. is applied at zero time between the cable and earth at the distant end. Such a curve is called an "arrival curve" and for an ideal loaded cable comprising only constant distributed resistance, capacity and inductance may have a form like that shown in Curve *b* of Fig. 3, which is to be compared with Curve *a*, which is the arrival curve of a non-loaded cable. The straight, vertical part of Curve *b* represents the "head" of the signal wave which has traveled over the cable at a definite speed and with diminishing amplitude. The definite head of the arrival curve is the most striking characteristic difference between the ideal loaded and the non-loaded cable. In the latter, as is evident from Fig. 3, the current at the receiving end starts to rise slowly almost as soon as the key is closed at the transmitting end. When an e. m. f. is applied to the sending end of the non-loaded cable, a charge spreads out rapidly over the whole length, the receiving end charging up much more slowly than the sending end on account of the resistance of the intervening conductor. Hence, if a signal train, consisting of rapidly alternating positive and negative impulses, is applied to the sending end, the effect at the receiving end of charging the cable positively is wiped out by the succeeding negative charge before there has been time to build up a considerable positive potential and the successive alternating impulses thus tend to annul each other. In the loaded cable the effect of inductance is to oppose the setting up of a current and to maintain it once it has been established, and thus to maintain definite wave front as the signal impulse travels over the cable. Hence, with inductive loading, the strength and individuality of the signal impulses are retained and a much higher speed of signaling is possible. It should be noted that by speed of signaling is meant the rapidity with which successive impulses are sent and not the rate at which they travel over the cable. This speed of travel is actually decreased by the addition of inductance, about one third of a second being required for an impulse to traverse the New York-Azores cable from end to end.

It should be noted that Curve *b* of Fig. 3 is for an ideal loaded cable in which the factors of resistance, capacity, and inductance are constant. In a real loaded cable none of these factors are constant and the arrival curve cannot be simply and accurately computed. Even the capacity which is usually assumed as constant for real cables, varies appreciably with frequencies in the telegraph range, and, owing to the fact that gutta percha is not a perfect dielectric material, its conductance, which is also variable with frequency, must be taken into account. Although the inductance of the cable is substantially constant for small currents of low frequency, it is greater for the high currents at the sending end of the cable on account of the increase of magnetic permeability of the loading material with field strength and is less at high frequencies than at low on account of the shielding effect due to eddy currents. The resistance is highly variable since, in addition to the resistance of the copper conductor, it comprises effective resistance due to eddy currents and hysteresis in the loading material, both of which vary with frequency and current amplitude. Furthermore, there is variable inductance and resistance in the return circuit outside the insulated conductor which must be taken into account. Although it is very difficult to compute the exact arrival curve of a cable subject to all of these variable factors, an approximate calculation in a specific case, like that of the New York-Azores cable, shows that the arrival curve has the general shape of Curve *c* of Fig. 3. It will be noticed that although this arrival curve lacks the sharp definite head, characteristic of the ideal loaded cable, it still has a relatively sharp rise and that the time required for the impulse to traverse the cable is not greatly different from that of the ideal loaded cable.

Although it is difficult to take exact account of the variable characteristics of the loaded cable in the solution of the transient problem, it is easy to take account of them in the steady state or periodic analysis by means of well-known methods. If a steady sinusoidal voltage, V_s, is applied at one end of the cable, the resulting voltage, V_r, at the distant end, will be given by the equation

$$V_r = k\, V_s\, \epsilon^{-Pl}$$

where l is the length, P the propagation constant of the cable, and k a constant which depends on the terminal

5. Theory of the Submarine Telegraph and Telephone Cable, London, 1917.

6. TRANS. A. I. E. E., Vol. 38, p. 345, 1919.

impedance and which is unity in case the cable is terminated at the receiving end in its so-called characteristic impedance. The propagation constant is given by the formula,

$$P = \sqrt{(R + i\,p\,L)\,(G + i\,p\,C)} = \alpha + i\,\beta$$

where R is the resistance, L is the inductance, G is the leakance, C is the capacity per unit length, and p is $2\,\pi$ times the frequency. The real part of the propagation constant, α, is called the attenuation constant and the imaginary part, β, the wave length constant. By separating α and β, the amplitude and phase displacement of the received voltage relative to the sent voltage may be computed for any particular frequency and the behavior of a complex signal train may be worked out by analyzing it into its Fourier components and treating them separately. The phase shift is, however, of importance mainly as regards the shape of the received signals, and their amplitude may, in general, be obtained from the attenuation constant alone. Thus if it is known that the signal shape can, in any case, be corrected by terminal networks, there is no need to be concerned with more than the attenuation constant to compute the speed of the cable.

In the case of a cable of the permalloy-loaded type, α is given with an approximation[7] sufficiently close for the purposes of this discussion by the equation

$$\alpha = \frac{1}{2}\sqrt{\frac{C}{L}}\left(R + \frac{G}{C}\,L\right)$$

For the purpose of computing R, it is convenient to separate it into its components, giving

$$\alpha = \frac{1}{2}\sqrt{\frac{C}{L}}\left(R_c + R_e + R_s + R_h + \frac{G}{C}\,L\right)$$

where

R_c = copper resistance per unit length
R_e = eddy current resistance per unit length
R_s = sea return resistance per unit length
R_h = hysteresis resistance per unit length

The copper resistance, R_c, is that determined by a direct-current measurement of the loaded conductor, since the resistance of the loading tape is so high and its length is so great that the current flowing longitudinally through it may be safely neglected.

The eddy current resistance, R_e, is given approximately by the formula,

$$R_e = \frac{m\,\mu^2\,t^3\,f^2}{\rho\,(d - t)}$$

where t is the thickness or diameter of the loading tape or wire, d is the outside diameter of the loaded conductor, f is the frequency, ρ is the resistivity of the loading material, μ is its magnetic permeability, and m is a constant which depends on the form of the loading material and is, in general, greater for tape than for wire loading. Although it is possible to compute a value of m, the value found in practise is always larger than the theoretical value, which is necessarily based on simple assumptions and does not take into account such a factor as variation of permeability through the cross-section or length of the loading material. Accordingly, it is necessary for any particular type of loaded conductor to determine m experimentally.

The sea-return resistance may safely be neglected in the computation of slow speed non-loaded cables, but it is a factor of great consequence in the behavior of a loaded cable. By sea-return resistance is meant the resistance of the return circuit including the effect of the armor wire and sea water surrounding the core of the cable. Although the exact calculation[8] of this resistance factor is too complex to be discussed here, the necessity of taking it into account may be quite simply explained. Since the cable has a ground return, current must flow outside the core in the same amount as in the conductor. The distribution of the return current is, however, dependent on the structure of the cable as well as on the frequencies involved in signaling. If a direct current is sent through a long cable with the earth as return conductor, the return current spreads out through such a great volume of earth and sea water that the resistance of the return path is negligible. On the other hand, if an alternating current is sent through the cable, the return current tends to concentrate around it, the degree of concentration increasing with the frequency. With the return current thus concentrated the resistance of the sea water is of considerable consequence. It is further augmented by a resistance factor contributed by the cable sheath. This may be better understood by considering the cable as a transformer of which the conductor is the primary and the armor wire and sea water are each closed, secondary circuits. Obviously, the resistances of the secondary circuits of armor wire and sea water enter into the primary circuit and hence serve to increase the attenuation. The presence of the armor wires may thus be an actual detriment to the transmission of signals.

To take account of the hysteresis resistance, R_h, and also of the increased inductance and eddy current resistance at the sending end of the cable, it is most convenient to compute the attenuation of the cable for currents so small that R_h may be safely neglected. The attenuation thus computed is that which would be obtained over the whole cable if a very small sending voltage were used. The additional attenuation at the sending end for the desired sending voltage may then be approximated by computing successively from the sending end the attenuation of short lengths of cable over which the current amplitude may be considered constant, the attenuations of separate lengths being added together to give the attenuation of that part of

7. For accurate computation of attenuation the complete formula for α must be used.

8. See Carson and Gilbert, *Jour. Franklin Inst.*, Vol. 192, p. 705, 1921, and *Electrician*, Vol. 88, p. 499, 1922.

the cable in which hysteresis cannot be neglected. In this computation, account must, of course, be taken of the increased inductance and eddy current resistance accompanying the higher currents at the sending end.

Having calculated or obtained by measurement the several resistance factors, and knowing the capacity, leakance, and inductance, the whole attenuation of a cable for any desired frequency may be computed and a curve drawn showing the variation of received current with frequency for a given sending voltage. This relation for a particular case is shown in Curve *c* of Fig. 4. Curve *a* shows, for comparison, the relation between frequency and received current of a non-loaded cable of the same size, that is, a cable having a conductor diameter the same as that of the loaded conductor and having the same weight as gutta percha. Curve *b* shows

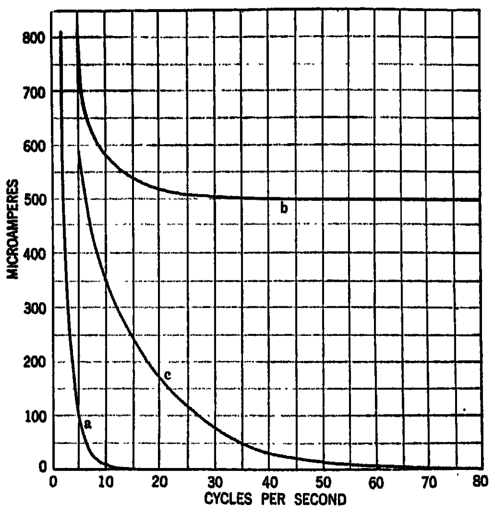


FIG. 4—RECEIVED CURRENT VS. FREQUENCY

a. Non-loaded cable
b. Ideal loaded cable
c. Real loaded cable

the behavior of an ideal loaded cable having the same inductance, capacity, and d-c. resistance as the real loaded cable of Curve *c*, but in which the leakance and alternating current increments of resistance are assumed to be zero.

Now, if the level of interference through which the current must be received is known, the maximum speed of signaling for the loaded cable may be obtained from Curve *c*. It is that speed at which the highest frequency necessary to make the signals legible is received with sufficient amplitude to safely override the superposed interference. Just what the relation of that frequency is to the speed of signaling cannot be definitely stated, since it depends on the method of opertion and code employed as well as on the desired perfection of signal shape. J. W. Milnor[9] has suggested that for cable code operation and siphon recorder reception a fair value is about 1.5 times the fundamental frequency of the signals, that is, the fundamental frequency when a series of alternate dots and dashes is being sent.

By referring again to the equation for α, it can now be explained why high permeability is a necessary characteristic of the loading material if benefit is to be obtained from continuous loading. The addition of the loading material has two oppositely directed effects; on the one hand it tends to improve transmission by increasing the inductance and consequently decreasing the attenuation, and on the other hand it tends to increase the attenuation by increasing the effect of leakance and by the addition of resistance. Not only are the hysteresis and eddy-current factors of resistance added by the loading material, but it must also be looked on as increasing either the copper resistance or the capacity on account of the space it occupies. Generally it is more convenient to look on the loading material as replacing some of the copper conductor in the non-loaded cable with which comparison is made, since by so doing all of the factors outside of the loaded conductor are unchanged. Now, if the loading material is to be of any benefit, the decrease in attenuation due to added inductance must more than offset the increase due to added resistance, including the added copper resistance due to the substitution of loading material for copper. In the limiting case the lowest permeability material which will show a theoretical advantage from this point of view is that which, as applied in a vanishingly thin layer, gives more gain than loss. For any particular size and length of cable there is a limiting value of permeability which will satisfy this condition, this limiting value being greater the longer the cable and the smaller the diameter of its conductor.[10] For transatlantic cables of sizes laid prior to 1923, the minimum initial permeability required to show an advantage is higher than that of any material known prior to the invention of permalloy. Actually a considerably higher permeability than this theoretical minimum was, of course, required to make loading an economic advantage, since there are practical limits to the thickness of loading material and since the cost of applying it has also to be taken into account. Further, there are limits on methods of operation, imposed by loading, which necessitate still higher permeability to make loading worth while.

Since the addition of loading has two opposite tendencies in its effect on attenuation, the practical design of the cable must be based on a compromise between them. Thus, to secure the maximum gain from loading a cable of a given size, the loading material should be chosen of such a thickness that the gain due to increased inductance from a slight increase of thickness just offsets the loss due to increased resistance and dielectric leakance. In practise, of course, economic considerations of the cost of various thicknesses of loading must also be taken into account.

9. JOURNAL A. I. E. E., Vol. 41, p. 118, 1922; TRANSACTIONS A. I. E. E., Vol. 41, p. 20, 1922.

10. O. E. Buckley, British Patent No. 184,774, 1923.

In designing the New York-Azores cable some assumption had to be made as to the extraneous interference which would be encountered. Theoretical considerations led to the belief that the loaded cable would be no more subject to external interference than non-loaded cables. It even appeared that it would be less affected by some types of interference, for, owing to the shorter wave length for a given frequency, a disturbance which affects a great many miles of cable simultaneously is less cumulative in its effect at the terminal of a loaded than a non-loaded cable. A reasonable assumption seemed to be that the total overall attenuation which could be tolerated for the loaded cable was at least as great as that which experience had shown to be permissible for simplex operation of non-loaded cables. Of course, this maximum permissible attenuation depends on conditions of terminal interference, and no fixed value can be given as applicable to all cables. However, for average conditions of terminal interference in locations free from power-line disturbances, and where the cable lies in relatively deep water near to its terminal landing, a reasonable value of total attenuation constant for the fundamental frequency of cable code is about 10 (86.9 T. U.) for recorder operation and about 9 (78.2 T. U.) for relay operation. These were the approximate values assumed for the New York-Azores cable and later experience has demonstrated that they were well justified.

Throughout all of the preceding discourse, it has been assumed that the relation between attenuation and terminal interference would limit the speed of simplex operation, rather than that distortion of signal shape would be the limiting factor. Although this is, in fact, the case with non-loaded cables,[11] it was not self-evident as regards the loaded cable, and to make reasonably certain that the speed could be determined from the attenuation-frequency relation required a demonstration that the signal distortion of a real loaded cable could be corrected by suitable terminal apparatus. One of the merits long claimed for loading was that it would reduce distortion and, indeed, an ideal loaded cable with constant inductance and without magnetic hysteresis, eddy current loss, dielectric leakance, and sea-return resistance would have very little distortion and would give a speed limited only by terminal apparatus. However, a real loaded cable, the inductance of which varies with both current and frequency and in which all the above noted resistance factors are present, may give, and in general will give when operated at its maximum speed, greater distortion of signals than a non-loaded cable.

To solve the question of distortion on a purely theoretical basis required consideration of the transmission of a transient over the loaded cable. This was made extremely difficult by the existence of numerous possible causes of signal distortion, the effects of which could only be approximated in the solution of the transient problem. In addition to the distortion resulting from the rapid increase of attenuation with frequency due to the various sources of a-c. losses, distortion peculiar to the magnetic characteristics of the loading material had also to be taken into account. There are several types of magnetic distortion worthy of consideration. First, there is the production of harmonics as a result of the non-linear magnetization curve of the loading material; second, there is a possible asymmetrical distortion due to hysteresis, and third, there is a possible modulation resulting from the superposition of one signal upon another, which is, in effect, a modulation of the head of the wave of one impulse by the tail of the wave of a preceding impulse. The first two of these are effective at the sending end of the cable and the third near the receiving end.

A computation of distortion, including the peculiar magnetic effects, by a steady state, a-c. method, based on measurements of short loaded conductors, indicated that the cable should operate satisfactorily with ordinary sending voltages. Further evidence that none of these various types of distortion would be of serious consequence, and that the distortion of a loaded cable could be corrected by terminal apparatus, was obtained by experiments with an artificial line constructed to simulate closely, as regards electrical characteristics, the type of loaded conductor with which experiments were then being made. This artificial line was loaded with iron-dust core coils, which admirably served the purpose, not only as regards inductance and a-c. resistance but also as regards magnetic distortion. Iron dust is, of course, very different from permalloy in its magnetic characteristics. But owing to the large number of turns on a coil, it is operated at much higher field strengths and on a part of the magnetization curve corresponding approximately to that at which permalloy is operated on the cable. In fact the case for magnetic distortion was a little worse on the artificial line than in the then proposed cable. Fig. 5 shows an illustration of the artificial line, the coils of which are in the large iron pots and the resistance and paper-condenser capacity units of which are in the steel cases. This line was equivalent to a 1700-nautical-mile cable, loaded with 30 millihenries per nautical mile, and over it, legible signals were secured at speeds up to more than 2600 letters per min. Such a speed of operation was quite beyond the range of the then available telegraph instruments, and accordingly special transmitting and receiving instruments were required. The multiplex distributor, of the Western Electric printing-telegraph system, proved an excellent transmitter for experimental purposes, and for receiving, use was made of a combined vacuum-tube amplifier and signal-shaping network, the signals being recorded on a string

11. Recent work of J. R. Carson (U. S. Patent 1,315,539—1919) and R. C. Mathes (U. S. Patent 1,311,283—1919) has shown that with the combined use of vacuum tube amplifiers and distortion correcting networks, distortion in non-loaded cables can be compensated to any desired degree.

oscillograph. Fig. 6 shows part of a test message received over the loaded artificial cable at a speed of 2240 letters per min.

The results of the tests with the artificial loaded cable were entirely in agreement with the author's calculations, and showed that it was possible to obtain satisfactory signal shape with a coil loaded cable having a-c. resistance and distortion factors approximating those of the permalloy loaded cable. The exact

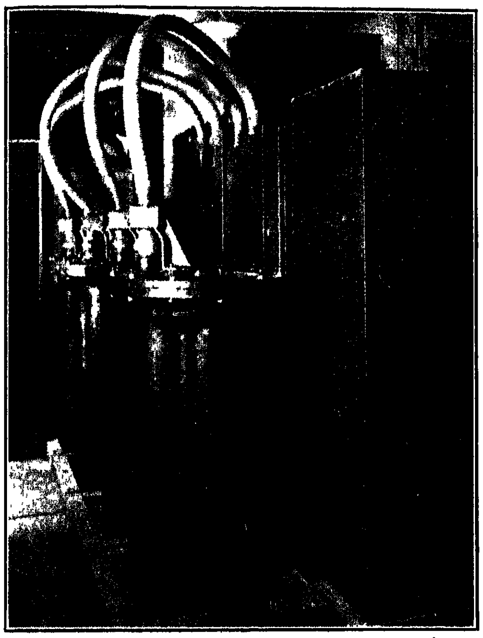


FIG. 5—LOADED ARTIFICIAL LINE

behavior of the proposed cable, including such factors as sea-return resistance and a somewhat variable distributed inductance, could not, of course, be duplicated without prohibitive expense. The approximation was considered, however, to be sufficiently good to justify proceeding with a loaded cable installation so far as questions of signal shaping were concerned. It is interesting to note that the factor which limited the operating speed of the artificial loaded cable was one which is not present in a continuously loaded cable but which possibly would be a serious factor in the operation of a coil loaded cable, namely, the oscillations[12] resulting from the finite size and separation of the inductance units.

With the completion of the artificial loaded cable tests, there was still one principal question of transmission which had to remain unanswered until a cable had been installed. This was the question of balancing the cable for duplex operation. Ordinary submarine cables are generally operated duplex, the total speed in the two directions being usually from about 1.3 to 2 times the maximum simplex or one-way speed. Except in cases where the external interference is very bad, the limiting speed of duplex operation is determined by the accuracy with which an artificial line can be made the electrical equivalent of the cable. Ordinarily, the artificial line is made up only of units of resistance and capacity arranged to approximate the distributed resistance and capacity of the cable. Sometimes inductance units are added to balance the small inductance which even a non-loaded cable has. In the actual operation of cables, artificial lines are adjusted with the greatest care and a remarkable precision of balance is obtained. This is necessary because of the great difference in current amplitude of the outgoing and incoming signals, the former being of the order of 10,000 times the latter. It is quite obvious that it will be much more difficult to secure duplex operation with a loaded cable than with one of ordinary type, since not only do the copper resistance and the dielectric capacity have to be balanced, but the artificial line must also be provided with inductance and a-c. resistance. Also the sea-return resistance and inductance which vary with frequency must be balanced.

In view of these difficulties, it will probably be impossible to get as great a proportionate gain from duplex operation of loaded cables as is secured with ordinary cables. However, it is quite evident that it will be possible to obtain duplex operation at some speed, since, with loaded as with non-loaded cables, the ratio of received-to-sent current increases rapidly as the speed is reduced, and on this account it is much easier to duplex the cable at low speeds than at high. To make duplexing worth while on a cable with approximately equal traffic loads in both directions, it is, in general, only necessary to get a one-way duplex speed half as great as the simplex speed. In fact, in some cases the operating advantages of duplex would warrant even a slower duplex speed. On the other hand, there are cables on which the traffic is largely unidirectional through most of the day and which would accordingly require a one-way duplex speed somewhat higher than half the simplex speed to justify duplex

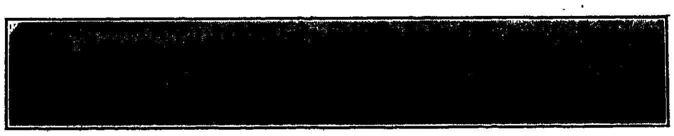


FIG. 6—TEST MESSAGE

Signals received April 16, 1920 over coil-loaded artificial line equivalent to a 1700 n. m. cable with 30 m. h./n. m. Speed 2240 letters per minute.

operation. Whether a sufficiently great speed of duplexing could be secured to justify designing a cable on the basis of duplex operation could not be judged in advance of the laying of the first cable, and accordingly it was decided to engineer that cable on the basis of simplex operation.

Although it was expected that the new cable might at first have to be operated simplex it should not be supposed that any great difficulty or loss of operating efficiency was anticipated on this account. The speed of the New York-Azores cable is so great that to realize its full commercial advantage practically requires

12. Carson, TRANS. A. I. E. E., Vol. 38, p. 345, 1919.

working it on a multi-channel basis, as, for example, with a Baudot code, multiplex system, similar to that used on land lines. Such a system may be conveniently adapted to automatic direction reversal and, with this modification, most of the common objections to simplex operation are removed. Indeed, simplex operation may in this case possess a real advantage over duplex, from the commercial point of view, since it permits dividing the carrying capacity of the cable most efficiently to handle the excess of traffic in one direction.

Although means for making efficient use of the loaded cable have been made available, it should be recognized that the method of operation best suited to satisfy commercial demands must be determined from future experience with cables of the new type. This is especially true with regard to relatively short cables. In this paper the discussion of the loaded cable problem has been confined wholly to the realm of long ocean cables where the limitations of the cable, rather than terminal equipment or operating requirements, determine the best design. This is the simplest case and the one which, at present, seems to show the greatest gain from loading. Where traffic requirements are limited and there is no prospect of ever requiring a speed higher than can be obtained with a non-loaded cable of reasonable weight, the advantage of loading is less and becomes smaller as the weight of non-loaded cable which will accomplish the desired results, decreases. It should not, however, be concluded that loading will not find important application to short cables. Many short cables are parts of great systems and must be worked in conjunction with long cables. In such cases it may pay to load short sections where loading would not otherwise be justified. Permalloy loading also offers great possibilities for multiple-channel carrier-telegraph operation on both long and short cables, and with this type of operation in prospect it is too early, now, to suggest limits to the future applications of permalloy to cables or to predict what will be its ultimate effect on transoceanic communication.

Discussion

W. C. Peterman: This paper treats of a phase of communication that apparently has not kept pace with the advances in other methods of communication. Numerous inquiries have been directed in recent years to the cause of this apparent lack of progress in the field of cable telegraphy. As a matter of fact, a very considerable progress has been made in the past 10 or 15 years. The improvements in this period, principally in the terminal apparatus, repeating apparatus, and operating methods, resulted in a substantial increase of speed and a reduction of operating cost. There were, among others, the introduction of a successful relay to automatically connect two cable sections, the development of an amplifier suitable for cable conditions, and the improvement in the design and use of artificial cables for duplexing.

During the past few years there has also been developed by the Western Union engineers, a system of repeating-cable signals which completely regenerates them as to shape and strength so that on leaving the repeating station they are as perfect as when originally sent. This system permits of the insertion of a number of repeaters in a circuit where previously one had been the limit. By this method, automatic through operation direct between New York and London was for the first time successfully accomplished. There has also been developed a special printing telegraph system which permits of a higher operating speed in letters per minute than can be obtained with the usual cable code which had heretofore been considered the fastest system for ocean cables. This printing method has proved highly successful on a through circuit from New York to London with duplex operation.

When the possibilities of this new type of cable, as outlined by Dr. Buckley, were presented to us, we were immediately interested. Our engineers went into the subject as presented to them, and came into entire agreement on all points. From then on our engineers checked over the designs and plans at every stage to satisfy themselves as to their correctness, for a submarine cable is such a large, long-time investment that every precaution must be taken to insure nothing having been overlooked. As an additional precaution a trial 120-mi. length of loaded cable was manufactured and laid in deep water off Bermuda. The results of tests made on this cable were in accordance with predictions from the previous laboratory tests and theoretical studies made by Dr. Buckley. The Western Union was satisfied with the results and proceeded to lay the 2300-mi. Azores cable. This was done without mishap and met all requirements. Dr. Buckley is to be congratulated on the conception of the application of a material of high permeability in ocean cables and on the successful development of the idea.

With this new cable, combining speed, accuracy, reliability, and the usual secrecy of cable messages, we are now, for the first time, linked by cable to Italy and Spain, and to these we hope in the near future to add Germany. At that time it is expected that this cable will be operated on the multiplex system with five or six channels. Three or four of these channels will be used by the Western Union, and it is planned that two or three channels will be used by the Commercial Cable Company under agreement with the Western Union. With the present plans, some of the channels will be operated directly from New York to Berlin and Hamburg without manual rehandling. One or more channels will terminate at the Azores. The channels assigned to the Commercial Cable Company will be operated directly from their office in New York to either Germany or the Azores; all this gives some idea of the flexibility of the multiplex system when used on such a cable.

But more important still, the confidence inspired by the performance of this cable has led the Western Union to order another similar loaded cable, this time to connect New York and London. The signals will be automatically repeated at Newfoundland and Penzance, England, so that there will be complete automatic, through operation between New York and London. This new loaded cable will have a still higher operating speed than the present loaded cable, being designed to transmit signals of frequencies up to 75 cycles per second, which corresponds to a speed of 2400 letters per minute with cable code.

We shall have to revise our ideas of cable telegraphy, for this speed is considerably higher than that at which most of the open-wire, land-line telegraph circuits in the United States are being operated today. This cable will probably require two overland circuits to carry its traffic from Penzance—the cable station on the coast of England—to London. Indeed, the traffic-carrying capacity of this cable will be so great that it will be nearly equal to the total capacity of our present seven cables to England. The addition of this cable alone will result in about a 40-per cent increase in the traffic-carrying capacity of all the present North Atlantic cable communication systems.

The operation of these cables with the Baudot code and with a number of channels, each with its own transmitter and printer, will place cable operation upon the same basis as our trunk land

lines, with all that that means in similarity of operating methods and apparatus. With this code there is also the possibility of automatically extending some or all channels of such cables by means of our existing land-line system directly to other parts of the United States. A new era is here; between points where the traffic is heavy, the old wavy line of the siphon recorder will gradually drop out of the picture.

E. B. Craft: I should like to correct one impression which may have been given by Dr. Buckley's paper. As he has described the problem of the loaded cable it looks too easy; it looks as if all that had been done was to wrap a permalloy tape around a copper wire and then make a high-speed cable of it.

I wish you might appreciate how much more there was involved in this accomplishment than can be brought out in such a brief paper as that to which you have just listened. Years of painstaking effort were required and difficulties were encountered at every step. Precautions had to be taken against any harmful mechanical and electrical effect which might be encountered in laying or operating a cable.

One very interesting feature of the New York-Azores cable which Dr. Buckley has not mentioned is the new type of balanced sea-earth which was developed after an extensive investigation of cable interference. This sea-earth almost completely eliminates the effect of local power disturbances and atmospherics and makes it possible to operate cables efficiently in the most unfavorable terminal locations.

When the development of the loaded cable was undertaken there were no means in sight for efficient operation of such a high-speed loaded cable, even if the problems of the cable itself could be solved. New operating methods and new methods to measure the electrical constants of a loaded cable had to be worked out and new instruments had to be developed for these purposes. When the cable was laid, these instruments were ready and within a few hours after the final splice was made, the successful operation of the cable was demonstrated. Almost every piece of apparatus used in the test and demonstration of the cable was new and specially developed for the purpose. A new type of cable transmitter, working with compressed air, was devised to send messages at speeds many times greater than would be permitted by any previously existing transmitter. A siphon recorder which would record messages at over 2500 letters per minute was devised and made ready for the test. But perhaps the greatest achievement in this connection was the signal-shaping amplifier. This represented an achievement comparable with that of the cable itself. In addition to apparatus for test and demonstration of the cable, operating systems suitable for commercial use were worked out and tested over an artificial line in the laboratory. All of this was done in advance of laying the first cable.

I hope that some of these accomplishments may later be subject to publication, and when all that was done to make the permalloy-loaded cable a success is known, I am sure that you will feel, as I do, no small satisfaction in its having been an American achievement.

Chas. A. Perkins: It has occurred to me to ask why the air-gap which occurs in this permalloy winding does not largely neutralize the effect of the high permeability of the material.

O. E. Buckley: Professor Perkins has asked why the air-gap between the adjacent turns of permalloy tape does not materially neutralize the effect of the high permeability of that metal. The answer is rather interesting and may possibly surprise some of you. The reason that the air-gap does not introduce much reluctance in the magnetic circuit is that the magnetic lines of induction are not single loops around the conductor, as has sometimes been assumed, but have the form of a helix which takes a large number of turns around the conductor before crossing an air-gap between adjacent turns of the permalloy tape. In the case of a conductor like that of the New York-Azores cable the lines of induction follow the permalloy tape very closely, the pitch of the screw of the lines of induction being only slightly less than the pitch of the tape, with the result that a line of induction follows the tape for about 20 turns around the conductor, then jumps an air-gap between two adjacent turns and continues following the tape in the same direction as before for another 20 turns, when it again slips back across an air-gap. This means that if the permalloy tape were strictly uniform and continuous from one end of the cable to the other and if the cable carried a steady, direct current and was not subject to the effect of the earth's field, the lines of induction would be continuous from one end of the cable to the other.

Law Description and Hypothesis in the Electrical Science[1]

BY M. I. PUPIN[2]

YOUR invitation to deliver the first Steinmetz lecture I consider a very great honor. The late Doctor Steinmetz was a dear friend of mine. I met him in Yonkers in 1889, and from that time on until his death we were tied to each other by bonds of personal sympathy and scientific interest, which was a source of uninterrupted pleasure to both of us.

This lecture is an attempt to describe briefly how Faraday and Maxwell, starting from definite laws which were discovered by experiment, created the modern Electromagnetic Theory by a prophetic use of description and hypothesis, and how this theory furnishes the foundation of the Science of Electrical Engineering.

Our knowledge of electrical phenomena began its career as a science when it started to build upon a foundation of a quantitative law. Coulomb's law marks, therefore, the beginning of the electrical science. It says that two electrical point charges in a vacuum act upon each other with a mechanical force which is equal to the product of the two charges divided by the square of the distance between them.

In its mathematical form Coulomb's law is identical with Newton's law of gravitational action. Many theorems which the mathematical physicists of the eighteenth and the beginning of the nineteenth centuries had developed in their analyses of gravitational fields of force were, apparently, directly applicable to the analysis of electrical fields. This was very fortunate, because it attracted some of the best mathematical minds of those days to the electrical science. This raised its standing among the sciences which it badly needed.

Newton's great essay, Principia Philosophiae Naturalis, published in the beginning of the eighteenth century, created a new school of natural philosophers which dominated during the eighteenth century the scientific mental attitude of the world. No natural philosopher of those days could expect to attract serious attention who departed from the rigorously mathematical methods of this school. Even so great a natural philosopher as Benjamin Franklin may be said to have been snubbed by the Royal Society, when it refused to publish in its transactions Franklin's communications describing his electrical experiments. These experiments, suggested by and clustering around Leyden jar discharges, had no obvious connection with the Newtonian school of natural philosophy of the eighteenth century and, therefore, the Royal Society failed to recognize their full significance. One may imagine how welcome Coulomb's law was to some natural philosophers of the eighteenth century, to whom Newton's Principia was as final as the book of Genesis is to some people of our own generation.

Faraday was the first to point out a fundamental difference between Newton's law of gravitational action and Coulomb's law of electrical action. The action of a gravitational mass upon another gravitational mass is not influenced by the medium separating the two, but the action of an electrical charge upon another electrical charge is influenced very much by the medium separating the two. Coulomb's law unaided by other considerations was unable to explain this difference. Faraday was the first to enter into these considerations, and his first guide may be said to have been a hypothesis which maintained that all electrical charges trace their origin to the molecules and atoms of material bodies, which in their normal state contain, according to Franklin, the same amounts of positive and negative charges. This hypothesis of the atomic origin of electrical charges was undoubtedly suggested by Faraday's classical studies of the behavior of electrolytes, which revealed a new truth, namely, that a definite electrical charge is attached to each valency of atoms. The granular structure of ordinary electrical charges and the whole modern electron theory was first foreshadowed in these experiments. But how did this hypothesis affect Coulomb's law of force between Coulomb charges which are surrounded by a material medium?

Consider the insulators. The hypothesis suggested that in an insulator each molecule contains a definite quantity of positive and an equal quantity of negative charge which can be separated from each other by the action of an external electrical force impressed upon them, but that the distance of separation cannot exceed the dimensions of the molecule. Adopting this picture of the electrical structure and behavior of insulators, there was readily deduced a modified form of Coulomb's law of force between charges separated by an insulating medium, and this modified form of Coulomb's law says: *The force between two point charges in an insulating material medium is equal to that in a vacuum divided by a constant, called the specific inductive capacity of the material medium.*

But experiment told us that the hypothesis mentioned above concerning the process of separating molecular charges and everything inferred from it can be only approximately true, because the

1. The first Steinmetz lecture delivered on May 8, 1925, before the Schenectady section of the American Institute of Electrical Engineers.

2. Of Columbia University, New York.

specific inductive capacity of material insulators is usually neither constant nor does it always have a definite meaning. This law, therefore, could not be taken as our infallible guide in the study of the electrical fields of force in material insulators. The question arose then: Is there any other law to which we can appeal for guidance? Faraday's study of the electrical action of insulators, a subject to which Benjamin Franklin first drew attention, showed a way leading to the answer of this question. This study suggested one of the two great foundation pillars of the modern electromagnetic theory, which I venture to describe here briefly.

Faraday's method of representing graphically the field of force of electrical charges is well known, and it finds its simplest illustration in the well known conical tubes of force drawn from a point charge as vertex and expanding into all space. We are also familiar with Faraday's tubes[3] of force for any distribution of electrical charges. Faraday's pictorial method of describing the field of force leads to the same numerical results as Coulomb's law when the surrounding medium is free space without any material bodies in it. When, however, the surrounding medium contains material insulators, Coulomb's law offers small assistance in our study when these insulators have a variable specific inductive capacity and deviate otherwise from the characteristics of an ideal dielectric. It will be pointed out below that there are electric and magnetic fields which are not due to charges and in which Coulomb's law is altogether inapplicable. Faraday's picture of the field in terms of the tubes of force suggested to Maxwell a new law of force which is broader than Coulomb's law both in its meaning and its applicability.

Faraday's ideas concerning the physical character of the tubes of force were a guide to Maxwell, whose earliest studies of electrical phenomena, while still an undergraduate at the University of Cambridge, related to Faraday's *Physical Lines of Force*. In these early studies Maxwell made wonderful attempts to show by imaginative description and ingenious mechanical models what he saw in Faraday's tubes. But all these things were only a temporary scaffolding around a new structure which Maxwell was building. When the structure was finished the scaffolding disappeared, and what do we see today? I shall try to answer this question.

In Maxwell's mind, just as in the mind of Faraday, the tubes of force were not mere geometrical pictures but represented physical entities capable of actions and reactions. Each volume element of a tube of electric force is, according to Faraday and Maxwell, the seat of an electrical reaction against the change of its density, that is, of the number of tubes per unit area. When the surrounding medium is a vacuum or an ideal insulator, that is, a dielectric with a constant specific inductive capacity, then the numerical value of this reaction can be calculated. According to Maxwell's hypothesis, the electrical reaction in this case per unit length and unit cross-section of the tube is equal to the density of the tubes in the direction in which the reaction is considered, divided by the specific inductive capacity. The hypothetical reaction had a most significant corollary; it located the energy of the field in the volume elements of the tubes of force and assigned to each element, per unit of volume, an amount proportional to the square of the density of the tubes of force at that volume element. Dynamically, therefore, there is a perfect resemblance between the field of electrical reactions in ideal insulators and the field of elastic reactions in the interior of an elastically strained body which obeys the so-called Hooke's law.

According to this view, the charges transmit their action through the volume elements of the tubes against the reaction of the tubes. *When the field of electrical force is in equilibrium the external actions coming from the electrical charges and the internal electrical reactions of the tubes are equal and opposite to each other at every point of space.* This form of statement is suggested by Newtonian dynamics and furnishes a law which conforms to Newton's third axiom. It is different from Coulomb's law in form and meaning, and it holds good no matter how the impressed forces are generated or what the physical character of the material bodies is upon which these forces are impressed. It is obtained from the hypothesis that the tubes of force are physical entities which react against a change of their density. There is nothing in Coulomb's law which suggests this hypothesis and there cannot be, because this law suggests nothing concerning the velocity or the mechanism of transmission of force between electrical charges, whereas a reacting tube of force was suggested to Faraday and to Maxwell by the intuition that electrical actions are transmitted through the tubes of force with a finite and definite velocity which depends upon the dynamical properties, that is, the reactions of the tubes. The tubes of force attached to electrical charges or otherwise generated are, according to this hypothesis, the transmitting mechanism reacting in every one of its elements by reactions which in the case of the vacuum and of ideal dielectrics are identical in form with the elastic reactions of an ideal elastic body. This view of the field of electrical force is one of the foundation pillars of the Faraday-Maxwell electromagnetic theory. I shall next describe briefly the second foundation pillar of this theory.

What has been said above about our knowledge of electrical phenomena is also true of our knowledge of magnetic phenomena. It started its career as a science when Coulomb's measurements succeeded in formulating a law of force between magnetic charges. Since this law is identical in form with that for elec-

3. The term "tubes" is preferable here to "lines" because it brings out clearly the three-dimensional character of these structures.

trical charges, and since the presence of material bodies affects a magnetic field as the presence of material insulators affects an electrical field it is obvious that the Faraday-Maxwell intuitive philosophy leads here to the same results as in the case of electrical fields of force. Coulomb's law can, therefore, be replaced by a law which is identical in form with the law formulated above for electrical fields. It is as follows: *When the field of magnetic force is in equilibrium the external magnetic actions and the internal reactions of the magnetic tubes of force are equal and opposite to each other at every point of space.* Description and hypothesis serve here the same object as in the case of the electric fields, namely, to point out that the magnetic tubes of force are the transmitting mechanism of the magnetic force and that the quantitative relation between the forces impressed upon the tubes and their reactions is one of the determining factors of the mode of propagation.

It is obvious that so far I have been endeavoring to show that Faraday's and Maxwell's views paved the way to the formulation of new concepts, the concepts of electrical and magnetic actions and reactions, which like ordinary material actions and reactions obey Newton's third law. These endeavors will be continued in that which follows.

The law of equality between electrical and magnetic actions and their respective reactions in fields which are in static equilibrium can, obviously, tell nothing definite about the velocity of propagation. Reactions brought into play when this equilibrium is disturbed must be considered. Do they exist, and if so, do they show that the velocity of propagation of electrical force is the same as, or different from, that of the magnetic force? The electrical science prior to Oersted's and Faraday's discoveries could not have answered this question. These discoveries supplied the necessary knowledge. Broadly stated, they revealed the following new truth: Oersted discovered that electrical charges moving through conductors produce magnetic tubes of force which are interlinked with the conductors; Faraday discovered that magnetic charges and their tubes of force produce by their motion or variation electrical forces in conducting circuits which are interlinked with these tubes. This description of the discoveries intentionally emphasizes two facts, namely, that Oersted made his discovery while experimenting with conduction currents, and that Faraday explored the electrical field in conducting wires which are interlinked with the magnetic tubes of force only. The laws resulting from these experiments, namely, Ampére's law and Faraday's law, were necessarily limited to the conditions of the experiments which led to their formulation. Neither one nor the other were sufficiently general to give direct information concerning the unknown reactions associated with the variable electric and magnetic tubes of force at any point of a dielectric. Oersted's and Faraday's experiments did not detect them, nor was it obvious how to detect them experimentally. New hypotheses were needed and Maxwell was the first to formulate them; they were as follows: First, a variation of the flux, that is, the total number of electrical tubes of force through any area, is equivalent to the motion of electrical charges through that area; in other words, the so-called displacement current produces according to Maxwell the same magnetic effect as the conduction or convection current; secondly, the variation of the flux of the tubes of magnetic force through any area produces an electromotive force around the boundary curve of this area which is independent of the material through which this boundary curve passes. These two hypotheses extended the meaning of the Ampère and of the Faraday law and gave them that symmetry which is expressed in the following statements:

The rate of variation of the electric flux through any area is equal to the magnetomotive force in the circuit which forms the boundary curve of that area.

The rate of variation of the magnetic flux through any area is equal to the electromotive force in the circuit which forms the boundary curve of that area.

The first statement represents Maxwell's generalization of Ampère's law, and the second that of Faraday's law. Mathematical physicists call them Maxwell's field equations. This name does not convey clearly their physical meaning, nor does it express fully their historical significance. Prior to the time of Oersted and Faraday there were only a few rather feeble processes of generating and impressing upon material bodies electric and magnetic forces: Frictional machines, galvanic cells, action of permanent magnets, etc. . . . Ampère's and Faraday's generalized laws describe new processes of generating and impressing magnetic and electric forces *upon any part of space.* They might be called Maxwell's laws of electrodynamic generation, or briefly *Maxwell's laws,* the rest of the proposed title being understood. These laws give the total sum of the electric and magnetic forces impressed by those processes upon any circuit; the energy principle tells us that this sum is equal to the sum of the electric and the respective magnetic reactions in the circuit. The parcelling out of the total impressed forces thus generated among the volume elements of the circuit and the character of the reactions of each volume element must be determined by the character of each problem and by the physical properties of each volume element of the circuit. Circuits in ideal isotropic dielectrics present the simplest illustration of the general procedure, and this was the subject which Maxwell considered first. In this case the reaction per unit cross-section and unit length of the circuit is, as already pointed out, equal to the ratio of the flux density to the specific inductive capacity, or permeability, respectively, and this reaction must be equal to the force generated by the variable fluxes and impressed per unit length of the circuit. This leads to a reciprocal relation between the electric

and magnetic reactions in variable fields which in an isotropic dielectric exhibits a process of propagation identical in form with that obtained by Newtonian dynamics for the actions and reactions in an isotropic, incompressible, elastic medium. Maxwell's greatest achievement is, in my opinion, his introduction into the electrical science of new concepts, electric and magnetic actions and reactions, which are subject to the same laws as the corresponding concepts in Newtonian dynamics. But it should be observed here that Maxwell's success was due to Faraday's suggestive description of the electric and magnetic fields in terms of tubes of force and to the intuition which created the epoch-making hypotheses endowing these tubes with dynamical attributes formerly belonging to material substances only. These hypotheses demanded experimental verification; Hertz seized the opportunity and furnished the epoch-making demonstration of the correctness of Maxwell's hypotheses.

The propagation of force through an ideal elastic solid makes the velocity of propagation depend upon two constants only, the density and the elastic constant. The first determines the inertia reaction and the second the elastic reaction per unit volume of the solid. Similarly in the propagation of the electric force through the electric and magnetic tubes of force in an ideal dielectric the velocity of propagation depends upon two constants only, the specific inductive capacity of the tubes and their magnetic permeability. One determines the reaction of the electrical tubes of force and the other the reaction of the magnetic tubes. These reaction constants determine the velocity of propagation through the electric and magnetic tubes in the same manner as density and elastic constants determine the velocity of propagation through ideal elastic bodies. The question arises, as to which of the two reaction constants of Faraday's tubes corresponds to the density and which to the elastic constant of material bodies. In other words, which of the two constants is characteristic of the inertia reaction of the tubes?

The generalized laws of Ampère and of Faraday, which I call the Maxwell laws, suggest a permissible answer to this question. They indicate a scheme which demands one primary or fundamental flux only, the electric flux. A variation or velocity of motion of the electric flux generates, according to the first Maxwell law, magnetic forces and corresponding magnetic fluxes which in an isotropic dielectric are proportional to the impressed magnetic forces, the factor of proportionality being the magnetic permeability of the tubes of the magnetic field. If, therefore, we consider the magnetic flux of the field, thus generated, as the momentum of the varying or moving electric flux, since it is proportional to its rate of variation or velocity of motion, then the electrical field generated, according to the second Maxwell law, by the variation of the magnetic flux will be due to the change of this momentum. According to this scheme the permeability constant in the electromagnetic theory would correspond to density in the theory of propagation through elastic solids.

Electron physics supports this scheme. It traces the origin of all magnetic forces of magnets to the orbital motions of electrons. This reminds us of the old Ampèrean conception. Magnetic tubes of force associated with so-called permanent magnets are, according to electron physics, the result of the motion of electric tubes of force attached to electrons. Maxwell always associated with magnetic tubes of force the momentum of some electric motions; what Faraday called the electrotonic state, he called the electro-kinetic momentum of a circuit, that is, the magnetic flux interlinked with the circuit. The reactions of varying magnetic tubes of force are, therefore, inertia reactions, and their reaction constant, the permeability, should, as already pointed out, be considered as corresponding to the density of elastic solids, whereas the reciprocal of their specific inductive capacity corresponds to the elastic constant. Faraday's tubes of force in free space have, in electromagnetic units, a permeability equal to unity and, measured in the same system of units, an exceedingly small specific inductive capacity. They behave, therefore, like incompressible elastic bodies of moderate density but of very high elastic constant for shearing strains. It is equal to 9×10^{20}. Hence the great velocity of propagation of electromagnetic disturbances through tubes of force in free space, as experimentally verified by Hertz.

Electrical propagation through ideal dielectrics, including the vacuum, demands, according to the above picture, nothing more than Faraday tubes of electric force (which I call here *primary flux*) capable of two distinct reactions, one an electrical reaction and the other a magnetic, that is an inertia, reaction. The tubes react like a material medium of reasonable density but of most extraordinary stiffness. But neither this similarity to material bodies nor anything else in our present knowledge of electrical phenomena justifies the hypothesis that they consist of a substance which has qualities of ordinary matter in bulk. One cannot resist the temptation of asking the question: What are these tubes made of? I venture, therefore, to offer the following pardonable suggestion.

Our ideas of these tubes are associated with our concepts of electrical charges which are the terminals of the tubes when they have a terminal. In this we follow in the footsteps of Faraday. It is not an unreasonable hypothesis to assume that they are made of the same fundamental substance of which the electrical charges are made. The name "electricity" may, therefore, be reserved for that substance, whatever it may be, so that we may say: The medium which transmits electrical disturbances is "electricity," meaning thereby the electrical tubes of force. Light is an electrical disturbance and it is, according to this view, transmitted by electricity. The concept suggested by the word "electricity" is much more definite than that suggested by the words "lumeniferous ether," because we associate

with electricity two perfectly well known and experimentally determinable reaction constants, that is, the reaction constants of the primary flux of force at rest and in motion. These are the only attributes that we can dynamically predicate of a material substance, hence the concept "electricity" is dynamically just as definite as the concept "material substance"; the concept "ether" is not.

Perhaps I have dwelt too much upon that part of the electromagnetic theory which is a little outside of the daily problems of the electrical engineer. Some people think that it is entirely outside of the theory which underlies electrical engineering problems. Permit me to show you, as briefly as I can, that this is not so, and that the same form of laws and the same dynamical methods apply to electrical engineering problems as to the problems discussed above. Electrical engineering problems deal with actions and reactions in electrical and magnetic circuits and so does the general electromagnetic theory. I have pointed out how starting with Coulomb's law a more general law was formulated for the field of force due to electrical or to magnetic charges at rest, the law of equality of actions and reaction in every volume element of the field in static equilibrium. The validity of this law was maintained for the dynamical equilibrium of variable fields when Ampère's and Faraday's laws were formulated by Maxwell in their most general form. The principle of conservation of energy demands that this law be always true irrespective of the physical character of the circuit or of the process of generating the impressed forces. This furnishes then the most fundamental basis in theoretical electrical engineering. It may be stated as follows:

In every circuit or part of a circuit the algebraic sum of electrical reactions is equal to the algebraic sum of the impressed electrical actions.

Omit the words "electrical" from this statement and you have the most fundamental law in Newton's dynamics, showing that "electricity" obeys the same fundamental law which ponderable matter obeys.

Take for an illustration an electrical circuit in which we have a constant electromotive force, generated by a voltaic cell, and a constant current flowing through a conducting wire. Consider any two points on the wire. Heat is generated in the wire between these two points and, therefore, there must be an electrical reaction in the wire between these two points. Heat is the result of the work done against this reaction by the impressed electrical force transmitted by the battery. This reaction may be called a *resistance reaction,* whereas the impressed action is the difference in potential between these two points. The law of equality of action and reaction says: The resistance reaction is equal to the difference of potential. This relation is independent of the so-called "Ohm's Law." When, however, the wire is maintained at constant temperature its resistance reaction is found by experiment to be proportional to the current; this empirically established characteristic of most metal wires is called Ohm's law. It really is not a law any more than Joule's rule for the rate of heat generation by a current flowing through a metal wire. Both are accurate empirical descriptions of a physical characteristic of most metal wires. It is occasionally stated with some show of disappointment that the flow of current through a gas does not obey Ohm's law, which really means that the resistance reaction is not proportional to the current, and that it cannot be described as simply as the resistance reaction of a metal wire. That a conducting gas should react differently from a conducting metal wire should not surprise anybody; but it seems that it does.

Consider, as another simple illustration, a toroidal magnetic circuit consisting of several different radial sections of different kinds of steel separated from each other by small air gaps and magnetized by a current flowing through turns of wire wound uniformly around the toroid. The total magnetomotive force generated by the current is given by Ampère's law. Each part of the magnetic circuit receives its definite share of the total magnetomotive force; this share is the magnetizing force impressed upon that part of the circuit. In each part of magnetic circuit the impressed magnetizing force is equal to the magnetic reaction of that part, so that according to the fundamental law the sum of the magnetic reactions is equal to the total impressed magnetic actions, which is the magnetomotive force. This is the fundamental law, whereas the usual method of calculating, roughly, the magnetic flux from impressed magnetizing forces and reluctances by making use of a new kind of Ohm's law for the magnetic circuit is, in my opinion, a misleading use of the word law. This spurious Ohm's law is abandoned, of course, as soon as we attempt to devise an experimental method for measuring hysteresis losses during a complete cycle of magnetization, but we do not abandon the dynamical law that in every part of the magnetic circuit the magnetizing force is equal to the magnetic reaction. On the contrary, we could not without it interpret dynamically the hysteresis losses during cyclic magnetizations.

When in a network of linear conductors alternating-current generators are located at various points of the network, the current distribution in the network can be calculated by setting up equations for each circuit which state the fundamental dynamical law that in each circuit the algebraic sum of electrical reactions is equal to the algebraic sum of impressed electromotive forces, generated by the alternators. To call these equations mathematical expressions of a Kirchhoff law, as some do, is unpardonable abuse of language. Kirchhoff gave the *rule* that for any circuit in a network of metalic wire conductors in which there are sources of constant electromotive force the algebraic sum of the electromotive forces is equal to the algebraic sum of the

products of current and Ohmic resistance, but he never suspected that this is a special case of the fundamental dynamical law given above.

It is true that in 1858 Kirchhoff, in his analysis of electrical propagation along an overhead telegraph wire, stated correctly the relation between the electrical reactions at any element of the wire, and in this statement he was guided by Thomson's discussion of electrical propagation over a submarine cable. But neither Thomson nor Kirchhoff were aware of the general law stated above. Maxwell's Electromagnetic Theory had not yet been published, and prior to that publication the general law implicitly contained in this theory, and which is today the foundation of electrical engineering, could not be formulated.

The several simple examples, cited above, suffice to illustrate clearly that electrical engineering problems on their pruely scientific side are formulated in the same way as the problems in the general electromagnetic theory. Their solutions are obtained by the application of the same form of the fundamental laws employing the same methods of reasoning and the same terminology which Newton had formulated when he created the science of dynamics. The possibility of describing electrical phenomena in terms of Newton's concepts and language is one of the greatest achievements of Faraday and Maxwell. Law, description, and hypothesis were never employed with greater effect than by the genius of these great prophets of the electrical science.

Corona Investigation on an Artificial Line

BY MURRAY F. GARDNER*

Associate A. I. E. E.

Synopsis—Accompanying corona on a transmission line, there has been found, in several cases, an appreciable increase in the line charging current over that indicated by the usual formula. This has been taken to indicate an increased line capacitance resulting from the presence of the corona envelope about the conductors, and has, for this reason, come to be described as the extra-capacity effect of corona. This explanation, by an envelope, however, has not been entirely satisfactory, and the question has arisen as to whether the current increase could not have been equally as well attributed to the current and voltage harmonics which the corona introduces.

The paper describes an investigation made of this extra-capacity effect on an artificial transmission line using artificial corona. In avoiding, by means of this method, the glow discharge of real corona, all envelope effects were eliminated. The operation of the artificial corona is described, and the results given of the tests which were made in checking its characteristics against those of real corona.

By means of the artificial corona it is shown that it is possible to obtain the extra-capacity effect of corona without having present an ionized envelope. The explanation is offered that at least the larger portion of this effect found on transmission lines under corona conditions is apparent only; that it is a result of the approximate method by which the line capacitance is calculated, and the increased admittance which the line offers to currents of harmonic frequencies.

The effect of corona leakage in altering potential rise on an open line is shown by representative voltage distribution curves taken on an artificial line with and without the corona leaks operating. From a comparison of these curves it appears that the leakage of corona plays a predominant part in the establishment of the resultant voltage, the harmonics caused by the corona in no case producing unusual resonance effects or extra-potential rises. This is important largely for the indication it gives that the present methods of predetermining voltage distribution on open lines for corona conditions are accurate within the limits of engineering accuracy.

Qualitative results are given showing the effectiveness of corona leakage in reducing traveling waves such as result from switching operations. Two oscillograms of the same transient are shown, one taken with the corona leaks operating and the other without. The much shorter duration of the transient in the former case is evidence of the corona's effectiveness in absorbing the energy of the disturbance.

INTRODUCTION

UNDER ordinary conditions air is a good insulator, but it loses this quality when subjected to an excessive potential gradient, such as occurs near the surface of the conductors of a transmission line when the voltage between lines is increased above a limiting value. In this region of maximum stress, ionization takes place and there is formed about each conductor an envelope of semi-conducting and luminous air, called corona. This envelope is believed to relieve the concentration of stress by increasing the effective diameter of the conductors.

There is a minimum corona-forming voltage, e_0, for every line, dependent upon the configuration of its conductors and the atmospheric conditions under which it operates. With an alternating voltage, the maximum value must exceed this critical voltage before corona forms. If much in excess, the corona starts at e_0 on the increasing portion of each half-voltage wave, and continues until a slightly lower value is reached on the decreasing portion. Corona under these conditions is pulsating, its frequency being double that of the voltage. This pulsation causes a cyclic change in line admittance which introduces harmonics, particularly the third.[1]

Accompanying corona, there is a power loss which increases with the square of the excess voltage over the critical value. This is expressed by the formula[2]

$$p = c^2 (e - e_0)^2$$

where p is in watts, c is a constant, e is the applied voltage, and e_0 is the disruptive critical value for the line.

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1. F. W. Peeks, Jr.; "Voltage and Current Harmonics Caused by Corona", TRANS. A. I. E. E., 1921, p. 1155.

2. F. W. Peek, Jr., "Dielectric Phenomena in High-Voltage Engineering", 1920, p. 122.

This quadratic law holds except in the neighborhood of e_0, where varied results are obtained due to surface irregularities on the conductors.

Due to corona loss it has been customary to operate lines well below their corona-forming voltage. But with the large blocks of power now transmitted, it becomes feasible, with certain systems, to raise the operating voltage practically to the corona limit. In seasons of unfavorable weather it is to be expected, therefore, that portions of these systems will operate for considerable periods in a state of corona.

This continual presence of corona, in addition to causing high energy loss, may be a source of serious interference between the transmission line and neighboring communication circuits. The corona introduces an appreciable third harmonic, and this is within the range of audibility in the case of a 60-cycle current. If the power system is three-phase and has its transformers connected in grounded-Y, these triple-frequency currents can flow through the three lines in parallel, combine in phase in the neutral, and complete their circuit through the ground. This induces troublesome voltage disturbances in paralleling telephone and telegraph lines. If the power system does not have a grounded neutral, it is impossible for the third-harmonic currents to flow. To compensate for this, a third-harmonic voltage appears between each line and neutral.[1] These voltages, being in phase, cause the whole system to pulsate with a triple frequency to ground. This pulsation induces a voltage between lines and ground in the paralleling circuit which even transposition will not eliminate. It appears, therefore, that interference due to corona harmonics presents a serious problem in any extensive operation above the corona-forming voltage.

On the other hand, the presence or ready formation of corona may be of some advantage. It may assist in damping high-voltage transients.[3] When operating near the corona limit, the rapidly increasing energy loss for slight rises in potential above normal is likely to be of considerable value in absorbing and quickly attenuating these disturbances. Corona's real effectiveness in this respect, however, has not been definitely determined.

The leakage effect of corona may also be of assistance in keeping down the magnitude of potential rise at the open end of a long line should the line accidentally become open-circuited. This reducing effect would be desirable, as the rise subjects the insulators to high potential strains, and causes the line to draw an excessive charging current from the connected generators.

A further interesting feature of corona is the discrepancy found between the measured charging current for a line and that indicated by the usual formula, when voltages are used which are above the critical corona value.[4] As the line potential is raised from low values up to e_0, the charging current increases proportionately. Slightly above, as corona forms, the current diverges from the linear relation, and for higher voltages, rises much more than proportionately. Since this is the same result as would be obtained were the line capacitance to increase above normal at these high voltages, the divergence has come to be described as the extra-capacity effect due to corona.

There are several explanations of this non-linear increase of current. Usually it is explained by assuming the line capacitance to be increased due to a cyclic change in effective conductor diameter caused by the corona. To account for the total test charging current in this way, however, requires in some cases an effective conductor of from fifty to eighty times normal diameter.

This explanation is also open to question due to the fact that it assumes that, were the capacitance to remain normal under corona conditions, the charging current would continue to rise linearly with the voltage. It is believed that the current might rise above the linear relation even with a normal capacitance, because of the corona harmonics. From this standpoint, it would seem more likely that the increase is the result:

1. Of the very approximate method by which the capacitance current is calculated.

2. Of the increase in admittance which the line, like any other circuit of relatively high capacitance, offers to currents of harmonic frequencies.

In these two explanations there is involved no action of the corona envelope, the current increase being considered the result solely of harmonics.

Purpose

The purpose of this paper is to present the results of an investigation carried on in the Research Division of the Electrical Engineering Department, Massachusetts Institute of Technology, aimed at the representation of corona effects on artificial lines. It describes the artificial corona which was developed, and gives the results of tests in which it was used. The latter tests were for the purpose of: First, obtaining experimental evidence in support of the "harmonic" explanations for the increased charging current which accompanies corona; second, determining the effect of corona in altering voltage distribution; and third, investigating its effectiveness in reducing traveling waves in the system.

Development of an Artificial Corona

As evidence was sought to show that the increased charging current is the result of harmonics rather than of the corona envelope, it was necessary to have a check upon the envelope effect. This being practically impossible with real corona, the envelope was eliminated

3. F. W. Peek, Jr., "The Effect of Transient Voltages on Dielectrics", Journ. A. I. E. E., June 1923, p. 623.

4. W. W. Lewis, "Some Transmission Line Tests", Trans. A. I. E. E., 1921, p. 1079.
F. W. Peek, Jr., "Voltage and Current Harmonics Caused by Corona", Trans. A. I. E. E., 1921, p. 1155.

from consideration by simulating the corona effects artificially on an artificial line.

The artificial corona consisted of a representation of the cyclic change in line leakage peculiar to corona. The leakage variation was obtained by the use of three-electrode vacuum tubes, the current characteristics of

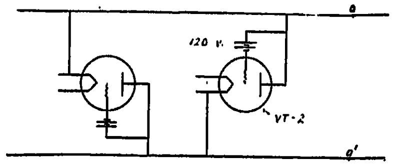


FIG. 1—CONNECTIONS OF ARTIFICIAL CORONA LEAK

which are in many respects similar to those of ionized air. As the tubes pass current only in one direction, two were necessary for one complete leak—one to operate on the positive half of the voltage cycle and the other on the negative half. Fig. 1 shows the interconnection used.

The operation of the leak depended upon the non-linear characteristics of the tubes. The filament of the first tube and the grid and plate of the second connected to one line wire, while the filament of the second and the grid and plate of the first connected to the other line wire. The breakdown voltage, that is, the e_0 of the leak, was governed by a bias potential of approximately 120 volts in each grid circuit. This, with the type of tube used (*V T*-2), kept the grid negative and the tube inactive until 115 volts, instantaneous between line wires, was reached on the ascending portion of the wave. With a plate potential of 115 volts and the grid 5 volts negative, conditions were favorable for the passage between line wires of a small plate current through one tube. As the maximum rose above 115 volts, the grid of this tube became less negative and finally positive, while its plate potential steadily

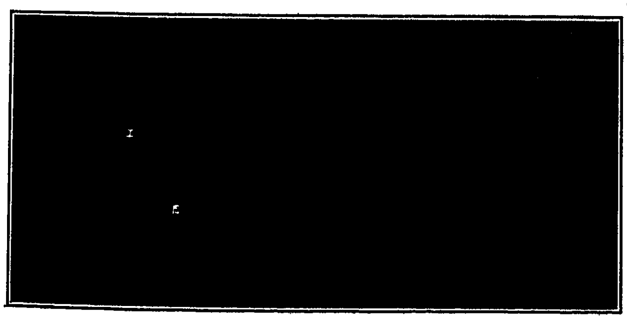


FIG. 2—DISCONTINUOUS LEAKAGE CURRENT GIVEN BY THE ARTIFICIAL CORONA LEAK

increased. This resulted in a rapidly increasing leakage current up to the crest of the voltage; beyond, on the descending portion of the wave, the action was reversed. The leak was then inactive until the second tube repeated the cycle under the negative voltage loop.

The leak may be likened to a two-way valve, in that it permitted leakage only for the intervals during which the instantaneous values of the voltage wave exceeded its breakdown value, e_0. It gave a small, discontinuous leakage current, Fig. 2, which was zero at all times except under the crests of the voltage wave. The shape and amount of this current could be controlled by adjustment of filament temperature and bias potential. By increasing the latter, the breakdown voltage was increased, and the leakage reduced. If the breakdown

FIG. 3—CURRENT OBTAINED WITH ARTIFICIAL CORONA LEAK AND CONDENSER IN PARALLEL

voltage had been established, the amount of the leakage could be controlled by the filament temperature. As used in the following tests the r. m. s. critical voltage, e_0, for the leaks was approximately 75 to 80 volts, representing $\frac{115}{\sqrt{2}}$ volts.

By shunting the terminals of the leak with a small

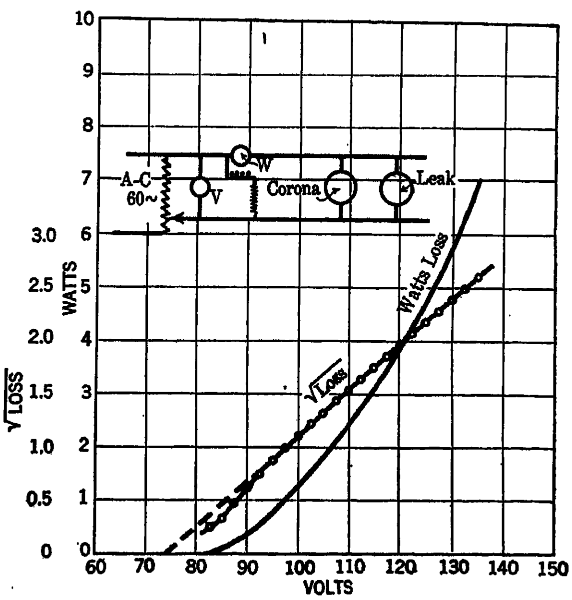


FIG. 4—CURVE SHOWING THAT THE ARTIFICIAL CORONA LEAK SATISFIES THE QUADRATIC LAW OF CORONA LOSS

paper condenser at $a\ a'$ in Fig. 1, the distorted current shown in Fig. 3 was obtained. As a voltage greater than e_0 was applied, there was a leakage current in addition to a charging current, the leakage current causing a hump on the latter slightly in advance of each voltage crest. The smaller irregularities in the current were due to tooth harmonics in the voltage wave.

In later tests these were eliminated by the use of filters.

That the leakage current of the artificial corona was similar to that of real corona may be observed by comparing Figs. 2 and 3 with the wave forms obtained of the latter by Whitehead[5] and by Bennett.[6] In both of these cases, the real corona was formed on a rod within a concentric cylinder. The oscillograms of Bennett show the distortion caused by corona of the rod-and-cylinder capacitance current when the applied voltage exceeded the critical corona-forming value for the apparatus. The corona introduced a hump into the charging-current wave just in advance of each voltage crest. In the duplication of this, with the artificial

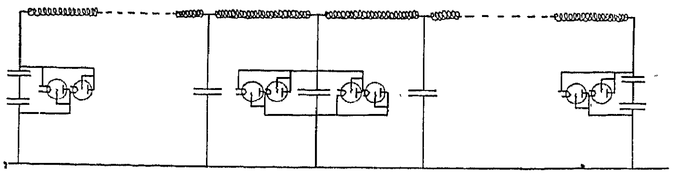


FIG. 5—CONNECTION OF CORONA LEAKS TO THE ARTIFICIAL LINE

corona, the capacity effect of the rod and cylinder was supplied by the small fixed condenser shunting the leak terminals. The wave forms reported by Whitehead were obtained by a separation of the total current, as found by Bennett, into two parts—a charging current and a corona current. For this the current wave was taken, first below and then above the critical voltage, the difference between the two giving the wave form of the corona current. This latter was pulsating. Its

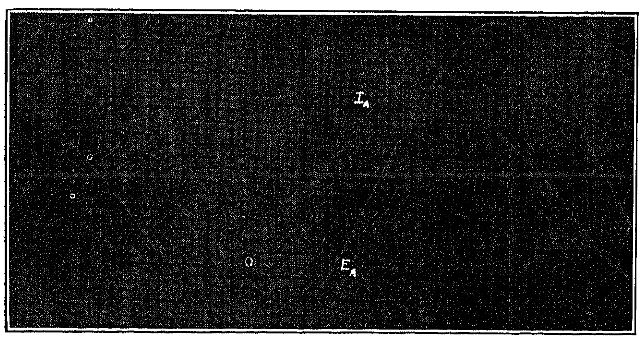


FIG. 6—CHARGING CURRENT FOR AN ARTIFICIAL LINE HAVING ARTIFICIAL CORONA LEAKS

humps were asymmetrical, but they attained their maximums approximately under the crests of the voltage.

The conformance of the artificial corona to the quadratic law of corona loss is shown by the typical power-loss curve given by a single leak (Fig. 4). The test for this type of curve consists of a plot of square root of the power against volts, the points of which should lie on a straight line. The leakage curve satisfied this requirement, as shown. The straight-line plot may be compared favorably with similar plots given by Peek[7] It is interesting to recall, in this connection, that the deviation from the theoretical curve in the region of e_0 is also a characteristic of real corona.

The artificial line to which the corona leaks were attached represented a single-phase line having 500,000-cir. mil conductors, spaced 9 ft. between centers.[8] One leak was placed at each end and two leaks in parallel at the middle. This was in accordance with the pi-construction of the line, and gave twice the leakage at the middle as at the ends. See Fig. 5. The critical voltage, e_0, for the line was established at 81 volts.

Using a 420-mile length of open line, oscillograms were taken of voltage and charging current at the generator

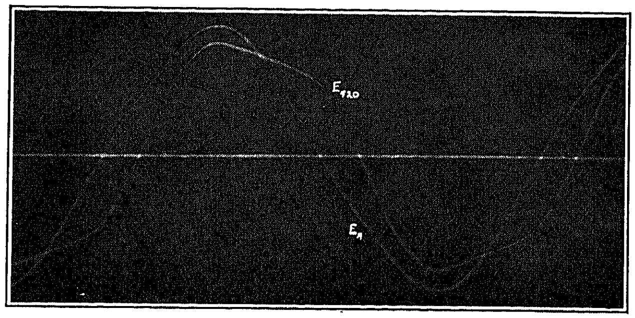


FIG. 7—VOLTAGE WAVE FORMS AT THE TWO ENDS OF A 420-MILE LINE WITH CORONA LEAKS OPERATING

end, both with and without the leaks operating. When they were attached, but inoperative, (filament currents zero), the current and voltage were normal sine waves. When placed in operation, they caused the distortion shown in Fig. 6. Analysis of the wave forms of the latter showed the voltage to have a three per cent third harmonic, and the current to have a nine per cent third harmonic, three per cent fifth harmonic, and one and three-tenths per cent seventh harmonic. These compare favorably with the voltage and corona-plus-capacity-current wave forms at the generator end of a real transmission line under corona conditions, as given by Peek.[9]

By these preliminary experiments the characteristics of the artificial corona were checked against those of real corona. It was shown that the leakage current was of approximately correct wave form, and that the power loss varied with the voltage in agreement with the quadratic law. Further, when operating on the artificial line, the distortion produced in the voltage and

5. Whitehead and Inouge, "Wave Form and Amplification of Corona Discharge", TRANS. A. I. E. E., 1922, p. 138.

6. E. Bennett, "An Oscillograph Study of Corona", TRANS. A. I. E. E., 1913, p. 1787.

7. F. W. Peek, Jr., "Dielectric Phenomena in High-Voltage Engineering," 1920, p. 128.

8. A. E. Kennelly, "Artificial Electric Lines", 1917, pp. 205-7. Kennelly and Nabeshima, "The Transient Process of Establishing a Steadily Alternating Current on a Long Line", *Pro. Am. Phil. Soc.*, 1920, p. 325.

9. F. W. Peek, Jr., "Dielectric Phenomena in High-Voltage Engineering", 1920, p. 118.

current was similar to that found with real corona on a real line.

Distortion of Voltage

Oscillograms were now taken of voltage wave forms at points out along an open line. The use of a vacuum-tube repeater[10] made it possible to attach the oscillograph to the unloaded line without disturbing voltage conditions. When the leaks were inoperative there was no distortion at any point on the line. But when they were placed in operation, they produced the distortion shown in Fig. 7. Only the two end voltages are given, as they represent the extreme conditions. Analysis of all the waves of the series was made, however, and it was found that the distortion was progressive. It consisted almost entirely of a third harmonic, the magnitude of which increased steadily from four to fourteen per cent between generator and open end.

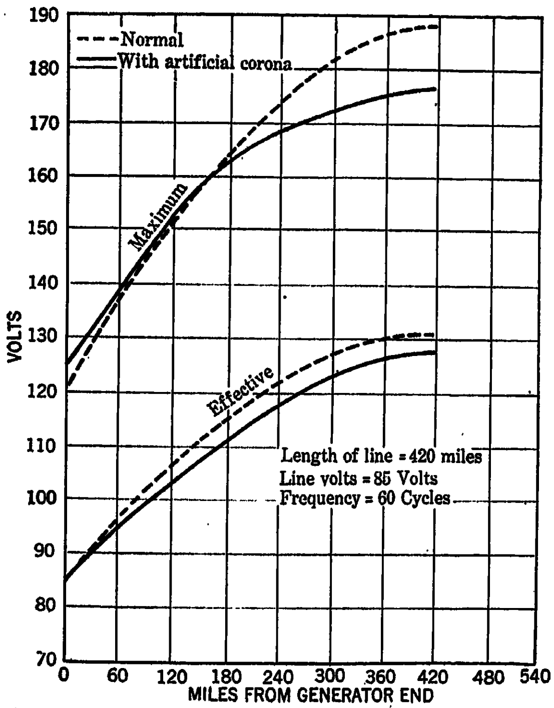


Fig. 8—Crest and R. M. S. Voltage Distribution on 420-Mile Line, with and without the Artificial Corona

Using a vacuum-tube, crest-reading voltmeter (also arranged to take negligible power), the maximum voltage values at positions along the line were found in similar manner. Under corona conditions the crest value was increased near the generator end, but decreased near the free end, as shown in Fig. 8.

This progressive increase in voltage distortion is due to the difference in resonance conditions for the fundamental, and for the various harmonics. The latter, as a result, become a larger percentage of the resultant voltage at the distant points.

This distortion of the voltage along a single-phase line is of interest, since it occurred under conditions which provided a circuit for third-harmonic currents. A three-phase line with grounded neutral is equivalent to three such single-phase lines in parallel. The results indicate, therefore, that even with a grounded-neutral system there can be a distortion of voltage between each line and ground. It is of interest also in connection with the explanation given later of the charging current which is found under corona conditions.

Any alteration in maximum voltage values, such as was found, would have an appreciable effect upon the distribution of corona loss, increasing it at the near end and decreasing it at the far end of the line.

Effect of Corona Leakage on Voltage Distribution

An excessive potential rise on an open line under corona conditions has been reported.[4] This has raised questions regarding the effect of corona in altering voltage distribution in cases of open circuit, and the possibilities of serious resonant voltages at triple frequencies.

These points were investigated with the artificial corona. Voltage distribution curves were taken with an electrostatic voltmeter over a wide range of line lengths and voltages, with and without the corona leaks operating. The curves shown in Fig. 8 are typical of the alteration in distribution found in every case. To this, even a 240-mile line proved no exception. This is important as this line represented approximately a quarter-wave length for the third harmonic of sixty cycles—a condition which would be most favorable for the formation of high-resonant voltages by the corona if resonance effects of the harmonics should prove to be present.

In these tests it was found that the normal voltage distribution, that is, for no corona leakage, held also for corona conditions unless the applied voltage was excessively high or the length of line extreme. With a long line and an applied voltage near the critical value, the distribution was appreciably lowered. It appears, therefore, that in the case of distributions on long lines the reducing effect of corona as a leakage will largely overbalance any tendencies toward high voltage resonance due to the harmonics it may introduce.

In making voltage distribution computations for high-voltage lines where corona must be considered, it has been customary to represent the probable corona loss by an equivalent ohmic leakance. In this, the assumption has been made that corona would tend as a leakage to reduce the rise of potential should the line accidentally become open-circuited. The above results appear to substantiate this assumption. Further, it was possible in the case of each of the tests on the artificial line to calculate a distribution which would agree within the limits of engineering accuracy with

10. F. S. Dellenbaugh, Jr., "*Artificial Transmission Lines with Distributed Constants*," Journ. A. I. E. E., Dec. 1923, p. 1293.

4. loc. cit.

the distribution obtained by test. For this the usual hyperbolic formulas were employed, using a leakance, g, obtained from the division of measured loss by the average of the squares of the end voltages.

For all these tests it was possible to check the adequacy of the corona representation. The critical corona voltage of the single-phase power line, which the artificial line represents,* is approximately 95.2 kv. to neutral, assumptions being made as to atmospheric conditions and state of conductor surface.[11] As this corona-forming voltage was represented on the artificial line by 81 volts, the ratio between corresponding line

FIG. 9—CURRENT TRANSIENT ON OPEN LINE WITHOUT CORONA SWITCH CLOSED 115 DEG. AFTER VOLTAGE PASSED THROUGH ZERO

voltages was $\frac{81}{95{,}200}$. It was thus possible to check the corona representation, as the corresponding losses on the artificial line and on the real line were to be to each other as the square of this voltage ratio. In all cases the loss measured was found to be within 10 per cent of the loss which was calculated for the condition by this method.

EFFECTIVENESS OF CORONA IN DAMPING TRANSIENTS

Tests were also made to show the possible effectiveness of corona in attenuating traveling waves, such as result from switching operations in the system. For this a single-phase artificial line with distributed constants was used. It represented 331 miles of No. 00 solid copper conductors having 8 ft. 9 in. spacing.[10]
A synchronous switch closed the circuit between generator and line at any desired point on the voltage wave. Two oscillograms for each point of switch-closing were taken—one to show the transient under normal conditions, and the other the same transient with corona present. Their difference represented the modification resulting from the corona leakage.

In Figs. 9 and 10 are given typical transients for the two cases. They show the generator voltage and the current entering the open line when the switch was closed on the descending portion of the voltage wave. In the first, without corona, the deviation from the sinusoidal state is apparent for approximately two cycles, and six reflections are distinguishable. In the second, taken with corona present, the steady-state condition was reached in half a cycle. In this latter, the normal distortion of the current due to the corona should not be confused with the transient.

Although these last results are only qualitative, they support the general belief that corona, as a leakage occurring with excessive voltage, acts to suppress abnormal voltage disturbances. In functioning as a safety valve, it serves to dissipate the excess energy of the traveling waves and reduce them rapidly to the steady-state condition.

EFFECT OF HARMONICS ON APPARENT CAPACITANCE

In corona loss tests on transmission lines, readings are taken of current, voltage, and power. From the ratio of watts to volt-amperes the power factor is calculated, and thereby the line current resolved into two components—one in phase with the voltage, and the other at right angles and leading. The latter, (considered the charging current for the line capacitance) increases linearly with the voltage up to the point of corona formation; beyond, it bends rapidly upward. Curves showing this are given by Lewis[4] and Peek. The variation from the linear relation above the critical voltage is generally attributed to the presence of the corona envelope increasing the line capacitance.

This feature of corona was investigated on the artificial line. Corona loss tests were made, and essentially the same procedure followed in measurement and calculation as outlined above. For each voltage the value of line capacitance corresponding to the quadrature current was computed. The results, given in

FIG. 10—CURRENT TRANSIENT AS SHOWN IN FIG. 9, BUT WITH CORONA

graph form in Fig. 11, show in the upper part the variation of volt-amperes, net watts of corona loss, and power factor with applied voltage. In the lower part are given the total current, corona current, charging current, and the apparent capacitance. There is an interesting similarity between these and the characteristic curves of real corona.

The true capacitance of the 189 miles of artificial line was 1.445 microfarads. The capacitance calculated to agree with the test charging current at a voltage 40 per cent above the critical value was 1.51 microfarads. Thus with only the leakage effect of corona represented,

*See page 816.
11. F. W. Peek, Jr., "Dielectric Phenomena in High-Voltage Engineering," 1920, p. 203.
10. loc. cit.

the capacitance of the line was apparently increased 4.6 per cent, and this with no actual change taking place in the fixed condensers which supplied it. In this connection, Lewis reports an increase of 32 per cent, found when the voltage was 70 per cent above the critical value. Although small in comparison with this, the increase which was obtained on the artificial line showed plainly that an ionized envelope is not an essential to the occurrence of this effect.

It was possible to show in another way that at least much of this capacitance change found on lines is apparent. The above test was repeated, measuring only the loss in a single corona leak shunted by a fixed condenser. The quantities, Fig. 12, were calculated as for

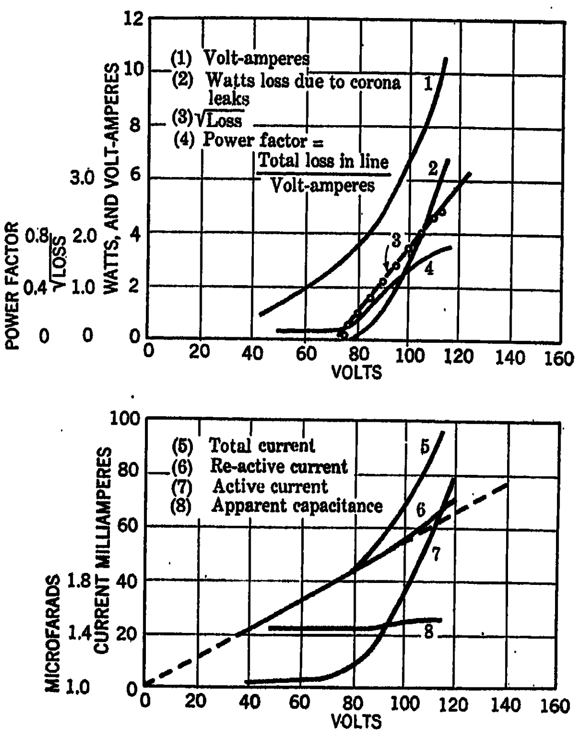


Fig. 11—Characteristic Corona Curves Obtained on an Artificial Line having Artificial Corona, Showing Typical Increase in Line Capacitance

a line. The curves are almost exact duplications in form of those given for real corona. The charging current and apparent capacitance of the condenser were distinctly increased at voltages exceeding the critical value of the leak. The capacitance changed from 1.06 to 1.25 microfarads, which is a 21 per cent increase for a voltage 67 per cent above the critical value. These results were obtained, using a voltage of approximately 100 volts; there was no corona envelope; and the true capacitance of the circuit was known to remain constant, being a fixed condenser.

In both of these tests the increase in charging current was *real*, but the increase in capacitance only *apparent*. This may be explained by the presence of harmonics.

In the transmission line case there was a distinct distortion of the voltage between line and ground out along the line due to the corona harmonics. (Shown in Fig. 7). Being impressed on the normal line capacitance, these harmonic voltages produce large charging currents due to the fact that there is an increased susceptance to currents of harmonic frequencies. The result is that the quadrature, or capacitance component of the line current, is increased more than proportionately to the applied voltage.

Further, as shown in Fig. 6, the current at the generator end of the line under corona conditions was highly distorted in comparison with the voltage at that point. The meters connected there for measuring the current and voltage indicated only the effective values of the

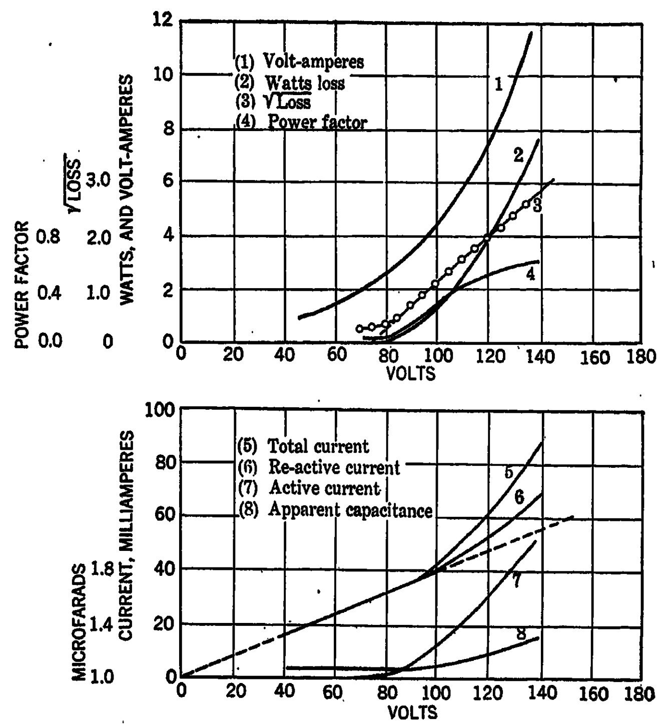


Fig. 12—Characteristic Corona Curves Obtained with an Artificial Corona Leak Shunted by a Fixed Condenser, Showing Apparent Increase of Capacitance

wave forms, and in making the subsequent computations, equivalent sine waves having these values were substituted for the distorted waves.

The error introduced by neglecting the harmonics in making the calculations is brought out by the results of the test on the leak and shunting condenser. In that case the current and voltage wave forms were as shown in Fig. 3; the current only was distorted, the voltage remaining practically sinusoidal. This produced a decrease in the power factor, for the harmonics which existed in the current and not in the voltage contributed nothing to the average power, but did increase the effective value of the current required to produce that power. The result of this is shown by the vector diagram of Fig. 13.

The fundamental I_1 of the current has an energy component I_e in phase with the voltage, V. As the

voltage is sinusoidal, this is the only part of the current that contributes to the power. The harmonics increase the effective value of the total current from I_1 to I, but its energy component must remain the same. Consequently, when an equivalent sine wave is used for I, as is done in making the usual computations, the result is an increased quadrature current I_q, and corresponding apparent capacitance calculated from $C = \frac{I_q}{\omega V}$.

The results of the above tests are evidence that the capacitance change attributed to corona is not at least entirely a phenomenon due to the presence of an envelope. The indications are that it is a direct result of distorted wave forms, and of a method of calculation in which only equivalent sine waves are considered. The "change in capacitance" accompanying corona is a means of describing a particular effect, that is, the upward bend of the charging current, and its explanation therefore is largely a matter of definition. Although capacitance is usually defined as the charge per unit rise of potential, it is not likely that the ions of the corona envelope constitute a charge in the true sense of the word, as probably little of the charge in the envelope can return to the conductor as the voltage drops from its crest value.

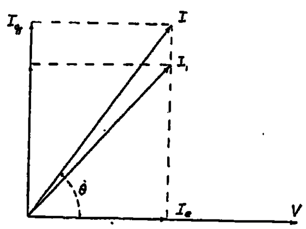


FIG. 13—ILLUSTRATING HOW INCREASE IN CAPACITANCE RESULTS FROM THE SUBSTITUTION OF EQUIVALENT SINE WAVES

The final proof of whether or not there is a real capacitance change may well lie in the actual measurement of a line capacitance both with and without corona present. Tests of this nature have been made at Massachusetts Institute of Technology by M. T. Dow. An exploring wire was placed in the field near one of the conductors of a laboratory span, upon which was formed direct-current corona, and the capacitance between the wire and conductor measured. This is a method laden with experimental difficulties due to the high voltages which must be used, the smallness at best of the quantities dealt with, and the essentially negative nature of the problem. Four different methods of capacitance determination were used, but no indications of any capacitance increase were found. This warrants the conclusion that the increase, if there is any due to the corona envelope, is very small. There seems, therefore, to be as yet no definite proof that corona does cause an actual change in the capacitance of a transmission line.

SUMMARY

1. The energy loss and distortion characteristics of corona can be closely approximated on an artificial line by simulating the cyclic change in line leakance peculiar to corona. This corona leakage can be lumped similarly to the resistance, capacitance, and inductance of the line.

2. Corona may cause a distortion of voltage to neutral even when there is a complete circuit for the third-harmonic currents provided by a grounded neutral.

3. The harmonics introduced by corona will not cause unusual resonance effects or extra-potential rises in the case of an unloaded line.

4. The results indicate that voltage distribution for usual corona conditions can be calculated within engineering accuracy by the methods at present in use. For this, the corona loss can be represented by an equivalent ohmic leakance.

5. Qualitative results were obtained which support the general belief that corona, in forming only at high voltages, can be effective in suppressing high-voltage disturbances.

6. The "extra-capacity effect" accompanying corona is largely a matter of definition. The presence of an envelope of ionized air about the conductors is not an essential to its occurrence. It is probable that at least the greater part of the capacitance change is apparent only, resulting from the method by which the capacitance is calculated, and from the increased line admittance to charging currents of harmonic frequencies.

In conclusion, the author wishes to thank Doctor V. Bush for suggesting and supervising this investigation, and also Messrs. M. T. Dow and R. Henriksen for their assistance in the experimental work.

Three-Phase, 60,000-Kv-a. Turbo Alternators for Gennevilliers

BY E. ROTH[1]

Associate, A. I. E. E.

THE Société Alsacienne de Constructions Mécaniques at Belfort (France), installed for the Société l'Union d'Electricité, in the Gennevilliers Generating Station near Paris, three 45,000-kv-a. sets in 1922 and another in 1924. The alternators of these sets were, at the time, the largest four-pole machines ever built, but they are now exceeded in power by the two new 60,000-kv-a. units, of 65,000-kv-a. overload capacity, that the Société Alsacienne is building for the Gennevilliers Generating Station. Like the 45,000-kv-a. generators, these new units are designed for delivering three-phase current at 6000 volts, 50 cycles, and run at 1500 rev. per min. direct-connected to 50,000-kw. steam turbines (Fig. 3).

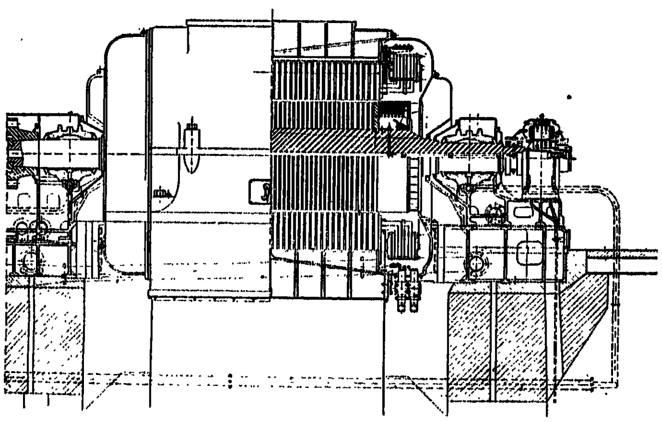


FIG. 1—LONGITUDINAL PART-SECTIONAL ELEVATION OF 60,000-KV-A. ALTERNATOR

The makers have sought to obtain the greatest strength, electrical and mechanical, in the construction of these machines, ensuring complete reliability of operation, and three years' service with the 45,000-kv-a. alternators has shown that this purpose has been fully attained. These machines have been running without accident, and tests have proved that they can even yield 55,000-kv-a. without difficulty. Indeed there have been cases where it was necessary to run them at this overload at very low power factors. In view of these good results, it is easy to understand that relatively small modifications had to be made in the design of the 45,000-kv-a. alternators to provide the 60,000-kv-a. units. It has even been possible to make their stators and rotors interchangeable.

The detailed description of the 45,000-kv-a. alternators which the author has published elsewhere[2] can thus apply to the 60,000-kv-a. units, and it seems unnecessary to repeat it here. It will be sufficient to point out, shortly, the modifications that the first alternators have undergone. However, it has occurred

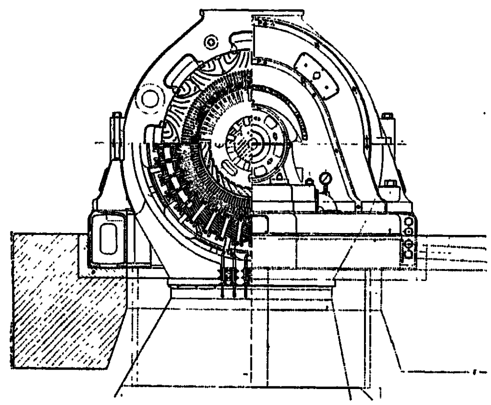


FIG. 2—TRANSVERSE PART-SECTIONAL ELEVATION

to the author that it would be interesting to reproduce in this paper, as an example of European practise, some of the photographs which have been reproduced in the

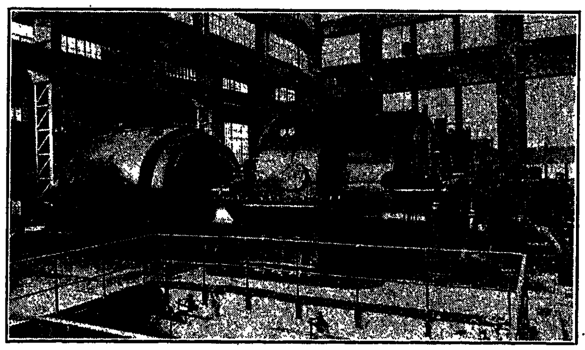


FIG. 3—GENERAL VIEW OF 50,000-KW. SET

1. Chief Electrical Engineer, Société Alsacienne de Constructions Mécaniques, at Belfort.

Presented at New York Section Meeting of the A. I. E. E., October 23, 1925.

2. E. Roth. The 40,000-kv-a. alternators built by the Société Alsacienne de Constructions Méchaniques for the Gennevilliers Power Plant of the Union d'Electricité. Revue Générale de l'Electricité, 24th February 1923, Vol. XIII, page 307, and Bulletin de la Société Alsacienne de Constructions Mécaniques, No. 2, April 1923, page 42.

Large Turbo Alternators at Gennevilliers; the 40,000-kv-a. alternators of the Société Alsacienne. The *Electrical Review* 21st and 27th April 1923, pages 604 and 646.

E. Roth. Advances in the Construction of Large Turbo Alternators. An account delivered to the International Congress at Léige, 1922, and Revue Générale de l'Electricité, 27th January, 1923, Vol. XIII, page 129.

papers to which reference has been made. These photographs show assembled views of the machines (Figs. 1 to 5) as well as detailed views of the stator (Figs. 6 and 7) and the rotor (Figs. 8 and 9) in the various stages of construction. But it is desired especially to call attention to some of the unusual features of these machines which may interest the American engineer, and to point out the methods and results of tests carried out on these machines.

Comparison between the Dimensions of 60,000-Kv-a. and 45,000-Kv-a. Alternators

Some of the numerous heat tests made on the 45,000-kv-a. alternators are given later in this paper. At the

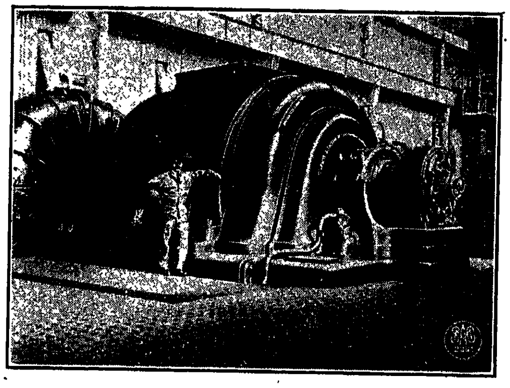


Fig. 4—View of 60,000-Kv-a. Alternator

50,200-kv-a. load (Test No. VIII) it will be noticed that the temperature rise of the copper, measured by thermocouples placed on the bare copper, inside the wrappings, was but 37.5 deg. cent., while the temperature rise of the rotor copper at the same load, as

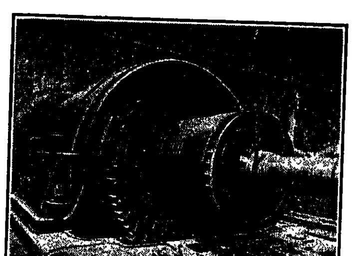


Fig. 5—Alternator in Course of Erection (Showing Details of Stator Winding and Construction of Rotor)

measured by the increase of its resistance, was only 67 deg. cent. although the power-factor was but 0.65. These small values of temperature rise, due to an excellent system of ventilation, have, as already stated, permitted the new machines to be given the same geometrical dimensions as the old, which have a total length of active iron of 278 cm. (Fig. 1).

The stator winding of the 45,000-kv-a. alternators was full pitch, while that of the 60,000-kv-a. units (Fig. 5) is a fractional pitch winding. The small increase of flux made it necessary to slightly increase the section of the rotor disks by reducing the air ducts. The resistance of the latter to the passage of the air has thus been somewhat augmented; this, however, is negligible, owing to the total section of the air ducts being very large.

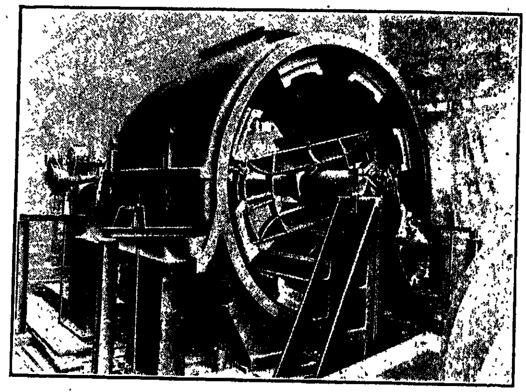


Fig. 6—Boring of Stator

For providing stability, the air-gap has been enlarged from 2.5 to 3 cm. The increase of the excitation which results therefrom is somewhat compensated for by the reduction of the armature reaction. The full-load excitation of the 60,000-kv-a. units requires 72,000 ampere-turns per pole while 65,000 ampere-turns are required in the 45,000-kv-a. units. It is obvious from the results of the test already mentioned and which corresponds to 75,000 ampere-turns, that the temperature rise of the rotor copper of the new machines will be less than 65 deg. cent. These 72,000 ampere-turns correspond to a current of 630 amperes with 19 conductors per slot or 114 turns per pole. (Fig. 9).

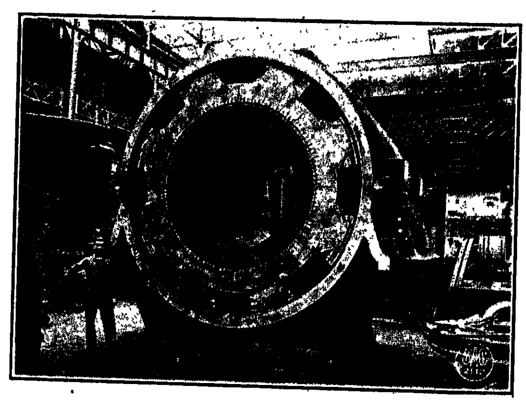


Fig. 7—Complete Stator Ready for Winding

Fig. 10 represents the stator slots of the two alternators and shows the proportions employed to provide for a very great leakage flux, constituting the most remarkable feature in these machines. Reference will be made to this feature later in the paper. The number of slots has not been altered but the section of the copper is slightly larger in the 60,000-kv-a. machines than in the 45,000 kv-a. units.

The bars in both machines are similarly constructed, and are made up of component conductors strongly insulated from each other and placed obliquely to the center line of the slot in order that they shall have identical positions with respect to the slot-field, (Fig. 11). The eddy current losses are thereby cut down to a negligible value. In fact, the examination of the reports of the heat test shows that the readings of the couples which are laid right on the copper (the first at the top of the lower coil side and the second at the top

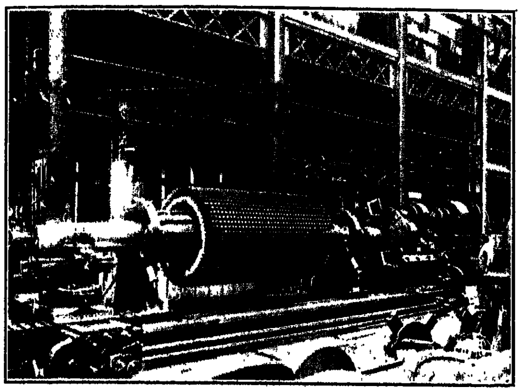


Fig. 8—Rotor on the Lathe

of the upper coil side), numbered 10 and 12 in Fig. 10, are very nearly the same. In badly designed bars, the stray losses due to eddy currents are very much greater in the upper conductor of a slot than in the lower. The fact that the temperature rise of both conductors is the same leads to the conclusion that the supplementary losses are completely eliminated.

Fig. 9—60,000-Kv-a. Rotor Being Wound

The dimensions of the air-cooler, a description of which has already appeared in the papers mentioned, have been somewhat enlarged. It will be recalled that the air describes a closed circuit. It is cooled by means of an air-cooler wherein the condensate circulates and which possesses the interesting characteristic that the amount of cooling water is maintained greater than a given minimum whatever the load may be. Thus a certain part of the water returns to the condenser immediately after leaving the air-cooler. This arrangement was made to overcome the difficulty of the losses in turbo alternators being but slightly variable with the load, whereas the amount of condensed water essentially depends thereupon; hence, there is a risk of overheating the air at light loads if the condensate alone is employed for cooling.

The total weight of the 60,000-kv-a. alternator is 162 metric tons (356,400 lbs.), without bearings and base-plate. The stator alone weighs 104 tons; the rotor, with the exciter, 50 tons; and the end-shields, 8 tons. It may be interesting to compare these weights with

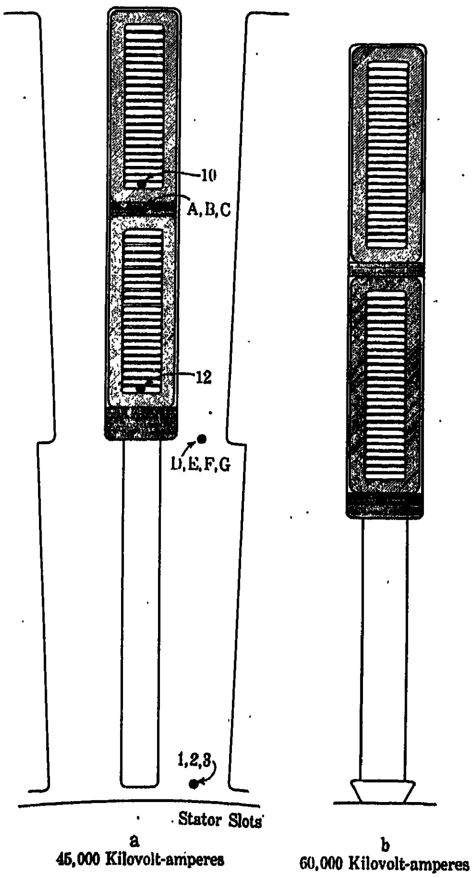


Fig. 10—Showing Arrangement of Thermocouples

those stated by Messrs. Foster, Freiburghouse, and Savage in their paper on *"Large Steam Turbine Generators,"* Journal of the A. I. E. E., October 1924, page 923. For a 62,500-kv-a., 1200-rev. per min., 60-cycle turboalternator, these authors give the following weights: stator, 93 tons without bearings and base-plate; rotor, 93 tons; end shields, 11 tons;—giving a total weight of 197 tons. The differences in weight are easily explained by the fact that the Société Alsacienne's alternator is a four-pole machine while the alternator described by those authors is a six-pole machine.

Consideration of the Advantage of Employing Leakage Slots

The leakage slots (Fig. 10) are the most interesting feature in these machines. They serve a double purpose since they first artificially increase the leakage for the stator, thus reducing the instantaneous short circuit current to a very low value, and then serve

advantageously as a channel for the cooling air. These two functions of the leakage slots will be examined in detail and it will be seen that the latter is by no means the least important.

The design of these machines has been largely influenced by the condition that, without the use of reactance coils, the instantaneous value of the symmetrical short-circuit current should not exceed four to five times that of the normal currents. The actual values are 4.17 for the 45,000-kv-a. alternator and 3.7 for the 60,000-kv-a. units. To obtain these exceptionally low values, special arrangements had to be employed.

It is a well-known fact that the instantaneous short-circuit current in turbo alternators is almost solely limited by the stator leakage, for, not only is the rotor

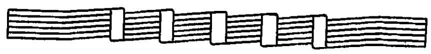


FIG. 11

leakage very small, but its effect is still further reduced by the presence of dampers. Therefore, neglecting the rotor leakage, the above mentioned values of the instantaneous short-circuit current in the 45,000-kv-a. and 60,000-kv-a. alternators correspond to respective inductive drops equal to 24 per cent and 27 per cent of the normal pressure with the normal current. But it is not recommended to obtain these high inductive drops by the usual means consisting of designing an alternator with a high armature reaction, because, as will be seen later on, serious disadvantages could result in the course of operation. But let us first examine the conditions which must be fulfilled in the dimensioning of the leakage slots and their advantages to the alternator itself.

The height of these slots may occasion surprise, since the same inductive drop could, in fact, be obtained by means of much smaller slots. Thus the supplementary leakage obtained by the leakage slots is the same in *a* and *b* of Fig. 12. But it should be borne in mind that on a short-circuit the normal path offered to the flux is checked and that the major part of the flux is obliged to seek its way across the slots. It is necessary, therefore, that the depth of the leakage slots be sufficient to avoid saturation of this path, since the magnetomotive force, and thus the short-circuit current which produces it, would be increased due to saturation.

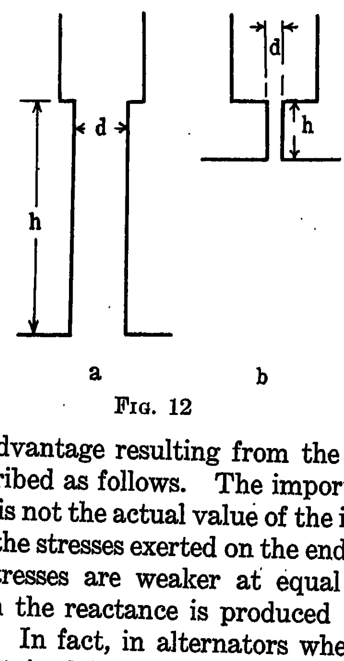
The following table shows how the inductive voltage drops caused by stator leakage are distributed in both machines; these figures are stated in per cent of the normal voltage for the normal current.

	45,000-kv-a. Alternator	60,000-kv-a. Alternator
Normal slots	2.8 per cent	4.1 per cent
End connections	9.5 per cent	12.0 per cent
Zig-zag leakage	1.5 per cent	1.9 per cent
Leakage slots	10.2 per cent	9.0 per cent
Total:	24.0 per cent	27.0 per cent

Therefore, on a sudden short-circuit, the flux which passes across the leakage slots in the 45,000-kv-a. alternator is $\frac{10.2}{24}$ or 42.5 per cent, and, depending on the instant of the short-circuit, even twice this value, or 85 per cent of the normal flux. In the 60,000-kv-a. alternator, the corresponding values are $\frac{9}{27}$ or 33 per cent and can reach twice this value, or 66 per cent of the normal flux.

Owing to these values being smaller in the 60,000-kv-a. machines, it has been possible to reduce the height of the leakage slots (Fig. 10) and thus increase the height of the core back of the slots with the same magnetic density in the core of both alternators, notwithstanding the greater total flux in the 60,000-kv-a. design.

Advantages of the Leakage Slots with Regard to the Alternator. It may be contended that these high teeth cause an increase of the losses in the teeth, but this possibility is easily compensated for by very slightly increasing the weight of the copper. And here the first advantage of these slots appears, in that they permit a greater weight of copper to be placed on the stator than that corresponding to the diameter of the bore. This greatly facilitates the construction of very powerful alternators.



a b

FIG. 12

Another advantage resulting from the leakage slots may be described as follows. The important point on short-circuit is not the actual value of the instantaneous current, but the stresses exerted on the end connections. But these stresses are weaker at equal short-circuit current when the reactance is produced by increased slot leakage. In fact, in alternators where the stator leakage is obtained by a high armature reaction, the major part of the leakage is due to the end-connections. It is consequently necessary that on short-circuit these should withstand the dynamical stress due to a flux nearly equal to the normal flux, or, depending on the instant of short-circuit, to even twice this flux. Now in the case of the 45,000-kv-a. alternator, the flux that, on short-circuit, is interlinked with the end-connections is but $\frac{9.5}{24}$ or about 40 per cent, and in the worst

conditions 80 per cent of the normal flux. In the case of the 60,000-kv-a. units, this value becomes $\frac{12}{27}$ or 44.5 per cent, and a maximum of 89 per cent. Furthermore, in machines with high armature reaction, the weight of copper to be placed in the end-connections is much greater than in a machine with a small armature reaction, and consequently, the involutes are longer and more difficult to secure and higher stress must be withstood by a part less capable of withstanding it.

We have already stated that the leakage slots perform an important function in the ventilation of these machines; reference to this will be made later.

Advantages of the Leakage Slots with Regard to the Operation. Having pointed out the advantages of the leakage slots with reference to the alternator itself, we will now examine the advantages they present with regard to operation.

One important condition with which the alternators have to comply relates to *stability of operation,* which requires a high pull-out torque; the alternator should not tend to hunt or run out of step under an accidental overload. The condition of stability is generally complied with when the machine operates on a system having a certain inductive reactance, but it is more difficult to satisfy when it may be called upon to supply a system with a capacitive load or even when the power factor is not much different from unity. Now, when the limiting instantaneous short-circuit current imposed is low, the required stability may be attained with an alternator having a high armature reaction, or, in other words, owing to stability refinements it may become impossible to design the alternator for the short-circuit current imposed. In this case, compliance with the conditions of stability leads to the design of an alternator with a rather high flux, and to the artificial increase of the leakage by means of special slots so as to obtain the proper value of the instantaneous short-circuit current.

But a small armature reaction is also necessary when it is desired to avoid *self-excitation.* This question is of sufficient importance to be dealt with in the special section following. We shall see that in this further particular, the leakage slots are extremely useful.

THE AUTO-EXCITATION OF TURBO ALTERNATORS

It is known that in certain conditions an alternator, when switched on to a capacity, may be self-exciting; that is, it may proceed to function as a generator, its magnetizing current being supplied by the capacity. This phenomenon has been described many times[3], especially by American engineers. These studies show that an alternator will be less self-exciting the larger the air-gap and the smaller the armature reaction.

Synchronous Self-excitation of Alternator with Salient Poles. Messrs. Blondel[4] and Bethenod[5] have shown that an alternator, when connected to a capacity, may be self-exciting under certain conditions if the reluctance of the path offered to the flux of direct reaction is different from that offered to the flux of transversal reaction. The current produced is at a frequency synchronous with the pulsation. We call this phenomenon "synchronous self-excitation." This difference between the reluctances of the paths of the two fluxes of reaction always exists in salient-pole alternators, but it is not so marked, being sometimes absent altogether, in turbo alternators. The synchronous self-excitation cannot therefore generally take place in these machines, and is even absolutely impossible when the rotor teeth are uniformly distributed over the periphery of the rotor.

Asynchronous Self-Excitation of Turbo Alternators. In this case, however, another kind of self-excitation may take place, but it is necessary that the rotor be provided with dampers or the rotor winding be a closed circuit. The frequency of the current produced by the machine is no longer synchronous with the pulsation when the field does not revolve synchronously with the rotor, and obtains a certain slip with regard to it. This phenomenon which we have called "asynchronous self-excitation" has been studied by Mr. Bethenod[6], who pointed out the danger it would present to turbo alternators.

The conditions of self-excitation, whether synchronous or asynchronous, are the same; the main cause is the residual magnetism. The self-excitation may in general be explained as follows:

3. P. Boucherot. Alternateurs Auto-excitateurs. Bulletin de la Société Internationale des Electriciens, Feb. 1898, Vol. XV, p. 79.

A. Blondel et Ch. Lavanchy. Rapport sur les réactions d'un réseau à haute-tension sur l'excitation des alternateurs. Effets de résonance et d'auto-amorcage sous charge réduité. Compte-rendu des trauvaux de le 1° Conférence internationale des Grands réseaux, 1921, p. 881.

J. Bethenod. Sur l'alternateur à résonance. La Lumière electrique, 25th December, 1909, Vol. VIII, p. 398.

F. D. Newbury. The Behavior of Alternators with Zero Power-factor Leading Current. The *Electric Journal*, 1918, p. 363.

R. W. Sorensen, H. Cox, and G. E. Armstrong. California 220,000-volt, 1,500,000-kw. Transmission Bus. PROCEEDINGS A. I. E. E. 1919, Vol. XXXVIII, p. 1027.

W. O. Morse. The Behavior of Alternating-Current Generators when Charging a Transmission Line. *General Electric Review*, Feb. 1920, Vol. XXIII, p. 109.

C. J. Fechheimer. TRANSACTIONS A. I. E. E. 1920, Vol. XXXIX, p. 1637.

G. Darrieus. Les réseaux de distribution et la transmission à trés haute fréquence. Exemples Américains. Bulletin de la Société francaise des Electriciens 1920, Vol. X, p. 411.

T. Labouret. Répercussion des lignes de forte capacité à vide nu le fouctionnement des alternateurs. Revue générale de l'Electricité, 17th December 1921, Vol. X, p. 875.

4. loc. cit.

5. loc. cit.

6. T. Bethenod. Auto-amorcage des machines, à rotors cylindriques associées à des condensateurs. Revue générale de l'Electricité, 8 September 1923, Vol. XIV, p. 307.

When starting an alternator connected across a condenser, the residual magnetism induces in the alternator a very small electromotive force. This, owing to the presence of the condenser, produces a current leading by $\frac{\pi}{2}$ which adds its magnetizing action to that of the residual magnetism. However, this effect can start the auto-excitation only at the speed by which the capacitive reactance of the condenser is slightly higher than the inductive reactance of the alternator. In fact, from this time on, the current in quadrature with the electromotive force is sufficient to magnetize the machine. As the pressure builds up, the current does likewise, one boosting the other, the phenomenon being limited only by the saturation of the alternator.

But any condenser has losses, however small they may be. Losses are also developed in the iron and copper of the alternator due to the passage of current, and these losses can be even higher than the normal losses. As the corresponding active power can be derived only from the motor driving the alternator, an electromagnetic torque has to be developed between the stator and the rotor.

The existence of this torque is rendered possible in two ways,—in the salient-pole alternators, by the dissymmetry of the magnetic circuit in which case the self-excitation is synchronous, and in the turbo alternators, by the presence of a damping circuit or the closed field circuit. In the latter case, the self-excitation is asynchronous; in fact, an active component of the current in the stator requires the presence of a corresponding component in the rotor winding for the production of which an electromotive force is required, and in order that it may be induced, it is necessary that the speed of the rotor be different from that of the revolving field, the rotor leading with regard to the field by the speed of the slip. Therefore the conditions of the existence of this component of the current in the rotor are the same as the conditions of existence of the current in the induced winding of an asynchronous generator.

The condenser is practically represented by the line which has but a small resistance. The value of the slip is then very small so that the frequency of the alternator running under the conditions of asynchronous self-excitation is not, at the normal speed, much different from the normal frequency; hence it ensues that the conditions of asynchronous self-excitation and of synchronous self-excitation are the same. The alternator is self-exciting when the capacitive reactance of the line is slightly superior to the inductive reactance of the alternator, the limiting case being where the characteristic of the line coincides with the straight part of the no-load characteristics of the alternator.

Formula Useful in Ascertaining the Conditions of Self-excitation. The conditions of self-excitation just stated can be expressed in a very simple form. An examination of the no-load and short-circuit characteristics of an alternator determines whether self-excitation will take place in the case of a line of, say, L km. length.

Study of a certain number of high-voltage, three-phase lines indicated that their capacitive reactances did not vary much from one line to another and that the value per phase (whatever the pressure), is, for 50-cycle lines, about 0.385×10^6 ohms per km. It is therefore approximately correct to multiply the value of the line pressure U, per phase, by the reciprocal of this capacitive reactance and by the length, L, in km., to obtain the charging current of the line, or:

$$\frac{10^{-6}}{0.385\sqrt{3}} \times L \times U = 1.5 \times L \times U \times 10^{-6} \text{ amperes.}$$

Multiplying this value by the transformer ratio gives the value of the charging current in the line, corrected to the pressure U_a of the alternator. It becomes

$$I = 1.5 \times \frac{U^2}{U_a} \times L \times 10^{-6} \text{ amperes.}$$

According to what has just been stated, the self-excitation will take place when the magnetizing current supplied by the line is slightly higher than the current necessary for exciting the alternator. Therefore, by reading, on the extension of the straight part of the no-load characteristic of the alternator, the number of ampere-turns corresponding to the value U_a of the normal voltage of the alternator, and determining the armature current I_a on the short-circuit characteristic corresponding to the same number of ampere-turns, the self-excitation takes place when

$$I > I_a$$

or when

$$1.5 \frac{U^2}{U_a} \times L \times 10^{-6} > I_a \text{ amperes.}$$

Application of Formula to the 60,000-kv-a. Alternator. The full lines on Fig. 13 represent the no-load and the short-circuit characteristics of the 60,000-kv-a. alternator. It will be noticed that a current $I_a = 2400$ amperes on the short-circuit characteristic corresponds to the normal pressure of 6000 volts read on the straight part of the no-load characteristic. Assuming a three-phase line at $U = 150{,}000$ volts at 50 cycles, self-excitation will take place when the length, L, of the line is such that

$$L \times 1.5 \times \frac{150{,}000^2}{6000} \times 10^{-6} > 2400 \text{ amperes,}$$

or

$$L > 426 \text{ km.}$$

Let us now compare this alternator with another of the same kv-a. output, but without leakage slots, one which also has to comply with the condition that the instantaneous short-circuit current be 3.7 times the normal current, corresponding to 27 per cent inductive

drop due to the stator leakage. As it is constructed, the leakage slots of the alternator produce 9 per cent inductive drop. Suppressing these leakage slots will give a total drop of only

$$27 - 9 = 18 \text{ per cent.}$$

It is therefore necessary that the alternator without leakage slots be designed with a sufficiently higher number of conductors to increase the reactance from 18 to 27 per cent. As the inductive drop increases as the square of the number of conductors, the numbers of conductors on the stators of the two machines should be in the ratio,

$$\sqrt{\frac{27}{18}} = 1.225$$

The flux in the second machine is reduced in the same ratio. There being imposed the supplementary condition that the magnetic circuits of both alternators be identical and all parts be submitted to the same induction, it ensues that their lengths should be in the ratio of these fluxes. Therefore, the length of the active iron of the alternator without leakage slots becomes

$$\frac{278}{1.225} = 227 \text{ cm.}$$

The copper on the periphery of the rotor of the second machine will have the same section and will then be capable of developing a magnetomotive force corresponding to 72,000 ampere-turns. Constructing the Potier's diagram for this new alternator shows that this total number of ampere-turns can be maintained only when the armature reaction is increased by 22.5 per cent by reducing the air-gap from 3 to 1.64 cm. But it is very possible that an alternator so designed will be unstable.

The no-load and short-circuit characteristics of such an alternator are traced in dotted lines on Fig. 13. Applying the above stated rule, it will be found that the self-excitation will take place if the length, L, of the line be such that

$$1.5 \times \frac{150{,}000^2}{6000} \times L \times 10^{-6} > 1140 \text{ amperes,}$$

corresponding to a line of

$$L > 202 \text{ km.}$$

This shows that with an imposed instantaneous short-circuit current of, say, 3.7 times the normal current, the alternator with leakage slots is self-exciting when the length of the line exceeds 426 km. while the one without leakage slots is self-exciting when the length of the line is only 202 km. This important advantage is obtained by a relatively slight increase of the length of the active iron from 227 to 278 cm. Let us further add that the alternator without leakage slots becomes unstable; also that in spite of an equal short-circuit current, the end connections are stressed twice as much as those of the alternators with leakage slots, and that, finally, it becomes very difficult to place the copper on the periphery of the stator.

Ventilation of 45,000-Kv-a. and 60,000-Kv-a. Alternators

The ventilating air of the rotor follows two courses: One part enters the holes in the plates supporting the fans (Fig. 1), passes into the end-connections, and escapes by the notches cut into the end-plates of the rotor, as shown in Figs. 5, 8, and 9.

The other part enters the rotor by conduits cut into the shaft (Figs. 1 and 5), thence is distributed to the air ducts, from which it passes into the air-gap and is expelled through some of the air ducts of the stator (Fig. 7), thus contributing to the cooling of the same, and is finally discharged into the frame. Therefore, the cooling air follows a radial path passing through the rotor at a rate of from 13 to 14 cubic meters per second.

We have already pointed out the importance of the leakage slots to the ventilation of the stator, as performed in the following manner. Each fan situated at the ends of the rotor delivers from 18 to 19 cubic meters per second, draws the air into the end-shields, where it penetrates between the end-connections. These latter are particularly well cooled, because of a definite spacing maintained between the involutes, permitting the air to circulate freely around them.

From the end-shields, the air enters by channels arranged in the frame (Fig. 6) and is distributed to the stator ducts. (Fig. 1). A certain amount of this air cools the core on its way across the ducts and passes into the frame; but the major part of the air penetrates between the teeth, then circulates axially in the leakage slots and follows the reverse path in the next duct, from which it also passes into the frame.

These various paths of the air and the air guide baffles may be seen in Fig. 2. It should be understood that the baffles of one type which guide the air at the intake are situated along the entire air duct and those of another type to guide the air at the exhaust are arranged all along the next air-duct.

This system of ventilation may, in a certain manner, be compared with that described by Mr. Fechheimer in his paper[7], but two essential differences exist between the two systems. First, in that of the Société Alsacienne, the air flows axially, not in the air-gap but in the leakage slots, the air-gap being, as already stated, a collector for the air of the rotor. The leakage slots are very close to the conductors and afford a much greater surface for the transmission of heat than the air-gap. The second point which distinguishes the two systems lies in the fact that in the one described by Mr. Fechheimer the air of several air ducts is discharged by a corresponding number of other neighbour-

7. *An Experimental Study of Ventilation of Turbo Alternators.* Journal of A. I. E. E., May 1924, page 416, Fig. 23.

Also see: Dr. Bratt. *The Multiple Radial System of Cooling Large Turbo Generators.* Journal of A. I. E. E., March 1924, page 185.

ing ducts, while in the system described herein, the air flowing across an air duct passes in a reverse direction in the next two. The speed of the air in the leakage slots is therefore small.

This arrangement has consequently all the advantages of the axial ventilation without the inconveniences of producing very high temperatures at the center of the machines, the cooling air being distributed uniformly along the whole length of the laminations. The heat tests are the best proof of the efficiency of this system of ventilation. One will notice the fairly uniform value of the temperature rises along the whole length of the stator read during the test No. III (Table I), where the alternator, excited at the normal voltage, was submitted to a no-load heat test.

The necessary air pressure was determined experimentally by tests carried out on the apparatus as represented in Fig. 14.

Measurement of the Temperature Rise

The heat tests to which the alternators have been subjected are also reported in Table I. One short-circuit test (No. I), one no-load test, with no-excitation, which gave the temperature rise due to the ventilating air (No. II), the test No. III already mentioned, and a number of tests at various loads and various power

TABLE I

Test No.	I	II	III	IV	V	VI	VII‡	VIII‡
Nature of Test	On short circuit	No load not excited	No Load			On Load		
Voltage V	..	..	6000	4800	5400	5350	5610	5820
Current A	4800	..	..	4450	4200	4600	4600	5000
Load Kw	..	..	..	22,500	26,500	32,500	32,000	32,800
Load Kv-a	..	..	..	37,000	39,300	42,700	44,600	50,200
Power factor	..	..	..	0.61	0.70	0.765	0.72	0.65
Exciting current A	310	..	136	415	430	432	455	545
Position and Indication of the Couples*	Increase of temperature above the temperature of the intake air, deg. cent.							
Core M	25	7	39	42.5	44	45.5	49	52
" L	18	4.8	31.5	28.5	34	29.5	31.5	34.5
" K	17	6	30.5	27.5	29.5	30	28.5	31
Teeth G	70	9	24	47.5	56.5	53.5	54.5	60.5
" D	44	10	22	38.5	38	40.5	39.5	46.5
" F	27	8	26	32.5	34.5	33.5	33.5	36.5
" E	26	8	23	30.5	30.5	32	27.5	29.5
Between Wrappings A	47	9	20.5	35.5	37.5	37.5	39.5	46.5
" B	32	10	26	32.5	31.5	†	36.5	40.5
" C	27	9	23	23.5	25.5	†	23.5	27.5
On the copper 10	40	12	26.5	33.5	32.5	33.5	31.5	37.5
In the slot 12	41	12	27.5	33.5	33.5	32.5	31.5	37.5

*The indications of the couples correspond to those given in Figs. 10 and 15.
†Couple art.
‡Tests Nos. VII and VIII were not carried out on the same group as Nos. I to VI.

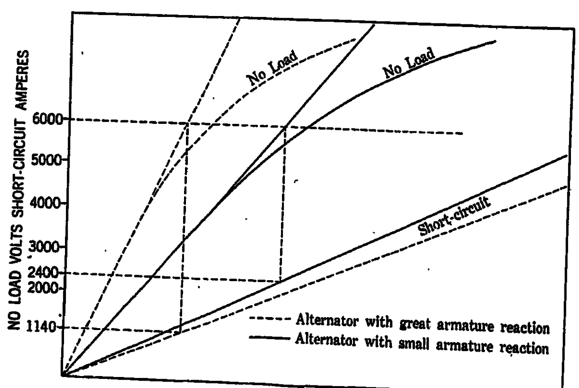


Fig. 13

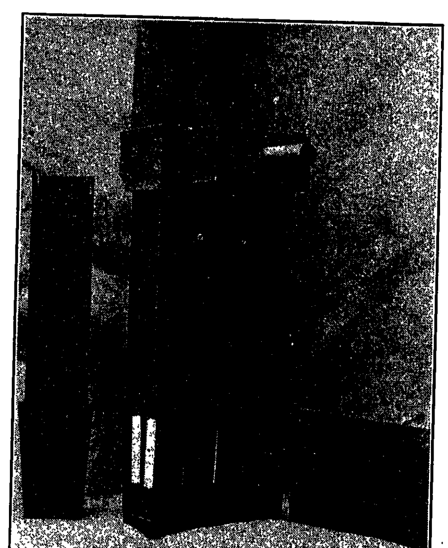


Fig. 14—Device for Measuring Pressure of Cooling Air

factors have been carried out, of which the tests No. IV to VIII have been reported.

The temperature rise of the rotor was determined by the measurement of the increase of the resistance, that of the stator by means of thermocouples. Many of these were installed throughout the stator, most of them during the construction; thus couples have been laid within the laminations in the core and in the teeth when piling up the plates. Two couples were embedded right on the copper inside the wrappings in the middle of the bars. Care was taken to connect the bars directly to neutral in order to avoid all danger during the reading.

In almost every case the couples were laid in duplicate, one serving as a spare to the other in case of breakage. The location of the couples is indicated in Figs. 10 and 15. With regard to the iron, the tables make a distinction between the couples in the core and those in the teeth, also for the measurement of the temperature of the copper by couples between the wrappings and on the bare copper.

MEASUREMENT OF THE LOSSES AND THE EFFICIENCY[8]

This article by the author contains data with regard to the efficiency of alternators at various loads. Tests have been carried out in order to measure the amount of the air by means of an anemometer and the temperature

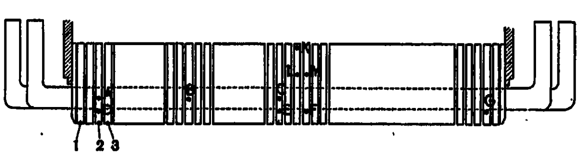


FIG. 15—ARRANGEMENT OF THERMOCOUPLES (SEE FIG. 10)

rise by means of thermo-electric couples laid at the inlet and at the exhaust of the air of the alternators. As the air circulates in a closed circuit the air chambers are not easily accessible, hence the measurement of the air speed by means of the anemometer is rather difficult. For this reason, the Société Alsacienne has elaborated the following method which has confirmed the accuracy of the first measurement. It will be an interesting matter to discuss.

Various methods have been proposed which obviate the necessity of measuring the amount of air[9]. Of these we shall mention the following: a. When it is possible to measure the losses p_0 in any running conditions (at no-load, for example), these losses are measured, as well as the temperatures t_0 at the intake and t_1 at the exhaust of the air. At the contemplated load the temperatures t_0' and t_1' of the air at the intake and the exhaust are also measured. The total losses of the machine at the considered rating will be

$$p = p_0 \frac{t_1' - t_0'}{t_1 - t_0} = p_0 \frac{\Delta t}{\Delta t_0}$$

b. It is also possible to install in the path of the air a heating resistance with which to calibrate the system. The power consumed by this resistance taking the place of the measurable losses p_0 mentioned in the first method, the total losses will be determined as indicated above.

It is obvious that these methods of measurement apply not only to the turbo alternators but also to any totally enclosed and ventilated machines. They can easily be employed on the test stands, but they often meet with certain difficulties when it is necessary, as is very often the case, to apply them to machines already erected. With particular regard to the method under the heading (b), it is easy to see that installing a heating resistance across the air channel is difficult; and it does not give the desired result because the heating is not produced at the same places as on normal running. In the method described under (a), on the contrary, the known losses, which serve for the calibration of the system, are produced within the machine, and for this reason this method should be preferred to the other one whenever applicable. However it also presents certain difficulties when it is desired to employ it for the measurement of the losses in machines already installed.

It often occurs that the no-load losses of the machine can be measured at the test stand where it is impossible to make a test under load. With these conditions, the method (a) is applicable to the machine when installed. The temperature rise corresponding to the known losses is first measured, the losses on load being deduced conversely from the temperature rise of the air. But when the machine is of such dimensions that the no-load test on the test stand is impossible, this method cannot be employed directly. This is particularly the case with the large turbo alternators. In order to measure in this case the known losses p_0 in the machine and to determine in a precise manner the temperature differences, the following method has been devised which is particularly applicable to machines the ventilation of which is performed in a closed circuit.

Calibrating the System. The known losses in an alternator may be determined by running it as a synchronous motor, but the power consumed by the latter includes not only the losses which contribute to heat the cooling air but also the friction losses in the bearings to which the losses in the driving motor (a steam turbine, for example) should be added when care has not been taken to disconnect it. However, two tests permit these constant losses to be eliminated, the alternator being run as a synchronous motor at two different voltages, the one e_2 as high and the other e_1 as low as possible. The constant temperatures being attained in each case, the temperature rises Δt_2 and Δt_1 of the air are measured, as well as the powers consumed p_2

8. E. Roth and G. Belfis. The Measurement of Losses in Totally Enclosed and Ventilated Electrical Machines, Specially the Turbo Generators. Bulletin de la Société Alsacienne de Constructions Mécaniques, No. 9, January 1915, page 20.

9. S. F. Barclay. The Determination of the Efficiency of the Turbo Alternator. The JOURNAL of the Institution of Electrical Engineers. August 1919, page 293.

and p_1. For a power $p_2 - p_1$ consumed in the machine, the temperature rise of the air would be therefore $\Delta t_2 - \Delta t_1$ and one may conclude conversely that, under the same running conditions as to rate of air flow, for which the temperature rise of the air would be Δt, the losses will be

$$p = (p_2 - p_1) \frac{\Delta t}{\Delta t_2 - \Delta t_1}$$

In order that the powers p_2 and p_1 may be accurately measured, it is advantageous to adjust the excitation to reduce to a minimum the current input.

Measuring of temperature rises. It is easy to understand that the value of the method will depend upon the

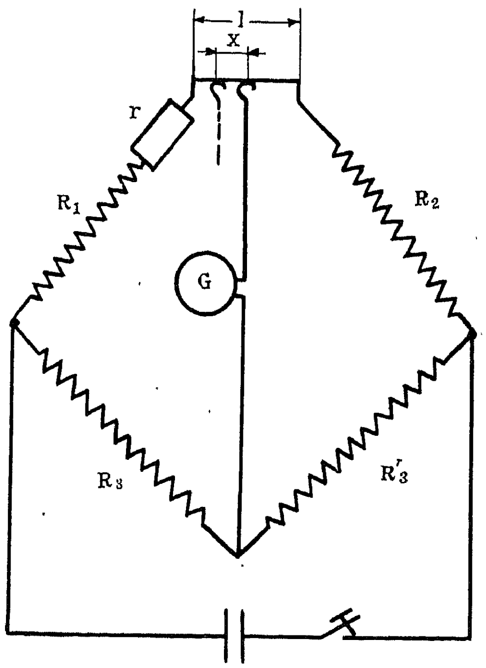


FIG. 16

accuracy with which the power and the differences of temperature, Δt, are measured. The industrial wattmeters permit of an approximation of 0.8 per cent; it will be seen later that, thanks to the test method adopted, the differences of temperature can be measured with such exactness that the accuracy of the method depends almost solely upon that of the wattmeters.

The temperatures can be measured by three different means, the thermometer, the thermoelectric couples, or the variation of the resistance of a metallic wire.

The thermometer is to be excluded from all methods of high precision; furthermore it is difficult to handle, as the apparatus for the measurement of the temperature must be installed at places highly inaccessible during the tests.

Thermocouples could be employed, but these devices also do not afford as high a precision as the method based upon the measurement of the increase of the resistance of metallic wires. It is a known fact that measurements of resistance are the most precise measurements in electrotechnics and even in physics in general. This last method has, therefore, been selected. It has the further advantage of greatly simplifying the measurements, owing to the fact that the increase of the resistance of a metallic wire in terms of the temperature

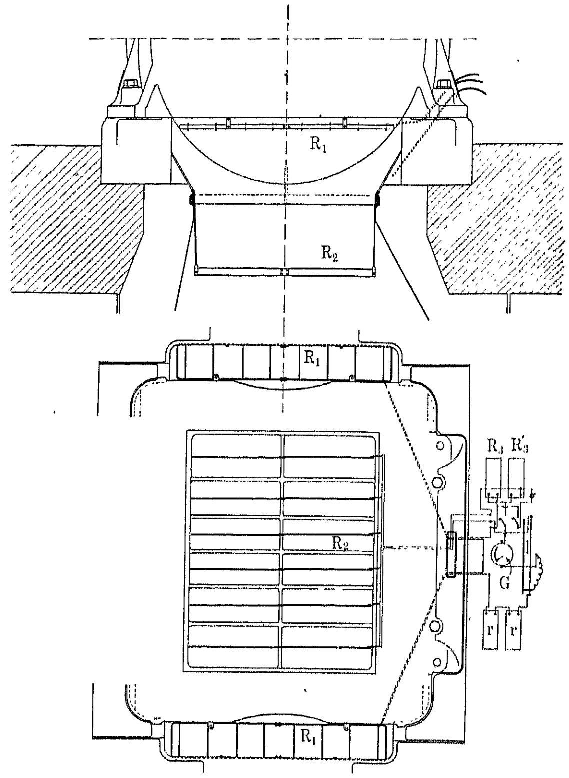


FIG. 17—ARRANGEMENT OF RESISTANCES AND APPARATUS FOR MEASURING EFFICIENCY

follows, at least in the limits that interest us, a rigorously linear law, which is not the case for the electromotive force of a thermocouple.

The variation of the resistance of a metallic wire is measured by means of a Wheatstone bridge. Direct measurements for the differences Δt between the tem-

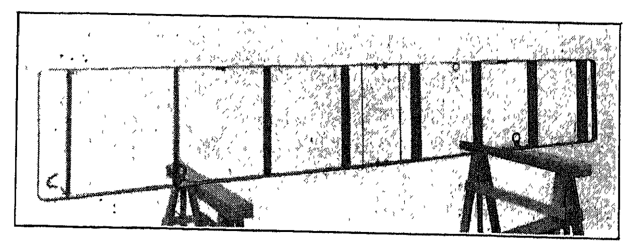


FIG. 18—ONE OF THE TWO RESISTANCES R_1 PLACED IN AIR ENTRANCE

peratures at the intake and the exhaust of the air are made, rather than measuring these temperatures separately. To this end, two resistances, R_1 and R_2, forming the two arms of a Wheatstone bridge (Fig. 16), are placed respectively at the intake, and the exhaust of the air R_1 and R_2, have equal resistances at equal temperatures.

As the temperature varies somewhat from one part of the section of the air conduit to another, the resistances R_1 and R_2 are placed so that their elements are uniformly distributed over the whole section of the conduit; the difference of the average temperature is thus directly measured (See Figs. 17 and 18).

Two other resistances, R_3 and R_4, rigorously equal, form the other two arms of the bridge. When the machine is running either as a synchronous motor or as an alternator, the values of the resistances R_1 and R_2 vary, the resistance R_2 increasing with respect to R_1. As soon as steady thermal conditions are attained the balance of the bridges is restored by means of a resistance box, r, and a wire of german silver, 1, provided with a slide which closes the circuit of the galvanometer.

The resistance, ρ, taken on the box, r, and the wire, 1, which restores the balance of the bridge, is proportional to the temperature difference desired. Designating by ρ_1 the correcting resistance for the calibrating test at the low pressure e_1, by ρ_2 that for the calibrating test at the higher pressure e_2, and by ρ the correcting resistance for the test under the considered load, there is obtained for the losses corresponding to this load:

$$p = (p_2 - p_1) \frac{\rho}{\rho_2 - \rho_1}$$

The application of this method requires, therefore, the measurement of two powers and three resistances.

Accuracy of method. It is possible to establish the values of the two resistances, R_3, and the resistance box, r, with a very high precision. The absolute values of the resistances, R_3, are of no importance. They need only be of the same order as the values of the resistances R_1 and R_2; but it is absolutely necessary that they should be rigorously equal to each other. Before being installed in the machine, the resistances, R_1 and R_2, are established with their definite connections and compared with each other. To this end, the elements being intermingled, they are placed for a sufficient length of time in closed boxes to make sure that the temperature of all elements is the same. This is ascertained by the fact that when replacing, R_1 or R_2, or conversely, the balance of the bridge is not disturbed. The accuracy of the values of the ratio

$\frac{\rho}{\rho_2 - \rho_1}$ is thus very high so that the accuracy of the method depends almost solely upon that of the wattmeters.

When making the calibration measurements it is advantageous, in order to increase the precision, to disconnect the alternator from the turbine; and this procedure should be followed every time the conditions of operation of the power plant permit it.

The possible error in the losses of one alternator, the efficiency of which is 96 per cent, or nearly so, varies from four to five per cent, with four per cent losses. The error in the efficiency therefore lies between:

$$4 \times 0.04 \text{ and } 4 \times 0.05 \text{ per cent,}$$

say, between 0.16 and 0.20 per cent.

Discussion

W. F. Dawson: The author is to be particularly congratulated on the production of a very fine machine. It has shown low heating, many novel and ingenious ideas, good mechanical engineering, and, I should say, very good efficiency. But there is one feature of the design that is quite startling to us in the United States. I know I speak for myself and for my immediate colleagues although I do not know that I speak for other manufacturers in this country.

Mr. Roth has laid particular stress on the fact that, by means of his leakage slot, he has achieved very low short-circuit current, current as a result of sudden short circuit. He has also pointed out that he has done this in preference to using very high armature reaction and a correspondingly low flux. Nevertheless, his armature reactions are proportionately higher than we would feel safe in using.

If we go back to the *Electrical Review* of London, April, 1923, we find there the saturation and impedance curves (page 646, issue of April 27th, 1923). The "saturation," 6000 volts is, expressed in field amperes, about 136 amperes. The short-circuit impedance at 4200 amperes is approximately 270 amperes. We feel that in designing alternators, particularly those to carry inductive load, the excitation required for normal ampere short-circuit impedance should not be much in excess of the excitation required at normal open-circuit voltage.

I understand that it is the practise of many European designers to allow a high short-circuit impedance, making a machine in which the full-load field excitation is three times the open-circuit excitation.

I also understand that somehow in general they are successful with it, probably because the increment of the load is small compared with the total load and perhaps, too, because of the more general use of automatic voltage regulators. I have had at least two or three glaring cases in my own practise, in which, before I realized the importance of designing for "voltage stability," the voltage would break down at or about full load. One case was in a cotton mill where one turbo alternator was carrying the entire load of the mill and, according to the calculations which I learned later how to make, the machine should have had a voltage breakdown at about 2300 kw.; it was rated at 2500 kw. I furnished a new armature which had lower armature reaction and, of course, higher flux, and the trouble disappeared.

I have had other cases into which it is not necessary to go; we have also noticed it particularly in the case of ship propulsion machines where one turbo alternator is to take care of the induction-motor load. That machine, or a group of machines, will be running along satisfactorily, but rough sea will increase the load on the motors, and the voltage falls faster than the amperes increase; hence the kv-a. is reduced and the motors break down.

I should like the author to tell whether he has had any such experience.

Philip Torchio: Mr. Roth's paper gives an excellent illustration of the progress in large turbo alternator design. Some of the special features of his machines will undoubtedly be commented upon by expert designing engineers. From an operator's standpoint, I wish to express my sincere appreciation of the remarkably low temperature rises of the copper in the stator and the rotor. The long narrow slots with an abundance of radiating area, the winding bars of the armature, with their careful assembling to eliminate eddy-current losses and a combination of axial and radial ventilation, have been used to secure results which are superior to any with which I am acquainted.

In making this statement I wish to add that I have not had the time to analyze closely how much the results are influenced by the differences in number of poles and the relative capacities of the machines compared. Mr. Roth has already mentioned how the difference in number of poles for machines of same capacities affects the total weights of such machines. Undoubtedly to some extent the number of poles imposes limitations upon the temperature rises. The employment of leakage slots may, however, be of a decided advantage in giving remarkably low temperature rises.

As to the advantage of reducing the short-circuit current, I shall be very much interested to learn of the comments of designing engineers. In our practise we have, for more than 13 years, considered it essential to the safe operation of a large system to install reactors between generators and buses so as to protect the main bus against sort-circuit failures in generator windings. In such installations, therefore, the reduction of short-circuit current on sound generators is limited by the external reactors, and designers may not find it necessary or desirable to employ leakage slots. This is a problem that should receive consideration in the discussion.

C. M. Laffoon: These generators are unique, in that certain design features are carried to rather extreme limits. The author states that the 45,000-kv-a. turbo generators, which were installed in the Gennevilliers station in 1922, were the largest four-pole turbo generators built up to that time, This statement is somewhat liable to misinterpretation because these are 50-cycle machines and the design and construction of a given size, four-pole generator, operating at 1500 rev. per min. are not so difficult as the design and construction of a four-pole, 60-cycle generator which operates at 1800 rev. per min. At the time these Gennevilliers generators were installed, they were rated at 40,000 kv-a. At the same time the Westinghouse Company was installing the first of the Hellgate generators which were rated at 43,750 kv-a., but were six-pole, 1200-rev. per min. The Westinghouse Company has since built generators of this same kv-a. rating but of four poles, 1800-rev. per min. Recent tests on these generators show that they will carry 50,000 kv-a. at 80 per cent power factor and do not exceed the standard guarantee of 60 deg. cent. temperature rise on the armature winding and 90 deg. on the field winding. The Westinghouse Company has also built 62,500-kv-a. generators for the Brooklyn Edison Company, but these were of six poles and 1200-rev. per min. At the present time a four-pole, 1800-rev. per min. 62,500-kv-a. 80 per cent power-factor turbo generator is being developed by the Westinghouse Company and 1800-rev. per. min. generators with ratings as high as 75,000 kw. and 90 per cent power factor appear feasible to build and operate.

It is to be noted that this generator is being wound for 6000 volts, which is a relatively low voltage for 60,000-kv-a. rating. In the United States the majority of central stations generate at voltages varying from 11,000 to 13,800, and with some companies there is a decided tendency to specify a final insulation test voltage of three times normal plus 1000 instead of the standard insulation test of twice normal plus 1000. However, some utility companies are considering the advisability of having generators wound for lower voltages and solidly connecting the generators to step-up transformers without any intermediate circuit breakers. In this case the final insulation test voltage would be on the order of four times normal plus 1000. On the other hand, other companies in the far west are considering 16,500-volt generators. At the present time there does not appear to be sufficient data available to determine the maximum voltage stresses due to switching, short circuits, and lightning to which the generator windings may be subjected during actual service conditions. Until these data are available, it will be difficult to determine the voltage and insulation strength which will give the greatest protection against over-voltages and still not seriously affect the cost and the reliability of operation from the standpoint of temperature.

In going over this paper the following outstanding features are of paramount importance:

1. Small physical dimensions.
2. Low short-circuit ratio, i.e., a low ratio of no-load field ampere-turns to the field ampere-turns which are required to maintain full-load sustained armature current.
3. High leakage reactance.

In comparing the physical dimensions of this 50-cycle generator with those of a four-pole, 60-cycle, 1800-rev. per. min. generator of the same kv-a. rating, it is noted that the overall length of the active iron including the ventilating ducts is considerably less than that required for the 60-cycle generator. This is partly due to the fact, as previously indicated, that, for a given rotor diameter, the design and construction of a 50-cycle generator are less difficult than they are for a 60-cycle generator of the same rating and number of poles. Since the mechanical stresses in the rotor body and retaining rings are proportional to the square of the peripheral speed, approximately 20 per cent more ampere-wires per inch of wound periphery can be obtained for a given rotor diameter for the 50-cycle generator than for the 60-cycle generator on the basis of the same number of poles. In either case, for a given short-circuit ratio, the kv-a. output is proportional to the product of the total flux, rotor ampere-turns, and rev. per min. Hence, the length of the active iron of the 50-cycle generator would be appreciably less than that of the 60-cycle generator and would be equal to that of the 60-cycle generator multiplied by the inverse ratio of the total flux in the machine. Since the iron loss varies approximately as the second power of the flux density and about as 1.2 power of the frequency, the relative total flux for the two cases will depend on the effectiveness of ventilation for each generator. An examination of the temperature data of Table I shows rather high stator-iron temperatures as compared to the temperatures on the bare copper. This indicates that the stator iron is worked at rather high magnetic induction and consequently this feature also tends toward a short machine.

From Fig. 13, the short-circuit ratio of this 60,000-kv-a. generator is approximately 0.475. This same value was obtained from the specified dimensions and design data which were given for the armature and field windings. The short-circuit ratio of a Westinghouse generator of the same rating would be on the order of 1. Similarly, the full-load field ampere-turns are approximately 3.1 times the no-load field ampere-turns, whereas, in the case of the Westinghouse generator, the ratio is about 2 to 1. This means that the armature ampere-turns are high as compared to the no-load field ampere-turns and a small change in the armature load current produces a large change in generated voltage and kw. output. Fig. 2 herewith shows the relation between armature current and voltage, armature current and kw. output, and kw. output and voltage for the 60,000-kv-a. Gennevilliers generators and for a 62,500-kv-a., 60-cycle, 1800-rev. per min. generator with the field excitation corresponding to full rated kv-a. at 80 per cent power factor. The curves for the 60,000-kv-a. Gennevilliers generators were determined from the specified design data and physical dimensions of these machines, and the no-load and short-circuit saturation curves of Fig. 1 herewith. The curves in Fig. 1. are the same as given in Fig. 7 of the paper for the 45,000-kv-a. generators but modified to suit the design constants of the 60,000 kv-a. generators. An analysis of the load curves in Fig. 2 shows the following comparison of the stability characteristics of the two generators when operating at 7 per cent power factor loads, and with a field excitation corresponding to full-load 80 per cent power factor conditions:

1. The maximum kilowatt output of the Westinghouse generator is 106 per cent, whereas that of the Gennevilliers generator is only 100.5 per cent.

2. The Westinghouse generator will deliver 100 per cent kilowatt output with an armature-current range of 100 to 138 per cent, whereas the Gennevilliers generators will deliver full kilowatt output over an armature-current range of 100 to 108 per cent only.

3. The rate of change of terminal voltage with respect to kilowatt output, at the point of 100 kw. output, is approximately five times as great for the Gennevilliers generator as for the Westinghouse generator.

This comparison shows that the Gennevilliers generators,

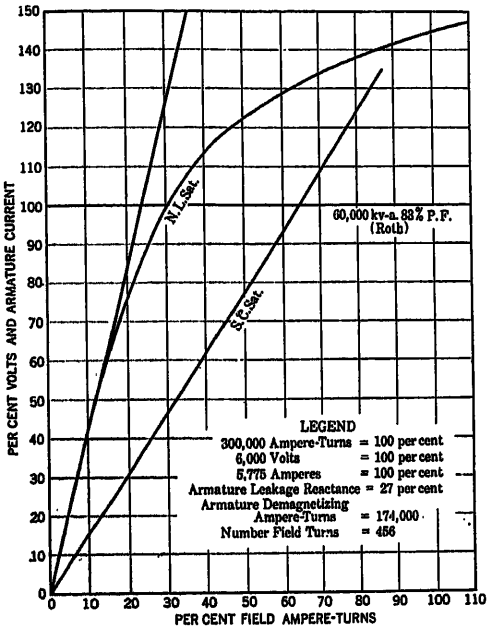


FIG. 1—NO-LOAD AND SHORT-CIRCUIT SATURATION CURVES 60,000-KV-A., 50,000-KW., 83 PER CENT POWER-FACTOR, 6000-VOLT, 50-CYCLE, 1500-REV. PER MIN., 5775-AMPERE GENNEVILLIERS GENERATORS

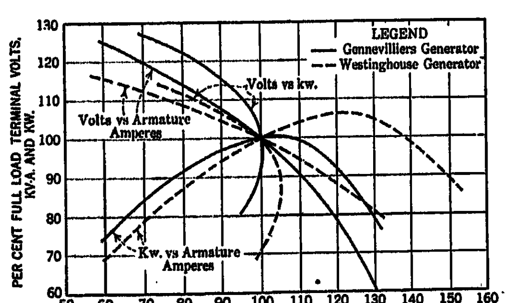


FIG. 2—LOAD CHARACTERISTICS FOR 50,000-KW. TURBO GENERATORS 80 PER CENT POWER FACTOR AND CONSTANT FIELD EXCITATION

which are designed with a low short-circuit ratio are much more sensitive to sudden changes in load than the Westinghouse generators which have a short-circuit ratio which is more than twice as large. Hence, when operating alone or in parallel with other generators which have the same characteristics, it would be necessary to provide voltage regulators in order to maintain reasonably constant voltage for rapidly changing loads on the system. The application of quick-acting voltage regulators to these machines is difficult because of the extremely wide range in field current required in going from no-load to full-load operating conditions.

If these generators were operated in parallel with other generators which have greater stability under changing load conditions, that is short-circuit ratios of one or more, it would be necessary not only to provide high-speed voltage regulators but the characteristics of the voltage regulators and of the governors on the driving turbines would have to be carefully designed to meet the particular conditions. But even then, there is always the possibility that hunting action may take place between the generators with poor and good regulating characteristics and reach such magnitudes as to cause the machines to pull out of step.

In the case of high-voltage transmission system, the design of the generators should be such that the armature ampere-turns are small as compared to the no-load field ampere-turns; that is, the short-circuit ratios should be high.

A generator designed with a low short-circuit ratio has considerably smaller dimensions than one designed for a high short-circuit ratio on account of the fact that the portion of the field ampere-turns which is used to give stability in the high short-circuit ratio machine is used to give kv-a. output in the case of the generator with the low short-circuit ratio. There is no doubt but that generators with short-circuit ratios as low as 0.475 can be operated satisfactorily with hand regulation on systems which have reasonably smooth load curves, as well as with automatic voltage regulators on systems with varying loads, provided the voltage regulators and turbine governors are designed with the necessary characteristics. However, there is no doubt that such generators require careful attention and are likely to give trouble during transient disturbances or sudden load changes. The extent to which it is desirable to reduce the short-circuit ratio, and hence increase the output for given physical dimensions depends, to a large extent, on the load characteristics of the system on which the generator is to operate, the characteristics of the generators which operate in parallel, the characteristics of the voltage regulators and turbine governors, and the amount of attention the operators give to the machine. Our own experience and observation indicate that turbo generators which have short-circuit ratios of approximate unity give satisfactory operation on the average central-station system in this country. However, this value of short-circuit ratio can and must be widely departed from in the case of generators which operate on systems or central stations in which greater or less generator stability is necessary.

The leakage reactance of the armature winding of the Gennevilliers generators is unusually high for turbo generators. A large portion of this reactance is obtained by providing leakage slots immediately above the slots for the main winding. If it is necessary to limit the initial values of short-circuit current to values comparable with those delivered by slow-speed waterwheel-type generators and the reactance must be within the generator, this is an attractive method of obtaining a high reactance. The mechanical forces on the end turns are reduced and the increase in reactance due to the air slots does not materially increase the iron loss in going from no-load to full-load conditions. However, it must be remembered that the use of these slots increases the over-all diameter of the machine, the cost, and the value of the iron losses. The increase in diameter and cost are partially offset by the fact that more room is obtained for the armature copper. An increase in diameter not only adds to the weight but also the shipping difficulties. The iron loss in the stator teeth is usually about one-half of the value of the loss in the core. The additional tooth projections increases the iron loss in the stator teeth about 60 per cent and this corresponds to an increase in the total iron loss of approximately 20 per cent. In the case of particular generator, the additional 20 per cent increase in iron loss corresponds to 0.15 to 0.20 per cent reduction in the generator efficiency.

The leakage reactance of a 60-cycle, 1800-rev. per min. West-

inghouse turbo generator of the same rating would be from 15 to 18 per cent, and the end turns of the armature winding are sufficiently well braced to withstand a three-phase short circuit at the generator terminals under no-load initial conditions and 110 per cent of normal rated voltage, as required by the 1925 A. I. E. E. Standards. So far as the initial values of the short-circuit currents are concerned, this condition corresponds very favorably with the actual conditions under full-load operation. In machines of this class the percentage of winding failures due to short circuits has been very small. In general we feel that the end turns can be sufficiently well braced to withstand short circuits that occur under usual operating conditions when the leakage reactance of the generator is 10 per cent and above. If still greater protection is desired it can be obtained by making further improvements in the bracing instead of increasing the leakage reactance by a method which involves an increase in cost and overall diameter, and a decrease in the generator efficiency of 0.15 to 0.12 per cent.

A comparison of the temperature rises obtained by detectors C, 10, and 12 show that when carrying 50,200 kv-a., the temperature rise on the bare copper at the midway axial position is only 37.5 deg. cent or 10 deg. higher than the temperature rise of detector placed between the conductor sections. Using this same temperature difference, the temperature rise of the bare copper near the ends of the machine would be 50 to 56 deg. cent. The iron temperatures vary over quite a wide range, being particularly high in the teeth at the ends of the machine and in the core at the middle of the machine. The temperature rises of the armature and field windings compare very favorably with corresponding temperatures obtained on Westinghouse generators which have a similar system of ventilation. This particular form of the multiple-path radial system of ventilation is exceptionally well worked out from the standpoint of utilizing the frame space behind the stator punchings, and gives excellent results in stator and rotor ventilation. The air requirements for the generator are approximately 105,000 cu. ft. per min. and the author states that approximately 26.5 per cent of the air passes through the rotor body. This is an unusually large percentage for the rotor and no doubt is responsible for the low temperature of the field winding.

In order to determine the losses of turbo generators when operating under normal load conditions, it is necessary to know the volume of cooling air and its temperature rise for any particular load condition. Various methods have been suggested and used for measuring the amount of air passing through the machine. When the air discharges to the atmosphere the air volume can be determined with a good degree of accuracy by measuring the velocity head at the discharge from a specially designed stack or nozzle. The discharge velocity can be made practically uniform over the entire discharge section by properly designing the stack and passing the air through fine-mesh screens as it leaves the generator. However, the most promising method is the one suggested by the author, in which the generator is operated as a synchronous motor under no-load conditions and the electrical input and final temperature rise of the cooling air are measured for two widely different values of voltage. If temperature detectors of the thermocouple or resistance type are properly placed in the inlet and outlet air ducts and due care exercised in making the measurements, the air volume can be determinded with a satisfactory degree of accuracy. Since most large turbo generators cannot be tested at the manufacture works under normal load conditions, it seems very essential that all such generator units should be so arranged that the steam end can be disconnected and the generator operated as a synchronous motor. The operating companies should have sufficient interest in the performance of the machines to be willing to cooperate with the manufacturers in making the tests.

Briefly summarizing, the 60,000-kv-a. turbo generators as described by Mr. Roth are exceedingly interesting on account of:

(1) The small physical dimensions which result from the following:

a. The generators are designed for 50 cycles at 1500 rev. per min., and consequently the stator iron can be worked at a higher magnetic induction than in a 60-cycle generator;

b. The short-circuit ratio is unusually low; that is, the portion of the field ampere-turns which is used to give stability in the case of a Westinghouse generator is used to give kv-a. output in the Gennevilliers generators; and

c. The ventilating system provides excellent cooling for both stator and rotor.

(2) Low short-circuit ratio, which results in a machine that is sensitive to load changes and is likely to be very unstable when operated on a system with rapidly changing load. The application of voltage regulators is more difficult on account of the wide range of exciting current which is required for a given load change.

(3) The high armature leakage reactance, which is obtained from the additional slots, provides short-circuit protection and the additional reactance which is secured does not materially increase the core loss under stable load conditions. However, this method of obtaining high reactance involves an increase in cost and overall diameter, and 0.15 to 0.20 per cent reduction in generator efficiency.

C. J. Fechheimer: A few points of interest in this paper will be pointed out in this discussion.

1. The scheme of stator ventilation is unique, as the circumferential system is combined with the axial system. The air which flows circumferentially does not pass through the stator teeth and into the air gap, but the air that flows axially first flows radially inward and then axially through the leakage slots.

Referring to Fig. 2, you will notice that at the back of the core through which the air passes first axially to get into the radial slots, there are eight openings distributed about. The air passes down, radially, in the vent ducts and then, in the same vent ducts, some passes around circumferentially and goes out. Some of the air goes farther radially between the coils and gets into the leakage slots and then moves axially. You see that on one side are shown one kind of guide for the air, and on the other side of the line the guides are arranged differently. Now on the right side of the line, the guides are for the air coming in, and to the left they are for the air going out; that is, the air moves axially and then out, radially, in the next vent duct, and that is the one at 45 deg. to the left of the division line. It moves axially through the leakage slot.

It would seem that the objections which were raised to the circumferential system of ventilation are believed not to apply to Mr. Roth's machine.[1] In the tests on the model for circumferential flow described in my A. I. E. E. paper in 1924, the air flowed radially through the vents and then circumferentially through the air gap. Although an analytical study of Mr. Roth's system of ventilation has not been made, the low temperatures obtained and the comparatively small dimensions indicate that the system is excellent. It would seem that the supply of air to the leakage slots is sufficient to maintain comparatively low temperatures in the tooth belt where the material is worked the hardest.

2. The method which Mr. Roth uses for measuring the losses and air volumes at full load is novel. Mr. Roth assures us that he can obtain accuracy by calibrating the system, operating idle as a synchronous motor with known total losses at two different voltages. In the equation on page 10, what is the

1. *An Experimental Study of Ventilation of Turbo Alternators.* TRANS. A. I. E. E., 1924, pp. 486-488.

order of magnitude of $\Delta t_2 - \Delta t^1$? I presume it is about 10 deg. cent., and to obtain accuracy these temperature rises must be measured with extreme care.

The measurement of the temperature rises of the air by means of resistance coils connected in a Wheatstone-bridge network is one that is frequently used, and if proper precautions are taken, it should lead to accurate measurements. With small temperature rises, it is necessary to measure resistances with extreme accuracy, as a one-degree change in temperature corresponds approximately to only 0.4 per cent change in resistance; or, if the temperature rise is 10 deg., it is necessary to read to 0.04 per cent if the error is to be not over 1 per cent. In some of our work, temperature rises of the order of only two degree are obtained, and then it is necessary to read to 0.008 per cent. While, with extreme care, such measurements of resistance can be made, there are possible sources of error, such as those due to contact resistance or those which might arise from the stretching of some of the wires, and this introduction of considerable errors.

In the early work on air-volume measurements by the thermal method at the Westinghouse Company, resistance measurements were made, but they were abandoned partly because of the necessity for great precision in measurement and partly because a few of the wires stretched and the resistance was consequently changed. We have since been using large numbers of thermocouples connected in series, and, for the most part, have been able to secure very reliable results. We can read our volumes within about 1 per cent. Of course, if only a few thermocouples are used, this method is not recommended, as a reliable average is not then obtainable.

3. The volume of air through the rotor seems to be very high. In conversation Mr. Roth explained to me the method which he used for measuring it. I think that the method is of sufficient interest to warrant his telling the members of the Institute what it is.

4. The device shown in Fig. 14, for measuring the pressure of the cooling air, is a model of part of the machine in which the flows are imitated as accurately as possible. I should inquire of Mr. Roth how close the agreement was between the pressure measured in the model and the pressure measured in the machine. I am a great believer in imitating in a model, conditions in the actual structure, but in this, care must be exercised, as sometimes the conditions are extremely difficult to imitate with sufficient accuracy.

5. Mr. Roth states that with a salient-pole alternator, the self-excitation is synchronous. It may be of interest to those who have not noted it before that it is possible to increase the load on a salient-pole alternator gradually when it is being excited by condensive reactance until, at a certain load, the alternator pulls out of step and runs at a speed slightly above the frequency of the line. In other words, in a salient-pole machine just as in a turbo alternator it is possible to have self-excitation and operate as an asynchronous generator. One of the oscillograms in the discussion referred to by Mr. Roth shows this very clearly.[2]

E. H. Freiburghouse: I agree in general with what Mr. Laffoon has said as to the principles governing the limit of electrical stability of alternators, and I, too, question whether these generators which have been described by Mr. Roth possess the necessary margin of stability.

Nevertheless, Mr. Roth has informed me that generators having these characteristics do operate successfully and that the power-station people do not find it necessary to employ voltage regulators. He states that voltage regulators have been installed but their use has not been found necessary.

Some years ago, the General Electric Company rebuilt a large foreign-made, turbine-alternator in which they even increased the originally high synchronous impedance. There have always been doubts about the stability of that generator; however, it has now been operating satisfactorily for seven years, although the excitation for synchronous impedance is 2.04 times that for normal voltage at no load. The above is an abnormal ratio which we do not advocate. Instead, we usually make the ratio less than unity.

The deep, partially closed leakage slots which Mr. Roth has employed to increase the reactance also inherently make it necessary to assemble the stator bars axially from the end of the core. Obviously, the fit of the coil cannot be so close in the slot as it would be if it were inserted radially under pressure from the air gap or open end of a slot. However, his machine is much shorter than we are building for that output.

We do not believe that 27 per cent reactance is necessary to insure the safety of the turbine-type alternator which has its stator winding laced at the ends to supporting rings. This machine as built in America is apparently flexible and strong enough to withstand many thousand dead short circuits without permanent distortion of the end structure. I recently witnessed a number of dead short circuits upon generators rated 35,300 kv-a. upon which there was no permanent distortion whatsoever.

Referring to the heating of these generators as given by Mr. Roth in Table I, we find that the temperatures obtained during the open-circuit run No. III were fairly uniform throughout the core; however, this was not the case during the short circuit test No. I. On tooth G, Fig. 15, the rise was 70 deg. cent.; on tooth F, 27 deg.; and on tooth E, 26 deg. G was in the second package of iron from the end of the core whereas the others were in the middle of the core. I venture to say that the extra heating of tooth G was caused by flux from the magnetomotive-forces of the stator winding outside of the core. We are interested to know what temperatures were obtained at the other end of the core by couples 1, 2, and 3.

W. B. Kirke: This paper brings out new methods of incorporating characteristics of high reactance without obtaining excessive armature reaction. At the same time the use of leakage slots provides an effective method to keep the temperature rise within low limits.

The first characteristic of high reactance is provided for in a great many systems in this country by the use of external reactors in the circuit connecting the generator to the bus. Such reactors not only aid in keeping the voltage near normal with a generator short circuit but they also reduce the interrupting duty on the generator circuit breakers. On the other hand, if the reactance is built into the machine by the use of leakage slots or other means, a generator short circuit means dropping the voltage of the bus section fed by that unit.

Increasing the stability of the machine is a very desirable feature. As systems are more extended and more power plants interconnected, one wonders if ever a condition will exist when the extreme ends of an interconnected system will start rocking, due to lack of stability of the interconnected system. Any measure tending to increase the stability of a single unit will aid in stabilizing a large system consisting of many such units.

The use of leakage slots would seem to indicate a considerable advantage in ventilating the unit. With the maximum size of turbo generators steadily increasing as the size of systems and interconnection facilities develop, capacities of 100,000 kv-a. will soon be required. Any method which indicates an improved means of ventilation should be given thorough analysis by designing engineers.

Robert Pohl: "The most interesting part of Mr. Roth's paper is his advocacy of leakage slots below the main stator slots. This design is for obvious reasons superior to the deep slot bridges advocated by Miles Walker many years ago, but somewhat similar in principle to the practise of some makers to use much deeper open slots than necessary for the winding. The author has convincingly demonstrated the advantages of

2. TRANS. A. I. E. E., 1920, Vol. XXXIX, p. 1637.

his design. I should like to say, however, that the properties of such leakage slots do not appear to me generally advantageous but only in special cases. In the first place, why employ so high a reactance as 27 per cent? If Mr. Roth's 60,000-kv-a. alternator had been made without leakage slots, its reactance of 18 per cent would have been fully high enough for all ordinary requirements and perhaps already too high for stations with feeder reactances and a poor power factor. In Europe we have often to deal with station power factors much below 0.8 down to 0.6. In such cases the increase of reactance makes itself seriously felt in the size and to some extent in the efficiency of the alternator.

As to the author's comparison of the two methods of obtaining high reactance, leakage slots or increased electric loading, one misses the third alternative, i.e., a separate reactor. It is obvious that an alternator with separate reactor will, as regards stability, auto-excitation, and stresses on end connections, behave exactly as a corresponding alternator with leakage slots and equal total reactance. A short circuit on the alternator terminals need hardly be feared. It seems to me that the leakage-slot design with its appreciable increase in the stator dimensions is the more expensive way of creating the desired additional reactance. If so, it can only be justified by the remaining advantage, i.e., the improvement in ventilation. This seems to me the decisive point. Here one has to distinguish between bipolar and multipolar designs. In bipolar turbo alternators the output is mostly limited by the temperature rise of the rotor. The stator winding is generally cool enough when placed in the ordinary way in close proximity to the rotor. Hence there appears to be no cause for changing this practise. The same applies to smaller four-pole machines with solid rotors. In the larger four-pole and even more so in the six-pole designs with built-up rotors the ventilation of the latter is more effective and the limiting temperature may be found in the stator winding. Here it may well be advantageous to employ an otherwise unnecessary depth of slot or leakage slots after Mr. Roth's proposal if there is no other way of improving the ventilation.

Another way of improving the cooling of altenators is the use of hydrogen or other gases superior to air as cooling media.

Franklin Punga: I congratulate Mr. Roth on his contributions to developments in the design and construction of large turbo generators, and in particular on the means by which he was able to increase to 60,000 kv-a. the rating of the 45,000-kv-a. generator without altering the external dimensions.

Regarding certain features I should like to make the following observations:

I thoroughly endorse the employment of a stator slot provided with a large open portion at the end opposite the air gap. This is very useful for ventilation purposes. In the design which I some time ago proposed to the Thyssen firm, and contrary to the design of Mr. Roth, the lower third of the slot was widened as shown in the accompanying illustration of the slot, Fig 3. The principal reason for this relates to winding construction technicalities. In the normal partly or entirely closed slot, the conductor has to be inserted axially. This makes it necessary to make a relatively great allowance for clearance between the side of the slot and the insulated conductor; otherwise the insulation on the conductor is liable to be harmed in the process of being inserted into place in the slot. Should any repair of the winding ever be necessary it is exceedingly difficult to remove such a conductor from the slot. In designs in which the conductor is inserted radially into an open slot, it is practicable to allow less clearance between the insulated conductor and the sides of the slot. This is desirable for several reasons, such as (1) obtaining a better space factor, and (2) decreasing the liability to corona phenomena. When employing my design of slot the insulated conductor is first introduced axially into the space in the lower third and is then pressed radially upward into place, the space available in the ventilating ducts being convenient for the application of the necessary radial pressure.

But this design of slot fulfils a second important purpose. It is well known that if the tooth saturation is too great, the flux passing parallel to the sides of the slot occasions a considerable copper loss. For this reason it is customary to employ a relatively low no-load tooth saturation (some 16,000 lines per sq. cm.), so that at full load the value of 20,000 lines per sq. cm. shall not be exceeded. Now the difference between the tooth saturation at full load and that at no load is chiefly dependent upon the distorting ampere-turns. The decreased tooth section in the lower third of the slot consequently serves as insurance against too great saturation in the portions where the copper is located and will be found especially valuable in turbo generators with a cos near ϕ unity.

Furthermore, the slot leakage is of course also decidedly increased, although not so much as in the case of the turbo generator designed by Mr. Roth. The advantages which Mr. Roth mentions, relating to relatively great slot leakage, are all of them correct, but a disadvantage of the great slot leakage ought not to be overlooked, namely, the radial saturation of teeth at particular parts of the circumference on the occurrence of sudden short circuits. For instance if we represent

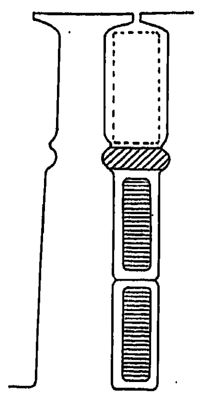


FIG. 3

the leakage of one slot by a vector, this will be of uniform magnitude and direction until we come to a place on the circumference where one phase winding is completed and the next phase winding begins. In a four-pole machine with two conductors per slot, and 100 per cent winding pitch, there are on the circumference twelve such places. The slot leakage vector moves through 60 electrical degrees and the consequence is that at these points a flux represented by a vector of equal magnitude passes through the tooth and into the laminations behind the slots. Consequently in such a machine at short circuit there would be complete saturation of twelve teeth equally distributed around the circumference. This would decrease the effectiveness of the great slot leakage. In this respect the use of fractional-pitch windings is of advantage since the number of these teeth is then increased from twelve to twenty-four so that the flux set up in each tooth is appreciably smaller.

Mr. Roth's observations about self-excitation have interested me very much. In German power houses it is now required that large polyphase generators shall be able to carry as leading load 80 per cent of their rating. This requirement has a close relation to the problem of self-excitation discussed by Mr. Roth. This problem will be of even more importance in the future if the lengths and voltages of transmission lines are increased.

In conclusion I should like to briefly mention that the principal progress which has been made in the development of

large turbo generators has been due to the avoidance of stray losses. These stray losses were due to (1) current distortion in the slot copper; (2) flux passing parallel to the slot sides and of too high tooth saturation; (3) flux passing from the field into the open slot; (4) variations in flux density around the circumference of the field, due to the slot openings; (5) the leakage flux from the end windings, (a) in the copper of the end windings, (b) in the end clamping parts (6) iron short circuits due to bad workmanship in slotting and assembling the cores.

From this paper of Mr. Roth's and from his previous publications it is very evident that the author has dedicated much study and research to the problem of decreasing the stray losses and of their exact measurement.

J. Rosen (communicated after adjournment): I would refer only to one section of Mr. Roth's paper in which he deals with the difficulty of alternator instability. It is sometimes forgotten that many of the difficulties lie in the excitation circuit. Some ten years ago, instability was experienced on an 18,000-kv-a. alternator, operating at unity power factor and sometimes with a leading power factor. The alternator air-gap was increased with entirely satisfactory results.

Later experience and tests on other plants showed that attention should also have been directed to the design of the exciter. One of the improvements adopted was the addition of a few series turns to the exciters which overcame entirely the alternator instability difficulties which had up to that time been experienced. The tests have proved that sudden changes in load, faulty synchronising, and faults in the transmission line are reflected in the alternator rotor by momentary increases in the value of the rotor current. I would refer to the papers, and the discussions upon them, on "Exciter Instability" by R. E. Doherty[3], and "Some Problems In High-Speed Alternators and their Solution"[4] by the writer, in which the whole problem is discussed in full.

Calculated figures are given by Mr. Roth of the total inherent reactance of the plant. It would be of interest to learn if they are confirmed by actual sudden-short-circuit tests at the normal operating voltage.

E. Roth: The most important observation presented in this discussion has been expressed by Messrs. W. F. Dawson, Philip Torchio, C. M. Laffoon, E. H. Freiburghouse, and W. B. Kirke, and regards the armature reaction to which American practise gives lower values than that which exists in our alternators.

Mr. Laffoon has given a very complete statement of this point and I am in general in agreement with him. His comments apply more particularly to the conditions of stability.

These machines were designed to run first at $41^2/_3$ cycles in parallel with the distributing system of Paris, and later at 50 cycles. In fact, the machines have never been operated at $41\,^2/_3$ cycles. Moreover the customer wanted a very low value of instantaneous short-circuit current, of about four times the normal current, which condition had never been required before from large turbo alternators. It was therefore necessary to design a machine in accordance with these requirements without neglecting, of course, the condition of stability.

Four years' experience has shown that these machines have been giving entire satisfaction to the users. Had difficulties arisen with regard to stability, it would have been easy to enlarge the air-gap of these machines sufficiently without exceeding the standard American temperature rise of 90 deg. cent., which up to the present has never been attained in our machines.

These machines have been designed for 35,000 kw. at 80 per cent power-factor, that is, 43,750 kv-a.; in fact, they are operated at 40,000 kw. and 50,000 to 55,000 kv-a.

Based upon this experience the 60,000-kv-a. machines have been built on the same principles but the flux has been increased slightly and the air-gap enlarged to maintain the same conditions of stability. Considering the operating conditions we are confident that they will run as satisfactorily as the former, and should difficulties arise these could be easily remedied.

As pointed out by Messrs. Torchio, Kirke, and Pohl, the low value of the instantaneous short-circuit current could have been obtained by reactance coils. This solution was examined very seriously but had to be rejected. Indeed, these reactance coils are very cumbersome, and the price of the alternator together with reactance coils has been found higher than that of the alternator with leakage slots alone. The efficiency using reactance coils is less than with the solution adopted; first, it is not possible to build reactance coils with a loss smaller than 0.15 to 0.20 per cent, and further, when external reactance coils are used the alternator has to be operated at a higher terminal voltage, which creates core losses which do not exist in the case of leakage slots.

With regard to operation, the use of reactance coils or of leakage slots leads exactly to the same result. Whenever it is desired that the voltage drop at the busses on short-circuit be not too high, individual reactance coils should be installed on every feeder.

I wish to emphasize what I have already stated in my paper, that using leakage slots in a given machine does not practically alter the conditions of stability; thus should a customer require from American manufacturers as small an instantaneous short-circuit current as in the Gennevillier's machines, they could obtain it, without changing any of the properties of their machines, by simply adding leakage slots.

I greatly appreciate the technical progress which the American manufacturers have made by designing their 62,500-kv-a. turbo alternator for 1800 r. p. m. I know very well the difficulties which they may have encountered and I think that this is a still more marked progress than that which the Socéité Alsacienne made when they built their 45,000-kv-a. alternators for 1500 rev. per min., which capacity was at that time only realized at 1200 and 1000 rev. per min. I have learned with great interest that Mr. Laffoon sees the possibility of constructing 75,000-kw. machines at 90 per cent power-factor and 1800 rev. per min. In Europe we think that it is possible to build 50-cycle turbo alternators for over 100,000 kv-a. at 1500 rev. per min. and for 40,000 kv-a. at 3000 rev. per min.

3. JOURNAL A. I. E. E., Vol. XLI, No. 10, p. 731.
4. *Journal* I. E. E., Vol. 61, No. 317, p. 452-3.

Hydrogen as a Cooling Medium for Electrical Machinery

BY EDGAR KNOWLTON[1] CHESTER W. RICE[1] E. H. FREIBURGHOUSE[1]

Members, A. I. E. E.

Synopsis.—*This paper presents the results of a large amount of theoretical study and a large number of tests to determine the advantages of hydrogen as a cooling medium in electrical machiney.*

The conclusions fall into two classes, some definite and some speculative. In the former class are:

1. For the same operating temperature a steam turbine-driven generator of a given size will have a capacity at least 30 per cent greater when operated in hydrogen than when operated in air.

2. The efficiency will be 1 per cent or more higher.

3. There will be no danger of fire destroying the insulation.

4. The detrimental effects of corona if present will be greatly reduced.

5. The machine may be protected by suitable devices from the formation of an explosive mixture of hydrogen and air.

6. The frame can be made sufficiently strong to resist an explosion. This is additional security in case of failure of the protective devices or negligence in operation.

7. The cooler for removing the heat from the hydrogen may be considerably smaller than a cooler for removing heat from air.

In the speculative class are:

1. A hydrogen pressure of several atmospheres will result in still greater possibilities provided a sufficiently gas tight enclosure can be developed.

2. The insulation may be made thinner and still have as long a life as the present insulations operated in air.

I. Introduction

SEVERAL years ago, Dr. W. R. Whitney suggested in a note to one of the writers the desirability of studying the problem of using hydrogen as a cooling medium for large electrical machinery. He pointed out that it is purely an accident that our machines are operated in air and that an atmosphere of hydrogen was much more favorable for the following reasons:

1. Less windage loss, because of the low density of hydrogen.

2. Cooler, because of high heat conductivity of hydrogen.

3. Less damage due to electrical failure, because the insulation cannot burn.

This suggestion of Dr. Whitney led to an extensive study of the characteristics of hydrogen and other gases as cooling media.[2,3,4] An investigation of the windage losses has also been reported.[5] A subsequent investigation showed that the idea of using hydrogen as a cooling medium in electrical machinery was proposed some years earlier by Max Schuler.[6]

The most desirable application of hydrogen as a cooling medium is to machines having both a relatively large windage loss and adverse cooling factors. These occur in all steam turbine-driven alternators, especially those which at present seem to be approaching the maximum desirable capacity for a given speed. The use of hydrogen as a cooling medium will in such cases allow a greater capacity for a given speed and obviate the necessity, for certain capacities, of reducing the speed or changing to methods of ventilating that are less favorably considered. For example, for an air-cooled machine of a given capacity and speed, it may be necessary to use external blowers for circulating the air, whereas if the same capacity of machine was cooled by hydrogen the fans could be placed on the rotor.

In certain cases of air-cooled machines, with fans mounted on the rotor, the power to drive the fans becomes excessive if the fans are designed to circulate the maximum quantity of air which can be obtained with the diameter and speeds chosen. It is customary in such cases to be content with a lesser quantity of air than is otherwise desirable. With a hydrogen-cooled machine a 25 or even a 50 per cent increase in power to drive the fans is of no relative importance.

II. Characteristics of Hydrogen

The important characteristics of hydrogen when compared with air as a cooling medium for large high speed electrical machines are:

A. Lower density
B. Higher thermal conductivity
C. Higher forced heat convection
D. Practically no damage to insulation by corona
E. Prevention of fire

A. Lower Density. Tests in hydrogen showed a windage loss of 10 per cent of that in air which agrees with the chemical analysis of commercial hydrogen. The total losses of large capacity alternators of 1800 or 3600 rev. per min. amount to approximately 2½ per cent and the windage loss is one or more per cent of the rated capacity. By operating such a machine in hydrogen the windage loss is practically eliminated and results in about 1 per cent increase in efficiency.

B. High Thermal Conductivity of Hydrogen. Hydrogen has approximately seven times the heat conductivity of air or as high a thermal conductivity as the

1. All of General Electric Co., Schenectady, New York.

2. Chester W. Rice: "Free and Forced Convection of Heat in Gases and Liquids," TRANS. A. I. E. E., Vol. XLII, p. 653, 1923.

3. Chester W. Rice: "Free Convection of Heat in Gases and Liquids, II," JOUR. A. I. E. E., p. 1141, Dec. 1924.

4. Chester W. Rice: "Forced Convection of Heat in Gases and Liquids, II," *Jour.* Industrial and Engineering Chemistry, Vol. 16, No. 5, p. 460, 1924.

5. Chester W. Rice: "Windage Losses in Air, Hydrogen, and Carbon Dioxide," *G. E. Review*, May, 1925.

6. Max Schuler, American Patent No. 1,453,083. Filed Oct. 25, 1916. Issued April 24, 1923.

Presented at New York Section Meeting of the A. I. E. E., October 23, 1925.

ordinary insulating materials. When a machine is operated in air any spaces in the insulation, or between the insulation and slot sides, appreciably increases the thermal drop from the copper to the surface from which the heat is to be removed by the air. With hydrogen in the machine, the gas spaces are no longer harmful in this respect because of the greater heat conductivity of the hydrogen. To illustrate the effect, consider an extreme case in which the armature insulation is 7/32 in. thick and the spaces aggregate 1/64 in. additional. The following table gives the thermal drops in both cases with an energy flow of 0.32 watts per square inch through the insulation, assuming a thermal resistance of 270 deg. cent. which is closely true for the type of insulation considered.

	Air	H_2
Drop through insulation	17.5 deg. cent.	17.5 deg. cent.
Drop through gas spaces	9.	1.25
Drop, total	26.5 deg. cent.	18.75 deg. cent.

An additional gain will be made in the thermal drop in those heat paths in the armature core and teeth which are parallel to the axis of the armature, since the spaces between the laminations will be filled with hydrogen. The ratio of the temperature drop in a hydrogen atmosphere to that in air would be about 65 per cent in this case.

C. Forced Heat Convection. The efficiency of heat removal by high velocity gas blowing over the surfaces is approximately 1.3 times as great in commercial hydrogen as in air. In other words, the surface film temperature drop required in the hydrogen machine to transfer one watt per sq. in. is 77 per cent of that required for the same machine in air.

The machine under consideration is totally enclosed, and the heat is removed from the circulating air, or hydrogen, by tube and fin surface coolers, provided with water circulation. The rate of heat transfer from hydrogen to a single tube in commercial hydrogen at the usual velocities is approximately three times as great as for the same tube in air at the same velocity. Somewhat the same ratio is expected in the case of built-up coolers, since the spacing between the tubes will be large compared with the film thickness, and the mutual influence of the tubes should be small. Accurate tests are, of course, needed to determine the exact ratio.

Effects of Lower Density and Greater Conduction and Convection. We may summarize the temperature conditions for the machine in hydrogen and air as follows:

1. The practicable absence of windage loss when hydrogen is used results in less heat to be removed from the machine.

2. The temperature drop required to transmit a given amount of heat from the copper to the iron, through the insulation, will be materially lower in hydrogen. The temperature drop required to transmit heat across the laminations will also be lower in hydrogen than in air.

3. The temperature drop required to transmit the heat from the surface to the high velocity ventilating gas will be less in hydrogen than in air.

4. When the gas reaches the cooler, less temperature difference will be required to remove the heat from hydrogen than from air, and, therefore, the hydrogen will be cooler when it returns to the machine.

The accumulative effect of all these factors is considerably lower copper and insulation temperatures, and the advantage may be used either to increase the life or the capacity of the machine as seems most desirable.

D. Corona in Air and Hydrogen. The potential gradient at which corona starts in hydrogen is approximately 60 per cent of that at which it starts in air.[7] Thus a machine designed to operate in air without corona might produce severe corona when operated in hydrogen at atmospheric pressure. The destructive nature of corona in air on insulation is well known and it was, therefore, feared that insulation troubles would soon develop, if a standard machine without corona shields were operated in hydrogen. To test this point some preliminary comparative endurance tests were made in hydrogen and air on two similar samples of yellow varnished cloth. These samples were made by winding the cloth around glass tubes approximately ¾ in. in diameter by 7 in. long, until thirteen layers were obtained. The inner electrode consisted of a ½ in. diameter copper tube which was slipped inside of the glass tube, and the outer electrode consisted of several widely spaced turns of 0.01 in. diameter cotton covered wire extending over approximately 1 in. of the central portion of the cloth. One sample was contained in a glass bell jar through which hydrogen at atmospheric pressure was slowly circulated, and the other was placed in the open air. The samples were connected in parallel across the terminals of a small testing transformer, and the voltage gradually raised. The tests were made in a moderately darkened room, and it was expected that corona would first be distinctly visible on the hydrogen sample, but this was not the case. On the contrary, needlelike purple streamers were first seen on the air sample. The corona in hydrogen was a very faint diffused glow. Probably in a completely darkened room the corona would have been visible in hydrogen before it appeared in air. The main point of interest was the strikingly different appearance of the corona in the two gases. In air there were vicious needlelike streamers coming out from spots along the wires, while in hydrogen the appearance of corona was soft and diffuse in character, and gave the impression that hydrogen corona would be harmless compared with air corona on the same sample and at the same voltage. A potential difference of

7. Wolf, *Weid. Ann.*, 37, p. 306, 1889.
Hayashi, *Ann. der Phys.* 45, p. 431, 1914.
Jensen, *Phy. Rev.*, Vol. 8, p. 433, 1916.

10 kv. at 60 cycles was applied for 76 hours. Fig. 1 shows the samples with the fine wire removed. In the air sample small holes may be seen along the imprint of the wire on the cloth. Photographs were taken of the samples when unrolled, but the variations in light reflections completely masked the effects of corona; therefore a contact print was made and this was photographed to obtain Fig. 2. The samples of cloth were about three feet long, and since a reduction to the size necessary for reproduction would render the punctures invisible, only the outer and inner layers are shown in this figure. These layers are numbered 1 and 13 respectively, No. 1 being the surface on which the wire was wound, and No. 13 the layer which was against the glass tube. In both cases the intervening layers resembled to a degree those shown.

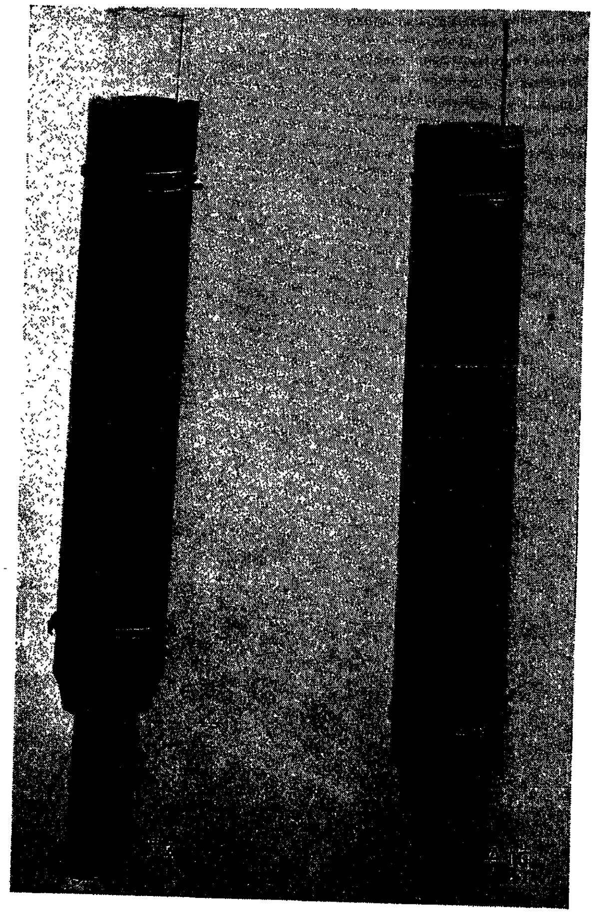


Fig. 1—Samples of Varnished Cloth Subjected to Corona in Air and Hydrogen

The sample in air was punctured through thirteen layers as shown in the figure. The hydrogen sample had no punctures. The only noticeable effect was the removal of the glaze on the varnished surface in the vicinity of the wire. This duller area was greatest on the outer layer, but was present on all layers. On the outer layer this removal of glaze extended over a considerably greater area in the hydrogen than in the air sample.

Different materials were tested in a similar manner for 56 hours. The damage in each case was not so great, but the same relative effects were obtained.

The appearance of corona in hydrogen resembles the appearance of corona in air at reduced pressure. It was, therefore, interesting to compare the relative destructiveness of corona in air at atmospheric pressure with that in air at reduced pressure. Some preliminary tests in air at one atmosphere and one-half atmosphere pressure indicate that the corona is not materially different in the two cases. In order to obtain more information on the question of whether corona destruction is due to mechanical or chemical effects, six layers of yellow varnished cloth were placed on a table and a drop of concentrated nitric acid placed in the center, with five drops of dilute acid surrounding it. An inverted beaker was placed over the drops to retain the acid fumes in contact with the cloth. After three days a visual inspection of the samples showed no difference between the part exposed to the acid fumes under the beaker and the portion exposed to the air. There was, however, a very marked reduction in mechanical strength in the area subjected to the acid fumes. A pencil point could be easily forced through all six layers anywhere within the beaker diameter,

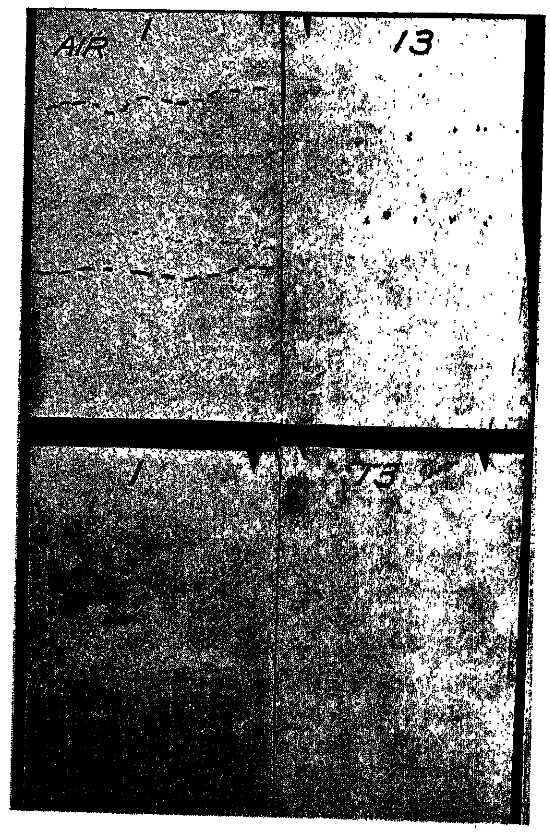


Fig. 2—Effect of Corona on Varnished Cloth in Air and Hydrogen

Two upper samples in air. Two lower samples in hydrogen. In both cases No. 1 was the surface on which wire was wound and 13 was the surface which was against the glass tube.

whereas outside of the beaker diameter the material retained its normal strength.

It may be inferred from these tests that the destructive action of air corona on insulation is due to the mechanical weakening of the insulation by the presence of nitric acid or ozone produced by the corona, followed by erosion of the corroded material by the mechanical or blast action of the corona streamers. In hydrogen, corrosion is not produced, as no chemical action takes place, and erosion is not of itself sufficiently severe to seriously impair the life of the insulation.

Following these preliminary tests, G. B. Shanklin made corona endurance tests on eight samples of armature insulation. Four were varnished cloth and four mica tape insulation, each having a thickness of 0.23 in. Horn fibre 0.015 in. thick was moulded over the insulations and over this were wound thin copper strips ½ in. wide with a similar space between the edges. Half of the samples were placed in hydrogen at atmospheric pressure and half in air and subjected to a potential of 22,000 volts which was sufficient to produce clearly visible corona at the edges of the copper strips, in a slightly darkened room.

The two cloth samples in air failed after 2500 and 2650 hours operation. The remaining samples were tested for 8000 hours without failure.

there have been cases of fire starting on the surfaces of insulation without any electrical failure. If ventilated with air, the fire may spread with great rapidity

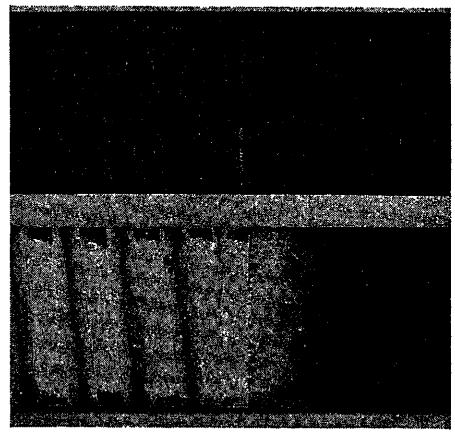


Fig. 3—Appearance of Armature Coil Insulation Subjected to Corona Endurance Tests in Air and Hydrogen

and injure the winding to the extent of requiring extensive replacement. Also, a slight electrical failure, involving initially one or two coils, may start an equally serious fire. With hydrogen no fire can occur and the

TABLE 1
RESULTS OF THE OPERATION OF ARMATURE INSULATION IN AIR AND HYDROGEN

	Mica Tape in		Cloth Tape in	
	Air	H_2	Air	H_2
	No failure		Failed	No failure
Length of Test in Hours.......	8000	8000	2500 2650	8000
Appearance See Fig. 3 with copper strip removed	Fibre bleached. Pitted through fibre and next layer of cloth tape.	Fibre retained original color. Fibre pitted only in a few places.	Fibre partly bleached. Pitted through fibre and two layers of cloth in quite a number of places.	Fibre slightly bleached. Pitted through fibre and through two layers of cloth in a few places.
Dielectric strength test. Test started at 30,000 and increased 3000 volts each minute	60,000	63,000	31,500	48,800

Table I gives a condensed statement of the results.

None of the mica tape samples showed any pitting of the mica tape. The horn fiber covering was entirely pitted through in numerous places in the air sample but examination of the mica tape disclosed no pitting. Dissecting the insulation revealed brown powder and white powder in some places between the layers of tape of the air sample but none in the hydrogen sample. Fig. 3 shows the appearance of the mica tape samples.

A visible examination of the cloth samples would not lead one to suppose that there should be so great a difference in puncture values as shown by the tests. It seems reasonable to conclude that the cloth in air was injured considerably more than the cloth in hydrogen but not to the extent indicated by the puncture values since unknown factors may also have considerable influence on puncture values.

E. Prevention of Fire. In steam turbine generators

damage due to an electrical failure will be confined to a few coils.

Most of the large capacity air-cooled steam turbine generators are provided with pipes for the injection of steam or water in case of fire. The pipes are usually located near the ends of the windings and when the steam or water is turned on the ends of the windings are enveloped or covered by the extinguishing substance as the case may be. When there is a suspicion of a fire, the operator naturally wishes to be sure that the situation is serious before a machine is taken off the line and subjected to steam or water, and the tendency is to wait so long that damage may be done either by the fire or by the amount of steam or water that is turned into the machine. With a hydrogen filled machine no such disturbing condition can exist. The only failure to be considered is an electrical short circuit or ground, and the well known automatic devices now

available should remove the machine from the line before any serious damage results.

III. Influence of Hydrogen on the Design of Insulation

A few tests were made on the puncture voltages of sheet insulating material in air and hydrogen and no appreciable difference was found in the two cases; therefore no conclusion could be reached.

In order to obtain some information on the effect of hydrogen on the life of fibrous insulation, three samples of black varnish cloth were tested for 1300 hours under the conditions given below. The effect on the flexibility is given in the last column

Sample Number	Surrounding Medium	Temperature	Flexibility
1	H_2	100 deg. cent.	100
2	Air	100 deg. cent.	25
3	Air	25 deg. cent.	90

The samples were in the form of strips supported so that the air or hydrogen had free access to both sides. This condition is much more severe than that existing in insulation wound tightly on a coil and protected by external coverings of varnish, but it gives results of value in forming conclusions as to the relative merits of the two gases in respect to the flexibility of the insulation. These tests of samples at about operating temperatures indicate that the life of insulation will be greater in hydrogen than in air as far as this life depends on brittleness of insulation.

The thickness of armature insulation for higher voltages is at present governed by the consideration of having a potential gradient that will not cause corona in minute spaces which may be present initially or developed later in the internal parts of the insulating material. The thickness necessary to withstand mechanical injury, high-potential tests, and the stresses of operation, is less than that required for the above consideration, and the removal of the more severe requirement first mentioned will allow a favorable decrease in thickness with the attendant benefits in design.

The thickness of field insulation is governed largely by the mechanical requirements of operation, but mechanical deterioration depends to a great extent on oxidation, and since this is not present in hydrogen, an increase in life may be confidently expected.

IV. Selection of the Most Suitable Hydrogen Pressure

If we neglect the question of leakage it surely would be a strange coincidence if it should be found that the most efficient pressure for hydrogen operation is at that of our surrounding atmosphere. To illustrate the properties which may be obtained in hydrogen under pressure, consider the case of ten atmospheres, or 147 lb. per sq. in. absolute.

1. The windage loss will be equal to that of the same machine in air.

2. The heat conductivity of a gas is independent of the pressure, and will therefore be the same as hydrogen at atmospheric pressure, which, as has been stated, is seven times as great as air.

3. The forced heat convection, or efficiency of heat removal by the high velocity gas flowing over the surface, will be approximately thirteen times as great as for the air machine. This condition would render the surface temperature drops negligible, whereas in the air filled machine, the surface drops are considerable.

4. The dielectric strength of hydrogen, for one-cm. spacing at ten atmospheres pressure, will be approximately 60 per cent as strong as transil oil in bulk.

Thus a machine running in hydrogen at ten atmospheres pressure would approach many of the properties of an oil immersed machine and still have the same windage loss as an air cooled machine.

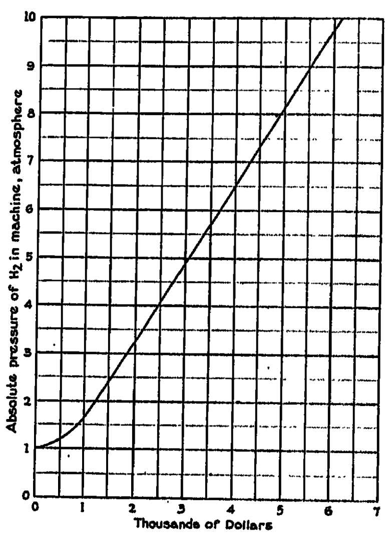


Fig. 4—Yearly Cost of Hydrogen Leakage per Pin Hole as a Function of Hydrogen Pressure

Fig. 4 shows the yearly cost of hydrogen leakage per pin hole of 0.025-cm. diameter (0.0005 sq. cm. area) as a function of the pressure of hydrogen in the machine. It will be noted that below a pressure of 1.9 atmospheres the curve is of a parabolic form and above that pressure it is a straight line. This difference is on account of the fact that the efflux of a gas is in conformity with one law up to a certain pressure and above that pressure it is in accordance with a different law, the point of transition being where the lower or outside pressure is 52.7 per cent of the higher pressure from which the gas escapes. At this point the velocity in the escaping jet is equal to the velocity of sound in the gas at atmospheric pressure and density or

$$V = \sqrt{\frac{\gamma P_0}{\rho}}$$

Where

V = velocity in cm. per sec.

γ = ratio of specific heat at constant pressure to that at constant volume

ρ = density of H_2 at the lower pressure in grams per c. c.

P_0 = External pressure in dynes per sq. cm. One atmosphere equals 1.0132×10^6 dynes per sq. cm.

At any pressure below 1.9 atmospheres, the transition point when the lower pressure is assumed to be one atmosphere, the amount of hydrogen passing through the orifice may be calculated from the following equation

$$W = 2.62\, A \sqrt{P\,\rho}\; \sqrt{\left(\frac{P_0}{P}\right)^{1.42} - \left(\frac{P_0}{P}\right)^{1.71}}$$

Where

W = gr. per sec.

A = area of orifice in sq. cm.

P = pressure of H_2 in machine in dynes per sq. cm.

ρ = density of H_2 in machine in gr. per c. c.

P_0 = external pressure in dynes per sq. cm.

For pressures above the point of transition the expression

$$W = 0.685\, A \sqrt{P\,\rho}$$

should be used.

In making calculations for the curve the area of the orifice was assumed to be as stated above, and the cost of hydrogen \$6.00 per kilogram, or approximately \$0.015 per cu. ft.

Derivations of the above equations are given by Lamb[8]. The determination of the most efficient pressure will depend on many design factors which are now unknown and therefore an estimate of the most suitable pressure, all things being considered, is impossible at the present time.

V. Tests of a Hydrogen Filled Machine

Object. Tests were made with a steam turbine type alternator operated as a synchronous motor to determine the capabilities of hydrogen as a cooling medium in a closed system of ventilation.

During the tests attention was directed primarily to the operating characteristics of the alternator which are inherent to the density, thermal conductivity, and convection of hydrogen. To emphasize these characteristics comparisons were obtained by making similar tests with air as a cooling agent.

While making these tests an opportunity was given to apply and observe the operation of protective devices, also to study some of the difficulties pertaining to the confinement of hydrogen under moderate pressures.

Description of Generator. A high-speed marine type alternator was selected for these tests, since it had fairly high windage losses, also because the outer shields which carried the armature were cast integral with the bearing supports, thus making a short closed circuit system of ventilation readily obtainable.

Ventilation System for Tests. Fig. 5 is a view of this generator arranged with closed circuit ventilating ducts and coolers for removing the losses. The gas was discharged from the top of the stator frame into a sheet iron chamber which was attached by gas tight joints to the discharge flange of the stator frame. The gas then divided and each half passed through two coolers located near each end of the chamber from which the heat was removed by water. After passing through the coolers, the gas was drawn downward from the ends of the chamber through diverging sheet iron ducts over each end of the stator frame, thence into the generator through the fans on each end of the rotor.

Coolers and Rate of Heat Transmission. Since it was the purpose of these tests to determine the relative temperature rises of the windings above that of the incoming cooling medium, little attention was given to the coolers other than to hold the water flow fairly constant.

The area of the cooling surface was 400 sq. ft., and the

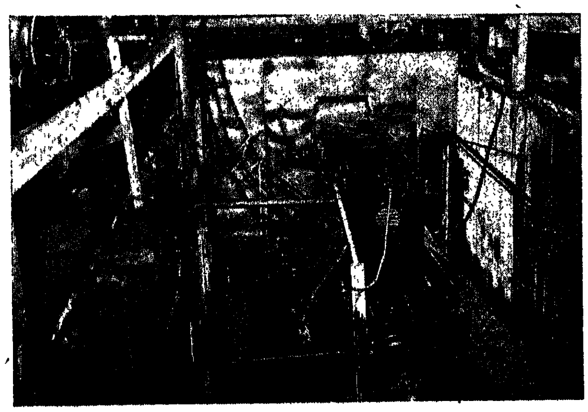


Fig. 5—Turbine Generator Arranged for Operation as a Synchronous Motor with Closed Circuit System of Ventilation Containing Hydrogen or Air as a Cooling Medium

net area of the air passages was 4 sq. ft. The velocity of water in the cooler tubes was 25, and the velocity of gas was 2100 ft. per min. Twenty-five gallons of water and 8400 cu. ft. of air were circulated per minute.

Although no accurate heat transfer tests were made on the coolers during operation it was found that they were much more effective with hydrogen than with air.

Protective Devices. Mechanical and electrical devices were used to detect explosive mixtures, automatically eliminate them, and prevent high pressures in case explosions should occur.

A small gasometer was connected to the point of lowest pressure in the machine, thereby maintaining a constant pressure slightly above that of the atmosphere, independent of temperature changes and leakage. Although leakage occurred on account of the crude joints between sheet iron and rough castings, the de-

8. Horace Lamb, Hydrodynamics, Cambridge University Press, 3rd. Edition, p. 23, 1906.

sired pressure was automatically maintained and contamination by air was prevented.

A recording device developed by C. Dansizen gave an accurate continuous log of the amount of impurity in the hydrogen. This device produced a graphic record of the difference in potential between the terminals of a pure metal filament placed inside the generator and through which a constant electrical current was automatically maintained. The resistance of the filament is a function of its temperature which depends upon the rate of heat transmission from the wire to the gaseous mixture, which, in turn, depends on the amount of impurity in the gas. With this apparatus a potential drop of 11.5 volts was observed with pure hydrogen and 12.5 volts with a mixture of hydrogen and one per cent of air impurity by volume.

Another safety device consisted of a metal filament, mounted upon a suitable support, placed within the generator and maintained at a temperature of from 700 deg. to 900 deg. cent. by means of an electric current. A mixture of air and hydrogen, having less than 15 per cent of air, will combine without explosion when brought in contact with the filament. The object of this device was to keep the air constantly being burned out of the hydrogen so that an explosive mixture could never be reached even though air should be continuously leaking into the machine.

Six diaphragms of varnished horn fiber ten inches in diameter and 15 mils thick were inserted in the outer walls of the ventilating ducts to relieve the pressure in case the other devices were ignored or prevented from functioning and an explosion occurred.

Two diaphragms were so located that the gas impinged directly on their inner surfaces. It was feared that the varying force of the gas would cause sufficient movement of the diaphragms to produce failure if these were made of fiber. Lead diaphragms were used in these locations and they were cut nearly through their thickness so that their strength compared favorably with those made of fiber. The inertia of the lead prevented any destructive flutter.

Test Results. A duplicate machine had been operated in air at a load of 3380 kv-a. and at 3000 rev. per min. The losses under these conditions were

	Kw.	Per cent of Output	Per cent of Total Losses
Windage	35	1.48	38
Open Circuit Core Loss	27.5	1.16	30
$I_2 R$	13.25	.56	15
$I_2 R$ Loss of Rotor	16.1	.68	17
Total	91.85	3.88	100

From this tabulation of losses it is seen that a loss of 1.48 per cent of the capacity of the generator is produced by windage. It may be said in general that the windage loss in large steam turbine generators will fall between 1.00 and 1.75 per cent of the full-load capacity.

Unfortunately, conditions made it necessary to run the heating tests at 2400 instead of 3000 revolutions per minute; hence the windage loss was only 51 per cent of normal. This should be kept in mind when considering the results of the tests.

Heating Tests Using Air and Hydrogen for Cooling Agents. Four tests were made, the results of which are shown by Table II. These tests were made at no

TABLE 2

RESULTS OF TESTS WITH AIR AND HYDROGEN AS THE COOLING MEDIUM

Test Number	1	2	3	4
Duration of test hours	6	6.5	7.5	8
Cooling medium	Air	H_2	H_2	Air
Input to motor kv-a.	2400	3200	2400	2107
Armature current amps	750	1000	750	657
Armature voltage	1850	1850	1850	1850
Speed rev. per min	2400	2400	2400	2400
Field current, amps	174	196.5	169.5	159.9
Field voltage at collector rings	114.2	129.6	100.5	98.7
Field input kw	19.9	25.5	17.0	18.8
$I_2 R$ loss in stator winding kw	10.9	19.5	9.9	8.0
Core loss kw	19.	19.	19.	19.
Windage, kw	17.9	1.8	1.8	17.9
Total loss in motor kw	67.7	65.8	47.7	60.7
Temperature rise of rotor winding above average temperature of the ingoing cooling gas	79.	79.2	55.	62.
Maximum temp. rise of armature winding above average ingoing gas	39.	42.5	27.	31.1
Average temperature rise of six temp. detectors between coils of armature winding	35.9	28.2	23.9	28.7
Pressure of gas on suction side of fan inches of H_2O above atmosphere	—	1.	1.3	2.4
Temp. rise of field in deg. cent. per kw. of loss	3.79	3.10	3.23	3.49

load, using the alternator as a synchronous motor, with a value of field current to obtain the desired kv-a. input. Two of the tests, No. 1 and No. 3, were taken with 2400 kv-a. input, to determine the relative temperature rises of the rotor and stator windings using air and hydrogen.

Test 4 was made to determine approximately the reduced load necessary to secure the same temperature rise with air as was obtained with hydrogen in Test 3. Being unable to predict the temperature rises which would exist at the end of the run, the final temperatures in Test 4 were somewhat higher than in Test 3. By a comparison of runs No. 1 and No. 2 it will be seen that one-third more kv-a. were carried on the motor, with the same rise in temperature of the field winding, by using hydrogen instead of air as the cooling medium. The maximum temperature rise of the stator winding, as determined by resistance temperature detectors located between the top and bottom armature coils, was only 3.5 deg. cent. higher during the hydrogen run at 3200 kv-a. than with air during the 2400-kv-a. run.

Comparing tests No. 3 and No. 4, the final temperature rise of the field winding was 62 deg. cent., with air as the cooling agent during Test No. 4 at 2107 kv-a., whereas the rise obtained by Test No. 3 at 2400 kv-a. with hydrogen was only 55 deg. cent. Based upon the data secured during Tests No. 3 and No. 4, it is estimated that a load of closely 2700 kv-a. could have been carried when using hydrogen cooling with an

average temperature rise in the field winding of 62 deg. cent. This is 1.28 times the load of 2107 kv-a. carried when air was used as the cooling agent. It is doubtful whether the temperature rise of the armature at 2700 kv-a. cooled by hydrogen would be more than 1 deg. or 2 deg. higher than the rise obtained at 2107 kv-a. when cooled by air.

The estimates of additional loads of 33 per cent and 28 per cent which have been made would be greater if the tests had been made at the normal speed of 3000 instead of 2400 rev. per min.

VI. Heat Transfer from Rotor Winding

Having determined the increased cooling and rating permissible by the use of hydrogen in the small turbine-driven generator, an estimate was desired of the relative temperature rises which would be obtained by the embedded rotor copper in a large steam turbine-driven alternator.

The temperature drops at heated surfaces cooled by forced convection of air and hydrogen were known, but no information was available by which an estimate could be made of the relative thermal drops from copper

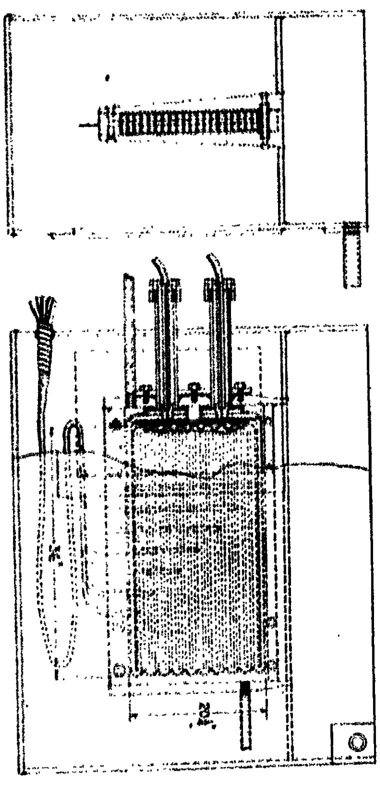


Fig. 6—Simulated Field Coil Arranged for Testing in Air and Hydrogen

through the insulation to the sides of the rotor slots.

To obtain this information, a coil having a resistance of 0.002 ohms at 25 deg. cent. was made of copper strip 0.875 in. by 0.03 in. in section, folded back and forth to obtain 20 layers in depth. The turns were separated from each other by layers of mica, and insulating armor 0.1 in. thick was moulded about the coil. This coil was actual size as regards the cross section but of course its length and shape differed from that of the coil which it represented.

The coil was enclosed in a retainer and seven thermocouples were soldered into the strips of the coil at different depths. The thermocouple leads were carried down through the coil and out of sealed holes in the steel bar at the bottom of the slot; see Fig. 6. As shown by the longitudinal and transverse sections, the coil container was welded into and hung from the edges of a

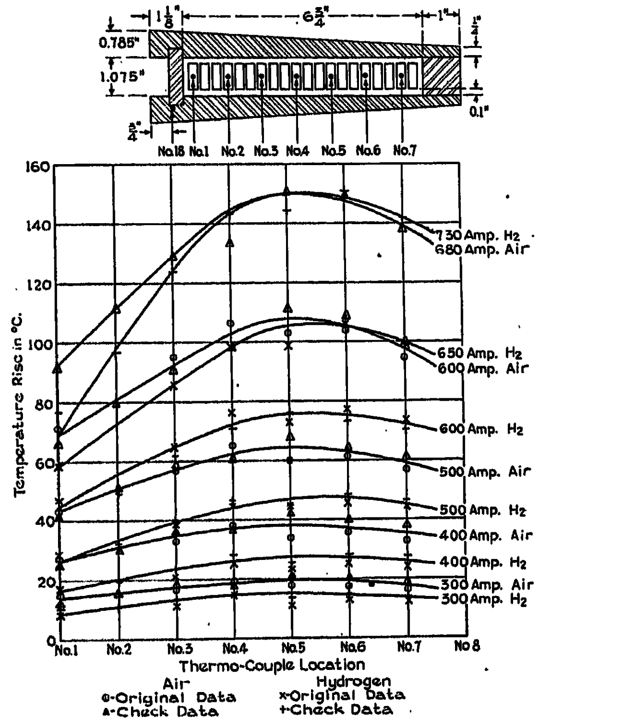


Fig. 7—Temperature Rise of Different Layers of Simulated Field Coil above Temperature at Location No. 18, when Cooled by Air and by Hydrogen

rectangular slot in a steel plate. The coil, its container, and supporting plate were mounted in a steel box. Below the supporting plate the space about the container was lightly packed with silox, a silicon oxygen carbon compound of very low thermal conductivity. This caused practically all of the heat to pass to those surfaces of the container which corresponded to the surfaces exposed at the air gap of an actual machine. The heat was removed from these surfaces by water. The gas used for impregnating the coil was admitted and discharged through the end plates of the container. Gas pressure was maintained constant at 3¼ in. of water by means of a gasometer. The hydrogen used in these tests was analyzed and found to have the following composition by volume:

Carbon dioxide	0.4
Ethylene	Nil
Oxygene	1.0
Carbon Monoxide	0.2
Hydrogen	90.5
Methane	Nil
Nitrogen	7.9
	100.0

The gas used in this test did not contain as high a percentage of hydrogen as is present in the average commercial hydrogen.

Thermometers were placed in the water in contact with the face of the teeth and on the water-cooled

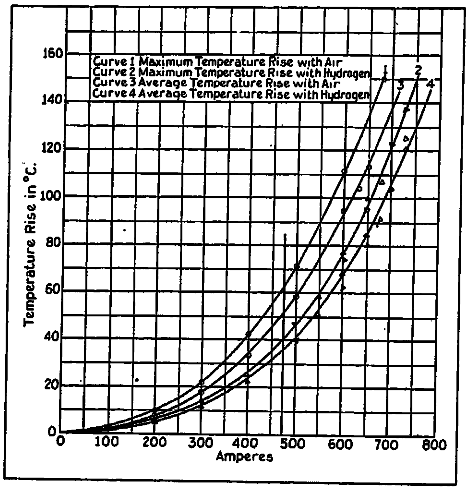


FIG. 8—TEMPERATURE RISE OF SIMULATED FIELD COIL WHEN COOLED BY AIR AND HYDROGEN

surface of the slot wedge. Three thermocouples were soldered to the bottom of ⅛-in. holes which were drilled ¾ in. below the water-cooled surface of the teeth. Their leads were protected from the water by small copper tubes soldered into the drilled holes.

The temperature rises were taken by resistance of the

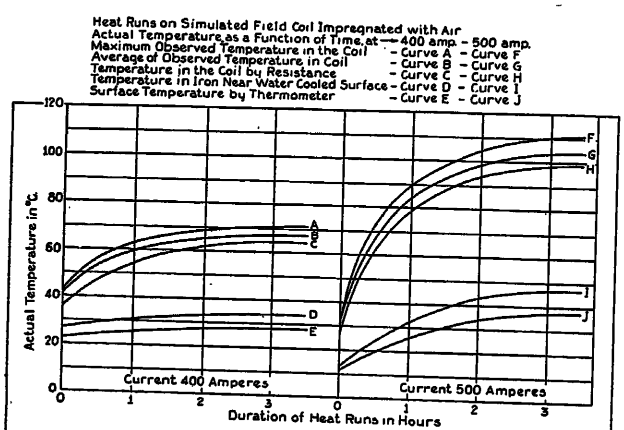


FIG. 9—RECORD OF TESTS ON SIMULATED FIELD COIL IN AIR WITH CURRENTS OF 400 AND 500 AMPERES

coil itself, and by the thermocouples in the turns of the coil. In Fig. 7 the temperature rises of the thermocouples are plotted above the temperature of No. 18 thermocouple which was embedded near the face of the tooth. From an inspection of temperatures plotted in this figure it will be seen that the bottom strips of the coil did not attain the highest temperatures. Sixteen heat runs were made at different values of current, the maximum being 730 amperes, with coil impregnated with hydrogen.

The final maximum and average temperature rises, above that of the thermocouple embedded in the tooth face, are shown in Fig. 8 as a function of current. With 475 amperes, which is normal exciting current, both the average and maximum temperatures of the

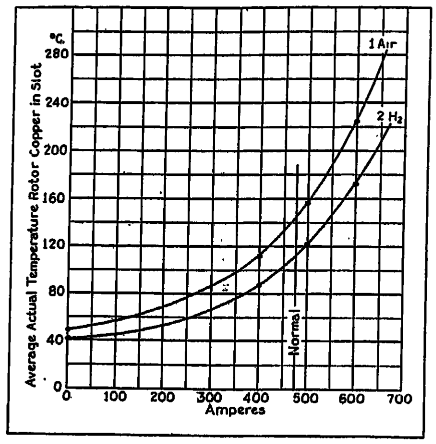


FIG. 10—ESTIMATED ACTUAL TEMPERATURES OF THE FIELD COIL OF A LARGE CAPACITY STEAM TURBINE-DRIVEN GENERATOR WHEN COOLED BY AIR AND HYDROGEN, INCOMING GAS TEMPERATURE ASSUMED TO BE 40 DEG. CENT.

coil were about 35 per cent less with hydrogen than with air.

In Fig. 9 are shown the actual temperatures during

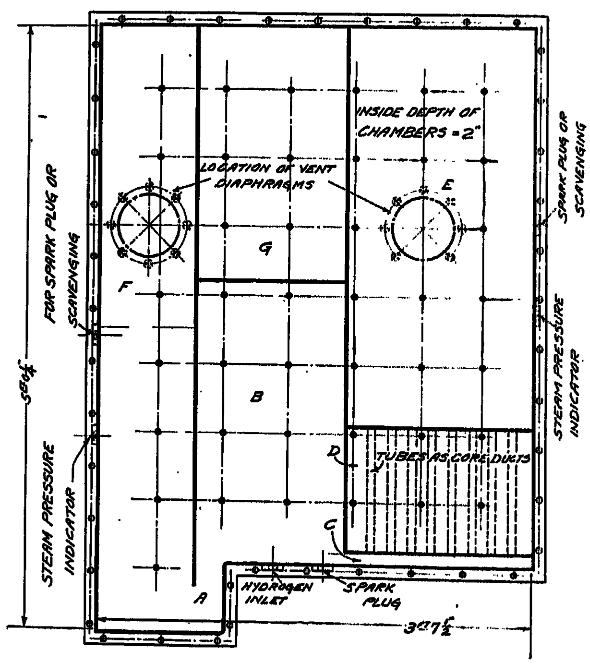


FIG. 11—MODEL OF GENERATOR CORE, FRAME, AND SHIELDS USED FOR EXPLOSION TESTS

A. Represents fan space
B. Represents end windings space
C. Represents air gap
D. Represents armature core
E. Represents armature frame space
F. Represents intake chamber
G. Represents connecting passage *E* to *F*

two heat runs at 400 and 500 amperes when the spaces were impregnated with air. Curves *D* and *I* refer to thermocouple No. 18 in Fig. 7. Curves *E* and *J* were

obtained by a thermometer located in the water close to the face of the tooth in which thermocouple No. 18 was placed. Referring to the final temperatures, the difference between D and E and between I and J are 7 deg. and 10 deg. cent. respectively.

Table III gives the final maximum and average temperature rises, watts input, and watts dissipated per square inch of coil surface during a number of the heat runs with the coil impregnated by air and hydrogen.

This data was used to estimate the field temperature rises in air and hydrogen of the machine whose field winding was simulated by the coil tested. See Fig. 10. The temperature of the medium entering the machine was assumed to be 40 deg. cent. It will be noted that at 0 amperes the temperature of the winding was 50 deg. for air and 41 deg. cent. for hydrogen cooling. These are the temperatures which would be attained by the field winding because of windage loss only. It will be seen that the temperature rises above the temperature of the medium entering the machine are 10 deg. and 1 deg., depending on which medium was assumed.

The normal exciting current was 475 amperes. Referring to Fig. 10, Curve *1*, it is seen that, at the current given, an actual temperature of 143 deg. cent. would be obtained in the embedded portion of the rotor body when cooled by air, whereas as shown by Curve *2*, the same load might be carried when cooled by hydrogen at an actual temperature of 110 deg. cent. This is a gain of 23 per cent. The armature temperature would also be reduced in approximately the same ratio.

Capitalized otherwise, the field might be operated at 143 deg. cent. with an excitation of 550 amperes when hydrogen-cooled, which is sufficient to carry a load of 25 per cent more than normal. This does not take into account the lesser temperature drop at the surface of the cooler. With the increased load and reduction in windage loss the efficiency of the machine would be 1.2 per cent higher. These values agree closely with the results of the tests on the 3380-kv-a. machine mentioned in Section V.

VII. Explosions

The question most frequently asked the authors by those interested in this subject is in regard to explosions.[9]

9. A Study of Explosions in Gaseous Mixtures, by Kratz and Rosecrans, *Bulletin* 133, Engineering Experiment Station, University of Illinois.

In order to answer this question some preliminary tests were made by E. W. Kellogg. The apparatus consisted of an iron pipe 4 in. in diameter and 24 in. long, provided with a steam-engine indicator and spark plugs. The following maximum pressure rises were observed:

Gas Mixture by Volume	Maximum pressure lb. per sq. inch gage
20 per cent H_2 in Air	53
30 " " " " "	61
50 " " " " "	54
60 " " " " "	50
65 " " " " "	50

It was impossible to ignite mixtures containing more than about 65 per cent hydrogen. On the basis of heat content, an ideal mixture of hydrogen and air should produce a temperature of 4100 deg. cent. with a pressure of about 180 lb. per sq. inch gage. That such pressures are not obtained in practise is due to dissociation and the cooling effect of the enclosure. To test the effect of an explosion on insulation, small pieces of double cotton covered wire were placed in the bomb. Examination showed the outer layer of cotton to be scorched and slightly weakened. When the pressure was relieved by a paper vent over the end of the bomb, no scorching was detected. It may be concluded from these tests that an explosion in a machine would have no detrimental effect on the insulation. To determine the feasibility of limiting the explosive pressure in a machine to moderate values by the use of suitably placed vents, the model shown in Fig. 11 was constructed. This represented as closely as convenient the various parts of the generator. The parts were full size except that the model was 2 in. thick in a direction normal to the surface of the page.

The various spaces simulated were:

A—fan space
B—space around ends of winding
C—air gap
D—armature core and ducts
E—chamber encircling armature core
F—intake chamber
G—connecting passage from *E* to *F*

Pressures were measured by a steam-engine indicator started automatically just before the ignition of the gas. In general, the thicker the diaphragm the greater was the pressure developed by the explosion. Volumes of air and hydrogen as nearly equal as possible were used, but in spite of the fact that the explosive range is considerably beyond this point in both direc-

TABLE 3
TEMPERATURE DATA ON FIELD COIL

Gas in Coil	H_2	Air	H_2	Air	H_2	Air	H_2	Air	H_2	Air	H_2
Amperes	200	200	400	400	500	500	600	600	650	650	730
Watts input	81.2	82.0	360.0	377.6	612.5	647.5	972.0	1054.8	1200.0	1280.2	1636.0
Watts per sq. in. of coil surface	0.142	0.143	0.626	0.657	1.067	1.128	1.693	1.835	2.090	2.230	2.845
Max. temp. rise above thermocouple in Tooth face	5.9	8.4	25.6	37.9	46.2	64.7	77.3	106.4	100.1	126.1	138.5°C
Average temp. rise above thermocouple in Tooth face	4.6	7.3	22.5	33.4	40.9	57.7	68.7	95.8	85:3	113.2	120.8°C

tions there were several times that the charge failed to explode, doubtless due to incomplete mixing. Twenty-five explosions were made and the highest recorded pressure (45 lb. per sq. in.) occurred when the venting diaphragms were replaced by steel plates.

There is no more reason to fear the explosion of a machine filled with hydrogen than that of other apparatus having considerable stored energy such as boilers, gas tanks, rotating elements, high-speed trains and automobiles, and various other familiar examples. Most of us fear a new condition if it has the possibility of accident, and accept as a matter of course the usual dangers of our daily lives.

VIII. Conclusions

To summarize previous statements, the advantages which may be realized in various proportions are: lower temperatures, greater capacity, lower losses, elimination of fire hazard, longer life, and greater reliability.

There seems to be no great difficulty or danger in the use of hydrogen in a properly designed and operated machine.

The automatic maintenance of hydrogen pressure slightly above atmosphere will prevent the ingress of air and the possibility of the explosive mixtures.

Several devices, each operating on a different principle, may be used to detect a change in the mixture and give an alarm long before the mixture reaches an explosive stage.

Finally, should all of these contrivances fail to operate and a spark be applied to an explosive mixture inside the machine the results would not be serious, since it is not difficult to vent the machine to reduce the force below a destructive value.

Grateful acknowledgment is made to Dr. W. R. Whitney and Dr. H. G. Reist for the keen interest which they have taken in the progress of the work.

Discussion

G. E. Luke: In determining the probable temperature rise of any electric machine, we may first start with the calculation of the volume of fluid necessary to be circulated through the machine in a given time.

Thus, comparing air and hydrogen as possible fluids, the volume of hydrogen required will be only 2 or 3 per cent more for hydrogen than for air, on the basis of the same temperature rise of the gas due to the same loss absorbed. This temperature rise in the gas used should not be over one-half of the maximum temperature rise of the ventilating surfaces.

The ventilating-surface temperature can be calculated when the surface-heat-transfer coefficient (K) is known. Considerable data regarding this constant (K) are available for air[1]. However, little experimental data are published for hydrogen. Some experimental tests by Rice[2] on a small cylinder (axial flow) gave a heat transfer for hydrogen 137 per cent of that for air at an average velocity of 5000 ft. per min. On the other hand Rice[3] gives for large plane surfaces this heat transfer (K) as proportional to $\frac{\rho\, k}{\mu}$, where ρ, k, and μ are the density, thermal conductivity, and viscosity of the gas, respectively. On this basis, the unit heat transfer from the surface will be about the same for hydrogen as for air.

Pohl[4] has calculated from Nusselt's[5] work this coefficient (K) to be about 50 per cent greater for hydrogen than for air. The equation given by Nusselt can be reduced to the form[6] of

$$K \,\alpha\, k^{n-1} \quad (\rho\, C_p\, V)^n$$

where

K = unit surface heat transfer
k = thermal conductivity of the gas
ρ = density of gas
C_p = specific heat of gas
V = velocity of gas

This equation is also practically the same as that given by Pohl. Thus, the ratio of the heat transfer with hydrogen to that obtained with air will depend upon the exponent (n) of the velocity factor. Thus, with (n) = 1, the ratio is about 98 per cent and with n = 0.786 as given by Nusselt the ratio becomes about 150 per cent. The writer has found that for turbulent gas flow where the gas path is straight and uniform, the heat loss varies as $V^{0.85}$ to $V^{0.95}$; and where the air path is irregular the unit heat loss varies as $V^{0.75}$ to $V^{0.85}$. Hence, it is estimated that in an average generator the unit heat dissipated with hydrogen will be about 25 per cent greater than with air, which is not far from the figure given in the paper.

The paper states that the rate of heat transfer in the coolers should be about three times greater for hydrogen than for air. This figure seems too high; it might be correct for the tubular surface but is probably too high or the fin surface, and this latter surface is usually several times greater than the tubular surface. The writer estimates the heat transfer in the usual type of cooler to be 50 to 100 per cent greater for hydrogen than for air. This, of course, is a big factor since the cost and size of the present cooler can be reduced.

The majority of the iron and copper losses in the core have to flow an appreciable distance through the iron to the ventilating surface. This necessarily requires a temperature drop. Where radial ventilation is used, the heat flow is mainly across the laminations, in which direction the heat flow has a high-resistance path due to the varnish and gas film between the laminations.

With one-watt flow across a 1-sq. in. section of the usual 0.017-in. varnished iron laminations, the temperature drops are approximately:

Deg. cent. drop through

Iron	Varnish	Gas		Total
1.0	11.0	(Air)	21.0	33 deg. cent.
1.0	11.0	(Hydrogen)	3.0	15 deg. cent.

Hence the rate of heat flow across the laminations should be at least doubled by using hydrogen as the cooling medium.

One of the most important portions of the heat-flow path is from the copper through the insulation. This part has more influence in limiting the rating than any other since the thermal conductivity of insulating materials is so low. The thermal resistance through ordinary insulation in air is about 3000 times greater than that through copper. All insulation in the built-up

1. See *Cooling of Electric Machines*, by G. E. Luke, Trans. A. I. E. E., 1923, p. 635-636.

2. See *Free and Forced Convection of Heat in Gases and Liquids*, Trans. A. I. E. E., 1923, p. 653.

3. Forced Convection of Heat in Gases and Liquids, Eng. & Ind. Chem., May, 1924.

4. "Fundamentals of Heating Calc.," by R. Pohl, *Arch. f. Electro*, June 30, 1923.

5. "Heat Transmission in Conduits," by W. Nusselt, Z. V. d. I., October 23, 1909.

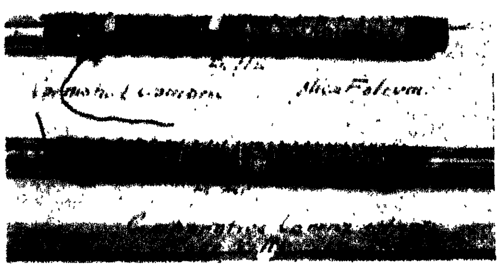
6. This was done on the basis of (K) being independent of the temperature of the gas and surface, which is practically true of the range in which we are interested.

wrapper will necessarily contain some small gas spaces; hence these gas spaces, if they are air, will offer considerable resistance to the heat flow, since the thermal resistance of air is about ten times that of the insulation itself.

To check the above, the Westinghouse Research Laboratory has made tests on the thermal conductivity of the insulation on turbo armature coils. The results show that the thermal conductivity of these mica insulations as used is from 150 per cent to 250 per cent as great with the coils in hydrogen as obtained on the same coils in air. The particular ratio depends upon the compactness of the insulation, that is, the percentage gas space in the wrapper. This increase in heat flow with the hydrogen cooling system will result in a considerable reduction of the conventional "hot spot."

The temperature of the rotor copper is usually the limiting temperature, with air as the cooling fluid. It will also tend to be the limiting factor when hydrogen is used, due to space limitations and to appreciable temperature drops through the solid iron core which will be unaffected by the gas used for cooling.

In the tests given by the writers, the apparatus simulated conditions found in a solid rotor where there would be a considerable temperature drop through the iron. However, in a ventilated rotor, even better results could be expected, since this drop through the iron would be reduced.



COMPARATIVE CORONA EFFECTS UNDER 15,000 VOLTS FOR 19 DAYS.

As to the insulation, the writer also agrees with the authors that its life would be materially increased in a hydrogen atmosphere. Oxygen in the air is the main factor which causes mechanical deterioration. Dr. C. F. Hill made tests regarding the corona action upon insulation in air and in hydrogen. The results were even better than those quoted in the paper in favor of hydrogen. Two kinds of insulation were tested, one varnished cambric, the other a mica wrapper. Both were wound on a glass tube and 15,000 volts a-c., 60 cycles, was applied for 19 days. Most of the stress was through the glass tube but a heavy corona could be seen covering the insulation. At the end of the test, the sample in hydrogen was unchanged, while the one in air was radically altered. The varnished cambric was bleached and was very brittle; the paper in the mica wrapper was completely destroyed. The hydrogen prevented the chemical action found on the sample in air.

As to the possibility of explosive mixtures with hydrogen, the average of nine investigations[7] gives the explosion limits as 7.9 to 69.4 per cent hydrogen in a hydrogen and air mixture.

An indicating or recording instrument for giving the purity of the mixture can be easily obtained by using the conductivity-cell bridge method[8]. This is exceedingly accurate and is well suited for such purposes.

Gases other than hydrogen can be used for a cooling medium. Thus helium (if made available in the future) is an inert gas with a density about $^1/_7$ of that of air. Its specific thermal capacity is about 73 per cent of that of air and its thermal conductivity is almost as high as that of hydrogen. Such a gas would be preferable to the operating men.

L. B. Bonnett: From the user's point of view, there are some very striking things in this Table II showing the results of the tests on the small 3000-kv-a. machine. If we can expect to get a one-third increase in capacity out of the same material, presumably at approximately the same price, we are getting something that is very interesting indeed. At that increased rating the total loss is practically the same, in fact, it has slightly decreased.

Looking at it from a little different point of view, many of us use stand-by machines that are in operation ready to take load, and the no-load losses are a very important factor. If the use of hydrogen can reduce those losses by, say, 50 per cent, that indeed is a very great advantage for this particular duty.

This light-load loss, too, has another rather interesting application. Turbo generators are commonly equipped with closed ventilation and air coolers and very frequently condensate is used for cooling. Since the light-load losses are usually more than half the full-load losses, the condensate at light loads has to be recirculated in a more or less complicated fashion or some other means of cooling supplied. This very great reduction in the fixed losses would mean that the losses would decrease more nearly in proportion to the load and the condensate itself might be perfectly adequate for cooling the machine all the way down.

With all the advantages mentioned in this paper—a really astonishing catalog of advantages—it behooves us, the users, not to be too sure that the one disadvantage, the possibility of an explosion, is an insurmountable defect. I believe our serious consideration is well worth while.

W. B. Kirke: It is hoped that further investigation on the life of insulating material when operated in hydrogen as compared to operation in air can be made. It might be found quite practical to operate at higher temperature limits in hydrogen than have been standardized for operation in air. It is also to be hoped that this paper will be supplemented by others which will indicate the installation cost of such a ventilating system and some idea of the equipment necessary for its operation.

C. J. Fechheimer: A few years ago the only media for cooling considered were air oil, and water. The use of oil or water has never met with favor in this country, even though certain important advantages could be secured by their adoption. It seems that electrical engineers were not aware until a few years ago that the gains to be obtained by means of some other gas were ufficient to warrant employing it instead of air. Even after the suggestion of the use of a lighter gas was offered to the designing engineers, they did not immediately consider its adoption. The gas proposed in the Schüler patent is hydrogen, and the first thought that entered the mind of the engineer was the danger of explosion. It was not until he learned that detonation will not occur if hydrogen constitutes more than about 70 per cent of a mixture with air that he felt that possible gains were great enough to warrant investigation. We now have records in this paper of the studies and researches given by three engineers of one of the leading electrical manufacturing companies on this subject. It is the first public presentation to a group of engineers of a systematic study of this advance which it is believed will considerably modify the design and construction of large electrical machines in the future.

Of the various gains to be obtained by means of hydrogen, there are two of prime importance. The first is the enormous reduction in windage loss due to the low density, and the other is the decrease in thermal drop through the insulation. In the large high-speed steam turbo alternator the windage is the greatest loss and may be as high as 50 per cent of the total. By substituting hydrogen for air this loss becomes almost negligible. In addition to the gain in efficiency, the temperature rise due to the windage becomes insignificant, whereas in the present day machine it is from 5 to 10 deg. cent.

It has been recognized for a number of years that the tiny

7. See article by C. J. Rodman, *Elec. World*, 6-24-22.

8. Thermal Conductivity for Analysis of Gases. Technical Paper, Bureau of Standards, No. 249.

voids in insulation reduce the net thermal conductivity of the wrapper to about 50 to 75 per cent of that which would obtain if the wrapper were solid throughout. Now we find that because hydrogen diffuses so readily hydrogen will supplant the air and the resistance to heat flow in the voids will be decreased to about one-seventh, and the net conductivity will be greatly improved. The authors find 30 to 58 per cent improvement in net conductivity for the field core, and they estimate about 42 per cent gain for the armature coil. Also, because the thermal conductivity is high, the transverse drop through a package of laminations is reduced. and the drop from the surface is decreased. So the authors find that as a result of all the gains, a certain turbo alternator can be rated about 30 per cent higher by substituting hydrogen for air. But that is not the final word; to take full advantage of the properties of hydrogen, the machine should be proportioned differently. For example, the velocities of the gas in the vent ducts can be increased, and the lamination-package thickness can be enlarged. Owing to the reduction in total losses, the volume per unit time of the gas may be lowered.

Also, as the authors state, there is a likelihood of reducing thickness of insulation wall in the stators when mechanical considerations do not enter. At present it is difficult to state how much the weight, cost and size of the generator may be decreased if full advantage of all gains is taken in the design. But certainly the cost will be reduced considerably.

There are a few points which are not covered in the paper. Two will be mentioned. In very large machines as designed at present, it is not feasible to evacuate in order to replace the air by hydrogen or vice versa, as the stresses arising from atmospheric pressure are prohibitive. While it is possible so to proportion those parts as to prevent collapse while evacuating, it is believed that an alternative plan which will maintain all parts at or near atmospheric pressure should be entirely satisfactory. The plan is to replace the air by an inert gas, such as nitrogen, and then to replace this inert gas by hydrogen. Tests are now being conducted for determining how satisfactorily this can be done.

Another feature is that to minimize leakage, suitable stuffing boxes should be provided where the shaft passes through the openings in the end bells. It seems at present that this is the most difficult part of the problem. Experimental work is now under way on a water-gland seal, and with this device it is believed that the leakage will be negligible. Ample precautions are being taken to avoid the escape of water into the generator.

The authors have used the thermal-conductivity method for determining the extent to which the hydrogen is contaminated. Another method consists of a small fan driven at constant speed, the inlet and outlet of which join into the system. The pressure which the fan generates is directly proportional to the density of the gas, and the pressure difference between the inlet and outlet can readily be indicated on an ordinary manometer. The relation between the percentage of hydrogen and the reading on the manometer is linear, assuming that air is the contaminating gas. The authors state that with the thermal conductivity method, 1 per cent impurity will change the potential drop from 11.5 to 12.5 volts or 8.7 per cent. With the density method, the same change in constituency will alter the manometer reading 13.3 per cent. Thus, there is greater sensitivity, and it is believed that the device is more direct and simpler than the thermal method. The density method can be used to operate a signal, or possibly to operate switches automatically.

While further experimental work must be done prior to the building of machines for service, the outlook is very bright, and it is believed that the time is not far distant when machines using hydrogen will be in operation.

J. Rosen (communicated after adjournment): The authors' investigations into the difficulties of ventilation of electrical machinery will be welcome as being of theoretical interest, more particularly as they have some bearing upon the conditions for generating at higher voltages than have been customary in the past. I do not think, however, that the use of hydrogen can be considered practicable at the present moment. It has the drawback of increased cost and complication in design. Further, I do not think that the danger of forming an explosive mixture can be altogether avoided. The closed-circuit system of ventilation has now been generally adopted for large alternators. The advantage with the use of hydrogen in avoiding the danger of fire also applies to the closed-circuit system using air, as, with the limited amount of air in the latter, the damage that can be done by fire is limited. I illustrate this by the following example:

The volume of air in the alternator and ducts of the closed-circuit system of a 25,000-kv-a. alternator at 3000 rev. per min. is approximately 2000 cu. ft. containing 40 lb. of oxygen. This quantity of oxygen could consume 15 lb. of carbon or 40 lb. of wood, but as the principal product of combustion is carbonic acid gas, and a flame is extinguished when only 4 per cent of carbonic-acid gas is present, the amount of wood consumed would only be about 2 lb. The total weight of combustible material in the alternator, including wood packing and insulation, exceeds 1000 lb. It is obvious therefore that the fraction of material that would be damaged or consumed by fire would be negligible.

To reduce the losses in the fans attached to the rotor body, I prefer to use separately driven fans, and to adopt a suitable system of ventilation to reduce the pressure drop through the alternator to a minimum. By this means, an improvement in efficiency of one per cent can be obtained. In the ventilation scheme described in the paper[9], "Some Problems in High-Speed Alternators and their Solution," the air-pressure drop through the alternator is reduced to approximately 3-in. water gage.

Robert Pohl: The valuable research which Messrs. Knowlton, Rice and Freiburghouse publish on this subject might with advantage be extended to the use of methane. In a paper published in 1923 (*Archiv. F. Elect.*, June 30, 1923, p. 361) I defined what one might term the cooling constant of various gases and showed that this constant is even higher for methane than for hydrogen. Since methane is also cheaply obtained as a by-product, its use may well be considered. Although the risk of explosion is not serious in any case the much smaller area of "exploibility" would be a practical advantage.

E. H. Freiburghouse: From the discussions it is evident that other engineers have also been giving active consideration to the subject and almost all of them seem very optimistic for the future use of hydrogen as a cooling medium for electrical machinery.

Opinions and data which have geen given seem, in the main, to agree quite well with those given in the paper. Although the points raised by Mr. Fechheimer were not covered in the paper, they have been carried out or considered during the investigation by the authors.

Mr. Fechheimer mentions the use of an inert gas, such as nitrogen, for replacing the air and hydrogen in the generator before and after the installation of the hydrogen. Nitrogen was used for this purpose during the heat tests which were made upon the 3380-kv-a. generator.

It was realized from the beginning that to prevent the leakage of hydrogen between the rotating and stationary parts of the generator was the most difficult and expensive problem to solve. Two different types of seals have been developed each of which reduces the leakage to a negligible value. In the liquid seal it is thought that oil is preferable to water.

A small fan driven at constant speed was used in some of our earlier investigations to determine the density of the gas mixture and, as Mr. Fechheimer states, it has several attractive features.

If hydrogen is used as a cooling medium, the authors believe that Mr. Rosen will agree that fans upon the rotor of the generator are preferable to separately driven fans. We believe that the pressure drop of the air through the generator should greatly exceed the 3 in. of water which Mr. Rosen mentions, if the necessary velocity of the air is obtained for a high value of unit-surface heat transfer.

9. *Journal I. E. E.*, Vol. 61, No. 317, p. 447-8.

Stored Mechanical Energy in Transmission Systems

BY J. P. JOLLYMAN[1]

Associate, A. I. E. E.

Synopsis.—The paper considers the performance of the stored mechanical energy in the moving masses connected synchronously to a transmission system during changes in load, changes in input, and changes of transmission capacity,

The stored mechanical energy greatly affects the performance of a transmission system during sudden changes in transmission line capacity occasioned by switching out or in a parallel circuit. Such switching operations lead to oscillations of input due to the interaction of stored mechanical energy and the altered difference in phase between generated and received voltages over a transmission line.

* * * * *

THIS paper will consider the performance of stored mechanical energy in a transmission system during changes in load, in input, and in transmission capacity.

Typical Transmission System

For this paper's purpose, a typical transmission system will be considered to consist of two or more generating plants connected to a network feeding load centers by double circuit lines of considerable length. Some generating capacity may exist near the load centers, but its existence or absence has little effect on the subject under consideration.

Mechanical energy is stored in every moving mass connected to a transmission system. The most important of these masses are those revolving elements the speed of which fluctuates with changes in system frequency. The rotors of synchronous machines, such as generators, synchronous motors, rotary converters and synchronous condensers, are of greater importance than the rotors of induction motors or the revolving parts of machinery driven by induction motors, because the synchronous machines must follow the system frequency precisely while induction motors have some slip. The stored energy in devices such as street cars connected to a transmission system in a non-regenerative manner does not influence the performance of the stored energy of the system.

Typical Flywheel Effects

The greatest flywheel effects per kv-a. of capacity will usually be found in the rotors of large generators. Typical flywheel effects are:

Large hydro unit – 27,000 kv-a., 225 rev. per min. $W R^2 = 5{,}000{,}000$.

Stored energy = 43,031,250 ft-lb. = 16.2 kw-hr. = 1590 ft-lb. per kv-a.

Small high speed hydro unit – 7500 kv-a., 514 rev. per min. $W R^2 = 200{,}000$.

Stored energy = 8,976,000 ft-lb. = 3.37 kw-hr. = 1195 ft-lb. per kv-a.

Synchronous condenser – 20,000 kv-a., 600 rev. per min. $W R^2 = 500{,}000$.

Stored energy = 30,600,000 ft-lb. = 11.5 kw-hr. = 1530 ft-lb. per kv-a.

Total Flywheel Effect of System

The total flywheel effect of a large transmission system is difficult to estimate but may be of the order of 2000 ft-lb. per kv-a. of connected generator capacity. Assuming 1.25 kv-a. of generating capacity for each 1.00 kw. of load, a five per cent increase in load will reduce the speed from 60 to 59 cycles in about two seconds, if the mechanical input is held constant.

Many classes of load decrease with a decrease of speed. Among these are centrifugal pumps and other motor-driven devices. Experience on a typical large transmission having a load of 300,000 kw. indicates a loss of about 3.5 per cent in load for a reduction in speed from 60 to 59 cycles and a corresponding gain for an increase of one cycle.

Performance of Stored Mechanical Energy During Load Changes

An addition of load to a transmission system does not cause an immediate increase in input of the governing generators. The governors are responsive to speed only; hence they cannot add input until the speed has dropped the 0.1 per cent to 0.2 per cent necessary to cause the governors to act. The additional load is supplied from the stored energy of the system while the speed is decreasing to a point where the governors start to act. The input is then increased until the speed returns to normal or until it returns to the speed corresponding to the load on the governing units, depending on whether the governor is adjusted to maintain a flat speed or a drooping speed.

Where two or more generating units are governing at the same time, their governors must be set with a drooping speed characteristic. In such cases the speed will not return to normal unless the synchronizing motors of the governors are readjusted.

A reduction in load causes a similar train of events.

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The governing unit does not immediately decrease its input. The excess input increases the stored energy with the increase of speed necessary to bring the governor into action, whereupon the speed returns to normal.

A consideration of the events attendant upon a change of load shows the fallacy of a speed governor which is controlled by the electrical output of the governing unit. The only change in output occasioned by a change in load is the share of the total stored energy contributed by the governing unit. If the governing unit is to maintain speed it must be controlled by speed.

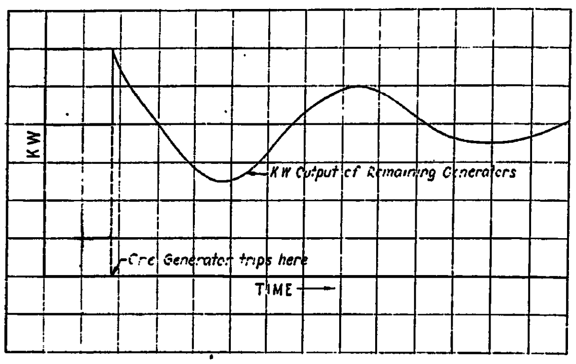

FIG. 1—OSCILLATION OF OUTPUT DUE TO SUDDEN LOSS OF GENERATOR

PERFORMANCE OF STORED MECHANICAL ENERGY DURING CHANGES IN INPUT

A generating unit may be separated from the system by the operation of a switch with the result of a sudden decrease of the input to the system.

The general effects are the same as when a load is suddenly applied. The stored energy of the system is drawn upon until the speed drops to a point where the governors begin to act.

A sudden decrease of input due to switching off a generator under load causes an oscillation of the stored energy of other generating units in the same or nearby plants with respect to the system at the receiving end of a transmission line. The flow of power over a transmission line is accompanied by a lag in time phase of the receiver voltage behind the impressed voltage. The difference in phase depends upon the length of the line and the amount of load carried. This difference in phase is analogous to the twist in an elastic shaft occasioned by a torque. The degree of twist between the ends of the shaft depends upon its length and the torque applied.

The sudden tripping of a generator under load leaves the remaining generators in the same plant in an angular position ahead of the angular position corresponding to the decreased input to the transmission line. These generators must drop back in angular position, hence, must slow down slightly and yield some of their stored mechanical energy. Finally, the generators must return to the system speed and the normal amount of stored energy. The process involves an oscillation of the angular position of the generators with respect to the synchronous equipment at the receiver end of the line with a corresponding oscillation of electrical output. This oscillation is independent of the mechanical output of the prime mover which will usually remain constant, since the changes in speed during the oscillation will usually be less than those required to cause the governors to act. Fig. 1 shows the character of this oscillation.

An increase in input as might be occasioned by a plant pulling up a block of load acts in a manner similar to a decrease of load—the stored energy is increased with the increase of speed to the point where the governors begin to act.

Since increases of input are usually less sudden than decreases due to the separation of a generating unit, the attendant oscillations are usually less pronounced. Such as occur are due to the increase in the phase angle over the line which must take place before the line can deliver additional load.

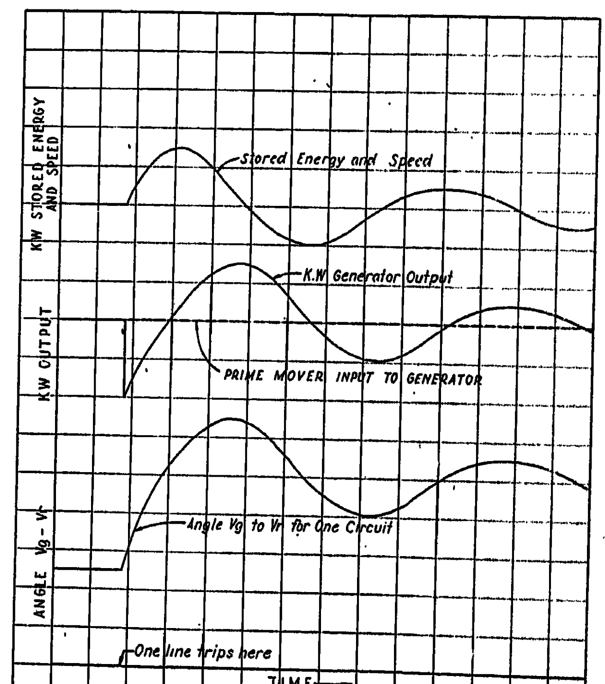

FIG. 2—OSCILLATIONS OF SPEED, STORED ENERGY, GENERATOR OUTPUT AND ANGLE BETWEEN GENERATOR AND RECEIVER VOLTAGE DUE TO ONE OF TWO PARALLEL TRANSMISSION LINES TRIPPING

THE PERFORMANCE OF STORED ENERGY DURING CHANGES IN TRANSMISSION LINE CAPACITY

Stored energy is an important factor in the performance of a transmission system during changes of transmission line capacity occasioned by switching out or in a parallel circuit. Such an operation results in a sudden change in the load on the line or lines remaining in service. The change of load on the line

requires a change in the angular position of the generator voltage with respect to the receiver voltage. This change in angular position requires a change in stored energy and the transition is accompanied by an oscillation.

The case most likely to cause difficulty is the loss of one of two parallel circuits when the load on the remaining circuit approaches the limit of synchronous stability. Under these conditions the oscillation in kilowatt input arising from the oscillation of the stored energy may result in an input which exceeds the stability limit even though the steady state input is considerably under the limiting load for the remaining line.

A better understanding of the subject may be had by considering the performance of the generators at the time one of two circuits is switched out. Reference is made to Fig. 2.

Preceding the instant the parallel line trips out, the angle between the generator and receiver voltage may be $n°$. When the circuit trips, the remaining circuit cannot carry the total load until its angle has become even more than 2 $n°$; hence the generator output drops to about half of its previous value and the excess of input from the prime mover increases the stored energy and the speed until the normal generator output is restored.

At this point the generator is running faster than the receiver; hence the output continues to increase until the excess stored energy is reduced to normal when the generator will be slowing down. Thus an oscillation of kilowatt output, angular difference between generator and receiver voltages and stored energy is started which may result in a maximum kilowatt input exceeding the limiting load the circuit can carry.

The performance of the transmission system under these conditions is greatly complicated by the fact that the sudden decrease in input attendant upon a line opening, starts an oscillation of the synchronous receiving apparatus with respect to the generators.

The period of oscillation of each synchronous unit will depend upon its $W R^2$ per kv-a. which will usually be different for each unit of different size or character. The resultant performance of the transmission system is extremely complicated and probably impossible to predict with accuracy. Observations indicate that the oscillations of energy flow in the transmission system are composed of several different frequencies superimposed.

Switching in a second parallel circuit also causes an oscillation of mechanical energy with the reverse effects to those observed for switching a line out. The generator output will be momentarily increased, followed by an oscillation. Such an operation is probably less likely to cause synchronous instability than switching out a parall elcircuit; however, a severe disturbance may be set up. A much better method of operation consists in transferring generators to the incoming line and equalizing the loads before the lines are paralleled at the generating plant.

Discussion

R. J. C. Wood: The outstanding point which struck me in this paper was the very small number of kilowatt-hours that are stored in the kinetic energy of the moving fly-wheel. Here is a 27,000-kv-a. unit and the total stored energy from full speed down to nothing is only 16 kw-hr. That seems to be such a small amount that it would not enter materially into any disturbances of the system, particularly as the whole of that 16 kw-hr. is not used since the speed will not drop from full speed to zero; the fluctuation will be very small and the amount of energy re eased will correspond to it.

L. N. Robinson: In connection with Mr. Jollyman's paper, it may be of interest to mention that it is intended to synchronize the 19,500-kv-a., hydraulic-turbine-driven generators at the Baker River Station automatically.

It is expected that this will facilitate operation and improve the service considerably. It will reduce the number of attendants to one on each shift. It is expected that automatic synchronizing will expedite the starting up and connecting of generators to meet changing load requirements and that it will be effective in restoring service quickly after serious interruptions.

Two speed switches are geared to the shaft of each of the generating units. One speed switch is closed when the speed of the unit is more than 95 per cent of normal speed and the other is closed when the speed is less than 105 pe: cent. The two speed switches are connected in series so that the circuit is closed only when the speed of the unit is within 5 per cent of normal. When he desires a unit to be synchronized automatically, the operator will turn a key switch on the main switchboard. This key switch connects the closing circuit of the generator oil circuit breaker through the speed switches and the oil circuit breaker will be closed automatically when the speed comes to within 5 per cent of normal. This procedure applies whether the machine is being started up or is coming back to normal after excessive speed due to sudden interruption of the load.

The generators are equipped with amortisseur windings, since they are intended to be thrown on the bus without d-c. field excitation, relying upon the induction-motor effect of the squirrel-cage winding to bring them nearly to synchronism. The automatic synchronizing control is interlocked with the generator field switches so that a generator cannot be thrown on the bus automatically unless its field switches are open.

As far as we know, these are the largest generating units to be equipped for automatic synchronizing up to this time, and they are of especial interest because each unit constitutes approximately 20 per cent of the present peak generating capacity of the system on which they are being installed.

J. P. Jollyman: While as Mr. Wood has pointed out, it is true that the fly-wheel effect of a large generator is only a few kilowatt-hours, still the total fly-wheel effect of a large system is sufficient to prevent the usual load changes from producing material speed changes within the time required for governor action.

Transmission Line Design

Mechanical Design of Spans With Supports at Unequal Elevations

BY G. S. SMITH[1]

Associate, A. I. E. E.

Synopsis.—The purpose of this paper is to derive a practical method of design based upon the catenary formulas for spans whose supports are not at the same elevation. It is furthermore intended completely to formulate the method for applying it to any cable which is suspended freely or is uniformly loaded, as well as for obtaining its characteristics after the temperature or loading conditions of the cable have changed.

After a brief survey of the present need for more accurate methods of mechanical design of transmission line cables, the several formulas which are adapted to spans with supports at unequal elevations are given, and the method of using them is fully explained. All such special formulas are fully derived in the appendix.

The first method given depends somewhat upon interpolated values from the symmetrical span data. A second pure interpolation method is next described which has the advantage of being shorter than the first as well as more accurate, since it depends upon fewer interpolated values from mathematical tables.

The use of these two methods is then fully illustrated by the solution of three typical spans. Data for the cable used in symmetrical spans are first given, and from these the solution for the span with supports at unequal elevation is obtained. For the purpose of comparison the computations used in obtaining the desired values by each method are given in full for one of the spans chosen.

These data are entered on a complete set of forms stating all formulas used and indicating by the headings of each column the operation performed to obtain the results recorded therein. These forms have been so arranged that by supplying the proper constants, they may be applied to any similar span problem, and thus will greatly reduce the work and the possibility of errors in the final results.

This affords a well defined method of making, to any desired degree of accuracy, computations which can be applied equally well to all types of spans with supports at any relative elevation.

* * * * *

IMPROVEMENTS in the methods of mechanical design of electrical transmission lines have scarcely kept pace with the advancement made in the electrical design. Most of the methods offered for the solution of the span whose supports are at the same elevation are based either upon the assumption that the cable takes the form of a parabola or upon some scheme of selecting the desired values from a set of curves. Few, if any, of these methods used in the past are capable of giving results with any great degree of accuracy. For the solution of the span problem, when the supports are not at equal elevations, the methods available are still fewer in number, and even more approximate in character.

Such methods were sufficiently accurate in the past, since the cost of the small lines used was not great in comparison with the total cost of the development. However, a very marked change has taken place in the last several years until now the cost of the transmission line is often a very important item. Both the increased capacity and more permanent type of construction demand larger spans than were previously used in order that the greatest economy may be obtained. Then again, we are now reaching out into the very rugged parts of the country for the water-power available there, and here also we find a stimulus toward the use of long spans to decrease the construction expense. Here also we find many, if not most, of the spans, with supports at unequal elevations. It is needless to say that economy and reliability in such construction demand the most accurate data obtainable.

In 1917 Professor Kirsten[2] presented before the Institute his paper on "Transmission Line Design," in which he offered a method of design for spans with supports at equal elevations, based upon the equation for the catenary curve. His method, as later improved[3], is capable of almost any degree of accuracy desired. This paper presents a somewhat similar method of design for those spans whose supports are at unequal elevations.

Since the formulas derived have a definite relation to those usually given for the catenary curve, this relationship will be next explained and the reasons why new formulas may be desired will be shown.

Catenary Equations with Higher Support as Origin of Coordinates

A flexible cable suspended from two points at an equal elevation forms a catenary symmetrical to these two points of support under all conditions of temperature or uniform loading. In this case the origin of coordinates may be taken at its lowest point where the tangent to the cable is horizontal. Thus only half of the span need be considered.

The value of X at the point the cable is fastened to the support will be equal to half the total span, and the corresponding value of Y will be the total sag of the cable for the load tempretaure and tension conditions existing. If we should raise or lower the supports so that the lowest point of the cable would always be at the same elevation, under varying load and temperature

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Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., Sept. 15-19, 1925.

2. F. K. Kirsten, Transmission Line Design, 1917 A. I. E. E. Trans.

3. F. K. Kirsten, University of Washington Experiment Station Bulletin No. 17. G. S. Smith, University of Washington Experiment Station Bulletin No. 29.

conditions, the cable would take positions similar to those shown in Fig. 1.

In the catenaries shown, the tension at any point on the cable x distance from the origin may be expressed by the simple catenary equation given in hyperbolic functions; thus

$$T = c\,w \cosh \frac{x}{c} \tag{1}$$

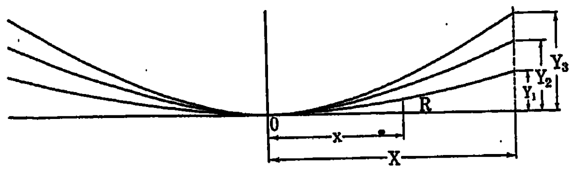

Fig. 1

Similarly the distance of the same point above the horizontal is expressed by

$$y = c \cosh \frac{x}{c} - c \tag{2}$$

and the equation for the actual length of the cable from O to R is

$$s = c \sinh \frac{x}{c} \tag{3}$$

The concept c is a constant for all points on any given catenary. It has a value equal to the length of cable whose weight is the same as the value of the horizontal tension, or the actual tension on the cable at the lowest point.

If the same cable is suspended from supports not at the same elevation, the point at which the tangent to the cable is horizontal does not lie midway between supports. Furthermore it changes its position for every change of temperature or loading. However, each of the various curves formed, is a portion of some symmetrical catenary similar to those in Fig. 1.

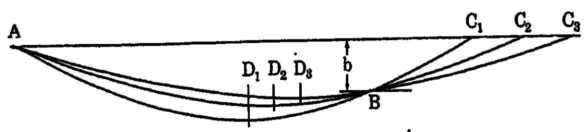

Fig. 2

Fig. 2 represents such a cable suspended between points A and B, showing three of its positions under three respective assumed temperature or loading conditions. Each curve is projected to the horizontal line $A\,C_3$. It is evident then, that each curve between A and B is a portion of a symmetrical catenary with the lowest points at D_1, D_2 and D_3 respectively. Obviously the simple catenary equations hold for any one curve referred to its lowest point as an origin, but this proves of little value since the origin shifts when the assumed conditions change. Thus the simple relations between the curves in Fig. 1 for various assumed conditions is lost for the similar curves in Fig. 2.

From the foregoing it is evidently desirable to derive the corresponding equations with the origin at one of the supports, which is a point on the resulting catenary under all conditions. Since the support A has the greater tension at all times, and will always be one of the ends of the resulting equivalent symmetrical catenary, it is the more desirable for the origin. Thus with the higher support as an origin, most of the values found in the tension-sag data for symmetrical supports can be substituted directly in the new formulas. The equations derived with A as the origin will hold equally well for B or D as the origin if proper substitutions are made.

Fig. 3 shows one of the curves in Fig. 2 with the various tensions, angles, etc., indicated. With the point A as the origin of coordinates, the following equations have been derived:

$$s = \frac{c}{\cos \beta_m} \left[\sinh \frac{x}{c} - \sin \beta_m \left(\cosh \frac{x}{c} - 1 \right) \right] \tag{13}$$

For the sag of the point p below the point A.

$$y = \frac{c}{\cos \beta_m} \left[\cosh \frac{x}{c} - \sin \beta_m \sinh \frac{x}{c} - 1 \right] \tag{14}$$

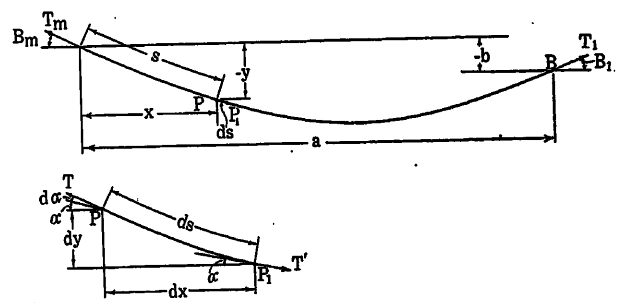

Fig. 3

and for the tension on the cable at point P

$$T = \frac{c\,w}{\cos \beta_m} \left[\cosh \frac{x}{c} - \sin \beta_m \sinh \frac{x}{c} \right] \tag{15}$$

The full derivation of these formulas is given in Appendix 1.

The following list will explain the symbols used in these formulas, as well as all others which will be used throughout the paper to specify special values.

w = weight per foot of cable in pounds
s = total length of the cable from A to any point P
x = any horizontal distance measured from A
X = half span length of the equivalent catenary with supports at the same elevation
y = sag at any point
Y = maximum sag corresponding to the value X on the equivalent catenary
T = tension at any point
T_m = tension at the upper support or at either support in the equivalent catenary
β = angle the tangent to the cable makes with the horizontal

β_m = value of β at the upper support

c = length of cable whose weight is equal to the horizontal tension on the cable

a = horizontal distance between supports of the span with supports not at the same elevation

b = difference in elevation of supports

d = maximum horizontal distance from the straight line between supports to the catenary

Equations (13), (14), and (15) are the generale quations for the catenary, assuming the origin of coordinates at the higher support. By substitution of the value of β at any point, P, for β_m, the formula will then be the equation for the catenary with the point P as the origin. Thus if the value of β is zero, which is true at the point where the tangent to the cable is horizontal, the formulas will reduce to the simpler expressions given in (1), (2), and (3).

The new equations derived might be used to solve spans with supports at any relative elevation, but the mathematics involved in finding the new span data after changing temperature or loading conditions would be very difficult. However, since the tension-sag data for spans of various lengths, with supports at the same elevation, are usually required for every line designed, certain parts of these data may be substituted in the foregoing equations to obtain the desired results for the span with supports on an incline.

Applications of Equations—First Method

In Fig. 4 two catenaries from the tension-sag data for symmetrical spans are shown as $A\,B_1$ and $A\,B_2$. These

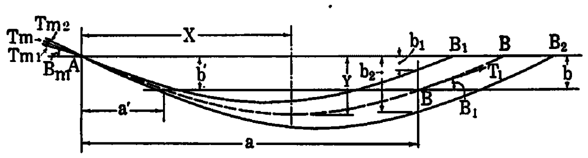

Fig. 4

are both at the same temperature and loading conditions but for different span lengths. The third catenary $A\,B'$ is the desired one, passing through its points of support A and B. Thus the part $A\,B$ is the actual cable suspended, and $A\,B'$ the equivalent symmetrical catenary.

All values of c, T_m, Y, X, and w are known for $A\,B_1$ and $A\,B_2$. Also the spacing (a) and the difference of elevation of supports (b) on $A\,B$ are known.

If in equation (12) the value (a) is substituted for x and also the values of c, $\cos\beta_{m1}$, and $\sin\beta_{m1}$ corresponding to the known curve $A\,B_1$, the resulting value of y will be the value indicated by b_1. If a similar substitution is made in (12), using values of c, $\cos\beta_{m2}$, and $\sin\beta_{m2}$ corresponding to curve $A\,B_2$, the resulting y will be the value indicated by b_2.

It is only necessary to compute the values of y for three or four of the known catenaries, and plot curves between these values of y, now given as b_1, b_2, etc., and the corresponding values of c, X and T_m. The desired values of c, X, and T_m will then be the points corresponding to b on these curves. The values of Y, w or any others may be found by a similar method, or may be computed.

The value X here is the half span length of the equivalent symmetrical catenary and Y the maximum sag of the same catenary.

It is obvious from Fig. 4 that there are two points on the curve $A\,B'$ which have the same difference of elevation from the support A. Thus b corresponds to span a, and b' corresponds to a'. In other words the desired catenary may have a total span a greater than the half span X of the equivalent symmetrical catenary, or a span a' which may be less. In the latter case the lowest point on the line is at the lowest support and the vertical component of strain upon this support will be upward.

An inspection of the data given later will show that no large amount of computing is necessary for obtaining the tension sag or any other data for a given span with supports at unequal elevations. Many of the desired values have already been given in the data for supports at equal elevations. Thus in equation (14) a is substituted for x and the result is the required value b.

$$b = \frac{c}{\cos\beta_m}\left[\cosh\frac{a}{c} - \sin\beta_m \sinh\frac{a}{c} - 1\right]$$

It is shown in Appendix 1 that

$$\frac{c}{\cos\beta_m} = \frac{T_m}{w}$$

But the tension equation for the symmetrical span is

$$\frac{T_m}{w} = c\cosh\frac{x}{c} = \frac{c}{\cos\beta_m} \qquad (17)$$

where x is now the half span. The value of $c\cosh\dfrac{x}{c}$ is given for each assumed condition in the data under Col. (21) in each table. Also from (41)—

$$\cos\beta_m = \frac{1}{\cosh\dfrac{x}{c}}$$

and

$$\sin\beta_m = \left[1 - \cos^2\beta_m\right]^{1/2} = \left[1 - \frac{1}{\cosh^2\dfrac{x}{c}}\right]^{1/2} \qquad (18)$$

The values of $\sinh\dfrac{a}{c}$ and $\cosh\dfrac{a}{c}$ may be interpolated from the tables of sinhs and coshs.

In carrying through the calculations for the method just described, it was found that the same results might be obtained with less than half the computations involved here, by a pure interpolation method. The designer will doubtless choose the second method,

TABLE I

Mechanical Design of Electrical Transmission Cables

Specifications and Constants

Base temp.	(t)	= 68 deg. fahr.	Ice Loading	(a)	= 0.5 in.
Minimum temp.	(t_1)	= 0.0 deg. fahr.	Wind Loading	(V)	= 8 lb. per sq. ft.
Freezing temp.	(t_2)	= 32 deg. fahr.	Wt. of Ice	(u)	= 57.37 lb. per cu. ft.
Maximum temp.	(t_4)	= 110 deg. fahr.			
Modulus of Elasticity	(E)	= 29,000,000			
Coefficient of Linear Expansion	(α)	= 0.000,006,62			
Elastic Limit (Stranded)	($T\ e$)	= 108,000 lb. per sq. in.			
Wt. per Cubic Inch	(W)	= 0.2833 lb.			

$[1 - \alpha(t - t_1)] = K = 0.99954984$ $\qquad 1 + \alpha(t_2 - t_1) = K_1 = 1.0002118$

$[1 - \alpha(t - t_1)]^2 = K^2 = 0.99909988$ $\qquad 1 + \alpha(t_4 - t_1) = K_2 = 1.0007282$

$[1 - \alpha(t - t_1)]^4 = K^4 = 0.99820057$ $\qquad 1 + \alpha(30° - t_1) = K_3 = 1.0001986$

$1 + \alpha(60° - t_1) = K_4 = 1.0003972$

(1) Cir. Mill Area	(2) No. of Strands	(3) Layers around central wire	(4) Circular Mills per Strand $\frac{(1)}{(2)}$	(5) Diam. of Strand-inches $\frac{\sqrt{(4)}}{1000}$	(6) Total Diam. of Cable-inches (5) [2 × (3) + 1]	(7) Area of Circle Enveloping Cable sq. ft. = A $\frac{\pi \times (6)^2}{4 \times 144}$
729646 76	37	3	19720.183	0.14042857	0.983	0.00527029
(8) Area of Strand Unstressed-sq. in. $\frac{\pi \times (4)}{4 \times 1000^2}$	(9) Elastic Limit per Strand—lb. (8) × $T\ e$	(10) Elastic Limit of Cable—lb. (2) × (9)	(11) Max. Tension on Cable—lb. T 0.75 × (10)	(12) Equiv. Modulus lb. per sq. in. $\frac{E \times (5)^2 \times (2)}{(6)^2}$	(13) Equiv. Modulus lb. per sq. ft. E_1 144 × (12)
0.01548819	1672.7251	61890.831		21897959	3153306102.
(14) Wt. per ft. free from Stress w W' × (8) × (2) × 12	(15) Total Modulus $E_1 A$ (7) × (13)	(16) $E_1 A w K$ (14) × (15) × K	(17) $E_1 A \times K^2 + T$ (15) × K^2 + (11)	(18) Wt. per ft.—Cable Only—Under Max. Stress = W'' $\frac{(16)}{(17)}$	(19) π × (17)
1.0481858	16618834	32362002.	16665766.	1.9418251	52357048.
(20) $\frac{(13)}{(19)}$	(21) $\sqrt{(20)}$	(22) a + [2 × (7) × K^2 × 21]	(23) Wt. of Ice Load per ft. of Cable $i = u \times a \times \pi \times (22)$ 7.5013162 × (22)	(24) a + [(7) × K^2 × (21)]	(25) Wind load per ft. of Cable-lb. 2 × 8 × (24)
60.226985	7.7606034	0.12339761	0.92675509	0.08253381	1.3205409
(26) $w'' + i$ (18) + (23)	(27) (26²)	(28) (25)²	(29) (27) + (28)	(30) Wt. per ft. of Cable Loaded $\frac{w_1}{\sqrt{(29)}}$	(31) $\frac{A^2}{(7)^2}$
2.8695802	8.2287521	1.7438283	9.9725804	3.1579393	27.77594 × 10^{-6}
(32) $E_1 A^2 K^4$ (13) × (31) × K^4	(33) Area of Cable only under Max. Load = A_1 $\frac{(32)}{(17)}$	(34) $2\ E_1 A_1$ 2 × (13) × (33)	(35) Constant for G_1 32° F $\frac{(30)}{(34) \times K_1}$	(36) Constant for H_1 32° F $\frac{(30)}{(34) \times K_1^2}$	(37) Constant for G_2 0.0° F $\frac{(30)}{(34)}$
87428.452	0.005245.99	33084428	95.430708 × 10^{-9}	95.410496 × 10^{-9}	95.450924 × 10^{-9}
(38) Constant for H_2 0.0° F. $\frac{(18)}{(34)}$	(39) Constant for G_3 110° F. $\frac{(30)}{(34) \times K_2}$	(40) Constant for H_3 110° F. $\frac{(18)}{(34) \times K_2^2}$	(41) Constant for $G_{30°}$ 30° F. $\frac{(30)}{(34) \times K_3}$	(42) Constant for $H_{30°}$ 30° F. $\frac{(18)}{(34) \times K_3^2}$	(43) Constant for $G_{60°}$ F. 60° F. $\frac{(30)}{(34) \times K_4}$
58.693021 × 10^{-9}	95.381467 × 10^{-9}	58.607634 × 10^{-9}	95.431971 × 10^{-9}	58.669715 × 10^{-9}	95.413026 × 10^{-9}
(44) Constant for $H_{60°}$ 60° F. $\frac{(18)}{(34) \times K_4^2}$					
58.646423 × 10^{-9}					

though both are given here in order that the reader may be able clearly to see the principles involved.

APPLICATIONS OF EQUATIONS—SECOND METHOD

This alternate method of solving the span with supports on an incline depends only upon formulas given in equations (1), (2), and (3) together with the usual tension-sag data for spans with supports at the same elevation.

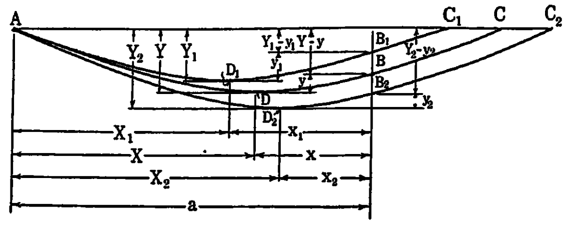

FIG. 5

In Fig. 5, let the catenary whose values are desired be represented by $A\,D\,B$ with supports at A and B. The difference in elevation of supports will be b and the total span length a. If the origin of coordinates is at point D, this will be a portion of an equivalent catenary $A\,D\,C$, symmetrical about an axis at D and with half span values of X and Y. Every point on this curve can be expressed by the equation,

$$y = c\left(\cosh\frac{x}{c} - 1\right)$$

The value c is constant for the whole length of the curve. If the value of X were known, the value of x could be found, since

$$x = a - X$$

The difference between the elevations of D and B could be found by

$$y = c\left(\cosh\frac{x}{c} - 1\right)$$

and

$$b = Y - y$$

However, the value of X is still unknown.

Now imagine an infinite series of catenaries, some shorter and some longer than the desired one, $A\,D\,C$. These were all suspended with the same maximum tension, under maximum loading and minimum temperature conditions, and they have all been subjected to the same loading and temperature changes.

If $A\,D_1\,C_1$ is one of the catenaries in this series with a total span length $A\,C_1$ an infinitesimal amount shorter than $A\,D\,C$, it will have half span values of X_1 and Y_1. The value y_1 may also be found for x_1 where

$$x_1 = a - X_1$$

TABLE II

Catenary at Minimum Temperature

Cable Under Maximum Loading Conditions

Working Formulas

$$\frac{T}{w} = c\cosh\frac{x}{c} \text{ or } c = \frac{T}{w\cosh\frac{x}{c}}$$

$$s = c\sinh\frac{x}{c}$$

T_m = Maximum allowable tension on cable = 61890.831 lb.

$$\frac{T_m}{w_1} = 19598.487$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Assumed Argument $\frac{x}{c_1}$	From Tables $\frac{1}{\cosh\frac{x}{c_1}}$	(2) × 19598.487 $c_1 = \frac{T_m}{w_1\cosh\frac{x}{c_1}}$	(1) × (3) $x = \frac{x}{c_1} \times c_1$	(3) × sinh (1) $s_1 = c_1\sinh\frac{x}{c_1}$	$\frac{(5)}{(4)}$ $\frac{s_1}{x}$	From Tables $\frac{\cosh\frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh\frac{x}{c_1}}$
0.04	0.99920053	19582.818	783.3127	783.5216	1.0002667	50.013338
0.06	0.99820270	19563.262	1173.7957	1174.5001	1.0006001	33.353346
0.08	0.99680851	19535.938	1562.8751	1564.5427	1.0010670	25.026698
0.10	0 99502075	19500.901	1950.0901	1953.3419	1.0016675	20.033394
0.12	0.99284295	19458.219	2334.9863	2340.5943	1.0024017	16.706772
0.14	0.99027940	19407.977	2717.1169	2726.0015	1.0032699	14.332549
0.16	0.98733513	19350.274	3096.0439	3109.2706	1.0042721	12.553584
0.18	0.98401586	19285.222	3471.3399	3490.1155	1.0054087	11.171467
0.20	0.98032799	19212.945	3842.5890	3868.2576	1.0066800	10.067155

Thus

$$y_1 = c\left(\cosh \frac{x_1}{c_1} - 1\right)$$

It is very evident from the sketch that $Y_1 - y_1$ will be less than $Y - y$, since Y_1 is less than Y, and y_1 is greater than y.

In a similar manner, a span an infinitesimal amount larger may be represented by $A\,D_2\,C_2$, and with similar reasoning $Y_2 - y_2$ can be proven greater than $Y - y$.

If $Y_1 - y_1$ is an infinitesimal amount smaller than b, and $Y_2 - Y_2$ is an infinitesimal amount larger than b, the point b must be one of a series of such sag differences, each corresponding to one of the series of catenaries having A as one support and the other support at a horizontal distance of a out on the curve. In other words, the catenary of this series, upon which b is one point, can be interpolated from the infinite series.

By choosing values of $\frac{x}{c}$ with differences as small as is desired, an infinite series of catenaries, related as was previously described[4], can be approached. However, if the point b and its corresponding catenary can be

TABLE III

Catenary at Freezing Point

Cable Under Maximum Loading Condition

Working Formula

$$\frac{\sinh \frac{x}{c_2}}{\frac{x}{2}} - 1 = F_1 - G_1 + H_1\left[\frac{\cosh \frac{x}{c_2}}{\frac{x}{c_2}} + \frac{1}{\sinh \frac{x}{c_2}}\right] - 1$$

$$F_1 = \frac{s_1}{x}(1 + \alpha\,[t_2 - t_1])$$

$$G_1 = \frac{w_1}{2\,E_1\,A_1\,(1 + \alpha\,[t_2 - t_1])}\,s_1\left[\frac{\cosh \frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh \frac{x}{c_1}}\right]$$

$$H_1 = \frac{w_1}{2\,E_1\,A_1\,(1 + \alpha\,[t_2 - t_1])^2}\,s_1$$

Const. for F_1 = 1.0021184
Const. for G_1 = 95.430708×10^{-9}
Const. for H_1 = 95.410496×10^{-9}

(8) (6) × 1.0021184 F_1	(9) (7) × 95.430708×10^{-9} $\frac{G_1}{s_1}$	(10) (5) × (9) G_1	(11) (8) − (10) − 1 $F_1 - G_1 - 1$	(12) (5) × 95.410496×10^{-9} H_1	(13) Values from Curves Plotted from Aux. Table $\frac{x}{c_2}$	(14) $\frac{(4)}{(13)}$ $c_2 = x + \frac{x}{c_2}$
	$\times 10^{-9}$			$\times 10^{-9}$		
1.0004786	4772.8082	0.00373960	−0.00326101	74756.188	0.042060	18623.698
1.0008121	3182.9335	0.00373836	−0.00292628	112059.64	0.062641	18738.458
1.0012791	2388.3155	0.00373662	−0.00245755	149273.79	0.082930	18845.714
1.0018797	1911.8009	0.00373440	−0.00185471	186369.32	0.103021	18929.054
1.0026141	1594.3391	0.00373170	−0.00111762	223317.26	0.122990	18985.172
1.0034824	1367.7653	0.00372853	−0.00024613	260089.15	0.142894	19014.912
1.0044849	1197.9974	0.00372490	+0.00075998	296657.05	0.162766	19021.441
1.0056217	1066.1010	0.00372082	+0.00190092	332993.65	0.182625	19008.021
1.0068933	960.7158	0.00371630	+0.00317697	369072.37	0.202486	18977.060

(15) Cosh (13) − 1 $\frac{y_2}{c_2} = \cosh\frac{x}{c_2} - 1$	(16) sinh (13) $\sinh\frac{x}{c_2}$	(17) (14) × (15) y_2	(18) (14) × (16) $s_2 = c_2 \sinh\frac{x}{c_2}$	(19) $\frac{(5)}{(18)}$ $\frac{s_1}{s_2}$	(20) (19) × 3.1579393 $w_2 = \frac{s_1}{s_2}\,w_1$	(21) (14) + (17) $\frac{\text{Max. } T_2}{w_2}$ $c_2 \cosh\frac{x}{c_2}$	(22) (20) × (21) Max. T_2
0.0008847	0.0420724	16.47	783.544	0.9999719	3.157850	18640.17	58863
0.0019626	0.0626819	36.77	1174.563	0.9999461	3.157769	18775.23	59288
0.0034407	0.0830251	64.84	1564.667	0.9999205	3.157688	18910.56	59714
0.0053114	0.1032033	100.54	1953.541	0.9998978	3.157617	19029.59	60088
0.0075728	0.1233003	143.77	2340.877	0.9998791	3.157557	19128.94	60401
0.0102267	0.1433808	194.46	2726.373	0.9998637	3.157509	10209.37	60654
0.0132757	0.1634856	252.52	3109.732	0.9998515	3.157470	19273.96	60857
0.0167224	0.1836418	317.86	3490.668	0.9998417	3.157439	19325.88	61020
0.0205704	0.2038725	390.37	3868.901	0.9998337	3.157414	19367.43	61151

4. That is, with the same maximum tension on all catenaries of the series at the maximum loading and minimum temperature conditions, and the same temperature and loading conditions for all of this series, which is the original series after changing from the critical to the desired condition.

interpolated from an infinite series, it can also be interpolated from a finite series.

The desired results are easily obtained from the usual tension-sag data by carrying out on several of the catenaries the operation described, obtaining values similar to $Y_2 - y_2$ greater than b, and others similar to $Y_1 - y_1$ less than b. These sag differences may then be plotted against their corresponding values of X, c, or T_m, and the value of X, c_1, or T_m, corresponding to the catenary upon which b is one point, may readily be found. Other values, such as Y and w, may be found in a similar manner, or may be computed.

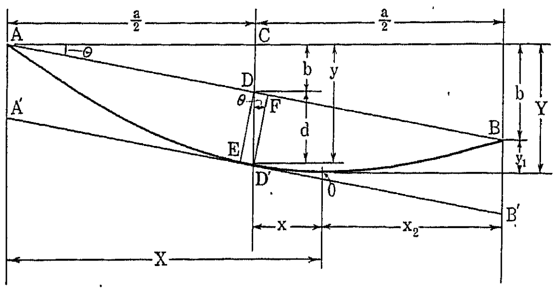

FIG. 6

METHOD OF OBTAINING MAXIMUM SAG OF CATENARY FROM STRAIGHT LINE BETWEEN POINTS OF SUPPORT

The two methods just explained will give all of the information necessary if the line is to be strung by means of a tension dynamometer, or if the sag Y can be used. However, in stringing cables over supports on an incline, it is often more convenient to measure off the

TABLE IV

Catenary at Minimum Temperature

Cable Without Ice or Wind Load

Working Formula

$$\frac{\sinh \frac{x}{c_3}}{\frac{x}{c_3}} - 1 = F_2 - G_2 + H_2 \left[\frac{\cosh \frac{x}{c_3}}{\frac{x}{c_3}} + \frac{1}{\sinh \frac{x}{c_3}} \right] - 1$$

$$F_2 = \frac{s_1}{x}$$

$$G_2 = \frac{w_1}{2 E_1 A_1} s_1 \left[\frac{\cosh \frac{x}{c_1}}{\frac{x}{c_1}} + \frac{1}{\sinh \frac{x}{c_1}} \right]$$

$$H_2 = \frac{w''}{2 E_1 A_1} s_1$$

Const. for F_2 = 1
Const. for G_2 = 95.450924×10^{-9}
Const. for H_2 = 58.693021×10^{-9}

(8)	(9)	(10)	(11)	(12)	(13)	(14)
(6) × 1 F_1	(7) × 95.450924 × 10^{-9} $\frac{G_2}{s_1}$	(5) × (9) G_2	(8) − (10) − 1 $F_2 - G_2 - 1$	(5) × 58.693021 × 10^{-9} H_2	Values from Curves Plotted from Aux. Table $\frac{x}{c_3}$	$\frac{(4)}{(13)}$ c_3
	× 10^{-9}			× 10^{-9}		
1.00026669	4773.8193	0.00374039	−0.00347370	45987.252	0.025667	30518.282
1.00060011	3183.6077	0.00373915	−0.003139040	68934.962	0.040427	29034.945
1.00106701	2388.8214	0.00373741	−0.00267041	91827.736	0.057157	27343.564
1.00166750	1912.2059	0.00373519	−0.00206769	114647.536	0.075836	25714.569
1.00240173	1594.6768	0.00373249	−0.00133076	137376.553	0.095984	24326.829
1.00326987	1368.0550	0.00372932	−0.00045945	159997.264	0.116980	23227.191
1.00427213	1198.2511	0.00372569	+0.00054644	182492.486	0.138348	22378.667
1.00540875	1066.3269	0.00372160	+0.00168715	204845.425	0.159808	21721.941
1.00668001	960.9193	0.00371708	+0.00296293	227039.725	0.181220	21204.001

(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
cosh (13) − 1 $\frac{y_3}{c_3} = \cosh \frac{x}{c_3} - 1$	sinh (13) $\sinh \frac{x}{c_3}$	(14) × (15) y_3	(14) × (16) $s_3 = c_3 \sinh \frac{x}{c_3}$	$\frac{(5)}{(18)}$ $\frac{s_1}{s_3}$	(19) × 1.9418251 $w_3 = \frac{s_1}{s_3} w''$	(14) + (17) $\frac{\text{Max. } T}{w_3}$ $c_3 \cosh \frac{x}{c_3}$	(20) × (21) Max. T
0.0003294	0.0256698	10.05	783.398	1.0001572	1.942130	30528.33	52290
0.0008173	0.0404380	23.73	1174.115	1.0003279	1.942462	29058.68	56445
0.0016339	0.0571881	44.68	1563.727	1.0005214	1.942838	27388.24	53211
0.0028769	0.0759087	73.98	1951.960	1.0007080	1.943200	25788.55	50112
0.0046100	0.0961314	112.15	2338.573	1.0008642	1.943503	24438.97	47497
0.0068500	0.1172470	159.10	2723.318	1.0009853	1.943738	23386.30	45457
0.0095854	0.1387900	214.51	3105.930	1.0010755	1.943914	22593.18	43919
0.0127965	0.1604891	277.97	3486.134	1.0011420	1.944043	21999.91	42769
0.0164654	0.1822136	349.13	3863.657	1.0011907	1.944137	21553.13	41902

vertical distances $A A'$ and $B B'$ shown in Fig. 6 and while sighting between points $A' B'$ to allow the cable to sag until it is tangent to the line of sight.

The usual method of finding the value of $A A'$, which corresponds to $D D'$, is to assume that the maximum

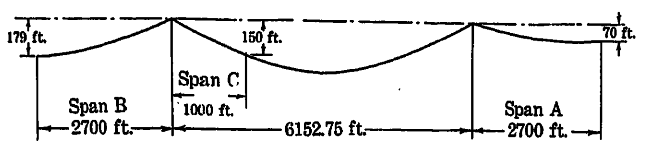

FIG. 7

deflection of the cable appears at a point E, which is also one point on the line perpendicular at $A B$ at its middle point. This assumption would be correct if the curve were a parabola, but it is not true for the catenary.

It is proved in Appendix No. 2 that the point of maximum deflection of the catenary from the line between the points of support occurs at the center of the span in certain cases, and so near the center in all other cases that a considerable degree of accuracy must be attained to detect the difference. Thus the value $D' F$ may be considered the true maximum deflection. The vertical distance, $D D'$, will be equal to $D' F \cos \theta$.

However, the value of $D D'$ may be computed directly from the data obtained by either method. Using equation (**14**), the value for y, which is shown as $C D'$, will be obtained by substituting $\frac{a}{2}$ for x. Also from the right triangle $A C D$, $C D = b' = \frac{b}{2}$ since it is at the center of the span. Then $D D' = d = y - b'$. This

TABLE V

Auxiliary Data to Tension—Sag Tables

Values for plotting curves to find $\frac{x}{c}$ in formula

Assumed Value $\frac{x}{c_1}$	(23) $\frac{x}{c}$ From Tables	(24) (12) × $\frac{\cosh \frac{x}{c}}{\frac{x}{c}} + \frac{1}{\sinh \frac{x}{c}}$	(25) (24) $\mp$ (11)	(26) $\frac{\sinh \frac{x}{c}}{\frac{x}{c}} - 1$ From Tables	(23) $\frac{x}{c}$ From Tables	(24) (12) × $\frac{\cosh \frac{x}{c}}{\frac{x}{c}} + \frac{1}{\sinh \frac{x}{c}}$	(25) (24 $\mp$ (11)	(26) $\frac{\sinh \frac{x}{c}}{\frac{x}{c}} - 1$ From Tables
0.04	0.041 0.042 0.043	0.0036477 0.0035609 0.0034781	0.0003867 0.0002999 0.0002171	0.0002802 0.0002940 0.0003081	0.025 0.026 0.027	0.0036794 0.0035379 0.0034069	+0.0002057 +0.0000642 −0.0000668	0.0001042 0.0001127 0.0001215
0.06	0.061 0.062 0.063	0.0036764 0.0036171 0.0035598	0.0007501 0.0006909 0.0006335	0.0006203 0.0006408 0.0006616	0.040 0.041 0.042	0.0034477 0.0033636 0.0032836	+0.0003086 +0.0002246 +0.0001445	0.0002667 0.0002802 0.0002940
0.08	0.082 0.083 0.084	0.0036449 0.0036011 0.0035583	0.0011874 0.0011435 0.0011008	0.0011210 0.0011486 0.0011764	0.057 0.058 0.059	0.0032238 0.0031683 0.0031146	0.0005534 0.0004978 0.0004442	0.0005416 0.0005608 0.0005803
0.10	0.102 0.103 0.104	0.0036606 0.0036252 0.0035905	0.0018059 0.0017705 0.0017358	0.0017349 0.0017691 0.0018036	0.075 0.076 0.077	0.0030601 0.0030199 0.0029808	0.0009924 0.0009523 0.0009131	0.0009378 0.0009629 0.0009885
0.12	0.122 0.123 0.124	0.0036700 0.0036404 0.0036111	0.0025524 0.0025227 0.0024935	0.0024825 0.0025234 0.0025646	0.094 0.095 0.096	0.0029272 0.0028965 0.0028664	0.0015965 0.0015657 0.0015357	0.0014733 0.0015048 0.0015367
0.14	0.142 0.143 0.144	0.0036756 0.0036501 0.0036249	0.0034295 0.0034039 0.0033788	0.0033641 0.0034117 0.0034596	0.115 0.116 0.117	0.0027887 0.0027648 0.0027413	0.0023293 0.0023053 0.0022818	0.0022056 0.0022442 0.0022831
0.16	0.161 0.162 0.163	0.0037012 0.0036785 0.0036562	0.0044612 0.0044385 0.0044161	0.0043258 0.0043797 0.0044341	0.137 .138 0.139	0.0026725 0.0026532 0.0026343	0.0032189 0.0031997 0.0031807	0.0031311 0.0031770 0.0032233
0.18	0.182 0.183 0.184	0.0036796 0.0036597 0.0036400	0.0055805 0.0055606 0.0055410	0.0055298 0.0055909 0.0056522	0.159 0.160 0.161	0.0025876 0.0025715 0.0025557	0.0042747 0.0042587 0.0042429	0.0042188 0.0042721 0.0043258
0.20	0.202 0.203 0.204	0.0036792 0.0036613 0.0036436	0.0068562 0.0068383 0.0068206	0.0068146 0.0068823 0.0069504	0.180 0.181 0.182	0.0025364 0.0025225 0.0025088	0.0054993 0.0054854 0.0054717	0.0054088 0.0054691 0.0055298

value of maximum deflection is given in the column with the heading d in the table.

The computations to obtain d may be simplified still further by the use of equation (2) instead of (14). The origin will then be at the lowest, or center point, of the equivalent catenary. This method will be evident from the calculations used in Appendix No. 2.

Demonstration of the Use of Methods Explained

The use of the methods described is illustrated by the solution of the following problem which arose in some preliminary work on the Tacoma Lake Cushman Power Project.

The transmission line crossing "The Narrows" for the Tacoma Cushman Power Project was originally planned with a main span of 6152.75 feet between two supports at the same elevation. On either side of this main span, a counter span was planned, using the same cable. This is shown in Fig. 7. The span A was assumed to have supports with a horizontal spacing of 2700 feet and with a difference in elevation of 70 feet. Span B has supports with the same horizontal spacing, but with a difference of elevation of 179 feet. To illustrate the solution on a short steep slope, a third span C was assumed with its supports 1000 feet apart horizontally, and with a difference of elevation of 150 feet.

This study was made on a one-inch plow steel cable with a maximum breaking strength of 240,000 pounds per square inch. The elastic limit of the cable was taken at one-half of the breaking strength and this

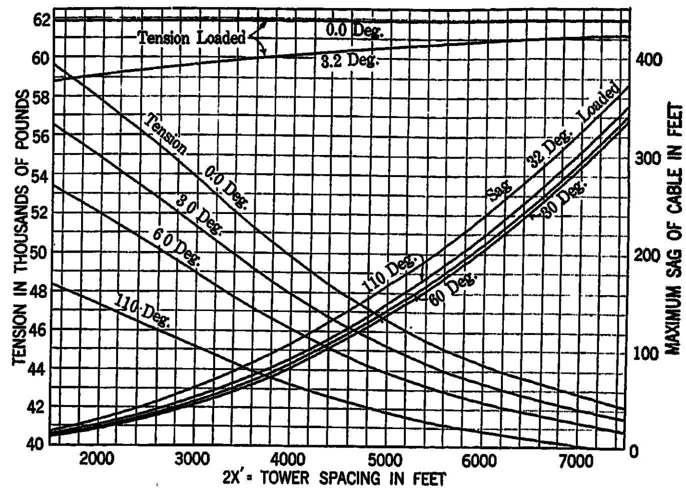

Fig. 8—Temperature-Tension Stringing Chart for One-Inch Diameter High Strength Steel Cable

again reduced 10 per cent for galvanizing, giving 108,000 pounds per square inch as the maximum allowable tension on the cable. All other specifications and constants are given in Table I.

TABLE VI

Design for Supports at Unequal Elevations

First Method

General Data for all Spans with Supports at any Difference of Elevation

Working Formula

$$\cos\beta_m = \frac{1}{\cosh\frac{x}{c}} \qquad \sin\beta_m = \sqrt{1 - \cos_1^2\beta_m}$$

Assumed Value $\frac{x}{c_1}$	32° F. Cable Loaded (50) $\frac{1}{(15)+1}$ $\cos\beta_m$	32° F. Cable Loaded (51) $\sqrt{1-(50)^2}$ $\sin\beta_m$	0.0° F. Cable Only (50) $\frac{1}{(15)+1}$ $\cos\beta_m$	0.0° F. Cable Only (51) $\sqrt{1-(50)^2}$ $\sin\beta_m$	30° F. Cable Only (50) $\frac{1}{(15)+1}$ $\cos\beta_m$	30° F. Cable Only (51) $1-(50)^2$ $\sin\beta_m$
0.08	0.9965711	0.0827405	0.9983688	0.0570936	0.9982107	0.0597947
0.10	0.9947167	0.1026581	0.9971312	0.0756919	0.9968929	0.0787694
0.12	0.9924841	0.1223736	0.9954111	0.0956909	0.9950979	0.988941
0.14	0.9898768	0.1419294	0.9931967	0.1164484	0.9928223	0.1195991
0.16	0.9868982	0.1613440	0.9905056	0.1374722	0.9900863	0.1404603
0.18	0.9835527	0.1806215	0.9873652	0.1584609	0.9869134	0.1612514
0.20	0.9798442	0.1997633	0.9838015	0.1792612	0.9833263	0.1818498

	60° F. Cable Only (50) $\frac{1}{(15)+1}$ $\cos\beta_m$	60° F. Cable Only (51) $\sqrt{1-(50)^2}$ $\sin\beta_m$	110° F. Cable Only (50) $\frac{1}{(15)+1}$ $\cos\beta_m$	110° F. Cable Only (51) $\sqrt{1-(50)^2}$ $\sin\beta_m$		
0.08	0.9980358	0.0626458	0.9977039	0.0677264		
0.10	0.9966369	0.0819441	0.9961723	0.0874115		
0.12	0.9947699	0.1021407	0.9941927	0.1076147		
0.14	0.9924375	0.1227509	0.9917738	0.1280031		
0.16	0.9896589	0.1434409	0.9889321	0.1483686		
0.18	0.9864559	0.1640267	0.9856836	0.1686057		
0.20	0.9828473	0.1844215	0.9820431	0.1886568		

All the data are recorded on forms, with the working formulas and constants given, and with the operation performed in each column clearly indicated at the head of the column. These may be used as forms for the

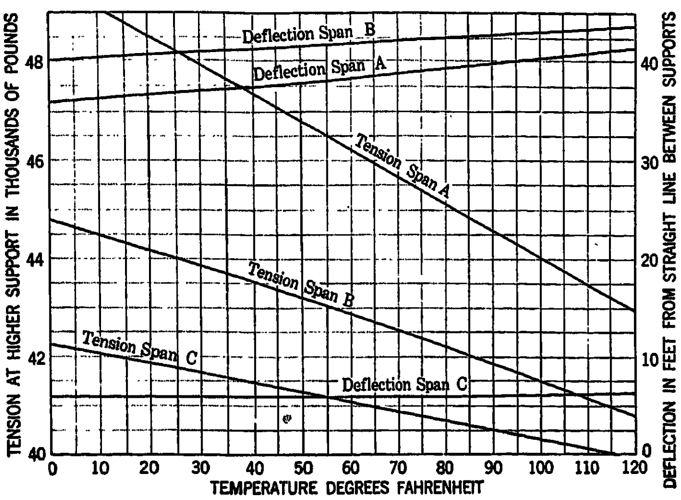

FIG. 9—TEMPERATURE-TENSION STRINGING CHART FOR SPANS WITH SUPPORTS AT UNEQUAL ELEVATIONS

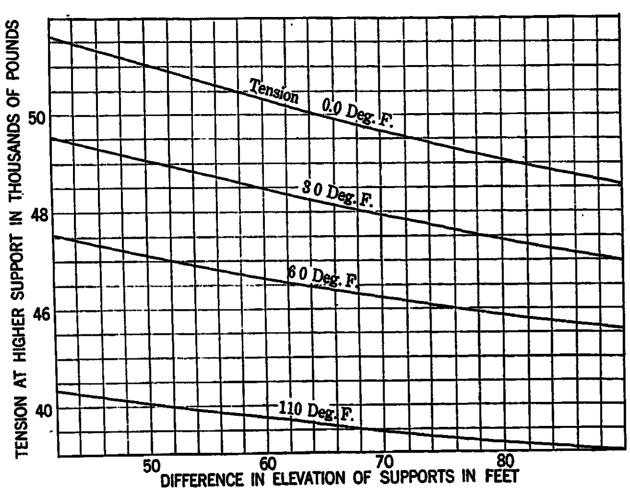

FIG. 10—TEMPERATURE-TENSION CHART FOR VARIOUS DIFFERENCES IN ELEVATION OF SUPPORTS—SPAN LENGTH 2700 FT.

TABLE VII

Design for Supports at Unequal Elevations

First Method

Working Formula

$$b' = \frac{c}{\cos\beta_m}\left[\cosh\frac{a}{c} - \sin\beta_m \sinh\frac{a}{c} - 1\right]$$

$a = 2700$ ft.
$b = 179$ ft.

Temp. and Load Conditions	Assumed Value $\frac{x}{c_1}$	(52) $\frac{2700}{(14)}$ $\frac{a}{c}$	(53) cosh (52) $\cosh\frac{a}{c}$	(54) sinh (52) $\sinh\frac{a}{c}$	(55) (51) × (54) $\sin\beta_m \sinh\frac{a}{c}$	(56) (53) − (55) − 1 $\cosh\frac{a}{c} - \sin\beta_m \sinh\frac{a}{c} - 1$	(57) (56) × (21) $-b'$
32 deg. fahr. Cable Loaded	0.10	0.142638	1.0101900	0.1431222	0.0146926	0.0045026	85.683
	0.12	0.142216	1.0101298	0.1426959	0.0174622	0.0073324	140.262
	0.14	0.141994	1.0100981	0.1424716	0.0202209	0.0101228	194.453
	0.16	0.141945	1.0100911	0.1424221	0.0228799	0.0128878	248.399
0.0 deg. fahr. Cable Only	0.12	0.110988	1.0061655	0.1112160	0.0106424	0.0044769	109.410
	0.14	0.116243	1.0067639	0.1165050	0.0135668	0.0068029	159.096
	0.16	0.120651	1.0072872	0.1209439	0.0166264	0.0093393	211.003
	0.18	0.124298	1.0077349	0.1246183	0.0197471	0.0120122	264.266
30 deg. fahr. Cable Only	0.12	0.114728	1.0065885	0.1149799	0.0113708	0.0047823	113.101
	0.14	0.119417	1.0071387	0.1197010	0.0143161	0.0071774	163.454
	0.16	0.123308	1.0076121	0.1236207	0.0173638	0.0097517	215.665
	0.18	0.126525	1.0080150	0.1268629	0.0204568	0.0124418	269.025
60 deg. fahr. Cable Only	0.12	0.118521	1.0070318	0.1187987	0.0121342	0.0051023	116.846
	0.14	0.122596	1.0075243	0.1229033	0.0150865	0.0075622	167.816
	0.16	0.125961	1.0079436	0.1262943	0.0181158	0.0101722	220.322
	0.18	0.128743	1.0082988	0.1290990	0.0211757	0.0128769	273.762
110 deg. fahr. Cable Only	0.12	0.124921	1.0078028	0.1252462	0.0134783	0.0056655	123.168
	0.14	0.127898	1.0081901	0.1282470	0.0164160	0.0082259	175.093
	0.16	0.130352	1.0085079	0.1307215	0.0193950	0.0108871	228.030
	0.18	0.132405	1.0087784	0.1327922	0.0223895	0.0136111	281.589

Values desired, taken from curves plotted with b' and X, c, T_m, Y, w

	X	c	T_m	Y	p	w
32° Cable Loaded	2608.8	19009	60587	179.2	48.10	3.1575215
0.0° Cable Only	2868.0	22863	44795	179.1*	40.21	1.9438121
30° Cable Only	2834.5	22368	43830	179.4*	41.06	1.9435410
60° Cable Only	2800.0	21880	42875	179.3*	41.84	1.9432702
110° Cable Only	2745.0	21078	41297	179.2*	43.31	1.9428130

*These values of Y lie on the equivalent catenary beyond the point of the lower support.

solution of any span problem by supplying the proper constants and performing the indicated operations.

Tables II to V show the usual tension-sag computations for a range of various span lengths and with several different temperature and loading conditions. These employ the previously mentioned Kirsten method

TABLE VIII

Design for Supports at Unequal Elevation

Second Method

Working Formulas

$$x' = a - X$$

$$y' = c\left(\cosh \frac{x'}{c} - 1\right)$$

a = 2700 ft.
b = 179 ft.

Temp. and Load Conditions	Assumed Value $\frac{x}{c_1}$	(50) From Col. (14) c	(51) 2700 − (4) $x' = a - X$	(52) $\frac{(50)}{(51)}$ $\frac{x'}{c}$	(53) cosh (52) $\cosh \frac{x'}{c}$	54) (50) [(53) − 1] $y' = c\left(\cosh \frac{r'}{c} - 1\right)$	(55) (17) − (54) $Y - y'$
32 deg. fahr. Cable Loaded	0.10	18929.054	749.90991	0.0396169	1.0007849	14.857	85.083
	0.12	18985.172	365.01368	0.0192262	1.0001849	3.510	140.260
	0.14	19014.912	− 17.11687	0.0009002	1.0000005	0.009	194.451
	0.16	19021.441	−396.04389	0.0208209	1.0002168	4.123	248.397
0.0 deg. fahr Cable Only	0.12	24326.829	365.01368	0.0150046	1.00011257	2.738	109.400
	0.14	23227.192	− 17.11687	0.0007369	1.0000004	0.008	159.097
	0.16	22378.667	−396.04389	0.0176974	1.0001566	3.505	211.003
	0.18	21721.941	−771.33990	0.0355097	1.0006305	13.606	264.270
30 deg. fahr. Cable Only	0.12	23533.898	365.01368	0.0155101	1.0001203	2.831	113.101
	0.14	22609.856	− 17.11687	0.0007571	1.0000004	0.009	163.451
	0.16	21896.417	−396.04389	0.0180872	1.0001636	3.582	215.666
	0.18	21339.636	−771.33990	0.0361459	1.0006534	13.942	269.025
60 deg. fahr. Cable Only	0.12	22780.799	365.01368	0.0160229	1.0001284	2.924	116.487
	0.14	22023.594	− 17.11687	0.0007772	1.0000004	0.008	167.814
	0.16	21435.255	−396.04389	0.0184763	1.0001707	3.659	220.321
	0.18	20972.076	−771.33990	0.0367794	1.0006765	14.187	273.761
110 deg. fahr. Cable Only	0.12	21613.639	365.01368	0.0168881	1.0001426	3.083	123.168
	0.14	21110.543	− 17.11687	0.0008108	1.0000004	0.008	175.093
	0.16	20713.203	−396.04389	0.0191204	1.0001828	3.787	228.030
	0.18	20391.937	−771.33990	0.0378257	1.0007155	14.591	281.589

Values desired, taken from curves plotted with b' and X, c, T_m, Y, w

	X	c	T_m	Y	p	w
32° Cable Loaded	2607.8	19009	60591	179.1	48.10	3.1575219
0.0° Cable Only	2863.2	22860	44800	179.0*	40.21	1.9438132
30° Cable Only	2829.7	22373	43818	179.5*	41.06	1.9435425
60° Cable Only	2800.0	21882	42871	179.4*	41.84	1.9432700
110° Cable Only	27450	21078	41298	179.1*	43.31	1.9428123

*These values of Y lie on the equivalent catenary beyond the point of the lower support.

TABLE IX

Tabulated Data for Various Types of Spans with Supports on an Incline

Span	Temp. and Load Condition	X ft.	c	T_m lb.	Y ft.	p ft.	w lb.
Span A a = 2700 ft. b = 70 ft.	32° Cable Loaded	1839.8	18910	59987	90.0	48.23	3.157636
	0.0° Cable Only	2012.0	25477	49626	79.3	35.89	1.943252
	30° Cable Only	1985.5	24586	47917	80.5	37.00	1.943074
	60° Cable Only	1965.0	23703	46220	81.5	38.52	1.942875
	110° Cable Only	1928.0	22295	43473	83.5	40.92	1.942566
Span B a = 2700 ft. b = 179 ft.	32° Cable Loaded	2608.8	19009	60587	179.2	48.10	3.157521
	0.0° Cable Only	2868.0	22863	44795	179.1*	40.21	1.943812
	30° Cable Only	2834.5	22368	43830	179.4*	41.06	1.943541
	60° Cable Only	2800.0	21880	42875	179.3*	41.84	1.943270
	110° Cable Only	2745.0	21078	41297	179.2*	43.31	1.942813
Span C a = 1000 ft. b = 150 ft.	32° Cable Loaded	3341.9	19016	60969	294.3*	6.67	3.157449
	0.0° Cable Only	3696.0	21403	42230	320.6*	5.85	1.944102
	30° Cable Only	3654.3	21115	41660	317.5*	5.97	1.943784
	60° Cable Only	3611.2	20826	41086	314.2*	6.05	1.943466
	110° Cable Only	3539.8	20341	40122	308.9*	6.22	1.942983

*These values of Y lie on the equivalent catenary beyond the point of lower support.

as it was later revised. Fig. 8 shows a complete set of tension-sag curves for various lengths of spans.

Tables VI and VII show the computations for span B obtained by using the first method described, while Table VIII shows the application of the second method to the same span. An inspection of the results obtained by the two methods shows no appreciable difference. The second method illustrated by Table VIII should be the more accurate since it depends upon fewer values interpolated from hyperbolic tables.

Table IX gives the data for all three spans in tabulated form, while Fig. 9 shows the temperature-tension and temperature-sag curves plotted from the data given. It might be noted that considerable more data were computed than would usually be required in a design problem.

Charts with curves showing data for a wide range of conditions may be made with a small amount of additional computations. Thus, Fig. 10 shows a set of tension curves for spans 2700 feet in length but with various differences in elevation of supports. Curves covering a range of differences in elevation of supports several times that shown in Fig. 9 may be interpolated from the data shown in Table VIII without further computations.

Other useful charts might show tension or deflection values for a range of span lengths, with each curve representing a single difference in elevation of supports as well as a definite temperature condition. Such a chart would require computations similar to Table VIII for four or five different span lengths. With this amount of data, a series of charts could be made which would cover all of the varying conditions of practically any line.

Since the method of design presented here requires the computations for the usual tension-sag data, it is somewhat longer than most methods given. However, in the usual design of a transmission line the tension-sag data for the various symmetrical spans are always required. Thus the computations for the spans with supports on an incline require little additional labor, while in the design of very long spans the additional labor required is more than compensated for by the complete, definite, and accurate results obtainable.

Appendix No. 1

The conditions for the derivation of the formulas, with their origin at the upper support, may be obtained from Fig. 3.

Let S = total length of the cable between A and B: Then the total weight of the cable suspended is

$$w\,S = T_m \sin\beta_m + T_1 \sin\beta_1 \qquad (4)$$

The horizontal tension must be the same at each end

$$T_m \cos\beta_m = T_1 \cos\beta_1 = c\,w \qquad (5)$$

where c is the length of cable whose weight is equal to the horizontal tension. From the above

$$\cos\beta_m = \frac{c\,w}{T_m} = (1 - \sin^2\beta_m)^{\frac{1}{2}} \qquad (6)$$

Other useful relations may be derived from the enlarged sketch. Thus, horizontal tensions are equal throughout the span;

$$T_m \cos\beta_m = T\cos(\alpha + d\,\alpha) \qquad (7)$$

Vertical tensions may be expressed thus:

$$T_m \sin\beta_m - T\sin(\alpha + d\,\alpha) = w\,s \qquad (8)$$

With $d\,s$ at its smallest limit

$$\cos(\alpha + d\,\alpha) = \frac{d\,x}{d\,s} \text{ and } \sin(\alpha + d\,\alpha) = -\frac{d\,y}{d\,s} \qquad (9)$$

From the triangle relation of forces

$$T^2 = [\,T\sin(\alpha + d\,\alpha)\,]^2 + [\,T\cos(\alpha + d\,\alpha)\,] \qquad (10)$$

Extracting sq. root and substituting (7), (8), and (9) in (10):

$$\frac{d\,x}{d\,s} = \frac{T_m \cos\beta_m}{[\,T_m^2 - 2\,w\,s\,T_m \cos\beta_m + w^2 s^2\,]^{\frac{1}{2}}} \qquad (11)$$

From (5)

$$\cos\beta_m = \frac{c\,w}{T_m} = [\,1 - \sin^2\beta_m\,]^{\frac{1}{2}} \qquad (12)$$

$$c^2 w^2 = T_m^2 - T_m^2 \sin^2\beta^2 \qquad (13)$$

Substituting (12) and (13) in (11),

$$d\,x = \frac{c\,w\,d\,s}{[T_m^2 - 2\,w\,s\,(T_m^2 - c^2 w^2)^{\frac{1}{2}} + w^2 s^2\,]^{\frac{1}{2}}}$$

Let $T_m = K\,w$

$$d\,x = \frac{c\,d\,s}{[\,K^2 - 2\,s\,(K^2 - c^2)^{\frac{1}{2}} + s^2\,]^{\frac{1}{2}}}$$

$$x = \int_0^s \frac{c\,d\,s}{[K^2 - 2\,s\,(K^2 - c^2)^{\frac{1}{2}} + s^2\,]^{\frac{1}{2}}}$$

$$\frac{x}{c} = \log \frac{s - (K^2 - c^2)^{\frac{1}{2}} + (K^2 - 2\,s\,[K^2 - c^2]^{\frac{1}{2}} + s^2)^{\frac{1}{2}}}{K - (K^2 - c^2)^{\frac{1}{2}}}$$

But $K = \dfrac{T_m}{w} = \dfrac{C}{\cos\beta_m}$ and $\dfrac{1}{\cos^2\beta_m} - 1 = \tan^2\beta_m$

Substituting and taking log

$$\epsilon^{\frac{c}{x}} = \frac{s\cos\beta_m - c\sin\beta_m + (c^2 - 2\,s\,c\sin\beta_m\cos\beta_m - s^2\cos^2\beta_m)^{\frac{1}{2}}}{c - c\sin\beta_m}$$

Squaring and collecting terms

$$s = c\left[\tan\beta_m - \tan\beta_m\left(\frac{\epsilon^{\frac{c}{x}} + \epsilon^{-\frac{c}{x}}}{2}\right) + \sec\beta_m\left(\frac{\epsilon^{\frac{c}{x}} - \epsilon^{-\frac{c}{x}}}{2}\right)\right]$$

$$s = \frac{c}{\cos\beta_m}\left[\sin\beta_m - \sin\beta_m\cosh\frac{x}{c} + \sinh\frac{x}{c}\right]$$

$$s = \frac{c}{\cos\beta_m}\left[\sinh\frac{x}{c} - \sin\beta_m\left(\cosh\frac{x}{c} - 1\right)\right] \qquad (14)$$

This is the general equation for the actual length of

cable between one support and any point on the catenary x distance from support. β_m is the angle between the tangent to the wire at the point of support and the horizontal, the value of which may be found by the relationship given in (12).

In a manner similar to that above, the equations for the sag $-y$ or tension T for any point on the catenary at x distance out may be derived as follows:

$$\frac{T_m \sin\beta_m - w\,s}{-\dfrac{d\,y}{d\,s}} = [(T_m \sin\beta_m - w\,s)^2 + (T_m \cos\beta_m^{2}]^{1/2}$$

$$-\frac{d\,y}{d\,s} = \frac{T_m \sin\beta_m - w\,s}{[T_m^2 - 2\,w\,s\,T_m \sin\beta_m + w^2\,s^2]^{1/2}}$$

And from (5)

$$-\frac{d\,y}{d\,s} = \frac{c\,w \tan\beta_m - s\,w}{\left[\dfrac{c^2\,w^2}{\cos^2\beta_m} - 2\,c\,w^2\,s \tan\beta_m + w^2\,s^2\right]^{1/2}}$$

$$d\,y = \frac{s - c\tan\beta_m}{\left[\dfrac{c^2}{\cos^2\beta_m} - 2\,c\,s \tan\beta_m + s^2\right]^{1/2}}\,d\,s$$

$$y = \int_0^s \frac{s\,d\,s}{\left[\dfrac{c^2}{\cos^2\beta_m} - 2\,c\,s \tan\beta_m + s^2\right]^{1/2}}$$

$$- c\tan\beta_m \int_0^s \frac{d\,s}{\left[\dfrac{c^2}{\cos^2\beta_m} - 2\,c\,s \tan\beta_m + s^2\right]^{1/2}}$$

$$y = \left[\frac{c^2}{\cos^2\beta_m} - 2\,s\,c \tan\beta_m + s^2\right]^{1/2} - \frac{c}{\cos\beta_m}$$

But the value of s is given by equation (14). Substituting and simplifying,

$$y = \frac{c}{\cos\beta_m}\left[\cosh\frac{x}{c} - \sin\beta_m \sinh\frac{x}{c} - 1\right] \qquad (15)$$

Again substituting (7) and (8) in (10),

$$T^2 = [\,T_m \sin\beta_m - w\,s\,]^2 + [\,T_m \cos\beta_m\,]^2$$
$$T^2 = T_m^2 - 2\,T_m \sin\beta_m\,w\,s + w^2\,s^2$$

Substituting values from (12) and (14),

$$T = \frac{c\,w}{\cos\beta_m}\left[\cosh\frac{x}{c} - \sin\beta_m \sinh\frac{x}{c}\right] \qquad (16)$$

Appendix No. 2

An investigation to find the position on the catenary of the point which has a maximum deflection from the straight line between the points of support of the catenary shows some interesting results.

Referring to Fig. 6, the catenary $A D' B$ will have a maximum deflection from line $A\,B$ at some point near the middle of the span. At the point of maximum deflection, the tangent line will have the same slope as $A\,B$, which is $-\dfrac{b}{a}$. The method to be used in finding the position of the point of maximum deflection will be to assume that the tangent at the middle point, D, has the same slope as line $A\,B$, and to compute the error found in making this assumption.

With the origin of the catenary at 0, the value of vertical deflection from point 0 is given by equation (2)

$$y = c\cosh\frac{x}{c} - c$$

The slope of the tangent at any point on the cable is the first derivative of the foregoing equation, or

$$\frac{d\,y}{d\,x} = \sinh\frac{x}{c} \qquad (19)$$

If x is the value at which the cable has a maximum deflection from line $A\,B$, then

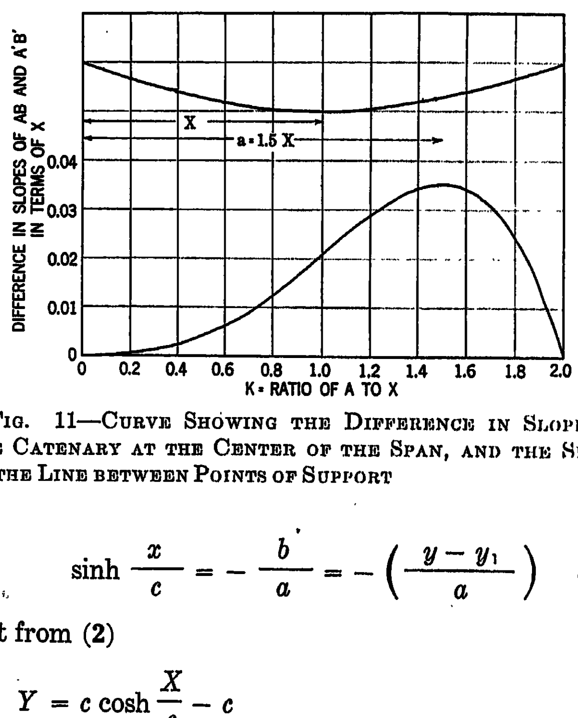

FIG. 11—CURVE SHOWING THE DIFFERENCE IN SLOPE OF THE CATENARY AT THE CENTER OF THE SPAN, AND THE SLOPE OF THE LINE BETWEEN POINTS OF SUPPORT

$$\sinh\frac{x}{c} = -\frac{b}{a} = -\left(\frac{y - y_1}{a}\right) \qquad (20)$$

But from (2)

$$Y = c\cosh\frac{X}{c} - c$$

and

$$y_1 = c\cosh\frac{x_2}{c} - c = c\cosh\frac{a - X}{c} - c$$

Substituting these values in (20)

$$\sinh\frac{x}{c} = -\frac{c}{a}\cosh\frac{X}{c} + \frac{c}{a}\cosh\frac{a - X}{c} \qquad (21)$$

The hyperbolic function may be expressed as a series. Thus if the first two terms for the sinh and the first three for the cosh are substituted, in (21), the error will be very small, since the next term in the series is of the

fifth power, and the ratio of $\frac{x}{c}$ is never greater than ⅓.

Then $\frac{x^5}{c^5\,/5}$ will be of the order .000034 +.

Evaluating equation (**21**) thus,

$$\frac{x}{c} + \frac{x^3}{6\,c^3} = -\frac{X^2}{2\,a\,c} - \frac{X^4}{24\,a\,c^3} + \frac{(a-X)^2}{2\,a\,c} + \frac{(a-X)^4}{24\,a\,c^3}$$

or

$$x^3 + 6\,x\,c^2 = 3\,a\,c^2 - 6\,X\,c^2 + \frac{a^3}{4} - a^2\,X + \frac{3\,a\,X^2}{2} - X^3 \quad (22)$$

If x_1 is substituted for x in (**22**), the portion before the equality sign will be the slope of the tangent at the middle of span a.

Thus from Fig. 6,

$$x = -\,x_1 = -\left(X - \frac{a}{2}\right)$$

Substituting and simplifying, but keeping all terms belonging to each value of slope on its side of the inequality sign, we have only the terms which do not cancel out.

TABLE X

K Ratio of a to X $a = K\,X$	Difference in slopes of lines $A\,B$ and $A'\,B'$
	In terms of X
2.	0.0
1.9	0.0143
1.8	0.0243
1.7	0.0307
1.6	0.0341
1.5	0.0352
1.4	0.0342
1.3	0.0320
1.2	0.0288
1.1	0.0248
1.0	0.0208
.8	0.0128
.6	0.0063
.4	0.0021
.2	0.0003
0	0

$$\frac{a^3}{8} - \frac{3\,a^2\,X}{4} = \frac{a^3}{4} - a^2\,X \quad (23)$$

A careful analysis of all the terms to the right and left of the inequality sign in the operation indicated above would disclose the fact that all terms would cancel if only the first term of the sinh series and the first two terms of the cosh series were used. In other words, using these first two terms as an approximation of the true value, the point of maximum deflection would always be at the center of the span. The terms in the inequality (**23**) constitute only a small portion of the total number of terms in the total slope value.

The numerical value of (**23**) would be the difference in the slope values for the two straight lines $A\,B$ and $A'\,B'$. Since the term on the left is the smaller, the desired point on the catenary must be a little nearer the higher support.

Evaluating (**23**) with a given in terms of X, the difference in slopes, also given in terms of X, may be found for any assumed value of a. The results are shown in Table X and the corresponding curve in Fig. 11. Thus (**23**) is an equality if a is equal to $2\,X$ or to zero. The point at which the difference is greatest occurs when $a = 1.5\,X$ appx.

TABLE XI

a 1.5 X

			To satisfy equation 24	% error in terms of
.333	1	3	−0.2525	0.17
.250	1	4	−0.2515	0.10
.200	1	5	−0.2509	0.06

To determine just how far from the center of the span the desired point on the catenary lies, values of c and a, both expressed in terms of X, may be substituted in (**22**). Then a value of x which will satisfy the equation may be found by inspection. Thus to find the greatest possible variation, let $a = 1.5\,X$. Then

$$x^3 + 6\,c^2\,x = -\,1.5\,X\,c^2 - 0.1563\,X^3 \quad (24)$$

The ratio of $\frac{x}{c}$ is a variable for different spans and increases in value as the tension is decreased or as the weight of the cable is increased, other conditions remaining constant. However, the value of the ratio $\frac{x}{c}$ will probably never be found as high as ⅓, and in all practical design problems will range between ¼ and 1/5.

Assuming $X = 1$, and $c = 4$, equation (**24**) will be satisfied by $x = 0.2515$ approx. From Fig. 6, if $X = 1$, and $a = 1.5$, $\frac{a}{2}$ will have the value of 0.75, and x_1 will be − 0.25. The difference between x and x_1 is a true measure of the error with the values assumed; that is,

TABLE XII

Span	Temp. deg. fahr.	K $a = K\,X$	Ratio $\frac{X}{c}$	At center	At point 5 ft. toward higher support	At point 5 ft. toward lower support
A	32	1.47	0.097	48.226	48.225	48.225
A	0.0	1.34	0.079	35.886	35.886	35.886
A	60	1.37	0.083	38.518	38.517	38.517
B	32	1.03	0.137	48.102	48.101	48.100
B	0.0	0.94	0.125	40.208	40.208	40.208
B	60	0.98	0.130	43.314	43.314	43.313
C	32	0.30	0.175	6.672	6.658	6.658
C	0.0	0.28	0.173	5.854	5.854	5.853
C	30	0.27	0.173	5.966	5.966	5.966

when $\frac{x}{c} = \frac{1}{4}$, the true point of maximum deflection lies 0.0015 X from the center point of the span. This is an error of 0.1 per cent when considered in terms of a.

Since the values of deflection are nearly equal for a considerable distance on either side of this maximum point, the per cent of error caused by assuming the maximum deflection at the center point is still smaller.

Table XI gives the distances of the true maximum points from the center of the span, as well as the per cent of error for $a = 1.5$, and for three assumed ratios of $\frac{x}{c}$.

Table XII gives the deflection of the catenary from the straight line between supports, as computed from the data given in Table IX. In each case three points on the catenary were selected, one at the center of the span, and one five feet on either side of the center. In almost every case these data clearly indicate the same facts proved by the foregoing discussion.

Discussion

F. K. Kirsten: I wish to congratulate Mr. Smith on his courage in using the catenary equations for the analysis of spans supported from points at unequal elevations, and I would also like to make some supplementary comments.

We know that the more closely we wish to describe a physical phenomenon by means of mathematical formulas, the more involved and unwieldy these formulas become. As an illustration, we might use the simple equation of a circle to express the location in space of a suspended cable, and describe actual conditions with sufficient accuracy although the assumptions involve considerable error. By a closer analysis of the forces in action we find, however, that the parabola describes actual conditions better than the circle and hence the application of the parabola yields greater accuracy, although the mathematical operations are more involved. But still a considerable error is made in the assumption that the weight per unit projected length of the span is uniform along the span. The demand for greater accuracy forces us to base our mathematical formulas upon the assumption that the weight per unit length of span, measured along the span, is constant. This assumption leads us to the catenary, a rather involved mathematical stratagem. This mathematical form is rather difficult to handle, especially if temperature changes accompanied by stress changes must be covered by its use. And still the catenary equation does not describe conditions with absolute accuracy. Since the tension changes along the span, the weight per unit length of cable cannot be uniform. But an attempt to involve this actual condition in the modification of the catenary form would lead to unmanageable expressions.

It will be apparent from a perusal of Mr. Smith's work that the chief difficulty in applying the catenary equation to spans of unequal elevations resides in the introduction of the slope of the suspension points into equations which are already complex enough for ready manipulation. Especially do we feel a naturally increasing reluctance to use the catenary analysis if changes of wind and ice loading together with temperature variations over a considerable range must be accounted for by proper mathematical operations with the catenary form. It is, therefore, in my opinion, a step in the right direction to adhere to the simple catenary forms which Mr. Smith ingeniously introduces into his second method instead of using the first method supported by equations **12**, **14**, and **15**.

We now have, thanks to Mr. Smith's work, a simple catenary analysis of spans of unequal elevation, and there cannot be any excuse for the use of parabolic forms in the future. It must be remembered that the catenary method is independent of fixed mass and space units; hence all span conditions which may occur in practise may be expressed by a series of curves from which by interpolation any span, at any slope, for any size or material of cable or any temperature range, may be read at once. This set of curves is as readily applied, without modifications by constants, in continental Europe where the metric system is used, as in the lands of inches, pounds and quarts.

Another important finding in Mr. Smith's paper, I believe, is the discovery that the point of maximum deviation from a straight line connecting any two points of unequal elevation of the cable in suspension occurs midway between the points of support in the middle of the span. This naturally facilitates a check upon the strain of a cable from points of unequal elevation by dropping targets from point B_2 to point A, of predetermined length, the line of vision touching the point of maximum deflection from the line joining these two points.

R. J. C. Wood: Our engineering department has done a good deal of work in calculating sags and we went into this question of catenary versus parabola quite fully and find that up to about a 1000-ft. span, the parabola is practically a satisfactory curve to use. It must be remembered that after the mathematician has figured out the sag of the line, some fellow in overalls is going to pull that line up as near as he can to either the predetermined sag or tension.

We have been a little undecided as to whether to use the sag or the tension in the line as the criterion. At one time we have used one and at another time the other. We have finally decided to use a dynamometer to measure the tension in the new line. The maximum tension under worst conditions will be about 12,000 lb. and under ordinary stringing conditions may be 6000 or 7000 lb. The dynamometer will probably read within 200 lb. so that any extreme refinement of calculation as to the desired tension is not necessary.

I do not wish to detract in any way from the fine work of mathematicians who calculate these things because, while we may be able to use the laws of a parabola up to a certain point, yet we depend upon the mathematician to tell us where that point is and from where on we should use the more accurate formula. We have ourselves used the catenary formula in all long-span work.

Our spans run up to about 3000 ft. and for that length of span the catenary is necessary. We have had to go a little further than indicated in Mr. Smith's paper, because we have found it quite necessary to calculate clearances to ground under wind conditions. In building the line on a hillside it is not only necessary to know the vertical clearance in still air, but it is quite essential to know that the line will not blow onto the ground when the wind is crosswise, so we have to calculate those conditions of load applied sideways.

It leads to some very interesting mathematics when there are, for instance, two spans which are dead-ended at the outer ends and hanging on a string of suspension insulators at the middle support, and the wind blows sideways, because the two spans then no longer remain in a plane but become a warped figure and the catenary calculations become quite involved.

C. E. Magnusson: Reference to the parabola by Mr. Wood leads me to bring out a point on the use of hyperbolic functions. Many engineers seem to think that using hyperbolic functions in connection with engineering problems is throwing out a smoke screen to prevent the reader from following the argument. They do not appreciate that for certain types of problems hyperbolic functions become a very convenient means for obtaining accurate solutions. A few years ago while discussing a problem with a

nationally prominent engineer, one who has presented many papers before the Institute, I suggested that for an accurate solution hyperbolic functions should be used, and was astounded to find that he had no idea as to what type of problems would require hyperbolic functions. As there may be others in the same predicament let me state that the basis for using circular or hyperbolic functions is simply this: A rotating vector of constant magnitude can be fully represented by circular functions, but if the radius vector varies in length while changing in phase position hyperbolic functions fit the case. For example, the voltage and current functions along a transmission line vary in magnitude as well as in time-phase position and hence require hyperbolic functions as they cannot be correctly expressed by circular functions.

R. W. Sorensen: I want to second the motion on hyperbolic functions. Dr. Kennelly of Harvard has been trying for years to get us to use them. I have been trying for about fifteen years to get engineering classes to use them, and they use them just as readily as they use trigonometric functions if you once start them off.

L. J. Corbett: The paper given by Mr. Kirsten[1] a number of years ago is, I think, a classic in its field of the application of hyperbolic functions to catenaries and long spans. I think Mr. Smith's paper is a very creditable companion paper to that one.

I like particularly Fig. 4 in which the common point A is used as a basis instead of the usual method, although I have not had time to check it and see whether there are particular advantages to be gained over the old method of calculating sag from the inclined line between A and B.

Mr. Smith, in his discussion, brought up also a point in which we were very much interested in the re-insulation of the Carquinez crossing of the Pacific Gas & Electric Company, and that is the difference in expansion between a long span and a contiguous short span. If you can, visualize that crossing. On the south side the anchors are an average of probably 80 ft. from what is called South Tower. Then comes the long span of 4427 ft., then a span of 1350 ft., and then a final span of 335 ft. to the other set of anchors. On the high tower between the 4427-ft. span and the 1350-ft. span, was placed a saddle with a sliding top. This was to allow for a difference in expansion between the two spans and to insure a vertical reaction on the tower. We calculated by a number of methods, but chiefly we went back to the original hyperbolic functions, and we thought that, taking all things into consideration, we might expect a possible travel of 5 in. We allowed for a travel of somewhat more than that, 8 in. being the final figure allowed, 4 in. in each direction.

The saddle was placed on roller bearings which were immersed in grease so as to offer the very freest possible travel for the cable. The operating department tells me that in the past year, according to the marks on the saddle, there has been a travel of only 1 in. so it is questionable how successful our calculations were. There is still room for improvement in our methods of calculation for long and important spans.

M. T. Crawford: I find Mr. Smith's paper very interesting in that he has gone into unequal supports, something which seems to have been more or less avoided in considerations of span problems. I would ask if he could suggest a method of approach toward the solution of a problem that we have. At one previous Pacific Coast Convention, a paper[2] was presented describing transmission-line construction in crossing Stampede Pass in the Cascade Range, where the loading conditions were very extreme at certains times of the winter, and where it was found advisable on account of the unequal loading which would come alternately on successive spans to change all dead-ends to a suspension form of conductor support, doubling up the insulator springs with yoked attachment at the suspension point.

The changes described in the previous paper have proved eminently successful in eliminating the operating troubles we had of jerking insulator strings in two. In trying to calculate the sags which would result from unequal loading in successive spans, we found a complication came into the matter where an entirely suspension form of construction was used, and where the towers were at different elevations, in that heavier loading in one span would pull the suspension strings out of the vertical position. This would make a change in the length of conductor in the span, location of points of support, and other factors, which are assumed constant in most of the ordinary methods of approaching the subject.

We worked it out fairly closely by making assumptions and trials, but found that the extreme condition which we might assume would occur when the insulator string was pulled out to a position approximately tangent to the catenary. This would result in the wire being down in the snow in winter conditions, but we have never found in practise that it went that far, because there was always some tension in the adjoining spans which would prevent the strings from pulling that far out.

I would like to have Mr. Smith add some discussion or suggestions as to how we might approach the problem of calculating the result of extremely unequal loading in the successive spans where the suspension form of construction is used.

G. S. Smith: I would like to thank Mr. Wood for mentioning a point I did not have time to bring up in the presentation and perhaps did not make clear in the paper, that is, that the method presented was intended primarily for long spans or special problems. It would usually prove too laborious for a single short span. However, where the Kirsten method is applied to the various symmetrical spans encountered in the usual line, it requires only a small amount of additional work to compute, by the method presented here, the remaining spans whose supports are not at the same elevation, since the cable used is commonly uniform throughout.

In discussions by Dr. Magnusson, Professors Kirsten and Sorenson, a more general use of hyperbolic functions was advocated. My experience has been somewhat similar to the instances mentioned. While most of us are more or less reluctant to use hyperbolic functions freely, I believe it is largely because of two reasons: first, we have never become accustomed to thinking in terms of such functions, and second, in attempting to use them we find it difficult because so few good tables of hyperbolic functions are available. In some previous work in connection with transmission-line design, I found it necessary to compute sufficient tables for this particular use. These tables will be found in one of the University of Washington Experiment Station bulletins referred to in the paper.

Messrs. Corbett and Crawford pointed out some very interesting problems in this same connection, problems which might be termed those of "variable spans." A similar problem was encountered in the design of "The Narrows" span of the Tacoma Lake Cushman Project mentioned in this paper. It was in an attempt to apply the Kirsten method to such problems that I found it desirable to first work out the problem of spans with supports at unequal elevations. Thus far I have found no direct method of attacking such problems, but the possibility of finding such a method seems entirely feasible.

1. Trans. A. I. E. E., 1917, p. 735.
2. Transmission-Line Construction in Crossing Mountain Ranges, by M. T. Crawford, A. I. E. E. Transactions, 1923, page 970.

The Long Span Across The Narrows at Tacoma

BY J. V. GONGWER[1]
Non-Member
and
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Associate, A. I. E. E.

THE double transmission lines now being built by the City of Tacoma, as part of its new Cushman Power Development, connecting Power House No. 1 and No. 2 (future) with the Cushman Substation in Tacoma, reaches 44 mi. from the North Fork of the Skokomish River in the foot hills of the Olympic Mountains around Hood Canal, across North and Henderson Bays, and over The Narrows to Tacoma.

The crossings at North and Henderson Bays are over shallow water, permitting of the installation of off shore towers, thereby materially reducing the length of spans that would otherwise have been necessary at these two crossings. Plans and profiles of the crossings at North and Henderson Bays showing the length of spans and the general features of the design are demonstrated herein.

Over The Narrows is being constructed what is believed to be the longest aerial span in the world, for the purpose of the transmission of electrical energy. The Narrows is a channel approximately 4800 ft. wide from shore to shore, 40 fathoms deep at the point of greatest depth, with a bottom strewn with large rocks. The entire tide run of lower Puget Sound must pass through this restricted channel, the current attaining a velocity of approximately 10 mi. per hour.

The use of submarine cable was never seriously considered for the crossing of The Narrows because of the danger of injury to cable by abrasion over the rocky bottom in the current of the ever changing tide, and further, because it would have been necessary to locate a step-down transformer station on the west side of The Narrows to reduce the line voltage from 110 kv. to a voltage at which submarine transmission can reasonably be expected to offer dependable service, at present probably not in excess of 50 kv. Both the initial and operating costs for the submarine crossing would be considerably in excess of those items for an aerial span.

An all-land line, avoiding the crossing of The Narrows, would, of necessity, have skirted the radial waterways of lower Puget Sound by way of Shelton and Olympia, a distance of 64 mi., 20 mi. longer than by the line of way of The Narrows, and through a district where a transmission line right-of-way would have been considerably more costly.

The natural conditions at The Narrows are very favorable for an aerial span, there being a high bluff on both sides of good bearing soil on which to locate supporting towers and cable anchorages, although the high ground is well back from the water's edge, requiring a single span of 6241 ft. 6 in. in order to take advantage of the tower sites.

CHOICE OF CONDUCTOR

The strength, size, and electrical characteristics of the conductor used are, to a large degree, the determining factors in the supporting tower and anchor design and considerable study was given to its choice.

Three general types of conductor cables were considered, viz: all steel, steel core with an aluminum envelope, and steel core with a copper envelope. Two circuits were to be built across The Narrows to operate at 110 kv., with grounded neutral, each circuit to be of sufficient capacity to carry 300 amperes under normal conditions and 600 amperes in the event that one circuit might be out of service and the entire load thrown on the other. The Narrows being a navigable body of water, the War Department required that a conductor clearance of 200 ft. above high tide be maintained. The Washington state laws governing electrical construction specify that transmission lines shall be designed to withstand a wind load of 8 lb. per sq. ft. on the projected diameter of the conductor when incased with ½ in. of ice, at a temperature of zero deg. fahr., and under these conditions shall have a factor of safety on the ultimate strength of the conductor of not less than two.

The inherent objection to a steel cable as a conductor arises because of the high electrical losses and the resultant heating in the cable which seriously impairs its strength if the temperature of the conductor is permitted to exceed the temperature at which the annealing of high strength steel begins. This temperature is quite generally agreed upon by steel cable manufacturers as being approximately 400 deg. fahr. if that temperature is maintained for long periods of time.

Concentric cable has the advantage of having more cross sectional area of material for a given diameter than any other form of stranding, thus presenting less diameter to the wind and offering less surface for the formation of ice for a given strength of cable. The electrical losses of such a cable may be minimized by reducing the size of the individual strands to the least practical diameter and reversing the direction of the spiraling of the successive layers.

Tests to determine the losses and heating were conducted on two samples of cable, each one inch in diameter and composed of 37 strands of galvanized steel having an ultimate strength of 195,000 lb. per sq. in., as a cable, after the galvanizing and stranding operations. One sample had all strands spiraled in the same direction, and the other had the successive layers spiraled in the opposite direction. These tests indicated that at 300 amperes, the normal full load of each

1. Both of the Cushman Power Project, Tacoma, Wash.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash. Sept. 15-19, 1925.

transmission line, the electrical losses in the cable having the reverse lay were only 60 per cent of the losses of the uniformly spiraled cable. Thermometer readings of the surface temperature indicated that even with the reverse lay cable to carry 600 amperes of current in one transmission circuit, using steel cable of the required strength, a diameter of at least 1¼ in. would be required to insure that the annealing temperature would not be dangerously exceeded.

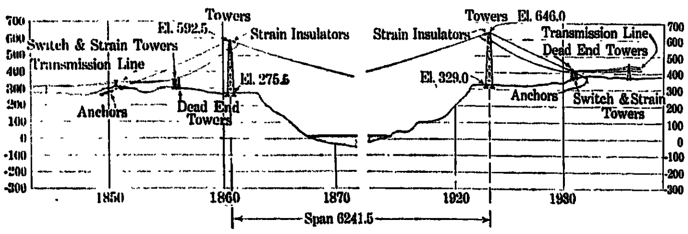


Fig. 1—General Diagram of Crossing

A 1¼ in., 35-strand, specially constructed, concentric, reverse lay steel cable having an ultimate strength of 180,000 lb. after galvanizing and stranding was chosen as offering the greatest economy in the first cost of the complete aerial crossing and its subsequent operation.

Circuit Arrangement

The general design of the crossing is shown in Figs. 1 and 2. The conductors are arranged in a horizontal plane 30 ft. between phases and 120 ft. between the two circuits. The four main towers are each 316 ft. 6 in. high from the tower footing to the point of cable support at the top of the tower and are of the simple prop type. The conductors proper terminate at a nest of 12 multiple strings of high-strength suspension insulators, each string consisting of 11 units, equalized by means of springs to insure the proper mechanical loading of the individual strings, and the entire assembly attached to the supporting cables which pass over large sheaves on the top of the main towers and thence down and back approximately 900 ft. to anchors imbedded in the earth.

Jumper cables of copper attach to the steel conductors of the long span on the water side of the main towers and are carried through and around the latter back to switching towers near the main cable anchors. Disconnecting switches are provided here to isolate either crossing circuit and to multiple the transmission lines through one crossing circuit, or if necessary any three of the six conductors crossing The Narrows may be selected for a circuit and tied in with both transmission lines.

General Structural Design

To reduce maintenance cost, and to attain the utmost dependability in service, simplicity of design was sought in all features.

Investigations were conducted over a period of several years and such information gathered as was available concerning similar installations, notably the crossings of the Pacific Gas & Electric Company and of the Great Western Power Company over Carquinez Straits, the Knoxville Power Company's crossing over the Little Tennessee River, and the crossing of the Shawinigan Power Company over the St. Lawrence River. Considerable correspondence was carried on and some interviews were had with the engineers and officials of the companies mentioned.

In the design of towers, foundations, and anchors it was possible to proceed along the lines of standard practise. Matters concerning the strength and other characteristics of cables—fittings and insulators—are now also fairly well established. However, the question of the behavior of this equipment under service similar to that proposed opens up a large field for discussion and research, probably the most troublesome consideration being the matter of vibration in long suspended cables.

It was desired to avoid, so far as practicable, all undesirable features affecting upkeep and continuity of service as experienced in similar installations.

It was early decided that a messenger span with 1⅜ in. concentric strands, working at a factor of safety

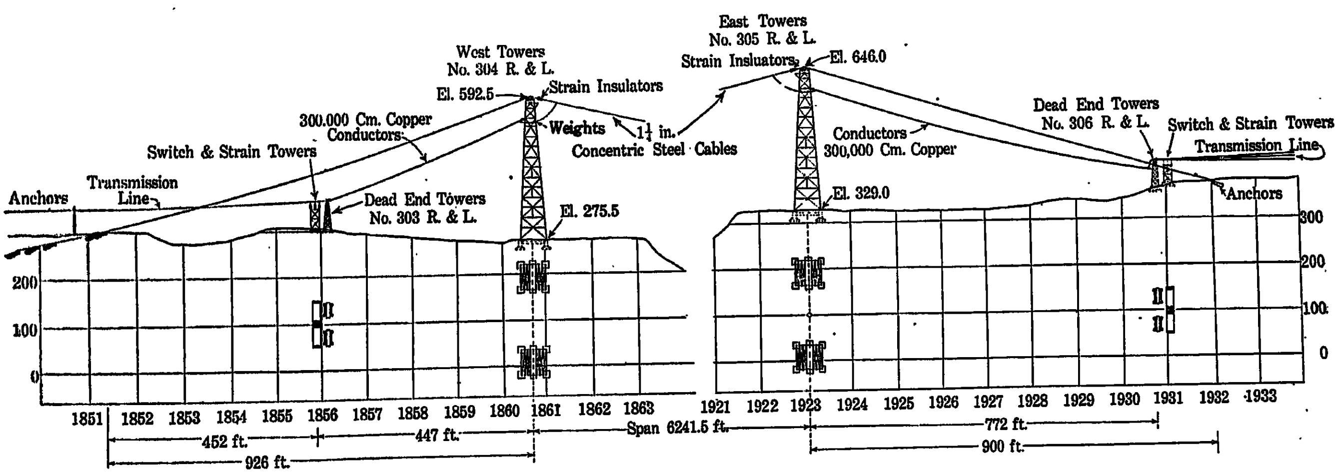


Fig. 2—Elevation and Plan of Crossing

of two and supporting separate conductors, would be the most desirable arrangement, the suspension insulators to be placed at irregular intervals to damp vibration. Further study, however, led to the final adoption of 1¼ in., 35-wire concentric steel strands, acting as self supporting conductors, using a factor of safety of three based upon the ultimate strength of the strand, and supplying at the supports and anchors devices in the way of fittings and relieving cables which would minimize the stresses and effects set up by vibration.

The combination of devices adopted are believed to be novel, but some of these features have been suggested by similar installations.

It has been the aim throughout, wherever indeterminate shocks and variations of stress are to be expected, to provide such ample factors of safety as to remove all possibility of failure from these sources.

Economical Considerations Affecting Sag and Tower Heights

The factor of safety to be provided in the cables having been decided upon, comparative estimates were prepared, based upon various cable sizes, and the corresponding sags, tower heights, anchors, etc.

The costs of cables, and fittings, towers, and anchors were interdependent and must be computed as a whole for each design.

It was found that, as size of cables and consequently tower heights and anchorages were varied within practicable limits, the total cost of the complete crossing remained comparatively fixed.

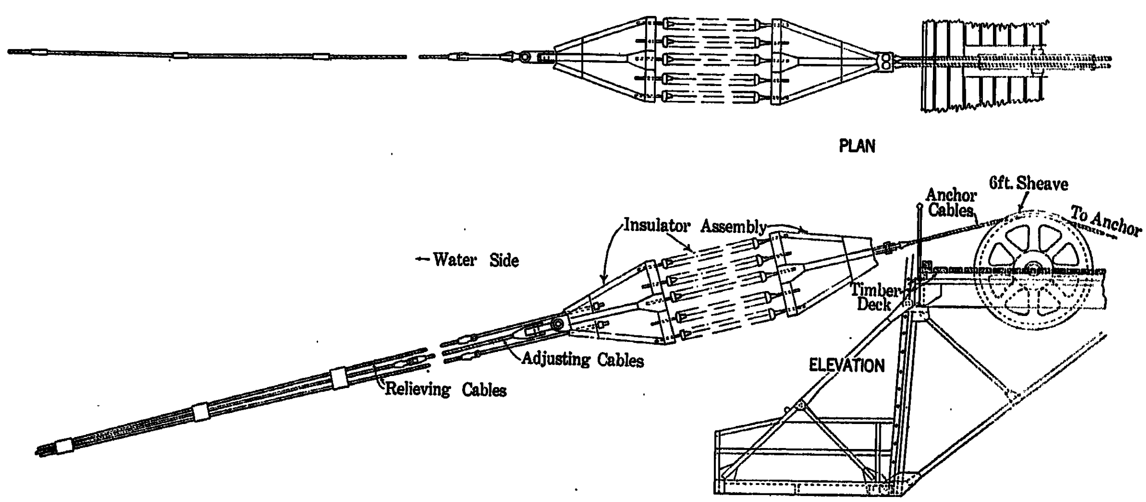


Fig. 3—Showing Assembling of Insulators

It was therefore decided to base the design upon 1¼ in. cable, as being the smallest size that would satisfy electrical requirements.

As to possible necessity for replacement, it is interesting to note that some of the cables of the Pacific Gas and Electric Company's span, under similar conditions, have been in continuous service for 24 years, with an unknown length of useful life still remaining. Examination of one of these cables showed the outside wires to be only slightly deteriorated and the interior wires to be in perfect condition, with galvanizing unimpaired.

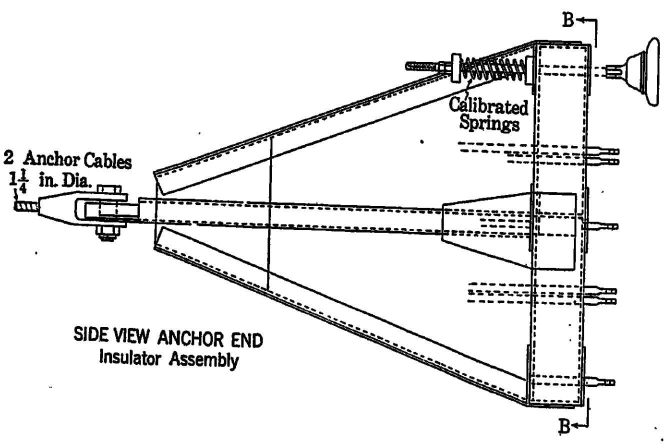


Fig. 4

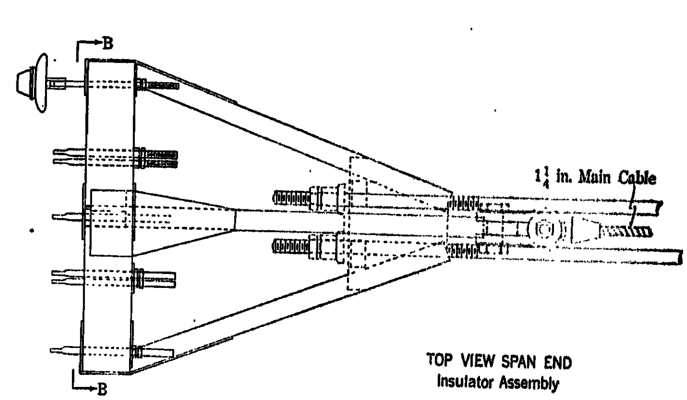


Fig. 5

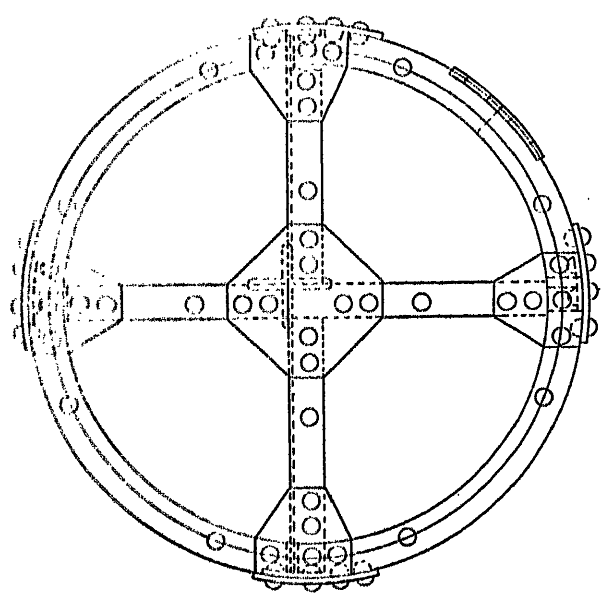
Cables and Fittings

The cables for this crossing will be 1¼-in., 35-wire, galvanized strand, consisting of acid, open hearth, plough-steel wire. The net area of steel in the strand is 0.93 sq. in., and it will weigh approximately 3.25 lb. per linear foot.

The guaranteed ultimate breaking strength of this strand is to be 180,000 lb., and tests already carried out show strengths well over this figure. Each coil of galvanized wire going into the make-up of the cables is being tested, the breaking strength running between 210,000 lb. and 220,000 lb. per sq. in.

Under maximum loading of ice and wind, the working stress in the cables was taken at 60,000 lb., affording a factor of safety of three, and the corresponding sag for determining clearance at 32 deg. fahr., when laden with ice, will be approximately 397 ft., with a stress of 56,500 lb. Stringing stress at 70 deg. fahr. would be about 43,000 lb.

The main cable, considering one cable only, is attached by an open socket and clevis to the front of the insulator assembly, as shown in Fig. 3. One 90-ft. and one 70-ft. relieving cable will extend out from insulator assembly along the main cable. The outer ends of these relieving cables are to be securely served to the main cable and also attached thereto by a series of evenly spaced U-bolt clamps of cast steel. While these clamps are being attached, the relieving cables will be stressed in successive increments by means of the long threaded eye-bolts attached to front of the insulator assembly, in order to put the same amount of stress on each clamp.



SECTION BB

Fig. 6—Cross Section of Insulators

From the back of the insulator assembly, two anchor cables will pass over the sheaves and attach separately to evener at front of anchorage by means of adjustable bridge sockets.

The purpose of the relieving cables, with their load of clamps, is to reduce stress in the main cable and damp vibrations, while the two anchor cables give double strength where passing over the sheaves and facilitate replacements or take-up.

All cables are identical, except that the main cables are reverse lay, while relieving and anchor cables are ordinary lay.

Insulator Assemblies

The question of insulation appears to be adequately solved by the design of insulator assembly adopted, see Figs. 3 to 6. This consists of two rigid steel frames or yokes, conical in general form, and constructed of light channels, angles, and bent plates.

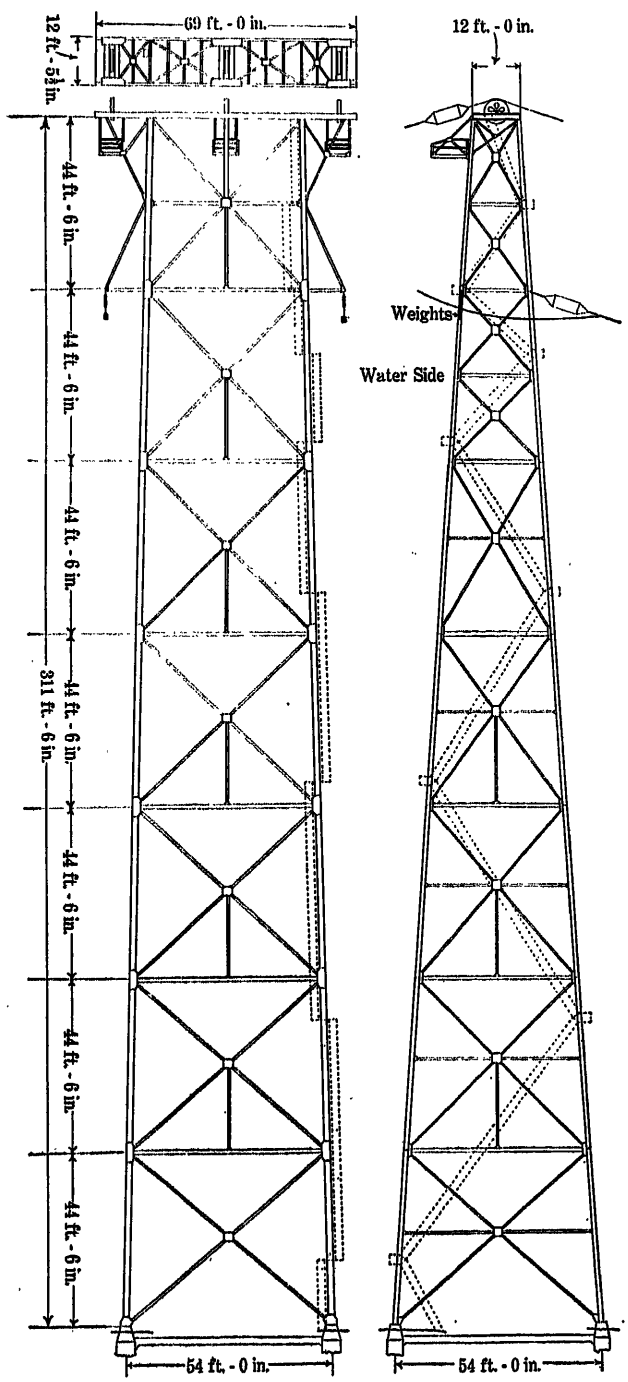


Fig. 7—Side Elevation of Tower

Between these yokes are placed twelve strings of high

strength suspension insulators, eleven units to the string. The minimum ultimate strength of these units is 18,000 lb. while the working stress will be 5000 lb., which closely parallels the conditions of service found to be satisfactory on the span of the Great Western Power Company.

The two yokes are identical except as to the arrangement for cable attachment.

Calibrated springs are provided at the rear end of each string to equalize stress and assist in eliminating shock.

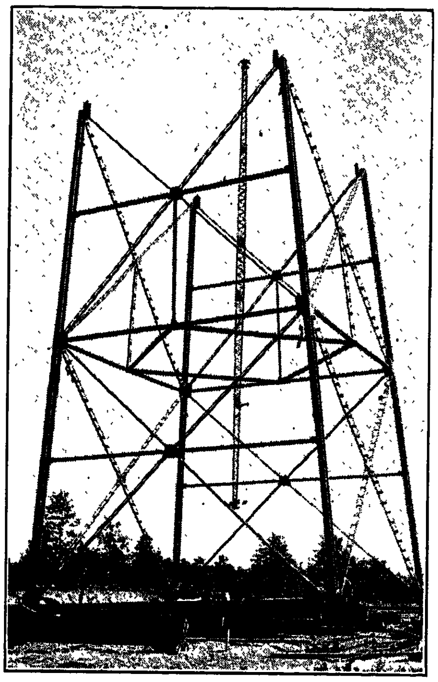


Fig. 8—Erecting Tower with Floating Gin Pole

With this rigid form of yoke, the breakage of any string would only slightly unbalance the loads carried by the other strings and a broken unit may be replaced and the string brought up to the proper working tension by means of the eye-bolts and springs.

During erection the insulators may be replaced by steel rods.

Towers

The towers are rectangular in form, 54 ft. square at the base, in which least number of members and ease of erection were considered together with economy of material; see Fig. 7.

Although the foundations on one side of the crossing are 53½ ft. higher than on the other side, it was decided to make the four towers of the same height, and identical in all respects.

The towers are 313 ft. from top of concrete to top of deck girders, consisting of seven 44-ft., 6-in. panels plus the depth of the 18-in. deck girders. The sheaves and bearings add another 3 ft. 6 in., making the total height from top of foundations to point of cable support 316 ft. 6 in.

The maximum computed reaction at base of the columns is 310,000 lb. and the greatest uplift 89,000 lb., the greatest stress in any of the diagonals being 33,500 lb.

Due to the importance of the structure as a vital link in the transmission line, and due also to legal regulations, the following conservative working stresses were adopted:

	Lb. per sq. in.
Tension:	
Rolled Steel	18,000
Bolts (net area)	12,000
Bearing:	
Pins, Shop and Field Rivets	20,000
Shear:	
Web Plates	10,000
Pins, Shop and Field Rivets	10,000
Extreme Fibre Stress:	
Rolled Steel Shapes	16,000
Rolled Steel Pins	20,000

Columns or Struts Axially Loaded:

Working stress not to exceed 18,000-701/r, and not to exceed 14,000 lb. per sq. in.

Slenderness Ratio 1/r:

Not to exceed 120, except for bracing and members resisting wind only, where 1/r is not to exceed 150.

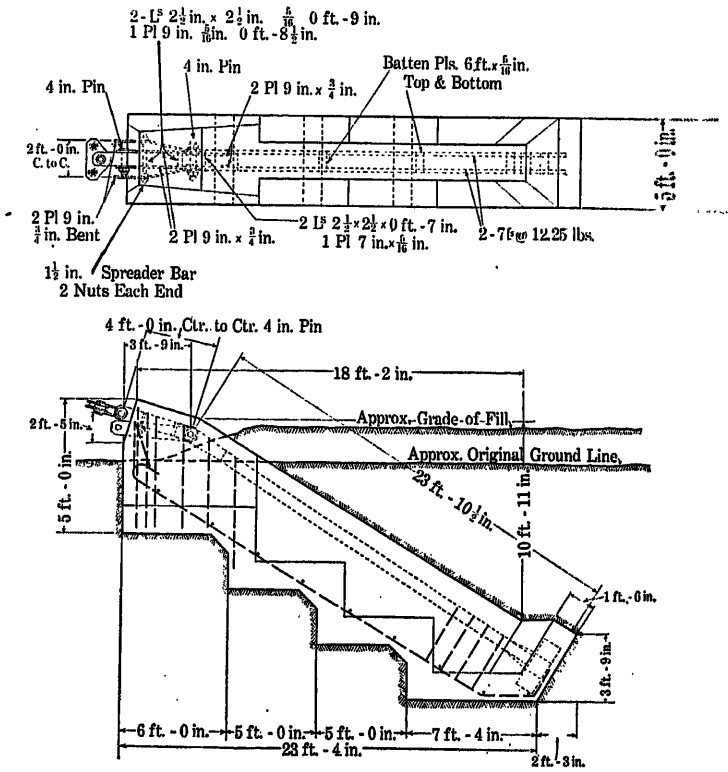


Fig. 9—Plan and Elevation of Anchor

A wind pressure of 20 lb. per sq. ft. was assumed on the projected area of all steel members.

The details of the towers are notable for the entire absence of latticing.

For economy of material, accessibility for painting,

and other considerations, the familiar built-up type of column was selected, consisting of an I-beam and two channels, the largest section being one 12-in. I of 31.8 lb. and two 12-in. [s of 30 lb. at the base of the tower, and one 9-in. I of 21.8 lb. and two 8-in. [s of 13.75 lb. for the upper sections. The lower diagonals are two [s 3½ in. by 2½ by 5/16 in., battened. The minimum thickness of metal used is 5/16 in., with possibly a few minor exceptions.

For a slight additional cost above that of a simple ladder, a light stairway with convenient landings is provided on each tower.

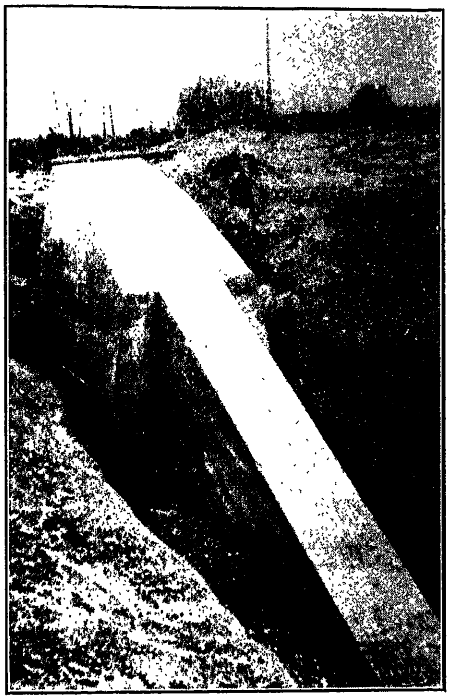


Fig. 10—Completed Anchor—Back View

Platforms extending out under the strain insulator assemblies facilitate inspection. These platforms are pin-connected to the main structure and can be swung down out of the way if desired at any time. The hand rails surrounding the platforms are also easily folded flat on the deck.

The total weight of each tower is 260,000 lb.

Tower Footings

The footings consist of simple concrete stepped footings, 12 ft. square, surmounted by 5-ft. square shafts. Four 2⅜-in. anchor bolts are embedded therein, extending to a steel plate 12 inches above the bottom of the footings. The footings are carried to a minimum depth of 9 ft. 6 in. below finished grade of the sites and are connected just below grade by 18-in. by 24-in. reinforced concrete struts.

Anchorages

From several preliminary designs, a somewhat novel type of cable anchor affording a large factor of safety and eliminating any apprehension from this feature of the structure was decided upon. (Figs. 9 to 11.) The lower faces of the anchors are stepped and the entire excavation is in firm compacted gravel, which gives them a power of resistance which is impossible to calculate with accuracy, and provides a large margin of strength for any possible future requirements.

The structural steel members to which the anchor cables are attached are carried to the extreme back end of the anchors.

The steel eveners are so designed as to permit one anchor cable to be completed slackened off and removed without cramping the terminal socket of the other cable.

Extra eye-bars are provided for haul-back purposes.

Sheaves

Built up structural steel sheaves 6 ft. in diameter, with cast steel rims, were originally designed for the support of the cables, but one-piece cast steel sheaves have been since manufactured for this purpose, the castings being exceptionally perfect and free from flaws.

The shafts are forged steel 7 in. in diameter, turned down to 5 in.—where they rest in bronze bushed bearings.

Each sheave weighs approximately 5000 lb. Calculations indicate that variations in loading may cause these sheaves to rotate approximately 13 in.

Construction and Erection

The foundations and cable anchors were constructed during the winter of 1924-25 at a cost of $33,000, and the contract for the fabrication and erection of the main towers together with the switching towers and stringing of cables was awarded Feb. 16, 1925, to the Star Iron and Steel Company of Tacoma, at a contract price of $149,000, the towers to be completed by Sept. 19, 1925, and the cables to be strung by Oct. 19.

Fig. 11—Group of Anchors at East End of Cables

The contract for furnishing cables, fittings, and insulator assemblies was awarded Feb. 13, 1925, to the John A. Roebling's Sons Company, at a contract price of $37,600, delivery to be made Aug. 25, making the complete cost of the crossing $219,600.00.

Erection was begun on the switching towers May 21, and the first steel column of the main towers was set June 6th, and the erection is progressing rapidly.

A 90-ft. basket or floating gin pole is being used, suspended in the center of the tower by four sets of falls attached to the columns at the panel points, the top being controlled by four lines operated by hand winches attached to the 18-in. by 24-in. concrete struts between footings.

Upon the completion of each panel the pole is raised or "jumped" 44 ft. for erecting the next panel. This method of erection has been used on many of the radio towers constructed in recent years.

Cost of Development

The cost of the development of the first 50,000 h. p. of the Cushman Power Project will be approximately $5,000,000.00, and is being carried on under the general supervision of J. L. Stannard, Chief Engineer.

The authors wish to express appreciation of the courtesies extended by J. P. Jollyman and L. J. Corbett of the Pacific Gas & Electric Co., J. A. Koontz of the Great Western Power Co., Theodore Varney of the Aluminum Co. of America, and S. Svenningson of the Shawinigan Power Co. Limited. C. C. Sunderland of the J. A. Roebling's Sons Co. also rendered valuable service when the matter of steel cables was under consideration.

Discussion

G. S. Smith: I should like to ask Messrs. Gongwer and Darland if any provisions are being made to obtain data on the cable movement over the two main supports? This could easily be accomplished by means of a graphic instrument to keep a record of the sheave movement, together with temperature. A record of the wind velocity and its direction, as well as a log of ice loading or other unusual conditions, would be very desirable.

The slow movement of the cable over the supports due to the temperature or loading changes, as well as the vibrations which may take place, would make a very interesting study in this unusually long span.

Such data would be highly instructive from the standpoint of investigation and future design, and might also prove valuable from the standpoint of maintenance.

G. R. F. Nuttall: I should like to bring up a point in connection with cables for long crossings. In our own transmission lines we have had to go into the details of sag calculations, and in all cases it is necessary to make quite a number of assumptions to start with, particularly in connection with cables which are formed of two different metals, aluminum and steel, for instance, where the combined modulus of elasticity and coefficient of expansion of the two metals has to be taken into account.

Have any of the cable manufacturers made any experiments with cables having a layer of aluminum strands on the outside, then a layer of strands—the modulus of elasticity and coefficient of expansion of which lie between those of aluminum and steel—and within the center the usual steel core? This would offer a more homogeneous cable in which the relative movement between layers would be less likely to occur.

R. J. C. Wood: For our use we have developed a dynamometer which is essentially a steel rod some 20 in. long, and we measure the extension of that rod under load with an ordinary dial extensometer.

L. J. Corbett: In this paper the point which I noticed particularly was the test of the two types of cable—that with the concentric layers oppositely wound and that with the layers wound in the same direction. The Pacific Gas and Electric Company long ago adopted for Carquinez the cable in which the layers are wound in the same direction, with the idea of getting greater strength for a given weight. This method produces a very compact strand, and this was the controlling feature in the selection of the cable. A new point is brought up when tests showed a difference of 60 per cent in resistance. This is certainly well worth considering.

A. F. Darland: Mr. Smith has brought up the point of keeping a record of the movement of the cable over the sheave on the Narrows crossing. We have computed the probable movement we shall get, and expect probably a foot. No definite means has been provided at this time to keep a record of that movement, but it will undoubtedly be noted and recorded. We have a calibrated spring on each string of the high-tension insulators whereby we may know the tension in the cable, and we shall in addition keep an accurate record of the sag, measuring it with a level.

220-Kv. Transmission Transients and Flashovers

BY R. J. C. WOOD[1]

Associate, A. I. E. E.

Synopsis.—*The conclusion has been reached that birds are the cause of flashovers on the Southern California Edison line. The frequency and location of flashovers is given for nine years of operation at 150 kv. and two years at 220 kv. The increase in the number of flashovers when first going to 220 kv. has been reduced so that now there is no greater number than there was at 150 kv. This has been done by installing bird guards which are, however, not yet completely bird proof. Other possible causes of flashovers are considered, including corona, standing and traveling waves of high voltage, harmonic resonance, sustained high-frequency effects, lightning, and highly ionized air. Investigations to discover the presence of such disturbances are described; they included the use of a homemade photographic surge recorder, the klydonograph, and oscillograph. The amplitude of voltage surges caused by various switching operations and the quantity of tertiary and residual current at the different stations on the system are tabulated.*

The conclusions reached are that there are not any voltage disturbances of greater magnitude than those produced by normal switching, and that such voltage rises as do occur are totally inadequate to cause flashovers or cause any damage to connected apparatus. The evidence is all against the existence of sustained high-frequency currents or voltages and it may be stated confidently that they do not exist.

* * * * *

FLASHOVERS

THE more or less occasional flashovers on the Big Creek Transmission System of the Southern California Edison Company have been the subject of study for several years in the effort to find their true cause, but none of those who studied this problem could find an electrical theory that would explain all the facts. Finally, a man unacquainted with mysterious resonance phenomena and dire high-frequency potentialities observed the one fact that had escaped the rest of us, and, as H. Michener describes in his paper[2], the bird theory was evolved. This theory, that flashovers on this system are due to the excrement of birds falling through the air and affording a conducting path between conductor and tower, is now firmly established and believed by those on the ground.

The frequency of these flashovers from January 1, 1914 to July 31, 1925 is shown in Fig. 1, the voltage of the line being 150 kv. prior to May 6, 1923 and 220 kv. thereafter. The immediate increase of flashovers is noticeable when the raise in voltage increased the size of the bullseye which the bird had to hit in order to score. It will also be seen that since the installation of various kinds of bird guards, which was done mainly during the last half of 1923, the occurrence of flashovers has been of about the same frequency as during 150 kv. operation.

When it was first determined to put on bird guards, two factors were underestimated: first, the lateral distance from the center line of the insulator string from which a roosting bird could cause a flashover when the wind was in the right direction to carry the stream of excrement into the vicinity of the conductor, and secondly, the bird's intelligence. He had used that tower for a roost and observation point for years and looked upon any effort to oust him as an invasion of his rights, and he proved quite clever in surmounting the difficulties first put in his way.

In the winter months of 1924-25 observation on the line showed that the birds, prevented from roosting on the more accessible parts of the tower, were accommodating themselves to new conditions and getting in on secondary members of the steel work that it had at first been considered unnecessary to guard, so that the line is not yet fully equipped with absolutely bird proof devices and the flashovers still occurring have in each case been traced to imperfect bird guards.

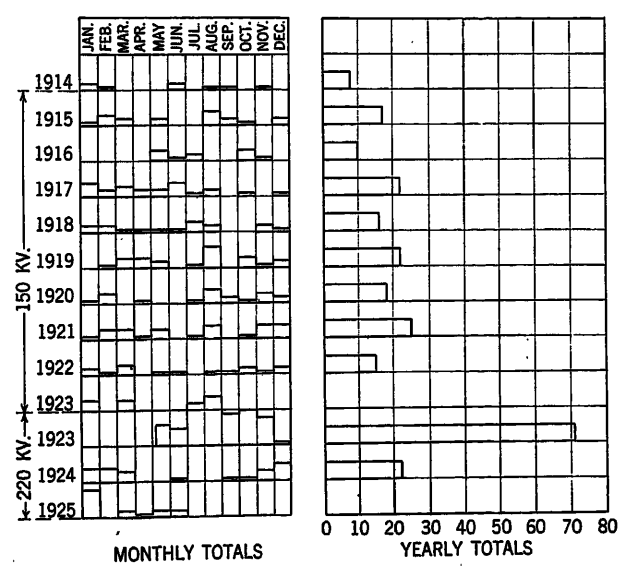


FIG. 1—RECORD OF THE NUMBER OF FLASHOVERS ON BIG CREEK LINES; TOTALS BY MONTHS AND YEARS

The commercial significance to service of the flashovers has become a vanishing quantity now that reliable automatic relays are in use, provided duplicate sectionalized lines are available which will not be overloaded upon the loss of a section. The effect of a flashover upon received voltage and frequency is shown in Fig. 2, and is seen to be inconsiderable and momentary.

The space distribution of flashovers along the line is given in Fig. 3; the remarkable similarity between the distributions for 150-kv. and 220-kv. operation is at once apparent. This in itself is significant of no new

1. Research Engineer, Southern California Edison Co.
2. Transmission at 220 kv., Section 1, by H. Michener.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash. September 15-19, 1925.

factor having arisen with the increase of voltage; in fact, it is very evident that the tendency to flashover chiefly depends upon the location of the insulator along the line.

Possible Causes of Flashovers

Considering other possibilities than birds, the exciting cause of flashovers might be either internal, such as standing or traveling waves of abnormal voltage, or sustained high-frequency capable of breaking down relatively large air spaces, or external, such as lightning or highly ionized air.

The three horizontal conductors are asymmetrical with regard to the ground wire so that the outer conductor most remote from it should suffer most from lightning disturbance. During 150-kv. operation, 66.3 per cent of flashovers were on the middle conductor;

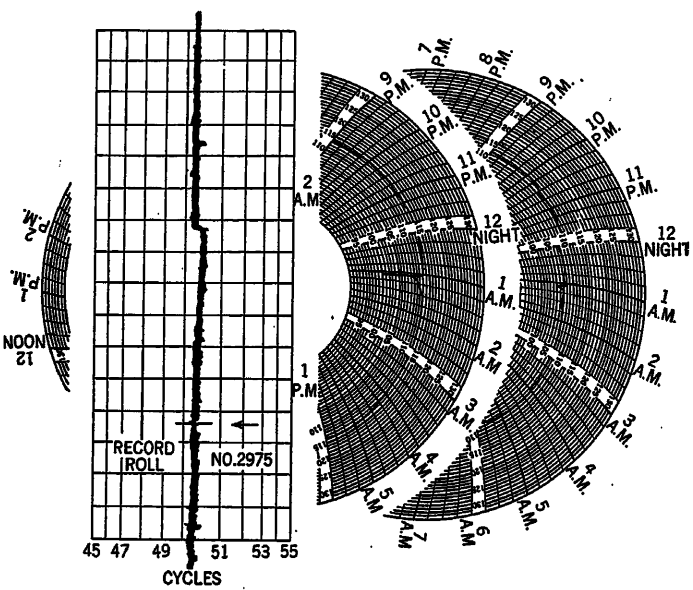


Fig. 2—Disturbance Caused by Flashover. to System Frequency and Voltage at Two Kv. Substations

after going to 220 kv. this percentage increased to 88.5 per cent.

Evidently, no distributed atmospheric effect is responsible. This eliminates ionized air and lightning, especially since practically all of the flashovers have occurred when there was no lightning.

The possibility or otherwise of internal disturbance is not so easily disposed of, and in an effort to clear up this question a few investigations were undertaken.

Corona

It has been suggested that corona on the arcing horns of the old 150-kv. equipment might be a starting point for the high-frequency streamer, in fact, that the least bit of corona was a dangerous thing to have around. In November 1921, hollow copper balls, some 3-in. and some 4-in. diameter, were put over the ends of the arcing horns on the middle conductor of the East Line from mile 105 to mile 163 where, as may be seen in Fig. 3, flashovers were above the average in frequency. These balls suppressed the corona, did not decrease the arcing distance, and improved the voltage distribution of the insulator string, but the three next flashovers in that month and three out of four in December were over ball protected insulators. Evidently, corona was not the cause of the breakdowns.

Photographic Surge Recorder

At the time these matters were being studied in 1921,

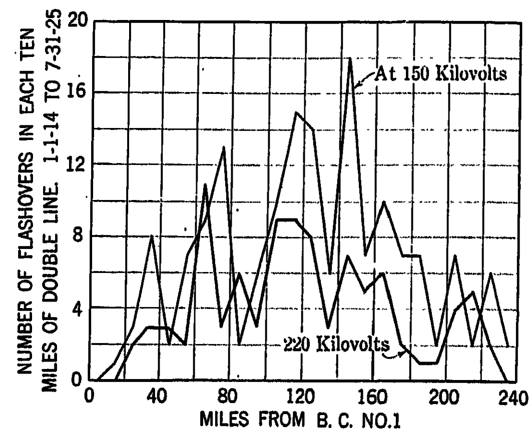


Fig. 3—Space Distribution of Flashovers Along Line

preparatory to increasing the line voltage to 220 kv., there did not appear to be available any commercial voltage surge recorders that were suitable to our purpose, so we combined a kodak film with the clockwork mechanism of a curve drawing wattmeter, and there resulted a somewhat crude daylight loading surge recorder using standard 3A film.

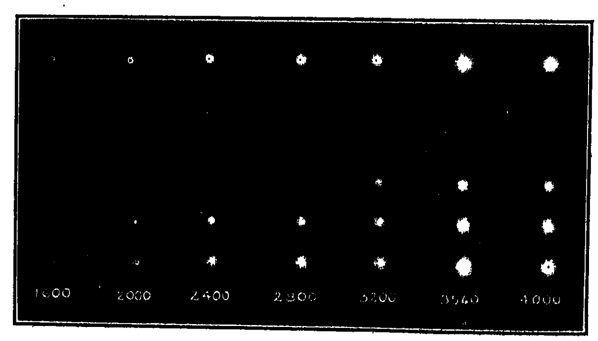


Fig. 4—Calibration of Surge Recorder on A-C. Voltage

Six hemispherically-ended metal wires, together with a grounded metal drum, formed six spark-gaps of different lengths through which the film passed. The spark-gap terminals were all connected in parallel across the ground-end unit of a string of insulators hung from the line under test, and so shielded as to provide a suitable voltage for the recorder. The parallel gaps broke down successively at increasing voltages as long as the insulating celluloid film was in them, as shown in Fig. 4, which is a laboratory calibration using a-c. voltage. The film was driven by a weight-actuated mechanism, the

clock regulating the speed. The whole device was enclosed in a wooden box supposed to exclude light. One of the spark-gaps was used to mark time, being in series with a commutator that broke circuit every two hours and was driven by the same clock that regulated the film speed. The instrument is shown in Fig. 5.

This instrument was in use on the Eagle Rock 150-kv. bus for three months and was then installed at Vestal. A record obtained by its use is shown in Fig. 6. The broad band at the left is the discharge from the

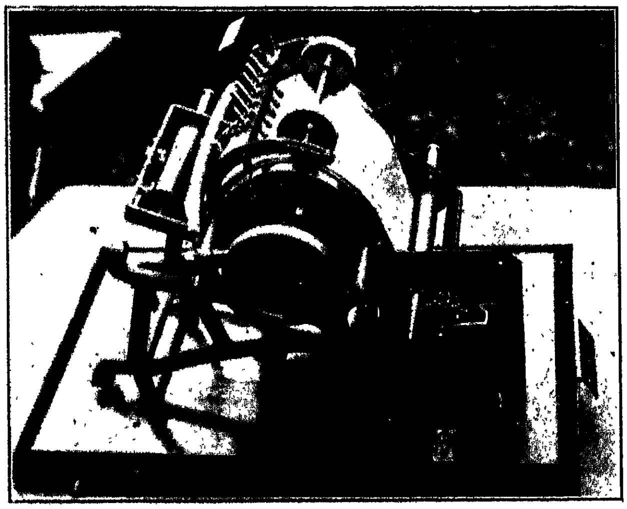


Fig. 5—Photographic Surge Recorder

first gap which was set at just below normal voltage, the second gap went over at approximately 1.25, and the third one at 1.50 times normal voltage. The right hand band is from the timing point working indifferently well.

It was the general rule to kill the bus before admitting

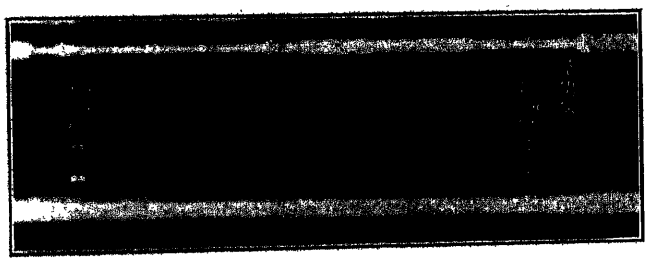


Fig. 6—Surges Recorded at Eagle Rock on 150-Kv. Bus

any one to the bus compartment for the purpose of changing films, and it was soon found that this station switching caused the highest voltages that were recorded, none of which, however, exceeded approximately double normal. The recorder was in operation upon the 150-kv. bus at Vestal after the line had been raised to 220 kv. but was never installed directly upon the 220-kv. line. It had satisfied us that there were no voltage surges on the 150-kv. system of anywhere near sufficient magnitude to cause flashovers.

This recorder suffered from many defects, largely of a mechanical nature; difficulty was encountered in getting the film to reel up smoothly on the receiving spool, and considerable fogging of the film took place when installed out of doors from leakage of light through the containing wooden box, the time-recording device was imperfect, and the indications were not at all

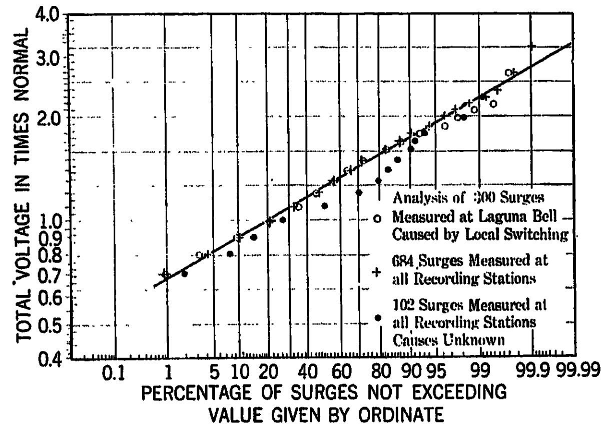


Fig. 7—Relative Percentage of Voltage Surges of Different Amplitudes

precise, increasing as they did by 25 per cent steps. Still it worked after a fashion and indicated conditions that have since been confirmed by more perfect apparatus.

Klydonograph Tests

Some time later, the Westinghouse Electric Company brought out its Klydonograph[3] surge recorder in which

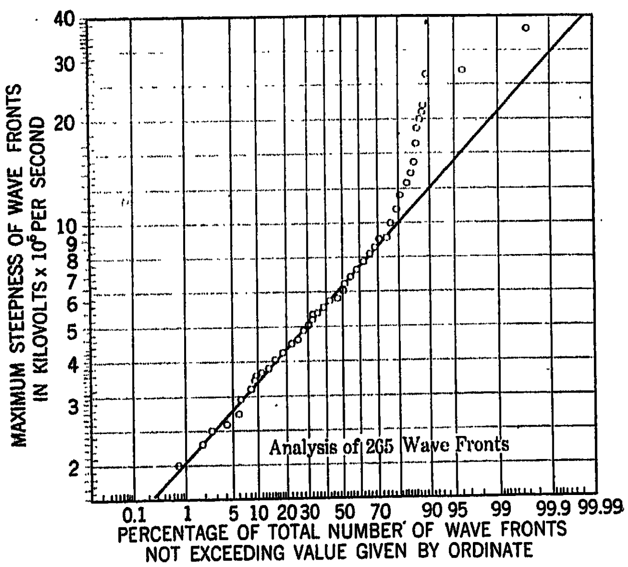


Fig. 8—Relative Percentage of Wave Fronts of Different Maximum Steepnesses

only one recording point is used and the magnitude and character of the voltage estimated from the size and nature of the picture produced. Through the courtesy of the Westinghouse Co., installations were made at Big Creek No. 1, Vestal, Magunden, and Laguna Bell.

3. The Klydonograph and its Application to Surge Investigation, by J. H. Cox and J. W. Legg.

These installations were in service 89 days at Big Creek No. 1, 75 days at Vestal, 71 days at Magunden, and 119 days at Laguna Bell. There were 684 indications of voltage rise and 556 indications of wave front slope recorded. More than a normal amount of switching was being done at the time as one line was taken out of service every night to reduce losses. The approximate number of oil-switch operations performed on the 220-kv. system averaged over 36.3 per day and about 18 daily oil-switch operations were made on the 150-kv. station busses which are conductively coupled to the 220 lines through auto-transformers. The average total number of surges per day was 8.71. The ratio of number of surges to switchings is 24 per cent if only the 220-kv. switching is considered and only 16 per cent if the 150-kv. switching is included.

The relative percentage of abnormal voltages of different magnitudes is shown in Fig. 7 plotted upon probability paper. The very close conformity of the plotted results with a straight line indicates both that a sufficient number of observations were obtained to get representative results, and also that no particular predominating cause exists that is responsible for surges of any particular value.

The curve for surges measured at Laguna Bell, due only to local switching operations, is practically coincident with the curve of all causes for the whole of the system. From this we may conclude that the magnitude of the voltages resulting from normal switching operations was, on an average, independent of the location of the switching.

The surges of unknown origin which could not be correlated with switching on the 220-kv. system average of lesser magnitude.

The steepness of wave fronts is shown in Fig. 8. In this case the probability law is followed up to wave fronts of about 10×10^6 kv. per sec. Above this value results are erratic. This is explainable by the fact that the greater part of the observations between 2×10^6 and 10×10^6 kv. per sec. were obtained at Laguna Bell as the result of a great number each of a number of different switching operations, whereas the steeper slopes were observed at Vestal and Magunden and are inherent to a few particular operations at those stations. The curves therefore seem to indicate that slopes of wave fronts originating at different stations are of different orders of magnitude, whereas the amplitudes of potential surges at different stations are of the same order. It may be pointed out for comparison that the maximum steepness of normal 50-cycle sinusoidal waves at 220 kv. is 0.0564×10^6 kv. per sec.

The evidence as to the extent that surges are transmitted over any considerable distance is rather hard to interpret depending somewhat upon the definition of the word "surge."

A surge might be considered to have traveled:

1. When a voltage rise is due to switching at a distance
2. When antenna indications of current waves show them to have traversed certain distances
3. When records of voltage rise are found at two or more separated points all due to a common localized cause

TABLE I
TRANSMISSION OF VOLTAGE DISTURBANCES

Recording Station	Number of Disturbances Recorded at			
	Laguna Bell	Magunden	Vestal	B. C. No. 1
Total Number recorded......	402	38	17	41
Number originating at Laguna Bell and also recorded as shown....................	..	0	0	0
Ditto originating at Magunden	3	..	0	0
	2	..	0	2
Ditto originating at Vestal....	1	0	..	0
Ditto originating at B. C. No. 1	0	0	0	..
Ditto origin unknown........	1	0	0	1

It has been thought best to confine the evidence to that under heading No 3, as this required direct proof of the transmission of a voltage disturbance, while Nos. 1 and 2 do not necessarily do so. Such evidence is given in Table I and shows to what an insignificant amount such action occurs over the distances in question.

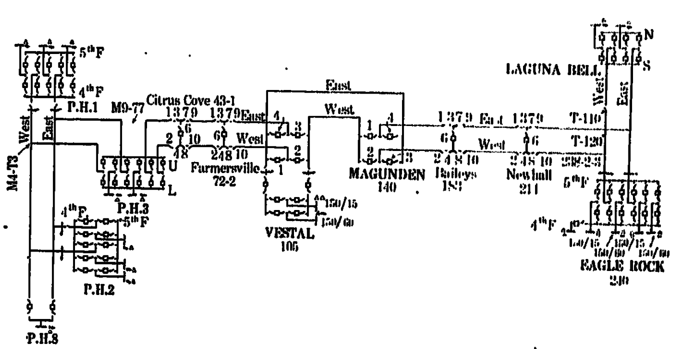


FIG. 9—SWITCHING DIAGRAM OF 220-KV. SYSTEM

The antenna instruments gave many indications of current rushes in the line without corresponding voltage records being obtained. Apparently there was not any definite relation between voltage amplitude and wave front steepness.

For those who care to analyze further, Fig. 9 presents a switching diagram of the system and Tables II and III show the different switching operations performed, together with the range of the resulting surges.

It should be noted that even though the disconnects at the main switching stations only make or break the charging current of a few feet of bus and one side of an open oil switch, that when so doing they cause voltage rises of as high a value as do other switching operations that involve 100 miles of line.

TABLE II
CAUSES AND MAGNITUDES OF SWITCHING SURGES

A. SURGES MEASURED AT LAGUNA BELL

Location of Switching	Nature of Switching	No. of Records Obtained	Total Voltage in Times Normal		
			Min.	Avg.	Max.
B. C. No. 1	Disc. closed	1		1.2	
	West Line closed	1		1.1	
B. C. No. 8	East Line closed	3	0.9	1.1	1.4
	West Line closed	1		1.5	
B. C. No. 3	East Line opened North	1		1.1	
	East " " South	5	1.1	1.3	1.6
	" " closed South	1		1.2	
	West " opened South	5	1.5	1.6	1.6
	" " closed South	1		1.3	
	Disc. closed	1		1.6	
Citrus Cove	Parallel opened	3	1.0	1.2	1.3
	" closed	1		1.7	
Farmersville	" opened	6	1.1	1.2	1.5
Vestal	East Line opened North	6	0.9	1.5	1.8
	" " " South	3	1.0	1.2	1.4
	West " " North	3	1.1	1.2	1.3
	" " " South	6	1.1	1.3	1.5
	" " closed North	1		1.3	
	" " opened South	4	1.0	1.4	1.7
	Disc. opened	2	1.4	1.7	2.1
Magunden	East Line opened North	4	1.1	1.6	2.3
	" " " South	3	1.3	1.5	1.8
	" " closed North	6	1.1	0.3	1.4
	West " opened South	27	0.8	1.3	2.4
	" " closed North	4	0.9	1.0	1.1
	" " " South	6	1.0	1.3	1.8
	Disc. opened	3	1.4	1.7	2.2
	" closed	1		1.1	
Newhall	Parallel opened	3	1.1	1.2	1.4
	" closed	1		1.0	
Laguna Bell	East Line Opened	32	0.7	1.3	1.9
	" " closed	10	0.8	1.0	1.5
	West " opened	34	0.8	1.3	1.9
	" " closed	26	0.8	1.3	2.2
	Changed from South to North bus	72	0.8	1.4	2.1
	" " North to South "	61	0.7	1.4	2.2
	Parallel busses	10	1.0	1.3	1.7
	North bus opened	2	1.0	1.1	1.1
	South " "	4	1.2	1.5	1.8
	" " closed	3	1.6	1.7	1.8
	Disc. opened	14	1.1	1.5	1.9
	" closed	1		1.6	
	Transformer put on bus	1		1.1	
	" taken off "	8	0.9	1.4	1.8
	Changed Transformers	7	0.9	1.2	1.8
B. SURGES MEASURED AT MAGUNDEN					
B. C. No. 3	East Line opened South	2	0.9	0.9	0.9
	" " closed "	1		1.5	
Vestal	West " " "	1		1.5	
Magunden	East " opened South	2	1.3	1.3	1.3
	" " closed North	1		1.2	
	" " " South	7	1.2	1.7	2.2
	West " opened South	10	0.8	1.1	1.5
	" " closed North	2	0.7	1.1	1.6
	" " " South	9	0.8	1.3	2.1
Newhall-Baileys	Disc. opened	3	1.2	1.6	2.0
C. SURGES MEASURED AT VESTAL					
Farmersville	Disc. closed	1		2.0	
Vestal	East Line opened South	2	1.0	1.0	1.0
	" " closed North	1		2.0	
	West " opened South	3	0.9	1.7	2.1
	" " closed South	5	1.3	2.0	2.4
	Disc. opened	1		1.2	

TABLE II (Continued)
CAUSES AND MAGNITUDES OF SWITCHING SURGES

Location of Switching	Nature of Switching	No. of Records Obtained	Total Voltage in Times Normal		
			Min.	Avg.	Max.
	D. SURGES MEASURED AT B. C. NO. 1				
B. C. No. 1	East Line closed	2	1.7	1.7	1.7
	West " opened	1		2.0	
	" " closed	4	0.9	1.5	2.2
	Disc. closed	1		1.3	
	Generator taken off	2	1.7	1.8	1.8
B. C. No. 2	East and West lines parallel	1		1.9	
B. C. No. 8	" " " " "	1		1.9	
B. C. No. 3	East Line closed North and South	1		1.9	
	West " " South	3	2.0	2.1	2.3
Farmersville	Parallel opened	1		2.0	
Vestal	East Line opened North	10	0.8	1.5	2.3
	" " " South	1		2.3	
	West " " North	1		2.0	
Tower 118-5	Flashover, East Line	1		1.9	
Magunden	East Line opened South	2	1.6	2.0	2.3
	" " closed North	3	1.2	2.1	3.2
	" " " South	1		1.7	
	West " opened South	2	1.7	1.7	1.7
	Disc. opened	1		1.7	
Laguna Bell	East Line opened	1		1.9	

Some explanation of the meaning of voltage rises of less than 1.0 times normal is due.

The klydonograph actually records a measured fraction of the line voltage, an electrostatic potentiometer being used to effect this. Between the klydonograph and potentiometer is a sphere-gap adjusted to breakdown for a slight increase in line voltage and to give a corresponding size of picture on the photographic plate.

TABLE III
MAGNITUDE OF SURGES
SURGES OF UNKNOWN ORIGIN

Measured at	No. of Records Obtained	Total Voltage in Times Normal		
		Min.	Avg.	Max.
Laguna Bell	86	0.7	1.2	2.3
Magunden	4	1.1	1.2	1.5
Vestal	3	1.1	1.1	1.2
B. C. No. 1	9	0.9	1.5	2.0

Records were occasionally found upon the plate corresponding in size to voltages less than the normal breakdown value of the sphere-gap, and these have been included in the tabulations, as found, as less than unity surges. The calibration of the photographic plate at these low voltages is somewhat uncertain especially when the duration of the surge is so short that only a few rays are formed in the picture and this is the probable explanation of pictures of a size less than corresponds to the series spark-gap. In any event these apparently subnormal surges are only of academic interest and have no bearing upon flashovers. At supernormal voltages the accuracy of the calibration is probably well within 15 per cent at twice normal voltage and better yet at higher voltages. Calibrations made at the factory, in the Southern California Edison laboratories and in the field, check very well and prove

FIG. 10—LINE VOLTAGE TO GROUND AT BIG CREEK NO. 3

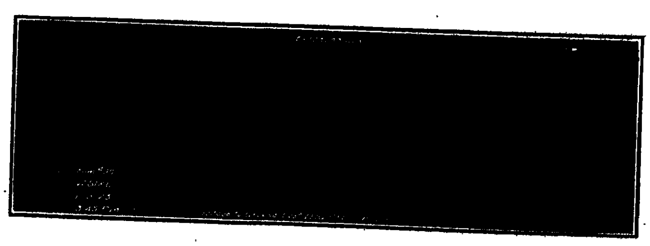


FIG. 11—LINE VOLTAGE TO GROUND AT VESTAL

that the obtaining of 0.7 times normal pictures does not mean that the whole calibration of the klydonograph is 43 per cent low.

TERTIARY AND RESIDUAL CURRENTS

To determine whether there were any abnormal residual, tertiary, or other odd harmonic currents or voltages, a number of oscillograms were taken, nothing

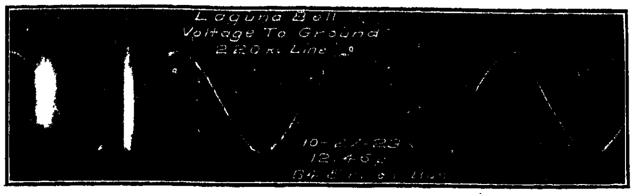


FIG. 12—LINE VOLTAGE TO GROUND AT LAGUNA BELL

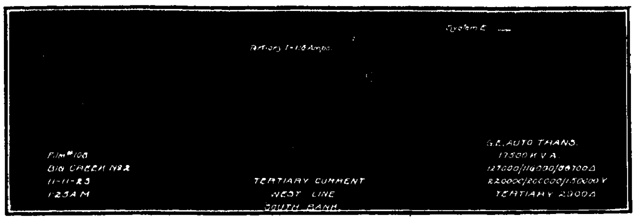


FIG. 13—TERTIARY CURRENT IN AUTO-TRANSFORMERS AND SECONDARY VOLTAGE AT BIG CREEK NO. 2

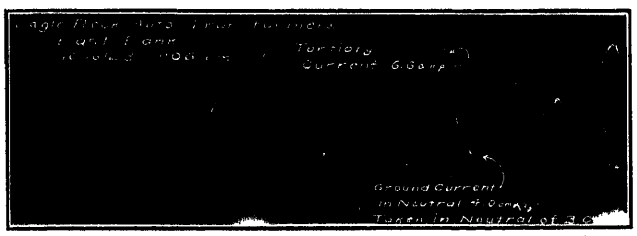


FIG. 14—TERTIARY AND RESIDUAL TO GROUND AT EAGLE ROCK

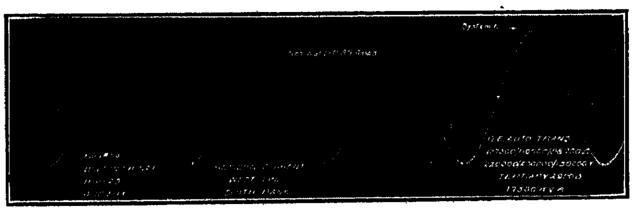


FIG. 15—RESIDUAL TO GROUND AND SECONDARY VOLTAGE AT BIG CREEK NO. 1

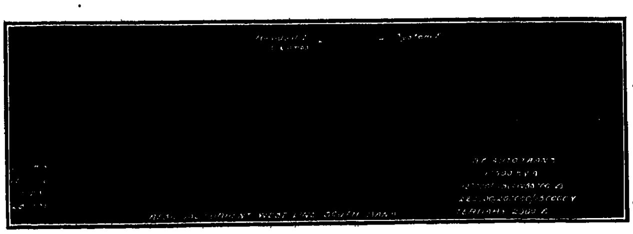


FIG. 16—RESIDUAL TO GROUND AND SECONDARY VOLTAGE AT VESTAL

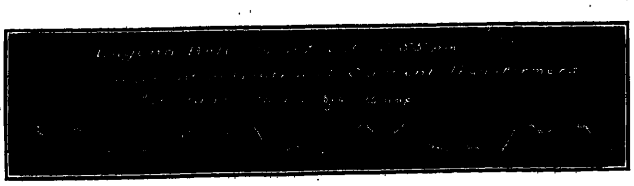


FIG. 17—RESIDUAL TO GROUND AT LAGUNA BELL

out of the ordinary being done except, perhaps, in obtaining oscillograms of line voltage direct without interposing potential transformers.

This was accomplished by using a 50-ft. length of ½-in. common garden hose filled with water as a resistance in series with the oscillograph element, with a precautionary shunt, in parallel with the element. No special difficulties were encountered except that in the

TABLE IV
TERTIARY CURRENTS IN AUTO-TRANSFORMERS

Station	Transf. Bank	Tertiary Amps.	Rated Voltage of Tertiary	Approx. Per cent of Rated Voltage on Transf.
Eagle Rock.....	East Line	6.6	11,000	86.4
	West Line	7.5	11,000	86.4
Vestal..........	North	128.	2900	96.9
	South	149.	2900	96.9
B. C. No. 1.....	North	119.	2900	100
	South	116.	2900	100
B. C. No. 2.....	East Line	122.	2900	100
	West Line	116.	2900	100

first trials the water in the hose lowered in level at the high voltage, due to leakage and evaporation, whereupon a small internal arc burned up that end of

TABLE V
RESIDUAL CURRENTS TO GROUND

Station	Transf. Bank	Residual Amps.	Rated Voltage
Eagle Rock......	East	4.0	220000/150000
	West	4.0	" "
Laguna Bell.....	No. 1	2.8	200000/72000
	No. 3	1.6	" "
Vestal..........	South	0.6	220000/150000
	North	0.6	" "
B. C. No. 1......	North	0.84	220000/150000
	South	0.88	" "
	No. 1 Zero load	0.96	6600/150000
	No. 2 Zero load	0.96	" "
	No. 3 16000 Kw.	0.20	" "
	No. 3 16000 Kw.	0.84	" "
B. C. No. 2......	No. 1	1.52	6600/150000
	No. 2	1.52	" "
	No. 3	1.52	" "
	Total No. 1, No. 2, No. 3	3.52	" "
	E Line	1.24	220000/150000
	W Line	1.44	" "
	No. 3 4000 Kw.	1.6	6600/150000
	No. 3 7000 "	1.6	
	No. 3 9500 "	1.6	
B. C. No. 8......	Total less than	.04	11000/222000
B. C. No. 3......	No. 1	2.72	11000/220000
	No. 2	2.88	" "

the hose. This difficulty was entirely overcome by having a small reservoir of water (old 5-gal. oil can) at the high-voltage and high-level end; this supplied losses

and kept the hose full of water. The current through the hose and oscillograph to ground varied from 37.0 milliamperes at Laguna Bell to less than 0.3 milliamperes at Big Creek No. 3, showing resistances of the 50 ft. of hose of 3.36 and 423 megohms respectively. This, it may be remarked parenthetically, is truly indicative of the relative difficulties in the two locations of obtaining good ground connections for transformer neutrals.

Efforts were also made to determine the crest voltage on the line by direct sphere-gap measurements putting the sphere-gap at the ground end of the 50-ft. hose full of water. The drop of voltage along the hose due to charging currents rendered the method useless and we had to be satisfied with the wave shape of voltage obtained as previously described.

The results of the investigations were negative so far as supplying any reason for flashovers was concerned. Third, fifth, and ninth harmonic currents were no larger than was to be expected, and voltage waves were apparently so nearly sinusoidal that we did not indulge in harmonic analysis.

Examples of the results are shown in Figs. 10 to 17 which are self-explanatory. Table IV gives a summary of the magnitude of tertiary currents and in Table V will be found the values of residuals to ground. These residuals are due to the impedance of tertiary windings, generator harmonics other than multiples of the third, unbalance of the lines which are not transposed, and possibility to slight transformer inequalities. Considering the magnitude of the system and the untransposed line, the residuals are small.

The search for abnormal and startling effects has been distressingly unsatisfactory from a spectacular point of view, but most reassuring to those who contemplate the use of high voltages for transmission. Not only have no unusual effects been discovered but not even the beginning or tendency towards them has been made manifest. We have particularly looked for evidence of sustained high frequencies; we are confident there are none.

Conclusions

1. Corona in the amount found upon arcing horns did not cause flashovers.
2. No voltage surges occur of sufficient magnitude to cause flashover or damage to apparatus.
3. Switching causes rises of voltage as great as any recorded.
4. Voltage rises recorded at the times of two flashovers may have been due to subsequent switching and were less than many known to arise from switching.
5. High-voltage switching as practised on this system may be done without fear of danger.
6. Only very occasionally did voltage surges travel even 35 miles without being greatly attenuated.
7. Such alternating surges as were found were highly damped, being usually of but one alternation.
8. There were no harmonic resonances or distortion of voltage wave shapes.
9. Residuals to ground were small.
10. There were no sustained high-frequency effects.

Acknowledgments

The author is pleased to acknowledge his indebtedness to Mr. W. W. Lewis of the General Electric Company for his kindly assistance and direction in the field in taking and interpreting wave shapes, and to Messrs. J. H. Cox and L. Gale Huggins for indefatigable work with the klydonographs and again to them and the Westinghouse Company for permission to borrow freely from their report, which has furnished the substance for a great portion of this paper. Mention must also be made of the unselfish consideration of Messrs. J. H. Cox and J. W. Legg who withheld a great deal from their own paper[3] so that this presentation of the search after the abnormal might be more comprehensive and complete in itself.

Discussion

Percy H. Thomas: This paper appears to clear away definitely the persistent suspicions that there is some source of mysterious overpotential inherent in 220-kv. or in very long lines that does not appear on the surface—something involving an unrecognized principle and jeopardizing reliability of service.

The subject of surges and high-voltage breakdowns has long been a *bête noir* for transmission operation. When power transmission first became of importance, one of the first serious sources of shut-down other than the ever-present lightning was the frequent short circuit of transformers in high-tension service. While this proved to be due partly to moisture and inadequate insulation, it was, nevertheless, necessary that the now well-known but then almost unknown effect of the steep-wave-front surge on windings be studied out[4] and this led to radical change in the method of insulating high-tension windings. While the theory of the propagation of surges or waves in transmission lines was academically known, nobody knew what the real effect in an actual line practically was, and this led to a certain feeling of uncertainty as to what might be expected with plant extensions. This led to a series of investigations in 1902 in the Middle West on a number of high-tension plants, similar in object to those reported by Mr. Wood and leading to a somewhat similar conclusion.[5] In the old investigation, the voltages were lower and the apparatus cruder, but the interest was much the same.

At a still later period, with the advent of larger systems and higher voltages, still another experimental study was made, this time based largely upon the oscillographs covering the action of switching and surges in the system of the Pacific Gas & Electric Company[6] and still again with the result of disproving the existence of any new or unexpected phenomena.

Mr. Wood's work goes over the research again, taking up all possible sources of excessive surges and using perfected apparatus. This makes clear the conclusion that the explanation of

3. loc. cit.

4. See paper, Percy H. Thomas, Trans. A. I. E. E., "Static Strains in High Tension Circuits and the Protection of Apparatus," Vol. XIX, 1902, p. 213.

5. See paper, Percy H. Thomas, *An Experimental Study of the Rise of Potential on Commercial Transmission Lines Due to Static Disturbances Caused by Switching, Grounding, etc.*, Trans. A. I. E. E., Vol. 1905, p. 317.

6. Guiseppe Faccioli, *Electric Line Oscillations*, Trans. A. I. E. E., Vol. XXX, Part III, p. 1803.

mysterious surges even on 220-kv. lines must be sought among the well recognized laws of electric phenomena.

Considering, now, the detail of the paper, it may be pointed out that the results of the klydonograph appear to agree with the old fundamental rules of wave motion, e. g.: (a) When a potential is abruptly applied to a transmission line, a wave equal in crest to the applied potential passes along the line until it reaches a reflecting point when it is reflected back toward the start, reaching *double potential* at the *reflecting point,* neglecting losses, (b) When a small-capacity branch line leads from such a reflecting point, the wave starts off in the branch line at the maximum potential reached at the reflecting point and is subject to reflection at the end of the branch line with another doubling of the potential, (c) Whenever a condenser is charged through reactance (neglecting resistance) the potential will rise to double the charging potential and only gradually settle down to equilibrium after a series of oscillations.

As Mr. Wood properly says, in considering surges in a line, the effect of distance of travel is only to weaken the surge. Other things being equal, the effect of a switching operation is greatest nearby.

It is particularly gratifying to find that no abnormal *sustained* high-frequency potentials exist in the line, as these might be exceedingly troublesome.

F. W. Peek, Jr.: Mr. Wood's findings, which are quite in accord with theory, show no mysterious high voltages at sustained high frequency causing arcs of indeterminate distance. The expected highly damped switching surges occur but never of sufficient voltage to cause trouble. The cause of many arc-overs is dirt. In the tests made most of the dirt trouble was a purely mechanical one due to birds and as easily understandable as the effect of a fuse wire dropped over an insulator string. This investigation shows conclusively that there need be little fear from transient voltages originating in a grounded-neutral system.

Thus, in the part of the country where the lines of the Southern California Edison Company are located, the chief cause of insulator arc-over is a weakening of the insulators by dirt or other mechanical means.

Other important points brought out are that high-voltage switching may be done without fear, and that with quick-acting relays insulator arc-overs may be cleared without interruption of service. Of course, in addition to quick-acting relays, it is important to have grading shields or rings on the insulator to direct the arc away from the string while the relays are operating.

While dirt is the principal cause of arc-over in certain parts of the country, in other parts its effect may be practically negligible and lightning may be a very serious factor.

To illustrate how well Mr. Wood's conclusions are in agreement with our laboratory investigation I would draw your attention to a recent discussion of mine[7] at the Annual Convention which shows that our conclusions are very similar to Mr. Wood's.

C. L. Fortescue: Klydonograph is derived from two Greek words which mean wave and record. It is a device for making a record of surges. The klydonograph is a very simple instrument which consists merely of an electrode which is connected to the line through a potentiometer device, and measures a voltage proportionate to the tension of the line. This electrode passes over a sensitized plate or film in contact with the insulated surface of a grounded plate. Now the oscillation passing over the line makes a record on this sensitized plate which, after development, shows a figure possessed of certain peculiarities. A figure for a positive wave in any direction is different from the figure for a negative wave, so that it is possible to tell what the polarity of the wave is. One can tell whether it is an oscillating wave and, with certain arrangements, know not only the character of the wave, but also the steepness of the wave fronts and the direction in which the wave is traveling. So, with this device, it is possible to obtain very complete information about surges.

The method of determining the amplitude of the wave consists in measuring the diameter of the figure. The diameter is very closely proportional to the actual amplitude of the wave within the range in which this measurement is useful. In other words, for the higher voltages where the surges are of some concern, the calibration is very close, within 15 per cent.

The first klydonograph consisted merely of an ordinary sensitized photographic plate rotated under the electrode and the plate had dials which would register time on the record. The complete rotation occupied one day. This, of course, was not very convenient, inasmuch as the plates had to be changed every day. The present klydonograph is a much more suitable instrument. It consists of the same arrangement except that it has a roll film which goes over an insulated grounded cylinder. The roll needs to be changed only once a week and provision can be made for longer or shorter periods as desired.

Now, so far as possible, we are giving klydonograph service to those who have problems requiring investigation. We have about twenty of these seven-day instruments in operation. We have two experts whose work is devoted solely to looking after the instruments and giving this service. We have investigated quite a large number of systems in the east.

Of course, in carrying out this work we aim to do our field work as much as possible during the summer months when we have lightning, etc., so that we have been able to make only a preliminary analysis of the results. The preliminary analysis shows that in no case were there any signs of high frequency, and, in fact, the highest frequency we obtained was something less than 30,000 cycles per second. The duration of these surges is very brief. They don't travel very far. They very quickly become damped so the bogey of sustained high frequency does not exist. We haven't found it anywhere. Our experimental work has extended particularly over such portions of the United States in which lightning is very frequent and severe.

Lightning has proved to be the most prolific source of high voltages, but even lightning has caused nothing we need fear. The highest voltage hasn't gone beyond the possibility of insulation.

We expect to carry on this service to the best of our ability, but of course we are limited as to men and also as to number of instruments. On a large system it is pretty hard to do with less than a dozen instruments; to carry out investigations properly, one should have more.

We expect to do some theoretical work in the winter months, analyzing results of the investigations of the previous months; and we probably shall have to do some work on cable systems during the winter. Cable systems have been very free from surges and that, of course, from theory is what we expect. We wouldn't expect to have the surges in cables due to effects outside, and surges from cables connected to outside lines don't amount to anything at all. I don't think that the trouble with cables can be attributed to outside sources. The troubles with cables are inherently inside the cables themselves; surges may come about due to the trouble in the cables, possibly.

I may state that grounded-neutral systems have been very free from surges due to short circuits and other abnormal operations.

J. H. Cox: Mr. Wood's instructive paper leaves little to be said to lay the ghost of alarming abnormal conditions on the 220-kv. lines of the Southern California Edison Company. As pointed out, the tests were sufficiently extensive to be truly representative of conditions on those lines. Since Mr. Wood's paper tells the whole story so far as his system is concerned, it seems appropriate to present information gathered more recently with the klydonograph on other lines.

7. A. I. E. E. JOURNAL, October 1925, p. 1151

During the year 1925 surge investigations have been made on quite a number and variety of systems, both open-wire and cable. The causes of abnormal voltages on transmission lines may be classed under three headings, switching, short circuits and grounds, and lightning.

Switching. For the most part the experience with surges resulting from operating activity has been much the same as that on the Southern California lines. Switching surges in general are less than two times normal in terms of crest voltage to ground. Only two types of operations caused higher surges. One of these was the opening of an idle but energized line on a 15-mi., free-neutral line and caused surges as high as 4.3 times normal. The other was synchronizing with a high-voltage switch at the end of a 150-mi., free-neutral line. These surges reached a maximum of 4.6 times normal.

Short Circuits and Grounds. Experience with short circuits and grounds on other grounded-neutral systems has agreed with that on the Southern California Edison lines. No major surges on such lines have resulted from short circuits and grounds either accidentally or intentionally produced. Short circuits and grounds on free-neutral systems have, in general, produced high-voltage oscillations reaching a maximum of 4.5 times normal. These voltages were recorded on the two ungrounded phases in the case of a single-phase ground.

Lightning. As would be expected, lightning has thus far proven to be the best generator of high-voltage surges though in the California Edison System high voltages due to this cause were absent. No differentiation is made by this source between types of systems but it varies widely with locality. In one case a voltage of 1000 kv. to ground was recorded. Many other surges caused by lightning were recorded, ranging from 400 kv. to 700 kv. Some of these were oscillating and others unidirectional. The unidirectional surges were positive in polarity. The oscillating surges were usually recorded at times when an interruption was caused by the stroke.

R. W. Sorensen: During the past twelve years I have been much interested in these line flashovers. Some of our first theories as to the cause included mechanical means, such as spider webs, dirt, soiled insulators, etc. As a basis of the spider-web hypothesis there were found many big spider webs attached to the lines and it was supposed that these webs might become wet and cause some of the flashovers. The possibility of these flashovers being caused by birds was also considered. But, at that time, there was never sufficient conviction to warrant the expense of erecting devices to keep the birds away from the towers as has been done recently.

It is my endeavor in this discussion to encourage the idea of trying to solve our difficulties by doing simple things first, although it seems to be human nature to first apply complicated methods and later simple methods to problem solutions. One of the objects of engineering education should be to teach us, as engineers, to avoid a complicated method of attacking problems.

It must be borne in mind that, although it has not been mentioned in Mr. Wood's paper, he is dealing with a line to which are connected transformers with delta connections so that on this line there is no probability of getting effects such as arcing grounds might produce on Y-Y connections if these connections are not properly supplemented with tertiary windings connected in delta.

G. R. F. Nuttall: Perhaps it would not be out of place to mention the 220-kv. tie line between the Great Western Power Company and the San Joaquin Light & Power Company.

The design and tests on the standard towers have just been completed and Mr. J. A. Koontz of the Great Western Power Company has taken particular care to shield all points on the towers so that no corona will form near the wires. In order to reduce eccentricity in the joints, it is better to place the bracing in the cage of the towers alternately inside and outside. This inside bracing ordinarily offers quite a sharp edge which is a point where corona might form. Therefore we have bolted a small angle to the outside brace which exposes its flat side to the wire.

Mr. Wood mentioned that they used a dynamometer in stringing their cable. I wonder if we could have information as to the type used, as the spring type is unreliable at the higher tensions.

I should like to present for discussion the question of insulation, not of the line itself but at the ends of the line. Mr. Wood's paper has given us useful information on transient voltages and 220-kv. operation and I wonder what the manufacturers' views are as to the rating of their bushings for oil switches and transformers.

In the case of the Pacific Gas & Electric Company and the Southern California Edison Company, I think I am right in saying that the A. I. E. E. ruling (Sept. 1922) has not been upheld. The switch bushing ought to be tested for 2¼ times the line-to-line potential which equals 495 kv. for 220 kv. operation, and the transformers (three single-phase, auto-transformers with grounded neutral) at an induced voltage of twice line-to-line potential plus 1000 which equals 481 kv. for 240 kv. at the sending end.

L. N. Robinson: In connection with Mr. Wood's paper, the most effective cure for the flashovers seems to be the bird guards. I wish Mr. Wood would give us a description of them.

D. I. Cone: I wish to comment on a by-product result of Mr. Wood's paper. The table on the seventh page and the oscillograms tells of the investigation of the normal residual currents of the system which were found to be without features that would aid in the explanation of transients. This record, if supplemented by data regarding the sizes of transformers, their characteristics and connections, will be of considerable value to the Joint Committee of the National Electric Light Association and the Bell Telephone System, which is studying the distribution of such residual currents in systems of this kind. A special project committee is doing work with a view to enabling us to predict these residuals and their resulting inductive effects upon neighboring lines.

Roy Wilkins: In the discussion on Mr. Wood's paper the question is brought up regarding the insulation on the oil switches and bushings and transformers used on the 220-kv. line. The transformers and switches themselves are tested for 2.73 times the normal voltages to ground, the bushings at 2.25 times line voltage. There is before the Institute's Standards Committee at the present time the proposition to change the requirements for the potential tests on grounded transformer equipment to some such value at 2.73 times line voltage.

H. Richter (communicated after adjournment): The paper states that, on the small overhead networks, the transformers are of the type having a network protector in the same case. This form of network protection has been abandoned on the two systems where it originated, because of faulty operation and lack of real protection. Experience shows that these devices are excellent transformer protectors but cannot be relied upon for network,—that is, service protection. I am rather inclined to ascribe the excellent record of 0.7 per cent transformer burn outs mentioned on the seventh page to the lightning protection offered by the common system neutral, as emphasized on the fifth page. As these protectors are so designed that they do not protect against trouble in the primary or secondary lines, their cost of about $3 per transformer-kv-a. makes a rather high insurance rate against transformer trouble. The substitution of carbon circuit breakers tripped by reverse-power relays in the underground area of Minneapolis is in line with the latest methods of network protection where real reliability of service is demanded.

In general it may be said that the loop method of primary

feed is particularly applicable to bulk loads of 300 kv-a. up and fed at the higher primary voltages of 11,000 volts and above. This is because of the high cost of spare capacity in feeders and substation apparatus, high rupture capacity, loop-sectionalizing switches, and pilot-wire control. For underground areas where transformer banks, serving miscellaneous distribution loads, range from 75 to 300 kv-a. each and are spaced on the average 500 ft. apart (but may be up to a maximum of about 1000 ft. apart) there is being installed in six large cities and planned for many others a very simple system of a-c. distribution. This is the secondary-network system with automatic network protection that was described by A. H. Kehoe[5] and by W. R. Bullard,[6] and that has been in successful operation in New York City for almost three and a half years.

In this network system the feeders are radial, no primary protective or sectionalizing devices being necessary. The secondary mains are spliced together to form a solid mesh which requires no junction boxes, and a compact triple-pole network unit inserted in the transformer-secondary connections to the network protects for every type of fault that might interfere with continuity of service except failure of the prime source of power. Advantage is taken of the elimination of primary switches, cut-outs, and disconnecting potheads, to employ higher primary voltages such as 13,200 volts. Thus may be saved the cost of either station step-down transformers and lower voltage switches, or even of the entire substation. By limiting the feeder capacity to 150 amperes, each feeder can carry about 3000 kv-a. and be confined to either a small or a large area depending on the load density.

The system of multiple street lighting introduced in Minneapolis is undoubtedly a step ahead both from an operating and economic point of view. However, some electric service companies object to pilot wires for control and others to mercury switches. One manufacturing company has developed a system of control by a form of carrier current over the primary feeders. This dispenses with both of these features and also the reenergizing contactors. The switching units are sturdy, comparatively simple, will not be expensive, and are small enough to mount in the vase of an ornamental post. They can also be used in conjunction with a primary switch for controlling pole-mounting constant-current transformers feeding series lamps. The sender at the station is likewise simple and substantial. Further, there are practically no losses in the switching units.

The system has had a successful trial equivalent to a year's service. It is anticipated that this method of control, together with multiple street lighting, will be the standard street-lighting system of the future.

R. J. C. Wood: There was a question asked about possible slipping of aluminum on steel with changes of temperature. I have given that matter quite a little thought and have come to the conclusion that there is no longitudinal motion between the two metals. Imagine a piece of cable, steel inside and aluminum outside; somewhere it has two ends clamped together, so that the two ends cannot move with respect to each other. If you make it sufficiently hot the aluminum expands away from the steel, but in an actual line, the tension of the cable is sufficient to stretch the steel so that the aluminum does not become loose. In actual construction, I doubt if you will find separation of the aluminum from steel. They will act together as a unit with this exception: with changes of temperature the stress passes from one to the other. The hotter the metal, the greater the stress in the steel; the colder the metal, the more stress in the aluminum.

In closing my paper: the design of the bird guards has been a matter of trial and error. We equipped a portion of the line with what is known as Mr. Barre's bird pans. The bird pans consist of a horizontal tray of metal lying on the lower member of the top crossarm, the idea being that it would form an efficient mechanical shield between the bird and the conductor. Apparently they have worked quite successfully, the only objection being that they are rather expensive, and unless they are very well anchored to the members of the tower, the ordinary vibration sets them imitating big drums, so that there has been some complaint from real estate agents trying to sell property in the immediate neighborhood.

There is another kind of bird guard which is simply an exaggerated saw-toothed, galvanized iron, the points being perhaps 1½ or 2 in. apart and with a height of some 6 in. so that it is a very acute point which is not comfortable to the bird. These were fastened along the members of the tower for a distance of approximately 5 ft. on each side of the center conductor. This line, by the way, is of horizontal construction, and similar pieces of metal were fastened to the sloping portion of the crossarm in the outer positions.

When we first put up these bird guards as mentioned in the paper, we didn't know what a clever fellow the bird was, and we put up one kind of a guard to protect the place we thought he was going to roost upon, but, driven out of there, he took the next best thing. He even went so far as to climb into a little piece of 6-in. channel which is underneath the main crossarm, a piece only about a foot long, which is a part of the structure from which the center string of insulators is hung. When prevented from getting on the main body of the crossarm, he flew underneath and got into this little cage place, so we have had to protect that too.

Regarding the residual currents and the situation of the transformers and auto-transformers, the information Mr. Cone asked for can be given him, but it really will not be of very much use since it refers to a line which is not transposed. We have been trying ever since the line was built to find time to transpose the conductors, but we have never had time to take the line out of service and do this work. As soon as the third line is in, we expect to be able to take out the other two lines, one at a time, and transpose them. This will balance them, statically, against ground, and will reduce to a considerable degree that residual current.

5. Underground Alternating-Current Network, by A. H. Kehoe A. I. E. E. TRANSACTIONS, 1921, p. 844.

6. A Study of Underground Distribution Systems, by W. R. Bullard, A. I. E. E. TRANSACTIONS, 1924, p. 856.

Fundamental Considerations of Power Limits of Transmission Systems

BY R. E. DOHERTY[1]
Associate, A. I. E. E.

H. H. DEWEY[1]
Associate, A. I. E. E.

Synopsis—At this time the power limit of transmission lines is a live subject and presents such complications as to require very careful analysis. The paper points out the essential features to be considered in a study of the problem, and calls attention to some outstanding results of an experimental investigation of the subject with a view to clarifying some of the points that have been under discussion in the past two years.

It is shown that the problem of stability is not necessarily confined to long-distance, high-voltage transmission, but may be present in any system where the impedance of the transmitting circuit is high compared with the load to be carried.

While the impedance of the transmission line and transformers plays an important part in establishing the breakdown point of a system, the characteristics of synchronous apparatus with the method of voltage regulation used are of equal importance.

It is shown that the synchronizing power of synchronous apparatus is largely dependent upon the field excitation at the time excess load is applied, that field excitation is determined by the circuit conditions under steady load, and, in order to provide for increase of excitation with increasing loads of considerable magnitude, some automatic means of controlling the field is essential.

The rate at which mechanical load in large quantities can be added to a system is limited on account of the necessity of change in angular displacement between the generators and receiving bus; this changing angle requires relative speed change, which takes time. This fact, together with the inherent tendency of synchronous machines to "stiffen" under sudden applications of load, makes it possible to rely on the usual vibrating-type voltage regulator working on the field of the exciter to provide the necessary field change. It is brought out that the maximum load that a system can carry under steady conditions at normal voltage can be suddenly thrown on, and the voltage regulator, with the assistance of the factors mentioned, will provide the necessary excitation.

Voltage regulators are practically a necessity where it is desired to approach, under operating conditions, the ultimate maximum power of the system.

Transient load changes that occur on the usual system, such as throwing on or off load, cutting in or out transmission circuits, etc., can be easily taken care of, providing such changes do not exceed the steady state limits of the system.

The effect of short circuits depends upon their nature, whether three-phase or single-phase, and upon the location and duration. This subject is discussed briefly and the conclusion drawn that successful operation can be obtained under usual short-circuit conditions if adequate relaying is provided.

The possibility of increasing the limit of power transmission by improving the apparatus and the characteristics of the transmission circuits is discussed, and it is pointed out that no great development may be expected from any scheme yet proposed regarding a modification in line characteristics. With reference to the apparatus, it is possible to make some changes in the design of synchronous machines tending to "stiffen" them, such as higher saturation, larger air-gap, etc., but in general no radical improvement may be expected here that does not materially increase the cost and decrease the efficiency of the machine. Attention is turned therefore toward such schemes of regulation, or compensation, of the synchronous apparatus as would increase the maximum power. Among these is the use of reactors for locally controlling power factor and thus too the field excitation of the more important synchronous machines. However, the possible additional power thus obtained is limited, and, as it now appears, other methods which have greater promise will be resorted to.

The use of a mercury-arc rectifier in the alternator field circuit seems to have great possibilities. By varying the field current in rigid proportion to the armature current, a very significant degree of compensation of the armature reaction is obtained—about 50 per cent, which is, of course, equivalent to almost doubling the inherent capacity of the generator. While this scheme is not yet in practical form, its efficacy has nevertheless been demonstrated in factory tests, and it is regarded by the authors as one of the most promising developments at this time.

* * * * * *

THE problem of power limits of transmission systems has within the last few years assumed importance in the study of long distance transmission of large blocks of power. Fundamentally there are no new elements entering into the use of long lines operating at high voltage that would be disturbing were it not for the fact that economic considerations require an approach closer to the maximum power which it is possible to transmit than has been the case in most of our existing systems.

In general, the problem is not confined to long distance transmission, but is one that may be met at any time where the impedance of the circuit from the scource of power to the point of consumption is comparatively high. There have been a number of isolated cases cited wherein the limit of stability has been reached and synchronous systems have pulled apart, where no great distance of transmission was involved. The growth of power networks is now so rapid, with interconnections that will require transfer of such large amounts of power, that it is essential that the principles on which stability of operation depends be thoroughly understood.

The rapidly increasing demand for electric power requires extensive development of our national resources with consequent growth of long distance transmission. The economical use of our water power, as well as mine mouth steam plants, makes it necessary to keep the investment in transmission lines as low as possible, which involves carrying the maximum amount of power feasible over each line with due regard given to continuity of service. Matters of efficiency and regulation can be met by well-known methods and the question of the maximum amount of power that can be transmitted over a single circuit becomes one of stability

1. Both of General Electric Co., Schenectady, N. Y.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., Sept. 15-19, 1925.

of the line with its connected apparatus. It is necessary, therefore, that all the factors entering into the problem be thoroughly well known and appreciated, as errors may be extremely costly both in investment charges and quality of service.

While great stress has rightly been laid on the limiting effects of long lines as a determining factor in stability, it is nevertheless true that no general conclusion can be drawn as to the probable limit of power from data regarding the line alone. There are many other elements that assume varying importance with different lengths of lines, such as different characteristics of generating and receiving apparatus, different types of regulating devices, etc.

The papers[2] that have been read and the discussions that have taken place on this subject during the past few years have shown such varied viewpoints that one who has not made a special study of the subject may easily be confused as to what the real problem is and its practical importance to the industry.

It is the purpose of the authors to outline the underlying considerations of the problem, with particular reference to the factors which determine power limits under various operating conditions, and to cite a few of the more significant results of an extended investigation. The details, both of the methods of analysis and of the calculated and test results, will be presented in forthcoming papers. In other words, the present paper is, to a great extent, merely a statement of the problem, a discussion of the factors involved, and an indication of the direction in which the investigation is leading.

Most of the studies of stability of transmission systems that have been made have been on lines of from 300 to 500 miles in length designed to operate at 220 kv. This fact has led, in some cases, to the erroneous impression that there is something inherent in the use of 220,000 volts for transmission purposes that may limit the amount of power which may be transmitted with stability. It should be borne in mind, however, that there is a definite limit of power which may be transmitted over any system regardless of voltage or length of line and it is just as possible to encounter unstable conditions in a line one mile long, operating at low voltage, as it is to encounter it in a long line at high voltage. It is merely a question of how close to the power limit normal operation may bring us.

With a given installation of synchronous apparatus, the amount of power that can be transmitted over a connecting circuit is, generally speaking, decreased by increased impedance and vice versa. With the growth in capacity of some of our large steam generating systems, requiring extensive use of power limiting reactors, it is necessary to take the matter of stability into account and avoid the possibility of too much reactance between large generating stations or even sections of the same station. There are a few well-known cases on record where the power limit has been reached between stations and loss of synchronism with consequent shutdown has resulted. This has occurred in systems operating at all voltages and varying lengths of connecting lines.

The stability problem, then, is not an entirely new one, though cases of trouble from this source have been sufficiently isolated and of such a special nature as to cause little stir among operating engineers.

On extremely long high-voltage lines there is another factor in addition to high impedance that has a tendency to limit the power to be transmitted with a given synchronous installation. The charging current of such lines becomes quite an item and, contrary to that which might be expected, may reduce the maximum power. This is brought about from the fact that, although the charging current reduces the effective impedance of the line, it also lowers the necessary excitation on the synchronous apparatus by reason of the higher power factor. This feature will be discussed later in greater detail. It is practically the only factor in long high-voltage lines tending to affect stability that may not be present in short low-voltage lines.

The matter of charging current may also become important in underground transmission, especially if the voltage of cables is raised materially. While this problem is not before us just now, the tendency toward higher voltage underground may place it there in the future.

An analysis of the problem of the determination of the limit of power on a given system involves a close study of not only the characteristics of the transmission line, transformers, generators, exciters, synchronous condenser and receiving load, but the method of operation and system of regulation as well.

The phenomenon of breakdown of a synchronous motor operated from a bus of large capacity is pretty well known. If the shaft load of such a motor is gradually increased it will finally reach a point at which no more electrical power can be supplied to the motor, even though the bus voltage remains constant, and the motor will drop out of step. The amount of electrical power which can be supplied to the motor at a given voltage depends upon its internal impedance and its excitation at the time the load is applied. If the excitation remains constant at its no-load value, breakdown will occur at a much lower load than if the excitation is increased.

A synchronous generator functions in a manner similar to a synchronous motor and can be driven out of step with the bus, if a prime mover of sufficient capacity is connected to it. The amount of shaft power necessary to drive the generator out of step with the bus depends upon its internal impedance and the excitation at the time.

If a synchronous generator is used to furnish power to a synchronous motor of the same size and characteristics, and with excitation on each machine correspond-

2. Groups of Papers presented at A. I. E. E. Convention at Philadelphia, Feb. 1924, and New York, Feb. 1925.

ing to no-load normal voltage, then when the shaft of the synchronous motor is loaded, both will drop out of step at a value of load that is approximately one-half of that which either would carry if connected to a bus of the same voltage and of infinite capacity. This is due to the fact that their impedances are in series and of twice the value of a single machine. As the motor is gradually loaded, it drops back in phase position with respect to the generator and drops out of step at a definite angle.

Going a step further, if a synchronous generator is used to supply power to a synchronous motor through a reactor or over a transmission line, there will likewise be

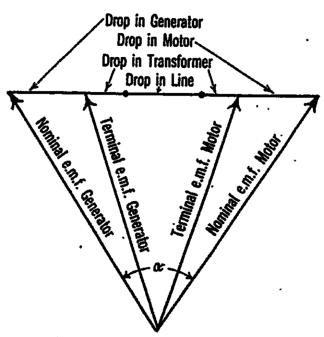


FIG. 1—SYSTEM VOLTAGE VECTORS

a definite breakdown point which, at a given excitation on each machine, will be less than before the reactor or transmission line was inserted. This is, of course, obvious, as the line or reactor causes in itself an added impedance drop and an added angular displacement between the generator and synchronous motor. In other words, the power first appears in electrical form at the generator, then successively passes through intermediary apparatus including transformers and line and finally through the motor. That is, it passes through a succession of electric circuits in each one of which, including the generator and motor, the power flow causes an impedance drop. This changes the magnitude and displaces the phase of the voltage as shown in Fig. 1. In the simple case considered, the components of impedance drop add in series, producing a total displacement between the internal, or nominal, voltage[3] of the generator and the internal voltage of the motor. The maximum power in this case is proportional to the product of these two voltages, *i. e.*, the field excitations, inversely as the total impedance between them, and occurs when they are displaced by a definite angle which is usually less than 90 deg.

The matter of excitation as noted above is thus one of the greatest importance in the problems of maximum power limit. Indeed, on short lines the difference between the breakdown point with no-load normal voltage excitation and with full load 0.8 power factor excitation may be as much as 100 per cent.

3. Corresponds to the value of field excitation.

At first glance, it may seem a simple matter to provide sufficient excitation to obtain high synchronizing force in the synchronous apparatus, but it is really a rather complicated problem. The excitation for any given voltage at the terminals of an alternator depends upon the power factor of the load. As mentioned above, in the case of the alternator driving a synchronous motor of equal size, there is no leeway at all in the control of the total field excitation of the two machines, if the terminal voltage is held constant. A reduction in the field excitation of the synchronous motor will require an equal addition in the field of the alternator and while one is weakened the other is strengthened a similar amount and no gain results.

In the case of long-distance transmission lines it is desirable, so far as the line is concerned, in order to keep the losses and regulation of the line at a reasonable figure, to deliver power at the receiving end at a high power factor. This results, however, in a high power factor on the generators at any given load and consequent lessening of their breakdown capacity. As a matter of fact, some local low power factor load on the bus at the generating end of a long-distance transmission system may actually increase the maximum power possible over the line by increasing the generator excitation.

In this connection it will be noted that a high power factor load is actually a detriment in so far as it affects synchronizing power, and it is certainly one of the

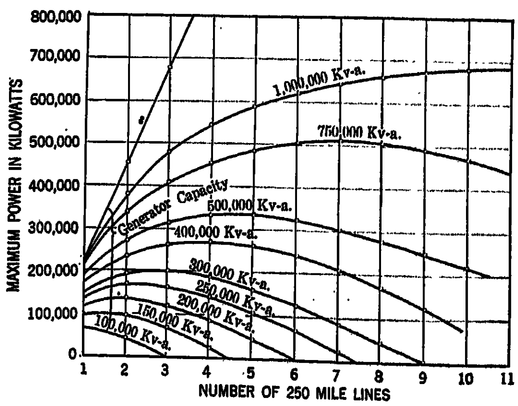


FIG. 2—MAXIMUM POWER WHICH CAN BE TRANSMITTED OVER A STRAIGHT-AWAY 250-MILE LINE AT 220,000 VOLTS, SHOWN AS A FUNCTION OF THE CAPACITY OF SYNCHRONOUS APPARATUS, AND THE NUMBER OF TRANSMISSION CIRCUITS. SYNCHRONOUS MOTOR LOAD ASSUMED.

phenomena that should be taken into account in the design of a long distance transmission system. It may come as somewhat of a shock to many engineers who have not made a close study of this problem, to learn that the inherent charging current of a long high-voltage line actually reduces the maximum power that can be transmitted with respect to that which could be carried over a line of similar reactance without charging current. The authors have made actual studies showing that more power could be carried over two high-voltage long-distance lines in parallel than over three with a given

installed generating capacity. This is merely an illustration of the effect of reducing the excitation on the generators by the excess charging current of the third line in a case where the impedance of the generators is an important factor.

Fig. 2 will serve to illustrate this point. It will be noted that for the 250-mile, 220-kv. line considered, there is a definite relation between the installed synchronous capacity and the number of transmission circuits that will give maximum power over the system. In the case of 300,000-kv-a. synchronous capacity of usual design at each end of the line, three circuits will carry approximately 200,000 kw., while five circuits will carry only 160,000 kw. and nine circuits, no power whatever. This effect would be still more pronounced in the case of a 500-mile line.

It has been suggested that the bus at the generating station end of the line be loaded with reactors to allow the transmission line a high power factor load but to lower the power factor on the generators themselves. This would increase the excitation and thus the maximum power that could be taken from the system. This plan shows some real possibilities and under certain circumstances may be employed, although there are other methods of obtaining similar results that may be used to better advantage.

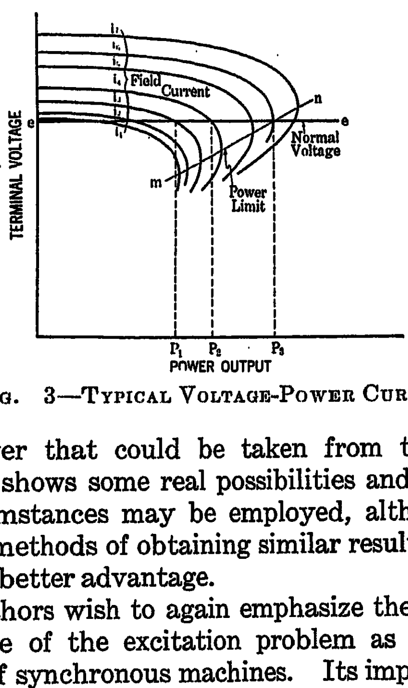


FIG. 3—TYPICAL VOLTAGE-POWER CURVES

The authors wish to again emphasize the very great importance of the excitation problem as a factor in stability of synchronous machines. Its importance has been recognized before by the authors of previous papers on this subject, but it has not been given quite the prominence that we feel it deserves. Were it possible to absolutely control the field current of synchronous machines as desired, the problem of stability would be very much simplified.

With the importance of field excitation in mind and the knowledge that under fixed conditions of load, voltage, and power factor, the strength of the field will also be fixed, it will be seen by reference to Fig. 3 that only a small increment of load may be added without breakdown unless the field excitation is changed. For instance at power p_1, corresponding to field current i_3, and normal voltage e, it would be possible to increase the load about twenty per cent, or if operating at power p_2, excitation i_4, the possible increase would be only six per cent. If means are provided, however, to automatically increase the excitation as load increases, the maximum power could be increased to p_3, which is the ultimate maximum that could be transmitted at normal voltage. This maximum could be reached regardless of what load was being carried at the time the increased load came on.

The authors of some of the early papers laid considerable stress on the point that voltage regulators in general use, while tending to follow the load requirements by increasing the field current as needed, were entirely too slow to do so adequately when the time transient of the exciter was taken into account. The argument looked entirely reasonable when it was suggested that increased load might be thrown on instantly, as by dropping off a large turbine generator at the receiving end—thus dropping its load on the transmission line. Fortunately there are compensating features in the characteristics of the system, and the linkage between the armature and field of the alternator make it possible for the ordinary vibrating type of voltage regulator and a properly designed exciter to supply the needed excitation at any rate at which load can come on to the line and generating station.

The explanation of this phenomenon which the authors feel is of the greatest moment lies in two important facts: first, that large blocks of system load cannot be thrown on the generators instantly, as the angle between the generators and receiving apparatus must increase—this requires a slowing down of the receiving system which, of course, takes time—and second, as load comes on the generators, the sudden increase in armature current induces a field current tending to hold them in step. These two phenomena consume sufficient time to allow voltage regulators and exciters to function, furnishing the necessary field current to maintain voltage at the greater load. A more detailed discussion of this point will be given later on.

This point brings out the very great importance of the use of voltage regulators in a long distance transmission system, as the maximum load safe to carry over a line, from the standpoint of stability, will be increased in some cases more than fifty per cent by their use. This important consideration, in the authors' opinion, has had practical demonstration in some existing systems that have operated for some years without difficulty from lack of stability. Many cases that have come to our attention would be on the ragged edge, if not impossible to operate, without voltage regulators.

The foregoing discussion covers, in a general way, the simple case of a generating station feeding a synchronous load approximately equal in kv-a. capacity. For such a case the maximum power is easily calculable and there are no serious problems when the matter of excitation and regulation are understood and properly provided for.

When complications arise, such as the generating

station feeding more than one independent transmission line or when the receiving end is a network with generating capacity and load of its own, the problem becomes difficult. In general, the more synchronous apparatus there is connected to either the generating or receiving-end bus, the more nearly will the maximum power limit of the line itself be approached, because the condition of infinite buses is being approached. Generating stations tapped into the middle of a long line, or synchronous condensers of large capacity used either at the receiving end or in the middle of the line, increase the maximum power that can be transmitted over a given system. The calculation of the actual power limits in such cases becomes very complicated.

During the past year the authors, in conjunction with other interested engineers, have been carrying on an extensive series of tests in the General Electric Company factory on an artificial transmission system equipped with synchronous generating and receiving apparatus, synchronous condensers, voltage regulators, transmission lines, adjustable as to the length, characteristics, etc. With this equipment it has been possible to set up almost any combination of conditions that might obtain in a practical installation and to vary the characteristics of individual pieces of apparatus in any way that might show promise of interesting results.

The experimental work on this artificial system has been paralleled by mathematical analysis throughout and not only has a mass of valuable data been obtained showing the effect of modifications in design or operation of the equipment under varying conditions, but methods have been developed which the writers feel will be of great assistance in the solution of any practical problem. The results of this comprehensive investigation, which is now being completed, will be published in the near future.

In the foregoing, the larger aspects of the problems which have arisen in connection with the development of power transmission in this country have been discussed in a general way, and the situation has been outlined as it exists today with respect to the use of standard apparatus. In that which follows, the problem of possible future increase in power limits will be stated and analyzed in terms of the various factors which determine the present limits, and possible solutions will be suggested.

Influence of Synchronous Machines

As already suggested, in those problems in which stability is a practical factor, the outstanding fact is the usually predominating influence of the impedance of the synchronous apparatus. For illustration, consider a 500-mile straight-away transmission line at 220,000 volts. If there were no power limitations in the electrical apparatus at the ends of the line (the conditions usually referred to as "infinite bus"), the power limit of the system, *i. e.*, of the line itself, would then be about 130,000 kw. With synchronous apparatus of the usual design and of the usual capacity with respect to the power to be transmitted, the limit becomes about 70,000 kw., or practically half. This point may be further emphasized by the fact that a few actual proposals have been studied in which the line, less than a hundred miles long, was practically a negligible factor. Hence the power limit is imposed in such cases largely by the impedance of the synchronous machines.

When this fact was first encountered, a number of plans for increasing the maximum power of the system were suggested, but much of the promise of success disappeared upon analysis. Larger generators might be used, but that was too costly. Another line might be added, but that was not only costly but, in the long lines, it actually decreased instead of increased the maximum power. Studies have been made of the use of series static condensers in the line of sufficient size to neutralize, at least partially, the reactance of the line and transformers. Theoretically, there are great possibilities in this scheme, as the total reactance of a line of any length can be reduced to a negligible value. The present high cost of condensers, together with certain difficulties of operation, render this plan out of the question for the present but it is one worthy of further attention for engineers studying this subject. A scheme[4] was proposed for decreasing the reactance and increasing the capacitance of the line, but although this might improve the line regulation, it did not gain favor, partly on account of the difficulty of construction, but principally because it would further weaken the generators by increasing the charging current. An exciter which would respond very quickly to the voltage regulator might, it seemed, strengthen the generators and thus increase the ultimate maximum power at normal voltage, *i. e.*, greater than p_3 (Fig. 3); but it was early recognized that this could not be accomplished by such an exciter and vibrating type regulator. Such regulation makes it possible, as already mentioned, to carry the ultimate maximum, corresponding to p_3 in Fig. 3, even if this is thrown on the system suddenly—but not significantly more than that. Thus, when all of the above proposals have been analyzed, the situation is not far different from the starting point with respect to increasing the ultimate maximum power under steady operation at normal voltage.

Yet this is what must be done, if it is hoped to transmit power over long lines—500 miles or more—in synchronous operation. There may be some other and better way to do it than in synchronous operation, but, if the latter is to be retained, the problem of increasing the maximum power reduces to one of stiffening up the synchronous apparatus.

Thus, the problem to which a number of interested engineers have been directing their attention has been along two general lines. One has been to "stiffen" the

4. "Output and Regulation of Long Distance Lines," Percy Thomas, A. I. E. E. Trans., Vol. XXVIII, p. 615, 1909.

synchronous apparatus, and thus approach the power limit of the line itself. The other, perhaps looking considerably to the future, has been toward finding some other means than synchronous operation. Regarding the latter, however, there is little of interest, to the authors' knowledge, other than that such studies are being made. In the light of the past growth and the probable future, such limits as that even of the line alone, cannot, of course, be accepted as final. Hence the situation demands such studies. However, the hope of the immediate future lies in the other direction —in finding such modifications and auxiliaries in connection with present synchronous apparatus, including the synchronous condensers, as will make possible a nearer approach to the power limit of the line alone.

The problem of stiffening the synchronous apparatus has been attacked along two general lines. One relates to schemes of regulation of the more important synchronous machines, the other, to modifications in design, which, in effect, would result in machines of higher *inherent* power capacity—that is, lower reactance. The former include the regulation of the field current, the use of shunt inductances across the generator terminals, and any other schemes which would apply to regulating currents from an exterior source. On the other hand, to lower the reactance of the machine itself is, of course, a problem in the design.

Conditions of Operation

Before taking up in detail the questions of design and regulation, it is well to consider the conditions of operation which affect the problem of maximum power. The study of this problem resolves logically into the consideration of two conditions of operation. In one, technically referred to as *steady state*, all forces involved in the entire system are in stable equilibrium. The power flow is everywhere steady, and in the apparatus the voltages, field excitation, magnetic flux, etc., are all constant. Everything is balanced and steady.

In the other, referred to as *transient state*, conditions are changing. Power flow, speed, magnetic flux, voltage—all of these are in a state of change.

Now the maximum power which can be transmitted, and, to a limited extent, the amount which it is necessary to transmit, depend upon the state. If, during steady state operation, a load is suddenly thrown on, or a loaded generator is dropped, or a short circuit occurs, a readjustment or transient must follow in speed, power, voltage, magnetic flux, etc., before steady conditions are again established. During this change, the synchronous apparatus all becomes inherently more powerful, or "stiffer," than in steady state, as explained under *transients*, but, due to possible "overshoot" of power on the swing of load following the shock, the power which must be transmitted may also be increased above that required after the system settles down. If a weight attached to a spring is suddenly dropped, thus stretching the spring, it will drop, on the first swing, not to the steady state position where the spring tension equals the pull of gravity, but will overshoot, throwing additional tension on the spring. If we imagine the stiffness of the spring to increase temporarily during this transient state, the analogy with the present problem would be fairly complete. The maximum power which can be carried is temporarily increased, but the amount which it is necessary to carry is also increased.

Steady State

The steady state operating characteristics of a generator supplying power to a transmission system are shown on Fig. 3. It will be noted that as the field current, shown as parameter, is increased, the maximum power—*i. e.*, the limit of stability, where the slope of the voltage-power curves is infinite—is progressively greater, and occurs at successively higher voltage as indicated by the line $m\, m$. Hence there is a particular value of field current, i_6, for which the maximum power occurs at normal voltage. This is obviously the ultimate maximum which the system can carry at normal voltage. While a still greater field current i_7 would give greater maximum power, it would nevertheless occur at a voltage higher than normal.

Influence of Field Excitation

In Fig. 3 it will be noted that operating at power p_1 and at normal voltage e requires a field current i_3. If the power were increased to p_2 without changing i_3, the system would break out of synchronism, since the maximum power with i_3 is less than p_2. Thus, when any system is operating as near the limit as in the above illustration, the field current must be promptly increased as the load is increased, to hold the voltage along the line $e\, e$. The question of the rate at which load can be applied, as well as other transient conditions, will be discussed later. The point to be observed here is that the degree of stability is indicated by the slope of the voltage-power curve; that the maximum power is greater, the greater the field current, and occurs at an ever higher terminal voltage; and that there is therefore one field current i_6 for which the maximum power occurs at normal voltage. And this is the ultimate maximum which can be transmitted under steady state at normal voltage.

It may be further stated as a very important fact bearing on the characteristics of synchronous generators that, roughly, the greater the field current *at the same voltage*, the greater the stability and maximum power, regardless of what means are used to obtain the larger field current. Thus, to lengthen the air-gap of the machine or increase the degree of magnetic saturation, or decrease the power factor of the load by connecting across the terminals a shunt inductance—any of such measures will require more field current at the same load and voltage, and therefore improve the stability and maximum power.

The steady state characteristics of synchronous condensers are, of course, very similar. These comprise a set of curves of the same general form as those in Fig. 3

and involve the same quantities, except that the abscissas are reactive kv-a. instead of active power. Similarly, the maximum value of reactive kv-a. which the synchronous condenser can deliver is progressively higher and occurs at successively higher voltage, as the field current is increased.

Parallel Operation

In both of the above cases, the characteristics are for a single machine. Suppose, for instance, that three or more generators, instead of one, are connected to the bus. Then what are the characteristics? If the units are all alike, including the governor characteristics of the prime movers, and the excitation currents of all units are kept equal, then a similar set of characteristics as in Fig. 3 could be drawn, taking field current as parameter. These would represent the voltage-power characteristics of the bus. In the successive steady state conditions after increments of load are added, the governors have placed correspondingly more power, the regulators correspondingly more field current, on all machines. So they all reach their respective power maximum at the same value.

However, if the units are different, additional parameters are necessary—the governor characteristics and settings for all prime movers, and also the distribution of field current on the several generators. Under this condition, the units do not reach their respective maxima at the same total load. Thus, when there are a number of units in parallel, the steady state limit of the combination is a function of the distribution of load and of field excitation, and also of the governor characteristics. And there is an optimum distribution which gives the greatest power, and this is such that all reach their respective maxima at the same total load.

The same is true of a number of branches of a power network connected to the same bus. Each branch may be from a power bus such as just discussed, involving the parameters mentioned. So while the problem of the extended combination is, of course, still definite, it is nevertheless greatly complicated by additional parameters.

So, in general, systems under steady state operation can be characterized by such curves as shown in Fig. 3. As generators are added, the slope of the curves becomes less, reaching zero with an infinite generator capacity. Up to three or four branches the characteristics are amenable to reasonably definite calculation. Beyond that, resort must be made to simplifying approximations or equivalent circuit tests. Methods of calculating the simpler cases of steady state conditions have been published[5]. Extensions and additional methods will be presented in the near future.

Transients

In transient conditions, many factors, in addition to those which influence steady state operation, are brought into play. Among these are the momentum of rotating masses, and the time element in both the electromagnetic circuits and in the governors.

Sudden Application of Load. In Fig. 4, if load is suddenly thrown on the shaft of the motor, the electrical power to the motor can not increase until the motor drops back in phase, which requires a temporary drop

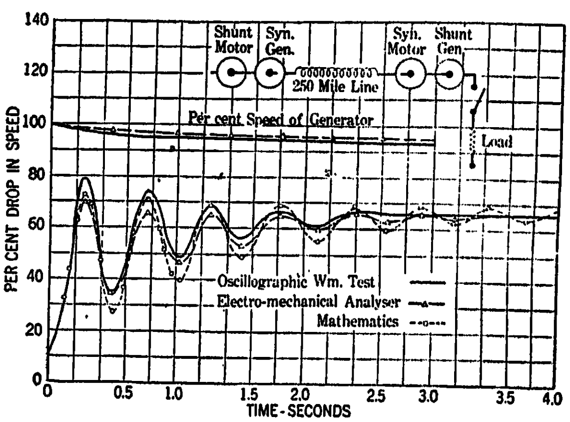


Fig. 4—Transient Conditions Following a Sudden Application of Load

in speed. This means that the initial increased demand was partially supplied from the momentum of the motor, causing the electrical power supply to increase more gradually, thus lessening the shock to the system. Moreover, the load which first falls on the generators is initially supplied from the momentum of the rotors, until the speed drops, after some oscillation, to that value at which there is balance, determined by the governor, between mechanical input and electrical output. Fig. 4 shows the transients for an analogous case as indicated.

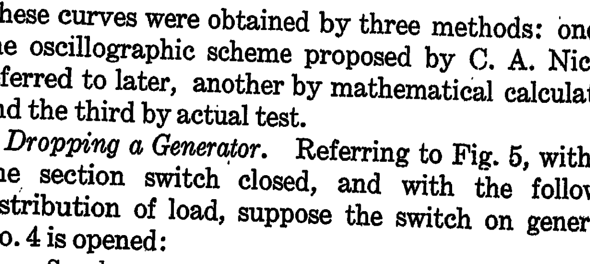


Fig. 5

These curves were obtained by three methods: one by the oscillographic scheme proposed by C. A. Nickle[6], referred to later, another by mathematical calculation, and the third by actual test.

Dropping a Generator. Referring to Fig. 5, with the line section switch closed, and with the following distribution of load, suppose the switch on generator No. 4 is opened:

Synchronous motor No. 3	400,000 Kw.
Generator No. 1	100,000 "
Generator No. 2	280,000 "
Generator No. 4	200,000 "

5. Group of papers and discussion by Edith Clarke and C. A. Nickle on this subject presented at the Midwinter A. I. E. E. Convention, Feb. 1924.

6. "Oscillographic Solution of Electromechanical Systems", by C. A. Nickle, A. I. E. E. Convention, Saratoga Springs, June 22-26, 1925.

Fig. 6 shows the nature of the resulting transients.

At first, the power flow and the electromagnetic torque of the various units are quite independent of the torque on the shaft of either the motor or generators which is determined completely by the electrical constants of the system, since due to the inertia of the rotating masses, the relative-phase positions of the rotors remain, for the moment, what they were before the switch was opened. After a transient, the system settles down to power conditions as determined completely by the governor on the prime movers. Thus, in the first moment the power flow is determined by the constants of the electrical circuits, and in the last, by the governors.

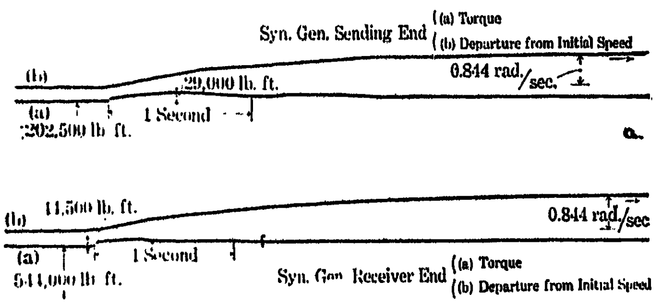


FIG. 6—TRANSIENTS DUE TO DROPPING GENERATOR NO. 4 FROM SYSTEM IN FIG. 5

Switching out a Section of Line. Another case is that in which a section of line is suddenly opened, as indicated in Fig. 5. The load distribution is the same as above, except that Generator No. 4 is disconnected. This sudden interposition of added reactance requires a phase adjustment which does not increase the steady state load on the system, but it does cause a power oscillation which usually overshoots the steady state value. This contains all of the transient elements discussed above, and is illustrated in Fig. 7.

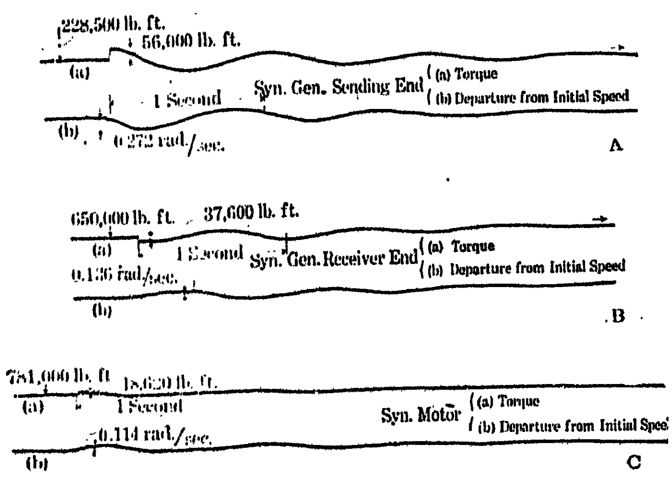


FIG. 7—TRANSIENTS FOLLOWING THE SWITCHING OUT OF A LINE SECTION INDICATED IN FIG. 5

During the above transients, the time elements of both the governor and the magnetic fields play a part. After load is thrown on a unit, it is a matter of seconds before the governor becomes adjusted to the new condition. Hence, regardless of the sudden load, the flow of water, or steam, as the case may be, is practically the same as before, and in studies of what happens in the first second or so, constant *flow* (not constant torque) is usually assumed.

The magnetic flux linked with the alternator field circuit also does not change in the first moment[7]. The armature currents due to sudden load automatically induce a corresponding m. m. f. in the field to sustain constant flux linkage. If there is no voltage regulator, the flux gradually dies down to steady state conditions, but during the meantime the reactance of the machine is less. It starts as transient reactance of relatively low value, which includes armature and field leakage, and ends as synchronous reactance, which is of relatively higher value and which includes armature leakage and armature reaction, but not field leakage. As already mentioned, this inherently stiffens the machine in the first moment. In other words, to speak of the spontaneous rise in alternator field current following a shock, whether it is a short circuit or a sudden application of load, or to say that the transient reactance applies, is merely to refer to the same phenomenon in different terms.

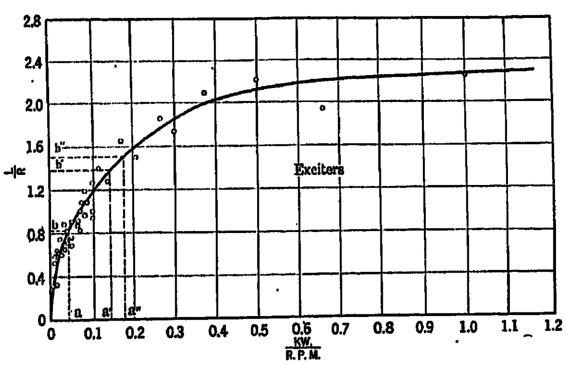


FIG. 8—CURVE INDICATING THE RELATION BETWEEN THE TIME CONSTANT AND THE SPEED AND CAPACITY OF EXCITERS

If there is a voltage regulator, then, as previously mentioned, it is a race between the rate at which the load comes on and rate at which the exciter can build up. As mentioned before, the combination of circumstances attending transients of this character are favorable in this connection. The alternator field current tends, by itself, to increase sufficiently to maintain constant magnetic linkages in the field circuit. So, the machine is not dependent, in the first moment of a shock, upon the exciter to increase the field. It does this itself, and the current tends to hold up for an appreciable time. This gives the exciter a chance. The period of oscilla-

7. "A Simplified Method of Analyzing Short-Circuit Problems", by R. E. Doherty, TRANS. A. I. E. E., Vol. XLII, p. 841, 1923.

tion of a system is of the order of one second, which means that the peak of the first overshoot in power would usually occur in about a half second. It is in this swing that the *inherent* field current rise, or the transient reactance, saves the situation and gives the exciter voltage a chance to reach the proper level by the time conditions have settled. And as already pointed out, the standard design of exciter has, in many cases, sufficiently low time element to adequately meet this condition. Where special attention is required in making the exciter more responsive—perhaps in the large, slow speed exciters—there are a number of ways in which this might be accomplished. The most obvious, and perhaps the simplest, is merely to decrease

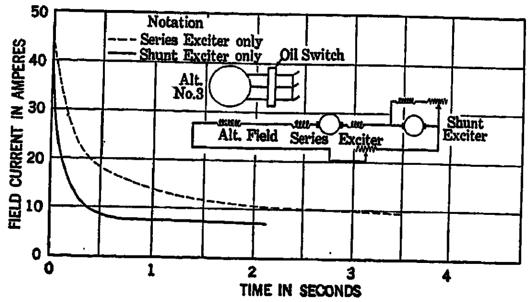


FIG. 9—EFFECT OF "SERIES EXCITER" IN LENGTHENING THE ALTERNATOR FIELD TRANSIENT

the duration of the transient in the same way the transient is shortened in any inductive circuit—by decreasing the value of the time constant L/R.

Such measures, if necessary, would apply to the larger exciters of low rotative speed—that is, machines of large volume. These have higher time constants because they have greater masses of copper and iron. To illustrate how these factors are related, Fig. 8 gives a number of points representing actual exciters over large ranges, showing the time constant plotted against kw./rev. per min., which, at given magnetic and current loadings, represents volume. It will be noted that these points lie well along the average curve. The exciters used in the tests fall at the points b a, b' a', and b'' a''.

Another and perhaps more efficient way of meeting the situation is to use the "series exciter" in the excitation circuit. This functions merely as a negative resistance, and thus neutralizes the effect of the ohmic resistance of the alternator field circuit. If it were not for the latter, the field flux of the alternator during a sudden swing in load would remain constant—that is, there would be no tendency for flux decay. So the alternator transient is lengthened by the extent to which the resistance is thus neutralized. This gives the shunt exciter a longer time in which to build up. Fig. 9 shows how the alternator field transient is lengthened by the series exciter. The total ohmic resistance was adjusted to the same value in either case.

Short Circuits. Short circuits are one of the most serious kinds of transients. This transient also calls into play all of the factors mentioned above. A three-phase short circuit on a branch of a system not only disturbs both the power and the magnetic balance of the generators, but also completely isolates the branch from the rest of the system until it is cleared. Loss of synchronism is usually expected in such cases.

A single-phase short circuit (which most of them are[8]) is not so serious, in that power flow is possible through the other phases past the point of short circuit. Whether the parts of the system break synchronism depends, among other things, upon the load at the time, the momentum of rotating masses, the duration of the short circuit, the electrical power transfer during the trouble, the restored voltage when the short circuit is cleared, and the amount of "induction motor" torque in the synchronous machines.

Long experience on existing systems is the most promising aspect of this question. Although now and then a case is reported where loss of synchronism follows a single-phase short circuit, it is rare where adequate relays with short time setting are used. This is a fact which cannot be overlooked in considering this important question. Calculations are usually made on the basis of conservative premises (as they should be) until basic data are complete; but even if such calculations indicate that loss of synchronism might occur oftener than good service could withstand, it must be remembered that experience rather indicates

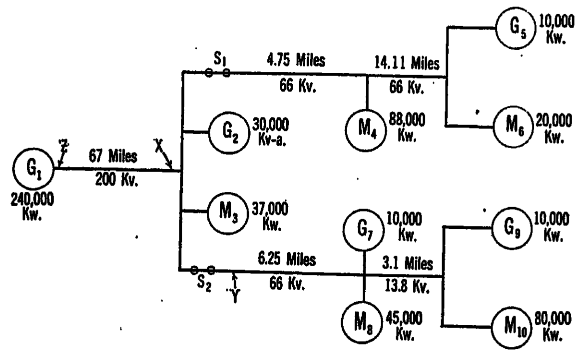


FIG. 10—TRANSMISSION NETWORK

the opposite. This is a problem on which sufficient detailed operating data have not been accumulated to afford adequate bench marks for calculation; and until this is accumulated, estimates must be no better than the premises on which they rest. Meantime, judgment, which is based on experience and such calculations as are possible, must be used in proposed undertakings.

An idea of the power oscillations following a single-phase short circuit at X on the system shown in Fig. 10 is given in Fig. 11. This is for a case in which the short circuit remains on the system, and the prime mover power is practically constant.

Many cases of transients similar to the cases referred

8. Particularly on high voltage systems in which the conductors are arranged horizontally.

to above, in which direct calculation is hopeless and step-by-step process is extremely difficult and often hopeless, can be effectively studied by Nickle's method, referred to above. In this, an equivalent electric circuit—involving no rotating apparatus—solves the equations, and the oscillograph plots the results.

With reference to all of the foregoing discussion of transients, while they may be much more difficult to solve than problems on steady state, methods of practical estimate and calculation are nevertheless available excepting for single-phase short circuits; but here it is less a question of method than of premises. Where direct mathematical methods fail, graphical methods can be used, as illustrated in the step-by-step processes devised by Bush and Booth.[9] Beyond this lies Nickle's equivalent circuit method.

Design

When the best estimates now available are made, it is found that, on the whole, the problem of increasing the

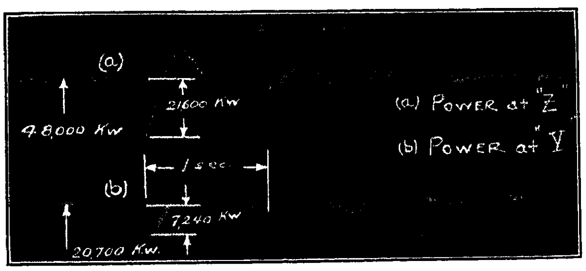


Fig. 11—Point Transient Following a Single-Phase Short Circuit at Point X on System Shown in Fig. 10

maximum power under all conditions of operation is principally one of designing apparatus and accessories that will give the greatest possible stability. It has been observed that this problem of stiffening the synchronous machines has been attacked along two lines, one relating to the design of the synchronous apparatus itself, the other to regulating schemes.

The problem of the designer of synchronous apparatus is to go to the limit in increasing the ratio of the field strength to armature strength, to the end that the armature reaction due to the load shall have the least possible effect in distorting or decreasing the magnetic field. The object thus sought is usually referred to, whether properly or not, as low synchronous reactance. From the discussion of field excitation under *steady state,* it follows that it is desirable to design the synchronous machines so that they will require at all times and loads as near as possible the field strength required by the maximum load, and to have this, in turn, as great as heating limits of the field will permit. Then, whatever the load, the machine is better prepared to take an additional load.

It is important to observe here that many generators for the usual commercial load of lagging power-factor are already designed to such a limit. These other measures for increasing the required field current are necessary in the present problem, because the power factor on long distance transmission lines is usually nearer unity than the ordinary commercial load, and therefore requires lower field excitation than the usual system load. In other words, what is thus attempted in design modification is, in reality, only to increase the stability or stiffness of the synchronous machines under this special condition of operation to that which it already possesses under the usual condition of lagging power factor operation.

The present problem, nevertheless, often demands more inherent stiffness than can be obtained by the above measures. This addition can be had only by increasing the size of the machine, which, of course, is both expensive and inefficient.

So the best that can be accomplished along the foregoing lines is hardly a satisfactory solution. It is an approach only, which, as it now appears, must be supplemented by effective regulating devices.

Regulation

This is the other general line of attack mentioned. The usual method of regulation is to automatically adjust the alternator field current by means of the vibrating type regulator operating on the exciter field to hold the bus voltage as near as possible to a constant value. Referring to Fig. 3, the function of this regulator is to cause the generator to pass from one curve to another along the line $e\,e$. There are, of course, an infinite number of such curves, and under steady state the alternator is functioning on some particular one of them. The *extremely important* point here is that although at any load under steady state the voltage is the same, *i e.,* along the line $e\,e$, nevertheless the machine is not operating on $e\,e$ as an inherent characteristic, but on one of the family of curves shown. In other words, although the machine operates at an intersection of $e\,e$ and one of the curves, it is on the latter, not the former. That is, the rate at which the voltage changes with respect to a power change is, for the moment, the slope $\frac{d\,E}{d\,P}$ of the particular curve, and this determines the degree of stability. The less the slope, the greater the stability, and, in the limit $\frac{d\,E}{d\,P} = 0$, there is the "infinite" generator. The slope of $e\,e$, of course, may be zero for any machine with a regulator, but that does not change the

9. "Power System Transients," Bush and Booth, Journal I. E. E., p. 229, March 1925.

inherent characteristic, on which the machine operates as illustrated in Fig. 3.

While it is not possible, in the nature of the case, to significantly change the slope of the characteristic by use of such a regulator, as explained later, it is possible to do it by other means of regulation. But before describing this, the general principle involved here which is not generally fully understood will be further discussed.

It is one thing to compensate for the effect of a phenomenon after it has occurred, and quite another to compensate *while* it is occurring. In the former, the thing happens and is then corrected; in the latter, it, in effect, does not occur. To illustrate the first: load is thrown on an alternator the voltage of which is controlled by a vibrating-type regulator. The voltage drops, then the regulator contacts close, which in turn starts the exciter voltage to build up, ultimately providing the field current necessary to restore the voltage. The time involved may be short, but the foregoing is nevertheless the sequence of events. The drop occurs and then it is neutralized. An exciter with low time constant may hurry the phenomena along, but the time-constant, even if decreased, still exists and it is interposed in the above sequence.

On the other hand, consider for instance the voltage drop in an inductive reactance of a feeder circuit. Add a series capacitive reactance of equal value. Then while the drop still exists across the reactance, it is, nevertheless, absolutely neutralized, at all instants, and at all loads, so far as the feeder circuit is concerned. The drop is compensated *as it occurs*, and the effect is the same as if it did not occur.

The difference between these two conceptions is the difference between what we have and what we should like to have in the principle of regulating alternators. If some one found such a scheme at once reliable and economically feasible for regulating alternators, there would be the equivalent of an "infinite" generator—one in which the change in voltage with change in load is zero, *i. e.*, $\frac{dE}{dP} = O$. And with apparatus so regulated at both ends of the line, the power limit of the line alone would have been attained. But such a scheme has not yet appeared in practical form.

To accomplish this in an alternator, two things are required. The armature reaction must be completely compensated by supplying opposing field ampere-turns of equal value in the proper space-phase and *at the time the armature reaction is occurring*. If this were accomplished it would be, in effect, as if there were no armature reaction.

The other point is that the leakage reactance must also be neutralized, which means that a voltage must be supplied which is in time-phase opposition to the voltage of the reactance. Theoretically, of course, this could be accomplished by series condensers of proper capacity, and this may sometime be practically feasible. Indeed, they may be used to neutralize the line reactance, as well.

To completely compensate for armature reaction, one is confronted by the difficulty both of applying the field ampere-turns in the right space-phase, and above all, of increasing it and decreasing it in exact conformity in time with the variations of armature reaction. However, while it is perhaps not to be expected that the ideal case will be realized practically, it is yet possible to partly compensate it by any scheme which will adjust the field current in rigid proportion to the armature current—which may be approximated by certain forms of self-excited machines.

But there appears to be a simpler way. Without expressing any assurance of practical application in the near future, it is noteworthy that a very substantial reduction in the effect of armature reaction has been actually obtained in a factory test by the use of the mercury-arc rectifier as an adjunct in the excitation

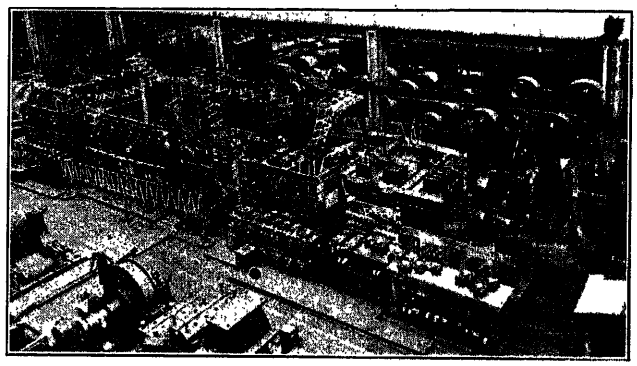


FIG. 12—GENERAL VIEW OF THE TEST

system. Its function is merely to supply field current in rigid proportion to the armature current, thus compensating for the latter *at the time it occurs*. By the nature of the circuits, and the variation of power factor, the space phase could not, in this particular case, be what was required. Nevertheless, the reduction in effective armature reaction, as evidenced by increased maximum power at the same voltage, under steady state, was of the order of 50 per cent.

Let the significance of this be clearly understood. Two maximum power (steady-state) tests were made on a minature 250-mile line with a 225-kv-a. generator supplying a synchronous motor of the same size. Fig. 12 shows a general view of the test set-up. One test was made with an ordinary vibrating regulator controlling the voltage of each synchronous machine, the other, with the rectifier as an adjunct. In the first, the power was gradually brought up to 120 kw. at 2000 volts at both ends of the line, 7.0 amperes on the motor field, 7.5 amperes on the generator field. This was the maximum power the system could carry. A further increase caused it to break out of step. Then the rectifier scheme was installed. Similarly the load

was brought up to the same point, *i. e.*, 120 kw., 2000 volts, 7.0 amperes field current on the motor, 7.5 on the generators. This power was *not* maximum. It was further increased at the same voltage to 154 kw., or an increase of 28 per cent.

This test is mentioned merely to illustrate the principle discussed above. The point involved is the difference between the slope of a curve and the value of the function itself. Analogously, if moving bodies were being considered, the interest would be not in the speed alone but also in the acceleration. It is not the value of voltage, but the slope of the voltage-power curve, that indicates the degree of stability. If a system controlled by vibrating regulators operating on the exciter is gradually loaded to imminent breakdown, the control can usually be taken over by hand and the same load held. In other words, although such regulators are highly effective in load transients in increasing the excitation as load is thrown on, they nevertheless do not significantly increase the steady state ultimate maximum power at normal voltage (p_3 in Fig. 3) over that corresponding to a fixed field current; and the reason is that, even with the regulator, the slope of the voltage power curve, in the first moment, is practically the same as if the regulator were not operating. And if it should be different during a sudden transient, it must ultimately return to steady state and thus to the slope corresponding to constant field.[10]

If this fact is not appreciated, it may be taken for granted, as frequently has been done before, that if the bus voltage is regulated, it may be assumed in calculations of maximum power that the voltage is *constant*. This assumption is justified only if $\frac{d\,E}{d\,P} = O$. It has even been proposed that by placing automatically regulated synchronous condensers along the line at given intervals, one may consider each of these sections as a unit, and that whatever power it is possible to transmit over one of them at the given voltage, can be transmitted, excepting losses, from one section to the other for a distance of, say, 1000 miles. Now this would be possible only with infinite generators and also infinite condensers, *i. e.*, such that the slope of the terminal voltage curve against reactive kv-a. supplied by the condenser is zero. To illustrate magnitudes, the maximum power of a 250-mile line at 220,000 volts, infinite bus at both ends, is about 225,000 kw.; of a 500-mile line, 130,000 kw. According to the proposal, with a condenser at the mid-point, it would be possible to transmit 225,000 kw., neglecting losses, over the entire 500 miles of line. Actually, a condenser at mid-point, of, say, 70,000-kv-a. capacity, would increase the maximum power only from 130,000 kw. to 156,000 kw. —not to 225,000 kw. This rather indicates what might be expected if the length were extended to 1000 miles.

10. An exciter with low time constant may expedite the regulation, but as long as the voltage must drop before the regulator contacts close, and the exciter field must be brought up before the alternator field current is changed, the compensation can not possibly occur at the time the armature reaction occurs.

Before concluding—regarding the foregoing discussion—it is well to mention again that consideration should not be confined to the cases of long distance straight-away transmission only. The problem in hand is one which, to a limited extent, relates also to power lines and networks of moderate distances, and the extent will undoubtedly become greater as the blocks of power to be transmitted continue to increase, or, indeed to tie connections of central station systems—to any, in fact, in which the power to be transmitted approaches the power limit of the system under the operating conditions. However, the problem involved in such systems, although more difficult to solve than those of single lines, is nevertheless the same, namely, to determine the power limits under various operating conditions, and to provide that these limits shall be as high as practically feasible.

What is the conclusion to be drawn from all of the foregoing discussion regarding synchronous machines? As to possible changes in design, the present indication is that no radical departure may be expected, that, at least so far as present proposals are concerned, reversion is made to the well established practises which have long since been followed in special cases where an increased stability or breakdown has been required: This is to to go to the limit in ratio of field strength to armature strength, and if that does not meet the required load, then get the remainder by merely increasing the size of the machine.

As to improvements by regulation, or more properly, compensation, it appears at this time that although there is possibly some hope for this in the future, the progress to date has been little more than the demonstration of the principle in actual test; but this affords sufficient incentive to actively pursue the investigation, when it is realized that for distances of transmission much above 300 miles the power limit with present equipment is such as to make these projects questionable from an economic standpoint, while, at the same time, the development of our resources will undoubtedly require the use of lines of this length in the near future.

ACKNOWLEDGMENTS

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Discussion

For discussion of this paper see page 994.

Transmission Stability

Analytical Discussion of Some Factors Entering into the Problem

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Fellow, A. I. E. E.

Synopsis.—*The subject of stability has been much discussed lately, because it has an important bearing on future large power developments. In the early stages of a large program, such as the proposed superpower program, good engineering and common-sense dictate that each step should be very carefully considered from all points of view, since a blunder or failure to give proper weight to some important factor, such as stability, might set back the development program for many years.*

A brief historical review of the subject of stability follows; for those who are not familiar with "static" stability there is a review of the subject in the Appendix.

A criterion of stability is suggested based on present operating conditions, namely, that for reliability each unit of the superpower shall be at least equal to the best that has heretofore been obtained with similar power systems.

The necessity of a careful study of the characteristics of all machinery connected to the transmission line is pointed out. The necessity of proper inherent characteristics in generators and synchronous condensers is emphasized, and particular stress is laid on the necessity of proper volt ampere characteristics both inherent and with the exciter.

The action inside a generator during the transient following a change in load is discussed; it is pointed out that the true field is a resultant due to several magnetomotive forces in addition to that of the field circuit; the combined effect is a marked tendency to self-excitation, and inherent self-excitation would take place if it were not for the damping effect of resistance in the different circuits.

A brief review of other factors entering into the problem is given. These factors comprise inertia of moving parts, mechanical torque, speed of relays, circuit breakers, etc. The difficulty of correlating all these quantities is pointed out, and a basis on which it is practical to make computation is suggested.

Those who have not studied the subject of stability are recommended to read the Appendix before proceeding with the subject of transient stability.

The subject of transient stability is opened with a definition of stability of a power system.

The elements of the problem are discussed in some detail. The problem is one of obtaining the conditions of equilibrium, taking into account mechanical or applied torque, electrical or counter torque, inertia torque and damping factors, in addition to the electrical characteristics of the system. The action of a generator under suddenly applied load is discussed in some detail.

The "transient" stability of a simple system is discussed, use being made of a new diagram known as the power angle diagram which may be derived from the circle diagram as obtained for static stability. Three diagrams are required for the simple investigation, but the method may be elaborated to include all the factors affecting the problem including the characteristics of governors, exciter systems, and so forth.

The difference between the problems of switching operations, load swings, and short circuits is pointed out. In the last case the effect of different values of ground resistance is discussed at some length and also the effect of length of time before circuit breaker opens.

The necessity of obtaining reliable data on ground resistance with faults is stressed.

Throughout this paper the essentiality of delivering the necessary kilovolt-amperes to the line either by adequate exciter systems or by proper modification of machine characteristics in order to maintain a high order of stability is insisted on.

It is pointed out in the Appendix that while inherently compensated generators, synchronous condensers, etc., are future possibilities, our main concern is the problem of getting the most out of present day designs as our present day problems depend on these and not on something that may be commercially developed five years from now. Consequently the conclusions refer to means that may be made quickly available and the two most important are:

a. Improved inherent regulation of machines.

b. Increased speed of excitation.

THE problem of stability of transmission systems has come up for considerable discussion during the last few years due to the fact that it has an important bearing on the future development of power in this country, particularly in localities where there already exist large concentrations of power which it is proposed to tie together by high voltage transmission lines.

The connecting links between these large distribution centers and generating points must be such that the resulting unified system has at least as good a perform-

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Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., Sept. 15-19, 1925.

ance as regards maintenance and reliability as the best of its constituents, and as a matter of fact to fulfill the hopes of its advocates this resulting system should show a better performance.

A principle which common sense seems to dictate in the early stages of a development, of such magnitude as this programme represents, is to carefully weigh all the factors which go to make a successful system. It is not incompatible with the optimistic spirit, which is part of the make up of the engineering profession, to weigh every factor carefully before taking the first steps because so much depends on the success of the first part of such a program. A blunder or failure to give the proper weight to such a factor as stability might have very grave effects and set back the development program for many years.

The old saying that "a chain is no stronger than its weakest link" may well be taken as a guide during the initial stages, and I think that good judgment dictates that these first links of our superpower chain, especially those that introduce new sources of power, should be made as strong as we know how, at the present state of the art.

I wish to emphasize this point because there seems to be an inclination on the part of some engineers to underrate the importance of a careful study of stability, for the reason that large utilities today are operating satisfactorily and instability does not seem to disturb them. This reasoning is all the more dangerous because it is partly correct. The true reason why existing utilities are not troubled by instability is that when conditions of interconnection are met that tend towards instability, they are avoided either by giving up the advantages of interconnection or by strengthening the tie lines.

It is not safe to take past experience as a criterion on which to base superpower developments because of the necessity of maintaining service over the interconnecting lines, greater extent of the lines, and greater amounts of power to be handled by the lines, all of which factors are such as to increase the tendency to instability. As a matter of fact we know of a number of cases where instability has actually occurred and large systems have pulled apart causing more or less lengthy interruptions to service. How much more might the effect of the breaking apart of a large generating station from a superpower system be felt, especially in the initial stages, before the complete unified system has been achieved! I repeat, therefore, that it is not only good engineering practise but also good sense to make the first links of a superpower program as perfect from all points of view as is possible with the present knowledge of the art.

It is the object of the present paper to analyze the factors entering into the stability problem, confining the discussion to the consideration of simple cases in order to avoid undue complexity.

General Review of the Subject

The attention of the engineering world has been drawn to the phenomena of instability as a result of actual experience in operation. The early cases naturally had to do with links between generating stations in distribution systems. I can recall some early cases of instability due to short circuits in feeders connecting large generating stations which were the cause of a good deal of speculation.

More recently there has been a tendency to connect large utilities together with a view to interchanging power in case of emergencies. Unfortunately these tie lines have been frequently of the type very aptly defined as "shoestrings" and have failed to carry out the purpose for which they were intended. In other words these shoestrings in themselves were unstable and could not tie the systems together. In many of these cases, fortunately, the reason for the tie was more sentimental than real, and the systems tied together being self sufficient, no harm was done when they broke apart. The case of a superpower interconnection would be another story and a break apart might be serious.

We first became interested in this question as a result of a study of the possibilities of troubles due to connecting large hydroelectric generating plants through long transmission lines with large public utilities distributing power. This study has thrown a new light on this whole class of operating phenomena and a number of happenings which were mystifying in the past, in the light of knowledge gained by this study have very simple explanations.

To cite a few examples, we have all heard of the fading out of power from a hydroelectric source supplying a utility. We have also heard of power surges which appear to be of more or less harmonic character; that is to say, a generating plant has a period during which it alternately acts as a motor or generator. These cyclic changes may be relatively slow and will be indicated by the wattmeter which will swing in synchronism with the load changes. The system in such a case has already passed the point of stability although not infrequently the generator ultimately falls in step again. The effect on a large system may be quite serious as synchronous motors may be thrown out of step during the period of unstable operation.

In the case where the generating station has not passed the point of instability, it may oscillate about the point of stable operation for several seconds, this being evidenced by oscillation of the station wattmeter until the ultimate operating position of the generator rotors is reached. These characteristics will be further elucidated after the elements entering into the problem of stability have been considered.

The first study of a phase of the problem of stability was made by the late Dr. C. P. Steinmetz. In his paper which was published in the 1920 TRANSACTIONS of the A. I. E. E., he confined himself to the analysis of system troubles but did not give consideration

to its aspect as a possible limitation in high-voltage transmission.

The consideration of a superpower network extending over the Eastern States and perhaps eventually over a great part of the United States and Canada has emphasized the importance of a careful study of the factors influencing the stability of large superpower systems with a view to controlling and improving the characteristics of the various units entering into the problem.

I take it that a criterion of operation must be set up for each portion of the superpower program. It cannot all be carried out at once; therefore each portion must conform to some standard of operation. What is this standard to be? I feel that all public utility engineers will be unanimous in insisting that each portion of the proposed program when completed and in service will perform this service with at least the same reliability as the best that has been obtained in similar existing power supply systems. By reliability I mean freedom from interruption and ability to deliver load at the proper voltage.

The nature of the standard set up for superpower lines will influence not only the type of line construction used but also the characteristics of the generators and synchronous condensers. It may be necessary to look into the secondary transmission lines to insure that they do not prove to be the weak link in our chain. Where long transmission lines are involved, intermediate synchronous condenser stations will prove economical provided that condensers of proper design are used.

It is necessary therefore in studying the stability of superpower systems to consider also the characteristics of generators, exciter systems, condensers, etc., to ascertain what type of design will prove most suitable for superpower extensions and will insure the highest degree of stability. Other apparatus such as circuit breakers, relays, etc., will have an influence on the problem, but the generator with its exciter and the synchronous motor are the fundamental elements in the problem of stability.

Electrically both generators and synchronous condensers are extensions of the main transmission line with somewhat different characteristics. But, as we know, transmission lines differ in characteristics, and no appreciable error is incurred by ignoring the distributed capacity in low-voltage transmission while in the case of 220 kv. it becomes of supreme importance. The same thing is true of generators and synchronous condensers; these may appropriately be considered as transmission lines having reactive impedance (although saturation will affect the constancy of the reactance), if we have to consider only slow changes in load without regard to matters of stability; but when studying matters of stability, whether static or transient, both must be considered with reference to their exciter and damper windings so that the action becomes that of coupled circuits. The result of this is that any change in armature current sets up in the field current and damper winding changes in the currents which tend to annul the magnetic effect of the change in armature current so that the field tends to remain constant for a substantial period after the change takes place. If it were not for the resistance of field and damper windings, these transient currents would persist and the generator or synchronous condenser would be to a certain extent self-regulating. There would be a drop in terminal voltage due to the increase in load, however, due to the imperfect magnetic coupling between field and armature and this drop may be termed the leakage drop.

Since it is not possible to get away from resistance in the damper and field winding, recourse is had to an exciter system which builds up the field current faster than this induced current dies down. Thus the effect is the same as, or better than, what would be obtained if the field and damper windings had no resistance. It is possible to carry this method of compensation to the point where the terminal voltage remains substantially constant during sudden changes in load or other disturbances.

There are other ways of course by which similar results may be obtained, but these involve new developments whereas that cited above follows along standard well-tried lines.

The above, together with the transmission line, the characteristics of which are well known, constitute the electrical elements entering into the problem. There are other elements which come into our picture due to the fact that our generator, synchronous condenser, and the load are not only electrical transmission systems but also mechanical transmission systems having inertia and these systems are coupled with the electric system by a torque-speed or power coupling. This adds very considerably to the complications when we try to compute what takes place with a sudden change in conditions. We shall return to this subject later on when we look into the problem of transient stability in more detail.

I have enumerated above the physical and electrical characteristics of machines that enter into the problem of stability. In addition to this the problem is tied up with the action of the various relays such as those controlling the exciter voltage, the hydraulic relay devices controlling the gate opening, and the steam throttle governors, the relays controlling the operation of circuit breakers in cutting out sections of transmission lines when a ground or short circuit takes place. It will be obvious to engineers that to take accurate account of all these factors would be a hopeless proposition, so we proceed on the basis of ignoring such factors as appear not to influence the problem to any appreciable extent. For example, the assumption is usually made for ransient stability that the power input to the generators remains unchanged during part of the disturbance; this is justified due to the extreme sluggishness of hydraulic governors.

Transient Stability

Much has been heard in recent years of stability but no clear and comprehensive definition of the term has been published. Stability may be defined as the capacity of a power system to remain in equilibrium under steady load conditions, and its ability to regain a state of equilibrium after a disturbance has taken place. The first part of this definition is referred to in this paper as "static stability," and the second part as "transient stability." It should be noted that after a transient disturbance, the system will not necessarily seek the original state of equilibrium.

The problem of stability is one of securing a proper balance between mechanical input to a generator and its electrical output, and the electrical input to a motor and its mechanical output, the electrical quantities being dependent on the characteristics of the machines and of the system as a whole. It is, therefore, an exceedingly complicated problem, involving mechanical factors such as inertia, governors, gate speeds, etc., and also electrical factors such as machine characteristics, line constants, breaker operation, etc.

The natural tendency of a generator connected to a power system is to deliver electrical power equal in amount to the mechanical power delivered to its shaft less its own losses. This condition of equilibrium is satisfied when the counter-torque due to armature currents and field is exactly equal to the mechanical torque applied at the shaft by the prime mover. When such a situation arises that there is a difference in the mechanical and electrical torque, the generator either speeds up or slows down until a new position of equilibrium is reached; and during the transition stage, the inertia forces due to the moving masses act in a manner which tends to prevent any change in speed.

In thinking over these questions, it is well to consider all the aspects a generator or system of generators may take. It may be purely a generator or take on the characteristics of a synchronous condenser, or even of a motor. The torque on its shaft working as a generator compels it to deliver sufficient power to equilibrate the applied power. In attaining this state of equilibrium, moving from the initial position where the machine acts as a synchronous condenser (no external load) to the position where it acts as a generator, the rotor is accelerated, so that when equilibrium is reached, the rotor, considered as a mechanical cyclic system, has advanced in phase. This advance in phase is just sufficient to cause the equilibrating current to flow as in a simple transmission line, so that the power input and output diagram of the generator would be exactly the same as that of a simple transmission line, if it were not for the effect of saturation and the effect of internal magnetic coupling during the transient state. The phase advance of the rotor is just equal to the phase angle between the terminal voltage of the machine and the component of internal voltage due to the direct current in the field. This phase angle, however, is not the true angle of advance for the reason that the electrical torque depends upon the action between armature current and the actual field—not a fictitious field. The actual field is produced not only by the field current, but also by the armature current.

When a change in input or output of a generator occurs, currents are set up in the field circuit and

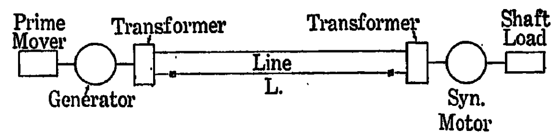

Fig. 1

dampers which tend to maintain field and the voltage generated thereby at their original value. At the same time, the rotor takes up a new position, and in so doing, inertia torques are introduced which must also be taken into account.

The proper value of voltage to take account of in matters of stability is therefore the voltage set up by the true field which is the resultant of the d-c. field current, induced field currents, damper currents, and the armature currents; this is the e. m. f. actually generated by the field and which is required to establish equilibrium between electrical and mechanical torques.

Let us analyze the problem with a simple system, assuming a generator supplying power to a synchronous motor over a two-circuit transmission line as indicated in Fig. 1. For the steady-state condition, there will be a phase angle between the internal e. m. fs. of the generator and motor. This angle is a function of the load, and in general varies in the manner shown in Fig. 3, which is known as a "power angle diagram." In the diagram, let P_0 represent a particular input to the generator. For this input, α_0 is the angle between the generator and motor e. m. fs. for the steady-state condition, or point of equilibrium.

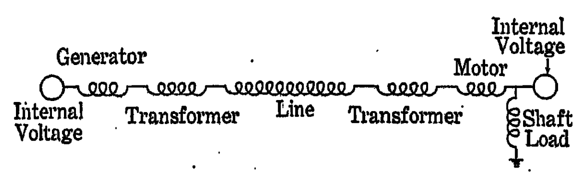

Fig. 2

It will be noted that it is possible to obtain another solution corresponding to α' which is, however, an unstable solution. Let us consider a small increase in the angle α beyond α'. This will cause a corresponding drop in the output, and consequently since the input is constant, the generator will accelerate and increase the angle of advance, the effect being cumulative. In the case of a small decrease in the angle α from α', the generator will slow down or move in the direction of α_0.

A similar analysis shows that α_0 is a point of stable equilibrium, because small displacements from that point will set up forces tending to restore the system to the point α_0.

It is evident from the diagram that the load can be increased to the value indicated by P_s which represents the static limit for the circuit assumed, and with the generator and motor voltages maintained. While

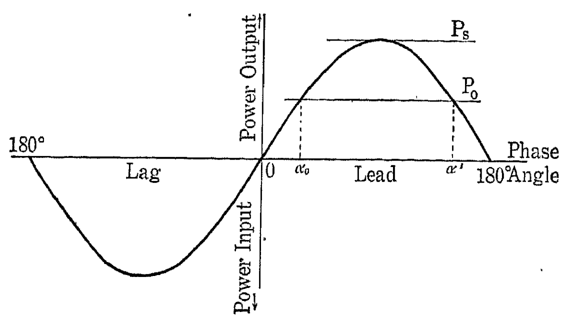

FIG. 3—ANGLE DIAGRAM OF SIMPLE POWER SYSTEM BASED ON CONSTANT VOLTAGES AT BOTH ENDS

there is a definite static limit dependent on the circuit characteristics and on the voltage conditions, there is no well defined limit for transient stability. In fact, it is necessary to specify both the load and the magnitude of the disturbance in order to determine the transient stability limit.

If the system is operating with the power input P_0 and a disturbance takes place, causing the angle α to increase beyond the value α', it will be unable to recover itself, even though the disturbing force has been re-

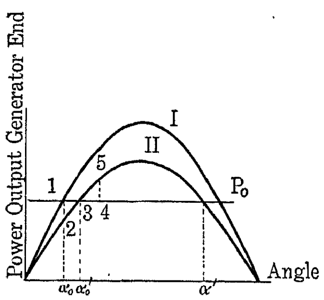

FIG. 4—ANGLE DIAGRAM OF SIMPLE POWER SYSTEM FOR SWITCHING OPERATION

Curve I Initial condition
Curve II Condition after switching

moved. In general, the disturbance will affect the generator and motor in different ways; consequently it will be necessary to employ two power angle diagrams in studying conditions during disturbances, one representing the generator end, and the other the motor end of the system.

The principal conditions of operation tending to produce instability in a power system are as follows:

a. Line switching
b. Load swings
c. System faults

As an example of line switching let us consider the simple system shown in Fig. 1 and Fig. 2 and assume that line L_1 is opened at either end. In this case the power angle diagram may be represented as shown in Fig. 4, for the condition before and after the switching operation. Let us assume that the power input to the generator is P_0 and that the original steady state angle is α_0. After the switching operation the new position of equilibrium will be reached at α_0'. When the system is operating at α_0 the opening of the line L_1 reduces the

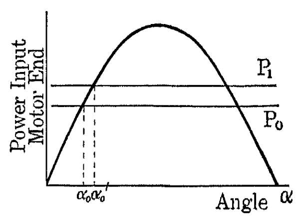

FIG. 5—ANGLE DIAGRAM OF SIMPLE POWER SYSTEM FOR SUDDEN INCREASE IN LOAD

power that will be taken over the system at that angle by the vertical distance between the power input curve and the new output curve. The power represented by the distance point 1 to point 2 represents that consumed in accelerating the generator rotor. Because of this acceleration the angle α will increase along the new curve towards point 3. As it approaches point 3 the accelerating force will decrease and the velocity will increase until at point 3 the accelerating force will be zero and the velocity a maximum. As a result the rotor

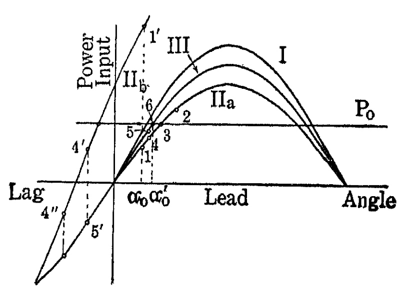

FIG. 6—POWER ANGLE DIAGRAM FOR SHORT-CIRCUIT CONDITIONS

Curve I Initial condition
IIa Low resistance fault
IIb High resistance fault
III Final condition after fault is cleared

will overshoot towards point 5, a retarding force being set up which will increase as the point 5 is approached. At point 5 the velocity will be normal and the retarding force a maximum tending to bring the rotor back to point 3. The amount of the overswing will be such that the area 1 2 3 equals the area 3 4 5 except for the damping action due to losses. Because of the losses the swings will become smaller and smaller, oscillating about

the final position α_0'. For the case considered the system is stable since the overshoot does not carry it beyond the point α'.

Load swings differ from switching operations in that the circuit constants remain substantially the same and the input and output vary, whereas, with switching, the input and output remain substantially the same and the circuits constants are changed. Considering again Fig. 1, and assuming that the load is increased as indicated in Fig. 5 from P_0 to P_1, if the increase is suddenly applied, the prime mover governor not responding immediately, the increment of power will have to be supplied by the kinetic energy of the system, slowing it down. This action will cause the governor to act and restore the speed to normal by increasing the input to the system to correspond to the new output. Initially, however, there will be a disturbance due to the falling back of the motor with respect to the generator increasing the angle α between them, and there will be an overswing similar to that described in connection with the switching operation in Fig. 4. The generator and motor will oscillate with respect to each other but together will slow down towards the new angle of equilibrium α_0'.

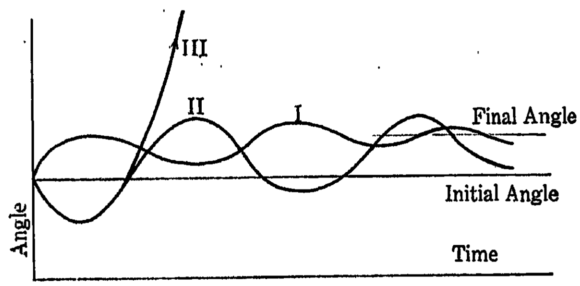

FIG. 7—ANGLE-TIME DIAGRAM OF SIMPLE POWER SYSTEM

I Switching Conditions—Stable
II High resistance fault—Stable
III High resistance fault—Unstable

The change of power input by governor action will in general necessitate the use of several power angle diagrams to properly represent the intermediate steps.

Short circuits present considerably more complications than switching operation and load swings due to the fact that three distinct networks are involved:

a. The original condition prior to the application of the short circuit.

b. A second condition while the short is on the system.

c. A third condition when the fault has been cleared, usually by a switching operation.

The second condition is what makes the short circuits radically different from switching operations or load swings. The short circuit may increase or decrease the power input according to whether it is a high- or low-resistance fault. A high-resistance short circuit usually occurs in the form of a fault to ground in a grounded neutral system. There is a certain resistance for which the power increase will be a maximum.

The effects of the two types of shorts are indicated in Fig. 6. Let Curve 1 represent the power angle diagram for the initial conditions with P_0, α_0 as the point of equilibrium. Curve I I_a will represent the conditions with a low resistance fault applied to the system. An oscillation will be set up about point 3 between 1 and 2 as described previously when considering switching operations. The circuit breaker may open at almost any point during this oscillation, for example at point 4. The opening of the breaker immediately modifies the circuit conditions and a third network is set up of which the power angle diagram may be represented by Curve III, the power output being changed from point 4 to point 5. In general the transition from the network of II_a to the final network of III will be accompanied by a second oscillation due to the fact that either the angle initially will not correspond to α_0', the angle of equilibrium, or the velocity will not be such as to satisfy the condition of equilibrium; the system will therefore oscillate about the point of equilibrium, point 6 in the diagram.

In the case of a high resistance fault it is possible for the power output of the generator to be greater than the input as indicated by the Curve II_b. In fact power may be drawn from the receiver end of the system to supply part of the energy absorbed in the fault, for which condition the angle becomes negative. The sequence of events is similar to that previously described for the low resistance fault. It will be noted that stability is indicated in case the breaker opens at the point 4′ because the energy absorbed during acceleration (area under P_0) is less than the energy that can be given up during retardation (area above P_0). Pull out would occur should the breaker open at 4″ instead of 4′.

It is obvious from these considerations that the fault resistance is of prime importance in the consideration of short circuits. Unfortunately there is very little information available at the present time concerning the resistance of faults under high current conditions.

In the preceding discussion an attempt has been made to give a general picture of the phenomena of stability. Certain simplifying assumptions have been made which in the more general cases will require modification; for example, the mechanical inputs and outputs of machine are assumed constant. When the actual values are known as a function of time, suitable correction can be made. In this connection it may be pointed out that the stability limit for transient disturbances could be increased if the governors were made to regulate the input to the generators to correspond more closely with their output. Another simplifying assumption made in the above discussion is that of constant internal voltage of the machines. The effect of this may be corrected for by drawing up several power angle curves for different voltage conditions.

In the determination of the angular swings of the system and the stability limit, it is necessary to employ the point by point method of analysis, all the forces for

each point being in equilibrium. The computation of the changes in the values of internal voltage during transient conditions is best accomplished by the "two reaction method" which permits the utilization of different time constants for the two components of main field flux. By this method it is possible to take into account the effect of cross-magnetization and also the effect of the excitation system. The two reaction method also permits the determination of the mechanical rotor angle, which must be taken into account in the consideration of the inertia effect.

Power angle diagrams do not show time; therefore, for the final results, it is preferable to use curves, with time as abscissas, since stability depends upon factors which are functions of time. The angle between the e. m. fs. at the two ends of a system or the variation in the angle as a function of time gives a very satisfactory criterion of stability. These angle time curves will have the general form indicated in Fig. 7. Curve I shows a switching operation; Curve II and Curve III show a short-circuit condition with a high-resistance fault. Curve II shows the breaker opening at such a point that stability is obtained and Curve III shows the breaker opening at such a point that instability results as indicated by the continuous increase in the angle.

The object of this discussion is to review the principal factors entering into stability and to provide an adequate visualization of the problem. It is beyond the scope of this paper to go into the detailed calculations of the magnitude of system disturbances and stability limits. In this respect it serves as an introduction to a more detailed paper by Messrs. Evans and Wagner which is being prepared for the Midwinter Convention in 1926.

In the above rather cursory analysis of the factors involved in the problem of transmission stability I have tried to stress several important points which I repeat below:

1. Stability is a characteristic of an entire power system—not of any particular tie line—and in the determination of limits, it is necessary to have a complete diagram of the system with full data on connected machines, loads, etc. Care should be taken to select for consideration operating conditions which are likely to be the most severe from the point of view of stability.

2. The standard of service in a superpower system must be of the highest order. While it is recognized that stability cannot be expected under all abnormal conditions, in my opinion the stabilty criterion should be that synchronism be maintained under single-phase faults to ground followed by the switching out of the faulty section.

A method of analyzing stability is indicated, based on so-called "power-angle" and "angle-time" diagrams. These diagrams take into account both the electrical and mechanical transients by a point to point method. The changes in machine fields are computed by Blondel's two-reaction method, as this permits the separate determination of the two components of flux, which have different paths and different time-constants.

CONCLUSIONS

In conclusion, particular attention is called to certain factors entering into the problem such as characteristics of machines, time required by relays and circuit breakers to open, speed of operation of governors, resistance in the ground faults, and earth connections.

As a result of the above analysis the following methods of increasing stability suggest themselves:

a. Improved inherent regulation of machines which can be obtained without greatly increased cost.

b. Increased speed of excitation; this possibility was discussed at the Philadelphia meeting of the A. I. E. E. in connection with the paper by Messrs. Bush and Booth.

c. Modification of the prime-mover governor more closely to regulate the input to the generator in accordance with the requirements of the transient, thus reducing the mechanical oscillations.

d. Development of a high-speed breaker and relay system to reduce the oscillations subsequent to the opening of the breaker.

e. Reliable data on ground resistance are very much needed, particularly under conditions of high fault currents. I would like to enlist the cooperation of public utility engineers in obtaining this information, as the value of ground is one of the controlling factors in the problem of transient stability.

f. The calculation of transient stability is, at best, a very tiring and long drawn out job. In view of the importance of making such computations in the case of every important branch or connection of the proposed superpower system, every effort should be made to reduce the labor involved in these computations. If an artificial model can be devised which will supply all the factors in their right relationship with one another, such a method of solving problems will be welcomed. A purely mathematical solution seems out of the question as the problem when reduced to the simplest form involves elliptic functions. Very few engineers are sufficiently familiar with these functions to be at ease in handling them.

I wish to express my appreciation of the help extended to me by Messrs. R. D. Evans and Powell in writing this paper and by Mr. Dovjikoff in making the sketches and checking the text.

Appendix

Review of the "Static Stability"

INTRODUCTION

In the papers presented at the 1923 Midwinter Convention at Philadelphia by my colleagues and myself we confined our attention to the phenomenon of stability which has been termed static stability. There was some criticism of these papers on the grounds that

they did not cover all there was to be covered in regard to stability of transmission systems. I think that I have said enough as to the elements that enter into the problem to indicate how hopeless would be any attempt to cover the subject generally in one paper and I may say in passing that this paper makes no pretense to go into details on any part of the problem but is more in the nature of an introduction to a paper by Messrs. Evans and Wagner which is in preparation for the next Midwinter Convention. The static stability problem appeared to be the stability problem in its simplest form and it seemed to us appropriate to begin the study of stability by a series of papers on this limit to the operation of transmission lines. That this is one of the important limits in operation under emergency condition is evident from the following statement by Mr. H. A. Barre in an article by him published in *The Electric Journal*, June, 1925.

In designing a long high-voltage transmission line, careful consideration must be given to its stability, because the economic load and the ultimate carrying capacity of the line are of the same order. Studies of the Big Creek line indicate that, with only a single line in service, the ultimate carrying capacity under steady load conditions should be about 180,000 kw. at the generator end. On one emergency, this condition was actually reached. One whole line was taken out of service on a Sunday for maintenance work. About six o'clock in the afternoon the load built up very rapidly to 183,000 kw. when the synchronous apparatus at the two ends of the line went out of step. The voltage dropped

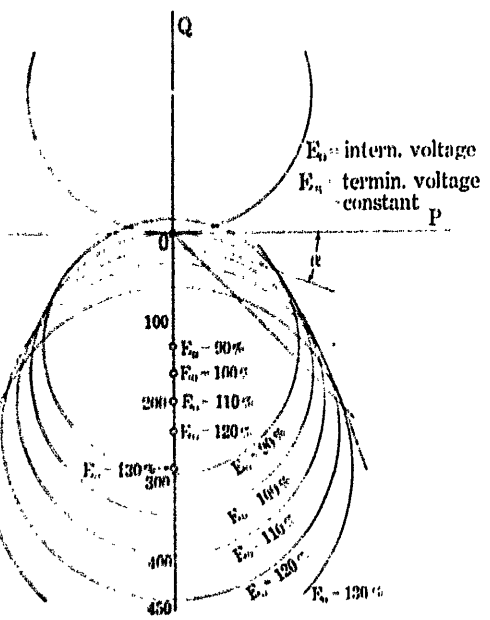

Fig. 8

nearly to zero, hovered there for awhile and then gradually built up to normal. About fifteen minutes later the load again built up to 183,000 kw. and the whole performance was repeated. Soon after the voltage came to normal the second time the other line was switched into service and there was no further trouble. So far as we know, this is the first time that a long high-voltage line has passed through this unstable condition.

The static stability limit will depend upon two conceptions. The first has to do with inherent stability and is based on the assumption that the regulator system is too sluggish to act before the transient current which tends to counteract the demagnetizing effect of the armature current has died out. This is the same as assuming constant excitation. The second considers the proposition that the dying out of the transient is so slow that the regulator has, in general, time to get into effect before the field has had time to change appreciably, so that the limitation is set not by the synchronous impedance of the generators motors and condensers but by the leakage impedance, which should also include the

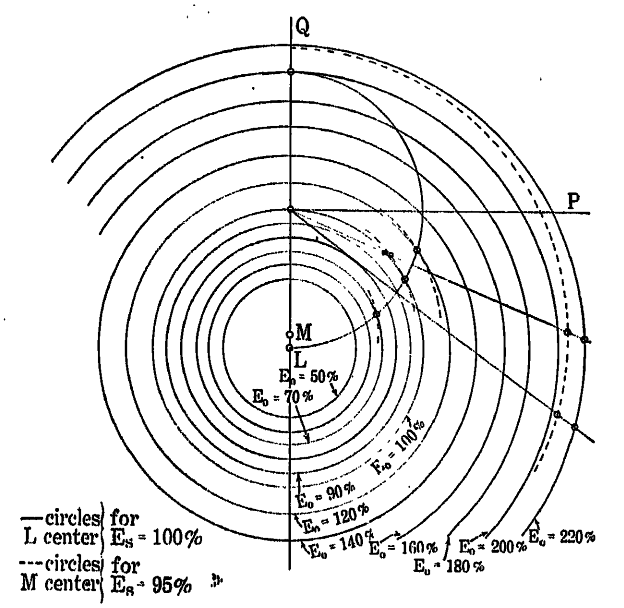

Fig. 9

leakage set up by the induced rotor currents. This is the same as assuming constant internal voltage.

There has been a great deal of discussion as to the proper basis on which the static stability should be determined. It has been argued that the synchronous impedance is the controlling influence but tests up to date indicate that this assumption is much too pessimistic; the second condition appears to be a reasonable assumption provided the rate at which excitation is required is not in excess of the ability of the exciter system. Several propositions that are necessary to a proper understanding of the subject will now be given consideration.

1. *An inherently compensated generator has no power limit.* This of course is an ideal that would be impossible in practise since internal heating would always be present in a practical machine. Several methods have been proposed for the internal compensation of generators, which are theoretically correct, but from a manufacturing point of view impractical. Even if such a generator were practicable the limitation of prime mover governors would make it of doubtful value. It is, however, a useful ideal to keep in mind when considering problems of stability.

2. *A generator having a fixed value of excitation will have a power limit depending on the nature of the load.* This power limit depends upon the value of the terminal voltage obtained under load and is therefore larger for leading power factor loads than for lagging power factor loads. The quantity entering into the terminal voltage regulation is the synchronous impedance which will vary with the field saturation and therefore is a rather crude quantity to use, but it serves quite well in illustrating the problem of stability qualitatively. If the load varies in power factor in such a way as to maintain constant terminal voltage as well, the load limit is quite definite and is obtained by a circle diagram of the same form as that for a simple transmission line. If we make a family of such circles (Fig. 8) using different values of terminal voltage, the loci for loads of constant power factor are lines drawn from the origin cutting the circles and stopping at the envelope returning back again to the origin. The power limit when the power factor is fixed is therefore the value of the load indicated by the intersection of this line with the envelope.

3. *A generator having its internal voltage fixed will have a load limit depending upon the nature of the load.* The same comments as in the case of proposition (2) applied here with the added statement that the actual load limit will depend upon the ability of the exciter system to maintain this constant field during changes of load.

Static Stability of Generator and Load. (*A dead reactive load*). A generator excited in the usual way will be stable for loads having only fixed resistance and inductance and the terminal voltage of the generator will be maintained constant within the limits of the capacity of the exciter system. During the transient stage, following an increase in load, there will be a fall in terminal voltage due to the leakage reactance of the generator, but this will be corrected as the exciter builds up the field. During the first transient stage, the field is kept practically constant by the corrective currents flowing in the field circuit and in the damper windings. The relation between internal voltage of the generator and the load may be easily obtained (Fig. 9) by drawing the power output circles for constant terminal voltage of the generator with varying internal voltage, drawing the locus for a constant power factor load which is a straight line drawn from the origin inclined at $\cos^{-1} \alpha$ to the axis of X. It will then be seen that, while the diagram indicates no limit, if we superimpose on this diagram a similar diagram for the same values of internal voltage but for a terminal voltage slightly less than the original value (Fig. 9), the power circles of the last diagram corresponding in internal voltage to that of the first will not always be outside the first, showing that an increase of admittance will result, after a certain load, in decreasing power at a given power factor, with the internal voltage considered constant. The condition when the first circle is wholly outside the second takes place in the neighborhood of $E_0 = 2\,E_s$ and for values of E_0 above this there will be a tendency for the voltage to dip on application of additional load of any power factor.

Dead Load with Leading Power Factor. In the diagram just given, the generator is assumed to have practically no resistance, so that the center of the concentric system of circles lies on the Y axis; the following construction, however, holds for generators and motors having resistance. If a circle is drawn having its center at the origin passing through the center for $E_s = 100$ per cent, this circle is the locus of intersection for the circles having the same value of E_0 but slightly lower value of E_s with the system of circles representing $E_s = 100$ per cent.

The Cartesian equation of the system of circles given in Fig. 9 is given by

$$(P_s - E_s^2\, g)^2 + (Q_s - E_s^2\, b)^2 - E_0^2\, E_s^2\, (b^2 + g^2) = 0 \tag{1}$$

In this equation the values $E_0\, E_s$ are parameters, fixed values of these quantities giving one of the members.

Let us consider the system E_s = constant; the system is a system of concentric circles with center at the point $P_s = -\,g\,E_s^2$; $Q_s = -\,b\,E_s^2$. If now we consider E_s to vary somewhat from its fixed value, let us say diminish slightly, the family of circles for the diminished value of E_s will intersect the other family of circles at certain points which will have significance because at these points the value of power delivered will be stationary because with increased admittance the voltage will tend to drop as fast as the current increases, the value of E_0 being considered fixed.

To obtain the locus of these intersections, differentiate (**1**) with respect to E_s and equate to zero; this gives us

$$2\,g\,(P_s - E_s^2\, g) + 2\,b\,(Q_s - E_s^2\, b) + E_0^2\,(b^2 + g^2) = 0 \tag{2}$$

Eliminating E_0^2 between (**1**) and (**2**) we have

$$P_s^2 + Q_s^2 + E_s^4\,(b^2 + g^2) - 2\,E_s^4\,(b^2 + g^2) = 0$$

or

$$P_s^2 + Q_s^2 = E_s^4\,(b^2 + g^2)$$

This is the locus of intersection of contiguous circles; E_s = constant when E_0 is the parameter and represents the loci of intersection of contiguous members of the family when E_s is considered to decrease slightly. It is a circle having the origin as center and which passes through the center of the family of concentric circles.

The circle touching the origin represents the condition of equality between internal and terminal voltage and all the circles that lie within this circle represent the condition of load in which the internal voltage is less than the external voltage; this condition can only occur at leading power factor. It will also be observed that where a line from the origin touches one of the circles, this represents a double solution and all such solutions lie within the circle locus. All points of intersections between constant power factor loci and the circles for $E_s = 100$ per cent contained in this region within the circle locus represent loads that are essentially stable

in character, whereas the values obtained outside this region are inherently unstable, for the reason that an increase of load does not take place naturally since the circle for a slightly lower terminal voltage and the same value of internal voltage as existed before the increase falls within the corresponding circle for the original terminal voltage and this same value of internal voltage. This illustrates the reason why under certain conditions the natural regulation of a system may become quite bad so that there is a tendency to produce a slump in voltage when additional load is thrown on, so fast that the exciter cannot keep up with it. If, however, the characteristic of the load is such as to give more leading current, or what is the same thing less lagging current, with decrease in terminal voltage the stability is greatly increased.

Induction Motor Load. This does not differ essentially from the dead inductive load discussed above but the tendency to slump in voltage after a certain load may be so marked that the exciter will not be able to cope with it and as a result the motor may fall in speed below its pull-out point.

Synchronous Motor Load. Assuming that the terminal voltage is maintained constant, we shall have three methods by which this may be done:

1. By generator field alone, synchronous motor excitation being kept constant.
2. By synchronous motor field alone, generator field being kept constant.
3. By varying both generator and synchronous motor field so as to get the best results.

The circle diagram for the input to a synchronous motor with constant terminal voltage is similar to that of a generator with constant terminal voltage except that the center of the circles is on the opposite side of the origin. In fact if the internal impedance of the two machines is the same, the motor circles are the images of the corresponding generator circles with respect to a line passing through the origin at right-angle to the line joining the centers of the generator and motor circles which also passes through the origin.

For the condition of constant internal field in the motor, the circle corresponding to this field strength is the locus for the power input to the motor. The generator circles passing through these points give the generator field required to maintain the terminal voltage at the given value. It will be readily seen that a limit to the amount of power that can be delivered is quickly reached.

For the condition of constant field of the generator the proper generator circle is taken, and the same comments apply as for the first case.

Next let us consider the case in which both machines are regulated; this must follow some specific scheme as both machines cannot regulate to maintain voltage.

For the condition of constant power factor, the power factor line gives the locus of the points of intersection of motor and generator power circles which will give the proper field strength for this condition with each value of load.

Mathematically, if unity power factor is taken, for an ideal generator and motor, that is to say, machines having no resistance, there is no limit to the amount of power. Actually the limit is set by the exciter system, for when the internal or field voltage reaches a certain value indicated by the circle of intersections, more power cannot be taken by the motor or delivered by the generator without an increase in the field strength of either or both machines. This may be shown graphically by drawing the circle diagram for the two machines based on this value of field voltage and varying terminal voltage; it will then be seen that the only solutions possible under these conditions give lower values of power. Therefore, further demand on the system must be supplied by an immediate increase in field strength or else the characteristics of the motor changed so that it delivers more leading current as the voltage decreases.

This condition does not necessarily imply instability but it indicates that without the aid of the exciting system a further increase in load admittance may be sufficient to cause the two machines to fall apart and this might also happen if the exciter system were too sluggish.

The point I wish to emphasize in this discussion is that the stability of such systems depends on the ability of the system to inherently supply the necessary wattless power required to meet the sudden demand for power; fortunately, except under abnormal conditions, the demand is relatively slow and plenty of time is given for building up the field to supply the necessary wattless power.

The interposition of a transmission line still further limits the load that can be transmitted between the two systems. I wish to take this opportunity to correct a statement that has repeatedly been made to the effect that the distributed capacity of a transmission line compensates the reactance of the line. It is true of course that at light loads it causes a rise of voltage along the line but it also increases the range of regulation and increases the tendency toward instability. It does not cut down the size of synchronous condensers as much as is popularly believed because the total range of regulation of the machine is increased, requiring lagging kv-a. at light loads and leading kv-a. at heavy loads.

To determine the static stability of a system consisting of generators, transmission lines, and synchronous motors on the assumption that the voltage is held constant at the sending and receiving end of the transmission line, the circle diagram of the transmission line at the receiving end is drawn, and the power circles of the input to the motor are also drawn to include the step-down transformer impedance on the same scale and superimposed on them. The same thing is done for the sending end with the generator. Several values of field voltage of the motor are then taken and the

corresponding values for the generator are obtained in the usual way from the second diagram. The circles for constant field voltage of this value with varying value of terminal voltage are then drawn with the corresponding value of transmission circles for both sending end and receiving end assuming simultaneous reduction in terminal voltage at each end. If the value of power input remains stationary for a slight decrease in terminal voltage the point of instability is indicated. Another procedure is to construct the circle diagram for the whole system including the generator and the motor leakage reactances checking up with the complete generator diagram, that is to say, the input of the generator considered as part of the transmission system and the output of the motor considered the same way. If maximum output is indicated for either machine with constant internal voltage this will be the point of instability.

I have dwelt some time on the problem of static stability because it is a necessary introduction to the main problem, that of transient stability, as it contains a number of the important elements required in the consideration of the transient stability problem. I have particularly considered the synchronous motor for the reason that the characteristics of most supply systems can probably be fairly well expressed by a modified form of a synchronous motor load.

Fig. 1 shows the generator, transmission line, and synchronous motor in the form of a diagram. Fig. 2 shows the simplest form of power system in the form of an electrical diagram. In this diagram the internal voltages of the collective machines, reduced to a common voltage, are expressed by a figure in a circle. It is apparently not a far cry from the first form of circuit to the second. *The main requirement is to obtain an equivalent system which will give the same voltage wattless power characteristics as the system under investigation.* It is important to obtain reliable data on the characteristics of systems that are to be connected to a high-voltage transmission line as the character of the loads will have a great deal to do with the stability of the transmission line.

Discussion

FUNDAMENTAL CONSIDERATIONS OF POWER LIMITS OF TRANSMISSION SYSTEMS

(Doherty and Dewey)

AND

ANALYTICAL DISCUSSION OF SOME FACTORS ENTERING INTO THE PROBLEM OF TRANSMISSION STABILITY

(Fortescue)

Seattle, Washington, September 16, 1925

P. H. Thomas: The paper by Doherty and Dewey emphasizes even more than those of previous date the part played by the terminal apparatus in stability of operation. Of course, it matters not what the electrical equations show as to the theoretical capacity of a long line if when terminal apparatus be applied to supply power and receive load the combination is unstable to operate, as might easily be the case with loads anywhere nearly approaching the theoretical capacity of the line should the usual present designs of synchronous apparatus be used.

However, we have this fact to remember: the capacity of the line as shown by equations is an absolute limit without power of change until some of the physical constants of the line are changed, while the limitations of the terminal apparatus are merely matters of economy and cost. The very long transmission line may well represent $125 per kilowatt of delivered power, while the terminal apparatus costs much less, at least so far as the securing of suitable stability characteristics is concerned. If the choice lies between limiting the maximum duty of the long line and adding to the terminal-apparatus cost, the latter course is to be chosen, generally speaking.

The proposal to correct by simultaneous and automatic support of the field magnetizing turns the falling field strength when a line is suddenly overloaded is a significant and important proposal and the analysis offered is clear and to the point. No doubt there are other ways of accomplishing this same result.

The 28 per cent greater load carried by the rectifier-excitation test as reported over the regulator test in the experimental line is very encouraging. At the critical point of the regulator test it is evident that a further increase in power transmitted would require a higher actual field magnetism both on account of the higher load and on account of the less favorable power factor that the line will demand and temporarily also greater on account of the initial drop in terminal voltage. It should be noted that the motor end must drop behind on the increased load before anything at all happens in the electrical circuit. But this regulator cannot act until the voltage actually has dropped and by that time the system has dropped out of step and the regulator never gets a chance to try. With the rectifier, however, an excess of exciting power is added in the field before the motor drops back to the new position for the greater load and the field is ready to support the necessary additional power current.

I think the authors are a little too severe in their reflections on the line charging current. While it may be true that with certain set-ups due to the limitations in machines, the total theoretical maximum power which can be delivered by the line will be slightly less over the line having its normal electrostatic capacity than over a hypothetical line with no capacity, nevertheless it is not likely that charging current will be a detriment under practical operating conditions.

The statement that a certain 300,000-kv-a. station may deliver more power over two lines than over three and none at all over nine lines is not so significant as it might seem. It simply means that, for the three or nine lines, the system is so proportioned that machine capacity which should be devoted to carrying kilowatts is absorbed in carrying charging current or that the ratio of synchronous condensers to number of lines is not favorable. Either the kv-a. of the station should be increased by building the machines to operate at a lower power factor or shunt reactance or other means used to neutralize a part of the charging current. It goes without saying that it would not be economical to use an unnecessary number of lines.

With regard to the authors' Fig. 3 and the discussion of the part played by field current, I should like to point out that, with a heavily loaded line, there is very little choice as to power factor, for this is definitely fixed by the load and the terminal voltages and will inevitably be high at the generator end. However, there is this advantage of power factor near unity: The effect of added lagging current due to increase of load on the drop from internal impedance within the machine is small with high power factor. This is an important matter.

Since the power factor changes with every change in load on the long line, the curves of Fig. 3 should be supplemented by other curves showing the effect of such change of power factor, and these modified curves might easily show a different best

relation between low-power-factor and high-power-factor loading from that indicated by the uncorrected curves.

One other point: the authors state that the scheme of using divided line conductors to reduce reactance and increase capacity has not found favor partly because of the increased charging current. As I see it, this increased charging current is, on the whole, no disadvantage, for on any useful loading it greatly improves the line power factor. While it is true that leading current will tend to reduce the field current setting in generators, this is a matter affecting only the performance of the generator, subject to correction in a number of ways. If the rectifier excitation scheme proposed in the paper or any equivalent scheme is available, this objection to the high charging current disappears. Meanwhile the reduction of line reactance which accompanies the increase in capacity with divided conductors means an increase of substantially the same proportion in the maximum load the line will carry, assuming the percentage of line resistance loss to be kept constant. With a 20 per cent increase in line capacity in a 350-mi. line and with a favorable line power factor, any generator difficulties from the additional charging current would no doubt be cared for in the same suitable way. As a matter of fact on any full loading no charging current would appear at the terminals as the reactance energy of the line would absorb it.

F. L. Lawton: In the paper by Messrs. Doherty and Dewey, considerable emphasis has been given the question of voltage regulation of the synchronous equipment of transmission systems. This, however, is in line with the papers[1] and discussions[2] at the last Midwinter Convention when the subject of high-speed excitation was given prominence.

As the method of regulation outlined by Messrs. Doherty and Dewey—viz., the use of mercury-arc rectifiers as adjuncts in the excitation circuits of transmission-system synchronous equipment—is probably the most promising development looking toward increased system stability, it seems wise to discuss it somewhat in detail.

During the course of various investigations of the stability of power-transmission systems, it was realized that considerably greater stiffness in a system would be desirable; also, that such greater stiffness could be secured by the use of excitation systems having a time constant much smaller than usual. As a consequence, Messrs. Fortescue and Wagner discussed results they had obtained with a so-called high-speed exciter, at the 1925 Midwinter Convention.[3]

While it is true that less voltage fluctuation will occur during load or short-circuit transients, when the synchronous machines of a power system are excited by high-speed exciters, it must be remembered that such exciters are inherently not different from any other exciter, so far as behavior under transient conditions is concerned. That is, when lagging load is suddenly added to a generator so equipped, the terminal voltage drops. The decrease in voltage energizes the Tirrill-regulator relay which short circuits the exciter field resistance, permitting the generator field current to increase.

After an appreciable time, the alternator terminal voltage is restored to the normal value. *Inasmuch as the alternator armature reaction is not compensated for at the time it occurs, the power limit*, for slowly applied loads, of any system equipped with high-speed excitation equipment, *is no greater* than for a system provided with normal exciters. Furthermore, as it is now realized that a power-transmission system is inherently stable for any load up to the steady-state power limit, no matter how added, there is comparatively little advantage in the use of such high-speed exciters beyond the reduction of voltage fluctuations.

1. Power System Transients, V. Bush & R. D. Booth, A. I. E. E. JOURNAL, March 1925, p. 229.
2. Discussion, A. I. E. E. JOURNAL, July, 1925, pp. 766-771.
3. A. I. E. E. JOURNAL, pp. 767-770.

Let us consider the case of an alternator equipped with an ordinary excitation system with the addition of a mercury-arc rectifier, excited from the line, as an adjunct. When a lagging load is thrown on such an alternator, the rectifier supplies an excitation current proportionate to the line current, varying simultaneously with it. *As a result of the simultaneity of action, the alternator armature reaction is at all times counterbalanced by a proportionate field current.* That is, *the armature reaction is effectively neutralized.*

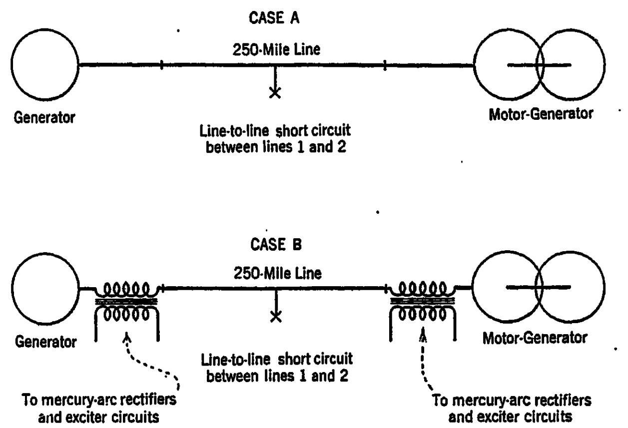

FIG. 1—SCHEMATIC DIAGRAM OF SYSTEMS USED FOR SHORT CIRCUIT TESTS

As Messrs. Doherty and Dewey have indicated, a reduction of about 50 per cent in effective armature reaction was obtained. While this reduction was not so great as theoretically possible, nevertheless it resulted in an increase of 28 per cent in the steady-state power limit of a 250-mi. miniature system.

It is worthy of note that this gain was maintained during tests involving the sudden addition of loads when, if ever, it might be expected that the rectifiers would be ineffective. Not only were increased power limits obtained, but practically no decrease in voltage occurred when a large load was suddenly added to a system equipped with mercury-arc rectifiers; but as much as a 20 per cent momentary drop in voltage occurred when the same load was added in the same way to the system using normal Tirrill-controlled excitation systems.

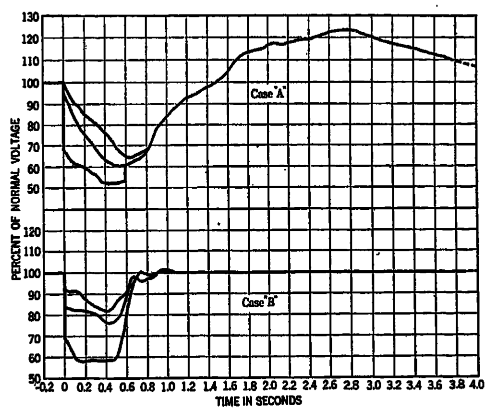

FIG. 2—VOLTAGE DISTURBANCE DURING A SINGLE-PHASE, LINE-TO-LINE SHORT CIRCUIT ON A 220-MILE SYSTEM

The advantages of rectifiers as adjuncts in the excitation circuits of transmission-system synchronous equipment are probably most marked in the case of system short circuits. Tests similar in all respects except the excitation circuits have been made on the systems illustrated by the accompanying Fig. 1 to determine the maximum amount of power which could

be transmitted with stable operation under the condition of a half-second, single-phase, dead, line-to-line short circuit at the mid-point. It has been found that the system with rectifiers could carry 50 per cent greater load with much less fluctuation in voltage. To illustrate the comparative voltage fluctuations, Fig. 2 herewith has been prepared. Case A, Fig. 2, shows the three receiving-end voltages for the system of Case A, Fig. 1, while Case B gives the corresponding voltages for the other system. While the duration of short circuit for Case B was somewhat less than for Case A, the load being carried prior to the short circuit was 35 per cent greater and the initial short-circuit current 50 per cent greater. In spite of these unfavorable factors, the decrease in voltage was less with the system of Case B; virtually no over-voltage occurred when the short circuit was cleared. With the system of Case A, considerable excess voltage occurred some time after the clearance of the short circuit; a much greater time was required for the restoration of normal voltage.

The above facts illustrate the very important advantages which may be gained by the successful application of mercury-arc rectifiers in the excitation circuits of the generators and other large synchronous equipment of power-transmission systems. There appear to be no disadvantages beyond the possible necessity for oil circuit breakers of a somewhat higher rating, at a few points.

In conclusion, it appears that the only high-speed excitation system which will enable the securing of increased system power limits, for all conditions of operation, must be such that the armature reaction of the synchronous machines is effectively neutralized by the addition of field ampere-turns in the proper space-phase simultaneously with the occurrence of the armature reaction.

I believe the mercury-arc rectifier, properly applied as an auxiliary in the excitation circuits of synchronous units, is the first development giving promise of *a real increase in the power-transmitting capacity of transmission systems.*

S. B. Griscom: The statement by Messrs. Doherty and Dewey that, during transients, "the synchronous apparatus becomes inherently more powerful" is not clear. The field transient accompanying an increase in armature current is of such a nature as to tend to prevent the main field flux from decreasing, but it does not strengthen it. Actually, the magnetic flux starts to decrease immediately upon an increase in armature current, and consequently the field becomes weaker. Another way of stating it is that the very presence of the additional field current, flowing through the field resistance, is *due* to a decreasing field flux.

Under "Regulation," it is stated that the slope $\frac{d\,E}{d\,P}$ of the voltage-power curve determines the degree of stability. I should like to point out that such a criterion does not take into consideration the mechanical transients which are always coincident with an unstable condition, and it may, therefore, lead to erroneous conclusions. In the region where $\frac{d\,E}{d\,P}$ approaches infinity, $\frac{d\,P}{d\,t}$ approaches zero, because $\frac{d\,P}{d\,\theta}$ is limited by the mass of the synchronous machines. For this reason $\frac{d\,E}{d\,t}$ becomes very small and, in the case of large systems having heavy masses, is undoubtedly much less than the combined time constants of voltage regulators, exciters, and generator fields, for a normal building up of load. The maximum load which may be carried under steady conditions is therefore considerably increased by the use of voltage regulators. This conclusion agrees in a general way with similar conclusions by the authors, although in some cases apparently contradictory statements and data are introduced.

In particular, it would be expected that the tests reported at the bottom of page 982 should show nearly equal maximum loads for the two forms of excitation used, provided the load was built up by increments that were small as compared with the stored energy released during a small shift in phase of the synchronous motor. It is also probable that a load consisting of a large number of small synchronous units would cause the power transmitted over the line to change more slowly, giving more time for the regulator to function.

In a similar manner, voltage regulators on synchronous condensers located at intermediate points on a transmission line, by holding the voltage constant under a gradually increasing load, should permit a much greater power to be transmitted than the same line with the same total condenser capacity located at the receiver end only. The maximum power limit of such a system, as given by the authors, appears to be entirely too low for a condition of steady loading. During transients, the maximum load that can be transmitted safely is considerably reduced but will still be much higher than a straight-away line for the same disturbance.

The use of synchronous condensers at intermediate points has been discussed a number of times, but the advantages apparently have not been fully appreciated. Condensers are usually installed for the purpose of voltage regulation and since the receiving end of the line is usually the only point where voltage regulation is needed, all of the condensers are located there. However, the real function of the condensers is to supply the reactive energy loss due to the flow of current through the line reactance. This loss is distributed practically uniformly over the length of the line and consequently a reduction in copper loss and a slight decrease in total condenser capacity may be effected by installing a portion of the condenser capacity at an intermediate point. Such an arrangement would tend to reduce short-circuit currents, particularly at the receiving end which is usually a point of high power concentration. Location of condensers at intermediate points should not prove unduly expensive or difficult because, in the majority of cases, switching stations and attendance will be required for line sectionalizing. Machines of suitable characteristics and equipped with a high-speed excitation system, or compensation, are of particular advantage for this application. It should be noted, however, that such features are made desirable principally by the conditions obtaining during transients and not for steady loading.

R. D. Evans: Probably the most interesting data submitted by Doherty and Dewey are the results of the calculations shown in Fig. 2 of their paper. This figure shows the "Maximum power which can be transmitted 250 mi. at 220,000 volts, shown as a function of the capacity of synchronous apparatus, and the number of transmission circuits."

The curves of Fig. 2 were presented for the purpose of showing the importance of the charging kv-a. of lines in reducing the power limit and they serve this purpose in an excellent manner. However, the important effects of the charging kv-a. in limiting the maximum power appear only when the synchronous capacity is small in comparison with charging kv-a. of the transmission line. This condition of operation would suggest that a lower transmission voltage would give higher actual power limits.

The condition in which the generating capacity is small per line is of relatively minor importance because the power to be transmitted per circuit at 220 kv., 250 mi., must be of the order of 75,000 to 125,000 kw. in order to be within the economical range at the present time. With this relation in mind, it is advantageous to compare the results shown in Fig. 2 for different kv-a. capacities of synchronous apparatus. In the first place, it will be noted that the characteristics of machines assumed by Doherty and Dewey are such that the rating of the machine can-

not be developed because the power limit of the system is approximately two-thirds of the nominal capacity of the synchronous machines; that is, 100,000-kv-a. capacity on this line will show a power limit of approximately 70,000 kw. per circuit. If the characteristics of the terminal equipment are altered so that the reactance is approximately two-thirds, and the field current approximately three-halves of the values assumed, the power limit would be increased to approximately 100,000 kw. This condition would correspond to the curve given in the paper for 150,000 kv-a. in synchronous apparatus. Similarly, if a power limit of 150,000 kw. were to be obtained, the machines should have approximately somewhat less than two-thirds the reactance and more than three-halves of the excitation of the machines assumed by Doherty and Dewey. In other words, the power limit per circuit can be increased up to at least 125,000 kw. per circuit by the use of machines of suitable characteristics. The significance of this discussion is that the desired static limit may be obtained by merely modifying the authors' assumptions so as to employ machines of lower reactance and higher excitation. In the absence of alternatives, which are still in the development stages such as special regulator and compensator schemes, the use of machines of suitable characteristics is a practical solution available at the present time for producing quite marked increases in the stability of systems.

In view of the position taken by the authors that static limits are the only limits that are worthy of computation, it is interesting to compare the calculated static stability limits as given in Fig. 2 of the paper with the practical results of experience on an actual 220-kv. system. For a single-circuit, 250-mi. transmission system operated at 60 cycles, with approximately 200,000 kv-a. in synchronous apparatus at each end, the static limit is calculated to be about 115,000 kw. In the June issue of the *Electric Journal*, H. A. Barre states that the static limit of the Edison System was reached under a particular emergency condition. For this condition, power was transmitted from 240 to 270 mi. over a single circuit at 220 kv., 50 cycles, with approximately 200,000 kv-a. in synchronous apparatus at each end, and the static limit was found under actual operating conditions to be 183,000 kw. One would expect that the static condition on the Edison system would correspond well with the other condition mentioned previously, the greater length of the 50-cycle system and the probably greater transformer impedance roughly compensating for the increased frequency upon which the calculations were based if the authors had assumed machine characteristics corresponding to those of the synchronous apparatus on the Big Creek system. What is the explanation of this discrepancy from 183,000 kw. on an actual system to 115,000 kw. as given in the calculated results? In the first place, the exact assumptions used by Doherty and Dewey are not stated, and it may be that the explanation lies in them. If such is the case, the Fig. 2 should be interpreted with care. A possible explanation may lie in the fact that probably synchronous motor load was assumed in the Doherty and Dewey calculations, whereas the actual system involves a certain amount of resistance load which would cause the power to fall off with drop in voltage. The influence of the load characteristics is not mentioned, so far as the writer can recall, at any point in the paper, and it may be that this factor is of importance in the particular case of the static limit on the Edison system and under certain conditions would undoubtedly be of great importance in other cases. The explanation of the discrepancy between the calculated maximum limit and the maximum limit obtained under actual operating conditions is, of course, very important. It is worth pointing out that actual static limits may be appreciably in excess of calculated limits which do not take into account all the factors affecting stability.

S. W. Copley: These two papers indicate that the methods of calculation used do not differ greatly fundamentally, but there is some difference between them in the assumptions made as to the values used for the characteristics of the terminal apparatus. This difference causes some important divergence in views as to the power-limit figures. Possibly Doherty and Dewey are too pessimistic in their assumptions with respect to the reactance of terminal apparatus, or they have not given enough credit to the action of voltage regulators in holding the system together. Both of these points warrant further investigation. Machines of lower reactance can be designed and a regulator which has higher speed characteristics is a possibility. There are certain drawbacks to the application of such machines and regulators, but if the limits of power transmission must be raised the disadvantages can without doubt be overcome.

C. L. Fortescue: Messrs. Doherty and Dewey have presented with clarity the characteristics of synchronous apparatus which are of importance in the problem of stability. In dealing with the stability of machines of this type, they have laid much stress on the fact that high power factor is detrimental to stability, as it involves low excitation or, what is the same thing, low internal voltage. While undoubtedly at times, transmission lines reach very low loads in which the excitation is correspondingly low, I do not know of a single case in which a system was thrown out of step by a sudden increase of load or a short circuit due to low excitation. I believe that the explanation which I shall give later will account for the fact that instability under such conditions is practically unknown.

While I believe we should keep such cases in mind as elements of the problem, I feel that the authors have over-emphasized their importance and may, therefore, produce a false impression in the minds of those who are not sufficiently familiar with the problem. A properly designed transmission system will make provision for the generators not to supply all of the charging current at light load, and, at heavy loads, the generator power factor will be normally lagging. Two transmission lines properly designed with the proper size of generating station and with proper provision at the receiver end and intermediate points to take care of the line reactive-volt-ampere requirements will always transmit more power than one line. I state this fact not because I believe the authors intended to convey the opposite impression but because the emphasis they lay on certain features of generators and synchronous motors might easily convey the opposite idea to the minds of those who are not closely in touch with the problem.

I had felt encouraged when I read the statement made by the authors on the fourth page, column one, and in the first part of column two as to the importance of excitation, though I will take issue with them in regard to part of the statement by referring to a discussion on excitation in last Midwinter Convention by several of my colleagues in which the possibilities of high-speed excitation were discussed at some length and with considerable emphasis. Later on in their paper I was disappointed to find that the authors had reached the conclusion that high-speed excitation would not fit the bill but inherent regulation was what would be required. Again I must take issue with them in this matter and state that in my opinion they have reached an erroneous conclusion and their error is mainly due to their failure to perceive that the so-called static-stability problem is in reality one of transient stability.

I shall first show by means of a simple mechanical model that if generator and motor are provided with perfect regulators, no matter what their characteristics, they will furnish power up to the stability limit of the line itself. The model which I have in mind is quite simple and was devised by S. B. Griscom. If two sticks pivoted about one point are connected at the two ends by an extensible elastic string such that its linear extension is proportionate to its tension, and if torque be applied to one member, the restraining torque on the other member will represent the power output of a line. The applied torque is the power input. The above applied to a line having no losses

but having reactance and distributed capacity. This assumption involves no appreciable error in considering the actions that cause instability because the resistance of the line and generators has only a small effect on the stability problem. One may picture one of these pivoted members attached rigidly to the shaft of a motor and the other to the shaft of a generator. The motor drives the generator at constant speed, using this device as a mechanical transmission. As the generator load increases, the elastic connection would be extended in such a way that the sine of the angle between the two members will increase in proportion to the load. If we keep on loading the generator, the angle between the two members will finally become a right angle, at which point the mechanical transmission system will have reached its maximum ability to transmit power and the system will become unstable.

Now the internal voltages of the generator and motor may be represented by two additional sticks (as shown in the accompanying illustration) as extensions of the transmission line beyond this angle. The terminal voltages are kept constant; that is to say, the length of the original two pivoted sticks. As the angle is increased, the elastic line is kept straight by increasing the sticks representing the generator and motor internal voltages. Using this model, it can be shown that the torque at the generator will reach its maximum when the two members representing the terminal voltage of the transmission line are at an angle of 90 deg. with each other.

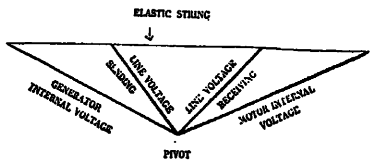

FIG. 3

I have made no restrictions as to the characteristics of the generators and motors except in the matter of losses as in the case of the transmission line, but it is easy to show by means of the model that the rate at which the internal voltage must be increased with increase of load is very much influenced by the internal characteristics of the machines, and if we make the internal leakage impedance low, we shall not require to have as great a range of excitation.

Objection may be raised that ideal excitation systems do not exist and that commercial regulators are too slow. The explanation resides in the fact that the problem is, in reality, not one of static stability but one of transient stability. You must remember that instability involves a change in angular position and this means a change in angular position of the generator rotor and the motor rotor as well as of the line terminal voltage. The electrical angle and mechanical angle are, you might say, irrevocably tied together and to deliver power to a motor through a line at a given excitation requires that they must take up a definite angular position with reference to each other. In my paper, I have pointed out that the angular relation is a continuous function of the power and is therefore suitable for analytical work whereas the voltage at the terminals is a discontinuous function and is not suitable for analytical work.

The fact that the angular positions of the rotors must change with change of load introduces the natural period of the system into the problem of voltage regulation and this means also that sluggishness of the hydraulic or steam governor may have some advantages. If the load is increased, the motor slows down and so does the generator. In slowing down, the motor supplies part of the increased load by inertia. The generator supplies part of the increase in transmitted load by inertia at a slightly lower frequency, so that matters are improved from a stability point of view, since, during this stage, only part of the increased load must be transmitted and also because it is transmitted at an appreciably lower frequency. While this is going on, the regulator has had time to get in its work and if the angular change is not too great, the voltage regulator will catch up before the angular displacement has reached the point where the system will pull apart at the increased value of load. Theoretically, this may be carried out close to the limit of stability of the line, providing the load increments are not so great as to cause large swings.

The authors have made no mention of the effect of the characteristics of the load and have merely touched on the improvement to be obtained by changing the generator characteristics, dismissing it with a statement that it will prove too costly. I wish to take exception to this statement and to say that such changes can be made, with a small increase in cost over that of standard generators and the increase in the ability of the system to transmit power will more than counterbalance this added cost. Such generators are immediately available and, with specially designed high-speed exciters, will permit of operation of transmission lines with a high stability limit.

In regard to the problems of internally compensated machines, much progress has been made along these lines, but since it is still in the development stage, very little can be said about it at present. It is possible to compensate a generator completely and even to extend this compensation to cover the impedance of transformers so that the generator has the characteristics of the infinite system of which Mr. Doherty and Mr. Dewey speak in their paper; but there are grave questions to be considered in the application of such machines. The rapid retardation and acceleration, caused by short circuits, and the subsequent clearing of the lines may produce mechanical stresses of great magnitude. In all probability some limiting device will be needed to limit the amount of current that can be delivered above a certain value.

Regarding the matter of transients due to short circuits, I agree with the authors that experience has shown that in existing systems with suitable relay protection, short circuits do not constitute a serious problem in operation. However, they make the statement that experience is the best guide and that calculations are always made on the basis of conservative premises and are therefore too pessimistic. The transient condition under short circuit will undoubtedly be the factor which will determine the ultimate rating of a transmission line and it behooves us to avoid being so conservative that our results are pessimistic.

I believe that we can or will be able to compute, with a fair degree of approximation, the results of a short circuit either to ground or between phases if we are provided with proper data as to ground resistance and load characteristics. I am further willing to go on record that we shall be able to install apparatus that will enable us to approach within a reasonable distance of the effect of an infinite system, not only on a straight-away transmission but also for one using intermediate synchronous condensers so that, with the latter system, the stability will be determined by the weakest section.

The automatic voltage regulator is not a hopeless problem for it is assisted by the fact that it takes time for the system to change its angular position whereas the terminal voltage changes instantly. Moreover, the internal reactions are resisted by inherent flow of exciting current. Therefore, the voltage regulator has time to act before the system gets beyond control.

P. H. Thomas: Mr. Fortescue, with his usual skill, has developed generalized equations for showing the theoretical limits of stability in electric systems containing synchronous machines.

The most difficult and illusive part of the problem of stability is the determination of the numerical value of the parameters of stability in particular cases and especially for the pendulum action or the tendency to overswing when a change of load in a

system or its equivalent requires a new position of equilibrium. The principles involved are well understood and at least one analytical and one electric analog method have been proposed. These, however, are merely methods of overcoming the mathematical difficulties of the solution and still leave untouched the difficulty of properly evaluating the factors of losses and damper currents.

As is well known, any damper current set up by an advance or retreat in the position of the rotor with regard to the revolving field of the stator will tend to reduce, temporarily, the forces driving the rotor toward its new position of equilibrium and similarly, if the rotor over-shoots, these damper currents tend to restrain the over-swing. If sufficiently powerful, these forces may render the swing of the rotor dead-beat. The conditions governing the effect of these damper currents, which will usually be of very low frequency, should be fully studied to see what can be done therewith to restrain the overswing.

These damper currents should be controlled by the resistance of the damper rather than its reactance, but subject to this limitation the damper resistance should apparently be as low as possible. The field winding, having by far the greatest weight of copper surrounding the field poles, would naturally have the most effective damper action.

While Mr. Fortescue has given a definition of stability which is entirely logical, we shall ultimately need something further, viz: a standard test or criterion to be satisfied by any particular system to insure reasonable continuity of operation.

It is yet too soon to offer such a criterion for actual adoption, but ultimately something in the nature of an overload to be taken at a certain power factor, or a drop of voltage (taking account of both ends of the line) that must be sustained without falling out of step, will be required.

It is gratifying that much has been done to render the terminal apparatus more responsive to its duties—but still more is desirable and no doubt possible, on account of the dominating importance of the cost of the line as compared with the cost of regulating and exciting means.

R. E. Doherty: I agree with the general point of view expressed by Mr. Fortescue—namely, that the lack of any great amount of trouble from instability up to the present time should not be interpreted as evidence that such a thing is not possible with longer lines and greater power, that, in any study of the problem, all elements of the system, including generators, exciters, condensers, lines, etc., and not merely one element of it, should be considered, and that the problem involves two fundamentally different "states" of operation—"transient" and "steady"—during which the operating characteristics of the apparatus are different. I also agree with the general exposition regarding the angle-power relations, only in so far as it gives a physical, qualitative picture of the phenomenon. I strongly endorse his appeal for reliable information regarding ground resistance and would add an appeal for recorded data and system experience during single-phase short circuits. And, finally, I endorse the view that the mathematical, or even graphical, determination of stability of such system networks as comprise our modern power systems is quite beyond the range of practical possibility. For these cases an equivalent system from which the required values can be obtained by test becomes necessary—just as for the same reason, it is necessary to thus determine short-circuit currents in complicated networks.

Such methods are available; the equivalent system proposed by C. A. Nickle[4] affords a valuable means of studying the power oscillations during transients, and the scheme proposed by Spencer and Hazen[5] affords a means of testing steady state power limits with practical accuracy. The latter method assumes sine-wave relation between power and angle, and is, therefore, roughly correct for maximum power studies. Nickle's method, however, in its present state of development, assumes linear relation between power and angle, and is, therefore, limited in this particular application to a study of the behavior of the various components of the system during those transients which do not involve power swings beyond the point where the linear relation ceases to hold. However, there are many interesting and important phenomena bearing on stability which can thus be determined, and if a circuit element giving the sine-wave relation is found, the maximum power can also be determined. The company with which I am associated is now completing the development of both of these facilities. While, of course, further improvements are ahead, the present development provides a very helpful aid, and, in my opinion, is a significant step forward.

There are a few points in the paper with which, if I interpret them correctly, I do not agree. They are relatively unimportant with respect to the general problem to which I have referred above, but have importance only in numerical calculations. I refer to the author's statement that the steady-state limit depends upon the leakage impedance and not upon the synchronous impedance. Making due allowance for saturation, which, in effect, reduces the impedance, the steady-state, ultimate power limit at normal voltage is determined by synchronous impedance, not by the leakage or "transient" impedance. A steady-state, hand-controlled test, giving a family of voltage-power curves (as in Fig. 3 of the paper by Mr. Dewey and myself) shows the same power maxima as determined by automatic regulator tests, and as calculated, using synchronous reactance. While it is possible with an automatic regulator, as it is not by hand control, to throw on suddenly loads equal to the ultimate maximum steady-state power, it is nevertheless not possible, according to our tests and conception of the problem, to carry significantly more than that by using a regulator actuated by the alternator terminal voltage and operating on a shunt- or compound-wound exciter of quick response. This is discussed fully in our paper.

In discussing the group of papers by Mr. Fortescue and his colleagues at the Midwinter Convention in 1924, I stated power limits which "in the present state of engineering knowledge" I considered to be justified. I said: "We must neither gamble that a voltage regulator will be able to insert a supporting prop under an otherwise falling system, nor depend for stability during load transients upon possible momentary favorable conditions due to momentary and field transients." Now the intensive study and investigation of the past year and a half has shown what we did not then know—that, up to the ultimate maximum power value, as determined by tests under hand control, the regulator can be depended upon to insert "the supporting prop," but not beyond that limit. I understand the author's statement to be that the regulator makes it possible to carry a constant load significantly greater than the above maximum; with this I disagree.

I do agree with the author that the object sought with an excitation system is to obtain the same characteristics as would be afforded by a machine which would inherently maintain constant flux linkages—one in which the field and damper resistances were zero. To say the same result would be obtained by an exciter of quick response which holds the alternator field approximately constant is in all respects parallel to the statement that if the terminal voltages at both ends of a line were held approximately constant by a regulator (as they can be, even by hand control, with gradual increases of load) the ultimate maximum power would be that of the line alone; *i. e.*, all limitations in the generator would thus be compensated, which obviously is not true. Yet the two cases essentially involve the same elements. It can be shown[6] that, following a load change, the flux change of

4. Oscillograph Solution of Electro-Mechanical Systems, by C. A. Nickle, JOUR. A. I. E. E., p. 1277, Dec. 1925.

5. The Artificial Representation of Power Systems, H. H. Spencer and H. L. Hazen, A. I. E. E. JOURNAL, January, 1925, page 24.

6. Exciter Instability, R. E. Doherty, A. I. E. E. TRANSACTIONS, 1922, page 767, eq. 30.

the alternator field, $\frac{d\phi}{d t}$, is not zero, but essentially negative, unless a series negative resistance is introduced in the alternator field circuit. A method for obtaining this is described in our paper. A shunt- or compound-wound exciter, of however quick response, does not have this characteristic. Fortunately, however, for any load up to the ultimate steady-state limit at normal voltage, the ordinary exciter and regulator usually suffice; and this steady-state limit cannot, so far as our study and tests show, be increased by speeding up the exciter magnetically.

R. J. C. Wood: The main reaction I get from these papers is a feeling that perhaps we are using the wrong term when we talk about instability. We are not, ourselves, putting up the money to build these big systems, and I think that anything which unduly suggests a weak point in transmission should be avoided. I do not mean by that we are to conceal the truth in any way, but there is a psychological effect produced by the word "instability" which I do not think is produced when we talk about power limit.

What we are getting at is the power limit of a system. There are various things which limit that power; the current may be so great that the wire will fuse and fall in half, or it may be this "instability" that we are talking about.

If you go out to buy an automobile you take the automobile out and try it and you run it up a hill and the hill is of increasing steepness. After a time that car just lies down and quits. You have not an unstable automobile; you simply have reached the limit of its ability. Perhaps you kill the engine; perhaps you spin the hind wheels. That might be an illustration of the generators falling out of step.

It would be better if we could think and speak of this more in terms of power limit; everybody is familiar with the idea of a limit of endurance, both humanly speaking and as regards apparatus and machinery.

Roy Wilkins: I am employed by a company owning an interconnected system of upwards 8000 mi. of line 60-kv. and over, and with a total generating capacity of a little over 880,000 kv.-a. The connecting rotating load is about 2,000,000 h. p. At different times there have been tested 110-kv. lines up to 500 mi. in length in operation and carrying load, and loops as long as 350 mi.

I should like to point out certain road signs in the line of "don'ts" for people who, in the actual power industry, take up the study of power limits or instability as it has been called. First, don't worry about anything except the actual operating conditions. You will find trouble enough without running into any weird combinations which are impossible operating conditions. Second, don't expect to simplify the problem and still check the performance of a complete transmission system, because in a transmission system, every piece of connected apparatus has certain characteristics. These characteristics are all in their proper places, proper order, and proper values. Any simplification means a certain amount of error.

At the present time, there is too little known about circuit-breaker operations, corona, WR_2, impedance, load character, and load power factor, together with certain cases of trouble as grounds, etc. As a passing note, nobody, at the present time, knows exactly what "load power factor" is. I haven't been able to measure it in a year and a half. The final result will come from a mass of accumulated operating data just as in the past for the final solution the twenty-year old problem of a grounded-neutral system came not from brilliant mathematics or special studies, but from the final check and the actual operating procedure of a great number of operating systems.

F. G. Baum: If you will read the TRANSACTIONS of the Institute of twenty years ago you will find that the problem of stability was then one of very great importance and a great deal of work was done on it at that time. The reason for its importance was this. The Stanley Company made for operation on the first long transmission systems an inductor type of generator which had no revolving coils of any kind. The generator had 100 per cent reactance. It was good for the conditions which then existed, since we had only air switches with which to open the circuit. If there was a short circuit, the voltage dropped to zero and you could open the circuit with the air switches.

Generators were actually advertised as being capable of being short-circuited without damage and only a few years ago many engineers were specifying that the short-circuit current should not be over so many times normal current.

We are now talking transmission-line stability again, and the generators are the main element in it. The transmission line will take care of itself if you will take care of the generators. The reason we are talking so much about instability is because we have flashovers, or expect them, and therefore want to be ready to take up and quickly replace the difficulty caused by the flashover. In other words, the trouble now is with flashovers, while twenty-five years ago it was that we didn't have any switches. At the time the Stanley generator was put out I was in charge of the operation of the Pacific Gas and Electric Company in which we had a number of these generators and the regulation was very poor. The voltage variation under normal operations was so bad that we couldn't operate lights at the same time as motors. Something had to be done so we applied to all the synchronous apparatus an excitation in proportion to the load. All the d-c. load current was taken around the exciters and the voltages built on the d-c. exciter in proportion to the load.

At that time I also worked out a regulating scheme for the a-c. generators which would build up the generator voltage in proportion to the drop in voltage due to the load, which as you know is practically $I \sin \theta$. I wasn't popular for proposing that because the generators were being sold because they had poor regulation, and I proposed to make them good.

For the last year and a half we have been making quite elaborate tests on power limits of transmission systems, and I agree with Mr. Wood that it is power limits and not instability which we are talking about. We don't get instability unless we have troubles outside of those that are expected and most of those come from flashovers. Stop flashovers and you won't get the instability.

The tests made have checked calculations very, very accurately. Early last spring we calculated the power limit on the Pit River System as 185,000 kw., using a power-angle diagram, which you will find in the *Electrical World* in 1902. The limit of the Edison system under actual operation has been found to be 183,000 kw. and if you will allow for the increased length of line and decreased frequency the results check.

I want to express surprise at the suggestion made in the Doherty-Dewey paper for the use of series capacity in the transmission line. You can do that in a radio system but to consider it seriously for a transmission line seems unreasonable.

The most important part of the transmission system today is the oil switch and relays. If we didn't have them today we couldn't operate our transmission systems. Any electric power system of high or low voltage and without proper relays, switches, and fuses to eliminate defective line sections is inoperative. I think you will agree with that, so I say work on the oil switches and relays. First work on the flashovers, and get rid of those so the switches won't have any more to do than necessary. Then when you get through with that, static stability will be the criterion of your power line and not transient stability.

A statement has been made with reference to the limitations of long-distance power transmission, which was quoted in a morning paper. The statement was that with the present

apparatus (and present state of mind, especially) 300-mi. transmission is questionable of economic results.

I challenge that statement. I think it should never have been made. Any man who sets a limit at the present time on power transmission either does not understand the problem or perhaps he is purposely making the statement for some other reason.

I wrote a paper for this meeting which I didn't submit because certain developments afterward made it advisable to add other information. The first sentence reads: "The natural and approximately the economic load per circuit for load transmission is given by the equation $P = 2.5\ E^2$."

If 220 kv. will not do the work, we can go to 330 kv., or some other reasonable voltage, and if 330 kv. won't do it, we can go to 440 kv. When I say that I am saying it in view of the intense study made in the last two years on insulation, coupled with the results obtained on the present system of the Pacific Gas & Electric Company, operated at 220,000 volts. It is the most successful piece of work we have undertaken, and the power is transmitted about 300 mi.

To have a real natural transmission system, you must balance the magnetic energy all along the line with the electrostatic energy. To develop that you get this equation, $I = E\sqrt{\frac{C}{L}}$. I want to call your attention to the importance of this equation. I think that equation is more important than Ohm's law or any other in electrical engineering. It is a fundamental equation. Nature tries to transmit power with the conditions given by that equation.

Now, if I want more current, I can do three things: raise the voltage, lower the reactance, or increase capacity. Most people will recognize that if you lower the reactance you will immediately get increased line capacity, but they do not appear to recognize that if you increase C you get the same results practically. I can reduce L to one-half and multiply the amount of power by $\sqrt{2}$; I can change C by doubling its value and also increase the power by $\sqrt{2}$.

Regarding line insulation, with the study we have been making in the last year and a half, I am satisfied that when we want the 330 kv. we can get it. We started this work early in 1924, not because of any troubles on our present 220-kv. transmission lines, but because we didn't want to be caught like we were in 1912 when we put in the 110-kv. lines, and later found troubles we didn't know anything about. We decided in 1924 that we would make a thorough study of insulation. The Westinghouse Company has supported that work on insulation, and the Pacific Gas and Electric Company is cooperating in the long-line tests and the practical tests which we find necessary.

So the first thing was to decide how to get at the matter of the mechanism of flashovers. I decided, after several years of study, taking probably thousands of flashovers and arriving at no mental picture of what was happening, that we had to get a reliable picture of what actually was happening whenever we had a breakdown. To get that we decided that we should probably have to get at it from a d-c. standpoint, projecting the electrons through the air, and if possible taking their pictures on the way. They are fast-moving and don't pose very long.

The pictures we have taken will, I believe, give a mental picture of the insulation of the air such as we have not had, and I believe the work done and being done will tell us the true story of line insulation.

R. E. Doherty: The extensive discussion indicates a keen interest in this important subject, and serves the very helpful purpose of focusing attention on those points which have not yet been generally agreed upon. The more they are discussed, the more they will be studied and the sooner will the various interested engineers agree upon the more important details. While perhaps there is not at this time *universal* agreement upon even the more fundamental aspects of the problem, there has been, I think, for the past two years general agreement on these fundamental aspects by those who have given the matter serious study. As I mentioned at the Midwinter Convention at Philadelphia, in 1924, the fundamental theory underlying the problem, and the equations arrived therefrom, are used by all informed engineers. Divergence of views enters only when assumptions are made regarding numerical values for a particular case. More specifically, disagreement centers, not about the transmission-line theory or the general equations of the system, but about faulty understanding regarding internal characteristics and constants of synchronous machines. Although different views regarding such machine characteristics apparently result in different estimates of maximum power which can be transmitted over a given system, I am not sure that this difference is as great as might be expected from the tone of the discussion. Messrs. Fortescue and Evans say that the calculated values in our paper are too low, but they do not indicate what these values should be. And I believe that in any definite proposed undertaking, their conclusions as to the practical feasibility of carrying out the given proposal would not be significantly different from ours. Indeed, in more recent proposals where such parallel studies have been made, the conclusions have not been widely different. All of this indicates, of course, that the protracted discussion regarding certain alleged weird behavior of synchronous machines is somewhat of a trifling character, and not of the importance which engineers not familiar with such details might be led to suspect.

I shall attempt to answer the questions raised regarding our paper, although most of them could have been answered by referring to statements in the paper. Mr. Thomas believes that it is not likely that the charging current of a long line will be a detriment under practical operating conditions. It may, indeed, be a great advantage provided the generating capacity at the end of the line is sufficiently large, as clearly shown in Fig. 2. But this is not a matter of opinion; the extent of its effect can be easily computed for any given case. It must not be concluded just because calculations made on the basis of constant terminal voltage show a larger power limit with the normal line capacitance than without it, that the same result would obtain with synchronous apparatus of a kv-a. capacity comparable with the load to be transmitted.

Mr. Griscom states that he does not understand why synchronous apparatus becomes inherently more powerful during transients. The reason is that, for the moment, the *transient* reactance, instead of the *synchronous* reactance, determines the power characteristic of the synchronous machines. The ratio of synchronous reactance to transient reactance in ordinary commercial synchronous machines is of the order of 5 to 1. It may be as low as 3 to 1, or as high as 10 to 1. In other words, in rough values the synchronous reactance is about 100 per cent, and the transient reactance about 20 per cent. Thus the machine in such a transient state is decisively stiffened up. Mr. Griscom's question would probably be removed by reading over the paper—under the heading "Transients."

He also questions whether the slope of the voltage-power curve determines the degree of stability. If the slope is zero at all values of power, it merely indicates that the voltage of the bus under consideration is not affected by any power change whatsoever, regardless of whether the synchronous machines constituting the infinite bus are of zero inertia or infinite inertia or any value between these extremes. Mr. Griscom's statement is correct that when $\frac{d\,E}{d\,P}$ approaches infinity, $\frac{d\,P}{d\,T}$ approaches O, but this is not, as Mr. Griscom states, because $\frac{d\,P}{d\,\Theta}$ is lim-

ited by the mass of the synchronous machines, but because at that moment the electrical characteristics are such that the power is not changing, although the angle Θ may be changing.

I wish to add a word about this 183,000-kw. story as related by both Mr. Evans and Mr. Baum, and which Mr. Dewey has answered. As the engineers of the country are eagerly waiting for every additional fact of operating experience bearing on this subject, it seems unfortunate, indeed, that when some real data do become available, their meaning should be so completely misunderstood or misinterpreted. When the sending end wattmeter reads 183,000 kw. and the receiving-end meter reads 135,000 kw., should we say or imply that 183,000 kw. is *transmitted over* the line? And I think that neither Mr. Evans nor Mr. Baum would, after serious thought, adduce that test (which our calculations check) as evidence that our calculated values are much too low. In Mr. Evans' published work, he calculates, as most every one does, the receiver-end, not the sending-end, power.

Mr. Evans raises two other points: One is that the maximum power can be increased by reducing the reactance of the generator; the other, that Fig. 2 might be misleading. As to the one, the authors heartily agree, as presumably every one else does. Nobody would question that. The question is *how* will you decrease the reactance. Mr. Evans is referred back to the paper to the heading "Design," where this matter is fully discussed. The generator capacities given are based on present-day practise —synchronous reactance approximately 100 per cent. Nothing would be gained by cutting the ratings to, say, one-half, thus reducing the per cent reactance to 50 per cent. That would not increase the maximum power. The question is how would one alter the design of a machine of given magnetic dimensions in order to lower the synchronous reactance, and to what extent could it be thus lowered. There could not be much disagreement among informed designing engineers on that point. After that is done, any further reduction must be obtained by increasing the active volume of the machine, or by adding more machines. Thus, as Mr. Evans says, and as the paper clearly points out, "quite marked increases in the stability" can be obtained in these ways, but there are perfectly obvious reasons, as the paper also points out, why this process cannot be extended far enough to satisfactorily solve the problem.

The other point raised by Mr. Evans is well founded. The authors acknowledge that the title of Fig. 2 should be more specific, and will revise it accordingly. The illustration refers to a 250-mi. straight-away line with synchronous-motor load.

Mr. Fortescue does not know of "a single case in which a system was thrown out of step by a sudden increase in load, or short circuit due to low excitation." There have been such cases, nevertheless. The September 1919 trouble of the Commonwealth Edison Company, described by Dr. Steinmetz in the 1920 TRANSACTIONS, is one notable example among others.

The authors naturally agree that if an additional line with duplicate sending and receiving apparatus be installed, the maximum power will be increased. Two power *systems* will obviously carry more than one. But that is not the point. Fig. 2 merely shows the relation between generating capacity, number of lines, and maximum power. To say, as Mr. Fortescue does, that two "properly designed" systems with "proper size generating stations," etc., etc., will always carry more than one, would hardly bear close scrutiny where costs are regarded, because "proper size" for maximum power may be prohibitive in cost.

It does not require a mechanical model to prove the platitude that if "the terminal voltages are kept constant," the power limit will be the limit of the line itself. But to the authors' knowledge the "perfect regulator" to hold this condition does not exist. When it shall exist, or else when some other method than synchronous operation is utilized, it will be time enough to talk seriously about the limit of the line alone. I have discussed this problem of regulation in my comments on Mr. Fortescue's paper.

Mr. Fortescue also bespeaks the gain from changing the generator characteristics. Such changed generators, he avers, will permit of operation with a "high stability limit," but he doesn't say how high. And the whole point, if there is any, depends upon *how high*. He, like Mr. Evans, is referred to the paper under the heading "Design."

They also mention that the authors have not discussed the effect of load characteristics. Loads which are functions of the voltage, such as lights and certain classes of converters feeding a constant voltage d-c. bus, are inherently stable. These have a maximum power, but that is not the limit of stability. Indeed, there is no stability limit with a plain impedance load. The shaft load of induction and synchronous motors is independent of voltage, and for these, the maximum power limit and stability limit coincide. Thus, a composite load would have greater stability than a pure shaft load of the same amount.

H. H. Dewey: In closing, I shall take up some of the points that have been brought up in the discussion of the paper by Mr. Doherty and myself.

Mr. Thomas in his discussion brought out a point in regard to the relation of the cost of the terminal apparatus to the cost of the line, pointing out that the line cost was very high per kilowatt. It might run to $125.00 per kilowatt, whereas the terminal apparatus, generators and motors, would be very much less than that, perhaps $8.00 or $10.00 per kilowatt, and that since the terminal apparatus was an important factor in the power limit of the completed system, the place to work to extend our power limits was on the synchronous apparatus. I agree with that thoroughly, and we can do considerable along that line.

Mr. Evans and Mr. Fortescue spoke of the possibilities of synchronous-apparatus improvement as a thing about which we were unduly pessimistic in our paper, that is, the question of what could be done to increase the power limit of terminal apparatus. Our paper did not stress that point for the reason that it seemed obvious. In presenting the curve given, showing the breakdown of a given generator and a given motor, it is quite apparent that the power limit is determined by the size of the generator. If we had a generator of double the size, or a motor of double the size, we would get double the power. In line with Mr. Thomas' suggestion then, since we can put on generating apparatus for $8.00 or $10.00 per kilowatt, we can obviously increase the size of the generator until we strike an economic balance between the size of the generator and the capacity of the line.

Now, the size of a generator is determined by its characteristics. We would fool ourselves if we tried to take a 10,000-kilowatt generator and cut its reactance in half, change its field, style, etc., and still call it a 10,000-kilowatt generator. It wouldn't be a ten-thousand any more; it would be fifteen or twenty, and it would cost more money to build it. That is an obvious thing that we did not stress in our paper. Since generating and receiving apparatus is a large factor in limiting the power that can be transmitted over a system, everything possible should undoubtedly be done to improve their characteristics.

The paper tried to cover the essential points in determining what a given set-up would give. There was a point that Mr. Evans brought up and Mr. Fortescue also, in their discussion, in regard to the effect of charging current of transmission lines. They took issue with Mr. Doherty and myself in stating that charging current was an actual detriment. They evidently did not read our discussion of that point very carefully.

With given apparatus, with a given generator, with a given receiving-motor load, and a given voltage at each end of the line, which we always have, charging current *is* an absolute detriment. We reduce the amount of power which can actually be transferred from the generator to the receiver end, no matter what the relation of charging current of the line to the generator. The charging current of the line on a high-voltage system will actually reduce the amount of power you can transfer from the generators

to the motors, because it reduces the excitation of the generators and motors.

Mr. Evans brought out a point in which he indicated that our calculations on a 250-mi. line were in error in that it was considerably less than the value that had already been obtained in practise by the Southern California Edison Company. He stated that our calculations showed the limit to be 115,000 kw. whereas they obtained 183,000 kw. The calculation is 120,000 kw., not 115,000 as read from the curve by Mr. Evans. We have made a calculation of the Southern California Edison System based upon data furnished by Mr. Barre which showed 183,000 kw. Our check calculations, made in the same manner as our calculations in the paper, came within 5 per cent or thereabouts of the same results.

The big difference in these values comes in this respect: The 120,000 kw. shown in the paper was receiving load. The 183,000 kw. obtained in actual practise on the Southern California Edison Company's system was at the generating end. At the time they had 183,000 kw. input they had 135,000 kw. output. Thus, we have an error of only 120,000 kw. to 135,000 kw. and there is a difference of 20 per cent in frequency, that is, 50 instead of 60 cycles, which brings it to almost exactly the same thing.

Mr. Wood spoke of the term that has been quite often used to describe "power limit," that is, "instability," and sounded a warning against its use. I agree with him thoroughly on that. It was for that reason that the title of our paper was made Power Limit. Power limit is something that is perfectly harmless, and I agree that the use of the word instability is likely to cause concern where concern is not necessary.

Mr. Fortescue's discussion was quite lengthy and very helpful. He took issue with some parts of our paper. Some of these points I did not quite digest. Some of them made me rather think that he was saying the same thing we were saying only in different words, particularly as he laid very great stress on the effect of voltage regulators. I agree with him thoroughly that the regulator is a very important thing, and is so much more important than we originally thought that it makes a great deal of difference in the ultimate capacity of the transmission system.

We have very few real things to worry about. I agree with Mr. Wilkins that the final solution of this problem is going to come in the data that we get from actual operation, but so far the data of actual operation, where we have been able to obtain data, have checked so closely with our present methods of calculation that we feel we are in a position where it will be possible to predict what will happen to any given system. The more complicated the system, the more difficult it is to calculate, and when we get such complications, we must resort to some scheme such as Mr. Nickle has described[9], just as we reach the limit of the possibility of calculating short-circuit current on a network such as described by Mr. Wilkins with 880,000 kw. of generating capacity and thousands of miles of interconnected transmission line. That would be a hopeless case to calculate the short-circuit current, but we have calculating machines that arrive at these values very closely. We can likewise use the Nickle calculator or some other device to arrive at our stability problem. We are not worrying about the question of stability or power limits; we know pretty closely how to calculate them, and it is very essential that we do so, as the amount of power that can be carried over a given line greatly influences the cost of delivered power.

9. "Oscillographic Solution of Electromechanical Systems," by C. A. Nickle, Jour. A. I. E. E., December 1925, p. 1277.

C. L. Fortescue: In regard to Mr. Thomas' discussion, he emphasizes the effect of damping factors. In our calculations we try to work in the effect of the damping factor as much as possible, but Mr. Thomas remarks that if you increase the damping factor sufficiently the machine will be critically damped. We know that in practise this condition is never met. We have records of power swings and they are as far as we know never critically damped.

As regards Mr. Doherty's discussion I think we are substantially in agreement. I think an explanation of the differences in view might be somewhat as follows:

Two investigators have approached this problem from somewhat different angles, and while there is no disagreement in the fundamentals of the problem, there is a little apparent disagreement in what might be termed derived ideas.

The problem of stability involves so many factors, the speed of the exciters, the speed of the regulators, the inherent tendency of the machines to correct themselves, etc., that it is quite excusable that there should be a slight difference of opinion.

Now, I think you might say that Messrs. Doherty and Dewey and myself agree on what should be. We probably agree pretty well on what actually is, but we disagree somewhat in regard to what may be or might be, Messrs. Dewey and Doherty putting the "might be" a little closer to the "is," and I myself putting the "might be" a little closer to the "should be."

I hope that the result of our investigations will finally bring us both to an agreement on the "might be" and that the "might be" will finally come closer to the "should be" than Messrs. Doherty and Dewey place it at present. That is my hope. I am quite open-minded about this.

The final decision about this particular question will undoubtedly come about from actual work in the field and in the laboratory. We shall, of course, make calculations and analyze these results and there is no doubt at all that we shall finally come to a substantial agreement. In that day I hope we will be very close to the "should be."

Application of Electric Propulsion to Double-Ended Ferry-Boats

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Synopsis.—*The double-ended ferry-boat propelled by means of a bow and stern propeller has become the recognized standard type, due to its maneuvering possibilities and general handiness in congested harbors.*

In all cases in which the prime mover is directly coupled to the two propeller shafts which must necessarily turn at the same revolutions per minute, the over-all propulsive efficiency is lowered due to the performance of the bow propeller.

The electric drive system permits of applying power when and where required and to any degree desired. Tests on the double-ended ferry-boat, W. R. HEARST, show a material gain in propulsive efficiency when driving the bow propeller electrically at a speed which gives neutral thrust. Later tests indicate, however, that there is no substantial difference in the propulsive efficiency whether the bow propeller is driven electrically at neutral thrust or is electrically disconnected and driven by the water. Sufficient tests have not been made, however, to show that this is true in all cases.

The reciprocating steam-engine or Diesel-engine type of drive, in which both shafts are direct connected, requires approximately 19 per cent more horse power at the propeller shafts than the electric system, due to the difference in propulsive efficiency.

The calculated fuel consumption of a typical reciprocating steam-engine drive with the direct-connected system shows that it requires approximately 40 per cent more fuel than the steam turbine electric system, due to the difference in propulsive and thermal efficiencies.

The electrical transmission losses are less than the propulsive efficiency losses of the direct systems. In addition to the more efficient method of power application, electric drive also has many inherent advantages, such as rapid maneuvering qualities and ease of control.

The Ward Leonard system, similar to that used on the Chicago fire boats which were put in operation in 1908, permits of the use of pilot-house control, eliminating the personal factor which is always present with the engine-room telegraph.

The operating records of ferry-boats in service prove electric drive to be reliable.

The respective field of application of turbine electric drive or Diesel electric drive for double-ended ferry-boats depends upon the relationship of first cost to the operating changes and needs of the service.

THE double-ended ferry-boat presents a problem unique in marine and electrical engineering and the data pertaining to the specific applications which follow bring out quite forcibly the manner in which electric drive overcomes the inherent losses of other systems and brings about a higher over-all efficiency than formerly attained in preceding types.

Briefly, the economic gains brought about through the electrification of the main propelling machinery consist of two things:

I. Gain in propulsive efficiency due to the method in which the power is applied to the forward and aft propellers.

II. Gain due to the higher thermal efficiency of the turbo electric and Diesel electric machinery as compared to the reciprocating steam engine.

It has long been recognized by naval architects and marine engineers that an inherent loss in propulsive efficiency exists when one prime mover is directly coupled to a bow and stern propeller, which must necessarily turn at the same rev. per min.

Compromise designs have been made in an attempt to decrease the bow propeller losses, but it is not apparent from published test records that this has resulted in an increased overall propulsive efficiency.

Several tests have been made on reciprocating steam engine-driven, double-ended ferry-boats, driving with the stern screw with bow screw removed, pulling with bow screw with stern screw removed, and driving with both screws at the same rev. per min. A comparison of the power input required to give the same boat speed with the various systems is given in Table VI.

Driving with the stern screw with forward screw removed consumes the least power at a given speed. Due, however, to the fact that ferry-boats operate in congested waters and usually on short trips, it is not practical to turn the boats around, and therefore the double-ended arrangement has come into use as being the most practical for this type of service. The double-ended ferry-boat may, therefore, be considered a compromise type, in which high propulsive efficiency is sacrificed for maneuvering ability and general handiness.

As electric power may be applied when and where required and in any degree desired, it is but natural that this type of propulsion finds a ready application in the double-ended type of ferry-boat, as the two propellers may be driven at such relative speeds that the entire work of propulsion is accomplished entirely with the after screw.

The general method adopted in the various electrically driven ferry-boats is the driving of the boat with the after screw and the operation of the bow screw at sufficient rev. per min. to overcome either a pulling or pushing affect.

Tests have been made on the electrically driven ferry-boats and the power input to the bow screw at neutral thrust noted. The results may, therefore, be compared

1. Both of the Marine Engineering Dept., General Electric Co., Schenectady, N. Y.

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with the tests previously made on the reciprocating steam-engined ferry-boats.

The electric system is comparable to the one screw arrangement plus the additional power required to drive the bow screw.

As will be shown later by records of tests, the increased power required by the two-screw arrangement, driving at the same rev. per min., is approximately 25 per cent as compared to driving with one screw, whereas with the electric drive, the bow screw may be operated at neutral thrust with an expenditure of power amounting to approximately five per cent.

The data which follows include a description of the electric equipment, control features, and various tests on the following electrically driven boats:—a-c. turbine electrically driven ferry-boats—*W. R. Hearst*, *Rodman Wanamaker* and *Geo. W. Loft*; d-c. Diesel electrically driven ferry boats—*Golden Gate* and *Golden West*; and d-c. turbine electrically driven ferry boats—*Hayward* and *San Leandro*.

The *W. R. Hearst*, *Rodman Wanamaker*, and *George W. Loft* are operated in New York harbor by the City of New York and were placed in service during 1923 and 1924.

The propelling machinery consists of an 8-stage horizontal Curtis steam turbine, direct-connected to an a-c. generator. This generator furnishes power to two double squirrel-cage induction motors, each rated 36/52 poles, 2100/100 h. p., 176/122 rev. per min. The 36-pole winding is used to drive the boat by means of the aft propeller, and the 52-pole winding to drive the forward propeller at or near neutral thrust. The rotor of one motor is direct-coupled to the forward propeller shaft, and the rotor of the other motor is direct-coupled to the aft propeller shaft.

Excitation and power for the electrically-driven auxiliaries, lighting, etc., is obtained from one 125-kw., 220/110-volt, d-c. generator, which is driven by a geared Curtis condensing steam turbine. There is a second set, of the same capacity, which is a spare. Practically all continuous duty auxiliaries excepting the boiler feed pump are motor driven.

The control station is located in the engine room. The operator normally controls the ship by means of an electric lever and a speed switch. Emergency levers are provided so that in case of the failure of electric control or the turbine variable speed maneuvering governor, the propelling machinery may be operated by the manually operated levers.

The electric lever controls the solenoid operated contactors and the speed switch controls the governor setting of the turbine.

Due to the exacting maneuvering requirements met on ferry-boat service, the 36-pole induction motor winding was designed to give normal full load torque with super-excitation of the generator field, irrespective of the turbine revolutions per minute or generator frequency. This was accomplished by using a double squirrel-cage rotor winding. This winding consists of a winding having a relatively high resistance and low inductance, electrically, in parallel with a winding of relatively high inductance and low resistance. When starting, the reactance of the low resistance squirrel-cage is high, which causes the current to flow chiefly through the high resistance winding.

At full speed or when the motor has pulled in step, the reactance of the low resistance winding is low and the current flows chiefly through this winding. This permits a relatively high torque at starting and an efficient motor when running in step.

The advantage of this type of motor is that it eliminates brushes, collectors, external resistors, and short-circuiting contactors. It is possible to maneuver without slowing down the turbine or generator revolutions, but if this is done, it requires a longer time for the motor to be pulled into step than if the turbine revolutions are reduced.

On July 28, 1923, several reversal tests were made and it was found that with the boat going full speed ahead, the propeller could be stopped in about five seconds from the time the signal was given from the pilot house, reversed and pulled into step in the opposite direction in about ten seconds, and the boat stopped dead in the water in from fifty to sixty seconds. These tests were conducted in New York harbor, and therefore, it was impossible to note the boat's speed and difficult to determine when the boat had come to rest in the water. These readings are therefore mentioned as being approximate and only of general interest.

Before starting tests on the propelling machinery the bow motor was electrically disconnected and readings taken of the stern and bow motor rev. per min. We were unable to estimate the speed of the boat. These readings give the relative rev. per min. of the stern and bow motor when the ship is being propelled by the stern motor, the bow motor being driven by the propeller. When the stern propeller was being driven at 171 rev. per min. the bow propeller drove the bow motor at 100 rev. per min. During and following these tests, the end play of the bow motor was noted. When power was applied, the bow motor end play was taken up in a forward direction, occasionally coming aft due to waves, etc. It is believed, therefore, that approximately neutral thrust or a slight pull was exerted when the bow motor was driven electrically.

During the tests on the propelling machinery the following readings were taken:

TABLE I

W. R. Hearst

A-C. Generator				Aft Motor		Forward Motor	
Line Volts	R.P.M.	Field Amps.	Field Volts	Amps.	R.P.M.	Amps.	R.P.M.
2500	3050	135	110	515	165	39	118
2275	3000	125	110	560	167	39	118
2300	3050	125	110	550	168	50	118
2350	3000	130	110	530	165	40	118
2350	2900	130	110	500	158	35	112

Based on factory tests, the calculated power delivered to the bow and stern propeller shafts is as follows:

TABLE II

Kv-a.	P.F.	Eff.	Kw. Input	Kw. Output
Aft Motor				
2230	74	94	1650	1550
2207	76	94	1675	1575
2190	76	94	1660	1560
2160	75	94	1620	1525
2036	73.5	93.9	1495	1405
Forward Motor				
169	53	83.5	90	75
153.5	57	84.5	87.5	74
199	64	84.0	127	107
162	57	84.3	92.5	78
143	49	82	70	57.5

The calculated cable loss between the generator and the propelling motors is approximately 1700 watts with full load on the bow and stern motor windings. The combined bow and stern motor full load efficiency is 93.7 per cent at 74.8 per cent power factor. From the results of these tests it appears that when the stern motor was delivering 2100 h. p., the bow motor was delivering approximately 107 h. p., or 5.13 per cent of the power delivered by the stern motor.

Test results check the estimates made by M. G. Kindlund, Naval Architect, based on model tests, as well as general estimates made by Commander S. M. Robinson, published in the December 1920, *Marine Engineering*.

The propellers on the *George W. Loft* were of a different design, giving a different relationship of the bow and stern revolutions per minute for neutral thrust. Based on these tests and tests conducted on other electrically driven ferry-boats, it was decided that it is practically as efficient to drive the bow motor by means of the bow propeller as it is to drive the bow motor electrically; that is, instead of having an output of the stern motor of 2100 h. p. and of the bow motor of 107 h. p., practically the same results can be obtained by using the same generator output on the stern propeller motor, which would be equivalent to slightly more than 2207 h. p. output of the stern motor, due to the difference in efficiencies, allowing the water to drive the bow propeller. This permits the use of a single speed motor instead of a double speed motor.

The *Golden Gate* was the first Diesel electric, double-ended ferry-boat to be placed in service and is therefore taken as an example of Diesel electric drive. This boat was placed in service in San Francisco Bay, July 4, 1922.

The electric propelling machinery consists of two Diesel driven, direct-current, separately excited generators, each rated 360 kw., 250 volts, 225 r. p. m., and two 35-kw., 115-volt exciters, each mounted on the shaft extension of the main generators. The generators are normally electrically connected in series. There are two separately excited propelling motors each rated 750 h. p., 145/180 r. p. m. The rotor of one motor is direct-coupled to the forward propeller shaft, and the rotor of the other motor is direct coupled to the after propeller shaft. Either one of the two exciters is used for exciting the generators, propelling motors, control, and furnishing power for the electrically driven auxiliaries, lighting, etc.

The control is of the Ward Leonard or voltage-current system, similar to that first used on the Chicago Fire Boats, *Joseph Medill* and *Graeme Stewart*.

The direction of rotation and rev. per min. of the propelling motors is controlled by varying the excitation of the generator fields and the relative rev. per min. of the bow and stern motor by changing the field excitation of the motors. It is possible, with this type of control, to change the relative bow and stern propeller rev. per min. at any predetermined position of the controller.

There are three control stations, one in each pilot house and the third station located in the engine room.

Normally, the generators are electrically-connected in series but cut-out switches are provided so that either generator may be disconnected, the boat being operated by the remaining unit. The motor field control permits utilizing full output of one engine, which corresponds to approximately 80 per cent propeller rev. per min.

The several severe tests, and also the operating records in normal service, have demonstrated the advantage of pilot house control as well as the reliability of this type of machinery.

This ferry-boat has been stopped from full speed ahead to dead in the water in 30 to 35 seconds.

More complete tests were conducted on the *Golden West*, which is a sister ship of the *Golden Gate* and has the same propelling equipment. We shall, therefore, use results of the tests conducted on the *M. S. Golden West*.

The following readings were taken April 6, 1923, when running over the measured mile off California City:

GOLDEN WEST

TABLE III

		Aft Motor			Forward Motor			Total Motor Excit.			
Run	Line Volts	Line Amps.	R.P.M.	Kw. Input	Line Amps.	R.P.M.	Kw. Input	Fld. Amps.	Fld. Volts	Knots	Remarks
1	475	1138.6	173.3	540	59.3	138.6	28.2	121.7	115	11.75	Against
2	470	1031.6	174	485	88.3	139.5	41.3	122.6	115	12.25	With
3	470	1142	174	537	77	143.2	36	124.2	115	11.5	Against
4	472	1000	172	472	130	144.2	61.2	121	115	12.4	With
5	511.6	1406.6	193.5	718	53.3	157.1	27.1	103.6	115	11.75	Against
6	511.6	1295.3	193.5	661	98.3	158	50.2	98.3	115	5.3	With

TABLE IV
FORWARD MOTOR ELECTRICALLY DISCONNECTED

	Aft Motor			Foward Motor		
Run	Amperes	Volts	R.P.M.	Amperes	Volts	R.P.M.
7	975	480	170.6	0	0	123.7
8	975	475	171	0	0	124
9	1000	475	170	0	0	126
10	1000	475	170	0	0	125
11	980	475	170	0	0	.126
12	960	475	170	0	0	126

Based on factory tests, the power delivered to the bow and stern propeller shaft is as follows:

TABLE V

	Aft Motor		Forward Motor		TotalMotor
Run	Kw. Input	Kw. Output	Kw. Input	Kw. Output	Excit. Kw.
1	540	507	28.2	22.5	14.
2	485	457	41.3	35.5	14.1
3	537	505	36	30.2	14.3
4	472	445	61.2	55.1	13.9
5	718	672	27.1	21.4	11.9
6	661	620	50.2	44.3	11.3

Results of these tests show that the average power output of the bow propeller motor was 6.8 per cent of the stern motor. The full load overall efficiency, including generator losses, exciter losses, motor losses, cable losses, and rheostat losses, when the stern motor is delivering 750 h. p. and the bow motor 51 h. p., is 83.9 per cent.

Unfortunately, there is no notation on the tests conducted that the bow motor was exerting neutral thrust or a forward pull, but based on the relative rev. per min. of the stern and bow motor when the bow motor was being driven by the water and when the bow motor was electrically driven, we believe that there was a slight forward pull, which may or may not account for the slightly higher power input to the bow motor than that shown on the tests conducted on the *W. R. Hearst*.

The *Hayward* and *San Leandro*, which were placed in service on San Francisco Bay in 1923, are of the d-c. turbo electric type.

The electric propelling machinery consists of a 1100-kw., 3600-rev. per min., horizontal, Curtis steam turbine direct-connected through a reduction gear to a 1000-kw., 900-rev. per min., separately excited generator. Mounted on the shaft of the main generator is a 75-kw., 115-volt exciter. The 1000-kw. generator supplies power to two separately excited, double armature motors, each rated 1200 h. p., 100 to 125 rev. per min. The armatures of one motor are direct-coupled to the forward propeller shaft and those of the other motor to the aft propeller shaft.

The 75-kw. generator supplies power for exciting the 1000-kw. generator, the two propelling motors, control, auxiliary power, and lighting.

The control station is located in the engine room, and the control is of the Ward Leonard or voltage-control system, similar to that described for the Golden Gate ferry boat.

The above boats have been in continuous operation since their inauguration into service. As an example of the severe duty required of these boats, the following data, covering a short period of their operation, are given.

Between May 31, 1923 and February 10, 1924 the *Hayward* made 10,500 trips, traveled 30,312, miles and carried upwards of 4,000,000 passengers. The two boats, in a year's time, carry upwards of 10,000,000 people and the largest number of people carried in one day by one boat has aggregated upwards of 30,000 people. They serve one of the most congested terminal traffic lanes in the world. Six hundred and fifty electric trains per day serve the traffic at the Oakland Terminal and during the one hour rush period between five and six o'clock in the evening, 48 trains are moved.

The operating record of these boats is given rather than a repetition of test data on the previous boats, due to the fact that it brings out one very essential point, that is, the absolute reliability of this type of machinery where continuity of service is the paramount factor.

Comparative Efficiencies

The following section is devoted to an analysis and economic comparison of the reciprocating steam engine driven and turbo electric driven types of ferry-boats.

This will tend to show in a concrete way the fundamental differences that exist due to both the propulsive and thermal factors.

The Diesel electric system has the same advantage as the turbo electric drive in the matter of gain due to the difference in propulsion characteristics. The high thermal efficiency of Diesel engines, as compared to either the reciprocating steam engine or turbo electric, is fully recognized, but due to the fact that this subject has been so fully covered by the Diesel engineers, it is omitted from this paper.

The selection of steam turbines or Diesel engines depends upon the relationship of first cost to operating costs and the needs and requirements of the service, and is also influenced, to a more or less extent, by the past experience of the operating companies.

Relative Power Requirements. The relative power required to maintain a given boat speed, as ascertained by tests made on the reciprocating steam engine driven type of double-ended ferry-boats, is given in Table VI.

Conclusions to be Drawn from Tests

A. Propelling with bow screw, stern screw removed: The tests show that over 50 per cent more power is required when pulling with the bow screw than when pushing with the stern screw.

B. Pushing with stern screw, bow screw removed: This method takes the least power and may, therefore, be considered the base with which to compare other

TABLE VI

Knots	A Pulling with Bow Screw, Stern Screw Removed I. H. P.	B Pushing with Stern Screw, Bow Screw Removed I. H. P.	C Propelling with Both Screws at the same R.P.M. I. H. P.
Ferry-Boat *Edgewater*			
8	255	178	222
9	370	245	312
10	560	345	440
11	850	500	612
12	1270	740	840
Ferry-Boat *Cincinnati*			
9	443	250	332
10	638	364	464
11	880	520	624
12	..	720	816

References: Trial trip data of *Edgewater* from paper presented by E. A. Stevens and Chas. P. Paulding before Society of N. A. & M. E., November 20, 1912.

Trial trip data of *Cincinnati* from paper presented by F. L. Du Bosque before Society of N. A. & M. E., November 12-13, 1896.

systems. In service, however, this system is not practical.

C. Driving with bow and stern screw at same rev. per min.: The tests disclosed that approximately 25 per cent more power is required to maintain the same speed of boat than in system "B". This is the usual method of propelling a double-ended ferry-boat in which the prime mover is directly connected to the two propeller shafts.

D. Driving with after screw only, bow screw operated to give neutral thrust; electric drive system only: The series of tests conducted on the electrically driven ferry-boats disclose the fact that the bow propeller may be operated to give neutral thrust with an expenditure of power amounting to approximately five per cent of the total.

In service, we are interested only in systems "C" and "D". As the relation between these two systems is of the order of 125 per cent to 105 per cent, taking the electric drive system as the base, the new relation becomes 119 per cent and 100 per cent, respectively.

In other words, the electric drive system requires but 84 per cent as much power as the reciprocating engine drive to attain the same boat speed, due to the gain in propulsive efficiency.

Analysis of Steam and Fuel Consumption

The 2200-s. h. p. turbine electric, a-c. driven ferry-boat has been selected to compare with a reciprocating, steam engine driven ferry-boat as water rate and efficiency tests were conducted at the factory on the electrical equipment. The s. h. p. required by the reciprocating engine driven type to be comparable will be 2620. On account of the long line shafting required in the latter type, a mechanical efficiency of 90 per cent is assumed, which gives an i. h. p. of 2910.

Comparative Water Rates of Main Propelling Machinery:

a. Reciprocating steam engine; so far as the authors are aware, there have been no data published regarding water rate tests of reciprocating steam engines installed on ferry-boats in which either the power developed or steam conditions are comparable to the turbine electric installations. Recourse, therefore, must be made to such formulas as Jenson's or the later methods deduced by E. A. Stevens, Jr.

Average values range from 14 to 17 lb. per i. h. p-hr. for saturated steam at 250-lb. gage pressure, and correcting for 200 deg. fahr. superheat, these values are reduced to 11.5 to 14 lb. per i. h. p-hr.

In the comparisons which follow the minimum value of 11.5 lb. per i. h. p-hr. is used.

b. The water rates of the turbo electric equipments furnished for the New York municipal ferries were determined by a very rigorous series of tests. This amounted to 10.135 lb. per s. h. p. per hr. including the generator and motor losses, with the hand valves open. Under the following steam conditions—pressure 250 lb. gage, superheat 200 deg. fahr., vacuum 28.5 in.—quite a throttling loss takes place at rated full load with

TABLE VII

	Reciprocating Engine	Turbine Electric a-c.
S. h. p.	2620	2200
I. h. p. (Mechanical efficiency 0.90)	2910	
Water Rate lb. per hp-hr.	11.5 (I.H.P.)	10.0 (S.H.P.)
Steam Consumption lb. per hr..		
Main propelling unit	33400	22000
Auxiliary turbine generator	1620	2194
Steam auxiliaries	2960	2625
Total lb. per hr	37980	26819
Lb. per I.H.P—All purposes	13.5	..
Lb. per S.H.P—All purposes	14.5	12.2
Evaporation per lb. oil	12.8	12.8
Lb. fuel oil per hour	2970	2100
Lb. fuel oil per i.h.p-hr	1.02	..
Lb. fuel oil per s.h.p-hr	1.135	0.955
Difference, lb. fuel oil per hr	870	
Difference, tons per 24 hrs	9.35	
Relative fuel consumption	142%	100%

TABLE VIII

AUXILIARIES	Reciprocating Engine	Turbine Electric
Varying with type	Lb. per hr.	Lb. per hr.
Feed Pump	1000	725
Steam for heating fuel oil	210	150
Constant for both types		
Service Pumps	1750	1750
Total lb. per hr	2960	2625
Electric Auxiliaries:	Kw-Hr.	Kw-Hr.
Varying with type:		
Circulating pump	15.0	30.0
Condensate pump	7.5	5.0
Excitation	0.0	15.0
Blower for motor ventilation	0.0	13.0
Constant for both types		
Lubricating oil pump, lights, sanitary pump, fresh water pumps, steering gear, ventilating fans	37.5	37.5
Total Kw	60.0	105.5
W/R	27.0	20.8
Lb. steam per hr	1620	2194

the hand valve open. With the hand valve closed, in which rated full power is available, the water rate is 9.98 lb. per propeller shaft horse-power hour. In the comparisons which follow a flat rate of 10.0 lb. is used. (The hand valve is used for overload conditions during rush hours.)

Conclusions

1. That the reciprocating steam engine or Diesel engine, in which the bow and stern propellers are operated at the same revolutions per minute, requires approximately 19 per cent more power than the electric system, due to difference in propulsive efficiency.

2. That the fuel consumption of the reciprocating steam engine drive with the direct connected bow and stern propellers requires approximately 40 per cent more fuel than the turbine electric system, due to the difference in propulsive and thermal efficiencies.

3. That the operating records of the boats in service prove electric drive to be reliable and a great step forward as a method of propulsion for the double-ended type of ferry-boat.

4. That in the comparison of turbo d-c. and a-c., the d-c. is superior in flexibility, simplicity of control, and general handiness afforded by bridge control. That the a-c., however, is slightly more economical as regards fuel consumption.

5. That both turbo electric and Diesel electric drive overcome the inherent propulsive efficiency loss of the reciprocating steam engine type of drive and that their respective spheres of application are dependent upon the relationship of first cost to operating charges and needs of the service.

Discussion

H. F. Harvey, Jr.: About the only comment which I have to offer is that sufficient emphasis has not been laid on the maneuvering feature. With direct control from each pilot house it is very evident that the one in charge can maneuver the boat more easily than by means of the ordinary signals to the engine room.

I believe that thus far most ferry-boats with electric drive are used on fairly long runs. Such runs do not show to advantage the speedier maneuvering, either when entering or leaving the slips. For short runs as between New York and Jersey City, or between Camden and Philadelphia, electric drive would show a decided advantage in this respect.

Ferry-boats are usually operated in congested waters where it is very necessary to have close control of the vessel in order to avoid accidents. Electric drive, I believe, affords quicker stopping and reversing than any other drive. Too much emphasis, therefore, cannot be placed upon the superior maneuvering qualties of electrically driven ferry-boats.

F. K. Kirsten: There has been no mention made in the paper as to the design of the propellers involved in ferry-boat propulsion. It seems that these boats are designed to travel in either direction with practically the same propeller showing. As a consequence, some design must be used on these particular ferry-boat propellers differing from that used in ordinary steamers. I would like to know if any particular statements could be made in that direction.

M. J. Whiteman: I should like to know if it is possible with a ferry-boat having four-propeller drive to rotate the boat on a center or pivot in order to make quick turns.

A. Kennedy Jr.: Mr. Whiteman asked if it is possible with an electrically-driven ferry-boat to pivot the boat in order to make quick turns, and also the number of electrically driven ferry-boats that are in operation. All electrically driven ferry-boats use the same method of steering as that used on reciprocating steam engine driven ferry-boats; that is, they use one rudder. I do not know of any way to make a ferry-boat pivot in order to make quick turns unless some change is made in the design of the boat.

At the present time there are eight electrically driven ferry-boats in operation, three in New York, four in San Francisco, and one at Poughkeepsie. These, I believe, are the only ones, but of course, others are being considered.

Professor Kirsten asked whether or not it was necessary to modify the design of the propellers for electrically driven ferry-boats. Normally, with steam engine driven ferry-boats, a compromised propeller design is used, as it is necessary to use the faces and backs of the blades. A good deal of work has been done trying to improve the over-all propulsive efficiency by decreasing the amount of power required to drive the forward propeller. As far as I know, no one has been able to improve the over-all propulsive efficiency by using this specially designed propeller, but they have reduced the power required to drive the forward propeller.

With electric drive it is possible to use a standard propeller for the simple reason that the bow propeller does not do any work. Only the face of the propeller is used for driving, whereas with the reciprocating steam engine connected to a through shaft a special design is made as the back of the blade is normally used on the forward propeller to assist in driving the ferry-boat.

Some Features and Improvements on the High-Voltage Wattmeter

BY JOSEPH S. CARROLL[1]

Associate, A. I. E. E.

THE high-voltage wattmeter herein described is the result of three years of study and experimental work carried on at Stanford University. Included in this report on the wattmeter is a description of a high-voltage voltmeter and a crest voltmeter. The operation of these instruments is entirely independent of any connection to the supply transformers. In other words the equipment in its present form can be connected directly in on the high voltage line, and simultaneous readings of the power, total effective voltage, crest factor, and line current can be taken at any instant and at any voltage up to 175 kv. to neutral.

The wattmeter consists of an ordinary low-voltage instrument located in an electrostatically shielded cage that is at high potential. The current coil of the meter is connected directly in on the line with an ammeter in series. The potential coil with its special high resistance multiplier is connected from the line to ground. There is also a milliammeter in this circuit. (See diagram of connections.) These instruments are all read with telescopes at a safe distance. This special multiplier consists of a column of ordinary tap water 16.5 ft. long and 3/16 of an in. in diameter; it has a maximum resistance of approximately three million ohms and a current carrying capacity of 65 milliamperes. A rubber air hose is used as container for this column of water. This hose is wound into a helix of five turns about one foot in diameter and the helix is placed with its axis vertical between two horizontal circular plates four feet in diameter and separated thirty inches. These plates are supported by three bakelite strips 1 in. by 3 ft. 1 in. long. The hose is held in place by means of a single bakelite strip with wooden pegs of 3/8 in. maple doweling projecting out radially, on the end of which the hose is fastened. (See Fig. 2.) The water is forced up the hose by means of a gear pump, the maximum pressure used being about 80 lbs. per sq. in. and the maximum flow being about 1.5 gal. per min. After reaching the top of the helix and passing the wattmeter connection, the water flows through about seven feet more hose and is then discharged to the ground in a spray, the spray completely breaking the circuit.

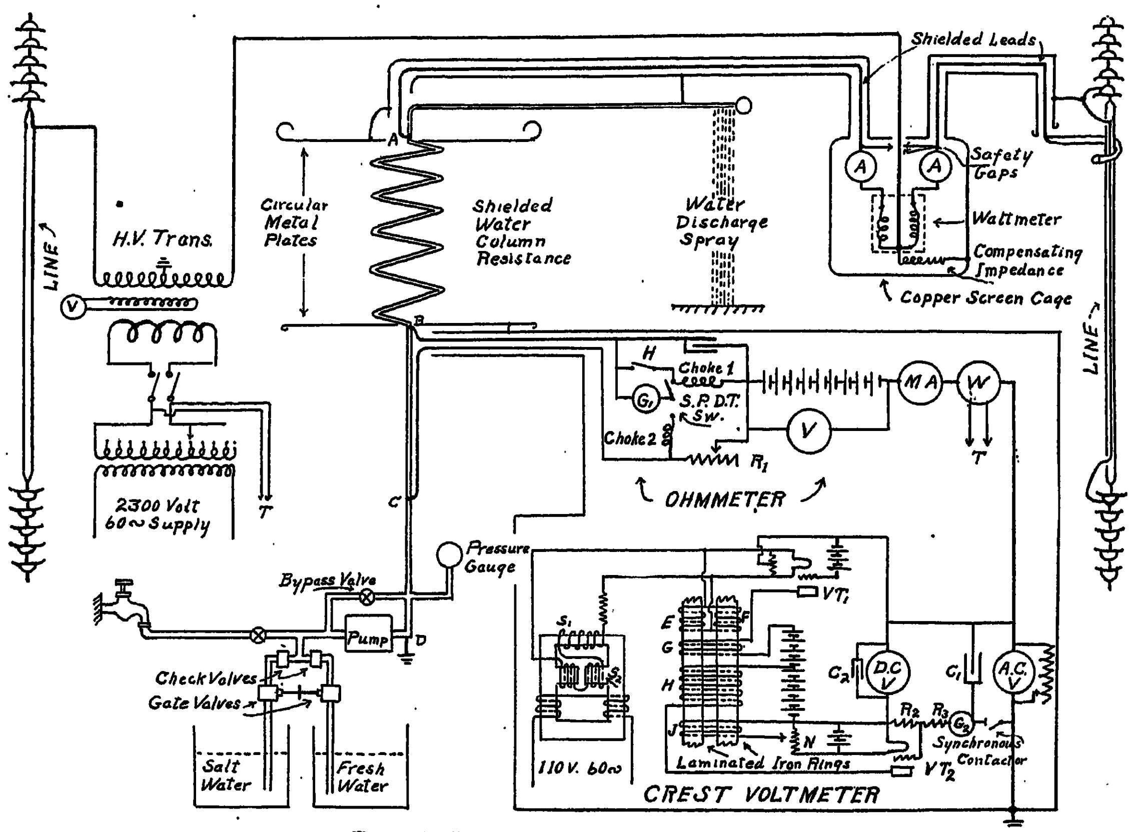

FIG. 1—WATTMETER CIRCUIT ASSEMBLY

1. Elwell Fellow in Electrical Engineering, Stanford University, Palo Alto, Calif.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., September 15-19, 1925.

When the current flows through the water resistance the water, of course, warms up, the heating being accumulative as the water moves up the hose. The effect of this is a somewhat complex change in resistance which in turn makes the voltage gradient down the column differ from a straight line function. This is a condition that must be controlled before correct electrostatic shielding can be accomplished. First of all a means must be had for keeping the voltage gradient down the column constant at all voltages. The next problem is then to match the potential of the water column at every point by the external field between the two plates.

Before going further, let us solve the problem of a current flowing through this column of water that is being constantly supplied at the lower end with cool water and wasting the warm water at the top. The resistance-temperature coefficient of ordinary tap water varies considerably for different temperatures. The

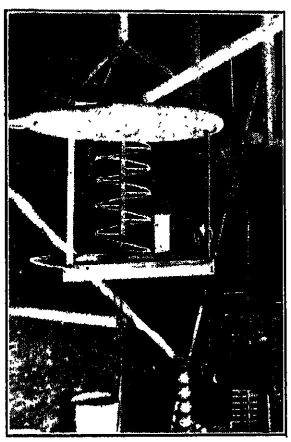

FIG. 2—THE SHIELDED WATER-COLUMN RESISTANCE USED AS THE WATTMETER MULTIPLIER

following has been found to be approximately the relation of resistance to temperature between zero and 100 deg. cent.[2]:

$$R_t = \frac{40\,R_{20}}{20 + t}$$

where R_t is the resistance at any temperature t (deg. cent.) between 0 deg. and 100 deg. and R_{20} is the resistance at 20 deg. cent.

Beginning at the ground end, the rise in temperature $d\,t$ in a differential length $d\,l$ of the column is as follows:

$$d\,t = \frac{I^2\,d\,r\,\dfrac{d\,l}{V}}{4.184\,A\,d\,l} = \frac{I^2\,d\,r}{4.184\,A\,V} \qquad (1)$$

where I = current in amperes through the w. c.

$d\,r$ = resistance of a length $d\,l$.

V = velocity of water through the column in cm. per sec.

A = area of column in sq. cms.

4.184 = mechanical equivalent of heat.

Integrating the above equation the actual temperature of this differential length is

$$t = \frac{I^2\,r}{4.184\,A\,V} + T \qquad (2)$$

T being the temperature of the water entering the column.

$$d\,r = R_t\,\frac{d\,l}{A} \qquad (3)$$

R_t specific resistance of the water at temperature t.

$$R_t = \frac{40\,R_{20}}{20 + t} \qquad (4)$$

R_{20} specific resistance at 20 deg. cent.

Substituting (4) in (3),

$$d\,r = \frac{40\,R_{20}\,d\,l}{(20 + t)\,A} \qquad (5)$$

Substituting (2) in (5),

$$d\,r = \frac{40\,R_{20}\,d\,l}{\left(20 + \dfrac{I^2\,R}{4.184\,A\,V} + A\right)A} \qquad (6)$$

Simplifying,

$$(83.68\,A\,V + 4.184\,A\,V\,T + I^2\,r)\,d\,r = 167.4\,V\,R_{20}\,d\,l \qquad (7)$$

Integrating,

$$\left(\frac{I^2\,A}{2}\right) r^2 + A\,V\,(83.7 + 4.184\,T)\,r - 167.4\,V\,R_{20}\,l = 0 \qquad (8)$$

Solving for r,

$$r = \frac{-\,A\,V\,(83.7 + 4.184\,T) \pm \sqrt{A^2\,V^2\,(83.7 + 4.184\,T)^2 + 335\,V\,R_{20}\,I^2\,l}}{I^2} \qquad (9)$$

This gives the resistance for any length equation (1) when the following are known: the area of the column, velocity of the water, specific resistance of the water at 20 deg. cent., the initial temperature of the water, and the current through the column.

Equation (9) shows something that was practically self-evident, that is, as the current I changes, it is possible to keep the resistance all along the column constant by changing V, the velocity of the water. As can readily be seen, the velocity must vary as the square of the current—as would be expected.

If the maximum effective value of the current to be used is fixed and the allowable temperature rise (if a glass tube were used instead of rubber this temperature could be 100 deg. cent.) of the water decided upon, then the distribution of voltage down the column can be computed. When this is done the pitch

2. From tests by Applequest and McKenny, M. I. T., 1912. *Pender Handbook*, (edt. 1922), p. 1356.

of the helix can be changed in such a manner that the potential of the water column at all points is the same as the space it occupies between the two plates—under these conditions the shielding is ideal. In order for the voltage gradient of the column to remain constant it is only necessary to keep the temperature of the ingoing and outgoing water constant. The temperature of the ingoing water is easily adjusted. The temperature of the outgoing water is controlled by regulating the velocity. To tell when the temperature was correct a thermometer could be placed in contact with the water at the discharge end and read by means of a telescope. However, a more direct method than this was employed. A wattmeter was connected in the ground circuit of the water column as shown in the diagram of connections. One coil carries the current that passes through the water resistance and the other coil of this wattmeter with suitable resistance is connected across the low voltage primary of the high voltage transformer. The wattmeter reading will be

FIG. 3—VIEW OF HIGH-VOLTAGE WATTMETER AND AUXILIARY EQUIPMENT

At center on right is insulated screen cage enclosing wattmeter instrument and milliammeters; at the top on right is shielded water column resistance; in center at bottom is hydraulic apparatus, and at left is cage containing crest voltmeter and ohmmeter

an indication of the amount of power absorbed by the water column. Thus, for example, if the conductivity of the supply water remains constant and the line voltage is doubled, the reading of this wattmeter will be increased four times which means that to keep the temperature constant there must be four times the quantity of water flowing through the hose. This, of course, will require a certain increase in water pressure. By knowing the constants of this wattmeter and the water flow in terms of the pressure-gage reading, a scale was made for the wattmeter so that instead of reading watts the wattmeter indicates the pressure necessary to keep the temperature of the outgoing water constant. This works out very well under actual test; with the conductivity of the supply water constant the resistance of the water column was determined at various voltages and found to be practically constant. This, of course, means that the voltage gradient down the column remains constant. Assuming that the temperature resistance equation is not altered by the introduction of salt into the water, the value of R_{20} can be changed, thereby uniformly increasing r throughout the length of the column. This will, of course, increase I; hence V must be correspondingly increased. This will not change the voltage distribution down the column and therefore will not alter the shielding. In other words, the resistance of this resistor can be uniformly changed throughout its length without changing its dimensions.

The necessity of the exact matching of the field established by the water column with that between the two plates has been found by integrity tests. On account of the relatively large charging current through the line coil of the wattmeter, it requires only an extremely small capacitance current through the other coil to produce a fair sized deflection, which of course is error-reading. Any field picked up or supplied by the water column will cause such an error. To test for correct shielding, a reading of the line wattmeter is taken at a certain water conductivity and line voltage; then with the line voltage kept constant, water of, say, twice the conductivity is used, which should double the original wattmeter reading if the shielding is correct. The only thing that is changed during this test is the conductivity; the physical dimensions of the circuit and the voltage gradient down the column remain unaffected. If the wattmeter reading is not proportional to the current in the potential circuit at constant line voltage, there is error-reading due to improper shielding. However, as discussed in a previous paper on the high voltage wattmeter,[3] an error-reading can also be the result of improper shielding of other parts of the circuit besides the water column. After the present water column was designed and built the above test was applied and the shielding was found to be correct within the limits of observations. Besides the above test, another obvious advantage of being able to change the resistance of this wattmeter multiplier is to greatly increase the wattmeter readings at the lower voltages.

The change in the resistance of the water column is accomplished by the introduction of common salt into the water. The water supplying the pump is drawn from two tanks, one fresh and the other a salt solution. The valves on each line from these tanks are connected together in such a manner that as one opens the other closes. When the valves are once set the mixtures remain constant to a surprising degree of steadiness. The gear pump no doubt aids considerably in the mixing process; also, the bypass valve is never completely closed when the salt solution is being used. The hydraulic system of controls must be such that the pressure, and hence, the flow, can be changed without affecting to any great extent the conductivity. One thing necessary in order to do this is to maintain the level of the water in the two supply tanks constant and

3. "Power Measurements at High Voltages and Low Power Factors," by Joseph S. Carroll, Thomas F. Peterson, and George R. Stray, JOURNAL A. I. E. E., p. 941, Oct. 1924.

equal at all times. Also the change in conductivity should not alter the flow; however, as the conductivity is changed the flow must be controlled to keep the temperature constant.

The Ohmmeter

The resistance of this wattmeter multiplier is determined in the following manner (see diagram of connections) while the high voltage is on: A storage "B" battery of about 100 volts is inserted in the ground end of the water column. This battery forces a direct current through the galvanometer G_1, the choke 1, up through the water column, through the secondary winding of the high voltage transformer, to ground and back to the other side of the battery. This direct current goes only through the galvanometer whereas the alternating current is allowed to pass only through the 10 m. f. condenser in parallel with the galvanometer and choke. Knowing the voltage of the battery and the calibration of the galvanometer, the resistance of the water column can be determined. Of course, the resistance of the transformer winding and choke must be subtracted and the drop across the condenser be corrected for if necessary. Knowing the resistance of the water column and the effective value of current through it, the total effective line voltage can be computed. The values of voltage obtained in this manner check on an average within 0.5 per cent of the voltage as measured by the voltmeter coil within the transformer. This difference at present is about the limit of our accuracy.

To obviate the necessity of insulating the apparatus at the ground end of the water column in order that the current through the galvanometer shall have only one path to ground, a connection is made from the battery through a resistance R_1 and to a point C on the lower end of the water column. The current in this circuit flows from C to the ground at D. The resistance of R_1 is made such a value that the drop across it is the same as the drop across the galvanometer and choke. This puts the point B at the same d-c. potential as C so that there is no current flowing between B and C and the only current through the galvanometer is that through the water column. To test for the equality of potential of B and C, a single-pole double-throw switch is connected in the circuit so that the galvanometer can be connected between these two points with a choke 2 in series. The resistance R_1 is then adjusted until there is no current through the galvanometer and then the switch is thrown back to connect the galvanometer in its normal position. The switch H is, of course, closed during this operation of balancing. This balance when once made remains the same throughout the test; the ratio of the resistance between C and D, and A and B is the same for all conductivities. The resistance of the water between B and C is never less than 50,000 ohms and for the higher voltages it is 500,000 ohms. The resistance from C to D is about half that between B and C and is sufficient to limit the current drawn from the battery to a reasonably small value. Since the resistance of the galvanometer G_1 is only 14 ohms and that of the choke 1 is about 7500 ohms, no correction is necessary when the galvanometer is shifted from one place to the other. The a-c. drop across the condenser is 16 volts with a current of 0.060 amperes at 60 cycles; however the secondary of an audio transformer keeps the galvanometer free from vibration due to this voltage.

The Crest Voltmeter

The crest values of voltage are determined by multiplying the effective values by the crest factor. This crest factor is determined as follows: The current through the water column has the same wave-form as the line voltage so that an ordinary voltmeter inserted in the ground end of the water column will give a replica of the total line voltage. By means of a synchronous driven contactor, a condenser is charged with the crest value of this effective voltage, the ratio of these two being the crest factor. The voltage of this condenser could be determined directly by means of an electrostatic voltmeter; however the time of taking readings does not permit the use of such slow acting instruments. In place of such a voltmeter, a practically instantaneous self-balancing potentiometer was used in which the voltage of the condenser is read by means of the ordinary quick acting d-c. voltmeter. Before the operation of this potentiometer is taken up, a brief description of the different parts will be given.

The current through the d-c. voltmeter is furnished by a storage "B" battery of about 135 volts. This current also passes through the plate circuit of a 201A vacuum tube, $V\,T_1$ (see diagram), and through the winding G on an iron core choke. The filament current of $V\,T_1$ is a-c. and is furnished by a specially designed constant current transformer. In parallel with this filament are the windings E and F of a choke made up of laminated iron rings; the winding E is around half of the rings and F is around the other half. The windings G, H and J are around all of the rings. The coils E and F are connected in parallel in such a manner that the flux set up by the current through them is in opposite directions and hence there is no voltage induced in the other windings. There is a positive bias put on the grid of $V\,T_1$ to reduce its filament to plate resistance. $V\,T_2$ is also a 201A tube and might be called the detector tube since it detects any difference between the potential of the condenser C_1 and the voltage at the terminals of the d-c. voltmeter. The filament of this tube is lighted with a four-volt storage battery. Sixty-five volts of the storage "B" battery are used on the plate of this tube; in this plate, circuit is the coil H consisting of 10,000 turns of No. 36 B. & S. d. s. c. copper wire. There are 250 turns in coil J which are used to produce a field almost equal and opposite to that of H.

With this description of the apparatus, the operation will now be taken through step by step. The alternating current through the water column resistance on its way to ground passes through the a-c. voltmeter and its non-inductive shunt—the shunt being necessary on account of the current carrying capacity of the voltmeter. The effective value of the voltage across the voltmeter is kept about 50 volts on the 75-volt scale by means of the shunt resistance; however, this is only for the convenience of reading and the operation over a range from 35 to 70 volts gives accurate results. The crest of this voltage is picked off by the synchronous contactor and charges the condenser C_1. If to begin with, this potential is not the same as that across the d-c. voltmeter there will be a current through the galvanometer G_2; in this case the current through the bias winding J is changed by means of the potentiometer N until there is no current through G_2. This is the only adjustment necessary and when once set it requires but very little changing. With the deflection of G_2 zero the d-c. voltmeter gives the potential of the condenser C_1. This reading divided by the reading of the a-c. voltmeter, is of course, the crest factor. Now, suppose that the a-c. voltage decreases; this will lessen proportionately the potential of C_1 and the d-c. voltmeter will momentarily be the same as before; a current then will flow through G_2, R_3 and R_2 for a very short time making the grid of $V\,T_2$ positive with respect to the filament. This will increase the plate current which is also the current through the coil H; this increases the saturation of the iron core and reduces the reactance of the windings E and F. Since these windings carry about 80 per cent of the constant current from the filament transformer, the filament current of $V\,T_1$ will be reduced. This increases the filament to plate resistance which will in turn diminish the current through the d-c. voltmeter and bring down the voltage across it until it is the same as the potential of C_1. In case the a-c. voltage rises, just the reverse of these operations will take place. The coil G in series with the d-c. voltmeter might be called a tickler winding; the number and direction of turns are such that the change in the current through the voltmeter produces just sufficient change in the saturation of the iron to keep the voltmeter where it is put without maintaining extra current through the coil H by a difference of potential between C_1 and the d-c. voltmeter. A very good test of this is to disconnect C_1 from the contactor while it is charged; the needle of the d-c. voltmeter stands practically still. If part of the charge is allowed to leak off the condenser, the voltmeter will immediately drop to that potential and remain there when the leak is taken off—thus demonstrating the minute amount of current necessary through R_2 to operate the system. One thing that aids considerably in the sensitiveness of this apparatus is the fact that the vacuum tube $V\,T_1$ is operated where the plate current is most sensitive to a change in filament current; for example, a change in filament current from 200 milliamperes to 215 milliamperes will change the plate current from about 3 milliamperes to 6 milliamperes with the 16,000 ohms of the voltmeter in series.

The condenser C_2 of 1 μ. f. is necessary in order to smooth out a small component of a-c. on account of using 60-cycle current to light the filament of $V\,T_1$. These ripples produced an a-c. potential between the grid and filament of $V\,T_2$ which was rectified leaving the grid negative and thus causing trouble. The resistance R_3 was found necessary to damp out very low frequency oscillations or hunting between the two condensers C_1 and C_2, caused mainly on account of the time lag in the change in temperature of the filament of $V\,T_1$.

The transformer furnishing the current for $V\,T_1$ was designed to take care of any reasonable fluctuation in line voltage. Since $V\,T_1$ is sensitive to slight changes in filament current, any change in line voltage would have to be adjusted for by $V\,T_2$ which is not the primary duty of this tube. This transformer furnishes practically a constant current in the secondary circuit with a 10 per cent change of primary voltage. One part of the secondary S_1 is wound on a practically saturated iron core. In series with this winding and 180 deg. opposite in phase is a winding around an air-gap magnetically in parallel with the saturated core of the other winding. The winding S_2 gives about half the voltage of S_1. With a change in primary voltage the flux across the air-gap changes at about twice the rate as that through the iron core; in this way the two changes just offset each other. The adjustment of the two secondary windings is made in an over-all manner by means of a milliammeter in the plate circuit of $V\,T_1$. Of course the secondary current of this transformer is far from a sine wave but this is in no way a handicap.

The crest voltmeter circuit may seem elaborate; however, it has been found worth while in order to have an electrostatic instrument with the advantages of the modern highly damped sensitive d-c. voltmeter. The operation of this crest voltmeter is simple and reliable and to further improve its simplicity, it is hoped that sometime the synchronous contactor can be replaced by a special vacuum tube.

Another feature that has been added to the equipment is a 230-ft. line of hollow copper conductor. With this conductor it is possible to electrically cut out both strings of insulators as shown in the diagram and hence measure only the losses from the conductor. By changing the connections, the insulator losses may be included with the loss from the conductor or else they may be measured separately. The insulators are cut out by supplying the power to them directly and not through the wattmeter. The power is supplied to the string at the far end by running an insulated wire through the hollow conductor and connecting it back of the first insulator.

The work this year has been largely development

work; however, several tests were made on the three 230-ft. lines in the rain and also in dry weather. The results of the tests substantiate very well the work done with the wattmeter last year. During the rain tests, the fact was again emphasized that a rain test does not mean much unless the rate of rainfall is determined because the power loss varies considerably with the rate of rainfall. Next year it is planned to have a rainfall indicator and to find the relation between the rate of rainfall and the power loss from the different conductors and insulators.

Encouraged by the success of the present wattmeter, the design of the million-volt wattmeter has been begun. The crest voltmeter and ohmmeter in their present form can be used with but very little change. The main problem will be the fifteen-million-ohm, shielded multiplier.

ACKNOWLEDGMENTS

The present high voltage wattmeter development was made possible through a grant of two hundred and fifty dollars made for the purpose by the National Academy of Sciences from the Joseph Henry Fund.

The cooperation of the Anaconda Copper Co. and the Southern California Edison Co., in furnishing free of charge 500 ft. of 1.125-in. outside diameter, flexible tubular-center stranded copper conductor, made possible the study of the complete segregation of insulator and corona losses from heavy line conductors.

The author wishes to express his appreciation for the assistance given him by members of the graduate class of the Electrical Engineering Department in taking observations and also in some of the actual construction work. Through the work of C. V. Litton and P. F. Schofield the solution of the pitch of the water column by an approximate method was replaced by the exact solution as given in this paper.

Discussion

R. W. Sorensen: When I first saw the diagram of connections shown in this paper I was awed by the apparent amount of large apparatus necessary to construct such a wattmeter. However, I have had the opportunity to stop at Professor Ryan's laboratory and become acquainted with the equipment described in this paper. An acquaintance with the equipment eliminated the impression of bigness which I had received from the diagram and which was perhaps due partly to the high voltages which the equipment will measure. In place of this impression I received a definite picture of the cleverness exhibited by Mr. Carroll and Professor Ryan in handling this problem. For example, the transformer of three windings and a split core, which seems in the diagram to be so large, is actually very small; in fact, it can easily be held in one hand.

The water column, about 16 ft. long, is so arranged between the shielding plates at the ends as to occupy a space approximately 3 ft. in height. The small transformer and the indicating instruments are all mounted on a small table surrounded by a wire cage the dimensions of which approximate 4 ft. high, 6 or 8 ft. wide and perhaps 12 ft. long.

With this knowledge, I found it much easier to read the paper and appreciate what has been done in Professor Ryan's laboratory toward producing practical wattmeters and voltmeters for high-potential measurements.

H. V. Carpenter: Mr. Carroll mentions the integrity tests by which he established the fact that he was able to read with accuracy loads of a watt or two, with a voltage of 150,000. It seems to me the condition is so critical there that a word in relation to his method of establishing the integrity would be interesting. Also regarding the formula given for the resistance of the water column, I would like to ask whether he established it over a wide range of densities and for any materials except common salt.

J. S. Carroll: Ordinarily the man in a measurements laboratory thinks of errors in measurements as a few tenths of a per cent. However, in this case where we are measuring one watt of power at 150,000 volts and the apparent power is of the order of 6000 watts, I frankly admit that we are very well satisfied with an accuracy within 25 per cent of true values. As the load increases the accuracy greatly increases so that at 40 watts we expect the error to be not over two or three per cent. One of the integrity tests used is described in the present paper; that is the double-conductivity test. Another test is described in the Oct. 1924, A. I. E. E. JOURNAL in the paper on Power Measurements at High Voltage and Low Power Factor, by Carroll, Peterson, and Stray. In this test a shielded resistance was inserted in the connection to the line; the value of this resistance was known as well as the line charging current through it from which the power absorbed by it was computed; this increase in power was also measured by the wattmeter. The agreement between the results of the two determinations was very satisfactory. The double-conductivity test was also used in connection with the above test.

In regard to Professor Carpenter's second question, I might say that we have so far tried only a common salt solution and have not gone farther than checking the formula given in the paper in an overall way for the purpose o finding any serious error. We measured the cold resistance of a solution and then calculated what the hot resistance of the water column should be; this result agreed very well with the value obtained under actual operating conditions. On some of these things we wish to make a closer follow-up as soon as we have time.

On the Nature of Corona Loss

BY CLARENCE T. HESSELMEYER[1]
Non-Member

and

JAROSLAW K. KOSTKO[2]
Associate, A. I. E. E.

INTRODUCTION

If a high voltage is applied between two electrodes, experience shows that in any given case there is a limiting voltage below which no loss occurs; a permanent current flows only if the voltage is an alternating one and is then a purely charging current. Above this voltage loss appears, accompanied by the familiar phenomenon of corona; a permanent current flows even in the case of continuous potential, showing that the air space between the electrodes becomes a conducting part of the circuit.

If E is the applied voltage and Q the quantity of

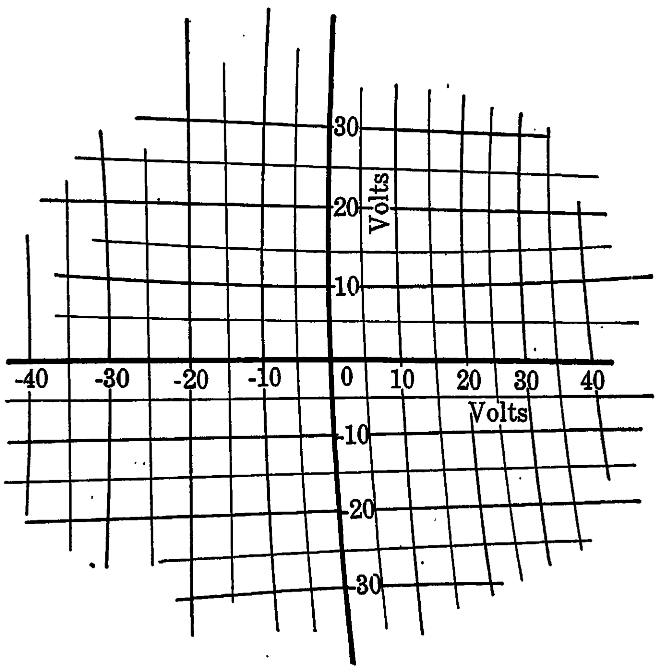

FIG. 1

electricity which has passed in the circuit, then with alternating potentials the relation between E and Q is represented in rectangular coordinates by a closed curve whose area is a measure of energy per cycle. The $E-Q$ relation is therefore of prime importance in the study of corona loss.

A number of such $E-Q$ diagrams of corona loss have been taken, and the results are presented in the first part of the paper. They lead to certain conclusions regarding the nature of corona loss discussed in the second part of the paper.

PART I

The $E-Q$ diagrams were obtained by means of the cathode-ray cyclograph. In this instrument a stream of electrons emitted by a hot filament of a vacuum tube is directed along the axis of the tube by means of a strong electric field. The inside of the opposite end of the tube is covered with fluorescent material which becomes luminous at the spot where the electrons strike it. Along its path, this stream passes between two consecutive pairs of parallel metal plates arranged at right angles to one another.

If the relation of two variables is desired, voltages proportional to these variables at every instant are applied between the corresponding pairs of plates. The electric fields set up by these voltages deflect the ray and cause it to describe a luminous curve on the fluorescent screen. For further detail, the reader is referred to an article by J. B. Johnson, *Bell System Tech. Jour.*, Vol. I, p. 142, Nov., 1922.

On account of the weakness and non-actinic qualities of the light produced by the luminous spot, it was necessary to make all observations visually and record them by tracing on a piece of tracing cloth held against the end

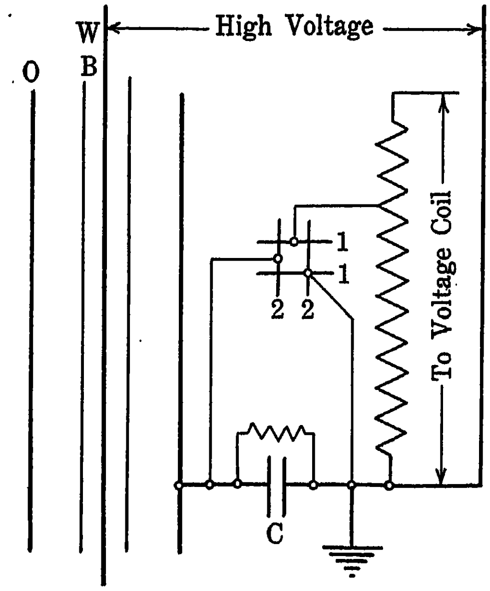

FIG. 2

of the tube by means of a special fixture. In order to calibrate the tube, two sets of readings were made using alternating voltages in one pair of deflectors and known values of direct current voltage in the other pair. From this test a set of coordinates shown in Fig. 1 was plotted, by means of which any $E-Q$ card can be corrected for distortion and replotted in rectangular coordinates. All areas, whether replotted or not, were corrected for distortion.

A typical connection diagram is shown in Fig. 2; W and O are the electrodes (in this case a wire and a cylinder); B is an electrically isolated cylinder the pur-

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pose of which will be explained later. 1—1 are deflectors connected across an adjustable portion of a resistance placed across the voltage coil of the high voltage transformer. These deflectors give a deflection proportional to the applied voltage, E. Deflections proportional to Q are obtained by connecting the other pair of deflectors,

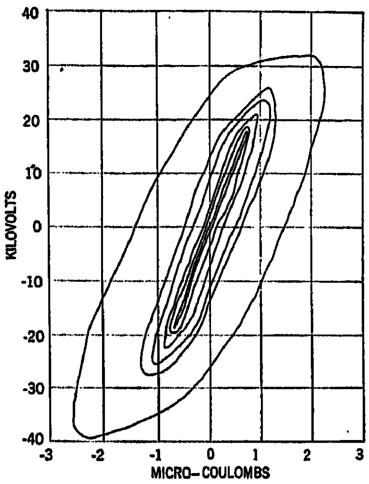

FIG. 3—BARRIER EXPERIMENT—60 CYCLES
0.034-in. diam. wire in 16-in. diam cylinder.
No barrier.

2—2, across a large capacitance C, connected in series with the outer cylinder. These deflectors are also shunted by a resistance of the order of a megohm. Without this precaution the plate not connected to the

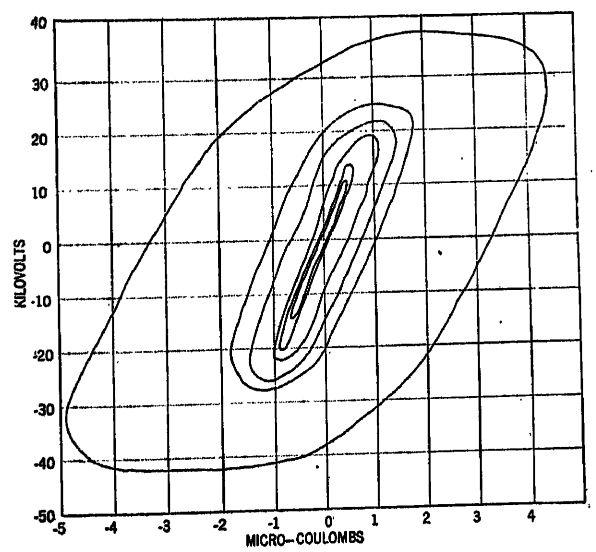

FIG. 4—BARRIER EXPERIMENT—10.5 CYCLES
0.034-in. diam. wire in 16-in. diam. cylinder.
No barrier.

anode would collect a charge and the spot would drift. Since the action of corona over positive and negative crests is unsymmetrical, there is a tendency for the capacitance, C, to accumulate a unidirectional charge, which is also drained off by this shunting resistance. The diagram itself gives the voltage across this resistance so that the current in it can be calculated and the diagram corrected for it. The effect of this resistance was found to be negligibly small.

Most of the tests were made with the classical arrangement of a wire along the axis of a cylinder. The connection between corona loss and the conduction of electric charges through the air space was studied by placing in the path of the charges, insulated barriers, B (Fig. 2), in the form of concentric cylinders of different diameters. The walls of these barriers were always thin so as not to modify the original field, and it was then of no consequence whether they were made of conducting or insulating materials.

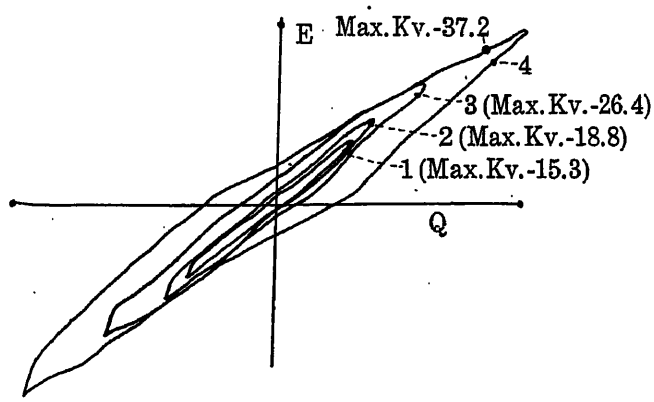

FIG. 5—BARRIER EXPERIMENT—60 CYCLES
0.034-in. diam. wire and 16-in. (mesh) cylinder
Barrier metal cylinder 1¼ in. in diam.

In all cases the wire was of copper 0.034 in. in diameter, and the outer cylinder O was formed of ½-in. mesh wire screen 16 in. in diameter. Figs. 3, 4, 5, 6 and 7 show some of the diagrams obtained with this wire and cylinder arrangement.

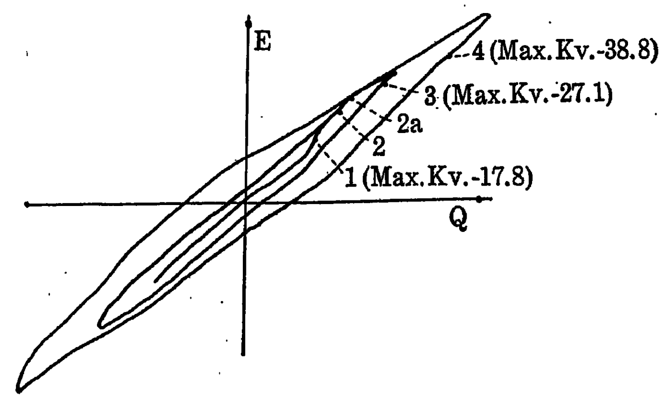

FIG. 6—BARRIER EXPERIMENT—60 CYCLES
0.034-in. diam. wire in 16-in. diam. (mesh) cylinder.
Barrier glass cylinder 1¼ in. diam.

In Figs. 3 and 4, the original cards (taken with no barrier at 60 and 10.5 cycles respectively) were replotted in order to reduce them to a common scale.

Figs. 5, 6 and 7, taken at 60 cycles and all at the same scale, are reproduced without any change; barriers

were of 1 ⅛ in. diameter metal tube, 1⅛ in. diameter glass tube, and 11 in. diameter metal tube, respectively.

Fig. 8 shows $E-Q$ areas (in joules) for the same maximum voltage (28.5 kv.) as a function of barrier diameters.

Fig. 9 is the reproduction of a card taken at 60 cycles on a 230-ft., single-phase line composed of No. 20 copper conductors spaced 36 in. apart. Figs. 10, 11 and

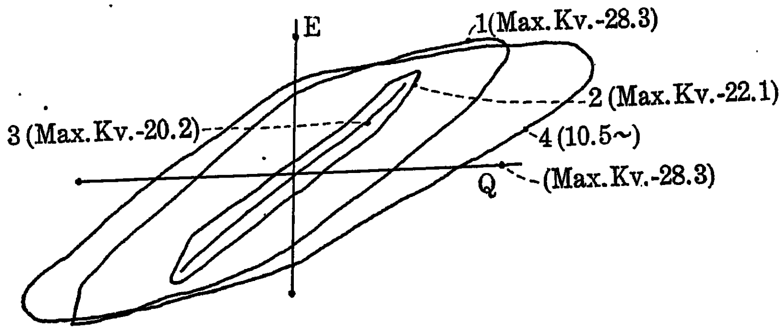

FIG. 7—BARRIER EXPERIMENT—60 CYCLES
0.034-in. diam. wire in 16-in. diam. (mesh) cylinder.
Barrier metal tube 11 in. diam.

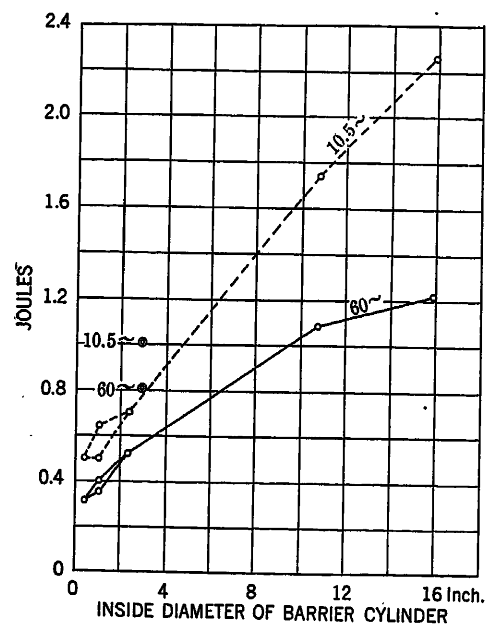

FIG. 8—E-Q CURVES AS A FUNCTION OF BARRIER DIAMETERS
0.034-in. diam. wire in 16-in. diam. (mesh) cylinder.
Maximum kv. = 28.5

12 show cards taken on the same line at 60, 120 and 10.5 cycles respectively, replotted to the same scale.

PART II

The general characteristics of the $E-Q$ curves suggest the following explanation of corona loss:[3]

When the electric field acting on a free electron exceeds a certain critial value, the velocity acquired by the electron becomes so great that when it collides with an atom of the air it dislodges other electrons from this atom; that is, it ionizes the air; and if this field is exceeded throughout a certain definite minimum distance from the conductor (critical corona striking distance), visible corona appears.

The liberated ions of opposite sign to that of the wire move towards the wire and can be assumed to reach it

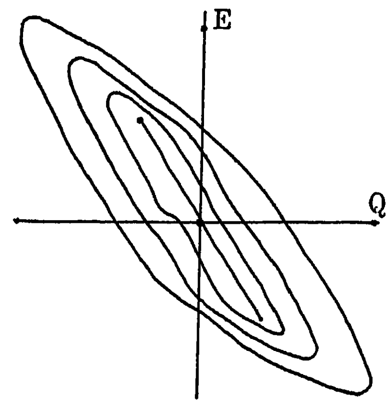

FIG. 9—OUTDOOR LINE—60 CYCLES
No. 20 copper, spaced 36 in., 230 ft. long.

instantly, the zone of active ionization surrounding the wire being always very narrow; the ions of the same sign as the wire move away from it, towards the outer cylinder. To move these ions, energy must be expended by the source of e. m. f. creating the field. *Corona loss is this energy*, regardless of the place where it is expended, near the surface of the wire, or on the boundary of the

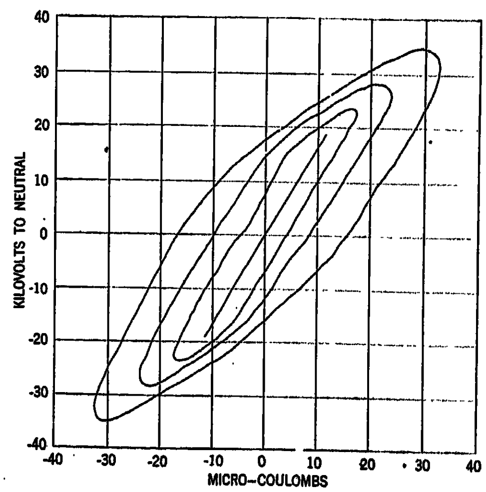

FIG. 10—OUTDOOR LINE—60 CYCLES
No. 20 copper, spaced 36 in., 230 ft. long.

zone of active ionization, or midway between the electrodes; the manifestations of corona perceptible to our senses—light, sound, heat—are caused simply by further transformations of the kinetic energy imparted to the ions by the electric field, and depend on the place

3. See paper entitled "The Hysteresis Character of Corona Formation" by Prof. Ryan and Prof. Henline, JOURNAL A. I. E. E., p. 825, Sept. 1924.

and the mode of these transformations; within the zone of ionization, the energy is mostly spent in the invisible process of ionization by collison; if the impact of an ion against an atom of the air is strong enough to disturb the equilibrium between the nucleus and an electron of the atom, but not sufficiently strong to separate them permanently, the electron returns to its original position by a sort of vibratory motion, giving up energy acquired from the colliding ion in the form of light. Outside of the ionizing zone, the path of an ion is marked by a series of collisions with the atoms of the air; at each collision a part or the whole of the kinetic energy of the ion is transferred to the atom; if the ion strikes a solid electrode, its kinetic energy is converted into heat. If the applied e. m. f. is alternating, the ions in the air space move as alternately positive and negative waves; an outgoing wave meets a return wave of opposite sign; in the ensuing process of recombination, the energy of the ions is radiated in the form of light.

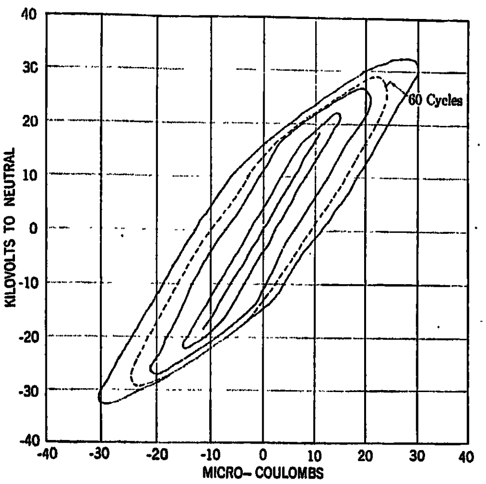

Fig. 11—Outdoor Line—120 Cycles
No. 20 coppor, spaced 36 in., 230 ft. long.

For simplicity, it will be assumed that positive and negative corona phenomena are identical.

In the corona caused by a continuous potential, the unidirectional flow of ions across the air space between the electrodes constitutes a true current in the circuit containing the applied e. m. f. The power expended in corona originates in the cooperation of this corona current with the applied e. m. f. In the corona excited by an alternating potential, a wave of ions may not cross the distance between the electrodes before the reversal of the field; in this case the to-and-fro motion of the waves of ions in the air space sets up a motion of electric charges induced by electrostatic induction in the circuit connecting the electrodes and containing the applied e. m. f.; the action of this e. m. f. on these induced charges corresponds to the corona energy loss.

If a barrier is placed infinitely close to the outer cylinder, we have an exact equivalent of the system of a wire in a cylinder, without any barrier; if there is no barrier, an ionic charge crossing the air space and arriving at the outer cylinder will neutralize an equal and opposite charge on this cylinder; if a barrier cylinder, infinitely close to the outer cylinder, is present, these two charges will remain separate but infinitely close to one another; in both cases their combined external action will be zero. For the sake of generality, it is convenient to assume that a barrier cylinder is always present.

As before mentioned, the action of corona is to release into the space a charge of ions; an equal charge of opposite sign goes to the wire. By analogy with the vacuum tube terminology, the former charge will be called "space charge;" at any given moment it may be located entirely in the air, or entirely on the barrier, or be distributed in the air and on the barrier. Let $-Q$ be the charge on the outer cylinder and q = charge on the wire; the total charge of the system being zero, the space charge x is given by the relation

$$x + q - Q = 0$$

hence $x = Q - q$. Let E be the e. m. f. applied between the wire and the outer cylinder; with the barrier cylinder placed between the wire and outer cylinder, the charge $-Q$ can reach the latter only through the circuit containing E; hence this charge on the outer cylinder is the same as the charge Q in the E-Q diagram.

The amount of space charge is controlled mainly by two factors: the variation—in time—of the field near the wire, and the initial ionization (number of free ions at the moment when the field reaches the ionizing value); the field here means the resultant of the field due to the applied e. m. f. and the field set up by the space charge.

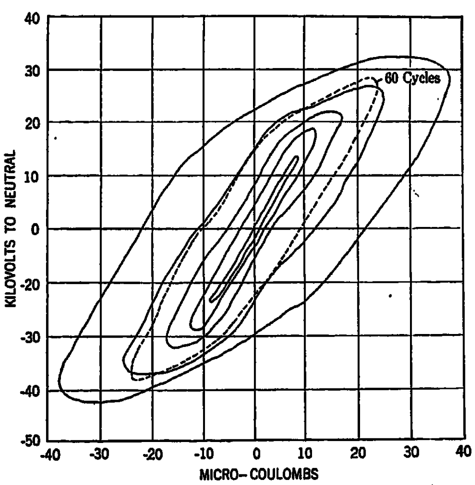

Fig. 12—Outdoor Line—10.5 Cycles
No. 20 copper, spaced 36 in., 230 ft. long.

It will be assumed[4] that, as a limiting influence, the second factor is negligible relative to the first, *i. e.*, that the antecedent ionization is sufficient to cause an instant and unlimited ionization by collision, unless checked by the drop of the field caused by the reaction of the space charge; in other words, as long as corona

4. See Footnote No. 3.

lasts, the action of the space charge is to maintain the same distribution of the field in the zone of active ionization as that which exists at the moment when corona starts for the first time. This hypothesis means that during the existence of corona, the charge on the wire must remain constant and equal to the charge which it has when corona first starts; if the initial critical corona voltage is E_0, and the capacity of the wire to the outer

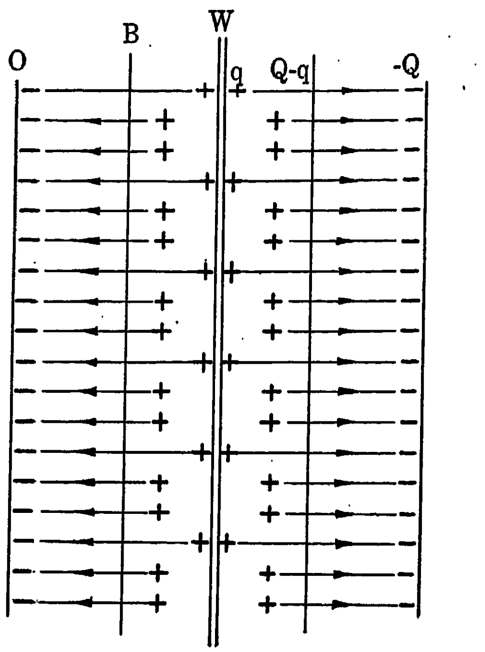

FIG. 13

cylinder is C, then this charge is $Q_0 = C\,E_0$; therefore,

$$q = Q_0 \qquad (1)$$

Fig. 13 shows the distribution of charges. The charge q on the wire, with an equal and opposite charge $-q$ on the outer cylinder, set up a potential difference q/C; the space charge $Q - q$ and an equal and opposite charge on the outer cylinder set up a potential difference E'; the sum of these potential differences must at all times be equal to the applied voltage E; therefore,

$$q/C + E' = E \qquad (2)$$

Equation (1) applies only during the portion of the cycle when corona exists; equation (2) holds good throughout the cycle.

For the purpose of experimental verification of the theory it is convenient to study the simple case when the distance between the wire and the barrier is so small that the interval of time required by the ions to reach the barrier is negligible relative to the period of the supply voltage. The entire space charge is then on the barrier; if the capacity of the condenser consisting of the barrier and the outer cylinder be denoted by C', the potential difference E' due to the space charge $Q - q$ is $(Q - q)/C'$, so that equation (2) becomes

$$q/C + (Q - q)/C' = E \qquad \textbf{(2a)}$$

The equation of the E-Q curve when corona exists is obtained by substituting $q = Q_0 = C\,E_0$ [equation (1)] into equation (**2a**); this gives

$$E = Q/C' + E_0\,(1 - C/C') \qquad (3)$$

This is a straight line, 1—1, Fig. 14, of slope $1/C'$, which passes through the point C of initial corona (*i. e.*, starting for the first time, without any space charge).

When the voltage reaches its maximum value at M and begins to decrease, corona immediately stops. For, beyond M, the continuation of corona would mean that in equation (2), q/C would remain constant by equation (1), while E' would increase on account of the increasing space charge, the net result being an increase, not a decrease, of the applied e. m. f. E.

After corona stops, the space charge remains constant, *i. e.*, $Q - q =$ (constant); the equation of the E—Q curve for this part of the cycle is obtained by substituting $q = Q -$ (constant) in equation (**2a**); this gives

$$E = Q/C - (1/C - 1/C') \times \text{(constant)}$$

This curve is a straight line $M\,N$ of slope $1/C$, *i. e.*, of the same slope as the precorona line $O\,C$ of the condenser formed by the wire and the outer cylinder.

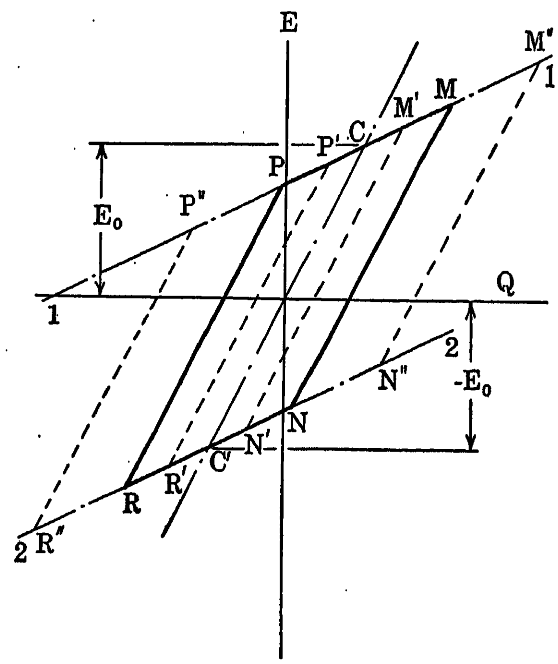

FIG. 14

By reason of symmetry, corona of the opposite sign will follow the straight line 2—2, parallel and symmetrical to the line 1—1. This line passes through the point C' symmetrical to C and corresponding to the critical voltage $-E_0$; but corona begins earlier, at N, because the space charge on the barrier now assists the applied voltage E in producing the ionizing gradient (field).

If the maximum of the voltage corresponds to the

points, M' or M'', the loops, $M'N'R'P'$ and $M''N''R''P''$, may have very different appearance.

Let E_m be the maximum value of the applied voltage; the area of the $E - Q$ loop is $4\,E_0\,(E_m - E_o)\,(C' - C)$.

The characteristic features of the loop of Fig. 14 are well in evidence in Figs. 5 and 6 where the diameter of the barrier cylinder is small enough to justify the assumption of the instantaneous transfer of ions from the wire to the barrier, yet large enough to minimize the effects of various irregularities such as eccentric location of the wire relative to the cylinder, vibration of the wire, dirt on its surface, etc.

The simplified diagram is independent from the frequency; it is also not affected by the wave shape of E so long as the latter has no multiple peaks.

If it is attempted to extend this construction to the case where the barrier coincides with the outer cylinder, the lines 1—1 and 2—2 become horizontal, and the loop extends to infinity. This absurd result is due to fact that so long as the charge on the wire is constant, the increment of the space charge is equal and opposite that of the outer cylinder, and, being infinitely close to the latter, completely neutralizes it so that there is nothing to prevent an unlimited amount of corona discharge. In this case the finite velocity of the space charge cannot be ignored.

Suppose, now, that the velocity of ions is not infinite; then a space charge may exist not only on the barrier but in the space between it and the wire. If the field were unmodified by the presence of the space charge it would be easy to predetermine the motion of an ion; the field at a distance x from the axis of the wire is

$$g_x = \frac{E}{x \log \frac{R}{r}}$$

where R and r are radii of outer cylinder and wire. Therefore, the velocity being proportional to the field, the distance x is determined from the equation

$$\frac{d\,x}{d\,t} = \frac{a\,E}{x \log \frac{R}{r}}$$

where a is a constant and E is a function of time t. This equation, integrated for the case where $E = E_m \cos \omega\, t$, gives

$$\frac{x^2}{2} - \frac{x_o^2}{2} = \frac{a}{\log \frac{R}{r}} \int_{t_0}^{t} E\,d\,t$$

$$= \frac{a\,E_m}{\omega \log \frac{R}{r}} \quad (\sin \omega\, t - \sin \omega\, t_0)$$

where $x_0 = x$ at the boundary of zone of ionization from which the ion starts and t_0 = time at which it starts. This relation is shown in Fig. 15 for a 0.034-in. diameter wire in a 11-in. diameter cylinder. The abscissas E_0/E_m give the ratios between the critical corona voltage E_0 and the maximum applied voltage E_m. The ordinates show the maximum travel of ions: the upper branch, for the ions discharged as soon as the voltage reaches the value E_0, increasing, and corona begins; the lower branch, for the ions discharged when the voltage passes the value E_0, decreasing, and the corona is on the point of stopping.

However, the study of the experimental $E-Q$ curves shows that the field is profoundly modified by the space charge; the analytical treatment of this case with alternating potentials is a difficult problem; but it is easy to explain all the peculiarities of the experimental curves, as affected by the voltage, frequency, etc. Since these peculiar features are of no importance except as a proof of the validity of the theory, only a few of them will be analyzed below.

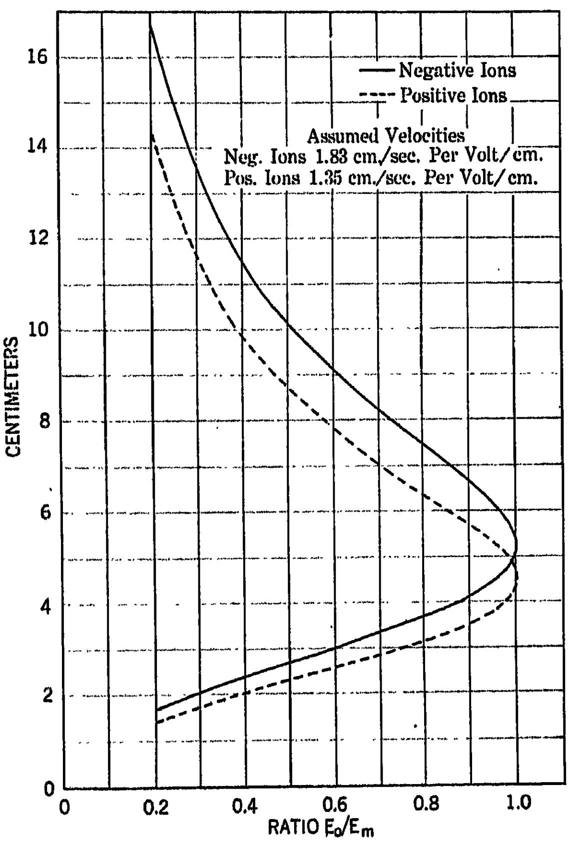

Fig. 15—Travel of Ions in a Radial Field

(0.034-in. diam. wire in an 11-in. diam. cylinder) E_o = critical corona voltage = 16.4 kv. at 60 cycles (maximum).

Experiment gives a loop such as shown in Fig. 16. Assume that at M, where the voltage is maximum, the polarities are as shown in Fig. 13. Corona stops at the point A, where Q passes through the maximum. For, if x denotes the space charge, we have $Q = x + q$; as long as corona exists, q is constant and x increases; therefore Q increases; as soon as corona stops, x remains constant and q decreases; therefore, Q decreases.

The applied voltage at A, where corona stops, is necessarily higher than the critical value E_0, because the action of the space charge is to oppose the field at

the wire due to the applied voltage. After corona stops, the space charge $x = Q - q$ remains constant, *i. e.*, $d\,Q = d\,q$, and the slope of the $E - Q$ curve results from equation (2):

$$\frac{d\,E}{d\,Q} = \frac{1}{C} + \frac{d\,E'}{d\,Q}$$

At first the potential difference E' decreases because the space charge approaches the outer cylinder. Be-

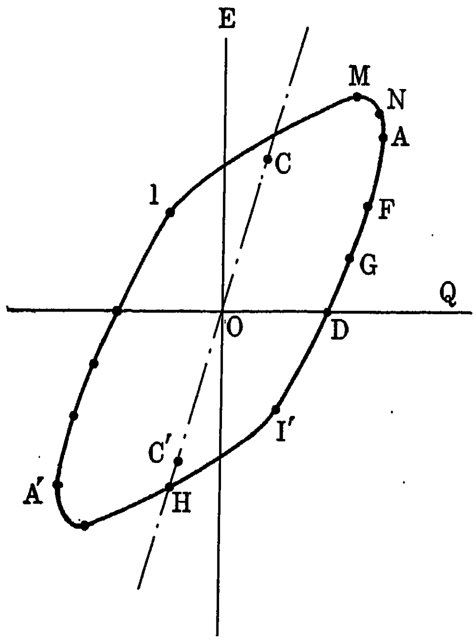

FIG. 16

yond M the decrease of E is small while that of E' is finite; there must be a compensating increase of Q; at A the decrease of E is just equal to that of E'; beyond A, E decreases faster than E', as shown by decreasing Q. At A, $q = Q_0$, and is positive; at D, where $E = O$, q is already negative, as otherwise the sum of q/C and E' could not be zero; therefore, there exists a point F where $q = O$; at this point the field near the wire reverses; with the increasing negative q the region of reversed field increases, gradually overtakes the ions of the space charge, and causes them to flow back to the wire; the rate of decrease of E' diminishes and soon E' begins to increase; since the space charge now cooperates with the applied e. m. f. in setting up the field near the wire, the ionizing gradient is soon reached and corona discharge begins at I' before E reaches the critical value E_0. At the point G, where E' ceases to decrease and begins to increase, $d\,E' = O$ and the slope is $1/C$; at this point the tangent is parallel to the precorona $E-Q$ line $O\,C$.

At I' the corona discharge suddenly releases into the air space a negative charge, while an equal positive charge goes to the wire; the negative ions meet and discharge the residual positive space charge so that the net result is as if this residual positive charge were suddenly transferred to the wire; the variation of E' caused by this sudden transfer must be balanced by an equally sudden variation of Q; this explains the more or less abrupt change of the slope of the $E-Q$ curve at I'. Beyond I' the new charge gradually discharges and supersedes in the air space the residual space charge, at A' the negative space charge is maximum; corona stops, as at A, and so on, throughout the cycle.

If the frequency is low, the ions of the space charge will have time to reach the barrier; at very low frequencies the amount of space charge in the air will be small relative to the charge accumulated on the barrier, and conditions of Fig. 14 will be approximated; at very high frequencies the ions will have no time to move from the wire beyond the corona-striking distance; the effect will be as if the barrier were close to the wire; the $E-Q$ loop will again be as in Fig. 14, but this time with a very small area. As at all frequencies the area

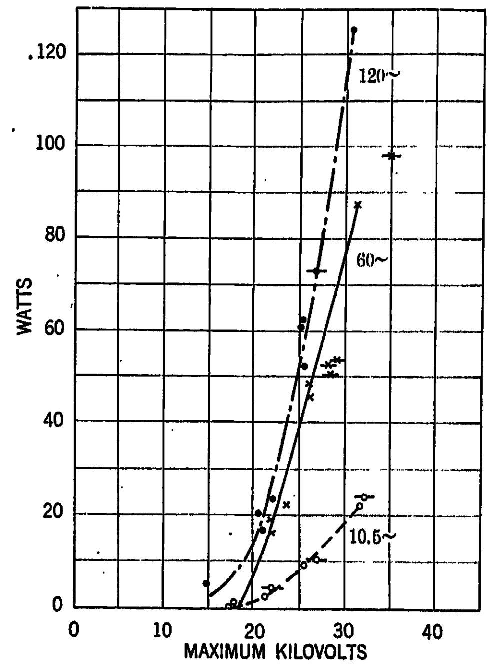

FIG. 17—CORONA LOSS—HIGH-VOLTAGE WATTMETER READINGS AND CYCLOGRAMS SHORT OUTDOOR, SINGLE-PHASE TRANSMISSION LINE

No. 20 copper conductor, spaced 36 in., 230 ft. long

Wattmeter Readings	Cyclograms
o 10.5 cycles	– o – 10.5 cycles
× 60 "	– × – 60 "
. 120 "	– . – 120 "

is controlled by the same factors, *viz.* the amount and the extent of travel of the space charge, we can say; from very high frequencies to very low ones the area increases from nearly zero to a maximum corresponding to the sharp-cornered loop whose construction was given in connection with Fig. 14.

In the practical case for high frequencies, because of the formation of a very open brush pattern wherein the brushes shield the intervening spaces preventing all corona, the ions returning to the conductor are few compared with the ions of both signs remaining in the brush-streamers, the space charge becomes negligible and the phenomenon changes over to one of ordinary

conduction. In air, at common pressures and temperature, laboratory studies to date indicate that the space-charge type of corona formation disappears at a frequency of about 5000 cycles.[5]

With the same outer cylinder, the greater the diameter of the barrier the greater is the extent of the travel, therefore, the greater the area of the loop; but only up to a certain limit: if the diameter of the barrier becomes so great that the number of ions reaching it begins to decrease, its influence on the area becomes less and less pronounced, and vanishes altogether when ions cannot reach it at all.

The curves of Fig. 8 confirm these statements: the low frequency curve lies above the high frequency curve; when the diameter of the barrier increases, the area increases, but at a gradually diminishing rate. Points denoted by ⊙ refer to a test made with the barrier cylinder formed of ¼-in. mesh wire screen; their displacement from the respective curves shows distinctly that the screen, like the grid in a vacuum tube, is not a perfect barrier in the path of the ions. Double points for 1⅛-in. diameter are taken from two tests.

The arrangement of a wire in a cylinder was used on account of its theoretical and experimental simplicity, but the foregoing conclusions are not specific to this arrangement. In all cases of corona there is a space charge whose action is always of the same general character. An $E-Q$ loop is bounded by two distinct kinds of curves, each of a more or less constant slope, corresponding to a continuous variation of physical elements, but with a sort of discontinuity and an abrupt change of slope at the junction points, due to the sudden appearance and disappearance of corona. Curves of Figs. 9, 10, 11, and 12 show these general characteristics; in Fig. 17 the loss computed on the basis of Figs. 10, 11, and 12 is compared with the loss directly measured with the high voltage wattmeter, and the agreement is very satisfactory.

Discussion

R. W. Sorensen: The peculiar curves for Fig. 1 of this paper have to be used in charting the results because a piece of tracing cloth, stretched over the end of the cathode-ray tube, must have lines, as shown, drawn on it to correct for the curvature of the tube and the distortion of the wave.

H. J. Ryan and J. S. Carroll: In the paper on The Hysteresis Character of Corona Formation, by Henline and Ryan,[1] presented a year ago at the Pasadena Convention of the Institute, the authors, in arriving at the existence of a space charge about a conductor in corona, worked at a disadvantage because they had not actually located the radial position of such space charge with respect to the conductor. During the past year two of our graduate students, Mr. Hesselmeyer and Mr. Kostko, proposed to study the radial location of the space charge by the simple plan of a concentric barrier that would limit the radial distance of the space charge from the conductor surface. They would use an isolated barrier cylinder mounted concentrically as specified in their paper; they would obtain the corresponding $E-Q$ relation and then change the radius of the barrier and obtain the $E-Q$ relation again. This plan would be continued over a wide range for the radius of the barrier in order to determine the manner in which the character of the $E-Q$ relation would change with the barrier radius. The purpose of this undertaking was to find out whether the character of the $E-Q$ relation, when barriers were used that bound the space charges to definite radial positions, would approach the character in form and area of the $E-Q$ relation as found for widely separated parallel conductors surrounded by no barriers.

When the work was completed and this paper was prepared by the authors and studied by us, the following point of view developed: The areas in units of energy, given by the Hesselmeyer-Kostko $E-Q$ diagrams, could be expressed in terms of the voltage and the capacitances of the conductor to the space charge and of the conductor to the grounded cylinder or neutral plane. The corresponding power would be the product of the energy by the frequency. Henline and Ryan, in their paper a year ago, had given the corresponding equation for the energy per cycle in terms of voltage and but one value of capacitance, viz., that of the conductor to the neutral plane. It was manifest that, by combining these equations, one could isolate the value of the capacitance of the conductor to the space charge.

Mr. Kostko then derived the equation for the power lost in corona using a barrier, as follows:

$$P = 4 f C (E E_0 - E_0^2) \frac{1}{\dfrac{C''}{C'} - 1} \tag{2a}$$

wherein

E, is the value of the crest voltage,
E_0, the value of the critical voltage,
C, the capacitance from conductor to neutral,
C'', the capacitance from conductor to the space charge, and
f, the frequency.

The corresponding equation given in the Pasadena paper a year ago, wherein the term C'', capacitance of conductor to space charge was not used, was:

$$P = 4 f C (E^2 - E E_0) \tag{2}$$

By combining and reducing these equations the value of the capacitance of the conductor to the space charge was found to be:

$$C'' = C \left(\frac{E_0}{E} + 1 \right) \tag{3}$$

If the radial distance from the conductor to the cylindrical space charge be D_r, and the radius of the conductor r, then the value of C'' will also be:

$$C'' = \frac{0.00368}{\log_{10} \left(\dfrac{D_r}{r} \right)} \tag{2d}$$

By combining equations (2c) and (2d)

$$\log \left(\frac{D_r}{r} \right) = \frac{0.00368}{C \left(\dfrac{E}{E_0} + 1 \right)} \tag{2c}$$

and

5. Eugene D'Hooghe, "The Influence of Frequency on Corona Discharges", Standford University Thesis, June, 1922.

$$\log D_r = \log r + \frac{0.00368}{C\left(\frac{E}{E_0} + 1\right)} \qquad (2f)$$

wherein

D_r and r are in inches

C and C'' are in farads per 1000 feet of conductor.

Equation (2f) was applied to one of the corona loss-voltage curves[1] given by Professor Harding's Pasadena 1924 paper[2] the following locations of the space charge were obtained:

e Kv., r. m. s. swe. to neutral	D_r in inches	Kv. per inch, r. m. s. Conductor to space charge	Between pointed electrodes*
140	9.5	14.8	10
165	12.0	13.8	10
220	18.9	11.6	9.9
260	24.1	10.8	9.8
300	29.8	10.	9.8

*A. I. E. E. Standardization Rules, 1912.

As a check upon this understanding of the distance of the space charge from a conductor in corona the following trial was made: In front of a pointed electrode at a distance of 9.5 inches a grid of fine wires was mounted. Alternate wires were electrically connected, thus forming two groups of wires each interlaced with the other. To the groups, a 20-volt, dry-cell battery was connected through a portable galvanometer; 60-cycle alternating voltage was then applied between the pointed conductor and grounded plate and the indications of the galvanometer noted as the value of the voltage was raised. The galvanometer indicated that no current was set up through the air between the two groups of wires in the grid until the voltage was raised to an effective value of 110 kv. Thereafter the current increased at the rate of 0.1 microampere per kilovolt until the value of three microamperes was attained at 140 kv. As a slight further increase of the voltage was made, the current through the air between the grid wires rose to eight microamperes. And then, as the voltage increase was continued, there was no corresponding continuation of increase of current. This is precisely the sort of thing that should happen if the foregoing understanding of the existence and position of a space charge about a conductor in corona is correct.

The matter was tried out by another plan: The space charge was reversed while the voltage increased from the critical value E_0 to the crest value, E, in a corresponding interval $\Delta t = t - t_0$. During such interval, Δt, electrons must travel from the conductor to the location of the space charge when the potential of the conductor is negative, and vice versa when positive. When the electrode in corona is the point of a conductor, the resulting luminosities produced by the migration of the electrons as just specified might be intense enough to be visible in full darkness to or near to the radial position of the space charge. On trial, such was found to be the case.

Another reasonable conjecture in regard to action due to the space charge was encountered: Voltage was applied between a pointed conductor and a grounded metal plate. As the voltage was raised, corona filled a conical space that expands from the point toward the plate through distances in relation to voltages that correspond roughly to those given in the above table. As the voltage crests occur the space charges and point potentials have the same sign, while the signs of the space charges and bound charges induced in the grounded plate as opposing electrode are opposite. The consequence is that the intensity of the electric field between the space charge and the point has been reduced and that between the space charge and plate has been correspondingly increased. The outcome must be that, as the voltage is raised, critical electric stress will be encountered in the air between the space charge and the plate beyond which the intervening air must be ionized and rendered conductive. On trial, this too was found to be the case. As the voltage is raised, the faintly luminous cone develops, attached to the point with rounded base thrust forward. Then, as the rise of the voltage continues and the growth of the cone moved its base to a position whereat it was somewhat nearer the plate, a faint pillar of light suddenly extended from the cone to the plate; the air column connecting the point to plate had been ionized and spark-over and arcing followed with slight further increase of voltage.

And so thus far every plan that has occurred for authenticating the existence and position of the alternating space charges established and maintained about a conductor in corona due to 60-cycle voltages when tried out, has resulted in corroboration of the understanding as given.

H. S. Bates: I should like to ask Mr. Hesselmeyer if there will be any means of accurately measuring corona loss? I wish to ask also what is the best method of preventing it?

C. T. Hesselmeyer and J. K. Kostko: The experiments of Prof. Ryan and Mr. Carroll are interesting not only because they prove the existence of a space charge, but also because they suggest experimental arrangements for a quantitative study of the distribution of the space charge and the field. It is easy to set up equations theoretically determining these two elements (Poisson's equation and equation of continuity); but numerical solutions could only be obtained by reducing these general equations to simpler types, based on the results of a preliminary experimental study of the problem.

In the author's opinion the most accurate method of measuring corona losses available at present is by means of the high-voltage wattmeter developed at Stanford University and described in several Institute papers by Mr. Carroll and others.[3] In Fig. 17 of the paper the losses measured with this wattmeter are compared with the losses obtained by a very different method—integration of the E-Q cyclograms—and the agreement is remarkably good.

As indicated by the theory and confirmed by experiment (Fig. 8), it is possible to reduce corona loss by setting up a suitable space charge around the conductor, for instance by enclosing it in a cylinder of a small diameter; it does not seem, however, that this method is suitable for practical applications, at least in the case of a transmission line. A radical reduction of the transmission frequency would result in a reduction of corona loss (Fig. 17), in addition to many other advantages, such as better regulation, etc.

1. In applying these equations it should be remembered that the voltage must be taken at a value sufficiently above the critical voltage to ensure that a fixed brush pattern has been formed and the value of C is a constant as presented in the Pasadena paper.

2. *Corona Losses between Wires at Extra High Voltages* by C. F. Harding, A. I. E. E. JOURNAL, October, 1924, page 932.

3. *Power Measurements at High Voltages and Low Power Factors* by J. S. Carroll, T. F. Peterson, and G. R. Stray, JOURNAL A. I. E. E., Oct. 1924, page 941.

Some Features and Improvements on the High-Voltage Wattmeter, by J. S. Carroll, JOURNAL A. I. E. E., Sept. 1925, page 943.

The Study of Ions and Electrons for Electrical Engineers

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THE present paper is presented as a contribution to the educational activities of the Institute. Education is often an art and never completely a science. With respect to ions and electrons the science is new and the development of the art scarcely begun,—an art that is bound to undergo rapid evolution. The value of the present paper at best can only exist temporarily.

Physicists and chemists in their studies of the foundation of matter during the last quarter century have been profound students of ions and electrons. Virtually all of their discoveries and results are of direct or indirect value to electrical engineers. The technical and practical uses of knowledge of this character today are extensive. Many important developments have been possible only by its means. And such developments have encountered difficulties that in turn have defined the great need of further knowledge of the same general character. The need for the electrical engineers is being formed, however, in an entirely different mold from that which shapes the requirement of the more general science worker.

In many of the problems originating nowadays in the electrical industries wherein ions and electrons are involved, the physicists and chemists are, in all ordinary circumstances, so loaded with necessary duty in the solution of their own problems that they can rarely afford the time and facilities to come to the aid of electrical engineers. The electrical engineers can, therefore, no longer depend largely upon the physicists and chemists for enhanced results that will enable them to solve their own problems of this class. They will have to do their full share from this time forth to extend knowledge of the facts in regard to ions and electrons and their behavior.

It is of corresponding importance that all advanced students among the incoming generation of electrical engineers be reasonably well equipped with an understanding of the present-day expediency for attacking problems encountered in the electrical industries that require for their solution a clear understanding of the behavior of ions and electrons. It is also recognized that advanced students are not always young men in the colleges. It is important that all,—old and young, wherever situated, should know that it will be most helpful to them and to the electrical industries to acquire a well-founded knowledge of ions and electrons and to learn how to take advantage of every opportunity to apply or to extend such knowledge.

Millikan has splendidly summarized existing knowledge of electrons and ions in his book on The Electron[1], more especially from the point of view of the physicist. J. J. Thomson[2] has rendered a similar service from the point of view of the chemist, though himself a physicist. To these recent classical treaties the advanced student, at the threshold of the subject, is referred.

The following are some of the fundamental facts in regard to electrons that must always hold the attention of many electrical engineers: There are two varieties of electrons, distinguished primarily by the signs of their respective electric charges. The quantities of these charges are alike for each,—4.78 by 10^{-10} electrostatic units, or 1.59 by 10^{-20} ampere-seconds. In respect to other attributes they differ decidedly. The mass of the negative electron is 1/1845 that of the hydrogen atom,—the lightest of all atoms. Correspondingly, the mass of the positive electron is approximately 2000 times the mass of the negative electron, or nearly the same as that of the atom of hydrogen. In atomic structure the positive electrons behave as though they and some of the negative electrons formed the atomic nucleus, while the rest of the negative electrons associated with an atom behave as though they were set in orbits about the nucleus. So far as is known, positive electrons do not exist in the free state. They exist only within the nuclei of atoms accompanied by some of the electrons that bind them in close proximity by electrostatic attraction. The numbers of positive and negative electrons present in the same neutral atom are equal.

The spontaneous breaking up of the nuclei of the heavy (radioactive) atoms into helium atoms and negative electrons is the nearest known approach to the existence of free positive electrons. On the other hand, free negative electrons exist in abundance. An almost endless variety of physical or physicochemical actions may break their orbital bonds to their corresponding atomic nuclei and set them free. The removal of a negative electron from an electrically complete or "neutral" atom results in the presence of one free negative electron, ordinarily called *electron,* and an atom carrying the positive charge of one electron. Such an atom is ordinarily referred to as a *positive ion.* Under all ordinary conditions approaching quiescence, free electrons adhere to atoms, otherwise neutral. The bond is weak and easily broken when the atom is

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1. For all references see bibliography appended hereto.

driven electrically or mechanically through gases, fluids, or near the walls of solids.

All conduction of every character is now known to be due to the movement of positive and negative electrons or more simply ions or electrons, or both. The electrons or ions may be moved mechanically, electrically or electromagnetically. An example of their movement electromagnetically is encountered in the electron jet cyclograph, wherein the electrons liberated from a hot cathode are driven forth in a jet by a strong electric field, and the jet is then deflected transversely by a magnetic field.

It follows that the mobilities of electrons and ions through solids, liquids, gases, and empty space are factors of the highest importance. Far too little is known about these mobilities. Physicists, however, have determined them as the velocities of positive and negative ions in electric fields of unit strength in air and in hydrogen at the usual density occurring at a temperature of 15 deg. cent. and a pressure of 76 cm. as follows:

	Mobilities in cm. per sec. in unit fields, *i. e.*, one volt per cm.	
	Positive	Negative
Air.............	1.35	1.83
Hydrogen........	6.1	7.8

For practical purposes the relations of these mobilities to their corresponding fields of strength may be assumed to be linear for the time being.

Correspondingly, all non-conduction must be due to one of two things,—the non-movement of all ions and electrons present or their total absence. Materials through which ions or electrons can be moved freely are designated as conductors. Materials through which ions and electrons can not be moved are designated as true insulators.

With the new understanding of electrical phenomena, it is helpful to distinguish three sorts of conductors and corresponding conductions:

I. Metallic conduction—due to the free movement of electrons from atom to atom, requiring no e. m. f. for their detachment and only that e. m. f. which is required to supply the heat absorbed through the increased atomic agitation that has been produced.

II. Electrolytic conduction—the free movement of ions through an ionized liquid (or salt solution) from anode to cathode and vice versa, using an e. m. f., part of which is consumed positively or negatively in detaching or attaching ions at the electrodes in dissociation and recombination of the electrolyte and for the rest in supplying the inevitable heat due to the increased mechanical molecular agitation.

III. The movement of free ions or electrons in a non-ionized fluid. Conduction of this type is dependent upon two factors: (1) A requisite source of ions or electrons, and (2) the e. m. f. required to overcome the counter e. m. f. of space charges and again to supply the inevitable heat.

Fluid conduction may be set up in every kind of fluid, liquid or gaseous. No fluid of any sort pervaded with a supply of ions or electrons can properly be regarded as an insulator. Correspondingly every fluid in which ions and electrons are absent must function as an insulator.

Amorphous bodies or the precooled liquids, such as glass, the matrix of porcelain, fused quartz, etc., should be remembered as belonging to fluid conductors. The hardness of these bodies is due to their high viscosities, occurring when they were cooled from the molten state without crystallization. Pure, normal sulphur is an example of a non-fluid or crystalline body free of ions that intercepts completely the flow of ions and electrons, and functions, therefore, virtually as a perfect insulator. Fluids can have no such dependable barrier quality. This is the great reason why fluid insulators must always be supplemented with substantial barriers that break up the threadlike channels occupied by moving ions when driven by applied e. m. f.

Many have a feeling that air is a well-nigh perfect insulator or barrier to the passage of current forced along by applied voltage. The fact is that air has little or no barrier quality. If ions are liberated into the air, as by the passage of X-rays, on the application of a few volts only, the air may be observed to conduct with relative facility. It is actually no insulator in the sense that sulphur is. The great reason why air appears to function as an insulator in all ordinary cases is because of the absence of virtually all facility for liberating ions or electrons into the air.[8] To detach an electron from a metal electrode into a gas requires an electrical field terminating on the wall of the electrode that has been formed by the application of a million volts per centimeter. Above such voltage gradient terminating upon a conductor, air ceases to function as an insulator because the applied e. m. f. is sufficient to expel ions copiously from the one metal electrode and drive them through the air to contact with the opposing electrode where they are discharged, thus completing the electric current circuit. At correspondingly lower voltages, the air will function as an insulator only because electrons can not escape from the conductor walls.

Because metals and carbon when raised to sufficiently high temperatures will radiate electrons and thus supply ions copiously, air in the presence of highly heated electrodes ceases to be an insulator and functions abundantly as a conducting medium.

We are thus compelled to recognize once for all that actually air and other gases are not really insulators; the thing that did the insulating, which was mistakenly attributed to the air, was actually a property of the wall of the conductor-electrodes by which electrons were confined within the conductor and not permitted

to escape into the air or other gases occupying the space between and surrounding the electrodes.

It is particularly in this "no-man's-land" of ions and electrons, wherein insulators are not insulators and conductors are not conductors, that the electrical engineer is much concerned today.

The most important of the expedients for liberating electrons are:

I. From metal electrodes immersed in air or other gases

a. by heating the electrodes,

b. by applying ultraviolet light to the electrodes, for which some metals are more effective than others,

c. by coating the electrodes with certain salts that emit electrons copiously when heated,

d. by intense electric charges, 1000 kv. per cm. in air—1250 kv. per cm. in vacuum,

e. by evaporation or boiling of the metal or carbon electrodes.

II. From gases:

a. by exposing the gases to X-rays or by the emanations from radium and other radioactive substances,

b. by collision ionization, commonly called corona.

III. From metals to fluids

Electrons pass out from the cathodes and into the anodes immersed in electrolytes by the phenomenon known as electrolysis, long since well understood through the activities of the chemist.

IV. From metals to solids

Herein little is known as yet. There appears to exist no general understanding of the phenomenon of the passage of electrons from a metal to a non-conducting solid. Nevertheless, among the classical experiments of a century ago there was the one in which the metallic coatings of a "Leyden jar" were made removable. With the electrodes mounted the jar was charged. The coatings were then removed and the jar and coatings were examined to determine the seat of the charge. The coatings were discharged and replaced and the jar thereafter discharged as a whole, when it was found that the discharge was virtually as strong as if the coatings had not been removed, discharged and remounted. Through the new knowledge, we now know that when metal electrodes make good contact with solid dielectrics, electrons pass easily from the metal electrode to the atoms of the contacting dielectric and vice versa. The conclusion is inevitable that the contact e. m. fs. between the metallic and dielectric walls are extraordinarily low, permitting the easy exit of the electrons from the negatively charged electrode to the adjacent dielectric, and conversely from the dielectric to the positively charged electrode. Because in solid dielectrics neither electrons nor charged atoms can migrate with any but the slightest degrees of freedom, the dielectric functions as a barrier; an excess of electrons in the superficial face of the dielectric under the cathode and the opposite condition under the anode occurs and develops until the counter e. m. fs. of the bound charge thus produced balances the applied voltage whereon the action rests in a potential state.

V. Liberation of ions and electrons is produced by friction, splashing of liquids and bubbling of gases through liquids.

Of the highest importance, likewise, are the facilities available for the quantitative observation of the causes and corresponding effects of the activities of ions and electrons. In occasional circumstances, the quantities to be measured are all suitably large, including the expenditures of power for which there is at hand a wealth of well-known measuring expediencies. Often, however, one or more of the essentials to be measured are relatively very small or very large and the corresponding facilities available are as yet few, if at all, and general experience in their use may be lacking.

For the detection and gaging of small free charges in the air and gases, the gold-leaf electroscope, the delicate electrometer or galvanometer are often necessary. New uses for the old expedient of the potential plate or potential electrode are being found for the determination of potentials due to position, potential gradients, voltage duties, and potentials as modified by the presence of space charges. Conducting or non-conducting barriers in plates, tubes or other forms as required to limit the migrations of ions or electrons are often most helpful. A metal woven mesh, coarse or fine, may have its uses as a kind of "grid" for high voltage studies of the migrations of the electric carriers in air, gases, and liquids. The modern cathode-ray oscillograph and electron jet recorder are of extraordinary value for the promulgation of these studies.

In many studies the time-relation wherein a thing happens within the duration of a cycle or transient is of dominating importance. In these cases some dependable criterion in time relation must be found. When the actuating voltage is cyclic or transient and maintained with the requisite power, the non-inductive, non-capacitive resistor for "tapping out" fractional replicas of the total actuating voltage is a valuable expedient herewith. The potential plate, judiciously used, is also a helpful expedient for the same purpose.

Studies of this character in one way or another require electric power supply-sources in almost every thinkable voltage-current-time relation. Herein for continuous high voltages the old electrostatic machine and modern kenotron (the latter with requisite accessories) are available. Below and above commercial frequencies, laboratory forms of alternators and arc-converter and electron-tube oscillators are available; the choice must be determined by the special circumstances.

In the aggregate, there must be provision for the use of almost every character of substance occupying the immediate space about the electrodes which in turn must be available in every required form taken with

respect to the method by which ions or electrons are to be detached from them. Among these electrodes there must be those which are made of boiling metals or carbon.

Henceforth, problems without number will come up for attention in the electrical fields the solution of which will be feasible only through the use of abundant knowledge of ions and electrons to be acquired only by orderly, persistent effort. In closing, by way of example herein, one may mention the problem of the reduction of the damage done by power line flashovers. In a considerable percentage of these flashovers the trouble is started by indirect lightning, or other forms of over-voltages or some sort of attenuated conducting material laid across the circuit, usually from conductor to tower. In a great many of these cases the trouble is known to have had a very small beginning that now and then is not augmented and clears itself. But in the majority of cases the local metal faces of the conductor and tower are heated with great rapidity to the boiling point in those spots that happen to be located at the termini of the thin ion-electron stream that inaugurated the action. The ionized vapor liberated by the boiling that ensues enormously augments the conductivity of the original stream of electric carriers, resulting in the rapid development of a heavy short. An effective procedure for the solution of the problem may be to cover the conductors at the towers suitably with a heat resisting barrier that will not permit the discharge to terminate on the conductor in a sufficiently narrow spot to produce boiling so as to permit the action to clear itself. It is not forgotten that in procedures of this sort not one but all perceivable options that promise a solution must be worked through at least to that point from which it is seen that they may or may not be given up effectively.

In conclusion, it should be helpful to all electrical engineers to acquire a knowledge of the more important factors in the behavior of ions and electrons; and for the maintenance of a well-balanced progress in the electrical industries, it is highly necessary that some of the electrical engineers acquire, augment, and apply the highest attainable knowledge of this subject.

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Discussion

R. W. Sorensen: Professor Ryan says, "There are two varieties of electrons, positive and negative." On this he can find much authority; in fact in his book, "The Electron," Dr. Millikan speaks of positive and negative electrons, but I much prefer in this particular to follow the practise of those who define the electron by saying it is a cathode particle such as is found in the cathode rays. I choose this path of thinking about electrons because we all know something about cathode rays. These rays have been found to be made up of negatively charged particles, and the particles are called electrons.

This definition makes the electron a single individual, always negative, rather than twins, one positive and one negative, and it is, as a consequence, more easily recognized. If the electron cannot be a twin, we must provide it with an affinity from which it wishes never to be separated, and in this form of nomenclature the term proton has been applied to the same individual that Professor Ryan has called the positive electron.

More frequently than not, these electrons and their protons are found in large groups, but, in the case of the hydrogen atom, they are alone, this atom being made up of one electron and a single proton nucleus. That points out one question to which we might refer in Professor Ryan's paper where he stated, I believe, that the positive electrons were not found alone; but this is the one exception. If one of these hydrogen atoms encounters a disturbing influence, such as an electric field powerful enough to detach an electron, the electron becomes free and there is left the single proton.

Proceeding in the direction of complexity of structure, we could discuss the helium atom which has two electrons attached to a nucleus that appears to have four protons.

When one or more of these electrons or negative particles is taken away from an atom, the remainder behaves in such a way as to indicate that it has a positive charge, and is known as a positive ion. Thus atoms become ions when they have a deficiency or excess of electrons.

I see no reason why protons as defined cannot exist in a free state in a hydrogen arc, but it is, of course, true that compared to the number of times one can find free electrons, such an occurrence is very limited.

I had planned to question the statement that "under all ordinary conditions approaching quiescence, free electrons adhere to atoms, otherwise neutral." But now, although a number of physicists think they are free, I am inclined to think that we shall get into less trouble if we take Professor Ryan's statement that these electrons are likely to attach themselves to neutral atoms, making them negative ions.

Something is said about electron travel. In regard to that, when we measure an electric current as so many amperes we are measuring the sum of the positive and negative ions passing through the ammeter.

I should like to suggest that perhaps as engineers we would clear up Professor Ryan's statements as to the movement of electrons by saying that the electrons or ions may be moved mechanically, electrically or magnetically; that is, you can move them by mechanical means by placing them in what we call electrostatic field, or in a magnetic field. I know that is exactly what Professor Ryan intends to say, but I think it would be better to say magnetically rather than electromagnetically.

On the second page Professor Ryan has divided conduction into three groups. I think we should add something to these. An electric current made up of these ions or electrons will also travel through a vacuum, and I do not believe that this has been included in these three sections. Also, I am not quite sure that these three groups as listed show conduction through an arc.

To the paragraph at the top of the second column on the second page I am inclined to add the idea that every atom has electrons; therefore, how can one have a fluid which does not have atoms and hence does not have electrons? Correspondingly how would it be an insulator under Professor Ryan's definition?

Considering practical engineering application, I am one of the many who have a feeling that air, pure and unadulterated, if not a nearly perfect insulator, is at least a pretty good one and one which will serve us for a long time. In the final analysis, all our transmission lines are air-insulated, the porcelain bead chains with which we decorate our transmission towers being, after all, only decorative suspenders which serve to keep the lines from falling. The insulator is the air.

I think the reason Professor Ryan and I differ in this is because he says a thing is not a thing unless it is at least 99.44 per cent pure. Air, then, is an insulator just as oil is an insulator. If we introduce into the air, free electrons or ions, the air as an insulator becomes defective in exact proportion to the amount of impurity introduced. In undertaking our engineering problems we call oil an insulator. It never is a perfect one and it ceases to be an insulator at all if moisture is added to it, its value as an insulator decreasing in proportion to the amount of water added.

I might also add that in the sense which Professor Ryan has spoken of an insulator, a vacuum is not an insulator. Current can and will go through it. To my way of thinking, a high vacuum is an insulator, but it is not a perfect insulator; therefore, Professor Ryan says it is not an insulator.

When two conductors are brought very close together, a potential of 1,000,000 volts. per cm. or even greater potentials may be required to break down the gap. Also, cold electrodes in high vacuum require potential gradients of this magnitude as ionizing potentials, but charged electric conductors in air at sea level and separated practical distances will arc over one to the other if the potential gradient in the air between them is 30,000 volts per cm.

However, on this point I must assure you that though we are using different words, Professor Ryan and I understand each other thoroughly in this matter, and in conducting experiments involving these things, we would use in many cases the same strategy and anticipate the same results.

C. E. Magnusson: There is one factor—in fact a vital factor in the electron theory—the physical characteristics of which are seldom discussed while the attention is focused on the several forms of mass units involved. I refer to the electric charge. What is the innate nature of positive and negative charges, which take possession of, or are possessed by, electrons, ions, protons, corpuscles, or by whatever name the mass units may be designated? How can the charge, if located on or attached to an electron or other mass unit of definite size, produce action at a distance or be attracted or repelled by other charges attached to far away mass units? What is back of Faraday's lines or tubes of force? May I ask Professor Ryan to give us his concept of the electric charge?

C. L. Fortescue: I think one of the reasons why electrical engineers have difficulty in following and applying the electron theories is because it is the first time they have come up against the subject of statistical mechanics, a subject with which physicists have become very familiar in their study of the dynamic theory of gases.

Now many laws of physics, dynamics, and physical chemistry were found before the kinetic theory of gases was well established, and these laws prove to be true under practical conditions.

Electrical engineers have been accustomed to think of air as an insulator which breaks down, under ordinary conditions, at about 30,000 volts per cm. This, of course, is a very convenient way of looking at it, but we know by the electron theory that this isn't at all true except under certain specific conditions.

The electron theorists tell us that the air will break down at any point where the rate of ionization and the rate of recombination are equal. I believe the rate of recombination depends upon the mobility and the rate of ionization depends upon the density of the air and also upon the total value of applied potential or the difference of potential between electrodes; but we have two quantities there that have to be taken into account. As a consequence we find for a very small separation, as Mr. Sorensen points out, a breakdown strength of the order of 1,000,000 volts per cm. In a larger space the breakdown strength of the air becomes less and less. In the ordinary spaces the engineer uses, we find it averages around 30,000 volts per cm.

In Professor Ryan's paper, I was a little disappointed when I read his remarks about the three sorts of insulators. Unfortunately we are likely to generalize and think of these things practically as hindering our methods of insulation. For instance, Professor Ryan makes his statement in such a way that one would think, reading it superficially, that barriers were absolutely indispensable in connection with all insulators. We know by actual experience if proper care is taken to prevent the formation of corona, or, putting it in terms of the electron theory, when local ionization is avoided, we can use air without applying barriers, and the strength of the air will follow the avarage law which I have mentioned, breakdown taking place at about 30,000 volts per cm.

Certain conditions occur when the bounding surface between solid dielectric and air apparently does not follow the law of breakdown in air. I think these discrepancies have been attributed to the effect of the absorption of gases or moisture on the surface, but if you have a perfectly clean surface of proper conformation, the path along the surface will have the same breakdown strength as the air has.

I should like to ask Professor Ryan to clear up this difficulty in the interpretation of this paper. We are sure as engineers that the air is still a medium for insulation.

F. G. Baum: For many years (since 1911) Dr. Ryan has exhorted us to study the electron. I am here as a missionary today to try to help show the importance of studying the electron. For many years the subject of electrostatics has been taught in schools. In my opinion, there is no such subject as electrostatics except as you get down to an extreme vacuum where you have no ions injected into the vacuum; otherwise you have "electron mechanics," and I believe in a very short time you will find that our textbooks will be rewritten and the term "electrostatics" practically eliminated. It is wrong and we must get another proper term and realize that we are dealing with objects moving at very high speed and causing entirely different conditions from those which we thought true when we studied electrostatics.

Ordinarily, we take two bodies and draw lines between and say that is an electrostatic field. It is an electrostatic field only because electrons are moving from one of those bodies to the other; and our higher voltage insulation problem is dependent on a knowledge of handling this electron flow.

H. J. Ryan: Replying to Professor Sorensen: I can accept, if necessary, the use of *proton* in lieu of *positive electron* as proposed by some physicists. It is simply a choice of terms. Personally, however, I like Doctor Millikan's use of *positive electron* to emphasize the fact that all matter is substantially made up of cathode and anode particles. As implied, it is true that experimental facilities are as yet more abundant or convenient for the liberation of cathode particles than for anode

particles. These cathode and anode particles surely are twins of just the character referred to. They carry elemental charges equal in amount and opposite in sign. The positive electron or proton is much smaller in diameter and has a correspondingly greater mass than the negative electron. The same elementary electric field or charge centers in the electron whether positive or negative,—the one and only known difference being that of direction or polarity. I do not feel that the use of the term "proton" is adapted to the presentation of these facts as well as the term "positive electron." It is helpful to have been reminded of the stripped hydrogen atom which can be produced and which must behave as an isolated positive electron, proton, or anode particle as we may variously call it.

I quite agree with the idea put forth in regard to the movement of ions and electrons mechanically, electrically or magnetically. However, it should be remembered that they may be moved also by any combination of these agencies. For example, ions in the air that is blown along between the poles of a magnet and between metal plates maintained at a difference of potential will be moved mechanically, magnetically, and electrically. It may be that it is not helpful to compound these terms and say that the ions were moved electromagnetomechanically. I am quite agreed to say that they were moved electrically, magnetically, and mechanically.

I am glad to accept *vacuum* for a place in the list of electrical conductors. I had left it out originally because in the first place a vacuum is not anything, anyway in the ordinary sense, and therefore can not assist or hinder the migration of ions or electrons; in the second place, as Mr. Wood brought out in the talk referred to, as soon as ions or electrons are admitted to the vacuum it may in a sense be thought of as having ceased to be a vacuum.

I can see no difficulty with the statement "No fluid of any sort pervaded with a supply of ions or electrons can properly be regarded as an insulator. Correspondingly, every fluid in which ions and electrons are absent must function as an insulator." Of course, each molecule of neutral transformer oil is made up of complete or neutral atoms that in turn are made up each of an equal number of positive and negative electrons bound together, forming neutral aggregates. Such oil is not a supply of ions and will not conduct under an impressed electromotive force of moderate value. If, however, the oil contains impure water in suspension it is pervaded with a supply of ions and will conduct.

I cordially admit the powerful revulsion of feeling that must come to one when first confronted by the fact that it is the wall of the metallic conductor when immersed in air that is really the insulator and not the air. Take away the air, as one may do in a vacuum, and the conductor will be insulated just as well as before. This is the fact that made me doubt the wisdom of putting "vacuum" in the list of conductors. It does not really matter, though, as long as we can agree as to the circumstances in which it does or does not conduct. Years ago, when the idea prevailed in my own mind that air is one of our best insulators, with dielectric strength greatly enhanced by compression, I undertook to provide a powerful dielectric by means of air compressed to 1500 lb. per sq. in. I was greatly perplexed by the results because I was wholly unaware of the fact that air will permit ions to pass through it freely if one will but provide a source thereof, such for example as a hot carbon electrode. We did not know then, as we know now, that in all ordinary circumstances electrons cannot escape from the wall of a conductor which is the basic reason why we were made to believe that the air was the real insulator. Furthermore, we did not know then, as we do now, that at extraordinarily high electric intensities at the surface of an electrode conductor (1,000 kv. per cm. approximately) electrons or ions will escape from the wall of the conductor and be driven freely through the air to the opposing electrode where they will be discharged. With a knowledge of these facts twenty years ago we would not have been perplexed by the anomalous behavior of air as an insulator when put to a real test.

I cannot agree that it is a matter of degree to be covered by such a small item as the departure of 99.44 from 100. It is not a question of purity or impurity any more than it is in the case of water. Water will conduct as long as it has ions suspended in it. Being a fluid it ceases to conduct only when the supply of ions has been eliminated. And this will cover also the reference to oil.

Doctor Magnusson asks the question "What is the innate nature of positive and negative charges which take possession of or are possessed by electrons, ions, protons, corpuscles, or by whatever name the mass-units may be designated?" This question and the form in which it is put are helpful even if one has not a ghost of an answer. I can only discuss this question; I cannot begin to answer it. I can only offer what appears to me to be a reasonable conjecture in regard to the perhaps most important attribute of the electron. This is that all electric fields are made up of unit-fragments alike in constitution. Each field fragment terminates on an electron from which it extends radially and expands uniformly, and so far as we know, indefinitely. These field fragments are the same, whether positive or negative, differing only in polarity and in radius of the electron surface at which the field terminates, being much smaller for the positive electron which must, therefore, have a correspondingly greater mass, the measure of the energy that was used in the extra concentration of the field. Whatever else they may be, electrons are surely these field fragments. All greater electric fields are merely aggregations of these unit-field fragments. The electric intensity through any field volume is the vector sum of the radial field fragments attached to the electrons that constitute the charges to which the field is attached. Maxwell understood the composition and resolution of electric fields and taught us to locate "tubes" of electric force by drawing diagonals through the parallelograms that are formed by the radial lines which represent the electric fields that extend uniformly in all directions from charges located at a point. It is the vector composition of superimposed fields terminating upon the positive and negative electrons that forms the "tubes of force" of an electric field. It is in the presentation of these facts that I find the term *positive electron* more helpful than *proton*.

It is also helpful to have Mr. Fortescue emphasize the importance of *statistical mechanics*. I trust that all who are interested in the new knowledge will read thoughtfully what he has said. He also refers to the extremely short space that must exist between metal electrode faces before electrons will leave them and the vacuum, or gas-filled space between them, will become conductive. If the fact is allowed to stand in that light I fear we shall give our more general audience the impression that this action is the result mainly due to the close proximity of the metal electrode-faces. In fact it cannot primarily be due to the nearness of such faces as Hayden and Steinmetz have shown in their A. I. E. E. paper on the dielectric strength of the vacuum.[2] The preparation of my paper was only possible by the use of old terms with new or modified meanings. I had to count, therefore, upon precisely such disappointment as that of Mr. Fortescue because I have referred to air as a conductor instead of as an insulator. I have no thought of proposing that we stop calling air an insulator. What I do want to see established is a better understanding as to how it can be made to conduct abundantly. With that, and with more of a background in the subject which will come with experience, the choice of terms with no doubts on important difference, will be readily accomplished. I am a hearty advocate of the high

2. High-Voltage Insulation, by J. L. R. Hayden and C. P. Steinmetz, A. I. E. E., TRANSACTIONS, 1923, page 1029.

value of the *Fortescue principle* wherein air insulation barriers of powerful solid dielectrics are applied, having boundaries coincident with those of the tubes of electric force in the air adjacent. Such barriers displace the air in regions of dense electric fields that would otherwise ionize and afford prolific conduction. We do not differ as to the facts and in the end we shall have put new meaning into old terms or adopted new terms by which all who use them will apply helpfully the new knowledge of these things.

Mr. Baum has declared rightly that electrostatics are hopelessly inadequate for understanding and for effective control of electrical states and actions. This I know to be the case even though we cannot agree as to facts when he says that a quiescent electric field between two charged bodies is such "because electrons are moving from one of these bodies to the other." Of course, this brings us face to face with the age-long problem of "action at a distance" and my question to myself is: "Has Mr. Baum made some progress toward the definition and solution of that problem?" Most of us have not,—we face a high wall and cannot see what is beyond. To me the static electric field between two bodies is the composition of the two field aggregates of opposing polarity that terminate on the corresponding free positive and negative electrons that are bound to the surfaces of such bodies.

Engineering Research—An Essential Factor in Engineering Education

BY C. EDWARD MAGNUSSON[1]

Fellow, A. I. E. E.

IT may seem trite to assert the interdependence of engineering research and engineering education, and no doubt most workers in the vineyard would consider the title of this paper as axiomatic,—the statement of a self-evident fact. *In theory* the great majority of the members of our engineering faculties, as well as practising engineers, who have given time and thought to the training of young men for the engineering profession, readily subscribe to statements emphasizing the importance of research as a factor in engineering education; but *in practise*—well, "the spirit is willing but the flesh is weak."

In the majority of our engineering colleges and technical schools very little if any research is in progress, and even in our leading institutions comparatively few members of the teaching staffs are actively engaged in worth-while investigations. As yet the research spirit is an attribute of individual members of the faculties and not of our engineering colleges as institutions. Professors who have gained recognition as investigators are looked upon as ornaments, and not as the bone and marrow of the university. Research remains a side issue, something very desirable and highly creditable to our technologic institutions, but not to be taken seriously as an essential factor in the training of engineering students.

The rapidly increasing importance of engineering research in the industries stands in striking contrast to the apathetic condition that exists in many of our educational institutions. In large manufacturing concerns the research departments have gained in recent years prestige and influence far beyond the fondest dreams of the pioneering investigators of thirty or forty years ago. The development of new ideas is by now an integral, and, in many cases, a vital part of industrial organizations. Research engineers are continually transforming new ideas into effective weapons of offense and defense in present day industrial warfare. The research departments provide the means for gaining new markets, and form the bulwarks of defense against competition in established fields. That industrial research will continue to expand is certain, as the work rests on a sound economic basis. Properly conducted research not only pays well—in fact brings large returns on the investment—but is a basic necessity in order to enable industrial organizations to live and prosper.

While, in the main, the expansion of research in the industries has had a salutary effect on our engineering colleges, still it has developed conditions adverse to the effective training of engineering students. Industrial research has emphasized the importance of giving students clear concepts of well established fundamental principles and created a new and highly attractive professional field, thus providing the studious, scientifically inclined, engineering student with a better appreciation of fundamental physical laws and a definite goal for his ambition—to become a research engineer.

Of adverse factors two are of special significance:

1. The type of mind required for becoming a successful research engineer is essentially the same as for gaining prominence as a professor in some field of engineering. As industrial concerns are able to pay larger salaries than educational institutions, and what is often of greater importance, can provide better facilities for carrying on investigations, the teaching staffs are being robbed of their best men, and, to a large extent, have been cut off from the supply of desirable recruits. This factor affects not merely the faculties of engineering departments but likewise those of physics and chemistry. Unless this movement can be checked and emoluments of professors be made as good or better than those given to research engineers, the results will

1. Dean, College of Engineering, Director, Engineering Experiment Station, University of Washington.

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soon prove disastrous to technologic training. For, in order to have a first class college of engineering, there must be first class men on the faculty.

2. Even under present conditions, the amount of research accomplished by industrial concerns is overwhelmingly greater than what is done in the universities and professional schools. With industrial research staffs having several thousand members—as, for example, the recently organized Bell Telephone Laboratories—and with adequate facilities at their disposal, is there any wonder that the research output of a college faculty, with its members giving practically all their time and effort to teaching, should be very small in comparison? The contents of our scientific and technical journals and the proceedings and transactions of our scientific and technical societies give evidence of the increasing preponderance of papers from industrial research departments, with a corresponding relative decrease in contributions from educational institutions. To those who are familiar with existing conditions for conducting research in our colleges of engineering and institutes of technology, as compared with the facilities provided by research departments of many industrial concerns, the wonder is not how little, but how surprisingly much, the professors actually accomplish.

Still the distressing fact remains that the relative importance of our institutions of learning as centers of research is rapidly decreasing, a situation that should be given serious consideration, not only by faculty members of our engineering colleges and technical schools, but likewise by all progressive engineers and captains of industry. It is, by now, quite generally recognized that during his four years in college, the main purpose of the prospective engineer is to acquire a mastery of engineering fundamentals and a working knowledge of mathematics and English, while comparatively little importance is attached to the gathering of detailed information on current practise. (In paranthesis it may be noted that while engineers and educators are generally agreed that the major part of the student's time and effort should be given to engineering fundamentals, there are apparently widely divergent views as to what the term implies. For the purpose of this paper let it be assumed that by *engineering fundamentals* is understood the *basic physical laws* with and under which the practising engineer works, lives, and has his being.) The student in college should gain a deep and full realization of the far reaching importance of the principle of the conservation of energy, Newton's, Joules', Ohm's and Kirchoff's laws, and similar established basic physical relations, and *acquire the ability to apply them correctly to quantitative engineering problems.*

In order to give the student a clear insight into the basic laws of engineering and their application to quantitative problems with that perspective of coming events, so tersely expressed by Mr. B. G. Lamme, "*it should be remembered that much of what these young men will work with has not yet been discovered*" and requires teachers who are continuously applying basic principles in their own search for new truths. The title "research professor," which, in the last few years, has made its appearance in several university catalogues and bulletins, indicates an unfortunate tendency. It gives official recognition to a separation of the teachers from the investigators—and thereby erects a new barrier between the students and the progressive thinkers on the teaching staffs. It would be better both for the students and institutions if successful research could be made a requirement for all members of the faculty, and that in rating our engineering colleges and technical schools, more emphasis could be placed on the quality and number of engineering bulletins and scientific papers published than on the number of students or available laboratory equipment and size of buildings. There can be no question that much more research must be done in our engineering colleges than obtains at present. Attempting to solve new problems is the best means for developing initiative and the ability to think.

If it be admitted that research surroundings are essential to the proper training of engineering students, and that present conditions in most engineering and technical schools make worthwhile investigations well nigh impossible, it is evident that the question of improving existing conditions must be given serious consideration.

Of course most of the difficulties that beset our engineering colleges would be eliminated if adequate funds were available for research facilities and pay for staff members comparable to those provided by industrial organizations; but there is little hope for general relief in this direction, as the number of students seeking training is, in most institutions, increasing at a greater rate than the material wherewithal. As a more likely method for bringing engineering students in touch with worthwhile investigations, the author would stress the importance of a closer and more extensive cooperation between the research divisions of industrial organizations and educational institutions. The following suggestions are submitted:

1. Industrial organizations should establish numerous *research fellowships in engineering colleges*, not as gratuities but as necessary investments against their own future need of properly trained additions to their staffs.

2. *Cooperative work between research departments of industrial organizations and educational institutions* should be greatly increased.

3. Some form of *bonus system* should be established by which engineering colleges would receive financial returns from industrial organizations for having discovered and trained exceptionally successful engineers.

4. Let educational institutions take out *patents on new ideas of commercial value* originated by faculty members; that is, expand and make effective the plan recently adopted by Columbia University.

5. Establish research foundations as integral parts of engineering colleges.

6. Let a larger share of the normal income of educational institutions be expended on investigational work. This would be in accord with the policy adopted by industrial organizations. Twenty or thirty years ago the budgets for research were insignificant in most of our industrial organizations, while today the research divisions in several manufacturing and operating companies expend millions of dollars annually.

7. There should be instituted an *exchange of engineers* between engineering faculties and engineering staffs of manufacturing companies and other industrial organizations; the plan might be somewhat similar to the *exchange of professors* between European and American universities. If a man for man exchange could not be effected, let the manufacturing companies donate the services (Sabbatical year) of some of their engineers, who would devote all of their time in selected engineering schools during a full academic year.

To properly train engineers is an expensive process. To obtain satisfactory results our engineering colleges must be provided with much *greater financial support* in the future than has been the case in the past. If the service rendered by our engineering colleges is of value to industrial development, let organized industries assist in providing adequate means so that the work may be well done.

If the basic necessity for research on a scale larger than hitherto exercised in our engineering colleges and technical schools is fully understood and appreciated, means and methods for improving existing conditions will no doubt be forthcoming.

Discussion

L. N. Robinson: Professor Magnusson emphasizes the point that engineering courses should teach students to think if nothing more. If that is so, why do not engineering curriculums, in general, include courses in the mental sciences, particularly logic and allied subjects? However, it is not my present purpose to argue for or against the inclusion of any particular subject in the curriculums. What should be pointed out is that present conditions indicate a general lack of scientific treatment in the design of engineering curriculums as well as in their application.

We insist that scientific methods should be employed in designing engineering structures. Engineering students have an equal right to insist that scientific methods be employed in designing the curriculums of engineering schools.

When electric generating stations are designed by the same methods that are employed in laying out many engineering school courses, hybrid plants are produced in about the same proportion as our engineering schools turn out bond and automobile salesmen.

In designing an engineering structure, it is customary to decide first what is to be designed,—whether it shall be a bridge, an office building, a locomotive or something else. Next we determine which parts shall be of steel, which of copper, etc., then we prescribe methods of fabrication and assembly. In other words, every step from the inception of the enterprise to the finished product is worked out with utmost scientific care.

Is this method followed in designing engineering curriculums and in training engineers? Before it can be said that engineering courses, themselves, are scientifically designed, we should at least first determine what the product of an engineering course should be. Then the elements that should constitute the course can readily be determined, with equal care to deciding how the courses shall be conducted.

Last June at a commencement the chancellor of a university said that the world is full of people who know all about education except what it is for. This is a challenge to the engineering fraternity and to the engineering schools in particular; it demands a scientifically sound justification for our college courses and especially for our engineering courses. And, since we profess to be scientific in our engineering work, we should be better able than the humanists to answer the challenge. In doing so, however, we must be especially careful not to mistake consensus of opinion for sound scientific truth else we shall class ourselves with those who scoffed at Christopher Columbus because his views differed from generally accepted notions.

J. C. Clark: It seems to me that Professor Magnusson has lost a grand opportunity in his paper to bring out one of the main reasons why we should have research in engineering colleges. He has stressed the value of the educational aspect of research almost exclusively, and has pointed out that it nearly goes without saying that research has an indispensable part in technical education. Professor Sorensen also emphasized very strongly the educational value of research in the colleges. In other words, it is for the good of the colleges and the students that research is carried on in the colleges.

I believe that it ought to be pointed out by Professor Magnusson that the industry needs engineering research in the colleges as much, if not more, than do the colleges. As Professor Magnusson has so emphatically stated, it is true that the large corporations of today have millions to spend in research. They have large and well trained staffs to carry on industrial research, and they carry it on successfully and efficiently. On the other hand a great many of the smaller concerns have very meager appropriations with which to work and carry on research.

It follows that most of us very much need to have a place to have scientific facts revealed. Not only is this true in the smaller manufacturing industries, but it is even more vital for the public to have these independent public laboratories of research which the colleges and universities can so well maintain. It really is not so much the amount of work done that matters, as it is the unbiased check that such laboratories can give upon the results that are revealed and published by the larger corporations.

We do not charge that the large corporations color what they print about their laboratory results, but it is human and almost inevitable that they show the best side of what they find out, and somewhat neglect the unfavorable aspects of those things they are trying to promote.

By doing a very little work sometimes with small funds, the smaller college laboratories can aid industry and the social body a great deal. The exact amount of money at their disposal is not of prime importance although it is true that they need a vastly greater support than they now have.

The professor who has one thousand dollars a year to spend for all his equipment, including its repair, and maintenance, is so handicapped that he can surely do but little with the energy that he puts into the work.

I should endorse what Professor Magnusson said: Industrial establishments of any size that do no research have "signed up for the exit." However, industrial research in smaller establishments may not be carried on because they have not the facilities, and thus they have to depend upon the colleges.

I find much in Professor Sorensen's paper which is of interest. Professor Sorensen starts out with an assumption which surely needs no argument,—that research and engineering are insepa-

rable. Research *must be* a part of engineering education. He takes the same attitude as Professor Magnusson does in pointing out the necessity for it from the standpoint of education. This is self-evident indeed.

Professor Sorensen gives the time honored definition of engineering that I believe was given by Tredgold, the first President of the Institution of Civil Engineers in England; "The art of directing the great sources of power and nature for the use and convenience of man," etc., running through many, many words. It starts as a catalogue of the functions of engineers and thus its weakness is obvious. It can, of course, only begin to catalogue the functions of engineers, but I think that in this very definition, a part of the weakness of modern engineering education lies. Educators and engineers have been more or less hypnotized by such definitions of engineering, emphasis being placed entirely upon the technical aspect of engineering, the scientific, and the big and glorious things to be done by engineers.

I should substitute a very much shorter definition for engineering. I define an engineer as one who directs the economic use of matter or energy. I should let it go at that because I believe that we have among us as great engineers who are directing the destinies of banks or steamship companies, from the standpoint of executive officers, as we have in any other capacity. In other words, technique should not be stressed exclusively in the work that is done in the colleges. There should be a little more emphasis placed on economics. There are very few colleges today that devote any great amount of time to the study of engineering economics. I do not see how it is possible to turn out men with a proper attitude toward engineering without giving them much study of the principles of engineering economics, not the old social economics, but the engineering student does need a very considerable amount of engineering economics if he ever becomes a true engineer.

I wonder how much deleterious effect this long-drawn-out definition of Tredgold's has had upon engineering education. Has it not hypnotized the profession somewhat into thinking too much about the imparting of information about technical things, perhaps information about the things that can best be learned by the man after he is out of college?

In Professor Sorensen's paper I found one thing with which I think many men in practical research would particularly take issue. That is the standpoint that the undergraduates' thesis was wisely eliminated. Professor Sorensen says "it was therefore wisely eliminated." The very next sentence takes the curse off, "But we should not forget that the thesis is symbolical of research—a function which must be the keystone of engineering education if the engineer is to occupy the place for which he is ambitious."

I think that is certainly true. The thesis is needed to support the idea of research, to give some little practise in the methods of research, with the object of bringing out latent research talent in the student.

Professor Sorensen reveals that he has some such idea in a latter part of his paper, since he speaks of the honor sections that are established at Pasadena where the better men are permitted some chance to help in advanced and special work, assisting research men. I think he has there revealed his true attitude toward the research that may be done by undergraduates.

It seems to me that the practise of segregating the students into the ordinary and the extraordinary,—the latter class including these so-called honor men,—is one of the very best developments I have ever seen. I wonder if the real reason for eliminating the undergraduate thesis was not rather the difficulties that were encountered in lack of equipment or lack of time and energy for the administration of the theses than the lack of educational and developmental value in them?

In the paper by Dean Pender and the one by Professor Sorensen there are two quite distinct methods given for the selection of engineering student material. Of the two, I think the one that is the more hopeful is the one that is being tried out in Pennsylvania. The statement that Harold Pender made is quite true; that a boy fresh from high school is usually just a boy,—and I think it does require the few years of college which are a part of the Pennsylvania plan to bring out the judgment of a student and too, to weed out the defective material found among students. It is not defective material except that it is unfit for engineering although possibly very fit for some other, perhaps higher, walk of life. At any rate, there is a great difference among students in their fitness for engineering.

I am somewhat in doubt as to the real value of the physical examination in the Pasadena plan because we have in our electrical history so many able men who might have had difficulty in passing the Pasadena physical test. Is it not going to eliminate an occasional man who has great latent ability in analytical ways?

H. V. Carpenter: This matter of engineering education is one that we shall always talk about I suppose. The Society for the Promotion of Engineering Education is at the present time spending something like a hundred thousand dollars, and I don't know how many dollars' worth of time, in analyzing methods of engineering teaching.

It seems to me that one of the most difficult problems we have before us in that study is this matter of research. You remember that some Roman ruler said, "All right, if we destroy the Greek statuary, we shall send some men out and have it replaced." If we aren't careful, we shall put our research work on that same mechanical basis. I believe that a research man is born. Maybe Dean Magnusson can make a research man, but I am afraid I cannot. I think it is more a matter with us of being able to discover the research man when he comes along in our classes, rather than being able to take an entire class of students and turning them all into first-class research thinkers.

What we need to do is to teach these men to think, and research is only one of the better methods of teaching a man to think. A large share of these boys will never be research thinkers, but they will be very effective engineering thinkers in the ordinary sense of being able to go out and size up a job and put through the best design for it. They may not be research men, but they will be thoroughly useful men on straight engineering propositions.

It is our business to give the man who can do original thinking the inspiration to go ahead to develop his abilities in that line, and to give him a chance in the laboratory. Professor Sorensen's scheme is a very good one; they have adopted a similar one at the Massachusetts Institute of Technology where the honor men are to be given a chance to do as much as they please. The smaller institutions have always done that to a considerable degree in an easier way, having a much simpler problem.

Dropping the thesis was a good thing, I think. I agree with Professor Sorensen on that as a requirement for every student, but for the student who shows interest, I think the thesis should be maintained, and carefully nursed along. We cannot expect revolutionary things from the senior, but perhaps if we start a real genius to thinking he may turn out a revolutionary piece of work in later years.

L. J. Corbett: Doctor Magnussen describes the tendency to take research away from the colleges. I think we need not fear for that, as times are changing. As he states, the large companies are doing a great deal of research, but there is another factor; they also need men—and one of the ways to develop men is through this very research work. I believe this is being recognized by some of our large companies, as evidenced by the assistance which has been given recently, to both the California Institute of Technology and Leland Stanford, Jr., University, in the way of aid in the establishment of high-voltage laboratories. These, no doubt, will give a good account of themselves in con-

tributions to engineering research, and some students will recivee valuable training in this field.

From Mr. Sorensen's paper I note that they are omitting the engineering thesis at the institution he represents. However, from the discussion which took place, I see that there is still some opportunity given for the honor men to investigate along the lines of their special inclinations, thus recognizing work on such theses.

To return to Doctor Magnusson's paper, we must recognize the tendency of the companies of today to require specialization in their men. This, I think, has been the inference in the advice which has often been given to engineering colleges to have actual departments of research where some men of the staff could go ahead and do research only, without the change in mental attitude which is necessary when a man does some research and also teaches a group of undergraduates, old, well-known material.

I think the work of the teacher, as Professor Carpenter has stated, is inspirational. I think that if you can inspire a student to make the best use of his faculties, and not to consider his education complete when he has his degree, but to go ahead with his studies and research—you will do a greater work than merely filling him with information and making an encyclopedia of him. I think this quality is particularly exemplified in the man we have with us in the person of Professor Ryan. I believe that all his students will vouch for the inspirational quality of his work.

There is one feature in Dr. Pender's paper that struck a discordant note. In one of his conclusions there is the statement that the student should recognize that engineering is a profession, and not a job. That is all very well for us to recognize, but I think we should not emphasize it too strongly to the undergraduate student, because it is due to such thoughts that we have men who are reluctant to don overalls after getting their degrees. I think a man would get farther in the engineering profession if he were willing to don overalls for a time after his graduation and learn the rudiments of practical work.

J. S. Bates: I once heard an engineer defined as a man who is skilled in the use of the word "approximately." There is really a great deal in that because no matter to how many decimal places we carry our calculations, there is always one more; one or a vast multitude makes no difference. For that reason we should say a very important part of an engineer's education is to determine what percentage of error is to be allowed.

G. S. Smith: In the University of Washington we still have the thesis work scheduled as an elective but it has been virtually discarded, since it is seldom chosen by the student. However, I believe we have found other ways more effective in obtaining the results aimed for in the usual thesis work.

Several of our courses scheduled for upper classmen and graduates, are presented in such a manner that each student, or more often a group of two or three students, selects or is assigned some individual problem to be worked out completely. These problems are usually of such a nature that they require a considerable amount of thought, reference work, or experimental work, and thus arouse in the student any latent inclinations toward research work.

As an example I should like to mention a course to which we have paid a good deal of attention in our institution; that of electrical transients as a prescribed course of undergraduate study. In the laboratory part of this course a certain number of topics are assigned to the student for which he must obtain representative oscillograms. This is the more or less routine or practise part of the course. To satisfy the remainder of the requirements, the student must select some problem or topic, acceptable to the instructor, to be investigated by the taking of oscillograms. This problem must be one which has not been previously chosen by other students who have taken the course. Thus their work is, to a certain extent, original.

The response on the students' part is usually more than gratifying. They not only put more energy and thought into this part of the work than in the routine portion, but they also show a strong tendency to attack it from the investigational point of view; that is, they try to find the best method of attack, try to determine the results they expect, and then attempt to verify them experimentally, watching all the while for unexpected results. At times, of course, they make complete failures, but more often they are very successful.

H. H. Henline (communicated after adjournment): I wish to emphasize the importance of full and frank discussion of the problems of engineering education by members of engineering faculties as well as by executives in industry. These authors have pointed out some very serious defects which are found in many curriculums.

Most of the engineering curriculums now in effect were planned twenty or more years ago, and changes made since have not altered in any essential details the general plans followed originally. Therefore, we find that most of the curriculums furnish excellent preparation for certain types of work which some of the graduates enter. However, this number seems small when compared with the total. On account of the extremely rapid progress which has been made in many branches of engineering during the past few years, we find ourselves living in a period when the applications of engineering knowledge are so many and diversified in character that any curriculum designed to meet directly certain needs in industry may indeed prepare men in a most excellent manner for those needs, but it fails utterly to prepare them for the great range of engineering problems, both executive and technical, which all graduates will be called upon to solve.

There is a strong and growing tendency to choose executives from the men with engineering training, since the problems which executives in industry must meet are becoming so complex and so closely allied with fundamentals of engineering that great dependence must be placed on the judgment of engineers in order to reach the correct solution. Obviously the schools cannot train men directly for executive positions, since qualifications for such positions depend greatly upon inherent characteristics, and any amount of training would not make capable executives of men who do not possess the necessary characteristics. However, if we wish the engineering graduates who do possess such characteristics to have opportunity to enter the management side of engineering, we must give them the broad, general foundation so absolutely essential.

One of the most noticeable features in the large amount of discussion of curriculums occurring in recent years is the fact that many of our largest employers of technical graduates now wish to secure men who have had a broad training in general subjects and the fundamentals of engineering rather than men who have specialized in a particular branch of engineering. Thus we find that the old situation in which teachers wanted to adhere to fundamentals, and many executives in industry advocated certain specialized courses, is rapidly reversing. Now we find the leaders in industry not only frowning upon specialized courses, but even considering many courses as too highly specialized which are given with the excuse that they are necessary from the standpoint of fundamentals.

In the planning of an engineering curriculum, certain decisions must be made as to the types of activity for which it should prepare men. In the present stage of development it seems necessary to recognize the needs of two distinct groups of students. Those who expect to spend their lives in highly technical design or research must have a more extended technical preparation than those who will be engaged in commercial or industrial phases of engineering. Both groups need a broad foundation on such subjects as English, economics, biology, geology, history, business law, etc., and a thorough training in chemistry, physics, mathematics, mechanics and other subjects which make up the heart of engineering. Such training should

be mixed with, and followed by, courses giving the fundamentals of all of the principal branches of engineering, and there should be a reasonable amount of time available for elective subjects. Thus far there is no serious difference between the wishes of executives in industry and teachers of engineering. It, therefore, seems that the chief cause of argument is the relatively small group of men who will engage in research and other highly technical phases of engineering. This group must have better opportunities for the development of research ability and specialized study than can be provided in any four-year course which makes necessary the provisions for training in the fundamentals.

It seems that the best solution of the problem may be a four-year course following the general outline mentioned above and leading to some such degree as Bachelor of Arts or Bachelor of Science in Engineering. This curriculum should be so planned that students may have good opportunity to develop powers of initiative and judgment to the maximum extent, and to determine the kind of intellectual effort for which they are best qualified. In order that they may see how they can best fit into that field, it should give them a broad outlook and an excellent perspective of the whole engineering field. Graduates of such a curriculum would be prepared for commercial pursuits and for many types of technical work in which employers prefer that specialization be deferred until after the entry into industry. It is believed that they would advance more rapidly in either commercial or technical employment than if they had graduated from the narrower four-year engineering curriculum.

For those who wish to prepare for research and other highly technical branches of engineering, there should be provided pportunities for two years of graduate study. Having completed the liberal four-year curriculum, they would have the broad foundation which should precede advanced study. The graduate curriculum should be planned with the idea of developing ability in research and permitting specialization in analytical studies, design, or other branches of engineering. Since the number of students in the graduate classes would be relatively small, the effectiveness of the training would be maximum. The progress would be much better than could possibly be made in the same work with undergraduate classes containing many students not really qualified for or genuinely interested in such subjects.

I believe this combination of four years' liberal curriculum for all, and two-year graduate curriculum for those who are properly endowed for and desire it, would result in producing men better qualified to take their proper places in the world than are those who complete most of the present-day curriculum.

All engineering students should realize that it is impossible to escape passing through an apprenticeship period of some sort. A university curriculum cannot possibly replace the apprenticeship period except in the cases of the comparatively small number of persons who are preparing for research and highly technical progress. Therefore, it is extremely important that the curriculum be such as will aid in choosing the proper kind of apprenticeship to fit the individual's mental endowment and his aptitudes. There should be fewer misfits because the first four years' training would give them an excellent foundation, and they could choose the type of work most interesting to them. This would eliminate a very real difficulty now in common existence. For instance, a boy before graduating from high school has built radio sets, worked in a local power plant, or had some kind of electrical experience. Perhaps electricity appeals to him more strongly than any other subject. He wants a university education because he has been told that it will enable him to earn a good salary immediately after graduation, and will go far toward insuring success in later life. What then is more natural than for him to register in electrical engineering, because here is the opportunity, in his opinion, to secure the university education he wishes and at the same time specialize in his favorite subject. Naturally he has considerable pride in his choice and is reluctant to make any change even though the first year or two may prove that he does not possess the necessary types of ability to succeed in engineering. He may eventually graduate and begin work, still determined to be a successful electrical engineer. The result is in many cases an employe who has not the ability and characteristics necessary for his work. He must then work on as best he can and be a failure or only a mediocre success in electrical engineering, or find something to which he is better adapted. In so far as possible, a young man's interests and abilities should be determined before he graduates. If this can be fairly well accomplished and he can be sent out either with the broad four-year training, or better, with both it and the two-year graduate study, his chances of success should be greatly improved, and he should be a happier man in later life.

R. W. Sorensen: I would like to add to the list of books given at the end of Professor Ryan's paper one entitled, "Ions Electrons, and Ionization Radiations" by J. H. Crowther, published by Edward Arnold, London. That book is easy to read and it presents in a reliable manner much of the information about ions that engineers wish to know.

In the three papers bearing directly upon education, you will, of course, find differences of opinion, and so there should be. The biggest crime educators could be guilty of would be that of making a standard curriculum, which would be the same for all engineering colleges. In fact, we often remind ourselves at California Institute of Technology that we must not simply add to the group of good engineering courses in California just another course like the one at our State University or like Professor Ryan has developed at Leland Stanford. We at the Institute are spending annually about $700 per year per student enrolled, to carry on our work. That money has been given us by individuals for a specific purpose and should not be used to duplicate the work of other institutions.

Dr. Magnusson has sounded a keynote when he says every faculty man should do enough research work to show that he has the ability to inspire students in that direction. Some undergraduates are qualified to do research work along with their regular undergraduate work. Such men should be given an opportunity to do that work. Also, there should be graduate students, the more the better, doing research work, and the college should make such provisions for such a plan. In this way, every student has a chance to come in contact with research work, learn what it is, and methods of procedure. Every engineer will, to a large extent, have his success determined by the amount of research enthusiasm which he can develop, even though that enthusiasm is not applied to the type of problem ordinarily classified as a research problem.

As to the senior thesis for all, the discontinuance of that plan was due to several factors, one being lack of time on the part of the faculty to supervise many students and assign each student a problem of just the proper magnitude to fulfill the requirements for graduation. In place of the thesis, certain problems are assigned students, the problems selected being tempered by the circumstances under which the student is working. Such a problem does not have to be written in thesis form, thus allowing all the available time for work on the problem.

Why do we include four years of English in our curriculum? We have had complaints that the engineers are underpaid, and underpaid chiefly for one reason, that the engineer confines most of his contact with men to those who talk only engineering language, and there is no use to try to sell engineering information to other engineers unless you are a much better engineer than the other man, which doesn't happen very often. On the other hand, there are thousands of people in the world who want engineering information and who would take it if presented to them so they could understand it. You can't present engineering data to the artist in a way that he will understand it unless you know his language; you can't present it to the doctor, lawyer, or

merchant in a way that they can understand, unless you know something of their language.

Therefore, we have a department of English which runs through five years of our five-year course, and we must have a large department in order to keep the type of instructors we have, instructors whose business it is to inspire men and cause them to make of themselves all-round men. That is the reason we think it worth while to put in a five-year humanity course right along with a five-year technical course. Many of us would not do the things we do if some seemingly insignificant thing had not inspired us to go in an inviting direction. Without the instruction we would have failed in finding the doorway to new and interesting fields of endeavor.

In reply to the comments made by Messrs. Robinson, Clark, Henline and others, there are many details about which we could argue, but each exception only makes plainer the fact that all individuals should not be required to follow the same course of preparation for engineering work. Some men should prepare a thesis, some should not, some should have five years of education in the field of English literature, including some history and economics, others should find it more profitable to plan a different use of their time, but in nearly every case, I would conclude that in the essential things there are no differences of opinion—except as to relative values. This condition could not be otherwise because engineers differ as to details even in designing machines, when a much smaller number of variables are encountered than is the case when we attempt to form the character of the youth who will in the future be our engineers.

Reference to values in character building prompts me to direct attention to the value of the training obtained on the athletic field as evidenced by the success of many of our engineering students who participate in athletics. A survey of the condition governing athletics at many of our educational institutions indicates to me that the advantages of athletics are for the most part lost to engineers because training for athletics has become such a time-absorbing specialty as to make it almost impossible for an engineering student to be on his college team. Our educators in the technical field, and those who are our friends, should help correct this condition of affairs.

C. E. Magnusson: Mr. Robinson's remarks lead me to think that I failed to state clearly what I had in mind. I meant to say that education is essentially training to think; that engineering education is training to think along engineering lines; that the main purpose of engineering students during their four years in college should be to gain clear concepts of the basic physical laws underlying engineering and acquire the ability to apply these laws to the solution of quantitative practical problems. The rest of their college work is accessory to this backbone of engineering education and may vary widely for different individuals. It goes without saying that all live engineering colleges frequently revise their curriculums. At the University of Washington, the revision of curriculums in the college of engineering comes regularly on the calendar every four years, and many changes both in regard to required courses and their content have been made during the past twenty years. I am grateful to Mr. Clark for emphasizing the value to industrial establishments of research in our engineering colleges although this phase of the problem does not come under the title of my paper.

Improvement in Distribution Methods

BY S. B. HOOD[1]

Member, A. I. E. E.

Synopsis.—*The object of this paper is to point out how protective measures which have been applied to a low-voltage distribution network can be utilized to improve the operating characteristics of the entire distribution system, as well as materially lower its installation cost.*

Many of the developments herein described are novel departures from more generally recognized practises. They have all, however, withstood the test of time and usage.

The common neutral system of distribution described has steadily increased in its application since first developed by the author for use in Toronto, Canada, about 16 years ago. It is at present the standard system with a number of large utilities with an aggregate capacity of some 300,000 kw. The particular system described is that of the Northern States Power Company in the Minneapolis territory, representing about 60,000 kw., or about one-half the total output of this widespread organization.

The heavy duty a-c. underground network and the remote-control multiple street lighting systems described are both later developments that are more or less inter-related with this common neutral development.

PROTECTIVE GROUNDING

In order that the lives and property of the community may be protected against the hazard due to the low-voltage utilization circuits becoming crossed with those of dangerously high voltage, it is now compulsory to install protective grounds.

Such compulsory grounding requirements are embodied in both the National Electrical Code and the National Electrical Safety Code.

The desirability of such grounding is no longer a matter for question or argument, as it has, in innumerable cases, proven its protective value.

However, where such grounding is not carried out strictly in accordance with both the wording and the spirit of the rules in both codes, it is worse than useless in that it creates a false sense of security.

There is little question but that the responsibility of seeing that these grounding rules are enforced rests initially with the electric utility engineer. Unless he can create a more equitable division of the responsibility by transferring a part to the consumer through municipal regulation or company rules, his company must not only shoulder the responsibility but also the burden of total cost involved.

In a number of our large cities, such distribution is effected through the local inspection authorities, requiring that each and every service be properly grounded to the water-supply piping in the buildings served, this being done by the customer whom it protects and at his expense.

It is particularly unfortunate that a number of communities still refuse to make such regulations, even after the American Water Works Association has recommended that such grounding be permitted without fear of injury to the water system. In still other cases municipal authorities are not only trying to evade this responsibility but are attempting to exact a toll from the electric utility for use of the water system as a means of protection to the community. Where such unfortunate conditions exist pressure should be brought to bear to not only permit, but require, the use of the water system for protective grounding, as it imposes no burden or expense.

Where the low-voltage secondary network is adequately protected by grounding, there can be no hazard from high-voltage crosses, either accidental or intentional.

Any system of protective grounding to be effective must be capable of safely passing the heaviest current required to trip a feeder circuit breaker. Where the nominal feeder rating is 300 amperes per phase, it follows that about 500 amperes will be the tripping value.

It must be self evident to any engineer that to ground a section of secondary by means of one or two driven pipe grounds is hopelessly inadequate. Such grounds are, as a rule, incapable of blowing a branch circuit house fuse when a ground occurs on an outer wire on the customer's premises. This type of grounding was originally installed in Minneapolis and later reinforced by installing at least one service ground in each city block, using the water service for this purpose. This effectually eliminated the troubles due to former inability to blow clear accidental grounds on the outer wires of the secondary bus section. It was not, however, sufficient to protect against a heavy feeder cross between primary and secondary. In order to get the benefit of all existing grounds, by connecting them in multiple, the various secondary section neutrals were interconnected. This was a much smaller undertaking than might be supposed, in that service requirements made most of the secondary sections terminate within one or two spans of each other. In order to avoid placing an anchor guy at each of these ends, or a continuous head guy, it was general practise to carry the neutral conductor through these gaps and insert a strain insulator at the midway point. All that was necessary to interconnect these sections was to place jumpers across these numerous neutral strain insulators, as when this was done there was automatically established

1. Of the Northern States Power Company, Minneapolis, Minn.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., September 15-19, 1925.

a system neutral in the form of a vast grid or screen that blanketed the entire area served. With thousands of line and service grounds connected to this, the resistance to earth was so low that the heaviest current obtainable through any form of cross could be carried to ground with absolute safety and without raising the potential to earth of the secondary system by any appreciable amount. In later years the City Electrical Department has required that every new service must have its own protective ground, installed by and at the expense of the building owner to water piping where available. This has assured a perpetual improvement of grounding conditions with growth of the system, as over 5000 new services, with their grounds, are connected to the system annually.

In order to assure low resistance to ground, periodic tests are made between system neutral and water mains and where the results show in excess of one-tenth ohm additional service grounds are installed.

In districts outside the water main areas, service grounds are attached to driven well casings in preference to using driven pipe grounds or other forms of artificial grounds. These are only used as a last resort.

Wherever economically possible, the secondary mains are extended for new customers in preference to extending the primary and installing additional small transformers. With this plan it is assured that the system neutral will always cover a greater area than the primary system and that there will be no isolated secondary sections not amply protected by connection to the system neutral.

Since Minneapolis is very similar in layout to other large cities, it follows that the plan of protection outlined above may be applied to almost any typical community, and has been applied by the author to three of our major cities.

Primary Distribution Systems

With the rapid growth of the industry, developing load densities far in excess of those previously thought possible, it has become very evident that the once commonly used 2300-volt, three-phase, delta primary system was not only uneconomical, but also impracticable unless substations were spaced much too close to make their size or operating costs reasonably efficient. It was therefore logical to change to some form of system that transmitted at a higher voltage, but still permitted the use of the vast amount of transforming equipment already in service. The star connection of the older equipment was the natural solution of this problem, and today we find the 2300/4000-volt, star-connected, four-wire primary system almost universal in cities of any size. By stringing in a fourth wire for the neutral, and reconnecting the transformer primaries, the old 2300-volt, delta system can be changed to transmit three times the energy over the same distance with equal loss, or the same energy over three times the distance. In practise, it has enabled us to carry a considerable amount of extra energy over each outgoing feeder and at the same time space our substations much further apart, these also naturally being of larger size than would have been practicable with the older system.

However, in order to safely use the same transforming equipment as for the older system, it was essential that this neutral wire be definitely stabilized in its voltage to earth. This is commonly done by grounding the primary neutral at the substation, connecting it directly to a water main or other known effective ground. Each outgoing feeder is then provided with its own isolated neutral over which any phase unbalance on the feeder returns. Under normal operating conditions this has worked out very well and there is no residual current to cause induction on adjacent low voltage signal or communication systems.

However, under certain abnormal operating conditions this method of stabilizing the neutral has proven far from effective. We are all familiar with the complex conditions that arise on the old Edison three-wire, direct-current systems when, for any reason, a neutral is opened. In this later four-wire, three-phase primary system, we get very similar conditions except that the maximum rise in potential on any phase can not exceed 73 per cent above normal. When a single-phase short-circuit occurs, there must be a 2300-volt drop of potential on the phase wire and the neutral combined. Assuming that both are of same size, this means that the system neutral at the point of fault will be shifted out of its central position and the other two phases will have their potential raised about 36 per cent. If for any reason the neutral should open during the occurance of such a single-phase short circuit, then the neutral at point of fault will be shifted 73 per cent. Since either of these conditions impose an overstress to ground on all the transforming and protective equipment, as well as induce corresponding voltage-increases on the utilization equipment of the customer, such a system is very hazardous and uncertain in its operation. In the case of heavy phase grounds, excessive passage of current through the earth may result in interference to adjacent communication circuits through both earth potentials and high residuals in the line.

In order to guard against such conditions it has become general practise, where this type of system is used, to install single gap lightning arresters at various points along the primary neutral. These are supposed to break down at about 350 volts and thereby reestablish the relation of neutral to earth. In practise these do break down, but unfortunately or otherwise, very often stay broken down. In other words there are established numerous more or less unreliable points of multiple grounding on the primary neutral. Since there are no ready means of determining such relief gap failures, these grounds usually are permitted to remain on the system, just as are accidental grounds

permitted to remain on the neutral of any extensive Edison low-voltage, three-wire network.

It follows, therefore, that many so-called single point protective grounded primary systems are today operating successfully with innumerable unadmitted, neutral grounds. Occasionally induction in adjacent telephone circuits leads to temporarily clearing these grounds, but they soon reoccur unless constantly checked up.

In the development of the system as used in Minneapolis, it has always been openly admitted and claimed

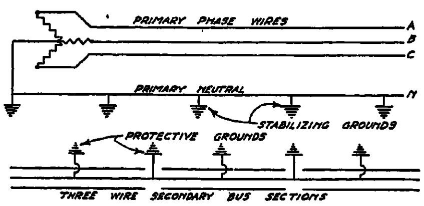

FIG. 1—DUPLICATION OF NEUTRALS

that a four-wire, star-connected primary system must have a large number of distributed multiple grounds on the neutral in order to make it practicable in operation.

By reference to Fig. 1 it will be noted that such a primary neutral is shown paralleling the system secondary neutral previously described. If this method is used, it gives two wires both held definitely at earth potential by numerous grounds. While the low-voltage

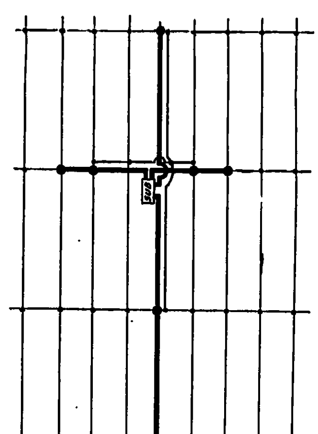

FIG. 2—NEUTRAL GRID

neutral can be adequately protected by means of service grounds, the primary neutral can not readily be so protected and, therefore, the necessary reliable grounds can only be obtained with difficulty, and ground wires must be carried down to the water mains buried under the streets. When, however, both neutrals are so protected, it follows that they are one and the same wire in so far as potential to earth is concerned, and, as such, may be combined physically. Where this is done we have what is now known as the "common neutral" system.

In Fig. 2 it will be noted how this common neutral forms many metallic return paths in addition to those through the earth. In order to avoid concentration of heavy currents on the system, due to abnormal operating conditions, neutral grid near the substation's heavy neutrals are carried out of the substation in several directions. These generally are extended for several blocks in each direction and tap the system neutral at each intersecting point. This arrangement is clearly shown by the heavy lines in Fig. 2, which represent the only actual primary neutral copper necessary where a common neutral system is used. All the other neutral is that required in the secondary system in order to obtain adequate protective grounding irrespective of the type of primary system used.

With this form of system, there are so many assured paths for both normal and abnormal neutral return currents that the entire primary neutral system is definitely held at earth potential. Normally the return current will take, by preference, the path closest to that of the primary phase wire, tests having shown that at least 65 per cent of the return will take this path irrespective of the number of other metallic paths established in addition to those through the earth. This is due to the close loop formed by the phase wire and its parallel neutral giving a far lower impedance than any of the wide open loops over other routes or through earth.

Since periodic tests show resistance values not to exceed one tenth of an ohm between system neutral and earth, it would appear somewhat inconsistent to find so large a percentage of the return current taking the metallic path closest to the phase wire unless one keeps in mind that this division is largely governed by impedance and not by resistance.

From the economic standpoint this system not only saves all the neutral copper required on the three-phase feeder of the ordinary four-wire system, except the small amount adjacent to the substations, but also eliminates one of the two wires on every single-phase branch taken off the three-phase line, and these form a considerable percentage of the total copper in any primary distribution system.

The transmission losses with this system are materially less than that of the more commonly used four-wire system in that the effective area of the neutral return is always greater. In general it would appear that for the portion distributed as single-phase load, including all branches and any unbalance on the three-phase routes, the loss will be from 25 to 40 per cent less.

Since the neutral is definitely stabilized at all points, it is no longer necessary to place protective equipment on both sides of the transformer primaries, the low side being tapped directly to the system neutral. This, of course, saves one-half the investment, troubles, etc., in such protective equipment.

In operation, the system has always proven far more reliable than either the usual form of four-wire system

or the older three-wire delta systems. This is partly due to the lower wire mileage involved, but more largely to the fact that only one primary phase-wire is required on all single-phase branches where most of the usual troubles occur. In such locations, the system neutral is spaced vertically and below the phase wire and can not be readily fouled with it. The general arrangement of the pole head on single-phase construction is shown

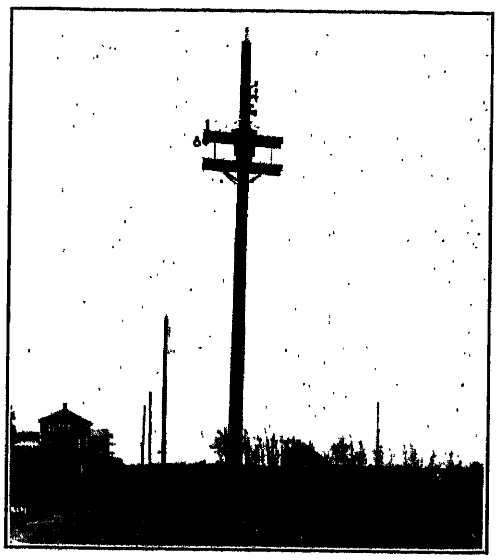

Fig. 3—Common Neutral Pole Head

in Fig. 3 which also shows the method of transformer mounting.

In order to permit of emergency operation of feeders, the general arrangement shown in Fig. 4 is adhered to as closely as typographical conditions will permit. Each feeder is provided with a single-pole, oil switch in each phase wire just before the loads are taken off. A series

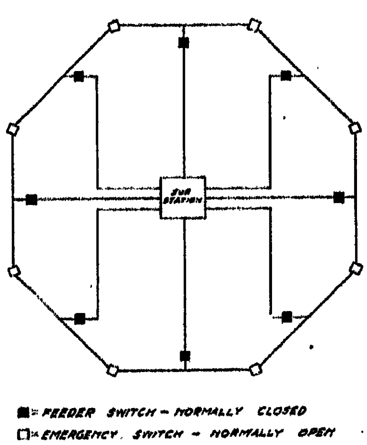

Fig. 4—Typical Feeder Layout

of emergency ties are installed between adjacent feeders, forming a ring in which each segment is sectionalized by groups of single-pole, oil switches, which are normally operated in the open position. This arrangement permits of taking the load from any one feeder and transferring it to at least two adjacent feeders.

Outside of this ring, the various phases are split up into small, single-phase areas, arranged as far as possible somewhat as shown in Fig. 5. The phase relations are distributed so that the minimum possible residual current is permitted along any one three-phase sub-feeder route. Since each feeder is of nominal 300-ampere rating per phase, it follows that each single-phase section is of about 240-kv-a. capacity. The section rings are adhered to wherever practicable in order to distribute the voltage drop as evenly as possible when covering a given mileage of street. A further advantage is that any break in a ring does not interrupt service, and when desired, such breaks can be cut out until repairs are permanently made under favorable weather conditions. The record of operation of automatic reclosing breakers has shown that it is very rare to have a feeder stay shorted after the third operation of the breaker, when it locks out. No fuses are used in any portion of the primary network as it has been found that these will blow under momentary overloads and will seldom blow under contact of a primary wire with

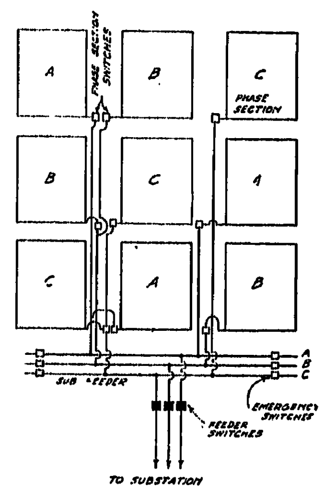

Fig. 5—Phase Area Distribution

the earth. Under such conditions they do not offer any material protection but do represent a hazard to continuity of service.

Lightning Protection

The ability of transformers connected to the common neutral system to resist damage from lightning is very marked.

On the Minneapolis system the total number of transformers burned out, from all causes, known and unknown, averaged eight-tenths of one per cent per annum, averaged over a period of three years.

Minnesota is noted for its severe and destructive lightning storms, and, therefore, this apparent immunity of the common neutral system can not be explained by lack of lightning hazard.

The use of the common system neutral, as a ground connection for lightning arresters, has not only fully

justified itself through favorable operating results, but was, in a large measure, adopted in self defense. Under the older and more generally used plan of having separate lightning arrester grounds, accidents due to arrester insulation breaking down and the baking out of the artificial ground occured far too often to justify the use of rules existing in defense of this method of lightning arrester ground connection.

For some unknown reason, the interconnection of secondaries is one of the factors in transformer immunity from lightning damage. It has been repeatedly noted that transformers feeding isolated secondary sections rather than part of the secondary network failed during lightning periods. Other transformers immediately adjacent to these isolated units showed 100 per cent immunity.

Inductive Coordination

The experience with the common neutral system in Minneapolis has very conclusively proven its desirability from the distribution standpoint, although under certain conditions which are now under investigation, it contributes to inductive disturbances occuring in adjacent communication circuits. It must not, therefore, be inferred that the common neutral system can be used without due consideration being given to proper coordination of the two types of circuits. Such coordination likewise implies a very close cooperation between the engineers of the two utilities.

In order to give full cognizance to the esthetic appearance of the streets of our communities it is advisable under certain conditions that the power and communication utilities use joint poles in overhead construction. It follows, therefore, that any practicable primary system of distribution must be such as will not appreciably impair the service of any well designed and maintained telephone system using the same poles as the power system.

It is not practicable to design or operate any power system that will be free from residual currents, neither is it practicable to design or operate a telephone system that will be so perfectly balanced that it will be void of susceptiveness to influences from the currents and voltages in the adjacent power system. The problem then narrows down to determining just how far either or both power and telephone systems can depart from the theoretical ideal and still give an entirely satisfactory service to all consumers served by both utilities.

To this end the engineers of both utilities have been working in very close harmony for some years. This cooperative work has culminated in the very elaborate and comprehensive tests that have been in progress for some months at the Minneapolis field laboratory and experimental line and which will not be completed until some time in the future.

Consequently, the results of these tests will probably not be available for general use for some considerable time; however, it is believed that within a short time the industry may look forward to an interim report covering the more important points of inductive influence and susceptiveness in the respective systems and the remedial measures that can be applied with a fair degree of assurance as to their effectiveness.

In actual practise it has been found that those remedial measures mentioned in the *Principles and Practises* of the Joint General Committee, such as reasonably close balancing of phases on the power system, the separation, where feasible, of the two classes of circuits, the grounding, where practicable, of aerial cable sheaths, and the use of high-impedance ringers, are valuable in reducing the respective influence and susceptiveness of the systems involved.

Other changes or additions tending to reduce interference are also possible at party line substations using grounded ringers.

Secondary Networks

Practically the entire Minneapolis system is operated

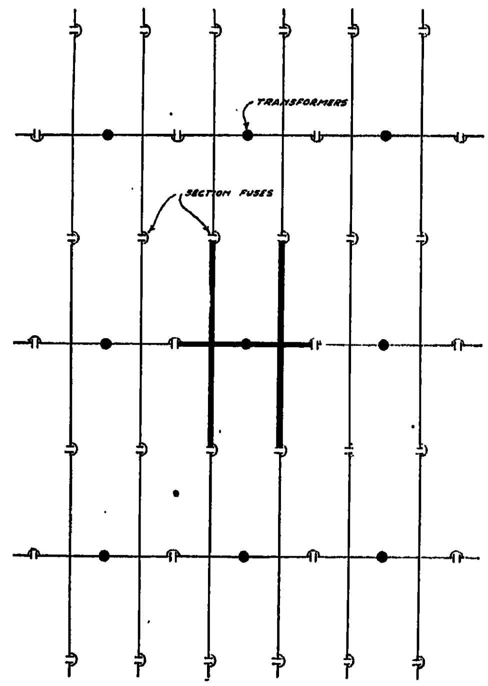

Fig. 6—Typical "H" Section Secondary Network

with the various secondary bus sections within any given phase area, interconnected. It has already been mentioned that the interconnection of the neutrals was easily accomplished, owing to the short gaps between sections. The outer wires of the three-wire secondary are extended in a similar manner and interconnected at the various midway points through copper-link limiting fuses proportioned so that they will blow if the current exceeds approximately 50 per cent of the capacity of the smallest adjacent transformer. Each transformer is also fused on its outer secondary leads by copper links which limit the current to about 200 per cent of transformer rating. The general arrangement of each secondary bus section is in the form of the letter *H* as

shown by heavy lines on Fig. 6. This also shows the adjacent secondary bus sections and the locations of the limiting section fuses between sections. It will be noted that any one transformer can drop out and the normal load on the section be easily carried by division between adjacent sections. In the event of a heavy

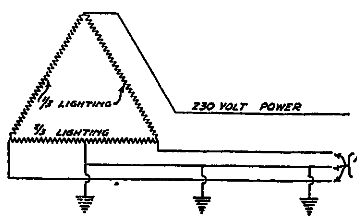

FIG. 7—FOUR-WIRE COMBINATION SECONDARY

secondary short circuit, both the transformer fuses and the section fuses will be blown and the defective section segregated from the network. This plan of interconnection not only assures reliability of service, and materially better regulation, but also takes advantage of about a ten per cent diversity factor between peaks on adjacent sections serving very similar loads. On

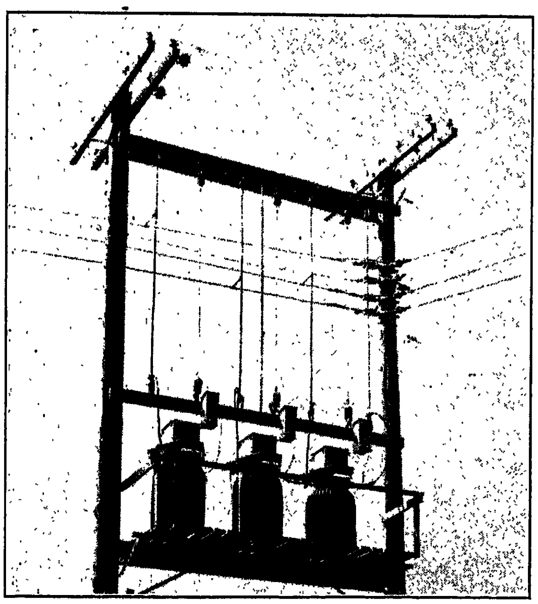

FIG. 8—COMBINATION TRANSFORMER INSTALLATION

ordinary residential service, transformers are regularly loaded to about 125 per cent full load rating on the winter system peak. This leaves sufficient reserve capacity to take care of emergency service to adjacent sections, as it has been found that transformers used for short-peak, winter service will safely carry up to 200 per cent rating. With this interconnection, it has been found that the primary neutral return current will split and a very considerable portion, at times as high as 60 per cent, will take the path through the secondary outside wires rather than through the common neutral. This materially increases the effective cross section of the common neutral and thus carries a greater percentage of the return current through the neutral in close parallel with the primary phase wire, thereby decreasing the extraneous return current for any given phase wire loading.

Where the power demand is a considerable portion of that for lighting, combination light and power transformer banks are used with the secondary delta tapped at the midpoint of one phase for the neutral. This connection is shown in Fig. 7 and a typical installation is shown in Fig. 8. With this connection one-third of the lighting load is carried by the two power transformers in vector series, the other two-thirds being carried by the lighting phase. It has been

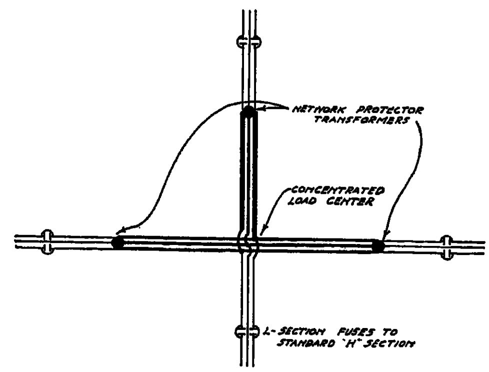

FIG. 9—THREE-UNIT NETWORK SECTION

found possible to carry both light and fluctuating power on these four-wire secondaries without material impairment of the regulation. Since the light and power peaks seldom directly superimpose, there is a very appreciable saving in investment where this connection is used.

For serving small commercial centers which develop close to intersecting car lines and which require a transformer capacity in excess of the average over the network, the secondary mains are of heavy copper, usually 4/0 gage, and the transformers are located at three or more points on the edge of the concentrated

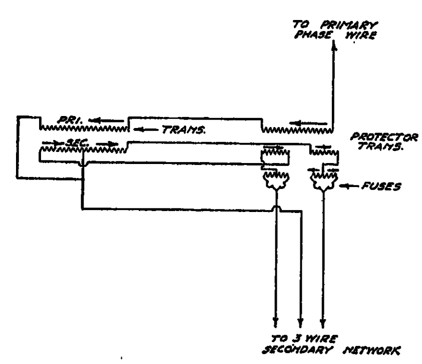

FIG. 10—NETWORK PROTECTOR

load district rather than near its center. This is necessary in order that there will be considerable impedance introduced between transformers in order to get proper division of load and selective sectionalization during times of trouble. The transformers used are provided with network protectors on the secondary

side and are thereby locked in on the system except in case of an actual failure within the transformer itself. The arrangement of these transformers is shown in Fig. 9 and the hook-up of the network protector feature in Fig. 10 the basic principle of which is simply neutralization of magnetic flux in the core of the auxiliary series transformer which is built into the standard transformer case. Any disturbance of this flux by reason of current reversal, due to internal faults, creates a potential in the tertiary windings across which the protective fuses are bridged. That this network system is not only economical but reliable is best shown by the fact that our transformer record for 1924 showed an average size transformer of 19 kv-a. and the burnouts were but seven-tenths of one per cent.

In the down-town, congested area of the city, there has recently been installed a heavy-duty, underground, secondary system, designed to take over all increased loads in this area, leaving the older d-c. system to continue in operation without growth until such time as it has depreciated to a point where its retirement can be justified. This sytem is served by transformers located in vaults built under the sidewalks. In general, each vault is provided with three 100-kv-a. units, although several have as high as 900-kv-a. capacity. The general arrangement is indicated in Fig. 11. The primary feeder serving this group of vaults is on the ring system, looping through each vault. On each side of the loop, automatic oil switches are installed together

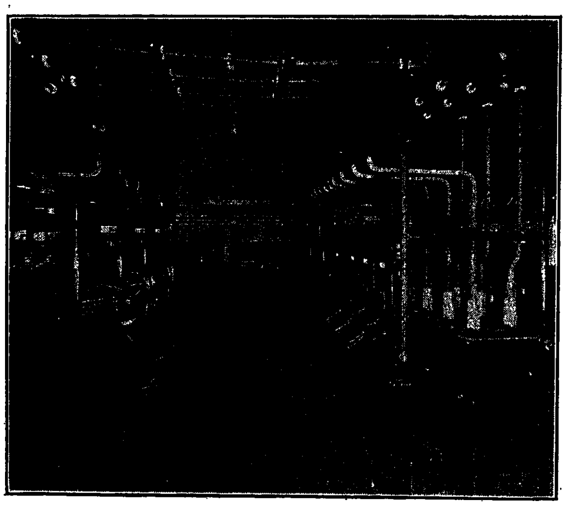

Fig. 11—Typical Transformer Vault

with relay trips. On the cable sections between vaults, the relays are differentially connected through pilot cables run in the same duct lines with the feeder. This locks the cable in except on faults in the cable itself. On the end sections of the loop the vault protection is by means of reverse power relays. At the major substation each end of the loop is provided with an overload-trip oil breaker and an automatic regulator. The control of the latter is cross connected to prevent hunting and hogging of the load.

In the two loops at present installed, as shown in Fig. 12, the substations are some distance outside the down-town area and normally serve a heavy demand in residential and apartment house service. By taking advantage of the peak diversity between this class of load and the down-town commercial and power loads, it has been possible to take on these down-town feeder loads without increasing the substation or transmission capacity. The use of 4000 volts has been justified in preference to direct use of the thirteen two-kv. trans-

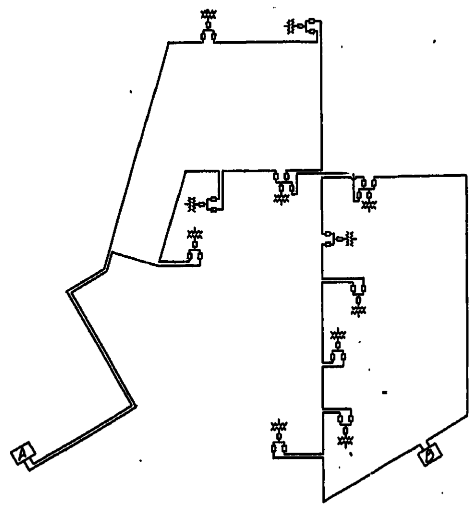

Fig. 12—A-C. Underground Primary System

mission voltage, by the fact that vault space is somewhat limited and it is difficult to get suitable spacing for higher voltage equipment. In addition, each loop can carry about 3000 kv-a. and distribute it over a substantial area. To make a higher voltage economical, a much greater area would have to be served from one feeder and it would be undesirable to take chances on an interruption so extensive.

The secondary of each transformer serves one phase of a four-wire, 115/200-volt, three-phase secondary. The ungrounded lead is protected by an air-break circuit breaker, controlled by a reverse power relay. The primary of each transformer is protected by either an overload oil breaker or a high-voltage fuse. In this network, both primary and secondary, the common neutral is used. Here in the underground district, the system neutral consists of not only copper conductors but all cable sheaths thoroughly bonded together. It also serves as the neutral for the Edison direct-current system, and has so served for many years.

In order to obtain standard 230-volt energy for power supply, rather than attempt to operate 220-volt equipment with 200 volts, small auto-transformers are installed in each vault. These are six to one ratio, dry type, and have ten per cent of the rating of the large transformers. This gives capacity for a 50 per cent division between lighting and power loads. From each vault, three-conductor, 500,000-cm. cables are carried out for secondary mains serving customers between

vaults. The use of separate cables for light and power has not proven uneconomical, as a single combined capacity cable could not be drawn into existing ducts.

In some instances where the power demand is small, dry-type auto-transformers similar to those in the vaults, but of only two-kv-a. rating, are installed on the building service panel and only the lighting mains carried in. Either two or three of these are used, depending upon the amount of power demand.

The operation of this system has been very successful and satisfactory. Heavy-duty, high-speed, passenger elevator service is carried on the same transformers as the lighting, with no appreciable interference with the lighting regulation. The auto-transformers appear to act as cushions for the heavy momentary power demands which have been found to cause considerable interference with lighting regulation where 200-volt service is supplied without these auto-transformers.

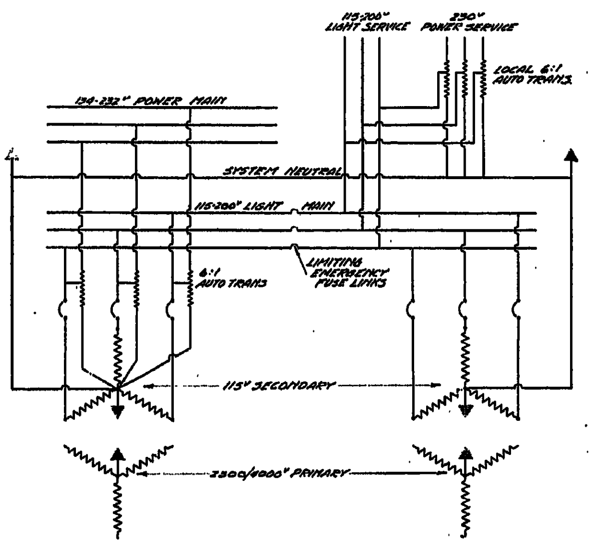

FIG. 13—TRANSFORMER AND SECONDARY CONNECTIONS

The general arrangement of the secondary system is shown in Fig. 13. As the system is extended, the lighting mains will be interconnected between vaults through limiting fuses in a manner similar to that which has worked out successfully on the overhead network. This will permit of emergency operation with any portion of a vault out of service.

STREET LIGHTING

Up to the end of 1923, street lighting in Minneapolis was by magnetite arc lamps suspended over street intersections, with the exception of gas lamps in some residential sections and five-light, ornamental cluster standards in the business districts.

For some years, we have been slowly developing a radically different system of street lighting distribution, using Type C incandescent lamps supplied with energy from existing distribution transformers and remotely-controlled by pilot wires. Late in 1923, when it was decided to abandon gas for street lighting, development work on this new lighting system was speeded up, and it was possible to start the first 500-high-c-p. units in service on January 1st, 1924. To these during 1924 were added additional lamps and this year some 500 of the old magnetite arcs are being retired, replaced with type C lamps. This will give about 2000 units in service.

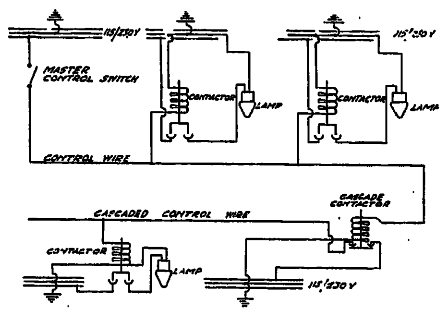

FIG. 14—REMOTE-CONTROL, MULTIPLE STREET LIGHTING SYSTEM

The general arrangement of this new system is shown in Fig. 14. The lamps are of two types,—those using the old form of mast arm suspension as shown in Fig. 15, with 1000-c-p. units at intersections and 400 c-p. for midblock lamps; the other is the new ornamental

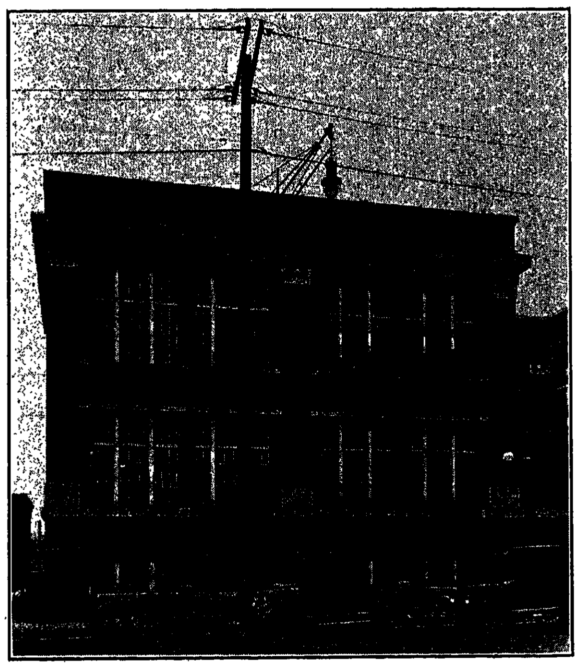

FIG. 15—MAST ARM INSTALLATION

system designed to supersede the ornamental clusters. The latter type, shown in Fig. 16, has been developed by the City of Minneapolis' engineering staff cooperating with the engineers of this Company. Electrically, the system is the same as that used for the general lighting.

The lamps are of the standard high amperage type as developed for series service. They are supplied from

small auto-transformers, using 115 volts on their primary, and installed in the lamp housing with suspended units, and pole bases for the ornamental standards. Each transformer is connected to the adjacent system neutral on one side, and on the other, to a section wire that in turn is connected to one of the secondary outside wires through a remote-control con-

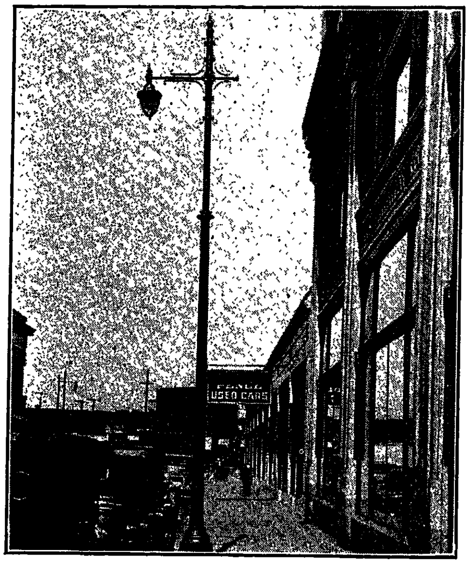

Fig. 16—Ornamental Standard

tactor. These contactors were specially designed for this system as they have to close by gravity and open electrically with a very small energizing current. They are of the mercury-cup type of the very simplest design and inherently free from trouble-making features. Their energizing solenoid coil consumes but 12 watts at 115 volts. The assembly of one of those is shown in Fig. 17.

For energizing these contactor windings, one side is connected to the system neutral and the other to a single-wire control circuit which is energized during non-burning daylight hours in order to hold the contactors in the off position. This assures that control system failures will not involve the lamps during burning hours. In the event of a control circuit or contactor failure, the lamps will burn during the daytime and trouble hunting becomes a daylight job at leisure rather than a rush night job in the worst of weather conditions. The control wires are carried out in each of four general directions from each substation. Here they are energized at 115 volts through a standard push button switch with indicating pilot lamps. In substations of the automatic type the energizing is done by electrically-wound clocks, fitted with astronomical dials which keep the schedule constant throughout the year.

When each control section has been extended to a distance and number of contactors where the voltage has dropped to about 80 per cent, a re-energizing contactor is inserted. These are similar to the other contactors except that they close the circuit when the solenoid is energized. These, in turn, take current from the nearest secondary mains and energize another section of control circuit. Since this can be repeated indefinitely, it follows that there is absolutely no limit to the area that can be covered with this type of multiple system using but 115 volts in any portion.

In operation, the system has proven so superior to any form of high-voltage series circuit that it is being rapidly adopted by other utilities and the various manufacturers are now prepared to furnish equipment for use with it at prices comparable with, or lower than, those for standard series operation. The average life of the lamps is running about 3000 hours with good sustained candle power up to the burnout point. With the low operating voltage and multiple operation throughout, failures are few and far between, the relative reliability of operation being about six to one in favor of the new system as compared to the older series circuits. The average lamp-hour outage is running about one-tenth of one per cent, which is remarkably reliable operation, meeting with the highest approval of the city authorities and the public in general.

In the selection of lamps for multiple street lighting,

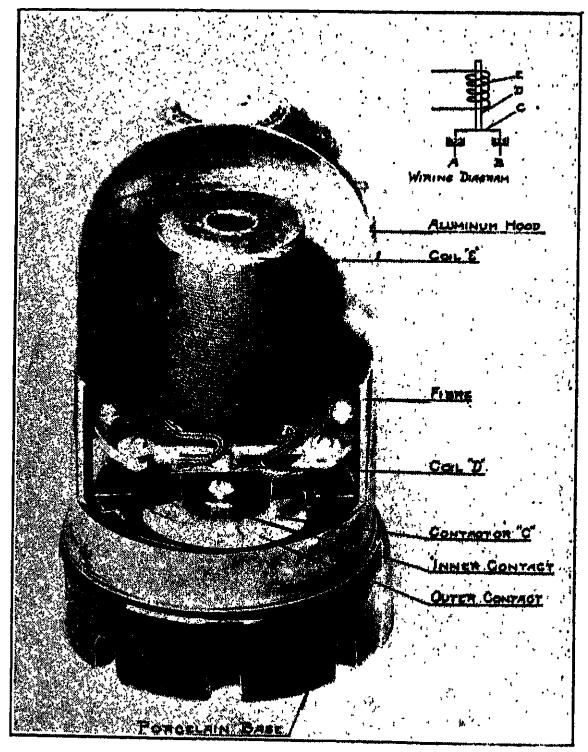

Fig. 17—Street Lighting Contactor

a much wider choice is available than with the series systems. Any of the usual series types of high amperage lamps can be coupled to the multiple circuit through suitable auto transformers, which are equivalent to the so-called compensator usual with series equipment.

While these series lamps are not specifically designed for constant potential service, they have given very

good satisfaction. The use of voltage taps on the auto-transformers permits of compensating for variable voltage loss in the 115-volt supply circuit. The long life of this type of lamp as compared to that of the standard multiple lamp is advantageous in that it lowers both lamp-renewal labor, costs, and lamp-hour outage. However, the lamp cost is approximately the same as that for the multiple lamp owing to the higher price put on the series type of lamp as compared to the standard multiple type.

Unfortunately, the lumens rating of multiple lamps and those of the series types do not match up, which makes it difficult to substitute one type for the other.

In the 10,000-lumen size, the series type can very well be replaced with the 500-watt multiple, giving 9400 lumens, thereby saving the cost of and losses in the auto-transformer. The possible disadvantage is that of more frequent renewals, about two and one-half times that for the series type lamp, and necessity of providing small auto-transformers for boosting excessive supply-circuit voltage drop on some long sections.

In the 4000- and 6000-lumen sizes, there is no multiple lamp substitute, as the 200-watt multiple gives but 3200 lumens and the 300-watt, 5100 lumens. However, where the series lamp lumen rating has not been established as a standard for any district, it would appear that the 300-watt multiple lamp could be used as a standard in place of a mixed standard of 4000 or 6000 lumens.

In the selection of lamps, it is desirable, where practicable, to use those of the multiple type as in that class the prospect for economic improvements is most to be expected.

If the series types are selected, and future improvements in lamp efficiencies are made, this type of lamp will change in its voltage rather than in lumens or amperage. Such a change would complicate the situation where auto-transformers of a definite, low-voltage rating are already in service.

Since the existing distribution transformers can usually carry the small addition of the street-lighting load, no transformer cost is involved and the existing high power factor is appreciably improved by this all-night steady loading. With the newer forms of series circuits the constant-current transformer cost is excessively high, its efficiency lower than for a standard transformer, and the power factor so low that they are very detrimental to the system.

Furthermore, the small auto-transformers used with high amperage lamps on series circuits will sometimes cause appreciable induction on adjacent communication and radio circuits when magnetically oversaturated when a lamp burns out. This effect is entirely eliminated with the multiple system. The comparative cost is roughly about 15 per cent in favor of the multiple system as compared to the latest types of series systems for either general or ornamental lighting. In addition, the over-all efficiency is about five per cent greater and the reliability and safety of operation much greater.

SUMMARY

The principal features which this paper describes are summarized in the following brief conclusions.

Protective Grounding. The maintenance of minimum impedance and ample current-carrying capacity from the secondary distribution system to earth, in conformity with the purposes of both the safety code and the fire code, is best effected by the adoption of a continuous network or wire grid for the secondary neutral covering the entire area served as one integral community with water-pipe grounds on every service connection and reinforced with bonds to water mains and driven-well casings where special circumstances justify it. Such use of the water pipes and mains for protective grounding purposes is not per se injurious to the water system, for extensive experience goes to prove that it is entirely harmless, and therefore municipal ordinances should require it as to the best interest of the public to be protected. Periodical surveys of the impedance to ground can be made very readily and constitute one of the advantages of this system of grounding.

Common Neutral System. In systems utilizing the 4000-volt, three-phase, four-wire type of primary distribution circuit, with single-phase lighting branches, there is substantial advantage and economy in combining the primary and secondary neutrals into one conductor, so as to serve both functions. It is a prime essential in the use of this system that the neutral be grounded in the general manner described in the previous paragraph, or its equivalent. With this system, the two outer secondary wires form an additional return path in parallel with the neutral, and the primary load current (in the single-phase instance) divides nearly in the ratio of copper cross-section. The return path via the neutral network is obviously of a complex nature in any given instance, but tests show that at least 65 per cent returns via the neutral on the same pole route with the primary. Each primary feeder district is served by a three-phase main which taps into a ring feeder through an oil switch, and the ring is sectionalized about midway between adjacent taps by means of oil switches which are normally open but can be closed in emergencies. The entire load area of each feeder district is divided into single-phase sub-districts, so associated and distributed among phases as to minimize the unbalanced load current in the neutral along any given three-phase route. The saving due to reduced investment is estimated at $8.00 per kv-a. of distribution transformer capacity and the energy copper loss is reduced variously from 25 to 40 per cent.

Inductive Coordination. The use of a common neutral system may be detrimental to the service rendered by adjacent telephone circuits unless proper coordination

is applied through close cooperation of the engineering personnel of the two utilities involved.

Pending completion of the joint tests now in progress, and a review of the results, definite recommendations can not be given other than for the use of such remedial measures as are now already known and recommended in the "Principles and Practices." The results of these tests, so far as some of the more important features of the problem of inductive coordination are concerned, will be available, it is expected, at a comparatively early date.

Secondary Networks. The secondary mains in any given phase area are interconnected by means of copper link fuses in the two outer conductors, which are proportioned so as to operate on approximately 50 per cent of the rated continuous capacity of the smallest adjacent transformer. The two outer secondary leads of each transformer are fused at about 200 per cent of rating. On ordinary residential service, the transformers are regularly loaded to about 125 per cent of rating on the winter system peak, but for short winter peaks will safely carry 200 per cent of rating. The typical layout of a secondary bus section resembles the letter *H*, as shown in Fig. 6. For mixed power and lighting loads a three-phase bank is used, as shown in Fig. 7. Local outlying commercial centers are served by means of the type of secondary layout and protection scheme which appears in Figs. 9 and 10.

Downtown A-C. Distribution. In the downtown area of Minneapolis an a-c. distribution system has been installed for ultimate relief of the three-wire Edison d-c. system. The 4000-volt primary feeders are arranged on the loop or ring system indicated in Fig. 12. Transformer installations are placed in vaults built under the sidewalks. At each substation supplying a loop there is overload protection on the outgoing feeder and also an automatic voltage regulator. Between vaults, the cable sections are provided with differential protection which functions only on a fault in the cable itself. The primary bus in each vault is equipped with reverse power protection and the primary of each transformer is provided with overload protection. The type of secondary layout is indicated in Fig. 13 and the common neutral system is used here also. Separate secondary mains are used for lighting and power. The three-phase lighting mains operate at 115/200 volts. The voltage on the power mains is raised to 133/230 volts by means of six to one-ratio auto-transformers and these mains are equipped, in each vault, with reverse power protection. In special instances, where the power demand is not large, these auto-transformers are installed at the customer's service connection, and supplied direct from the lighting main. As this downtown system is extended in future, the lighting mains will be interconnected between vaults, through limiting fuses, in the same manner that has proven successful in overhead distribution.

Street Lighting. The constant-current series type of distribution circuit is being replaced with multiple distribution under remote control by means of a pilot wire and individual contactors at each lamp. The type of circuit layout is shown in Fig. 14. This system is made possible by the fact that 115/230-volt secondary mains cover the lighted area of the city very completely. This additional load on the secondary system proves very desirable from an economic standpoint. Experience also proves that the multiple system is more economical, efficient, and reliable than the series system which it is replacing.

Discussion

R. E. Cunningham: I have particularly noted the first paragraphs of Mr. Hood's paper regarding the necessity of thoroughly grounding secondaries and I want Mr. Hood to know that I concur with him in his statements.

I believe that too many of us have been content to drive a pipe or two connected to each secondary system and call it good enough. We all know what may happen to such grounds, particularly in dry districts. I think we should arrange through local ordinances to have a ground connection made at each customer's service which would be installed at the same time the house is wired by the contractor. This should be a water-pipe ground.

Now, as to using a common neutral, Mr. Hood, no doubt, has a condition where, as he has stated, his plan has worked satisfactorily. Whether it would work under conditions obtaining in Southern California is a little doubtful. We have a long, dry season and a good ground is hard to get; in some cases it is impossible. There are very few districts where we have continuous water-piping systems and in some cases cement is used in making the joints in the pipes.

Thus far on our 4-kv., four-wire systems we have adopted the practise of grounding the primary neutral at frequent intervals, generally by the use of a driven pipe. We have not tried the plan of using the same wire for a secondary neutral.

I might say that in most cases we do not have secondary systems continuing throughout the primary circuits, so that possibly there is not the same opportunity for economy as exists with Mr. Hood. With the system as I stated, using the driven-pipe grounds connected to our neutrals, we have had a number of cases during the dry season when the phase wire has fallen and lain on the ground without kicking out the circuit breaker at the station. We are particularly concerned as to how to take care of such a hazard.

We have recently built a new substation in a district where the system was being changed over to 4-kv. four-wire and are trying out the scheme as shown in Fig. 1 herewith. Here the neutral wire is isolated from the ground except at the station and there the connection is through a current transformer, the secondary of which is connected to an ammeter and relay. This relay in turn can actuate an alarm bell. Any current in the neutral due to unbalanced load will not indicate on the ammeter or cause the relay to function, but any current returning to the station from a phase wire which might fall on the ground must flow through this current transformer.

The current transformer is provided with taps and the relay so adjusted that it will operate when one ampere flows through the ground connection. Tests have been made which show that about that amount of current would flow when a coil of bare wire connected to a phase wire was thrown into a patch of green grass at a distance of about one mile from the station. Normally adjustments are for a much higher current so as to protect against a

full ground and then once each hour the operator makes a test using the one-ampere adjustment.

We hope with this device to discover cases of phase wire on the ground under such conditions as not to cause sufficient return current to trip the circuit switch.

I would like Mr. Hood's advice on our situation and also wish to inquire whether with a phase wire down he usually gets enough current through the ground on his 4-kv. circuits to release the circuit switch.

E. R. Stauffacher: The ground relay mentioned by Mr. Cunningham will ring an alarm only when a current of one ampere or more goes back to the station from the line lying on the ground, whereas, our test indicates that only 0.3 or 0.4 ampere actually flows back to the station when a line is down on a 4000-volt distribution circuit about one mile from the substation. This, or course, applies to the dry soil condition. The equipment would indicate properly and give an alarm if the wire happened to be lying in green grass or in wet soil or against a green tree. However, there is every reason to expect that it will be rather difficult to develop equipment which will indicate when a line is lying down in dry soil or on a concrete or asphalt pavement. This leads me to believe that it would probably be better practise to install numerous grounds along the distribution line in addition to the ground at the substation, and to make every effort to see that the circuit breaker tripped out in case a wire dropped to the ground, rather than attempt to develop some type of ground detector which would indicate this hazardous condition.

C. A. Heinze: In Los Angeles, we have made use of grounds to water pipes for some years past. Our method is to select at least three services on each secondary and ground the neutral of each service to the local water pipe on consumers' premises.

It seems that ten or fifteen years ago the water department of our city experienced considerable trouble and difficulties with electrolysis. I imagine that a number of members know the full meaning of that word when applied to water systems, and the difficulties with electric railroads. To protect itself, the water department in Los Angeles constructed mains with cemented joints, thus preventing the possibility of getting a good ground by attaching directly to the main itself. However, on a three-wire service if we can obtain permission from the consumer, we bring down the neutral, connecting it directly to the water service. For mechanical reasons sometimes we use a piece of conduit, bonding it at each end.

I am a bit surprised to learn that Mr. Hood would like to have us go back to the multiple street-lighting system. It seems to me off-hand that we would be taking a step backward. I remember quite well that years ago we all, more or less, had multiple street-lighting systems, and we gave them up for the supposedly more efficient series system. Now, I find there is a tendency to go back to the multiple system again. I don't know whether the telephone engineers have had anything to do with this or not. At the same time I can't believe that it is a step in the right direction. Mr. Hood proposes and does install a contactor for connecting each lamp to the secondary bus nearest to the location of the lamp. From his paper I gleaned that it will require twelve watts of energy to keep this contactor energized during the daylight hours, and of course, during the hours when the lamp burns, the contactor is out of circuit or de-energized.

In Los Angeles, excluding the ornamental post lamps, we have 10,000 street lights of the suspension type. Now, if the contactor were to be provided for each one of these lamps in order to operate it on a multiple system as recommended by Mr. Hood, our loss in energy, valued at one cent per kilowatt hour would amount in a year to practically $7000. I don't know what Mr. Hood's company receives for energy for street-lighting purposes in his city, but surely none of us in the West are making sufficient money on street lighting to stand a yearly loss of any such amount, in our case amounting to $7000.

I can't see what the future holds for us if we go to a multiple system. I would like Mr. Hood to explain the reason for going to the multiple system. I have always been given to understand that the multiple lamp, toward the end of its life, greatly decreases in candle-power, while the series type tends to burn at full candle-power until it burns out. I feel that this will result in a large number of lamps being continued in service at reduced candle-power and not being replaced until they actually burn out, resulting in decreased illumination on the streets. I think we, as utilities, should strive to give the public all they are paying for and the best street lighting possible.

M. T. Crawford: I notice that the neutral return path described by Mr. Hood is apparently largely in the secondary neutral grid. I would like to ask if single-pole switches are used or three-pole switches on the outgoing feeders at the substation. It would appear that if single-pole switches were used and single-phase short circuits occurred, opening one or two switches, the neutral would be called on to carry the full-load current of the phase which remained in service, greatly increasing the duty of the neutral path, and I should think increasing the troubles that might come from using only a relatively light-capacity neutral grid.

Rather extensive use is made of fuses in the secondary main for sectionalizing in case of trouble. I would like to ask Mr. Hood what method he has for finding out when these secondary sectionalizing fuses blow. If secondary sectionalizing fuses should blow and no knowledge was had of the fact, they might stay open for some time and interfere with the proper interchange of load current.

Mr. Hood refers to the fact that the primary-neutral return current sometimes will split and take the paths of the secondary outside wires, rather than all go through the common neutral. I would like to ask if he has made any test which would determine whether or not this disturbs the voltage regulation on the secondary at such points. It would seem that it might have considerable effect on the voltage drop on the secondary bus and would affect the consumer's service.

In regard to the underground system, I would like to ask if Mr. Hood has experienced any operating difficulty in connection with the relay contacts. On the underground distribution system in Seattle where a large number of power-directional relays are installed, we have found it necessary to periodically inspect and clean the relay contacts. The gases and other substances in the subways appear in some way to cover these contacts so that they do not always function.

D. K. Blake: I find a large number of people who are strongly in favor of the common neutral and just as many who are strongly opposed to it, but the chief cause for opposition seems to be the telephone interference.

As to the multiple lighting circuit, there are a large number of eastern companies who are going into that, studying it, and applying it. I was very much surprised to find in Denver that the business section was supplied with multiple circuits with cascade pilot-wire control.

Mr. Hood referred to his polyphase secondary network for the business section which is a seven-wire system. There is a somewhat different type of system used by a large southern city of about 200,000 population. They have the same idea, that is, they do not want to supply an off-standard voltage to their customers' devices so they take the secondary winding of the transformer and extend it to 133 volts, which gives 230-star volts for the motors, and of course, a seven-wire system.

As to the matter of neutrals carrying load current, you might be interested in knowing that there is one large company which makes the practise of connecting distribution transformers from one wire to the grounded cable sheath and in that way

they have the cable sheath carrying a number of amperes of load current.

D. I. Cone: As already stated, the subject of distribution systems is of great interest to telephone engineers; one item in particular in Mr. Hood's paper presented today is of importance from that point of view; that is the question of the use of a common neutral for the primary and secondary ground connections and, also, the question of the multiple grounding of primary neutrals. I shall not undertake a discussion of that problem because we have attempted to cover the subject pretty thoroughly in the paper by Dr. Trueblood and myself.[1]

Mr. Heinze referred to the interest of the telephone engineers in the street-lighting problem. There is a paper by Mr. McCurdy which discusses series street-lighting systems from this point of view[2]. As to the multiple street-lighting arrangement, I think, without having had experience with it, that the multiple scheme would make coordination with the telephone plant much easier.

G. H. Smith: I would like to say a word in discussion of Mr. Hood's multiple street-lighting system. We have been living with an overhead series system in Seattle for twenty years and for the last five or six years we have been trying to find some way to dispose of it. The underground lighting system in Seattle is multiple, and the lamps are low-voltage, fed by transformers in the pole bases. We are averaging about 4000 hours' life on these lamps. It is hard for us to believe that we should use a 120-volt multiple lamp for that service although I believe that is the lamp manufacturer's recommendation. We hope before long to install an overhead multiple lighting system very similar to the one described by Mr. Hood. We have been working on it for years, and believe that it will justify itself from the standpoint of safety alone. Also, our figures seem to show that it will be fully as cheap and more reliable.

S. B. Hood: Mr. Cunningham has brought up the point as to whether the common-neutral system was suitable for California. I think probably the best way to find out would be to try it. The main thing in the common-neutral system is to have sufficient neutral copper so that the earth does not form a part of the return, not an essential part. Therefore, if the air conditions are very dry, all that is necessary is to equalize the potential between the neutral system and the earth, so I rather think that the point of safety would be just about on a par with the dryness of the earth. If the earth is absolutely dry, it is a perfect insulator, so if you maintain your neutral at earth potential at the interconnection, you never would get any appreciable difference between that neutral system and the earth, even though the earth conditions give very high resistance.

On the question of the opening of the circuit breakers in case of a fault to ground, I suppose what Mr. Cunningham meant by ground was the conductor lying on the ground. My experience has been that it doesn't make any difference what kind of a system you use, isolated-neutral or common-neutral, you cannot depend on opening a circuit breaker on contact to ground. The artificial ground connection such as a driven pipe will not have less than 100 ohms resistance. Now, you can readily see that 8 or 10 ft. of 3¼-in. pipe will not give a good ground. The contact resistance of a wire on the ground may be up in the thousands of ohms. So I don't think it makes any difference; you can't depend on opening the breakers through accidental contact with the earth.

Mr. Heinze brought up the question of the individual house grounds, and the difficulty of getting continuous grounds through the water main. I think that is probably common all over the country, possibly not to as great an extent as in Los Angeles, but you get the benefit of the buried pipe which forms a service pipe from the main, including the section of main in which that service pipe is tapped. So in the aggregate you have a fairly good surface of water pipe in contact with the earth, certainly much better than you could ever expect by an artificial ground. When you interconnect all of those grounds by a system neutral, you can count on pretty good protection.

I am sorry I can't agree with Mr. Heinze about the multiple system being a backward step. We think it is the greatest step forward we have ever made. I have felt for a great many years that the practise of running series circuits, which may have potentials up to 6000 volts, through alleys, and out to every street corner, networking the whole area much more highly than the primary circuits can, comes pretty near to being a crime, and there are probably more accidents to the public and to the utility's operating staff through those high-voltage series circuits than any of the other circuits you operate. The only trouble in the past has been to get the same efficiency in transmission with the multiple system. That can be solved by utilizing the distribution transformers, and the relay control system. Our experience with the maintenance of candle-power in the lamps has been remarkably good. I have always criticized the series system on a basis that the lamps, unless you break them up, will get so dim that you can't see whether they are burning or not.

Regarding the contactor losses in the multiple system, I think you ought to get the right point of view. You must consider that the series constant-current transformer at best will have only an efficiency of about 85 per cent, whereas the contractor uses 12 watts only for initial action, and as soon as the core rises it chokes the consumption down to 8 or 9 watts. That represents less than 2 per cent on a 500-watt lamp, and in a great many cases a group of five or six lamps may be found on one contactor. You can readily see the contactor losses are almost negligible compared to the losses in the series type of transformer.

The principal advantage we found in the multiple lighting system has been the better service during storm conditions. It frequently happens during a bad storm that lamp outages on the series circuits will run 17 to 20 times as high as on the multiple. As soon as the storm starts you will see the multiple lamps winking like stars all over the city. It can readily be seen with the series system that each accident or effect of the storm which has lit the multiple lamps would have put out of business the series type of circuit.

The question of maintained candle-power is one which I think we shall have to leave to the engineers of the lamp association. We picked the series type of lamp and adopted the multiple system, believing that the series lamp was the best of the two lamps. Now, they tell us that the series lamp was a poor lamp at its best, and we should use the multiple lamp, so I don't know which is right. Experience will have to show. I think, however, that one feature in favor of the series type of lamp, particularly in our large cities, is its longer life. They claim series lamps used on multiple system will give 2160 hours' life. We are getting 3000 hours on ours and better, and very well maintained candle-power. They also claim the multiple type of lamp is good for 1300 hours. My experience with the large systems is that if you get 1000 hours life out of it you are lucky.

Now, where you have cars parked along the streets all day and most of the night, when are you going to get a maintenance car up to the ornamental lamps? The men must go around before sunrise, and even then you will find many cars.

Mr. Crawford asked whether we use single-pole or three-pole switches on our feeders at the substations. Originally all our lighting feeders, before we converted to the common neutral, were the two-pole. Thus the only change we made was to change the original double-pole switches; the two poles were put in

1. Power Distribution and Telephone Circuits, by H. M. Trueblood and D. I. Cone, A. I. E. E. JOURNAL, December, 1925, page 1353.

2. Induction from Street-Lighting Circuits, by R. G. McCurdy, A. I. E. E. JOURNAL, October, 1925, page 1088.

series, giving twice the breaking capacity. Those circuits, however, were almost entirely lighting circuits and we found that the action was substantially that of a one-wire single-phase circuit. They didn't act as three-phase circuits at all. However, all our three-phase power circuits had three-pole switches, and as we cut to combination feeders, we found the single-pole switch had no advantage, due to the fact that among the other peculiar things we do, we always ground the neutrals of our three-phase power transformers. When you get a single-phase short circuit, the closed secondary delta transfers the short circuit to a very considerable extent to the other two phases, so that all three switches would go out in the same way as though they were on a three-pole switch. For that reason, as we rebuilt our substations, we conserved substation space by putting in entirely three-pole switches. In the modern substation with automatic closing equipment, it is the fact that almost always a circuit will not lock-out but will burn the fault off.

The matter of locating blown section fuses is largely checked up by periodic inspections made just before the fall peaks. During the summer season, most of the load in the interconnected districts being residential load, it doesn't make much difference whether those fuses are in or out except in some places where the range load is heavy. We make careful inspection just prior to the fall peak, and from then until we pass the overlapping period, the spring of the year, we depend on the customer to let us know when those section fuses are out. Generally he doesn't waste much time in doing so. In other words, our transformers will carry the load with those fuses out as well as with them in, but the regulation would be very much poorer; therefore, we almost invariably get a complaint from a customer when a section fuse is out.

We have never had any trouble with the effect on secondary regulation caused by flow of current in the neutral. There is under certain rather abnormal conditions a tendency to unbalance voltage, but you must have a very abnormal condition to bring out that effect.

Regarding Mr. Crawford's question as to relays in the underground a. c. system, our condition is possibly just a little different from what he may have in mind in that our transformer vaults are to all intents and purposes substations. We don't have the dampness and the gases found in an ordinary manhole vault. We are particularly fortunate in Minneapolis because we can put vaults under the sidewalk, and they are just as dry as the basement of a building; in fact, they are virtually the front of the basement. We use the same type of relays and equipment as used in the ordinary substation, and they are inspected periodically, in most cases once a week.

In Mr. Blake's remarks, he referred to a system in the South which used 133-volt transformers. That is the type of transformer with the 133-volt secondary tapped at the 115-volt point. We had considered that, but the objection we saw was that it made a semi-special transformer which is objectionable from the standpoint of simplicity in warehouse stock where it is necessary to stock one type of transformer for overhead distribution and another for underground. If the underground required a special type of transformer, probably that tap would be all right, but in our case we have adhered to standard equipment throughout.

Power Distribution and Telephone Circuits

Inductive and Physical Relations

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Synopsis.—*Consideration of the relations between power distribution and telephone systems is naturally involved in any comprehensive review of the problems of the rapidly expanding power-distribution networks in this country.*

Avoidance of contact and provision of suitable working conditions for employees in situations of close proximity are dealt with in the National Electrical Safety Code and in State Regulations which provide arrangements for safety where complete separation is not feasible.

Induction from distribution circuits has heretofore had less general attention than induction from power-transmission lines. Recently the Joint General Committee of the National Electric Light Association and the Bell Telephone System has undertaken comprehensive investigations of these problems. Of particular interest is the study of induction under joint use conditions now progressing actively at Minneapolis. Pending completion of this and other studies, a preliminary and qualitative discussion is here given.

Situations of exposure fall into three groups determined by the character of the area served: (1) "downtown" districts; (2) residential urban districts; (3) rural districts. The major problems arise in the second group. A wide variety of arrangements characterize both systems, and require consideration.

Among technical features, coefficients of induction for close exposures, shielding action of metallic cable sheaths for both power and telephone circuits, and "ground potential" effects are distinctive problems. Where both classes of circuits are in cable with suitable precautions as to grounding, interference is rarely to be anticipated.

Low-frequency induction due to transient power-system disturbances is rare at the lower voltages but may assume great importance with higher voltage circuits, unless the amount of exposure or fault currents are suitably limited. Various methods for reduction of disturbing effects are discussed.

Noise induction from power-distribution circuits is chiefly from residuals which occur on single-phase branches of polyphase circuits, or where triple harmonics or load-current unbalances are introduced by grounding neutrals, or where admittances to ground of phase wires are unequal. Residual currents are largest in systems having multiple-grounded neutrals, both load-currents and triple harmonics occurring. Approximate resonance at triple harmonic frequencies between the inductance of station apparatus and power cable capacitance has characterized several situations. Various single, two- and three-phase arrangements are compared from the induction standpoint.

The closely related matter of unbalances in the telephone plant is briefly discussed. This is one of the subjects on the joint research program.

With the great variety of specific situations encountered and the growth to be expected, joint study by power and telephone engineers of existing cases and future plans furnishes the only practicable solution.

INTRODUCTORY

In any comprehensive consideration of the varied problems of our rapidly expanding systems of power distribution, there is naturally involved a discussion of the problems of relationship between these systems and the communication systems, which to a very large extent serve the same customers. Such a discussion is undertaken in this paper in the hope that a better understanding of the physical and inductive relations of different distribution circuits and neighboring telephone circuits will promote the coordination of the two classes of circuits without imposing undue restrictions on the development of either.

To appreciate the magnitude of the problem, it will suffice to recall that the number of power consumers in the United States has now passed 16 million. There is no present indication of abatement in the rate of growth, now about 15 per cent annually in number of consumers and in total load supplied. The number of telephones is likewise approximately 16 million. The growth of this companion electrical industry is today less striking (about six per cent per year) only because its period of rapid expansion began some years earlier. To a very large degree the customers of the two services are the same people; hence it is apparent that the combined growths present a problem of continually increasing proportions.

In and near the great centers of population, this growth of the two kinds of service to the public has led to the development of two highly specialized electrical systems of which large parts are necessarily in close proximity to each other. The most obvious consideration in this relationship of power and communication circuits is to avoid contacts between conductors at crossings and conflicts, and in the joint use of poles. A closely related matter in such situations is that of providing climbing and working spaces so that men can work on conductors and equipment of either class in a safe and convenient manner. These factors, and the necessity of limiting the numbers of poles on streets, led early to the development of specifications to govern clearances and strength of construction, and to extensive arrangements for the joint use of poles.

Contacts between power and communication circuits must be carefully guarded against, since they not only cause interruption of service on either or both systems, but may result in injuries to persons or damage to property such as (1) electric shock to employees or subscribers; (2) acoustic shock or noise; (3) breakdown of lines and equipment, with attendant fire hazard and destruction of plant. The intimate handling of telephone instruments in daily life requires the greatest

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precautions to prevent their energization at dangerous potentials.

Standards for minimum clearances, for strength of construction, for insulation and other protective measures in both classes of circuits have been gradually established through long experience, the various features being continually under review as the arts progress. These standards include provisions against breakage or excessive sag of the upper conductors or breakage of supports at wire crossings, and against accidental contacts between wires of a given circuit, or failure of insulation.

In working out the relations between overhead lines of different utilities where the higher voltages are involved, the natural first line of action is to endeavor so to locate the lines as to eliminate the possibility of contact. This is in recognition of the increased hazard to persons and the impracticability either of building the communication plant to withstand damage by breakdown or of providing protective devices to limit effectively the amount of plant exposed to such breakdown. The problem of avoiding hazardous conditions is one not only of current- and voltage-limiting devices at circuit terminals, but of the insulation, clearances and operating practises over the entire exposed plant. Where it is found impracticable to avoid proximity, particular attention is required to the provision of adequate strength and arrangements of construction.

Preferences and minimum strength and clearance requirements for crossings, conflicts and joint use of poles by power and communication circuits have been embodied in the National Electrical Safety Code[1] and in the regulations of many of the states. The Joint General Committee of the National Electric Light Association and the Bell Telephone System[2,3], proceeding with these as a basis for further study, has subcommittees at work on both the contractual and technical phases of the problem, in recognition of the changing conditions of distribution practises.

Until recently, recognition of the importance of the problem of the inductive relations between power distribution and telephone networks has not been so general as that accorded a good while ago with respect to the association of long power and telephone lines. It is true that specific situations arose, some of them of rather large importance, sufficiently critical to require the development of measures of relief. However, the problems involved in the close inductive relations of power-distribution circuits and the telephone plant were not attacked on a large scale until they were taken up by the Joint General Committee of the National Electric Light Association and the Bell Telephone System, as part of the wider question of methods of coordinating the facilities generally of the two services for the avoidance of interference. This Committee has already published[3] certain recommended practises, mainly qualitative, which are applicable to the situations with which this paper is concerned. A Subcommittee on Development and Research is now engaged in comprehensive investigations of the various technical questions connected with this matter, and one of the "projects" under which the work of this Subcommittee is organized, is devoted to a single important aspect of the problem,—the question of induction under joint-use conditions.

1. Reference numbers apply to Bibliography appended hereto.

At the present time, the Committee in charge of this project is working in cooperation with the Northern States Power Company and the Northwestern Bell Telephone Company, using a most complete temporary field laboratory and experimental line in the outskirts of Minneapolis. While this work is progressing favorably, it involves many problems of a fundamental nature, and therefore does not permit of giving quantitative results until further advanced towards completion, which will require a considerable period of time. It is believed, however, that within the next six months an interim report can be issued dealing with those factors and remedial measures which are of prime importance to a solution of the problem.

This paper is thus of a preliminary and primarily qualitative character. It undertakes to present a view of the problems arising from inductive effects which are peculiar to the close association of extensive networks of the two utilities in and near centers of population, and discusses certain phases in some detail, with illustrative references to operating experience.

It is not proposed to deal with the subject of electrolytic corrosion of underground structures, a problem which involves not only the power and communication systems but also the gas, water, and electric railway systems. The underground cables of electric power companies are, of course, in common with telephone cables, susceptible to electrolytic injury, but such injury is usually caused by stray current from electric railways rather than from electric power distribution circuits.

General Survey

Problems, arising primarily from situations in which long power transmission lines and toll telephone lines parallel each other, assumed great importance about fifteen years ago, and led to the extensive studies of the Joint Committee on Inductive Interference in California.[4] Much of the work of that committee is fundamental to the present discussion. When the subject of the inductive relations of power distribution circuits and telephone circuits is set off as a topic in itself, it is not because the underlying principles are different from those which apply to the longer lines of the two services. It is rather the characteristics of the two systems in the thickly populated districts within and near the modern American city, and to some extent in rural districts, which give individuality to this problem.

While the primary concern is here with power distribution and local telephone circuits, nevertheless long

power transmission lines and telephone toll lines require consideration, if only indirectly, for in the final analysis these long circuits exist purely for the purpose of serving the public, and the sole links between them and the consumers are the local distribution systems. In the broad view, it is the public whose interest in the solution of these problems is fundamental. The vital importance to the two industries of finding the best engineering answer rests ultimately on this public interest.

In approaching the subject from a technical standpoint, perhaps the most convenient method of division is by reference to the physical relationships of the two utilities as determined by the character of the areas served. We may thus distinguish three general groups:

1. *Dense Urban or "Downtown" Districts.* In these localities which are principally in the larger cities, it is usual to find the circuits of both utilities in underground cable, carried, however, in separate conduit systems. There is, of course, a great deal of parallelism under these conditions in our large cities, from which, so far as the authors are aware, no cases of serious inductive effects have resulted, and except in rare instances, no physical damage has occurred. The problem of electrolysis has its chief focus in these districts but a discussion of this question is beyond the scope of this paper.

2. *Residential Urban Areas.* This term is used here to denote those districts in the smaller towns as well as the large cities in which the local telephone circuits are found mainly in aerial cable, with occasional fairly long runs of twisted pair or open wire, while the power circuits are chiefly in open wire. In the more thickly settled sections, long exposures at roadway separation or under joint-use conditions are common, and for shorter distances, in alleys, joint use or conflicting construction is frequent. Important questions arise here, both from the standpoint of physical hazard and from that of induction. The closeness of association and the large numbers of service connections involved, combined with the lack of the effective shielding afforded by cable sheaths and earth in the dense urban districts, make these residential urban areas the most important in respect to induction in the local telephone circuits.

Telephone toll circuits in these districts may be either in cable or in open wire. Owing to the relatively small numbers of such circuits and their limited space-occupancy, it is possible by suitable cooperative planning to keep them comparatively free of exposure.

3. *Rural Distribution Areas.* These are regions in which the population is scattered and the service drops of both utilities are relatively infrequent. Space is no longer a vitally essential factor, though it may still be an important one. Telephone toll circuits as well as subscribers' circuits are usually in open wire. These open-wire subscriber lines are likely to be long and to serve a considerable number of subscribers. By cooperative planning in advance, it is practicable to keep the telephone circuits comparatively free of exposure.

A large variety of power systems and voltages is found in different parts of the country, in the situations covered by these three broad classes. A classification on the basis of the purpose which the power circuit serves would give such groups as trunk circuits at voltages of 6600, 10,000 to 15,000, and higher, in both open wire and cable; primary distribution at voltages in the 2300 to 4600 range; secondary distribution at from 110 to 440 volts; and circuits for such services as street-lighting and street-railway power supply. Series street-lighting circuits are, under some conditions, of much importance as sources of interference and are discussed in another paper.[5] Railway circuits are also important from the point of view of inductive relations, but the discussion of their special characteristics is beyond the scope of this paper. Secondary-distribution circuits will come into the discussion in only an incidental way; the indications of experience are that interference from these low-voltage circuits is rare, though cases have been known to occur, due to accidental circuit conditions causing excessive currents in earth connections, or to unusual or poorly designed apparatus.

A classification along somewhat similar lines may be made of telephone circuits, having regard to the purposes to which the circuits are assigned. The subscribers' lines radiating from the central offices may be likened to the secondary power distribution circuits. Corresponding to the primary distribution system is the intricate network of trunks between central offices, used in local communication between subscribers in different exchanges. Other trunks, used in establishing connection between a subscriber and a toll center, when toll service is required, may be regarded as the counterparts of the higher voltage circuits. As the development of the art of power distribution has led to several kinds of arrangement such as direct-current, two-phase, three-phase and their varieties, so also in the telephone system, there are three main types of local circuit, according as the service is handled manually, or by the step-by-step type, or the panel type, of machine switching system. At present, in many of the large cities, the manual and one of the machine switching systems will both be found, together with systems of the intermediate semi-mechanical character, designed to give service between subscribers where one of the central offices is on a machine switching and the other on a manual basis. Many minor differences in equipment for signaling and supervision also exist, and there are several different arrangements of subscribers' sets and cord circuits, depending upon the type of service given; for example, individual line, or party line with full-selective or semi-selective signaling, with further differences according as the service is unlimited or on a measured basis, or according to the method of collection in the case of pay stations. In smaller communities, many other variations are encountered, including local battery arrangements with magneto ringing and sundry combinations of the local and common battery schemes.

Technical Features

Coupling and Shielding. A distinctive problem in the field of this paper is that of determining the coefficients of induction between power and telephone circuits under the conditions of close exposure that obtain in joint use and in conflicts. The fundamentally important work on this subject, done under the direction of the California Joint Committee, applies only to open wires at horizontal separations in excess of about

20 feet.‡ While the determination of the coefficients at closer distances is possible by calculation, (this method was used by the California Committee in their work at the wider separations), the labor required for the complete solution in the case of a large telephone line would be excessive. The experimental method is shorter and probably cheaper for these conditions and some experimental work is desirable in any event as a check. Such experimental determination is a part of the program of the Joint Development and Research Committee.

A power circuit having its conductors enclosed in a grounded metallic sheath will have very little external electric field, and if the currents are confined to the enclosed conductors and sheath there will be negligible external magnetic field. The result is the practical elimination of inductive coupling. It is only where part of the current returns by other paths than the sheath that induction results. While there are few data from direct experiment on the inductive effects of power circuits in cable, there is a great deal of evidence from operating experience in the case where both power and communication systems are underground, and a small amount where the power circuits are in aerial cable. This experience is based upon cables having all conductors in one sheath, and, under this condition, may be accepted as showing that inductive interference is rarely to be anticipated where the power system is entirely in underground cable with continuous metallic sheath. Experience to date with exposures in which the power circuits are in aerial cable with continuous metallic sheath, effectively grounded, justifies similar expectations. It might perhaps be thought that a breakdown of insulation with grounded sheath (either underground or aerial) would give rise to currents in the earth which would induce sizeable low-frequency voltages in closely exposed communication circuits. The absence of such occurrences in operating experience is probably due to the facts that a fault in a polyphase cable must usually involve all phases and that the sheath possesses fairly low resistance, thus tending to keep the fault current out of the earth. It is possible that the use of separate single-core cables for polyphase circuits may produce sufficient field to cause interference under certain conditions of proximity.

Both as to space occupied and shielding from induction, cabling the telephone conductors in metallic sheaths is advantageous. However, the magnetic fields, due to residual power currents and the "ground potential" effects, may be of great importance. If the cable sheath is continuous and is either underground or bonded to underground cable with an additional effective ground on the side of the exposure remote from the underground cable, the resultant electromagnetic shielding is of much value in reducing inductive effects. Where aerial cables are to be bonded to underground cables, however, careful attention to electrolysis conditions is required.

Another feature characteristic of inductive relations in distribution areas is that kind of coupling usually referred to as "ground potential." This term implies a difference in potential between two specified points in the earth. The term is convenient, but it should be understood that it does not connote a different kind of phenomenon from that where two grounded circuits are involved in a parallel exposure. The total voltage in a grounded circuit, a, due to current in another grounded circuit, b, (see Fig. 1) is produced in a distributed manner, partly in the metallic part of a and partly in the ground part of a. This latter part is, in general, the effect of both resistive and inductive coupling, and in practical cases its magnitude will depend almost entirely on the relative positions of the ground connections of a and b, and the route followed by the wire part of b. If, then, this part of the total voltage in a that is produced in the earth is considerably the larger part, the total voltage is frequently referred to as a

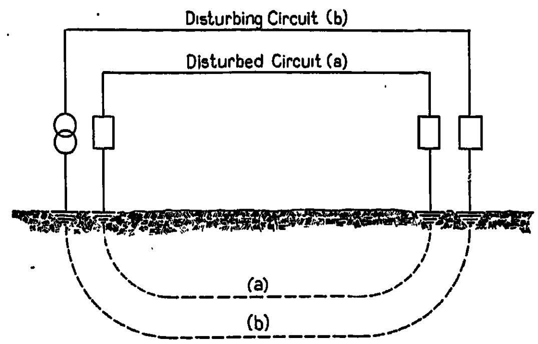


FIG. 1—GROUND-POTENTIAL
Inductive and Resistive Coupling

"ground potential." Used in this way, the term means simply that the effects observed are not due, to an important extent, to coupling involving the metallic part of the disturbed circuit a. Such situations, of course, are usually found where the amount of parallel exposure is relatively small.

The local telephone plant, particularly subscribers' lines radiating in all directions from a central office, is more likely to be affected by "ground potentials" than the toll plant. With some types of power distribution systems, the use of ground connections to the same water pipes or other underground structures as are used for telephone station ground connections has a tendency to increase these effects.

Owing to the numerous discontinuities of exposure that are inherent in distribution circuits, it is often impracticable to make effective use of transpositions of open-wire power or telephone circuits. The continual change of load connections and circuit layout in a growing plant tends to nullify a plan designed to fit a specific arrangement. There are, of course, exceptions to this, as in

‡. Refer to Item No. 4 of Bibliography, Technical Report No. 65.

long rural exposures where transpositions[6] are a suitable means of coordination.

Low-Frequency Induction Due to Transient Power System Conditions. Aside from questions of hazard through physical contact, the principal concern of the telephone engineer is usually noise, where the power-circuit voltage is not greater than 10,000 to 15,000 volts. For voltages higher than this, the effects of low-frequency induction under conditions of fault to ground on the power circuit are also important, often more serious than noise. It is quite possible that even with lower voltage circuits interference may arise under some conditions of transient disturbance. Operating experience, however, shows that such cases have been unusual.

With the higher voltage circuits, cases of serious trouble in exposed telephone circuits at times of power-system disturbance have been by no means uncommon. This is particularly true when the neutral is grounded, in which case the agency is magnetic induction due to current in the earth.

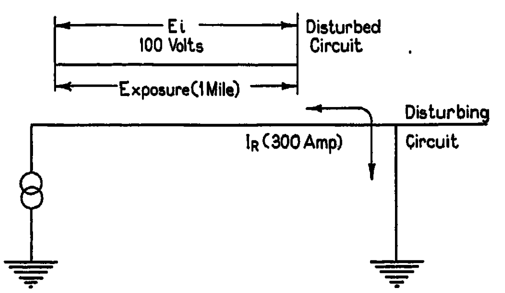


FIG. 2—MAGNETIC INDUCED VOLTAGE ALONG CONDUCTOR IN EXPOSURE AT HIGHWAY SEPARATION TO GROUND-RETURN CIRCUIT (AT 60 CYCLES)

To indicate the magnitude of this effect, it may be noted that the voltage induced between the ends of a conductor, paralleling at highway separation a circuit having ground as one side, is of the order of one volt for three ampere-miles of exposure, at 60 cycles per second. Thus, in the case shown in Fig. 2, with a fault current (I_R) of 300 amperes and an exposure one mile in length, the induced voltage (E_i) is about 100 volts. The value decreases less rapidly than the separation increases with change of separation up to several hundred feet§, and is, of course, dependent upon the current distribution in the earth. In the most favorable case the induced voltage is attenuated and appears as voltage to ground divided equally between the terminals of the disturbed circuit, but very often practically the full value of the induced voltage appears between conductors and ground at one terminal.

The forms in which this interference appears are substantially the same whether the exposed telephone plant is local or toll, *i. e.*, the induced low-frequency voltages may cause interference with telegraph and, if high enough, the operation of protective devices, false operation of signals (including subscribers' bells), acoustic or electric shock and damage to apparatus or equipment. These effects may be very serious and the development of generally satisfactory methods for their prevention is one of the outstanding problems confronting the engineers of the two industries. The work which has been done to date on this question has been called forth by the occurrence of more or less critical situations where relief by some means finally became a necessity to the telephone company. The measures which have been considered have ranged from removal of one circuit or the other to the use of resistance or reactance in the connections from power transformer neutrals to ground, and to the use of drainage on the telephone circuits. Good results have been secured in several cases where resistances in neutral ground connections have been installed. This was made practicable by the development of sensitive relays, arranged to operate on comparatively small residual (or ground) currents. However, there are situations in which the use of impedance in the neutral, no matter how large, cannot by itself afford a complete remedy. These are cases in which, even though the power neutral were isolated, the exposure is of such length and closeness that the voltages which would be induced electrically between the telephone wires and ground, when a fault occurs on the power system, are themselves sufficiently high, and the corresponding charges sufficiently great, to cause interference. An exposure of this character is most troublesome and any one who has had experience in attempting to deal with such a case would probably agree that all reasonable alternatives should be exhausted before such a condition is created.

The use of drainage on commercial telephone circuits for the relief of high-voltage induction, while not impossible, involves complications of operation and maintenance so great as to make it impracticable in the majority of cases. Drainage of the ordinary balanced retardation coil type cannot be used on any type of circuit employing d. c. signaling, and this excludes it at once from a large part of the class of circuits with which this paper is primarily concerned. Conditions under which drainage is feasible are most likely to be met on relatively short circuits where the induction is electric in character. This last condition means that few drainage coils, perhaps only one per circuit, would be required. Where the induction is magnetic and the induced voltages are not relatively small, difficulties are encountered in making drainage effective without causing excessive transmission loss or interference with signaling. The possibilities of drainage on telephone circuits as a measure for the reduction of interference are being considered in detail by the Joint Development and Research Committee.[2]

An apparatus for application in the telephone plant

§. For method of calculation, etc., see Item No. 4 of Bibliography, pp. 656 and 682. Taylor, Ref. 7, gives values that are low due to assuming current in earth returning very close to the surface.

known as the "acoustic shock reducer" is under development and is now being tried out in a few places. It is designed to reduce the intensity of acoustic shocks and is thus purely a specific remedy, without bearing on the several other kinds of important low-frequency interference. The trial installations now under study have been made at operators' switchboard positions. These devices must be regarded as expedients serving to alleviate, though not to eliminate, one of the most troublesome types of interference.

The provision of adequate separation between the power and telephone circuits, is, of course, the most effective remedy where it is feasible. Where the situation involves local service connections, separation is often not an available remedy, and there are other situations involving toll lines and power circuits where, owing to the nature of the terrain, separation of lines is, practically speaking, out of the question. These situations often present problems of very great difficulty.

Noise Induction. With the great variety of relationships, normal and accidental, which can exist between power distribution and telephone circuits, instances where noise interference occurs in the telephone circuits range from the heavy, low-toned rumble, caused by temporary contact of a 110-volt house service on a cable suspension strand, to the steady, high-pitched, single-frequency note sometimes found with apparently little exposure, coming from a grounded-neutral generator. It is not practicable to discuss here many very interesting but unusual cases. The great bulk, however, are found to be traceable to residual currents, and, in less degree, to residual voltages, in the power system. Induction from balanced voltages and currents is of importance, as a rule, only in exposures long enough to make practicable the use of transpositions as a means of relief.

It is well known that induction from residuals is much greater in proportion to the magnitudes of the inducing currents and voltages than that from balanced components. Some discussion of their origin and possible means of control is therefore given.

For convenience and clearness, and without intent to suggest a definition for more permanent use, the term "residual" is used in this paper to refer to voltages or currents that are residual with respect to all the metallic conductors properly belonging to the distribution circuit. In the case of a particular line, for instance, this includes the phase-wires and any associated neutral wire carried on the same poles. With cables, the sheath is included. It should not be overlooked, however, that voltages and currents in the circuit having the neutral as one side and the line wires as the other, are a distinct class, just as residual voltages and currents are distinct from those belonging to the line conductors only, and are likely to be characterized by large induction coefficients.

The situations of chief importance, in which residual currents and voltages exist in distribution circuits, are:

1. Along single-phase branches from polyphase circuits,
2. Where there are multiple grounds on neutral wires,
3. Where triple harmonics are introduced by grounding the neutrals of generators or transformer banks in three-phase circuits,
4. Where admittances to ground of the phase-wires are unequal.

To illustrate, consider Fig. 3, which shows a single-phase branch on a three-phase, four-wire circuit. This single-phase circuit is, of course, entirely unbalanced from a voltage standpoint. That is, the star voltage of supply is also very closely the residual voltage. If all three phases were present, there would be no such intrinsic residual voltage, but with only two phase-wires the effect is the same as with one. Not only at the higher voltages, such as 11,000 or 13,000, but also at the usual voltages of primary distribution, this residual voltage may give rise to serious noise in directly exposed open-wire telephone circuits.

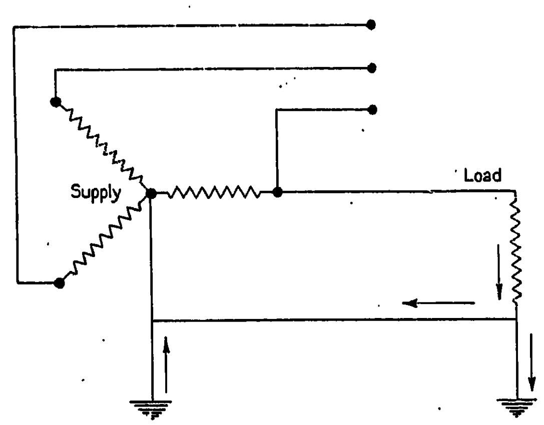


Fig. 3—Single-Phase Branch of Three-Phase Circuit

The amount of residual current in the case of Fig. 3, with neutral wire grounded at both ends, depends on the division of the load-current between the neutral wire and the earth, which is, of course, governed by their respective impedances. With the neutral grounded at only one point there is no residual current (neglecting capacitance to ground of the phase-wire). Were the neutral wire absent, the current would be entirely residual.

With all three phases present, the residual load-current will be reduced by balancing of the load among the phases, but with multiple grounds, there often remains ample residual load-current to cause interference. With only two phases present, equally loaded, the effect is the same as with one. It is a well established principle that the ground should not be used as a load-current path.

The arrangement illustrated in Fig. 3, with all phases present or with some of them lacking, may also give rise to triple harmonic residuals (item 3 above). The

presence of a single neutral ground causes the appearance of triple-harmonic residual voltages and charging currents. With one or more additional grounds, these residual currents are often much greater.

In some cases the system may be nominally grounded at only a single point, the additional grounds appearing more or less incidentally, as when the neutral wire is carried along as the sheath of a cable in sections where the circuit goes into underground or submarine cable. In other cases the system is designed to have multiple neutral grounds. An example is the three-phase, four-wire system in which a single wire serves as both the primary and secondary neutrals. This arrangement introduces a large number of grounds on the primary neutral because of the grounding at customer's premises or on the secondary side of transformers. It is the most difficult to deal with of all polyphase distribution systems employing a neutral wire. Owing to the impracticability of establishing and maintaining balance of loads, this arrangement results in residual load-current even when all phases are present in an exposure.

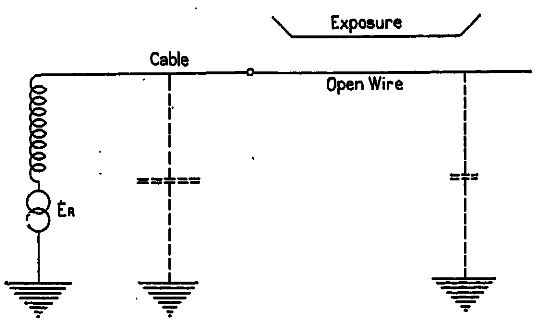


FIG. 4—EQUIVALENT CIRCUIT TO SHOW POSSIBILITIES OF RESONANCE OF TRIPLE-HARMONIC RESIDUALS IN THREE-PHASE, GROUNDED-NEUTRAL GENERATOR AND LINE

The distinctive difficulty is due to the residual current which as a rule contains both triple and non-triple harmonics.

Another type of situation involving triple-harmonic residuals which has recently appeared in several places, involves circuits at higher voltages. In these cases the power system is usually partly in cable and partly in open wire, with the generator connected directly or through auto-transformers to the cable plant. The generator neutrals and also those of the auto-transformers, where used, are directly grounded. In several particularly interesting cases, it appeared that a condition of resonance, or of an approach to resonance, existed at one of the frequencies of the triple-harmonic series between the direct capacitance to ground of the underground-cable system and the inductance of the generators or generators and transformers. The interference was not due to parallels with the underground cables, but in these instances appeared in circuits exposed to open-wire power circuits connected directly to the cables. The situation is sketched diagrammatically in Fig. 4 which shows the equivalent single-phase circuit of the three phases in parallel with "ground-return." It is to be observed that, with appreciable direct capacitance to ground beyond the exposure, the induction may be due to currents as well as voltages. Tests have indicated, in some of the cases which have been studied, that both types of induction were of importance.

A remarkable feature about one of these "resonance" cases was that the wave form of the generator voltage was unusually pure. Analyses of the generator voltage showed only a very small percentage of the particular harmonic (in this case the ninth) which was prominent in the noise in the telephone circuits—so prominent at one time that other frequencies could not be noticed on direct listening. The condition in which this harmonic was so strongly the dominating frequency in the noise passed with certain changes in the power system, although this frequency still continued as a prominent feature.

In another case, in which the evidence of resonance was less striking, the fifteenth harmonic was the dominant frequency in the noise, although the ninth and some of the upper triples were also present. The third harmonic was the strongest triple in the neutral current of the generator, accounting, in fact, for nearly all of the r. m. s. neutral current. The fifteenth harmonic was well marked in the line to neutral voltage of the machine. The telephone interference factor[8] of the neutral current wave was about 2000, in contrast with values of the order of 50 commonly found for load voltages and currents.

Another particularly interesting example, in which the power-circuit voltage was 13,000, involved an aerial power cable connecting at both ends to open wire and a telephone subscribers' cable, also aerial and at highway separation. The power cable went underground at one end of the exposure and tests showed that the interference, which was of severe character, was due to residual charging current, principally of the ninth harmonic frequency, which returned to the source of supply by way of the earth rather than along the sheath of the cable. This condition was due to the lack of an effective ground for the cable sheath between the exposure and the supply station. The case is of especial interest as illustrating the fact that shielding, even by metallic sheaths around both systems of conductors, may, in some cases, be quite ineffective, and also because it brings out the difficulties which may arise when dealing with induction from residual currents.

These cases and others resembling them have led to the application of various means of suppressing or reducing the magnitudes of the triple-harmonic residuals. A familiar and generally effective scheme is the provision of a grounding bank of transformers with a low-impedance delta and preferably the removal of the ground connection from the machine or transformers giving rise to the residuals. For effective results, such a bank should be designed for relatively

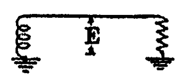
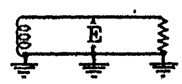
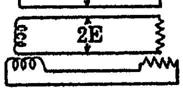
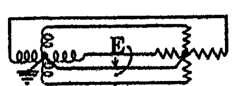
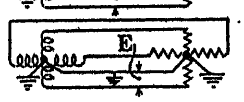
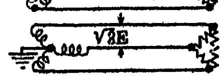
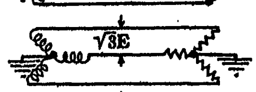
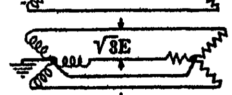
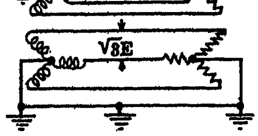
low magnetic flux density. Other methods which have been found effective involve the introduction of impedance in the connection from neutral to ground. This impedance may consist of a tuned circuit[9], where only a single harmonic is important, or it may consist of a simple reactor. In either case, its magnitude may be made small at the fundamental frequency. The Petersen reactor or earth-coil, in considerable use in Europe, is an effective appliance for the reduction of triple-harmonic residuals.[10]

When a circuit is designed to be isolated entirely from ground, residuals arise in it only through differences of admittance of the phases to ground. A common example is a single-phase tap on a three-phase line. The tap has residual voltage approximately equal to the star voltage to ground. In addition, it causes residual voltage on the main line. An isolating transformer to separate the branch from the main line prevents both effects. Serious induction has been experienced where tree grounds, defective insulation, etc., cause partial phase grounds. One ground, and sometimes more, may persist for long periods without affecting the power-circuit operation, meantime causing noise. Even high-frequency carrier current systems may be greatly affected by such intermittent ground connections.

Two-phase systems have characteristics in general

TABLE I

COMPARISON OF CERTAIN PRIMARY DISTRIBUTION SYSTEMS FROM STANDPOINT OF INDUCTIVE COORDINATION*

System				Residuals†				
				All Wires Present		One or Two Wires Missing		Facility of Coordination (all wires present)
No. of Phases	No. of Wires	No. of Grounds	Typical Diagram	Voltage	Current	Voltage	Current	
Single-Phase	1	Multiple		E	Total Load	..	..	Very difficult
	2	None		0	0	.	..	Good
	2	Single		E	Charging	..	..	Difficult—Residual voltage may be important
	2	Multiple		E	Part Load Plus Charging	E	Total Load	Difficult
Two-Phase	3	None		0	0	0.47 E or 0.75 E	0	Good
	4	None		0	0	..	..	Good—Identical with single-phase two-wire
	5	Single		0	Charging	0, E or $\sqrt{2}\ E$	Charging	Good
	5	Multiple		0	Part Load	0, E or $\sqrt{2}\ E$	Part Load and Charging	Difficult. Possible even harmonic residuals
Three-Phase	3	None		0	0	E	Charging	Good
	3	Single		Triple Frequency		E	Charging	Fair
	3	Multiple		Triple Frequency		E	Load	Difficult
	4	Single		Triple Frequency		E	Charging	Fairly Good
	4	Multiple		Triple Frequency and Part Load		E	Part Load and Triples	Difficult

*For high voltage three-phase systems factors not here considered may be important. (See text).

†Residuals arising from differences of admittance to ground, whether due to the configuration of conductors or to accidental leaks, are not included in the tabulation. By residual current is meant that part which is in the earth or in remote wire paths.

similar to the single- and three-phase systems just described. The two-phase, four-wire system with no connection between the phases or to ground amounts, in effect, to two independent single-phase systems. The problem of inductive coordination is thus reduced to the consideration of the effects of a single-phase circuit having no intrinsic residuals other than those due to line unbalances to ground which, for the voltages and lengths of circuit in common use, are negligible, providing, of course, that the line is maintained free of leaks to ground. This system is the simplest of all polyphase distribution systems to deal with, from the standpoint of inductive coordination.

The two-phase, three-wire system isolated from ground has no intrinsic residuals other than those due to line unbalances or to the omission of a conductor. The residual voltage, with one phase-wire lacking, is approximately 0.75 E (E being the voltage to neutral) while with the neutral lacking, it is 0.47 E. In at least one instance where a circuit, partly four-wire and partly three-wire, was operated as two-phase, three-wire by paralleling two wires in the four-wire portion, large admittance unbalance and induction resulted.

The two-phase, five-wire system, (really a four-phase symmetrical system), is similar to the three-phase, four-wire arrangement except that triple harmonics cannot appear as residuals, which is, of course, an advantage. If, however, even harmonics are present, the fourth and its integral multiples may appear as residuals, while the second and its odd integral multiples may appear in the metallic circuit, each side of which consists of two phase-wires in parallel. This system, with multiple neutral grounds, has essentially the same possibilities as the three-phase, four-wire system as regards residual currents due to load unbalances. The omission of two phase-wires of opposite phases causes no residuals. The omission of one or three produces a residual voltage equal to the star voltage.

To summarize this comparison of the various types of distribution circuit, Table I has been prepared to show the residuals encountered in thirteen different cases and the corresponding problem of coordination. Balanced currents and voltages are not included, as their treatment is much the same for the several systems described. The relative desirability of the several arrangements from the standpoint of power-system operation exclusively is not taken into account, and it is realized that the preferences indicated are doubtless not the same as would be arrived at by consideration from that point of view.

Although the paper is concerned primarily with a consideration of different kinds of power-distribution systems in an attempt to develop their characteristics which react on telephone circuits, (a point of view in keeping with a general discussion of power distribution, of which this paper forms a part), a brief statement relating the properties of the telephone plant to the problem may be helpful. This subject is being studied in detail by the Joint Subcommittee on Development and Research, previously mentioned.

The benefits and limitations of metallic cable sheaths for reducing the susceptiveness of telephone circuits have already been discussed. In cables, both local and toll, direct induction in the metallic circuit is negligible; noise appears in the metallic circuit as the result of the action, upon unbalances in lines or equipment,[11] of induction in "ground-circuits" (*i. e.*, circuits having the metallic circuit conductors as one side and ground as the other). These are self-unbalances to ground of the particular circuit under consideration, or unbalances between it and the other ground-circuits.

Telephone-line conductors, whether in cable or open wire, are designed to have the two sides of the circuit closely alike, so that the accidental unbalances inherent under actual construction and maintenance conditions are the only ones requiring consideration. Intricate transposition systems for open wire,[6] and elaborate arrangements for twisting cable pairs and splicing cable sections together, are examples of this design. Complete elimination of these accidental unbalances is, of course, impracticable.

In the cord circuits used in connecting subscribers together, there are small self-unbalances at the central office, but these are not sufficient to be a factor in the production of noise except in unusual cases of severe exposure. The necessity of guarding against noise and crosstalk from sources within the telephone plant involves many of the same considerations that are pertinent to this problem and has led to the limitation of these unbalances.

At subscribers' stations, there are unbalances due to grounded ringers on some types of party-line circuits and to coin-collect relays in some classes of service. Unbalances, due to ringers, may be reduced by the use of ringers of higher impedance than the standard, developed for this purpose, and some application of these higher impedance ringers has been made in working out specific cases of interference. Other possibilities for party-line stations exist, involving additional relays or other apparatus at the subscribers' stations.

Conclusion

1. The relationships of power distribution and telephone circuits are extremely intricate and varied, involving many considerations of the details of design of both systems. Both systems are expanding rapidly, having a common goal of universal nation-wide electric service.

2. The location, design, construction, operation, and maintenance of both systems require careful consideration from the standpoint of safeguarding properly the public, the employees, and the physical plant as well as suitable provisions for meeting present and future requirements for both services in an adequate and convenient manner.

3. Residual currents in power-distribution systems

are an outstanding source of inductive influence both at voice frequencies, and, under transient conditions, at the fundamental frequency. Under normal operating conditions, the limitation of residual currents appears to be largely a matter of the type of distribution system used. Under abnormal conditions, their limitation is a more difficult problem but still very much dependent on the type of distribution system.

4. With telephone toll-circuits, avoidance of severe exposure by choice of locations for both classes of lines is usually possible and the most satisfactory measure of prevention or mitigation.

5. The use of telephone cables with continuous metallic sheaths, grounded continuously or at terminals, is a very helpful measure.

6. Experience indicates that interference is not produced by exposures to power circuits in underground cable. The combination of underground cable in grounded-neutral systems with an open-wire power circuit in exposures has produced severe interference conditions in a number of cases. Aerial power cable with sheath adequately grounded is similar to underground cable from the standpoint of inductive effects but may give rise to interference if the sheath is not grounded in the proper manner.

7. A well-organized scheme of joint study and research covering the numerous engineering problems of coordination is in progress. While this study should result in the development of technical information of general applicability, the solution of existing interference problems is not attainable by any simple general rule, but only after careful analysis of the respective plants as actually constituted, to find the most practicable measures for relief applying to the specific case. Such solutions must be adequate to provide reasonable margins of flexibility in operation.

8. The design and construction of new plants for both classes of service, and the choice of measures of relief in existing cases, should be guided by a sense of common responsibility to insure successful coordination in the period of great growth that is anticipated. To this end, the exchange of information and joint planning by power and telephone engineers, in advance of the crystallization of designs, is obviously the only practicable method of handling the problem.

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Reference should be made also to the Bibliography in the Final Report of the Joint Committee on Inductive Interference to the Railroad Commission of the State of California. This report is included (pp. 23-84) in reference 4, above.

Discussion

H. S. Phelps: Messrs. Trueblood and Cone have very ably and clearly pointed out the importance of the type of power-distribution system in the matter of inductive coordination. This importance is recognized, and as the problem is better understood, more and more attention is being given it. It seems to me, however, that they have not placed sufficient stress upon another equally important feature—namely, the characteristics of the telephone system.

The ideal arrangement, of course, would be, first, a power system without residual currents or voltages, with the conductors very close together; such a system would be closely approached by using multi-conductor cable, where all the current was forced to return in the conductors; second, a telephone system without any impedance unbalance in the metallic circuit or admittance unbalance between the two sides of the circuit to ground. In this desirable telephone system the subscriber's set should not require utilization of an unbalanced ground connection for ringing. Connecting such an instrument to the system by means of a well designed and carefully installed cable circuit to a well balanced central-office cord circuit would likewise materially improve the situation.

An ideal power system would not produce longitudinal voltage in neighboring telephone circuits, and therefore could not cause noise in the telephone system, regardless of telephone circuit unbalances. On the other hand, an ideal telephone plant would not be susceptible to induced longitudinal voltages, and therefore could not be affected by any distribution system during normal or abnormal conditions, unless the induced voltages were of a magnitude to constitute danger to life or property.

Since neither ideal system is necessary or practical, it remains to determine how far either system may depart from the ideal without placing serious limitations and burden on the other. In order to ascertain the technical facts underlying this problem, the joint Development and Research Committee of the National Electric Light Association and American Telephone and Telegraph Company has been carrying on work under one of its projects at Minneapolis for over a year.

Although many interesting and useful results have been obtained, it would be premature to attempt outlining any of these at this time. However, it is expected that an interim report will be issued in the early spring covering the major findings of this work.

The considerations outlined for coordination of the telephone circuits with a distribution system apply in the same manner to

the problem of induction from a lighting system discussed in the paper by R. G. McCurdy.[1]

P. D. Jennings: I gather from reading this paper and from an informal talk with Mr. Cone that the residual higher harmonic currents are the ones that give the most trouble. I should like to ask why resonance shielding on substation ground circuits might not take care of most of this trouble. As an example, suppose in a particular substation, our oscillograph records show that the ninth and fifteenth harmonics are predominant. It occurred to me that a resonant shield might be used to resonate out the ninth and the fifteenth harmonics on the substation ground and in that way eliminate most of the trouble.

A. A. Williamson: Mr. Cone referred to the relatively greater freedom from inductive interference that is usually experienced when both the power and the telephone circuits are in cable. Cases sometimes arise, however, where interference is experienced between the two classes of circuits when they are both in cable, and such cases are usually of somewhat more than ordinary interest. A very brief reference is made to such a case in the paper and it seems to me that a few words of additional description of the conditions in that case would be interesting.

In this instance, power at 13,000 volts was supplied directly by a three-phase generator, connected in Y and with the neutral grounded, to two open-wire circuits, branching and each supplying 13,000-volt power to a substation. The power company decided to provide a tie between the two substations and concluded that in this case the tie should be of cable. The tie when installed was approximately 1.65 mi. in aerial cable, and about 0.7 mi. in underground cable. The sheath of the underground cable was, of course, well grounded, but was not connected in any way to the neutral of the generators. This cable tie paralleled in its aerial portion an aerial telephone cable.

As soon as the tie was energized, interference was noted in the telephone circuits. An investigation showed that the sheath of the aerial power cable was also grounded at intervals with driven grounds but that these grounds were of rather high resistance so that the residual charging current flowing into the cable sought ground and returned to the generating station through the low-resistance ground afforded by the sheath of the underground portion of the cable. Thus the charging current flowed through the parallel as residual current and produced interference. By installing a low-resistance ground on the sheath of the aerial portion of the power cable, at the end more remote from the underground portion, the induction was very greatly reduced. The reason for this was that with the additional ground in place, the charging current flowed through the capacity of the wires to the sheath and back over the sheath. Thus, in each part of the parallel, there was a return path for the charging current flowing out over the three-phase wires and instead of acting as a residual current, it became a balanced component. This case illustrates the effect of residual current in causing induction sometimes of serious magnitude even when both classes of circuit are in cable.

I should like also to cite very briefly a case with which I happen to have had intimate contact on the Atlantic Seaboard. This case illustrates the importance on the induction problem of unbalanced load current flowing in the ground. In this instance, a 4000-volt, three-phase, four-wire distribution system with the neutral grounded at the point of supply paralleled in open-wire construction an aerial telephone cable. The parallel was about 8000 ft. in length; it was joint construction and at the end of the parallel more remote from the point of supply to the power system, the three-phase, four-wire circuit entered underground power cable. At that point, the neutral wire of the three-phase, four-wire system was connected to the underground cable sheath. As the sheath of the underground cable was well grounded, it could be seen that any current in the neutral due to unbalance of load between the three phases would return to the power station both by way of the neutral and by way of the ground, the division of current in the two paths being in approximately inverse proportion to the impedance of these two paths.

In this case, measurements were made of the interference on party line subscribers' circuits in the telephone cable, and approximately 900 standard noise units were found at the subscribers' receivers. Owing to the excellent cooperation of the power company in this case, it was possible during the investigation to disconnect temporarily the power cable from the aerial portion. When this was done, the path for the unbalanced load current to return through the ground no longer existed. Under this condition the induction was reduced to about one-third of its former value.

This case seems to illustrate very well the effect of unbalanced load current when it can return through the ground. The use of power cable with the sheath connected to the neutral wire is only one of the ways in which a path may be provided for unbalanced load current to return through the ground. Whenever the neutral wire of a three-phase, four-wire system is grounded at several points, this same opportunity exists and experience has indicated that it is usually one of the most important features from an induction standpoint.

K. L. Wilkinson: In practically all cases telephone subscribers are users of electric light and power service. Therefore, the companies, in order to serve these customers in distribution areas, must of necessity have their overhead lines in close proximity to each other. The problems arising therefrom are mutual ones since they involve the rendering of both services to these common customers in a safe, adequate, and economical manner. It seems to be now generally appreciated that these problems require cooperative consideration by the two utilities in order to be successfully solved. The advantages of this cooperative treatment have generally been recognized throughout the country, and the splendid cooperation between the operating utilities in the field is producing most satisfactory results.

Now, in order that these individual efforts in the field may be most successful, it is desirable that all parts of the country know what is going on in all other parts of the country and be in a position to have made available to them all of the data and information which would shed any useful light on the problem. To this end, some four years ago there was organized the Joint General Committee of the National Electric Light Association and the Bell Telephone System. This Committee was to investigate the physical relations between electric supply lines and communication lines, and to develop principles and practises for the guidance of the operating companies in solving their day-by-day problems.

The Joint General Committee has at its disposal all the operating experience of the country and has, as you know, published *Principles and Practises for Inductive Coordination of Supply and Communication Systems.*

The principles which have been developed are nothing new; they are based on the operating experience in their day-by-day work in coordination of the two systems.

One of the most important, I think, is the principle of cooperation and the advance notice. I thought of that particularly when Mr. Heinze mentioned the growth and development of the power systems in the distribution areas, the increasing load density and the fact that nearly everybody now has a telephone and every telephone must be a part of a system that operates throughout the United States, so that any one telephone in any one part of the country can be connected with any other telephone in the system.

If we are going to have the best and most economical power system to supply the people with electric light and power, and if we are to achieve the ideal of universal communication throughout the country, it is absolutely essential that locally and nationally we establish and maintain the closest contact

1. Induction from Street-Lighting Circuits, by R. G. McCurdy, A.I.E.E. JOURNAL, Vol. XLIV, October, 1925, p. 1088.

between the two utilities in order that the public may obtain the fullest benefits of all of our engineering knowledge and experience.

The Joint General Committee, in approaching the problem, established one thing clearly and that was that each party should be the judge of his own service requirements and what was necessary to serve his customers. Next to that was the duty of coordination; that is, each party should so conduct his business as to be less productive of adverse influence on the other system; and, the system should be as free as practicable from things which would make it capable of being adversely affected.

I think that if we bear these major thoughts in mind,—first, that each is the judge of his own service requirements, and second, that we have a mutual duty toward the public to see that they get safe, adequate, and economical service,—then the necessity of planning well in advance so that the situation does not get out of hand will be fully appreciated and we shall be promoting the best interests of the public in getting these two necessary services.

F. O. McMillan: Would it not be advisable to include in this paper under the three-phase transformer connections, some reference to both the primary and secondary winding connections, because of the fact that the third-harmonic magnetizing current and all multiples of the third harmonic in Y-delta-connected transformers are very nicely cared for when the delta connection is used on either side of the transformer?

S. B. Hood: In the paper I note, in the tabulation of the relative ease of coordination of the different systems, that almost without exception, the power system which is easiest to coordinate is the very system which the power man does not want to use.

Now that means that we must have a cooperative spirit of give and take. At some place in the list is the system which is just as good for the telephone man as for the power man. Just where it is, I don't know. I don't think it has ever been discovered, but we certainly have to work in a true cooperative spirit toward that end.

M. T. Crawford: Mr. Cone refers to the possibility of a slight ground which persists for some length of time as being a very serious source of interference when it occurs on the primary system.

I believe that practical experience will bear me out that on a grounded-neutral primary system it is almost impossible for a slight ground to persist for any great length of time. The ground on the grounded-neutral system is a short circuit and very soon develops into something that will trip the switch out. That would be an argument in favor of the grounded-neutral system as being superior from the point of telephone interference.

I should like to ask Mr. Cone what he considers the principal objection to raising somewhat the 5000-volt limitation which is at present observed in connection with joint-pole construction of light-and-power and communication circuits. This, of course, is an old question, but it seems today to assume a new aspect inasmuch as there is here evident such a willingness to cooperate. Perhaps the difficulties of joint-pole construction on voltages over 5000 have been where the distribution work of the light and power companies was not planned far enough in advance to take into account the telephone company problem.

The Puget Sound Power and Light Company now operates on a part of its system a commercial telephone system which was taken over in connection with the purchase of a smaller company. On this system we have 6600-volt primary distribution on our own poles in combination with our own telephone service lines. We have been able to live very well with ourselves under such conditions.

L. J. Corbett: I wish to second Mr. Hood's remarks in regard to cooperation in spite of the suggestion which has been made that a wave of propaganda is upon us.

I have observed that the telephone men have studied the theoretical part of power transmission and distribution rather thoroughly. But very few power men study the telephone problems thoroughly. If we did and suggested to the telephone companies how to operate their systems, it might be taken in the same spirit as that in which the power men receive suggestions from the telephone men as to how they might operate their systems.

The ideal manner of handling the inductive-coordination problem would be realized if the same interests owned both the communication system and the power system. If this were true they would be compelled to get together and determine the accurate economic solution in each case.

In California we act under the California Railroad Commission. The order of the commission is a state law, and under that law we are "required to cooperate." We find that this cooperation really works both ways. It is not always merely doing that which is requested by the telephone company; we do a little telephone engineering ourselves, although not in a very aggressive way. When the telephone company suggests coordinating measures, as a rule we put them in when it is possible without unreasonable expense.

If, from our standpoint, they do not appear reasonable, or if some construction or operating difficulty is involved, or a higher unjustifiable expense is indicated, we raise the question as to whether or not the benefits to be gained by these measures are worth the expense and trouble involved. When such communications reach telephone company officials, the requests are usually modified or dropped.

The beauty of the California law is that public interest comes first, and the cost of any of these coordinating measures is reflected in the rate. In this manner the public is the final unifying agent or manager and the holder of the purse-strings.

F. H. Mayer: Mr. Cone raised some objection to the grounding of the neutral return on 4-kv. distribution systems. It is the practise of the Southern California Edison Company to ground the neutral at numerous points throughout the distribution system to enable the secondary voltage to remain somewhere near a safe value should the return cable become broken. The driven pipes will tend to span the gap and thus prevent the Y connection from straightening out to a straight delta connection. If the loads on the different phases are carefully balanced there ought not be any communication disturbances.

H. M. Trueblood: I am sure we can all endorse the attitude expressed in Mr. Phelps' discussion, namely, that the solution of the problem consists essentially in finding the degree to which ideal systems must be approximated.

Mr. Phelps' reference to the joint investigation at Minneapolis should, I feel, include mention of the Northern States Power Company and the Northwestern Bell Telephone Company as participants. The progress which has been made in that work is due in no small degree to the effective cooperation and assistance of these two companies.

As regards the suggestion that the paper does not lay sufficient stress on the characteristics of the telephone system, I wish to say that there is no desire to ignore this phase of the problem. The paper has been presented as one of a group dealing with various aspects of power distribution and, as such, it does not purport to discuss telephone-system characteristics except incidentally.

With reference to Mr. Jennings' inquiry, I know of at least one case in which the measure which I believe he has in mind was applied successfully. In this instance, there were two harmonics to be taken care of, the 15th and the 33rd, and two antiresonant elements, each consisting of a condenser and an inductance in parallel, tuned respectively to the frequencies of these two harmonics. They were connected in series between the neutral

of a 13,000-volt generator and a grid resistance of low value, the other terminal of which was grounded. The exposure involved was about 3 mi. long at a separation of some 30 ft. I have been informed that effective reductions were obtained in the noise, previously quite severe, and that the arrangement has proved satisfactory from a power-operating standpoint.

Mr. McMillan refers to the omission of reference to the effects of delta windings. Because of the rather extensive ground covered in Table I, it was necessary to simplify the table and to make some selection among the different features that might be included. Of course, delta windings on transformers do affect the magnitudes of the triple-harmonic voltages and currents that appear on the lines; but this effect may be either to increase or to diminish the magnitudes of the line residuals of these frequencies, depending upon the locations of the transformers concerned and the conditions of grounding. Of the systems summarized in the table, those presenting the greatest difficulties in coordination are ones in which the load currents are more important than excitation currents, and, of course, the transformer connections are immaterial, so far as load currents are concerned.

Mr. Hood and others have remarked that the distribution systems classified in the paper as presenting the greatest facility of coordination are not those which a power company would ordinarily adopt if nothing more than the distribution of power were involved. Without attempting to pass judgment on the relative merits of different systems from the latter standpoint, I believe we should not be surprised that a conflict of this character is found to exist. This is an essential feature of the situation with which we are confronted at the present time. In fact, we have an inductive-coordination problem largely because types of systems which are deemed advantageous from one standpoint may not be so from the other. Mr. Hood's inference from the situation to which he refers is substantially the same as that arrived at by Mr. Phelps, and I find no difficulty in concurring in it.

Mr. Hood has referred to the use of multiple grounds on the neutral as a stabilization proposition. While stabilization may be the principal purpose in using the multiple-grounded neutral, it is unfortunately true that this does not prevent the setting up of residual load currents.

In his remarks applying to Mr. McCurdy's paper, as well as to that by Mr. Cone and myself, Mr. Heinze refers to the question of cooperation between the telephone company and the other electric utilities. It is true that most telephone engineers who have had to do with the inductive-coordination problem keep prominently before them the idea of cooperation. That is because it appears to us that no other method of approach can be successful. While this is more nearly self-evident now than it was some years ago, it is so important that one feels justified in laying stress upon the idea. The same thought has been expressed more than once by power engineers in the discussion of these papers.

Mr. Heinze asks how far the telephone company will go in this cooperative endeavor. As to the general spirit and attitude of the Bell System, I will merely remind you that for a number of years it has gone to considerable trouble and expense to adapt itself to circumstances which have arisen because of situations of proximity between its circuits and power circuits. That this willingness to cooperate in the fullest way will be maintained in the future seems to be sufficiently evidenced by the adoption of principles and practises by the Bell System and the N. E. L. A., under which cases are now being handled generally throughout the country, and by the undertaking of a joint research investigation to determine the fundamental physical and engineering factors which enter into the inductive-coordination problem. The work at Minneapolis is one project in this general research program, and other projects are under way in different parts of the country. As to division of cost, it must be recognized that after the correct engineering solution of a given case has been found, the question of an equitable division of the expenditure necessary to put it into effect will arise. This phase of the problem has so many ramifications that any attempt to summarize it here might be misleading, and a comprehensive discussion would carry us much beyond the field with which the paper is concerned.

In conclusion, I wish to make it clear that Mr. Cone and myself have attempted nothing more than to analyze the problem in a preliminary way without going into detail, and to bring out the technical facts known to us as we see them. Mr. McCurdy, I am sure, would agree with this attitude.

D. I. Cone: Mr. Crawford spoke of the interference that arises from accidental grounding of one phase wire. On a three-wire system normally isolated from ground, such conditions may persist for days at a time, or longer. This can be prevented by suitable maintenance measures. On the other hand with the grounded-neutral system the tendency is for such accidental contacts to develop into short circuits and to operate the circuit breakers. From the discussion by Messrs. Crawford and Cunningham it is evident that the local conditions immediately surrounding an accidental ground contact cause great variations in contact resistance and resulting ground current.

Mr. Crawford has raised the question of joint use at higher voltages than have been customary in the past. Many of you are doubtless aware that the subject of conditions of joint use is under active study by the Joint General Committee of the N. E. L. A. and the Bell Telephone System. We do, of course, recognize that there are some special cases where joint use of poles at higher voltages is the best engineering solution to meet the conditions of a particular problem and in such cases joint use is being approved.

Mr. Crawford also stated that his company had lately acquired a combination power and telephone distribution system with the power lines operating at 6600 volts. Our experience has not been encouraging as to the conditions of noise and hazard that obtain in such circumstances.

There are, of course, differences of opinion in respect to the question of protection from the hazards of power distribution. I wish to suggest, first, that higher voltages are ordinarily employed in distribution for longer distances and larger loads and, second, that this inevitably means either extraordinary measures to prevent hazard and impairment of service or else lowered standards. It is our view that advance planning will enable us to avoid to a great extent the necessity of joint use at the higher voltages. Meanwhile, any specific situation that arises we are more than ready to consider.

Mr. Corbett has described the working out of cooperative consideration of specific cases in California. As he states, the best results are obtained when the reactions of proposed measures are thoroughly considered by both parties. Suggestions arising from the study of these problems by the power engineers are cordially welcomed. I think it fair to say that the working together in California is not merely a matter of compliance with regulations but a realization that it is the rational way for the power and communication utilities to solve these problems. Mr. Mayer's discussion brings out the fact that there are conflicts to be adjusted between the requirements of the communication and power distributions and that the degree of detriment from multiple grounding depends upon several factors in the layout of the systems. Mr. Mayer's point about the use of multiple grounds on the primary neutral for stabilization has also been brought out by Mr. Hood. While recognizing that accurate balance of loads might prevent detrimental induction in communication circuits so long as all phases are present, we must not overlook the facts that setting up and continuously maintaining such close balance is not a simple matter and that exposures often occur where only one- or two-phase wires are involved.

Induction from Street Lighting Circuits

Effects on Telephone Circuits

BY R. G. McCURDY[1]
Associate, A. I. E. E.

Synopsis.—This paper discusses series street lighting circuits from the point of view of their relations to nearby telephone circuits. These lighting circuits often have a much greater inductive influence in proportion to the amount of power transmitted than have most other types of power distribution or transmission circuits. This is due to the relatively large distortion in wave shape of voltage and current on certain types of these lighting circuits, and to the unbalanced voltages to ground which occur with series layouts. Three general types of lighting circuits are discussed. These are a-c. arc circuits, d-c. arc circuits supplied by mercury arc rectifiers, and alternating-current incandescent circuits. Of these, the incandescent type of circuit, in which the lamps are equipped with individual series transformers or auto-transformers, is the most important in this respect. Measures for reducing interference from these circuits are discussed. It is hoped that the information given in the paper will be useful to power and telephone engineers in their cooperative efforts to solve these difficulties.

* * * * *

INTRODUCTION

INDUCTIVE interference from series street-lighting circuits was one of the first interference problems which confronted the telephone engineer and arose very early in the development of the power and telephone industries. Then, as now, these lighting circuits contributed a much larger amount of interference to exposed telephone circuits in proportion to the amount of power transmitted than most other types of power distribution[2] or transmission circuits. This is due to the relatively large distortion in wave shape of voltage and current on certain types of these lighting circuits, and to the unbalanced voltages to ground which occur with series layouts. Exposures of telephone circuits to this type of power circuit occur more frequently in or near towns which are naturally the terminals or repeater points of toll telephone circuits, which tends to emphasize the importance of this kind of exposure. Moreover, these exposures are often irregular, the layouts being frequently changed due to adding and removing lamps, which makes it difficult, and often impracticable, to coordinate transpositions in the lighting and telephone circuits, a remedy commonly employed in other types of inductive exposure.

Three types of series-lighting circuit are of interest in this problem and are discussed in detail later in the paper. These are a-c. arc circuits, d-c. arc circuits supplied by mercury arc rectifiers and a-c. incandescent circuits. The last type of circuit may also be divided into two classes in one of which the lamp filaments are metallically in series with the line and the second in which they are connected to the secondaries of individual series transformers or auto-transformers. In the straight series circuit the lamps are bridged by film cut-outs which break down and close the circuit when the lamp fails; in the individual transformer or auto-transformer type the transformer is usually operated with open-circuited secondary in case of lamp failure. High-current lamps of the higher candle-power ratings are normally used in connection with the individual transformer type of circuit.

The a-c. arc is now practically obsolete and very few installations remain. In the present state of the art, the d-c. arc and the high candle-power incandescent circuits with individual transformers are used mainly for high-intensity "white-way" lighting in downtown districts, where both the supply and telephone circuits are in cable. Lighting circuits in the outlying districts of cities where both the lighting and telephone circuits are of open-wire construction are more often of the straight series incandescent type. This is a fortunate circumstance with respect to inductive interference which will be apparent from the detailed discussion of the characteristics of the various systems. However, a serious problem is presented by the number of cases which occur where both the lighting and telephone circuits are in open-wire and the lighting circuits are either of the arc lamp type supplied by mercury arc rectifiers, or of the incandescent-lamp type equipped with individual transformers. As the general trend of development is toward the use of higher intensities and toward the extension of highway lighting, the importance of the problem seems to be increasing.

While there are a few cases in which these series circuits are supplied directly from constant potential sources, as a rule a constant-current regulating transformer or regulating reactor is used to maintain a constant current, the terminal voltage varying with the load. The reactance of this transformer is normally large as compared to an ordinary constant potential transformer, particularly when the circuit is only partly loaded, which is often the case in order to provide for flexibility and growth. The reactance of this regulating transformer has an important effect on the wave shapes in the a-c. circuits.

In the discussion which follows, the different types

1. Department of Development and Research, Am. Tel. & Tel. Co., New York, N. Y.

2. A discussion of the inductive effects of distribution circuits generally is given in a paper, "Power Distribution and Telephone Circuits—Induction and Physical Relations," by H. M. Trueblood and D. I. Cone; see page 1052.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., Sept. 15-19, 1925.

of circuit are treated separately. Multiple street-lighting systems are not discussed in this paper as it is considered that the problems involved are not materially different from those arising with other multiple lighting systems.

A-C. Arc Circuits

Distortion of wave-shape in a-c. arc circuits is chiefly due to the non-linear characteristics of the arc.[3] On account of the high reactance of the constant-current regulator, the current wave is maintained approximately sinusoidal. Every time the current wave passes through zero, it is necessary for the voltage to build up in order to reestablish the arc; the voltage then decreases as the current increases to its maximum. The voltage again increases as the current decreases, a second maximum of voltage being reached just before the current passes through zero. The voltage wave thus contains two peaks in each half wave occurring immediately before and after the current goes through zero. As a result, the voltage wave on the circuit contains a complete series of odd harmonics, the magnitudes decreasing with increasing order but continuing large within the voice-frequency range.

Each lamp on the circuit contributes to the distortion. If the lamps are distributed approximately uniformly along the circuit, the harmonic voltages to ground at the regulator terminals will be equal and opposite, and the voltage to ground at the middle of the circuit will be zero. Aside from the length and other physical dimensions of the exposure, it is evident that the magnitude of the induced effects will depend upon whether or not both wires of the lighting circuit are present, the total voltage of the circuit, and the location of the exposed section with respect to the terminals of the lighting circuit.

If the exposure involves only one wire of the lighting circuit near either of its terminals, the induced effects are liable to be severe; if near the middle of the circuit, the effects are relatively smaller. If two sides of the circuit are present, the voltages to ground are equal and opposite provided an equal length of wire and an equal number of lamps are in each side within the exposed section and between the exposed section and the constant-current regulator. The magnitude of the harmonic voltages between the two wires depends upon the number of lamps beyond the exposed section. With such symmetrical lamp circuit layouts it is generally practicable to reduce the induced disturbances to tolerable magnitudes by means of transpositions in both lines, provided the separation is not less than the width of a highway. Since each lamp contributes to the disturbance, lamps occurring within the exposed section must be treated as discontinuities. This makes coordination difficult, particularly if the number of telephone circuits is large. Such an arrangement also makes for inflexibility in the lighting circuits, as in adding or removing lamps the symmetrical arrangement must be maintained.

Another method of coordination which is more satisfactory when conditions permit its application, is the use of a series transformer to isolate the exposed section from the remainder of the circuit. The harmonic voltages acting on the exposed section will then be only those due to the lamps on the secondary of the series transformer and dissymmetry on the primary side will not contribute to the induction. The symmetrical layout need then be applied only to the secondary side. Replacing the arc lamps on the series transformer secondary with incandescent lamps without individual transformers and equipped with film cutouts, constitutes an effective remedy.

With open-wire lines it is generally difficult, at highway separations, to reduce the noise to tolerable magnitudes unless the exposures are short. Under joint use conditions it is ordinarily impracticable to reduce the noise satisfactorily without replacing all or part of the arc lamps. It is fortunate, therefore, that the use of the a-c. arc lamp for street lighting is rapidly disappearing.

D-C. Arc Circuits

Distortion of wave-shape in d-c. arc circuits is principally due to the characteristics of the mercury arc rectifier.[4] As there are no reversals of current through the arc lamps and the changes in line current are comparatively small, changes in the resistance of the lamp arcs with change in current have a practically negligible effect in causing wave-shape distortion. The voltage and current waves set up by the rectifier contain a ripple of double the frequency of the a-c. supply with single-phase rectifiers and of six times the a-c. supply with three-phase rectifiers. Odd and even harmonic frequencies of this fundamental ripple are also present.

In order to maintain the arc, a sustaining reactance is required in the single-phase rectifier. This reactance damps out the ripple and its harmonics to a large extent. If the lighting circuit contains an appreciable length of cable between the rectifier and the open-wire section, the harmonics will be still further reduced.

Since the harmonic voltages and currents are impressed at the lighting-circuit terminals, if the lamps are distributed uniformly around the circuit, the harmonic voltages to ground at the circuit terminals will be equal and opposite and the voltage to ground will be zero at the middle of the circuit. The voltage distribution is thus the same as with the a-c. arc circuit.

The discussion given above in connection with the a-c. arc circuit of symmetrical layouts and two-wire circuits within exposures applies also to the d-c. arc

3. Tobey and Walbridge, "Stanley Alternate Arc Dynamo," Trans. A. I. E. E., 1890, Vol. VII, p. 367.

4. Steinmetz: "Transient Phenomena and Oscillations," pages 264 and 265.

circuit, except that the series transformer cannot be used and that symmetry must therefore extend through the whole circuit.

While the wave-shape distortion in these circuits is less serious than in a-c. arc circuits, coordination is very difficult when the exposures are severe, as when open-wire lines are joint.

A-C. Circuits with Straight Series Incandescent Lamps

If it were not for the distortion in lamps and associated equipment the wave-shape of any series a-c. circuit, regulated for constant current, would be better than that of the constant potential source from which it is supplied. This is due to the reactance of the constant current regulator which interposes a high impedance to the harmonics in the constant-potential supply. With straight series incandescent lamps this condition is substantially realized. While the resistance of the filaments changes with the temperature and thus with the current, the changes within a cycle are very small and the resulting distortion is negligible. A number of instances have been noted[5] where the telephone interference factors of the voltage waves of such circuits were from one-fourth to one-half the factors of the constant-potential supply sources.

Unless rather severe exposures with single-wire lighting circuits are involved or the telephone interference factor of the constant-potential supply is very high, induction into telephone circuits from circuits of this type is small. Where induction does exist it is usually practicable, by revising the circuit layout as discussed in connection with the a-c. arc circuit, to reduce the voltage to ground. Obviously the departure from symmetry of layout can be much greater for a given exposure than for either type of arc circuit.

A-C. Circuits with Incandescent Lamps Individually Equipped with Series Transformers, Auto-transformers or Bridged Reactance Coils

Under normal conditions, with all lamps in service, this type of circuit, from the induction standpoint, is practically the equivalent of the straight series circuit discussed above. Harmonic exciting currents required for either the individual transformers, auto-transformers, or bridged reactance coils are largely supplied by the local circuits through the lamp filaments. Important wave-shape distortion in this type of circuit occurs only when the secondary circuit of the individual transformer opens due to failure of the lamp filament. When the secondary circuit opens, the full line current becomes exciting current in the primary, greatly over-exciting the core. Because of the high impedance of the series circuit including the reactance of the constant-current regulator, the line current remains approximately sinusoidal. Practically the full amount of the harmonic voltage generated by the individual transformer appears across the terminals of the constant-current regulator.

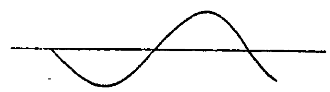

Fig. 1—Current in Transformer Primary 6.6 Amperes

Figs. 1, 2 and 3 show tracings of the voltage and current waves of one of these individual auto-transformers. Fig. 1 shows the line current in the primary

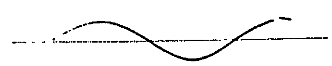

Fig. 2—Voltage Across Primary with Rated Lamp in Secondary

winding at its normal value of 6.6 amperes, Fig. 2 the primary voltage with the secondary closed through its normal lamp load, and Fig. 3 the primary voltage with

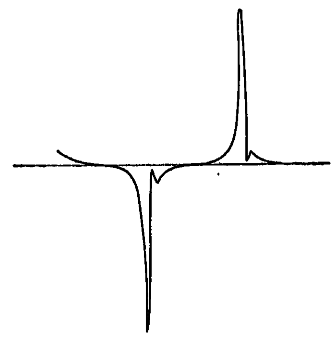

Fig. 3—Voltage Across Primary with Lamp Out

the secondary open. Following is an approximate analysis made by the 36-ordinate method of the voltage wave shown in Fig. 3:

Order of Harmonic	Frequency Cycles per Second	Volts Effective
1	60	57
3	180	40
5	300	34
7	420	31
9	540	28
11	660	26
13	780	22
15	900	19
17	1020	16
19	1140	13
21	1260	10
23	1380	8
25	1500	6
27	1620	4
29	1740	3
31	1860	2
33	1980	.2
35	2100	1

In the following tabulation is given the analysis of the voltage wave at the lighting-circuit terminals and

5. Osborne, "Wave-Shape Standard," Trans. A. I. E. E., Vol. XXXVIII, p. 261, 1919.

the corresponding telephone interference factors with all lamps in service and with one lamp out:

Order of Harmonic	Frequency Cycles Per Second	Volts Effective	
		All Lamps In	One Lamp Out
1	60	1380	1380
3	180	60	78
5	300	29	31
7	420	2.3	36
9	540	0.7	36
11	660	2.0	34
13	780	*	33
15	900	*	27
17	1020	*	29
19	1140	*	18
21	1260	*	20
23	1380	*	20
25	1500	*	20
27	1620	*	18
29	1740	*	16
31	1860	*	13
33	1980	*	13
35	2100	*	12
37	2220	*	10
39	2340	*	9
41	2460	*	8
43	2580	*	7
T. I. F.		13.6	447

*Indicates a value of 0.5 volts or less.

The effect of the one individual transformer, with open-circuited secondary, in increasing the harmonic voltages and the telephone-interference factor of the voltage wave of the circuit, is evident. The presence of other transformers with open secondaries increases the harmonic voltages slightly less than in direct proportion. That the increase is not linear is undoubtedly due to the effect of the small harmonic charging currents and to changes in phase of the harmonic voltages along the circuit from one individual transformer to another.

In respect to the distribution of the harmonic voltages around the circuit, an important distinction is to be made between the a-c. circuit with incandescent lamps equipped with individual transformers and the two types of arc circuit discussed above. In the arc circuits, the distribution of harmonic voltage is fixed and the voltages to ground on the exposed section remain the same as long as the circuit layout is unaltered. With this incandescent lamp circuit the harmonic voltage to ground depends both upon the number of individual transformers having open secondary circuits and upon their locations.

In discussing the effects of "out-lamps" at different locations along the circuit, it is convenient to replace the transformer by an equivalent generator with terminal voltage, e, of complex wave form and equal to the harmonic voltage drop caused in the circuit by the transformer with rated line current in the primary and with the secondary open. The impedance of the constant-current regulator at the harmonic frequencies involved is high compared to the impedance of the line and of the individual transformers having closed secondary circuits. Hence, the actual circuit may be replaced by the equivalent circuit shown in Fig. 4.

In this figure, e represents the generator equivalent to the individual transformer with open secondary. The capacitances C_{1g} and C_{2g} represent the capacitances to ground of the two wires of the circuit between this individual transformer and the constant-current regulator. It is evident that the relative values of C_{1g} and C_{2g} will vary with the position of the individual

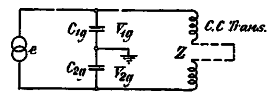

FIG. 4

transformer along the circuit, but that the sum of the two capacitances (the total capacitance to ground of the circuit) will be a constant. Letting V_{1g} indicate the voltage to ground of the section of wire having the capacitance C_{1g}, and V_{2g}, the voltage to ground of the section having the capacitance C_{2g},

$$V_{1g} = e\frac{C_{2g}}{C_{1g} + C_{2g}}$$

$$V_{2g} = -e\frac{C_{1g}}{C_{1g} + C_{2g}}$$

$$V_{1g} - V_{2g} = e$$

$$V_{1g} + V_{2g} = e\frac{C_{2g} - C_{1g}}{C_{1g} + C_{2g}}$$

If the construction of the lighting circuit is uniform, the capacitances to ground will be directly proportional to, and the voltages to ground inversely proportional to, the respective lengths of wire on the two sides of the individual transformer with open secondary.

As an example illustrating the effect of the relative positions of the constant-current regulator, the individual transformer with open-secondary circuit and the exposure section, consider the arrangement shown in

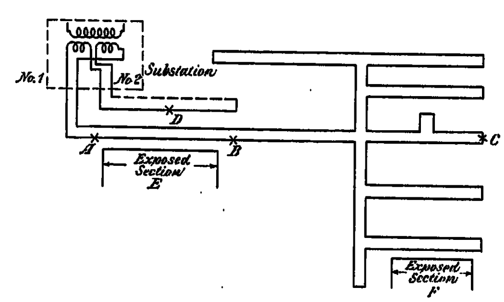

FIG. 5

Fig. 5. This includes a constant-current transformer with double secondary feeding two lighting circuits. It is assumed that the circuits are two-wire throughout their whole length and that the wire arrangements in the two exposed sections are identical.

If a lamp fails at D or at any point along the circuit (No. 2) not involved in the exposure, the harmonic

voltages impressed on circuit No. 1 will be relatively small due to the high impedance of the constant current transformer. The effects in causing noise in either exposure indicated will be correspondingly small. If a lamp fails at A, the wire on one side of the individual transformer with open secondary will be a small fraction of the total length of the circuit, while the other wire will be nearly equal to the total length. The voltages to ground on the wire involved in either exposure and the corresponding noise effects will then be relatively low, the wire having the high voltage to ground not being within the exposures.

If a lamp fails at B, one of the wires involved in the exposure section E will have a relatively large harmonic voltage to ground and the other a relatively low voltage, while both wires in section F will have the relatively low voltage to ground. Thus, considerable noise may be caused in telephone circuits involved in the exposed section E, but relatively little in section F.

C indicates a lamp at the middle of the circuit. In case of lamp failure at C, equal and opposite harmonic voltages to ground will be caused on the two wires. The voltages to ground within exposure E will then be approximately balanced. The voltage between the wires will be equal to the full harmonic voltage caused by the individual transformer. If the circuits in this section are in close proximity, as on a joint line, the noise effects are apt to be as severe as when the lamp failure occurs at the point marked B, especially if the two wires of the lighting circuit are at opposite ends of the crossarm. With the lines separated by the width of the highway, the noise effects will be much less than with an outage at B.

In exposure section F, when the outage occurs at C, the two wires of the lighting circuit will have equal voltages to ground both in magnitude and in phase, and each equal to one-half of the total harmonic voltage generated by the individual transformer. Under these conditions, the noise effects are apt to be severe whether the lighting and telephone circuits are on the same or opposite sides of the highway.

It will be evident from the preceding discussion that it is impracticable to obtain adequate relief from induction merely by symmetrical arrangements of the lighting circuits. While such an arrangement might be effective for one location of the out-lamp, it might be quite ineffective for some other location. More practicable methods involve the use of group series transformers in combination with circuit rearrangements.

Since the harmonic voltage drop along the lighting circuit is small, the harmonic voltage across the terminals of a series transformer supplying a group of individual transformers, and the voltage to ground on the secondary, are small when lamp failures occur at points other than on the secondary of the series transformer. When a failure occurs, opening the secondary circuit of one of the individual transformers in the group supplied by the series transformer, the harmonic voltages to ground along the secondary circuit of the series transformer are distributed as though the series transformer were replaced by the constant-current regulator. Similarly, the distribution of harmonic voltage along the main lighting circuit is as though the series transformer were replaced by the individual transformer having the open secondary circuit. Thus, when a number of groups of individual lamp transformers are supplied by series transformers, lamp failures cause an important effect only on the secondary of the particular series transformer in which the outage occurs and on the main circuit and not on the secondaries of the other group-series transformers. If the telephone exposures involve only the secondary sides of the group series transformers, the number of lamps which may cause noise due to outage will be much less than if all individual transformers were directly in series with the main circuit. Thus, a large reduction in noise interference due to lamp outage may often be obtained by rearrangement of the lighting circuit so that the sections involved in telephone exposures are secondary circuits of series transformers supplying a limited number of individual lamp transformers.

Failure of a lamp on the particular secondary loop involved in the exposure will introduce the harmonic voltages on this section of circuit. If the lighting and telephone circuits are on joint lines, or if there is but a single lighting-circuit wire present at highway or closer separations, the noise effects may be very severe when a lamp fails on the section involved. With two-wire lighting circuits at highway separations having a comparatively small number of lamps the noise effects due to one lamp being out will usually be much less than a similar lamp failure on a circuit having a comparatively large number of lamps. This is due chiefly to the shorter length of the circuit having a limited number of lamps as compared to one of a large number. When a failure occurs on one of these shorter circuits at such a point that the voltage to ground on one side of the individual transformer with open secondary is large, as compared to that on the other, the length having the higher voltage will be so short as not to constitute a very serious exposure. If the failure occurs at a point on the circuit, so that the exposed section is between the series transformer supplying the circuit and the individual transformer with open secondary, the lengths of wire on the two sides of this individual transformer will be nearly equal, and therefore, nearly equal and opposite voltages to ground will be set up on the two wires within the exposed section.

Similar results may be obtained by the use of small pole-type constant-current regulators, controlled by time-clocks or by series relays in another lighting circuit (the so-called cascade arrangement). As compared to the circuit supplying a number of group series transformers, distortion of wave-shape on a main series circuit is avoided. This facilitates coordination in many cases as it is often difficult to avoid exposures

with the main series circuit when the group series transformers are employed. Failure of lamps on the individual circuits will affect only the wave-shape of the circuit upon which they occur.

Effects of out lamps on the individual circuits where supplied by group series transformers or separate regulators may be avoided by replacing the lamps on the circuits involved in the exposures with *straight* series lamps without individual transformers.

It is very difficult to obtain satisfactory conditions from the standpoint of induction from this type of circuit by lamp maintenance alone. A very high degree of maintenance is required, necessitating careful and frequent inspections and prompt replacements of the failed lamps. On circuits containing 100 lamps each, an average outage of only one per cent corresponds to one lamp out on each circuit which as already shown may cause considerable noise in exposed telephone circuits. In some instances, where less careful maintenance routines have been followed, an average outage of between three and four per cent has been noted, resulting in severe noise conditions in exposed telephone circuits.

As an aid to obtaining a very high degree of maintenance, a device known as an "Out Lamp Indicator" has been developed. The device consists of a filter in combination with a milliammeter which is connected to the secondary of a potential transformer, the primary of which is connected across the line terminals. The filter cuts off from the meter substantially all current of fundamental frequency, but permits the transmission of the higher harmonics which are generated by an individual transformer with open secondary circuit. The magnitude of the reading obtained with one lamp out depends upon the rating of the individual transformer and the ratio of the potential transformer. In order to avoid an excessive fundamental frequency voltage on the network and saturation effects on the potential transformer, it is necessary to limit the secondary voltage to approximately 120 volts. Readable deflections with one individual transformer with open secondary circuit may be obtained with potential transformer ratios up to 50 to 1.

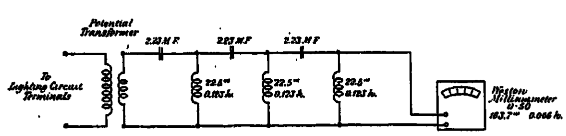

Fig. 6

A diagram of the filter with constants of the elements is given in Fig. 6. Fig. 7 shows a frequency response curve of the combination indicating the milliamperes in the meter per volt at the potential transformer secondary terminals at various frequencies.

In employing this device in maintaining the circuits, readings are taken on the lighting circuits indicating whether or not lamps are out and how many. The circuits may then be patrolled and the out lamps replaced. Experience has indicated that lamps near the end of their life fail immediately after a surge such as when the circuit is deenergized or when energized after having been out of service. If readings are taken

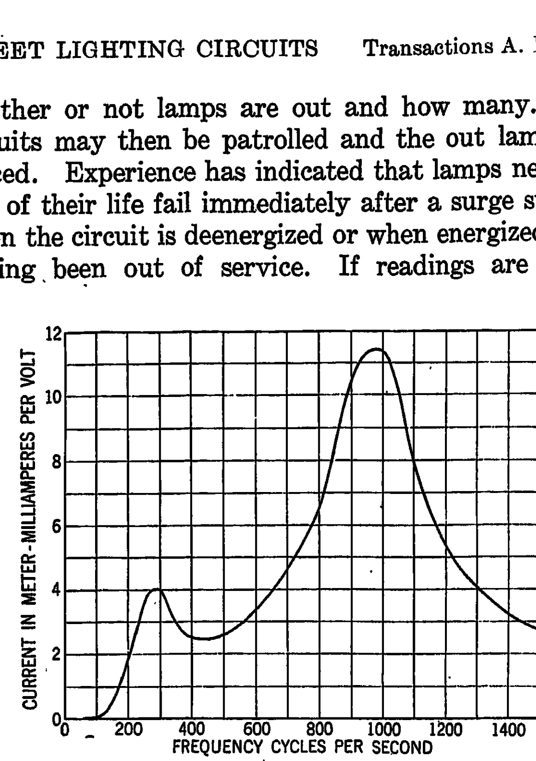

Fig. 7—Frequency Response Curve of out Lamp Indicator Milliamperes in Meter per Volt at Secondary Terminals of Potential Transformer

shortly after the circuits are energized and the out lamps replaced, all lamps are likely to remain in service for the rest of the night. If indications are not obtained on a given circuit, patrolling is, of course, not necessary. Where a circuit is made up partly of straight series lamps with film cutouts and partly of individual-transformer type lamps, no indication will be obtained of an outage of one of the straight series lamps.

Another valuable aid to maintenance is the practise followed by one large company of replacing lamps at definite intervals less than full life in connection with the cleaning schedule. The replaced lamps are then submitted to a photometer test and if not up to standard are destroyed. The lamps indicating additional useful life are then reinstalled in circuits that are not involved in exposures with telephone circuits. This method, in conjunction with the use of the out-lamp indicator, has, except for an occasional outage, eliminated inductive trouble from these circuits.

While great improvement may be effected in many cases by the application of the measures discussed above, the most direct and fundamental remedy for noise interference from this type of circuit would be the provision on each fixture of a device which would short-circuit the individual transformer when the associated lamp fails. However, unless such a device can be secured at a cost which would not sacrifice the economies obtainable by the use of the high-current lamps with the individual transformers, it would become cheaper to place the lamps directly in the series circuit omitting the individual transformer. Due to saturation of the individual transformer, only a small increase in the effective voltage across the secondary takes

place, a much larger increase occurring in the value of the peak voltage. Some device the operation of which depends upon the peak voltage seems most promising and least likely to be subject to false operation due to surges.

Conclusion

Close cooperation between the power and telephone utilities, as in all matters involving the relations between these utilities, is needed to prevent or overcome interference from lighting circuits, particularly in planning extensions or new construction. It is hoped that the information given in this paper will prove useful to power and telephone engineers in their cooperative efforts to solve these difficulties.

Discussion

R. R. Cowles: The present tendency of the Pacific Gas & Electric Company is toward the use of small series street-lighting loops with incandescent lamps only. These series loops are 6.6-ampere and are supplied from a multiple-series constant-current transformer installed on the pole in the same manner as any other distribution transformer. These constant-current transformers are of the moving-coil type, fundamentally similar to those formerly used inside of stations. In size they range from 5 to 20 kw. each, depending upon the average size of the series incandescent lamps which are connected to the circuit. It is the aim to avoid long loops and to restrict the number of lamps per circuit to approximately 60.

These transformers are connected on the primary side to a 4000-volt, four-wire, three-phase, star-connected feeder circuit, said circuit being switched from the substation just as any other 4000-volt commercial feeder. Where the primary neutral is available no additional neutral is extended for the street-lighting feeder. Single-phase lines are run in various directions to supply the territory in the same manner as single-phase lighting feeders are branched from a three-phase feeder for commercial light and power consumers. The effect of the above arrangement is greatly to improve the reliability of street-lighting service through the elimination of long series loops and the resulting reduction in potential on these series loops. Multiple 4000-volt feeders have proven their reliability on the system of the Pacific Gas & Electric Company; hence no new features are involved in this type of feeder circuit. The constant-current street-lighting transformers are furthermore removed from the substations, thereby providing room for other apparatus which cannot conveniently be placed outside of the station.

The 6.6-ampere series street-lighting incandescent lamps are standard for 4000-lumen lamps and less. This is a slight departure from previous practise which indicated the desirability of using series transformers or auto-transformers with lamps larger than 2500 lumens. An improvement in lamp manufacture, however, has made it practicable to use 4000-lumen lamps at 6.6 amperes. Lamps larger than 4000 lumens are operated at 20 amperes, necessitating the use of series transformers or auto-transformers installed on the same pole or in the same fixture which supports the lighting unit. The film cut-outs are used with all 6.6 ampere lamps, operating without auto-transformer or series transformer.

The use of series circuits with currents in excess of 6.6 amperes has been considered by the engineering department of this company but no action has as yet been taken. Small series loops supplied from individual constant-current transformers, with lamps of higher intensity spaced fairly close, would permit of operation at 15 or 20 amperes if the characteristics of these lamps demanded it. Consideration should also be given to methods of switching street-lighting circuits from a remote point which would make possible the use of even smaller loops and considerably simplify the apparatus involved therein. There are a number of types of remote-control apparatus already developed and a number more in process of development. It appears to the writer that the use of radio control for this purpose might be worked out to a practical and economic application at some future time.

C. A. Heinze: Any power engineer who has had relations with the American Bell Telephone Company will notice that the telephone engineers all tell the same story. The main thought in all of their papers is cooperation between the power engineers and the telephone engineers.

I want to state, first, that the electric utilities want to cooperate with the telephone company. We recognize the fact that we have mutual services to render, but I should like to ask Dr. Trueblood just how far the American Bell Telephone Company will go in cooperating with the electric utilities in sharing part of the expense in safeguarding the telephone companies' equipment.

S. B. Hood: Mr. McCurdy recommends some of the usual remedial measures, principally isolating transformers. That is the 100-per cent remedial measure of telephone engineering, and in most cases it is a 100-per cent remedial measure, but in very few cases is it the measure which the power man wishes to adopt. It adds to the investment, adds to the losses in the system, more or less interferes with the regulation, and has a great many objectionable features.

Another recommended cure on series circuits is the straight-series lamp. That is very nice until we get to the higher candle-power. We are all developing "white ways" that require higher densities of lighting. Therefore, recommending the type of lamp which the lamp manufacturers are not prepared to furnish is looking far into the future.

It seems to me that possibly a better recommendation which they could make—particularly since Dr. Trueblood is so enthusiastic over the multiple system—the better cure where inductive exposure is used, would be to change the lamps in that particular exposure to multiple, using a series relay for controlling. Of course, the recommendation for closed loops and balancing the lamp on a loop is very effective, but when you put a series street-lighting system with both wires looped on the same street, as far as the investment goes, it is practically getting back to a multiple system.

It seems to me, however, that in all these papers the indication is that no matter how far apart our past differences of opinion have been, we are all gradually coming closer together; the telephone men and power men are gradually getting down to a uniform system which I think is a very promising outgrowth.

R. G. McCurdy: The tendency of the development towards the use of smaller series street-lighting loops as described by Mr. Cowles in his discussion of the Pacific Gas and Electric Company's practises, from the standpoint of inductive coordination, is in the right direction. Because of the smaller number of lamps per circuit and since the constant-current transformer is closer to the lamp load, the length of any given circuit is much reduced and in case of failure of lamps equipped with individual transformers, the length of circuit upon which the harmonic voltages are impressed, and which may be involved in inductive exposures, is much less than when circuits of a large number of lamps are employed. In many cases also, this method of operation would facilitate supplying separately "white ways," where lamps of high candle-power equipped with individual transformers are used, and outlying sections where in many cases, only straight-series lamps are employed.

Mr. Hood referred in his discussion to the disadvantages of the straight-series lamp, especially in districts where high-density lighting is required. As brought out in my paper, how-

ever, it is very often the case that these high lighting intensities occur in the densely populated sections of cities and towns, where both the telephone and lighting circuits are in cable. In such cases the occurrence of "out" lamps equipped with these transformers is unimportant from the inductive standpoint. In other cases, where the "white-way" section may be of limited extent, it will often be practicable to connect the high-current lamps to circuits not involved in telephone exposures having as far as possible only straight-series lamps on the circuits involved in the inductive exposures.

Many of the difficulties of coordination discussed in the paper are inherent with the series system, and it would doubtless be less difficult to coordinate with multiple systems. The remedy suggested by Mr. Hood, therefore, of changing the lamps in a particular exposure section to multiple, using a series relay for controlling, would probably be an effective one. As far as the incandescent systems are concerned, however, it is felt that the difficulties existing with the series circuit would be overcome by the use of a reliable form of cut-out with lamps equipped with individual transformers or auto-transformers.

Opportunities and Problems in the Electric Distribution System

BY D. K. BLAKE[1]
Associate, A. I. E. E.

Synopsis.—An illustrative electric service system is analyzed to show the importance of the distribution system for the benefit of those not familiar with distribution problems. The diversity factor and load factor are employed to show their effect on investment and losses. The important subject of a-c. secondary networks is discussed relative to switching, induction regulators and a-c. elevator equipments. The circumstances which make single pole switching preferable are outlined. A new, simple and effective connection of the control circuits of induction regulators to ensure stability in parallel operated circuits is described. The present status of a-c. elevator equipments is outlined with emphasis on the merits of Ward-Leonard control where high class service is required.

The new translator network is analyzed at length in order to show its performance and inherent difficulties. It gives a three-phase, four-wire, 115/230-volt system which in no way effects the consumers meters or utilization devices. The author's opinion is that it may be adopted on the grounds of company policy rather than on technical and engineering economic grounds.

Other distribution problems are briefly mentioned. Recent developments in carrier-current control for street lighting equipments are encouraging. Static condensers located at the load center of distribution circuits prove economical in some cases. Various automatic and remote control sectionalizing schemes are under consideration in order to restore service on feeders quickly.

INTRODUCTION

THE purpose of this paper is two-fold: first, to present material to attract the attention and arouse the interest of young electrical engineers in the technically-neglected subject of electric distribution systems; secondly, to discuss several important problems appertaining to a.-c. secondary networks which have become, at the moment, of vital interest to the advanced distribution engineers. It seems to be quite generally admitted that the distribution system is in need of engineering study and of good engineers to do the work. There is nothing spectacular about the conventionalized distribution system and, therefore, little to attract the attention of a young engineer who is looking for a field that is not overcrowded and that will give him an opportunity to do some original and constructive work. There seems to be a general but erroneous opinion that there are not many technical problems involved in distribution. On the contrary, there are plenty of technical and even mathematical problems. But the emphasis herein is not placed on the technical side but on the need of initiative and judgment in the attacks on the problems. What is needed now are engineers who can bring an old distribution system up to date and appropriately plan it for the future growth. This work requires engineers of the same high caliber as those who have accomplished such marvelous results in generating stations, transmission lines, and sub-stations.

RELATION OF THE DISTRIBUTION SYSTEM TO THE ENTIRE SYSTEM

It is the purpose of the following discussion to consider the relation of the distribution system to the entire system. It is not claimed that the divisions and figures given represent accurately any particular system, but it is believed that this method gives a comprehensive picture of a large system and the important position of the distribution system in it.

Large modern electric service systems may be conveniently divided, for the present purpose, into four major divisions: viz., generation, transmission, transformation, and distribution. It is important to observe that this system is virtually a four-link chain and to remember that "a chain is no stronger than its weakest link." In an ordinary chain, each link carries as much load and is, therefore, just as important as each of the other links. Similarly the same load is carried by each major division of the electric service system and, logically, the distribution system is therefore just as important as any of the other divisions.

Service and *economy* must always be kept in mind

1. Central Station Engineering Department, General Electric Company, Schenectady, New York.

Presented at the Pacific Coast Convention of the A. I. E. E., Seattle, Wash., Sept. 15-19, 1925.

by the designer in every part of the system—Service first and economy second. Rarely can these two factors be separated. *Reliability* may reach its peak in generation, transmission and transformation improving the service as a whole, yet the service rendered the consumer is no better than the reliability of the distribution system. Maximum *economy* may be obtained in generation, transmission, and transformation, but there is still a long way to go in the whole system economy if the distribution system is neglected. It is evident from the foregoing review that to render excellent service at maximum economy a well laid out distribution system is necessary.

The distribution system itself may be analyzed in a manner similar to the foregoing main analysis and subdivided into the following parts: the primary feeders

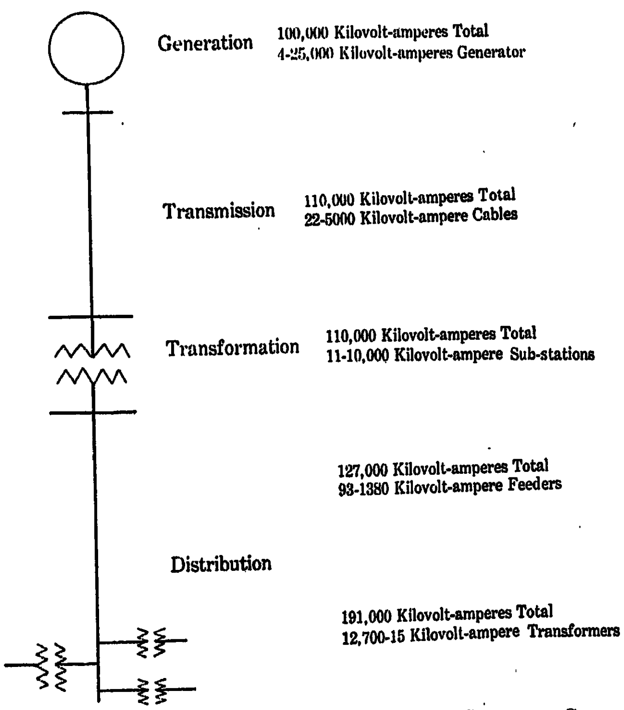

Fig. 1—Major Divisions of Electric Service Systems

and mains; the distribution transformer and related devices; and the secondary mains and service wires. This method of subdivision is made according to the *electrical* functions. Another subdivision may be made according to *physical* characteristics, namely, poles and fixtures, copper wire, and distribution transformers with their related devices such as lightning arresters, fuse cutouts, and temperature indicators.

With these methods of dividing the distribution system in mind, it may be asked how the distribution system differs in character from the rest of the system and also how it compares in the matter of money invested. Suppose it is assumed that Fig. 1 represents a radial system with one main generating station rated at 100,000 kv-a. and that the transmission lines are underground 13,200-volt cables, radiating from the generating station. In an actual system an appreciable amount of power may be supplied to electric railways and sold to other public utilities as well as to large power users. However, for the purpose of illustration, it is assumed that all of the 100,000 kv-a. is supplied to general light and power loads and that there are no inter-connections between substations to form a transmission network as is frequently the case.

The total kv-a. rating of the transmission cables exceeds the total kv-a. rating of the generators because the peak load on most transmission cables does not occur at the same time the peak load on the generating station occurs. When some cables are carrying full load, others will be carrying less than full load. If the peak loads of all the cables are added and divided by the peak load on the station, the well-known diversity factor is obtained. Therefore, the rating of the generating station may be multiplied by an average diversity factor of 1.1 for this radial system, to derive the approximate rating of the transmission cables which is 110,000 kv-a. The usual rating of each cable is about 5000 kv-a. which means that there are 22 cables. There is no diversity between substations and cables in this case because the same power that enters the transmission cables enters the substations. The total substation rating is then 110,000 kv-a. Since 10,000 kv-a. is a good average size substation, there will be 11 of them. It is well to point out here that the relation between generator, transmission cable, and substation ratings will be improved by the use of tie lines between the substations, because the diversity between these parts is changed by the transfer of power through the tie cable. Knowing the substation rating, the distribution feeder ratings can be determined by using an average diversity factor of 1.15 which will give about 127,000 kv-a. in feeder circuits. The usual distribution circuit for such a system will be rated 200 amperes, 1380 kv-a., 2300/4000 volts, three-phase, four-wire. Dividing this into 127,000 kv-a. gives about 93 distribution circuits. Notice how, as depicted in Fig. 1, the system is split up into small kv-a. units which increase in number and total kv-a. rating proceeding from the generating station outward. This is further emphasized when determining the number of units and rating of the distribution transformers. Multiplying the total feeder rating of 127,000 kv-a. by an average diversity factor of 1.5, 191,000 kv-a. is obtained as the total rating of the distribution transformers. This rating is almost double the generator rating to supply them. The average size of distribution transformer may be chosen at 15 kv-a.; therefore, there will be in this illustrative system, about 12,700 transformers. These distribution transformers will not be located in one building as is the equipment in a generating station but the transformers will be mounted on poles and are thereby scattered over a large area. In addition to the transformers there will be mounted with them such related devices as the fuse cut-outs, lightning arresters, and temperature indicators.

A cut-out with each transformer means 12,700 cut-outs, while a lightning arrester with each transformer means 12,700 arresters. Now, if these devices are not applied to the system with care and thought, a great deal of money will be wasted and losses incurred which might be saved. A standardized method of mounting these devices which would save a dollar is multiplied 12,700 times when the whole system is considered. If it were possible to connect all the distribution transformer secondaries together to form a secondary network, then, theoretically, all of the diversity factors back to the generators are removed and the system rating throughout is 100,000 kv-a. Of course, such networks can be formed only in limited areas of high load density.

It is easy to see from the foregoing analysis that the investment in the distribution system must be a large part of the total investment in the system. In some systems, it may be 30 per cent, in others 60 per cent. Distributing companies which purchase energy from

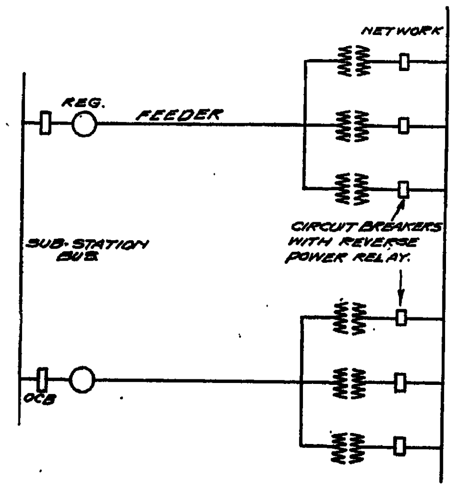

FIG. 2—MULTIPLE FEEDER NETWORK RECEIVING GENERAL FAVOR

large systems have practically all of their investment in the distribution system. These facts clearly indicate that a distribution system which is well laid out will effect large savings in investment and appreciable improvement in service.

The load factor on the distribution transformers may be taken as 20 per cent. There is a relation between load factor and diversity factor such that the load factor of 20 per cent on the distribution transformer may be multiplied by the transformer diversity of 1.5 to obtain 30 per cent which is the load factor of the feeder. Carrying this method all of the way back the load factor for transformation is 34.5 per cent, transmission 34.5 per cent, and generation 38 per cent. Observe that the distribution transformers are used about one-half as much as the generators.

The losses which occur in the distribution system have to be transmitted through the rest of the system and for that reason it is important that losses in the distribution system be reduced to a minimum. A kilowatt loss in the distribution system means more than a kilowatt at the generating station switchboard because some additional loss is incurred in transmitting it. Obviously, the kilowatt lost in the distribution system costs more than a kilowatt lost at the generating station. The distribution transformer has lower losses as well as lower cost per kv-a. for a large size than for a small size; therefore, a well laid out system should have the highest possible average size distribution transformer.

There are many other points which may be added, but it is believed that the foregoing discussion sufficiently emphasizes the importance of and opportunities in the distribution system.

A-C. SECONDARY NETWORKS

General. One of the foremost problems now confronting distribution engineers is the a-c. secondary network for areas of high load density. These are now generally supplied by the well known d-c. Edison three-wire system, which is very expensive. The purpose is to obtain the more economical polyphase a-c. network which will permit supplying lighting and power loads from the same set of mains and which will approach the Edison system in reliability.

Single-Pole versus Three-Pole Subway Switches. Where networks are under consideration the multiple feeder type of network shown in Fig. 2 is meeting with general favor. The circuit breakers in the low-voltage side of the transformers trip on reversals due to short-circuit currents or transformer core loss and are automatically reclosed when voltage conditions on the incoming feeder are correct for carrying load. This type of system is in use at New York City and New Orleans and is described in A. H. Kehoe's article, *Underground A-C. Network Distribution for Central Station Systems*, JOURNAL of the A. I. E. E., June 1924; and W. R. Bullard's article, *Study of Underground Distribution Systems for the City of New Orleans*, JOURNAL of the A. I. E. E., Nov. 1924. All of these systems under consideration, except one, anticipate using Y-connection on low-voltage side of the transformer. If the secondary switches used with this system are too large, it may be necessary to rebuild the opening of many vaults and in quite a few cases new vaults will be necessary because wall space is not available for large triple-pole switches. It is believed that a small, compact single-pole switch has many advantages over a larger triple-pole switch. The following discussion is intended to show the practical advantages of three small single-pole switches over one large triple-pole device.

The single-pole switch may be conveniently arranged for the lead from transformer to enter at the bottom and the lead to the network arranged to leave from either side of the box at the top. The three-pole switch is not readily designed with leads out either side but with leads out through the top which requires an extra bend

in the cable and more head room. The dimensions of the subway box may be made small enough to permit the switch to be lowered through any manhole opening that will admit the transformer it is to be used with. In many vaults a large wall space cannot be found on which to mount a large triple-pole switch but three small spaces may be found to mount three single-pole devices. The single-pole switch may be designed small enough to permit mounting it on the side of the transformer tank where wall space is not available. The box may be supported from the floor by a pipe and braced to the wall by angle irons. The wall surface back of the box may be used for racking cables.

It is frequently necessary to build transformer vaults on side streets or in alleys because other sub-surface structures make it impossible to obtain a space large enough for a vault to contain three transformers. Three small vaults large enough to contain one transformer and one single-pole switch may in many cases be located on the main thoroughfare, thereby avoiding the cost of a duct line from the feeding point to the transformer vault. The three smaller vaults will have the additional advantage of increased wall surface per transformer for heat radiation and this may be sufficient to make special ventilating systems unnecessary. The complete switch with an aluminum subway box may be designed with a weight low enough to permit two men to handle it without the use of tackle. A three-pole switch would be much heavier and more difficult to install. After the box is mounted inspection will be easy because of the small cover. The design could be made so that the panel, on which the circuit breaker and relay are mounted, may be easily removed from the box after removing some small bolts without disturbing the mounting or cables. This feature permits all testing and repairwork to be done at the shop where complete units in working order may be taken to a vault and readily placed in the box after the defective unit is removed. One company anticipates using a single-pole device with four outgoing leads at the bottom. Each lead is connected in the box through a fuse or copper link. This arrangement makes a combined switch and junction box which is economical in cost and space requirements.

The foregoing discourse emphasizes the advantages from an installation standpoint but there are also a few advantages from an operating standpoint. The substation breakers in the primary feeder may be arranged for single-pole switching as is now the practise in radial feeders on some systems. The single-pole switch in the transformer secondary removes only one-third of the transformer capacity when there is a transformer failure or a single-conductor cable failure. The loss of this phase does not cause single-phase operation of induction motors because the three-phase relation is maintained by the other feeders. It is not probable that a failure which may occur on another feeder will be on the same phase and it is, therefore, quite an advantage to keep unaffected phases of a feeder in service since they may be of help when trouble comes on other feeders.

Transformer Load Division. The use of a reactance in series with the transformer is the method being used to improve the division of load between transformers. Some companies are using special transformers with the reactance increased to 10 per cent and others are using external reactors consisting of laminated iron punchings slipped over the lead cable on the low-voltage side of the transformer. This latter method has the advantage of using standard transformers and permits the location of the reactance at points where they are needed. The dimensions of such a reactor to increase the reactance to 10 per cent on a 100-kv-a. transformer with 115 volts secondary isa pproximately 16 in. by 6½ in. by 6½ in. For a 75-kv-a. transformer the length will be

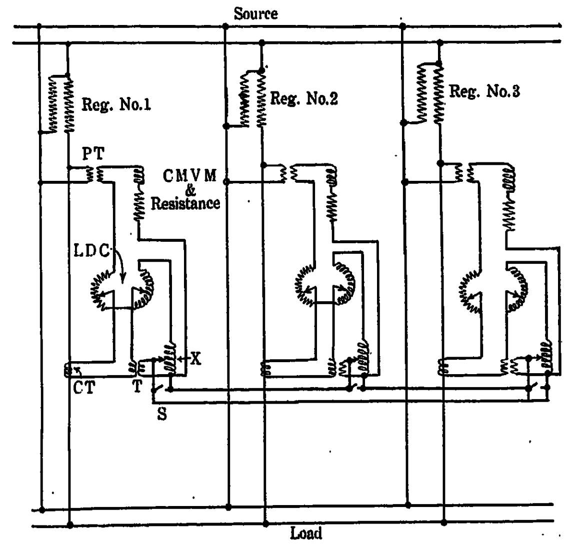

FIG. 3—NEW CONNECTION OF INDUCTION REGULATOR CIRCUITS TO ELIMINATE CIRCULATING CURRENTS

increased to 18 in. on account of there being less current to give the same voltage drop. One reactor is used with each transformer by looping both leads through the punchings to obtain the magnetic effects of both leads.

A New Regulator Connection. When the network is supplied by several high voltage feeders with induction regulators, it is necessary to take some precautions to eliminate circulating currents caused by improper operation of the induction regulators. Several schemes have been proposed but the one shown in Fig. 3[2] seems to be the most promising because of its simplicity and ease of installation. This scheme can be applied to a varying number of regulators, it being possible to connect in additional feeders or disconnect them as may be desired without changing the adjustment of any of the apparatus.

To the conventional regulator connections, there is added in Fig. 3 a transformer T in the current trans-

2. Devised by Mr. F. J. Champlin of Pittsfield, Mass.

former circuit, a reactance X in the potential transformer circuit, and a switch S which serves to short circuit the reactance. This switch S is interlocked with the feeder breaker so that when the feeder switch is opened, the switch S is closed and vice versa.

When conditions are normal the load current circulates through the secondaries of T but none through X. If there is a circulating component in the feeders which flows out through the No. 1 feeder and returns over the other feeders in parallel then two-thirds of this component flows through No. 1 X and one-third through the other two reactances in series but in a direction opposite No. 1 X. By properly arranging the polarities of the current and potential circuits the voltage set up across X will be opposed to the unbalanced potential set up across the line drop compensator by the circulating current. This voltage set up across X will assist the secondary voltage of the potential transformer thus tending to operate the contact making voltmeter so that the regulator will be returned to a stable point, thereby eliminating the circulating current.

No mechanical interconnection or additional current transformers in the feeder are required. The transformer T and variable reactance X may be contained within a single housing, mounted on the switchboard. A certain amount of resistance may be used in combination with the reactance. Three regulators using the exact connections shown in Fig. 3 have been connected in parallel and tests were made including change in voltage, change in load, and change in position of the individual regulators. No matter what change was made, the regulators would always adjust themselves to eliminate the circulating current which is the general source of trouble in the parallel operation of single-phase regulators.

A-C. Elevator Equipments. Practically all companies contemplating networks expect it will not be economical to change all the d-c. motors to a-c. and, therefore, it will be necessary to maintain an appreciable part of the d-c. for many years after the a-c. secondary network is in successful operation. Where 60-cycle power is available[3], a-c. motors can be applied directly to elevator engines and give quite satisfactory operation up to maximum car speeds of 450 ft. per min. A large number of these a-c. equipments are being installed, even in such buildings as hotels, office buildings, and apartment houses, and are giving service which is entirely satisfactory to the owners. Power consumption on these a-c. driven elevators is generally less than with any form of d-c. driven elevator as there are no shunt field or dynamic braking resistor losses.

When a high-grade elevator is required and the power supply is alternating current, Ward-Leonard control with separate motor generator set for each elevator is the ideal. Such equipments are being installed on both geared and gearless equipments in fairly large numbers even for car speeds as low as 250 ft. per min. with geared machines and as high as 700 ft. per min. with gearless machines. For gearless machines the demand is almost entirely for Ward-Leonard control even where d-c. power supply is available, due to smoother and more refined control and to reduced power consumption where the service is heavy and the number of stops per car mile is large. Where the service is light and the stops per car mile is low, the power consumption may be slightly greater.

When all charges against an elevator are considered, such as interest on investment, depreciation, maintenance, operator's salary, rentable area of building occupied by elevators, etc., it is readily seen that power consumption is a relatively small percentage of the total. Such features, then, as reliability, maintenance, smoothness, refinement of control, accuracy of landings, and speed regulation may play a more important part in determining the type of drive to use than power consumption. For example, where it can be shown that the above features will so speed up the service as to permit one less elevator to be used with Ward-Leonard control than would be necessary with rheostatic control,

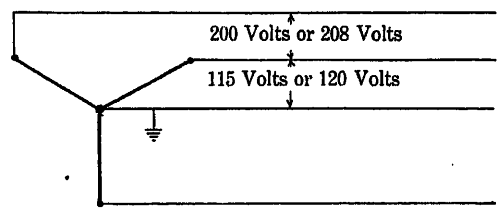

FIG. 4—Y-CONNECTED NETWORK NOW IN USE

the use of Ward-Leonard would be justified on account of the saving effected, even though the operating conditions were such that power consumption on Ward-Leonard might be slightly greater. Such cases have actually arisen.

Rates of acceleration and retardation can be made substantially quicker with Ward-Leonard control than with d-c. rheostatic control without discomfort to the passengers, since they follow a smooth curve instead of being jerky due to cutting out the steps of resistance on a rheostatic control.

It is well recognized that the greatest percentage of elevator outages with rheostatically controlled elevators are due to control troubles and that most of the maintenance expense occurs on the control panel on account of the heavy currents handled and the size of the devices. With the Ward-Leonard system, the control panel contains only small devices handling small field currents. Consequently troubles from burning are negligible and mechanical wear and tear is reduced to a minimum. The motor-generator set, while a rotating piece of equipment, is simple and easily understood by the average electrical maintenance man and its maintenance

3. Discussion on elevators contributed by J. A. Jackson of Schenectady.

consists of an occasional oiling, keeping it clean, and an occasional change of brushes. The reduction in maintenance charges should be a considerable item in favor of Ward-Leonard control.

This type of system will cause practically no voltage fluctuations on the mains since there are no a-c. motors switched on the line every time the car is started. The motor generator set is started without load and should cause no voltage dip.

The New Translator Network. Most companies that do not have two-phase distribution are considering the three-phase, four-wire system and some which have two-phase distribution are giving more consideration to the two-phase, five-wire system.

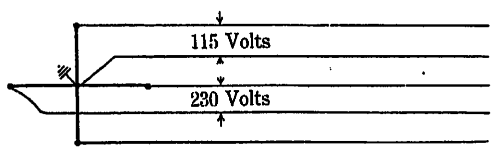

Fig. 5—Proposed Two-Phase, Five-Wire Network

The three-phase, four-wire system shown in Fig. 4 will have either 115 volts for lamps and appliances and 200 volts for motors or 120 volts for lamps and appliances and 208 volts for motors. In the first case standard appliances and fractional horse power motors are suitable while the 200 volt is just on the lower limit for standard 220-volt polyphase motors. Some engineers believe this system will be successful at 115/200

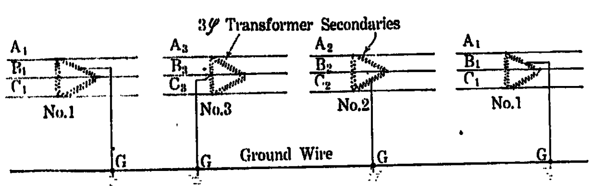

Fig. 6—Delta-Connected Transformers for Supplying Lighting and Power Loads

volts because the majority of motors are not generally applied to devices that require rated starting, pull-in or pull-out torque. Where difficulty is anticipated or encountered they expect to install auto-transformers to step the voltage up to 220 volts. This would require the separation of the motor circuits from the lighting circuits in the building or the use of auto-transformers with each motor. Other engineers believe 200 volts is too low for motors and expect to use the 120/208-volt system. It is believed that the 208-volt is sufficiently above the lower voltage limit (198 volts) of successful motor operation so that with a system regulated for lighting no trouble should be experienced. On the other hand, standard 110-volt appliances and fractional horse power and fan motors are not recommended for operation at 120 volts particularly when it is likely that the voltage will exceed 120 volts occasionally. Lamps and some heating appliances rated at 120 volts are considered as a standard rating while all fractional horse power and fan motors are standard at 110 volts with a permissible variation of plus or minus five per cent. However, some companies are supplying the 110-volt devices on 120-volt radial circuits. With the three-phase, four-wire system it will be necessary to use polyphase metering with the extra testing difficulties

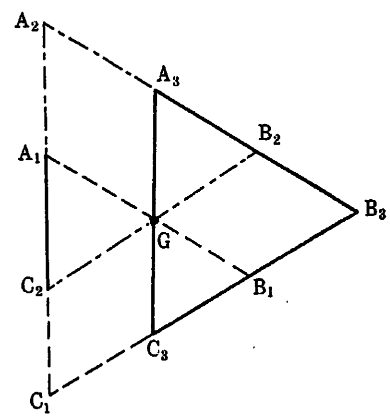

Fig. 7—Vector Diagram of Voltage Relations of Transformers in Fig. 6

or two single-phase meters with the extra bookkeeping when supplying three-wire services since the voltages from each line to ground are not in phase with each other.

The two-phase, five-wire system shown in Fig. 5 gives standard voltages for lamps, appliances and

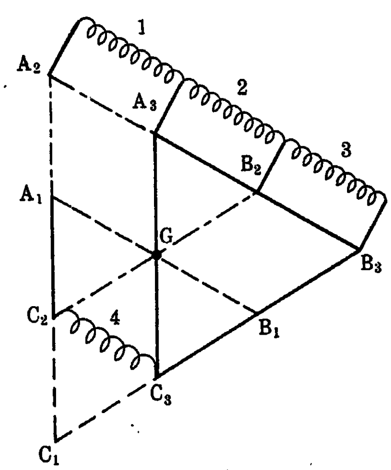

Fig. 8—Vectorial Relation of Combined Transformer and Auto-Transformer Used to Form Tie Between Transformers in Fig. 6

motors. The single-phase, three-wire meter may also be used on three-wire services. The two phases are easier to balance than the three phases of the other system. The two-phase motor, four-pole control equipment and T-connected transformers are objections to this system. Three-phase is ideal for generation and transmission and while two-phase may be ideal for

distribution the combination is undesirable to the manufacturers from the point of view favoring the standardization of three-phase motors and ultimate disappearance of two-phase motors.

It will be observed that either of the foregoing systems affects the consumer's equipment. If it is the Y-connected three-phase system the polyphase motors operate at sub-normal voltage and the fractional h. p. motors and some appliances operate above the permissible voltage limit in case of 120/208-volt system.

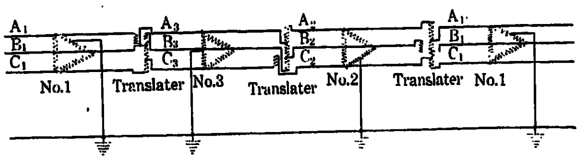

FIG. 9—SAME AS FIG. 6 EXCEPT THAT TRANSLATOR IS ADDED TO PERMIT TRANSFER OF POWER BETWEEN SECTIONS

With the two-phase, five-wire system the consumer may have difficulty in obtaining short shipments on two-phase motors and will have additional investment in control equipment and wiring.

Mr. John C. Parker, of Brooklyn, has devised a three-phase, four-wire network system which gives 115 volts for lamps and appliances and 230 volts for polyphase motors. The lighting load may be balanced on all three phases. This system may be best understood by first referring to Fig. 6. Each bank of transformers

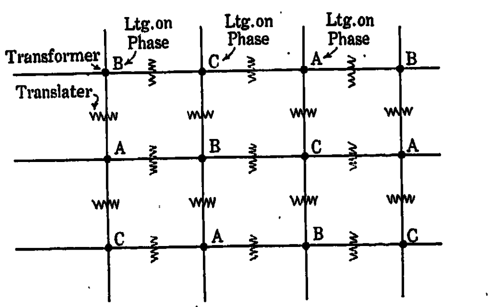

FIG. 10—SECTION OF NETWORK SHOWING MOST ECONOMICAL LOCATION OF TRANSLATORS

supplies a set of three-phase, four-wire mains; but the mains cannot be tied together directly to form a network, since the grounds on the different phases would short circuit the transformers. The grounds are necessary and limited to 150 volts to ground on lighting circuits in order to protect human life. The ground is common to all of the transformers and, therefore, the vector diagram shown in Fig. 7 may be drawn. It will be observed that the voltage difference between A_3 and A_2 (also between B_2 and B_3) is equal to ½ the delta voltage and is in phase with the voltages $A_3 - B_3$ and $A_2 - B_2$. The voltage difference between C_2 and C_3 is equal to $A_2 - A_3$ and in phase with it. An auto-transformer consisting of a coil with taps which divide the coil into three equal parts may be connected to A_2, A_3, B_2 and B_3 as shown in Fig. 8. This auto-transformer is excited from $A_2 - B_2$ at 230 volts and also from $A_3 - B_3$ at 230 volts. Load may be transferred through the auto-transformer with $A_2 - B_2$ as the input circuit and $A_3 - B_3$ as the output circuit or $A_3 - B_3$ may be the input circuit and $A_2 - B_2$ the output circuit. In order to complete the connection between the No. 1 bank and No. 2 bank, it is necessary

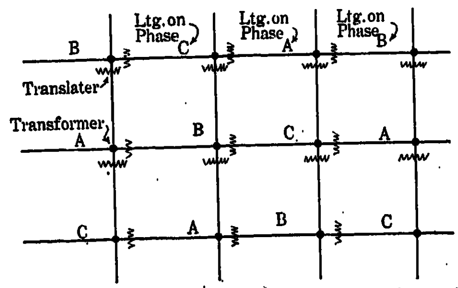

FIG. 11—SECTION OF NETWORK SHOWING MORE PRACTICAL LOCATION OF TRANSLATORS

to make a connection between C_2 and C_3. This is done by a coil which is wound on the same core as the auto-transformer.

With the extra coil added as in Fig. 8 we have a combined transformer and auto-transformer which can be used to transfer single-phase or three-phase power from bank No. 1 to bank No. 2 or from bank No. 2 to bank No. 1. In case of a transfer of three-phase power coils, No. 1, No. 3, and No. 4 carry the current between banks. Under this condition the flux due to the load of the one coil is equal and opposite to the resultant flux due to the load of the other two coils. Therefore, the load current does not magnetize the iron to cause an excessive drop. The device may be designed with a regulation as good or better than the best distribution transformers. Single phase lighting load is carried by coils No. 1 and No. 4 when it is transferred from section No. 2 to No. 3 and coils No. 3 and No. 4 when it is transferred from section No. 3 to No. 2. In this case also the load fluxes are opposed and do not magnetize the iron core. It is, therefore, possible to operate such a network and

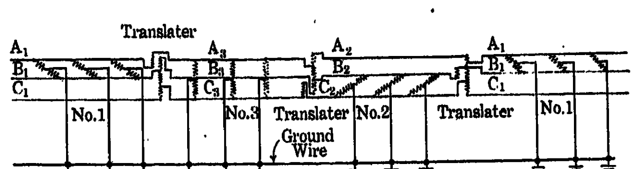

FIG. 12—METHOD OF CONNECTING TRANSFORMERS TO AVOID TRANSMITTING LIGHTING LOAD THROUGH TRANSLATORS

supply lighting load from all three phases. In order to transfer load between sections No. 3 and No. 1 and sections No. 1 and No. 3 a translator (combined transformer and auto-transformer) is connected similar to the one between sections No. 2 and No. 3. Fig. 6 is redrawn in Fig. 9 to show the translators connected.

The most economical application of the translator

occurs when it is located so as to transfer a minimum amount of load. This location is midway between transformers with a uniformly distributed load. Fig. 10 shows the appearance of a network with translator applied in this manner. In this location the translator will have to carry half the load connected to a section of main in the event of a transformer outage. To avoid changing the translator with load growth, it would seem best to design it with one-half the current carrying capacity of the cable. However, it does not seem probable that the translator can be applied in this manner for best economy because the load is not uniformly distributed and it is frequently desirable or necessary to connect transformers to mains at other points than the junction point. It seems best, therefore, to design the translator with a current carrying capacity equal to the current carrying capacity of the cable. It is believed that three standard sizes such as 325 amperes for 4/0 cable, 450 amperes for 350,000 cm. cable, and 600 amperes for 500,000 cm. cable would cover most underground networks. With the cable and translator designed for the same capacity it would be unnecessary to make any changes in translators until it became necessary to change the cable mains. Load growth or transformer locations will have no effect on the translator capacity.

The translator may be located in the same manhole with the transformers as shown in Fig. 11 or may be installed in the middle of the mains as in Fig. 10. The location of the translators fixes the phase areas for the lighting load. The letters *A*, *B*, and *C* in Figs. 10 and 11 refer to the phases which supply the lighting load. After the translators are once installed the phase to which lighting loads are to be connected may be plainly marked in order to avoid confusion when making connections. In case it should be necessary to change the location of a translator it will not be necessary to change the lighting services from one phase to the other, but only to reconnect the cable mains so as to shift all the lighting load affected to the proper phase.

The primaries of the distribution transformers may be either Y- or delta-connected. Transformer $A_3 - B_3$ in Fig. 9 will supply through the translator some of the lighting load connected to $A_1 - B_1$, and transformer $B_3 - C_3$ will supply some of the lighting load connected to $B_2 - C_2$, so that with a Y-connected on the high voltage side, no appreciable transformer capacity will be lost by these units ($A_3 - B_3$ and $B_3 - C_3$) supplying part of the lighting load connected to $A_3 - C_3$.

It is obvious that the transformer connected to $A_3 - C_3$ will supply more of the lighting load connected to $A_3 - C_3$ than either $B_2 - C_2$ or $B_1 - C_1$ because the latter have to supply the lighting load through a longer distance as well as through the translator. It seems that Fig. 9 would be preferable where the power load is more important but where, as is usual, the lighting load is more important, it may be better to use the connections shown in Fig. 12. The lighting load is then supplied direct from the transformers. The motor load is supplied through the translators on two phases and direct from the transformer on the third phase. If there were no lighting load the motor would receive unbalanced voltage because of the extra drop in the phases supplied from the translators; but the lighting load on the third phase will bring these voltages more nearly equal and the result will probably be that the motor will receive no more of an unbalanced voltage than it would on any other system. Even if the full drop through the translators unbalanced the voltages applied to the motors, the unbalanced voltage should not be sufficient to be serious. When the primaries are delta-connected it would seem that this method of connecting the transformers would cause serious unbalanced currents when supplying motor loads because the translators and an appreciable length of mains are inside the delta causing unequal impedances in each side, with the result that the transformers nearest the motor would supply much more current to the motor than the other phases. While this may be true for a particular section, it should not be

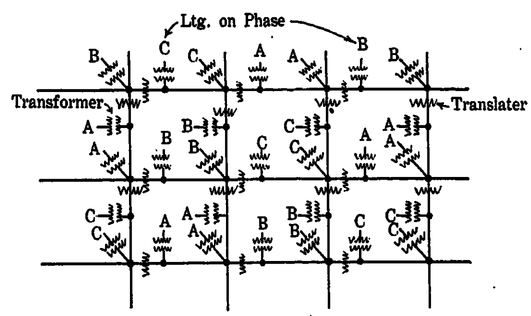

FIG. 13—APPEARANCE OF NETWORK WITH TRANSFORMERS CONNECTED AS IN FIG. 12

true for several sections because the unbalanced current occurs on different phases for adjacent sections and, therefore, the result would be an approximately balanced load. With the primaries Y-connected the division of load between phases is not affected because unequal impedances do not cause an unequal division of load in the Y-delta connection, but an unequal division of magnetizing current. Where the power load is heavy it will be best to add additional units for the other two phases. Fig. 13 shows the appearance of a network with the lighting supplied direct from the transformers. In each manhole there may be three transformers and two translators or they may be placed in different manholes. It is not necessary to have a translator for each main as in Figs. 11 and 12, but the network may be divided into phase areas as shown in Fig. 14. Transformers may be added on the other phases where the power loads are heavy or located far from the other phase areas. The areas, of course, need not be equal geographically but equal electrically.

This system may be used with single-phase switching by connecting the neutral of the Y-connected prim-

ary to the fourth wire and using double-pole switches for each transformer secondary. With single-conductor primary cable it will be possible to operate the system with a minimum capacity removed from service during a cable failure. The remaining transformers on this feeder would operate in open delta with transformers on the remaining feeders.

It is obvious that use of the translator necessitates unbalanced currents in the mains of each section because of the connection of the lighting load to one phase. However, the Y system also has unbalanced load in each section of the mains. While it is possible to balance the lighting load on a Y-connected system so that the feeders supplying the network are well balanced, it is not possible to have each section of secondary mains so well balanced. The translator system, of course, will be appreciably more unbalanced in each section than the Y-connected system; but the translator system also may be operated with the feeders supplying the network as well balanced as the other. In the Y-connected system the loads are balanced over the three-phases by individual loads; but in the translator system the loads may be balanced over the three-phases in groups, which should be much easier. Assume that in the Y-connected system the lighting load is balanced over the three phases and that the current in each phase is 100 amperes for lighting at unity power factor and 100 amperes for power at 86.6 per cent power factor. The resultant current is 194 amperes for each phase. Then with the translator system when the lighting load is supplied from phases *A-B* the current is 237 amperes for *A* phase, 207 for *B*, and 87 amperes for *C* phases. The power factor of 86.6 per cent for the power load was chosen because this gave the condition of maximum current carried by one phase. When the power load is one-half the lighting load the current in each phase of the balanced Y-connected system is 145 amperes; but with the translater system the current in *A* phase is 194 amperes, *B* phase 181 amperes, and *C* phase 44 amperes. When the power load is one-quarter the lighting load the current in each phase of the balanced Y-connected system is 122 amperes; but with the translator system the current in *A* phase is 172 amperes; *B* phase, 162 amperes; and *C* phase, 22 amperes. The effect of this on the losses in the mains is to decrease the total loss 5.5 per cent below the Y-connected system in the first case and increase the losses 14 per cent and 26 per cent in the second and third cases. These values refer to the case where no lighting load is transmitted through the translator under normal conditions. Where some lighting load is carried on the other 2 phases through the translators, phases *A* and *B* would carry relatively less current and phase *C* relatively greater current, so that the resulting unbalanced current would not be so bad as previously indicated. Furthermore, the power loads in many cases will be almost equal to the lighting load which is not a bad condition. The annual charges of the energy losses in the secondary mains is about 12 per cent to 17 per cent of the total losses in substations, feeders, transformers, and mains. Therefore, whatever change occurs in the secondary main losses, when using the translator system, will not have an appreciable effect on the total losses.

In the system shown in Figs. 9, 10, 11 and 12 the effect of the cost of the translaters when designed to

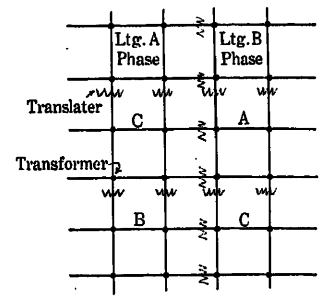

FIG. 14—LOCATION OF TRANSLATERS TO FORM LARGER PHASE AREAS FOR LIGHTING LOAD

carry full cable capacity on the total investment from substations to secondary mains depends on the relation of the current carrying capacity of the secondary mains to the transformer capacity. A system with 500,000 cm. cable has a much higher percentage increase investment when three 100-kv-a. units are used than when three 200-kv-a. units are used. A study of some systems indicates extremess of 7 per cent to 15 per cent

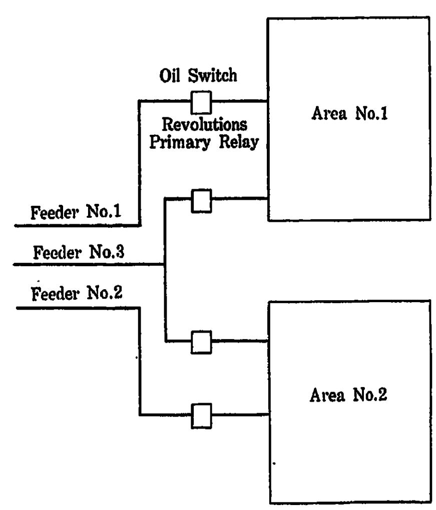

FIG. 15—PROPOSED TYPE OF PRIMARY DISTRIBUTION NETWORK

increase for systems having 13,200-volt primaries and 5 per cent to 12 per cent increase for systems having 2300/4000-volt Y primaries. Where the translators are used to define phase areas as in Fig. 14 instead of one in each section of secondary mains the increase in investment then, of course, depends on the number of areas for each phase. When the losses are added, the increase in annual charges ranges from 6 per cent to 20 per cent. The lower figure applies to a case where the vaults are large enough to receive the translators without any changes or new vaults.

It is apparent from the foregoing analysis of the translator system that there are technical and practical difficulties as well as increased cost and losses of the translator system over the Y-connected and two-phase systems. All of these difficulties and extra costs are borne by the electric service companies and none by the consumer. The consumer is not called upon to operate his single-phase devices at excessive voltage or his polyphase motors at sub-normal voltages, to increase his investment in control equipment, nor to contend with possible delays in shipment of two-phase motors. Three-wire lighting services may be metered with the standard 3-wire meter instead of two single-phase meters or one polyphase meter. It seems, therefore, that the selection of the translator system must be upon the grounds that the electric service company does not want to run any risk of inconvenience to, or complaint from, the consumer nor to cause the consumer any extra cost but prefers to accept the extra cost and complications. The Y-connected or two-phase network represents about one-third the annual cost of an Edison system, and, therefore, it does not seem that the extra cost of the translator system would be an unreasonable burden to assume. This extra cost, of course, is borne by the industry but the cost would be less than the cost to the industry to properly take care of off-standard equipment used on Y-connected and two-phase networks.

Notes on Other Important Problems

Street Lighting Control. The methods now in use to control either pole-type, constant-current transformers or multiple lamps are time clocks, pilot-wire circuits, and cascade systems. The latter may include time clocks and pilot wires or may consist only of a succession of lighting circuits connected through suitable switching equipment. In some cases pole-type, constant-current transformers are supplied and controlled by a special 2300-volt feeder which covers the substation territory.

In addition to the foregoing control schemes, there is the possibility of carrier current control which at present seems very promising. The ordinary primary feeder conductors may be used to conduct the carrier wave to the receiving equipments which may control either a pole-type, constant current transformer or a multiple lamp circuit. The development of a successful carrier-current system low enough in cost may be the deciding factor in the extensive use of multiple street lighting circuits supplied from the usual distribution transformer.

Power Factor Improvement. One large eastern company has installed a few pole-type static condensers at the load center of their distribution circuits. This practise may prove a very economical practise in a number of cases by postponing, for a few years, the building of a new circuit in a growing territory. After the new circuit is built and some of the load transferred from the old to the new circuit, the condensers may be removed to another circuit.

It may also be shown that it is more economical to improve the power factor of some systems at the distribution feeder load center with special pole-type static condensers than to improve it at the substation. Twelve circuits were studied assuming the pole-type condenser cost $5.00 kv-a. more than substation corrective equipment. The extra annual charge was found to be 67 per cent of the saving in $I^2 R$ loss from the substation to the load center. The location of standard condenser equipments by the electric service company on the consumer's premises would be somewhat more economical, since larger standard outfits may be used at a lower cost per kv-a.

Feeder Service Restoration and Maintenance. Considerable study is being given to a number of schemes to restore service on feeders quickly. The extensive use of automatic reclosing feeder equipments has accomplished considerable in quickly restoring service. The records of one large eastern company show that for a period of over a year, 58 per cent of the operations tripped once, reclosed once and stayed in. On the second reclosure 11 per cent stayed in. These circuit trippings resulted in practically no interruption to service. This case indicates that there must be a remainder of around 30 per cent in actual interruption to service on feeder circuits to be restored quickly by other means. Various automatic and remote control sectionalizing schemes are under consideration.

One large middle-western company proposes to try out a form of primary network shown in Fig. 15. The advantage gained by the use of this scheme over simple radial operation of feeders is the automatic preservation of service in case of a short circuit on a feeder cable. In case of a short circuit on a primary ring or its branches the service in that area is interrupted the same as it would be with radial feed. Under normal operation advantage is gained in using the reserve feeder to carry load, in improved regulation, and reduction in losses.

Discussion

C. E. Carey: I wish to discuss one or two details in Mr. Blake's paper. He mentions primarily the single-pole protective unit. He goes into detail and attempts to establish the justification of the multiplicity of parts. However, the question of single-phase or single-pole units cannot be separated from the whole network. Personally, I believe that the multiple unit is the real solution. We have seen that there are certain state laws requiring that we open the entire circuit, rather than allow an unbalance on the networks and produce inductive interference. It is much better, I believe, to clear the entire trouble as quickly as possible rather than take out one phase at a time. The three-pole unit will not occupy as much space in the manhole as three single-pole units. The automatic a-c. network protector is a device which does everything you can expect it to do. It is designed to go into the standard 35-in. manhole cover. The covers of the units are of aluminum, are light, and can be handled by one man. Furthermore, if inspection

of these devices is to be made the time consumed will be less with three-pole units than with three single-pole units.

In regard to the translator, I believe that anything we put in to complicate the network is defeating the purpose of the a-c. system. The whole thing resolves itself back into continuity of service, simplicity, and low cost, and the only way we can arrive at that service with a reasonable cost is to trim it down to just the bare necessities.

Practically everyone will agree that if we had it to do over again, we would never put in a d-c. network. We have it, but what are we going to do with it? If we attempt to change to an a-c. network we are confronted with the cost of changing over the customer's equipment, which will be so high in some cases that the interest and other fixed charges on that additional cost will more than carry the high losses in the d-c. network. However, one solution is being used to bring the economics of the d-c. very close to those of the a-c. network, the use of the automatic substation. The automatic substation gives primarily the same type of distribution as the a-c. system, that is, high voltages to the converting stations and low voltage for distribution. The automatic substation is reliable and economical. A real analysis of the problem will often show that it is better to hold to the d-c. network, remove the concentrated manually operated stations, and distribute that conversion power over a network to a large number of small conversion stations, approaching extremely close to the transformers on the a-c. network in losses and other factors which come into the distribution problem.

Henry Richter: In considering a-c. secondary networks, it is well to recognize that the network systems mentioned by the United Electric Light and Power Co. as used in New York City—and also those in New Orleans—use triple-pole automatic network units exclusively, and that similar systems and network units will shortly be in operation in Dallas, Memphis, Knoxville, and Atlanta. Minneapolis will install these units as the load grows, preparatory to forming a network. These comprise all the companies that have completed or started to construct a network like that of Fig. 2, using automatic network units.

In practically none of the installations in these cities, where space might permit single-pole units to be installed, has it been necessary to add to the dimensions of new manholes in order to accommodate the triple-pole network unit; or to construct new manholes, or enlarge old ones, due to any extra space required because single-pole units were not available; or even to take up space in an existing vault that could be ill spared. These networks are considered justifiable only in heavily loaded areas where the loads are too great to be supplied in the single-phase manner. Three-phase transformer banks are, therefore the rule, ranging in size from three 25-kv-a. single-phase transformers up to three 100-kv-a. transformers. The manholes and building vaults to accommodate such banks, to allow for growth of load, to ensure no excessive temperatures, and to permit proper racking and maintenance of cables, must be of such size that there is no lack of wall space for a triple-pole network unit. It is customary to locate this unit on the wall opposite that along which the three transformers are placed, and close to a corner of the manhole. In existing manholes having three-phase banks formerly part of a radial primary and secondary system, the triple-pole network units can usually be put into the space formerly occupied by the large secondary junction boxes. In most cases, this space is not otherwise useful, for the network systems employing these automatic network units are so simple that nothing else is necessary in the manhole besides the transformers with or without small reactors, the cables and the network unit. Further, it is an erroneous idea that single-pole network units, particularly in a submersible housing, can be made so tiny that they can be installed in any odd corners of a manhole. The desirability of giving them proper maintenance makes such procedure far from advisable even if they were so small.

If it is desired to have the leads to the secondary mains leave the triple-pole network unit horizontally, to either side, it is extremely simple to provide a small terminal box where the outgoing leads emerge from the top of the submersible housing. This small box can have wiping nipples mounted horizontally at either side. The extra bend in the cable is thus eliminated. However, in none of the cities enumerated has this been necessary. Seven and a half feet is an average height for these transformer manholes as determined by good subway construction practise and even with a triple-pole network unit for a bank of three 100-kv-a. transformers there is no cramping of cables entering and leaving the housing.

When the triple-pole network unit was being designed, it was recognized that the submersible housing must pass through a certain minimum-size manhole opening. A country-wide survey was made and it was learned that the great majority of companies have adopted 35-in. diameter round as the smallest opening for transformer manholes in existing or contemplated construction. Apparently this minimum was governed by the dimensions of 100-kv-a., single-phase subway transformers. Accordingly the triple-pole units of all sizes were designed to pass through a 35-in. diameter round opening. In none of the six network cities mentioned (and together they are typical of all other cities) will it be necessary to rebuild the opening of any manhole to permit the unit to pass through; and careful investigation shows that in none of the cities where three-phase networks are being considered for the future will there be any difficulty in this regard.

If it becomes necessary to rack cables along the wall back of the network unit, it is just as easy to mount the triple-pole unit on a pipe framework braced to the wall as for the single-pole unit, since the two types are not very different in depth. However, just as with junction boxes, subway oil-circuit breakers and such apparatus, electric service companies do not consider this good practise in manholes. In only one case has a company mounted on a transformer tank a piece of auxiliary apparatus of such size as a single-pole network unit might be, and there the conditions are unlike those anywhere else in the country. In general, the place for such equipment is against a wall.

Where subsurface conditions make it impossible to build a manhole for three transformers at any particular location in the street on a main thoroughfare, three solutions have been found to be applicable: (1) obtain a vault in the basement of a building; (2) install a manhole under the sidewalk, or (3) locate the manhole in the street as close to the desired location as possible. In almost no case has it been impossible to use one of these methods. Where the third method must be employed, the distance from the most desirable location is usually so short that the cost of the extra length of duct line is small compared with the extra cost of three smaller manholes over one larger one. It is also doubtful whether city authorities will permit subsurface obstruction at three neighboring points.

The triple-pole network unit for manholes is constructed with a window of heavy, wired glass, amply strong, in the cover; this makes inspection of the principal parts easier than by taking off the cover of a single-pole unit. If it is necessary to get at the parts inside, the time to remove the few extra bolts or lugs holding the cover on the triple-pole unit is negligible. The aluminum cover of the triple pole unit can be handled by one man without difficulty.

The design of the single-pole unit to permit easy removal of the panel from the housing when it is desired to make repairs at the shop follows the identical idea that is incorporated in the triple-pole network unit design. Similarly, right from the start, fuses have been installed on the panel of the triple-pole unit, in series with the outgoing leads, to make a separate

fuse box unnecessary. These last two points, therefore, do not apply exclusively to single-pole units.

The paper claims that the single-pole unit conforms with the method of single-pole switching inherited from the radial system of distribution. In a properly designed three-phase network system single-pole switching is no longer necessary and may be abandoned. In a radial system it is better to have some light than no light, when trouble occurs on a feeder, and hence the value of single-pole switching. Networks are designed so that in no case will trouble on any feeder cause interruption of any service fed from the secondary network. Among such troubles there must always be included the putting out of service of all three conductors of a feeder, either by phase-to-phase short circuit in a three-conductor cable, or, where three single-conductor primary cables are in the same duct, by the melting down of all the conductors by a severe fault to ground on one of them. It may even be necessary to provide for the possibility of a manhole fire taking two feeders out of service simultaneously. Thus, there is no necessity in complicating the system to get the insurance that goes with keeping two of the phases in service when the third goes out. One large company even plans to use three-phase regulators on three-phase feeders serving an important network, and some are thinking of using triple-pole oil-circuit breakers at the station and three-phase distribution transformers where these can be passed through the existing manhole openings.

The paper overemphasizes the importance of manhole installations. While the difficult conditions encountered in manholes must be met and are being met, it should be remembered that of a total of over 600 triple-pole network units that will be in operation by the end of this year, less than one third will be in manholes.

Three single-pole units have three operating mechanisms in place of one for the triple-pole unit, and this means more parts to maintain. Three units together have more surface to gasket than a triple-pole unit, which gives more chance for water to leak in with any given type of construction. Three units also occupy a greater total space in the manhole, and every cubic foot is valuable. When the design of automatic network units was first under consideration, all these factors were weighed and the single-pole type was abandoned as inferior to the triple-pole type.

The simplest method of ensuring stable operation of regulators on feeders operating in parallel on a network[1] does not require any extra apparatus such as the transformers T and reactors X of Fig. 3. It employs only a transfer switch, corresponding to switch S for each feeder. This scheme was given a thorough test on the network system of the United Electric Light and Power Company, where it originated, and was shown to be entirely satisfactory.

One of the greatest problems in connection with secondary networks is the type of combined light and power secondary system to employ. Mr. Parker has blazed the trail in an effort to devise some scheme whereby the advantages of the combined system may be obtained and the disadvantages of the star connection avoided. It cannot be too strongly urged that others follow in his steps. However, it must be pointed out that the translator scheme may introduce disadvantages that outweigh those of the simple four-wire, star system, and these disadvantages may so handicap the development of the network idea as to result in a loss to all concerned.

In the star system of Fig. 4 the only voltage unbalance at the motor terminals is caused by unbalance of load in the secondary mains due to varying sizes of the loads as encountered along the street. This unbalance may be reduced to a negligible amount by care in connecting two-wire and three-wire services on alternate phases. From the analysis of the translator scheme it is evident that it may easily result in voltage unbalances of at least 10 per cent at the motor terminals. For the same per cent voltage unbalance on a motor as per cent voltage reduction, both the heating and starting torque are affected to a worse degree in the case of the unbalance. Hence the effect on motors would be worse with the translator scheme than by operating them at 199 volts on a 115/199-volt star-connected secondary.

Adding extra transformers on other phases to obtain a better balance of voltage where power loads are frequent not only increases the number of transformers on the system, but also calls for larger or more manholes to house them. This would materially cut down the savings in transformer capacity gained by the diversity that networking makes possible.

Where the 115/199-volt star-connected secondary system is employed it has been necessary to use auto-transformers on not more than 15 to 20 per cent of the motors connected to the system in order to supply satisfactory voltage. On the basis of power load equal to half the lighting load these auto-transformers represent less than 7 per cent of the capacity of the secondary system. If the translators, equivalent in capacity to the total capacity of the secondary system, cause an increase of investment averaging 10 per cent of the entire distribution system cost, the auto-transformers necessary to ensure satisfactory operation of motors on a star system involve an increase in investment of less than 0.7 per cent.

The unbalance of current for the case of power load half the lighting load requires that the wire carrying the maximum current in the translator system use about 60 per cent more copper than for the star system. It would be highly inadvisable to proportion the size of the three-phase wires in the translator scheme according to the loads in those wires, because the sizes would have to be changed from block to block all over the system, and this would complicate the system even more than by adding the translators. Hence all secondary wires would be as heavy as the largest one, and this might mean a 60 per cent increase in the total amount of secondary copper required. Where 500,000-cir. mil copper is taken as the largest size, the extra copper would in many cases require a second main and duct. To these extra costs must be added those of the translators, value of manhole space they would occupy, and losses in secondaries and translators. Even the seven-wire, separate-light-and-power secondary system would not cost more and would be simpler. The increase in cost due to all these items might easily overcome any saving gained by combining power and light mains and defeat the very purpose of the translator scheme.

If the electric service companies want to inconvenience the fewest number of customers using a star-connected system, they will adopt the 115/199-volt three-phase system, as only the polyphase motor users will be affected. In the few cases where tests show insufficient voltage, the simple auto-transformers can be employed to boost the voltage. In Memphis, a 115/199-volt system of this kind has been in operation for over ten years and the customers are entirely satisfied. In Rochester, for several years, light and power loads in large buildings downtown have been supplied by 120/208-volt transformer banks in the basements and, even though motors up to 40 h. p. are connected at the end of long risers and no auto-transformers are employed, the customers praise the service.

It must be recognized that for most systems an a.-c. network system, even though fed at 13,200 volts, is just a little less than half the annual cost of a d-c. system and hardly less than 80 per cent of the cost of an a-c. radial system. These ratios have been checked independently by the engineers of five large systems. Hence, we would be deceiving ourselves as to the economic value of the translator scheme for it introduces such elements of additional cost as would wipe out the balance now in favor of a-c. networks.

The control of multiple street lamps or pole-mounting, constant-current transformers supplying series street lamps, by

1. Described in the *Electric Journal*, July, 1925.

means of carrier current over the primary feeders, will represent a great step ahead. One company has tried out such a system, but the operation has been faulty and the relay units on the poles are of such nature as to be relatively expensive. The sender at the substation is also complicated. Another company has developed a simpler form of relay unit, and tests of a number of these, equivalent to at least a full year's service, have proved them satisfactory. This relay unit is compact, substantial and inexpensive. The sender unit is also simple and strongly built.

M. T. Crawford: Mr. Blake's paper refers to a method shown in Fig. 5 of which the title is "New Connection of Induction Regulator Circuits to Eliminate Circulating Currents." I think I am correct in stating this connection has been used recently by our company in our new Union Street substation and has proved very successful in regulating 4500-volt feeders in a multiple-feed network. One practical point in connection with it has been that some means has been found desirable to automatically disconnect the regulator control circuit on low voltage, so that regulators will not assume, during system trouble, different positions after fluctuations. I mean by that if a short circuit comes on the transmission network, and the voltage of the system as a whole oscillates back and forth, perhaps reaching very low values for brief moments, there is a tendency for the regulators to attempt to follow these voltage variations up and down. By the time matters have settled down again, some of the regulators are in one position and some in another, due to their slightly different characteristics. That has resulted at times in tripping out of network switches on the underground distribution system due to reverse flow of power for brief periods from one feeder into another where the voltage conditions were slightly different. By the simple expedient of providing a low-voltage release on the regulator control circuit, this trouble has been eliminated. The operator at the substation can reset the control circuit after the system has quieted down to normal.

The translator referred to is a very clever development. The Puget Sound Power and Light Company's underground distribution system employs single-phase, three-wire mains for lighting and alternate blocks with longitudinal alleys are placed on alternate phases, so the first alley is on one phase, the second alley on another, and so on. By that method the phases are relatively well balanced at the substation. In places, we have recently installed a fourth wire paralleling the single-phase, three-phase mains to provide small polyphase service. The result has been three-phase, four-wire mains in each alley similar to the ones referred to by Mr. Blake, and the respective alleys in the same phase relation as those shown in his diagram.

The translator, therefore, in our case offers something to look forward to as a possibility of permitting interconnection of the three sets of secondary mains for purposes of phase balancing or load protection if it should be found desirable. However, the addition of apparatus is always to be very closely scrutinized, and its necessity must be proven before it is added, as the simpler a system is, the better the service will always be.

A. H. Kehoe (communicated after adjournment): Regarding Mr. Blake's discussion of single-pole versus three-pole a-c. subway network switches I consider the proper switch to use is the one which will give minimum cost over an extended period. The cost of revising a few existing locations will be negligible on the total installation cost. Tripping and closing elements in these switches represent a major item of cost and single-pole units will nearly triple this cost for each installation. Space, that is, cubical contents, naturally will be less in a three-pole unit than in three single units. These conditions seem to make the three-pole unit the one to be adopted generally. However, I do not favor certain of the existing three-pole switches of the so-called "battleship" construction. If the principal installation advantages set up by Mr. Blake for single-pole switches are used are specifications for three-pole switches, none of the latter's many advantages need be considered except the greatly reduced cost.

Under operating advantages there are several references to separating the phases either in the physical location or for operation of the transformers. I believe this is a case of confusing what *can* be done with what *is likely* to be done. It is possible to separate phases in three transformer vaults but it is probable that one vault three times as long will be cheaper and better if polyphase load is to be supplied. The system of polyphase secondary distribution is primarily for better universal utilization of electric service, as methods of balancing are now successful without it. However, that balanced polyphase loads are desirable at all points on a system, is a design axiom. All progress up to this time points to polyphase rather than single-phase distribution transformers for ultimate use. If the standard three-wire, single-phase system is taken as analogous to the polyphase system, it is evident that two single-phase, two-wire transformers could be placed in separate transformer vaults supplying a three-wire system. This is not likely to be found, however, in standard practise as all the capacity is found in one unit and a vault is built to take this larger unit, as this design gives minimum cost. I believe similar conditions will hold as the polyphase system develops.

Concerning the operating situation with radial distribution, the single-pole substation switches had economical advantages which could be charged to reliability. Since, in network distribution, there must be, and is always, sufficient reserve on each phase to allow for a failure of that phase, in each locality where it exists it is certain that the remaining phases will have sufficient reserve capacity to be eliminated in case of fault. Such operation does not have any effect upon service which can be equated against the increased cost, when the money, if necessary, could be expended to obtain an increased reserve upon all phases so that the particular phase in trouble would benefit by having a greater reserve. I doubt whether single-pole network switches are proper even with those systems already operating with three single-pole switches on four-wire, 4000-volt service, as new load will cause a higher voltage supply to be used than 4000 volts. It is possible to superimpose the new load on to the existing system and avoid the double transformation which is otherwise required.

Mr. Blake describes a method of cross-current compensation as the "most promising." It is, however, neither simpler nor easier of installation than others now in use. The several such connections should be given consideration before applying a definite arrangement to a particular system. The method described makes constant current and power factor the major considerations, while constant service voltage becomes a secondary one. Constant and correct service voltage is one of the most important elements in the business. While the network insures reliability and better voltage regulation than on radial systems this latter should not be compromised unnecessarily. It seems to me to be preferable to design the system for proper balance of load and have the regulating equipment give proper voltage under all conditions. The connection proposed does the opposite. Applying such a connection to a number of long feeders on an extended network makes other connections preferable due to the voltage variation under the ordinary load shifting.

It should be noted that a distribution network does not simulate a bus in respect to voltage or to concentration of load as indicated in Fig. 3 of the paper.

I fail to find in the translator description any reference to the use of this connection on certain distribution transformer secondaries to join and supply the different mains. If such a device ever has a practical application I believe it will be necessary to make it up in such a form, rather than as an auto-transformer, owing to cost and losses involved. Mr. Blake's statement that each section of secondary mains on a Y-connected system is not likely to be well balanced is not in accord with the

purpose of adopting polyphase distribution. In some existing installations a certain amount of unbalance may exist due to the utilization having been designed for three-wire, single-phase service, but new installations are easily balanced and it is only in districts where growth of loads occur that any considerable amount of three-phase, four-wire mains are likely to be installed.

D. K. Blake: Mr. Carey questions the application of the single-pole switch. Mr. Richter's discussion also differed as to the application of the single-pole switch. It is recognized that the objection to the multiplicity of units for maintenance is valid. It is also recognized that a number of large companies will be able to utilize building and sidewalk space and, therefore, for these places the triple-pole switch is preferable. In the synopsis of the paper is this statement: "The circumstances which make single-pole switching preferable are outlined." It was my impression that there was a similar statement in the text, but on reading it over I notice there is not, and, therefore, some wrong conclusions might be drawn because of this omission. In his discussion Mr. Richter seemed to deny that these circumstances exist at all. He mentioned the cities where the large switches are being used. I agree with him that the triple-pole switch is preferable in these cities with one exception. Mr. Richter's references to the design of the switches indicates he misunderstood the paper. No exclusive features are claimed. To include all of these features in a triple-pole switch presents serious difficulties which are expensive to overcome. The operating advantages of single-pole switching may be questionable. Some engineers believe it desirable.

Mr. Richter mentions that the transfer switch shown in the *Electric Journal*, July 1925, corresponds to switch S shown in Fig. 3 of my paper. Switch S is a single-pole auxiliary switch whereas the transfer switch consists of four auxiliary switches or else one auxiliary switch to control an electrically operated double-pole double-throw switch. The cross-connected scheme is not as simple in its operation. All regulators are not adjusted at the same time according to the amount of circulating current passing through them but adjustment is obtained in a sequence. Mr. Kehoe's statement that the proposed connection does not "give proper voltage under all conditions" is not clear to the author. The line-drop compensator in each feeder maintains constant voltage at the load center with varying load. The impedances of the feeders may be different; some feeders may be long and some short. The proposed connection, by substituting a phase shifter at T, can also be used with three-phase regulators which is not true of other connections now in use.

I want to thank Mr. Crawford for telling us of his experience with the new regulator connection. We shall have to learn something about the operation of this connection on the systems where we use the sensitive reverse-power relay. I am glad to learn that Mr. Crawford has found a simple way of correcting the trouble.

It is not evident to the author why the translator system "may easily result in voltage unbalances of at least 10 per cent at the motor terminals." The translator system is simply the tying together of four-wire combined light-and-power mains which are supplied by delta-connected transformers. The unbalance on these mains is due to the lighting load on one phase and since this is limited to 3 per cent regulation that is also the extent of the unbalance on a 230-volt circuit. Heavy power loads are usually supplied with individual transformer banks and, therefore, it is not evident why larger or more manholes are necessary for this purpose on the translator system.

The translator system may be used with single-conductor cable with two of the phase wires larger than the third—just

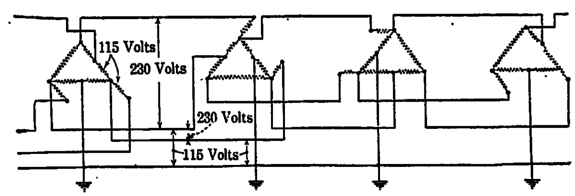

Fig. 1

as is done in radial practise. These sizes would not have to be changed from block to block but would be uniform over the system. The case of power load equaling half the lighting load required that the wire carrying the maximum current in the translator system use about 34 per cent more copper than for the star system instead of 60 per cent as given by Mr. Richter.

Mr. Kehoe refers to a scheme using transformer secondaries. Such a connection is shown in Fig. 1, herewith, utilizing four standard distribution transformers. The main objection to this system is that it is tied up with the primary feeder making it necessary to provide two triple-pole network switches for protection. The translator is independent of the primary voltage. The fourth transformer is greater in kv-a. rating than the translator and very much more expensive in case of 13,200-volt or 11,000-volt primaries. The fourth transformer should be located in the same manhole with the other three while the translator may be located where convenient. Part of the lighting load is supplied from open-delta connections.

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1925

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Forty-first Annual Report, for the fiscal year ending April 30, 1925. A general balance sheet showing the condition of the Institute's finances on April 30, 1925, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held nine meetings during the year; seven were held in New York and one each in Chicago, Ill., and St. Louis, Mo.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees, and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

President's Visits.—During his term of office, President Osgood has presided at the principal conventions of the Institute in Pasadena, New York City, and St. Louis. In addition he has visited, and addressed, the members of numerous Sections and other engineering groups throughout the country, his itinerary having included Vancouver, Seattle, Spokane, Portland, San Francisco, Panama, Havana, New York, Boston, Lynn, Pittsfield, Troy, Rome, Syracuse, Schenectady, Philadelphia, Washington, Atlanta, St. Louis, Milwaukee, and Madison. He has also addressed meetings of engineering students at California Institute of Technology, Leland Stanford Jr. University, Massachusetts Institute of Technology, University of Pennsylvania, Rensselaer Polytechnic Institute, Syracuse University, and the University of Wisconsin.

Other meetings that President Osgood is scheduled to attend in May and June are: Regional Meeting, Swampscott, May 7-9; Cleveland Section, May 21; Utah Section and Utah Engineering Council, Salt Lake City, May 25; Society for the Promotion of Engineering Education, Schenectady, June 16; Annual Convention, Saratoga Springs, N. Y., June 22-26.

Meetings.—The policy of holding in addition to the Annual business meeting four general meetings of the Institute each year was continued. The meetings held were as follows: Annual, Pacific Coast, Midwinter, and Spring Conventions.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 16, 1924. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1924, was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1924.

Following the business meeting President Harris J. Ryan gave a lecture on "The Atmosphere as a Factor in Electrical Engineering." This part of the meeting was held under the auspices of the New York Section.

Annual Convention.—The Fortieth Annual Convention was held at Edgewater Beach, Chicago, Ill., June 23 to June 27, 1924. Seven technical sessions were held including parallel sessions on three mornings. Thirty-two papers were presented. One session was devoted to standardization including reports by subcommittees of the Standards Committee. The entertainment provided included sightseeing and inspection trips, golf, tennis, dancing, cards, etc. The attendance was about 1000. The annual conferences of the Sections Committee were held on Monday, occupying the entire day and evening; forty-two Sections were represented. A report of the conference was published in pamphlet form.

Pacific Coast Convention.—The Thirteenth Pacific Coast Convention was held at Pasadena, Cal., October 13th to 17th, 1924. Eight technical sessions were held at which twenty-six papers were presented. The total attendance at the convention was 363.

Midwinter Convention.—The Thirteenth Midwinter Convention was held in New York, N. Y., February 9 to 12th, 1925. Twenty-five technical papers were presented at six sessions. Among events of particular interest were the ceremonies in connection with the presentation of the Edison Medal to John White Howell, the Smoker, and the numerous inspection trips. Registration reached approximately 1500.

Spring Convention.—The Fourth Spring Convention was held in St. Louis, Mo., April 13th to 17th, 1925. Seven technical sessions were held and twenty-six papers were presented. The attendance was 700.

Regional Meetings.—During the year two meetings of a new type were held. These were the Conventions or Regional meetings of District No. 1 and District No. 2 held respectively at Worcester, Mass., June 5th and 6th, 1924, and Washington, D. C., January 23rd and 24th, 1925. These Regional meetings were planned and carried out by the Sections located in the respective Districts through suitable District Committees organized in each case under the guidance of the Vice-president of the district. The Meetings and Papers Committee cooperated in obtaining many of the technical papers presented. These Regional Meetings are likely to increase in accordance

with the policy of decentralization formulated by the Officers of the Institute and of the various Districts and Sections.

Abstracts of the reports of the chairman of many of the Institute committees and delegations are included herein under various headings.

Meetings and Papers Committee.—The Meetings and Papers Committee during the past year has concerned itself with carrying to completion work so well started by the last committee including formulation of policies and procedure. The Committee has seriously studied the possibilities of regional conventions at the request of the Board and has formulated a policy with respect to the preparation of papers and programs for these meetings. This policy in brief recognizes the regional meetings as accepted Institute practise, places responsibility for the initiation of the meetings and the papers on the regional organizations, outlines a scheme of regional committee organization for cooperating successfully with the national committee organization, and endorses the principle that regional and sectional papers are eligible to the same treatment as papers presented at national conventions. As worked out with respect to the regional meetings in Districts No. 1 and No. 2 this year this policy has produced good results and the regional meeting promises to be a permanent feature of Institute activities in certain districts.

The national conventions of the year were outlined and the programs were prepared with the idea of reducing the number and improving the quality of papers, securing diversity in topics treated, and obtaining good discussions. The first meeting of the year in Pasadena was unusually successful as it had a large and well diversified program and an attendance of members from all parts of the country. The cooperation of the local committee on the Coast enabled the Meetings and Papers Committee to help greatly in making the convention successful.

The Midwinter Convention in New York was notable for the high-grade papers presented, the good discussions, and the absence of parallel sessions.

The Spring Meeting in St. Louis embodied a well diversified program dealing very largely with application aspects of electrical engineering. A notable feature of this meeting was the papers and discussions on modern power stations.

In the light of developments during the year it would seem that the present arrangement and number of conventions of the Institute meet the needs of the members, provide an adequate outlet for the technical paper production, and meet the budget requirements as to expenditures. A judicious culling of papers and scheduling of papers at more than one meeting solves the publication problem and gives better discussion, and the development of regional meetings permits of a selection and a scheduling of papers with greater flexibility than has hitherto existed. The cooperation of the Meetings and Papers Committee with Sections and Geographical Districts has been continued and developed.

It seems that the Institute organization and plans for the cooperation necessary to obtain good papers and programs at all Institute meetings are perfected and only constructive work along present lines is required to maintain the present degree of excellence in this aspect of Institute activity.

Publication Committee.—The principal task of the Publication Committee is that of managing the publication of the JOURNAL, the TRANSACTIONS, and periodical indexes.

The views of the Institute's officers and members relative to publication policies have been developed through a series of more or less related committee actions.

The Committee on Development, of 1919, in its Report presented various constructive suggestions which have enabled us in the main to meet the wishes of the majority of the members. Broad consideration was given to publication problems by the Editing Committee of 1919, and by a committee of the Section Delegates at the Swampscott Convention of 1923.

Regulations adopted by the Publication Committee, approved by the Executive Committee, were published in the October, 1923, JOURNAL. With practically negligible complaint the policies announced at that time have been continued.

It is difficult to print an equal number of pages of reading matter in each monthly issue of the JOURNAL, but the endeavor has been to limit the amount to about eighty pages of technical material, and twenty pages of all other material, including "Institute and Related Activities." Appropriation of funds for publication of the JOURNAL is made on this basis.

In September, 1924, it became necessary to take advantage of the authority to abstract the longer papers, without invalidating the rule that every paper presented at a meeting shall have full publication in pamphlet form, in the JOURNAL, or in the TRANSACTIONS.

It was so managed that all of the papers presented at meetings during the year 1924 were printed either in full or in abstract in the JOURNAL by the close of the year 1924, so that the January 1925 and following issues of the JOURNAL were available for the printing of papers presented at the Midwinter Convention, 1925, and at subsequent meetings.

The dates of publication of discussion have been advanced considerably, thus bringing the dates of publication of papers and discussions closer together.

All papers presented at Regional meetings of the Institute are considered for publication on the same basis as papers presented at National meetings.

Rapid progress is being made in preparing for early

publication an Index to the TRANSACTIONS covering the years 1911-1921, inclusive. The determination of the end of the period covered was made in view of the fact that the Index should conform in book size with that of the TRANSACTIONS of that period. The size of the TRANSACTIONS page was changed to uniformity with the JOURNAL page beginning with January, 1922.

The Publication Committee welcomes communications from the membership bearing on publication matters and the character of the JOURNAL. All such communications receive careful attention at meetings of the committee.

Sections and Branches.—The great increase in Institute membership during the last few years and the wide distribution of that membership has necessitated the development of plans providing for participation in A. I. E. E. activities of that large portion of the membership who, because of their location, cannot conveniently attend the national meetings. The Regional Meetings which have been held at Worcester, Mass., Washington, D. C., and Swampscott, Mass., (the latter having been held May 7-9, 1925) are indicative of this development. The more thorough organization and the activity of District Executive Committees and a closer affiliation of all Sections with the national Meetings and Papers Committee, are still other steps in that direction. Each year these problems on Section developments are thoroughly discussed at the Sections Committee Conference during the Annual Convention. During the year two new Sections have been authorized, Nebraska and the Niagara Frontier.

A similar development of Student Branches has also occurred with plans under way for a more effective supervision and direction of their work. New Branches were authorized at New York University, South Dakota State School of Mines, and Missouri School of Mines and Metallurgy.

	For Fiscal Year Ending			
	May 1 1919	May 1 1921	May 1 1923	May 1 1925
SECTION				
Number of Sections......	34	42	46	49
Number of Section meetings held.............	217	303	344	386
Total Attendance........	25,837	37,823	46,672	49,029
BRANCHES				
Number of Branches.....	61	65	68	82
Number of Branch meetings held.............	156	443	503	548
Attendance.............	6,441	21,629	26,893	27,603

Standards Committee.—During the past year, the Standards Committee has made great progress in the revision of the Institute's standards. Five sections of the revised standards have been adopted by the Board of Directors and the work on numerous other sections has been carried so far along that it is expected before the end of the Institute year the revision will be substantially complete as regards sections dealing with types of apparatus or branches of the art which are covered in detail in the 1922 Standards of the Institute. With the issuance of these sections a large amount of work will remain in the completion of additional sections covering standards for other types of apparatus or branches of the art; these are now in course of preparation, or are contemplated in the general scheme of the revision of the Institute Standards.

The Standards relating to each type of apparatus are condensed into a small pamphlet making it immensely easier to determine beyond doubt what are the standards applying to that type of apparatus, a determination which, with the growth of the Institute Standards in their old form to a sizable volume, had become a task, in spite of an excellent index.

Furthermore, the subdivision of the Standards has an essential effect on the case with which they can be revised, it being now possible to make revisions in the Standards dealing with each class of apparatus, without involving changes in a large book.

Perhaps the most important effect of the revision, however, is the extent to which it facilitates cooperation with other organizations interested in electrical standardization. There is no other single organization actively interested in all of the Institute Standards as the Institute alone covers broadly the field of electrical industry. However, with regard to almost any individual section of the revised Standards, there are one or more organizations doing standardizing work in an allied field or so closely identified with that field, as to form desirable co-partners with the Institute in the formulation of standards. The revised Standards contain a large amount of new material of value and there are included standards for types of apparatus covered slightly or not at all in the earlier work. Outstanding examples of this are the standards for insulator tests, the standards for electrical measuring instruments, and the standards for arc and resistance welding apparatus.

During the year the Standards Committee has continued its work in coordinating the numerous standardizing activities of the Institute. The Institute is sponsor for nine projects organized under the rules of the American Engineering Standards Committee and has representation on twenty-three additional committees. The Standards Committee works also in very close cooperation with the U. S. National Committee of the I. E. C., and has done a large amount of work preparing information for the guidance of the delegates at the Hague Meeting in April of this year. Attention has been given by the Standards Committee to the closest possible coordination between its work and that of the technical committees so that the Standards Committee might take advantage of all suggestions which the technical committees could make as to desirable fields and scope of new standards in accordance with the recommendations of the Committee on the Review of the Technical Activities of the Institute.

The present organization of the Standards Committee was set up by the Board of Directors in 1922 in view of the increase in complexity of the standardization work in which the Institute is directly concerned, in order to to bring about a complete coordination of the Institute's standardizing activities. This is carried out by having on the Standards Committee representatives of all the different standardizing committees or commissions in which the Institute is cooperating. These organizations are now 58 in number and include in their membership a total of over 900 men, not excluding as duplicates men who are on more than one committee. In order to enable this large committee to function efficiently, the administrative work is carried on by a small executive committee. It is believed that this organization is admirably suited to the requirements of the present situation and has resulted in excellent progress in the Institute's standardizing work.

American Engineering Standards Committee.—The American Engineering Standards Committee which was organized in October 1918 by the A. I. E. E. and four other societies serves as a national clearing house for engineering and industrial standardization, and provides an information service on engineering and industrial standardization matters. The ultimate responsibility for and control of the work rests with the organizations whose representatives constitute the Committee.

The American Engineering Standards Committee is primarily for the purpose of insuring adequate procedure in the formulation of standards and establishing rules that must be observed by the cooperating bodies in order that the approval of the Committee may be given.

Thirty-two national trade, technical, scientific, and regulating organizations are now cooperating bodies; a total of 160 projects in different fields have official status, 19 of these having been approved as American Standard, and 41, as Tentative American Standard. The Institute is sponsor or joint sponsor for nine projects as follows: Safety Code for Elevators, lightning protection, rating of electrical machinery, wires and cables, symbols for the electrical equipment of buildings, electrical properties of aluminum, electrical installations on shipboard, scientific and engineering abbreviations and symbols, and radio. The Institute is also participating in 18 others by representation on Sectional Committees of other sponsors. The Institute is also participating in the revision of two important electrical codes, the National Electrical Code and the National Electric Safety Code. Agreement has been reached to undertake the unification of overhead line material, including certain classes of insulation, and the unification of specifications for poles, bare and insulated copper conductors, in which work participation by the A. I. E. E. will also be given.

In carrying out the work of the Committee an information service has been established. A complete file of American and foreign standards is being gradually compiled and is available to the Institute and its Standards Committee. Close cooperation is maintained with foreign organizations which is of value to the A. I. E. E. in lines not covered by the I. E. C.

U. S. National Committee of I. E. C.—The past year has been a very active one for the U. S. National Committee, inasmuch as a meeting of the Advisory Committees of the I. E. C. was held in London, July 15th to 18th, 1924. The U. S. National Committee was represented at this meeting by the following delegates: C. L. Collens, E. C. Crittenden, L. L. Elden, H. M. Hobart, A. E. Kennelly, C. O. Mailloux, Clayton H. Sharp, C. E. Skinner.

A considerable amount of very important work was transacted at this meeting. An extensive report on this has been prepared and is available at A. I. E. E. headquarters.

It was arranged at the London meeting of the Committee on Action, held subsequent to the meetings of the Advisory Committees, that the next meeting of the Advisory Committees should be held in The Hague, April 16th to 23rd, 1925.

Preparation for this meeting and for representation by a suitable delegation has involved a large amount of work on the part of the U. S. National Committee. The Standards Committee of the A. I. E. E. has been particularly helpful in preparing matter for submission to the U. S. National Committee.

U. S. National Committee International Commission on Illumination.—The past year has been an important one for the U. S. National Committee, I. C. I., inasmuch as a plenary session of the Commission was held in Geneva, Switzerland, July 22nd to 25th, 1924. The meeting was attended by a number of prominent A. I. E. E. delegates.

The work of this Commission is divided into two parts. One consists of papers and the other in the adoption of standards. It is gratifying to note that at Geneva the Commission adopted a number of the proposals laid before it by the U. S. National Committee and that this Committee was represented also by some very able and interesting papers. The meeting was reported in the October 1924 JOURNAL of the Institute.

One gratifying conclusion which the Commission reached was to hold its next plenary meeting in the United States in 1927. In the meantime the work of the National Committee is being directed toward the development of a representative and creditable program for this first American meeting of the Commission.

Committee on Safety Codes.—The Committee on Safety Codes of the American Institute of Electrical Engineers is made up of representatives of the Institute assigned to cooperate with various standardizing committees in matters relating to National Safety Codes.

The chief work of the committee during the past year has been in connection with the revisions of the National Electrical Code. The Institute has been represented on the Electrical Committee of the National Fire Protection Association and has assisted in the revision of the Code through its representation on that Committee and on its various technical sub-committees.

The Institute will be represented by regularly appointed delegates at the annual meeting of the National Fire Protection Association, to be held in Chicago in May, at which time the recommended changes in the Code will be considered for adoption.

American Committee on Electrolysis.—The American Committee on Electrolysis has been practically inactive during the past year, pending the decision by the various organizations represented on the Committee regarding the continuance of the work.

The expectation outlined in April 1924, to the effect that these organizations would financially support the work of the Committee, has not been realized, although a majority of such organizations are in favor of such support. As a result the work of the main Committee has been inactive, although the work of the Research Sub-Committee has gone forward as well as it could without such financial support.

Technical Committees.—Reports on Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the Annual Convention and printed in the JOURNAL.

Membership.—The results of the Membership Committee's efforts this year are shown in the following table:

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1924	6	594	2,359	13,496	16,455
Additions:					
Transferred		11	64		
New Members Qualified		1	80	1,901	
Reinstated		3	7	61	
Deductions:					
Died	2	7	13	62	
Resigned		3	23	316	
Transferred			10	65	
Dropped		2	28	734	
Membership, April 30, 1925	4	597	2,436	14,281	17,318
Net increase in Membership during the year					863

Deaths.—The following deaths have occurred during the year:

Honorary Members: C. E. L. Brown, Oliver Heaviside.

Fellows: Henry M. Byllesby, Henry J. Crowley, John L. Harper, James A. Lighthipe, Charles B. McLeer, Charles O. Poole, Hubert S. Wynkoop.

Members: Lawrence Birks, Fred A. Bryan, Earnest L. Buttler, Patrick B. Delany, Henry C. Egerton, Frederick A. Hall, Walter E. Harrington, Arthur Jacob, Nathaniel S. Keith, Benjamin G. Lamme, T. Commerford Martin, Harry F. Randolph, F. J. T. Stewart.

Associates: Chester T. Allcutt, August Andrew, John R. Bainton, Charles H. Baker, Charles L. Baker, Ferdinand N. Bechoff, Ernest F. Bliss, William W. Bradfield, Harry Bruns, William T. Burns, Gordon Cameron, James G. Cockran, James A. Crawford, Edward H. De Witt, John M. Elwell, James B. Foote, Eugene Frank, John M. Frase, William H. Funk, Thomas A. Furlong, Edward R. Gorman, George H. Groenke, Joseph M. Harrison, Lafayette B. Hedge, William E. Holcombe, Theodore R. Johansen, Henry H. Lyon, James E. Macauley, Elmer K. McDowell, George S. McLaren, Samuel H. McLeary, Henry F. Mitchell, Nicholas G. Morrell, Geoffrey C. Nicholson, Thomas W. O'Reilly, Charles H. Parker, Frederick W. Pastor, Claiborne Pirtle, Herbert S. Potter, Clarence Renshaw, Ralph C. Rodgers, Edgar Russell, A. Gero Schmidt, Max Schmidt, Henry K. Sellers, Oscar D. Smalley, Earle H. Shive, Charles S. Sperry, Stanley S. Stevens, Charles R. Sturdevant, Walter D. Sultan, Howard W. Thomas, Robert W. Thompson, Stephen N. Tillman, Claire P. Upson, Foster Veitenheimer, Theodore Vladinoroff, Mabel Weil, Leslie W. Wilcox, John E. Wilson, Santaro Yamato, James B. Yeakle.

Board of Examiners.—The Board of Examiners during the year held ten meetings, averaging about three hours. It considered and referred to the Board of Directors a total of 3912 applications for admission or transfer to the higher grades.

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate	2006	
Not recommended	14	2020
Recommended for grade of Member	75	
Not recommended for admission to this grade	29	104
Recommended for grade of Fellow	1	
Not recommended for admission to this grade	3	4
Recommended for enrolment as Students		1669

APPLICATIONS FOR TRANSFER

Recommended for grade of Member	63	
Not recommended for transfer to this grade	18	81
Recommended for grade of Fellow	12	
Not recommended for transfer to this grade	5	17

PROPOSALS FOR TRANSFER

Informally Recommended	17	17
Total number of applications considered		3912

Proposed Constitutional Amendments.—In the spring of 1924 the Board of Directors appointed a committee to consider a revision of the election procedure of the Institute. This committee formulated and recommended a new plan, which was referred to a Committee on Revision of the Constitution.

The latter committee considered the above plan relating to election procedure and also numerous other suggestions from various sources relating to the constitution, and finally made recommendations to the Board of Directors at a meeting in January 1925, embodying proposed amendments which in accordance with the action of the Board were sent to the membership with a letter ballot, under date of February 25, 1925.

The proposed amendments relate to the following subjects: qualifications for membership; admission, transfer, and expulsion of members; dues; life membership; election, terms, and changes in titles, of officers.

The result of the voting upon these proposed constitutional amendments is not known at this writing, but it will be reported at the Annual Meeting of the Institute on the evening of May 15, 1925.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute two scholarships in electrical engineering in addition to the one previously granted. In consequence, the Institute is now authorized to award a scholarship each year so that there may be one man in each class holding a scholarship on the nomination of the Institute; these scholarships will continue until further notice. Each scholarship pays $350 toward the annual tuition, and reappointment for completion of course is conditioned upon the maintenance of good standing.

The second award was made to Mr. T. Horiuchi of the University of Colorado, who holds a scholarship for the academic year 1924-1925.

Institute Prizes.—At the meeting of the Board of Directors of the Institute of April 16, 1921, recommendations were approved establishing two Institute prizes to be awarded yearly to authors of worthy papers. Each prize consists of $100 in cash with suitable certificate.

The Transmission Prize for 1923 was awarded to J. L. R. Hayden and Charles P. Steinmetz for their paper entitled "High Voltage Insulation." Honorable mention was given to Howard S. Phelps and E. D. Tanger for their paper entitled "A New Method for Routine Testing of Alternating-Current High-Voltage Paper Insulated Cables" and to F. S. Dellenbaugh, Jr., for his paper "Artificial Transmission Lines with Distributed Constants." No eligible paper was entered in competition for the First Paper Prize.

On September 26, 1924 the Directors approved the establishment of yearly "First Paper" and "Best Paper" prizes of $25.00 in each of the ten geographical districts.

Edison Medal.—The Edison Medal for 1924 was awarded to John White Howell, Harrison, N. J., "for his contributions toward the development of the incandescent lamp." The presentation ceremonies took place Wednesday evening, February 11, 1925, at the Midwinter Convention, New York, N. Y.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers, awarded the 21st medal to John Frank Stevens of New York for great achievements as a civil engineer, particularly in planning and organizing for the construction of the Panama Canal; as a builder of railroads, and as administrator of the Chinese Eastern Railway." The medal was presented at New York, at a meeting held on the evening of March 23, 1925.

Lamme Gold Medal.—A bequest of $6000 was made by the late B. G. Lamme to cover the annual award by the Institute of a gold medal to a member who has shown meritorious achievement in the development of electrical apparatus. Details are to be arranged.

Kelvin Medal Award.—On December 14, 1923 the Committee of Award of the Kelvin Medal Trust named Dr. Elihu Thomson of Swampscott, Mass., as the recipient of the second triennial award. The Trust Fund represents the surplus obtained in connection with a call for subscriptions to erect a memorial window to Lord Kelvin in Westminister Abbey. The award is made by a committee composed of the Presidents of the principal representative British Engineering Institutions, as a mark of distinction to a person who has achieved eminence as an engineer or investigator in the kind of work applicable to the advancement of engineering with which Lord Kelvin was especially identified. The presentation of the medal took place at the Kelvin Centenary Celebration, London, England, on the afternoon of Thursday, July 10, 1924. After the presentation by Sir Charles L. Morgan and response by Dr. Thomson, the Kelvin oration was delivered by Sir Joseph J. Thomson.

Commission of Washington Award.—The Washington Award for 1924 was voted to Arthur Newell Talbot, Professor of Municipal and Sanitary Engineering, University of Illinois, and the presentation was made at the annual meeting of the Western Society of Engineers, held June 9, 1924. This award was made "for his life work as a student and teacher, investigator and writer, and for his enduring contribution to the science of engineering."

Jonas Waldo Smith has been named as the recipient of the 1925 award.

The award is made annually by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E. and the A. I. E. E. The award of the medal was established in 1917 by Past President J. W. Alvord of the Western Society "to be

annually presented to an engineer whose work in some special instance, or whose service in general, has been noteworthy for its merit in promoting the public good." The endowment has been increased $2000 by its founder.

Employment Service.—The employment service which the Institute has maintained for many years is now conducted as a cooperative bureau in conjunction with a similar service maintained by the National Societies of Civil, Mining and Mechanical engineers under the title "Engineering Societies Employment Service." It is supported by the joint contributions of the societies and their individual members who are benefited.

As in the past the Service consists principally in acting as a medium for bringing together the employer and the employee. In addition to continuing the publication of the "Employment Service Bulletin" in the monthly Journals and the weekly subscription bulletin, the Secretaries of the four societies in their capacity as members of the supervising board are arranging for the establishment of branch offices of the service in other cities. It is hoped to continue this development from year to year as conditions warrant.

American Engineering Council.—This organization of which the Institute is one of the constituent societies has had an active year in the service of the engineering profession. The Annual Meeting of the Council was held in Washington, D. C., in January 1925, and the Administrative Board held three meetings during the year. Council was active in connection with the passage of the Clarke McNary Act providing a comprehensive forestry policy; it sponsored the Temple Bill calling for the completion of the topographical map of the United States; assisted in securing additional space, personnel and equipment for the Patent Office; appeared at hearings for providing increased salaries for Federal judges, hearings on the Muscle Shoals project, and in connection with government reorganization and centralization of engineering functions in one division. Among other actions taken were:—authorization of a committee to study the Reclamation Report and make recommendations; appointment of a committee on Aeronautics; authorization for publication of manuscript on "Industrial Coal, Purchase, Delivery and Storage." An active part was also taken in many other national and international conferences and the staff furnished a large amount of direct service to member societies including obtaining of speakers, making appointments with government officials, and recommending qualified engineers for important public and private work.

The representatives of the Institute upon the Assembly and Administrative Board of the Council have continued to take an active part in the work.

World Power Conference.—The first World Power Conference was held in London, June 30 to July 12, 1924. It was held at the British Empire Exhibition, Wembly, London, and was promoted by the Council of the British Electrical and Allied Manufacturer's Association, in cooperation with Technical and Scientific Institutions in Great Britain and other countries. A large delegation of members of the A. I. E. E. and other American engineering societies was in attendance.

The Conference was called for the purpose of discussing the technical and economic problems of power development, transmission and utilization, and for promoting general interest in power development. The question of how the industrial and scientific sources of power may be adjusted nationally and internationally was also considered.

A condensed classification of the topics discussed indicates the wide scope of the Conference: Power Resources, Power Production, Power Transmission and Distribution, Utilization of Power, and General—including Economics, Financial and Legal Considerations, Research, Standardization, Education, Health and Publicity. Forty papers were presented by American authors. Numerous receptions, luncheons, banquets, and other social features were arranged by the various National Committees and other organizations. The complete Proceedings will soon be available in book form.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical and Electrical Engineers certain specific acts which are governed by contracts, the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1925 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc. A lending department is also maintained.

A copy of the annual report of the Engineering Societies Library covering the calender year 1924 may be obtained by applying to Institute headquarters.

Engineering Foundation.—Engineering Founda-

tion is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a Department of United Engineering Society.

The Foundation has made appropriations for various research projects and has cooperated in others.

The annual report of the Foundation is available in printed form.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has accepted sponsorship and appointed representatives upon a number of new Sectional Committees of American Engineering Standards Committee. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is appended.

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary.*

New York, May 15, 1925.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
37 WEST 39TH STREET
NEW YORK

ATLANTA	KANSAS CITY	SALT LAKE CITY
BALTIMORE	LOS ANGELES	SAN DIEGO
BIRMINGHAM	MINNEAPOLIS	SAN FRANCISCO
BOSTON	NEWARK	SEATTLE
BUFFALO	NEW ORLEANS	TULSA
CHICAGO	NEW YORK	WATERTOWN
CINCINNATI	PHILADELPHIA	
CLEVELAND	PITTSBURGH	BERLIN
DALLAS	PORTLAND	LONDON
DENVER	PROVIDENCE	PARIS
DETROIT	SAINT LOUIS	SHANGHAI

CANADA - CUBA - MEXICO
DELOITTE, PLENDER, HASKINS & SELLS

May 12, 1925.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

Pursuant to engagement, we have audited your books and accounts for the year ended April 30, 1925, and submit herewith our certificate and the following described exhibits and schedule:

Exhibit "A"—General Balance Sheet, April 30, 1925.
Schedule No. 1—Reserve Capital Fund—Securities.
Exhibit "B"—Summary of Income and Profit & Loss for the Year ended April 30, 1925.

Yours truly,
HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1925, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly exhibits the financial condition of the Institute at April 30, 1925, that the Summary of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS

NEW YORK,
May 12, 1925.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A.

GENERAL BALANCE SHEET, APRIL 30, 1925

ASSETS

REAL ESTATE:			
One-Fourth Interest in United Engineering Society's Land, Building, and Building Equipment, 25 to 33 West 39th Street (Depreciation carried on Books of United Engineering Society)			$491,642.36
EQUIPMENT:			
Library—Volumes and Fixtures		$40,539.85	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures	$16,611.46		
Less Reserve for Depreciation (including $4,500.00 funded)	11,458.52	5,152.94	
Total Equipment			48,694.14
WORKING ASSETS:			
"Transactions, etc."		$12,655.45	
Paper and Cover Paper		3,612.34	
Badges		2,024.30	
Total Working Assets			18,292.09
CURRENT ASSETS:			
Cash		$22,013.18	
Accounts Receivable:			
Members—For Dues		16,372.70	
Advertisers		2,079.27	
Miscellaneous		1,057.10	
Accrued Interest on Investments		438.52	
Accrued Interest on Bank Balances		360.76	
Total Current Assets			42,321.53
FUNDS:			
Reserve Capital Fund—Securities—Schedule No. 1.		$58,046.09	
Life Membership Fund:			
Cash	$ 2,038.67		
Chicago, Burlington & Quincy Railroad Company 4% Registered Bonds, 1958, par Value $5,000.00	4,868.75		
Accrued Interest	33.33	6,940.75	
International Electrical Congress of St. Louis—Library fund:			
Cash	$ 623.88		
New York City 4½% Corporate Stock, 1957, Par Value $2,000.00.	2,204.05		
New York Telephone Company 4½% Registered Bond, 1939, Par Value $1,000.00	878.75		
Accrued interest	67.50	3,774.18	
Mailloux Fund:			
Cash	$ 9.47		
New York Telephone Company 4½% Registered Bond, 1939, Par Value $1,000.00	1,000.00		
Accrued Interest	22.50	1,031.97	
Midwinter Convention Fund—Cash		155.62	
Depreciation of Furniture and Fixtures Fund—Cash		4,500.00	
Total Funds			74,448.61
Total			$675,398.73

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable	$12,150.01	
Dues Received in Advance	3,158.25	
Entrance Fees and Dues Advanced by Applicants for Membership	541.00	
Subscriptions for "Transactions" received in Advance	113.00	
Total Current Liabilities		$ 15,962.26
FUND RESERVES (NOT INCLUDING DEPRECIATION RESERVES):		
Reserve Capital Fund	$58,046.09	
Life Membership Fund	6,940.75	
International Electrical Congress of St. Louis—Library Fund	3,774.18	
Mailloux Fund	1,031.97	
Midwinter Convention Fund	155.62	
Total Fund Reserves (Not Including Depreciation Reserves)		$ 69,948 61
SURPLUS, Per Exhibit "B"		589,487 86
Total		$675,398.73

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT & LOSS FOR THE YEAR ENDED APRIL 30, 1925

EXHIBIT B.

INCOME:			
Dues		*$184,140.59	
Students' Dues		10,664.96	
Entrance Fees		10,297.50	
Transfer Fees		705.00	
Advertising		67,833.91	
Journal Subscriptions		7,294.87	
"Transactions" Subscriptions		8,833.00	
Miscellaneous Sales		5,962.46	
Badges Sold	$ 5,027.00		
Less Cost	3,949.96	1,077.04	
Interest on Securities in Reserve Capital Fund		2,612.93	
Interest on Bank Balances		1,450.83	
Total			$300,873.09
EXPENSES:			
Publications:			
Journal	$99,993.80		
"Transactions"	15,489.44		
Year Book	7,448.23	$122,931.47	
Meetings		17,861.63	
Administrative Expenses		50,721.76	
Sections Committee		27,309.52	
Membership Committee		8,317.94	
Standards Committee		2,830.96	
Finance Committee		275.89	
Headquarters Committee		96.07	
Code Committee		60.00	
Law Committee		526.97	
Edison Medal Committee		261.51	
Geographical Districts Expense:			
Traveling Expense—Executive Committees	$ 857.84		
Traveling Expense—Vice-Presidents	115.30		
First Paper Prize	25.00	998.14	
American Engineering Standards Committee		1,500.00	
International Electrotechnical Commission		480.68	
United States National Committee of International Commission on Illumination		300.00	
President's Special Appropriation		1,975.01	
Board of Directors—Mileage		3,133.90	
Honorary Secretary		4,000.00	
John Fritz Medal Award		323.02	
Transmission Prize, 1923		100.00	
Engineering Societies Library—Maintenance		8,000.00	
United Engineering Society Assessment		4,860.00	
American Engineering Council		15,887.00	
Engineering Societies Employment Service		1,635.00	
International Annual Tables		100.00	
"Transactions" Index		200.00	
World's Power Conference		1,200.00	
Total			275,886.47
NET INCOME (FORWARD)			$24,986.62

*Includes $82,275.00 allocated to subscriptions for the JOURNAL.

NET INCOME (FORWARD)		$ 24,986.62
PROFIT & LOSS CREDIT:		
Adjustment of Inventory of Library Volumes and Fixtures		62.58
GROSS SURPLUS FOR THE YEAR		$ 25,049.20
PROFIT & LOSS CHARGES:		
Uncollectible Dues Written Off	$ 6,744.00	
Provision for Depreciation of Furniture and Fixtures	709.79	
Adjustment of Inventory of Furniture and Fixtures, April 30, 1925	157.30	
Adjustment of Inventory of "Transactions," April 30, 1925	143.00	
Total		7,754.09
SURPLUS FOR THE YEAR		$ 17,295.11
SURPLUS, May 1, 1924	$586,031.50	
Less Transferred to Capital Fund Reserve in Accordance with Resolution of Board of Directors	13,838.75	572,192.75
SURPLUS, APRIL 30, 1925		$589,487.86

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

RESERVE CAPITAL FUND—REGISTERED SECURITIES

APRIL 30, 1925

EXHIBIT A.

SCHEDULE NO. 1.

	Par Value	BookValue
The Detroit Edison Company 1st and Refunding, 6% Series "B", Gold Bonds, Due 1940	$ 5,000.00	$ 5,178.13
The New York Central Railroad Company, 5%, Refunding and Improvement Mortgage Bonds, Series "C," Due 2013	6,000.00	5,742.50
Chicago, Burlington & Quincy Railroad Company 5%, 1st and Refunding Mortgage, Gold Bond, Series "A," Due 1971	1,000.00	1,010.00
Great Northern Railroad Company 5½%, General Mortgage, Gold Bonds, Series "B," Due 1952	10,000.00	9,847.50
Southern Railway Company 5%, 1st Consolidated Mortgage, Gold Bond, Due 1994	1,000.00	980.00
City of Wilmington, Delaware, Sinking Fund Loan, 4¼%, Series 159, Due 1934	15,000.00	15,469.21
The Western Electric Company 5% Bonds, Due April 1, 1944	10,000.00	9,818.75
United States Third Liberty Loan 4¼% Bonds, Due 1928	10,000.00	10,000.00
Total	$58,000.00	$58,046.09

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GEORGE A. HAMILTON

SECRETARY

F. L. HUTCHINSON

HONORARY SECRETARY

RALPH W. POPE

GENERAL COUNSEL

PARKER & AARON

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*Norvin Green, 1884-5-6.
*Franklin L. Pope, 1886-7.
*T. Commerford Martin, 1887-8.
Edward Weston, 1888-9.
Elihu Thomson, 1889-90.
*William A. Anthony, 1890-91.
*Alexander Graham Bell, 1891-2.
Frank Julian Sprague, 1892-3.
*Edwin J. Houston, 1893-4-5.
*Louis Duncan, 1895-6-7.
*Francis Bacon Crocker, 1897-8.
A. E. Kennelly, 1898-1900.
Carl Hering, 1900-1.
*Charles P. Steinmetz, 1901-2.
Charles F. Scott, 1902-3.
Bion J. Arnold, 1903-4.
John W. Lieb, 1904-5.
*Schuyler Skaats Wheeler, 1905-6.
*Samuel Sheldon, 1906-7.
*Henry G. Stott, 1907-8.
Louis A. Ferguson, 1908-9.
Lewis B. Stillwell, 1909-10.
Dugald C. Jackson, 1910-11.
Gano Dunn, 1911-12.
Ralph D. Mershon, 1912-13.
C. O. Mailloux, 1913-14.
Paul M. Lincoln, 1914-15.
John J. Carty, 1915-16.
H. W. Buck, 1916-17.
E. W. Rice, Jr., 1917-18.
Comfort A. Adams, 1918-19.
Calvert Townley, 1919-20.
A. W. Berresford, 1920-21.
William McClellan, 1921-22.
Frank B. Jewett, 1922-23.
Harris J. Ryan, 1923-24.

*Deceased.

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E. T. Brandon,	Peter Junkersfeld,	M. M. Samuels,
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